On the Role and Effects of IMF Seniority

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Documento de Trabajo N° 317

Santiago, Septiembre 2006

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On The Role and Effects of IMF Seniority.

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This Version: September 2006

Abstract

We analyze the IMF as a lender to countries in financial distress highlighting the fact that it is a senior creditor. An advantage of delegating senior lending in a single institution rather than on competitive markets is that it would be able to reach the socially optimal solution. This would require the IMF not to intervene when the crisis is severe enough. However, a commitment device might be needed to achieve the socially optimal solution. If IMF lending were done for all shocks, the country would be always ex-post better off but lenders would be worse off when the country situation is either good or weak, which is consistent with empirical evidence. Anticipation of senior lending might make the country better off by preventing inefficient liquidation. However it might actually hurt the country ex-ante and too much rescuing in the future could lead to too little lending in the present which is contrary to the moral hazard critique.

JEL classification: F30, F34, F40, E00.

Keywords: Seniority, Sovereign Debt, IMF, ex-ante, ex-post, welfare effects.

*E-mail address: dsaravia@faceapuc.cl. This paper is a modified version of the first chapter of my PhD dissertation at Maryland (2004). I am grateful to Fernando Broner, Guillermo Calvo, Enrique Mendoza, Michael Pries, Carmen Reinhart and John Shea for discussions and suggestions. I have also benefited from comments by Eduardo Ganapolsky, Pedro Rodriguez and seminar participants at the Bank of England, Columbia University, IDB, Latin American Econometric Society, Royal Economic Society Annual Meetings (2006), University of Guelph, UC Riverside and the University of Maryland.
1 Introduction

The role that the IMF should play in the International Financial Architecture and the effects of its interventions are important issues in the policy and academic debate, especially after the crisis that hit emerging market economies in the recent past beginning with Mexico in 1994. Arguably, the IMF has some special characteristics that make it a special player in international financial markets. Some argue that it may have more information than other lenders (e.g. Rodrik 1995) and that could be used by other lenders as a screening device (Marchesi and Thomas (1999)). Others claim that the IMF could act as a delegated monitor through its conditionality and surveillance functions and serves as a country’s commitment device to behave well (e.g. Tirole (2002)). Others highlight the role of the IMF as a liquidity provider to countries in financial distress or as an International Lender of Last Resort (e.g. Rogoff (1999) and Fisher (1999)). There is no consensus about the relevance of these characteristics, the way they affect capital flows to a country and how they help countries overcome financial crisis.

This paper also focuses on the role of the IMF as providing liquidity to countries in financial distress but focusing on another aspect of IMF lending specifically its status as senior lender.

The IMF is a de facto preferred creditor. Countries have shown a higher aversion to default on IMF loans than on loans from private creditors. Using Eichengreen (2003) words “The IMF typically gets paid back (instances of arrears to IMF loans are the exception to the rule).” Several commentators rationalize this preferred creditor status arguing that it allows the IMF to provide funds to countries in financial distress when other lenders are not willing to lend and countries respect this status because they know that they would need IMF assistance in the future. In turn, it is also argued, that the lower rates on IMF loans respond to its senior status. In principle, countries could assign this de facto preferred creditor status to other types of lenders. However, it is argued, that this status relies on the IMF because it is a big and identifiable creditor. The preferred creditor status of the IMF relies on the belief that it is better to maintain a relationship with the IMF because of its willingness in the future to provide emergency assistance, than it is with other lenders.

In this paper we analyze how the introduction of this fact in the analysis affects the way that

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1The argument is similar to the argument of giving seniority to new money in case of bankruptcy.

2See, for example, Roubini and Sester (2004), for a detailed discussion on these issues.
IMF lending interacts with private capital flows and how it affects countries and other lenders welfare. We conduct the analysis comparing a situation where senior lending is allowed with one where lending is left to atomistic lenders acting in competitive markets. We do this from an ex-post point of view, i.e. once initial lending decisions have been taken and from an ex-ante point of view, i.e. when the initial lending decisions are taken.

The presence of senior lending may introduce a conflict of interest between the country and non-senior lenders. Since a senior lender has a greater probability of being repaid, it will be able to lend in a crisis where other non-senior lenders are not willing to do so because of debt overhang and credit ceiling considerations. This would allow the economy to overcome the crisis but it may affect negatively non-senior lenders that see their debts diluted. In turn, private non-senior lenders would take into account these opposing forces of future senior interventions when making their initial loans to a country.

Since we are interested in the seniority issue, we will study the IMF as a deep-pocket investor with seniority rights that chooses to make zero profits in expectation despite its monopolistic situation; which, arguably, is a reasonable assumption about IMF lending. The relevant distinction in the model is, therefore, between senior and non-senior lending. As noted above this senior lending could also be interpreted as done by private lenders. However, the interpretation of the IMF as the senior lender is supported by the de facto seniority observed in the real world.

The paper presents a model with three periods: a planning period, a period when a shock hits the economy and a final period where a random level of output is obtained and consumption and debt repayment take place. In the planning period, the country borrows from international markets to invest in capital, which is used in the production process in order to maximize expected utility. In the middle period the country has already installed the capital and potentially has to borrow new money to cope with a liquidity shock. When the liquidity shock is high enough non-senior lenders would expect losses on new loans and, assuming that they are atomistic and cannot coordinate efforts to make “emergency loans” to protect initial claims, they will not be able to lend; in contrast, senior lenders would make nonnegative profits for a higher range of shocks since they have priority in case of default. If the economy is able to cope with the shock, a random level of output is obtained in the final period and if it were not able to do it, the country cannot continue with the project and has to default on its debts at that moment.
Borrowers are always ex-post better off if senior lending were available because it is always in their interests to cope with the shock and a senior lender lends at a lower interest rate than other lenders. However, the effects of senior lending on other existing lenders depend on the size of the shock and what they expect to get if the country were not able to cope with it (a scrap value in the model). When the shock is small enough such that non-senior lending is available a senior intervention makes existing lenders worse off independently of the scrap value since continuation occurs anyway but senior lending dilutes their debts. When the shock is high enough such that non-senior lending is not available, existing lenders would be worse-off with senior lending if the scrap value is high enough. The reason is that they would prefer liquidation rather than seeing their debts diluted. On the other hand, when the scrap value is not too high, existing lenders would prefer a senior intervention if the shock is not too high. The reason is that they prefer continuation although their debts are diluted. When the shock becomes high enough existing lenders prefer liquidation rather than seeing their debts diluted.\footnote{When the scrap value is an intermediate range, the prediction of the model is consistent with empirical evidence presented in Mody and Saravia (2006).}

In the planning period, lenders are aware of the nature, senior or non-senior, of future lending and take this into account when making their pricing decisions in that period. When the scrap value is small, continuation of the project is socially optimal. However, when the scrap value is high enough liquidation is optimal when the shock is also high enough. In this case, a social planner willing to maximize borrowers’ welfare ex-ante will allow liquidation to obtain the scrap value.

This solution could be obtained if senior lending were available only it is socially optimal to continue with the project. Since senior lending make nonnegative expected profits in the crisis period, this would imply that senior lending would not be available in cases when it is profitable to do it. This might be a reason why it is optimal to delegate senior lending in a single institution rather than in atomistic lenders acting in competitive markets. It is likely that these ones would lend automatically in the crisis period as long as they do not expect losses in their lending; while, it is more likely that a single lender would be able to discern about whether to lend in a given situation. However, even in this case there might be incentives to lend once the shock occurs. The reason is that it might be a dynamic inconsistency problem since there is a potential conflict between ex-ante and ex-post borrowers’ interests. Suppose senior lending is realized to maximize borrower welfare. Although it might be optimal ex-ante not to allow senior lending in the future for
high shocks, it is always in borrowers interests to continue ex-post. Thus, once capital is installed and the initial lending decisions have been taken there would be incentives to borrow from senior lender(s) to continue for all sizes of the shock. A senior lender willing to maximize borrowers’ welfare would need a commitment device to maintain the optimal policy from an ex-ante point of view. Without this commitment device, lenders in the planning period would expect senior interventions in the future and would price this in their initial loans making the country ex-ante worse off. Of course this tension does not exist when senior lending is optimal also from an ex-ante point of view.

Thus, the potential distortion that senior lending introduces is that it would allow for over-continuation of the projects. As shown in the paper this would imply that there would be less lending in the planning period when a senior intervention is expected in the future. This is contrary to the standard moral hazard critique that says that too much rescuing in the future would lead to too much lending in the present. Here too much rescuing in the future could lead to too little lending in the present.

In the paper we recognize that the first best outcome could be reached with senior lending; however, for the reasons given above, we analyze the case where senior lending occurs for all sizes of the shock and compare the effects of this potentially suboptimal policy with one where lenders are non-senior. In fact, some argue that there were cases in which the IMF has intervened too much in a country. In the case of Argentina, for example, anecdotal evidence suggests that even private lenders at some point preferred the IMF to stop lending, making the country face the “unavoidable” debt crisis. For example, when referring to the Argentine crisis, a financial industry official said that members of the lending community have told Argentine and IMF authorities:

“The game is up you need a broad-based restructuring that involves a reduction in the value of our claim” (Blunstein (2005) pp. 163)

Next subsection relates this work to the literature. Section 2 describes the elements of the model. Section 3 solves the model backwards. We compare a situation where senior lending is not allowed, and creditors have equal sharing in case of default, with one where it is. Analyzing period 1, when capital is installed and the shock hits the economy, we will examine ex-post effects of senior intervention on the country and private creditors’ welfare. In period 0, when borrowing and lending decisions are made, we study how the possibility of a senior intervention affects the
initial level of investment and the country’s welfare ex-ante. Section 4 concludes.

