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## A Model of the Confidence Channel of Fiscal Policy

This article presents a simple macroeconomic model where government spending affects aggregate demand directly and indirectly, through an expectational channel. Prices are fully flexible and the model is static, so intertemporal issues play no role. There are three important elements in the model: (i) fixed adjustment costs for investment, which create an inaction zone; (ii) noisy idiosyncratic information about the aggregate economy; and (iii) imperfect substitution among private goods and goods provided by the government. An increase in government spending raises demand for private goods and may prevent a coordination failure. The optimal level of government expenditure is high when the desired level of investment is low, which we interpret as a time of low economic activity.

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THERE IS A WIDESPREAD PERCEPTION among policymakers, the business community, and the media that governments should intervene to induce confidence in the economy. However, this “confidence channel” is met with skepticism among economists owing to the lack of theoretical backing. Mankiw (2009) is sympathetic to the view that “confidence is the key to getting the economy back on track” but adds that “we economists don’t know very much about what drives the animal spirits of economic participants. Until we figure it out, it is best to be suspicious of any policy whose benefits are supposed to work through the amorphous

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channel of confidence.” Kotlikoff (2012) discusses the possibility that “thanks to the government’s intervention, the economy will psyche itself up, coordinate on good times, produce good times” and adds that “the coordination failure models do contain this magic as a real possibility,” but asks “how, precisely, can the government coordinate beliefs on good times?” Cochrane (2009) complains that commentators “say that we should have a fiscal stimulus to ‘give people confidence’ even if we have neither theory nor evidence that it will work.”

This article presents a simple model where fiscal policy affects agents’ beliefs about others’ investment and production decisions. We see these beliefs as natural model counterparts of what is often meant by confidence in surveys and policy discussions. Beliefs about others’ actions affect a producer’s demand expectations and, consequently, her own investment and production decisions.

The mechanism can be summarized as follows. Firms choose between inaction and investment. Incentives for investment depend positively on government spending and on beliefs about other firms’ investment. An increase in government spending directly incentivizes investment; hence, it affects a firm’s beliefs about others’ investment decisions, which, in turn, provides a further boost to investment. This indirect effect of government spending through beliefs is what we call the confidence channel of fiscal policy.

The model is simple and standard in many respects. Three ingredients are key for the results: (i) fixed adjustment costs for investment, (ii) noisy information about aggregate productivity, and (iii) public goods as imperfect substitutes for private goods. The interaction of these three features of the model gives rise to the confidence channel of fiscal policy.

Firms produce differentiated goods and operate under monopolistic competition. Hence, an increase in the production of a given variety increases the demand for other varieties in the economy, raising their prices. Fixed adjustment costs for investment generate an inaction region, because it is not worth paying the fixed costs if the desired level of investment is small. This gives rise to a coordination game: one’s decision about paying the fixed cost for investing depends on her expectations about demand, and consequently, about whether others will choose to leave the inaction region as well. In case of complete information, this would yield equilibrium multiplicity.

Second, firms receive an idiosyncratic signal about the aggregate productivity level. This creates strategic uncertainty among firms and removes an undesirable feature of models with strategic complementarities and complete information, namely, that firms know what others are doing in equilibrium. When deciding whether to invest or not, firms have to try to forecast what others will be doing—and hence to forecast the forecast of others. Although firms choose investment from a continuum of actions, the model is tractable and we can apply standard results from the literature of global games presented in Morris and Shin (2003).

In equilibrium, different outcomes can be achieved as two regimes of a unique threshold equilibrium. For some parameters, coordination failures in investment arise: firms do not invest because they expect others to be stuck in the inaction region, even though it would be optimal for everyone to invest if they believed that others would do

so as well. The private sector fails to achieve the optimal level of aggregate demand owing to pessimistic beliefs about others' decisions—a justified lack of “confidence.”

The third important feature of the model is that fiscal policy has a direct effect on incentives for investment. In the model, goods provided by the government and those produced by private agents are imperfect substitutes. Hence, a larger level of government spending increases the utility from consumption of private goods in a way that resembles a preference shock in standard macroeconomic models. In consequence, larger government spending leads to a larger demand for private goods.

In the absence of the fixed cost for investment, the optimal level of government spending would equate its marginal cost and its marginal effect on agents' utility. In this model, larger government spending also raises incentives for producers to incur the fixed cost and invest. It thus has a direct positive effect on the demand for private goods and an indirect effect by affecting beliefs about whether others will stay in the inaction region. When this extra benefit is relevant, the optimal level of government spending is larger.

The model is static for analytical convenience but captures some essential features of the economy that would also be present in a fully fledged dynamic framework. Firms are endowed with some initial level of capital, that can be interpreted as the level of capital inherited from previous periods, after depreciation has taken its toll. Usually, firms would invest because capital has depreciated and productivity has increased. However, when the desired level of investment is low enough, firms might choose inaction—which is free—instead of investment. Hence, coordination failures arise in the model when the desired level of investment is low, which we interpret as times of low economic activity.<sup>1</sup>

Empirical work highlights the importance of nonconvex adjustment costs for firms' decisions. Cooper and Haltiwanger (2006) build a structural model that incorporates convex and nonconvex adjustment costs and use microlevel data on investment in the United States to understand which kind of adjustment costs are needed to explain firms' behavior. They show that fixed adjustment costs are important for explaining investment decisions, which is consistent with one of our main assumptions.

Nonconvex adjustment costs generate a region of inaction that is key for the results of this paper. Bloom (2009) uses microlevel data to understand the effect of shocks to macroeconomic uncertainty on firms' decisions. He finds evidence for “substantial fixed costs of investment and a large loss from capital resale.” He also shows that the “region of inaction” plays a key role in explaining the reaction of firms to uncertainty shocks. In our model, fiscal policy can affect this region of inaction.

There are also empirical results supporting the confidence channel of fiscal policy. Bachmann and Sims (2012) employ a nonlinear vector autoregression (VAR) using three variables: output, government expenditure, and a measure of confidence.<sup>2</sup> Their

1. The static framework also makes clear that fiscal policy is not operating in the model through channels that rely on intertemporal considerations.

2. The confidence measure is the Index of Consumer Expectations from the Michigan Survey of Consumers.

main finding is that government expenditure has larger impacts on output during recessions than in expansions. Moreover, when the confidence channel is shut down in their VAR, impulse responses in recessions become very similar to their counterparts in expansions. These findings resonate with the results in this article. Here, in periods where productivity is relatively low so that coordination failures might lead to inaction (recessions), government spending has a positive effect on agents' beliefs about others' actions (confidence), but this effect is not present when everybody is confident that others will be choosing a positive level of investment (booms).<sup>3</sup>

The remainder of this introduction explores the relation between this article and the literature. The next section presents the model, Section 3 shows the results and Section 4 considers extensions to the basic model. Section 5 concludes.

## 1. RELATED LITERATURE

In our model, as in Kiyotaki (1988), monopolistic competition induces strategic complementarities among firms because an increase in the production of a given variety increases the demand for other varieties in the economy, raising their prices. The assumption of a fixed cost generates increasing returns to scale in a region of the production function, but eventually an extra unit of output becomes increasingly costly.<sup>4</sup> In Kiyotaki (1988), that gives rise to multiple equilibria. In a similar vein, Cooper and John (1988) show how strategic complementarities lead to multiple Pareto-ranked equilibria and Farmer and Guo (1994) explore similar ideas in an infinite-horizon framework.<sup>5</sup>

Since Keynes, it has been argued that investment decisions might be driven by "animal spirits," that is, shifts in expectations for no apparent reason. A key question though is about whether, how and when policy interventions could affect demand and lead agents to coordinate in a good equilibrium. Models with multiple equilibria do not allow us to understand what drives beliefs (as long as there are multiple equilibria, beliefs about which one will be played are not pinned down by the model).

There is a growing literature focused on the role of noisy information in business cycles in a framework with imperfect common knowledge. In Woodford (2002) and Nimark (2008), noisy information leads to inertia in pricing decisions, which is particularly relevant for monetary policy.<sup>6</sup> Closer to this article, the models in Lorenzoni (2009), Angeletos and La'O (2010), and Angeletos and La'O (2013) aim

3. Bachmann and Sims (2012) do not interpret their results as evidence for sentiment-induced increase in production. They interpret their findings as capturing increases in productivity induced by government expenditures in infrastructure and education. Future research might help us to assess more accurately the effect of each factor on the confidence index.

4. The specific assumptions on production are different in both models: here, there are fixed costs for investment but a technology with decreasing returns, whereas in Kiyotaki (1988), there are increasing returns to capital and labor, but a maximum level of labor.

5. See also Benhabib and Farmer (1994) and the survey in Benhabib and Farmer (1999). Multiple equilibria also arise in Gali (1996) and Ball and Romer (1991) but for different reasons.

6. See also Adam (2007).

at incorporating “demand shocks” and “sentiments” in a standard macroeconomic framework. In Chamley (2012), a preference (demand) shock might lead firms to choose a less efficient but flexible technology (which can be interpreted as inaction). In Guimaraes and Machado (2015), there is complete information but owing to timing frictions in investment decisions, expected demand plays a key role. None of those models study how government spending might affect confidence or sentiments. In independent work, Schaal and Taschereau-Dumouchel (2015) develop a quantitative model of business cycles with coordination failures and study the effect of government spending. However, though their model is suitable for quantitative analysis, our model is able to deliver analytical results on how fiscal policy can mitigate coordination failures.<sup>7</sup>

Much of this literature considers models with weak complementarities, where the equilibrium is unique no matter the structure of information (see Morris and Shin 2002, Angeletos and Pavan 2004). Here, in contrast, the fixed adjustment costs imply strong complementarities in investment decisions. From a technical point of view, our model builds on the global game literature started by the seminal papers of Carlsson and Van Damme (1993) and Morris and Shin (1998).<sup>8</sup> In this sense, the article is related to Sákovic and Steiner (2012), who study who should benefit from subsidies in an economy with heterogeneous agents and strategic complementarities.

