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Adoption Technology Targets and Knowledge Dynamics:  
Consequences for Long-Run Prospects

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# Adoption Technology Targets and Knowledge Dynamics: Consequences for Long-Run Prospects

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## Abstract

When targeting frontier technologies, less developed economies usually face obstacles to achieve high growth in the long run, because of their low level of knowledge relative to the adoption technology target. If the intensity in which the adoption activity uses knowledge is high, then the less developed economy may end up trapped in a low growth equilibrium. We show that in this case it is beneficial to target less advanced technologies, which helps to compensate the scarcity of knowledge during the transition. Nevertheless, polarization is possible. If knowledge intensity in the adoption activity is low, then possessing a low stock of knowledge allows targeting the technology frontier even in a poor R&D environment. In this case, all economies achieve a high growth equilibrium in which only income level differences persist in the long run.

This paper builds an analytical model that encompasses three insights of the economic development process: (1) technology adoption is a key determinant of economic growth in developing and developed countries, (2) the domestic stock of knowledge and the intensity in which it is used in R&D activities largely determine the optimal adoption technology target, and (3) openness and overall internal and external R&D environment affect the ability of countries to accumulate knowledge and to benefit from R&D investment.

**Keywords:** R&D, adoption, innovation, growth, development, transitional dynamics.

**JEL:** O30, O33, O40

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# 1 Introduction

Medium-term per capita GDP growth rates and per capita income trajectories have shown significant differences among different groups of countries and periods. According to Maddison (2001), the per capita income ratio between the traditional developed world (Europe and Western offshoots<sup>1</sup>) and non-developed regions (Africa, Western Asia, Eastern Asia, and Latin America) were between 1.5 and 2.5 in 1820. In the following 130 years, this ratio increased for all non-developed groups, but has shown decoupled patterns afterwards. African countries continued increasing their per capita income gaps with the developed world while Latin America slightly worsened its relative position. In contrast, Eastern Asian countries dramatically reduced this gap in the last 60 years. What can we expect for the future?

The literature has taken two positions regarding this question.<sup>2</sup> One approach states that every economy eventually starts a process of development that ends up with the economy sharing a common long-run growth rate with all other countries. This corresponds to the literature of convergence in growth rates. Almost all papers that study technological transfers and focus on explaining per capita income differences share this view. Some exceptions discussed later are Howitt and Mayer-Foulkes (2005); Acemoglu and Zilibotti (2001); Basu and Weil (1998). Empirical studies coherent with this framework are Barro and Sala-i-Martin (1992); Mankiw et al. (1992); Evans (1996); and Rodrik et al. (2003), among others. A second group of models emphasizes the existence of growth traps that produce per capita income polarization and growth differences in the long run. These models generally point out some market failure or externality that leads to multiple equilibria. In particular, countries grow at different rates in the long run. Consequently, widening income ratios can characterize the long run. Theories that include technological transfers are scarce in this type of models. Pritchett (1997); Mayer-Foulkes (2002); and Feyrer (2007) give empirical support for this approach.

This paper presents an analytical model of innovation and technology adoption that encompasses both approaches and discusses conditions that make one or the other framework more likely. The argument builds on two assumptions. First, adoption and innovation activities require a domestic input to be produced. We assume that this input is domestic knowledge, which is accumulated in a one-to-one basis with costly R&D investment in the R&D sector. Second, the productivity of the stock of knowledge in the adoption and innovation activities depends on the technology level that the economy is targeting (in particular, the higher the technology target, the lower the productivity of a given stock of knowledge). We argue that the type of development challenge that the economy faces depends crucially on how intense the adoption activity (and not innovation) uses knowledge for achieving a technology improvement and whether the country's R&D incentives allow to maintain knowledge updated as for adopting frontier technologies. The

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<sup>1</sup>Western offshoots correspond to the following countries: Australia, New Zealand, Canada, and the US (Maddison, 2001).

<sup>2</sup>We focus on models that explain long-run growth discrepancies through differences in the technological progress process. Although empirical studies have pointed out many other variables that affect economic growth, productivity seems to be the ultimate factor that explains long-run growth (e.g. Easterly and Levine, 2001; Hall and Jones, 1999; Klenow and Rodriguez-Clare, 1997; Parente and Prescott, 2002).

mechanism is the following: R&D investment depends on the technology improvement that the firm can achieve. This improvement, in turn, depends on the stock of knowledge of the economy. If knowledge is an important input for adopting new technologies and this stock is relatively low, potential technology gains will be low, and so will be incentives to invest in R&D. As a consequence, knowledge accumulation will be slow. If foreign technologies are advancing rapidly, the country may find it difficult to maintain its stock of knowledge sufficiently updated.

We identify two situations: First, if knowledge intensity in the adoption activity is low, then possessing a low stock of knowledge is not an obstacle to start developing in the sense that all countries share a common world growth rate in the long run. Even though the stock of knowledge is small, the economy maintains its adoption capacity and the incentives to invest in R&D. Second, If this intensity is high, then the economy may fall in a growth trap if its knowledge stock and overall R&D efficiency are low. In such an environment, the lack of knowledge reduces severely expected benefits of R&D. In this case, the economy may fall into a vicious circle of low R&D investment, low technology improvement and, consequently, low knowledge accumulation. This reduces the adoption capacity further producing even lower incentives to invest in R&D.

Each case provides different implications for development. In the first case (low intensity), all countries share the same growth rate in the long run independently of their institutions, policies or endowments. For countries with bad economic conditions this rate is determined by external conditions, particularly by the growth rate of the world technology frontier. Consequently, income gaps remain constant in the long run and the widening of income per capita ratios is a transitory phenomenon. As in the neoclassical model, policies and economic conditions explain the differences in the level of per capita income. The timing of development matters for when to start enjoying a higher per capita income, but does not harm the adoption and innovations capacities in the long run. These capacities are solely determined by the R&D environment that the country exhibits in steady state.

In the second case (high intensity), a common long-run growth rate is not guaranteed. Particularly, if an economy is in a low growth path, it has to improve its economic efficiency to access better growth paths as long-run growth depends on domestic conditions. In such a case, making reforms that improve R&D efficiency may help to overcome the shortage of knowledge in the early stages of development, although they do not ensure it. We show that in these cases, it is optimal to copy less advanced technologies during some periods. Moreover, once in a good path, the economy has to do continuous efforts to maintain this path as changes in domestic or external conditions may change the long-run growth equilibrium path. For example, if technological advances speed up, the country need to increase its knowledge accumulation process if it intends to copy the more advanced technologies.

The paper is organized as follows: Besides this introduction, section 2 frames the paper in the literature and discusses intuitively the components, mechanisms, and results of the model. The next two sections present the base model and analyze conditions for achieving a high-type or low-type growth equilibrium in the long run. Section 5 extends the base model to analyze the determinants of an optimal adoption target. Section 6

presents some concluding remarks.

## 2 An overview of the model

The framework is a multi-sector and multi-country model of Schumpeterian growth in the spirit of the models of Aghion and Howitt (1992 and 1998) and Howitt (2000). Technological improvements result from costly and risky R&D, which are undertaken by R&D firms in different sectors in the economy. The size of the technological change depends on two factors: 1) the potential technological achievement through innovation and adoption and 2) the capacity of the firm to attain these technology improvements.

In the case of innovation, the potential improvement is proportional to the technology level currently in use while in the case of adoption, it is proportional to the technology gap. The technology gap is defined as the difference between the technology to be copied and the level of technology currently in use.<sup>3</sup> In particular, we are interested in analyzing how an optimal target is determined.

