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The Delegated Portfolio Management Problem: Reputation and Herding

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por

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Abstract

The Delegated Portfolio Management Problem: Reputation and Herding

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This work studies the effects of the possibility of building a reputation in a delegated portfolio management context where financial intermediaries may herd. Reputation is modelled as investors' Bayesian beliefs regarding the ability of intermediaries given their past performance. Unlike previous works, we characterize reputational equilibria in which intermediaries' decision to invest in reputation depends on their current reputation. We find that intermediaries with good reputation are prone to invest in information, whereas those with poor reputation herd. Also, the presence of implicit incentives provided by reputation allows this market to operate using simple remuneration schemes (i.e. a percentage of assets under management). The empirical predictions of the model are discussed and are found to be broadly consistent with previous evidence.

> Felipe Zurita Chair

Resumen

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Este trabajo estudia los efectos de la posibilidad de construir una reputación en un contexto de administración delegada de portafolio, en el cual los intermediarios financieros pueden imitar a otros. La reputación se modela como las creencias Bayesianas de los inversionistas respecto a la habilidad de los intermediarios, dado su desempeño pasado. A diferencia de trabajos previos, caracterizamos equilibrios en los que la decisión del intermediario de invertir en reputación depende de su nivel actual de reputación. Encontramos que intermediarios con buena (mala) reputación tienden a invertir en reputación (imitar). La presencia de incentivos implícitos provistos por la reputación hace que este mercado pueda funcionar usando remuneraciones simples (i.e. un porcentaje del valor de la cartera administrada). Se discuten las implicancias empíricas del modelo y se encuentra que estas son en su mayoría consistentes con evidencia previa.

> Felipe Zurita Profesor Guía

I would like to dedicate this thesis to

Claudia, Sebastián and Juan Pablo. Without your love, support and encouragement I couldn't have gotten this far. I love you and I'm thankful for having you with me. This work is for you, with all my love.

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Chapter 1

Introduction

Financial intermediaries (FI) play a very important role in the economy as they channel resources from agents with liquidity surpluses (both individuals and firms) towards those with liquidity needs. Banks are one of the more traditional types but there are other intermediaries whose importance has increased over time, namely insurance companies, pension funds and investment companies (such as open and closed end mutual funds and hedge funds).

Table 1.1 shows how the stock of delegated assets under management for the second group of intermediaries in OECD countries has averaged an annual growth rate of 9.2% during the last five years. Some analysts (see BIS, 2003, IMF, 2004) have suggested that one of the main factors behind this considerable growth would be the social security reforms undertaken during the past few years in Latin American and Central Europe countries. Other important factors that explain this industry's growth would be financial liberalization, technological advances and an economic environment characterized by low inflation, which increases the attractiveness of financial asset holdings.

As a result of the previously mentioned reforms there has been a rise in demand for portfolio management services to invest the sizable funds that have been accumulated.

(USD Billions)	2002	2003	2004	2005	2006	$\Delta\%$
Investment Funds	$11,\!546$	$13,\!910$	15,922	18,239	19,712	10.7
Insurance Firms	10,100	$12,\!034$	13,877	15,141	15,781	10.6
Pension Funds	9,696	$11,\!876$	13,387	14,782	13,837	5.8
Others	868	986	1,257	1,480	$1,\!636$	14.2
Total	32,186	38,771	44,400	49,586	50,966	9.3

Source: Author's calculations based on OECD data. OECD countries are: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, United Kingdom and United States.

(USD Billions)	2002	2003	2004	2005	2006	$\Delta\%$
World	11,324	14,048	16,165	17,771	21,823	14.0
Americas	6,776	7,970	8,792	9,764	11,485	11.1
Europe	3,463	4,683	5,640	6,002	7,804	17.6
Asia and Pacific	1,064	1,361	$1,\!678$	1,939	2,457	18.2
Africa	20.9	34.5	54.0	65.6	78.0	30.2

Table 1.1: Financial Intermediaries' Assets Under Management.

Source: Investment Company Institute 2008 Factbook.

Table 1.2: Mutual Funds' Assets Under Management

Additionally, in countries with defined-benefits pension schemes there has been a tendency by sponsors (e.g. the State or the affiliates' companies) to professionalize the portfolio management of accumulated funds as means to meet in a better way future obligations with affiliates. Table 1.2 shows the world-wide evolution of assets under management for the mutual fund industry.

As Table 1.3 shows, the Chilean mutual fund market has also experienced a remarkable growth in the last years, managing assets in the order of USD 18 billions by December 2006. This figure is considerably smaller than the amount of assets managed by

(USD Billions)	2002	2003	2004	2005	2006	$\Delta\%$
Mutual Funds	6.349	8.295	11.496	13.564	17.964	23.1
Pension Funds	36.358	48.940	58.818	74.491	89.440	19.7

Source: Author's calculations using data from the Chilean Mutual Funds Association, the Superintendency of Pension Funds and the Central Bank of Chile.

Table 1.3: Chilean Mutual and Pension Funds Assets Under Management

pension funds, which were introduced in the early 80's. Both types of intermediary have experienced sustained growth in the 2002 - 2006 period, with growth rates surpassing those of European and American countries.

The growth of these intermediaries has been perceived as a positive development for several reasons: they have greater diversification capacity than individual investors, plus they reduce transaction costs; if insurance companies and pension funds have long term liabilities, they can help to develop and give stability to long-term financial assets markets; the presence of intermediaries can help to improve transparency and the corporate government of financial markets (IMF, 2004); and, due to their information processing capacity, FI can improve the efficiency of financial markets, exploiting arbitrage opportunities and preventing financial securities' prices to deviate from their fundamentals.

However, there are also reasons to monitor and study carefully the development and behavior of FI given that the size of the portfolios managed by these institutions imply that their trading decisions may have significant impact on financial stability and resource allocation. Some financial intermediaries, such as pension funds, tend to show similar investment strategies and portfolios, a phenomenon that is called *herd behavior* and which could increase excessively volatileness in financial markets¹; legal restrictions to investment in foreign assets for some FI, such as pension funds, could create price distortions in some domestic securities in countries where financial markets are small in relation to the size of these funds, besides limiting the investors' diversification opportunities; in a financial market where managers worry about their perceived ability by the market; this is, their labor market reputation, it's possible that they ignore useful information to make investment decisions and decide to imitate the decisions taken by other managers (Dornbusch et al, 2000).

This possibility, which has theoretical grounds in the works of Scharfstein and Stein (1990), Avery and Chevalier (1999) and Graham (1999), sharply contrasts with the view that reputation can act as a market mechanism to align incentives between investors and intermediaries, therefore allowing the delegated portfolio management market to operate even in the presence of informational asymmetries. This latter argument is made by Heinkel and Stoughton (1994), Arora and Ou-Yang (2001) and Farnsworth (2003).

We make a contribution by studying the relationship between reputation and herding in a delegated portfolio management market applying a new methodology from the reputation literature which considers aspects left aside by previous works. In particular, we study how the incentives to herd change as a financial intermediary's reputation evolves over time, explicitly accounting for the fact that there must be permanent uncertainty about the intermediary's characteristics for a steady state reputational equilibrium to be feasible. This point is made by Hölmstrom (1999), Mailath and Samuelson (1998), (2001), Cripps et al (2004) and Vial (2008), although in a different context.

¹See the following Chapter for a survey on this and other delegated portfolio management market stylized facts.

Our main results are that as an intermediary's reputation improves his incentives to herd decrease. As we show in Chapter 3 there are two situations in which an intermediary may disregard the effects of his actions on his reputation; when his reputation is very bad or when it's very good. In these cases it is possible that the intermediary may try to cheat investors by shirking. Moreover, if the remuneration scheme is given by a percentage fee of the final value of assets under management (which as we will show in the stylized facts Section, is the case for most mutual funds); and if this fee is increasing in the intermediaries' reputation, then an intermediary with bad reputation that chooses to herd instead of incurring the costs of gathering private information will experience an expected loss in final value of assets under management but since his profits are given by a small percentage of this value, he will herd. On the other hand, an intermediary with good reputation that decides to herd will experience an important loss in expected profits, since these are given by a larger fee of the final value of assets under management. In order to avoid this loss the intermediary will acquire private information. This prediction is also made by Avery and Chevalier (1999). However, in their case an agent with good reputation in fact chooses a contrarian strategy, disregarding his private information and making the opposite decision from other agents in order to signal to principals that he is skilled. Of course, this behavior is inefficient from the investors' point of view. The work by Graham (1999) makes the opposite prediction: as the initial reputation of agents improves they will herd more because they want to avoid the large drop in profits associated with a fall in reputation, which in this model occurs if an agent's decision is different from that of other agents. However, in a long run equilibrium the initial reputation of an intermediary may be of limited importance in terms of determining his reputation several periods ahead. Therefore, this work is unsuitable to study how an agent's incentives to herd change as his reputation endogenously evolves over time.

We view our work as an alternative rationalization for the empirical evidence found in Chevalier and Ellison (1999) and Hong et al (2000) regarding the existence of a negative relationship between reputation and herding. However in our setup the mechanisms operating in equilibrium are different. In particular due to our modelling decision of using a continuum of intermediaries, the portfolio choice of a particular intermediary contains no information regarding the possible type of another intermediary. This is the basic mechanism affecting the behavior of managers in Scharfstein and Stein (1990), Avery and Chevalier (1999) and Graham (1999). Additionally, while our model makes predictions regarding reputation and herding like those of Avery and Chevalier (1999), our findings are much more optimistic in the sense that lack of herding by intermediaries with high reputation is associated with efficient investment and use of private information. This is important because the works by Avery and Chevalier and Scharfstein and Stein assume a positive relationship between reputation and profits for intermediaries in a two period setup. However, in the presence of the pathological behavior implied by these models, endogenously deriving a long-term, positive relation between reputation and willingness to pay seems like a harder task (Ottaviani and Sørensen (2006) also make a similar observation). Moreover, rationalizing the increasing importance of institutional investors in financial markets is difficult if all intermediaries, regardless of their reputation, make little or no use of private information. While we believe that the cases described by these authors may be of great relevance in determined time periods or situations, we argue that it is difficult to imagine that the delegated portfolio market could have experienced such successful growth if pathological behavior was always present, since intermediaries would have a hard time competing with investors who trade on their own behalf and presumably always make good use of their private information.

We also show how the size of the percentage fee that must be paid to intermediaries in order to align incentives can be considerably smaller if investing in reputation is possible relative to a situation in which this isn't feasible. Moreover, we illustrate how the possibility of investing in reputation may allow the delegated portfolio management market to operate when more sophisticated remuneration schemes cannot be used. However, there is a cost involved since in equilibrium the intermediaries' types are never revealed to investors. Therefore, it is possible that some skilled unlucky intermediaries are punished by investors through low fees while some lucky unskilled intermediaries may be paid high fees. Nevertheless, since skilled intermediaries that acquire information have a greater probability of making good investment decisions, which results in having a better reputation, this type of undesirable situations are rather unlikely to occur.

Additionally, we show that for a reputational equilibrium to be feasible, the gains from investing in reputation can either be obtained through higher fees or through larger assets under management. In both cases the intermediaries' expected profits are increasing in their reputation and the main features of the equilibria remain unchanged.

The rest of this work is organized as follows. Chapter 2 presents a series of stylized facts regarding the delegated portfolio management market and presents a selective review

of the literature studying this problem. Chapter 3 presents the theoretical model, first in a static setup and then in a dynamic one. Sufficient conditions are given for reputational equilibria (i.e. equilibria in which at least some intermediaries acquire information) to be feasible and we partially characterize such equilibria. Chapter 4 presents numerical exercises used to study the comparative statics properties of the model and to compare some of the properties of the static and dynamic economies. Chapter 5 discusses the models' empirical predictions and a comparison is made with previous evidence from other authors. We also discuss an estimation strategy in order to validate our predictions. Finally, Chapter 6 concludes and discusses some possible research areas.

Chapter 2

Stylized Facts and Literature Review

In this Chapter we introduce from a theoretical perspective the features of the delegated portfolio management market. We then proceed to survey the existing literature, both theoretical and empirical, summarizing the main findings and highlighting some interesting research areas.

The delegated portfolio management problem is an agency situation in which an investor (the principal), who has some initial wealth level W, chooses to delegate the task of managing and investing his resources to a financial intermediary (the agent). The investor would like the portfolio to be managed so as to maximize his expected utility for some future period. However, the intermediary will take his investment decisions with the objective of maximizing his own expected utility.

This conflict of interests is worsened because investors don't know with certainty whether an intermediary has the skills needed to make investment decisions and he can't verify that the intermediary has made effort to take sound investment decisions. Additionally, in most cases intermediaries have superior knowledge about financial markets compared to that of the investors whose portfolio they manage. Moreover, investors usually don't have a reliable mechanism to evaluate the manager's performance. Thus, an investor who observes a poor result (e.g. a low return of the portfolio) won't be able to know with certainty if this was due to incompetence, negligence or bad luck.

2.1 Delegated Portfolio Management Stylized Facts

The delegated portfolio management market has been the subject of research by several studies. These works have found some stylized facts in the data, which we briefly summarize.

2.1.1 Fees and Expenses

In the USA the types of remuneration schemes that can be used by investment companies is regulated by the Investment Advisers Act of 1940. Under this Act the use of performance based compensation is allowed as long as this is a "fulcrum" type compensation. This means that the compensation must include a bonus if the investment company's performance exceeds that of a benchmark and also a penalty if the performance is below the benchmark. This type of compensation is also referred to as symmetric.

Even though the USA law allows for the use of compensation-based fees, in practice such fees are not always used. Golec (1992) presents data on 476 open end mutual funds for 1985. Out of this funds only 29 used incentive or performance based fees, which represents a 6.9% of the sample. Also, Blake et al (2003) report that in 1999 only 108 out of 6,716 bond and stock mutual funds, that is only 1.6%, used this type of fee. Also, in the 108 funds using compensation based fees, 44% used the S&P 500 index as their relevant benchmark. Finally, Cuoco and Kaniel (2007) report that as 2004 50% of USA corporate pension funds with assets of more than US\$5 billion; 35% of all USA pension funds and 9% of all USA mutual funds employed performance based fees. Overall the data suggests that the use of performance-based fees is not uniform across institutional investors. Also, larger funds seem to be more likely to use such fees. However, for a large share of pension and mutual funds performance-based fees are not widely used.

Fung and Hsieh (1999) describe how hedge funds are allowed by the SEC to use asymmetric performance fees. The authors show that at the end of 1997 83% of the hedge funds in their sample used an incentive-based fee. For 51% of these hedge funds this fee ranged between 1500 to 2000 basis points, while average management fees range between 100 and 200 basis points. Surveying data from 1994 to 2006, Ang et al (2008) find hedge funds charge an average management fee of 150 basis points and a mean performance fee of 2000 basis points. All these authors document that one type of incentive based fee that is widely used is the "high-water mark". Under this arrangement the hedge fund manager receives a percentage of the increase in assets under management in excess of the last registered maximum. If the fund value doesn't exceed this high-water mark then the manager only gets the management fee.

In the introduction we documented how the number of mutual funds has increased over recent years. In many cases there is a large number of funds competing in the same category. Given this, it could be possible to expect fees in this narrowly defined markets to show little to no dispersion if the offered service is fairly homogenous. However, the work by Khorana et al (2008) challenges this view. The authors gather data on mutual fund fees for 46,580 mutual fund classes offered for sale in 18 countries. These funds account for 86% of the worldwide fund industry in 2002. The authors' measure of fund fees includes two components. The first one is the expense ratio, which encompasses management fees and charges made to cover investment management, administration, servicing, transfer agency, etc. The second element are distribution fees, such as front-end or back-end loads. Using this variable the authors find substantial dispersion in funds fees from country to country. For example mean fees vary from 63 basis points in Sweden to 189 basis points in Dublin for bonds funds. In the United States this fee averages 105 basis points, while the full sample mean is 121 basis points. For equity funds, fees range from 82 basis points in the Netherlands to 300 basis points in Canada. In the United States the average fee is 153 basis points, while the full sample mean is 180. To determine the causes of this dispersion the authors explore fund, sponsor an national characteristics. While the investment objective of funds¹ seems to be important for the size of fees charged, it seems that fees tend to be lower in countries with stronger investor protection. For the Chilean mutual funds market, Maturana and Walker (1999) report that the average fee charged by equity mutual funds from 1990 to 1997 was 600 basis points, while long term and short term bonds mutual funds charged average fees of 310 and 240 basis points, respectively, for the same time period. The authors don't provide evidence on fees' dispersion.

While the degree of fee dispersion across countries may not seem surprising as the mutual funds market conditions vary from nation to nation, Hortaçsu and Syverson (2003) find evidence of dispersion of fees even in narrowly defined categories such as index funds replicating the S&P 500 index. For instance, the authors study fees charged by 1,267 mutual funds operating in the International Equities sector. Funds in this sector charged an average annual fee of 225.5 basis points. The 90th to 10th percentile ratio is 3.2 while the 75th to 25th is 1.9. Even in a narrowly defined sector, such as S&P 500 index funds

¹The authors define 122 investment objectives based on fund category (e.g. bonds, bonds and cash, equities, money market, real state, etc.) as well as the region where the funds invest (e.g. Danish equities or Eurozone bonds) and the type of securities held (e.g. small cap stocks).

the authors find that in 2000 there were 85 funds in this category, charging an average fee of 97.1 basis points. For this sector the 90th to 10th percentile ratio is an outstanding 8.2 while the 75th to 25th is 3.1. These numbers certainly seem to be large, specially given all the information available for investors in the mutual fund market. The authors find evidence suggesting that this dispersion could be related to the existence of search costs, which would prevent investors from investing in the cheapest funds.

Overall, the evidence the evidence suggests that intermediaries such as mutual funds make little use of performance-based fees, although this type of fees is widely used in the hedge fund industry. Moreover, there is a considerable degree of fee dispersion in the mutual fund industry, even for narrowly defined categories.

2.1.2 Herding and Impact on Prices

There is substantial evidence regarding the existence of a certain degree of correlation in portfolio decisions amongst institutional investors who seem to make similar buying and selling decisions. This phenomenon has been called "herding". For USA pension funds Lakonishok et al (1992b) find weak evidence of funds buying or selling in herds. Grinblatt et al (1995) study the behavior of 155 mutual funds over the 1975 to 1984 period, finding evidence of momentum investment strategies. This is, funds tend to buy stocks with good recent performance while also selling stocks with poor recent performance (although in less degree). The authors also find evidence of herding examining stock prices. This finding could be explained by the use of momentum strategies. Nevertheless, the authors find that the level of herding, this is funds buying and selling the same stocks at the same time, is rather small. Wermers (1999) performs a comprehensive study of mutual fund behavior for virtually all mutual funds based on the USA which held equities from 1974 through 1994. The author finds evidence of low levels of herding consistent with those documented by Grinblatt et al (1995) and by Lakonishok et al (1992). Also, the evidence suggests that mutual funds are equally likely to herd as buyers or sellers of stocks. Additionally, the stocks that funds buy in herds seem to have higher abnormal returns that stocks that funds sell in herds. Finally, Wermers finds that most observed stock price adjustments following trading by herds appear to be permanent which favors the hypothesis that mutual funds herds speed the price adjustment process and are not destabilizing. This is interpreted as evidence supporting theories of herding based on private information on fundamentals (see Bikhchandani et al, 1992) as opposed to herding based on reputational concerns (see Scharfstein and Stein, 1990) and the works on reputational herding surveyed in the following Section).

Regarding evidence for other countries, Maturana and Walker (2002) find evidence of herd behavior in Chilean mutual funds for the 1990-1998 period. Walter and Moritz Weber (2006) find evidence suggesting the existence of moderate herding by German mutual funds in the 1998-2002 period. Wylie (2005) documents the existence of moderate herding for United Kingdom mutual funds covering from 1986 through 1993. Following the methodology developed by Lakonishok et al (1992b), Lobao and Serra (2002) study the Portuguese mutual funds market over 1998 through 2000 period. The authors find evidence of strong herding behavior in order of magnitude of 4 to 5 times stronger than herding documented in the USA and United Kingdom markets. Finally, Voronkova and Bohl (2005) study a sample of Polish pension funds from 1999 to 2001. They find stronger evidence of herding than for mature markets. The authors attribute this finding to more stringent investment regulation and high market concentration.

The works surveyed above focus mainly on finding evidence of herding by studying the behavior of stocks' prices rather than the decisions of individual funds. The works by Chevalier and Ellison (1999), Hong el al (2000) and Graham (1999) document how herding changes over the agents' careers². Chevalier and Ellison study the behavior of a mutual fund managers sample from 1992 to 1994. The authors find that the probability of being terminated due to bad performance is higher for younger managers. Also, they find that these managers tend to take on less unsystematic risk than older mangers and they also hold more conventional portfolios. Hong et al (2000) report similar findings using a sample of 8,421 security analysts producing earnings forecasts from 1983 to 1996. The authors find that more inexperienced analysts are likelier to be terminated if their forecasts differ too much from the consensus and that these analysts tend to make predictions closer to the consensus. These two studies proxy reputation by agents' age. Finally, Graham (1999) studies the relation between reputation and herding for investment newsletters. Contrary to Chevalier and Ellison and Hong et al, this author finds that investment newsletters with higher reputation tend to herd more.

Overall, there is evidence of herding from institutional investors, as well as analysts, although it doesn't seem to be too pervasive. Also, incentives to herd seem to change over time for individuals but there isn't a consensus on whether herding increases or not with agents' age or experience.

²This three studies cite the work by Scharfstein and Stein (1990) which explains how an agent's incentives to copy others' decisions instead of using his own information may be related to reputational concerns. This is, the degree of ability the agent is perceived to posses by the rest of the market. We discuss this and other related literature in the following Section.

2.1.3 Flows and Performance

Starting with Ippolito (1992), several works such as Gruber (1996), Chevalier and Ellison (1997), Sirri and Tufano (1998), Lynch and Musto (2003), and Olivier and Tay (2008) find evidence of an asymmetric relationship between past performance and net inflows in the mutual fund industry. For instance, Ippolito studies 143 mutual fund performance starting in 1965 and up until 1984. He finds that funds which perform better than the market by 100 basis points experience a growth in assets under management equal to 0.90% in the following year, while funds which under perform the market by 100 basis points suffer a decline of 0.35% in assets under management. Chevalier and Ellison (1997) use a mutual fund sample spanning from 1989 to 1994. The authors are able to derive the performanceflow relationships for young funds (funds that have operated for less than 2 years) and old funds (which have existed for more than 10 years). In the case of young funds, having a return 10 basis points above the market return means an average growth of 55% in assets under management, while having a return 10 basis points below the market means an expected reduction of less than 30% in fund size. For older funds, beating the market by 10 basis points leads to an expected increase of almost 15% in assets under management while trailing the market by 10 basis points means an expected decrease of little more than 10%in assets under management. Therefore, the performance-flow relationship appears to be convex for all funds and gets flatter for older funds. Sirri and Tufano (1998) use mutual fund data from 1971 through 1990. They find that while performance is positively associated with flows, this relationship is significant only for good performers, while it is statistically weak for the lower quintiles. Finally, the evidence provided by Gruber (1996), Lynch and Musto (2003) and Olivier and Tay (2008) is also consistent with the existence of a convex relation between funds performance and flows.

This findings imply that fund managers face asymmetric incentives since even if their profits are a fee of assets under management that doesn't change over time, they can raise their earnings if the size of the portfolio they manage grows. Given the response of investors to mutual fund performance it is possible that managers take on excessive risk in an attempt to outperform the market since succeeding would greatly increase their profits, while failing would reduce their gains by a smaller amount.

Regarding the value of active investment by mutual funds, Gruber (1996) finds evidence suggesting that average mutual funds performance between 1985 and 1994 is 65 basis points below that of market indexes. However, Wermers (2000) finds evidence supporting the theory that active investment by mutual funds is valuable. The author decomposes the returns and costs of mutual funds into: stock-picking skills; stocks holdings; trade related costs of stock-picking; fund expenses and management fees; and differences between gross stock portfolio returns and net fund returns that are due to holdings of cash and bonds. Wermers finds that mutual funds hold stocks that outperform the market index by 130 basis points, which amounts to the sum of their expenses rate and transaction costs. Of this 130 basis points 71 are due to stock-picking skills, while 55 to 60 are explained by stock holdings. For the Chilean mutual funds market Maturana and Walker (1999) report that equity mutual funds underperformed the authors' benchmark by 80 basis points between 1990 and 1997. These works are meant to be indicative of this topic, but do not constitute and exhaustive list. For further papers on this area see the references on Gruber (1996) and Wermers (2000). For more recent papers see Lo (2007), who suggests the existence of value in active investment. Also, see French (2008) who presents evidence supporting the opposite view.

While the existing evidence suggests that there is some value in active management, some authors have found evidence of little persistence in mutual fund performance. Cuthbertson et al (2006) survey the extensive literature that studies mutual fund performance and persistence. The existing evidence suggests that there is some persistence amongst the top decile of USA funds. Using a risk-adjusted gross returns measure persistence may last up to four years for a small number of growth funds and for up to one year when the top decile is formed using all funds categories. Also, there is strong evidence that poor performance persists across deciles.

2.1.4 Trading Volume

Dow and Gorton (1997) document that there appears to be a consensus in that the trading volume by institutional investors in inexplicably high, although it's difficult to rigorously prove this assertion as there is a lack of models that predict just how big trading volume should be. The authors gather data on this topic for foreign exchange market and for the New York Stock Exchange. For the first market, daily trading volume of foreign exchange transactions in all currencies in 1992 was US\$880 billion. Meanwhile, the total value of annual world trade in 1992 was \$3,646 billion which means that 24% of the annual trade was traded each day in this market. On the other hand, turnover for the NYSE was 49% in 1992. Moreover, Glaser and Weber (2007) report that annualized monthly turnover on the NYSE was roughly 100% in 2004. During this year 367,098,489,000 shares were



Figure 2.1: NYSE Turnover Rate

Source: NYSE website

traded in this market. The figure below shows the evolution of the NYSE turnover rate since 1944 until 2003. The existing turnover rate in this market seems to be large given the existing consensus that a 49% turnover rate was high back in 1992.

2.1.5 Summary

We have presented studies documenting several stylized facts in the delegated portfolio management market. We summarize these facts in the following list.

- Fact 1 Performance based fees and bonuses are not widely used (at least in the mutual fund industry).
- Fact 2 There is considerable dispersion in mutual fund fees, even for narrowly defined categories.

- Fact 3 There is a certain degree of correlation between institutional investors' portfolio decisions.
- Fact 4 The degree of correlation changes with the fund managers' age.
- Fact 5 There seems to be some value -although with low persistence- in active investment management.
- Fact 6 The use of asymmetric contracts has been restricted by regulators in the USA.
- Fact 7 There seems to be an excessive volume of financial transactions.
- Fact 8 There is a convex relation between institutional investors' performance and inflows. This relationship turns flatter for older funds.

2.2 The Delegated Portfolio Management Problem Literature

The literature studying the delegated portfolio management problem is vast and deals with many issues present in this agency problem. Some of these works attempt to rationalize one or several of the stylized facts discussed in the previous section, while others attempt to derive closed form solutions for optimal contracts between investors and intermediaries. A selective review of these and other works is made in the Appendix where we also select and discuss works that belong to the reputation and the herding literature. Some authors such as Heinkel and Stoughton (1994), Arora and Ou-Yang (2001) and Farnsworth (2003) argue that if managers are able to build a reputation this may help to lessen the problems caused by moral hazard. In particular, by being diligent in making investment decisions managers may favorable influence investors' opinions about their ability which may allow them to charge higher fees or manage larger portfolios in the future. On the other hand, some authors attempt to rationalize the finding that sometimes agents' decisions seem to be correlated; this is, that agents exhibit herd behavior. In particular, Scharfstein and Stein (1990) suggest that one possible reason for managers to herd is that they could opt to ignore private information and follow others if investors doubt more about their skill when they make a bad decision that is different from others' as opposed to a bad decision that is equal to others'. This phenomenon has been called reputational herding and it has been explored by Avery and Chevalier (1999), Graham (1999) and Ottaviani and Sørensen (2006). Given the goals of our work, we discuss the reputation and herding literature in the Appendix and in this section we focus on the reputational herding literature. We also review a strand of the reputational literature that focuses on long-run equilibria in which agents have incentives to invest in their reputation. For this, we discuss the works by Hölmstrom (1999), Mailath and Samuelson (1998), (2001), Cripps, Mailath and Samuelson (2004) and Vial (2008) as they will be methodologically important for our work. Throughout the literature review we will use the terms principals and investors interchangeably and the same applies to the terms agents, financial intermediaries and managers.

2.2.1 Reputational Herding Literature

Scharfstein and Stein (1990)

Scharfstein and Stein suggest that the presence of reputation in the delegated portfolio management market could cause phenomena like herd behavior, distorting the investment decisions of agents. The basic argument is that when there is uncertainty regarding a FI's skill he may choose to ignore his private information when taking investment decisions and instead, he would imitate the decisions of other managers if investors doubt more the skill of a manager when he makes a bad choice, that is different from the one made by the rest of agents, as opposed to a bad choice that is similar to the one made by other agents. In fact, this is one of the explanations suggested by Dornbusch et al (2000) for the contagion observed during the Asian crisis: during this period there was and exit of mutual funds from the majority of emerging markets, even though these markets' fundamentals did not justify such a decision.

In this model there is uncertainty regarding the managers' true type: neither investors nor managers know if they are skilled or unskilled. Managers must decide between investing or not in a new technology for a firm. It's possible that the investment turns out to be beneficial for the firm, although it may result useless too. Once the investment decisions are made and their results are known, agents update their beliefs about the managers' type given the quality of the investment decision they made and also based on how different the decision is relative to the decision made by other managers. The managers' remuneration is assumed to be a positive function of his reputation.

The authors make a critical assumption about the information received by managers. An unskilled manager receives no useful information about the convenience of adopting the new technology. On the other hand, skilled managers receive an informative signal useful to predict the investment's payoff. This information is correlated (e.g. skilled managers receive a noisy observation about the new technology's real value). Therefore, if a manager makes a wrong investment decision and his decision is different from other managers, the principal will be inclined to think that he is unskilled (i.e. he received a signal
uncorrelated with the rest of managers which made him take a wrong decision that is different to the one taken by the rest of managers). On the other hand, if the agent takes a bad investment decision but he acts like the rest of managers, the principal will think that he is skilled (i.e. he received a signal correlated with that of other agents and therefore took a wrong investment decision) but was unlucky.

This beliefs updating process will cause managers to ignore private information (which is available to them free of charge) useful to predict the investment's returns and rather choose to copy the investment decisions of other agents since doing this enhances their reputation. The authors point out that herd behavior could be partially avoided if: managers' utility functions include the profits of the firms they manage; they have limited liability; their remuneration depends on their relative rather than absolute perceived ability; or alternative definitions of ability are used by investors.

It is important to emphasize that the work by Scharfstein and Stein studies the behavior of firm's managers who must decide sequentially between undertaking an investment project or not³, so it is possible that their conclusions are not entirely valid in a DPMP context. In particular there are two important considerations in this context. Namely, the investment opportunities prices are assumed to be exogenous by the authors, while in a portfolio management setup this isn't necessarily the case (particularly for large investors). Also, portfolio managers may not be able to perfectly observe and copy other managers' decisions as this implies knowing the share of the portfolio invested in hundreds of assets which is a difficult task to achieve in real time. Regulation in the USA and Chile requires

 $^{^{3}}$ Zwiebel (1995) develops a similar model, which studies the relationship between reputation and herd behavior among firm's managers who must decide whether to adopt an innovation with uncertain results.

some intermediaries such as mutual fund managers to report their portfolios' composition, but this is done with a lag of three months in the USA and several weeks in Chile.

Avery and Chevalier (1999)

This work builds on Scharfstein and Stein's model in an attempt to rationalize the empirical finding that herding and reputation are negatively related (see Chevalier and Ellison, 1999 and Hong et al, 2000). Unlike Scharfstein and Stein, the authors assume that managers learn about their abilities as they make investment decisions. In this case, once a manager learns enough about his type the herding equilibrium will give way to a signaling equilibrium in which managers who have learned enough about their type will take a different decision from the rest of managers in order to show their self-confidence to the rest of agents.

The reason for this is that if a manager receives a private signal suggesting to deviate, i.e. making a different decision from other managers, this will be taken by the rest of agents as a signal that he is a promising manager (the authors argue that this interpretation is the most intuitive one for out-of-herding-equilibrium beliefs). On the other hand, by herding the manager can hide the fact that he received a different signal and therefore either he or one of the other managers is unskilled. At the beginning of the manager's career he knows little about his own type, so the incentives to herd will be stronger. However, as time goes by he will come to be more confident about his abilities (given that he is indeed skilled) and he will decide to take a contrarian behavior.

It should be emphasized that both the herding and signaling equilibria are inefficient in this model, since in both cases managers disregard private information and follow others decisions in the former equilibrium or take the opposite decision in the latter.

Graham (1999)

Graham further builds on the Scharfstein and Stein' model, studying it's comparative statics properties and empirical predictions. The author studies pure strategy Bayesian Nash equilibria in a sequential decision setup. Emphasis is placed on equilibria in which the first manager to make his investment decision chooses according to the private signal received, investing if he receives a bad signal and doing nothing if he receives a bad signal. The second manager, on the other hand, dismisses his private information when this suggests to make a different decision from the first manager. In other words, the second manager herds due to reputational concerns. The author demonstrates that there are parameter values for which these type of equilibria exist.

The model has four key parameters. First, the intermediaries' ability, which is measured as the probability of receiving a good (bad) signal given that the investment opportunity is good (bad) and given that the manager is skilled. The second parameter is the informative signal correlation, which is assumed to be strictly positive.⁴ Third is the managers' initial reputation; and last is the strength or prior information or the ex-ante (unconditional) probability that the investment opportunity is attractive.

The author studies how both managers' incentives are affected by changes in the model's parameters. Remarkably the effects are different for both managers, even though they are both ex-ante alike and differ only in the order in which they decide. For the first manager ,incentives to make efficient use of his private information are increasing in

⁴As Scharfstein and Stein (1990) show, for reputational herding to arise it is crucial that skilled agents' information is at least partially correlated. This is the reason that explains a smaller fall in reputation for managers or intermediaries that make bad investment decisions that are identical to that of other managers. Scharfstein and Stein assume that the skilled managers' information is perfectly correlated.

his ability since as his information is more accurate it is more likely that the investment outcome will be consistent with his private information. Also, if the correlation degree is higher this manager's incentive to use his private information increase, since it's likelier that the second manager will not make a different choice thus affecting his reputation. If the manager's initial reputation is higher his incentives to use his private information increase. This is so because the manager doesn't know his true type but if initial reputation is higher he will be more confident that the signal received is indeed informative about the investment opportunity's true value. Finally the incentives to use private information increase with the strength of the unconditional probability that the investment is attractive if this is consistent with the managers information and are lower otherwise.