1.1 Relation to the Literature

The paper contributes to the discussion about the role of the IMF as an International Lender of Last Resort (ILOLR). On the one hand, some have defended the intervention of the IMF as a LOLR (e.g. Fischer (1999)). On the other, some are more skeptical about its ability to perform this role (e.g. Rogoff (1999), Calomiris (1998)). An important point in this debate is the tradeoff between ex-ante moral hazard and ex-post efficiency. Some argue that having an ILOLR institution able to fill liquidity needs reduces the probability of crisis and ameliorates their effects once they occur. Other claims that an ILOLR would trigger debtor and other creditors’ moral hazard, because knowing that there will be a future bailout would make them take riskier strategies. Our model abstracts from coordination and moral hazard issues and adds to the literature by considering the nature of senior lender of the ILOLR. In our context, although the IMF might allow the country to cope with the crisis improving its situation ex-post, it might hurt other lenders if it lends when it is not socially optimal to do it causing, in this case, over-continuation of the project. The IMF would be able to reach the first best solution, although this might mean not lending when the crisis is severe. However this solution might not be reached because of the conflict between ex-post and ex-ante incentives and the expectation of future interventions might lead to a lower level of borrowing in the present. Thus too much rescuing in the future would lead to too little borrowing in the present which is contrary to the standard moral hazard critique.

Recent theoretical work by Corsetti et al. (2003) studies the role of the IMF in catalyzing capital flows by providing liquidity in a model with coordination problems between creditors having asymmetric information about the state of the economy. In one of the extensions to their model, they consider the case where the IMF is a senior lender. They conclude that since a senior lender is more willing to intervene, the probability of a crisis would be reduced, but since the return to junior lenders is lower they would be less willing to roll over their debts. As noted above, in our paper, we are not concerned with coordination problems and roll-over of short term debt issues although

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4 See Roubini and Sester (2004) for a summary of the literature on this issue.
5 Morris and Shin (2003) use a similar analysis to Corsetti et.al. to analyze the IMF’s ability to catalyze capital flows. Penalver (2004) reaches similar conclusions to Morris and Shin’s work with a different modeling strategy. None of these works analyzes the role of IMF seniority.
we recognize they are important. Rather, our framework allows us to analyze the impact of senior interventions on borrowers’ and lenders’ ex-ante and ex-post welfare, highlighting the conflict of interest between borrowers and lenders that a senior intervention may imply.

The issue of seniority was more studied in the corporate finance literature than in the context of sovereign debt. In an early contribution, Fama and Miller (1972) recognize that lenders can protect themselves from future dilution of their debts by issuing senior debt. Generally, the objective of these works is to explain the observed capital structure of the firm and derive the optimal one. For example Diamond (1993), in a context of asymmetric information about borrower’s type, argues that short-term debt would be senior to long-term debt. In his model issuing senior short term debt increases the sensitivity costs to new information for a given control rent and would be the contract that the best type chooses. Hart and Moore (1995) argue that long term senior debt is useful to constrain management from “empire building” investment (overinvestment). Berkovitch and Kim (1990) also argue that allowing for senior lending may cause over investment. In our model allowing for a senior intervention may cause continuation of the project when it is socially optimal to liquidate ex-post. However, the level of investment may be lower ex-ante. Detriagache (1994) highlights the fact that seniority clauses are not used in sovereign debt. She argues that this lack of seniority clauses (and the presence of equal-sharing) leads to excessive borrowing. In our paper we compare two scenarios, one where senior lending is allowed with one where all lenders have equal sharing. As mentioned above, in our context, equal-sharing would lead to less borrowing ex-post, although it may be higher ex-ante, than when senior lending is allowed. Dolley (2000) argues that countries optimally choose debt instruments that are difficult to restructure in order to maintain the punishment necessary to avoid strategic default. This would imply that International Financial Institutions would not have to intervene in crisis; otherwise, initial lending would not be possible in the first place. We obtain that in some cases the IMF should abstain from lending if it were interested in maximizing countries’ ex-ante welfare. However, our reasons and circumstances are different. In a recent contribution Bolton and Jeanne (2005) study the effects of debt dilution in choosing the optimal structure of sovereign debt. They argue that, in equilibrium, countries would issue debt that is difficult to restructure in order to avoid future dilution. In this paper we also have, in a different context, debt dilution when allowing for senior interventions. Both papers differ in motivation, framework and conclusions.
2 Model

*Time.* There are three periods, indexed by $t=0,1,2$. In period 0, agents make real investment and borrowing decisions. In period 1, the economy can be hit by a shock that affects the production process. In order to cope with this shock, agents have to borrow again. In period 2, output is realized, debt issued in period 0 and 1 is repaid and consumption takes place.

*Agents and production.* The economy is populated by a continuum of identical consumer-producers with linear preferences over consumption of a single good at date 2; i.e their utility function is $U(c_0, c_1, c_2) = c_2$. The production process has a time-to-build aspect: investment is realized in period 0 and 1 and output is realized in period 2. It is assumed that agents do not have any endowment of goods in period 0 and 1, so they have to borrow from abroad in order to import goods used as inputs in the production process. In period 0, agents borrow to install capital, $k_0$, which will be depreciated totally at the end of period 2.

To avoid borrower’s moral hazard considerations, we assume that investment is verifiable, or alternatively, that there is no storage technology available, so that the amount borrowed has to be invested in the production process.

Following Holmstrom and Tirole (1998) and Caballero and Krishnamurthy (2001) we introduce a liquidity shock in period 1 as a production shock that the economy has to cope with by borrowing additional funds.

Let $\rho$ be the aggregate liquidity shock that hits the economy in period 1. Agents will need a reinvestment of $\rho k_0$ to continue the project. If they do not reinvest this amount, then the project cannot continue and a scrap value, $S(k_0)$, is obtained in period 2. $S$ is assumed to be quasiconcave, increasing in $k_0$ and satisfies $S \leq k_0$.  

Assume $\rho$ is a random variable distributed between $[0, 1]$ with cumulative distribution function $G(\rho)$. In order to introduce market incompleteness, we assume that $\rho$ is observable but not verifiable, so that contracts in period 0 cannot be made contingent on realized values of the shock in period 1. 

We do not consider idiosyncratic shocks since we are interested in cases in which the economy as a whole needs liquidity, and we are not concerned with heterogeneity between

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6The scrap value could be interpreted as what lenders expect to get if the country is not able to overcome the crisis; for example, what they expect to get in a debt restructuring process.

7This assumption prevents countries signing insurance contracts (or pool liquidity risk) in period 0.
residents.\footnote{The assumption that the liquidity shock has to be reinvested rules out the possibility that new lending done to cope with the shock is used for other purposes, like paying existing lenders which is, according to some commentators, a possibility with IMF funds.}

If reinvestment is made in period 1, then the project continues and output in period 2 is $\lambda f(k_0)$, where $\lambda$ is a random productivity shock distributed between $[0, \bar{\lambda}]$ with cumulative distribution $F(\lambda)$, and where $f(k_0)$ is a concave function. It is assumed that $E(\lambda)f(k_0) > k_0$; otherwise, investors will not invest in period 0. So, the model presents two kind of shocks; $\rho$ is needed to introduce the demand for liquidity and $\lambda$ is introduced to allow for the possibility of default that, as will be explained later, is needed for the relevance of the seniority issue.

\[
\begin{align*}
\text{period 0} & \quad \text{period 1} & \quad \text{period 2} \\
    k_0 & \quad \rho k_0 & \quad \lambda f(k_0) \\
\text{reinvest} & \quad & \text{not reinvest}
\end{align*}
\]

$\rho \sim G[0, 1]$

$\lambda \sim F[0, \bar{\lambda}]$

$S(k_0)$

Financial contracts. As noted above, residents have to borrow from abroad in order to produce. This is an ability-to-pay model with no deadweight losses associated with bankruptcy. That is, when realized output is lower than debt face value or when the project is discontinued, lenders can seize output or the scrap value.

It is assumed that debt issued in period 0 and debt issued in period 1 both mature in period 2. International lenders are risk neutral, act in a competitive environment and have enough wealth to provide liquidity to the country when needed. Clearly, for any amount lent they will charge a positive interest rate since the default risk is positive (remember that the minimum value that $\lambda$ can take is zero).