The adoption and innovation capacities (i.e. the capacity to copy and to create technologies, respectively) are endogenously determined by i) R&D efficiency parameters that condition the productivity of R&D; ii) adoption and innovation barriers that condition the potential technology improvement<sup>4</sup>; iii) the productivity of the stock of knowledge, which depends on the complexity of the technology that is being copied and created; and iv) by the intensity in which this knowledge is used in these activities. The evolution and determinants of the stock of knowledge are crucial for the results emphasized in this paper.<sup>5</sup>

We distinguish two types of economies: Leading and non-leading countries. Leading countries have best R&D parameters and consequently show highest average productivity and largest stock of knowledge. These countries produce systematic improvements in the technology frontier and have enough knowledge to adopt any existing technology. For simplicity, we assume that these countries are in steady state. Non-leading countries, in contrast, have worse R&D parameters and may be transiting to or already have achieved their steady state. We focus on countries that have relatively lower productivity and low stock of knowledge. We show that these economies rely mostly on adoption activities to sustain positive growth rates.<sup>6</sup> The issue for the latter countries is how to provide a highly enough R&D reward to encourage R&D investment and whether this investment

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<sup>3</sup>Models that incorporate an adoption activity usually assume that the adoption target is the world technology frontier or a fraction of it, so that this target follows an exogenous trajectory for the domestic economy. We study two cases. The base case follows the literature and assumes that the adoption target is the technology frontier. In a second exercise, we determine the optimal technology target. Technology dynamics change significantly in this second case.

<sup>4</sup>Adoption and innovation barriers are assumed to be parameters and comprise all restrictions, policies, institutions or incentives to copy and to create new technologies.

<sup>5</sup>Back in 1952, Gershenkron pointed out the relevance of social capabilities for catching up. The classical paper of Nelson and Phelps (1966) stated that adoption capacities depended on domestic conditions and, particularly, on the stock of human capital.

<sup>6</sup>Innovation is not crucial for non-leading countries to achieve high growth in the long run.

is capable to sustain a high growth path.<sup>7</sup>

Within this framework, leading economies grow in steady state at their innovation rate. This equilibrium is unique. We define this long-run growth rate as the high-growth case. Unlike many models that jointly analyze leading and non-leading countries (e.g. north-south type models), these economies do not specialize in innovation. Moreover, we show that adoption is necessary to maintain the leading position.

In the case of non-leading countries, two situations arise: First, if knowledge is not intensively used in the adoption activity, then all these countries achieve the high-growth equilibrium in the long run. In this case, R&D rewards are always high enough to sustain a minimum level of R&D investment and to keep the adoption capacity of the country. The economy can start at any time its development race.<sup>8</sup> Consequently, only income differences remain in the long run, which are explained by differences in the economic structure (in our case, in the R&D environment)<sup>9</sup>. These results happen independently of the values of all remaining parameters.

The second case characterizes situations in which the knowledge intensity in the adoption activity is high. In this case, an economy may fall in a growth trap. The mechanism is the following: if knowledge intensity is high and the stock of knowledge is relatively low, then the adoption capacity can follow a decreasing path if R&D rewards are not high enough. This produces less knowledge accumulation and reduces R&D rewards even more, hindering the accumulation of knowledge and reducing R&D investment further. The consequence is that the economy is not able to produce enough technology adoption (and innovation) to catch-up with the leading economies remaining laggard and transiting to a polarized equilibrium. Once in a path to a low growth equilibrium, improving institutions and economic policies may help to compensate the scarcity of knowledge. Better policies and less adoption and innovation barriers may help to accomplish this goal. In this scenario, different policy parameters can explain growth rate differences in addition to income level disparities.

However, improving policies and institutions may be a difficult task, particularly for non-leading countries (see, among others, Acemoglu et al, 2006; Persson and Tabellini, 2000; Drazen, 2000). In this setting, a country with low efficiency, that is trying to implement technologies that are not coherent with its developing stage, may harm its growth prospects. A way to preserve—and eventually to increase—the country’s adoption capacity consists in targeting less advanced technology during some periods. The reason

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<sup>7</sup>Dynamics of the adoption absorptive capacity is as follows: The technology gap provides a reward to invest in R&D. This R&D investment produces a raise in the technology level and in the stock of knowledge. The latter increase, though, does not ensure a raise in the adoption capacity as it depends on the stock of knowledge relative to the technology target. For the former not to decrease, it is necessary that the stock of knowledge increases sufficiently to remain updated with the technology frontier. If this capacity declines, then R&D firms—and the economy—reduce their potential to benefit from adoption.

<sup>8</sup>This start is understood as an opening to technological transfers. Lucas (2000) argues that this situation characterizes the developing process.

<sup>9</sup>Despite policies and economic environment in non-leading and leading countries being identical, technology in the former countries will not jump instantaneously to the leading countries’ level. Reasons are: R&D is risky at the idiosyncratic level (in the line of Grossman and Helpman, 1991; Aghion and Howitt, 1992, 1998, and subsequent Schumpeterian growth models); and even though knowledge intensity is not a determinant factor for long-run growth, it does condition short- and medium term growth.

is that the bottleneck for developing is that the economy does not have enough knowledge to implement frontier technologies. As a result, it does not invest sufficiently in R&D and does not accumulate the necessary amount of knowledge to remain in line with the advances of this frontier in the long run. However, the economy's stock of knowledge may be large enough for targeting a less complex technology and for continuing profiting from adoption. The idea is that R&D firms with low stock of knowledge copy non state-of-the-art technologies making the existing stock of knowledge more productive (as the productivity of the stock of knowledge depends on the adoption/innovation target). When following such a strategy, R&D rewards increase and it is possible to sustain higher levels of R&D investment. Note, however, that if an economy wishes to achieve the high growth rate in the long, it has to copy eventually (a fraction) of the technology frontier.

We show that the optimal adoption target is a positive function of the knowledge stock of the economy and the development stage of the economic sectors. Thus, there will be sectors targeting frontier technologies and sectors aiming at less advanced technologies. This target is also negatively related to the knowledge intensity in the adoption activity. Moreover, the higher the knowledge intensity in the adoption activity, the longer the periods that the economy copies a technology that is below the technology frontier. Finally, only when knowledge intensities are very high and initial conditions very low, it may be necessary to do both, improve the economic environment and target lower technologies.

The two cases described (namely the low and high knowledge intensity in the adoption activity) complement other frameworks and mechanisms in the literature. The first case is related to the literature that explains income disparities in the long run. These models assume that all countries share a common growth rate in the long run. In general, these papers focus on the obstacles faced by non-leading economies to benefit from adoption.<sup>10</sup> Contributions in this line are Parente and Prescott (1994, 1999); Basu and Weil (1998); Acemoglu and Zilibotti (2001), among others. For instance, Parente and Prescott (1994, 1999) argue that economic and legal restrictions are the main factors that prevent technological transfers. In our model, parameters associated to adoption and innovation barriers play the role of these restrictions and, together with low knowledge intensity in the adoption activity, explain an important fraction of per capita income disparities. In our model, R&D barriers play additionally a second role: they affect R&D rewards and thus the path of knowledge accumulation. This characteristic will be critical for the case that we discuss next.<sup>11</sup> Basu and Weil (1998) and Acemoglu and Zilibotti (2001) provide a complementary explanation. These authors sustain that difficulties for benefiting from technological transfers arise because technologies developed by leading economies are not appropriate for non-leading countries. In both models, leading economies' technologies are created for an input mix that is not available in the non-leading country. In Basu and Weil (1998), the non-leading economy is short of physical capital; in Acemoglu and Zilibotti (2001), the shortage is skilled labor. These type of models explain different rates of development, but countries never fall in growth traps.

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<sup>10</sup>Slow technological diffusion can be obtained by including explicit costs to the adopting activity (e.g. Barro and Sala-i-Martin, 1995; Aghion et al., 1997; Segerstrom, 1991) or by assuming that some time must elapse for an economy to be able to copy a more advanced technology (Segerstrom et al., 1990; Eeckhout and Jovanovic, 2002).

<sup>11</sup>This second effect produces that adoption barriers do not only explain differences in per capita income levels, as in the models of Parente and Prescott, but also differences in long-run growth.