For the second manager, who observes the first manager's decision before making his own, the incentives to use his private information also increase with his ability; with the informative signal's correlation; and with the strength of the unconditional probability that the investment is attractive given that this is consistent with the managers information. However, the incentives to herd increase with the manager's initial reputation. The reason for this is that this parameter determines the manager's profits if he herds, since in equilibrium the rest of agents will be aware of his behavior and therefore his investment decision won't affect his initial reputation. If initial reputation is low the manager's profits will be low in case he doesn't make use of his private information. However, when this parameter is high the manager will prefer to abstain form making different investment decisions, as this may cause him to have a lower reputation therefore losing high profits. Using data for investment news letters Graham finds evidence consistent with the model's predictions. This work makes an important contribution as it makes a prediction regarding the relationship between a managers' reputation and his incentives to herd. We emphasize how this prediction is actually the opposite of that provided by Avery and Chevalier (1999).

Dasgupta and Prat (2005)

This work's objective is to study the dynamics of assets' prices in a delegated portfolio management context in a setup in which managers care about their reputation.⁵. Specifically, the authors study an economy with multiple discrete time periods. There is an asset whose final liquidation value is unknown. There is also a large number of fund managers and noise traders or agents that take random decisions due to exogenous liquidity shocks. Every period a manager or noise trader is randomly selected to submit orders to a market maker to either buy or sell one unit of the asset. This market maker adjusts the price using all available public information. A manager may be one of two types. A skilled manager receives private information about the asset's final value, while an unskilled manager receives less accurate information. Neither the manager nor investors are aware of the former's type. After several periods have passed the asset's true value is known. Investors update their beliefs about managers' abilities and each manager is paid. The remuneration scheme is assumed to be exogenous and is given by a weighted average between the trading profits made by the manager and a reputational payoff.

Under this setup Dasgupta and Prat show that prices never converge to the asset's true liquidation value. The reason for this is that as more information about the true value is gathered, incentives appear that make managers stop using their private information.

⁵Other works that study asset pricing in a delegated portofolio managment context are Cuoco and Kaniel (2007) and Goldman and Slezak (2003). However, this authors do not explore the subject of reputational concerns.

First, as prices become more precise, there are less opportunities for managers to profit from trading. Also, when prices are close to their true value, managers who receive a signal that contradicts current beliefs (e.g. getting a good signal when the asset's price is very low) may choose to ignore their private information because it they use it and the investment decision turns out to be bad the reputational cost will be high. This leads to a situation in which private information stops flowing in to the market and the asset's true value in never fully known. However, as long as the asset's price is not too low or too high, it is possible that managers trade on their private information, since there is no reputational cost associated to trading for intermediate asset prices. This findings are used to make predictions about the long term return for assets: if an asset has been persistently bought (sold) by managers then it is likely to experience negative (positive) corrections when uncertainty is resolved, leading to low (high) long-term returns. The authors report that these predictions are consistent with existing empirical evidence.

Finally, it is shown that the model's results are robust when signals are continuous rather than binary and if managers care about their relative reputation. However, the mispricing doesn't survive if managers are aware of their type.

Ottaviani and Sørensen (2006)

Ottaviani and Sørensen study the incentives faced by an expert who cares about his reputation when he is evaluated on the basis of the advice given and the realized state of the world. The authors show that in equilibrium no more than two messages are reported by the expert, even though he has a continuum of possible messages to report. This means that the expert will only be able to truthfully report the direction of his private information, e.g. the state will be "good" or it will be "bad", but he is unable to give information regarding how good or bad the state is. Also, in the long run there will be incomplete learning and herding.

In this model there is an expert and an evaluator. The expert's ability is unknown even by himself and he receives a private signal that is informative regarding the true state of the world. The authors generalize the commonly used binary signal (see for example Scharfstein and Stein, 1990 and Avery and Chevalier, 1999). The expert's signal is assume to be a multiplicative linear experiment, this is, a mixture between an informative and uninformative experiment. The expert's ability determines the probability that his signal is from the informative experiment. This allows to consider a continuum of states, signals and ability types in an analytically tractable way. Once the signal is observed the expert sends a message, chosen as to maximize his posterior reputation since it is assumed, as in Scharfstein and Stein (1990), that profits are an increasing function of reputation. Once the expert sends the message and the true state of the world is revealed, the evaluator updates his prior beliefs about the expert's ability and the expert is rewarded.

Given the model's structure the authors show that if the evaluator believes that the expert is truthfully reporting his signal the latter will have incentives to lie. In particular, if prior beliefs about the true state are biased towards a bad realization, then experts will want to report messages that are ex-ante likely to be similar to the prior. This is due to the fact that the evaluator will lower his assessment of the expert's reputation if his reported message is different from the realized state. Therefore, regardless of the signal received, the expert's reported message will be biased downwards. For similar reasons, when prior beliefs about the true state are biased upwards, the expert will be over optimistic with his report. On the other hand, if the prior beliefs are in a middle range any message reported is likely to result in a positive or negative impact on the expert's ex-post evaluation. Therefore, the expert will bias his report upwards or downwards depending on the message observed.

Using the previous results the authors show that it is possible that no informative equilibria exist if priors about the state are too biased upwards or downwards. Moreover, when priors are not too biased informative equilibria exist but this will be binary. Moreover, the lower message sent by experts will be negative and the higher message will be positive. This means that the expert will be able to report the direction of his information, but not the intensity. The authors also find that there are cases in which experts with better initial reputation report messages that are less informative than experts with lower initial reputation. In this case it would be difficult for monotonic reputational profits to exist in a dynamic version of the model with more than two periods. Regarding the robustness of their results, the authors argue that all informative equilibria continue to be binary even if experts profits are not only affected by their reputation but also through their decisions, provided that reputational payoffs are sufficiently important to experts. However, if experts can learn about their own ability there will always be informational equilibria.

Finally, the model is extended to a multiple period setup in which different experts sequentially give their report about the same state of the world. The ordering is exogenous. After all experts give their messages the evaluator observes the true state and updates reputations. Unlike Scharfstein and Stein (1990), Ottaviani and Sørensen assume that experts' private signals are independent, which means that each expert message carries information about his ability but not about the ability of others. However, the authors show that once the beliefs about the true state become sufficiently concentrated experts are no longer informative and learning stops before the true state is revealed.

2.2.2 Non-Finance Reputation Literature

There is a group of works that study the effects of reputation, although not in a delegated portfolio management context. The works by Hölmstrom (1999), Mailath and Samuelson (1998, 2001), Cripps, Mailath and Samuelson (2004) and Vial (2008) differ from the previously discussed papers because they recognize that investing and building a reputation is a slow process, and if there isn't some source of permanent uncertainty about the FI's characteristics the existence of a long-run equilibrium with investment in reputation wouldn't be feasible. The reason for this is that investors would be eventually convinced that the FI is competent, so he would lose interest in making effort to maintain his reputation, because investors will attribute a bad outcome to bad luck, rather than to the intermediary being negligent. This characteristic is not present in the existing models of reputation in a DPMP context reviewed in the Appendix, such as Heinkel and Stoughton (1994) and Farnsworth (2003).

Hölmstrom (1999)

Hölmstrom builds upon the argument of Fama (1980), who claims that in a relationship in which there is moral hazard, time should have a beneficial impact since it allows to have more information about agents' behavior, thus allowing the principal to make more accurate inferences about agents' actions. This work's contribution lays in that it studies under what circumstances the possibility of investing in reputation will have a long term impact on agents' behavior.

The author models an economy in which a manager sells labor services to a principal. The manager has the possibility of making costly effort to help obtain higher output. However, it is not possible to write contracts contingent on results. Therefore, in a one period context it won't be possible to induce the manager to make effort. If there are multiple time periods the manager could be willing to make effort if his present performance transmits information about future performance thus leading to higher wages being paid by the principal.

Since the agent's true ability is unknown, the principal will infer this variable through his observation of each period's output. Hölmstrom shows that as time goes by, in the limit, the agent's type will be fully known. In the meantime, the agent will make a lot of effort (more than he would make in a first base case without moral hazard) as he will try to make a good impression in order to be paid higher wages. However, as his ability is revealed his actions will have little impact on the principal's beliefs, so he will make no effort and his labor supply will fall below that of a first base case. Therefore, in the long run there will be no role for the possibility of investing in reputation.

However, if there is permanent uncertainty about the agent's true ability, then his actions will always have an effect on his reputation, thus a long run equilibrium with investment in reputation will be feasible. Hölmstrom assumes that the agent's ability is not fixed, but rather changes over time. This modification allows him to show that there will be a higher level of effort in the long run compared to a case where ability is eventually fully known. Moreover, if the agent doesn't discount utility from future periods, this effort level is efficient, but if the agent has a discount factor of less than one, the long-run effort level will be lower than the first-best level.

Hölmstrom's comparative statics results show that reputation will work more effectively if there is greater uncertainty about the agent's ability and if the observations on results are less noisy. In these two cases learning will be faster and investing in reputation will be more profitable. Also, the steady-state equilibrium will be stable, and will feature an overinvestment in labor supply for earlier periods since investing in reputation is more attractive when the principal's information about ability is still obscure. Since early labor supply levels are higher than the first-base case this transition to the steady state will be inefficient.

Finally, even though the possibility of investing in reputation is beneficial in terms of aligning incentives, the author points out that if there is little alignment between higher reputation and higher outputs, the fact that agents' care about their reputation may introduce inefficiencies, since they could take actions to improve the principal's opinion about their ability but this could be detrimental for output. One way in which this problem could be solved is by giving the agent some participation on output.

Mailath and Samuelson (1998)

Mailath and Samuelson study a model in which a firm's reputation is gradually built, can be managed, and slowly disappears if it isn't maintained. There is a continuum of consumers buying an experience good from the firm, whose type is unknown. There is a moral hazard problem since a skilled or competent firm can incur in costly effort in order to raise the probability that the consumers' receive high utility from buying the good. Therefore, outcomes act as signals that help consumers infer whether the firm is making effort or not; however, they are imperfect signals since it's possible that a firm makes high effort but bad luck causes the consumer to receive low utility. Also, each consumer's experience is unique and unobservable to other consumers. This means that this is a model of imperfect private monitoring. The authors focus on this kind of models instead on those of perfect public monitoring (such as Fudenberg et al, 1990) and imperfect public monitoring (such as Abreu et al, 1990 and Fudenberg and Levine, 1992) because in this games the means by which incentives are aligned are trigger strategies. This is, since there is public monitoring, coordination between consumers is feasible. As a result, equilibria exist in which consumers initially believe that the firm is competent and makes effort and the will firm behave accordingly if making effort is not too costly and the firm is patient enough. Any deviation on part of the firm (intentional or not) triggers a punishment from all consumers, thus aligning incentives. This means that reputations spring to life and end suddenly. which prevents the study of a situation in which firms gradually invest in reputation and this asset's value slowly changes over time. Also, in previous reputation literature like Kreps et al (1982) and Kreps and Wilson (1982) there is a good or Stackelberg type of firm who always chooses high effort and ordinary firms, who may choose to make effort in order to make consumers believe that the firm is good. Again, the existence of equilibria with effort by ordinary firms relays on the use of trigger strategies which means that reputations spring to life and may steady decline later on. Mailath and Samuelson make the assumption that there is an additional type of firm: unskilled or inept ones, who never make effort. Now an ordinary firm makes effort in order to make consumers believe that they are not bad firms, thus they must gradually build and then manage their reputation.

An additional key ingredient in this model is the introduction of a permanent source of uncertainty about the firm's characteristics. In this sense, this work is related to Hölmstrom (1999). However, Mailath and Samuelson assume the existence of a continual possibility that an existing firm might be replaced by a new firm (consumers cannot observe when this replacement takes place). Also, Hölmstrom's model is one of imperfect public monitoring where neither the market nor the agent himself knows his true ability. This means that the agent's effort cannot depend on his talent, so the agent's evaluation about the profitability of effort reflects only market beliefs and he doesn't think that his effort will affect the average market beliefs about his ability.

As stated before, the authors study an economy with a continuum of infinitely lived consumers and a single firm. The consumers buy an experience good from the firm and may receive a high or low utility from this purchase. Each consumer's experience is private information. The firm may be competent or inept. A competent firm can choose to make costly effort in order to raise the probability of consumers obtaining a good outcome. The firm knows its true type, but consumers can only infer it from the history of private results obtained when buying the good. Each period there is an exogenous probability that the current owner of the firm is replaced and the new owner's type will be competent with some positive exogenous probability. Consumers do not observe if a replacement occurred. This means that they will never be completely sure that a firm is competent or incompetent, so there will always be incentives for the firm to make effort in order to invest in reputation.

Like Hölmstrom (1999), the authors show that if the replacement probability is

zero, then the only possible long-run equilibrium features the firm always making low effort. The reason for this is that once consumers are convinced that the firm is competent, the realization of bad outcomes will be attributed to bad luck rather than to negligence. This induces the firm to stop making effort since it's reputation won't be substantially affected, thus destroying the reputational equilibrium.

With a positive replacement probability bad (good) outcomes will always have a negative (positive) impact on firm's reputation and the authors show that if the cost of making effort is not too high there will be an equilibrium in which the competent firms always choose to make effort. The authors then extend their analysis to consider the existence of multiple effort levels. In particular, they study how the equilibrium properties change when firms can make intermediate effort levels, which improve their reputation but are inefficiently low, and excessive effort levels, which have high impact on their reputation but are inefficiently high. The possibility of the firm making high effort is negatively affected by the fact that this choice implies a resource expenditure now but only future rewards, since in this models the firm's actions affect its profits only by changing consumers' perception about its type. Therefore, if a firm is to choose an inefficient effort level it's likelier that it will involve too little effort.

Cripps, Mailath and Samuelson (2004)

This work provides further results showing that in long-run equilibria with imperfect public monitoring it's impossible for players to maintain a permanent reputation unless there is some mechanism by which the uncertainty about types is continually replenished, as in Hölmstrom (1999) and Mailath and Samuelson (1998). The authors argue that the assumption of imperfect public monitoring is crucial for their results. The reason for this is that if monitoring was perfect then it is not difficult to construct equilibria that exhibit permanent reputations. In this case, any deviation from the commitment strategy reveals the type of the deflector and triggers a punishment, which prevents the deviation from occurring. However, under imperfect monitoring, any deviation by the long run player doesn't reveal his type nor triggers a punishment. Rather, as beliefs about the long-run player type converge over time, this guarantees that any deviation will have only small effects on the short-lived players' beliefs. Therefore, there won't be a cost from deviating for the long-run player and the final effect of this situation will be to eliminate uncertainty from the equilibrium, thus revealing the long-run player's true type. The authors prove this result under the use of simple Markov strategies and under more complicated commitment types.

Mailath and Samuelson (2001)

This work is similar to Mailath and Samuelson (1998). There is, however, a key difference since the authors now turn to study the properties of a market for reputations. Each period there is an exogenous probability that the firm exits the market. While the probability that the new firm is competent is exogenous as in Mailath and Samuelson (1998) in this model potential entrants include both competent and incompetent firms who compete to buy the right to use the existing firm's name and reputation. Therefore the authors are able to study what kind of firms will buy which kind of reputations.

In order to have a tractable model the authors assume that consumers' experiences after buying the good from the firm are observed by all participants. This means that the model is one of public imperfect monitoring. Since this introduces the existence of multiple equilibria, such as ones featuring trigger strategies which may depend on the history of consumers' results and where reputations are not gradually built, the authors require that all behavior is Markov, guaranteeing that all strategies are based only on history's length and the current value of state variables. The rest of the model maintains the assumptions and structure of Mailath and Samuelson (1998).

The authors show that the result of no reputational equilibrium in the long run in the absence of positive replacement probability continues to hold with public monitoring. When there is a positive replacement probability reputational equilibria are feasible. The authors assume that, when a firm exogenously abandons the market, the owner sells the name to a new firm. The potential entrants include both competent and incompetent firms, where the former type is relatively scarce and has higher opportunity costs than the latter type. The exiting firm sells the right to use its name using a second price auction, which guarantees that the firm's name is sold to the entrant with the highest valuation. If the cost of making effort and the probability that there is a competent firm with no opportunity costs amongst the potential entrants are small enough then the existence of a reputational equilibrium is feasible. That the cost of making effort shouldn't too high has a clear intuition. On the other hand, the requisite that there is a small probability that of one of the potential entrants is competent and has no opportunity cost is necessary to avoid situations in which, for some values of the current firm's reputation, consumers and potential entrants coordinate on an equilibrium in which entrants are likely (unlikely) to be competent because the value of a competent firm is high (low), only because consumers expect entrants to be competent (incompetent). This would render meaningless the notion that higher

reputations are good and not just a product of the coordination between consumers' and firm's beliefs.

With regards to what kind of firms buy which kind of names in a reputational equilibrium, the authors find that average reputations are likelier to be bought by competent firms, while incompetent firms are likelier to buy very good or very bad reputations. The intuition behind this result is that competent firms find it too expensive to build a good name "from scratch" and, while getting a good name is attractive, they find it more convenient to buy a cheaper, more average reputation, and then make effort to improve it. On the other hand, inept firms won't value average reputations too much since they don't have the means to improve them. Very good names, however, are much more attractive since they can guarantee high profits while slowly depleting the value of reputation.

Finally, the authors discuss the implications of allowing firms to announce consumers that a replacement has occurred. If this announcement is costless, it will be ignored by consumers, since they know that both competent and incompetent firms with low reputations will be interested in announcing a change. On the other hand, if the announcement is costly (e.g. the firm remodels or introduces a limited-time offer for costumers) then it's possible that it modifies consumers' beliefs. Specifically, if the firm can choose how much to spend on sending a costly signal after the consumers' utility is received, but before the replacement is realized, then an equilibrium exists in which costly signals will be sent only by competent firms. In this equilibrium competent firms always make effort. However, if bad luck causes the competent firm's reputation to fall below some critical value, then the signal is sent. This signal convinces consumers that the firm has been replaced by a competent firm and thus they adjust their beliefs accordingly. Incompetent firms don't make effort and eventually end up having the lowest reputation possible, but do not send a signal unless they are eventually replaced by a competent firm.

Vial (2008)

This work studies the properties of reputational equilibria in an imperfect public monitoring context using a similar setup as Mailath and Samuelson (1998), (2001). A key difference between this and previous works is that instead of the firm being a monopoly, there is a continuum of firms. This raises the issue of whether the existence of a competitive equilibrium can be reconciled with the fact that firms investing in reputation should be able to charge higher prices for their goods or services. The author addresses this question and also studies the properties of the distribution of firms reputations finding that in the long run the aggregate distribution for reputations is constant even though the reputation of each particular firm changes each period. This makes it possible to study the steady-state equilibrium of the model, where it's feasible to analyze which consumers will buy from which firms and how large is the improvement in profits associates with having a higher reputation.⁶

In this model there is a continuum of short-lived consumers and a continuum of long-lived firms, each capable of serving at most one consumer. Firms can be either competent or inept, with the former type being able to make effort in order to improve the odds of its consumer having a good experience, while the latter doesn't have this option. If a firm makes effort it ensures that the quality of the good provided is high. This is observable

⁶Hörner (2002) also studies reputational equilibria with many firms and consumers. However, he uses a different framework in which all firms charge identical prices and consumers stop buying from a firm after obtaining a poor result. In order to avoid losing consumers, competent firms make effort. Under this conditions all firms share the same reputation.

only to the consumer. However, at the end of the period a public signal is observed by all agents. The chances of a firm getting a high signal are increased if effort was made. As in Mailath and Samuelson (1998) there is an exogenous replacement probability for firms, which guarantees the existence of permanent uncertainty about the firms' characteristics so long run reputational equilibria are feasible.

A key element of the model is that consumers' willingness to pay for a firm's good is increasing in the firm's reputation, if they conjecture that such firms make effort. The author points out the existence of a low quality equilibrium in which no firm makes effort and consumers adjust their believes accordingly. Therefore, reputations are irrelevant since competent firms offer the same quality as incompetent ones. This means that all firms charge the same price and thus there are no incentives to invest in reputation. However, if the cost of making effort is bounded from above, there is also an equilibrium in which all competent firms make effort. In this case Vial proves the existence of a steady state distribution of reputations for firms. This is, even though each firm's reputations changes over time, improving after delivering a high-quality good and declining after bad results, the distribution of aggregate reputation for firms, which evolves deterministically due to the continuum assumption, is invariant. This makes it possible to study a steady-state equilibrium, where prices and assignments are independent of time. In particular, since the quality of the good is appreciated by consumers, an interior solution to their decision problems requires prices to be increasing in reputation. This provides firms with the incentives to invest in reputation. Moreover, if consumers are heterogenous in wealth and a personal attribute which negatively affects the costs of providing the good, then there will be stratification by wealth and personal attributes. More precisely, holding the personal attribute constant, richer consumers will be served by firms with higher reputations. Also, holding wealth constant, consumers with higher endowment of the personal attribute will be served by firms with higher reputations. Vial applies this findings to the schooling markets, where there is evidence of the existence of stratification by wealth and ability (if a student is more able then it's cheaper for the school to educate him).

2.2.3 Summary

As we have discussed, one of the reasons for intermediaries to herd that has received attention in the literature is that of reputational concerns. The works by Scharfstein and Stein (1990), Avery and Chevalier (1999), Graham (1999) and Ottaviani and Sørensen (2006) show how intermediaries worried about their reputation may herd instead of using their private information. In fact, Dornbusch et al (2000) suggest that this could be one of the contagion mechanisms that operated during the Asian crisis. This view contrasts with that of Heinkel and Stoughton (1994), Chemmanur and Fulghieri (1994) and Farnsworth (2003), who argue that the presence of implicit incentives provided by reputation may alleviate the inefficiencies caused by informational asymmetries even without the use of bonus of performance-based fees (we discuss these three papers in the Appendix). Also, the predictions about the relationship between reputation and incentives to herd are mixed; Avery and Chevalier (1999) predict a negative relationship while Graham (1999) makes the opposite prediction. There is also mixed evidence with Chevalier and Ellison (1999) and Hong et al (2000) validating the prediction by Avery and Chevalier (1999) and Graham (1999) presenting evidence supporting his own predictions. Given the existent lack of consensus regarding the effects of the possibility of investing in reputation in a delegated portfolio management context, we make a contribution by studying the relationship between reputation and herding in such a context, recognizing that investing in reputation is a slow process that takes place over several periods and that, absent some source of permanent uncertainty about the intermediaries' characteristics, steady-state reputational equilibria cannot exist. We thus follow the methodology developed by Mailath and Samuelson (1998), (2001) and Vial (2008), which hasn't been applied before in a delegated portfolio management context with herding.

CHAPTER 3

The Relation Between Reputation and Herding in a Delegated Portfolio Management Problem Context

The fact that the delegated portfolio management remuneration schemes -at least in the mutual funds case- tend to be rather simple and do not exhibit sophisticated properties suggested by the literature leads us to explore the possibility that reputation building by financial intermediaries plays a key role as an incentive-aligning device that substitutes for more sophisticated remuneration schemes.

Also, we seek to shed light on the effects of reputation on agency problems in the existing delegated portfolio management problem literature. In this sense this work makes a contribution by studying the relationship between reputation and herd behavior, applying the methodology of reputation models such as Mailath and Samuelson (1998) and to Vial (2008). Our work is also related with those of Heinkel and Stoughton (1994), Arora and Ou-Yang (2001) and Farnsworth (2003) with regards to their basic argument that views reputation as an implicit incentive that can help to align incentives. In addition, like Scharfstein and Stein (1990), Avery and Chevalier (1999) and Graham (1999) we seek to unite the reputation and herd behavior literature, but in a delegated portfolio management context.

In building our model we improve over some of the limitations of the existent

literature, which we discuss in the Appendix. For example, Arora and Ou-Yang (2001) assume the existence of a linear relationship between the agent's future revenue and his present performance. While this may be consistent with the existence of reputation as a market mechanism to align incentives, it may also be product of pre-specified contracts between investors and intermediaries. Moreover, investors beliefs are not explicitly modeled. On the other hand, the work by Heinkel and Stoughton (1994) does model investors' beliefs but makes extreme assumptions about how the negotiation power shifts form investors to intermediaries. Also, by modeling a two-period economy they overlook the steady state issues studied by Hölmstrom (1999) and Mailath and Samuelson (1998) and (2001). The work by Farnsworth (2003) makes exogenous assumptions about investors' pre-commitment to delegate more wealth to intermediaries' if they make good investment decisions and, like Heinkel and Stoughton, doesn't study how the presence of permanent uncertainty about intermediaries' types affects the strategies and nature of the long-run reputational equilibria.

Like most of the reputational literature, we will refer to a financial intermediary's *reputation* to the probability assigned by agents (investors and possibly other intermediaries) to the possibility that this intermediary is skilled¹. Furthermore, we will state that an intermediary presents *herd behavior* if he prefers to imitate the decisions of others instead of obtaining private information to make his investment decisions. We emphasize that in this sense our definition of herding is different from the traditional reputational herding literature such as Scharfstein and Stein (1990), Avery and Chevalier (1999) and Graham (1999) and is closer to the definition used by Calvo and Mendoza (2000), because in the former agents imitate the decisions of others even though they have free access to private

¹Later we will specify the characteristics of a skilled intermediary.

information.

We predict the existence of a negative relationship between reputation and herding. This prediction is also made by Avery and Chevalier (1999). However, in their case an agent with good reputation not only doesn't herd, but chooses a contrarian strategy, ignoring his private information and making the opposite decision from other agents in order to signal to principals that he is skilled. Of course, this behavior is inefficient from the investors' point of view. The work by Graham (1999) makes the opposite prediction: as the initial reputation of agents improves they will herd more because they want to avoid a large drop in profits associated with a fall in reputation, which in this model occurs if an agent's decision is different from those of other agents. However, in a long-run equilibrium the initial reputation of an intermediary may be of limited importance in terms of determining his reputation several periods ahead. Therefore, this work doesn't focus on studying how an agent's incentives to herd change as his reputation endogenously changes over time.

We also show how the size of the percentage fee that must be paid to intermediaries in order to align incentives can be considerably smaller if investing in reputation is possible as opposed to a situation in which this isn't feasible. Moreover, we illustrate how the possibility of investing in reputation can allow the delegated portfolio management market to operate when the use of more sophisticated remuneration schemes is not possible. Of course, there is a cost involved since in the reputational equilibrium the intermediaries' types are never revealed to investors. Therefore, it is possible that some skilled unlucky intermediaries are punished by investors through low fees while some lucky unskilled intermediaries may be paid high fees. Nevertheless, since skilled intermediaries who acquire information have greater probability of making good investment decisions, this type of scenario is unlikely to occur.

Additionally, we show that for a reputational equilibrium to be feasible, the gains from investing in reputation can either be obtained through higher fees or larger assets under management. In both these two cases intermediaries' expected profits are increasing in their reputation.

3.1 A Static Model in a Risk Neutral Economy

We proceed to model a static delegated portfolio management problem where financial intermediaries cannot build a reputation. We explore what type of remuneration schemes would be needed in order for skilled intermediaries to separate out form unskilled ones. As we will show, a remuneration scheme with both a fixed monetary payment and a percentage of the final value of assets under management is needed for a separating equilibrium to be feasible. However, this type or scheme calls for a negative fixed pay (i.e. financial intermediaries pay investors in order to have the right to invest their wealth). This is not confirmed by the stylized facts. We now proceed to describe the model's setup.

3.1.1 The Economy

Financial Securities

In the economy there is a risk-free asset, that pays r_f for each unit of wealth invested and has a price q_f . There is also a risky asset, which pays r for each unit of wealth invested. This return can take on two values: r_G with probability π , or r_B with probability $(1 - \pi)$, where $r_G > r_B$. The risky asset has a price of q. The life of both assets lasts one period; their short sale is not allowed for investors nor for intermediaries; and their prices are exogenous, in the sense that all investors and intermediaries are price-takers and prices don't reveal the intermediaries' private information. If this assumption is not fulfilled it is possible that, in equilibrium, intermediaries would not be willing to obtain information, because they wouldn't be able to benefit from this action if their investment decisions reveal their information to the rest of intermediaries and investors through price changes in assets. This assumption could be justified by the existence of unmodeled noise traders in the economy (on this see Grossman and Stiglitz, 1980).

We make the following assumption:

$$R \equiv \pi \left(\frac{r_G}{q}\right) + (1 - \pi) \left(\frac{r_B}{q}\right) = \left(\frac{r_f}{q_f}\right) \tag{3.1}$$

We note that if (3.1) holds, a risk-neutral investor will be indifferent (ex ante) between buying the risky or the risk-free asset.

Note that rather than an assumption, (3.1) could be seen as an equilibrium condition for the prices of both assets in an economy in which investors are risk-neutral.

Also, Equation (3.1) implies:

$$\frac{r_G}{q} > \frac{r_f}{q_f} > \frac{r_B}{q} \tag{3.2}$$

Therefore, in the good (bad) state the risky asset's gross return is higher (lower) than that of the risk-free asset. This means that the agent's optimal portfolio composition would be different if he knew which state was to materialize.

We have assumed a Bernoulli distribution for the risky assets return. This is nonstandard in the DPMP literature, which tends to assume a normal distribution. However, our choice allows us to have an analytically tractable multi-period model.²

Investors

There is a continuum of risk neutral investors with measure 1, indexed by i. These investors live for one period. At the beginning of their lives, in period t, they are endowed with initial wealth W_t , identical for all i.

Investors may choose to manage their own investment portfolio or they may choose to delegate the portfolio's management to a FI. Also, let R_j represent the gross expected return for the portfolio managed by intermediary j. Therefore, if D represents the delegated wealth to a FI we have that the decision problem faced by investor i is:

$$\max_{D} E(W_{t+1}) \tag{3.3}$$

Since investors are risk-neutral we have:

$$D = \begin{cases} W_t & \text{if } R_j \ge R \\ 0 & \text{if } R_j < R \end{cases}$$
(3.4)

An investors delegates the management of all his wealth only if R_j is greater than the gross expected return that he would have if he managed the portfolio by himself, denoted by R. If this condition is not met, the amount of delegated wealth is 0. The expression for R_j is given by (3.50), whereas R is given by (3.1).

Financial Intermediaries

There is a continuum of risk-neutral financial intermediaries, with measure 1,

indexed by j. These financial intermediaries may be hired by investors to delegate their

²We will return to this point in the dynamic version of the model.

wealth management.

There are two types of intermediary. Skilled FI ($\tau = s$) may pay for information that is useful to predict the risky asset's return. An unskilled FI ($\tau = u$) doesn't have the ability to acquire information. Although the FI's type is not known to investors, they know there is a mass θ of skilled FI in the population. Additionally, the effort made to obtain information can only be observed by the FI that makes it. This investment has a cost c and allows the intermediary to receive a signal ϕ , which can take two values: good ($\phi = \phi_G$) or bad ($\phi = \phi_B$). That is to say, $\phi \in {\phi_G, \phi_B}$, where:³

$$\Pr\left(\phi = \phi_G | r = r_G\right) = P_G \tag{3.5}$$
$$\Pr\left(\phi = \phi_G | r = r_B\right) = P_B$$

Furthermore,

$$P_G > P_B \tag{3.6}$$

Therefore, if intermediary j receives a good signal, he will revise upwards his previous estimate of the risky asset's return, because the Bayesian update of r given $\phi = \phi_G$ is:

$$\Pr(r = r_G | \phi = \phi_G) = \frac{P_G \pi}{P_G \pi + P_B (1 - \pi)}$$
(3.7)

On the other hand, if he receives a bad signal, he will revise downwards his previous estimate, since:

$$\Pr\left(r = r_G | \phi = \phi_B\right) = \frac{(1 - P_G)\pi}{(1 - P_G)\pi + (1 - P_B)(1 - \pi)}$$
(3.8)

³The information structure se use differs from standard DPMP models, such as Bhattacharya and Pfleiderer (1985) and is similar to the one used by Chemmanur and Fulghieri (1994). Also, note that we do not index the FI's signal, because all skilled FI that gather information receive the same signal. In other words, skilled FI's information is perfectly correlated.

Based on the previous equations, we can see that the intermediary decides to invest all the portfolio in the risky asset if he receives a good signal, whereas if he receives a bad signal he decides to invest all in the risk-free asset. We can see that acquiring information is useful, because it is more likely to receive a good signal when the return of the asset is higher.

Therefore, letting α_I represent the portfolio share that the skilled intermediary invests in the risky asset when he acquires private information, the optimal investment strategy is given by:

$$\alpha_I = \begin{cases} 1 & \text{if } \phi = \phi_G \\ 0 & \text{if } \phi = \phi_B \end{cases}$$
(3.9)

Equation (3.9) incorporates the fact that short selling is not allowed.

From the discussion above we can conclude that the gross expected return of a portfolio managed by a skilled FI that acquires information is given by:

$$R_{I} = \pi P_{G}\left(\frac{r_{G}}{q}\right) + (1 - \pi) P_{B}\left(\frac{r_{B}}{q}\right) + (\pi (1 - P_{G}) + (1 - \pi) (1 - P_{B}))\left(\frac{r_{f}}{q_{f}}\right)$$
(3.10)

This is so because with probability π the risky asset has a high return. Nevertheless, this asset will be bought only if the intermediary receives a good signal, which will happen with probability P_G . In that case, the gross rate of return received by the investor will be $\frac{r_G}{q}$. On the other hand, with probability $(1 - \pi)$ the risky asset's return is low. Given this, there is a probability P_B that the intermediary receives a good signal and buys the risky assets so the return to the investor will be $\frac{r_B}{q}$. The third term corresponds to the probability that the intermediary receives a bad signal and buys the risk-free asset, so gross return is $\frac{r_f}{q_f}$.

Using (3.1) it's possible to rewrite the previous equation:

$$R_i = R + \pi \left(1 - \pi\right) \left(P_H - P_L\right) \left(\frac{r_H - r_L}{q}\right)$$
(3.11)

In (3.11) we show how the expected return of the portfolio managed by a FI that acquires information is greater than the one of a portfolio managed without information (e.g. managed by the investor himself).

