INFLATION AND WELFARE: AN APPLICATION TO CHILE

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ABSTRACT

This paper develops a monetary endogenous growth model where money is no longer supernormal. Economic growth is propelled by the accumulation of human capital and money enters into the optimization problem of the individual through a shopping-time technology. A higher rate of inflation induces the agent to increase the allocation of time devoted to transaction activities, reducing time available to more productive activities and hence, affecting the growth rate of the economy and the welfare of the individuals.

1. INTRODUCTION

Inflation was and is an important issue in many countries. Many governments have relied on it to finance levels of expenditures well above taxes collected. The recent experience of Latin America in this regard is singular. Many countries in the region endured, over several periods, levels of inflation not observed since the hyperinflations in European countries during the 1920s and 1940s. For example Argentina, Brazil, Bolivia, Chile, Nicaragua and Peru all had inflation levels in excess of 200% per year in the 1970s or 1980s. In fact, considering the period between 1973 and 1988, levels of inflation in Latin America were above those observed in other regions of the world. As reported by Fischer (1991), mean inflation during the periods 1973-80 and 1980-88 was, respectively, 14.1% and 25.7% for countries in Africa, 6.0% and 6.9% for those


Key words: Endogenous growth model, human capital, shopping-time technology, welfare.

in Asia, and 24.1% and 111.2% in Latin America. The experience of Latin American countries thus provides an interesting experiment with which to study the effects of high inflation in the economy.

Today there is a consensus that inflation is "bad" or that it is the most "unfair" of taxes. As such, the majority of countries experiencing high levels of inflation have, under the advice of international organizations such as the International Monetary Fund and the World Bank, implemented stabilization programs aimed at stopping inflationary processes in the economy. But why is inflation bad? Why do people feel worse in an economy with inflation? How does inflation affect agents in an economy?

The number of papers studying the welfare costs of anticipated inflation is large. From the neoclassical point of view, based on the work by Bailey (1956), the welfare cost of a given inflation level could be estimated as the area of the deadweight loss triangle under the demand curve for real balances (see Lucas (1981, 1996), Fischer (1981)). In these modes, when inflation is fully anticipated, the only costs it imposes are those associated with the community's attempts to economize on non-interest-bearing money balances. Inflation constitutes a tax on the holding of currency, and it imposes welfare costs as agents alter their behavior in response. The welfare costs found in these studies are usually not very big. For example, Lucas (1981) found the welfare costs of 10% annual inflation to be around 0.9% of annual GDP, while in Fischer (1981) the cost amounted to 0.3% of GDP. Other approaches use computable general equilibrium models to calculate the welfare costs of inflation. Imrohoroglu (1992) introduces money as a store of value in a stochastic economy where agents can face unemployment. She found that the area of the deadweight loss triangle under the demand curve is a poor measure of the costs of inflation: the costs in her model are 1.07% of GDP for a 10% rate of annual inflation. In a real business cycle model, Cooley and Hansen (1989) introduce money with a cash-in-advance constraint on consumption goods in an effort to assess the welfare costs of inflation. In steady state, they report that a 10% inflation rate results in a welfare cost of about 0.4% of income relative to an optimal monetary policy.

However, inflation is a monetary phenomenon and as such it will affect all the roles (functions) that money plays in the economy: store of value, medium of exchange and unit of account. The conventional approach mentioned above has been subject to criticism since the analysis is based on a theory which treats money as a temporary abode of purchasing power, thus emphasizing only its role as a store-of-value. As argued by Laidler (1990):

"(the conventional view) does not properly represent the nature of those costs because it concentrates on the role of money as a store of value for the individual economic agent, to the exclusion of considering the effects of anticipated inflation on the role of money as a means of exchange and a unit of account".

This paper broadens the analysis by modelling the transaction role of money. As in McCallum and Goodfriend (1990):

"the term (money) will be taken to refer to an economy's medium of
exchange: that is, a tangible asset that is generally accepted in payment for any commodity".

In this paper, the demand for money will be motivated by the fact that real money balances help to reduce transaction costs. The transaction cost function will depend on the consumption level of the individual and the amount of real money balances held. The higher the consumption level, the higher the transaction cost for a given level of real balances. Moreover, real balances help to reduce transaction costs for a given pattern of consumption.

This paper also develops an alternative model to study the effects of anticipated inflation on the welfare of individuals: I analyze the effects of high inflation in an endogenously growing economy. I present a perfect-foresight monetary growth model to examine the issue of "superneutrality" of money. The model belongs to the class previously studied by Uzawa (1965) and Lucas (1988), where the accumulation of human capital is the reason for sustained growth. The model predicts a negative relation between the level of inflation and the growth rate of the economy. The intuition is the following: an increase in the rate of inflation produces a reduction in the amount of real money balances held by the representative agent. A lower quantity of real money balances, due to the inflationary environment, implies more time wasted realizing transactions and so a reduction in time available to work and accumulate human capital. Through this channel money invokes not only level but also real growth effect since inflation decreases the marginal product of both kinds of capital (human and physical). The result is a lower steady state level of growth per capita income.