The second case presented is related to models that study growth traps. This literature is extensive;<sup>12</sup> however, models that include technological transfers are less abundant. For instance, Howitt (2000) presents a model in which the high-growth equilibrium is always reached provided that there is some investment in R&D. Implicitly, the model has a constant adoption capacity, so that long-run low growth occurs when the R&D sector fully disappears. Aghion et al (2005) and Howitt and Mayer-Foulkes (2005) extend this model to emphasize two channels that can lead to growth traps. The first paper focuses on credit constraints that impede that the less developed economy gets enough funding for financing R&D activities. The second paper, more in line with this paper, highlights the problems of skills acquisition, which are needed for R&D. The authors describe the current income distribution as a result of a positive one-time R&D productivity shock that only benefit workers that surpasses an exogenous skill threshold. The model, however, does not address a mechanism that links R&D productivity with skills needs or how the skill threshold is determined. Our paper encompasses this case, but it is more general. In particular, we study how/when increases in that productivity lead to growth traps or just to lower per capita income in steady state. We also study conditions under which different R&D environment (in particular, with low or high knowledge requirements) lead to low- or high-growth in the long run. In a complementary view, Acemoglu, Aghion, and Zilibotti (2006) argue that technological advances depend on the economy's capacity to generate adequate institutional arrangements that maximize growth in every development stage. If economies do not change institutions in line with the development requirements, polarized equilibria may arise.

Finally, models that incorporate an adoption activity usually assume that the technology goal is independent of the development stage of the economy. Particularly, the literature normally assumes that this target is (a fraction of) the technology frontier. By optimally determining the adoption technology target, we explicit the trade-off between choosing a high target that increases the size of the technological change due to a larger technology gap and a low target that increases the adoption capacity. Easing the restriction of copying (a fraction of) the technology frontier can change results significantly.

### 3 The Model

The model builds upon Howitt (2000). Consider one benchmark economy out of  $J$  countries of a world economy. The economy is small and open. Economies trade only the final consumption good and are open to capital flows.

The economy is composed of two types of sectors: An homogenous and competitive final goods sector and an intermediate sector producing different qualities of inputs. The

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<sup>12</sup>These models generally introduce an economic friction or an externality that impedes the accumulation of a productive factor, such as physical or human capital. These factors enter directly the production function or are inputs of the technology production function. See, for instance, Becker, Murphy and Tamura (1990), Galor and Weil (1996), Becker and Barro (1989), Azariadis and Drazen (1990), Durlauf (1993), Benabou (1996), Galor and Zeira (1993), Galor, Moav and Vollrath (2008), Galor and Tsiddon (1997), Murphy, Shleifer and Vishny (1989), Galor (2005), McDermott (2002). Feyrer (2008) contrasts stylized facts with the implications of several of these models.

intermediate sector comprises a continuum of monopolies producing inputs and R&D firms trying to improve the technologies embedded in these inputs. Every firm is aware of the technologies available elsewhere. This awareness, however, does not imply that technologies can be implemented or mastered for free. Everyone using a technology has to have attained it through costly R&D. Technologies are general, and not rival. Technological progress is endogenous at the country and world levels.

There is a continuum of households that live infinitely. Households derive utility from the consumption of the final good only and supply inelastically their endowment of labor. There is no population growth. The framework used ensures that there is no aggregate risk. We further assume that markets are complete and that there is perfect access to foreign capitals. Under this setting, consumption and production decisions are independent. Optimal financial wealth allocation, in contrast, implies some arbitrage conditions that are used to derive some equilibrium relations.<sup>13</sup>

Thus, the complete development path of an economy can be characterized by studying the productive and R&D decisions and by exploiting the mentioned arbitrage conditions.

All decisions are made at the disaggregate level and follow the Schumpeterian growth literature. As this paper focuses on the implications for the aggregate equilibrium, we present only a sketch of the standard sectoral relations in the main body of the paper. The complete model is presented in appendix A. However, we discuss in detail the R&D market as it contains the main features that yield the aggregate results. In particular, we discuss how the technology improvements are determined. We start presenting the firms' problem in the final and in the intermediate production sectors.

### 3.1 Producers

The final goods sector is competitive. The representative firm in this sector produces a perishable good  $Y(t)$  using labor flows  $L$  and inputs  $x_i(t)$  according to equation (1). Input  $i$  embeds a productivity level of  $A_i(t)$ . The higher the productivity embedded in input  $x_i(t)$ , the higher the quantity of  $Y(t)$  that one unit of  $x_i(t)$  generates.

$$Y(t) = L^{1-\alpha} \int_0^1 A_i(t) x_i^\alpha(t) di \quad (1)$$

Every input sector comprises a monopoly that is producing the correspondent input (the incumbent) and an R&D firm that is out of the market, but that is investing in R&D to contest this producer. Inputs differ in the productivity that they provide. Monopolies choose the optimal quantity of inputs by marginalizing their relevant demand and by taking into account the cost of capital, which is the only input of production. As there is perfect capital mobility, the domestic risk-free interest  $r_B$  rate equals the foreign one. The world is in steady state so that  $r_B$  is constant. Consequently, the cost of capital is

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<sup>13</sup>In particular, net return on physical capital, return on foreign bonds, and expected return on stocks are all equal in equilibrium.

given by  $r_B + \delta$ , where  $\delta$  denotes the depreciation rate. The corresponding flow of profits are given by:

$$A_i(t)\pi(t), \text{ where } \pi(t) \equiv \alpha(1 - \alpha) (\alpha^2 / (r_B + \delta))^{\alpha / (1 - \alpha)} L \quad (2)$$

The flow  $\pi(t)$  is equal across sectors as it depends only on aggregate variables. Differences in monopolists' profits are solely explained by differences in the productivity provided by the input.

## 3.2 The R&D Market

Every sector comprises an R&D firm that is trying to displace the incumbent monopoly producing the inputs. Displacement occurs only if the R&D firm accomplish a better technology to embed in input  $x_i(t)$ . If the R&D firm tries to contest the incumbent embedding the same level of technology into the intermediate input, both firms engage in a Bertrand competition that leaves each firm with zero benefits. If the R&D firm is successful, the previous monopoly stops producing and starts engaging in R&D activities.<sup>14</sup>

### 3.2.1 R&D's decision problem

R&D firms engage in R&D activities to improve the current technology embedded in input  $i$  to get the monopoly profits (equation 2). This activity is risky. The R&D firm chooses the amount to invest by considering the expected profits that it will get if it is successful and the expenses in R&D. Expected profits have two components: i) the probability of success  $p_i(t)$  and ii) the potential technology improvement  $\bar{A}_i(t)$ . We assume that investment in R&D only affects the probability of success.<sup>15</sup> The technology improvement depends on the adoption and innovation capacities of the firm.

**R&D Investment.** The probability of success is defined as  $p_i(t) \equiv n_i(t)\beta$ . The first component is  $n_i(t) \equiv I_i(t)/\bar{A}_i(t)$  and corresponds to the resources invested in R&D,  $I_i(t)$ , scaled by the technology goal  $\bar{A}_i(t)$  that the R&D firm can accomplish. Scaling R&D expenditures by the technology goal accounts for increasing difficulties of mastering more advanced technologies. The probability also depends on parameter  $\beta$ , a measure of the productivity of R&D investment. However, risk is idiosyncratic; there is no aggregate

<sup>14</sup>When the R&D firm is successful, it becomes the new monopoly and the previous monopoly starts contesting the new incumbent in the same sector. The restriction of contesting the same sector is not restrictive as, under free choice R&D, firms are indifferent about which sector to contest. The reason is that projects in every sector render the same expected profit per unit of R&D invested.