Also, each financial intermediary has a fixed capacity to serve one investor. This assumption is consistent with the decentralization observed in the market of financial intermediation⁴. Berk and Green (2004) develop a model in which the financial intermediaries (mutual funds) choose to limit the size of the portfolios they manage, because larger funds limit the intermediary's capacity to generate above normal returns. In this sense, our assumption of fixed capacity is consistent with the work of these authors, but it is not totally equivalent, because Berk and Green's limited capacity refers to the amount of assets under management, whereas in our model it is related to the mass of investors served.

The Herding Possibilities

There is an exogenous probability $\eta \in (0, 1)$ that any given intermediary succeeds in copying the portfolio decision of an intermediary j'. That is to say, if j' buys the risky (risk-free) asset, with probability η the intermediary j will observe that j' bought the risky (risk-free) asset, and with probability $(1 - \eta) j$ will not be able to observe the other manager's portfolio. This assumption is introduced to consider, in a simple manner, the fact that in a DPMP context it is not possible to observe with certainty the decisions of other intermediaries.⁵

⁴According to our own calculations, the Herfindahl-Hirschman Index for the stocks category mutual fund market in Chile is 1,268, whereas for USA emerging markets category mutual fund it's 1,262. According to the Department of Justice of the United States, markets with indices between 1,000 and 1,800 present a "moderate" degree of concentration.

⁵An alternative assumption would be that an intermediary makes a noisy observation of the portfolio composition of another intermediary. However, this does not seem very suitable in our context of binary investment decisions.

Additionally, we will limit the number of intermediaries at which j can try to imitate in the following way: every period j can try to observe the portfolio of only one intermediary, and if j is competent, once he decides to imitate he cannot obtain private information during that time period. The fact that j can only try to observe the portfolio of one intermediary per period could be justified by means of the existence of a cost of observing the portfolio of rivals. On the other hand, the assumption of the irreversibility in the decision to imitate for a competent intermediary could be due to the fact that once j decides to imitate, he doesn't have enough time left to collect information on the risky asset. These two assumptions considerably simplify the model.

Given that there is a continuum of intermediaries a FI that decides to imitate must also decide from whom to do so. Since we limit to one the number of intermediaries to imitate and since the type of each FI is known to the rest of intermediaries, the imitator will try to observe the portfolio of any given skilled FI.

Finally, given (3.1), we will assume that, in the event that an intermediary doesn't acquire information and isn't able to observe the portfolio of a rival, he will randomize between investing his portfolio in the risky or risk-free asset with 50% probability.⁶ Therefore, following the same steps used to derive (3.11), the expected gross return of the portfolio managed by an intermediary who herds is:

$$R_H = R + \eta \pi \left(1 - \pi\right) \left(P_G - P_B\right) \left(\frac{r_G - r_B}{q}\right)$$
(3.12)

In view of the fact that $\eta < 1$ the expected return for the portfolio managed by a skilled intermediary who acquires information will be greater than the expected return for

⁶This assumption is important since the investment strategy of an uninformed intermediary will affect the way in which investors update their beliefs regarding intermediaries' characteristics in the dynamic version of the model. We will return to this point in Section 3.2.

the portfolio managed by an imitator. Also, R_H will be greater than the expected return for the portfolio managed by an uninformed FI that doesn't imitate:

$$R_{I} - R_{H} = \pi \left(1 - \pi\right) \left(1 - \eta\right) \left(P_{G} - P_{B}\right) \frac{\left(r_{G} - r_{B}\right)}{q} > 0 \tag{3.13}$$

$$R_H - R = \pi \left(1 - \pi\right) \eta \left(P_G - P_B\right) \frac{(r_G - r_B)}{q} > 0 \tag{3.14}$$

This leads us to the following proposition.

Proposition 3.1 Whenever $\eta \in (0, 1)$, for any linear remuneration scheme consisting of a fixed monetary payment plus a percentage of the final value of the assets under management, a financial intermediary who does not acquire information will prefer to imitate the portfolio of another intermediary over randomly making his investment decisions.

Proof. For any given remuneration scheme investors' willingness to pay will be increasing in his portfolio's expected return. Therefore, a FI prefers to imitate rather than to invest without information if $R_H > R$, which holds by Equation (3.14).

It's important to emphasize that, although the assumptions made about the portfolio imitation are ad hoc, they allow to have an analytically tractable model. In addition, the results obtained later are robust, in the sense that the only necessary condition for some FI to be willing to acquire information is that his choice can't be perfectly imitated by other intermediaries. If this were true, the expected return of the portfolio that he manages will be greater than those of a FI that imitates. Otherwise investors would not be willing to pay more by their services and they would not be able to compete with imitators, who don't incur in information acquisition costs.

The Remuneration Scheme

We will assume that the FI's revenue consists of a fixed monetary payment \bar{p} plus a percentage p of the final value of assets under management, W. Therefore, the expected utility for an intermediary who acquires information will be given by the expected difference between the revenue from fees and the investment costs.

3.1.2 Pooling Equilibria

We begin by characterizing an equilibrium in which there is a single fixed payment and percentage fee for both types of intermediaries. We study a situation in which investors offer the same contract to all intermediaries. If the intermediaries accept the offer they are hired and those who are skilled acquire private information to make portfolio decisions, while those that are unskilled copy the portfolio of some skilled intermediary. After that, assets' return are realized and payoffs are made to all participants. We emphasize that in order for an equilibrium to be feasible investors should not be tempted to hire an intermediary different from the one that is serving them.

The feasible values for \bar{p} and p must satisfy all agent's participation constraints. Additionally, skilled FI should have incentives to acquire information. These constraints are given by:

$$W_{t+1} = (1-p) R_{\theta} W - \bar{p} \ge R W$$
(3.15)

$$\Pi_I = pR_I W + \bar{p} - c \ge 0 \tag{3.16}$$

$$\Pi_H = pR_H W + \bar{p} \ge 0 \tag{3.17}$$

$$pR_IW + \bar{p} - c \ge pR_HW + \bar{p} \tag{3.18}$$

where

$$R_{\theta} = \theta \left(R_I - R_H \right) + R_H \tag{3.19}$$

Equations (3.15) through (3.17) state that the contract offered must be such that investors and FI (both skilled and unskilled) receive at least their reservation utility. Equation (3.18) states that skilled FI must be better of investing than herding, if this Equation does not hold, there won't be spending in private information and therefore there will be no imitation either, and investors would be indifferent between delegating their wealth management or investing themselves.

Figure (3.1) depicts the participation and incentive constraints and the combination of fees p and \bar{p} that satisfy investors, skilled and unskilled FI's participation constraints; and the incentive compatibility constraint. Since investors' expected utility is higher for schemes closer to the origin, if there is competition among FI the equilibrium contract should be given by point A, where intermediaries' expected profits are at their lowest.

With this contract investors would have an expected utility of $R_{\theta}W - \theta c$, while both skilled and unskilled FI would have zero expected utility. We note that the offered contract features a fixed payment $\bar{p} = -\left(\frac{R_H}{R_I - R_H}\right)c$ and a percentage fee $\frac{c}{W(R_I - R_H)}$. We focus on a case in which c is not too high relative to W so that this fee is strictly less than 1. The commission is an increasing function of the investment cost and of the probability of making a successful imitation, since if these parameters are higher, skilled FI would find it more attractive to herd. Also, the commission is a decreasing function of investors wealth, the quality of the information received by skilled FI (measured by $(P_G - P_B)$) and of the difference in gross expected return between the good and the bad state (measured



Figure 3.1: Pooling Equilibrium

by $\left(\frac{r_G - r_B}{q}\right)$). An increase in any of these variables makes acquiring information more attractive for skilled FI.

However, a pooling equilibrium will not be feasible since skilled FI will have incentives to signal their type to investors it they can offer a contract lying on their participation constraint and that is to the left of point A in figure (3.1). Note that such a contract, which features a higher percentage fee and a lower and negative fixed payment would not be attractive for unskilled FI since they would have negative expected utility.

3.1.3 Separating Equilibria

Having established that a pooling equilibrium is not feasible we proceed to characterize equilibria in which there is a menu of remuneration schemes for intermediaries to choose from. Again, contracts offered must satisfy investors' and FI's participation constraints. Additionally, skilled FI must be given incentives to obtain information, and no type of FI should envy contracts designed for other types of FI. The constraints are given by:

$$W_{t+1}^{I} = (1 - p_{I}) R_{I} W - \bar{p}_{I} \ge R W$$
(3.20)

$$W_{t+1}^{H} = (1 - p_{H}) R_{H} W - \bar{p}_{H} \ge R W$$
(3.21)

$$\Pi_I = p_I R_I W + \bar{p}_I - c \ge 0 \tag{3.22}$$

$$\Pi_H = p_H R_H W + \bar{p}_H \ge 0 \tag{3.23}$$

$$p_I R_I W + \bar{p}_I - c \ge p_H R_H W + \bar{p}_H \tag{3.24}$$

$$p_H R_H W + \bar{p}_H \ge p_I R_H W + \bar{p}_I \tag{3.25}$$

$$p_I R_I W + \bar{p}_I - c \ge p_H R_I W + \bar{p}_H - c \tag{3.26}$$

Equations (3.20) through (3.23) are participation constraints for investors and FI. Equation (3.24) states that skilled FI must be better off investing than herding. If Equation (3.24) does not hold, there will be no spending in private information and therefore there will be no imitation either and investors would be indifferent between delegating their wealth management or investing themselves. Finally, equations (3.25) and (3.26) are self-selection or no-envy constraints which state that unskilled and skilled FI should prefer the contracts designed for they own types

Figure (3.2) shows the participation and incentives constraints. Feasible contracts should offer skilled FI a fee p_I of no less than $\frac{c}{W(R_I - R_H)}$ so that they are willing to acquire information. Also, the self-selection constraints imply that unskilled FI should be better off by selecting the pair (\bar{p}_H, p_H) than selecting the contract showed in point A. This is


Figure 3.2: Separating Equilibria

accomplished by offering the unskilled FI a contract that lies on their isoprofit curve passing trough point A. Therefore a contract offered to skilled FI given by A and a contract offered to unskilled FI given by zero fixed payment and zero fee (shown in point B in figure (3.2)) satisfy the intermediaries' participation and self-selection constraints, while also giving incentives to skilled intermediaries to acquire information and make investors indifferent regarding which intermediary they hire. Thus, these contracts form part of a feasible separating equilibrium. Although there are infinite pairs of contracts (\bar{p}_I, p_I) (\bar{p}_H, p_H) that could achieve a separating equilibrium, the pair proposed has the property of giving all FI their reservation utility, while also giving incentives to skilled FI to acquire information and satisfy the self-selection constraints. Moreover, the proposed contracts leave all investors with the same expected utility, whether they hire a skilled or unskilled FI.

The contract offered to unskilled FI features no fixed payment ($\bar{p}_H = 0$) and zero percentage commission ($p_H = 0$). On the other hand, the contract offered to skilled FI has a fixed payment $\bar{p}_I = -R_H W$ and a percentage fee $p_I = 1$. Given this, it is easy to see that, in equilibrium, all investors will have expected utility of $R_H W$. Unskilled FI's will have zero utility, and skilled FI will have positive expected utility as long as the investment cost isn't too large. Namely:

$$W_{t+1} = R_H W \tag{3.27}$$

$$\Pi_H = 0 \tag{3.28}$$

$$\Pi_I = (R_I - R_H) W - c \tag{3.29}$$

We note that, in order for the described separating equilibrium to exist, it must be true that $\bar{p}_I < 0$. If for some reason the FI are not allowed to make payments to investors in exchange for managing their portfolios, then the existence of a separating equilibria will not be possible. The reason for this is that for investors to be indifferent between both types of intermediaries and for the contracts to satisfy the self-selection constraints it must be true that $p_I = 1$. But this implies that \bar{p}_I must be negative, since otherwise the investors' participation constraints will not be satisfied. We formalize this argument below.

Proposition 3.2 For a separating equilibria to be feasible, skilled intermediaries must charge a negative fixed fee to investors. This is, \bar{p}_i must be negative.

Proof. In a separating equilibrium it must be true that the self-selection constraints are satisfied for both types of intermediary. Also, investors must be indifferent between both types of intermediary, since otherwise they would have incentives to outbid for the intermediaries they find more attractive. Additionally, the participation constraints of all market participants must be met, as well as the incentive compatibility constraint for skilled intermediaries. We note that for an unskilled intermediary to (weakly) prefer the contract (\bar{p}_H, p_H) rather than the contract (\bar{p}_I, p_I) it must be true that their expected utility under the former contract is at least as high as he would get under the latter. This is:

$$p_H R_H W + \bar{p}_H = p_I R_H W + \bar{p}_I$$

$$\bar{p}_H = p_I R_H W - p_H R_H W + \bar{p}_I$$
 (3.30)

On the other hand, for investors to be indifferent between both types of intermediary, it must be true that:

$$(1 - p_H) R_H W - \bar{p}_H = (1 - p_I) R_I W - \bar{p}_I$$
$$\bar{p}_H = (1 - p_H) R_H W - (1 - p_I) R_I W + \bar{p}_I$$
(3.31)

Combining both conditions we have:

$$p_I R_H W - p_H R_H W + \bar{p}_I = (1 - p_H) R_H W - (1 - p_I) R_I W + \bar{p}_I$$
(3.32)

After some algebraic manipulation we obtain:

$$p_I = 1 \tag{3.33}$$

If we assume that \bar{p}_I must be nonnegative then it must be set to zero in order to give investors the highest possible expected utility. Therefore, expected utility for investors will be:

$$W_{t+1} = (1 - p_I) R_I W - \bar{p}_I$$

= $(1 - 1) R_I W - 0$
= 0,

which is lower than investors' reservation utility. \blacksquare

The following proposition summarizes our results regarding the static equilibria:

Summary 3.1 Under the assumptions made about assets' returns and agents preferences, there won't be a pooling static equilibrium in the delegated portfolio market. Moreover, in the static separating equilibrium unskilled intermediaries will have zero expected utility, while skilled intermediaries' expected utility will be $(R_I - R_H)W - c$. Also, investors' expected utility will be R_HW . However, if it's not possible for intermediaries to charge negative fixed payments, a separating equilibrium will not be feasible.

3.2 A Dynamic Model in a Risk-Neutral Economy

We use a model with infinite periods to capture the fact that the construction of reputation is a slow process, with features that cannot be studied in finite- horizon models. In the economy there is a risky asset and a risk-free asset, whose returns are the same as those assumed in the previous Section. Investors live for one period and have an initial wealth endowment, which they can invest by their own means or delegate to a FI. Financial Intermediaries have (potentially) infinite lives. Some FI (the skilled ones) have the ability to obtain private information that is useful to predict the risky asset's return. This characteristic is not observable to investors. In addition, the acquisition of information is observable only to the FI that makes it. We now proceed to detail the characteristics of the assets and agents in the economy.

3.2.1 The Economy

Financial Securities

We maintain or previous assumptions regarding the existence of a risky and riskfree asset in the economy, as well as the assumptions made about their returns and prices.

Investors

At every date, there is a continuum of risk-neutral investors with measure 1, indexed by *i*. Each generation lives for one period. Let *F* denote the cdf over initial wealth for investors, which we assume invariant each period and is degenerate if all investors have the same wealth level. Investors choose to manage their own investment portfolio or to delegate the portfolio's management to a FI. Letting *D* represent the amount of wealth delegated to a FI, and $R(\mu)$ denote the gross-expected return for the portfolio managed by an intermediary with reputation μ , we have that the optimal delegating policy for investors is given by:

$$D = \begin{cases} W_t & \text{if } R(\mu) \ge R \\ 0 & \text{if } R(\mu) < R \end{cases}$$
(3.34)

where $R(\mu)$ is given by Equation (3.50) and R is given by (3.1).

Financial Intermediaries

There is a continuum of risk-neutral financial intermediaries, with measure 1, and indexed by j. These FI have potentially infinite lives and are hired by investors to manage their portfolios. We maintain our assumption regarding the fixed capacity of intermediaries which means that each FI may serve at most one investor. There are two types of intermediary. Skilled FI ($\tau = s$) may obtain information useful to predict the risky asset's return. An unskilled FI ($\tau = u$) doesn't have the ability to obtain information. Although the FI's type is not known to investors, they know there is a mass θ of skilled FI in the population and they assign a certain probability to the event that FI j is skilled, given his investment decisions record up until t. This probability, denoted by μ_j is the FI j's reputation, and we detail its analytical expression below.

Also, let G_t denote the cdf of reputation at time t for the population of intermediaries. This cdf has support $U_t \subset [0, 1]$. Moreover, if G_t^S , respectively G_t^U , denotes the cdf of the populations of skilled, respectively unskilled, intermediaries then $G_t = \theta G_t^S + (1 - \theta) G_t^U$.

The acquisition of information can only be observed by the FI that makes it. This investment's characteristics are modified from the static model assuming that:

$$P_B = 1 - P_G \tag{3.35}$$

$$P_G > \frac{1}{2} \tag{3.36}$$

The importance of these assumptions will become clear when we discuss the evolution of investors' beliefs. The skilled FI's investment strategy, given that he acquires information, will be given by (3.9). Also, the gross-expected return of a portfolio managed by a FI that acquires information and for a FI that herds are given by:

$$R_{I} = R + \pi \left(1 - \pi\right) \left(2P_{G} - 1\right) \left(\frac{r_{G} - r_{B}}{q}\right)$$
(3.37)

$$R_{H} = R + \eta \pi \left(1 - \pi\right) \left(2P_{G} - 1\right) \left(\frac{r_{G} - r_{B}}{q}\right)$$
(3.38)

We will restrict to Markov strategies for intermediaries so agents make their decisions based only on their current reputation and type. With this restriction we ignore more complex equilibria in which strategies are contingent on the histories of prices, strategies or distributions of reputation. If this was a game of private monitoring this requirement would be unnecessary as coordination between firms and/or investors would not be feasible⁷. Therefore, the strategy of the intermediaries will take the form:

$$\sigma^{S} : U \to \{0, 1\}$$

: $\sigma^{S}(\mu) = \beta$ (3.39)

This is, a skilled intermediary will choose to obtain information ($\beta = 1$) or herd ($\beta = 0$) based only on the value of his reputation. As we will see later, the unskilled intermediaries' strategy is trivial since they will always herd.

3.2.2 Steady-State Equilibria

As a result from the interaction between intermediaries and investors there will be an equilibrium price function p_t , and an equilibrium assignment rule $\tilde{\mu}_t$, that determines which intermediaries serve which investors. First, we define the equilibrium in the stage game and since intermediaries have (potentially) infinite lives we then proceed to define the equilibrium in the repeated game. Special attention will be paid to the conditions under which, in a steady-state equilibrium, all the unskilled intermediaries imitate the portfolio decision of a rival, whereas some skilled intermediaries also imitate. Therefore, it's important to guarantee that a steady state does exist. A sufficient condition for this is that there are stable distributions G^S , G^U , so that the cdf for the population of intermediaries is constant. This will guarantee that the price and assignment function, which are functions of

⁷On this topic see also the discussions in Mailath and Samuelson (2001) and Vial (2008).

these cdf are also constant. That such an ergodic distribution exists for equilibria in which skilled firms always make effort is proved in Vial (2008). The equilibria we study share common elements with those studied by Vial. However, there is an important difference since we focus in equilibria in which not all skilled intermediaries will make effort. Since proving that an stationary cdf for the reputation of intermediaries exists is not trivial, here we will assume that such a distribution exists.

Definition 3.1 A Walrasian Equilibrium (WE) for the economy $\langle F, G_t^S, G_t^U, \beta_t \rangle$ of a stage t of this game is constituted by a price function p_t and assignment function $\tilde{\mu}_t$, such that:

- **WE1** The assignment function $\tilde{\mu}_t$ is optimal for investors, given the price function p_t and the investment strategy β_t .
- **WE2** The assignment function $\tilde{\mu}_t$ is optimal for intermediaries, given the price function p_t and the investment strategy β_t .
- **WE3** All investors are served by intermediaries. This is, there is market clearing since the mass of intermediaries coincides with the mass of investors.

Note that this definition doesn't use optimality of the investment strategy β_t , since this can only be determined in the repeated game as the decisions made by an intermediary in this period will affect the value of his reputation and thus his expected profits for future periods.

Definition 3.2 A Markov Sequential Equilibrium (MSE) of the economy $\langle F, G_0^S, G_0^U \rangle$ is a sequence of price lists $\{p_t\}$, Markov investment strategies $\{\beta_t\}$, assignment correspondences

- $\{\tilde{\mu}_t\}$ investors beliefs about investment strategies $\{\tilde{\beta}_t\}$ and reputation distribution pairs $\{G_t^S, G_t^U\}$ such that for all t and μ :
- **MSE1** Each tuple $\langle p_t, \tilde{\mu}_t \rangle$ conforms a WE of the stage game given β_t .
- **MSE2** The investment strategy β_t is optimal for skilled intermediaries with reputation μ for all $\mu \in U_t$.
- **MSE3** Investors' beliefs about investment strategies are consistent with intermediaries' strategies. This is $\beta_t = \tilde{\beta}_t$.
- **MSE4** Intermediaries' reputation and the population cdfs of reputations (G_t^S, G_t^U) evolve according to Bayes' rule and equilibrium strategies, defining a dynamic system $(G_{t+1}^S, G_{t+1}^U) = T(G_t^S, G_t^U)$.

Vial (2008) focuses on a high quality MSE where all skilled firms make effort regardless of their reputation. However, we will be interested in a Herding MSE where skilled intermediaries may or may not choose $\beta = 1$ depending on their reputation. We do this in order to study the relationship between reputation and herding. Specifically, we want to investigate how do the incentives to herd change as an intermediary's reputation improves.

Additionally, if the MSE exhibits stable distributions (G^S, G^U) then p and $\tilde{\mu}$, which are functions of G, will also be stable. In this case, the tuple $\langle p_t, \tilde{\mu}_t, \beta \rangle$ is a steady state WE of $\langle F, G^S, G^U \rangle$.

Equilibrium Elements

The Remuneration Scheme We will consider linear remuneration schemes, i.e. schemes given by a percentage p of the final value of assets under management. In a static model with asymmetric information about intermediary's type and where investment in information is unobservable, a scheme that contains a fixed pay and a percentage p of final value of assets under management would be needed for the existence of a separating equilibrium. The reason is that if the scheme consisted only of a fixed payment, it wouldn't be possible to give incentives to a skilled intermediary to acquire information. Indeed, given a fixed payment, a skilled intermediary will always reach a greater expected utility if he doesn't obtain information. On the other hand, if the remuneration scheme contemplates only a percentage of the final value of assets under management, it wouldn't be possible to separate the skilled intermediaries from the unskilled ones, because if a contract with higher commissions is offered to encourage skilled FI, who must incur in the expenses of acquiring information, such contract would also be attractive to unskilled intermediaries.

We discard a mixed remuneration scheme, based on the stylized fact that observed contracts often do not contemplate the use of a fixed payment, and if they do, this is positive, not negative as the existence of a separating equilibrium would require.⁸ Indeed, the fixed payment must be negative because in this way the skilled FI manages to separate himself from the unskilled ones: he charges a high fee p and at the same time offers a payment to the investor in exchange for investing his portfolio. The unskilled FI cannot offer this type of contract without having a negative expected payment. Below we discuss the implications

⁸See ICI (2008) for the case of USA mutual funds. For the case of Chilean mutual funds, the data of investment costs is available in the web page of each of the 20 fund management companies.

of the use of different types of contracts on the feasibility of the proposed equilibria.

The Beliefs Updating Rule Recall that β_t denotes the probability that an intermediary acquires information, given that he is skilled, in period t,⁹ and let μ_t denote the probability that investors assign to an intermediary being competent, given the history of returns obtained by the intermediary up until period t.

We follow Mailath and Samuelson (1998) in assuming that there is an exogenous probability λ that an intermediary of any type is replaced in a certain period. If the FI is replaced, with probability θ the replacing FI is skilled. The authors demonstrate that, in the absence of this assumption, the existence of an equilibrium in which the intermediaries invest in their reputation is not possible. The reason is that eventually investors would be convinced that the intermediary is skilled, causing him to lose interest in making effort when managing the portfolio, because he knows that investors will attribute a poor outcome to "bad luck" and not to the possibility that the intermediary has been negligent in his work. Regarding the plausibility of this assumption, Berk and Green (2004) find evidence of expected survival rates for mutual funds that are lower than those predicted by their model. They argue that one factor behind this finding could be that good fund managers are promoted or move to other firms, which renews the market's learning process about managers' abilities.

An intermediary's investment decision has four possible outcomes: he can invest in the risky asset when its return is high ($\alpha_t = 1$ and $r_t = r_G$); invest in the risk-free asset when the risky asset's return is high ($\alpha_t = 0$ and $r_t = r_G$); invest in the risky asset when

⁹In equilibrium, β_t will also be the probability assigned by investors to a FI investing in information, given that he is competent.



Figure 3.3: Investment Decisions with Investment

its return is low ($\alpha_t = 1$ and $r_t = r_B$); and invest in the risk-free asset when the risky asset's return is low ($\alpha_t = 0$ and $r_t = r_B$). The first and the fourth cases are good ex-post investment decisions (d = g), whereas the second and third cases would be bad ex-post investment decisions (d = b). This is shown in figure (3.3) for a skilled FI that acquires information.

We have assumed that $P_G = 1 - P_B$, which implies that the change in reputation for an intermediary is the same if d = g for cases one and four; and the change in reputation will be the same if d = b for cases two and three. This assumption makes the model more tractable. Nevertheless, relaxing it doesn't affect our main results.

We define an intermediary j's reputation in t, given his investment decision up until t and his previous reputation as:

$$\mu_{j,t} \equiv \Pr\left(\tau_j = s | d_t\right) \tag{3.40}$$

Applying Bayes' rule we have that if j makes a good investment decision his reputation will become:

$$\mu_{j,t} \left(d_t = g \right) = \frac{\Pr\left(d_t = g | \tau_j = s \right) \Pr\left(\tau_j = s \right)}{\Pr\left(d_t = g \right)}$$
(3.41)

We have to derive the expression for the probability of a good investment decision being made by a skilled and an unskilled intermediary. From figure (3.3) and given our definition of a good investment decision, we see that:

$$P_{I} \equiv \Pr(d_{t} = g | \tau_{j} = s) = \pi P_{G} + (1 - \pi) P_{G}$$
$$= P_{G}$$
(3.42)

For an unskilled intermediary who herds figure (3.4) shows that:

$$P_{H} \equiv \Pr(d_{t} = g | \tau_{j} = u) = \pi \left(\eta P_{G} + \frac{1}{2} (1 - \eta) \right) + (1 - \pi) \left(\eta P_{G} + \frac{1}{2} (1 - \eta) \right)$$
$$= \eta P_{G} + \frac{1}{2} (1 - \eta)$$
(3.43)

We point out that the probability of making the correct investment decision will be greater for an intermediary who acquires information than for one who herds if $P_G > \frac{1}{2}$:

$$P_I - P_H = \frac{1}{2} \left(2P_G - 1 \right) \left(1 - \eta \right) > 0 \tag{3.44}$$

Therefore we have:

$$\mu_{j,t} \left(d_t = g \right) = \frac{P_I \mu_{j,t-1}}{P_I \mu_{j,t-1} + P_H \left(1 - \mu_{j,t-1} \right)} \tag{3.45}$$



Figure 3.4: Investment Decisions with Herding

Where P_I is given by (3.42) and P_H is given by (3.43).

In a similar fashion we derive the expression for j's reputation if he makes a bad investment decision:

$$\mu_{j,t} \left(d_t = b \right) = \frac{\left(1 - P_I \right) \mu_{j,t-1}}{\left(1 - P_I \right) \mu_{j,t-1} + \left(1 - P_H \right) \left(1 - \mu_{j,t-1} \right)} \tag{3.46}$$

Taking into consideration the replacement probability λ and equations (3.45) and (3.46) we obtain the dynamic system that describes the evolution of beliefs:

$$\mu_0 \equiv \theta \tag{3.47}$$

$$\mu_{1} = \begin{cases} \lambda \mu_{0} + (1 - \lambda) \frac{(P_{I}\beta_{0} + P_{H}(1 - \beta_{0}))\mu_{0}}{(P_{I}\beta_{0} + P_{H}(1 - \beta_{0}))\mu_{0} + P_{H}(1 - \mu_{0})} & \text{if } d_{0} = g \\ \lambda \mu_{0} + (1 - \lambda) \frac{((1 - P_{I})\beta_{0} + (1 - P_{H})(1 - \beta_{0}))\mu_{0}}{((1 - P_{I})\beta_{0} + (1 - P_{H})(1 - \beta_{0}))\mu_{0} + (1 - P_{H})(1 - \mu_{0})} & \text{if } d_{0} = b \end{cases}$$
(3.48)

$$\mu_{t+1} = \begin{cases} \lambda \theta + (1-\lambda) \frac{(P_I \beta_t + P_H (1-\beta_t))\mu_t}{(P_I \beta_t + P_H (1-\beta_t))\mu_t + P_H (1-\mu_t)} & \text{if } d_t = g \\ \lambda \theta + (1-\lambda) \frac{((1-P_I)\beta_t + (1-P_H)(1-\beta_t))\mu_t}{((1-P_I)\beta_t + (1-P_H)(1-\beta_t))\mu_t + (1-P_H)(1-\mu_t)} & \text{if } d_t = b \end{cases}$$
(3.49)

For reputation to improve (worsen) when the FI makes a good (bad) investment decision it's necessary that $P_G > \frac{1}{2}$. Namely, the probability of making a good investment decision when acquiring information must be greater than the probability of making the right decision using the method of "flipping a coin" that is employed by an intermediary without information and that could not imitate the portfolio decision another FI.

The dynamic system that describes the evolution of beliefs will be affected by the investment strategy of the financial intermediaries, since if they acquire information, $\beta_t = 1$ so beliefs will evolve according to the solid continuous lines in figure (3.5), whereas if he imitates, $\beta_t = 0$ and beliefs will evolve deterministically, according to the discontinuous straight line.

It is worth emphasizing that, for the intermediaries who don't gather information, the beliefs updating rule does not distinguish if the investment decisions made were good or bad. The reason for this is that investors are sure that these FI are herding. Therefore, the quality of the investment decision doesn't provide useful information to distinguish the true type of the intermediary.

In addition, the reputation of the FI that doesn't acquire information increases in a deterministic fashion, until it reaches θ . It may seem odd that reputation for an intermediary who doesn't acquire information increases even if he consistently makes bad



Figure 3.5: Evolution of Reputation



Figure 3.6: Evolution of Reputation with Policy Function

investment decisions. The reason for this is the existence of the replacement probability $\lambda > 0$, which guarantees that, even though a FI shows a poor performance in some given period, there is a probability λ that he will be replaced by another FI in the following period, and with probability θ the new intermediary will be skilled.

It will be useful to consider that, for policy functions in which the FI acquires information if $\mu \ge \mu^*$ and herds otherwise, the dynamic system that describes the evolution of beliefs will be deterministic for $\mu < \mu^*$ and stochastic otherwise. Furthermore, the system will present a discontinuity in μ^* , as shown in figure (3.6).

We note that, due to the existence of a positive replacement probability $\lambda > 0$



Figure 3.7: Evolution of Reputation: Two Decision Sequences

and if $\beta = 1$, an intermediary j's reputation in period t it will depend not only on j's record of ex-post results (i.e. number of good and bad investment decisions) but also on the order in which these decisions occurred. This is to say, an intermediary's reputation will be different if his ex-post record is $\{g, g, b\}$ instead of $\{b, g, g\}$. In particular, the intermediary's reputation will be higher in the second case, as shown in figure (3.7),where $\mu_2 > \mu_1$, due to the fact that more recent investment decisions are more important in order to asses the intermediary's reputation. Indeed, distant investment decisions are less important since it is more probable that the intermediary has been replaced since the time they were taken. The Assignment Rule The assignment rule is a function that determines which intermediary will attend which investor. This rule is obtained from the condition that, in equilibrium, the mass of served investors must coincide with the financial intermediaries' installed capacity. If all investors had the same level of wealth, W_t , FI would be indifferent between serving any investor. In addition, under certain conditions (which are specified in the following Section) investors will be indifferent between being served by any FI. Therefore, the assignment of investors to FI will be undetermined by the model, and given the assumptions about installed capacity, the FI industry will serve all the investors. Note that since the assignment is undetermined it is not necessary that a stationary cdf for reputations exists in order to characterize the steady state equilibria.

The Equilibrium Price List The fee paid to a FI will depend on its reputation, that is to say, on the probability assigned by investors to the intermediary being skilled. Therefore, p will be a function of $\mu_{j,t}$.

We derive the function $p(\mu_{j,t})$ from the condition that in all periods, in equilibrium, investors must be served by the FI that maximizes their expected utility. Since we focus on steady-state equilibria the price function won't depend on time directly. That is to say, p will not be a function of t, meaning that it will not change between periods, except through changes in μ .

Also, note that the gross expected return for a portfolio managed by an intermediary with reputation μ is given by:

$$R(\mu) = \mu\beta \left(R_I - R_H\right) + R_H \tag{3.50}$$

Thus, maximizing the investor's expected utility with respect to μ :

$$W_{t+1} = (1 - p(\mu)) \left(\mu\beta \left(R_I - R_H\right) + R_H\right) W_t$$
(3.51)

The first order condition is given by:

$$\frac{\partial W_{t+1}}{\partial \mu} = (1 - p(\mu)) \left(R_I - R_H \right) W_t - \frac{\partial p}{\partial \mu} \left(\mu \beta \left(R_I - R_H \right) + R_H \right) W_t = 0$$
(3.52)

Also, the second order condition is:

$$\frac{\partial^2 W_{t+1}}{\partial \mu^2} = -W_t \left(2 \frac{\partial p}{\partial \mu} \left(R_I - R_H \right) + \frac{\partial^2 p}{\partial \mu^2} \left(\mu \beta \left(R_I - R_H \right) + R_H \right) \right)$$
(3.53)

For this problem to have an interior solution, it's necessary that $\frac{\partial p}{\partial \mu} > 0$, this is to say, FI with higher reputations charge larger fees. The reason is that if the investor chooses a FI of higher reputation, his expected utility increases, because the expected return of his portfolio will be greater. If in addition this FI charges lower fees $(\frac{\partial p}{\partial \mu} < 0)$ the problem won't have an interior solution, because all investors will seek to be served by the FI with the highest reputation. But this FI has a fixed capacity to attend investors, so there would be an excess demand for his services.

