On the cyclicality of liquidity in a consumption-based asset pricing model with search frictions*

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Abstract

For markets characterized by search frictions, data suggests that asset liquidity, defined as the speed at which one can sell an asset, is procyclical. We introduce search frictions in the market for assets of a consumption-based asset pricing model with idiosyncratic income shocks. We study the endogenous behavior of liquidity and asset prices. We find that liquidity is independent of dividend shocks, implying acyclical liquidity in the model. We illustrate that the independence is a consequence of the agents self-selection into buyers and sellers.

We model search costs as the opportunity cost of time, measured by labor income. When dividend shocks are accompanied by asymmetric changes in search costs, liquidity is no longer acyclical. Liquidity is procyclical if the dispersion of income is countercyclical, an empirical feature that has been documented using micro data.

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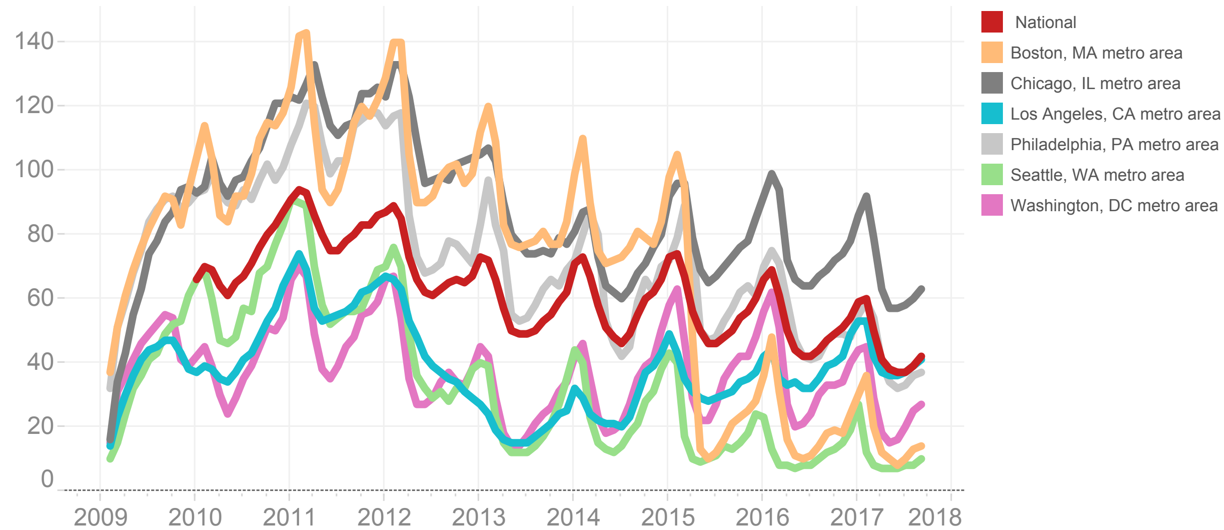
We also show that the presence of search frictions amplifies the impact of asset demand shocks on asset prices, while it dampens the impact of supply shocks.



Source: Redfin

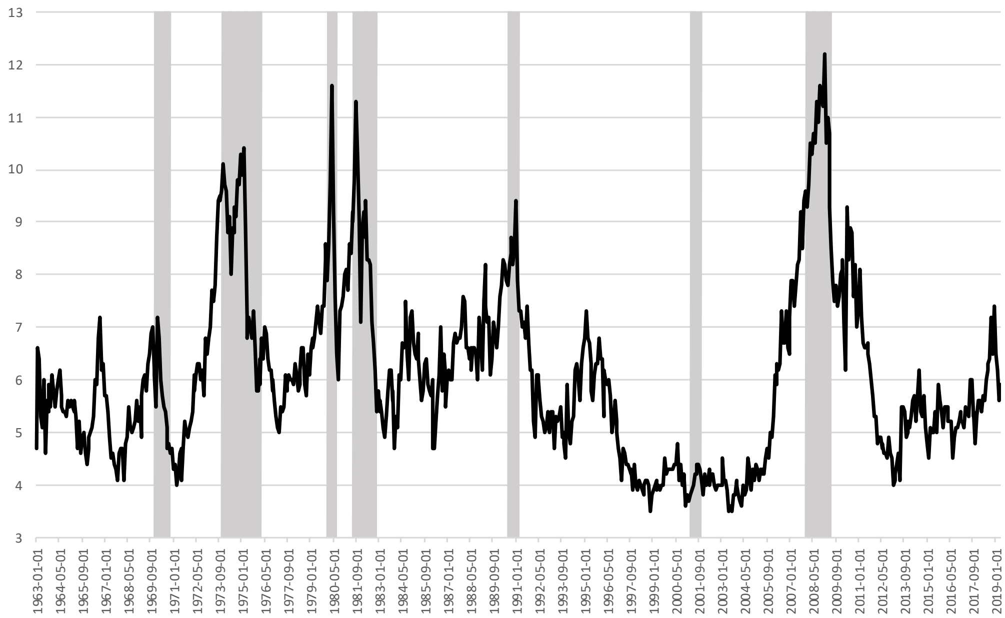
Figure 1: Median days on market (home sales)

1 Introduction

Some asset markets are characterized by procyclical liquidity in the sense that these assets sell more quickly in an expansion than in a recession.[1] This is typically the case of markets that suffer from search frictions. For example, in the case of the housing market, Figure 1 depicts the evolution of the average number of days it takes to sell a house advertised on Redfin—a residential real estate company that provides web-based real estate database and brokerage services.[2] The graph shows the monthly evolution from 2009 until now for six major cities in the US as well as the country wide average. It displays an increase in the time to sell following the recession, which is reversed after 2011, turning the evolution into a progressive decline.

[1]This definition of liquidity is sometimes explicit in asset pricing models with search frictions such as Duffie, Gerleanu, and Pedersen (2005), Weill (2008), Lagos and Rocheteau (2009), Lester, Postlewaite, and Wright (2012), Guerrieri and Shimer (2014), Dusha (2015), Geromichalos and Herrenbrueck (2016), Cao and Shi (2016), Cui and Radde (2016) and Chang (2018), among others. We do not explore alternative measures of liquidity such as the sensitivity of return changes to trading volume (Amihud, 2002) or exogenous shocks to bid-ask spreads (Gazzani and Vicondoa, 2016).

[2]The data was downloaded from the Redfin webpage: https://www.redfin.com/blog/data-center. Similar data can be downloaded from the Zillow website (https://www.zillow.com/research/data/). We choose to report the Redfin data on Figure 1 because the time period is longer. The Zillow data has a larger cross-section. This aspect is exploited in Appendix A.1, which provides regression results at the US State and MSA level using data from Zillow.



Source: U.S. Census Bureau and U.S. Department of Housing and Urban Development. Note: Shaded areas depict recessions according to the NBER Business Cycle Dates.