The article is also related to a strand of the literature focused on the effects of fiscal policy in economic activity.<sup>9</sup> In standard real business cycle (RBC) models, government spending generates a negative wealth effect that brings down household’s income and induces them to work more. However, the size of government expenditure multipliers is likely to be low. Bouakez and Rebei (2007) build an RBC model with competitive firms where goods provided by the government and private goods are imperfect complements, and obtain results that are closer to VAR evidence on the relation between consumption and government expenditures.

In New Keynesian models with nominal rigidities, government spending has a higher effect on output than in RBC models, but still in general the fiscal multiplier is way below one. As pointed out by Linnemann and Schabert (2003), the effect of fiscal policy depends on monetary policy as the demand channel depends crucially on the real interest rate. Building on this insight, a recent literature emphasizes how fiscal multipliers can be large when the economy is in a liquidity trap. As argued in Christiano, Eichenbaum, and Rebelo (2011), if the economy is stuck at the zero lower bound (ZLB), an increase in government spending can lead to an increase in expected inflation, which, in turn, helps to lower the real interest rate, boosting private spending (see also Eggertsson 2011, Woodford 2011). In Eggertsson and Krugman

7. There are also substantive differences between both models. In Schaal and Taschereau-Dumouchel (2015), government expenditure is a waste of resources, and a negative wealth effect on labor supply is necessary for government spending to enhance coordination. Here, in contrast, government spending affects the household valuation of consumption goods.

8. Morris and Shin (2003) provides a comprehensive review of this literature.

9. See, for instance, Rotemberg and Woodford (1992) and Blanchard and Perotti (2002). For a survey of the results and new evidence, see Ramey (2011). For evidence considering several different countries, see Ilzetzki, Mendoza, and Vegh (2013).

(2012), Ricardian equivalence does not hold because some agents are facing their borrowing limit. Moreover, due to an increase in prices, government spending can reduce the debt burden for credit-constrained agents. Hence, fiscal policy can be effective if a debt crisis pushes the economy against the ZLB. Erceg and Lindé (2014) show that once we take into account that fiscal policy affects the duration of a liquidity trap, we get that the multiplier decreases with the amount of government spending. Mertens and Ravn (2014) argue that the effect of government spending on output is high only if the economy got to the ZLB because of a fundamental shock and not due to pessimistic beliefs. Rendahl (2016) shows that equilibrium unemployment dynamics can significantly increase the fiscal multiplier as government spending can put a halt to a downward spiral of self-reinforcing thrift.

Those papers show that fiscal policy can be effective when the economy is in a liquidity trap but have not much to say about economies that are far away from the ZLB—which was the case of many emerging economies during the last recession. Canzoneri et al. (2016) present a model with costly financial intermediation that generates large fiscal multipliers in recessions without relying on the ZLB. Our model provides a different reason for why multipliers might be large. The mechanism is not related to any kind of nominal rigidity (prices are perfectly flexible): government spending might trigger a switch from an “inaction regime” to an “investment regime.” The fiscal multiplier can be large in our model even though all the burden from taxation falls in the current period.

## 2. MODEL

### 2.1 Households

There is a representative household who derives utility from leisure and from the consumption of private and public goods. His preferences are given by

$$U = u(C_p, G) - \chi L,$$

where  $\chi > 0$ ,  $L$  is the total amount of labor supplied, and utility from consumption depends on the amount of private and public goods consumed according to

$$u(C_p, G) = \left( \omega C_p^{\frac{\theta-1}{\theta}} + (1-\omega)G^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad (1)$$

where  $G$  is the amount of public goods,  $\omega \in (0, 1)$ ,  $\theta > 1$  represents the elasticity of substitution between private and public goods and

$$C_p = \left( \int_0^1 c_i^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where  $c_i$  denotes the amount consumed of good  $i$  and  $\theta > 1$  is the elasticity of substitution among private goods.<sup>10</sup>

The expression in (1) shows the first departure from standard macroeconomic models: the good provided by the government and private goods enter the consumer's utility function in the same way. Consumption aggregators of macroeconomic models usually comprise all types of goods from oranges to mobile phones. Here, parks are also combined in the constant elasticity of substitution (CES) aggregator.<sup>11</sup> The elasticity of substitution  $\theta > 1$  between private goods is equal to the elasticity of substitution between a private good and the public good  $G$ .<sup>12</sup>

Although we refer to  $G$  as a public good, it does not need to be nonrival and nonexcludable. Strictly speaking,  $G$  is the good provided by the government and plays no special role in the agents' utility function.<sup>13</sup>

The household buys private goods from firms. Public goods are provided by the government and financed through a lump-sum tax on the representative household. Therefore, the household budget constraint is given by

$$\int_0^1 c_i p_i di = wL + \Pi - T,$$

where  $w$  is the wage,  $\Pi$  is the amount of profits received from firms,  $p_i$  is the price of good  $i$ , and  $T$  are lump-sum taxes. We define the price index as

$$P_p = \left( \int_0^1 p_i^{1-\theta} di \right)^{\frac{1}{1-\theta}},$$

which turns out to be the minimal amount of spending needed to get one unit of the composite private good  $C_p$ . Therefore, we can replace  $\int_0^1 c_i p_i di = P_p C_p$  in the household budget constraint.

The total amount of labor supplied is given by

$$L = L_K + L_G + \int_0^1 l_i di,$$

where  $L_K$  is the amount of labor supplied to capital producers,  $L_G$  is the amount supplied to the government, and  $l_i$  is the amount supplied to firm  $i$ .

10. The assumption of linear utility on labor is similar to the one in Kiyotaki (1988) and implies a constant ratio between the wage and the hedonic price of the consumption aggregator.

11. The assumption that utility is not separable in public and private goods is common in the public finance literature since Samuelson (1954).

12. We relax this assumption in Section 4.2.

13. Fiscal policy might affect incentives for investment for other reasons, for example, government spending could increase productivity in the private sector as in Barro (1990). We present one alternative model of this direct effect of fiscal policy in Section 4.1 and show that an indirect effect through beliefs is present as well.

## 2.2 Private Goods Firms

There is a continuum of firms indexed by  $i \in (0, 1)$  that produce private goods consumed by the household. Each intermediate good  $y_i$  is produced by a monopolist with the following technology:

$$y_i = A k_i^{\alpha_k} l_i^{\alpha_l},$$

where  $A$  is a productivity shifter that is the same for every firm;  $k_i$  and  $l_i$  denote the amount of capital and labor used by firm  $i$  and  $\alpha = \alpha_k + \alpha_l < 1$ , so production is subject to decreasing returns.<sup>14</sup>

A firm is endowed with a level of capital  $k_0 > 0$ , which is the same across firms. Although the model is static, we think about  $k_0$  as the level of capital firms inherited from a previous period, after depreciation has been considered. The level of capital is given by

$$k_i = k_0 + I_i.$$

Firm's profits are given by

$$\pi_i = p_i y_i - w l_i - P_K \mathcal{C}(I_i),$$

where  $P_K$  is the price of capital,  $\mathcal{C}(\cdot)$  denotes the cost of investment, and  $I_i$  denotes the amount invested by firm  $i$ . A firm maximizes households' valuation of its profit, given by real profits times the marginal utility of consuming one extra unit of the composite good, that is,  $(\pi_i/P_p)(\partial u/\partial C_p)$ .

Positive investment entails a fixed adjustment cost, which captures the nonconvex costs highlighted in Cooper and Haltiwanger (2006), Bloom (2009), and others. Hence,  $\mathcal{C}(\cdot)$  takes the form

$$\mathcal{C}(I) = \begin{cases} I + \psi & \text{if } I > 0, \\ 0 & \text{if } I = 0, \end{cases}$$

where  $\psi > 0$  are the fixed cost associated with a positive level of investment. For simplicity, we impose irreversibility of capital, that is,  $I_i \geq 0 \forall i \in [0, 1]$ .<sup>15</sup>

This is the second important departure from a standard macroeconomic model. The fixed cost introduces locally increasing returns that may lead to strategic

14. The assumption of decreasing returns in production captures, in a simple way, the idea that at some point, increasing output becomes increasingly costly.

15. This assumption greatly simplifies the analysis because the strategic interaction among firms becomes a binary-action global game model. We can thus rely on previous results from the literature of global games that guarantee equilibrium uniqueness in the model. If instead firms were allowed to sell some of their capital, then either incentives to pay the fixed adjustment costs would not be monotonic in  $A$  (because firms would choose to pay the cost for very high and very low  $A$ ) or the strategic interaction among firms would become a three-action global game model (if firms also had the option of paying a "disinvestment cost"). Any of these options would complicate the analysis without necessarily adding any insights to the problem we study.



complementarities in investment decisions. But for large levels of production, there are decreasing returns.

Prices are fully flexible. Intermediate good firms choose the amount of labor they hire and the amount of capital they buy. The decision on capital can be decomposed in two choices: a firm chooses whether to pay the fixed cost and if yes, the amount of capital  $k_i$ .

### 2.3 Capital Goods Firm

There is a competitive representative firm producing a capital good. The production function is given by

$$K = \frac{1}{\kappa} L_K, \quad (2)$$

where  $\kappa > 0$ .

### 2.4 The Government

The government has access to a linear technology that transforms labor in the public good  $G$ , so that

$$G = \frac{1}{\gamma} L_G, \quad (3)$$

where  $\gamma > 0$  and  $L_G$  is the amount of labor hired by the government. Given the static nature of the model, the government has to run a balanced budget, hence  $T = wL_G$ , where  $w$  is the equilibrium wage. The level of government spending  $G$  is chosen in order to maximize the expected utility of the household.

We make the following assumption to guarantee that the planner's problem has a solution:

$$(1 - \omega)^{\frac{\theta}{\theta-1}} < \gamma\chi. \quad (4)$$

This expression provides an upper bound for productivity in the public goods sector, which guarantees an interior solution for the equilibrium amount of public goods.

### 2.5 Information and Timing

The only exogenous source of uncertainty in the model is the productivity factor  $A$ . Agents and the government have a prior over  $A$  with pdf  $f(\cdot)$ . Each firm  $i$  receives a signal

$$x_i = \log A + \sigma \varepsilon_i,$$

where  $\varepsilon_i$  is a random variable with pdf  $q(\cdot)$  and  $\mathbb{E}(\varepsilon_i) = 0$ . We study the case  $\sigma \rightarrow 0$ . Although there is very little uncertainty about  $A$ , agents face strategic uncertainty:

they do not know what others will choose in equilibrium, so they try to forecast each others' actions—which requires forecasting others' forecasts as well. That is the third important feature of the model.