From (3.52) we obtain the following differential equation:

$$\frac{\partial p}{\partial \mu} = \frac{\beta \left(R_I - R_H\right)}{\left(\mu\beta \left(R_I - R_H\right) + R_H\right)} \left(1 - p\left(\mu\right)\right) \tag{3.54}$$

Solving this equation:

$$p(\mu) = 1 - \frac{K}{\mu\beta (R_I - R_H) + R_H},$$
(3.55)

where K is an integration constant that must be positive for $p(\mu)$ to be increasing in μ . From here on we will assume K > 0. This equation shows the maximum fee that an investor is willing to pay if he is to be attended by an intermediary with reputation μ . However, this solution is not consistent with consumers obtaining maximum utility. This can be seen from plugging (3.55) into (3.53) which turns out to be exactly zero. Therefore, the price function proposed implies that investors are at their saddle points. Nevertheless we stick to using (3.55) because it has the property of making investors indifferent between all financial intermediaries. Indeed, substituting (3.55) in (3.51), we see that the investor's expected utility will be equal to a constant, KW_t .

Regarding the price function's properties, its slope is given by:

$$\frac{\partial p}{\partial \mu} = \frac{\beta \left(R_I - R_H\right) K}{\left(\mu \beta \left(R_I - R_H\right) + R_H\right)^2} > 0 \tag{3.56}$$

Moreover, $p(\mu)$ is a concave function of μ :

$$\frac{\partial^2 p}{\partial \mu^2} = -\frac{2\beta \left(R_I - R_H\right)^2 K}{\left(\mu\beta \left(R_I - R_H\right) + R_H\right)^3} < 0 \tag{3.57}$$

Additionally, for a FI who doesn't acquire information and/or for those whose reputation equals 0, the fee is:

$$p(0) = 1 - \frac{K}{R_H} \tag{3.58}$$

Whereas for a FI whose reputation is the highest possible ($\mu = 1$), the fee is given by:

$$p(1) = 1 - \frac{K}{R_I} \tag{3.59}$$

It is reasonable to assume that (3.58) and (3.59) are both nonnegative, which implies that the value of the integration constant K would be bounded from above by R_H .¹⁰ Moreover, since investors' expected utility must be at least as high as their reservation utility

¹⁰The empirical evidence shows that mutual funds don't charge negative fees for their services.

if they are to hire an intermediary, it must be true that K is bounded from below by R. Therefore $K \in [R, R_H]$.

Investors and Unskilled FI's Strategies

Since investors only live during one period, the only decision they make is whether to delegate the management of his wealth and to which intermediary to do so, although given (3.55) in equilibrium investors will be indifferent between being served by any FI. Unskilled financial intermediaries will also have a simple strategy, because even though they may live for infinite periods, they will always (try to) imitate the portfolio decisions of some skilled FI.

Skilled FI's Strategies

Every period t a skilled FI with reputation μ_t must choose between acquiring information (a = I) or imitating another FI's portfolio, this is, herding (a = H). Therefore, we can write the skilled FI's Bellman equation as:

$$V(\mu_{t}) = \max_{a \in \{I,H\}} \left\{ p(\mu) R_{i}W - c + \rho E\left[V(\mu_{t+1}) | a = I\right], p(\mu) R_{H}W + \rho E\left[V(\mu_{t+1}) | a = H\right] \right\}$$
(3.60)

The first term is the FI's expected utility if he acquires information in period t. This utility is given by the expected payment for the end of this period, which is equal to a percentage $p(\mu)$, of the expected final value of assets under management, R_IW , minus the costs of acquiring information c. The second element of the first term is the expected present value of future utilities, conditional in acquiring information in t.¹¹ Since $p(\mu)$ is

¹¹The term ρ is an adjusted discount factor. Letting δ denote a standard discount factor, then $\rho =$

an increasing and concave function of μ , the first term of the Bellman equation will be an increasing function of μ . The reason is that $V(\mu_{t+1})$ contains the sum of infinite terms of the type *pRW*. Although the sum of concave functions is concave, function V is a sum of the maxima between concave functions and therefore it's not possible to assure that it will be concave, but we do know that it will be a strictly increasing function in μ .

The second term of (3.60) is the expected utility if the FI herds in the present period. This utility is given by the expected payment for the end of period, $p(\mu) R_H W$, plus the present value expected future utilities, conditional on herding in t. The form of this term will be similar to the expected utility if the FI acquires information in the present period (i.e. it will be a strictly increasing function of μ).

When an intermediary is deciding whether to invest or to imitate, he faces the following trade-off: if he acquires information today, he must incur a cost c, but in return, it's more likely that his utility at the end of this period is greater, since $R_I > R_H$, and in addition, he has a greater probability of having a better reputation next period, because $P_I > P_H$,¹² which will allow him to charge higher fees in future periods. On the other hand, if he doesn't obtain information, he saves c, but faces a more unfavorable scenario for future periods.

It's important to note that in models like those of Mailath and Samuelson (1998), (2001) and Vial (2008), the decision between investing or not in reputation doesn't affect expected payoffs for the current period. This characteristic will be crucial when determining the feasibility of the existence of equilibria in which intermediaries with high reputation $\overline{\delta(1-\lambda)}$.

¹²Recall that P_I is equal to P_G , whereas P_H is equal to $\eta P_G + \frac{1}{2}(1-\eta)$.

acquire information. We will return to this point later.

We will proceed to establish that the Bellman operator associated to Equation (3.60) is a contraction. In order to do this, we first prove that (3.60) is a bounded and nondecreasing function in the interval [0, 1]. Then we will prove that the metric space composed of bounded non-decreasing functions equipped with the sup norm is a complete metric space, so that the sequence defined by the Bellman equation converges to a bounded nondecreasing function. Finally, we prove that (3.60) satisfies Blackwell's sufficient conditions for a contraction, which guarantees that the Bellman equation has a unique solution.

Lemma 3.1 The function given by (3.60) is bounded and non-decreasing in the interval [0,1].

Proof. Equation (3.60) depends on function $p(\mu)$ and on the constants R and W. The function $p(\mu)$ is concave and therefore bounded in the interval [0, 1]. Also, it is non-decreasing in μ . By assumption RW is finite. On the other hand, V contains the sum of infinite terms of the form $p(\mu) RW$ discounted by the factor $\rho < 1$.

Imagine an intermediary who always acquires information and always makes good investment decisions. This intermediary's reputation will converge to μ_{max} , the highest possible value given by the fixed point of the Bayes' rule used to update beliefs conditional on good investment decisions being made. We know that $\mu_{\text{max}} \leq \lambda \theta + (1 - \lambda) < 1$. For this intermediary we have that:

$$\bar{V} \equiv V(\mu_{\max}) = \frac{1}{1-\rho} \left(p(\mu_{\max}) R_I W - c \right) < M$$
 (3.61)

where \overline{V} denotes the upper bound for V and M > 0 is an arbitrarily large finite number. Now imagine an intermediary who always acquires information and makes bad investment decisions. This intermediary's reputation will converge to μ_{\min} , given by the fixed point of the Bayes' rule used to update beliefs conditional on bad investment decision being made. We have $\mu_{\min} \ge \lambda \theta > 0$. Then:

$$V(\mu_{\min}) = \frac{1}{1-\rho} \left(p(\mu_{\min}) R_I W - c \right)$$
(3.62)

Alternatively, consider an intermediary who always herds and makes bad investment decisions. In this case:

$$V'(\mu_{\min}) = \frac{1}{1 - \rho} \left(p(\mu_{\min}) R_H W \right)$$
(3.63)

Let $\underline{\mathbf{V}}$ denote the lower bound for V. Then:

$$\underline{\mathbf{V}} = \min\left\{ V\left(\mu_{\min}\right), V'\left(\mu_{\min}\right) \right\}$$
(3.64)

Since $p(\mu)$ is bounded and the term RW is finite, we have:

$$|\underline{\mathbf{V}}| < M \tag{3.65}$$

Therefore, V is bounded.

On the other hand, since R_I, R_H, W, ρ and c are parameters, whether V is increasing or not in μ depends on $p(\mu)$. Recall that this function is given by:

$$p(\mu) = 1 - \frac{K}{\mu\beta (R_I - R_H) + R_H}$$
(3.66)

Therefore, if in equilibrium skilled intermediaries make effort regardless of μ , $\beta = 1$ and $p(\mu)$ will be increasing. In this case, $V(\mu)$ will be increasing. On the other hand, if for some values of μ skilled intermediaries herd then $\beta = 0$ for this reputations and $p(\mu)$ will be a constant. Therefore, $V(\mu)$ may present regions in which it is constant. However, $V(\mu)$ will not be decreasing. The following lemma establishes that the metric space $(B(\mu), d)$ composed by the set of bounded non-decreasing functions in the interval [0, 1], along with the sup norm is complete.¹³

Lemma 3.2 The set $(B(\mu), d)$ of bounded non-decreasing functions in the interval [0, 1]along with the sup norm defined as $d(f, g) = \sup_{0 \le \mu \le 1} \{|f(\mu) - g(\mu)|\}$ constitutes a complete metric space.

Proof. Let $\{V_n\}$ be a Cauchy sequence in $(B(\mu), d)$. For each $\mu \in U_t$ let $V(\mu)$ be the limit of the sequence $(V(\mu))$.

First, since $\{V_n\}$ is a Cauchy sequence, then we know that $\sup_{0 \le \mu \le 1} \{|V_n(\mu) - V_m(\mu)|\} < \epsilon$ for all $n, m \ge n_0$ with $n_0 \in \mathbb{N}$. In particular, setting $\epsilon = 1$ and $m = n_0$ we have:

$$\sup_{0 \le \mu \le 1} \left\{ |V_n(\mu) - V_{n_0}(\mu)| \right\} < 1 \text{ for all } n \ge n_0$$
(3.67)

Also, since the difference of the supremum is smaller than the supremum of a difference:

$$\sup_{0 \le \mu \le 1} \left\{ |V_n(\mu)| \right\} - \sup_{0 \le \mu \le 1} \left\{ |V_{n_0}(\mu)| \right\} \le \sup_{0 \le \mu \le 1} \left\{ |V_n(\mu) - V_{n_0}(\mu)| \right\} \text{ for all } n \ge n_0$$
(3.68)

But if this is true for the supremum then it is true for all μ so:

$$\{|V_n(\mu)|\} - \{|V_{n_0}(\mu)|\} \le \{|V_n(\mu) - V_{n_0}(\mu)|\} \text{ for all } n \ge n_0, \text{ for all } \mu \in U_t$$
(3.69)

Rearranging terms:

$$\{|V_n(\mu)|\} \le \{|V_n(\mu) - V_{n_0}(\mu)|\} + \{|V_{n_0}(\mu)|\} \text{ for all } n \ge n_0, \text{ for all } \mu \in U_t$$
(3.70)

¹³The proof for this lemma is adapted from Etgü (2008).

Note that the first term on the right hand side is smaller than 1 for $n \ge n_0$, while the second term on the right hand side is smaller than M since V_{n_0} is bounded. Therefore

$$\{|V_n(\mu) - V_{n_0}(\mu)|\} + \{|V_{n_0}(\mu)|\} < M + 1 \text{ for all } n \ge n_0, \text{ for all } \mu \in U_t$$
(3.71)

And this in turn implies:

$$\{|V_n(\mu)|\} < M+1 \text{ for all } n \ge n_0, \text{ for all } \mu \in U_t$$

Moreover, since $|\lim_{n\to\infty} V_n(\mu)| = |V(\mu)|$, then:

$$|V(\mu)| < M+1 \text{ for all } \mu \in U_t \tag{3.72}$$

Therefore, $V(\mu)$ is bounded.

Second, given some $\epsilon > 0$ and since $\{V_n\}$ is a Cauchy sequence there exists $n_0 \in \mathbb{N}$ such that $d(V_n, V_m) < \frac{\epsilon}{2}$ for all $n, m \ge n_0$. Since this holds for the supremum, then:

$$\{|V_n(\mu) - V_m(\mu)|\} \le \frac{\epsilon}{2} \text{ for all } n, m \ge n_0, \text{ for all } \mu \in U_t$$
(3.73)

Taking the limit as n goes to infinity in the last inequality we obtain:

$$\{|V(\mu) - V_m(\mu)|\} \le \frac{\epsilon}{2} \text{ for all } m \ge n_0, \text{ for all } \mu \in U_t$$
(3.74)

Therefore:

$$d(V, V_m) = \sup_{0 \le \mu \le 1} \left\{ |V(\mu) - V_m(\mu)| \right\} \le \frac{\epsilon}{2} < \epsilon \text{ for all } m \ge n_0$$
(3.75)

This proves that the sequence $\{V_n\}$ converges to V in $(B(\mu), d)$. This, together with the fact that V is bounded completes the proof that $(B(\mu), d)$ is a complete metric space.

The following lemma establishes that V is an operator, mapping bounded nondecreasing functions in the interval [0, 1] into the set of bounded non- decreasing functions in the interval [0, 1].

Lemma 3.3 The function V defined in (3.60) is an operator.

Proof. That V is an operator is evident from (3.60). We have already established that V is a bounded non-decreasing function. Equation (3.60) shows that V chooses the maximum between two bounded and non-decreasing functions. Therefore, V is an operator.

The following two claims are used to prove that the operator V satisfies the monotonicity and discount properties.

Claim 3.1 The operator V has the monotonicity property.

Proof. Let T denote the operator V defined in (3.60) and suppose that we have two functions $f(\mu)$ and $g(\mu)$ such that $f(\mu) \ge g(\mu)$ for all $\mu \in U_t$. Then:

$$Tg = \max \left\{ p(\mu) R_{I}W - c + \rho E \left[g(\mu_{t+1}) | I \right], p(\mu) R_{H}W + \rho E \left[V(g_{t+1}) | H \right] \right\}$$

$$\leq \max \left\{ p(\mu) R_{I}W - c + \rho E \left[f(\mu_{t+1}) | I \right], p(\mu) R_{H}W + \rho E \left[V(f_{t+1}) | H \right] \right\}$$

$$= Tf \qquad (3.76)$$

Therefore, T satisfies the monotonicity property for operators.

Claim 3.2 The operator V has the discount property.

Proof. We have that for any positive constant m:

$$T(f+m) = \max \left\{ p(\mu) R_I W - c + \rho E \left[f(\mu_{t+1}) + m | I \right], p(\mu) R_H W + \rho E \left[V(f_{t+1}) + m | H \right] \right\}$$

$$= \max \left\{ p(\mu) R_I W - c + \rho E \left[f(\mu_{t+1}) | I \right] + \rho m, p(\mu) R_H W + \rho E \left[V(f_{t+1}) | H \right] + \rho m \right\}$$

$$= \max \left\{ p(\mu) R_I W - c + \rho E \left[f(\mu_{t+1}) | I \right], p(\mu) R_H W + \rho E \left[V(f_{t+1}) | H \right] \right\} + \rho m$$

$$= Tf + \rho m$$
(3.77)

This means that T satisfies the discount property for operators. \blacksquare

Finally, the next proposition proves that V has a unique solution.

Proposition 3.3 The Bellman equation $V(\mu)$ defined in equation (3.60) is a contraction.

Proof. We have already proved that equation $V(\mu)$ is an operator that has the monotonicity and discount properties. Therefore, it satisfies Blackwell's sufficient conditions for an operator being a contraction. This guarantees that $V(\mu)$ has a unique solution and completes the proof.

Before discussing which type of equilibria are feasible in this model, it will be useful to note that the skilled FI will be indifferent between investing in reputation and herding if both terms of the Bellman equation are equal. That is to say, if:

$$\rho\left(P_{I}-P_{H}\right)\left[V\left(\mu_{(d=g)}\right)-V\left(\mu_{(d=b)}\right)\right]=c-p\left(\mu\right)\left[R_{I}-R_{H}\right]W$$
(3.78)

Let $v(\mu)$ denote the left hand side of (3.78), which represents the change in present expected value of future utility if the FI acquires information instead of herding. Additionally, let $w(\mu)$ denote the right hand side of (3.78), which is the change in expected utility for the present period if the FI decides to imitate rather than to obtain information. Thus, for the intermediary with reputation μ to acquire information in the present period, it's necessary that the change in expected utility for future periods that is obtained by acquiring information is greater than the change in expected utility for this period if the intermediary herds.

Change in Expected Utility for the Present Period The analytical expression for this term is given by:

$$w(\mu) = c - \left(1 - \frac{K}{\mu\beta(R_I - R_H) + R_H}\right) [R_I - R_H] W$$
(3.79)

It will be useful to note that, if $\beta = 1$, and if $\mu = 0$, w will be equal to:

$$w(0) = c - \left(1 - \frac{K}{R_I}\right) [R_I - R_H] W$$
(3.80)

If the cost of acquiring information is 0, then (3.80) will be positive (negative) if p(0) < 0 (p(0) > 0). Moreover, if c is large enough (3.80) will be positive, and in this case the intercept of (3.79) will be positive.

On the other hand, when reputation is 1, we have:

$$w(1) = c - \left(1 - \frac{K}{R_I}\right) [R_I - R_H] W$$
 (3.81)

If c is equal to 0, (3.81) will be negative, given that p(1) > 0, which should hold so that there are incentives to obtain information. In addition, if c is high enough, the value of (3.81) could be positive.

Finally, we obtain the analytical expression for the value of μ when (3.79) is equal to 0. We denote this value by $\hat{\mu}$:

$$\hat{\mu} = \frac{R_H c - (R_I - R_H) W (R_H - K)}{(R_I - R_H) ((R_I - R_H) W - c)}$$
(3.82)



Figure 3.8: Function w(mu)

When c is equal to 0, $\hat{\mu} < 0$, and if c is large enough, $\hat{\mu} > 1$, so it will always be possible to find a value of c such that $\hat{\mu} \in (0, 1)$.

Based on the previous discussion, in Figure (3.8) we plot Function (3.79) for the case $\beta = 1$ and $\beta = 0$. In the following figures we plot all functions in the interval [0, 1]. Although in the steady-state equilibria the value for intermediaries' reputation will be contained in the interval $[\mu_{\min}, \mu_{\max}] \subset [\lambda \theta, \lambda \theta + (1 - \lambda)] \subset [0, 1]$. However, note that by making the replacement probability λ arbitrarily small all these intervals can be arbitrarily similar.

It will be useful to consider that for policy functions in which the FI invests if his reputation is equal or greater than a critical level μ^* , and herds otherwise, the Function w will look like the one in the Figure (3.9).



Figure 3.9: Function w(mu) for Policy Function

Change in Expected Future Utility This term is equal to:

$$v(\mu) = \rho(P_I - P_H) \left[V\left(\mu_{(d=g)}\right) - V\left(\mu_{(d=b)}\right) \right]$$
(3.83)

The first two terms are positive and their analytical expressions are simple.¹⁴ However, the last term depends on the value function $V(\mu)$, whose analytical form is not known. We will partially characterize this term.

Assuming $\beta = 1$ and since V is an increasing function of μ , we know that the term $\left(V\left(\mu_{(d=g)}\right) - V\left(\mu_{(d=b)}\right)\right)$ will be positive or equal to zero. The reason is that if $\mu \in (0, 1)$, and if investors assign probability 1 to the skilled FI making effort, reputation for the following period will always be greater if a good investment decision is taken (d = g) as opposed to a bad decision (d = b). Therefore, given the dynamic system that describes

¹⁴These terms are given by:

 $[\]frac{1}{2}\delta(1-\lambda)(1-\eta)(2P_G-1) > 0$



Figure 3.10: Function v(mu)

the evolution of beliefs, $\mu_{(d=g)} > \mu_{(d=b)}$. In addition, since V is increasing, we will have $V\left(\mu_{(d=g)}\right) > V\left(\mu_{(d=b)}\right)$. If $\mu = 0$ or $\mu = 1$, then $\mu_{(d=g)} = \mu_{(d=b)}$ and $V\left(\mu_{(d=g)}\right) = V\left(\mu_{(d=b)}\right)$, so $\Omega(\mu)$ will be equal to 0 in these cases.

Below we graph $v(\mu)$ for the case in which investors assign probability 1 to the skilled FI acquiring information ($\beta = 1$) and for the case in which this probability is 0 ($\beta = 0$). In the latter function v is equal to 0 for every μ .

As with function w, it will be useful to consider that for policy functions in which the FI invests if his reputation is equal or greater than a critical level μ^* and imitates otherwise, function v will look like Figure (3.11). It is important to note that $v(\mu)$ is a discontinuous function of μ , due to the fact that the dynamic system describing the evolution of beliefs is discontinuous for the previously described policy functions, which will



Figure 3.11: Function v(mu) for Policy Function

cause function $p(\mu)$ to be discontinuous. Although it is not trivial to predict the way in which $v(\mu)$ will be affected by changes in μ^* , the fact that it presents discontinuities will not affect the feasibility of the equilibria we study below.¹⁵

Equilibria with (almost) no Investment in Reputation

Let us suppose that investors assign probability 0 to the skilled FI acquiring information. That is to say, $\beta = 0$ for $\mu \in [0, \mu_{\max})$. For this equilibria to be feasible it must be true that no skilled FI prefers to acquire information, given the investors' beliefs.

From figure (3.12) we see that, if there is a positive cost of acquiring information, w will be negative and v will be equal to 0, so in equilibrium no skilled FI will obtain information, validating the investors' beliefs.

¹⁵In the numerical examples section we will graph Function $v(\mu)$ for various parameters values.



Figure 3.12: Equilibrium with (almost) no Investment

The following lemma formalizes this finding.

Lemma 3.4 Equilibria with almost no investment in reputation are feasible in the economy $\langle F, G^S, G^U \rangle$.

Proof. Suppose investors assign $\tilde{\beta} = 0$ for all $\mu \in [0, \mu_{\max})$. From equations (3.80) and (3.83) we know that $w(\mu) > v(\mu)$, whenever $\tilde{\beta} = 0$ for μ . This means that the incentive compatibility constraint fails to hold and skilled intermediaries with reputation μ will choose to herd, validating investors beliefs.

Equilibria with Investment in Reputation

We now explore a situation in which investors believe that all skilled FI acquire information and proceed to verify if these beliefs are consistent.



Figure 3.13: Equilibrium with Investment 1

Starting from a situation in which $\beta = 1$ for all values of μ , we consider the following policy function:

$$a = I \quad \text{if} \quad \mu \in [0, 1]$$
 (3.84)

According to (3.84) the FI would acquire information independent of its reputation level. However, observing figure (3.13) we see that this policy function doesn't constitute an equilibrium, because for reputation values $\mu < \mu^*$, FI would prefer not to acquire information.

Based on these results we conjecture the existence of equilibria in which FI with high reputation will obtain information, whereas those of low reputation will herd.

We have established that the existence of equilibria in which all FI invest in reputa-


Figure 3.14: Equilibrium with Investment 2

tion is not feasible even for investment cost values such that $\hat{\mu} \in (0, 1)$. Now we demonstrate the feasibility of equilibria in which FI whose reputation is higher than a certain level acquire information.

We assume that investors beliefs are such that $\beta = 0$ for every $\mu < \mu'$, and $\beta = 1$ otherwise. Furthermore, we propose the following policy function for skilled FI:

$$a = \begin{cases} I & \text{if } \mu \ge \mu' \\ H & \text{if } \mu < \mu' \end{cases}$$
(3.85)

From figure (3.14) we can see that a FI with reputation greater than μ' will indeed obtain information, whereas those whose reputations are smaller than μ' will herd. Therefore, the proposed policy function is optimal, given the investors beliefs. There is a numerous set of this type of equilibria. Indeed, if we lower the value from which investors think that FI will invest in reputation we will find that FI with reputation greater than that level will indeed acquire information. In fact, the smaller level of reputation for which we can be sure that the previous conclusions holds is $\hat{\mu}$, since we know that from this value onwards w will always be positive, whereas v is always negative or equal to 0. Therefore it should be possible to apply some refinement criterion, such as evolutionary stable equilibria or the intuitive criterion to reduce the size of the equilibria set.

It's possible that intermediaries whose reputation are smaller than $\hat{\mu}$ choose to obtain information. This is clear from figure (3.13). In this figure we see that for values of μ between μ^* and $\hat{\mu}$ function w is negative, but v is yet more negative, meaning that FI with reputation located in this interval would prefer to acquire information rather than to imitate. Thus, if investors assign probability 1 to a FI with reputation between μ^* and $\hat{\mu}$ acquiring information, the optimal policy function would be given by:

$$a = \begin{cases} I & \text{if } \mu \ge \mu^* \\ H & \text{if } \mu < \mu^* \end{cases}$$
(3.86)

This is to say, for reputation levels greater than a critical level μ^* skilled FI will acquire information, whereas if reputation is below μ^* they will imitate. The characteristics of this type of equilibria depend on the form in which v changes if μ^* is modified. Thus, if vmoves toward the horizontal axis when $\mu^* > 0$, the critical level will increase but it must be true that $\mu^* < \hat{\mu}$. On the other hand, if v moves towards the horizontal axis when $\mu^* > 0$, the critical level will fall. However we know that μ^* will never be greater than $\hat{\mu}$, because w is zero for this value, whereas v is always positive for $\mu \in (0, 1)$. Also, we know that μ^* will never be equal to 0 either because even if investors assign $\beta = 1$ for $\mu = 0$, function wwill be positive for such reputation, whereas function v will be equal to zero. Therefore it must be true that $\mu^* \in (0, \hat{\mu})$.

The following proposition proves the existence of equilibria in which skilled intermediaries with reputation $\mu < \hat{\mu}$ acquire information.

Proposition 3.4 In the economy $\langle F, G^S, G^U \rangle$ for suitable c values such that $\hat{\mu} \in (0, 1)$, there are equilibria in which skilled financial intermediaries with reputation $\mu \in [0, \hat{\mu})$ acquire information.

Proof. If c is such that $\hat{\mu} \in (0, 1)$ then we know that the intercept of function $w(\mu)$ will be positive. Moreover, for $\mu < \hat{\mu}$, $w(\mu) > 0$, while for $\mu < \hat{\mu}$, $w(\mu) < 0$. We also know that, given $\tilde{\beta} = 1$, for all $\mu \in (0, 1)$ function $v(\mu)$ will be strictly positive, while $v(\mu) = 0$ when $\mu \in \{0, 1\}$. This means that, given $\tilde{\beta} = 1$ for all μ , functions $w(\mu)$ and $v(\mu)$ will cross at least once in the $(0, \hat{\mu})$, interval. Let μ^* denote the value of μ such that to the left of μ^* , $w(\mu) < v(\mu)$ and to the right of μ^* , $w(\mu) > v(\mu)$. Additionally, note that by construction $w(\hat{\mu}) < v(\hat{\mu})$. This means that for all reputation values above or equal to $\hat{\mu}$ the incentive compatibility constraint holds. Also, w(0) > v(0) so the incentive compatibility constraint or value. This means that the reputation value for which functions $w(\mu)$ and $v(\mu)$ cross will be in the interval $(0, \hat{\mu})$. That is $\mu^* \in (0, \hat{\mu})$.

The existence and characterization of these steady state equilibria doesn't depend on the existence of a stationary cdf for reputations since the price function $p(\mu)$ is such that investors are indifferent between intermediaries. Moreover, since investors are homogeneous in their initial wealth level intermediaries are indifferent between investors. This implies that the assignment rule is not determined in the model.

Note that the equilibrium we study has the implication of a negative relationship between herding and reputation. This is, intermediaries with higher reputation are less prone to herd. This is in contrast with Graham (1999) who makes the opposite prediction. Graham argues that the reason for this is that if an intermediary's initial reputation is high and he herds, the rest of agents will not revise their opinion regarding the intermediary's type since they know that he herds and he will enjoy high profits, which are a linear function of his reputation. In our model, on the other hand, even if an intermediary herds investors will revise upward the intermediary's reputation due to the existence of a replacement probability. On the other hand, the dynamic system that describes the evolution or reputation shown in Figure (3.5) shows that the intermediary's decision has low impact on his reputation when his initial reputation is low and when it's high. Also, his remuneration will be low when μ_{t-1} is low, and therefore he will herd. However, if μ_{t-1} is high and the intermediary herds he will forfeit expected return of a larger percentage of assets under management. In order to avoid this loss the intermediary with high reputation will choose to acquire private information instead of herding.

Non Monotonic Policy Functions

Under certain conditions, the policy function given by (3.86) may be non-monotonic in the interval $\mu \in [0, \hat{\mu})$. This is shown in figure (3.15).



Figure 3.15: Equilibrium with nonmonotonic Policy Function

In this case for very low values of reputation ($\mu < \mu'$) the FI imitates, whereas if $\mu \in [\mu', \mu'']$, he prefers to acquire information. Moreover, if his reputation increases ($\mu \in (\mu'', \mu''')$), he will imitate again.

In order to avoid this situation it's necessary that the value function V doesn't display slope changes that are too steep for values of μ smaller than $\hat{\mu}$. In terms of the price function $p(\mu)$, this shouldn't have slope changes that are too steep for reputation values smaller than $\hat{\mu}$. This would be fulfilled if the value of the parameters P_G and $q(\eta)$ are not too great (small).¹⁶

$$\frac{\partial^2 p}{\partial \mu^2} = -\frac{2\left(R_I - R_H\right)^2}{\left(\mu\left(R_I - R_H\right) + R_H\right)^3}K$$

¹⁶The second derivative of $p(\mu)$ is given by:

and it is a negative (positive) function of parameters P_G and $q(\eta)$. Thus, for example, if we increase the value of P_G the second derivative's value falls, that is to say, it becomes more negative, and so function $p(\mu)$ becomes more concave. The concavity of $p(\mu)$ also depends negatively on the value of the integration constant K, which doesn't have a readily interpretation as the parameters previously mentioned, but whose value is limited by R_H since otherwise the fee charged by a FI with reputation 0, or that does not invest in

While we cannot rule out this type of situations from a theoretical perspective, the numerical exercises showed in the following Chapter suggest that this wouldn't be a relevant case, at least for the set of parameters studied.

3.2.3 Importance of the Investment Cost

As we discussed before, the cost of acquiring information plays an important part in the determination of $\hat{\mu}$. In addition, this cost will also affect the value of μ^* and the feasibility of the policy function (3.86). If *c* increases, function *w*'s intercept will be lower, whereas *v* will change slightly. Therefore, μ^* will be greater. If *c* is 0, *w* will be equal to $\left(1 - \frac{K}{R_H}\right) [R_I - R_H] W$. If this term is positive, $\mu^* = 0$. That is to say, all skilled FI will obtain information since they obtain greater expected utility for period *t* and a greater probability of having a better reputation (thus charging higher fees) in the future. Therefore, it's always possible to guarantee the existence of equilibria with spending in information if *c* is not too high.

3.2.4 Absorbing States

The equilibria in which FI with reputation greater than a critical level obtain information whereas those with lower reputation herd may not be sustainable if the dynamic system that describes the evolution of reputations presents absorbent states. Specifically, if the critical reputation μ^* is too low or too high, the proposed equilibria will not be feasible. These levels are called μ^*_{\min} and μ^*_{\max} , and are shown in Figure (3.16). Note that $\mu^*_{\min} > \lambda\theta$, while $\mu^*_{\max} = \theta$.

information, would be negative.



Figure 3.16: Absorbing States

In the first case, if $\mu^* < \mu_{\min}^*$ once a FI leaves the "punishment zone" (i.e. the zone where he earns a constant fee given by Equation (3.58)), he will never return to it, because even if he always makes bad investment decisions, his reputation will never be lower than μ_{\min}^* and therefore it will not fall below the critical level. In this case the equilibria with policy function given by (3.86) will be irrelevant because, even though there is a punishment zone in which the fee falls, the FI will always acquire information regardless of his reputation. Therefore herd behavior will not exist among skilled FI.

On the other hand, if $\mu^* > \mu^*_{max}$, once a FI enters the punishment zone he never leaves it because his reputation will evolve deterministically until it is equal to μ^*_{max} , but it will never increase above this level, independent of the FI's investment decisions. Therefore the FI will never reach the zone where he prefers to invest in reputation instead of herding.

3.2.5 Reputation Effects under other Remuneration Schemes

In this model, unlike Mailath and Samuelson (1998), (2001), (2004) and Vial (2008), changes in the strategy of the FI will affect not only their utility in future periods through changes in reputation (impact captured by v), but also their payments for period t, which is reflected in the term w. Indeed, if the intermediary decides to herd, it's more probable that his reputation falls lowering his expected utility for future periods, and also his expected revenue for the present period is lower, since this is a percentage $p(\mu)$ of the final value of assets under management.

If the FI's revenue consisted of a fixed payment $a(\mu)$, the set of feasible equilibria would be different. In this case, it's possible to demonstrate that FI's revenue will be given by:

$$a(\mu) = (R_I - R_H)W\mu + \kappa \tag{3.87}$$

where κ is an integration constant. Under this scheme, if a FI herds this will affect his expected utility for future periods (the term v corresponding to this situation would still be given by (3.83)) but their revenue in period t would not be affected; in fact, his expenses fall in c, so his utility for this period would increase. Now w would be equal to -c, and we can no longer assure that a policy such as (3.86) is optimal. In the best case, as seen in figure (3.17), we would have a situation in which the effects of reputation on future utility are sufficiently important to guarantee the existence of a reputation interval for which the FI invest in reputation. Nevertheless, for values of μ that are too low ($\mu \in [0, \mu']$) or too



Figure 3.17: Reputation Effects with Alternative Contract

high $(\mu \in [\mu'', 1])$ the FI will herd.

On the other hand, if the remuneration scheme contemplates the payment of a percentage $p(\mu)$ of the *initial* value of assets under management, W, the results in terms of optimal strategies would also be different, because once again function ω would be equal to -c, so equilibria in which all FI of high reputation obtain information wouldn't be feasible.¹⁷ This possibility is present in Farnsworth (2003) who argues that in this case explicit incentives would be needed to solve the moral hazard problem between intermediaries and investors.