I find that the negative effect on the growth rate of real variables is small even for high rates of inflation. However, the welfare effects on the individuals are significant at these levels and non negligible at low rates of inflation. Simulations show that, decreasing the rate of inflation from a benchmark level of 20.4% to 0% per year increases the growth rate of real variables in the economy from 1.1% to 1.15% per year. In terms of welfare, the consumers would require a 0.96% and 1.67% across-the board income increase to accept voluntarily an increase in the rate of inflation from 0% to 10% and 20% respectively, while they would demand an 3.24% increase if the inflation rate goes up to 50%. So even though inflation is a nominal occurrence, living with high levels of inflation implies the use of resources (output and/or time) that could have been directed to more productive activities. This will have real effects on output and costly welfare effects on individuals.

The remainder of the paper is organized as follows. Section 2 considers some relevant empirical facts. In Section 3 the model with human capital and shopping time transaction costs is presented. Section 4 shows the results of the simulation of the model. Finally, Section 5 presents some conclusions.
2. **Empirical Facts**

2.1. **Cross-country data**

There is a large number of econometric studies that used cross-country data to test the hypothesis that inflation is negatively related to the growth rate of the economy. In a number of studies where the dependent variable is a measure of the growth rate of the economy, the coefficient of inflation is negative and statistically significant, even other explanatory variables are included\(^1\). I do not attempt to reproduce these econometric, but as illustration I plot the relevant data from 31 different countries in Latin America and Asia\(^2\).

The horizontal axis plots the inflation rate; each observation corresponds to the average rate for a particular country over one of the three periods considered: 1971-80, 1981-90 and 1991-95. The vertical axis plots the growth rate of GDP per capita. Figure 1 presents these data\(^3\): the figure illustrates the presence of a negative (and non-linear) relation between the growth rate of GDP per capita and inflation. Alternatively, Figure 2 presents a scatter plot of the data where the horizontal axis is the logarithm of the inflation rate during the periods instead of its level. In this case the coefficient in a simple regression of growth of GD per capita on the logarithm of the inflation rate could be interpreted as its semi-elasticity with respect to the inflation rate\(^4\). The figure shows again a clear negative relationship between the two variables considered.

A relevant question is if those particular observations with very high level of inflation and low growth rate account substantially for the observed negative relation between inflation and per capita GDP growth. Figure 3 plots only observations with a rate of inflation less than 100%. The panel shows that the fit is not driven by the inverse relation between growth and inflation at high rates of inflation. That is, the negative relation between the two variables is consistent even when considered only the relatively low-inflation countries.

Finally, the coefficient estimates of different simple regressions of growth of GDP per capita on inflation are shown in Table 1. These coefficients estimates are those for the regressions lines shown in Figure 1 to 3. In all the cases, the coefficients of inflation (or log inflation) in this simple regressions are significant at 5% level\(^5\).

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1. For a review of these empirical studies see Gyllfason and Herbertsson (1996).
2. See Appendix A for a description and sources of the data. The 17 Latin American countries are: Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Paraguay, Peru, Uruguay and Venezuela. The 14 countries for Asia are: Bangladesh, Burma, Hong Kong, Indonesia, India, Korea, Malaysia, Nepal, Pakistan, Philippines, Singapore, Sri Lanka, Taiwan, and Thailand.
3. Observation for Nicaragua for the period 1971-80 was not available. Thus, with the exception of Nicaragua, we have 3 observations for each country giving a total of 92 data points.
4. We can also interpret the coefficient as an approximation of the elasticity of the gross growth rate of GDP per capita with respect to the inflation rate.
5. These regression results are presented only as an illustration. Other explanatory variables should be incorporate into a multiple regression set up to find a more accurate estimation of the (negative) effect of inflation on the growth rate of the economy.
FIGURE 1
31 LATIN AMERICAN AND ASIAN COUNTRIES
Periods 71/80-81/90-91/95

FIGURE 2
31 LATIN AMERICAN AND ASIAN COUNTRIES
Periods 71/80-81/90-91/95
TABLE 1. DEPENDENT VARIABLE: GROWTH OF GDP PER CAPITA
(t-statistics in parenthesis)

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.85</td>
<td>3.71</td>
</tr>
<tr>
<td></td>
<td>(9.55)</td>
<td>(8.93)</td>
</tr>
<tr>
<td>Log Inflation</td>
<td>-1.18</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(-6.07)</td>
<td>-</td>
</tr>
<tr>
<td>Inflation</td>
<td>-</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.89)</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.28</td>
<td>0.08</td>
</tr>
<tr>
<td>Sample Size</td>
<td>92</td>
<td>82</td>
</tr>
</tbody>
</table>

Observation: In regression (2) all observations with inflation rate above 100% were eliminated.

2.2. Time series data

Although cross-country data seems to support the hypothesis that high inflation is correlated with a lower level of economic growth, it is relevant to test if this hypothesis holds for a single country over time. However a test on a time series data for a single country could be difficult to carry out. One element of the problem could be to obtain a suitable approach to defining the long run and detecting long run relationships. In this part of the paper the basic proposition that the growth rate of the economy and the inflation rate are negatively correlated is examined from a nonstructural, low frequency point of view.
To obtain the low frequency components of the time series I use the approximate band-pass filter developed by Baxter and King (1995). For the empirical applications, I adopt the definition of the business cycle suggested by the procedures and findings of NBER researcher like Burns and Mitchell (1846), that specified that business cycles were cyclical components between eighteen months and eight years. I adopt these limits as the definition of the business cycles, so to isolate the trend or low frequency of the data I consider those frequencies with periodicity of eight years or higher.

