A Theory of Noncontributory Pension Design

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A theory of noncontributory pension design\textsuperscript{1}

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Abstract

Noncontributory subsidies for the old poor (first-pillar pensions) affect the welfare of hundreds of millions around the world. Their benevolent rationale is to redistribute progressively, subject to efficiency considerations. This paper focuses on a critical efficiency issue: first pillars may affect another, even bigger program, namely contributory pensions for the middle classes, by inducing a reduction in the density of contributions. A major source of concern with contributory pensions in emerging economies is that the total replacement rate is too small for participants with low density, which are prevalent. The paper develops a model where density of contribution is endogenous, because for a substantial subset of jobs, the State is unable or unwilling to impose a mandate to contribute. Thus, the job selection decision is bundled with a saving decision. The first finding is that bundling modifies the effective rate of return on contributions, raising it without bound as earnings in uncovered jobs become smaller (relative to earnings in covered jobs). Another finding is that the standard designs of first-pillar pensions reduce the equilibrium density of contributions. Thus, standard first-pillar designs do crowd out contributory pensions for the middle classes. The paper then analyzes two second-generation designs. The “proportional” minimum pension is found to create horizontal inequity and inefficiency. In contrast, a subsidy with a small withdrawal rate applied to contributory pensions minimizes the loss of contribution density. Optimal income taxation theory suggests that the latter also provides the most efficient progressive redistribution.

\textsuperscript{1} This is the conceptual paper of a pair in which the companion paper discusses the pros and cons of the 2008 pension reform in Chile. I appreciate comments from Alejandro Micco (Ministry of Finance), Rafael del campo (INP Director), Jaime Ruiz-Tagle (Ministry for the Presidency), Ignacio Irarrázabal and Rosario Palacios to a precursor paper, which appeared as Valdés, S. (2007) “Pensión solidaria: ventajas, defectos y propuestas”, Temas de la Agenda Pública series Nº 13, December, Santiago de Chile, www.puc.cl/agendapublica/. None is responsible for the claims made in this paper. I also thank the World Bank for translating to English a previous version.

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1. Introduction

Alleviation of insufficient density of contribution for old-age pensions has paramount implications for hundreds of millions of middle-class people around the world. When the State is unable or unwilling to provide high density, contributory systems are prevented from providing adequate replacement rates. If a substantial share of participants have a low density, the contributory systems promoted by the State, either through mandates or fiscal incentives, fail to meet their fundamental aim, namely to alleviate (partial) neglect of old age. Too many among the middle class may fall into poverty at old age.

Separately, poverty among the old creates a demand on the political system to grant progressive subsidies to the old poo, different from subsidies to the poor of any age. These are "first-pillar" subsidies. One possible design is a universal flat subsidy for all the old. However, this is so expensive fiscally that poor countries have been unable to rely on it.

One way to reduce this fiscal cost is to exclude from benefits those older people that have built observable contributory pensions of sufficient size, or those that have an observable consumption level financed by other means. This is feasible only if the State can and wishes to promote contributory pensions with sufficient coverage and sufficient density of contribution, and if the State can measure personal consumption with little error and at a modest incremental cost. Under these two conditions, targeting allows both a reduction of the fiscal cost and stronger redistribution.

Many States do not meet these stringent conditions. Coverage is limited in South Korea - only 58% of the labor force contributes- and in Chile - 65% of employment contributes each month (World Bank, 2000). Many emerging countries exhibit uneven density of contribution, since large groups of self-employed (most of them poor) are exempt from the mandate, in addition to women engaged in home production. What can be done in countries where some conditions are met, but others are met partially or unevenly? Development experts say that “The greatest challenge is to design and manage the link between tax-financed and contributory social security schemes”(van Ginneken, 2007).

Beyer and Valdés-Prieto (2004) were the first to emphasize that insufficient density of contribution to old-age pensions for the middle classes (second and third pillars), can be driven by the inadequate design of first pillar programs. To be sure, the literature showed long ago that the marginal link between contributions and benefits in contributory old-age pensions for the middle classes affects the supply of hours to the labor market (Auerbach and Kotlikoff, 1987), but was silent on choice between covered and uncovered jobs. Another literature argues that the design of mandatory old-age pensions for the middle classes ("second pillars") affects choice between covered and uncovered jobs, but is silent on the effect of first-pillar design. Valdés-Prieto (2002, p. 241-57) showed how the presence of uncovered jobs allows workers to cap the implicit tax imposed by a second pillar, but is silent on the impact on density. Hubbard, Skinner and Zeldes (1995) argue that a high rate of withdrawal of subsidies to the old poor reduces the voluntary saving of the
poor, but are silent on the impact on density of contribution to second pillars. The literature on optimal income tax schedules stresses the disincentive effect on supply of hours of violent withdrawal of subsidies, identifies the optimal two-bracket schedule (Slemrod et al, 1994), and more general optimal tax schedules (Diamond, 1998), but is silent on job choice. The idea that when a participant already meets the vesting requirement for the Minimum Pension Subsidy, she finds that additional contributions do not increase her total contributory pension is very old, but is not explicit about job choice and density.

This paper contributes the first formal model for the link between first-pillar design and density of contribution to second-pillar (mandatory) pensions. It shows that at given wages, changes in the design of first-pillar subsidies can have a large impact on the choice between jobs covered by the mandate to contribute and uncovered jobs, implying a large elasticity of labor supply to the covered sector to the relative wage paid by that sector. If the demand for labor in the covered sector has some elasticity, these changes in job choice reduce contribution density in equilibrium. This implies that badly designed noncontributory subsidies may crowd out contributory pensions and crowd in self-employment, informal jobs and home production among those that find the first-pillar subsidies significant.

The policy implication is a new approach to raise density of contribution: to improve the design of the first pillar. Results in optimal income tax theory by Slemrod et al (1994) and Diamond (1998) recommend a withdrawal rate at low income levels, larger than the rate for median income levels (Valdés-Prieto 2002, p. 67-71). The simulations by Poblete (2005) for a simplified model found that the optimal level for withdrawal rates is modest – about 20% - . This is much lower than in the second tranche of the “minimum pension supplement” of the new Swedish system, where the withdrawal rate is 48% (Scherman, 1999). An optimal withdrawal rate above zero also implies that universal flat pensions are not recommended. The design called “proportional minimum pension”, used by Switzerland, is shown to create large horizontal inequities and to have lower efficiency than a program with a small withdrawal rate.

The Chilean reform of 2008 adopted this approach, as it replaced existing non-contributory subsidies whose withdrawal rates range from 100% to 61%, for a unified new subsidy with a withdrawal rate of about 32% in response to mandatory contributory pensions. However, a controversial cliff withdrawal for other income was retained and the size of first pillar subsidies was increased too lavishly because of fiscal abundance (Valdes-Prieto 2008, companion paper).

Section 2 presents a benchmark model for the theory of non-contributory subsidies developed in the next sections. Section 3 uses the model to show how two standard non-contributory subsidy schemes induce a reduction in contribution density and crowd out contributory systems. Section 4 uses the same model to analyze a new generation of designs for first-pillar pensions: a proportional minimum pension, and a subsidy with a

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3 Although the empirical literature on the impact of tax incentives on voluntary saving finds modest elasticities for the poor (Attanasio and Browning, 1996), violent withdrawal of subsidies create unusually large implicit tax rates, so the effect on voluntary saving can still be sizable.

4 A similar argument may apply to underreporting of earnings.
small withdrawal rate. Section 5 evaluates the impact of low density of contributions on the social value of a mandate to save for old age. The final remarks point out that the social value of a contributory system may depend on the quality of non-contributory subsidies.

2. A theory of first-pillar pensions: the benchmark

This section presents a model where each individual chooses between jobs covered and uncovered by a mandate to contribute to an old-age pension plan. Therefore, coverage of contributions in the contributory plan is endogenous. In the benchmark case, noncontributory subsidies are not available in old age.

2.1 Assumptions about the labor market

The State is unable to enforce a mandate to contribute based on all labor productivity. This is obvious for home production. Self-employment also facilitates underreporting of work and earnings, for two reasons: (i) there is no employer interested in maximizing reported labor costs in order to minimize corporate income taxes; and (ii) there is no employer interested in minimizing the penalties applied by enforcers of employment protection regulations, which include compliance with social security. Thirdly, informal activities, defined as those that evade taxes and regulations, also evade the mandate to contribute. The informal employer loses by reporting the earnings of his employee to the State, because this betrays its presence. Thus, there are several reasons why a mandate to participate in contributory old-age pensions is never fully enforced by the State. Moreover, the State may be unwilling to enforce a mandate to contribute on the self-employed that are poor. This can originate legal exemptions to the mandate to contribute.

The premise of this paper is that uncovered jobs are a significant job option, and not a marginal or irrelevant exception. The proportion of the active labor force not covered by contributory systems is above 25% in South Korea and Chile. Empirical work for Chile finds that uncovered jobs are a realistic option for many workers (Torche and Wagner, 1998). In most emerging economies uncovered jobs are closer to 50%, and in most countries in Africa and South Asia it is closer to 80%. (van Ginneken, 1999).