Nevertheless, it's remarkable that the possibility of constructing a reputation substantially modifies the FIs' strategies and therefore, the type of feasible equilibria in this economy. Indeed, if it weren't possible to construct reputation and investors assigned prob-

¹⁷In the case of the Chilean mutual funds, the law establishes that the remuneration will be given by a percentage of the value of assets under management. This percentage is accrued daily.

ability θ to any FI being skilled, function w would be equal to $(R_I - R_H) pW - c$, whereas v would be equal to 0 since the investment decisions would not modify investors' beliefs. In this case, depending on how expensive it is to acquire information, either all or none of the skilled FI would make this investment. Thus, for this market to exist contracts would have to include the use of explicit performance bonds or a more complex remuneration scheme would have to be used in order for skilled FI to obtain information.

3.2.6 On the Existence of Sticky Fees

So far we have focused on a situation in which changes in reputation affect financial intermediaries' profits through changes in the fees they charge to investors. However, even if fees do not change across intermediaries and/or over time, for instance due to the existence of price controls or menu costs, a reputational equilibrium may still be feasible if intermediaries with good reputation manage larger portfolios than intermediaries with poor reputation. In this case, investing in reputation would be worthwhile because it increases the expected future utility through increases in assets under management. Sirri and Tufano (2008) study how investors react to changes in mutual fund fees but do not provide evidence of whether there is substantial variation in fees charged by individual funds in their sample. ICI (2008) presents evidence of falling mutual fund fees over time but again, at an aggregate level. Even though we are not aware of any works studying the evolution of fees over time for mutual funds or other institutional investors, the available data suggests that fees do not present substantial variation. For example, Chilean mutual funds are forced by regulation to establish fees in their prospectuses which usually are changed every four to five years. This would suggest that fees remain fixed for substantial time periods. However, the fees reported in the prospectuses are maximum fees to be charged by funds and not necessarily effective ones. Therefore, this is a topic that requires further research.

In order to asses the feasibility of reputational equilibria under sticky fees we modify our model by assuming that all intermediaries charge the same percentage of assets under management p. Now this will be a parameter rather than a function of intermediaries' reputation. Also, we assume that investors are heterogeneous in their initial wealth level. Thus there is a distribution of wealth across investors, with cdf given by F(W) which has a support $[\Psi, \bar{W}]$. We maintain the rest of our previous assumptions.

We make the following proposition regarding the assignment rule in the economy with sticky fees and heterogeneous investors.

Proposition 3.5 In an economy with fixed fee $p \leq 1 - \frac{R}{R_H}$ and heterogenous initial wealth across investors, if a stationary distribution G for reputations exist, then the assignment rule will be given by:

$$F\left(W\left(\mu\right)\right) = G\left(\mu\right) \tag{3.88}$$

where $F(W(\mu))$ is the cdf of initial wealth across investors, whereas $G(\mu)$ is de cdf of intermediaries' reputation; each intermediary will serve one investor and the intermediaries as a whole will serve all investors. Furthermore, there will be positive matching between investors and intermediaries. This is, investors with high levels of wealth will be served by intermediaries with good reputation.

Proof. If fees are such that $p \leq 1 - \frac{R}{R_H}$ then investors will be willing to hire any financial intermediary, even those who are known to herd in equilibrium. This can be seen

from the investors' participation constraint:

$$(1-p) R_H W \geq R W$$

$$p \leq 1 - \frac{R}{R_H}$$
(3.89)

Therefore, in equilibrium intermediaries will all investors. Given the existence of capacity constraints, each intermediary will serve one investor. Also, investors won't be indifferent between all intermediaries due to the fact that their expected utility is increasing in the intermediaries' reputation and p is fixed. This means that there will be an excess demand for the services of the intermediary with the highest reputation, $\bar{\mu}$. However, this intermediary can only attend one investor which means that he will choose to manage the portfolio of the investor with the highest wealth endowment, \overline{W} . Note that intermediaries will not be indifferent between investors. However, the intermediary with the second highest reputation will have to conform with attending the investor with the second highest wealth level. There is no way to persuade the investor with the highest wealth level to hire him since discounts can't be offered (recall that p is fixed). This is also true for the investor with the second highest level of wealth: he cannot persuade the intermediary with the highest reputation to work for him offering him a higher fee since p is fixed. Therefore, the investor with the second highest level of wealth will hire the intermediary with the second highest reputation and so no. This means that the assignment rule will be given by equation (3.88). Finally, note that:

$$\frac{\partial W}{\partial \mu} = \frac{f(W)}{g(\mu)} > 0 \tag{3.90}$$

where f and g denote the pdf for investors' initial wealth level and intermediaries' reputation, respectively. This is, there will be positive matching between investors and intermediaries. \blacksquare

In this case, the skilled intermediaries' Bellman equation will be given by:

$$V(\mu_{t}) = \max_{a \in \{I,H\}} \left\{ pR_{I}W(\mu) - c + \rho E\left[V\left(\mu_{t+1}\right)|a=I\right], pR_{H}W(\mu) + \rho E\left[V\left(\mu_{t+1}\right)|a=H\right] \right\}$$
(3.91)

This equation is very similar to equation (3.60), indeed, the set of feasible equilibria will resemble that of the case with homogeneous investors and variable fees. In particular, functions $w(\mu)$ and $v(\mu)$ will now be given by:

$$w(\mu) = c - pW(\mu)(R_I - R_H)$$
(3.92)

Without loss of generality we assume that the lower bound of the wealth distribution is equal to zero. Then we have that:

$$w\left(0\right) = c \tag{3.93}$$

$$w(1) = c - p\bar{W}(R_I - R_H)$$
 (3.94)

$$\hat{\mu} = \frac{c}{p\bar{W}\left(R_I - R_H\right)} \tag{3.95}$$

For suitable investment cost c, it will be true that $\hat{\mu} \in (0, 1)$. We plot function $w(\mu)$ in Figure (3.18).

Note that unlike the case with flexible fees, function w doesn't depend on the probability assigned by investors to the intermediary acquiring information. Therefore, wwill be continuous regardless of the intermediary's strategy.

On the other hand, function $v(\mu)$ is still given by (3.83) and looks like the one in figure (3.10). This is due to the fact that the dynamic system that describes the evolution



Figure 3.18: Function w(mu) with Sticky Fee

of beliefs will present a discontinuity if the skilled intermediaries' policy function is given by (3.86).

Finally, note that all the arguments given for the case with flexible fees still apply when we have sticky fees, in the sense that for all reputation values for which investors assign $\beta = 0$, intermediaries will find it optimal to herd, validating investors' beliefs. Also, there will be some value $\mu^* \in (0, \hat{\mu})$ such that, for reasonable investors' beliefs, intermediaries will herd whenever their reputation is less than μ^* and will acquire in information otherwise. Figure (3.19) below shows this equilibrium.

We have shown how a reputational equilibria may arise even if fees are fixed and equal for all intermediaries provided that investors are heterogeneous in their initial wealth level so that the reward for investing in reputation is managing larger funds thus obtaining higher expected utilities.



Figure 3.19: Equilibrium with Investment and Sticky Fees

CHAPTER 4

Numerical Examples

We use a MATLAB routine, which is showed in the Appendix, to study the model's comparative statics properties. Specifically, we are interested in establishing how herd behavior changes when the model's parameters are modified. We first focus on the case with flexible fees and later on the case with sticky fees. Later on we compare some of the equilibrium properties for the static version of the model with those of the flexible and sticky fee versions.

4.1 The Case with Flexible Fees

We will explain the procedure to determine the value of μ^* , the critical reputation level for which skilled intermediaries are just indifferent between acquiring information and herding. The parameters for our baseline case are specified below.

The values chosen for the model's parameters imply an ex-ante net expected return of 60% for both the risk free and risky asset. Also, the critical reputation values for which the model presents absorbing states are $\mu_{\min}^* = 0.1251$ and $\mu_{\max}^* = 0.5$. This is, if $\mu^* < 0.1251$ in equilibrium no skilled FI will herd, whereas if $\mu^* > 0.5$, once a skilled FI lands in the "punishment zone" he will no longer acquire information. The implied investment costs for intermediaries are given by the ratio $\frac{c}{W}$ and is equal to 50 basis points¹. For the parameters

¹Wermers (2000) reports that mutual funds have mean investment costs of 100 basis points but we choose

Parameter	Description	Value
η	Successful imitation probability	0.45
P_G	Information quality	0.85
c	Information cost	3
W	Assets under management	600
λ	Replacement probability	0.15
δ	Discount factor	0.75
r_G	Risky asset's return in good state	2.2
r_B	Risky asset's return in bad state	1
q	Risky asset's price	1
π	Good state (unconditional) probability	0.5
θ	Mass of skilled FI	0.5

Table 4.1: Flexible Fees Model's Parameters: Baseline Case

chosen $\mu^* = 0.1792$. Also, we set $K = R_H$, which implies that a FI that herds will charge a fee of zero. Also, our parameters imply that $\hat{\mu} = 0.6638$.

4.1.1 Determination of μ^*

First, we plot the price function in Figure (4.1) and the beliefs updating rule in Figure (4.2). In order to determine μ^* we initially assume it's value to be equal to zero. We then proceed to obtain the optimal policy function given our guess. The MATLAB routine plots the functions w and v in Figure (4.3) and the value function V in Figure (4.4).

We see that $\mu^* = 0.1792$ which is different from our initial guess. Therefore we update our guess and determine that μ^* is indeed equal to 0.1792. Also, we update our plots to reflect this. The updated plots are shown in Figures (4.5) through (4.8).

our values so as to maintain reasonable fees range and interesting μ^* values.



Figure 4.1: Price Function: Baseline Case



Figure 4.2: Reputation Evolution: Baseline Case



Figure 4.3: Functions w(mu) and v(mu): Baseline Case



Figure 4.4: Value Function: Baseline Case



Figure 4.5: Price Function: Baseline Case with Updated Mu*



Figure 4.6: Reputation Evolution: Baseline Case with Updated Mu^*



Figure 4.7: Functions w(mu) and v(mu): Baseline Case with Updated Mu*



Figure 4.8: Value Function: Baseline Case with Updated Mu^*

4.1.2 Comparative Statics

The following expressions will be useful in explaining our results:

$$\Delta R \equiv R_I - R_H = \pi \left(1 - \pi\right) \left(1 - \eta\right) \left(2P_G - 1\right) \frac{(r_G - r_B)}{q} > 0 \tag{4.1}$$

$$\Delta P \equiv P_I - P_H = \frac{1}{2} (1 - \eta) (2P_G - 1)$$
(4.2)

$$\Delta \mu \equiv \mu_g - \mu_b = \frac{(1-\lambda)\,\Delta P\mu\,(1-\mu)}{(\Delta P\mu + P_h)\,(1-\Delta P\mu - P_h)} \tag{4.3}$$

Equation (4.1) shows the difference in expected return between a portfolio managed by a skilled intermediary who acquires information and one managed by an intermediary who herds. Equation (4.2) shows the difference in the probability of making a good investment decision between an intermediary who invests and one who herds. Finally, Equation (4.3) shows the difference in next period's reputation if the intermediary makes a good investment decision as opposed to a bad one.

We now proceed to report the results of our comparative statics analysis, detailing how changes in the model's parameters affect the incentives faced by FI and how this, in turn, affects the degree of herd behavior in the delegated portfolio market. Throughout our exercises we change the parameter values, thus affecting μ^* , however, we focus on equilibria in which $\hat{\mu} \in (0, 1)$, so functions $w(\mu)$ and $v(\mu)$ cross only once in the [0, 1] interval.²

Changes in the Probability of Successful Imitation

An increase in η will have a negative effect on ΔR . This, in turn, will lower investors' willingness to pay, causing a decrease in the price function $p(\mu)$. This means that skilled FI will find acquiring information less attractive since they would get a smaller

²For all the following excercises μ^* was found using the iteration procedure described above. Although the numer of iterations needed to find the critical reputation value varied from case to case we always obtained convergence.



Figure 4.9: Mu^{*} Comparative Statics: Eta

percentage of end-of-period assets under management. In terms of $w(\mu)$, this function will move towards the horizontal axis.

On the other hand, the increase in η will also affect ΔP , since now the probability of making a good investment decision if a FI herds will be higher. Finally, $\Delta \mu$ will also be reduced because given a lower ΔP , the result of the investment decision will be less informative to investors in terms of guessing if a given type of decision was made by a skilled intermediary with information or by an intermediary who herd.

These changes will affect function $v(\mu)$, reducing its value, which will make herding more attractive than before in terms of the change in expected utility for future periods. Therefore, both in terms of this period and future periods' expected utility, skilled investors will find herding more attractive than before, resulting in an increase in μ^* . Figure (4.9) shows how changes in η affect μ^* .

Changes in Information Quality

An increase in P_G will have a positive effect on ΔR . This, in turn, will raise investors' willingness to pay, causing an increase in $p(\mu)$. This means that skilled FI will find acquiring information more attractive since they will get a higher percentage of endof-period assets under management. In terms of $w(\mu)$, this function will move away from the horizontal axis.

Also, ΔP will change since now the probability of making a good investment decision if a FI invests will be higher. Finally, $\Delta \mu$ will also be increased because given a higher ΔP , the result of the investment decision will be more informative to investors

These changes will affect function $v(\mu)$, rasing its value, which will make herding less attractive than before in terms of the change in expected utility for future periods. Therefore, both in terms of this period and future periods' expected utility, skilled investors will find herding less attractive than before, resulting in a reduction in μ^* . This is shown in Figure (4.10).

Changes in Information Cost

An increase in c won't affect neither ΔR or $p(\mu)$. However, function $w(\mu)$ will decrease, meaning that skilled FI will find acquiring information less attractive since it's costlier to do so.

Also, the increase in c won't change ΔP or $\Delta \mu$. Nevertheless, function $v(\mu)$ will move slightly towards the horizontal axis, reflecting the impact of higher investment costs on profits.

Therefore, both in terms of this period and future periods' expected utility, skilled



Figure 4.10: Mu* Comparative Statics: PH

investors will find herding more attractive than before, resulting in an increase in μ^* . This is shown in Figure (4.11) below.

Changes in Assets Under Management

An increase in W doesn't affect ΔR or $p(\mu)$. Function $w(\mu)$ will move towards the horizontal axis since now if the skilled FI herds he will obtain some percentage $p(\mu)$ of a larger portfolio. For the same reason, function $v(\mu)$ will move away from the horizontal axis, even though ΔP and $\Delta \mu$ haven't changed. Therefore, both in terms of this period and future periods' expected utility, skilled investors will find herding less attractive than before, resulting in a reduction in μ^* , which is shown in Figure (4.12).

Changes in Replacement Probability

If λ increases, it is clear from equations (4.1) - (4.3) that the only impact on intermediaries' behavior will come through a reduction in $\Delta \mu$. Indeed, with a higher replacement



Figure 4.11: Mu* Comparative Statics: c



Figure 4.12: Mu* Comparative Statics: W



Figure 4.13: Mu* Comparative Statics: Lambda

probability, building a good reputation becomes less attractive, since it's more likely that the intermediary won't be in the market in future periods to benefit from his investment. This effect will cause function $v(\mu)$ to move towards the horizontal axis, increasing μ^* , as Figure (4.13) illustrates.

Changes in Discount Factor

If δ increases then function $v(\mu)$ will be increased, since profits in future periods will have a higher present value. Therefore, in equilibrium there will be less herding, as Figure (4.14) shows.

Changes in Good State Return

An increase in r_G will have a positive effect on ΔR . This, in turn, will raise investors' willingness to pay, causing an increase in $p(\mu)$ and $w(\mu)$, which will make acquiring information more attractive.



Figure 4.14: Mu^{*} Comparative Statics: Delta

Even though ΔP will not change, function $v(\mu)$ will increase, since profits in future periods will be higher due to the increase of the risky asset's pay in the good states, which will also make herding less attractive. Figure (4.15) illustrates the results.

Changes in Bad State Return

Even though an increase in r_B raises the risky asset's ex-ante expected return, it decreases the attractiveness of acquiring information since the difference in payment in the good and bad states is reduced. Therefore investors' willingness to pay, reflected in $p(\mu)$, is reduced, which has a positive effect on $w(\mu)$, making herding more attractive. For the same reason function $v(\mu)$ will move towards the horizontal axis, even though ΔP and $\Delta \mu$ haven't changed. The result will be an increase in μ^* , which is showed in Figure (4.16).

Changes in Asset's Price

Our numerical simulations show that an increase in q has no significant effect



Figure 4.15: Mu* Comparative Statics: rH



Figure 4.16: Mu^{*} Comparative Statics: rL



Figure 4.17: Mu^{*} Comparative Statics: q

on $p(\mu)$. However, ΔR will be reduced, which will increase $w(\mu)$, making herding more attractive relative to acquiring information. The reduction in ΔR will also bring about a reduction in $v(\mu)$, further increasing the incentives to herd. As a result herd behavior will be more frequent, as Figure (4.17) shows.

Changes in States Probabilities

In this case, the effects on μ^* are not straightforward to analyze. This can be seen from (4.1) which shows that an increase in π will raise ΔR only if $\pi < \frac{1}{2}$, otherwise the effect will be negative. The numerical simulations show that for low π values, an increase in this parameter will affect negatively $p(\mu)$ (recall that this is a function of both ΔR and R_h). However, ΔR will increase, offsetting the first effect and causing function $w(\mu)$ to move towards the horizontal axis. For similar reasons, function $v(\mu)$ will be slightly increased. The result in this case will be a reduction in μ^* . However, as π increases, both $p(\mu)$ and



Figure 4.18: Mu* Comparative Statics: pi

 ΔR will fall, therefore increasing function $w(\mu)$. Accordingly $v(\mu)$ will be reduced and the result will be an increase in μ^* . Figure (4.18) summarizes these results.

Changes in Mass of Skilled Intermediaries

A change in θ will have no effect on $p(\mu)$ or $w(\mu)$. Also, it won't affect the probability of making good investment decisions or the difference in reputation between making good or bad investment decisions. However, θ will affect the intermediaries reputation's absolute value, thus affecting his incentives through changes in $v(\mu)$. Our results suggest that the final effect on μ^* will be small and non-monotonous, as Figure (4.19) shows. However, even if this parameter doesn't have substantial direct effects on the amount of herd behavior, its role is important since it determines the reputation values for which the dynamic system describing the evolution of beliefs presents absorbing states. Figure (4.20) shows how increases in θ widen the range of reputation values for which our proposed equilibrium



Figure 4.19: Mu^{*} Comparative Statics: Theta

is feasible.

We summarize our findings regarding the effects of changes in the model's parameters in Table (4.2) below.

4.2 The Case with Sticky Fees

Now we proceed to solve for μ^* in the case where intermediaries charge a fixed fee and investors differ in their initial wealth level. For the purposes of our example we assume that intermediaries' reputation follow an uniform distribution with support $[\mu^*_{\min}, \mu^*_{\max}]$ to be determined below. Also we assume that investors' initial wealth level also has uniform distribution with supports $[W_{\min}, W_{\max}]$ specified below. The parameters for our baseline case are given in Table (4.3).

We maintain or previous parameter values from the flexible fee case in order to



Figure 4.20: Mu*max and Mu*min Comparative Statics: Theta

Parameter	Description	Effect on μ^*
η	Successful imitation probability	+
P_G	Information quality	-
c	Information cost	+
W	Assets under management	-
λ	Replacement probability	+
δ	Discount factor	-
r_G	Risky asset's return in good state	-
r_B	Risky asset's return in bad state	+
q	Risky asset's price	+
π	Good state (unconditional) probability	Non-monotonous
θ	Mass of skilled FI	Non-monotonous

 Table 4.2: Comparative Statics Summary: Flexible Fees

Parameter	Description	Value
p	Fee (basis points)	330
η	Successful imitation probability	0.45
P_G	Information quality	0.85
c	Information cost	3
W _{min}	Lower support W	100
W _{max}	Upper support W	1100
λ	Replacement probability	0.15
δ	Discount factor	0.75
r_G	Risky asset's return in good state	2.2
r_B	Risky asset's return in bad state	1
q	Risky asset's price	1
π	Good state (unconditional) probability	0.5
θ	Mass of skilled FI	0.5

Table 4.3: Sticky Fees Model's Parameters: Baseline Case

make both cases as comparable as possible. As before, the model's parameters imply an ex-ante net expected return of 60% for both the risk free and risky asset. Also, the critical reputation values for which the model presents absorbing states are $\mu_{\min}^* = 0.1251$ and $\mu_{\max}^* = 0.5$. The sticky fee p is set equal to 330 basis points. This means that for an investor with initial wealth level of 600 that is served by an intermediary with average reputation ($\mu = \theta$) expected utility will be the same as for investors in the flexible fee case. For the parameters chosen the critical μ value that determines whether FI herd or acquire information is $\mu^* = 0.1862$.

4.2.1 Determination of μ^*

The procedure to determine μ^* is the same as in the sticky fee case. The plots also turn out to be very similar to the previous case; therefore we only show the graphs of functions w and v for the initial guess $\mu^* = 0$ and for the correct guess $\mu^* = 0.1862$. The



Figure 4.21: Functions w(mu) and v(mu): Baseline Case with Sticky Fees results are shown in Figures (4.21) and (4.22).

4.2.2 Comparative Statics

The comparative statics exercises for the sticky fee case shows that μ^* behaves in a similar way to the flexible fees case. The results suggests that herding increases with investment costs; the probability of successful imitation; the risky asset's payment on the bad state; and the risky asset's price. On the other hand, herding decreases with information accuracy; the discount factor and the risky asset's return in the good state. However, we obtain somewhat different results for the rest of parameters. Namely, the replacement probability, the good state unconditional probability and the mass of skilled intermediaries. For the replacement probability we find a non-monotonous effect: increases in this parameter lead to an increase in μ^* . However, for high replacement probabilities (more than 80%) further increases reduce μ^* . This is shown in Figure (4.23). In the case of the good state



Figure 4.22: Functions w(mu) and v(mu): Updated mu* with Sticky Fees

unconditional probability, for the range of parameters used we find a negative effect of increases in π on μ^* . But as Figure (4.24) shows, the effect becomes very small as π increases. Finally, for the mass of skilled intermediaries the results showed in Figure (4.25) suggest a positive, although weak relationship between θ and μ^* .

4.3 The Dynamic versus the Static Equilibria

In our static version of this economy we demonstrated that, if a separating equilibria exists, then investors' expected utility will be given by:

$$W_{t+1} = R_H W \tag{4.4}$$

We also noted that, for a separating equilibria to be feasible, it was necessary that skilled intermediaries charged a negative fixed fee to investors in order for them to credibly "reveal" their type. In this way, skilled intermediaries offered a fixed monetary payment to investors


Figure 4.23: Mu* Comparative Statics with Sticky Fees: Lambda



Figure 4.24: Mu* Comparative Statics with Sticky Fees: pi



Figure 4.25: Mu* Comparative Statics with Sticky Fees: Theta

and in return kept all the portfolios returns, i.e. the charged percentage fee was 100%. An unskilled intermediary would not benefit form offering such a contract, since he has lower probability of making good investment decisions and therefore he would have negative expected utility under such an arrangement. However, if this type of contracts, which contemplate the use of negative monetary payments is not available, for example due to liquidity constraints for intermediaries or due to legal restrictions, the separating equilibrium would unravel. An additional reason for remuneration schemes that present a negative fixed payment to be prohibited could be the existence of limited liability as in Dow and Gorton (1997) and Bhattacharya (1999). These authors emphasize how limited liability prevents investors from screening skilled and unskilled intermediaries. In this sense reputation could be seen as a mechanism to allow portfolio delegation to take place even without perfect screening. The stylized facts suggest that the type of remuneration schemes just described isn't used by financial intermediaries (at least by mutual funds). Therefore, an additional mechanism may be in place so as to make some type of equilibria feasible in the delegated portfolio management market. It turns out that in a dynamic equilibrium of the model where it is possible for financial intermediaries to build a reputation, a partially separating equilibria is feasible. Indeed, investors are never completely sure about the type of intermediary they hire. However, skilled intermediaries with reputation higher than a critical value will obtain information. All that is needed for this is the use of a contract featuring a percentage fee of the final value of assets under management. This fee is given by:

$$p(\mu) = 1 - \frac{K}{\mu\beta (R_I - R_H) + R_H}$$
(4.5)

Moreover, if $K = R_H$, which implies that an intermediary who doesn't acquire information (or one with reputation equal to zero) will charge a fee p = 0; then investors' expected utility will be equal to $R_H W$, just as in the separating static equilibrium. In this sense the possibility of investing in reputation acts as a substitute for the use of more sophisticated remuneration schemes.

Comparing expected utility for intermediaries between the static and dynamic equilibria is not straightforward since we have to account for the replacement probability λ , which is not present in the former. Moreover, we would also have to compute the numerical value of the Bellman equation for intermediaries in the dynamic model which was made using numerical examples.

In our baseline case $R_H = 1.6945$ and W = 600, implying that investors' expected utility will be equal to 1016.7 both in the static and dynamic equilibria. Also, for a skilled intermediary the present value expected utility in the static case equals 116.6, while for a unskilled intermediary it is $0.^3$ In the dynamic case, a skilled intermediary with average reputation ($\mu = \theta = 0.5$) has expected utility of 95.4, while an unskilled intermediary's present value of expected utility is strictly positive since p(0) = 0.

We highlight the fact that, given the parameters' values, in the static case, for skilled investors to be willing to obtain information, the percentage fee of assets under management paid must be no less than $\frac{c}{W(R_I-R_H)}$. This turns out to be 433 basis points. On the other hand, in the dynamic case with investment in reputation and flexible fees, paid fees will vary between 121.3 basis points for an intermediary in the brink of falling in the "punishment zone" (i.e. an intermediary whose reputation is $\mu^* = 0.1792$) and 486.8 basis points for an intermediary with the highest (theoretically) possible reputation (which is equal to 0.75 for the baseline case). For an intermediary with average reputation (equal to $\theta = 0.5$) the fee paid is 329.9 basis points.

Therefore we see that a skilled intermediary will acquire information if he is paid 121.3 basis points of assets under management, which is considerably less than the fee that he would have to receive in the static case, i.e. a situation in which investing in reputation is not possible. In fact, the minimum required fee in the static case is 3.5 times the minimum required fee in the reputational equilibrium. Of course, investors are no better in the dynamic scenario, since they don't receive a fixed payment from intermediaries in return from managing their portfolios. However, it's remarkable that the possibility of building a reputation makes aligning incentives "cheaper" in the sense of requiring remuneration schemes

³In order to calculate the expected present value for the skilled intermediaries we use the adjusted discount factor ρ , which takes into account both the intermediaries' discount factor δ and the replacement probability λ .

that require less power (i.e. a lower share of assets under management is paid to investors) and are far simpler (i.e. they don't require negative fixed monetary payments to intermediaries). Nevertheless, there is a cost involved, since in the reputational equilibria a complete separation between types isn't achieved, hence some skilled (unskilled) intermediaries are paid less (more) than they should; and also some skilled intermediaries herd whereas in the static equilibrium they always acquire information. Mailath and Samuelson (1998) also point out that the use of implicit incentives such as reputation may be a cheaper way to align incentives instead of firms offering guarantees to compensate consumers receiving bad outcomes or poor service. They argue that this could be prohibitively expensive if exerting high effort didn't always guarantee a good result. This argument is particularly valid in a delegated portfolio management context in which the final value of an investment portfolio is beyond the firm's control.

In the case of the sticky fee equilibrium, fees are fixed at 330 basis points. This implies that an investor with initial wealth equal to 600 and that is served by an intermediary with average reputation (equal to 0.5) will have an expected utility of 1016.7. Also, a skilled intermediary with average reputation has an expected utility equal to 90.8, while an unskilled intermediary's present value of expected utility will be positive. We note that in this case intermediaries with reputation as low as 0.1862 acquire information. Such intermediaries manage portfolios with an initial value of 176.3. In the static equilibrium, if fees were fixed at 330 basis points, the minimum amount of assets under management that a skilled intermediary must receive in order to obtain information is equal to 788.1. Therefore, skilled intermediaries must have 4.5 times more assets under management in the

Type of Equilibrium	Static	Rep. w/flexible fees	Rep. w/sticky fees
Investors' Expected Utility	1016.7	1016.7	1016.7
Skilled FI's Expected Utility	116.6	95.4	90.8
Unskilled FI's Expected Utility	0	> 0	> 0
Minimum required fee	433	121.3	NA
Ratio of minimum fees	1	3.6	NA
Minimum required AUM	788.1	NA	176.3
Ratio of minimum AUM	1	NA	4.5

Table 4.4: Summary: The Static Equilibrium versus Reputational Equilibria

static case compared to the reputational equilibrium with sticky fees in order to be willing to acquire information.

Table 4.4 summarizes the comparison between the static and the reputational equilibria with flexible and sticky fees.

CHAPTER 5

Empirical Discussion

In this Chapter we summarize the models' empirical predictions. We discuss to what extent these predictions are supported by previous evidence and point out areas in which further empirical research is needed. Then we propose an estimation strategy in order to obtain further evidence to evaluate the validity of our predictions.

5.1 Empirical Predictions

Based on the theoretical results and numerical exercises obtained in the previous Sections we obtain the following empirical predictions:

- **Prediction 1.** There will be a negative relationship between imitation or herd behavior and the FIs' reputation.
- **Prediction 2.** There will be cross-sectional dispersion in fees charged by the FI and/or the amount of their AUM.
- **Prediction 3.** There will be a positive relationship between the FIs' fees/AUM and his reputation.
- **Prediction 4.** There will be variation in the time series of fees charged by any given FI and/or the amount of their AUM.

Prediction 5. Herding increases with higher investment costs.

Prediction 6. Herding increases when imitation is easier.

Prediction 7. Herding increases with higher replacement or exit probabilities.

Prediction 8. Herding decreases with more accurate private information.

Prediction 9. Herding decreases with more uncertainty about the value of assets.

Prediction 10. Herding decreases when intermediaries are more patient.

Prediction 11. An increase in the mass of skilled intermediaries in the market will have a nonnegative effect on herding.

Regarding the first prediction, the work by Chevallier and Ellison (1999) studies the USA mutual fund managers market. Using age of manager as a proxy for reputation, the authors find that older managers tend to imitate less the decisions of the rest. As theoretical support for this finding the authors cite the work by Scharfstein and Stein (1990). However, this work simply postulates the existence of a relationship between herd behavior and reputation, whereas our work predicts that this relation will be negative (i.e. a greater the reputation, lowers the probability of incurring herd behavior). In a similar line, Hong et al (2000) find a negative relationship between herd behavior amongst financial analysts and their age (proxy for reputation). We emphasize how Graham (1999) presents evidence contradicting this prediction for a sample of investment newsletters' recommendations.

Hortaçsu and Syverson (2004) study the dispersion in fees charged by mutual funds in the USA. The authors find evidence of substantial dispersion in fees even in closely defined categories (e.g. mutual funds that replicate the S&P 500 index). The authors find that these differences can be explained partially by the existence of search costs for investors. Although this study's objective is not to determine if reputation plays a part in explaining the existing dispersion, it's suggestive that one of the authors' findings is that investors are willing to pay larger fees to older mutual funds (age is a commonly used proxy for reputation). It's worth emphasizing that our prediction of dispersion in fees charged is not shared by the model of Heinkel and Stoughton (1994), who, in fact, make the opposite prediction, sustaining it with evidence provided by Lakonishok et al (1992). However, see Khorana, Servaes and Tufano (2008), who find evidence that suggests the existence of considerable dispersion in fees charged by mutual funds world-wide. Regarding the existence of cross-sectional dispersion in AUM, Sirri and Tufano (1998) provide some supportive evidence for this prediction; the mutual funds in their sample have a mean value of AUM of \$588.2 millions with a rather large standard deviation of \$2267.7 millions. However, Sirri and Tufano's focus isn't to relate the size of AUM with mutual funds' reputation.

On the other hand, the work by Carter and Manaster (1990) presents preliminary evidence suggesting the existence of a positive relationship between the fees charged by investment banks when making IPOs and their reputation. The measure of reputation used in this case is built on the basis of the investment bank's relative position within publicity made to advertise the IPO, since generally there are several banks participating simultaneously and it's presumed that the most prestigious one occupies the first place in the list. Concerning the prediction of AUM dispersion over time, in the stylized facts Section of Chapter 2 we already discussed some works such as Chevalier and Ellison (1997), and Sirri and Tufano (1998), who document how the size of mutual funds varies over time. While these authors show that the relationship between flows and performance is asymmetric and less steep for more experienced funds, our prediction simply states that as intermediaries' reputation changes over time the size of AUM should also change. In our model, whether these changes are of bigger magnitude depends on the change in reputation resulting from making good or bad investment decisions. In particular, if the improvement in reputation resulting from good decisions is higher than the fall resulting from bad decisions, then there will be an asymmetric impact in the change of AUM. However, we cannot guarantee that this occurs for all parameter configurations.

The work by Graham (1999) finds supportive evidence for our 8th and 9th predictions. Namely, as the quality of intermediaries' information improves, herding should decrease as investing in reputation becomes more attractive. Also, when there is increased uncertainty about assets' value having private information is more useful, therefore reducing herding.¹ Regarding the relationship between reputation and herding, we find theoretical support for Graham's evidence and are able to reconcile his findings with our first empirical prediction. Indeed, Graham's evidence suggests that as a manager's initial reputation increases he will tend to herd more. However, as time goes by, the agents' reputation will endogenously evolve and change, and as it gets better, his incentives to herd will weaken. Therefore, prediction number 1 should be interpreted as relating herding with the agent's

¹In our setup it's difficult to disentangle the effects of changes in assets' unconditional expected return and variance, since for a Bernoulli distribution this two variables are intrinsecally correlated. Therefore we state our prediction in terms of assets' volatility instead of expected return, as done by Graham (1999).

current reputation, whilst prediction 11 relates herding with the agents' initial reputation.

Lastly, according to our knowledge no systematic empirical evidence exists regarding the existence of a positive relationship between reputation and fees/AUM for financial intermediaries who manage portfolios such as mutual funds. Also, there is lack of evidence regarding the existence of time-series dispersion for financial intermediaries' charged fees; ICI (2008) presents evidence of falling mutual fund fees over time but at an aggregate level. Also, even though we are not aware of any works studying the evolution of fees over time for mutual funds or other institutional investors, the existing data seems to suggest that fees do not present substantial variation. For example, Chilean mutual funds are forced by regulation to establish fees in their prospectuses which usually are changed every four to five years. This would suggest that fees remain fixed for substantial time periods. However, the fees reported in the prospectuses are maximum fees to be charged by funds and not effective fees². Also, there is further empirical work needed to asses how herding changes when acquiring private information is costlier; when imitation is easier; and when intermediaries are more patient due to higher discount factors or lower exit probabilities.