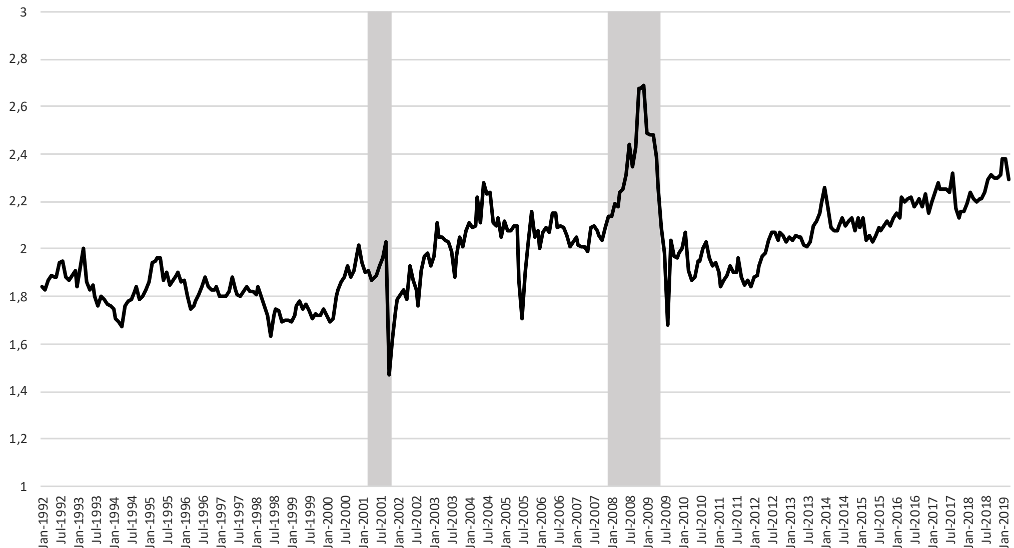
Figure 2: Monthly Supply of Houses in the US: Inventory-Sales Ratio

A commonly used proxy for time to sell is the inventory-sales ratio.[3] Figure 2 illustrates the procyclicality of liquidity of the US housing market for a longer time horizon—from 1963 to 2019—in the case of new homes only. Figure 3 shows the inventory-sales ratio for dealers of both used and new cars, other vehicles and automotive parts, while Figure 4 focuses on the market for kerosene jet fuel.[4] The latter two graphs also document a worsening of liquidity during the Great Recession. Finally, Cao and Shi (2016) focus on the secondary market for physical capital and how it reallocates between firms in booms and recessions. They define capital reallocation as acquisitions plus sales of property, plant and equipment and show that it is procyclical. Capital acquisition is not a direct measure of the probability to sell such an asset, but, given that reallocation is procyclical, their results suggest that the probability to sell should increase in a boom.

In this paper, we augment a consumption-based asset pricing model (CCAPM) with

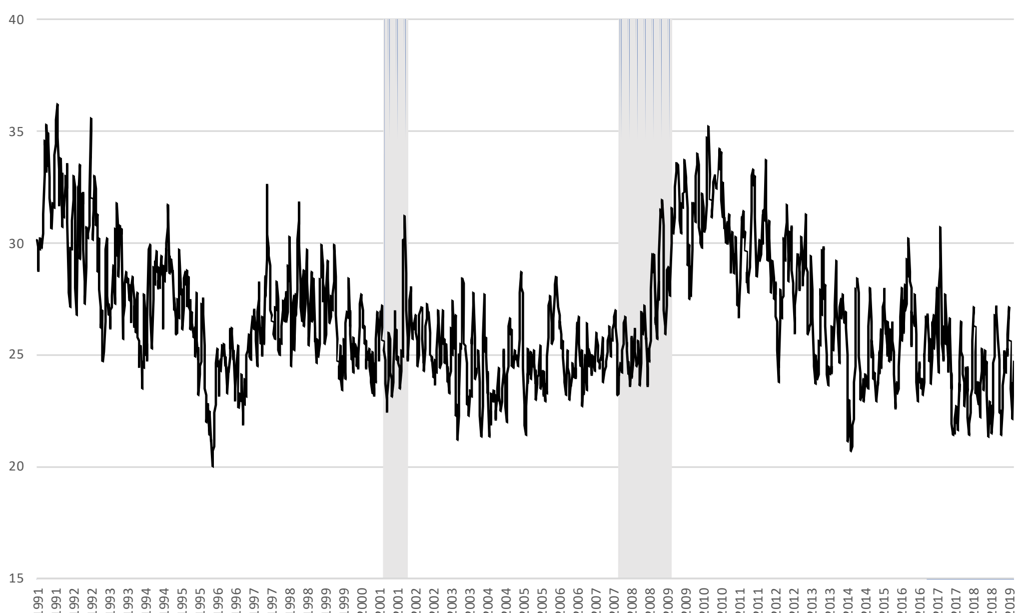
[3] Inventory-sales ratio is time to sell of a memoryless stochastic process.

[4] The market for kerosene jet fuel seems to be much less centralized than that for other commodities such as crude oil. It is characterized by significant price volatility. Airlines tend to write 1-2 year contracts with multiple suppliers to avoid interruptions. The price is typically negotiated directly between the buyer and seller. See Davidson, Newes, Schwab, and Vimmerstedt (2014) and the references therein. Appendix A.2 illustrates that the inventory-sales ratio of the crude oil market is nearly acyclical.



Source: U.S. Census Bureau. Note: Shaded areas depict recessions according to the NBER Business Cycle Dates.

Figure 3: Monthly Supply of Motor Vehicle and Parts in the US: Inventory-Sales Ratio



Source: Energy Information Administration. Note: Shaded areas depict recessions according to the NBER Business Cycle Dates.

Figure 4: Daily of Supply of Kerosene-Type Jet Fuel in the US: Inventory-Sales Ratio

search and matching frictions on the asset market and idiosyncratic income shocks. We focus on a CCAPM because it is a natural framework to study asset pricing issues. Households trade in order to smooth consumption. For trade to occur, some time must be spent searching. We study the endogenous behavior of liquidity and asset prices. Liquidity depends on the ratio of buyers to sellers in the market, which is commonly referred to as market tightness.[5] We ask if the model is able to generate procyclical liquidity according to the measures used in Figures 1-4.

We find that liquidity is independent of aggregate shocks, even though asset prices respond to the cycle as they do in the basic CCAPM model. We illustrate that this counterintuitive result is a consequence of the fact that agents freely choose whether to buy or sell—an assumption that we maintain from the traditional CCAPM. This is also a characteristic feature of search models with two-sided free entry: intuitively, market tightness does not respond to an increase in dividends because both buyers and sellers have larger incentives to enter, leaving the ratio of buyers to sellers unchanged.[6] As a consequence, market tightness depends only on the ratio of search costs.[7]

Time spent searching for an asset is costly because it distracts from both productive activity and leisure. Since labor income is the opportunity cost of search, we find that the acyclical nature of liquidity breaks down if the dispersion of idiosyncratic income is correlated with aggregate shocks. More specifically, if the dispersion of idiosyncratic shocks increases in downturns, liquidity is procyclical. The intuition for this result is as follows. Agents care about both the price of the asset and the time loss resulting from trade. Consequently a consumption-leisure decision drives the price-liquidity tradeoff: if consumption is marginally more desirable than leisure, a seller would be willing to wait longer for a higher price. In equilibrium, this consumption-leisure tradeoff is captured by the realization of the income shock. In short: if in a recession the wage of a seller drops relative to the wage of a buyer, the seller cares more about consumption than leisure, implying that she is willing to wait longer on the asset market, resulting in a fall in liquidity.