“Contribution density” is defined as the share of (the present value of) earnings in the active phase of life on which the individual contributes to some second pillar old-age system. Average density falls when self-employment expands and when activity outside the labor force (mainly home production) rises, for any given rate of turnover between covered jobs and other uses of time. Density also falls when underreporting of earnings rises.

This is a model under certainty. It collapses a continuum of dates and ages into just two separate periods, called active and passive phases of life. The “passive” label is inexact because work in old age is allowed. The labor market and tax variables are:

\[ y^c = \text{gross earnings in the covered sector per unit time, in the active phase.} \]
\[ t_a = \text{other net taxes applicable only to covered jobs in the active phase.} \]
\[ y^e = z^e, \quad y^c = \text{earnings per unit time in self-employment and jobs that are exempt of or evade the mandate to contribute, in the active phase.} \quad z^e = \text{the ratio } y^e / y^c. \]

\( D = \text{density}, \ v [0,1]. \) It is the proportion of (the present value of) earnings in the active phase of life on which the individual contributes to some second pillar old-age system.

\( e_p = \text{earnings in old age, expressed as a proportion of } y^c. \) Exempt from taxes.

\( t_p = \text{other net taxes that are applied only to contributory pensions in old age.} \)

Labor supply in active life is assumed to have a zero income elasticity. Although this is restrictive, it makes it transparent that the results of this paper do not rely on the income effect on leisure triggered by the introduction of non-contributory subsidies for old age.

The tax rates \( t_o \) and \( t_p \) are significant, because contributory pensions and covered earnings are subject to substantial health insurance taxes, while exempt jobs and the product of voluntary saving are free from those taxes. Health insurance taxes are large and highly redistributive, in the sense that they have almost no compensatory benefits at the margin. Covered earnings are also subject to the taxes that finance other branches of social insurance, and to mandatory housing contributions (as in Brazil and Mexico), whose marginal benefits are way below the marginal tax rate.

### 2.2 The contributory old-age pension system for the middle classes

Consider a contributory system in the absence of non-contributory pensions. The main benevolent justification for State promotion of contributory old-age pensions (mandates, fiscal incentives) is to help those that neglect old age. Many workers have difficulty visualizing the long-term future. Others suffer from excessive optimism and neglect old age and its pessimistic overtones until late in their lives. This justification is not modeled explicitly here (for an explicit model, see Valdes-Prieto 2002, section 3.3).

Workers may choose jobs without giving due credit to some benefits linked to work in covered jobs, not available in uncovered jobs. These include access to on-the-job learning, to employer-financed training, to social advancement and access to networks, and access to the future growth of the modern economy driven in part by general knowledge externalities. Covered jobs also increase consumer-credit limits, since covered earnings can be verified by the lender. So even if there is no undervaluation of old-age pensions available in covered jobs, undervaluation of these other benefits would justify some intervention.

Any contributory system for the middle classes can be described by parameters related to contributions, pension ages, the initial benefit (pension) and the indexation rule for ongoing pensions. Since only two phases of life are considered, several of these parameters do not apply here. Because the function of progressive redistribution is performed by the first-pillar subsidy, the second pillar is assumed to be purely earnings-related i.e. that avoids intra-generational redistribution.\(^5\) This cuts down the number of parameters to just four:

---

\(^5\) This division of roles maximizes transparency and thus minimizes the risk of capture by pressure groups. This division does not impair the ability of the first pillar and of the tax system to redistribute on the basis of lifetime wealth, since subsidies and taxes in old-age are conditional on income flows that depend on lifetime earnings, such as contributory pensions and capital income.
θ = contribution rate for old age, applicable only in covered jobs and uniform for all participants. This rate is applied to each individual’s contribution base.

$\text{CB} = \text{contribution base for old age, defined as } CB(yc) = \min[MTE; \max(0; yc - LTE)]$, where $LTE$ is Least taxable earnings and $MTE$ is Maximum taxable earnings.

$R^c = \text{internal rate of return (real terms) paid by the contributory system to each generation of participants. The financing method used by the contributory pension system affects the average } R^c \text{ per generation: full funding should make } R^c \text{ equal to the returns on saving offered by voluntary saving vehicles, while mature pay-as-you-go finance under a rule where at least one parameter is adjusted to insure financial independence from the fiscal budget, makes } R^c \text{ equal to the growth rate of the real covered wage bill.}$

$D_{2p} = \text{minimum density required to vest second-pillar old-age benefits. The individual that does not comply with density } D_{2p} \text{ loses all contributions (those made in the name of the employer and on the name of the worker). Equivalently, } R^c = -100\% \text{ if } D < D_{2p}.$

Vesting conditions are standard in traditional contributory systems. In the U.S. Social Security system, $D_{2p} = 120 \text{ months (10 years). In contrast, notional and fully funded defined contribution systems usually set } D_{2p} = 0.$

To simplify the model, it is assumed that $LTE$ is zero, that $MTE$ is large enough to be irrelevant for interactions with noncontributory subsidies, and that $D_{2p}$ is zero. For the pertinent range of earnings, the replacement rate is $\theta(1+R^c).$  

In the case where the State gives fiscal incentives to old-age saving (individual or employer-based, or “third-pillar saving”), the tax regimes affects $t_a, R^c$ and $t_p.$ Each of this rates should be interpreted as net of the advantage granted by the tax regime.

In the case of mandatory systems (second-pillar), tax rates $t_a$ and $t_p$ represent implicit taxes. In the accumulation phase, the stock of accumulated mandatory contributions for old age is not available to cover legitimate emergencies, owing to the inalienability of pension rights in a mandatory system (a creditor is not allowed to collect a lien on pension rights; Andolfatto, 2002). In contrast, in exempt jobs all saving can be held in liquid form, if needed, so saving is more valuable. The difference in value is an implicit tax on mandatory saving, which can be included in $t_a$ (Valdés-Prieto 2002, p. 223-229). Similarly, the illiquidity created by mandatory annuitization can be represented with a higher $t_p.$

2.3 Income identities and budget constraints

In each phase of life, the income identities before any voluntary saving are:

\begin{align}
(1a) \quad y_a(D) &\equiv (y^c \cdot D) \cdot (1 - \theta - t_a) + (z^{ex} \cdot y^c) \cdot (1 - D) \\
(1b) \quad y_p(D) &\equiv (y^c \cdot D) \cdot \theta \cdot (1 + R^c) \cdot (1 - t_p) + e_p \cdot y^c
\end{align}

It is instructive to obtain from (1) the marginal rate of transformation between $y_a$ and $y_p$:

---

6 More generally, the replacement rate is $\min[\theta(1+R^c); \theta(1+R^c)(MTE/yc)]$ if $D > D_{2p}$ and $\theta$ if not.
Equation (1c) shows that the effective gross rate of return on contributions can be much higher than \((1 + R^c)\), because the denominator \(\frac{z^{ex} - (1 - \theta - t_a)}{\theta}\) can be small, or even negative. This is the critical feature of the model presented in this paper.

In the case of workers that (partially) neglect old age due to excessive optimism, an old-age pension is valued at less than if old age is fully visualized, but is not valued at zero. Although job choice focuses to a larger degree on maximizing current job conditions when those preferences rule, pensions still have some weight. If the effective gross rate of return on contributions is high enough (if \(\frac{z^{ex} - (1 - \theta - t_a)}{\theta}\) is small enough) for a given participant that neglects old age to some degree, she may choose an \textit{interior} solution.

The argument that the poor never save for old age because their current needs outweigh any gain from saving for the long term (Titelman and Uthoff 2005, Van Ginneken 2007) is not generally valid, because the effective rate of return on contributions can be large enough to make old-age benefits valued in active life. In the same way, the argument that illiquidity must lead to null valuation of contributory pensions (Diamond and Valdes-Prieto 1994) is also invalid, because the effective rate of return can be large enough to compensate.

Remark: a tradeoff between job choice and saving arises only if the denominator in (1c) is positive, i.e. if \(z^{ex} > (1 - \theta - t_a)\). If not, covered jobs are productive enough, relative to uncovered jobs, to yield more income in \textit{both} phases of life. The extra productivity of covered jobs (over exempt jobs) makes up for contributions \(\theta\) and the tax differential \(t_a\), so that the contributory pension comes for free. This leads to choose the maximum possible density, \(D^* = 1\), regardless of the individual’s subjective discount rate.

Now add the possibility of pure voluntary saving in amount \(S\), which may be negative. The rate of return in pure saving is \(r\) (in real terms, after tax if any), and may depend on the sign of \(S\). Empirically, the rate of return is larger when the individual is indebted with consumer loans, than when he is a net saver \((r(-) > r(+))\). The period budget constraints are:

\begin{align*}
(2a) \quad c_a &= y_a(D) - S \\
(2b) \quad c_p &= y_p(D) + S \cdot [1 + r(\text{sign}(S))] 
\end{align*}

\subsection*{2.4 Individual optimization}

The individual maximizes lifetime utility. As usual, utility is assumed to be additive separable across phases of life. As explained before, labor supply is assumed to be independent from income in the active phase to avoid interaction with the allocative effects of the introduction of non-contributory subsidies, which create income effects. To achieve this, the utility function must be quasi-linear in consumption (Diamond, 1998). To simplify
further, the utility of leisure in old age is assumed to be additive separable from the utility of consumption in old age \((U_{cp,l_p} = 0)\). The individual solves the following program:

\[
\begin{align*}
\text{(P1)} & \quad \max_{[n,s,l_p]} U \equiv c_n + u(l_n) + v(c_p) + n(l_p) \\
& \quad \text{subject to } (2), \text{ to } D \in [0,1] \text{ and to } l_p, v \in [0,1].
\end{align*}
\]

where \(l_n\) and \(l_p = 1 - (e_p y^c / w_p)\) is the proportion of hours taken as leisure in the corresponding phase of life, and \(w_p\) is the net wage per hour available for work in old age. As usual, \(u', v'\) and \(n'\) are positive, while second derivatives are negative. The utility discount factor for old age is incorporated into the functions \(v\) and \(n\).