Without loss of generality, it is assumed that the gross international interest rate is equal to 1. At date 0 domestic agents borrow an amount $L_0$ (equal to $k_0$) and agree to pay a total amount of $D_0$ (i.e. initial amount borrowed plus interest) in period 2. At date 1 they borrow an amount $L_1$ (equal to $\rho k_0$) whose face value in period 2 is $D_1$. 

\[
\begin{align*}
\text{period 0} & \quad \text{period 1} & \quad \text{period 2} \\
    k_0 & \quad \rho k_0 & \quad \lambda f(k_0) \\
\text{reinvest} & \quad & \text{not reinvest}
\end{align*}
\]
3 Equilibrium

In what follows we will solve the model backwards beginning with period 2. In period 1, when the shock hits, we will consider what happens when senior lending is allowed in that period. Then we will consider period 0.

3.1 Period 2

In period 2, if reinvestment has been made in period 1, output is realized, debt is repaid, and consumption takes place. Consumption will be greater than zero if and only if output is greater than the total face value of debt contracted in period 0 ($D_0$) and in period 1 ($D_1$), which occurs when:

$$\lambda f(k_0) - D_0 - D_1 > 0$$

or, equivalently:

$$\lambda > \frac{D_0 + D_1}{f(k_0)} \equiv \lambda^*.$$  \hspace{1cm} (1)

Thus, total debt will be repaid and consumption will be positive if and only if the productivity shock is higher than a threshold value $\lambda^*$.

**Assumption 1.** In case of default (i.e. $\lambda < \lambda^*$) the proportion of output that goes to each creditor equals the share of his loan in total loans, i.e.

$$L_i / (L_i + L_{-i}).$$

That is, absent seniority, creditors have equal footing on output in case of bankruptcy. In the real world most sovereign debt restructuring are complicated processes with special characteristics of their own. However, although their interpretation and effectiveness is somewhat controversial, sovereign debt instruments usually have pari-passu clauses to avoid discrimination against them and to prevent their dilution in the event of default (see for example the Financial Market Law Committee (2005)). Also, when doing their restructuring proposals, many countries claim that they are based in the principle of equal treatment between private debt instruments.

The assumption that partial repayment in case of default is done without considering the interest rate is important for the analysis. We have not assumed that output is divided using the loan plus interests, i.e. $\frac{D_i}{D_i + D_{-i}}$, because new lenders would dilute more heavily existing debt for high shocks.
and could become effectively senior. In the context of our model this would imply that borrowers would always find fresh founds to cope with the shock by offering an interest rate that is high enough in detriment of existing lenders. This is because borrowers always want to continue for all sizes of the shock and, consequently, they would have incentives to offer an interest rate high enough to dilute existing lenders’ debt. Argentina constitutes a real world example where the total amount of interests accrued were not taken into account to determine the eligible debt to be restructured.

Under the assumption we use in the paper, there is also dilution of existing debt. The higher is the shock in period 1 the higher is the dilution of period 0 debt. However, this dilution is less severe than in the case where interests are taken into account to make the partial payments in case of default.

Since \( \frac{L_i}{L_i + L_{-i}} \) need not be the same as \( \frac{D_i}{D_i + D_{-i}} \), it is possible that the output due to a creditor in case of default is higher than his debt face value. To rule this out, assume:

**Assumption 2.** In case of default, if \( \frac{L_i}{L_i + L_{-i}} \lambda f(k_0) \) is greater than \( D_i \) then lender \( i \) gets \( D_i \).

Thus, a creditor’s repayment in period 2 will be the maximum of his contractual value of debt and his share of output under the equal footing scheme.

If reinvestment has not taken place in period 1, the scrap value of the project, \( S(k_0) \), is divided between creditors, and consumption is equal to zero (remember that by assumption \( S(k_0) < k_0 \) and, consequently, \( S(k_0) < D_0 \)).

### 3.2 Period 1

At the beginning of this period the random variable \( \rho \) is observed, there is installed capital \( (k_0) \), and the economy inherits a stock of debt contracted in period 0 \( (D_0) \). Agents need to borrow \( \rho k_0 \) in order to continue the project. Since it is assumed that if reinvestment is not made the project ends and consumption is zero, the borrower country will always want to reinvest as long as the highest possible output level is higher than the total value of debt. So the demand for loans is determined by the size of the shock.

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9 This ratio converges to one when the interest rate, and thus \( D_i \) converges to infinity.
10 Argentina defaulted on December of 2001. It was not until January of 2005 that the offer of new instruments opened. The eligible debt would include the interest accrued and not paid until December of 2001 (see the Exchange Offer presentation on January 12th 2005. Available at www.argentinedebtinfo.gov.ar)
3.2.1 Supply of loans under equal footing

As noted above, international capital markets are competitive and the international gross interest rate is equal to 1. Competition between lenders will ensure that expected profits from lending to the country will be zero.

Define \( \lambda^1 \) as the threshold productivity level above which period 1 lenders’ output share, computed under equal footing, is greater than their contractual debt value,

\[
\lambda^1 = \left[ \frac{L_0 + L_1}{L_1} \right] \frac{D_1}{f(k_0)},
\]

or equivalently, since \( L_1 = \rho k_0 \) and \( L_0 = k_0 \):

\[
\lambda^1 = \left[ 1 + \frac{\rho}{\rho} \right] \frac{D_1}{f(k_0)}.
\]  \(\text{(2)}\)

Similarly, define \( \lambda^0 \) as the threshold value above which period 0 lenders’ output share is greater than \( D_0 \):

\[
\lambda^0 = [1 + \rho] \frac{D_0}{f(k_0)}.
\]  \(\text{(3)}\)

This last expression follows from the fact that \( [1 + \rho] \) is equivalent to \( \left[ \frac{L_0 + L_1}{L_0} \right] \).

Note that \( \left[ \frac{\rho}{1 + \rho} \right] \lambda^1 + \left[ \frac{1}{1 + \rho} \right] \lambda^0 = \lambda^* \), so that the threshold productivity shock above which all debts are repaid \( (\lambda^*) \) is a weighted average of \( \lambda^1 \) and \( \lambda^0 \). When \( \lambda^1 \) is lower than \( \lambda^* \), it means that \( D_1 \) is totally repaid when the productivity shock is at least \( \lambda^1 \); for productivity shocks between \( \lambda^1 \) and \( \lambda^* \), \( D_0 \) holders get output in excess of \( D_1 \); and when the productivity shock is higher than \( \lambda^* \), output is enough to repay both \( D_0 \) and \( D_1 \). A comparable analysis holds when \( \lambda^0 \) is lower than \( \lambda^* \). Also, note that \( \lambda^0 \) will be higher than \( \lambda^1 \) if and only if the interest rate charged on period 0 loans is higher than the interest rate charged in period 1; both interest rates are determined in equilibrium below.

Thus, period 1 lenders’ zero profit condition under equal footing satisfies:

\[
\rho k_0 = \left[ \frac{\rho}{1 + \rho} \right] \int_0^{\min(\lambda^1,\lambda^0)} \lambda f(k_0) dF(\lambda) + \int_{\min(\lambda^0,\lambda^*)}^{\lambda^*} [\lambda f(k_0) - D_0] dF(\lambda) + \int_{\min[\lambda^1,\lambda^*]}^{\lambda^*} D_1 dF(\lambda).
\]  \(\text{(4)}\)

The right hand side is period 1 lenders’ expected repayment from investing in the country and
the left hand side is the amount lent. Alternatively, we can express the same condition in terms of each unit lent:

\[
1 = \left[ \frac{1}{1 + \rho} \right] \int_0^{\min(\lambda^1, \lambda^0)} \frac{\lambda f(k_0)}{k_0} dF(\lambda) + \frac{1}{\rho} \int_{\min(\lambda^0, \lambda^*)}^{\lambda} \left[ \frac{\lambda f(k_0)}{k_0} - \frac{D_0}{k_0} \right] dF(\lambda) + \int_{\min(\lambda^1, \lambda^*)}^\lambda r_1 dF(\lambda),
\]

where \( r_1 = \frac{D_1}{\rho k_0} \) is the gross interest rate charged to the country by international lenders.

**Lemma 1.** The interest rate \( r_1 \) is increasing in the amount lent.

Proof in the appendix.

So, the higher period 1 shock is, i.e. the higher the amount needed to continue the project, the more expensive, per dollar, it will be for the borrower to continue.

**Proposition 1.** There is a set of liquidity shocks sufficiently close to 1 for which no credit is supplied in period 1 under equal footing if and only if

\[
\int_0^{\tilde{\lambda}} \frac{\lambda f(k_0)}{2k_0} dF(\lambda) + \int_{\min(\lambda^0, \tilde{\lambda})}^{\tilde{\lambda}} \left[ \frac{1}{2} \frac{\lambda f(k_0)}{k_0} - \frac{D_0}{k_0} \right] dF(\lambda) < 1,
\]

(5)

**Proof.** A necessary and sufficient condition to have lending in period 1 that satisfies the zero profit condition under equal footing is:

\[
\rho k_0 \leq \frac{\rho}{1 + \rho} \int_0^{\tilde{\lambda}} \lambda f(k_0) dF(\lambda) + \int_{\min(\lambda^0, \tilde{\lambda})}^{\tilde{\lambda}} \left[ \frac{1}{1 + \rho} \lambda f(k_0) - D_0 \right] dF(\lambda).
\]

(6)

This is because, given the loan size (\( \rho k_0 \)) and the value of debt issued in period 0 (\( D_0 \)), period 1 lenders’ expected repayment is increasing in \( D_1 \); and the right hand side of (6) is lenders’ expected repayment when the value of \( D_1 \) is high enough that total debt (\( D_1 + D_0 \)) is greater than or equal to the highest possible repayment (\( \tilde{\lambda} f(k_0) \)).\(^{11}\) If condition (6) is not satisfied then period 1 creditors will expect losses on any loan of size \( \rho k_0 \). The set of values for \( \rho \) satisfying (6) is not empty. The right hand side is unambiguously greater than the left hand side for values of \( \rho \) near zero since \( \int_0^{\lambda} \frac{\lambda f(k_0)}{k_0} dF(\lambda) \) is greater than one.