The asset pricing literature based on heterogenous agent models with uninsurable idiosyncratic risk, such as e.g. Constantinides and Duffie (1996) and Storesletten, Telmer, and Yaron (2007), uses countercyclical income risk as a solution to the equity premium puzzle, as it simply increases systematic risk. We show that the same assumption also generates procyclical liquidity as suggested by the data. More recently, Guvenen, Ozcan, and Song (2014) and Salgado, Guvenen, and Bloom (2016), using

[5]We will use the terms "liquidity" and "tightness" interchangeably throughout the paper.

[6]Appendix C.1 illustrates with a simple search model that two-sided free entry and a standard pricing rule generate acyclical tightness.

[7]Interestingly, using similar reasoning, market tightness does not depend on the distribution of assets.

more comprehensive data, argue that idiosyncratic risk is not countercyclical. Instead, they find that left-skewness increases during recessions. The mechanism that generates procyclical liquidity in our model is also robust to such a statistical framework because it only needs sellers to suffer the consequences of recessions relatively more than the buyers.

We also find that the presence of search frictions makes asset prices more sensitive to changes in the asset valuation of buyers, while reducing their sensitivity to sellers' valuations. The intuition for this result is reminiscent of Pissarides (2009). Indeed, a difference of our model with respect to the standard CCAPM is that a buyer has to pay a search cost on top of the price to acquire an asset. Hence, while a one percent increase in valuation yields a one percent increase in the price in the standard model, the same increase in valuation would imply a more than proportional increase in the price, keeping the search cost fixed. Hence, search frictions amplifies the impact of demand shocks in our framework, while it dampens the impact of supply shocks. Evidence of this asymmetry has been documented for the housing market for example: Genesove and Mayer (2001) show a positive correlation between prices and sales volume in the Boston real estate market.

As we mentioned above, our paper is related to the asset pricing literature that relies on search frictions to obtain measures of liquidity. Many of the papers in this literature are spin-offs of the earlier money-search literature originated by Trejos and Wright (1995), Shi (1995) and Lagos and Wright (2005).[8] Our contribution is to analyze a framework that we think closer to the CCAPM. We maintain the two-sided free-entry assumption by modeling households using the large-family framework by Shi (1997).[9] An appealing feature of this framework is that risk associated with the random matching process for individual buyers and sellers is diversified away at the household level. This gives households tools to actively engage in a process of consumption smoothing rather than let their consumption fluctuate depending on how lucky they are while searching.

The strand of literature discussed above includes papers that generate procyclical liquidity. Guerrieri and Shimer (2014) introduce adverse selection in an asset pricing model, reducing the probability of selling an asset in a recession. Cao and Shi (2016) model an economy with search frictions in the market for used capital and obtain pro-

[8]See Nosal and Rocheteau (2011) for further references.

[9]The large-family framework allows the model to converge to a standard CCAPM when search frictions collapse. This framework also simplifies the solution of the model in the sense that the distribution of prices and liquidity is degenerate in equilibrium. It has also been applied to labor market business cycles in Andolfatto (1996). More recently, Shi (2015) has applied this framework to asset markets. A difference with our framework is that the fraction of equity an agent can sell in Shi (2015) is exogenous.

cyclical liquidity in the capital market through the entry of buyers. Moreover, they discuss how liquidity affects the dispersion of Tobin's q across firms (which is countercyclical in the data), while we rely on countercyclical idiosyncratic income dispersion to generate procyclical liquidity. Our paper is also closely related to Cui and Radde (2016), who introduce search on the market for assets in an RBC model. They also use a large-family framework. In their model, firms sell assets to workers through financial intermediaries. The presence of intermediaries implies a bid-ask spread, allowing them to study how this variable responds to the business cycle. Intermediation shocks imply positive comovement between the convenience yield and asset prices (i.e. procyclicality). Our paper offers a benchmark to understand the issue of acyclical liquidity in asset pricing models with search frictions. We illustrate the importance of three elements for liquidity to be acyclical: i) two-sided free entry, ii) standard pricing rules and iii) a symmetric response of search costs to the cycle. The mechanism we study that generates procyclical liquidity is based on idiosyncratic income risk, but any deviation from the above three elements could in principle produce cyclical liquidity (e.g. limited participation, price rigidity, etc...).

The acyclicality of market tightness being the consequence of two-sided free entry is a result that is encountered in other search models. Wasmer and Weil (2004) develop an undirected search model with search characterizing both the credit and the labor markets. In their model, firms search for banks on the credit market to finance the search cost that they then need to pay on the credit market. They assume two-sided free entry on the credit market, a feature of the model that delivers a credit-market tightness that does not respond to aggregate fluctuations in productivity.[10] Also related is Shimer (2013). He introduces a labor-market participation decision into an otherwise standard search and matching model of the labor market and shows that market tightness becomes almost acyclical in this context, unless some form of wage rigidity is also introduced. The complete-information version of some directed-search asset-pricing models—such as in Guerrieri and Shimer (2014), Chang (2018) and Dusha (2015)—also imply acyclical liquidity.[11] However, this characteristic is not due to two-sided free entry, but it is a consequence of equal valuation of the asset by buyers and sellers. Once the flow utility of enjoying an asset varies from an agent to another—as it is the case in a CCAPM model where the marginal utility of consumption is not

[10]In Appendix C.1, we introduce a toy model with search and two-sided free entry. The model considers either Nash bargaining with undirected search or competitive search. We show that two-sided free entry implies that market tightness does not depend on aggregate fluctuations in this model as long as search costs are unaffected. The framework of the toy model is very similar to Wasmer and Weil (2004) and yields the same formula for market tightness.

[11]This is also true for the version with adverse selection when equilibria are separating.

necessarily the same across agents—the acyclical property of liquidity disappears.[12]

The idea that agents use their time endowment to search for better price deals when leisure is abundant has been used in frameworks of the goods market. Kaplan and Menzio (2016) studies the business cycle dimension. They build a model where unemployed workers in a recession have more time to search on the goods market. Aguiar and Hurst (2013) studies the life cycle dimension. Their results suggest that old consumers have more time to search for cheaper products. A similar idea is also explored in Alessandria (2009), where deviations from the law of one price can be explained through differences in the opportunity cost of time across countries. We illustrate that these mechanisms can be applied to asset markets as well and can explain the procyclicality of asset liquidity.

Given our motivation from Figures 1-4, our paper is also related to the literature on housing that incorporates search frictions. A seminal contribution to this literature is Head, Lloyd-Ellis, and Sun (2014), who show that search frictions can generate persistence in housing prices. Hedlund (2016a) and Hedlund (2016b) is interested in the cyclicality of the housing market. He introduces real estate agents that make a framework with heterogenous agents tractable. Albrecht, Gautier, and Vroman (2016) explore the microfoundations of pricing in the housing market when sellers have private information in a model with directed search. We think our framework is more general than the literature cited above, but given that it is not specifically about the housing market, it lacks aspects of this market such as for example default.