Since in this model the budget constraint is the result of competition between linear options, many corner solutions are possible. Rather than going through all possible combinations in the Kuhn-Tucker conditions, these corners are ordered using the following identity, obtained from (P1), which can be written in two ways:

\[
\begin{align*}
(3a) & \quad \frac{\partial U}{\partial D} \equiv \left\{ \frac{\partial U}{\partial S} \cdot \left( z^{ex} - (1 - \theta - t_a) \right) \right. + v \cdot \left[ \theta (1 + R^c) (1 - t_p) - \left( z^{ex} - (1 - \theta - t_a) \right) \right] \cdot (1 + r(\text{sign}S)) \left\} \cdot y^c \\
(3b) & \quad \frac{\partial U}{\partial D} \equiv \left\{ \frac{\partial U}{\partial S} + v \cdot [MRT_{cs} - (1 + r(\text{sign}S))] \right\} \cdot \left( z^{ex} - (1 - \theta - t_a) \right) \cdot y^c
\end{align*}
\]

Equations (3) show that the relationship between pure saving and job choice (selection of density) depends on the sign of two terms: the term \(\left( z^{ex} - (1 - \theta - t_a) \right)\), which is the relative productivity of the uncovered sector, and the term in the square bracket of (3b).

If expression \(\left( z^{ex} - (1 - \theta - t_a) \right)\) is negative, meaning that covered jobs are productive enough to dominate uncovered jobs, version (3a) must be used, where it can be seen that both terms in the square brackets are positive. If in addition pure saving \(S\) is an interior solution (\(\partial U/\partial S\) is zero), the positive square bracket makes \(\partial U/\partial D\) positive, even in the polar case where \(R^c = -1\). Thus, \(D^*\) is in the corner with \(D^* = 1\). This is intuitive, because a negative \(\left( z^{ex} - (1 - \theta - t_a) \right)\) makes choosing covered jobs dominant.

When expression \(\left( z^{ex} - (1 - \theta - t_a) \right)\) is positive, version (3b) can be used and is more revealing. If voluntary saving \(S\) is in an interior solution (if \(\partial U/\partial S\) is zero), the sign of \(\partial U/\partial D\) is governed by the sign of the term in square brackets of (3b). This term is the difference between the net return of saving through the contributory system (eq. 1c) and the return on saving through voluntary vehicles. Either return may dominate. If the term in square brackets is negative, saving through the mandatory system is inferior to voluntary saving and the labor optimum is at the corner with the lowest possible density (\(D^* = 0\), case F1 below). If the term in the square bracket is positive, the labor optimum is with \(D^* = 1\).
(case F3 below). An interior solution for D applies to a range of intermediate cases ($D^* \in [0,1]$, case F2 below).

**PROPOSITION 1:**
The worker’s optimum can be in one of only four situations, labeled F1 to F4:

a) F1: covered jobs do not dominate uncovered jobs, and pure saving for old age has a higher net effective return than saving in the contributory system. Therefore, the individual prefers zero density ($D^* = 0$) and channels any desired saving through pure saving vehicles.

b) F2: covered jobs do not dominate uncovered jobs. The rate of return on saving in the contributory system is intermediate between the return on pure saving $r(\cdot)$ and interest on consumer credit $r(\cdot)$. If $D^*$ is interior then $S^* = 0$. If $D^* = 1$ then $S^* > 0$.

c) F3: covered jobs do not dominate uncovered jobs. The return on contributory-system saving is higher than the interest in consumer credit $r(\cdot)$. $D^* = 1$.

d) F4: covered jobs dominate uncovered jobs. Thus $D^* = 1$ and $S^*$ can have any sign.

Proof: A combination of eq. (3) and the global analysis provided in figure 1 below.

Figure 1 shows the consumption opportunity sets. The heavy line from A to F represents incomes achievable in the absence of voluntary saving ($S = 0$). Extreme A corresponds to the case in which the worker chooses zero density (absence of contributions to the second pillar), while extreme F corresponds to full contribution density, i.e. to $D = 100$ percent. The equation for AF is obtained by eliminating $D$ from equations (1a) and (1b). The slope of AF is the net return of mandatory saving, $(1 + R^c)(1 - t_p) / [[z^{ext} - (1 - \theta - t_a)] / \theta]$, which is the $MRT_{CS}$ that appears in equations (1c) and (3b). Recall that this return can be much higher than $(1 + R^c)(1 - t_p)$, because the denominator $[z^{ext} - (1 - \theta - t_a)] / \theta$ can be small. The height of AF in the vertical dimension is $\theta \cdot (1 + R^c)(1 - t_p) \cdot y^c$, so it is zero if $R^c = -1$.

In figure 1, opportunities for extra saving are represented by the dashed line that starts at F4 and goes Northwest. The individual may also choose to save less than 0 by issuing consumer debt, an option represented by the dotted line that starts from F4 and goes Southeast. The higher slope of the line going Southeast represents a higher interest rate on consumer debt.
Figure 1: Budget constraints created by a second pillar in the absence of noncontributory pensions (left panel: vase F1; right panel: cases F2 and F3)

The left panel shows the case F1, where covered jobs do not dominate uncovered jobs, and in addition, the slope of line AF1 is smaller than \((1+r(+))\) in absolute value (and also smaller than \(1+r(-)\), since \(r(-) > r(+)\)). This means that voluntary saving for old age is more attractive than the contributory saving associated to covered jobs. The implication for contribution density is dramatic: the individual is better off by moving to the corner with zero density \((D^* = 0)\) and making any desired saving through voluntary vehicles. This is of direct policy interest: if the contributory systems offer a relatively low net rate of return, all workers prefer zero density.

The case labeled as F2 (right panel) shows that when the return offered by the second pillar (the slope of line AF2) is intermediate between the return offered by voluntary saving vehicles \((1+r(+))\) and the cost of consumer credit \((1+r(-))\), the individual may choose an interior density of contribution \((0 < D^* < 1)\), rather than the corner with \(D^* = 1\). This happens when the contribution rate \(\theta\) is larger than the desired saving rate: in this situation, excess saving must be undone if full density is selected, and a limited reduction in density is cheaper than incurring expensive consumer debt. However, if the desired saving rate is higher than \(\theta\), then \(D^* = 1\) and \(S^* > 0\), as in point J.

In case F3, also in the right panel of figure 1, and interior contribution density is never optimal. When the contribution rate \(\theta\) is larger than the desired saving rate, a limited reduction in density is more expensive than consumer debt. Thus \(D^* = 1\).

The case labeled F4 is not shown for simplicity. Its graph is similar to case F3, with one difference: the line AF has a positive slope, because covered jobs dominate.
The model also yields interesting comparative statics results. For example, if the rate of return paid by the contributory system rises, this reduction will have no effect initially on decisions by workers, because they still prefer the same set of densities of contribution. However, when this rate of return (given by the slope of AF) rises above a threshold given by the return on voluntary saving \( r(+) \), then all the workers in case F1 switch to case F2 and raise density discontinuously from \( D^* = 0 \) to \( D^* \in (0, 1] \). Further increases in the rate of return paid by the contributory system attract continuous adjustments to the density of contribution if there was an interior solution, but density may rise or fall, falling if income effects compensate substitution effects. When additional increases in the rate of return paid by the contributory system equalizes the slope of AF to the interest on consumer credit \( r(-) \), all workers in case F2 jump to case F3. Density jumps discontinuously to \( D^* = 1 \). Thus, contribution density can be quite elastic to the net rate of return paid by mandatory saving.

If the demand of uncovered jobs is elastic, so that wages do not change much in response to these changes in job choice, equilibrium contribution density changes.\(^7\)

3. Standard first-pillar pensions in the presence of a contributory system

First-pillar pensions are noncontributory subsidies for the old poor. The standard rationale for them is progressive redistribution. This rationale encompasses the rationale of avoiding horizontal inequity and risk aversion from behind a veil of ignorance.

This section demonstrates the proposition that non-contributory subsidies for the old (first pillars) crowd out contributory pensions (second and third pillars). This proposition is valid only for those workers that have low labor productivity. This is because the amounts of first pillar pensions must be attractive for their presence to make a difference. This is certainly the case for the lifetime poor, but it may also be the case for the lower middle classes.

Two prevalent designs for first-pillar pensions are assistance pensions and minimum pension subsidies. They are analyzed separately.