\(^{11}\)If \( D_1 + D_0 > \tilde{\lambda} f(k_0) \), then \( \lambda^* > \tilde{\lambda} \) and \( \lambda^1 > \tilde{\lambda} \). Thus, the left hand side of (6) follows from replacing \( \lambda^* \) by \( \tilde{\lambda} \) in the left hand side of (4), taking into account that the third term vanishes.
Since the first term of the right hand side of (6) is a continuous, increasing and concave function of $\rho$ and the second term is continuous and decreasing in $\rho$, a necessary and sufficient condition to have a range of liquidity shocks where expected profits are negative is that (6) is not satisfied when $\rho$ is equal to one. So, if condition (5) holds, there will be a threshold value of $\rho$ strictly less than one above which expected profits to lenders are negative. Since the expected repayment function is increasing and continuous in $D_1$, there will be a value of $D_1$ such that expected repayment equals the loan size.

In what follows we assume that condition (5) holds, in which case there is a $\hat{\rho}$ less than 1 that satisfies:

$$\hat{\rho}k_0 = \frac{\hat{\rho}}{1 + \hat{\rho}} \int_0^\lambda \lambda f(k_0)dF(\lambda) + \int_\min[\lambda_0, \lambda]^\lambda \left[\frac{1}{1 + \hat{\rho}} \lambda f(k_0) - D_0\right] dF(\lambda)$$

such that for $\rho > \hat{\rho}$ there will be no lending under equal footing. A sufficient condition to have $\hat{\rho} < 1$ is that (5) is true even in the case where $D_0$ is equal to $k_0$, which is the lowest possible interest rate on period 0 debt and thus the case most likely to favor lending in period 1. Therefore, a sufficient condition is:

$$E(\lambda)\frac{f(k_0)}{2k_0} + \int_\min[2k_0, \lambda]^{\lambda} \left[\frac{\lambda f(k_0)}{2k_0} - 1\right] dF(\lambda) < 1.$$  

Note that it may be in the interest of period 0 lenders, as a group, to lend in period 1 at an expected loss in order to protect their initial claims. However, any individual lender will be better off if the other lenders provide liquidity allowing the project to continue. That is, there is a conflict between private and collective interests; each period 0 lender has incentive to ‘free-ride’.\footnote{This free rider problem has been acknowledged and discussed in the sovereign debt literature; see for example Krugman (1988) and Eichengreen (2002).} The lack of coordination between creditors is a characteristic of actual sovereign debt markets and the way to overcome the free-rider problem is an important unresolved issue in current policy and academic debate.\footnote{In a recent speech Anne Krueger states: “...These far-reaching developments in capital markets over the last three decades have not been matched by the development of an orderly and predictable framework for creditor coordination. Because the creditor community is increasingly diverse and diffuse, coordination and collective action problems result when scheduled debt service exceeds a country’s ability to pay” (see IMF survey April 2000).}

Clearly, creditors that have not lent in period 0 do not have any incentive to lend at an expected
loss in period 1. In this paper we assume that lenders are atomistic, act in a purely competitive market and cannot coordinate actions to pursue their collective interests (i.e. the free-rider issue is severe).\footnote{If we had assumed that period 0 lenders act in their interest as a group, then they might want to lend when the shock is so big that lending under equal footing is at a loss and continue with the project.}

### 3.2.2 Senior Lending allowed in period 1

Consider the case where senior lending is allowed in credit markets in period 1. The concept of seniority is relevant when contractual obligations cannot be totally satisfied; i.e. in the case of default. If this is not the case, there is no conflict of interest between creditors and the concept of seniority is not important. As noted above, in this model, we can think of this senior lending as done by competitive markets or as done by a single senior lender (such as the IMF) that chooses to make zero profits in expectation despite its monopolistic situation which is, arguably, a reasonable assumption about IMF lending. Also, as mentioned in the introduction, the interpretation of the IMF as the senior lender is supported by the \textit{de facto} seniority that has historically shown over other types of sovereign debt instruments.

Since senior creditors have priority on output in case of default, they do not have to consider the stock of existing debt when making their own lending decisions.

**Lemma 2.** Senior lenders do not expect negative profits for any loan in period 1.

*Proof:* The maximum payment that senior lenders could get is $E(\lambda)f(k_0)$, which is greater than $\rho k_0$, for all $\rho$, by previous assumption.

Thus, senior lending would be available in more states of nature than non-senior lending. If the only criteria used by senior lenders to make their loans in period 1 is the nonnegativity of their expected profits, they would lend for all sizes of the liquidity shock. As will be noted below when discussing the period-0 decisions, if senior lending is done in order to maximize expected utility ex-ante it would not be realized when the liquidity shock in period 1 and scrap values are high enough.

In the rest of this section we analyze how senior lending done to cope with a given sized shock in period 1 affects borrowers and period-0 lenders’ situation once the initial borrowing and investment decisions have been taken (i.e. ex-post).
Let \( D^s_1 \) be the value of debt owed to a senior creditor; the threshold productivity shock above which senior lenders are totally repaid is:

\[
\lambda^s \equiv \frac{D^s_1}{f(k_0)}.
\]  

(8)

If the productivity shock is lower than this threshold value, senior creditors will not be totally repaid and non-senior creditors will get nothing. The interest rate charged by a senior lender satisfies:

\[
\frac{1}{L^s_1} \int_0^{\lambda^s} \lambda f(k_0) dF(\lambda) + \int_{\lambda^s}^{\hat{\lambda}} r^s_1 dF(\lambda) = 1,
\]

(9)

where \( L^s_1 \) and \( r^s_1 \) are the amount lent by a senior creditor and the interest rate charged, respectively. The interest rate charged by a senior lender will not be the same as that charged by a non-senior one. In particular:

**Lemma 3.** For a given sized loan, the interest rate charged by a senior lender is lower than that charged by a lender without seniority rights.

Proof in the appendix.

This result implies that total expected consumption in period 2 is higher when a senior lender intervenes and, consequently, the country is ex-post (i.e. conditional on \( k_0 \)) better off under seniority. Obviously, borrowers prefer to pay less for a given amount lent.

At the beginning of period 1 there is a stock of debt issued in period 0 (\( D_0 \)) that matures in period 2. The period 1 value of this stock of debt will be affected by the size of the liquidity shock and by the nature (senior or non-senior) of period 1 lenders.

To see the impact of a senior intervention on the period 0 lenders’ position, we have to consider whether the liquidity shock is greater or less than \( \hat{\rho} \), the threshold value above which non-senior creditors are unwilling to lend.

Consider first the case when \( \rho < \hat{\rho} \). In this situation non-senior lenders are willing to lend to the borrower country and a senior intervention will make period 0 lenders worse off. To see why this is the case note that output is divided in period 2 between the country, period 0 and period 1 creditors. At the beginning of period 1, the expected value of output is given, since with \( \rho < \hat{\rho} \) the project will continue whether period 1 lenders are senior or not. Meanwhile period 1 lenders,
independent of their seniority rights, set the price of the new debt \((r_1 \text{ or } r^*_1)\) so that expected repayments in period 2 are equal to the size of the loan \((\rho k_0)\), by the zero profit condition.

Since expected output and expected repayment to period 1 lenders are the same with and without senior lending, but expected consumption is higher in the first case, it must be the case that period 0 lenders’ expected repayment (or, equivalently, the period 1 value of their claims) is lower under a senior intervention. A senior lender does not add value when the country is able to finance the liquidity shock using non-senior sources, but instead merely transfers resources from period 0 debt holders to the country. So, a senior intervention when \(\rho < \hat{\rho}\) reduces the period 1 price of the debt issued in period 0.

Consider now the case where \(\rho > \hat{\rho}\). In this case, the only way to finance the liquidity shock is by issuing senior debt.

To see how senior lending affects existing creditors in this situation, we compare the period 1 value of existing debt with and without seniority. When senior lending is not allowed, the project is canceled and the scrap value is obtained. Since this is an ability-to-pay model, period 0 lenders get the entire scrap value (remember that we have assumed that the scrap value is less than \(k_0\)). Let \(\text{V}^n\) be the period 1 value of \(D_0\) when there is no refinancing, that is:

\[
\text{V}^n(k_0) = S(k_0)
\]

and let \(\text{V}^s\) be the period 1 value of \(D_0\) when a senior intervention is allowed,

\[
\text{V}^s = \int_{\lambda^s}^{\lambda^B} \left[ \lambda f(k_0) - D^*_1(\rho) \right] dF(\lambda) + \int_{\lambda^B}^{\hat{\lambda}} D_0 dF(\lambda)
\]

where

\[
\lambda^B \equiv \frac{D_0 + D^*_1}{f(k_0)} \tag{10}
\]

and

\[
\lambda^s \equiv \frac{D^*_1(\rho)}{f(k_0)}
\]

The period 1 value of debt issued in period 0 is equal to the face value \((D_0)\) times the probability of being fully repaid, which occurs when the productivity shock is higher than the threshold value \(\lambda^B\), plus what existing creditors expect to get when output is not enough to cover total contractual
obligations. When the productivity shock is between $\lambda^s$ and $\lambda^B$ output is enough to cover senior debt in full but covers only part of non-senior debt. When the shock is less than $\lambda^s$, output is not enough to cover senior debt, and non-senior creditors get nothing.