The timing of the model is the following:

- (1) Based on the prior information about productivity  $A$ , the government chooses its expenditure  $G$ .
- (2) Firms receive their private signals and decide whether or not to pay the fixed cost of investing.
- (3) Firms choose their desired level of production and markets clear.<sup>16</sup>

The assumption that government expenditures are decided first captures the institutional frictions that prevents fiscal policy from reacting quickly to new information.<sup>17</sup>

### 3. EQUILIBRIUM

We start by solving the model in the last stage. We find the optimal decisions of firms and households taking the proportion of firms that have chosen to pay the fixed cost and government spending as given. We then solve the problem of firms in the second stage. Last, we find the optimal level of government spending.

#### 3.1 Equilibrium in the Third Stage

We will call *investing firms* those that have chosen to pay the fixed adjustment cost. For a given level of government expenditure  $G$  and a given proportion  $h$  of investing firms, an equilibrium in the third stage is defined as prices and quantities such that (i) households choose labor and consumption taking wages and prices as given; (ii) capital goods firms choose production taking wages and prices as given; (iii) each private good firm chooses inputs taking their prices and the demand schedule for its own good as given, and only firms that have chosen to invest can adjust capital; and (iv) markets clear.

*Optimal choice of the household.* For a given wage  $w$  and consumption  $C_p$ , equilibrium in the labor market implies

$$\frac{w}{P_p} = \frac{\chi}{\frac{\partial u}{\partial C_p}}. \quad (5)$$

16. For expositional simplicity, we assume that  $A$  and  $h$  are observed at this stage, but this is an inessential assumption, because agents' forecasts about these variables would almost always be very accurate.

17. This assumption is in line with the VAR literature on the effects of government spending that builds on Blanchard and Perotti (2002). The usual assumption is that government expenditure does not respond to contemporaneous shocks to other variables.

Given a level of consumption  $C_p$ , the optimal demand for each intermediate good is

$$p_i = y_i^{-\frac{1}{\theta}} (C_p)^{\frac{1}{\theta}} P_p. \quad (6)$$

*Optimal choice of capital goods firms.* Zero profit for capital producers implies that they supply capital inelastically as long as the price of capital  $P_K$  equals its marginal cost:

$$P_K = \kappa w. \quad (7)$$

*Optimal choice of private goods firms.* Using the demand schedule (6) and the equilibrium conditions in (7) and (5), we can write the firm value as

$$\hat{\pi}_i \equiv \frac{\pi_i}{P_p} \frac{\partial u}{\partial C_p} = (A k_i^{\alpha_k} l_i^{\alpha_l})^{\frac{\theta-1}{\theta}} \omega Y^{\frac{1}{\theta}} - \chi (\kappa C(I_i) + l_i), \quad (8)$$

where  $Y$  is GDP in this economy, given by

$$Y \equiv \left( \omega C_p^{\frac{\theta-1}{\theta}} + (1-\omega) G^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \quad (9)$$

Firms' profits are increasing in  $Y$ . Differently from a standard macroeconomic setting with monopolistic competition, in our model, the profit shifter is a composite of private and public goods ( $Y$ ), not the usual composite of private goods ( $Y_p$ ). Without government spending in the consumption aggregator (i.e., with  $\omega = 1$ ),  $Y = Y_p$  and the expression for output reflects only the usual demand externality effect: an increase in the production of others shifts up the demand schedule of each firm by increasing households' valuation of a good. With  $\omega < 1$ , an increase in government expenditure has an analogous effect, as it also increases households' valuation of private goods.

Intuitively, an increase in government spending raises the marginal value of private goods. This can be seen as a decrease in the right-hand side of (5). Hence, households are willing to exchange more hours of work for private goods.<sup>18</sup>

We now find firms' choices of inputs given aggregate output  $Y$  and productivity  $A$ . Firms that have paid the fixed cost of investing are able to choose labor and capital. Their optimal choices are given by

$$k_H(Y, A) = \max \{ X A^{(\theta-1)\eta} Y^\eta, k_0 \}$$

18. In contrast, government purchases of private goods (financed by lump-sum taxes) that are transferred to households have no effect in the model: households will work to "top up" their consumption of private goods up to their desired level.

and

$$l_H(Y, A) = \begin{cases} \kappa \left( \frac{\alpha_l}{\alpha_k} \right) X A^{(\theta-1)\eta} Y^\eta & \text{if } k_H(Y, A) \geq k_0, \\ l_L(Y, A) & \text{otherwise,} \end{cases}$$

where  $X \equiv \left[ \frac{\alpha_k \omega}{\chi \kappa} \left( \frac{\theta-1}{\theta} \right) \right]^{\theta \eta} \left[ \kappa \left( \frac{\alpha_l}{\alpha_k} \right) \right]^{\eta \alpha_l (\theta-1)} > 0$ ,  $\eta \equiv \frac{1}{(1-\alpha_l-\alpha_k)\theta + \alpha_l + \alpha_k} \in (0, 1)$  and  $l_L(Y, A)$  is the optimal choice of labor for firms that did not pay the fixed cost. These firms cannot adjust their capital level, so they choose labor  $l_L$  taking as given that their capital is equal to  $k_0$ . Their optimal choice of labor implies

$$l_L(Y, A) = Z k_0^{\alpha_k (\theta-1) \varphi} A^{(\theta-1) \varphi} Y^\varphi, \tag{10}$$

where  $Z \equiv \left[ \frac{\omega}{\chi} \left( \frac{\theta-1}{\theta} \right) \alpha_L \right]^{\theta \varphi} > 0$  and  $\varphi \equiv \frac{1}{\alpha_l + (1-\alpha_l)\theta} \in (0, 1)$ .

Labor and capital decisions pin down how much investing and noninvesting firms will produce:

$$y_H(Y, A) = \begin{cases} \left( \kappa \left( \frac{\alpha_l}{\alpha_k} \right) \right)^{\alpha_l} X^\alpha A^{\theta \eta} Y^{\alpha \eta} & \text{if } k_H(Y, A) \geq k_0, \\ y_L(Y, A) & \text{otherwise,} \end{cases}$$

and

$$y_L(Y, A) = Z^{\alpha_l} k_0^{\alpha_l \alpha_k (\theta-1) \varphi + \alpha_k} A^{\theta \varphi} Y^{\alpha_l \varphi}. \tag{11}$$

Note that investing firms' production and input demand are more responsive to increases in aggregate demand  $Y$  than the choices of noninvesting firms (since  $\eta > \varphi$ ).

The payoffs of each type of firm can be written as

$$\hat{\pi}_H(Y, A) = \omega y_H(Y, A)^{\frac{\theta-1}{\theta}} Y^{\frac{1}{\theta}} - \chi (\kappa (k_H(Y, A) - k_0 + \psi) + l_H(Y, A))$$

and

$$\hat{\pi}_L(Y, A) = \omega y_L(Y, A)^{\frac{\theta-1}{\theta}} Y^{\frac{1}{\theta}} - \chi l_L(Y, A).$$

Investing firms have a better balance between capital and labor, which implies higher production for the same increase in aggregate demand and higher revenues. That comes at a fixed cost equal to  $\chi \kappa \psi$ . The only endogenous variable in the expressions for firms' profits and input choices is  $Y$ . Profits of both types of firms  $\hat{\pi}_H(Y, A)$  and  $\hat{\pi}_L(Y, A)$  are increasing functions of  $Y$  and  $A$ .

*Market clearing.* We now show that there is a unique level of aggregate output  $Y$  that clears private goods markets, given firms' best responses. More specifically, we will show that the expression for  $Y$  in (9) with  $C_p$  given by

$$C_p = \left( h y_H(Y, A)^{\frac{\theta-1}{\theta}} + (1-h) y_L(Y, A)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

has a unique solution. The next lemma proves and derives some properties of  $Y$  in equilibrium. All proofs omitted from the text are in the Appendix.

LEMMA 1. *Equation (9) has a unique solution. In equilibrium,  $Y$  is an increasing function of  $G$ ,  $A$ , and  $h$ .*

Not surprisingly, aggregate output is increasing in the productivity level, in government expenditure in public goods, and in the proportion of investing firms. A larger  $A$  means firms will both choose a larger level of inputs and produce more with their inputs; a larger  $h$  means a larger fraction of firms will invest, and thus, a larger share of firms will produce more; and an increase in the supply of public goods  $G$  affects output directly (equation (9)) and also indirectly, because a large  $Y$  affects the choices of producers and leads to a larger  $C_p$ . Although government spending raises output, it may not increase welfare because the production of public goods requires that agents work more (or, analogously, it requires taxing agents to pay for the labor costs).

Last, it is useful to pin down the amount of labor supplied given  $h$ ,  $A$ , and  $G$  in equilibrium. The amount of labor supplied to the government is simply  $L_g(G) = \gamma G$ . The amount of labor supplied to capital goods firms is

$$L_K(h, A, G) = \kappa h (\psi + k_H(Y(h, A, G), A) - k_0). \quad (12)$$

Finally, the amount of labor supplied to private goods firms is

$$L_P(h, A, G) = h l_H(Y(h, A, G), A) + (1 - h) l_L(Y(h, A, G), A). \quad (13)$$

We write the total amount of labor supplied as  $L(h, A, G) = L_g(G) + L_K(h, A, G) + L_P(h, A, G)$ .

### 3.2 Strategic Complementarities

In the second stage, each firm chooses between investing or not. That will determine whether the firm's profits will be given by  $\hat{\pi}_H(Y, A)$  or  $\hat{\pi}_L(Y, A)$ . What matters for a firm's decision about paying the fixed cost of investment is the expected difference between  $\hat{\pi}_H(Y, A)$  or  $\hat{\pi}_L(Y, A)$ . Define  $D(Y, A)$  as the relative gain from investing:

$$D(Y, A) \equiv \hat{\pi}_H(Y, A) - \hat{\pi}_L(Y, A).$$

Proposition 1 shows that firms that can adjust their capital-level benefit more from an increase in  $Y$  than firms that cannot. This result is important for the supermodularity of the investment game at this stage and for the optimal choice of government spending in the first stage.