5.2 Empirical Strategy

This Section discusses an empirical methodology to obtain new evidence regarding the relationship between reputation and herding in delegated portfolio management markets. We will propose variables to proxy for financial intermediaries' reputation; the extent to which they herd; as well as information regarding investment costs, assets' return uncertainty, successful imitation probability, replacement or exit probability and discount

 $^{^{2}}$ For evidence on fee-waiving by money managers see Chirstoffersen (2001).

factor. We will first discuss how these variables could be proxied and then we briefly sketch an empirical strategy to find evidence which allows us to validate or reject our models' predictions.

5.2.1 The Variables

Reputation

Usually, reputation is proxied by a manager's age. This variable is used for example by Chevalier and Ellison (1999) and Arora and Ou-Yang (2001). The use of this variable is justified by arguing that it closely proxies a manager's stage in his career and the amount of information the market has gathered about his skills. In this same spirit, Hong et al (2000) proxy analyst's reputation by their experience. One disadvantage of these proxies is that it's possible for low skilled managers to keep their jobs for long time periods, for instance alternating between good and bad investment decisions. According to our model this kind of intermediary should have lower reputation than, say one that keeps making good investment decisions all periods. However, if tenure or age is used as a proxy, then both managers will seem to have the same reputation.

One way to avoid this could be the use of cumulative returns to proxy for reputation. This measure is used by Ippolito (1992) as a measure of quality for mutual funds. The model predicts that, on average, managers investing in private information should make better investment decisions than those who herd, thus obtaining better reputation. Of course, it's possible that in any given time period a lucky manager who herds makes a larger return than one who acquires information. However, over larger time intervals this should not be the case. The use of cumulative returns, for example for the last 3 or 5 years, could then be a reasonable proxy for reputation. In the spirit of Graham (1999) it could even be possible to construct a synthetic variable by defining that a manager has made a good investment decision if his portfolio return on this period exceeded some threshold, such as a stock index or the sample's mean return. This kind of proxies have the advantage of improving only if the manager shows a good performance.

Herding

One of the statistical measures of herding used by numerous authors is the Lakonishok, Shleifer and Vishny (LSV) measure, presented in Lakonishok, Shleifer and Vishny (1992b). This measure tries to identify the degree of correlation in trading patterns for a group of traders, such as pension or mutual funds, assessing if they tend to either buy or sell some particular stock in groups. If there is no herding between investors one could expect that trading is independent between them so, as the number of investor increases, the number of sellers of the stock should be roughly equal to the number of buyers. In this case the value of the LSV measure will be close to zero; otherwise the results are interpreted as suggesting the existence of herding. Bikhchandani and Sharma (2001) further elaborate on this measure and possible improvements. In our case the LSV measure won't be of much use since we are interested in assessing the degree of herding at the intermediary level and not only finding evidence of the existence of herding among a group of intermediaries.

Graham (1999) studies the recommendations of Investment Newsletter which consist in advising to increase or decrease the portfolio weighting of a certain stock. The author argues that the Value Line Investment Survey is the best known investment newsletter, whose advice is freely observable to investors. Moreover, this Survey has been well studied by the literature and would have a high stature. Therefore, it's assumed that this Survey has no reputational concerns and acts as the "market leader", which means that other newsletters may want to imitate its advice. Thus herding is modeled as a dummy variable taking the value of one if a newsletter makes the same recommendation as the leader and zero otherwise.

Chevalier and Ellison (1999) use three distinct measures to evaluate the degree in which a mutual fund manager's actions differ from the average. The first variable measures boldness in the sense of a manager having concentrated his portfolio in sectors that differ from those that are more popular at the time. This variable is the square root of the sum of squared differences between the share of fund i's assets in each of the industry sectors defined in this study and the mean share in each sector in year t among all funds in fund i's objective class (the authors study the growth and growth and income classes). The second variable measures boldness in terms of the unsystematic risk level in fund i's portfolio versus that of a typical fund. The variable is equal to the absolute value of the difference between unsystematic portfolio risk for fund i in period t and the mean of this variable for the rest of funds in i's category in period t. Additionally, the third variable measures whether fund i takes a large bet on the direction of the market and is defined as the absolute value of the difference between fund i's beta in year t and the average beta in that year for the rest of funds in i's category.

Hong et al (2000) measure the degree in which security analysts herd when making earnings forecasts by estimating the absolute value of the difference between analyst's i earnings estimate for stock j in year t and the average value of the forecast for the rest of analysts. Since analysts cover several stocks at the same time, the authors use the previously describe measure of deviation for all stocks and then estimate a score to summarize the information.

The first step to build the score is to rank all analysts covering stock j in year t according to the degree of deviation. The boldest analyst is assigned the highest rank and so on. After that, the authors scale each analyst's rank for a stock by the number of analysts who cover that stock since analysts who cover stocks that are thinly followed are more likely to have lower average ranks than those who follow stocks with high coverage. Finally the deviation measure is the average of deviation scores for analyst's boldness scores in year t and the previous two years.

The authors also explore the use of two additional measures of herding. The first variable measures whether an analyst is the first person to issue an earnings estimate for stock j in year t. This analyst is assume not to herd, since there wouldn't be anyone else to imitate. By doing this the authors then estimate the probability of an analyst making the first forecast as a function of their independent variables. The second proxy for herding is the frequency of revisions of earnings forecasts. A higher frequency of revisions may be interpreted as evidence of the analyst changing his mind many times to accommodate the opinions of others (although as the authors recognize this could also be due to new private information arriving to the analyst). Specifically, the revision variable is estimated counting the number of times in a year an analyst revises his earnings forecast of a stock and comparing that to how often other analysts covering the same stock in the year change

their estimates.

Arora and Ou-Yang (2001) proxy the degree of herding by a mutual fund manager with two variables. The authors make the assumption that a manager's performance is measured against the average performance of funds in his objective group. Given this, the first variable measures the degree of correlation of fund i's monthly returns in year t with the monthly mean returns in the objective group. The proxy is then estimated as the absolute value of the difference between fund i's correlation and the average correlation for all funds in the objective group. The more a fund herds, the closer this variable should be to one. The second proxy for herding is the correlation coefficient between the monthly returns of fund i in year t and the monthly mean returns of the objective group. Higher values of correlation are considered as evidence of a higher degree of herding.

Maturana and Walker (2002) take a different approach to measure herding. They assume that there will be mutual funds that act as leaders in the market and conjecture that the rest of mutual funds will try to imitate the leaders' portfolio decisions. Therefore, the authors make pair-wise estimations of Granger causality between funds buying/selling decisions. For example, the authors determine whether fund i's portfolio decisions led the portfolio decisions of fund j or viceversa.

As the previous summary shows, there are several ways to proxy for herding. In our current context, perhaps the most suitable variables would be those that study the degree of correlation of portfolio weights across intermediaries' portfolios. Since portfolios are composed of hundreds of assets it's very unlikely to find correlation at the individual asset's level. Rather than that, it could be possible to assess the degree of correlation at broader levels such as 3 or 4 digits NAICS classifications for industries or at country levels for international portfolios.

There is also an issue of how herding should be interpreted and defined. In our theoretical model we assumed that financial intermediaries copied the portfolio decisions of another intermediary. In empirical work however, herding is usually defined as the degree of correlation between an intermediaries' portfolio or decisions and those of the rest of intermediaries. We note that since we study a situation in which all skilled intermediaries receive exactly the same information, the intermediary that herds will be effectively copying the behavior of a group of peers rather than just one rival.

It's also important to stress the fact that in a situation in which financial intermediaries are aware of their own type but not of their rivals' type there is a non trivial decision to be made regarding which intermediary or which group of intermediaries should they attempt to imitate if it isn't feasible to try to observe all intermediaries' actions. In this case it can be shown that the probability of making good investment decisions and therefore having larger expected returns and better reputation in future periods is increasing in the reputation of the intermediary being copied. In this case one could study whether one intermediary or group of intermediaries in the sample seems to act as a leader in the market. An alternative to examine this possibility is testing for Granger causality as Maturana and Walker (2002).

Other Variables

While the reputation and herding variables are the main focus of our model, in order to fully evaluate if the existing data validates our predictions we also need to come up with proxies for the rest of variables. Below we briefly discuss how to construct these proxies.

Successful imitation probability

A candidate for this variable would be legal requirements on portfolio disclosure. As argued by Wermers (2001), an increase in the frequency of the portfolio disclosure could have the negative effect of making imitation easier to free-riders. One disadvantage is that it seems unlikely that these requirements are changed frequently. Therefore we would have to find the latest regulatory change on this subject and proxy the imitation probability as a dummy variable taking the value of one for periods in which regulation is tighter.

Private information quality

Chan et al (2005) study the relationship between herding and private information quality for analysts' earning forecasts. The authors proxy the quality of private information by the dispersion in analysts' forecasts. A stock with high dispersion would be one in for which there is little reliable information that can help analysts forecast the future, therefore suggesting poor quality of private information. Since we are interested in studying financial intermediaries this proxy cannot be directly applied. Nevertheless it could be possible to identify intermediaries that belong to a larger financial group which includes analysts forecasts. If the branches of this group support each other and share information, then the dispersion in forecasts made by the corresponding unit could be used to proxy for quality of private information. Given that analysts' forecasts follow multiple stocks, an option for summarizing these information into a single variable could be used to build a quality score using the methodology employed by Hong et al (2000) for forecasts' boldness.

Private information cost

This variable could be proxied with management costs reported by financial intermediaries in their balance sheets and income statements. Before using this proxy, the data should be expressed as a percentage of assets under management in order to be comparable with variables such as fees, which are also usually reported in this fashion.

Discount factor

While this variable is widely used in calibration exercises and various estimates are available, these estimates are built at a more aggregate level. Given the difficulty in finding a reasonable proxy to be used in econometric applications this variable could be omitted as it plays a similar role to the replacement probability, whose estimation we discuss below.

Replacement probability

In our model we have assumed the existence of an exogenous replacement probability which takes the same value for all managers. However, Chevalier and Ellison (1999) and Hong et al (2000) estimate actual survival probabilities for samples of mutual fund managers and security analysts. If we define the unit of our analysis as managers then a similar methodology could be used, employing probit models to estimate this variable as a function of, say, manager's performance measured by generated excess return.

Uncertainty about the value of assets

The uncertainty about the value of assets has a rather natural empirical counterpart. Namely, the historic volatility of assets. However, since portfolios are composed of several financial assets, an aggregate measure of volatility should be used, such as last period's portfolio's standard deviation. Alternatively, a more forward-looking variable could be used, predicting the portfolio's future risk given its current composition using a GARCH methodology.

Mass of skilled intermediaries

Our model assumes a constant mass of skilled financial intermediaries. The empirical counterpart of this variable could be the proportion of intermediaries with returns in excess of the fees they charge. This is the definition of a skilled manager used by Berk and Green (2004). According to their calibration about 80% of managers in their sample satisfy this criterion.

5.2.2 The Estimation

Once all the previous variables are constructed the empirical estimation strategy would consist in running regressions relating the herding variable to the reputation proxies and the rest of variables. Since financial intermediaries such as mutual funds usually differ in their investment objectives and styles it's important to include controls for this in the regressions. Additionally, the possibility of an intermediary belonging to a larger financial group must also be taken into account. For example, many Chilean mutual funds are associated with commercial banks. In this case there are tied sales considerations that are absent for stand-alone mutual funds. As an example, of the relevance of this possibility, Table (5.1) below shows the top nine mutual fund companies by market share and for private commercial banks (when applicable). These rankings turn out to be fairly similar.

Firm	Ranking in Market		
	MF	Banking	
B. de Chile	1	2	
B. Santander	2	1	
BCI	3	3	
BICE	4	9	
B. Security	5	8	
L. Vial	6	NA	
BBVA	7	4	
Scotiabank	8	5	
Penta	9	NA	
Source: Author's calculations based on data of the Chilean			

Superintendency of Securities and Insurance and the

Chilean Superintendency of Banks and Financial Institutions.

Table 5.1: Mutual Fund Managers Ranking by Market Share.

Chapter 6

Conclusions

We have studied the delegated portfolio management market using a simple model in which financial intermediaries have the possibility of investing in reputation. The existence of equilibria with investment in reputation would allow this market to subsist even with the use of simple remuneration schemes.

It was demonstrated that in our model different types of equilibria may exist. In particular, there are equilibria in which no FI invest in information/reputation. Nevertheless, we also demonstrated that there are equilibria in which all FI with reputation higher than certain level μ^* will acquire information, whereas those with smaller reputation may imitate. Although from a theoretical point of view it's not possible to rule out that the FIs' policy function is non monotonic in this interval, our numerical exercises suggest that this isn't a common case. Therefore, one of our main results is that as an intermediary's reputation improves his incentives to herd decrease. As we showed in Chapter 3, there are two situations in which an intermediary may disregard the effects of his actions on his reputation. Namely, when his reputation is really bad or when it's really good. In these cases it is possible that the intermediary may try to cheat investors and shirk. Additionally, if the remuneration scheme is given by a percentage fee of the final value of assets under management (which is the case for most mutual funds, as we argued in the stylized facts Section); and if this fee is increasing in the intermediaries' reputation, then an intermediary with bad reputation that chooses to herd instead of acquiring private information will experience an expected loss in final value of assets under management, but since his profits are given by a small percentage of this value, he will choose to herd. On the other hand, an intermediary with good reputation that decides to herd will experience an important loss in expected profits, since these are given by a larger fee of the final value of assets under management. In order to avoid this loss the intermediary will acquire private information.

This prediction is also made by Avery and Chevalier (1999). However, in their case an agent with good reputation actually chooses a contrarian strategy, disregarding his private information and making the opposite decision from other agents in order to signal to principals that he is skilled. Of course, this behavior is inefficient from investors' point of view since it implies a misuse of private information. The work by Graham (1999) makes the opposite prediction: as the initial reputation of agents improves they will herd more because they want to avoid a large drop in profits associated with a fall in reputation, which in this model occurs if an agent's decision is different from that of other agents. Moreover, we reconcile Graham's findings with our own by noting that in our model there is a clear difference between an intermediary's initial reputation and his incentives to herd in the current period as opposed to his reputation in this period and his incentives to herd today. We find that it's possible that as intermediary's initial reputation increases herd behavior also increases. However, as his current period reputation gets better, the incentives to herd always decrease. The infinite time horizon of our model allows us to neatly point out the distinction between these two cases. In this sense we view our work as an alternative rationalization for the evidence found in Chevalier and Ellison (1999) and Hong et al (2000) regarding the relationship between reputation and herding. However, in our setup the mechanisms operating in the reputational equilibria are different. In particular due to our modelling decision of using a continuum of intermediaries, the portfolio choice of a particular intermediary contains no information regarding the possible type of another intermediary. This is the basic mechanism affecting the behavior of managers in Scharfstein and Stein (1990), Avery and Chevalier (1999) and Graham (1999). Additionally, while our model makes similar predictions regarding reputation and herding as that of Avery and Chevalier (1999), our findings are much more optimistic in the sense that lack of herding by intermediaries with high reputation is associated with efficient investment and use of private information. This is important because the works by Avery and Chevalier and Scharfstein and Stein assume a positive relationship between reputation and profits for intermediaries in a two-period setup. However, in the presence of the pathological behavior implied by these models, endogenously deriving a long-term positive relationship between reputation and willingness to pay seems a harder task (Ottaviani and Sørensen, 2006 make a similar observation). Moreover, rationalizing the increasing importance and presence of institutional investors in financial markets documented in Chapter 1 is difficult if all types of intermediaries, regardless of their reputation, make little or no use of private information. While we believe that the cases described by these authors may be of great relevance in determined time periods or situations, we argue that it's difficult to imagine that the delegated portfolio markets could have experienced such strong growth if pathological behavior was always present, since intermediaries would have a hard time competing with investors who trade on their own behalf and presumably always make good use of their

private information.

We also show how the size of the percentage fee that must be paid to intermediaries in order to align incentives can be considerably smaller if investing in reputation is possible as opposed to a static model. Our numerical exercises suggest that the minimum percentage fee necessary to align incentives in a static model is 3.6 times the minimum fee required in a reputational model. Moreover, we illustrate how the possibility of investing in reputation can allow the delegated portfolio management market to operate when the use of more sophisticated remuneration schemes is not possible. However, there is a cost involved since in the reputational equilibrium the intermediaries' types are never revealed to investors. Therefore, it's possible that some skilled unlucky intermediaries are punished by investors through low fees while some lucky unskilled intermediaries may be paid high fees. Nevertheless, since skilled intermediaries who acquire information have a greater probability of making good investment decisions, the risk of this type of scenario is bounded.

Additionally, we show that for a reputational equilibrium to be feasible, the gains from investing in reputation can either be obtained through higher fees or through larger assets under management. In both cases the intermediaries' expected profits are increasing in their reputation. If higher reputation is rewarded with more assets under management most of the qualitative characteristics of the reputational equilibrium are maintained. In this case, for a fixed fee, the minimum amount of assets under management that skilled intermediaries must receive in order to obtain information in a static equilibrium turns out to be 4.5 times the minimum amount of assets under management necessary to align incentives in the reputational equilibrium. On the other hand, since there is no complete separation between intermediaries in the reputational equilibria there is some loss in efficiency since unskilled intermediaries will have strictly positive expected utility, while skilled intermediaries will tend to have lower expected utility. However, the probability of these types of situation occurring are small since intermediaries who acquire private information are more likely to make good investment decisions in our model.

From a policy standpoint, our model points out the existence of a policy trade-off between demanding more transparency from institutional investors, such as requiring more frequent portfolio disclosure by mutual funds and making portfolio imitation easier, which encourages herd behavior. This point has been made before by Wermers (2001) and our model provides theoretical support for this argument.

We have also identified some areas for future empirical research such as the degree of time-series variation in mutual fund fees and we briefly outlined an empirical estimation strategy that would allow us to obtain further evidence to validate our models' predictions.

Regarding areas for future research, the most straightforward extension for our model would be to consider the case where asset's prices are endogenously determined. The methodology typically used is the one developed by Kyle (1985) and Glosten and Milgrom (1985) which features a risk-neutral market maker who receives trading orders from investors and sets bid and ask prices using all available information. In order to maintain incentives to acquire private information, it would be necessary to introduce noise traders in the economy in order to avoid prices reflecting all private information. This extension would allow us to study equilibrium price dynamics and properties for assets in the long-run reputational equilibrium.

An interesting extension would be to include the possibility of an unexpected negative shock in our model and study how financial intermediaries are affected. For example, consider the occurrence of a shock that affects the economy before the risky asset's return is due. Further, suppose that this is an event whose materialization was assigned probability zero by all agents. If this shock takes the form of an alternative investment opportunity for investors, then those financial intermediaries with lowest reputation would be most affected, since the portfolios they manage are a less attractive investment opportunity compared to those managed by intermediaries with high reputation. This would have direct empirical predictions regarding, for instance, the relationship between the amount of outflows from an intermediary during a financial crisis or panic and its reputation. In this sense, having a good reputation would be valuable not only because it means charging higher fees and managing larger portfolios, but also because it acts as a shock absorber for intermediaries.

In our model we have assumed that entry and exit probabilities for financial intermediaries are exogenous. Relaxing this assumption would allow us to gain insights upon the behavior of survival rates for intermediaries in a reputational equilibrium. Mailath and Samuelson (2001) study a market for reputations for a monopoly and provide a characterization of what types of firms buy good, average and bad reputations. However, an analysis for the case of perfect competition among many firms remains to be done.

On the other hand, considering a model with risk-averse investors and intermediaries would make our model more comparable to previous delegated portfolio management literature. In particular, it would be possible to study whether the irrelevance result proposed by Stoughton (1993) holds if investors have information about an intermediary's behavior in the past. Moreover, it would also be possible to evaluate whether implicit reputational incentives are a cheaper and more efficient way of aligning incentives as opposed to more sophisticated (e.g. quadratic) remuneration schemes. The difficulty of working with risk-averse investors and assets whose returns follow a normal distribution is that the Bayes' rule that describes the evolution of beliefs may no longer have a tractable form. Consider for example an intermediary who chooses the portfolio weights of two risky assets conditional on receiving stochastic private information. If this information follows a normal distribution and the intermediary has a lineal investment strategy, then the portfolio weights will follow a normal distribution. However, the return of the portfolio is the result of the sum of products of portfolio weights and assets returns. Such product need not have a normal distribution. Moreover, to our knowledge the sum of these products has no closed-form solution. Therefore, if investors update beliefs based on the portfolio return of the intermediaries, as in our current model, numerical methods would then have to be used in order to solve the model.

Finally, an overlooked topic is the existence of related markets for financial intermediaries. As Table (5.1) showed, often financial intermediaries are part of a larger group offering many services to investors. It would be interesting to characterize the types of incentives that arise in these situations. It's possible that multiple agency layers arise, a topic studied by Gervais et al (2005), or that the decisions made in one market such as commercial banking have an impact on other markets such as mutual fund management market.

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Appendix A

Delegated Portfolio Management Literature Review

We know proceed to discuss some of the delegated portfolio management literature. Some of these works attempt to rationalize one or several of the stylized facts discussed in Section 2, while others attempt to derive closed-form solutions for optimal contracts between investors and intermediaries. We also make a selective survey of the herding literature and the reputation literature, which are phenomena we are interested in exploring. For an alternative survey of the theoretical delegated portfolio management problem literature see Stracca (2006); for works that survey the theoretical and empirical research on herding see Bikhchandani and Sharma (2001) and Hirshleifer and Hong Teoh (2003). Throughout the survey we will use the terms principals and investors interchangeably and the same applies to the terms agents, financial intermediaries and managers.

A.1 Optimal Contracts

Finding closed-form solutions for optimal contracts between investors and intermediaries is attractive since it facilitates gaining insights on whether the use commonly observed remuneration schemes can be rationalized, while possibly providing theoretically based advice on how this schemes could be improved. The following works explore this topic.

Bhattacharya and Pfleiderer (1985)

The seminal work in the DPMP literature is the study by Bhattacharya and Pfleiderer (1985). The authors' purpose is to determine which characteristics should an optimal contract have in order to align the investors' and financial intermediaries' incentives when the latter's skill is not know by the former. This contract should only attract intermediaries with certain degree of skill, giving them at least their reservation utility and should be incentive compatible (i.e. FI should be better off by reporting accurate information to the investor rather than lying). Moreover, the setup studied is static.

The FI has the option of investing in a risk-free asset or a risky asset whose gross rate of return follows a normal distribution. The FI receives a private signal that is informative to predict the risky asset's return. This signal is received at no cost. The intermediaries differ in the quality of the signal they receive, which is measured by their precision (i.e. the inverse of the conditional variance of the risky asset's gross return). The more accurate the information, the closer the posterior on the risky asset's return will be to the signal received. On the other hand, if the information is rather noisy, the posterior will be close to the unconditional expected gross return.

The investor himself has the ability to receive signals and has some precision level. This means that if the investor randomly hires some intermediary, he may well be worse off than if he managed the portfolio by himself. Therefore, the optimal contract must attract FI whose precision is no lower than that of the investor.

Additionally, the authors assume that FI have reservation wages that are increasing in their precision. Finally, both investors and agents have negative exponential Bernoulli utility functions.¹

¹This type of preferences is also referred to as Constant Absolute Risk Aversion preferences or CARA,

Under this setup the authors examine the incentive alignment properties of a linear contracts. However, this linear remuneration scheme won't be successful in aligning incentives since given that the intermediary has CARA utility function, he may lie about his precision and cover this lie by reporting a false information. Moreover, it's possible that under the linear remuneration scheme the investor fails to attract FI with a high precision. The reason is that if an FI's precision is higher, this has two opposing effects on the minimum ex-ante payment he must receive in order to work for the investor. The first effect is an increase in his reservation wage which raises the minimum ex-ante payment he must receive from the investor. The second effect is an increase in expected payment if he works for the investor since if the FI realizes he has high precision, the expected utility from receiving a percentage of assets under management increases, which lowers the minimum exante payment he must receive form the investor. If the former effect is larger than the latter, then any contract offered to a high- precision FI will also be attractive to less skilled intermediaries. This would be the case if the reservation wage increases rapidly with the intermediary's precision.

In this case, the authors show that a non-linear remuneration scheme is necessary to screen out unskilled intermediaries and align incentives. Nonlinearity assures that the agent can no longer cover lies about his private information. The authors show that under the proposed remuneration scheme the payoff distribution that the FI obtains if he tells the truth stochastically dominates (in second order) the payoff distribution he would get if he lied, meaning that incentives would be aligned for all slightly risk-averse intermediaries. since it has the property that the amount of wealth that the individual wishes to invest in a risky asset is independent of his total wealth level.

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One drawback of this scheme is that it doesn't achieve optimal risk sharing between the investor and the FI since given CARA utility functions, in order to achieve optimal risk sharing the remuneration scheme should be linear, as demonstrated by Wilson (1968).

There is one important element overlooked by this work: the possibility that it may be necessary for an intermediary to make (costly) effort in order to receive private information. If effort is non observable this would aggravate the informational asymmetries between investors and intermediaries. The following work studies this possibility.

Stoughton (1993)

The work by Stoughton (1993) addresses the question of whether non-linear remuneration schemes continue to be optimal under a moral hazard setup. The author uses a framework similar to that of Bhattacharya and Pfleiderer (1985). There is one investor and one intermediary. Additionally, effort is modeled as a linear increase in the FI's posterior precision. Therefore, if the intermediary makes greater effort, his signal's quality increases, which in turn makes him pay more attention to the signal received than to the risky asset's ex-ante expected return². However, exerting effort is costly for the FI. This is modeled as a monetary cost.

Stoughton considers initially the first-best problem where effort is observable for the investor. In this case, the optimal remuneration scheme is linear (recall that this is due to the fact that both the investor and the FI have negative exponential utility functions). Also, the FI makes effort up to a point where the marginal disutility is equal to the marginal gain. Moreover, the author shows that as the investor's risk aversion decreases, the optimal

 $^{^{2}}$ Although the author maintains Bhattacharya and Pfleiderer's notation, he uses net assets returns instead of gross returns

effort level increases, since in this case the value of having more accurate information is higher given that a larger proportion of wealth is invested in the risky asset.

If effort is non observable, the linear remuneration scheme is no longer optimal, as there will be underinvestment in effort relative to the first-best case. Moreover, as long as the FI is risk averse, the share of the portfolio's final value promised in the remuneration scheme is irrelevant. The reason for this result (sometimes referred to as the *irrelevance result*) is that in a DPMP context, the FI decides both the effort level, and the portfolio composition. Therefore, a risk averse agent can shirk and then invest all the portfolio in the risk free asset, claiming he received a bad signal and obtaining a risk-free payment.³

The author then proceeds to examine the properties of a nonlinear remuneration scheme. Such a contract manages to align incentives (i.e. the agent doesn't lie about the signal received). Also, as the investor's risk aversion approaches zero, the outcome using the nonlinear remuneration scheme resembles the fist-best case.⁴

Carpenter et al (2001)

Carpenter et al (2001) make use of the mechanism design theory and, appealing to the direct revelation principle, find mechanisms in which the intermediaries reveal their private information (a private signal useful to predict the risky assets' return); the portfolio choices are then made by a third party using preestablished rules.

In order to have an analytically tractable model the authors assume logarithmic utility functions for the investor and the financial intermediary. Also, there is a single period,

 $^{^{3}}$ Since the irrelevance result is derived under the assumption of negative exponential utility functions for the investor and the FI it is not clear whether it holds under more general risk averse preferences. See also the discussion on Gómez and Sharma (2005) ahead where under a similar setup the irrelevance result doesn't hold.

⁴However, see the work by Carpenter et al (2001) for a challenge on this claim.

and unlike the works by Bhattacharya and Pfleiderer (1985) or Stoughton (1993), the effort made by intermediaries doesn't affect their precision but rather modifies the probability of receiving a signal from an "informed" density function as opposed to an "uninformed" density function.

This work then proceeds to explore the characteristics of an optimal remuneration scheme that gives incentives to intermediaries to make effort and make good use of the signal received, while sharing risk efficiently. Such contracts are characterized under two different scenarios. In the first one the investor observes both the signal and effort level made by the intermediary. In this case the optimal contract turns out to be a proportional sharing rule (i.e. a fixed fee of assets under management). In the second scenario the investor observes the signal received by the agent, but not his effort level. In this setup the optimal remuneration scheme for the manager consists in a proportion of assets under management plus a fraction of the excess return of the portfolio over a benchmark (in this model the benchmark is given by the portfolio that the investor would choose if he made his own investment decisions). Finally, if investors cannot observe the signal either, an analytical contract cannot be derived. However, numerical examples show that the optimal contract rewards intermediaries for taking extreme or risky portfolio choices. The reason for this is to prevent intermediaries from slacking off, failing to receive an informed signal and then making conservative portfolio choices (a practice known as "closet indexing", i.e. intermediaries that pretend to follow active investment strategies charging high fees, when they are actually following passive strategies).

The authors point out that even though contracts such as the ones described in

cases one and two are used in practice this is not the case for contracts described as optimal for the third case, namely non-observable effort and signals. If this is so, it may be that there is another mechanism in play which guarantees that reasonable good investment decisions are made by intermediaries even if no explicit incentives are given to avoid closet indexing. One mechanism suggested by the authors is that the financial intermediaries' care about their reputation. Exploring this possibility is beyond this paper's scope as it uses a static framework.

Ou-Yang (2003)

In this work the author seeks to obtain optimal contracts between investors and intermediaries in a continuous-time economy with multiple periods. Ou-Yang claims that some of the previous principal-agent models in a continuous time setup are not suited for a delegated portfolio management context, since they don't take into account the fact that intermediaries control both the drift and diffusion terms of the portfolio process at the same time. This is explicitly modeled in the intermediary's decision problem.

In order to have closed-form solutions the author assumes negative exponential utility functions for investors and intermediaries. It should be stressed that Ou-Yang's work doesn't feature asymmetric information regarding stock returns. This is, unlike most of the delegated portfolio management literature surveyed so far, in this model intermediaries don't have superior information to make investment decisions. Therefore, investors hire them either because they can reduce transactions costs; because they have greater diversification possibilities; or because investor themselves don't have time to make their own portfolio decisions. In this model investors observe the evolution of the risky asset's prices, but they cannot observe the evolution of their portfolio's value. This is the source of asymmetric information between them and intermediaries.

Also, intermediaries must incur in costs in order to manage portfolios. The author assumes that this cost is increasing in the size of the portfolio. Under the described setup the author derives the closed form of optimal contracts; i.e. contracts such that intermediaries make portfolio decisions according to investors' preferences, subject to intermediaries' participation constraints, which are binding in equilibrium. The optimal contracts feature a fixed payment, plus a percentage of assets under management and a bonus or penalty depending on the portfolio's performance relative to a benchmark. The optimal contract turns out to be *symmetric* since it rewards (penalizes) the intermediary if his performance is above (below) some benchmark. Also, the appropriate benchmark is constituted by an active rather than a passive index. If the intermediaries' costs are constant, the optimal contract turns out to be a simple linear sharing rule, i.e. a percentage of assets under management. Finally, the author explores the case of more general preferences for investors. In order to obtain closed form solutions for contracts a simple cost function, consisting of a fixed cost only, is assumed and the optimal contract continues to be symmetric.

This work shows how obtaining closed form solutions for optimal contracts is no easy task and requires special assumptions about agents' preferences, cost functions, etc. In particular, the author assumes that there are no net inflows to the portfolio before intermediaries are paid. One important drawback is that no adverse selection or moral hazard problems are present and therefore it is not clear that the remuneration schemes derived continue to be optimal in these cases.

A.2 Asymmetric Contracts

One justification for the USA regulators to limit the use of asymmetric compensation contracts is the desire to avoid excessive risk taking by fund managers. Indeed, if managers receive a performance-fee in case of high returns, but no penalties in case of low returns they may desire to increase the riskiness of their portfolio, since their potential loses would be bounded. However, the following works challenge this intuition and make a case for the use of asymmetric contracts.

Carpenter (2000)

In a continuous-time economy the author derives the optimal dynamic investment strategy for a risk averse-manager. Investors pay this manager a fixed fee plus a call option on the assets he controls. Thus, it's possible that the manager may invest in overly risky positions as his loses are limited due to the characteristics of his remuneration scheme.

The author shows that following his optimal investment strategy, the manager will either outperform his benchmark or severely underperform it. However, his convex remuneration scheme wouldn't necessarily cause him to increase the portfolio's risk too much since as the value of assets under management grow or if the evaluation date is far, the manager will moderate the portfolio's risk. This result is robust to different specifications of the manager's utility function: constant relative risk aversion and hyperbolic absolute risk aversion. In the first case the manager determines the level of volatility for his personal portfolio. Investors then can increase the number of options thus increasing the volatility of the manager's portfolio. This would lead him to reduce the volatility of assets under management to offset the increase in borne risk.

Ross (2004)

Ross also challenges the traditional view that convex or option like contracts increases risk taking by otherwise risk-averse intermediaries or managers beyond efficient levels. The author argues that this common belief is due to option pricing theory, which demonstrates that the value of an option is increasing in its volatility. However, this doesn't imply that the option is more desirable to a risk averse manager. In a static setup Ross shows that the introduction of an option-like remuneration scheme doesn't necessarily reduce the managers risk aversion. For example, if the fee schedule is convex then for bets near the strike price the induced utility function for the manager may be less risk averse, but for bets to the right of the strike price the relevant domain of the agent's utility function changes and therefore his risk aversion can either increase or decrease.

It is shown that the total effect on the manager's risk aversion equals the sum of three effects. Namely: the translation, magnification and convexity effect. If the manager is offered a convex remuneration scheme, then the convexity effect is negative, since it makes the manager more risk loving (this is consistent with the option pricing intuition). However, if the managers' utility function exhibits increasing risk aversion, the relevant domain after the remuneration scheme is implemented will feature a higher degree of risk aversion. This means that in this case the translation effect would be positive. Finally, if the increase in remuneration associated to an increase in the value of the managers' output (for example the stock price of the company he runs) is large enough, then even a small gamble at the stock price with certain standard deviation will be magnified, thus exposing the risk averse manager to more risk and making him less willing to undertake it. This would also increase the manager's risk aversion. Therefore, if the sum of the translation and magnification effect offsets the convexity effect, the introduction of the option like remuneration scheme will make the manager more risk averse.