3.1 Introduction of an Assistance Pension in the presence of a contributory system

A standard assistance pension is defined here to be a flat subsidy with cliff withdrawal:

\[
NCS_{std}^d(D) = \begin{cases} 
A & \text{if } (y^D) \cdot \theta(1 + R^c) < T \\
0 & \text{if not}
\end{cases}
\]

where \( T \) is a threshold for the size of the contributory pension measured in $/month. An assistance pension may also have analogous thresholds, referred to the size of labor earnings in old age, to the size of per capita household income (which is indicative of

\(^7\) The effects on individual saving do not have an impact on the aggregate capital stock, because the affected workers receive a modest fraction of national income.
intrahousehold transfers) and to the size of capital consumption. If all thresholds are raised without bounds, a universal flat pension obtains.

The point is that a higher density of contribution D raises the contributory pension, and this may trigger the full loss of the subsidy, of size $A$. The budget constraint for old age must be modified to:

$$c_p = y_p(D) + S \cdot [1 + r(sign(S))] + NCS^A_{nd}(D, T)$$

The results of going through the new budget equations are presented in detail for case F2, and to save space the results are merely summarized for cases F1, F3 and F4.

The introduction of the Assistance Pension changes the budget constraint from (2b) to (2b’) and this twists the thick solid line AF2 in figure 2, by adding the thick dotted line AA’DEF2. This line falls in point D, to register the fact that when density rises sufficiently to make the contributory pension exceed the statutory threshold $T$, the worker loses the whole subsidy $A$ at once. Thus, at E his old age income is limited to income from work and the contributory pension.

The thick budget lines do not consider pure saving $S$. Pure saving and dissaving opportunities at point F2 create, respectively, the dashed line that goes from F2 to the Northwest (net saver status), and the narrow dotted line that goes from F2 to the Southeast (net debtor status). Similarly, saving opportunities around point D create the dashed line that starts from D to the Northwest, and opportunities for consumer credit create the narrow dotted line that starts from D to the Southeast. Although point F2 is not dominated, it now has serious competitors along line A’D.

Figure 2: Contribution density falls when an Assistance Pension is introduced (case F2 with $S^* = 0$ in the absence of noncontributory pensions)
If the original optimal decision for density was at point B, the introduction of the assistance pension can change the preferred density to a point like C. The new density of contribution is definitely below the original one. This is shown by the thick horizontal arrow, which measures the increase in consumption in the active phase due to the choice of longer spells in uncovered jobs, brought about by the introduction of the assistance pension. The optimal contribution density falls discontinuously due to the introduction of the assistance pension.

The abrupt withdrawal of the assistance pension at point E explains the large incentive to reduce contribution density, that is, to reject covered jobs and give preference to self-employment and home production once D has been reached. If the demand of covered jobs is elastic, so that wages do not change much in response to these changes in job choice, equilibrium contribution density will change. The conclusion is that the design of the assistance pension drives a reduction in equilibrium contribution density.

In case F1 uncovered jobs are more desirable and all saving for old age is made through pure saving vehicles. Since the assistance pension is offered to individuals with zero density, its presence is a further reason to keep density at zero. The introduction of the assistance pension does not affect the optimal density, which remains at zero in case F1. ($D^*=0$).

In cases F3 and F4, the introduction of an assistance pension can affect the choice of density ($D^*=1$). This happens if the size $A$ of the subsidy is large enough to be similar in size to the contributory pension. In this subcase, there is a tradeoff between reducing density below 100% in order to gain access to the subsidy $A$, and losing income from a covered job. The introduction of an assistance pension can reduce density discontinuously.

### 3.2 Introducing a Minimum Pension Subsidy in the presence of a contributory system

A “minimum pension subsidy” is defined as:

\[
NCS^{MP}(D) = \begin{cases} 
0 & \text{if } D < D_T \\
(1-t_p) \left[ MP_{Goal} - (y^c D) \cdot \theta(1+R^c) \right] & \text{if } D \text{ is in-between} \\
0 & \text{if } (y^c D) \cdot \theta(1+R^c) > MP_{Goal}
\end{cases}
\]

where $D_T$ is a threshold of contribution density required to have access to (vest) the subsidy. $MP_{Goal}$ is the statutory level of the minimum pension, which may be adjusted annually by ad-hoc legislation to follow the evolution of median or average earnings. This is the target that must be reached when adding the noncontributory subsidy to the self-financed contributory pension. The maximum subsidy is $[MP_{Goal} - y^c D_T \theta(1+R^c)]$.

This design has a double rationale. On the one hand, a higher contributory pension is taken as an indication of less need, so the noncontributory subsidy is reduced. On the other hand, stimulating the attainment of a positive level of contributions ($D_T$) seems to help alleviate neglect of old age. This rationale is discussed further in section 5.
Because of the subsidy, the budget constraint in old age is modified to:

\[(2b'') \quad c_p = y_p(D) + S \cdot [1 + r(sign(S))] + NCS^{MP}(D, D_T, MPGoal)\]

For the “in-between” range of D, when voluntary saving is held constant:

\[
(6) \quad \frac{\partial c_p}{\partial D} \bigg|_{D \text{ in-between}} = 0
\]

Equation (6) shows that consumption in old age does not increase at all when density rises! Each $1 of additional contributory pension is used by the state to withdraw its own subsidy by $1, leaving nothing extra to the worker. The “withdrawal rate” of the minimum pension subsidy is 100%, in this range. The withdrawal rate is an implicit tax on covered earnings.

Of course, with twisted budget constraints a local analysis such as the one in equation (6) is insufficient. In case F2, the global situation is described by figure 3. Unlike the case of the assistance pension, no subsidy is obtained with zero density, because the contribution requirement of \(D_T\) years is not met. Accordingly, there is no point A’. Only when the worker increases contribution density to point B, does he increase his total old age income, including the subsidy, to point B’.

Further contributions after point B’ are fully taken away by the 100% withdrawal rate. For example, total income in old age at point C is the same as the one in point B’, despite higher density. This is against the worker’s interest, because income during his active phase is less at C than at B’ (he has a lower take-home wage owing to the fact that his contributions are higher). In figure 3, the bold dotted line is flat in the region in which the State reduces the subsidy. The flatness indicates that the withdrawal rate is 100 percent.

We have not considered pure saving \(S\). From any given income point where pure saving is zero, positive saving opportunities create the dashed line to the Northwest, and consumer credit creates the dotted line to the Southeast. Figure 3 presents a subcase where F2 is not yet dominated by B’.
A participant finds optimal to turn down covered jobs once he has reached or expects to reach point B’. If he wants extra consumption at any age, it is always more desirable to attain it through pure saving and dissaving than through contributory pensions. For example, pure saving allows a move from B’ to E, which dominates a move from B’ to point C, because the return on pure saving is not taxed at the expropriatory implicit rate of 100 percent.

For preferences marked D and E in figure 3, the new optimal density of contribution (at point E) is $D^* = D_T$, well below the one chosen in the absence of the minimum pension subsidy (point D). The thick horizontal arrow in the upper half of figure 3 shows a large discontinuous reduction in density of contribution, from point D to B. Some of its consumption consequences are undone by increasing voluntary saving from B’ to E. However, for the preferences marked A and G in figure 3, the new optimal density (which supports point G) is higher than the one chosen in the absence of the minimum pension subsidy (point A). This is due to the density requirement $D_T > 0$: the participant observes that by increasing density up to point B, i.e. by choosing $D^* = D_T$, he attains the maximum subsidy. This strategy requires sacrifice of some take-home wage, but the consumption loss when active can be fully mitigated by taking consumer debt in amount BG (measured in the horizontal axis), along the dotted line that starts from B’ to the Southeast. In figure 3 preferences are chosen so that points G and A yield the same consumption when active. Underlying this constancy in consumption there is a large discontinuous increase in density of contribution, marked by the thick horizontal arrow from A to B. Thus, a minimum pension subsidy alone has ambiguous consequences for chosen contribution density.
An unintended consequence of a minimum pension subsidy is that it may favor the consumer credit industry, rather than the old poor. Participants that want point G must pay \( [r(-) - r(+)](c_p B' - c_p G) \) in interest for consumer debt. If the supplys of consumer credit is less than perfectly elastic, then some of the incidence of the subsidy reaches the owners of consumer credit operations. This reduces the efficiency of first pillar subsidies.\(^8\)

For completeness, consider cases F1 and F3. In case F1 exempt jobs are preferred and all saving for old age is made through pure saving vehicles. In case F1, the minimum pension subsidy raises the density of contributions from \( D^* = 0 \) to \( D^* = D_T \), in those subcases where the size of the maximum subsidy is attractive enough. The figure that proves this is omitted for brevity, but the intuition is straightforward: getting to a point like B', where the full subsidy is obtained, can be worth losing the advantages of reducing density to zero.

In case F3, the introduction of a minimum pension subsidy can reduce density from \( D^* = 1 \), but only in a rather extreme subcase. Cutting density to \( D^* = D_T \) gives access to the maximum subsidy, and this may be larger than the earnings loss due to the lower labor productivity of uncovered jobs. This subcase is extreme because \( D = D_T \) can only be better than \( D = 1 \) if, at the margin, the present value of the subsidy discounted at rate \( r(-) \) rises more than what is lost from a more productive job.

### 3.3 Simultaneous presence of an assistance pension and a minimum pension subsidy

A number of countries offer both subsidies together. One reason is that with a minimum pension subsidy alone, women engaged in home production fail and individuals that neglect old age choose a density \( D \) smaller than \( D_T \) and can fall into dire poverty when old in the absence of intrafamily transfers.