PROPOSITION 1. *The relative payoff of investing  $D(Y, A)$  is an increasing function of aggregate demand  $Y$  and productivity  $A$ .*

The return from paying the fixed cost of investment depends on (i) productivity and (ii) aggregate demand. The effect of  $A$  on  $D(Y, A)$  captures the supply-side incentives

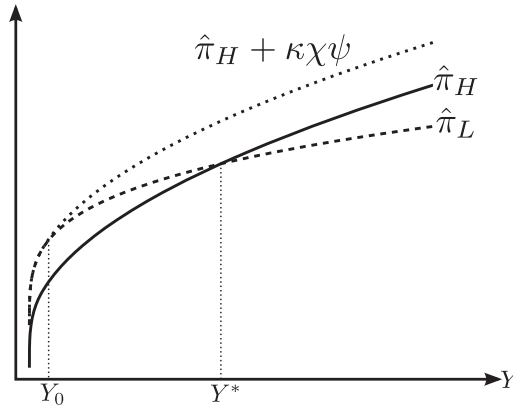


FIG. 1. Demand Externalities and Fixed Cost.

for investment. Higher productivity leads to a larger desirable level of capital and higher profits from investing.

As for aggregate demand, a larger  $Y$  raises the demand for a given variety. The effect of more production of other private goods can be seen from (6) and is standard. Government spending plays a similar role, as  $G$  and  $C_p$  enter in the consumption aggregator in the same way.<sup>19</sup> A larger  $G$  increases the relative price of the bundle of private goods  $P_p$ , as it raises  $\partial u / \partial C_p$ . For a firm, that is analogous to a reduction in the real wage, as shown in (5).<sup>20</sup>

A shift in aggregate demand increases the optimal level of production by firms. Firms that did not pay the fixed cost of investment can only increase production by hiring more labor. When the desired increase in production is large enough, investing firms are better off, as they have an extra margin for raising output.

This is illustrated in Figure 1. The picture shows, as functions of  $Y$ , (i) profits of firms that choose not to invest  $\hat{\pi}_L$ , (ii) profits of firms that choose to invest  $\hat{\pi}_H$ , and (iii) profits of firms that choose to invest without subtracting the adjustment costs  $\hat{\pi}_H + \kappa\chi\psi$ . The difference between (i) and (iii) is that investing firms have two margins of adjustment to raise output (they can increase capital and labor). The difference between (ii) and (i) is the relative profitability of investing  $D(Y, A)$ .

19. The effect of  $G$  on the price of a given variety in terms of the hedonic price of  $Y$  is equivalent to the effect of other private goods.

20. Here, the assumption of linear disutility of labor plays a role. In case of convex disutility of labor, an increase in  $Y$  would also lead to higher wages, which would contribute to a reduction in the return to investment. Were this effect large enough, larger production from others could actually reduce incentives for an individual firm to produce more. However, most papers in macroeconomics assume a relatively high elasticity of labor supply (see the discussions in Chetty et al. 2011 and Peterman 2016).

When aggregate demand is  $Y_0$ , (i) and (iii) coincide because the optimal change in capital is zero. Hence, profits of noninvesting firms are larger than profits of investing firms and the difference is  $\chi\kappa\psi$ . For a relatively low level of aggregate demand ( $Y \in [Y_0, Y^*]$ ), in the absence of fixed costs, the desired level of investment would be positive. However, the additional profit from adjusting the level of capital does not compensate the fixed cost (hence,  $\hat{\pi}_H < \hat{\pi}_L$ ). Higher aggregate demand implies that the possibility of adjusting both inputs is more important. For large enough aggregate demand ( $Y \geq Y^*$ ),  $\hat{\pi}_H > \hat{\pi}_L$  owing to the large gain from adjusting the level of capital.

The next corollary follows directly from Proposition 1 and from  $Y(h, A, G)$  being increasing in all arguments (Lemma 1).

**COROLLARY 1.** *Let  $\hat{D}(h, A, G) \equiv D(Y(h, A, G), A)$ .  $\hat{D}(\cdot)$  is increasing in all arguments.*

Corollary 1 implies that a higher level of government expenditure in public goods leads to more demand and hence more incentives for investment. Another implication is that firms that expect a larger  $h$  will be more willing to invest. In other words, agents' decisions about investing are strategic complements.

### 3.3 The Investment Game

Owing to the fixed cost for investment and the strategic complementarities in investment decisions, agents essentially play a two-action coordination game when choosing whether to invest or not.<sup>21</sup> In case of complete information, the model would display multiple equilibria in a range of parameters.<sup>22</sup>

At this stage, firms do not observe aggregate productivity  $A$  and output  $Y$ , so they form expectations about those variables. Although the former is exogenous, the latter depends on  $A$ ,  $G$ , and  $h$ .

Government spending  $G$  is observed. Beliefs about  $A$  are given by the signal received by each firm. Beliefs about  $h$ , the proportion of firms that will choose to invest, are endogenously determined by the model. An agent's belief about  $h$  depends on  $G$ , on its idiosyncratic signal about  $A$ , and on the strategy played by others in equilibrium. In our view, those beliefs capture the meaning of confidence in the discussions of Mankiw (2009), Cochrane (2009), Kotlikoff (2012), and others. They are crucial in the determination of expectations about the demand for a producer's goods.

The following proposition shows that there is a unique equilibrium in the model.<sup>23</sup>

21. The fixed adjustment costs induce strong strategic complementarities between investment decisions, as defined in Angeletos and Pavan (2004).

22. We briefly analyze the model with multiple equilibria in Appendix A.

23. The model has multiple possible outcomes as Benhabib and Farmer (1999) and others, but beliefs are pinned down by the model. The advantage of this approach is that we can understand how policies affect beliefs.

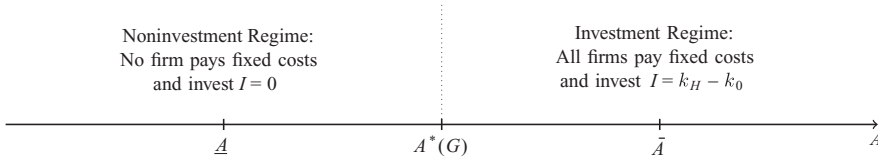


FIG. 2. Threshold Equilibrium.

PROPOSITION 2. As  $\sigma \rightarrow 0$ :

- (1) In every strategy that survives iterated deletion of strictly dominated strategies, firms pay the fixed cost whenever  $x_i > A^*(G)$  and do not pay the fixed cost whenever  $x_i < A^*(G)$ , where the threshold  $A^*(G)$  is given by

$$\int_0^1 \hat{D}(h, A^*(G), G) dh = 0. \tag{14}$$

- (2)  $A^*(G)$  is a decreasing function of  $G$ .

The proof of the first statement is an application of the proposition 2.2 in Morris and Shin (2003). Because investment decisions are strategic complements (Corollary 1) and there is strategic uncertainty, firms need to form beliefs about others’ investment decisions ( $h$ ). Each firm understands that others’ signals are likely to be similar to its own information and that all agents are trying to forecast what others will do. As well known in the literature of global games, that leads to a unique equilibrium.

The assumption of vanishing uncertainty ( $\sigma \rightarrow 0$ ) greatly simplifies finding an expression for the productivity threshold and ensures that all firms make the same investing decision. Consider a firm that got a signal  $x_i$  equal to the productivity threshold  $A^*(G)$ . As discussed by Morris and Shin (2003), when  $\sigma$  is sufficiently small, the posterior belief over other firms’ investment in this pivotal circumstance converges to a uniform distribution in  $[0, 1]$ . That is reflected in the equilibrium condition (14):  $A^*(G)$  is the productivity level such that a firm is indifferent between investing or not in this pivotal contingency. The integral in (14) is the expected difference in payoffs from investing and not investing, given that the distribution of  $h$  for a firm indifferent between both actions is uniform in  $[0, 1]$ .

The threshold determines two regimes: an investing regime in which all firms are investing (if  $A > A^*(G)$ ,  $h = 1$ ) and a noninvesting regime (if  $A < A^*(G)$ ,  $h = 0$ ), as shown in Figure 2.

Denote by  $\bar{A}$  the productivity level such that it is optimal for an individual firm to invest regardless of others’ decisions and  $\underline{A}$  the productivity level such that it is optimal for an individual firm not to invest regardless of what others do.<sup>24</sup> When  $A < \underline{A}$ , firms choose not to invest and would choose so even if they expected others

24.  $\underline{A}$  and  $\bar{A}$  are defined in Appendix A.



to invest. When  $A > \bar{A}$ , all firms choose to invest because the productivity level is high enough to persuade firms to incur the fixed adjustment cost regardless of others' decisions. When  $A \in (A^*, \bar{A})$ , firms coordinate in the "good" outcome. Fundamentals are not strong enough for investing to be a dominant strategy but are sufficiently strong to induce firms to invest when beliefs about others' actions are taken into account. For  $A \in (\underline{A}, A^*)$ , firms stay in the region of inaction due to a coordination failure. The economy gets stuck in the noninvestment regime with a low level of economy activity even though it would be individually optimal for firms to invest if they were confident others would do so as well. Pessimistic beliefs about others firms' investment prospects induced by low economic fundamentals—lack of confidence—lead to an inefficiently low level of investment.<sup>25</sup>

The second part of Proposition 2 shows that an increase in government expenditure shifts down the productivity threshold. To understand this result, consider first the effect of a larger government expenditure on a firm's decision, holding constant beliefs about  $h$  and the signal about  $A$ . A larger  $G$  implies a larger expected  $Y$  and hence raises the expected profits from paying the fixed cost (Proposition 1 and Corollary 1). This is the direct effect.<sup>26</sup>

On top of this direct effect, there is also an indirect effect through beliefs, which we dub "the confidence channel of fiscal policy." A firm knows that other firms are more likely to invest owing to a larger  $G$ . Hence, the increase in government spending also affects beliefs about  $h$ . From Corollary 1, that provides another boost to investment. Intuitively, a firm expects a larger  $h$ ; hence, a larger demand for its own goods and thus is more likely to leave the inaction region.<sup>27</sup> In consequence, for a given realization of  $A$ , firms will be more inclined to incur the fixed adjustment cost for investment.