Define the function $\psi(S, \rho)$ as the difference between the period 1 value of debt when a senior intervention is allowed and when it is not:

$$\psi(S, \rho) \equiv V^s - V^n.$$ 

That is, positive values of $\psi$ imply that period 0 lenders are better off with a senior intervention. $\psi$ is a function of the liquidity shock and of the scrap value, since both parameters affect the present value of debt with and without senior lending. We have:

$$\frac{\partial \psi}{\partial \rho} = - \int_{\lambda^s}^{\lambda^B} \frac{\partial D^s}{\partial \rho} dF(\lambda) < 0$$

and

$$\frac{\partial \psi}{\partial S} = -1 < 0.$$ 

Thus, $\psi(S, \rho)$ is a decreasing function in both arguments.

Note that when there is no scrap value (i.e. $S = 0$), $\psi(0, \rho)$ is greater than zero for all values of $\rho$. This is because cancelation leaves existing creditors with zero, while continuation leaves existing creditors with strictly positive expected returns.\(^{16}\) Also note that if the scrap value were equal to $D_0$, $\psi(D_0, \rho)$ is strictly negative for all values of $\rho$ since cancellation gives period 0 debt holders the full value of debt with certainty, while a senior intervention reduces the probability of repayment below one.

Since $\psi(S, \rho)$ is a continuous and decreasing function in both arguments, and since $\psi(0, \rho) > 0 \ \forall \rho$ and $\psi(D_0, \rho) < 0 \ \forall \rho$, there is for each $\rho$ a unique value of $S$, denoted by $S^0(\rho)$, where $\psi(S, \rho) = 0$. The higher the liquidity shock, the lower the value of $S^0$. We can express this in the following figure:

\(^{15}\)The terms derived from the differentiation of the integration limits cancel each other out.

\(^{16}\)The only case when period 0 debt holders expect to get nothing in case of continuation is when $D^s_1$ is equal to $\bar{\lambda} f(k_0)$; but in this case senior lenders’ expected profits will be strictly positive (since $k_0$ is lower than $E(\lambda)f(k_0)$) contradicting the zero profit condition.
Thus, existing creditors’ view of senior intervention depends on the size of the liquidity shock and the project’s scrap value. We can distinguish three situations. First, when the scrap value is lower than $S^0(1)$, a senior intervention will raise the value of existing debt for all $\rho > \hat{\rho}$. In this case, the value of liquidation is so low that even in the worst possible scenario (highest senior debt) period 0 lenders prefer to continue the projects.

Second, when the scrap value is between $S^0(1)$ and $S^0(\hat{\rho})$ there is a set of liquidity shocks in the vicinity of 1 where a senior intervention makes period 0 debt holders worse off. Moreover, there is a set of liquidity shocks close enough (from the right) to $\hat{\rho}$ where a senior intervention makes period 0 debt holders better off. So, in this zone seniority has ambiguous effects on existing creditors depending on the size of the liquidity shock. In particular, there is a nonlinear effect of senior intervention on the price of the debt issued in period 0. When the shock is small ($\rho < \hat{\rho}$) a senior intervention reduces this price (i.e. increases spreads over the international interest rate); when the shock is not too far above $\hat{\rho}$, a senior intervention increases this price; and when the shock is close to 1 the price is reduced by senior intervention again. This effect is consistent with the empirical evidence in Mody and Saravia (2006) where they show that IMF interventions increase spreads when countries’ solvency and liquidity situation is either good or weak and reduce spreads...
when it is in an intermediate range.\textsuperscript{17, 18}

Finally, when the scrap value is higher than $S^0(\hat{\rho})$, a senior intervention always makes period 0 debt holders worse off. Because the scrap value is so high, initial lenders prefer to get that value for sure rather than continuing the project and taking the risk of not being repaid.

We can summarize the findings of this section in the following proposition:

**Proposition 2.** Conditional on $k_0$ a senior intervention will improve debtors’ situation in all cases since it allows a higher level of consumption. The effect on period 0 debt holders depends on $\rho$ and $S$:

- If $\rho < \hat{\rho}$ a senior intervention will always make existing creditors worse off.
- If $\rho > \hat{\rho}$ we have three possible scenarios:
  1. If $S < S^0(1)$ senior lending makes existing creditors better off for all values of $\rho$.
  2. If $S^0(1) < S < S^0(\hat{\rho})$ existing creditors’ situation will improve if $\rho$ is close enough to $\hat{\rho}$ and will be worsened if $\rho$ is close enough to 1.
  3. If $S^0(\hat{\rho}) < S$ senior lending always makes existing creditors worse off.

That is, senior lending may affect borrowers and lenders differently; in some cases, it will allow for the continuation of projects when existing creditors would prefer to liquidate them. In these cases, there is a conflict of interest between the borrower and existing lenders since the former is always willing to finish the project.

### 3.3 Period 0

Period 0 is the planning period. Borrowers decide how much to invest and borrow in order to maximize their expected utility (expected consumption in period 2), and lenders set the price of their loans in order to attain zero expected profits.

In period 0 individuals have uncertainty about two shocks: the liquidity shock ($\rho$) and the productivity shock ($\lambda$). That is, expectations have to be taken over two random variables. We consider the case where all agents have perfect foresight about the nature of future interventions.

\textsuperscript{17}In this model, since the international gross interest rate is one, spreads in period 1 are equal to the ratio between the contractual value of debt issued in period 0 ($D_0$) and its market value in period 1 ($V^s$ or $V^n$).

\textsuperscript{18}The higher is the liquidity shock the worse is the solvency situation.
That is, borrowers and lenders take their decisions knowing whether interventions in period 1 will be senior or equal footing.

### 3.3.1 Equal footing in period 1

Agents make their decisions taking into account that if the liquidity shock in period 1 is high enough the project will have to be discontinued and there will be no consumption and only partial debt repayment.

In equilibrium, borrowers in period 0 decide the amount they want to borrow in order to maximize their expected utility, taking into account how their decisions affect the credit conditions they face. Borrowers maximize:

$$V_0 = \max_{k_0} \int_0^{\hat{\rho}(k_0)} \left\{ \int_{\lambda^*}^{\hat{\lambda}} [\lambda f(k_0) - D_0 - D_1(\rho k_0)] dF(\lambda) \right\} dG(\rho)$$  \hspace{1cm} (11)

subject to

$$k_0 = \int_0^{\hat{\rho}(k_0)} \left\{ \left[ \frac{1}{1 + \rho} \right] \int_0^{\min(\lambda^1, \lambda^0)} \lambda f(k_0) dF(\lambda) + \int_{\min(\lambda^1, \lambda^*)}^{\lambda^*} [\lambda f(k_0) - D_1] dF(\lambda) + \right. \right.$$ 
$$+ \int_{\min[\lambda^0, \lambda^*]}^{\lambda} D_0 dF(\lambda) \right\} dG(\rho) + \int_{\hat{\rho}(k_0)}^{1} S(k_0) dG(\rho)$$ \hspace{1cm} (12)

and

$$\rho k_0 = \left[ \frac{\rho}{1 + \rho} \right] \int_0^{\min(\lambda^1, \lambda^0)} \lambda f(k_0) dF(\lambda) + \int_{\min(\lambda^0, \lambda^*)}^{\lambda^*} [\lambda f(k_0) - D_0] dF(\lambda) + \int_{\min[\lambda^1, \lambda^*]}^{\lambda} D_1 dF(\lambda).$$  \hspace{1cm} (13)

$V_0$ is borrowers’ expected utility, and $\lambda^*$, $\lambda^1$ and $\lambda^0$ are as defined above in (1),(2) and (3) respectively. The outer integral of (11) corresponds to expectations taken over the liquidity shock, recognizing that if $\rho > \hat{\rho}(k_0)$ consumption is zero under equal footing. The inner integral corresponds to expectations taken over the productivity shock, knowing that consumption will be positive if output is enough to cover the total value of debt contracted in period 0 and in period 1. That is, consumption will be positive if and only if $\rho > \hat{\rho}(k_0)$ and $\lambda > \lambda^*.$

Equation (12) is the zero expected profit condition for period 0 lenders who face uncertainty
about both the liquidity shock and the productivity shock. They know that if \( \rho > \hat{\rho}(k_0) \), the project will not continue and they will get the scrap value. If \( \rho < \hat{\rho}(k_0) \) (i.e. there is no liquidation in period 1) what they expect to get in period 2 depends on the productivity shock. Analogously with the period 1 lenders’ zero profit condition in equation (4), if output is not enough to cover either \( D_0 \) or \( D_1 \), period 0 lenders receive a share \( \frac{1}{1+\hat{\rho}} \) (i.e. \( \frac{L_0}{L_0+L_1} \)) of output. If the proportion of output that corresponds to period 1 lenders allows \( D_1 \) to be repaid for output levels lower than that required to cover total debts (i.e. \( D_0 + D_1 \)), then period 0 debt holders get output minus \( D_1 \) until output is enough to pay also \( D_0 \). When output is higher than this amount, they are repaid in full.