The paper is organized as follows. Section 2 introduces the model. The equilibrium is analyzed in Section 3, while the main results are exposed in Section 4. Section 5 tries to quantify the asymmetry between the price impact of the asset valuation of buyers and and the one of sellers.

2 Model

2.1 Assets and output

Time is discrete. We will use the index t to refer to each time period, but, in most of the presentation below, we will simply drop time indices and use primes to refer to variables in period $t+1$, while variables without prime are evaluated at time t.

There are two agents indexed by $i \in \{1, 2\}$ who discount time by a factor $\beta_i \in (0, 1)$.

[12] Appendix C.2 introduces another toy model to understand this framework. We show that if buyers and sellers have equal valuation of the asset, liquidity does not depend on dividends, while liquidity increases with an increase in dividends if buyers value the asset more and decreases if sellers value it more.

The discount factor is stochastic, as described below.[13] They derive instantaneous utility $u(c_i, l_i)$ from consumption c_i and leisure l_i in each period, with u being subject to standard properties.

Each agent is the owner of a farm that produces w_i units of real output per hours worked in the farm each period. The w_i's are subject to idiosyncratic shocks and we will sometimes refer to them as the *wages* of the economy. We denote by n_i the hours the agent i chooses to work in the farm. Agents can sell goods produced by the farm in the goods market, but they cannot sell claims on the farm.

There is a Lucas (1978) tree delivering dividends π every periods. The vector $z \equiv (\beta_1, \beta_2, w_1, w_2, \pi)'$ follows a Markov chain with transition matrix Γ and ergodic mean $(\bar{\beta}, \bar{\beta}, \bar{w}, \bar{w}, \bar{\pi})'$. An agent i holds claims on the tree in quantities A_i. As we discuss below, the markets on which these assets are traded are characterized by search frictions, making them partially illiquid in the sense that it takes time to buy or sell an asset.

Goods are perfect substitutes whether they come from the tree or farms and exist to cover consumption needs. Aggregate output is the sum of the output realizations of all farms and the tree:

$$Y = \sum_{i=1}^{2} w_i n_i + \pi.$$

Through market clearing, Y is also the value taken by aggregate consumption,

$$C = \sum_{i=1}^{2} c_i,$$

since agents do not hold physical capital and no government habits this economy.

2.2 Search frictions

Agents trade assets in markets characterized by search frictions. In particular, we assume competitive search in the sense of Moen (1997). An advantage of using this framework as opposed to a context with undirected search and Nash bargaining is that it eases the comparison of some results with the benchmark of the consumption CAPM model since agents are price takers in both frameworks.

[13] Asset prices are empirically more volatile than their fundamental value. Shocks to the discount factor allow us to reproduce this fact. Moreover, considering shocks to the discount factor permits us to analyze how asset prices may respond differently to exogenous changes in the asset valuation of buyers as compared to exogenous changes in the valuation of sellers. See Section 4 below.

A continuum of submarkets opens every period, where agents can exchange the assets. Since the amount of markets cannot be determined in equilibrium, we normalize this amount to one. Each submarket may in principle be characterized by a different price, which we denote by p. Agents take these prices as given.

Each agent is endowed with L units of time every period. Agents can use their time to search in the asset markets, work on the farm or for leisure. Trade can occur only when search is successful. When a unit of time dedicated to search is successful, they can trade x units of assets. Search is directed: one freely chooses to spend time searching as a buyer or a seller in each one of the submarkets. Agents can potentially choose to search in several submarkets at the same time. We denote by b_i (and s_i) the time spent by agent i as a buyer (and a seller) on a submarket. Hence, each agent takes the following restriction into account:

$$l_i + b_i + s_i + n_i \leq L. \tag{1}$$

We denote by $\lambda_s(\theta)$ (and $\lambda_b(\theta)$) the probability that a seller (buyer) finds a partner to trade assets with in a given submarket (per unit of time effort invested in the search process).[14] It is an increasing (decreasing) function of the tightness $\theta = \frac{b}{s}$ and the property $\lambda_s(\theta) = \theta\lambda_b(\theta)$ holds. θ is a measure of liquidity: when θ is large, assets are sold quickly. Hence, agent i sells $s_i\lambda_s(\theta)$ assets and buys $b_i\lambda_b(\theta)$ assets on the same market.

We can write the value function characterizing the behavior of each agent i as follows:

$$V(A_i, z) = \max_{c_i, l_i, b, s, n_i} u(c_i, l_i) + \beta_i E\left[V(A_i', z')\right] \tag{2}$$

subject to (1), the budget constraint, i.e

$$n_i w_i + s_i x \lambda_s(\theta) p + A_i \pi \geq c_i + b_i x \lambda_b(\theta) p, \tag{3}$$

and the law of motions for asset holding:

$$A_i' = A_i - x s_i \lambda_s(\theta) + x b_i \lambda_b(\theta) \tag{4}$$

[14] These probabilities are obtained through a standard constant-return-to-scale matching function $m(b,s)$: $\lambda_s(\theta) = \frac{m(b,s)}{s} = m\left(\frac{b}{s}, 1\right)$ and $\lambda_b(\theta) = \frac{m(b,s)}{b} = m\left(1, \frac{s}{b}\right)$.

3 Equilibrium

3.1 First-order conditions

Agents buy or sell assets in order to smooth consumption. In equilibrium, each agent does not spend time searching as a buyer and as a seller at the same time. Depending on whether agents need to consume or save, they self select into sellers or buyers respectively.

Denote by $\xi_{i,t,t+j} \equiv \beta_i^j \frac{\partial u(c_{i,t+j}, l_{i,t+j})/\partial c_{i,t+j}}{\partial u(c_{i,t}, l_{i,t})/\partial c_{i,t}}$ the stochastic discount factor that discounts units of output in period $t+j$ from the perspective of agent i in period t. In Appendix B.1, we show that the following two asset pricing equations describe the behavior of sellers and buyers respectively:

$$p_t - \frac{\partial u(c_{i,t}, l_{i,t})/\partial l_{i,t}}{\partial u(c_{i,t}, l_{i,t})/\partial c_{i,t}} \frac{1}{x\lambda_s(\theta_t)} = \sum_{j=1}^{\infty} E_t \left[\xi_{i,t,t+j}\pi_{t+j}\right], \tag{5}$$

$$p_t + \frac{\partial u(c_{i,t}, l_{i,t})/\partial l_{i,t}}{\partial u(c_{i,t}, l_{i,t})/\partial c_{i,t}} \frac{1}{x\lambda_b(\theta_t)} = \sum_{j=1}^{\infty} E_t \left[\xi_{i,t,t+j}\pi_{t+j}\right]. \tag{6}$$

In the standard consumption CAPM model, the price of an asset is equal to the discounted sum of the expected dividends that the asset delivers: this benchmark corresponds to the case when x tends to infinity. When x is finite, the first-order conditions (5) and (6) show that search frictions act as a tax on financial transactions. This generates a gap between the effective price paid by buyers and the effective price received by sellers. For example, equation (5) subtracts the disutility of searching from the price a seller receives after a transaction has been made (the second term on the left-hand side). This cost considers the time spent searching for a buyer to sell the asset (the duration $1/\lambda_s(\theta^a_{m,t})$) weighted by the marginal rate of substitution between leisure and consumption (the seller sacrifices leisure for consume). A similar interpretation applies to equation (6), with the difference that, in this case, buyers have to pay an extra disutility cost on top of the price they pay to sellers.