<sup>15</sup>Assuming that R&D investment only affects the probability of success and not the technology improvement simplifies the discussion of the mechanisms and makes the model more tractable. The cost is that the framework does not allow to analyze how the technology improvement in a particular sector is affected by the resources invested. However, resources invested in every sector affect the average technology improvement of the economy (section 4 discusses implications for aggregate relations).

risk.<sup>16</sup> This implies that in equilibrium R&D firms will maximize the expected net benefit from R&D. Optimal  $I_i(t)$  is presented in equation (3).<sup>17</sup> See appendix A for the derivation of these solutions.

$$I_i(t) = \bar{A}_i(t)\pi(t) - \bar{A}_i(t)r_B/\beta \quad (3)$$

R&D investment depends positively on the expected flow of profits that a successful R&D firm obtains when displacing the current monopoly and positively on the country's productivity parameter  $\beta$ . As expected, it depends negatively on the interest rate  $r_B$ . Finally, the larger the expected technology improvement  $\bar{A}_i(t)$ , the more resources are invested in R&D. R&D investment in relative terms  $n_i(t)$  is presented in equation (4). This measure does not depend on any technological or sectoral variable and the probability of success is equal in all sectors.

$$n_i(t) = n(t) = \pi(t) - r_B/\beta \quad (4)$$

**The technology goal.** We assume that R&D comprises adoption and innovation which are jointly undertaken.<sup>18</sup> Adoption corresponds to the copy of existing technologies while innovation to the creation of new ones. We further assume that adoption and innovation are separable as described in equation (5). This separation implies that adoption and innovation improvements are independent of each other. The R&D firm has a technology goal denoted as  $\bar{A}_i(t)$ , which is defined as:

$$\begin{aligned} \bar{A}_i(t) &= A_i(t) + \lambda (kn(t)/kn^*(t))^\gamma [A_i^T(t) - A_i(t)] + [kn(t)/kn^*(t)]^\varepsilon A_i(t)s\mu \quad (5) \\ \gamma, \varepsilon &\geq 0 \\ \lambda, \mu &\in [0, 1] \end{aligned}$$

The second and third terms of the RHS of equation (5) conform the adoption and innovation components, respectively. Both activities depend on the sector's level of technology and on the country's characteristics. Next, we discuss each component separately.

*Technology adoption.* Adoption depends on the technology gap ( $A_i^T(t) - A_i(t)$ ), where  $A_i^T(t)$  corresponds to any of the technologies that already exist in the world. The R&D firm knows the pool of existing technologies. In particular, the world technology frontier  $A_{\max}(t)$  is defined as the highest technology in all sectors in all countries:  $A_{\max}(t) = [\max(A_{ij}(t)) | i \in [0, 1], j = 1, \dots, J]$ .

Adoption also depends on barriers, policies, institutions, or incentives to copy foreign technologies.<sup>19</sup> Parameter  $\lambda$  comprises all these effects. This parameter reflects the kind of barriers emphasized by Parente and Prescott (1994) and fluctuates in the range  $[0, 1]$ . We

<sup>16</sup>The continuum and independence of R&D firms ensure that all idiosyncratic risk can be diversified and that R&D firms can raise funds in the financial market at the risk-free rate.

<sup>17</sup>Interior solutions and probability bounded in the range  $[0, 1]$  require  $r_B/\pi(t) \leq \beta \leq (1 + r_B)/\pi(t)$ .

<sup>18</sup>Treating them separately produces no relevant differences for the aggregate results.

<sup>19</sup>For example, access to internet and to communication systems, economic and legal regulations, adoption-related policies (e.g. opportunities to attend seminars and congresses) and all variables that affect the overall efficiency of the adoption activity.

will refer to  $\lambda$  as the adoption barriers' parameter. No barriers to adopt new technologies implies a value for  $\lambda$  equal to one and, conversely, maximum barriers imply a  $\lambda = 0$ . The value of this parameter can vary across countries.

The term  $[kn(t)/kn^*(t)]^\gamma$  accounts for the role of knowledge in the adoption activity. Adopting a new technology is not an automatic process in the sense that it requires local knowledge to be performed. The relevant measure of knowledge adjusts the stock of knowledge ( $Kn(t)$ ) in two dimensions.<sup>20</sup> First, the productivity of the stock knowledge depends on the difficulty of the targeted technology. We assume, that the more advanced is a technology, the more complex it is to implement and the more knowledge it requires to be mastered. In other words, knowledge has to be kept updated. We account for this effect by scaling the stock of knowledge by the adoption technology target  $A_i^T(t)$ . This relative measure is the relevant variable for the firm and is defined as  $kn(t) \equiv Kn(t)/A_i^T(t)$ . Second, likewise the policy parameter  $\lambda$ , the lack of relative knowledge acts as a constraint for achieving a higher technology improvement. The relative scarcity of knowledge is measured relative to  $kn^*(t)$ , which corresponds to the highest stock of knowledge in the world adjusted by the complexity given by the the technology frontier; i.e.  $kn^*(t) \equiv Kn^*(t)/A_{\max}(t)$  and  $Kn^*(t) = [\max(Kn_{ij}(t)) | i \in [0, 1], j = 1, \dots, J]$ .<sup>21</sup> As a consequence, the effect of the overall knowledge component is bound at one ( $0 \leq kn(t)/kn^*(t) \leq 1$ ). This assumption, however, is not crucial.<sup>22</sup> Note that when the economy targets the technology frontier so that  $A_i^T(t) = A_{\max}(t)$ , the measure of scarcity is given by the gap of knowledge between the domestic economy and the economy with the highest knowledge stock.

Knowledge requirements may be determinant for adopting new technologies or non-essential. Parameter  $\gamma$  denotes the intensity in which the knowledge component is used in the adoption activity. This parameter is equal for all countries. A value of  $\gamma = 0$  implies that knowledge is not needed as an input for adopting and, consequently, does not affect the adoption possibilities. As the value for this intensity increases, knowledge as an input for adoption becomes more relevant.

Consequently, the complete term  $\lambda [kn(t)/kn^*(t)]^\gamma$  corresponds to the adoption capacity of the R&D firm and is jointly determined by adoption barriers, the relevant stock of knowledge, and the intensity in which this knowledge is used in the adoption function. If there are no barriers to adopt new technologies ( $\lambda = 1$ ) and adoption requires no specific knowledge ( $\gamma = 0$ , such that  $[kn(t)/kn^*(t)]^\gamma = 1$ ), then the R&D firm is capable to fully copy any existing technology. An analogous situation happens if the economy has accumulated enough relative knowledge so that  $kn(t) = kn^*(t)$  and  $kn(t)/kn^*(t) = 1$ . In both cases, the adoption capacity is at its maximum. In these cases, the R&D firm can fully copy the technology frontier and successful adoption provides its highest technology improvement. Yet, if adoption barriers or knowledge are binding restrictions, then R&D

<sup>20</sup>The absolute stock of knowledge corresponds to the economy's stock built from domestic experiences in adopting and innovating.

<sup>21</sup>In the next section, we show that the economies that systematically move the technology frontier have the largest stock of knowledge. These economies will be the leading economies. This assumption implies that leading economies never are knowledge-constrained.

<sup>22</sup>This assumption is included mainly to maintain the knowledge component bounded between 0 and 1. We could have also assumed that the restriction becomes not binding at an exogenous threshold. Results would hold.

firms will only be able to copy a fraction of this frontier.