Therefore, an increase in government spending shifts down the threshold  $A^*(G)$ . In the case of very accurate information, either almost all firms invest or almost no firm invests for almost all realizations of  $A$ . Nevertheless, beliefs and strategic uncertainty play a key role in determining the equilibrium of the model because when the economy is close to the equilibrium threshold  $A^*(G)$ , beliefs about others' actions are very diffuse.<sup>28</sup>

### 3.4 Optimal Government Spending

Firms' market power leads to well-known monopoly distortions. The fixed cost for investment might lead to coordination failures: firms would find optimal to pay the fixed cost and produce more if they expected others to do so but might get stuck in an inaction region if productivity is not high enough and they anticipate others will

25. Pessimistic beliefs in this article means "beliefs that others will not invest." The term does not imply that beliefs are somehow biased or incorrect, which is never the case in the model.

26. The role played by  $G$  is similar to the role played by demand shocks in standard macroeconomic models.

27. Higher order beliefs also play a role, because this boost in beliefs leads to a second-order increase in confidence because firms know that others' investment decisions depend on their beliefs as well. For more on the effects of higher order beliefs, see Morris and Shin (2003).

28. For more on strategic uncertainty, we refer the reader to Morris and Shin (2003).

not invest as well. The optimal fiscal policy considers not only the direct benefit of provision of public goods but also its potential impact on those market failures.

The planner chooses  $G$  to maximize welfare of the representative household given its prior distribution over  $A$ ,  $f(A)$ :

$$V(G) \equiv \int_0^{A^*(G)} (Y(0, A, G) - \chi L(0, A, G)) f(A) dA \quad (15)$$

$$+ \int_{A^*(G)}^{\infty} (Y(1, A, G) - \chi L(1, A, G)) f(A) dA.$$

Taking the first-order condition with respect to  $G$ , we get that the solution to the planner's problem must satisfy<sup>29</sup>

$$DMU(G) = MEC(G), \quad (16)$$

where the direct marginal utility of public goods  $DMU(G)$  is given by

$$DMU(G) = \int_0^{A^*(G)} \left( \frac{\partial Y(0, A, G)}{\partial G} - \chi \frac{\partial L(0, A, G)}{\partial G} \right) f(A) dA \quad (17)$$

$$+ \int_{A^*(G)}^{\infty} \left( \frac{\partial Y(1, A, G)}{\partial G} - \chi \frac{\partial L(1, A, G)}{\partial G} \right) f(A) dA,$$

and the marginal effect of public spending on coordination  $MEC(G)$  is given by

$$MEC(G) = \frac{\partial A^*(G)}{\partial G} f(A^*(G)) \begin{pmatrix} [Y(1, A^*(G), G) - \chi L(1, A^*(G), G)] \\ - [Y(0, A^*(G), G) - \chi L(0, A^*(G), G)] \end{pmatrix}. \quad (18)$$

In a usual social planner's problem, the optimal government spending is found at the point where its direct marginal utility  $DMU(G)$  is equal to 0. The flip side of a larger government spending is that the representative agent works more. The expected increase in households' utility through a larger provision of public goods has to compensate its opportunity cost, which is the disutility from labor. That effect is basically the same in the noninvesting regime and in the investment regime. Were this the only way through which government spending affected welfare, fiscal policy would aim at equalizing the marginal benefit of public goods and its marginal cost.

However, in this economy, fiscal policy might be able to switch the economy from a noninvestment regime to an investment regime. The term  $MEC(G)$  captures the expected gains from a marginal increase in government spending owing to a regime switch. Proposition 2 shows that  $\partial A^*(G)/\partial G < 0$ , that is, an increase in government spending reduces the threshold for investment. The second term  $f(A^*(G))$  is the prior probability that a marginal shift of the threshold will affect investment decisions. The

29. In the proof of Proposition 3, we will show that the solution must be interior.

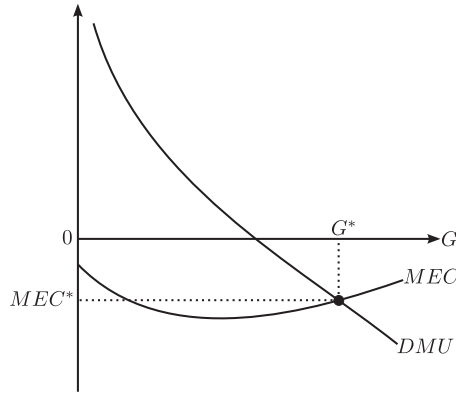


FIG. 3. Marginal Value of Public Goods.

last term multiplying this derivative is the welfare gain resulting from a switch to the investment regime.<sup>30</sup>

Without the fixed costs of investment, this extra effect of fiscal policy would not exist. The economy would always be in the investment regime (even if the desired level of investment was very small). The fixed cost essentially turns investment decisions into a coordination game. Government spending has an effect on coordination by affecting beliefs about other firms’ actions in this coordination game.<sup>31</sup>

Proposition 3 shows that the confidence channel of fiscal policy leads to a higher optimal level of government spending.

**PROPOSITION 3.** *For any prior with support on  $\mathbb{R}_+$ , at the optimal  $G$ , the direct marginal utility of public goods  $DMU(G)$  is negative.*

A corollary to Proposition 3 is that when  $A$  is large and the probability of a coordination failure is negligible, optimal fiscal policy equates its marginal cost and its expected direct marginal benefit. Fiscal policy should be more expansionary when the prior probability of a coordination failure at the margin,  $f(A^*(G))$ , is larger. We interpret a large perceived probability of a coordination failure as capturing times of low economic activity, when firms might be willing to choose inaction over investment owing to an intrinsic bad state of the economy, which is fueled by low demand expectations.

In this situation, the provision of public goods with a marginal value below its cost is welfare improving. Figure 3 illustrates this result. By increasing demand for

30. When  $A = A^*(G)$ , firms’ payoffs are higher in the investment regime than in the noninvestment regime. In the investment regime, firms produce more, so for a given  $G$ ,  $\partial Y/\partial C_p$  is smaller. Hence, real profits and real wages are higher. Thus, welfare of the representative household must be higher in the investment regime when  $A = A^*(G)$  and the gain from switching is positive.

31. The assumption of fixed costs of investment plays a key role in the model but could be replaced by alternative assumptions as long as investment decisions depended on coordination among firms (i.e., as long as there were multiple equilibria in an analogous model with complete information).

private goods and affecting beliefs about others' investment decisions, the seemingly inefficient increase in government spending can induce firms to leave the inaction zone and improve welfare.

As an illustration, the financial crisis of 2008–9 and the ensuing reduction in credit to the private sector can be captured in the model by a negative shock to  $A$  that reduces firms' productivity. Knowing the shock has affected all firms in the economy, an individual firm now expects lower demand for its goods. The low levels of productivity and demand reduce their desired level of investment. In the absence of fixed adjustment costs, investment in the economy would be small, but positive. However, owing to the fixed costs, firms prefer inaction. In this scenario, an increase in government spending works as a demand shock, increasing households' demand for private goods. The optimal level of production for firms has increased and workers find optimal to work more. Everyone knows others reason this way, and that might be enough to switch the economy to the investment regime.

### 3.5 Optimal Policy with More Instruments

In this section, we provide another policy instrument for the government. Now the government can also pay a subsidy  $s$  to firms that choose to invest. For a firm, the fixed cost of investment becomes  $\psi - s$ . The subsidy is financed through a lump-sum tax on the household, as in the basic model.

The equilibrium threshold now becomes a function of  $G$  and  $s$ , so we write  $A^* = A^*(G, s)$ . One can easily verify that this threshold will be decreasing in  $s$ , because  $\hat{D}(\cdot)$  will be increasing in  $s$ . Naturally, a higher subsidy increases the gain from investing. Hence, the government chooses  $G$  and  $s$  to maximize (15), but now it takes into account that the threshold also depends on  $s$ . The first-order conditions become:<sup>32</sup>

$$DMU(G, s) = MEC(G, s), \quad (19)$$

and

$$MEC(G, s) = 0, \quad (20)$$

where the direct marginal utility of public goods  $DMU(G, s)$  and the marginal effect of public spending on coordination  $MEC(G, s)$  are defined as in (18) and (17) with  $A^*(G, s)$  instead of  $A^*(G)$ .

The first-order condition for  $s$  implies that at the threshold  $A^*(G, s)$ , welfare must be the same in both regimes. Plugging (20) in (19), the first-order condition for  $G$  becomes  $DMU(G, s) = 0$ . Hence, the result in Proposition 3 does not hold when the

32. An argument similar to the one used in the proof of Proposition 3 shows that the solution for  $G$  must be interior. To verify that we also have an interior solution for  $s$ , note that if  $s = 0$ , when  $A = A^*$  the household will be better off in the high regime (for the same reasons explained in footnote 30). Hence, the planner is better off by increasing  $s$ . When  $s = \psi$ , agents always incur the fixed cost, but for low enough values of  $A$ , the gain in welfare must be negative, because firms will not increase its capital. Thus,  $s \in (0, \psi)$ .

government can also pay a subsidy to firms conditional on investment. Government spending is no longer used to shift the threshold. The optimal expenditure simply equates the (expected) marginal benefit of public goods with its (expected) marginal cost. Intuitively, the investment subsidy is a better instrument to enhance coordination because it provides the right amount of incentives for firms to invest without distorting the provision of public goods.

In practice, however, owing to the costs of verifying investment of every firm, the implementation of a subsidy policy might be too costly. Increases in government spending entail no extra implementation costs and hence might be preferred.<sup>33</sup>

#### 4. EXTENSIONS

This section extends the model in different directions. Section 4.1 shows that the same insights apply to a model where government expenditure affects aggregate productivity. Section 4.2 relaxes the assumption of equality of all elasticities of substitution. Section 4.3 shows that results are robust to the inclusion of heterogeneity in investment costs.