Equation (13) is lenders’ zero profit condition in period 1 for a given \( \rho \), as analyzed above in equation (4).

Integrating equation (13) from zero to \( \hat{\rho}(k_0) \) and adding this expression to equation (12) we get:

\[
k_0 + \int_0^{\hat{\rho}(k_0)} \rho k_0 dG(\rho) = \int_0^{\hat{\rho}(k_0)} \left\{ \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) + \int_{\lambda^*}^{\hat{\lambda}} [D_0 + D_1] dF(\lambda) \right\} dG(\rho) + \int_{\hat{\rho}(k_0)}^1 S(k_0) dG(\rho).
\] (14)

Adding and subtracting \( \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) \) in equation (11) we get:

\[
V_0 = \max_{k_0} \int_0^{\hat{\rho}(k_0)} \left\{ \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) - \left[ \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) + \int_{\lambda^*}^{\lambda^*} (D_0 + D_1) dF(\lambda) \right] \right\} dG(\rho). \] (15)

Inserting equation (14) into (15) we can express the borrower value function as:

\[
V_0 = \max_{k_0} \int_0^{\hat{\rho}(k_0)} \left[ \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) \right] dG(\rho) - k_0 \left( 1 + \int_0^{\hat{\rho}(k_0)} \rho dG(\rho) \right) + \int_{\hat{\rho}(k_0)}^1 S(k_0) dG(\rho). \] (16)

For simplicity, assume, from now on, that the scrap function is linear in the investment level; i.e. \( S(k_0) = sk_0 \). Then, the optimal investment (and borrowing) level under equal footing, denoted by \( k_0^e \), satisfies the following first-order condition:

\[
\int_0^{\hat{\rho}(k_0^e)} \left[ E(\lambda) \frac{\partial f(k_0^e)}{\partial k_0} \right] dG(\rho) + \int_{\hat{\rho}(k_0^e)}^1 s dG(\rho) = 1 + \int_0^{\hat{\rho}(k_0^e)} \rho dG(\rho) - \left\{ E(\lambda) f(k_0^e) - \hat{\rho} k_0^e - sk_0^e \right\} G'(\hat{\rho}) \frac{\partial \hat{\rho}}{\partial k_0},
\] (17)
where $\frac{\partial \hat{\rho}}{\partial k_0} < 0$; that is, the higher the level of investment, the lower the range of liquidity shocks for which continuation in period 1 will be possible without senior lending. See the appendix for the proof.

To set the optimal investment level borrowers balance the marginal benefit, given by the marginal productivity of capital and by the effect that one more unit invested has on the scrap value; and the marginal costs, given by the cost of investing in period 0, the expected cost of reinvesting in period 1 and the negative effect that one more unit of investment has on the threshold value $\hat{\rho}(k_0)$. Since higher scrap values allow period 0 lenders to offer better terms (see equation (12)), the optimal level of investment increases in $s$.19

### 3.3.2 Senior lending in period 1

Assuming that senior lending is allowed in period 1, the objective function is:

$$V_s^0 = \max_{k_0} \int_0^{\rho^s} \left\{ \int_{\lambda^B}^{\lambda^S} \left[ \lambda f(k_0) - D_0^s - D_1^s(\rho k_0) \right] dF(\lambda) \right\} dG(\rho)$$

(18)

subject to:

$$k_0 = \int_0^{\rho^s} \left\{ \int_{\lambda^B}^{\lambda^S} \left[ \lambda f(k_0) - D_1^s(\rho k_0) \right] dF(\lambda) + \int_{\lambda^B}^{\lambda^S} D_0^s dF(\lambda) \right\} dG(\rho) + \int_{\rho^s}^{1} s k_0 dG(\rho)$$

(19)

and

$$\rho k_0 = \int_0^{\lambda^S} \lambda f(k_0) dF(\lambda) + \int_{\lambda^S}^{\lambda^A} D_1^s dF(\lambda),$$

(20)

where the superscript “s” implies that senior lending is allowed; and $\lambda^B$ and $\lambda^S$ are as defined in (10) and (8) above. $\rho^s$ indicates the value of the liquidity shock above which there is no senior lending. If the only criteria that senior lenders use to lend in period 1 is the nonnegativity of expected profits they would lend for all sizes of the liquidity shock implying that $\rho^s$ will be equal to one. However, we are allowing for the possibility that senior lender(s) have other criteria to make their loans like maximizing borrowers’ welfare ex-ante. In this case, as will be shown below, continuation of the project would not be optimal for high liquidity shocks and $\rho^s$ would be less

19Analytically, this follows from applying the implicit function theorem to (17), taking into account that the second order condition is satisfied.
than one.

Equation (19) and equation (20) are the zero profit conditions for period 0 and 1 respectively. Period 0 lenders know that there will not be liquidation in period 1 and, consequently, they do not consider the scrap value in their zero profit condition. They know that senior lenders will have priority on output and they will begin receiving repayment if and only if senior debts are totally repaid. Equation (20) is the same as equation (9) above.

As before, integrating equation (20) over all possible values of $\rho$ and adding this expression to equation (19) we obtain:

$$k_0 + \int_0^{\rho_s} \rho k_0 dG(\rho) = \int_0^{\rho_s} \left\{ \int_0^{\lambda^B} \lambda f(k_0) dF(\lambda) + \int_{\lambda^B}^{\lambda^T} (D_1^s + D_0^s) dF(\lambda) \right\} dG(\rho) + \int_{\rho_s}^{1} s k_0 dG(\rho).$$

Adding and subtracting $\int_0^{\lambda^B} \lambda f(k_0^s) dF(\lambda)$ in equation (18) and plugging equation (21) in the resulting expression, the borrowers’ value function is:

$$V_{s0} = \max_{k_0} \int_0^{\rho_s} E(\lambda) f(k_0) dG(\rho) - k_0 \left( 1 + \int_0^{\rho_s} \rho dG(\rho) \right) + \int_{\rho_s}^{1} s k_0 dG(\rho).$$

Optimal investment under seniority, denoted by $k_0^{s}$, satisfies the following first order condition:

$$\int_0^{\rho_s} \left[ E(\lambda) \frac{\partial f(k_0^s)}{\partial k_0} \right] dG(\rho) + \int_{\rho_s}^{1} s dG(\rho) = 1 + \int_0^{\rho_s} \rho dG(\rho).$$

The left hand side of (23) is the expected marginal return of investment, given by the expected marginal productivity of capital times the probability of having senior lending in period 1 plus the scrap value times the probability of liquidation. The right hand side is the marginal cost of investment, given by the marginal cost of period 0 investment plus the expected reinvestment cost.

Clearly, when $\rho^s$ is equal to one, (23) and (25) reduce respectively to:

$$V_{s0} = \max_{k_0} E(\lambda) f(k_0) - k_0 - E(\rho) k_0$$

and

$$E(\lambda) \frac{\partial f(k_0^s)}{\partial k_0} = 1 + E(\rho).$$
3.3.3 Comparison

In this section we study how allowing for senior lending in period 1 affects the level of welfare and investment in period 0 (ex-ante). Since lenders set their prices in such a way that make zero profits in expectation, allowing senior lending in period 1 does not affect their welfare ex-ante as long as they are informed about the nature of future lending.

To have the benchmark case we introduce the solution of the social planner willing to maximize borrowers’ welfare in period 0. It is socially optimal to continue with the project in period 1 when the value of doing so is higher than the value of liquidating it getting the scrap value. This is when $E(\lambda)f(k_0) - \rho k_0 \geq sk_0$.\footnote{A social planner chooses the level of investment and the cutoff value of $\rho$ in order to maximize borrowers’ welfare in period 0. Analytically, this condition follows from maximizing an expression like (22) with respect to $k_0$ and a cutoff value of the liquidity shock.} The first-best cutoff value of the liquidity shock, above which it is socially optimal to liquidate the project is given by:

$$\rho^* \equiv \frac{E(\lambda)f(k_0)}{k_0} - s. \quad (26)$$

For liquidity shocks higher than $\rho^*$, continuation of the project will reduce its social value and make borrowers ex-ante worse off.\footnote{The loss is borne by the borrowers because, as noted above, the lenders price their loans to make zero profits in expectation.} Continuation of the project may not be optimal for high scrap values.

**Proposition 3.** There is some $\bar{s} < 1$, such that for $s > \bar{s}$, continuation is not optimal if and only if $\frac{E(\lambda)f(k_0)}{k_0} < 2$.