The approximate band-pass used in this analysis is the BP\textsubscript{6}(8) filter described in Baxter and King (1995), where the notation reflects the fact that the filter passes through components of the data with cycles higher than 8 years, and the subscript "6" means that 6 leads and lags of the data were used in constructing the filter (i.e. 6 annual observations are lost at the beginning and ends of the sample period for the filtered data).

Description of the data used is contained in Appendix A. For every country the original annual data runs from 1969 to 1992, so given de number of leads and lags used in constructing the filter, I have 12 observations for the filtered data\textsuperscript{6}. Figure 4 plots the relationship, for eight different countries, between the growth rate of GDP per capita and the inflation rate using the filtered data. For comparison, for each country I have also plotted the raw (original) data for the period 1975-86, also a total of 12 observations for every country. For all the countries considered, the plots of the original data illustrate the absence of a clear relationship between inflation and growth over time. However, after filtering the data and extracting only its long run components, a negative relation between these two time series emerges.

Table 2 gives the correlations coefficients, for the different countries, of the growth rate of GDP per capita and inflation. This table confirms the impression from Figure 4: the correlation between the original series is small and sometimes even positive, but it is clearly negative at the low frequencies.

<table>
<thead>
<tr>
<th>Country</th>
<th>Raw Data</th>
<th>Filtered Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.12</td>
<td>-0.89</td>
</tr>
<tr>
<td>Bolivia</td>
<td>-0.10</td>
<td>-0.94</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.06</td>
<td>-0.82</td>
</tr>
<tr>
<td>Colombia</td>
<td>-0.34</td>
<td>-0.98</td>
</tr>
<tr>
<td>Chile</td>
<td>-0.48</td>
<td>-0.89</td>
</tr>
<tr>
<td>Mexico</td>
<td>-0.77</td>
<td>-0.96</td>
</tr>
<tr>
<td>Paraguay</td>
<td>-0.03</td>
<td>-0.98</td>
</tr>
<tr>
<td>Peru</td>
<td>-0.09</td>
<td>-0.73</td>
</tr>
</tbody>
</table>


\textsuperscript{6} For Argentina data available for GDP per capita covers only until 1990, so in this case there are 10 filtered observations.
FIGURE 4
INFLATION AND GDP GROWTH
FIGURE 4
INFLATION AND GDP GROWTH

[Columbia graphs showing growth GDP per capita vs. inflation and filtered inflation]

[Chile graphs showing growth GDP per capita vs. inflation and filtered inflation]
FIGURE 4
INFLATION AND GDP GROWTH

[Two scatter plots showing the relationship between inflation and growth per capita for Paraguay and Peru, with filtered inflation on the x-axis and growth per capita on the y-axis.]
FIGURE 4
INFLATION AND GDP GROWTH

BRAZIL

Growth GDP per capita

FILTERED INFLATION

MEXICO

Growth GDP per capita

FILTERED INFLATION
2.3. Size of the financial sector

In order to simulate the theoretical model developed in this paper, I need an estimation of the amount of time that the average agent spends realizing transactions (the shopping time effort). Traditionally, the cost of expected inflation has been viewed primarily as the "shoeleather cost" of going to the bank more often (Baumol (1952), Tobin (1956). In this view, households reduce their average holding of cash by making smaller withdrawals with greater frequency and the cost of inflation is the utility loss associated with trips to the bank, waiting in line, etc.

Here I focus on the other side of these transactions: in order to satisfy the increased customer activity, banks may need to hire additional tellers, build more and larger branches, or make other costly investments in automation or technology. Thus, as a result of higher inflation, more resources are transferred to the financial sector to accommodate the increased number of transactions chosen by household as they attempt to shift the cost of holding currency onto others.

These resources are a social loss because if inflation were lower, the resources could be used directly to increase production of consumer goods. In our model the resource wasted in trying to diminish the negative effects of inflation is time. If there were no inflation, more time could be directed to more productive activities such as production of goods or accumulation of human capital.

Following English (1996), I test the hypothesis that the financial sector gets larger the higher the rate of inflation using cross-country data. I use data on 27 different countries from Latin America and Asia\textsuperscript{7}. The data is averaged over two periods: 1981-85 and 1986-90, so there are two observations per country giving initially a total of 54 observations\textsuperscript{8}. The independent variable is the size of the financial sector as percentage of GDP (measure as its value-added product), while the explanatory variables are a constant, per capita income, inflation and, in some regressions, a dummy variable if the country is a regional financial center (Hong Kong and Singapore).