The equilibrium is also characterized by a standard labor supply condition:

$$\frac{\partial u(c_i, l_i)/\partial l_i}{\partial u(c_i, l_i)/\partial c_i} = w_i \tag{7}$$

This relation will be a useful tool below to identify the marginal rate of substitution between consumption and leisure from the exogenous variable w_i.

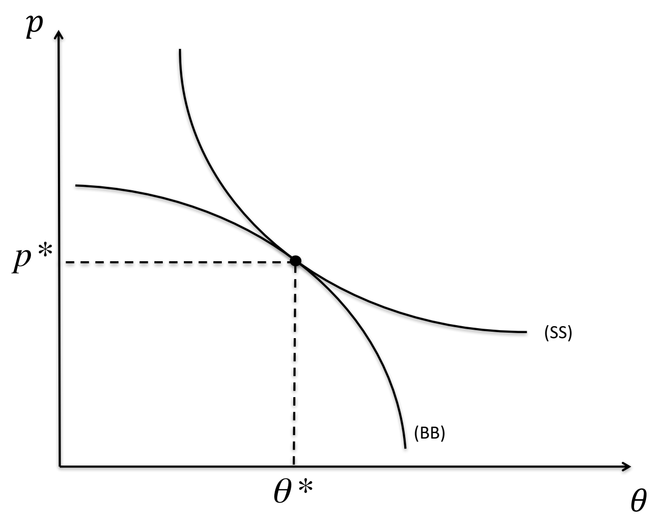


Figure 5: Joint determination of price and liquidity

3.2 The price-liquidity tradeoff

The Euler equations (19) and (20) describe indifference curves between θ and p for buyers and sellers respectively taking marginal utilities as given. These are shown in Figure 5. The (SS) convex locus refers to the implicit relation between p and θ given by equation (20), while the (BB) concave locus corresponds to the relation given by (19). Sellers prefer indifference curves characterized by high price and high liquidity: these curves would be located in the upper-right corner of the graph. Buyers prefer indifference curves characterized by low price and low liquidity (located in the lower-left corner of the graph).

Under competitive search, the set of prices in active submarkets is exhaustive: there exist no other price such that a submarket would have participants at that price. This implies that the two indifference curves (SS) and (BB) have to be tangent in equilibrium, as shown on Figure 5. In the Appendix B.2, we show that the tangency implies the condition given in the following Proposition:

Proposition 1. *Denote by c_s and l_s the consumption and leisure levels of sellers in a submarket and by c_b and l_b those of buyers. Denote by $\eta(\theta)$ the elasticity of the* *matching function with respect to the mass of sellers. Liquidity reads as*

$$\theta = \frac{1-\eta(\theta)}{\eta(\theta)} \frac{\partial u(c_s,l_s)/\partial l_s}{\partial u(c_s,l_s)/\partial c_s} \frac{\partial u(c_b,l_b)/\partial c_b}{\partial u(c_b,l_b)/\partial l_b}. \tag{8}$$

The liquidity of a given submarket thus depends on two elements: i) the ratio of the marginal rate of substitution between leisure and consumption of sellers and buyers; and ii) the elasticity of the matching function with respect to the mass of buyers relative to the elasticity with respect to that of sellers. The intuition for the presence of the first factor is the following. When sellers choose towards which type of market to direct their search, they compare the utility they obtain through consumption by selling the asset with the disutility in terms of leisure they suffer through search. If the marginal utility of consumption is relatively large, they are willing to spend a large amount of time searching to sell the asset. A similar reasoning applies to the behavior of buyers. This is why the ratio of marginal rates of substitution appears in equation (8).

Liquidity is also decreasing in the elasticity of the matching function with respect to the mass of sellers. When $\eta(\theta)$ is low, the probability of selling an asset quickly converges towards zero as more sellers enter the market. This means that it quickly becomes attractive for sellers to create an extra market and start searching there. This explains why only a few sellers operate in a market and liquidity is high when $\eta(\theta)$ is low. A similar reasoning applies when the elasticity of the matching function with respect to the mass of buyers $(1-\eta(\theta))$ is low: the incentive for buyers to operate in another market quickly increases as new buyers enter the market, explaining why only a few buyers stay in a market when $(1-\eta(\theta))$ is low.

Another interpretation of condition (8) is the following. In a context with frictionless trading, a social planner would like to equalize the marginal utility of consumption between agents. Markets could achieve this, but frictions generate a wedge that does not allow such equalization. This wedge depends on congestion in the market and the relative leisure loss of transporting a unit of output from one agent to another.[15] This is suggested by simply rewriting (8) as follows:

$$\frac{\partial u(c_b,l_b)/\partial c_b}{\partial u(c_s,l_s)/\partial c_s} = \theta \frac{\eta(\theta)}{1-\eta(\theta)} \frac{\partial u(c_b,l_b)/\partial l_b}{\partial u(c_s,l_s)/\partial l_s}.$$

An appealing characteristic of (8) is that it is a static condition. In directed search models with free-entry of buyers such as e.g. Dusha (2015), the liquidity of a market also depends on the value of reselling the asset in the future. With two-sided free entry,

[15] We thank Guido Menzio for pointing this out to us.

as in our case, this component disappears, making the condition a static one.[16]

4 Income, liquidity and asset prices

The intra-temporal condition (7) is useful as it allows to identify all the equilibrium marginal rates of substitution between consumption and leisure with the exogenous variables w_i's. The equilibrium tightness and the Euler equations can be rewritten as

$$\theta = \frac{1-\eta(\theta)}{\eta(\theta)}\frac{w_s}{w_b}, \tag{9}$$

$$p - \frac{w_s}{x\lambda_s(\theta)} = E(v_s') \tag{10}$$

and

$$p + \frac{w_b}{x\lambda_b(\theta)} = E(v_b') \tag{11}$$

respectively, where $E(v_s')$ and $E(v_b')$ are the present-discounted values of dividends when considering the stochastic discount factors of the seller and the buyer respectively (as they appear in the right-hand sides of (5) and (6)).

It is straightforward to see from (9) that the cyclicality of the wage ratio between sellers and buyers is what drives the cyclicality of liquidity in the model. For example, a procyclical ratio generates procyclical liquidity as long as the elasticity of the matching function is not too decreasing in the tightness. This would happen when wage dispersion is correlated negatively with the aggregate shock π. This result is summarized in the next paragraph:

Result 1. *Define* $\Gamma(\theta) \equiv \theta\frac{\eta(\theta)}{1-\eta(\theta)}$. *Assume* $\Gamma'(\theta) > 0$. *If* $COV\left(\frac{w_s}{w_b}, \pi\right) \lesseqgtr 0$, *then* $COV(\theta, \pi) \lesseqgtr 0$.