When both subsidies are offered simultaneously, an incompatibility rule is standard: both subsidies cannot be received simultaneously (in old age). Provided that the contributory system pays an annuity that is constant over time, incompatibility is resolved at pension age, as follows: given that density \( D \) and pure saving \( S \) are pre-determined as of pension age, each individual calculates the sum of contributory pensions and each of the subsidies, and chooses the highest sum. Since the options will be the same at any time after pension age, choices will remain constant over time too.

Figure 4 shows the empirically relevant case where the size \( A \) of the assistance pension is well below the size of the maximum subsidy offered by the minimum pension.

---

\(^8\) Since take-up of first pillar subsidies is voluntary, this loss cannot exceed the maximum subsidy. This is not the case for mandatory second pillars, where the loss in interest payments in consumer credit can leave the participant worse off than in the absence of the second pillar (Valdes-Prieto 2002, p. 215-218).
Figure 4: Contribution density falls when both designs are offered simultaneously
(case F2 in the left panel and case F1 in the right panel)

The left panel of figure 4 shows the simultaneous offer of both subsidies under the incompatibility rule, for case F2. The presence of the assistance pension makes selection of \( D^* < D_T \) attractive for the individuals with preferences such that, in its absence, would have chosen \( D^* = D_T \) (as in point G in figure 3). Thus, the vesting condition of the minimum pension subsidy is crowded out by an assistance pension with no vesting condition.

Of course, some individuals would still be attracted to \( D^* = D_T \), for example those with the indifference curve shown in the left panel of figure 4. However, the figure also shows that in the absence of both subsidies, that individual would have chosen a density higher than \( D_T \). Therefore, density always falls (in case F2) when both first-pillar programs are offered.

The right panel of figure 4 shows case F1, where in the absence of subsidies uncovered jobs are preferred and all saving for old age is made through pure saving vehicles. The case shown is one where pure saving that starts from point A’ dominates pure saving that starts from point B’. In this case, the optimal density \( D^* \) remains at zero despite the presence of a minimum pension subsidy that if acting alone would have attracted an increase in \( D^* \) to \( D_T \). Again, the presence of an assistance pension trumps the vesting requirement of the minimum pension subsidy.\(^9\)

\(^9\) This may explain why coverage of contributions fell in Chile in the late 1970’s as compared to the 1960’s: an assistance pension was introduced in 1975.
In conclusion, the simultaneous offer of both subsidies eliminates most of the cases where minimum pension subsidies increase the density of contribution.

3.4 Implications of the multiperiod nature of old age

This model does not capture the multiperiod nature of old age. A multiplicity of periods opens further opportunities to reduce evade the density condition $D_T$ and still get subsidies.

For example, participants that do not meet the threshold $D_T$ required to access a minimum pension subsidy, can contribute for a few years after reaching pension age, until the required number of contributions is reached. Some may even declare a fictitious job, whose only purpose is to declare contributions and improve the density record (Beyer and Valdes-Prieto, 2004). If past employers are empowered to supply density records, it is possible to request help to raise artificially the reported density, to reach $D_T$. The officials in charge of the records may also be bribed to reach $D_T$. Registry officials may also take the initiative in offering adulteration of records. The ability to increase density of the vesting condition in the minimum pension design is severely undermined by these strategies.

A density requirement like $D_T$, which is defined in terms of time and not in terms of the amount of money contributed, is also vulnerable to underreporting of earnings. Consider the case of a fictitious job. In order to minimize the expense of contributing $(\theta + t_a)y^c$, this fictitious job can be declared to be part-time and with a minimum wage. Public sector unions in many countries have lobbied successfully to legalize underreporting, by relabeling a part of earnings as “not taxable for social security purposes”.

The multiperiod nature of old age also allows the replacement of cliff withdrawal of assistance pensions for a withdrawal at a smaller rate. Consider the notion that the threshold $T$ must be compared with the current contributory pension, rather than with the contributory pension that would have existed if the participant had taken a constant lifetime annuity as of pension age.

Consider the subset of participants whose constant for life contributory pension is smaller than some level $L$. If the lifetime annuity that can be financed with the accumulation (contributions plus interest) is larger than $L$, the participant is excluded from the assistance pension for life.10 In the model, this implies considering the subset of participants who choose density smaller than $D_L = L/\dot{y}^c \theta(1+R^c)$. If density $D^* > D_L$ the participant is excluded from the assistance pension.

Introduce now a rule whereby the participant with contributory pension smaller than $L$ is allowed to withdraw a pension of amount $L$ from his own accumulation. Because the constant-for-life contributory pension is smaller than $L$, it is clear that the accumulation

---

10 In Chile this exclusion is implemented in two steps: (i) Mandate the purchase of a lifetime annuity to all participants whose accumulation is between 1.0 and 1.5 times the one needed to purchase a lifetime annuity equal to the Minimum Pension Goal; and (ii) choose $L$ to be equal to the Minimum Pension Goal.
will be exhausted if the participant survives to the age of life expectancy as of pension age. Call this life expectancy $LE$.

Because of the requirement that the current contributory pension be below threshold $T$, the subsidy $A$ cannot be claimed before the age of exhaustion. However, when and if exhaustion occurs, the contributory pension falls to zero. At exhaustion age the participant can file for an assistance pension of amount $A$.

An individual that expects a contributory pension below $L$ may recognize the following: if he chooses a higher density of contribution, and attains a higher contributory pension, then he will reach exhaustion date at an older age, and will be able to claim subsidy $A$ for a shorter period (before death). He will get fewer subsidies in total.

The threshold $T$ fades away in this multiperiod setting: even if his accumulation is unable to finance a lifetime annuity equal to $T$, it will certainly finance a pension equal to $L (> T)$ for a few months. While the accumulation is dispersed, the State saves paying the assistance pension that it would have paid under the “pure” design presented in section 3.1.

In a model where old age is collapsed into a single period, the life expectancy as of pension age has unit duration ($LE = 1$). In those units, the exhaustion age when the accumulation is dispersed at rate $L$ is $D \frac{y^\theta (1+R^c)}{L}$. The total amount of subsidy paid out in old age is:

$$
NCS^A_{Chilean}(D) = \begin{cases} 
A \left(1 - \frac{y^\theta D \cdot \theta(1+R^c)}{L}\right) & \text{if } y^\theta D \cdot \theta(1+R^c) < L \\
0 & \text{if not}
\end{cases}
$$

Therefore, choosing a higher density $D$ reduces total subsidies in old age at rate of $(A/L)y^\theta \theta(1+R^c)$ per unit of density. Cliff withdrawal is smoothed away by the multiperiod nature of old age, if the state adapts the rules of the assistance pension as indicated here.

In the case of Chile, the present value (as of pension age) of accumulated assistance pensions during old age is subject to a withdrawal rate estimated at 61 percent (Beyer and Valdés-Prieto 2004, p. 25-6). This implicit tax on contributions is much lower than the infinite (local) withdrawal rate of a standard assistance pension (section 3.1).\textsuperscript{11}

A potential drawback in this modified assistance pension is that the flow pension in $$/month drops from $L$ to $A$ at the age in which the accumulation in the second pillar is exhausted. Since this drop is announced in advance to the participant, he can predict the fall and can save in anticipation. Therefore, this design is compatible with efficient consumption smoothing. However, if the participant is unable to save, the drop in benefits is not optimal, because consumption is not smoothed (Valdes-Prieto, 2006). The drop also creates risk for participants if the date in which this fall occurs depends on the financial return earned by a pension fund, which is stochastic.

\textsuperscript{11} The combined effect on density of this modified assistance pension and the minimum pension subsidy is discussed in Beyer and Valdés-prieto (2004).
Summing up, section 3 finds that standard first pillar subsidies reduce the equilibrium density of contribution. The implication is that part of self-employment among the low-productivity workers can be explained by the poor design of these social programs. They create a “low contribution density trap.” This trap severely punishes, through subsidy withdrawal, those who increase their contribution density. At the same time, these programs have been successful at improving the material well-being of their beneficiaries.

4. Second-generation designs for first-pillar pensions

This section analyzes two second-generation designs: a “proportional” minimum pension, and a subsidy with a single withdrawal rate in proportion to second pillar pensions.

4.1 A proportional minimum pension

Consider a “proportional” minimum pension like the one that Switzerland had in its AHV system (Valdés-Prieto 2002, p. 62). In this design, the goal used by the minimum pension subsidy is not a fixed number, but is proportional to the number of years (or months) of contribution attained by each individual participant. This design can be represented by the following expression for the subsidy amount:

\[
PMPS^i = \max\left[0; \left(\frac{N^i}{N^R}\right) \cdot MPGoal_R - CP^i (D(N^i))\right]
\]

($/month)

where \(PMPS^i\) is the amount of the monthly subsidy in old age for participant \(i\), \(N^i\) is the number of periods of contributions by participant \(i\) (when the period is made smaller, an almost continuous version is obtained), \(MPGoal_R\) is the minimum pension goal for a reference number of periods of contribution, namely \(N^R\), and \(CP^i\) is the contributory pension built by participant \(i\), which depends on density, which also depends the number of periods of contribution. Note that the minimum density requirement \(D_T\) is zero.