*Technology innovation.* Similar variables determine the innovation improvement. Innovation builds on the technology in place  $A_i(t)$ . We assume that innovation is proportional to this level, particularly equal to  $A_i(t)s$ , where  $s$  corresponds to a fixed jump. Such specification is standard in models of endogenous innovation (see models and references in Aghion and Howitt, 1998, Grossman and Helpman, 1991, Barro and Sala-i-Martin 2006, Acemoglu, 2009).<sup>23</sup> Analogously to parameter  $\lambda$ , parameter  $\mu$  reflects economic conditions (barriers, incentives, policies) that affect the innovation activity.<sup>24</sup> We will refer to it as the innovation barriers parameter. Maximum innovation barriers imply a value for  $\mu$  equal to zero implying that no innovation is possible. No barriers imply  $\mu = 1$ . The relevant stock of knowledge affects the innovation activity in a similar way as it affects adoption. In this case, both stock of knowledge  $Kn$  and  $Kn^*$  are adjusted by the level of technology over which the firm is trying to innovate; i.e., the relevant gap of knowledge is just determined by the gap of the stocks of knowledge  $Kn/Kn^*$ . The intensity in which this component is used in the innovation activity is captured by the parameter  $\varepsilon$ . The higher this intensity, the more important is the relevant stock of knowledge to produce innovations.

*Technology improvement.* With the previous elements we can characterize the technology improvement in sector  $i$ . As stated in equation (6), technology improves by  $\lambda (kn(t)/kn^*(t))^\gamma (A_i^T(t) - A_i(t)) + A_i(t)s\mu (Kn(t)/Kn^*(t))^\varepsilon$  if the R&D firms succeeds and remains unchanged otherwise.

$$A_i(t) = \begin{cases} \lambda (kn(t)/kn^*(t))^\gamma [A_i^T(t) - A_i(t)] + A_i(t)s\mu (Kn(t)/Kn^*(t))^\varepsilon & \text{prob } p(t) \\ 0 & \text{prob } 1-p(t) \end{cases} \quad (6)$$

The probability of success is determined by the optimal value of  $n(t)$  according to equation (30). Now, we can analyze the technology growth of the aggregate economy.

## 4 Development paths and steady state

This section discusses the different development paths that an economy can follow. These paths are determined by the evolution of the average productivity (technology) and the law of motion of the relative stock of knowledge. We analyze the dynamics and steady state of both state variables and derive implications for long-run growth. Before continuing, we introduce some notation. Technology variables in lowercases are defined relative to the technology frontier, that is  $a_x(t) = A_X(t)/A_{\max}(t)$ . Variables without the sectoral index  $i$  correspond to sectoral averages. In all, notation is only used when strictly necessary to avoid confusion.

<sup>23</sup>Changing the assumption of a proportional fixed jump does not alter significantly results for the aggregate relations as long as the new assumption does not depend (inversely) on the technology level of the sector.

<sup>24</sup>This parameter captures, for example, infrastructure such as laboratories, public research centers, or promotions of innovation regions (e.g. Silicon Valley).

The literature usually assumes that  $A_i^T(t)$ , the technology adoption target, is (a function of) the world technology frontier. In the base case, we follow the standard assumption that all R&D firms always target the technology frontier so that  $A_i^T(t) = A_{\max}(t)$ . In the next section, we relax this assumption and allow R&D firms to target a different technology level.

*The knowledge stock.* Following the seminal work of Romer (1986), we model the accumulation of knowledge  $Kn(t)$  as an externality resulting from R&D investment.<sup>25</sup> Thus, the stock of knowledge is accumulated at the country level. We assume that while performing R&D activities, the economy learns independently of the result of this research. Knowledge accumulation is obtained by aggregating all R&D investment of the economy as in the next equation (7).

$$Kn(t) \equiv \int_0^1 I_i(t) di = I(t) \quad (7)$$

Replacing optimal  $I_i(t)$ , equation (3) in equation (7), aggregating across sectors, and recalling that optimal  $I(t)$  can be expressed as  $n(t)\bar{A}(t)$ , we get the equilibrium path for the stock of knowledge (equation 8). Hereafter,  $n(t)$  R&D resources relative to  $\bar{A}(t)$  refers to its optimized value according to equation (30). The accumulation of knowledge depends on the incentives to perform R&D.

$$Kn(t) = n(t)\bar{A}(t) \quad \text{where } \bar{A}(t) = \int_0^1 \bar{A}_i(t) di \quad (8)$$

*Average productivity level.* The probability of a successful innovation is given by the term  $n(t)\beta$ , which is equal for all R&D firms and thus for the aggregate economy. As there is no aggregate risk, the economy's average absolute technology  $A(t)$  evolves as

$$\dot{A}(t) = n(t)\beta \int_0^1 [\bar{A}_i(t) - A_i(t)] di = n(t)\beta [\bar{A}(t) - A(t)] \quad (9)$$

Replacing the definition of  $\bar{A}(t)$ , equation (5), in equation (8) and dividing it by  $A_{\max}(t)$ , we obtain the law of motion of the stock of knowledge in relative terms presented

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<sup>25</sup>Griliches (1992) and Branstetter (2001) support empirically that R&D has significant spillovers.

in equation (10), where  $g(t)$  corresponds to the growth rate of the technology frontier.<sup>26</sup>

$$\dot{kn}(t) = n(t) [a(t) + \lambda (kn(t)/kn^*(t))^\gamma (1 - a(t)) + a(t)s\mu (kn(t)/kn^*(t))^\varepsilon] - g(t)kn(t) \quad (10)$$

$$g(t) = \frac{A_{\max}(t)}{A_{\max}(t)} \quad (11)$$

Knowledge accumulation in relative terms depends positively on the reward to perform R&D activities and negatively on the growth rate of the technology frontier. The last term in equation (10) indicates the relative obsolescence of the stock of knowledge due to advances in this frontier. This equation also shows that even if there is a large technology gap to close, when relative knowledge is sufficiently low, then R&D will be low and knowledge accumulation will be slow. In such scenarios, a growth trap, defined as being in or transiting to a low long-run growth equilibrium, is possible if knowledge accumulation cannot cope with its obsolescence.

To obtain the average productivity relative to the technology frontier, we replace the definition of  $\bar{A}(t)$  in equation (9) to get

$$\dot{a}(t) = n(t)\beta [\lambda [kn(t)/kn^*(t)]^\gamma [1 - a(t)] + a(t)s\mu [kn(t)/kn^*(t)]^\varepsilon] - g(t)a(t) \quad (12)$$

When there is a large technology gap to close, then the economy obtains large benefits from copying more advanced technologies. As the average technology is low, innovation explains a low fraction of the technological change. On the contrary, when average productivity is high, then the relative weight of adoption falls and innovation becomes more relevant. Thus, in the early stages of development, adoption will be the more significant source of growth. Consequently, the adoption capacity is crucial in these stages. This capacity depends on the degree of openness of the economy, captured by parameter  $\lambda$ , and on the relative stock of knowledge. As we assume that  $\lambda$  is fixed, the evolution of the knowledge stock becomes fundamental.

Next, we characterize the transition and steady state for a leading economy. Afterwards, we analyze different scenarios for non-leading countries.

## 4.1 Leading economies and the technology frontier

A leading economy is defined as one that systematically moves the technology frontier. Suppose that all economies were initially endowed with an equal stock of absolute knowledge and that technologies were equal worldwide, but that R&D parameters (adoption and

<sup>26</sup>As in the base case, firms are copying the technology frontier, we divide each innovation knowledge component  $Kn(t)$  and  $Kn^*(t)$  by  $A_{\max}(t)$  [i.e.  $(Kn/A_{\max})/(Kn^*/A_{\max}) \equiv kn/kn^*$ ] to describe the knowledge and productivity dynamics only in terms of  $a(t)$  and  $k(t)$ .

innovation barriers) were different across countries.<sup>27</sup> Under these conditions, economies with best R&D parameters will become the leading economies (a star denotes a variable corresponding to a leading economy). We assume that this stand-in leading economy is in steady state and that its R&D parameters are  $\lambda^* = u^* = 1$ .<sup>28</sup> Another way to read it, is that the leading economy is the country with best practices and we measure the policy parameters of all other countries in relation to these best practices. This stand-in leading economy shows the highest productivity average (not necessarily in every sector) and the largest stock of absolute knowledge.