Equations (10) and (11) now have the interpretation that the economic price that a seller receives (or a buyer pays) is net of all the income she gives up while searching for a buyer (or a seller) for the asset. This is because a negative term appears on the left hand side of equation (10) next the price, which is the income of the seller would have received (per unit of time she would have worked) multiplied by the average search

[16] Interestingly, Wasmer and Weil (2004) obtain a similar condition in a context with undirected search, with some differences though. First, since the Hosios-Pissarides condition is not necessarily met, their condition includes the bargaining power of sellers instead of the elasticity of the matching function. Second, because agents are risk-neutral in their model, the marginal utility of consumption is absent and the opportunity cost of search (the marginal utility of leisure in our case) is constant.

duration for a seller. A similar interpretation applies to equation (11) from a buyer's perspective.

Define by $\kappa_s \equiv \frac{w_s}{x\lambda_s(\theta)}$ and $\kappa_b \equiv \frac{w_b}{x\lambda_b(\theta)}$ the search costs incurred by the seller and the buyer respectively. As it is clear from (9), these two terms only depend on the exogenous variables w_i's. By log-linearizing the conditions (10) and (11) around the steady state, one can see that the presence of search frictions increases the elasticity of the price with respect to the buyer's valuation of the asset, while it reduces the elasticity with respect to the seller's valuation. Indeed, denote with hats log deviations from steady state and with asterisks steady state values, one gets

$$\hat{p} = \frac{p^* + \kappa_b^*}{p^*}\hat{v}_b' - \frac{\kappa_b^*}{p^*}\hat{\kappa}_b$$

and

$$\hat{p} = \frac{p^* - \kappa_s^*}{p^*}\hat{v}_s' + \frac{\kappa_s^*}{p^*}\hat{\kappa}_s.$$

In a standard consumption asset pricing model, these elasticities would both be equal to one because the price is simply equal to the agent's valuation of the asset in the standard framework. However, the presence of search frictions here pushes the buyer's elasticity above one and the seller's below unity. This result is summarized in the following paragraph:

Result 2. *Denote by $\epsilon_b \equiv \frac{p^*+\kappa_b^*}{p^*}$ and $\epsilon_s \equiv \frac{p^*-\kappa_s^*}{p^*}$ the elasticity of the price of the asset with respect to the buyer and the seller's valuation. The presence of search frictions imply $\epsilon_b > 1$ and $\epsilon_b < 1$.*

The aim of Section 5 is to evaluate quantitatively these elasticities.

5 Quantifying the multiplier

We now attempt to evaluate quantitatively the elasticities ϵ_b and ϵ_s mentioned in Result 2. The idea is to try to understand the importance of p^* with respect to κ_b^* and κ_s^*. We use two data sources for this exercise. The first one is data on time to sell homes, as described for example in Figure 1. This allows to identify the search duration for sellers in equation (10). We show below elasticities for average durations of 2 and 4 months. The second source is data on price to income ratio. According to Numbeo, home prices are about 3 years of median wages in the US. A disadvantage of this data is that we do not identify wages of buyers or sellers specifically. Wages of individuals trading houses are likely to be higher than the median. In this case, because the opportunity cost of

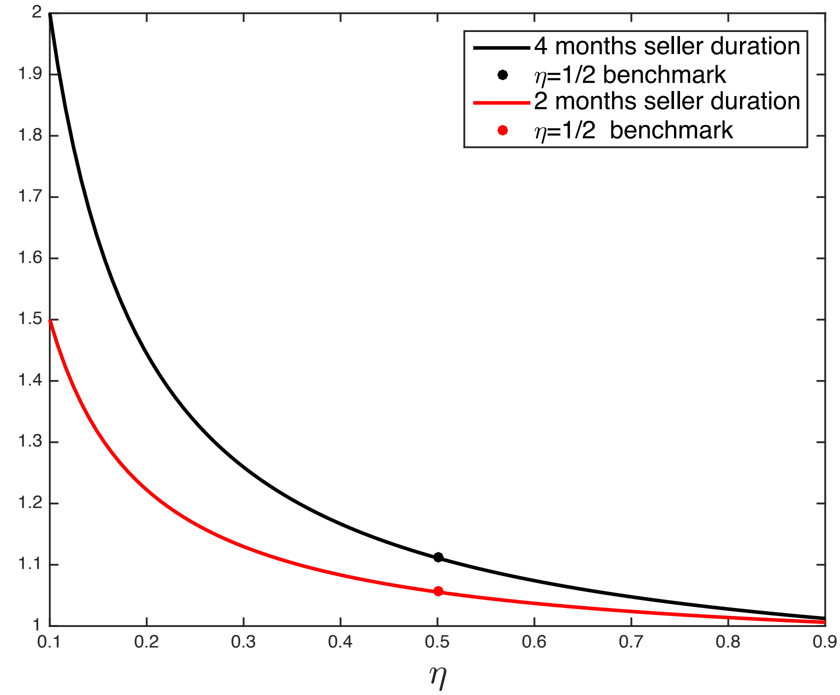


Figure 6: ϵ_b elasticity as a function of the elasticity of the matching function η

search would be higher, one would obtain higher values for ϵ_b and lower values for ϵ_s exacerbating the difference.

The data allows to obtain the importance of κ_s with respect to p^*. In particular, if the average search duration for sellers is four months, then the price response to a change in the valuation of the seller is attenuated by 11.1% ($\epsilon_s = 0.889$) as compared to a standard CCAPM framework, while it would be attenuated by 5.6% in the case of a two-month duration.

Unfortunately the data does not allows to identify κ_b^* directly. We thus rely on a Cobb-Douglas specification of the matching function for this matter: $\sigma(b, s) = b^{1-\eta} s^{\eta}$. This specification allows to rewrite the Euler equations as

$$p - \frac{1}{x}\left(\frac{\eta}{1-\eta}\right)^{1-\eta} w_b^{1-\eta} w_s^{\eta} = E(v_s')$$

and

$$p + \frac{1}{x}\left(\frac{1-\eta}{\eta}\right)^{\eta} w_b^{1-\eta} w_s^{\eta} = E(v_b').$$

A common benchmark from the search and matching literature is $\eta = 1/2$. In this case, the equations above imply that $\kappa_b = \kappa_s$. Hence, under this benchmark, the price response to a change in the valuation of the buyer is amplified by 11.1% as compared to a standard CCAPM framework if the average search duration for sellers is four months, while it would be amplified by 5.6% in the case of a two-month duration. Lower values of η increase ϵ_b, while higher values tend ϵ_b to one. Figure 5 depicts how ϵ_b varies with η. For example, if one considers a value for η as low as 0.1, then ϵ_b reaches 2 if the search duration of sellers is four months.