Equation (8) highlights the fact that the proportion of periods in which contributions are made is different from density, which is the proportion of the present value of earnings that is subject to contribution. The distinction is important because \(N^i\) affects the contributory pension exponentially: a higher \(N^i\) allows the internal rate of return of the second pillar system to operate for a longer time. For the admittedly restrictive case of a flat age-earnings profile with constant per-period density of contribution, the contributory pension is proportional to the future value function:

\[
CP^i = k^i \left[(1+R^e)^{N^i} - 1\right]/R^e
\]

where \(k^i\) is a proportionality constant that depends on the declared earnings of individual \(i\), the per-period density and life expectancy, and \(R^e\) is now the per-period internal rate of return paid by the contributory system to each generation.
In contrast, the minimum pension goal grows linearly with $N$. The discrepancy between exponential and linear growth has three implications: 12

a) When the number of years of contribution is small, the minimum pension goal grows faster than the contributory pension. Since the subsidy grows with $N$, there exists a marginal subsidy to increase the number of periods of contributions.

b) For a larger number of years of contribution, the subsidy falls with $N$, and thus there is a marginal tax on raising density. The marginal tax is zero when $PMPS^i$ reaches 0.

c) The amount of the subsidy has a different interior maximum for each $k^i$. The $N$ that maximizes the subsidy is larger for lower declared salaries. 13

Valdés-Prieto (2006) argued against a proportional minimum pension on the grounds of inefficiency and horizontal inequity. Inefficiency occurs because the marginal tax/subsidy rate on additional contributions varies in an unintended and very wide range. The value depends on the size of individual earnings ($k^i$) and on $N^i$, even for cases with constant per-period density of contribution. According to simulations, the marginal subsidy can be as high as 350% for low values of $k^i$ and falls continuously as $N^i$ rises until very negative rates are reached, according to a curvature that depends on the size of earnings. A 350% subsidy rate is large enough to justify fraud. At the other extreme, the very high implicit tax rates on additional contributions applied to many participants means that one of the less desirable aspects of the minimum pension subsidy is retained.

A withdrawal rate that varies continuously with ($k^i$, $N^i$) also has a high level of complexity, relative to a subsidy with a unique withdrawal rate, and this raises the risk of planning errors for individuals. Finally, the proportional minimum pension remains vulnerable to underreporting of earnings.

Moreover, the proportional minimum pension fails the test of horizontal inequity as soon as density of contribution ceases to be constant. The following illustration was offered by Valdés-Prieto (2006).

**Case A:** A seasonally employed woman works intensively on the fruit harvest for 3 months a year, earning a wage of CLP$ 200,000 a month in exchange for 12-hour workdays sweating in the sun. It is assumed that she works seasonally from the ages of 20 to 39. Then at age 60 she meets the legal age for starting a contributory pension, which may last until she dies at age 82. She postpones consumption of her contributions for 41 years on average.

**Case B:** Another (middle or upper class) woman works as a part-time secretary for a wage of CLP$ 132,665 a month, 5 hour a day, 12 months a year, between the ages of 40 and 59. Her contributory pension may last from age 60 until she dies at age 82. She postpones consumption of her contributions for only 21 years on average.

---

12 This discrepancy cannot be captured by the model of sections 2 and 3, where all periods in the active phase collapse into one. This prevents distinction between density D and the number $N$ of periods of contribution.

13 From (8), the marginal subsidy is $\frac{\partial \text{MPMS}^i}{\partial N^i} = \frac{\text{MPGoal}}{N_R} \left( \frac{k^i}{R^c} \right) \ln(1 + R^c) \cdot (1 + R^c)^{N_R}$ if $PMPS^i > 0$. 
In this illustration the numbers are chosen to insure that the account balance at age 60 (pension age) is the same for both women, if the real interest rate is 5 percent a year (the balance is CLP$5.4 million). Therefore, both self-finance a contributory pension of CLP$31,105 per month (if annuities yield 4 percent real). Since the accumulation as of pension age is a measure of the overall saving effort, both should get the same subsidy.

The seasonal fruit-picker has only five years (60 months) of contribution. If the minimum pension goal ($MPGoal_R$) is CLP$90,000 for a reference period of $N_R = 20$ years, (close to Chilean reality as of 2006), the proportional minimum pension goal for her is $(5/20)\cdot CLP$90,000 $= CLP$ 22,500. As this is smaller than her contributory pension of CLP$31,105/month, her subsidy is zero!

In contrast, the middle class secretary has 20 years of contributions. Her proportional minimum pension goal is $(20/20)\cdot CLP$90,000. Given her contributory pension of CLP$31,105/month, her subsidy is CLP$ 58,895 per month. This subsidy lasts about 22 years on average, counted from age 60. The present value of this subsidy as of age 60, discounted at a real interest rate of 4 percent is CLP$ 10.2 million. Comparing with her accumulation at age 60, the average rate of subsidy for her is 190 percent.

Although the seasonal fruit-picker can collect the assistance pension, the amount of the assistance pension is about half the CLP$90,000. The inequity is obvious.

The general lesson is that a proportional minimum pension suffers from defining effort in terms of the number of years (or months) of contribution, rather than in money terms. Because of these inequities, a proportional minimum pension cannot be Pareto efficient, or optimal according to any social welfare function.

4.2 A subsidy with a small withdrawal rate

This section evaluates a family of noncontributory subsidies where the withdrawal rate is set at a much smaller value. Valdés-Prieto (2002, pp. 57 and 70) summarized this design with the following formula:

\[ NCS(D)^{SW} = \max[0; \ BP - \gamma \cdot CP(D)] \]

where $CP(D)$ is the contributory pension financed by the participant, BP is a basic pension or subsidy given to those that never contributed to the second pillar, and $\gamma$ is the “withdrawal rate” of the subsidy, with $\gamma \in (0,1)$. This withdrawal rate can also be defined as $\gamma = BP/MCPWS$, where MCPWS is the “maximum contributory pension with subsidy”.

Equation (9) includes the minimum pension subsidy of section 3.2, as a polar case, by making $\gamma = 1$ and $D_T = 0$. In the other polar case where $\gamma = 0$ (if MCPWS $= \infty$), this formula generates a flat, universal pension (paid to all), as in Denmark and New Zealand. In the nonlinear case where $\gamma = 0$ when $CP(D) < T$, $\gamma = \infty$ at $CP(D) = T$, and $\gamma = 0$ thereafter for $CP(D) > T$, this formula reproduces the standard assistance pension of section 3.1.
Therefore, the “small withdrawal rate” family described by equation (9) is between the two extremes of a standard minimum pension subsidy and a universal flat pension. An imperfect example is the second tranche of the new Swedish “minimum pension supplement”, which was legislated in the 1990s and implemented in 2003. This subsidy is nonlinear and has two tranches, identified by $\gamma = 1$ when $CP(D) < 1.26$ base amounts, and by $\gamma = 0.48$ when $CP(D)$ is between 1.26 and 3.07 base amounts (Scherman, 1999). The basic subsidy is 2.13 base amounts, or 77,532 SKR per year in 1998. According to Palme (2003), the Swedish design is based on older Finnish and Norwegian designs. Another example of “small withdrawal” is the new “solidarity pension” created by the 2008 pension reform in Chile (Valdés-Prieto 2008, companion paper), where the withdrawal rate is $\gamma = 0.32$ and there is a single tranche.

In terms of the model of this paper, a small withdrawal rate subsidy is defined by:

\[
NCS^{SW}(D) = \begin{cases} 
(1 - t_p) \cdot \left[ BP - \gamma \cdot (y^c \cdot D \cdot \theta \cdot (1 + R^c)) \right] & \text{if } (y^c \cdot D \cdot \theta(1 + R^c)) < MCPWS \\
0 & \text{if not}
\end{cases}
\]

with $\gamma < 1$. The budget constraint for old age is modified to:

\[(2b^*) \quad c_p = y_p(D) + S \cdot [1 + r(sign(S))] + NCS^{SW}(D, BP, \gamma, MCPWS)\]

For the applicable range of $D$, when voluntary saving is held constant:

\[
(11) \quad \frac{\partial c_p}{\partial D} \bigg|_{\text{const}t} = \frac{\partial y_p}{\partial D} + 0 + \frac{\partial NCS^{SW}}{\partial D} = (1 - \gamma) \cdot (1 - t_p) \cdot y^c \theta(1 + R^c)
\]

Equation (11) says that consumption in the old age increases in proportion to $(1 - \gamma)$ when density rises. Each $1 of additional contributory pension is used by the state to withdraw its subsidy by $\gamma$ cents, leaving $(1 - \gamma)$ cents to the participant, to increase his total pre-tax pension (which combines subsidies and contributory pensions). In present discounted value terms, the implicit tax rate on covered earnings is $(\gamma + t_p + \gamma t_p)\theta(1 + R^c)/(1 + r(+))$.