The technology frontier expands by any innovation that produces a globally new technology. This expansion can occur in leading as well as in non-leading economies and depends on the innovative capacities of the countries. Under the assumption that  $\lambda^* = u^* = 1$ , the highest expansion is constant and occurs with probability one in the leading economy and is equal to  $s$ . If every country were only capable to adopt technologies, even in the most efficient way, but were not able to create a single new technology ( $\mu^* = 0$ ), then the frontier would stagnate. Consequently, there would be no growth in the long run and all countries would converge to the same technology level and the same per capita GDP in the long run.

Steady state values for relative knowledge and relative average productivity are obtained by making  $\dot{kn}(t)$  and  $\dot{a}(t)$  equal to zero (equations 10 and 12) and by replacing the values  $\lambda^* = u^* = 1$  and  $g = s$  in these equations. These steady states values are given by equations (13) and (14), where  $n_{ss}^* = \alpha(1 - \alpha) [\alpha^2 / (s\sigma + \rho + \delta)]^{\alpha/(1-\alpha)} - (s\sigma + \rho) / \beta^*$ .<sup>29</sup> This equilibrium is unique and stable (see appendix B).<sup>30</sup> The stand-in leading economy grows at the growth rate of the innovation rate  $s$ . This rate also corresponds to the growth rate of the technology frontier. Long-run growth is driven by the world capacity of expanding this frontier. Equations (13) and (14) describe, respectively, the average relative technology and the relative stock of knowledge in steady state:

$$a_{ss}^* = \frac{n^* \beta^*}{s + n^* \beta^* (1 - s)} \leq 1 \quad (13)$$

$$kn_{ss}^* = \frac{n^*}{s} \left[ \frac{s + n^* \beta^*}{s + n^* \beta^* (1 - s)} \right] \geq 0 \quad (14)$$

The stand-in leading economy does not specialize in adoption or innovation ( $a_{ss}^* < 1$ ) in steady state (i.e. this economy always performs a mix of both activities) and the

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<sup>27</sup>If all countries had equal policy parameters, then all economies would follow the same development path.

<sup>28</sup>This assumption allows to follow the leading economy analytically. Changing this assumption allows to analyze how a leading economy can lose its position and how a non-leading economy can overcome a leading one. However, this richer environment does not alter the conclusions for convergence and polarization analyzed in this paper.

<sup>29</sup>The optimal R&D level relative to the technology goal trades-off average profits obtained by the monopolies  $[\alpha(1 - \alpha) [\alpha^2 / (s\sigma + \rho + \delta)]^{\alpha/(1-\alpha)}]$  and the opportunity cost of the resources  $(s\sigma + \rho) / \beta$ .

<sup>30</sup>Supposing that the leading economy is in steady state simplifies the analytical solution and permits to analyze each country independently.

economy never has all its sectors at the frontier.<sup>31</sup> Even the most advanced economy relies on adoption for maintaining a high productivity average in the long run.

A higher innovation jump ( $s$ ) decreases the steady state values of the average relative productivity and the relative stock of knowledge. The intuition is that a fraction  $n\beta$  of the R&D firms are successful in acquiring the leading technology; however, the unsuccessful sectors are now relatively further laggard. As a result, average productivity relative to the technology frontier falls. Respecting the steady state relative stock of knowledge, a higher frontier growth rate increases the R&D reward and the knowledge obsolescence rate. The latter effect offsets the raise in the R&D reward producing that relative knowledge decreases in steady state.

An increase in the average productivity of R&D ( $\beta^*$ ), increases R&D investment ( $n^*$ ) and the probability of success in R&D activities. The higher probability of success translates into a larger technology improvement in the aggregate, producing an increase in  $a_{ss}^*$ . Two opposing effects influence R&D investment and thus knowledge accumulation. A higher productivity parameter  $\beta^*$  produces an increase in the yield of R&D stimulating its investment, but it also produces a higher relative productivity (and consequently a reduction in the technology gap and R&D reward) discouraging thereby this investment. The first effect dominates, so that an increase in  $\beta^*$  produces a raise in  $kn_{ss}^*$ .

## 4.2 Non-leading economies

A non-leading economy is not moving systematically the technology frontier. Particularly, we focus on economies with low relative average productivity and low stock of relative knowledge that are transiting to their steady state. We study their growth potential by analyzing the two-dimensional system in  $a(t)$  and  $kn(t)$ . Laws of motion for the average relative productivity and relative knowledge are given by equations (10) and (12).<sup>32</sup> From these two equations we derive the loci for  $\dot{a}(t) = 0$  and  $\dot{kn}(t) = 0$  presented in equations (15) and (16), which provide the necessary relations to describe the dynamics and steady state of this type of economy.

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<sup>31</sup>North-south type models traditionally produce specialization, with the north specializing in innovation and the south in adoption (Segerstrom et al., 1990; Grossman and Helpman, 1991; Helpman, 1993; Barro and Sala-i-Martin, 1995; Acemoglu and Zilibotti, 2001; Basu and Weil, 2001).

<sup>32</sup>These equations show that a higher relative knowledge produces a higher relative productivity as it increases the adoption and innovation capacities in the economy. Moreover, higher relative productivity increases the knowledge stock as it raises the R&D reward and thus R&D investment (the source of knowledge accumulation). Consequently, relative productivity and relative knowledge stock are complementary processes.

$$a(t)|_{a(t)=0} = \frac{n(t)\beta\lambda (kn(t)/kn_{ss}^*)^\gamma}{g + n(t)\beta [\lambda (kn(t)/kn_{ss}^*)^\gamma - s\mu (kn(t)/kn_{ss}^*)^\varepsilon]} \quad (15)$$

$$a(t)|_{kn^*(t)=0} = \frac{gkn(t) - n(t)\lambda (kn(t)/kn_{ss}^*)^\gamma}{n(t) [1 - \lambda (kn(t)/kn_{ss}^*)^\gamma + s\mu (kn(t)/kn_{ss}^*)^\varepsilon]} \quad (16)$$

A non-leading economy can achieve two types of long-run growth equilibria: the high one ( $a_{ss} > 0$ ), in which the economy grows at the rate of the technology frontier (which equals the growth rate of the leading economies, but not necessarily the same level) and the low one ( $a_{ss} \rightarrow 0$ ), in which the economy grows at a rate given by their R&D conditions.<sup>33</sup> As the origin  $(a, kn) = (0, 0)$  constitutes one possible steady state, the issue is to determine under which conditions this equilibrium is stable and whether there are other equilibria that leave the economy with higher long-run growth. Achieving one or the other equilibrium depends on the economy's ability to maintain the economy's R&D capacity.

Equations (15) and (16) show that adoption activities are necessary to maintain a high growth rate and to catch-up in the long run (adoption capacities are expressed in the term  $\lambda [kn(t)/kn_{ss}^*]^\gamma$ ). If the economy imposes full barriers to adoption ( $\lambda = 0$ ), then it will transit to the low-growth equilibrium.<sup>34</sup> If the economy performs no innovation ( $\mu = 0$ ), it may still achieve a high-growth rate ( $a > 0$  in equation 15). The condition of performing some adoption has to be fulfilled independently of the innovation capacities of the country. The reason is that innovation, when successful, allows at most maintaining productivity in line with the technology frontier. However, as successes do not occur in every sector, productivity relative to the technology frontier inevitably falls. Innovation is important, but technology adoption is the main vehicle for a non-leading economy to reach higher productivity. Minimum adoption to achieve high growth varies according to the knowledge intensity and the other parameters conditioning the R&D environment. In the next section, we analyze two distinct types of economic dynamics that an economy may be inserted in. We characterize them by the intensity in which knowledge is used in adoption.