Table 3 presents the econometric results. The first column shows the results of a regression of the logarithm of the share of the financial sector in GDP on a constant, per capital output and inflation. In columns 2 and 3, I have dropped all the observations with inflation rates higher than 200% and use (in column 3) the share of the financial sector (instead of its logarithm) as the dependent variable. In all the cases, the results show that higher levels f per-capita income and average inflation are reflected in a bigger increased financial sector. Both coefficients are economically as well as statistically significant.

\textsuperscript{7} There is a small change from the original set of 31 countries. Countries like Nicaragua, Burma, Nepal and Taiwan were excluded due to the lack of available data.

\textsuperscript{8} Appendix A contains a description of the data employed.
### TABLE 3. DEPENDENT VARIABLE: LOG. SIZE OF THE FINANCIAL SECTOR
(t-statistics in parenthesis)

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.72</td>
<td>-2.80</td>
<td>5.90</td>
</tr>
<tr>
<td></td>
<td>(-30.1)</td>
<td>(.28.29)</td>
<td>(5.97)</td>
</tr>
<tr>
<td>Per-capita Income</td>
<td>0.13</td>
<td>0.13</td>
<td>1.17</td>
</tr>
<tr>
<td>(in thousand)</td>
<td>(6.55)</td>
<td>(6.42)</td>
<td>(3.74)</td>
</tr>
<tr>
<td>Dummy</td>
<td>-</td>
<td>-</td>
<td>8.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.71)</td>
</tr>
<tr>
<td>Inflation</td>
<td>.0002</td>
<td>.0038</td>
<td>.05</td>
</tr>
<tr>
<td></td>
<td>(1.91)</td>
<td>(2.06)</td>
<td>(3.05)</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.45</td>
<td>0.48</td>
<td>0.72</td>
</tr>
<tr>
<td>Sample size</td>
<td>54</td>
<td>49</td>
<td>49</td>
</tr>
</tbody>
</table>

Observation: Size of the financial sector is measured as its percentage share of GDP. Regression 3: Dependent variable is the size of the financial sector.

Figure 5 depicts graphically the relation between the size of the financial sector and inflation. The horizontal axis plots the inflation rate; each observation corresponds to the average rate for a particular country over one of the time periods considered (1981-85 and 1986-90). The vertical axis plots the size of the financial sector, measured as its share on GDP, net of the fraction of the size of the financial sector that is explained by all of the explanatory variables aside from the inflation rate. Thus, the panel illustrates the relation between the size of the financial sector and inflation after all the other determinants of the size of the financial sector have been held constant. As it can be seen in the figure, the size of the share of the financial sector in the total product of the economy is increasing in the rate of inflation. As an illustration, Figure 6 presents the results for Chile for the period 1978-92. Again, the data shows a positive relation between inflation and the size of the financial sector in the economy.

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9 The residuals are constructed from the coefficient estimates shown in column 3 of Table 3. However, the results are similar if the coefficients from any other column of Table 3 are used.
FIGURE 5
OBSERVATIONS WITH LESS 100% INFLATION RATE

FIGURE 6
CHILE: PERIOD 1978/92
Equation (3) - Table (3)
3. **The Model**

3.1. **Basic Model**

In the model, I assume convex preferences and technology. The assumption that labor supply is exogenous (inelastic) is relaxed and it is assumed that there are external effects in the accumulation of human capital. The economy is populated by homogeneous agents and they solve the following optimization problem\(^{10}\):

\[
\max W = \int_0^\infty U(c) e^{-\rho t} \, dt
\]

subject to

\[
c + k + m = F(k, l, h, \bar{h}) - \delta k - \pi m + \tau
\]

\[
h = \phi[l-S(m, c)-1]h
\]

where \(c\) and \(m\) are (per capita) consumption and real money balances, respectively; \(k\) represents (per capita) physical capital, \(l\) is the amount of time devoted to production; \(\tau\) is a (per capita) lump-sum transfer from the government; \(\pi\) denotes the inflation rate; \(\rho\) and \(\delta\) are the (constant) rate of time preference and depreciation rate of physical capital, respectively. Individuals accumulate human capital, \(h\), where its law of motion depends linearly on time spent at this activity and its previous level. Also \(\bar{h}\) denotes the average level of human capital in the economy. Money enters into the optimization problem through a shopping-time technology, \(s\), with transaction effort depending on (per capita) real money balances and the level of (per capita) consumption.

The first constraint is the resources constraint. Output, net of depreciation, could be use for consumption, accumulate more physical capital or increase the amount of real money balances. Notice that the individual is paying an "inflation tax" for holding real money balances, \(\pi m\). The government is printing money at a constant rate, \(\mu\), and these resources are transferred to the agent in a lump-sum way, \(\tau\). The second constraint is the law of motion for human capital. In this economy individuals are endowed with one unit of time and they can distribute it in time to work, accumulate human capital or realize transactions.