As before, local analysis is insufficient. The optimization program $P_1$ is modified by introducing $(2b^*)$ to incorporate the first-pillar pension. The modified program $P_1'$ applies to the combined system, and not just to the contributory part. The budget constraint remains the result of competition between linear options. Again, the many corner solutions can be ordered using the following identity obtained from $(P_1')$:

\[
(12) \quad \frac{\partial U}{\partial D} = \left\{ \frac{\partial U}{\partial S} + v^t \left[ MRT_{cs} \cdot (1 - \gamma) - (1 + r(sign(S))) \right] \right\} \cdot (z^c - (1 - \theta - t))^c
\]

\footnote{See PowerPoint presentation to the PACPR (Valdes-Prieto, 2006).}
Equation (12) differs from (3b) only because the net rate of return offered by the combined contributory-noncontributory system is multiplied by \((1-\gamma)\), before comparing with the return of pure saving. This affects the frontier between cases F1 and F2, and it increases the range of parameters in which case F1 applies, where \(D^* = 0\). The presence of \((1-\gamma)\) also affects the frontier between case F2 and F3, by reducing the range of parameters where case F3 applies, where \(D^* = 1\). The range of cases where F4 applies is still given by the sign of \(\left(z^{ex} - (1 - \theta - t_a)\right)\), which is not modified by \(\gamma\).

Figure 5: First-pillar subsidy with small withdrawal rate (case F2)

Figure 5 provides a global analysis when case F2 applies before and after the introduction of a first pillar with a small withdrawal rate \(\gamma\). The basic subsidy BP is paid to persons who make no contribution to the second pillars, as indicated by points A and A’. The subsidies are the vertical distance between the solid line, AF2, and the dotted line A’C. The maximum contributory pension that receives a subsidy, the MCPWS, is the contributory pension at point C. The slope of A’C is the net return of the combined two-pillar system, which is the slope of AC, \(MRT_{CS}\), times the new factor \((1-\gamma)\). The slope of CF2 is not affected by \((1-\gamma)\).

As before, pure saving and dissaving opportunities are indicated in Figure 5 with dashed and dotted lines. The assumption that case F2 applies both before and after the introduction of this first pillar is reflected in the slope of the dashed and dotted lines: the dashed lines are flatter than both AC and A’C, while the dotted lines are steeper than both AC and A’C. In this case, figure 5 shows that the introduction of this noncontributory subsidy reduces density from point D to E. Therefore, a small withdrawal rate also crowds out contributory pensions.
Figure 5 shows that when case F2 is preserved, the reduction in density is smaller than in the standard first pillar designs reviewed in section 3, if $\gamma$ is smaller than 100%, in the case of the standard minimum pension. Although section 3 did not show in detail the cases when the introduction of a first pillar changes the case from F2 to F1, and from F3 to F2, a detailed analysis also shows that these case changes apply to a smaller range when the withdrawal rate is smaller, and thus that these drops in density are less likely.

Other configurations are possible: First, if the slope of the dashed line (return $1+r(+)$) is steeper than A’C (but not of AC), then the introduction of this first pillar subsidy changes the situation from case F2 to case F1, and density drops discontinuously from $D^* \in (0,1)$ to $D^* = 0$. If the slope of the dotted line (return $1+r(-)$) is flatter than AC (but not of A’C), then the introduction of this first pillar subsidy changes the situation from case F3 to case F3, and density also drops discontinuously from $D^* = 1$ to $D^* \in (0,1)$. \(^{15}\) Despite the approximate preservation of a range where case F2 applies, the introduction of this first pillar subsidy always creates a discontinuous drop in density when a case change occurs.

Summarizing, the gain offered by a small withdrawal rate is that density of contribution to the second pillar is reduced by less than with the standard designs. As the withdrawal rate is reduced towards zero, the crowding out of the contributory system is alleviated by more.

*The optimal withdrawal rate*

Why not reduce the withdrawal rate to zero, and adopt a universal pension? This appears to eliminate all crowding out of the second pillar. The fiscal cost of a universal pension is larger than in a scheme with a positive withdrawal rate, and this requires higher tax rates and additional distortions.

Alternatively, why not reduce the withdrawal rate to zero for the poor, by starting withdrawal only after some threshold in second pillar pensions has been reached? It may be argued that the distortions caused by the withdrawal rate would affect only the middle classes, where they matter less, not the low productivity workers. This is indeed the design of the Social Security system in the United States and the age pension in Australia.

The optimal income tax literature introduced by Mirrlees (19719 offers some answers. The simulation results of Slemrod et al (1994) for an economy with given factor prices\(^{16}\), and whose second pillar is a perfect substitute for voluntary saving, show that the optimal two-bracket income tax has two features: (i) all the marginal tax rates are smaller than 100%; (ii) the marginal tax rate for the low-productivity workers should be larger than the one for the high-productivity workers. The analytical work by Diamond (1998) for more general

\(^{15}\) In case F4 covered jobs are more productive than exempt ones and $D^* = 1$ in the absence of a first pillar. The introduction of a subsidy with a small withdrawal rate can reduce density only in an extreme subcase. Cutting density to $D^* = 0$ gives access to the maximum subsidy, and if this subsidy is larger than the earnings loss due to the lower labor productivity of exempt jobs. This subcase is extreme because $D = 0$ can only be better than $D = 1$ if, at the margin, the present value of the subsidy, which is affected by $(1-\gamma)$ and discounted at rate $r(-)$, rises more than what is lost from a more productive job.

\(^{16}\) This is an appropriate assumption for an open economy, where substitution with foreign saving prevents the impact on domestic saving from affecting capital accumulation and labor productivity.
income tax schedules also finds that for the empirical distribution of earnings in the U.S., it is optimal to have larger marginal tax rates for low productivity workers than for workers near the median of the earnings distribution.

Thus, public finance theory recommends that the “low density trap” be mitigated by reducing $\gamma$ well below 100%, but not to zero (Valdés-Prieto 2002, p. 67-71). There is less consensus about the level of the contributory pension at which withdrawal should begin. It is unknown whether this level should be zero, or should be positive as in the earned income tax credit (Diamond, 1998).

Poblete’s 2005 thesis (Universidad Católica de Chile) offers two additional findings, also based on simulations in an economy with given factor prices, where only a noncontributory subsidy is present. A major difference with Slemrod et al (1994) is that exempt jobs (whose saving is not observed by the government in old age) are available to individuals. First, the extreme with $\gamma = 0$ never maximizes social welfare, because the fiscal cost of the program is large and exacerbates tax distortions. Therefore, a universal flat benefit is suboptimal, even if feasible. Second, small reductions in $\gamma$ from an initial level of 100% are irrelevant, because participants remain in corner solutions. This is important for first-pillar policy, since it suggests that the welfare difference between choosing $\gamma = 0.48$ (Swedish second tranche) and $\gamma = 0.32$ (2008 Chilean reform) may be substantial. Poblete (2005) finds that the optimal social value for the withdrawal rate $\gamma$ is approximately 20 percent, for a substantial range of parameters.

A different way to evaluate a small withdrawal rate scheme is to compare it with a minimum pension subsidy that has the same fiscal cost. To simplify, let us assume also that $\Delta_T = 0$ in the minimum pension subsidy. The equal fiscal cost condition implies that the Basic Pension is smaller than the Minimum Pension Goal, and that the small withdrawal rate scheme is paid to workers that have a contributory pension larger than the Minimum Pension Goal. When the scheme with a small withdrawal rate replaces the minimum pension subsidy, workers with a contributory pension larger than the Minimum Pension Goal, but smaller than the MCPWS, begin to pay an implicit tax at rate $\gamma$ in covered jobs, which they would not have paid it in the absence of this replacement. Thus, this group reduces its density of contribution. On the other hand, the low-productivity participants receive some subsidy under both designs. They always reduce their density of contribution, but do so to a greater extent with the minimum pension subsidy. Since this group is poorer, their utility increases are valued by more. In addition, this group should be smaller in number as compared to the first group. What is the balance of these considerations? the simulations by Slemrod et al and by Poblete give an answer: social welfare is higher in a scheme with a small withdrawal rate.

5. Other policies to increase the density of contributions

The model presented in this paper and the literature on optimal income taxation quoted in the previous section assumes that mandatory contributive pensions would be a perfect substitute for voluntary saving if net rates of return were equal and pension rights were
alienable (Andolfatto, 2002). This assumption means that a mandate to contribute (second-pillar) would be irrelevant in this setting, because the individual would be able to undo any mandate using the financial market. This assumption implies also that fiscal incentives for old age savings (“third pillars”) do not have a benevolent justification in this setting. Moreover, the best way to achieve efficient density levels in this setting is to eliminate all first-pillar programs \((BP = 0)\). Concern with the lifetime poor should be addressed with a subsidy for all the poor, regardless of age. This is indeed the framework in which most of optimal income tax theory has been built.

The psychology literature has amply documented the substantial prevalence of “excessive optimism” among individuals, which does not mean a majority is affected. An instance of excessive optimism is neglect of old age. This is different from the hyper rational time-inconsistency in preferences discussed in Laibson (1997), because the latter excludes surprise when the high cost of old age is unveiled as excessive optimism dissipates over the ageing process.

Neglect of old age provides benevolent justifications for second and third pillars of limited size (see summary in Valdés-Prieto 2002, chapters 3.3 and 3.4). Neglect of old age also explains concerns with low contribution densities. The positive externalities of covered choices are an independent justification of this concern.

Neglect of old age also provides a benevolent justification for a separate noncontributory subsidy for the old poor, in addition to support for all the poor regardless of age, which is to alleviate the consequences of neglect of old age among formerly middle-class workers that failed to contribute enough.