### Case 1: Low knowledge intensity

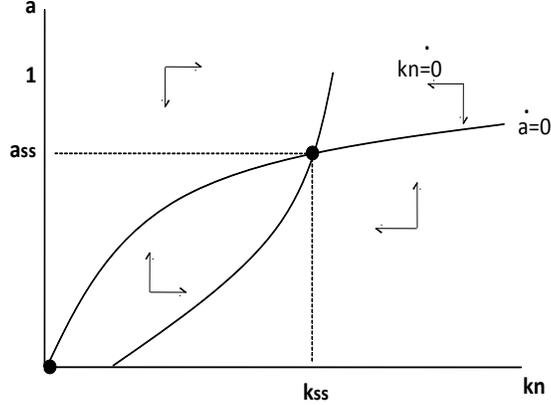
An economy follows low knowledge intensity dynamics when the economy achieves a high-growth equilibrium, regardless of the values of the rest of the parameters (provided that  $\lambda > 0$ ). In our model, this happens when  $\gamma < 1$ . The long-run growth rate does not

<sup>33</sup>Recall that  $a(t)$  and  $kn(t)$  define the average productivity and the stock of knowledge relative to the technology frontier. If these variables are positive in steady state, then average productivity and the stock of knowledge grow at the rate of technology frontier. In contrast, if  $a(t)$  tends to zero in the long run, then the average productivity is growing at a lower rate than the technology frontier, thus drifting away.

<sup>34</sup>If  $\lambda = 0$ , equation  $\dot{a} = 0$  has two solutions:  $a = 0$  and  $n(t)\beta su (kn(t)/kn_{ss}^*)^\varepsilon = g$ . However, as  $g > n(t)\beta su (kn(t)/kn_{ss}^*)^\varepsilon$ , the second solution can be discarded. See appendix B3 for the description of this case.

depend on domestic conditions.<sup>35</sup> Barriers to copy technologies or the R&D environment, in contrast, affect the medium-run growth and the long-run level of per capita income. As a result, these countries converge in growth rates, though not necessarily in levels. Figure 1 presents such case.<sup>36</sup>

Figure 1: A non-leading economy: The case of  $\gamma < 1$



Higher relative productivity (relative to  $kn(t) = 0$ ) implies larger R&D investment and higher relative knowledge growth in relation to the obsolescence rate. On the other hand, higher relative productivity (relative to  $a(t) = 0$ ) reduces the potential technological improvement producing a fall in its relative level. Steady state values for the relative average productivity and the relative stock of knowledge increase with lower adoption and innovation barriers (higher  $\lambda$  and  $\mu$ ) as they ease copying and innovating, thus raising the amount of technology that every unit of R&D generates.<sup>37</sup> The increase in average productivity, in turn, raises the resources invested in R&D, producing more knowledge accumulation in equilibrium. In contrast, both state variables decrease with the growth rate of the technology frontier ( $g$ ), as it enlarges the average technology gap, since sectors in which R&D is not successful remain further lagged. As a higher frontier growth increases the knowledge obsolescence rate, it reduces the accumulation and the steady state

<sup>35</sup>Excepting the case in which the non-leading economy becomes a leading one.

<sup>36</sup>For  $\gamma < 1$  and  $\varepsilon$  not too large, both functions  $\dot{a}(t) = 0$  and  $\dot{kn}(t) = 0$  are positively sloped in their relevant domain.

$$\frac{\partial a(t)}{\partial kn(t)} \Big|_{a(t)=0} = \frac{n(t)\beta\lambda \left[ \frac{g\gamma kn(t)^{\gamma-1}}{(kn(t)/kn_{ss}^*)^{2\gamma}} + n(t)\beta s\mu (kn(t)/kn_{ss}^*)^{\varepsilon-\gamma} (\varepsilon-\gamma) kn(t)^{\varepsilon-\gamma-1} \right]}{\left[ \frac{g}{(kn(t)/kn_{ss}^*)^\gamma} + n(t)\beta (\lambda - s\mu (kn(t)/kn_{ss}^*)^{\varepsilon-\gamma}) \right]^2} \geq 0$$

$$\frac{\partial a(t)}{\partial kn(t)} \Big|_{kn(t)=0} = \frac{a(t) [1 + (1-\varepsilon) s\mu (kn(t)/kn_{ss}^*)^\varepsilon] + \lambda (kn(t)/kn_{ss}^*)^\gamma (1-\gamma) (1-a_t)}{kn(t) [1 - \lambda (kn(t)/kn_{ss}^*)^\gamma + s\mu (kn(t)/kn_{ss}^*)^\varepsilon]} \geq 0$$

<sup>37</sup>Graphically, a higher  $\lambda$  produces a rightward movement of the function  $\dot{kn}_t = 0$  and a higher slope of the function  $\dot{a}_t = 0$  in figure (1). A higher  $\mu$  produces an increase and a decrease of the slopes for  $\dot{a}_t = 0$  and  $\dot{kn}_t = 0$ , respectively.

stock of relative knowledge and reinforces the negative effect on relative productivity.

Starting from every pair of points  $(a, kn)$ , the economy achieves the high growth steady state.<sup>38</sup> This case is consistent with the view that all countries can start developing at any time and will share the same growth rate in the long run (Lucas, 2000). Low growth rates or widening per capita income gaps are only transitory. In this case, all economies can benefit from adopting technologies independently from their own developing stage. Achieving high growth in the long run occurs even in the presence of high adoption and innovation barriers. Development can start at any time without harming growth possibilities in the long run. Achieving high per capita income, in contrast, requires an efficient R&D environment and minimum (if any) barriers to perform adoption and innovation.

## Case 2: Medium and high knowledge intensity

This second case characterizes situations in which knowledge requirements are important to perform R&D. Therefore, the dynamics of knowledge accumulation relative to the technology frontier becomes relevant. That means that adoption and innovation capacities are not only determined by domestic conditions, but also by external forces; i.e., factors that affect the expansion of the technology frontier. In this case, economies can achieve both high and low long-run equilibria and domestic conditions can affect long-run growth. In the model, this happens when  $\gamma \geq 1$ . We define knowledge intensity as medium when the value of  $\gamma$  restricts the set of parameter values that leads to high growth and this set of parameters does not depend on initial conditions. This occurs when  $\gamma = 1$ . Knowledge intensity is high when initial conditions also matter for the type of equilibrium that an economy can achieve. In the model, this case arises when  $\gamma > 1$ . Next, we present both cases:

**Medium knowledge intensity.** Contrary to the low knowledge intensity case ( $\gamma < 1$ ), when knowledge is relatively important for copying technologies, there is a minimum R&D environment required for achieving high growth in the long run. When  $\gamma = 1$  (and  $\varepsilon = 1$ ), we can obtain the analytical solution for this threshold as presented in equation (17).<sup>39,40</sup> This threshold is expressed in terms of the maximum adoption barriers (minimum  $\lambda \equiv \underline{\lambda}$ ) that are compatible with high growth.<sup>41</sup>

<sup>38</sup>Appendix B proves the instability of the equilibrium point  $(a, kn) = (0, 0)$  when  $\gamma < 1$ .

<sup>39</sup>When  $\varepsilon = 1$ , the corresponding steady-state levels are:

$$a_{ss} = \frac{\lambda n(g+n\beta) - kn_{ss}^* g}{(g+n\beta)n(\lambda - s\mu)}; \quad kn_{ss} = \frac{\lambda n(g+n\beta) - kn_{ss}^* g^2}{\beta g n(\lambda - s\mu)}; \quad n_{ss} = \frac{\alpha(1-\alpha)\alpha^2}{(r_{Bt} + \delta)^{\alpha/(1-\alpha)}\beta} - \alpha(1-\alpha)r_B.$$

<sup>40</sup>The condition stated in equation (17) does not depend on the innovation capacities of the economy. Innovation plays only a role when the economy has a positive productivity level. However, when  $\gamma = 1$ , the positive effect of larger innovation flows on knowledge accumulation does not compensate the negative effects of a smaller technology gap and a larger obsolescence flow. Graphically, in terms of panel b of figure 2, lower innovation barriers  $\mu$  put both curves nearer, but never produces that both curves cross each other.