Denote \(\lambda_1, \lambda_2\) and as co-state variables of the current Hamiltonian associated with (1) and the slack variable identity \(m = z\). These co-states variables can be thought of as the values (shadow prices) of the rate of increase of capital (physical and human) and real money balances respectively. The Hamiltonian of the control problem is given by:

\(^{10}\) Time subscripts have been dropped to simplify notation.
\( H = U(c) + \lambda_1 \left[ F(k, l, h, \bar{h}) - \delta k - \pi m + \tau c - z \right] + \lambda_2 \left[ \phi(1 - 1 - S)h \right] + \lambda_3(z) \)

Straightforward application of Pontryagin's Maximum Principle yields the FOC's:

\( U_c(C) = \lambda_1 - \lambda_2 \phi S_c(m, c) h \)

(3.1)

\( \lambda_1 F_t(k, l, h, \bar{h}) = \lambda_2 \phi h \)

(3.2)

\( \frac{\lambda_1}{\lambda_1} = -F_K(k, l, h, \bar{h}) + \delta + \rho \)

(3.3)

\( \frac{\lambda_2}{\lambda_2} = \frac{\lambda_1}{\lambda_2} = F_K(k, l, h, \bar{h}) + \rho - \phi(1 - 1 - S(m, c)) \)

(3.4)

\( \frac{\lambda_3}{\lambda_3} = \frac{\lambda_1}{\lambda_3} \pi + \frac{\lambda_2}{\lambda_3} \phi S_m(m, c) h + \rho \)

(3.5)

\( \lambda_1 = \lambda_3 \)

(3.6)

together with the constraints and the transversality conditions. The first equation is the efficient condition for a good's allocation: it must be allocated at each date so as to be equally valuable, on the margin, as either consumption or investment. The second one states that time has to be equally productive in two of its uses: as a productive input for goods or as an input in the accumulation of human capital. The third is a modified Keynes-Ramsey rule determining the optimal capital accumulation. It establishes that the after tax return to investment (net of depreciation) has to be equal to the real interest rate. The fourth and fifth equations determine the optimal accumulation of human capital and real money balances. In the margin, the benefit of the last unit of human capital has to be equal to its cost (shadow price). In the case of real money holdings, the optimal condition equates the return of a marginal unit (it frees up time that could be devoted to other activities) to its costs (the nominal interest rate). Finally, the last equation implies that the shadow prices of capital and money must be equal.

3.2. Balanced growth equations

To study the long-run behavior of the model, I use the solutions to the maximization problem of the agent to calculate what are known as balanced growth
equations. Along a balanced growth path, all variables are growing at a constant rate and the time allocated to different activities remains constant. In general, for the economy to follow such a path, both the production function and the preferences must take on special forms. I specialize the instantaneous utility function $U$ to exhibit constant intertemporal elasticity of substitution and the production function technology to adopt the Cobb Douglas form. Specifically it is assumed:

$$U(c) = c^{1-\alpha} / (1 - \alpha)$$

$$F(k(t),l(t),h(t)) = A(k)^\gamma (lh)^{1-\gamma} \eta$$

where $\alpha$ is the inverse of the intertemporal elasticity of substitution, $\gamma$ is the share of capital in production and $h$ is an exogenous and constant technology parameter. The term $\eta$ captures the external effects of human capital.

Following Drazen (1979), I assume that the shopping-time function is homogenous of degree zero so it can be written as depending on the ratio of (per capita) real money balances to consumption, $S(m,c) = \xi \left( \frac{m}{c} \right)$. This function is decreasing in its argument at an increasing rate, $S' < 0, S'' > 0$. A higher amount of real money balances for a given consumption level, allows individuals to have more time to be devoted to productive activities. I further specialize the shopping time function to be a linear function of the ratio of real money balances to consumption, so:

$$S(m,c) = \xi \left( \frac{c}{m} \right) + \kappa$$

To facilitate the use of computational techniques, it is convenient to consider the balanced growth path for the economy. From the system of equations implicitly defining the allocation functions and pricing, the balanced growth equations of the model are:

$$\frac{c}{\kappa} = \frac{\delta(1-\gamma) + \theta(\alpha-\gamma) + \rho}{\gamma}$$

$$1 = \frac{\rho + (\alpha-1)\theta}{\phi}$$

$$\theta = \frac{(1 - \gamma + \eta) \left[ \phi(1 - \xi \left( \frac{c}{m} \right)) - \rho \right]}{\alpha(1 - \gamma + \eta) - \eta}$$
\[
\frac{F}{\kappa} = \alpha \theta + \delta + \rho \\
\mu = (1 - \alpha) \theta + (1 - \gamma) \xi F \frac{1}{\xi} \frac{F}{\kappa} \frac{k}{c} \left( \frac{c}{m} \right)^2 - \rho
\]

\[
\frac{h}{h} = \nu = (1 - \alpha) \theta - \rho + \phi \left( 1 - \xi \frac{c}{m} \right)
\]

where $\theta = \frac{c}{k} = \frac{m}{m}$ is the growth rate of output; $\nu$ is the growth rate of human capital; $l$ is the balanced growth level of the labor supply; $\frac{c}{k}$ and $\frac{F}{k}$ are the steady state ratios of consumption and output to capital respectively; and $\frac{c}{m}$ is the steady state ratio of consumption to real money balances. This gives a system of six nonlinear equations describing the six, endogenous, variables of the model: $\theta, \frac{c}{k}, \frac{F}{k}, l, \frac{c}{m}$ and $\nu$, that can be solved given values of the parameters and the policy variable, $\mu$, to trace the long run reaction of the system to a change in policy.