This section briefly outlines policies to increase density of contribution, different from improving the design of first-pillar subsidies. In this section neglect of old age is recognized as a problem that merits some (limited) State intervention.

An interesting method to raise density is to allow each participant to share contributions with the spouse (Kotlikoff, 1997).\(^{17}\) This method raises density among home workers and second earners in a household. This is different from allowing judges of divorce cases to divide pension rights between spouses, because the contribution sharing raises density and reduces first pillar subsidies even in the absence of divorce. Contribution sharing also has the advantage of operating when divorce occurs after pension age.

Subsidies to contributions to the second pillar are another option to increase density. Of course, subsidies have fiscal costs and thus raise tax distortions or force a reduction in other government expenditure. In addition, such subsidies create incentives to raise density of contribution to socially excessive levels among those that visualize old age correctly (no excessive optimism). Despite this, a limited subsidy to increase density may be desirable,

\(^{17}\) Kotlikoff and Sachs (1997) propose to mandate a 50-50 split of contributions. To see why such a division should not be mandatory, consider a couple where she is a working nurse and he is a medical student with little earnings. These problems may be avoided and density may still rise if the State offers mutual division of contributions to all couples, subject to written approval by both members.
since higher density allows the contributory system to help those workers that neglect old age.\(^{18}\)

Subsidies to contributions are delivered immediately, they are likely to be more effective than first-pillar subsidies, which are delivered in old age. This is because persons that neglect old age -are excessively optimistic- are unlikely to give much credence to the pessimistic event of their own poverty in old age. Persons that visualize old age correctly also discount the expected value of those subsidies because of illiquidity and inalienability.

One problem of subsidies delivered at the time of contribution is that some of them would be given to individuals who subsequently accumulate above-average wealth, either because of luck or effort, and thus do not value the subsidy when old age arrives.

Interactions with employment protection regulations, such as a relatively high minimum salary, must also be taken into account. When a subsidy to contributions attracts more individuals to covered jobs, they will not find jobs in the presence of a minimum salary. In this case the subsidy to contributions merely increases the unemployment rate, and the fiscal cost is dissipated in more search effort. This consequence can be avoided if the law stipulates that some share of the subsidy must be paid to the employer. This provision reduces the effective minimum salary. This portion of the subsidy may increase the employment rate of the target group or may be passed along to the workers as a higher contractual wage which is paid immediately. In both cases density is stimulated.\(^{19}\)

Another tool capable of increasing density of contribution among persons that neglect old age is to lighten up the mandate. When the contribution rate \(\theta\) is reduced, this reduces \(z^{ex} - (1 - \theta - t)\) making cases F3 and F4 more prevalent, where D* = 1. Workers that visualize old age correctly would still save voluntarily for old age if \(\theta\) is reduced. However, if \(\theta\) is reduced, middle-class workers that neglect old age will accumulate a smaller amount of pension rights, and this aspect raises the chances of old age poverty. The elasticity of density to \(\theta\) needs to be above 1 in absolute value for this method to make sense.

Another way to lighten up the mandate, proposed by Beveridge (1942, p. 139), is to exempt the first portion of earnings from the mandate, i.e. to make the least taxable earnings positive (\(LTE > 0\); see definition in section 2.2). This approach has been applied for decades in the Netherlands and in Brazil. A positive \(LTE\) means that a beneficiary who subsequently does better than average would contribute more later in life, and this would exclude him from first-pillar subsidies if they are targeted. In contrast, a participant that is lifetime poor is allowed to consume more in the active phase, which is indeed desirable. This scheme provides relief when it is needed, at modest fiscal cost. This scheme also prevents the perverse situation where a mandate leaves a lifetime poor participant worse off than in the absence of all pillars (Valdés-Prieto 2002, p. 150-1). However, since a higher

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\(^{18}\) The same applies to a universal flat pension that pays the same basic amount to all, since it helps alleviate neglect of old age of middle class persons. However, a universal flat pension also subsidizes middle class workers that visualized old age and saved enough voluntarily. More efficient schemes may be possible.

\(^{19}\) This innovation is incorporated in the 2008 pension reform in Chile. See articles 82 to 84 of the 2008 pension reform law (Chamber of Deputies, 2008).
LTE with a given contribution rate means that the middle class worker that neglects old age saves a smaller amount, old age poverty looms closer, not farther.

Another approach to increase density is to minimize legal exemptions from the mandate to contribute. These exemptions are important only for workers whose earnings are at least partially observed by the State. This may be the case of the self-employed who declare honorarium income to the tax authorities, which may be called “formal self-employed”, but are legally exempt from the mandate to contribute. However, this measure would increase the incentives for the formal self-employed to go fully informal.

In response to these concerns, Berstein, Reyes and Pino (2005) proposed the following default scheme: the formal self-employed would keep the option not to contribute, but the tax authorities would assume that they want to contribute – and would direct the statutory portion of declared honoraria towards the old-age individual account, drawing on the retention of honoraria sent by the employer – unless the participant comes forward and actively requests not to contribute.

Fiscal incentives for voluntary old-age savings (“third pillar” programs), such as the tax incentives granted to IRAs, 401(k) plans and defined-benefit employer pensions in the United States, also contribute to increase the density of contribution. When a country exhibits uneven density of contribution and major groups in the population have low density, the third pillar’s performance in terms of replacement rates is likely to be evaluated as unsatisfactory, because neglect of old age is not alleviated. Moreover, the fiscal incentives fail the test of vertical equity if the groups that use the incentives tend to exhibit higher incomes or earnings than the groups that do not use the incentives. The benevolent justification for the fiscal incentives dissolves.

A design that reduces the degree of vertical inequity associated to standard tax incentives is to redefine the fiscal incentive as a State-financed matching fund for the individual’s contribution, at a constant rate independent of income, and subject to an absolute ceiling. A more radical approach is to define the fiscal incentive as an absolute amount per period.

This section reminds us that density of contribution can be increased with a number of policy tools, and not just by improving the design of first-pillar subsidies.

6. Final remarks

This paper offers a model where the contributory system is not mandatory, even if a mandate exists, because uncovered jobs are available, either legally or in practice. The contributory system bundles covered earnings with saving. This bundling means that job sector choice and saving decisions interact. In the benchmark case some individuals change

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20 Other benevolent justifications may remain, such as avoiding the double taxation of saving associated to income taxes. However, restring fiscal incentives to “retirement” saving is hard to justify with this rationale.
21 This design was adopted in the 2008 pension reform in Chile. See article 91 No. 13 of the 2008 pension reform law (Chamber of Deputies, 2008).
their contribution density very significantly in response to modest perceived reversals in
differences in the rates of return of contributory saving and pure saving vehicles. This result
is also valid for individuals that neglect old age or suffer from the illiquidity of pension
rights, if the effective return offered by the contributory system is enhanced by the
productivity premium of covered jobs over uncovered jobs.

The paper finds that the design of first-pillar subsidies can create those reversals. A small
withdrawal rate minimizes reversals. Still, this withdrawal rate should not be reduced to
zero because it is necessary to serve at the same time the fundamental goals of first-pillar
subsidies, which is to support the old poor despite tax distortions.

This paper suggests two topics for future research. One is whether the withdrawal rate
should be the same in response to different types on income when old. One such income
comes from mandatory pensions (second pillar), another is the one coming from tax-
favored pensions (either individual or employer-based: third pillar), a third source of
income is comes from pure saving, a fourth is the one coming from family transfers
(spouses, children) and a final type of income is the one coming from labor earnings (work
when old). The proper order of these withdrawal rates is not addressed in this paper.

The second topic concerns the design of second pillars. When the State does not provide a
universal mandate, low density ensues and the contributory system pays inadequate
pensions relative to former earnings, and thus fails to alleviate neglect of old age.

Public opinion considers low density of contribution to be a defect of the contributory
system. In the debate that led to the 2008 reform in Chile, the argument that low density is a
result of underdevelopment, which is associated with widespread self-employment among
low productivity workers, was considered an excuse, since this does not solve the problem
and rather helps preserve an unsatisfactory status quo.

In a situation of low density and low contributory pensions, where some middle-income
people fall into poverty in old age, it is tempting to introduce pay-as-you-go finance to
grant large supplements to all the current old, including the unionized higher middle class,
to be paid by future generations. This is part of the explanation of the widespread adoption
of pay-as-you-go finance in countries where incomes are rising, a fact that rules out
intergenerational insurance motives. However, pay-as-you-go grants pure subsidies to the
initial generation and finances them with hidden taxes on the covered earnings of future
generations, this policy reduces the efficiency of the economy (Abel et al, 1989), induces a
decline in fertility (Cigno and Werding, 2007) and may favor populist competition in
offering subsidies (Godoy and Valdés-Prieto, 1997).

The argument that inadequate design of non-contributory subsidies is one culprit of low
contribution density changed the scenario in the Chilean reform debate, by showing a
constructive method to raise densities. A small withdrawal rate first-pillar strengthens
contributory systems, as it alleviates a major failing in terms of its fundamental mission.
References


Chamber of Deputies (2008) Pension Reform Law; Oficio Nº 7,259 from the Chamber of Deputies to the President of the Republic of Chile, Valparaíso, January 16.