In all extensions, the technology for producing capital goods is given by (2), the technology for producing the public good is given by (3), and the assumptions on timing and information structure are as in Section 2.5.

##### 4.1 *Productive Government Expenditure*

This section presents a variation of our model where government expenditure affects aggregate productivity instead of boosting demand for private goods. Household utility is now given by

$$U = C_p - \chi L,$$

where  $C_p$  is as defined in (2.1). The household budget constraint is the same as in the problem of Section 2.1.

The production function of each intermediate firm is now given by

$$y_i = Ag^{\alpha_g} k_i^{\alpha_k} l_i^{\alpha_l},$$

where  $\alpha_g + \alpha_k + \alpha_l < 1$  and  $g$  denotes the amount of public goods produced in this economy. Government expenditures affect the supply side of the economy by making firms more productive. This is a common assumption in the literature since Barro (1990) and captures the positive effects of public services and infrastructure on productivity.

33. This point is not specific to our model. In general, if it were costless to act on the source of the market failures, government spending should not be an instrument to smooth fluctuations in output.

We can show that the amount consumed by the representative household in this economy is an increasing function of  $g$ ,  $A$ , and  $h$ , as before. Specifically, for given  $g$ ,  $A$ , and  $h$ , we can write household consumption as

$$C_p(h, A, g) = Y(h, Ag^{\alpha_g}, 0), \tag{21}$$

where  $Y(\cdot)$  refers to the equilibrium GDP in the model of Section 2. To see this, notice that once government has announced a level of expenditure  $g$ , this model is essentially the same as the previous model if we set  $G = 0$  and set the productivity as  $Ag^{\alpha_g}$ .<sup>34</sup> We will refer to  $\hat{A}(g) \equiv Ag^{\alpha_g}$  as the effective productivity from now on. The amount of labor supplied to the government is still given by  $L_g(g) = \gamma g$  and the total amount of labor supplied to capital and private goods firms are now given by

$$\hat{L}_K(h, A, g) = L_K(h, Ag^{\alpha_g}, 0)$$

and

$$\hat{L}_P(h, A, g) = L_P(h, Ag^{\alpha_g}, 0),$$

where we use hats to denote the labor supplied to the private and capital goods firms in the equilibrium of this model and  $L_K(\cdot)$  and  $L_P(\cdot)$  are the same as in the previous model (see the expressions in (12) and (13)). As in Section 2, we define the total amount of labor supplied as  $\hat{L}(h, A, g) = L_g(g) + \hat{L}_K(h, A, g) + \hat{L}_P(h, A, g)$ .

Proposition 1 and its corollary still hold in this model, because this is just a special case of the previous model with no government expenditures and productivity replaced by its effective counterpart  $\hat{A}(g)$ . Thus, there are strategic complementarities and dominance regions as before, meaning that in equilibrium, firms will decide whether to invest according to a threshold, denoted by  $A^{**}(g)$ . But notice that the following relation must be satisfied:

$$A^{**}(g) = A^*(0)g^{-\alpha_g},$$

where  $A^*(\cdot)$  is the threshold of the previous model. Therefore, government expenditures reduce the threshold  $A^{**}(g)$ , turning the high-investment regime more likely.

We can then show a result on optimal fiscal policy analogous to the one in Section 3.4. The government chooses  $G$  to maximize

$$\begin{aligned} V(g) \equiv & \int_0^{A^{**}(g)} (C_p(0, A, g) - \chi \hat{L}(0, A, g)) f(A) dA \\ & + \int_{A^{**}(g)}^\infty (C_p(1, A, g) - \chi \hat{L}(1, A, g)) f(A) dA. \end{aligned}$$

34.  $G$  refers to the amount of government provision of public goods in the model of Section 2.

If an interior solution exists, then it must satisfy the first-order condition

$$DMU(g) = MEC(g),$$

where  $DMU(g)$  and  $MEC(g)$  are defined as before.

As in the basic model,  $DMU(g)$  represents the direct benefit of government spending (which now relies on a different channel), while  $MEC(g)$  captures the expected gains from a marginal increase in government spending owing to a regime switch.

We show in Appendix B that an interior solution exists. Because government expenditures reduce the threshold  $A^{**}(g)$ , a result similar to Proposition 3 holds in this model.

In the model of Section 2, a larger provision of public goods induces higher demand for private goods for changing the household’s valuation of goods. Here, instead, government expenditures stimulate the supply of private goods by raising aggregate productivity. However, for the purposes of this paper, these differences are unimportant. In both cases, government expenditures have a similar positive indirect impact on the economy through their effects on agents’ beliefs and coordination.

#### 4.2 Different Elasticities of Substitution

We now relax the assumption of equality between the elasticity of substitution across private goods and the elasticity of substitution between private and public goods. The household’s utility over goods is given by

$$u(C_p, G) = \left( \omega C_p^{\frac{\rho-1}{\rho}} + (1 - \omega) G^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}},$$

where  $\rho$  is the elasticity of substitution between public and private goods and  $\theta$  denotes the elasticity of substitution among private goods (the expression for  $C_p$  remains unchanged).

*Equilibrium in the third stage.* Most equilibrium conditions are as in Section 3, but now we write firms’ profits as

$$\frac{\pi_i}{P_p} = (A k_i^{\alpha_k} l_i^{\alpha_l})^{\frac{\theta-1}{\theta}} Y_p^{\frac{1}{\theta}} - \frac{w}{P_p} l_i - \frac{P_K}{P_p} C(i).$$

Replacing equilibrium condition (7) for the price of capital above and solving for the optimal choice of capital and labor of firms in each regime, we find expressions for capital and labor in each regime as functions of  $w$ ,  $Y_p$ , and  $A$ .<sup>35</sup>

35. A firm maximizes its value, which is a constant time its profits (from the point of view of an individual firm).

To find the equilibrium values of  $Y_p$  and  $w$ , we use the household optimality condition (5) and the definition  $Y_p = C_p$ . That yields two equilibrium conditions:

$$\frac{w}{P_p} = \frac{\chi}{\left(\omega Y_p^{\frac{\rho-1}{\rho}} + (1-\omega)G^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}} \omega Y_p^{-\frac{1}{\rho}}} \tag{22}$$

and

$$Y_p = A \left( h(k_H(w, Y_p, A)^{\alpha_k} l_H(w, Y_p, A)^{\alpha_l})^{\frac{\theta-1}{\theta}} + (1-h) \left( k_0^{\alpha_k} l_L(w, Y_p, A)^{\alpha_l} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \tag{23}$$

This system has a unique solution. The pairs  $(w, Y_p)$  satisfying the equilibrium condition in (22) are an increasing function in the space  $w \times Y_p$ , whereas the pairs satisfying the equilibrium condition in (23) are a decreasing curve that passes through every  $w$  and  $Y_p$ . Thus, we can write  $w(h, A, G)$  and  $Y_p(h, A, G)$ . By the definition of  $Y$ , we can also write  $Y(h, A, G)$ .

Inspection of the effects of exogenous variables on the curves (22) and (23) in the space  $w \times Y_p$  reveals that  $Y(h, A, G)$  and  $Y_p(h, A, G)$  are increasing in all arguments.

*The investment game.* We can write the firm value as

$$\hat{\pi}_i \equiv \frac{\partial u}{\partial C_p} \frac{\pi_i}{P_p} = \left( A k_i^{\alpha_k} l_i^{\alpha_l} \right)^{\frac{\theta-1}{\theta}} \omega \tilde{Y}^{\frac{1}{\theta}} - \chi (l_i + \mathcal{C}(I_i)),$$

where  $\tilde{Y} \equiv (Y^{\frac{1}{\rho}} Y_p^{\frac{1}{\rho} - \frac{1}{\rho}})^{\theta}$ . Notice that this expression is very similar to (8), only with  $\tilde{Y}$  instead of  $Y$ . Thus, we can write  $k_H, l_H, l_L, y_H, y_L, \hat{\pi}_H$ , and  $\hat{\pi}_L$  as functions of  $\tilde{Y}$  and  $A$  (in the exact same way, we wrote them as functions of  $Y$  and  $A$  in Section 4). An argument analogous to the proof of Lemma 1 shows that the difference in payoffs

$$\tilde{D}(\tilde{Y}, A) \equiv \hat{\pi}_H(\tilde{Y}, A) - \hat{\pi}_L(\tilde{Y}, A)$$

is increasing  $\tilde{Y}$  and  $A$ .

Because  $Y(h, A, G)$  and  $Y_p(h, A, G)$  are increasing in  $h$  and  $G$ , we are left to show that  $\tilde{Y} = m(Y_p, G) \equiv (u(Y_p, G)^{\frac{1}{\rho}} Y_p^{\frac{1}{\rho} - \frac{1}{\rho}})^{\theta}$  is increasing in  $Y_p$  and  $G$ . As long as this holds, the function  $\tilde{D}(h, A, G) \equiv \tilde{D}(m(Y_p(h, A, G), G), A)$  is increasing in all arguments and the qualitative results of the basic model go through here.

A sufficient condition to guarantee that  $m(Y_p, G)$  is increasing in both arguments is  $\rho > (1 - \omega)\theta$ . Notice that this sufficient condition nests the case where private and public goods are almost perfect substitutes ( $\rho \rightarrow \infty$ ). Hence, we do not need complementarity between public and private goods for our results to hold. Moreover, in case  $\omega$  is large (high proportion of private goods in the economy), the condition is not very restrictive.



The confidence channel of fiscal policy relies on investment decisions being strategic complements. That might not be the case if  $\rho$  is low,  $\omega$  is low, and  $\theta$  is high. That is because the increase in wages followed by an increase in  $h$  (which is large for low values of  $\rho$ ) might dominate the demand externality effect (which is small for high values of  $\theta$ ).<sup>36</sup> Hence, an increase in the production of a given firm might actually reduce incentives for other firms to increase production. The magnitude of the effect on wages is high for low values of  $\omega$ .

### 4.3 Heterogeneous Firms

We now extend the basic model to allow for differences across firms with respect to the fixed cost of investment.<sup>37</sup> There is still a measure-one continuum of firms but now there are  $J$  types of firms. There is a mass  $\mu_j$  of type- $j$  firms that face fixed costs  $\psi_j$ ,  $j \in \{1, 2, 3, \dots, J\}$ .