6 Conclusion

How imperfect markets are? This is a recurrent question in economics. The welfare properties of models and their policy recomendations depend on the answer. This is also true for positive analysis: the response of some variables to shocks change depending on how frictional markets are. However, it is a question hard to answer because frictions are not observable in many contexts. In this paper, we have studied a consumption-based asset pricing model with a certain amount of search frictions, but we have modeled them such that agents are given a chance to smooth consumption even though frictions act like a tax on trading. In this framework agents are not totally passive and do not have to let their consumption fluctuate depending on how lucky the search process is. We found that the speed at which agents sell an asset—which we call liquidity—does not depend on dividends, while asset prices behave similarly as in the frictionless asset pricing model. Given that liquidity seems to be procyclical empirically, this is a negative result. This result is also robust, provided that the asset market is characterized by two-sided free entry and prices are determined by a standard price sharing rule. From there, to recover procyclicality, one may choose to add further frictions. A possibility is to limit entry—restricted market participation has solved some asset pricing puzzles. Unobserved asset quality and adverse selection can also generate illiquidity in recessions. Our model suggests that additional frictions are not necessary though. If the dispersion of idiosyncratic income shocks is countercyclical, then liquidity becomes procyclical because sellers, who suffer a severe negative income shock in a recession, are willing to wait longer to sell an asset to get a better deal.

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Table 1: Evidence on procyclical liquidity

	Coef.	Time effects	State effects	MSA effects	Obs.	# of groups	Period	Source
(1)	2.12***	Yes	Yes	No	3,332	49	2012-2017	Zillow
	(0.297)							
(2)	2.18***	Yes	No	Yes	22,916	337	2012-2017	Zillow
	(0.183)							
(3)	1.11***	Yes	Yes	No	4,230	47	2010-2017	Zillow
	(0.226)							
(4)	0.98***	Yes	No	Yes	39,864	443	2010-2017	Zillow
	(0.105)							
(5)	2.80***	Yes	No	Yes	39,864	443	2010-2017	Zillow
	(0.129)							

A Appendix: Data

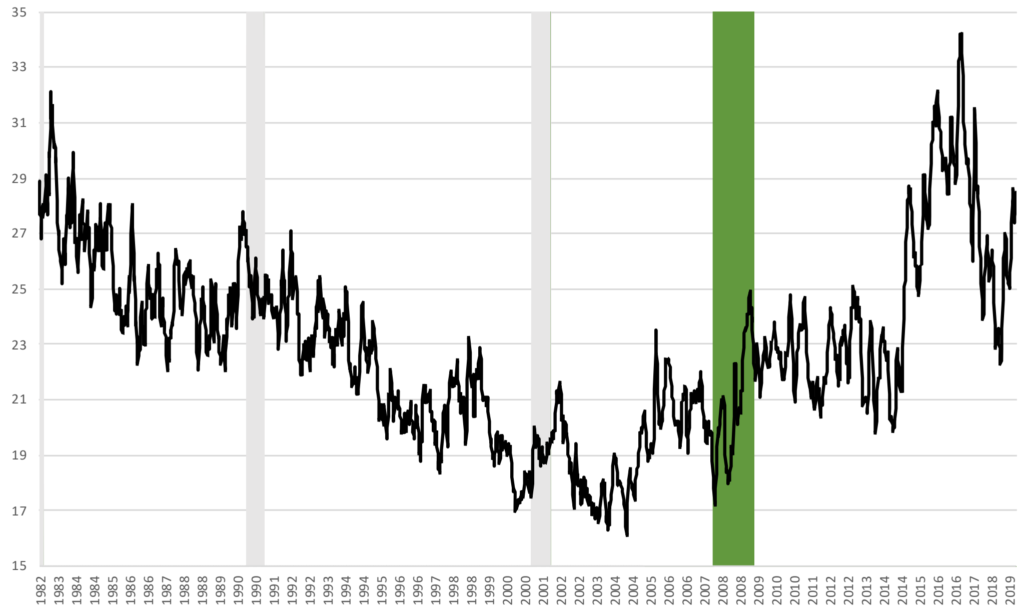
A.1 Additional evidence on procyclical liquidity of the housing market

In this Appendix, we show that the evidence reported in Figure 1 is robust when looking at more disaggregated data, such as at state or MSA level, controlling for time and fixed effects, using data from Zillow.

Table 1 reports regression results for five specification. In each regression, we use the unemployment rate as a proxy for the business cycle. A positive coefficient is thus associated with procyclical liquidity given the measures of liquidity we consider below.

Regressions (1) and (2) consider the average number of days homes have been on the market (on the Zillow website). The first one considers the average in each US State, while the second one considers liquidity data at the MSA level (for 337 cities). In both regressions, the unemployment rate is the State-level one from the Bureau of Labor Statistics.

Because of incompatibilities in terms of MSA coding between Zillow and the BLS, we were unable to match the liquidity data from regressions (1) and (2) with the unemployment data from the BLS at the MSA level. However, regressions (3) to (5) considers a liquidity variable that can be matched with unemployment data at the MSA level. The liquidity variable is the average number of days it took to sell a home. A disadvantage of this variable is that it excludes information for homes that cannot be sold—unlike the liquidity variable in regressions (1) and (2)—but it can be combined



Source: Energy Information Administration. Note: Shaded areas depict recessions according to the NBER Business Cycle Dates.

Figure 7: Daily Supply of Crude Oil in the US (excluding the Strategic Petroleum Reserve): Inventory-Sales ratio

with unemployment at the MSA level. Regression (3) is the equivalent of regression (1) with this alternative liquidity variable. It delivers a lower coefficient because the liquidity variable excludes information for homes that cannot be sold. Regression (4) considers data at the MSA level, with an estimated coefficient similar to regression (3). Finally, regression (5) is the weighted version of regression (4), where each MSA is weighted by its employment level.

A.2 The crude oil market

Figure 7 simply illustrates that the inventory-sales ratio for crude oil is much less cyclical than the kerosene-type jet fuel. Crude oil behavior is more in line with standard commodity cycles.

B Appendix: proofs

B.1 First-order conditions

Denote by μ_1, μ_2 and μ_3 the multiplier associated with (1), (3) and (4) and write the following lagrangian:

$$\begin{aligned}\mathcal{L}(c_i, l_i, A_i', b_i, s_i, n_i) = u(c_i, l_i) + \beta_i E\left[V(A_i', z')\right] \\ +\mu_1\left[L - l_i - b_i - s_i - n_i\right] \\ +\mu_2\left[w_i n_i + \left(s_i\lambda_s(\theta) - b_i\lambda_b(\theta)\right)xp + A_i\pi - c_i\right] \\ +\mu_3\left[A_i - A_i' - xs_i\lambda_s(\theta) + xb_i\lambda_b(\theta)\right].\end{aligned} \tag{12}$$

Derive with respect to each argument in the Lagrangian and set the derivative to zero:

$$\frac{\partial u(c_i, l_i)}{\partial c_i} = \mu_2 \tag{13}$$

$$\frac{\partial u(c_i, l_i)}{\partial l_i} = \mu_1 \tag{14}$$

$$\beta_i E\left[\frac{\partial V(A_i', z')}{\partial A_i'}\right] = \mu_3 \tag{15}$$

$$-\mu_1 - \mu_2 xp\lambda_b(\theta) + \mu_3 x\lambda_b(\theta) = 0 \tag{16}$$

$$-\mu_1 + \mu_2 xp\lambda_s(\theta) - \mu_3 x\lambda_s(\theta) = 0 \tag{17}$$

$$\mu_1 = \mu_2 w_i \tag{18}$$

B.1.1 Labor supply

The intratemporal condition (7) can easily be obtained by combining (13), (14) and (18).