4. **Numerical Calculation of the Model**

4.1. Calibration

Next, I provide estimates of the quantitative magnitudes of the growth effects of inflation. To provide these estimates, I must specify the parameter values that appear in the model. The values were selected using a combination of figures from previous studies and facts about the growth experience of Chile between 1980 and 1990.

Taking data from the Inter American Development Bank and the Penn World Tables, the following auxiliary relationships were used: the average annual growth rate in per capita gross domestic product (GDP), $\theta$, is $1.1\%$; the average annual rate of inflation, $\pi$, is $20.4\%$; and the annual investment rate (that is the same as the saving rate in our model) as a proportion of GDP, $s$, is around $21\%$. Also, the literature found that the capital share parameter is usually higher in Latin America countries than in more developed economies, so I set $\gamma$ equal to $0.45$ (see, for example, De Gregorio (1992)).

For the preferences parameters, I assume initially that the inverse of the
intertemporal elasticity of substitution, $\alpha$, is equal to 2 and the parameter of time preference, $\rho$, is set equal to 0.04. This parameterization imposes a restriction in the value of $\delta$, the depreciation rate. Using the social budget constraint it can be seen that the saving (or investment) rate in the economy is

$$s = \frac{\gamma(\theta + \delta)}{\delta + \alpha \theta + \rho}$$

so that $\delta$ has to satisfy the following restriction:

$$\delta = \frac{s(\alpha \theta + \rho) - \gamma \theta}{\gamma - s}$$

given this, $\delta$ is set initially equal to 0.0336.

I will consider an economy with spillover effects in the accumulation of human capital and I set the externality parameter, $\eta$, equal to 0.417 as in Lucas (1988). In order to reproduce the benchmark economy the value of $\phi$ has to be set equal to 0.0672. The calculation of this parameter is explained in Appendix B.

Initially the individual is endowed with an unit of time that he/she can use to accumulate human capital, produce goods or realize transactions. As mentioned before, the cost of expected inflation has been viewed traditionally as the "shoeleather cost" of going to the bank more often. In an high inflation economy, households reduce their average holding of cash by making smaller withdrawals with greater frequency and a higher rate of inflation means, among other things, more trips to the bank or to the ATM machines, waiting in line, etc. That is, the fraction of time the average agent allocates to transaction activities increases with the rate of inflation. The other side of these transactions is that in order to satisfy the increased customer activity, banks may need to hire additional tellers, build more and larger branches, or make other costly investment in automation or technology. Thus, a result of higher inflation more resources are transferred to the financial sector to accommodate the increased number of transactions chosen by households.

I approximate the amount of time that the average agent spend realizing transactions with the size of the financial sector in the economy. The justification for this is the following: I consider that there are two sectors in the economy, one producing goods and the other financial services. Assuming that both sectors have the same technology, the production of financial services will require the

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11 Given this preference parameters specification the real interest rate in the economy is 5.8%.

12 For example, Aiyagari, Braun and Eckstein (1997) found that empirically, for the US, the labor share in the banking and credit sector is not much different from the labor share in GNP. Assuming a Cobb-Douglas technology for the production functions, this fact implies that both sectors have approximately the same technology.
use of labor (time) and capital that are taken away from the good’s sector. Thus, the shopping time effort could be re-interpreted as labor utilized by the financial sector\(^{13}\). The amount of labor (time) used by the financial sector could then be approximate with the size of this sector in the economy.

In a previous chapter, I have estimated how the size of the financial sector as percentage of the GDP, increases with the rate of inflation. In the benchmark economy I am considering with an inflation rate of 20.4%, the size of the financial sector is around 14.8% of GDP. Decreasing the inflation rate of 0% per year, would decrease this share to 13.6% of GDP\(^ {14}\). To reproduce this allocation of time as a function of the rate of inflation I need to set the value of $\xi$ equal to 0.0021 and $k = 0.1235$.

Table 4 displays the values of the parameters chosen for calibration\(^ {15}\).

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Technology</th>
<th>Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 2.00$</td>
<td>$\gamma = 0.4500$</td>
<td>$\theta = 0.0110$</td>
</tr>
<tr>
<td>$\rho = 0.04$</td>
<td>$\phi = 0.0672$</td>
<td>$\pi = 0.2047$</td>
</tr>
<tr>
<td>$\eta = 0.4170$</td>
<td>$s = 0.2100$</td>
<td>$\delta = 0.0336$</td>
</tr>
<tr>
<td>$\xi = 0.0021$</td>
<td>$K = 0.1235$</td>
<td></td>
</tr>
</tbody>
</table>

4.2. Results

To calculate the growth effects of inflation, I solve the system of steady state equations for a variety of levels of the rate of monetary expansion, $\mu$, I use a range of value for this variable between zero and 55.

Table 5 below presents the results for seven different money supply rules. It can be seen that the relation is monotone, so over the whole range of inflation considered the higher the rate of inflation the lower is the growth rate of the

\(^{13}\) I can also impose in the economy a pecuniary transaction cost function and interpret it as capital use by the financial sector. However, since this will not affect the steady state growth rate, it will not have implications for the welfare of the individual and for simplicity I omit it.