One of the utility functions commonly used in the delegated portfolio management literature is the constant absolute risk aversion function. For this class of utility functions there is no translation effect, since manager's risk aversion doesn't change with his wealth level. However, the magnification effect is still present and, if the fee schedule increases faster than output value, this effect will be positive and the manager may become more risk averse. On the other hand, if the fee increases slower than output value the magnification effect will reinforce the convexity effect and the popular belief of less risk averse managers will hold.

To conclude, it's not obvious that introducing a convex remuneration scheme increases risk taking by managers. This will depend on the characteristics of the managers' utility function and also on the characteristics of the remuneration scheme itself.

Panageas and Westerfield (2007)

Like Carpenter (2000) and Ross (2004), the work by Panageas and Westerfield (2007) also shows that a convex remuneration scheme doesn't always imply more risk taking by intermediaries. The authors study a dynamic setup in which a risk neutral intermediary who seeks to maximize the present value of future fees is offered an option-like remuneration scheme called a high-water mark contract. Such contracts are widely used in the hedge funds industry (see Fung and Hsieh ,1999, and Ang et al, 2008). Under this remuneration scheme the intermediary receives a fixed percentage of assets under management plus a fraction of the increase in fund value in excess of the last recorded maximum, which is called the high-water mark. If there is no increase the intermediary only gets the fixed percentage.

It would seem that such contracts may induce excessive risk taking by intermediaries, specially if they are risk neutral. The authors show that this intuition is correct in a finite-time horizon setup. In this case intermediaries will increase the volatileness of the portfolio they manage without bound as the final date approaches. However, in an infinite horizon context even though a bolder portfolio in the current date increases the probability of crossing the last recorded high-water mark, it also increases the probability that the value of assets under management will be substantially lower in the next period, which given the unchanged last maximum, will lower the value of future options. This creates a trade-off for the intermediary making him behave like a risk-averse agent.

The authors also study some of the model's comparative statics properties and find that as the manager discounts more the future, he will tend to make bolder portfolio decisions. While this seems intuitive, Panageas and Westerfield also find that an increase in the high-water mark increases the present value of expected fees. The reason for this is that while an increased high-water mark means that it will take more time to receive the performance fee, the expected gain to an intermediary that has just reached the highwater mark is proportional to the value of assets under management, which means that once the manager reaches the high-water mark assets under management will be larger thus increasing the intermediary's profits. It turns out that for a risk-neutral intermediary the latter effect dominates the former explaining the counter intuitive result. However, the authors show that this result doesn't always hold for risk-averse intermediaries. It's important to emphasize that this model doesn't study the effects of informational asymmetries such as unknown ability or unobservable effort between investors and intermediaries.

A.3 Churning

Amidst the works concerned with the delegated portfolio management problem there is a strand of the literature that studies to what degree the existence of the principalagent relationship in this market is able to explain the relatively high volumes of securities transactions observed in the mutual funds market.

Dow and Gorton (1997)

Dow and Gorton (1997), argue that the existing volume of transactions seems to surpass the transactions needed to rebalance portfolio or those motivated by hedging needs. The authors suggest that this stylized fact could be due to investors failing to distinguish if a financial intermediary that didn't trade chose to do so out of negligence or because the information he collected suggested that this was the best course of action. If investors believe that the first possibility is valid, then intermediaries could trade even if they don't have real reasons to do so in order to avoid being punished by investors.

In Dow and Gorton's model there are two types of intermediaries, skilled and unskilled. An intermediary's type is private information. A skilled intermediary has the ability of making effort in order to receive private information about the economy's risky asset. This asset can give a high return or a low return with equal probability. Although making effort is costless per se, managers do have the option of shirking in their work, which gives them a positive payoff. This payoff is also equal to both types of managers' reservation wage.

Investors will be interested in offering a contract that attracts only skilled intermediaries since unskilled ones would only shirk on their job and make random investment decisions. Also, the contract must give skilled intermediaries incentives to get information. Unlike the works surveyed so far, in Dow and Gorton's paper it's not certain that a skilled intermediary that makes effort receives useful information. There is a probability that he will receive information useful to predict the risky asset's return but it's also possible that he will find himself with no useful information.

Since the contract stipulates no payment in case the intermediary doesn't make any transaction (otherwise unskilled intermediaries would find it attractive) the skilled intermediary will make a random investment decision, i.e. he will churn, in case he doesn't have useful information to make his investment decisions.

Nevertheless, in this model the existence of churning may be beneficial to all agents in the economy. The reason for this is that, in order for skilled intermediaries to make above-normal expected profits, there should be someone making below-normal expected profits; namely hedgers, or uninformed investors who receive random income shocks which they want to hedge against, even if it means buying (selling) overvalued (undervalued) financial securities. The existence of churning makes hedging cheaper, since it implies that the transaction's counterpart will not always be informed about the security's true value. This means that hedgers' transaction volume will increase, which will increase the amount of transactions intermediaries can make. This is due to the fact that, in equilibrium skilled intermediaries' volume of transactions will be equal to that of hedgers since otherwise they would reveal their type to the price-setting market maker, thus revealing the asset's true value and diluting their expected return. Therefore, financing cost for hedgers would be lower, expected return for investors would be higher, and intermediaries would be no worse (since the contract gives them their reservation utility) thus improving the economy's welfare.

An important element in this work is the existence of limited liability for financial intermediaries. This implies that, if the intermediary churns and makes a good investment decision out of luck he will receive a positive payment, whereas if he churns and makes a bad investment decision, he will receive zero payment, even though it will be clear to investors that he churned. If the limited liability assumption was removed it may be possible that the intermediary was charged with a fine if he churns, which my prevent this behavior. The following work elaborates on this point.

Bhattacharya (1999)

This paper is closely related to Dow and Gorton's (1997) model. The author demonstrates that it's possible to design contracts in such a way as to screen out unskilled financial intermediaries, while giving skilled ones incentives to acquire information and avoiding churning, even with the existence of limited liability for intermediaries.

In a setup almost identical to that in Dow and Gorton (1997), Bhattacharya introduces changes regarding the set of alternative activities at the intermediaries' disposal and also about the existence of trading costs. The existence of trading costs implies that unskilled intermediaries won't necessary find it attractive to work for an investor and churn if this means giving up their outside option. This is the assumption made by the author, who goes on to show that screening of skilled intermediaries can be carried out without inducing churning. However, for this to be feasible, it's necessary that the value of the outside options for skilled and unskilled intermediaries doesn't differ too much, since otherwise it would be too expensive to give skilled intermediaries their reservation utility (which is strictly higher than the value of reservation utility of unskilled intermediaries) while preventing churning.

The author acknowledges that the limited liability constraint emphasized by Dow and Gorton does limit investors' ability to screen agents. The reason is that it prevents investors from punishing bad investment decisions (which are evidence of churning in this model) by, say, charging fines. Bhattacharya then shows that such a restriction could be relaxed if the investor hires multiple intermediaries. If the skilled intermediaries information is correlated, then they should make very similar investment decisions. Therefore, if one of the intermediaries' ex-post return is very different (i.e. higher than the rest of agents ex-post return) then investors would punish such intermediary because the higher-thanaverage return would be evidence of churning. This unorthodox use of a benchmark based compensation would enhance investors' screening possibilities since it allows investors to effectively raised the expected cost of churning for intermediaries.

A.4 The Use of Benchmarks

Up until now we have seen that benchmarks may be useful as part of the remuneration schemes for financial intermediaries (e.g. Bhattacharya,1999, and Carpenter et al, 2001). However, other authors are critical about rewarding intermediaries' performance in relation to some benchmark, at least the ones used in practice.

Admati and Pfleiderer (1997)

This work probes the use of benchmarks on the remuneration schemes used to compensate financial intermediaries who have superior information that allows them to make (potentially) better investment decisions than investors. Using a model with riskaverse investors and intermediaries the authors find that commonly used benchmarks are generally inconsistent with optimal risk sharing between agents and generally fail to align incentives leading to suboptimal portfolio choices. Moreover, these benchmarks tend to lessen or have no effect on the effort level made by the intermediary; they aren't useful to screen unskilled intermediaries from skilled ones; and they fail to align incentives when the investors don't know the intermediaries' risk aversion degree.

One of the causes behind these findings is the irrelevance result present also in Stoughton (1993): if the intermediary has the ability of controlling the scale of his response to the incentives provided by the remuneration scheme, then he will always choose his most preferred portfolio, regardless of the incentives provided by investors.

Admati and Pfleiderer conclude that a series of benchmarks can be useful in mitigating the effects of the agency problems present in a DPMP context. However, the form of this benchmarks differ from those observed in practice. For example they will depend on the intermediaries' risk aversion degree.

Gómez and Sharma (2005)

Gómez and Sharma show how the irrelevance result obtained by Stoughton (1993) and Admati and Pfleiderer (1997) fails to hold when intermediaries face short-selling constraints. The authors study an economy in which investors are assumed to hire an intermediary to make investment decisions. This is, investors' decision regarding the amount of wealth to delegate is not modelled. In line with previous literature, the intermediary has the ability to make costly effort in order to obtain information useful to make better investment decisions. Also, the intermediary is assumed to have negative exponential utility and is paid a fixed monetary amount plus a percentage of assets under management.

If no constraints are placed on the intermediary's investment opportunities, then his effort level will be independent from the percentage of assets under management that he is paid by the investor and from the fixed amount he receives. Therefore, the irrelevance result holds and the contract will fail to align incentives between investors and intermediaries. Next, a case of bounded short-selling is considered. This means that the intermediary is able to invest an arbitrarily large fraction of the portfolio on either a risky or risk-free asset; however, this fraction must be finite. Under this conditions, when an intermediary makes effort the quality of his information increases, but the existence of bounded short-selling means that for some signals he will no longer be able to form his preferred portfolio. This would lessen the intermediary's incentives to make effort since it is possible that in some cases the information received won't be put to use. Therefore, by increasing the percentage fee paid to the intermediary, the investor will be able to marginally relax the portfolio restrictions faced by the intermediary thus increasing the attractiveness of making effort.

Under bounded short-selling and observable effort Gómez and Sharma show that the intermediary's optimal effort will be smaller than in a first base case with no agency concerns. Also, optimal effort will be increasing in the percentage fee paid to the intermediary. Finally, the difference between effort level in the bounded short-selling scenario versus the first base cased will be increasing in the bound placed on portfolio decisions and decreasing on the managers' risk aversion. The first result has a straightforward interpretation. As for the second one, if the intermediary is more risk averse then he will make less extreme portfolio decisions, which means that it is less likely that the short-selling bounds will affect him, thus validating the irrelevance result.

If there is bounded short-selling and effort is unobservable no analytic solution to the problem is derived, but using numerical methods the authors show that the percentage fee paid to the intermediary will be higher than in a first base case, causing an inefficient risk sharing between agents. Nevertheless this is necessary in order to align incentives. The analysis made by the authors show that the level of this distortion will be increasing in the intermediary's risk aversion and for tighter short-selling bounds.

A.5 Effects on Securities' Prices

A relevant issue is to what extent the existence of agency problems between intermediaries and investors affects assets' prices. This is of great practical relevance as institutional investors account for a large share of market participation. For instance, ICI (2008) reports that USA mutual funds were the owners of 24% of outstanding equity in 2007. Thus, the portfolio decisions made by these investors are likely to impact the prices of financial assets. Understanding this topic is the main objective of the following works (Dow and Gorton, 1997 also provides insights on this subject).

Cuoco and Kaniel (2007)

This paper studies the asset pricing implications of typical delegated portfolio management contracts which use relative compensation. The authors model a dynamic continuous-time economy which features a risk-free asset and two risky assets. There are three types of agents: active investors, fund investors and fund managers. Active investors make their own portfolio decisions, choosing the investment strategy that maximizes their expected utility of the final value of their portfolio. Fund investors delegate the choice of an investment strategy to fund managers. This could be due, for example to the fact that this investors face higher transaction or information costs. In this sense this work differs from the more traditional delegated portfolio management literature in which investors hire a manager because of his superior stock picking abilities. Finally, fund managers are hired by investors to make portfolio decisions. Managers make this decisions so as to maximize the expected utility of the value of their fees.

Cuoco and Kaniel assume an exogenous remuneration scheme given by a fixed payment; a percentage of the final value of assets under management; and a performance based fee which is relevant whenever the manager's performance is different from that of the benchmark. Initially the benchmark is exogenously chosen to be composed of a percentage of each of the two risky assets. By solving the investors' maximization problems the authors are able to show that when the managers' remuneration scheme includes symmetric performance fees and no fixed payment, in equilibrium managers will hold more (less) units of the first risky asset than of the second at a given period if and only if they are benchmarked against a portfolio holding more (less) unites of the first risky asset than of the second asset. The result of managers investing more heavily in the assets more weighted in the benchmark is to rise the equilibrium price of such assets. This is reported to be consistent with empirical evidence regarding prices of stocks that are included or dropped from the S&P 500 index. Also, if the benchmark portfolio is buy-and-hold, then the equilibrium strategies are also buy-and-holdso in this case the portfolio's turnover is not increased by the use of performance based fees. Using numerical examples Cuoco and Kaniel also show that under asymmetric performancebased fees risk-averse managers can either over weight their portfolio with assets that have high correlation with the benchmark, in order to make their remuneration less risky, or select assets with low correlation with the benchmark, attempting to maximize the variance of the excess return of the managed portfolio over the benchmark, thus increasing the expected value of their remuneration, which is a convex function of the excess returns.

The authors also show that in their model, even though investors and managers share the same preferences and risk aversion, a linear remuneration scheme fails to achieve first best decisions, contradicting the results obtained by Ross (1973). The reason for this is that in this model fund investors have direct access to the risk-free investment opportunity and they take the remuneration scheme structure as given when making their investment decisions. For this reason individual fund investors do not choose the level of delegation that ensures that a linear remuneration scheme achieves first base. This stems from the fact that individual investors fail to internalize that fees will have to increase if they underinvest in mutual funds in order to guarantee a certain reservation utility level for managers. Moreover, the authors show that for low managers' reservation utilities a contract that uses asymmetric performance fees or fixed payments doesn't generate welfare improvements compared to a contract that features only a fixed percentage fee. This is due to the fact that for this reservation utility levels, investors delegate enough wealth to managers so that the linear remuneration contract almost matches the first-best results. Nevertheless, if managers' reservation utilities are higher, performance based fees are preferred to fixed payments; and both this two contracts are preferred to contracts that only use a fixed percentage fee.

Goldman and Slezak (2003)

Goldman and Slezak show how managers' private information mail fail to be reflected in assets' prices when there is a mismatch between managers' tenure and the time needed for their private information to become public. This may lead to prolonged mispricing of assets, replicating a bubble.

In this model there are three types of risk neutral agents: Fund managers, noise traders and a market maker. There is also a risk-free asset and a risky asset. In the initial period a manager is entrusted to make investment decisions. The manager collects private information about the risky asset's future value and makes his investment decision. At the same time, noise traders make random investment decisions. The market maker observes only the net order flow and adjusts the asset's price to its expected value conditional on public information. After this there is a part of the first manager's private information that becomes public. The asset's price is adjusted and the manager is rewarded a percentage of the difference between the final and initial value of the portfolio. Then a second manager inherits the portfolio, receives private information and makes investment decisions. With an exogenous probability, called the revelation probability, the second manager's private information becomes public before his tenure ends. This means that it's possible that this doesn't occur. After this, the manager is paid a percentage of the difference between the portfolio's value when he leaves and its value before his first trade.

Under this setup, the change in the portfolio value for the second manager will be given by two parts. The first one is the change in the value of the inherited portfolio and the second one is the value of the second manager's trade profit. This means that if the current manager has negative information and inherited a long position on the risky asset, the trade that maximizes his trade profit will lower the value of his inherited position. Moreover, if his private information doesn't become public, the second effect may dominate and thus unless he receives sufficiently negative information he will not sell, causing the asset to be mispriced.

The authors show that there are four situations in which there will be no prolonged mispricing: if there is no noise in the first manager's signal; if there is no noise trading in the first period; if the revelation probability is one; and if, on average, it turns out that securities are priced without bias. When there is mispricing, its size will increase with the inaccuracy of the part of the first manager's private information that never becomes public since in this case this manager will trade more aggressively, thus inheriting a more extreme portfolio to his successor. Mispricing decreases with higher revelation probability, since even though the second manager will trade more aggressively, which increases the probability of mispricing, his current orders will be less dependent on the inherited portfolio so the first manager's signal error will have less persistent effects on asset's prices.

When managers' decision to acquire costly information is endogenous, Goldman and Slezak use numerical examples to show that for a fixed quality of private information, as the revelation probability falls, the informational efficiency of prices fall, since managers are less likely to benefit from their private information and therefore have less incentives to acquire it. Also, for a fixed revelation probability, price efficiency improves as the quality of private information decreases. This is so because prices will be less responsive to investment decisions so managers may invest more aggressively and make higher profits, which encourages them to acquire private information.

A.6 Reputation

This literature is sometimes referred to as career concerns. Reputation is typically modeled as the probability assigned by investors and intermediaries to a particular intermediary being skilled, conditional on some information set, which usually includes the intermediary's previous decisions record. While some of these works emphasize the positive role of reputation as an incentive aligning mechanism (e.g. Heinkel and Stoughton, 1994, Chemmanur and Fulghieri, 1994, and Farnsworth, 2003) there are also studies suggesting that reputational or career concerns may have undesirable side effects (e.g. Scharfstein and Stein, 1990, Huddart, 1994, Prendergast and Stole, 1996, Avery and Chevallier, 1999, Dasgupta and Prat, 2006).

Heinkel and Stoughton (1994)

So far we have seen that optimal remuneration schemes in a DPMP context are symmetric (e.g. Ou-Yang, 2003), may be non-linear (e.g. Bhattacharya and Pfleiderer, 1985) and contemplate the use of non-standard benchmarks (e.g. Admati and Pfleiderer, 1997, Bhattacharya, 1999). However, Heinkel and Stoughton (1994) argue that, even without the use of remuneration schemes previously described, the delegated portfolio management market could exist. The reason this is possible would be that the presence of implicit incentives, such as the FI's reputation, is sufficient to align incentives, even if the contracts used are simple (e.g. a fixed percentage of the value of assets under management). Indeed, if the agent's future remuneration depends on his reputation (e.g. greater fees, more clients), he could be willing to make a greater level of effort to manage the portfolio, thus avoiding possible future loses in revenues.

Heinkel and Stoughton's work is amongst the first ones to extend the DPMP literature to a dynamic context. This work considers the existence of adverse selection (i.e. investors are not aware if the intermediary they hire is skilled or unskilled) as well as moral hazard (i.e. it's not possible for investors to observe whether the intermediary makes effort to manage his portfolio).

In this model both investors and intermediaries are risk-neutral. The economy has two periods. In the first period the principal offers the agent a menu of contracts for him to choose from. Even though the principal may design this menu as to screen skilled from unskilled agents he will find it optimal not to do so, maintaining some degree of doubt about the intermediary's type. The reason for doing this is that in an intermediate date the investor may hire a third party to evaluate the intermediary's performance. Moreover, the investor can observe the result of the investment decisions made by the intermediary. With this information the principal decides whether to fire or retain the intermediary.

If the intermediary is not fired he offers the investor a contract which gives all the portfolio earnings to the intermediary, while giving the investor a fixed pay, equal to his reservation utility. The intuition behind this is that once the intermediary's type is known, several investors will compete for their services. However, given the characteristics of the contract offered in the first period (i.e. low degree of response of the intermediary's pay to his performance during the period), the investor cannot be certain about the intermediary's type. Instead, he uses the end of period performance evaluation to decide if he fires or retains the intermediary. Therefore, a skilled agent will make a greater level of effort in the first period, since he knows he will be fired if his performance is poor and he will miss the opportunity of getting attractive contracts in the second period.

The authors argue that this model is capable of explaining some stylized facts: long term contracts are not frequently used (Heinkel and Stoughton assume this type of contracts don't exist since the parts cannot credibly commit to avoid renegotiation of the contract in future periods); there is a weak relation between fees paid to intermediaries and their past performance; most contracts do not feature performance-based fees⁵; and most institutional investors pay professional evaluation services in order to have performance reports about their portfolio managers.

This work presents some shortcomings. In particular, it contains some arbitrary assumptions about the principal-agent relation and how negotiation power shifts completely from the former to the latter if he isn't fired. Although this assumption allows to obtain a tractable model it casts doubt on the general validity of their conclusions.

Chemmanur and Fulghieri (1994)

This work studies how reputational concerns modify agents' behavior in non DPMP context. In this model there is an entrepreneur who whishes to make his firm public, i.e.

⁵The authors cite Golec (1992) to assert that most mutual funds contracts don't contemplate the payment of performance-based commissions. However, this type of arrangement is used in hedge funds. On this see Panageas and Westerfield (2007).

he wants to make an IPO. For this he has the option of hiring an investment bank. The investment bank realizes an evaluation about the project's profitability and this information is used by investors to value the project. Moreover, the quality of the investment bank's evaluation depends on the degree of effort he makes. This effort is non observable to investors. Therefore, if the investment banks concludes that the firm is profitable, but the firm turns out to be a bad investment, investors cannot verify if the investment bank shirked or if he was unlucky.

In a two period setup the authors demonstrate that investment banks will make more effort than in a situation in which reputation was absent. This is, the presence of reputation helps to mitigate the inefficiencies caused by moral hazard in this economy. The reason is that banks face a dynamic trade-off: making more effort in their evaluation is costly in the short run but it may be beneficial in the long run, since investors will assign a lower probability to banks recommending bad investment projects. Therefore, in this model reputation is equal to the probability assigned by investors to the event that the firm recommended by the bank is profitable, conditional on the result of the bank's recommendation in the previous period. This will raise investors' willingness to pay for firms recommended by investment banks with good reputation, which in turn will increase these banks' profits, as they are given by an exogenous percentage of the IPO's raised funds.

The authors claim that the models predictions are supported by existing evidence. Namely: investment banks with better reputations charge higher fees; IPO's underperformance due to informational asymmetries will be lower if the investment banks' reputation is higher; and more prestigious investment banks will work with less risky firms. Finally, Chemmanur and Fulghieri don't assume a specific relation between revenue and reputation; instead, they model reputation as a Bayesian updating process of beliefs regarding the agent's skill. However, their model studies the behavior of the investment banks, and not a DPMP situation.

Arora and Ou-Yang (2001)

This work builds on Ou-Yang (2003) and studies the incentives that arise when the financial intermediaries' performance in the present period affects their reserve wage and portfolio inflows in following periods. In order to simplify their analysis the authors assume that there is a linear monotonic relation between future portfolio inflows and this period performance. They also assume there is a linear monotonic relationship between future reserve wages and present performance. In both cases performance is measured both in absolute terms and in relation with a benchmark.

In a multiple period economy the authors characterize the form of the optimal contracts between investors and intermediaries as well as the intermediaries' optimal portfolio policy. Optimal contracts are found to be symmetric; i.e. the agent is rewarded if he has a good result but is punished if the performance is poor. Also, the authors find that the agent will invest part of the portfolio actively while the remaining portion is used to mimic the benchmark; i.e. the portfolio will incorporate a herding component, whose existence is due to the fact that the agent is risk averse and in part desires to avoid having a performance worse than that of the benchmark, since his results will be partially measured against the benchmark.

An interesting finding is that the herding component of the portfolio is larger

during the first periods of the intermediary's life and its importance declines over time during the last stages of his career. The authors present empirical evidence that supports this prediction using data for USA mutual fund managers.

Even though the assumptions about monotonic relationships between reserve wage and fund inflows allow to have closed form solutions, they are certainly ad hoc and could be consistent with the existence of mechanisms like reputation (although investors' beliefs are not modeled) but also with the existence of long term binding contracts between intermediaries and investors.

Farnsworth (2003)

Farnsworth analyses a DPMP problem in a multiple period model similar to that of Carpenter et al (2001) but he allows for financial intermediaries to build a reputation. In this model there is a moral hazard problem between investors and intermediaries since the former can't observe the degree of effort made by the latter when managing their portfolio. The author argues that, even though this problem could be solved using performance bonus, these don't seem to be frequently used in practice. Instead of this type of explicit incentives, the author suggests that reputation could be a mechanism that manages to align incentives.

The model assumes the existence of two types of intermediaries: skilled and unskilled. The first type can make effort to receive an informative signal about the pays of the assets in the market; the second type has no such possibility. In this model a signaling equilibria is not feasible since unskilled intermediaries are convinced that they are skilled. This allows reputation to play a part as a mechanism to align incentives.

Specifically, reputation succeeds in avoiding the intermediaries' incentive compat-

ibility constraints to be binding. If this is the case explicit incentives would have to be provided in order to align incentives. Farnsworth assumes that investors' have committed ex ante with intermediaries to increase the amount of assets under management if the intermediaries' reputation improves, therefore increasing intermediaries' profits which are given by a constant percentage of assets under management. However, as the intermediaries make effort and their reputation grows, the subsequent increases in reputation are smaller since there is a limit as how good reputation can be. When this happens the incentive compatibility constraint will bind and additional incentives would have to be given to overcome the moral hazard problem. The author argues that this would be the case of large investment funds, which already have high reputation and must resolve to include explicit incentives in their contracts.

One shortcoming of this work is the assumption that the principal has committed ex-ante to increase the flow of delegated funds if the agent's performance has been good. This suggests that binding contracts should be used between investors and intermediaries when in fact many mutual fund contracts can be ceased on very short notice even if the relationship between the parts has only lasted a few months.

Huddart (1994)

This work studies the effects that reputation can have on the behavior of skilled financial intermediaries. In this model there are risk averse investors who hire financial intermediaries to make investment decisions. There are two intermediaries in the economy. One is skilled and the other unskilled and this is private information. The difference between these intermediaries is that the skilled one has private information useful to make better portfolio decisions while the unskilled one has no such information. The economy has two time periods. At the beginning of the first period half of the investors invest with one intermediary and the remaining half with the other intermediary. Both intermediaries make their investment decisions in a risky and risk free asset. At the end of this period all agents know whether the decisions made by the intermediaries were good or bad. Also, since it is more likely that a skilled intermediary made a good investment decision, investors will delegate their portfolio management to such an intermediary. Since the intermediaries' profits are given by a percentage of assets under management changes in their reputation will affect their expected utility.

Huddart assumes that investment decisions are taken simultaneously. This means that the unskilled intermediary won't be able to copy the investment decision of his rival. Since he has no information he should make his investment decision based on the unconditional expected return and variance of the risky asset, which given the author's assumptions means that he should invest all the portfolio in the risk free asset. However, since the structure of the information received by the skilled intermediary is known to all agents (although the specific realization of the signal received is only known to the skilled intermediary) the unskilled intermediary knows that the portfolio choice of his rival can take two possible values. Therefore, by choosing one of this values at random he has a chance of making a good investment decision. It is possible that the unskilled intermediary tries to make this guess rather than choosing to invest all the portfolio in the risk free asset, since under the first choice if he gets lucky he won't be fired by investors while under the second choice he knows for sure his type will be revealed and he will be fired.

The author explores the feasibility of two types of equilibria. In a pooling equilibrium the skilled intermediary makes his investment decisions according to his private information, while the unskilled intermediary attempts to copy this behavior. Sometimes the unskilled intermediary will succeed and investors initial beliefs won't change, while sometimes the unskilled intermediary will fail and his type will be revealed. It is even possible that the skilled intermediary invests according to his information, which turns out to be misleading, while the unskilled intermediary makes a lucky guess and investors mistakenly conclude that he is the skilled intermediary. For this equilibrium to be feasible it is necessary that the unskilled intermediary does indeed prefer to make a random investment decision rather than reveal his type (this decision is not trivial since he is risk averse); also, the skilled intermediary must prefer this type of equilibrium instead of a separating one in which he attempts to signal his type to investors through his investment decisions. In a separating equilibrium the skilled intermediary will choose more extreme portfolio positions (e.g. if his information suggests to invest 40% or 60% in the risky asset he will invest 30%or 70%). By doing this, he makes copying less attractive to his rival, thus revealing his type to investors and obtaining greater profits in the second period. To do this, however, he makes suboptimal use of his private information.

The separating equilibrium will prevail if the unskilled intermediary is relatively risk averse (in this case making random investment decisions is less attractive because of the risk involved) and also if the fee received by intermediaries is larger (in this case even if the unskilled intermediary invests all the portfolio in the risk free asset he will receive high profits at the end of this period, while receiving none in the second period). Huddart shows
that the inefficiencies caused by reputational concerns can be mitigated using performance fees rather than fixed fees, since the former do a better job at aligning incentives. Also, a commitment by investors not to reallocate their investment among intermediaries would enhance investors' ex ante expected utility.

Prendergast and Stole (1996)

The work by Prendergast and Stole examines how a manager's concern for his reputation can have an impact on his investment decisions. In this model each period managers must decide the amount of resources to devote to an investment project. The productivity of the project is unknown, but managers receive an imperfect private information of this parameter. Each manager is different in their talent, with more (less) skilled managers receiving more precise (imprecise) private information. The managers' type is unknown to the rest of agents in the economy.

The authors assume that managers' objective function includes only current profits (which are function of the investment project's profitability) and his immediate end of period reputation. It is also assumed that the market updates its beliefs about a manager's ability using only information on his investment decisions and not on the profits earned. This allows greater tractability of the model.

In this setup the authors show that during the first period that the managers are employed, the more talented managers will exhibit a higher variation of the posterior about the market's prior regarding the productivity of the investment. This happens because managers who are very talented (and have very precise information) will give less attention to the common prior and place more weight on their private information. On the other hand, for later periods the variance of the posterior has two components acting in opposite directions. There is an element that measures the weight placed on the most recent observation. This element is increasing in the manager ability because for skilled managers the current observation is trustworthy and therefore it should considerably change the posterior. This could lead managers to induce excess variability in their investment decisions in an attempt to appear well informed. However, there is a second element related with the variability of the current observation. This is decreasing in ability because as ability decreases this means that the previous observation did not yield a precise measure of true profitability and also the noise associated with the current observation is large. This second effect could lead managers to be too conservative since they would want to give the impression of already having made correct decisions. Whether a manager is conservative or overreacts to his private information for later periods depends on which of these two effects dominate.

Focusing on separating equilibria, Prendergast and Stole show that managers will exaggerate their decisions with respect to a case with no asymmetric information regarding ability, when they are working for the first time. Also, once enough time has passed since they were hired, they will be too conservative, making little use of their private information. This is not due to concavity of the rewards function, which is assumed to be lineal, but rather because the manager wishes to act like he already has made good investment decisions because changing his opinion too much will result in a decrease in reputation. In between the first time period and the date when the manager becomes conservative the authors do not fully characterize the manager's behavior although they find conditions under which separation arises. Finally, the authors show that when the parameter reflecting the productivity of investments changes over time, managers will not be conservative even in the long run. The reason for this is that if the environment evolves enough managers will be recriminated if they don't change their investment decisions, no matter how good their previous decisions were. On the other hand, if beliefs about ability are updated using public observation, such as past periods' profits it is possible that managers will not be conservative even in the long run.

Dasgupta and Prat (2006)

Dasgupta and Prat study the equilibrium features of an economy in which financial intermediaries are concerned about their reputation. The authors are able to study the properties of prices and transactions volume, showing how reputational concerns lead to excessive trading -i.e. churning- and a skewed relationship between fund returns and net fund inflows.

In the economy there are four types of agents. Investors are risk neutral and since they cannot trade directly they have to delegate this task to fund managers. These managers are also risk neutral and they can be skilled or unskilled. The managers' type is not known by investors nor managers. Managers make portfolio decisions on behalf of investors. At the end of the first period all parties observe the result of the investment decisions. Based on this information investors then decide whether to retain or fire the manager.

There is also a number of noise or liquidity traders, who make random investment decisions. The last type of agents are risk neutral uninformed rational traders. These act as

market makers who face Bertrand competition and therefore set prices equal to the expected value of the asset being bought or sold, conditional on the orders received. The authors assume that trade is anonymous and therefore market makers observe only transaction orders but they are not aware of the issuer's identity.

Skilled managers receive an informative signal, useful to predict the asset's true liquidation value. An unskilled manager receives no such information. We stress the fact that managers have to make no effort in order to receive their information. This is, the authors do not model a moral hazard situation. The remuneration scheme is assumed to be some percentage fee p of assets under value plus a fixed positive payment \bar{p} . The values of pand \bar{p} are exogenously given and therefore, the authors assume that they satisfy investors' participation constraint.

If there are no career concerns, i.e. the probability that the manager is fired at the end of first period is exogenous, managers will only trade if they have information. Therefore, only skilled managers will trade and there will be no churning. The reason for this is that skilled managers face positive expected returns if they invest due to the presence of noise traders. As market makers are aware of this they will charge a positive bid-ask price, which means that unskilled managers won't trade because they face negative expected returns.

On the other hand, the authors demonstrate that if career concerns are present, in equilibrium investors will retain their manager if he makes a good investment decision in the first period and they fire managers if they make a bad investment decision or do not trade, since either one of this events reveals the manager is unskilled. Faced with this, an unskilled manager will have to choose between no trade, which results in getting fired, or churning (i.e. making a random investment decision), which has an uncertain outcome: if he is lucky and makes a good investment decision he is retained but if the investment decision is a bad one, he will be fired. If the manager's contract features a low p the unskilled manager will prefer to churn. As a result trading volume will be higher in the presence of career concerns. Moreover, in this setting the reputational reward for good investment decisions will be higher (in absolute terms) than the reputational cost of bad investment decisions, which is consistent with the flow performance relationship documented in the stylized facts Section. Additionally, Dasgupta and Prat show how their results are robust to somewhat more general assumptions about the information received by managers and endogenous contracts.

One subject overlooked by this work is how the relationship between reputation and churning evolves over time. That is, as time goes by and managers' reputation improves or get worse, how do the incentives to churn change? Also, the authors do not deal with the possibility that obtaining information is costly to managers.

A.7 Herding

The correlation observed in portfolio holdings of financial intermediaries such as mutual funds and pension funds was documented in the stylized facts Section. This phenomenon is called herding. We briefly discuss some of the reasons for intermediaries to herd, beginning with earlier papers that explain herding as a result of informational cascades (see Banerjee, 1992, and Bikhchandani et al, 1992); this phenomenon is also called statistical herding. Then we proceed to discuss the works by authors who relax some of the assumptions made by the earlier literature such as exogenous prices and exogenous decision sequences and study whether herding can still occur in equilibrium. Other authors (see Arora and Ou-Yang, 2001, whose work was surveyed above, and Maug and Naik, 1995) discuss how herding can result from compensation schemes that rely on relative performance. Emphasis will be given to works discussing the possibility that intermediaries herd due to reputational concerns (see Scharfstein and Stein, 1990, Avery and Chevalier, 1999, Graham, 1999, and Ottaviani and Sørensen, 2006) . For exhaustive surveys of theoretical and empirical works studying the herding phenomenon in financial markets see Bikhchandani and Sharma (2001) and Hirshleifer and Hong Teoh (2003).