B.1.2 Euler equations

By combiging (13) to (16), one gets:

$$-\frac{\partial u(c_i,l_i)}{\partial l_i} - \frac{\partial u(c_i,l_i)}{\partial c_i} xp\lambda_b(\theta) + \beta_i E\left[\frac{\partial V(A_i', z')}{\partial A_i'}\right] x\lambda_b(\theta) = 0,$$

which can be rewritten as

$$\frac{\partial u(c_i,l_i)}{\partial c_i} p + \frac{\partial u(c_i,l_i)}{\partial l_i} \frac{1}{x\lambda_b(\theta)} = \beta_i E\left[\frac{\partial V(A_i', z')}{\partial A_i'}\right], \tag{19}$$

By combiging (13) to (15) with (17), one gets:

$$-\frac{\partial u(c_i,l_i)}{\partial l_i} + \frac{\partial u(c_i,l_i)}{\partial c_i} xp\lambda_s(\theta) - \beta_i E\left[\frac{\partial V(A_i', z')}{\partial A_i'}\right] x\lambda_s(\theta) = 0,$$

which can be rewritten as

$$\frac{\partial u(c_i,l_i)}{\partial c_i} p - \frac{\partial u(c_i,l_i)}{\partial l_i} \frac{1}{x\lambda_s(\theta)} = \beta_i E\left[\frac{\partial V(A_i', z')}{\partial A_i'}\right], \tag{20}$$

By applying the envelope theorem, one can calculate the right-hand side in (19) and (20):

$$\frac{\partial V(A_i, z)}{\partial A_i} = \mu_2 \pi + \mu_3,$$

which can be rewritten as

$$\frac{\partial V(A_i, z)}{\partial A_i} = \frac{\partial u(c_i,l_i)}{\partial c_i} \pi + \beta_i E\left[\frac{\partial V(A_i', z')}{\partial A_i'}\right].$$

By iterative substitution, one can rewrite the equation above as:

$$\frac{\partial V(A_{i,t}, z_t)}{\partial A_{i,t}} = E_t\left[\sum_{j=0}^{\infty} \beta_i^j \frac{\partial u(c_{i,t+j} l_{i,t+j})}{\partial c_{i,t+j}} \pi_{t+j}\right].$$

Replace the formula above into (19) and (20) and rearrange terms to get (6) and (5) respectively.

B.2 The price-liquidity tradeoff

A pair θ-p is determined such that the two curves (19) and (20) are tangent. Calculate the slope of the (SS) locus on Figure 5:

$$\frac{\partial p}{\partial \theta}|_{SS} = -\frac{\partial u(c_s,l_s)/\partial l_s}{\partial u(c_s,l_s)/\partial c_s} \frac{\lambda_s{}'(\theta)}{x\left[\lambda_s(\theta)\right]^2},$$

as well as the slope of the (BB) locus:

$$\frac{\partial p}{\partial \theta}|_{BB} = \frac{\partial u(c_b, l_b)/\partial l_b}{\partial u(c_b, l_b)/\partial c_b} \frac{{\lambda_b}'(\theta)}{x \left[\lambda_b^a(\theta)\right]^2}.$$

The subscripts s and b allow to respectively refer to the consumption and leisure levels of the sellers and buyers of the specific market.

By equating these two slopes we obtain

$$-\frac{\partial u(c_s, l_s)/\partial l_s}{\partial u(c_s, l_s)/\partial c_s} \frac{\partial u(c_b, l_b)/\partial c_b}{\partial u(c_b, l_b)/\partial l_b} {\lambda_s}'(\theta) = \theta^2 {\lambda_b}'(\theta).$$

Call $\eta(\theta)$ the elasticity of the matching function with respect to the mass of sellers. We can rewrite the equation above as:

$$\frac{\partial u(c_s, l_s)/\partial l_s}{\partial u(c_s, l_s)/\partial c_s} \frac{\partial u(c_b, l_b)/\partial c_b}{\partial u(c_b, l_b)/\partial l_b} = \frac{\eta(\theta)}{1 - \eta(\theta)} \theta.$$

By rearranging the terms in this equation, one gets condition (8).

C Appendix: toy models

C.1 A model with two-sided free entry

In this appendix, we show that market tightness only depends on the ratio of opportunity costs of search in a basic model characterized by free entry of both buyers and sellers and is independent of other aggregate measures. The model considers only the extensive margin of trade and does not endogenize the intensive margin as in the model of this paper.

The value for a buyer and a seller are respectively

$$rV = -c + q(\theta)(1 - \eta(\theta))S$$

and

$$rU = -b + p(\theta)\eta(\theta)S.$$

with b and c the flow opportunity of costs for seller and buyers respectively. $q(\theta)$ and $p(\theta) \equiv \theta q(\theta)$ and the rate at which agents can buy and sell assets respectively. If trade is successful, buyers obtain a share $(1 - \eta(\theta))$ of the surplus S, while sellers obtains a share $\eta(\theta)$. In competitive search models, $\eta(\theta)$ is the elasticity of $q(\theta)$ with respect to θ, while it is the seller bargaining power in undirected search models and is independent of θ in these models.

The model may contain other equilibrium relations. In particular, one should consider an equation that defines the structure of the surplus S, but, because the result on market tightness is quite general, we are not presenting this part of the model here and rely only on the two Bellman equations above.

Consider free entry of buyers and sellers, i.e.,

$$V = 0 \quad ; \quad U = 0$$

Combining these two conditions with the Bellman equations leads to a system of two equations and two unknowns (θ and S). The solution for market tightness is

$$\theta = \frac{b}{c}\frac{1-\eta(\theta)}{\eta(\theta)}.$$

If we had free entry only on one side of the market, say buyers, we would have gotten the following equilibrium condition:

$$\frac{c}{q(\theta)} = (1-\eta(\theta))S.$$

S then becomes a free variable in this case—which needs to be pinned down by other equilibrium conditions—allowing θ to potentially depend on other aggregate variables.

C.2 A directed-search asset pricing model with buyer free entry

In this appendix, we illustrate that, in a standard directed search asset-pricing model (with one sided free entry), liquidity is independent of aggregate shocks because buyers and sellers have the same value for the asset.

Consider a model with buyer free entry in which the buyer's flow cost of entry is k and the seller's flow cost of holding the asset is c. Denote the value to the buyer for holding the asset as s and the value to the seller as xs, with $x > 0$. Consider the Bellman equations for buyers and sellers respectively.

$$rU = -k + q(\theta)\left(\frac{s}{r} - p - U\right)$$

$$\max_{\theta,p} rV = xs - c + m(\theta)(p - V)$$

Buyer free entry gives us $U = 0$ and the relationship between price and tightness:

$$p = \frac{s}{r} - \frac{k\theta}{m(\theta)}$$

The seller's problem gives us the equation for liquidity:

$$\frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)} = \frac{(1-x)s + c}{k}$$

Note that for $x = 1$ the value of the asset does not affect liquidity, while θ depends on s when x is different from 1.