Nothing substantial changes in the third stage of the model. In the second stage, the investment game is now a two-action, many-player game. The difference in profits from investing and not investing can still be written as a function of  $h$ ,  $A$ , and  $G$ , so investing pays off for firm  $i$  if

$$E [\hat{D}_i(h, A, G)] > 0.$$

Because Proposition 1 and Corollary 1 hold in this setting, we can use the results in Frankel, Morris, and Pauzner (2003) to show that in the limit of arbitrarily small noise, there is a unique rationalizable equilibrium and a firm invests if its signal is larger than a type-specific threshold  $a_j^* = \log(A_j^*)$ .

Let  $A$  be the realization of aggregate productivity and  $a = \log(A)$ . The proportion of type- $j$  firms that invest is

$$h_j = 1 - Q\left(\frac{a_j^* - a}{\sigma}\right),$$

where  $Q$  is the cumulative distribution function of the error term  $\varepsilon_j$ . Hence,

$$h = 1 - \sum_{j=1}^J \mu_j Q\left(\frac{a_j^* - a}{\sigma}\right).$$

36. The direct effect of government expenditures is higher for lower values of  $\rho$ . Intuitively, if public and private goods are complements, an increase in the provision of public goods has a large effect on the demand for private goods. However, a lower  $\rho$  reduces the degree of strategic complementarities in investment decisions, so the confidence channel is less pronounced.

37. Heterogeneity  $\psi$  is particularly tractable because a firm's problem does not depend on which firms have chosen to pay the fixed cost, only on the fraction  $h$ . In case of heterogeneity in productivity, that would not be the case.

Consider a firm that received signal  $x_i = a + \varepsilon_i$ . Because  $a = x_i - \varepsilon_i$

$$h = 1 - \sum_{j=1}^J \mu_j Q \left( \frac{a_j^* - x_i + \varepsilon_i}{\sigma} \right),$$

so  $h$  is decreasing in  $a_j^* - x_i$  for all  $j$ .

Let  $\hat{D}_i(h, a, G)$  be the relative payoff from investing for the firm that got signal  $x_i$ . Its expected payoff from investing is

$$\int_{-\infty}^{\infty} \hat{D}_i(h, a, G) q(\varepsilon_i) d\varepsilon_i = \int_{-\infty}^{\infty} \hat{D}_i(h(a_i^* - x_i + \varepsilon_i, a_{-i}^* - x_i + \varepsilon_i), x_i - \varepsilon_i, G) q(\varepsilon_i) d\varepsilon_i,$$

where we wrote  $h$  as a function of  $a_i^* - x_i + \varepsilon_i$  and  $a_{-i}^* - x_i + \varepsilon_i$  (with a slight abuse of notation,  $a_{-i}^*$  denotes the set of thresholds for all types of firms, followed by all other firms).

Now define  $\hat{D}_i^*$  as the relative payoff from investing for a firm that got signal  $x_i = a_i^*$ . Then,

$$\int_{-\infty}^{\infty} \hat{D}_i^*(h, a, G) q(\varepsilon_i) d\varepsilon_i = \int_{-\infty}^{\infty} \hat{D}_i^*(h(\varepsilon_i, a_{-i}^* - a_i^* + \varepsilon_i), a_i^* - \varepsilon_i, G) q(\varepsilon_i) d\varepsilon_i.$$

The function  $\hat{D}_i^*$  is increasing in  $G$ ; it is decreasing in  $a_j^*$  for  $j \neq i$  because  $h$  is decreasing in  $a_j^* - a_i^*$ ; and it is increasing in  $a_i^*$  because  $\hat{D}_i$  is increasing in  $a$  and in  $h$ , and  $h$  is decreasing in  $a_{-i}^* - a_i^*$ . Type- $i$  firms must be indifferent between investing or not at  $a_i^*$ , hence

$$\int_{-\infty}^{\infty} \hat{D}_i^*(h(\varepsilon_i, a_{-i}^* - a_i^* + \varepsilon_i), a_i^* - \varepsilon_i, G) q(\varepsilon_i) d\varepsilon_i = 0.$$

This expression implicitly defines  $a_i^*$  as a function of  $G$  and  $a_j^*$  for  $j \neq i$ . Using the implicit function theorem, we get that  $a_i^*$  is decreasing in  $G$  and increasing in  $a_j^*$  for  $j \neq i$ . If the government spends more or others are more willing to invest (smaller  $a_j^*$ ),  $a_i^*$  goes down, type- $i$  firms require a lower productivity to invest.

As in the basic model, an increase in  $G$  has a direct effect on all  $a_i^*$  (keeping other thresholds  $a_{-i}^*$  fixed), and then an extra indirect effect through the decrease in all other thresholds.

### 5. FINAL REMARKS

This article proposes a model of the confidence channel of fiscal policy. Fixed adjustment costs for investment generate an inaction zone. Noisy idiosyncratic information about the economy generates strategic uncertainty. In this setup, variables that have a direct effect on incentives for investment also affect beliefs about whether

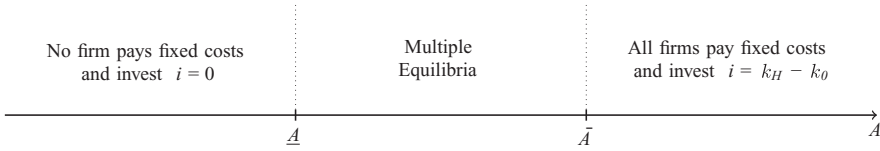


FIG. 4. Multiple Equilibria.

other firms will leave the inaction zone. In particular, an increase in government expenditure that induces households to demand more private goods also raises beliefs about investment by other firms. The effect on beliefs provides further incentives for an individual firm to leave the inaction zone.

The article shows the confidence channel of fiscal policy in a simple setup that allows for analytical results. Simplicity comes at a price, as we miss some important features of the data. To evaluate the quantitative importance of the confidence channel, a more realistic modeling of adjustment costs, firm-specific shocks, and a fully dynamic structure would be needed. Future research might incorporate the main aspects of this model into a quantitative dynamic macroeconomic framework.

APPENDIX A: THE MODEL WITH COMPLETE INFORMATION

In case  $A$  is common knowledge, the economy may feature multiple equilibria. Figure 4 shows the region where investing and not investing are equilibria.  $\underline{A}$  is the technology level that implies a single producer is indifferent between investing or not if everyone is investing, whereas  $\bar{A}$  is the technology level that implies a single producer is indifferent between investing or not if everyone is choosing not to invest.  $\underline{A}$  satisfies  $\hat{D}(1, \underline{A}, G) = 0$  and  $\bar{A}$  satisfies  $\hat{D}(0, \bar{A}, G) = 0$ .

Because  $\hat{D}(h, A, G)$  is increasing in  $A$  and  $h$ ,  $\bar{A} > \underline{A}$ . In the region satisfying  $A > \bar{A}$ , it is optimal to invest and there is only one possible outcome. Conversely, when  $A < \underline{A}$ , agents never invest in equilibrium. However, if  $A \in (\underline{A}, \bar{A})$ , there are multiple equilibria as individual investment decisions depend on (self-fulfilling) expectations about others' actions.

Lemma 1 implies that  $\underline{A}$  and  $\bar{A}$  are decreasing functions of  $G$ . Hence, an increase in government spending affects the region where coordination failures might occur, reducing the minimum level of productivity required for firms to investment conditional on both optimistic and pessimistic beliefs.

APPENDIX B: PROOFS

PROOF OF LEMMA 1. *First sentence:* The LHS of (9) is just the 45-degree line. Notice that the RHS is larger than zero as long as  $G > 0$ . Taking the derivative of the LHS with respect to  $Y$  and doing some algebra yields

$$\frac{\partial RHS}{\partial Y} = \omega \left[ \left( h \left( \frac{y_H(Y, A)}{Y} \right)^{\frac{\theta-1}{\theta}} \varepsilon_{y_H} + (1-h) \left( \frac{y_L(Y, A)}{Y} \right)^{\frac{\theta-1}{\theta}} \varepsilon_{y_L} \right) \right],$$

where  $\varepsilon_{y_H} \equiv \frac{\partial y_H}{\partial Y} \frac{Y}{y_H}$  and  $\varepsilon_{y_L} \equiv \frac{\partial y_L}{\partial Y} \frac{Y}{y_L}$ . Let  $\tilde{Y}$  be such that  $y_H(\tilde{Y}, A) = y_L(\tilde{Y}, A)$ . The expressions in (2.1) and (11) imply that for every  $Y$  larger (smaller) than  $\tilde{Y}$ , these elasticities do not vary with  $Y$ . Assume  $Y > \tilde{Y}$ . Notice that  $y_H(Y, A)/Y$  and  $y_L(Y, A)/Y$  are decreasing in  $Y$  (the exponents of  $Y$  in both functions  $y_H$  and  $y_L$  are smaller than 1). Therefore, this derivative is decreasing in  $Y$  and thus this function is concave for  $Y > \tilde{Y}$ . Same reasoning shows that this is also concave for  $Y < \tilde{Y}$ . Now, as  $Y \rightarrow \infty$ ,  $y_H(Y, A)/Y$  and  $y_L(Y, A)/Y$  converge to zero, implying that  $\partial RHS/\partial Y$  goes to zero as well, and thus the RHS crosses the 45-degree line at some point. Because the RHS is concave, it cannot cross it more than once. Therefore,  $Y$  is pinned down in equilibrium and we write it as  $Y(h, A, G)$ . In consequence, a unique  $Y_p$  is pinned down in equilibrium as well.