**Proof.** When $s = 0$, $\rho^*$ in (26) is greater than one, otherwise investment does not take place in period 0. Thus, it is socially optimal to continue for all sizes of the shock since the project has no value in the case of liquidation. When $s = 1$ and $\frac{E(\lambda)f(k_0)}{k_0} < 2$, $\rho^*$ is less than one. Since $\rho^*$ is continuous and decreasing in $s$ and it is equal to one when $s = 0$ and less than one when $s = 1$, it follows that there is only one value of $s$, called $\bar{s}$ above which liquidation is optimal for high enough period-1 shocks.

A senior lender willing to maximize borrowers’ welfare ex-ante would not lend when the liquidity shock and the scrap value are high enough, that is when $\rho > \rho^*$. However, if the senior lender(s) is
willing to lend as long as they make nonnegative profits there would be continuation of the project in all cases even though this reduces the social value; thus having senior lending available for all shocks is potentially suboptimal. This would be a reason why delegating senior lending to a single lender or institution rather than to atomistic ones acting in a competitive environment would allow to reach the ex-ante optimal solution. Atomistic lenders acting in a competitive environment would automatically lend for all sizes of the liquidity shock.

Next we compare borrowers’ welfare under the two potentially suboptimal cases: equal footing lending and when senior lender(s) lends for all sizes of the shock in period 1. Which scenario gives the higher level of welfare depends on the size of the scrap value.

When $s = 0$, outcomes under seniority are first best. Obviously, this means that, in this case, welfare is higher under seniority than under equal footing. The higher is $s$ the higher is the level of welfare under equal footing,\(^{22}\) while the utility level under seniority does not depend on the scrap value (since there is always continuation). When $s = 1$, we can express equation (16) as:

\[
V_0 = \max_{k_0} E (\lambda) f(k_0) - (1 + E(\rho)) k_0 - \int_{\hat{\rho}(k_0)}^{1} [E (\lambda) f(k_0) - (1 + \rho) k_0] dG(\rho). \tag{27}
\]

A comparison of (27) and (24) yields that the ex-ante utility level may be higher under the equal footing scheme than under seniority. A sufficient condition to have this inequality is that $\hat{\rho}$ evaluated at $k_{0s}^s$ (the level of investment that maximizes (22)) were less than one.\(^{23}\) This is because the definition of $\hat{\rho}$ in (7) implies that the last term in (27) is negative. As a consequence, when evaluating (27) at $k_{0s}^s$ we have that (27) is higher than (22). So, when evaluating (27) at the optimal level of investment under equal footing ($k_{0s}^s$), the difference between (27) and (22) is even higher. Since when $s = 0$, $V_0 < V_0^s$, $\frac{\partial V_0}{\partial s} > 0$ and $V_0 > V_0^s$ when $s = 1$, there is only one threshold value of $s$ such that for higher scrap values, equal footing yields a higher level of utility.

We can summarize this in the following proposition:

**Proposition 4.** If condition (5) holds when senior lending is allowed, there is some $\hat{s} < 1$, such that for $s > \hat{s}$, welfare is higher under equal footing than in a case where senior lending is available for all sizes of the liquidity shock.

---

\(^{22}\) Analytically, follows from deriving (16) with respect to $s$ applying the envelope theorem.

\(^{23}\) This is the case if condition (5) holds also in the case of senior lending.
To see how optimal investment is affected in the case that senior lending is available for all shocks, compare equation (17) and equation (25). First, assume that there is no scrap value in case of liquidation (i.e. \( s = 0 \) in (17)). In this case, the term in brackets that multiplies \( \frac{\partial \hat{\rho}}{\partial k_0} \) in (17) is positive (otherwise there will be no investment in period 0), implying that \( \int_0^\hat{\rho} \left[ E(\lambda) f'(k_0) - \rho \right] dG(\rho) < \int_0^\hat{\rho} \left[ E(\lambda) f'(k_0) - \rho \right] dG(\rho) \). This inequality can be expressed as:

\[
E(\lambda) f'(k_0) \left[ 1 - \frac{f'(k_0)}{f'(k_0)} \Pr(\rho \leq \hat{\rho}) \right] < E(\rho/\rho) \left[ 1 - \Pr(\rho < \hat{\rho}) \right].
\]

Since the first term on the left hand side is greater than one (by (17)), while the first term on the right hand side is less than one by definition, it must be the case that \( f'(k_0) > f'(k_0) \) implying that \( k_0 < k_0 \).

Note that in this model the expectation of senior lending does not make individuals take riskier actions, so the increase in borrowing and lending in period 0 is not the consequence of moral hazard but of avoiding inefficient liquidation.

Now consider the case where the scrap value is different from zero. As noted above, the scrap value makes period 0 credit conditions under equal footing less onerous, because it represents a positive payoff in case of liquidation. From equation (17) we can see that the higher is \( s \), the higher the level of investment under equal footing. When \( s \) is equal to one, the term in brackets on the right hand side of (17) is less than or equal to zero (see equation (7)), and a comparison of (17) and (23) yields

\[
E(\lambda) f'(k_0) \left[ 1 - \frac{f'(k_0)}{f'(k_0)} \Pr(\rho \leq \hat{\rho}) \right] > E(\rho/\rho) \left[ 1 - \Pr(\rho < \hat{\rho}) \right].
\]

In this case we can not rule out the possibility of \( k_0 \) being lower than \( k_0 \).

**Numerical exercise.** We present a numerical example to show that for scrap values sufficiently high it is possible to have a lower level of investment and welfare when a senior lending is available for all sizes of the liquidity shock. Consider the case where \( f(k_0) = k^{0.8} \), \( \lambda \) is uniformly distributed in \([0,3]\), \( \rho \) is uniformly distributed in \([0,1]\), and \( s = 1 \). In this case we obtain that \( V_0^s = 0.12 < V_0 = 0.15 \) and \( k_0^s = 0.32 < k_0 = 0.59 \).

To conclude this section assume that borrowers are able to set institutions in period 0 that
govern the availability of senior lending in period 1. In this case it would be optimal from an ex-ante point of view to liquidate the project when the shock and the scrap value are high enough. However, since borrowers always prefer continuation in period 1 there might be a conflict between period 0 and period 1 incentives. From an ex-ante point of view borrowers may maximize utility by committing not to allow senior lending in period 1 for high enough shocks. However, this promise is not time consistent since, once initial lending decisions were taken and the shock occurs, borrowers do have an incentive to allow for senior lending. Thus, in this case, a commitment device would be needed to maintain the promise. Without this device, lenders will set their debt prices in period 0 knowing that senior lending would be available for all shocks in period 1 and the country would be worse off. As noted above, in this case seniority may yield a lower level of welfare than that obtained in an equal footing scheme. Of course, this time consistency problem does not exist if continuation for all shocks is optimal ex-ante.

4 Conclusions

This paper presents a model that emphasizes the effects of seniority in international financial markets on borrower countries and on creditors' welfare. The model can be interpreted as referring to atomistic senior lenders that act in a competitive market or to a senior lender that chooses to make zero profits in expectation despite its monopolistic situation (such as the IMF). However, the interpretation of the IMF as the senior lender is supported by the preferred creditor status that has shown in international capital markets.

We compare a situation under an equal footing scheme and one where senior lending is allowed. We do this from an ex-post point of view, i.e. once the initial borrowing decisions have been taken and from an ex-ante point of view, i.e. when the borrowing and investment decisions are taken.

Ex-post, a senior intervention always makes the borrower country better off since it provides cheaper funds. However, existing lenders might be worse off with a senior intervention depending on the size of the shock and what they expect to get in case of liquidation (the scrap value). When the liquidity shock is small such that non-senior lending is available to cope with it, a senior intervention makes existing lenders worse off since the project would continue anyway and senior lending dilutes existing debt. When the shock is high enough such that non senior lending is not
available a senior intervention might improve existing lenders’ situation depending on the size of the scrap value. When the scrap value is small existing lenders would prefer continuation and a senior intervention make them better off, while the opposite occurs when the scrap value is high. The possibility of lenders being ex-post worse off with a senior intervention affects the interest rate they charge on their initial loans.

From an ex-ante point of view senior lender(s) could maximize borrowers’ welfare by lending if and only if it is socially optimal to do so. This might imply that senior lending would not be available when the shock and the scrap value are high enough. However, if the criteria that senior lenders use to lend is the nonnegativity of their expected profits, they would lend for all sizes of the shock although this might reduce the social value of the project. In this case, senior lending causes over-continuation of the project and initial lenders would charge a higher interest rate on their initial loans. As a consequence initial lending could be lower when a senior intervention is expected in the future which is contrary to the standard moral hazard critique that says that too much rescuing in the future would lead to too much lending in the present. In this model we obtain that too much rescuing in the future could lead to too little borrowing in the present.

In the real world coordination issues between lenders are likely to be important in international markets. This might be a reason why it would be optimal to delegate senior lending in a single institution rather to delegating it to atomistic lenders acting in competitive markets. These ones are more likely to lend in all cases where the expected profits are nonnegative.