\(^{14}\) To estimate the share of the financial sector for the benchmark economy under zero inflation. I use the estimated coefficients for inflation found in the empirical part of the paper (regression 2 on Table 3).

\(^{15}\) Simulations were performed changing the values of the parameters to see the sensitivity of the results to the values chosen in the calibration. In all the cases similar results were obtained regarding the growth and welfare effects of inflation, even when using as a benchmark economy the economic experience of Latin America and Asian countries with $\pi = 177\%$ and $\theta = 0.9\%$. 
economy. Moreover, the growth effects of inflation are similar to those typically obtained in the literature. According to the model, reducing the rate of inflation in the economy from 20.4% to 10% or 0% would increase the rate of growth of real variables to 1.12% or 1.15% respectively, instead of the observed 1.1% growth rate.

Notice also the effects of different rate of money growth on the distribution of time by the representative agent. In an economy with a rate of inflation of 20.4%, 14.8% of the time has to be wasted realizing transaction activities. This leaves 75.9% and 9.3% of time remaining for labor supply and human capital accumulation. Reducing the rate of inflation to 0% per year, will change this composition: 13.6%, 76.7% and 9.7% of the unit of time available to the individual will be dedicated to shopping effort, labor and human capital accumulation respectively. Finally observe that, as supported by empirical data, the velocity of circulation, measured as the ratio of consumption to real money balances, increases with the rate of inflation.

<table>
<thead>
<tr>
<th>TABLE 5. MONETARY EXPERIMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady states values and welfare, ( \alpha = 2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \pi = 0.00 )</th>
<th>( \pi = 0.10 )</th>
<th>( \pi = 0.15 )</th>
<th>( \pi = 0.20 )</th>
<th>( \pi = 0.30 )</th>
<th>( \pi = 0.40 )</th>
<th>( \pi = 0.50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>0.0115</td>
<td>0.0112</td>
<td>0.0111</td>
<td>0.0110</td>
<td>0.0108</td>
<td>0.0107</td>
<td>0.0105</td>
</tr>
<tr>
<td>( l )</td>
<td>0.7667</td>
<td>0.7622</td>
<td>0.7608</td>
<td>0.7589</td>
<td>0.7562</td>
<td>0.7539</td>
<td>0.7518</td>
</tr>
<tr>
<td>( h )</td>
<td>0.0975</td>
<td>0.0949</td>
<td>0.0941</td>
<td>0.0931</td>
<td>0.0915</td>
<td>0.0902</td>
<td>0.0890</td>
</tr>
<tr>
<td>( S )</td>
<td>0.1358</td>
<td>0.1429</td>
<td>0.1451</td>
<td>0.1480</td>
<td>0.1523</td>
<td>0.1559</td>
<td>0.1592</td>
</tr>
<tr>
<td>( c/k )</td>
<td>0.1697</td>
<td>0.1686</td>
<td>0.1683</td>
<td>0.1679</td>
<td>0.1672</td>
<td>0.1667</td>
<td>0.1662</td>
</tr>
<tr>
<td>( y/k )</td>
<td>0.2148</td>
<td>0.2135</td>
<td>0.2130</td>
<td>0.2125</td>
<td>0.2117</td>
<td>0.2110</td>
<td>0.2104</td>
</tr>
<tr>
<td>( c/m )</td>
<td>5.8720</td>
<td>9.2305</td>
<td>10.283</td>
<td>11.683</td>
<td>13.692</td>
<td>15.440</td>
<td>17.003</td>
</tr>
</tbody>
</table>

4.3. Welfare effects

In this section I derive a welfare cost measure of having different levels of inflation. To these end, I rewrite the discounted sum of the utility at \( t = 0 \) as:

\[
V(k_0, m_0; T, \pi) = \int_0^\infty U(c_t + Ty_t) e^{-\rho t} \, dt
\]

where \( T \) is a lump-sum equivalent variation payment to the individual (expressed as a ratio to output) with the inflation rate \( \pi \) for this regime. As in relayed studies, the welfare cost, \( T \), is defined in such a way that individuals would be as well off under this monetary regime as if there were no inflation. Let the superscript "a" denote the respective value of that variable under an alternative monetary regime \( a \). The welfare cost, \( T \), can the be obtained from the following
equation:

$$V(k_0, m_0; 0, 0) = V(k_0, m_0; T, \pi^a)$$

when the CES utility function is used this can be expressed as

$$\frac{\left(\frac{c}{k}\right)^*}{1 - \alpha} \frac{1}{\theta^*(1 - \alpha) - \rho} = \frac{\left(\frac{c}{k}\right)^a + T\left(\frac{y}{k}\right)^a}{1 - \alpha} \frac{1}{\theta^a(1 - \alpha) - \rho}$$

where "+" denotes the balanced growth path value of a variable under zero inflation. It is observed that there exist two effects on the costs of inflation: consumption and the real growth rate. The simulation of the model assess each individual effect as well as the overall effect.