*Second sentence:* Because  $y_H(Y, A) \geq y_L(Y, A)$ , the RHS of (9) increases in  $h$ . It follows that  $Y(h, A, G)$  is increasing in  $h$ . Same reasoning applies to  $G$  and  $A$ .  $\square$

PROOF OF PROPOSITION 1. We start by showing that  $D(Y, A)$  is increasing in  $Y$ . Let  $\Delta > 0$  be a small variation in  $Y$ . We want to show that

$$\hat{\pi}_H(Y + \Delta, A) - \hat{\pi}(Y, A) \geq \hat{\pi}_L(Y + \Delta, A) - \hat{\pi}_L(Y, A).$$

Let  $\tilde{\pi}_H$  be the profit from investing when output is  $Y + \Delta$  and a firm follows the following strategy: it keeps the level of capital at  $\tilde{k}_H \equiv k_H(Y, A)$  and optimally chooses labor with  $k_i$  replaced by  $\tilde{k}_H$ . Thus,  $\tilde{\pi}_H$  is a lower bound for  $\hat{\pi}_H(Y + \Delta, A)$ . Using (10) and (11), we can write

$$\begin{aligned} \tilde{l}_H &= \left( \frac{\tilde{k}_H}{k_0} \right)^{\alpha_k(\theta-1)\varphi} l_L(Y + \Delta, A), \\ l_H(Y, A) &= \left( \frac{\tilde{k}_H}{k_0} \right)^{\alpha_k(\theta-1)\varphi} l_L(Y, A), \\ \tilde{y}_H &= \left( \frac{\tilde{k}_H}{k_0} \right)^{\alpha_l\alpha_k(\theta-1)\varphi + \alpha_k} y_L(Y + \Delta, A), \end{aligned}$$

and

$$y_H(Y, A) = \left( \frac{\tilde{k}_H}{k_0} \right)^{\alpha_l\alpha_k(\theta-1)\varphi + \alpha_k} y_L(Y, A).$$

After some algebra, we get

$$\begin{aligned} \tilde{\pi}_H - \hat{\pi}_H(Y, A) &= \left( \frac{\tilde{k}_H}{k_0} \right)^{\alpha_k(\theta-1)\varphi} \left[ \omega \left( (Y + \Delta \text{big})^{\frac{1}{\theta}} y_L(Y + \Delta, A)^{\frac{\theta-1}{\theta}} \right. \right. \\ &\quad \left. \left. - Y^{\frac{1}{\theta}} y_L(Y, A)^{\frac{\theta-1}{\theta}} \right) \right] - \chi (l_L(Y + \Delta, A) - l_L(Y, A)) \end{aligned}$$

and

$$\begin{aligned} \hat{\pi}_L(Y + \Delta, A) - \hat{\pi}_L(Y, A) &= \omega \left( (Y + \Delta)^{\frac{1}{\theta}} y_L(Y + \Delta, A)^{\frac{\theta-1}{\theta}} - Y^{\frac{1}{\theta}} y_L(Y, A)^{\frac{\theta-1}{\theta}} \right) \\ &\quad - \chi (l_L(Y + \Delta, A) - l_L(Y, A)). \end{aligned}$$

Because  $\tilde{k}_H \geq k_0$ , one can verify that  $\tilde{\pi}_H - \hat{\pi}_H(Y, A) \geq \hat{\pi}_L(Y + \Delta, A) - \hat{\pi}_L(Y, A)$ . Because  $\tilde{\pi}_H$  is a lower bound for  $\hat{\pi}_H(Y + \Delta, A)$ , it must be that  $\hat{\pi}_H(Y + \Delta, A) - \hat{\pi}_H(Y, A) \geq \hat{\pi}_L(Y + \Delta, A) - \hat{\pi}_L(Y, A)$ .

The proof for  $D(Y, A)$  increasing in  $A$  is analogous.  $\square$

**PROOF OF PROPOSITION 2.** We just need to show that our investment game satisfies the assumptions in Morris and Shin (2003). We have already shown supermodularity in Corollary 1. We need to show that there are dominance regions: for any given  $G$ , there must exist  $\epsilon > 0$ ,  $\underline{A}$ , and  $\bar{A}$  such that  $\hat{D}(h, \underline{A}, G) < -\epsilon$  and  $\hat{D}(h, \bar{A}, G) > \epsilon$ . To see this, notice that if  $h = 1$ , there is a sufficiently low  $\underline{A}$  such that  $k_H(Y, \underline{A}) = k_L(Y, \underline{A})$ , implying that  $\hat{D}(1, \underline{A}, G) = -\chi\psi$  (and therefore  $\hat{D}(h, \underline{A}, G) < -\chi\psi$  for any  $h \in [0, 1]$ ). Finally, one can verify that  $D(Y, A)$  is an unbounded function of  $A$  for any  $Y$ .

The argument in Morris and Shin (2003) yields the expression in (14). The proof that  $A^*(G)$  is decreasing in  $G$  then comes directly from the fact that  $\hat{D}(\cdot)$  is increasing in  $G$ , as shown in Corollary 1.  $\square$

**PROOF OF PROPOSITION 3.** We need to show that the planner's problem has an interior solution, so that the first-order condition (16) characterizes the optimal level of  $G$ . The cost of an infinitesimal increase in  $G$  is given by

$$\chi \frac{\partial L}{\partial G} = \chi \left[ \gamma + \frac{\partial Y}{\partial G} \left( h \frac{\partial l_H}{\partial Y} + (1-h) \frac{\partial l_L}{\partial Y} + h\kappa \frac{\partial k_H}{\partial Y} \right) \right],$$

which is the disutility of the extra work imposed on the representative agent. The benefit is given by the increase in GDP ( $\partial Y / \partial G$ ). Therefore, the net benefit of an increase in government spending is given by

$$\frac{\partial Y}{\partial G} \left[ 1 - \chi \left( h \frac{\partial l_H}{\partial Y} + (1-h) \frac{\partial l_L}{\partial Y} + h\kappa \frac{\partial k_H}{\partial Y} \right) \right] - \chi\gamma. \quad (\text{B1})$$

We first show that as  $G \rightarrow \infty$ , the expression in (B1) is negative. It suffices to show that  $\lim_{G \rightarrow \infty} \partial Y / \partial G < \chi\gamma$ . If  $h = 0$  or  $h = 1$ , we get  $Y_p / Y = T_i Y^{-\xi_i} \equiv H(Y)$ ,

where  $T_i$  and  $\xi_i$  are constants such that  $T_i > 0$  and  $\xi_i \in (0, 1)$ , for  $i \in \{L, H\}$ . Using (9) yields an expression for  $G$  as a function of  $Y$ :

$$G = \left[ \frac{1}{1 - \omega} Y^{\frac{\theta-1}{\theta}} \left( 1 - \omega (H(Y))^{\frac{\theta-1}{\theta}} \right) \right]^{\frac{\theta}{\theta-1}}.$$

Taking derivatives of  $G$  with respect to  $Y$  leads to

$$\begin{aligned} \frac{\partial G}{\partial Y} &= \left[ \frac{1}{1 - \omega} \left( 1 - \omega H(Y)^{\frac{\theta-1}{\theta}} \right) \right]^{\frac{1}{\theta-1}} \\ &\quad \times \left\{ \frac{1}{1 - \omega} \left[ \left( 1 - \omega H(Y)^{\frac{\theta-1}{\theta}} \right) - Y \omega H(Y)^{-\frac{1}{\theta}} H'(Y) \right] \right\}. \end{aligned}$$

Taking the limit of this expression as  $Y \rightarrow \infty$  and using the inverse function theorem, we get

$$\lim_{G \rightarrow \infty} \frac{\partial Y(h, A, G)}{\partial G} = (1 - \omega)^{\frac{\theta}{\theta-1}}$$

and (4) implies that as  $G \rightarrow \infty$ , the expression in (B1) is negative, so the planner problem has a solution.

We now need to show that (B1) is larger than zero when  $G = 0$ , so the planner's problem has an interior solution. As  $G \rightarrow 0$ , we have  $\frac{\partial Y}{\partial G} \rightarrow \infty$ . Hence, it suffices to show that at  $G = 0$

$$\chi \frac{\partial l_H}{\partial Y} + \chi \kappa \frac{\partial k_H}{\partial Y} < 1 \tag{B2}$$

and

$$\chi \frac{\partial l_L}{\partial Y} < 1. \tag{B3}$$

First, assume  $h = 1$ . The wage and the price of capital at  $G = 0$  are simply  $w = \frac{\chi}{\omega} P_p$  and  $P_K = \kappa \frac{\chi}{\omega} P_p$ , respectively. Therefore, the LHS of (B2) is just the increase in production costs given an infinitesimal increase in  $Y$  times  $\omega$ , taking everything else constant. Because  $\omega < 1$  and profits are increasing in  $Y$ , we know that this is smaller than the infinitesimal increase in revenues after an increase in  $Y$ , which is given by taking derivatives of  $\omega y_H(Y, A)^{\frac{\theta-1}{\theta}} Y^{\frac{1}{\theta}}$  with respect to  $Y$ . Substituting  $Y = \omega^{\frac{\theta}{\theta-1}} y_H(Y, A)$  in the derivative shows that the increase in revenue is smaller than 1 (or  $1/\omega$ ) and so is the LHS of (B2). A similar reasoning applies to (B3).  $\square$

*Proof that Proposition 3 holds for extension of Section 4.1.* First, notice that  $g = 0$  is not a solution here, because in that case, firms produce nothing and households gets zero welfare. In order to show that an interior solution exists, it suffices to show that the increase in  $C_p$  after an infinitesimal increase in  $g$  goes to zero when  $g \rightarrow \infty$

for  $h = 1$  and  $h = 0$ . But given (21), this is equivalent to show that in the model of Section 2,

$$\lim_{g \rightarrow \infty} \frac{\partial Y(h, Ag^{\alpha_g}, 0)}{\partial g} = 0, \quad (\text{B4})$$

for  $h = 1$  and  $h = 0$ . Doing the algebra, one can see that when  $h = 0$ , we can write  $Y$  explicitly as  $Y = C(Ag^{\alpha_g})^{\frac{1}{1-\alpha_l}}$  with  $C > 0$  and when  $h = 1$ , we get  $Y = D(Ag^{\alpha_g})^{\frac{1}{1-\alpha_l-\alpha_k}}$  with  $D > 0$ . Because  $\alpha_g + \alpha_k + \alpha_l < 1$ , the marginal effect of government spending on household consumption vanishes as  $g$  goes to infinity, while the disutility of labor of an additional unit of government spending is at least  $\gamma\chi > 0$ .  $\square$

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