However, even a single senior creditor might be tempted to lend ex-post even when it is not optimal ex-ante. Since, once the initial borrowing decisions have been taken, borrowers always want to continue with the project there might be a dynamic inconsistency problem when liquidation is optimal from an ex-ante point of view. This single institution caring about borrowers welfare would have incentives to make senior loans once the initial decisions have been taken. Thus credible and clear rules are needed to reach the optimal outcome from an ex-ante point of view. Without these devices period 0 lenders would anticipate senior lending in the future and leave the country with lower welfare.
A Proof of Lemma 1

From zero expected profit condition we write the implicit function

\[ Q(\rho, r_1) \equiv 1 - \left[ \frac{1}{1 + \rho} \right] \int_0^{\min(\lambda^1, \lambda^0)} \frac{\lambda f(k_0)}{k_0} dF(\lambda) - \frac{1}{\rho} \int_{\min(\lambda^0, \lambda^*)}^{\lambda^*} \left[ \frac{\lambda f(k_0)}{k_0} - \frac{D_0}{k_0} \right] dF(\lambda) - \int_{\min[\lambda^1, \lambda^*]}^{\hat{\lambda}} r_1 dF(\lambda) = 0 \]

First consider the case where \( \lambda^0 < \lambda^* \); applying the implicit function theorem we have that \( \frac{\partial r_1}{\partial \rho} = -\frac{\partial Q(.)}{\partial \rho} \)

\[
\frac{\partial Q(.)}{\partial \rho} = \frac{1}{(1 + \rho)^2} \int_0^{\lambda^0} \lambda f(k_0) \frac{dF(\lambda)}{k_0} + \frac{1}{\rho^2} \int_{\lambda^0}^{\lambda^*} \left[ \lambda f(k_0) - \frac{D_0}{k_0} \right] dF(\lambda) + \left[ \frac{\lambda f(k_0)}{k_0} - \frac{D_0}{k_0} - \frac{1}{1 + \rho} \right] \left[ \frac{\lambda^0 f(k_0)}{k_0} \right] F'(\lambda^0) \frac{\partial \lambda^0}{\partial \rho} + \left[ r_1 - \frac{\lambda^* f(k_0)}{k_0} + \frac{D_0}{k_0} \right] F'(\lambda^*) \frac{\partial \lambda^*}{\partial \rho} \]

Taking into account that \( \lambda^0 = \frac{(1 + \rho)D_0}{f(k_0)} \) and that \( \lambda^* = \frac{D_0 + D_1}{f(k_0)} \) we have that the last two terms are both equal to zero. Thus, \( \frac{\partial Q(.)}{\partial \rho} > 0 \). Moreover,

\[
\frac{\partial Q(.)}{\partial r_1} = -\int_{\lambda^*}^{\hat{\lambda}} dF(\lambda) < 0.
\]

Thus, \( \frac{\partial r_1}{\partial \rho} > 0 \).

Proceeding in the same way we can show that this is also the case when \( \lambda^1 < \lambda^* \).

B Proof of Lemma 3

To simplify the exposition of this proof consider the special case when \( \lambda^0 = \lambda^1 = \lambda^* \). Without seniority, the interest rate is pinned down by:

\[
\int_{\lambda^*}^{\hat{\lambda}} r_1 dF(\lambda) + \left[ \frac{1}{L_0 + L_1} \right] \int_0^{\lambda^*} \lambda f(k_0) dF(\lambda) = 1
\]

and with seniority by

\[
\int_{\lambda}^{\hat{\lambda}} r_1^s dF(\lambda) + \left[ \frac{1}{L_1^s} \right] \int_0^{\hat{\lambda}} \lambda f(k_0) dF(\lambda) = 1
\]

The proof proceeds by contradiction. Assume that \( r_1 = r_1^s \). This implies that \( R_1^s = R_1 \) since \( L_1^s = L_1 \), and this implies that \( \hat{\lambda} < \lambda^* \) for sure. Splitting the integral limits and equating both
expressions:

\[
\int_{\lambda^*}^{\hat{\lambda}} r_1 dF(\lambda) + \left[ \frac{1}{L_0 + L_1} \right] \left[ \int_{0}^{\hat{\lambda}} \lambda f(k_0) dF(\lambda) + \int_{\lambda}^{\lambda^*} \lambda f(k_0) dF(\lambda) \right] = \\
= \int_{\lambda}^{\lambda^*} r_1^* dF(\lambda) + \int_{\lambda^*}^{\hat{\lambda}} r_1 dF(\lambda) + \left[ \frac{1}{L_1} \right] \int_{0}^{\hat{\lambda}} \lambda f(k_0) dF(\lambda)
\]

Rearranging we get:

\[
\int_{\lambda^*}^{\hat{\lambda}} (r_1 - r_1^*) dF(\lambda) = \int_{\lambda}^{\lambda^*} r_1^* dF(\lambda) + \int_{0}^{\hat{\lambda}} \lambda f(k_0) \left[ \frac{1}{L_1} - \frac{1}{L_0 + L_1} \right] dF(\lambda) - \left[ \frac{1}{L_0 + L_1} \right] \int_{\lambda}^{\lambda^*} \lambda f(k_0) dF(\lambda)
\]

The second term of the right hand side is positive and the first term is greater than the third one under the assumption that \( r_1^* = r_1 \). So the right hand side is unambiguously positive. So, the left hand side should be positive and not zero as it is under our original assumption.

There is a contradiction.

Now we have to show that \( r_1^* \) cannot be greater than \( r_1 \). Again we proceed by contradiction. Assume \( r_1^* > r_1 \), which implies that \( R_1^* > R_1 \). There are two possible cases: \( \hat{\lambda} < \lambda^* \) and \( \hat{\lambda} > \lambda^* \). In the first case the proof is the same as before. In the second case, split the integral limits as above, but now with \( \hat{\lambda} > \lambda^* \). We get

\[
\int_{\lambda}^{\hat{\lambda}} (r_1 - r_1^*) dF(\lambda) = \left[ \frac{1}{L_1} - \frac{1}{L_0 + L_1} \right] \int_{0}^{\lambda^*} \lambda f(k_0) dF(\lambda) + \int_{\lambda}^{\hat{\lambda}} \left[ \lambda f(k_0) - r_1 \right] dF(\lambda)
\]

The second term of the right hand side is positive under our assumption that \( \hat{\lambda} > \lambda^* \). Conditional on \( \lambda \) being greater than \( \lambda^* \) and lower than \( \hat{\lambda} \) output is greater than \( r_1 \). This is because output is higher than the necessary to totally repay the contractual interest rate \( r_1 \) (i.e. \( \lambda > \lambda^* \)). So, the left hand side is unambiguously positive and so should be the left hand side. But this contradicts our initial assumption. We conclude that \( r_1^* \) must be lower than \( r_1 \).
C Proof that $\frac{\partial \hat{\rho}}{\partial k_0} < 0$

From equation (7), define the function $F(k_0, \hat{\rho})$:

$$F(k_0, \hat{\rho}) \equiv \hat{\rho} - \frac{\hat{\rho}}{1 + \hat{\rho}} \int_0^{\lambda} \lambda \frac{f(k_0)}{k_0} dF(\lambda) - \int_{\min[\lambda^0, \bar{\lambda}]}^{\bar{\lambda}} \left[ \left( \frac{1}{1 + \hat{\rho}} \right) \lambda \frac{f(k_0)}{k_0} - \frac{D_0}{k_0} \right] dF(\lambda) = 0$$

Applying the implicit function theorem to this expression:

$$\frac{\partial \hat{\rho}}{\partial k_0} = -\frac{\frac{\partial F(.)}{\partial k_0}}{\frac{\partial F(.)}{\partial \hat{\rho}}}$$

$$\frac{\partial F(.)}{\partial k_0} = -\frac{\hat{\rho}}{1 + \hat{\rho}} E(\lambda) \frac{\partial A}{\partial k_0} - \int_{\min[\lambda^0, \bar{\lambda}]}^{\bar{\lambda}} \left[ \left( \frac{1}{1 + \hat{\rho}} \right) \lambda \frac{\partial A}{\partial k_0} - \frac{\partial B}{\partial k_0} \right] dF(\lambda) > 0$$

Since $A$ is a concave function and $B$ is a convex function (analogous to Lemma 1), this expression is greater than zero.

$$\frac{\partial F(.)}{\partial \hat{\rho}} = 1 - \frac{1}{(1 + \hat{\rho})^2} E(\lambda) \frac{f(k_0)}{k_0} + \int_{\min[\lambda^0, \bar{\lambda}]}^{\bar{\lambda}} \frac{1}{(1 + \hat{\rho})^2} \lambda \frac{f(k_0)}{k_0} dF(\lambda)$$

This expression will have the same sign as:

$$(1 + \hat{\rho}) - \frac{1}{(1 + \hat{\rho})} E(\lambda) \frac{f(k_0)}{k_0} + \int_{\min[\lambda^0, \bar{\lambda}]}^{\bar{\lambda}} \frac{1}{(1 + \hat{\rho})} \lambda \frac{f(k_0)}{k_0} dF(\lambda),$$

from the definition of $\hat{\rho}$ (equation (7)) we have that:

$$\frac{1}{1 + \hat{\rho}} E(\lambda) \frac{f(k_0)}{k_0} < 1$$

so that,

$$\frac{\partial F(.)}{\partial \hat{\rho}} > 0$$

These imply that $\frac{\partial \hat{\rho}}{\partial k_0} < 0$. 

31
References