Table 6 presents the welfare costs of inflation under the same seven alternative money supply rules as before. A 10% inflation rate results in welfare costs of 0.96% of GDP not a negligible amount. Increasing the rate of inflation to 29% or 50% per year would require a compensating transfer to the individuals equivalent to 1.67% and 3.24% of their income respectively. The same table also shows the contribution of each of the two factors that affect the costs of inflation in the model. For a 10% inflation, changing only consumption generate costs, $T_c$, of 0.49%, while changing only the real growth rate the costs, $T_\theta$, are 0.47%. Across the range of inflation considered, around half of the costs are generated by the consumption effect and half by the real growth effect. Thus, the model estimates sizable effects on individual welfare of increasing the rate of inflation in the economy.

| TABLE 6. MONETARY EXPERIMENTS |
| Welfare effects, $\alpha = 2$ |
|--------------------------|-----------------|----------------|----------------|-----------------|-----------------|----------------|
| $\pi = 0.10$             | $\pi = 0.15$    | $\pi = 0.20$   | $\pi = 0.30$   | $\pi = 0.40$    | $\pi = 0.50$    |
| $T$                     | 0.0096          | 0.0126         | 0.0167         | 0.0225          | 0.0277          | 0.0324          |
| $T_c$                   | 0.0049          | 0.0064         | 0.0085         | 0.0114          | 0.0141          | 0.0164          |
| $T_\theta$              | 0.0047          | 0.0061         | 0.0081         | 0.0109          | 0.0131          | 0.0157          |

Note: Transfers are in percentage of the original steady state's income for indifference. Original: $\pi = 0.00$

As in Gomme (1993), these individual costs do not usually sum to the total welfare cost of a given level of inflation, in that each change is considered in isolation by maintaining the other variable at its balanced growth path value under zero inflation.
5. Conclusions

Empirical researchers have found that the average long-run rate of inflation in a country is negatively associated with the country's long run rate of growth. Similarly, the paper presented some relevant empirical facts about inflation and economic growth. I considered not only cross-country data on countries from Latin America and Asia but I also evaluated time series data from different countries, using the tools of spectral analysis. In both cases it was found a negative correlation between growth of the economy and the inflation rate.

The long run effects of inflation has been examined in a framework with endogenous growth where the demand for money was motivated by the fact that real money balances help to reduce transaction costs. The model belongs to the class previously studied by Uzawa (1965) and Lucas (1988), where the accumulation of human capital is the reason for sustained growth. The model predicts a negative, but small, relation between the level of inflation and the growth rate of the economy. However welfare effects are important at high levels of inflation and not negligible at low ones. These results are not sensitive to changes in the parameterization of the model.

A. The data

Here I describe the data use to analyze the empirical facts in Chapter 2 of the paper.

Cross Country Data: I have used data for inflation and growth in the GDP per capita from the Interamerican Development Bank’s Annual Report and the Asian Development Bank’s Asian Economic Outlook. Each observation correspond to the average for a particular country over one of the three periods considered: 1971-80, 1981-90 and 1991-95.

Times series data: data from 1969 to 1992 were used for every country. The inflation rate data were taken from the United Nations Anuario Estadístico de America Latina y el Caribe. The data for the growth rate of GDP per capita were calculate using the GDP per capita data taken from the Penn World Table. The data is that corresponding to real GDP per capita (Laspeyres Index) (1985 international prices) (RGDPL).

Size of the Financial sector: the data is average over two periods 1981-85 and 1986-90. The data on GDP by sector is from the United Nations National Accounts: Main Aggregates and Detailed Tables (Table 1.11). The inflation data is taken from the Interamerican Development Bank’s Annual Report and the Asian development Bank’s Asian Economic Outlook. The per capita income at world prices is from the Penn World Table (RGDPL).
B. Numerical calculation of the parameter $\phi$

Remember that the saving equation impose a restriction on the preferences parameters:

$$\alpha \theta + \rho = \frac{\gamma \theta + (\gamma - s) \delta}{s}$$

By definition:

$$\frac{m}{m} = \mu - \pi$$

so that in steady state:

$$\mu = \pi + \theta$$

From the expression for growth I can get that:

$$S(m, c) = \frac{(1 - \gamma + \eta) [\phi - (\alpha \theta + \rho)] + \eta \theta}{(1 - \gamma + \eta) \phi}$$

Also the expressions for labor supply, $l$, and those for the ratios $\frac{F}{k}$ and $\frac{c}{k}$ can be expressed as functions of $\alpha \theta + \rho$. Thus the balanced growth equations, originally a system of six equations and six unknowns, could be reduced to:

$$\mu = \theta - (\alpha \theta + \rho) - \frac{(1 - \gamma) (\delta + \alpha \theta + \rho) [(1 - \gamma + \eta) [\phi - (\alpha \theta + \rho)] + \eta \theta]^2}{(\alpha \theta + \rho - \theta) [\delta (1 - \gamma) - \gamma \theta + \alpha \theta + \rho] \phi \xi (1 - \gamma + \eta)^2}$$

This is a non linear equation that could be solved for $\phi$ once I specify the value for the parameters of the economy and after making use of the restriction on the preferences parameters. In setting these values I use those for the benchmark economy.
REFERENCES