Banerjee (1992) and Bikhchandani et al (1992)

The works by Banerjee (1992) and Bikhchandani et al (1992) rationalize the existence of herding as a result of informational cascades. In a situation in which individuals must sequentially decide whether to take a decision or not, the decisions of early players will have a determinant impact on the behavior of the rest. Indeed, if each individual receives a signal that can take a good or a bad value, he will decide whether to buy a good or abstain from buying it basing his decision on his prior beliefs about the attractiveness of owning the good; his received private information; and also on the decisions made by those individuals who decided earlier. For example, suppose that the individuals share a common prior belief suggesting that they should buy the good when in fact the truth is that they would be better off not buying it. Further, suppose that all the individuals' signals are of the same quality and that the first individual receives a good signal. Given his prior beliefs and the signal received he updates his beliefs via Bayes' rule and buys the good. Now suppose the second individual receives a bad signal (which is the correct signal in this example). He knows that the first person received a good signal since otherwise he wouldn't have bought it. Since all signals have equal quality the second individuals' beliefs equal his prior and thus he buys the good. From then on, even if all following individuals receive bad signals, they will ignore their private information and follow the herd. Thus, the informational cascade that forms will prevent private information from reaching the market and the equilibrium outcome will be inefficient. Bikhchandani et al (1992) further discusses how robust herds are and they show that the release of accurate private information can stop individuals from herding.

Avery and Zemsky (1998)

This work studies the conditions under which herding would arise in financial markets focusing on the role market prices have preventing the occurrence of herds. The authors study a model in which a security's true value is unknown to agents. In the economy there is a continuum of risk-neutral traders who can either be informed or uninformed about the security's value. An informed trader receives private information and submits orders to the market maker seeking to gain positive expected profits. An uninformed trader is actually a noise trader who randomly submits orders to a market maker. In this model an informed trader herds if he ignores his private information to place orders. The authors show that if the only source of uncertainty in the economy is about the asset's true value then herding won't be an equilibrium outcome since adjustments in the security's price prevents this and guarantees that all private information this will be decisive for taking his final investment decision. In this fashion private information will keep flowing in to the economy. If there is an additional source of uncertainty; namely the quality of the agents' information (or their type) is unknown then it is possible that prices are no longer efficient and herd behavior may surge. The reason for this is that the security's price has only one dimension and is therefore unsuited to handle uncertainty in more than one dimension (assets' true value and investors' type). Based on this the authors hypothesize that multiple dimension prices, such as options prices, may be better suited to prevent herding.

Beaudry and González (2003)

The work by Beaudry and González studies the plausibility of herding in discrete investment decisions in an economy where information is costly to acquire and prices are endogenous in the sense that they react to agents' investment decisions. In this model there are intermediaries who make investment decisions consisting in acquiring some good and selling it at a later date. In deciding the amount of their investment intermediaries form expectations about future demand for the good they sell. They can incur in some cost in order to receive private information useful to determine the state of demand for future periods. This state can be either good (i.e. high demand) or bad (i.e. low demand). The cost of the investment is endogenously determined by the intermediaries' decisions and the same is true for the price at which intermediaries sell in future periods. In the economy there is also a small mass of noise traders.

The authors show that in an economy with no private information prices do not convey any information about the state of the economy. This is due to the fact that no agents have information about states. Also, there will be no randomness in aggregate investment decisions even though noise traders are present in the economy. This is due to intermediaries accommodating movements in noise trading since this is the only way in which they will be indifferent between investing or staying out of the market. Of course, since noise traders decisions are random, this means that the decisions of the intermediaries will also be random, although the aggregate investment decisions will be deterministic. This result depends on the assumption about noise traders being small, since otherwise intermediaries wouldn't be able to compensate their decisions.

When there is private information the equilibrium price distribution will have two points and the aggregate level of investment will also be given by a two point distribution: one for the good state and one for the bad state. Also, even though prices are informative in this case they will be noisy regardless of the amount of noise trading. The existence of this noisy price function is what gives intermediaries incentives to invest in private information. Those intermediaries who are uninformed will try to extract information about the demand state from prices. However, in addition to the existence of noise traders they must also take into account the presence of informed intermediaries. Therefore, in equilibrium, if the price is high an uninformed intermediary will not be sure if this is due to a high demand by uninformed intermediaries or a large demand by informed ones. Underlying this results is the assumption that investment decisions are discrete and bounded which results in prices never being fully informative. This means that it is possible that sometimes prices are high as a result of the decisions of uninformed intermediaries because many of them will be investing, and they are investing because they believe that the high price is a signal of a high future demand. This phenomenon is interpreted by the authors as herding. Finally, the authors show that as the cost of acquiring information gets smaller, is less likely that intermediaries make wrong investment decisions at the aggregate level. However, in turn, the size of this errors will be larger since they are what give intermediaries incentives to acquire information.

Cipriani and Guarino (2003)

This work also examines the herding phenomenon in an economy with endogenous prices studying whether herding can be a source of financial contagion. Cipriani and Guarino argue that a reason for assets' prices not to reflect fundamentals is that information about the fundamentals is spread across investors and if the herd, that is ignore their private information, it won't be possible for prices to aggregate this information.

The authors study an economy in which there are two assets traded in two markets, each one with a risk neutral market maker. Each period an investor is randomly selected from a continuum to trade in one of the two markets. Investors can be either informed or uninformed and trade on their own behalf. Therefore, in this model there are no agency problems. An informed investor receives information about the asset's value and since the two assets' fundamental values are correlated he also learns something about the asset in the market in which he is not currently trading. A crucial assumption is that informed investors are heterogeneous in the sense that part of them enjoy an extra utility while some suffer a disutility from holding the asset. The investors' type is private information. Under this setup there will be gains form trade. This means that it is possible that investors who enjoy the asset buy it even when the market makers ask price is higher than the asset's expected value while investors who experience disutility sell the asset even when it's expected value is higher than the market makers bid price. Regarding market makers, they set bid and ask prices for the assets according to the information available, which consists on current assets' prices; the history of prices and transactions for both assets; and the current transaction.

Cipriani and Guarino show that informational cascades can occur and in this case the investors' actions will be independent of their private information. Moreover, the authors define herding as a situation in which all investors of the same type choose the same action (i.e. all those who enjoy holding the asset buy and those who dislike holding it sell), ignoring their private information. Herding will occur in this economy whenever private information is not too informative about the assets' true value. This is contrast with Avery and Zemsky (1998) result of no herding with endogenous prices and is due to the existence of gains from trade.

Importantly, the authors show that having two markets instead of one is a doubleedged sword. On one hand, if there was only one asset in the economy, once a cascade starts it would never end. However, if information continues to gather in the other market and since assets' value are correlated, information will also arrive to the cascading market and eventually the cascade will break. On the other hand, with two markets the possibility of contagion arises. That is, it is possible that an informational cascade occurs in a market only because agents observe the history of trades in other market and this makes the price of the first asset to deviate from fundamentals, thus starting a cascade. Unfortunately, the authors are not able to conclude on which of these two effects is likelier to predominate.

Finally, using simulations methods the authors show that in their model the unconditional correlation between prices is always greater than the correlation between fundamentals, which is consistent with defining contagion as an excess correlation between assets' prices relative to fundamentals. This occurs even in the long run because with gains from trade informational cascades will arise and won't always break which implies that assets' true values are never discovered.

Chari and Kehoe (2000)

One of the criticisms made to the herding literature is that agents make their decisions sequentially and, as shown by Bikhchandani et al (1992) the final equilibrium usually depends on the order in which the agents decide, which is treated as exogenous. Chari and Kehoe (2000) relax this assumptions endogenizing the order in which agents act. The authors model an economy in which a group of risk neutral agents must decide between investing in a risk free asset or a risky asset. The risky asset's pay will be high in the good state and low in the bad state. Each period an informative signal is randomly received by one of the agents, who must make a discrete choice of whether to invest a fixed amount in the risky asset, which is an irreversible decision, or to wait. All agents observe the number of investments made in each period. In this model waiting to invest is beneficial to agents because they can gather information about the risky asset's true value from other agents' decisions. However, there is also a cost of waiting since agents forgo the flow return from investing. The authors show that in this model with endogenous moving order there are equilibria which exhibit informational cascades and herds which are from the same kind of those present in exogenous timing models such as Banerjee (1992) and Bikhchandani et al (1992). In particular, both herds of investment and herds of no investment will be present.

Next, Chari and Kehoe relax the discrete investment assumption, allowing agents

to make a once and for all investment of any nonnegative amount in the risky asset, which is assumed to have decreasing returns. Not only do the authors prove that herds are possible, but this are likelier to occur compared with a case with exogenous timing. The reason for this is that with continuous investment decisions agents will tend to forego the option of waiting and gathering information before investing because now they can optimally adjust the size of their investment according to what little information they may posses. On the other hand, if timing was exogenous herds of investment would not be present since each agent would have a take it or leave it option to invest and therefore as long as the prior of the risky asset's attractiveness is above some cutoff level the agent will invest some positive amount. However, this investment decision will be used to deduce the agents' private information. Therefore, with exogenous ordering the only form of herding that could exist are no investment herds.

Finally, the authors show that even if agents are given the option of sharing information thew will not have incentives to do so, because truth-telling equilibria will not be feasible. This is the result of assuming that there is an advantage of being early movers for informed investors which gives incentives to mislead other investors.

Zhang (2006)

Zhang (2006) also relaxes the exogenous ordering assumption of the earlier herding literature and studies whether herding persists and also if it is more or less error prone. In this model individuals must decide between adopting an irreversible investment decision or waiting. If they opt for the latter they can still adopt the decision at any future time period. Thus, the author names this a one-sided commitment decision problem. If both decisions faced by the individual were irreversible then this would be a two-sided commitment decision problem.

In this model there is a benefit from waiting because new information may be released about the attractiveness of the irreversible decision. However, there is also a cost since agents who wait gain only their reservation utility. Zhang shows that there is an equilibrium with the property that each period there will be a critical type of agents who make the irreversible investment decision with probability less than one; also all agents who receive good private signals investment and all others wait. In this equilibrium there is a strategic phase where agents wait or invest, depending on their information, followed by a herding phase in which all remaining agents invest immediately or keep waiting forever regardless of their private information. Interestingly, the author shows that in this case the disclosure of public information is of less use than in a model with exogenous ordering. The reason for this is that with exogenous ordering cascades break down once public information is released. This was established by Banerjee (1992) and Bikhchandani et al (1992) and is due to the fact that public information need only to offset the information form the last agent's action before the cascade began. However, with endogenous ordering, once the herding phase is reached all agents act simultaneously and there is no time for public information to break the cascade.

Additionally, Zhang shows that if agents are patient enough, there are equilibria in which almost no one makes decisions, since all prefer to wait for others to reveal their private information. This in turn means that almost no information will be made public. Therefore the expected number of correct decisions will be strictly lower than in a case with exogenous ordering, in which individuals are force to decide once and for all whether to invest or not using their private information and public information available at that moment. This ensures that more private information is revealed to the rest of agents.

Maug and Naik (1995)

This work seeks to rationalize herding by financial intermediaries in a delegated portfolio management setup as a result of performance evaluation contracts. Under this relative performance contract the authors show that it may be optimal for a skilled manager to ignore his private information and herd with other manager even if he knows that this manager is unskilled or less informed.

Maug and Naik study a single period economy in which an investor hires an intermediary of manager to make investment decisions. Both agents have CARA utility functions. The manager has the ability of obtaining superior information informative of the risk asset's return. After receiving his information the manager places an order with a market maker, who sets prices equal to the expected value of the asset, conditional on orders received. In the economy there is also another trader, who manages his own portfolio.

The authors assume that the investor offers the manager a linear relative performance contract which stipulates a fixed payment plus a percentage of the portfolio's final value minus a percentage of the other manager's final portfolio value. Optimal parameters for this remuneration scheme are derived maximizing the investor's expected utility for the case in which there is a moral hazard problem, i.e. the investor cannot verify that the manager makes effort to receive private information; and for cases in which the investor whishes to screen out relatively unskilled managers. In both cases it is shown that the relative performance parameter will be different form zero. The reason for this is that the manager is risk averse and thus giving him a payment that depends on the uncertain final value of assets under management exposes him to risk. For the manager to be willing to work for the investor, the latter must offer the former either a significant risk premium or, he can provide partial insurance by making final payment a function of the other manager's profits. Since both managers' information is partially correlated this rules out the possibility of punishing the manager in circumstances in which all managers perform poorly due to for example misleading private information.

However, it is shown that giving the manager a relative performance contract will lead him to change his investment strategy. Indeed, now when making his portfolio choices he will give a disproportionately small weight to his private information and a disproportionately bigger weight to the information he shares in common with the other manager. The authors show that even if the manager who trades on his own behalf has less information, the manager that works for the investor may herd with him in the wrong direction. This is, if the hired manager's private signal suggests that he should buy the risky asset, but the self employed manager's less reliable information suggests to sell the asset, the former may change his decision from buying to selling, thus herding with the less informed manager in the wrong direction.

Finally, the authors show that even if the investor can design the contracts to make his manager trade in a market different form the self-employed manager and thus making bigger profits from his monopolistic private information, in some cases he will prefer the manager to herd or trade in the same market as the other manager, since this substantially reduces the risk premium that must be paid thanks to the use of a relative performance contract.

Gümbel (2005)

A good part of the literature treats herding as an undesirable phenomenon mainly because it induces intermediaries to disregard private information useful to make investment decisions. The work by Gümbel extends the work by Maug and Naik (1995) exploring the possibility that herding is a useful tool for investors, who may deliberately induce intermediaries to herd , i.e. trade in similar assets, in order to make relative performance contracts feasible.

In this model there are two investors, two risk averse intermediaries with CARA utility functions, two market makers and noise traders. All agents live for one period. Each investor is assigned one intermediary. There are two risky assets in the economy and investors assign their intermediary to trade in one of these assets. This decision becomes public knowledge. Upon making the assignment investors offer intermediaries a linear contract that features a fixed payment, plus a percentage of the intermediary's trading profits minus a percentage of the other intermediary's trading profits. Once the intermediaries accept their contracts they have the option of making effort to receive a private signal that is informative to predict the risky assets' return. Based upon the information acquired intermediaries submit their trading orders to market makers, who setup prices and execute orders. There is one market maker for each asset and they are risk neutral and subject to Bertrand competition which means that they set the assets' prices equal to their expected value conditional on the orders received. The presence of noise traders who make random investment decisions guarantees that it is profitable for intermediaries to acquire information, i.e. assets' prices won't perfectly reveal their costly information.

In equilibrium each intermediary will make portfolio decisions in order to maximize their expected utility; given a price function used by market makers; given his contract and his rival's contract; and given the investors' choice about in which asset they will trade. Investors in turn make the asset choice and design contracts taking into account the intermediaries' participation and incentive compatibility constraints.

The author defines herding as a situation in which both investors assign their intermediary to trade in the same asset. If investors assign their intermediary to trade in different assets then each intermediary will act as a monopolist and will have better chance to exploit their private information. However, the existence of moral hazard means that contracts will have to satisfy the intermediaries' incentive compatibility constraints. Since intermediaries are risk averse increasing the percentage fee of profits they gain will induce intermediaries to take more conservative portfolio decisions. This is related to the irrelevance result studied by Stoughton (1993) and Admati and Pfleiderer (1997). In these two works increasing the contract's percentage fee has no effect on intermediaries behavior. However, in Graham's model the irrelevance is not complete since the intermediary's portfolio choice will have an effect on assets' prices. This means that intermediaries will face a trade off between hedging the risk implied by their contract and exploiting their private information.

In order to avoid this type of behavior from intermediaries managers have the option of making some of the formers' profits a function of their rival's performance. The author assumes that if two intermediaries trade in the same asset the signal received is correlated. This means that by making remuneration depend in relative as opposed to individual performance, intermediaries can be hedged against the risk of receiving useful although not perfect information. For example, if the intermediary's information suggests buying the asset and the decision turns out to be wrong, the investor can observe if the other intermediary also made a bad decision, thus restraining from punishing his manager. Of course, in order to use relative compensation both intermediaries must trade in the same asset, i.e. there must be herding.

There is a downside in making the intermediaries herd, since this will induce competition between intermediaries and principals. In particular, it is possible that one investor sets a relative performance contract strategically to induce aggressive trading by his intermediary, if this makes the other intermediary to reduce his trading intensity.

Whether the investors choose to induce herding or not will depend on a tradeoff between alleviating the risk aversion effects that absolute compensation causes and the competition effect which makes the intermediaries' private information less profitable. The form of the optimal contract under herding is not derived but the author uses numerical examples to show the situations in which inducing herding is desirable. This would be the case if intermediaries' risk aversion is high , if their information's quality is poor and if the cost of acquiring information is high. In the first case the inefficiencies of using compensation based on absolute performance are exacerbated and herding is desirable. In the second case something similar happens but this time due to the fact that intermediaries are subject to greater risk resulting from their information lack of accuracy. In the last case since acquiring information is costlier the intermediaries would have to receive a greater percentage of trading profits which, given that they are risk averse, will lead to suboptimal decision making.

Gümbel shows that if investors could, they would choose not to make their intermediaries' performance information available to the rest of agents in order to avoid the use of relative performance compensation and the increased competition that it brings about. This could be avoided by regulatory requirements such as those existing in the mutual funds market.

A.8 Other Topics

A.8.1 Mutual Fund Performance and Persistence

Berk and Green (2004)

Berk and Green provide a rationalization for several stylized facts observed in the delegated portfolio management market, such as asymmetric flow-performance relationships and lack of persistence in returns. In order to explain this regularities the authors model an economy where all participants are symmetrically informed. There are investors who delegate wealth to managers, who are heterogeneous in terms of the ability they posses to generate positive excess expected returns. This ability is initially unknown to investors and managers. Instead, they both update their beliefs regarding this treat as time goes by and each manager builds a record of returns earned by their portfolios.

A key aspect of this model is that skilled managers are able to generate abnormal returns through their stock picking skills. However, as the size of assets under management increases, managers' ability to deliver high return to investors will decrease. This is an exogenous assumption that captures the notion that as portfolios get bigger a manager will be forced to spread his stock-selecting activities too thin and also larger trades will be associated with a larger unfavorable price impact and higher execution costs. This means that, even though managers are skilled, it is possible that the final return paid to investors is lower as assets under management are bigger.

Additionally, Berk and Green assume that investors are willing to supply capital with infinite elasticity to funds that have positive excess expected return. This means that if a manager has a good investment record then all agents will assign a higher probability to the manager being skilled, and thus capable of generating positive excess returns. Therefore investors will supply new capital to this manager, who will be in charge of a bigger portfolio. This, however decreases his ability to deliver high excess return. In fact, in equilibrium all investors will end up obtaining the same expected excess return across managers, and this will be equal to zero. This means that there will be low persistence in performance, as perceived by investors (this is consistent with the evidence provided by Gruber, 1996). Also, note that this means that a good performance will lead to inflows for the managers, while a bad performance results in outflows. Moreover, the authors show that in their model this relationship is stronger for younger funds than for mature ones, which is also consistent with the stylized facts (see e.g. Chevalier and Ellison, 1997).

The model also allows the authors to study the life cycle of funds. In fact, it is shown that as funds survive and grow the manager will invest an increasingly larger portion of his portfolio passively. This will lead to the fund presenting less idiosyncratic volatility and lower attrition rates. Performing calibration exercises Berk and Green's model predicts that the survival rate for fund decreases with age, which is a feature corroborated by the data. However, for older funds the model predicts much higher survival rates than those observed in their mutual funds sample. The authors suggest that this could be due to managerial turnover within mutual funds, such as good managers being promoted or defecting to other firms. As a result, the low survival rates shown by the data could reflect a renewal in the learning process about managers' abilities. Finally, the authors also find evidence of a large percent of managers (about 80%) in their sample who show some degree of ability to generate positive excess return. Of course, the final return perceived by investors is much lower since as they compete to hire skilled managers they dissipate away such returns.

A.8.2 Multiple Agency Layers

Gervais, Lynch and Musto (2005)

So far the discussed works have focused on an agency problem between investors and financial intermediaries. In practice, however, investors usually make a contract with a fund management firm, which in turn hires some manager to make the portfolio decisions. This implies the existence of two delegation layers. Gervais, Lynch and Musto model this kind of situation in a setup where portfolio managers have the option of associating himself with a fund family. The authors assume that initially neither the managers themselves know their true types. As time goes by investors, managers and fund families gain knowledge about managers' abilities. However, investors take more time to learn than fund families. Therefore, the existence of families is useful if they adopt an appropriate firing policy because they could increase the credibility of the managers they hire. For instance if the family employs to managers and commits ex ante to fire one of them, investors know that the family will be better off keeping the more skilled managers, since family's revenue are positive function of the managers it hires. This reduces the problems caused by the existence of adverse selection.

A.9 Summary

As we have seen the literature on the delegated portfolio management problem is vast. There are several works (e.g. Bhattacharya and Pfleiderer, 1985, Stoughton, 1993, Carpenter et al, 2001, Ou-Yang, 2003) that attempt to derive closed-form solutions for optimal contracts between investors and financial intermediaries. It is clear from the above discussion that such contracts are derived under special conditions, such as particular utility functions. Hence the results lack generality. Moreover, sometimes the predictions made are not robust. In particular, the prediction that contracts should use symmetric performancebased fees is made by the above mentioned authors, such as Bhattacharya and Pfleiderer (1985) and Stoughton (1993). However, the works by Carpenter (2000), Ross (2004) and Panageas and Westerfield (2007) highlight the fact that asymmetric contracts do not necessarily lead to inefficient portfolio decisions by financial intermediaries. Also, a number of works such as Ou-Yang (2003) suggest that using benchmarks and relative compensation may be beneficial for investors. On this point the stylized facts suggest that this type of performance-based contracts are not widely used, at least in the mutual fund industry (see Golec, 1992, Blake et al, 2003, and Cuoco and Kaniel, 2007). Also, the benchmarks are typically simple indexes such as the S&P 500 while the theoretical models suggest the use

of more sophisticated benchmarks that are functions of the intermediaries' promised return (Bhattacharya and Pfleiderer, 1985, and Stoughton, 1993), depend on the intermediary's degree of risk aversion (Admati and Pfleiderer, 1997) or are used to punish those intermediaries with performance above the benchmark (Bhattacharya, 1999). An exception on this line is Gómez and Sharma (2005), who do suggest that simple benchmarks could be useful to align incentives if the intermediaries' trading strategies are restricted.

Nevertheless the theoretical works have been successful in rationalizing stylized facts such as high transaction volume by institutional investors (Dow and Gorton, 1997). There is also substantial effort being done to understand how the existence of delegated portfolio management may affect assets' prices (Cuoco and Kaniel, 2007, Goldman and Slezak, 2003).

The existing literature also suggests several reasons for intermediaries to imitate the investment decisions of rivals, such as infering private information (Banerjee, 1992, Bikhchandani et al, 1992) and payoff externalities when the intermediary is evaluated against some benchmark (Maug and Naik, 1995). Also, there are several works showing that herding is a robust phenomenon and would prevail even in more realistic setups with endogenous prices (Avery and Zemsky, 1998, Beaudry and González, 2003, and Cipriani and Guarino, 2003) or endogenous ordering (Chari and Kehoe, 2000, and Zhang, 2006). While herding is often associated with inefficiencies since it implies that intermediaries make no use of their private information, it may be the case that this phenomenon is beneficial, as suggested by Gümbel (2005), since it may make rewarding intermediaries less costly using relative performance remuneration schemes. One of the reasons for intermediaries to herd that has received particular attention in the literature is that of reputational concerns. The works by Scharfstein and Stein (1990), Avery and Chevalier (1999), Graham (1999) and Ottaviani and Sørensen (2006) show how intermediaries worried about their reputation may herd instead of using their private information. In fact, Dornbusch et al (2000) suggest that this is one of the contagion mechanisms that operated during the Asian crisis. This view contrasts with that of Heinkel and Stoughton (1994), Chemmanur and Fulghieri (1994) and Farnsworth (2003), who argue that the presence of implicit incentives provided by reputation may alleviate the inefficiencies caused by informational asymmetries even without the use of bonus of performance based fees. Also, the predictions about the relation between reputation and incentives to herd are mixed; Avery and Chevalier (1999) predict a negative relation while Graham (1999) predicts that this relation is positive. There is also mixed evidence with Chevalier and Ellison (1999) and Hong et al (2000) validating the prediction by Avery and Chevalier (1999) and Graham (1999) presenting evidence supporting his own predictions.

Given the existent lack of consensus regarding the effects of the possibility of investing in reputation in a delegated portfolio management context we make a contribution by studying the relation between reputation and herding in a delegated portfolio management context recognizing that investing in reputation is a slow process that takes place over several periods and that absent some source of permanent uncertainty about the intermediaries' characteristics steady state reputational equilibria cannot exist. We thus follow the methodology developed by Mailath and Samuelson (1998), (2001) and Vial (2008), which hasn't been applied before in a delegated portfolio management context with herding.

APPENDIX B

MATLAB Code

%This program solves de skilled financial intermediary's decision problem %between investing in information or herding.

%The program asks for an initial guess for critical reputation value below %(above) which intermediaries choose to herd (invest in information). On %output the program gives the correct critical value as well as information %regarding the economy's risk free return, the reputation for which %function omega equals zero (Mu hat); as well as the lowest and highest %values reputation can take (Mu min and Mu max respectively). The program %also plots the value function, price function, dynamic system that %describes the evolution of beliefs, and functions wmu and vmu. clear;

%Setting parameter values. Values in parentheses correspond to baseline %case

global k W p PH y rh rl q mucrit Eta Ri Rh Lambda Theta c Ph yh rh rl q Lambda = 0.15; %Replacement probability (0.15) Theta = 0.5; %Skilled FI mass (0.5) Eta = 0.45; %Successful imitation probability (0.5)

PH = 0.85;	%Good signal	l probability	given	r=rH	if FI	invests	(0.85))
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y = 1-PH;

p = 0.5; %r=rH unconditional probability (0.5)

$$rh = 2.2;$$
 %Risky asset pay in good state (2.1)

rl = 1; %Risky asset pay in low state (1.2)

q = 1; %Risky asset price (1.1)

W=600; %Wealth under management (600)

$$c = 3;$$
 %Information investment cost (1.25)

 $Ph = 0.5^{*}(1+Eta^{*}(2^{*}PH-1));$ %d=g probability given FI herds

yh = 1-Ph;

Delta = 0.75; %Discount factor (0.75)

 $Phi = Delta^{*}(1-Lambda);$ %Discount factor adjusted by replacement prob.

 $R = p^{*}(rh/q) + (1-p)^{*}(rl/q); \qquad \% Ex \text{ ante expected return (1.55)}$

 $Ri = R + p^{*}(1-p)^{*}(2^{*}PH-1)^{*}((rh-rl)/q);$ %Expected return given FI invests

 $Rh = R + Eta^*p^*(1-p)^*(2^*PH-1)^*((rh-rl)/q);$ %Expected return given FI herds

k = Rh; %Price function constant

Mumax = Theta; %(0.925)

%Discretizing Reputation

points = 1000;

Mu = linspace(0,1,points)'; %Initial reputation values

N = size(Mu);

%Reward matrix

f = zeros(points, 2);

% Transition probability matrix if FI invests in information

Mi = zeros(points, points);

 $\% {\rm Transition}$ probability matrix if FI herds

Mh = zeros(points, points);

%Initial Mu^{*} guess

 $mucrit = input('Mu^* Guess: ')$

%Obtaining reputation value one period ahead with "mub" and "mug" functions

for i = 1:N(1,1)

Mugraw(i,1) = mug(Mu(i,1)); %d=g

Mubraw(i,1) = mub(Mu(i,1)); %d=b

 end

%Nearing next period's reputation to closest value in grid

for i = 1:N(1,1)

[Mugindex(i,1), Mugindex(i,2)] = min(abs(Mu-Mugraw(i,1)));

Mug(i,1) = Mu(Mugindex(i,2));

[Mubindex(i,1), Mubindex(i,2)] = min(abs(Mu-Mubraw(i,1)));

Mub(i,1) = Mu(Mubindex(i,2));

 end

%Filling Mi matrix

for i = 1:N(1,1)

if Mugindex(i,2) == Mubindex(i,2)

else

Mi(i,Mugindex(i,2)) = PH;

Mi(i,Mubindex(i,2)) = y;

 ${\rm end}$

 ${\rm end}$

%Filling Mh matrix

for i = 1:N(1,1)

if Mugindex(i,2) == Mubindex(i,2)

Mh(i,Mugindex(i,2)) = Ph + yh;

else

Mh(i,Mugindex(i,2)) = Ph;

Mh(i,Mubindex(i,2)) = yh;

 ${\rm end}$

 ${\rm end}$

 $\% {\rm Obtaining}$ transition probability matrix

M = cat(1,Mi,Mh);

%Filling reward matrix using "fee" function

for i = 1:N(1,1)

$$f(i,1) = (fee(Mu(i,1)))^*Ri^*W-c;$$

 $f(i,2) = (fee(Mu(i,1)))^*Rh^*W;$

 ${\rm end}$

%Packing model structure

model.reward = f;

model.transprob = M;

model.discount = Phi;

%Solving model using Competon Toolbox

[v,a] = ddpsolve(model);

%Obtaining function wmu

for i = 1:N(1,1)

$$wmu(i,1) = c - (fee(Mu(i,1)))^*(Ri-Rh)^*W;$$

 ${\rm end}$

%Obtaining function vmu

for i = 1:N(1,1)

$$VMug(i,1) = v(Mugindex(i,2));$$

$$VMub(i,1) = v(Mubindex(i,2));$$

 ${\rm end}$

vmu = (VMug - VMub).*(Delta*(1-Lambda)*(PH-Ph));

%Obtaining Bayes rule

```
for i = 1:N(1,1)
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 $\mathrm{Mudg}(i,1) = \mathrm{mug}(\mathrm{Mu}(i,1));$

$$Mudb(i,1) = mub(Mu(i,1));$$

 ${\rm end}$

%Finding mutat (Reputation value for which wmu=0)

 $Muhat = ((W^{*}(Ri-Rh)^{*}(k-Rh)+c^{*}Rh))/((Ri-Rh)^{*}((Ri-Rh)^{*}W-c));$

%Obtaining price function

for i = 1:N(1,1)

bmu(i,1) = fee(Mu(i,1))*10000;

 ${\rm end}$

%Plotting

figure (1);

plot(Mu,v);

xlabel('Reputation')

ylabel('\$')

title('Value Function')

axis([0, 1, -Inf, Inf])

figure(2);

plot(Mu,wmu,Mu,vmu,'-');

xlabel('Reputation')

ylabel('\$')

title ('Functions w(mu) and v(mu)')

legend('w(mu)', 'v(mu)', 1)

axis([0, 1, -Inf, Inf])

figure(3);

plot(Mu,Mudg,Mu,Mudb,Mu,Mu);

xlabel('Reputation in t')

ylabel('Reputation in t+1')

title('Bayes Rule')

legend('Good decision', 'Bad decision', '45°', 4)

axis([0, 1, 0, 1])

figure(4);

plot(Mu,bmu);

xlabel('Reputation')

ylabel('Basis Points')

title('Fee')

axis([0, 1, -Inf, Inf])

%
Finding Mu* using the policy function

j=1;

while a(j,1) == 2;

j=j+1;

 ${\rm end}$

MuCritic = Mu(j,1);

% Finding Mumin

difmumin=Mub-Mu;

jjj=1;

while difmumin(jjj,1) > 0;

jjj=jjj+1;

 ${\rm end}$

 $\mathrm{Mumin} = \mathrm{Mu}(\mathrm{jjj}, 1);$

Table = [R MuCritic Muhat Mumax Mumin];

disp(' R Mu* Muhat Mumax Mumin')

 $\operatorname{disp}(\operatorname{Table})$

Appendix C

Additional MATLAB Functions

C.1 Mug Function

function [mug] = mug(x)

%This function updates initial reputation "x", conditional on a good

%investment decision being made (d=b).

%Specifically, we have $x(t+1)=(A^*x(t)+B)/(C^*x(t)+D)$

%Setting parameter values

global Lambda Theta mucrit Eta PH

A = Lambda*Theta*(1-Eta)*(PH-0.5)+(1-Lambda)*PH;

B = Lambda*Theta*(0.5+Eta*(PH-0.5));

 $C = (1-Eta)^*(PH-0.5);$

 $D = (0.5 + Eta^{*}(PH-0.5));$

%Defining function

if
$$x \le mucrit$$

mug = Lambda*Theta+(1-Lambda)*(x);

else

$$mug = (A^*(x)+B)/(C^*(x)+D);$$

 end

C.2 Mub Function

Mub Function

function [mub] = mub(x)

%This function updates initial reputation "x", conditional on a bad

%investment decision being made (d=b).

%Specifically, we have $x(t+1)=(Ap^*x(t)+Bp)/(Cp^*x(t)+Dp)$

%Setting parameter values

global Lambda Theta mucrit Eta PH

 $\label{eq:approx_appr$

Bp = Lambda*Theta*(0.5-Eta*(PH-0.5));

 $Cp = -(1-Eta)^*(PH-0.5);$

 $Dp = (0.5-Eta^{*}(PH-0.5));$

%Defining function

 ${\rm if}\; x <= {\rm mucrit} \\$

mub = Lambda*Theta+(1-Lambda)*(x);

else

$$\mathrm{mub} = (\mathrm{Ap}^*(\mathrm{x}) + \mathrm{Bp}) / (\mathrm{Cp}^*(\mathrm{x}) + \mathrm{Dp});$$

end

C.3 Fee Function

function [comision] = comision(x)

% This function estimates the fee p(x) charged by a financial intermediary

% with reputation x.

%Setting parameters' values

global k Ri Rh mucrit

%Defining function

 ${\rm if}\; x <= {\rm mucrit}$

comision = (1-k/(Rh));

else

 $\operatorname{comision} = (1-k/((\operatorname{Ri-Rh})*x+\operatorname{Rh}));$

 ${\rm end}$