<sup>41</sup>Equation (17) is derived in appendix B and conditions that the origin is not a stable equilibrium.

$$\lambda > \underline{\lambda} = \frac{gkn_{ss}^*}{n} \left[ \frac{g}{n\beta + g} \right] = \left[ \frac{n^* n^* \beta^* + s}{n} \right] \left[ \frac{s}{s + n^* \beta^* (1 - s)} \right] \quad (17)$$

Maximum adoption barriers increases ( $\underline{\lambda}$  decreases) with lower growth of the technology frontier  $g$ <sup>42</sup> and with higher R&D productivity  $\beta$  as both situations relax the restriction on knowledge accumulation compatible with high growth. Consequently, maximum adoption barriers compatible with high growth increase. A lower  $g$  makes it easier for countries to achieve the high growth equilibrium as it reduces the knowledge obsolescence rate decreasing thereby the amount of R&D investment required to maintain the stock of knowledge updated. A higher  $\beta$  increases the probability of success and thus the average productivity improvement. This results in a higher path of knowledge accumulation. On the contrary, a better R&D environment in the stand-in leading economy translates into a higher  $kn_{ss}^*$ , making it more difficult for other countries to achieve the high growth equilibrium. The reason is that in a better R&D environment, the stand-in leading economy produces larger technology improvements, invests more in R&D and accumulates more knowledge with the result of a higher average productivity and knowledge stock in steady state. If an economy wants to keep pace with the stand-in leading country, it has to produce larger technology improvements and accumulate a larger stock of knowledge than previously. These two goals reduce the maximum level of adoption barriers that is acceptable. Finally, if the critical value is higher than one ( $\underline{\lambda} > 1$ ), the economy achieves the low-growth equilibrium only (panel b in figure 2). This happens when the R&D productivity  $\beta$  is too low relative to  $\beta^*$ . In this case, the economy is not productive enough to successfully adopt better technologies (even when adoption barriers are extremely low,  $\lambda = 1$ ).<sup>43</sup>

The dynamics of both type of equilibria is presented in figure 2. Economies that are above the threshold achieve high growth in the long run (panel a, figure 2), while an economy with bad R&D environment transits to a low growth equilibrium (panel b, figure 2).

Finally, even if knowledge and technologies could be transferred to an economy that is in a low-growth path, this would not change the long-run growth rate. The economy would show a transitory raise in its growth rate, but would inevitable stay behind in the long run (see panel b in figure 2, where knowledge and technological transfers are presented as passing from point A to point B). The only way to raise the economy's long-run growth rate is to improve adoption and innovation capacities permanently.

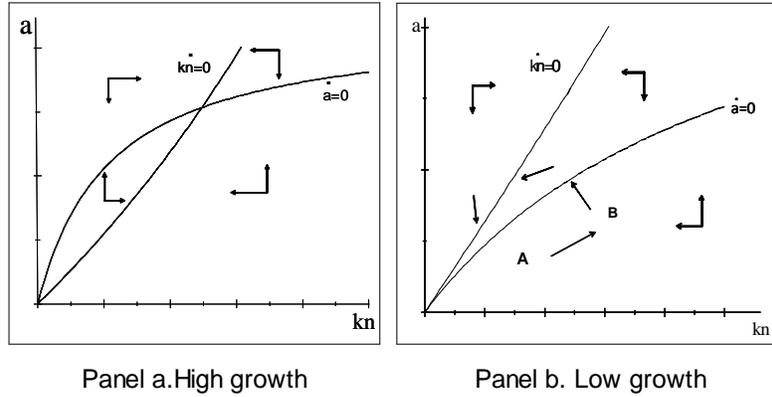
**High knowledge intensity.** In this case, the particular steady state that the economy achieves does not only depend on the economy's R&D environment, but also on initial conditions. That means that a given R&D environment, that led to high-growth in the previous case, does not necessarily ensure achieving that path if adoption knowledge-intensity is high ( $\gamma > 1$ ). Figure (3) presents an example of this case.

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<sup>42</sup>  $\frac{\partial \lambda}{\partial g} = \left[ \frac{n^*}{n(n\beta + s)} \right] \left[ \frac{s}{s + n^* \beta^* (1 - s)} \right] + \lambda \left[ \frac{n\beta}{(n\beta + s)s} + \frac{n^* \beta^*}{s + n^* \beta^* (1 - s)} \right] > 0$ .

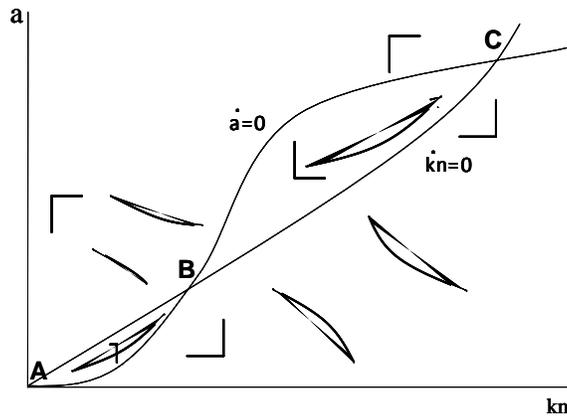
<sup>43</sup> Recall that the probability of success is given by  $n(t)\beta = (\pi(t)\beta - r_B)$  in equilibrium.

Figure 2: Long-run equilibria



As knowledge is intensively used to produce technology improvements, a relatively small stock of knowledge (combined with a low relative productivity) may produce small technology improvements which, in turn, cause low investment in R&D and slow knowledge accumulation. This might lead to a vicious circle, where the economy ends up growing at a low rate (point A in figure 3). In contrast, if the economy trespasses a threshold of knowledge and productivity, it may remain in a high growth path in steady state (achieving point C in figure 3). Point B corresponds to an unstable equilibrium. In this case, it is not possible to obtain closed analytical solutions. We present in appendix C numerical exercises that show how changes in the values of the parameters affect the technology level achieved in steady state. This appendix also studies the implications of changing initial conditions.

Figure 3: Various steady states



When adoption knowledge intensity is high, lower R&D barriers, that enhance adop-

tion and innovation, improve the chances to achieve a high-growth path. However, there are cases in which even sharing the same policies and R&D environment with the stand-in leading economy may not guarantee reaching the high growth equilibrium.<sup>44</sup> The reason is that a low stock of relative knowledge inhibits the economy to adopt better technologies even in the absence of adoption barriers. Escaping the low growth equilibrium would require an unlikely increase in both state variables.

This scenario is coherent with the view that countries may remain in a growth trap in the long run (Howitt et al., 2005). Contrary to the previous cases, the longer an economy waits for starting its developing process, the more difficult it is to achieve the high growth equilibrium (in accordance to Howitt and Mayer-Foulkes, 2005). The reason is that for a given stock of initial knowledge, the more time passes by, the more obsolescent becomes the absolute stock of knowledge. In this case, changes in the external environment may change the equilibrium path that an economy is following. Development requires continuous efforts to maintain low R&D barriers and efficient economic institutions: Better R&D conditions in the leading economies require a matching upgrade in domestic conditions to sustain growth.

## 5 Optimal technology adoption target

In the previous sections, we assumed that the adoption technology target was the technology frontier.<sup>45</sup> We showed that an economy may transit to a low-growth equilibrium if it failed to maintain its stock of knowledge updated with this frontier. Targeting the technology frontier, nevertheless, may put a heavy burden on the adoption capacity of the country as it requires a high stock of knowledge. This can considerably hinder benefitting from and growing by adoption as the technology improvement can be severely reduced. However, even though absolute knowledge may be scant for acquiring the technology frontier, it may be sufficient for attaining less advanced ones. In fact, the lower the technology that the R&D firm is targeting, the more productive is the absolute stock of knowledge. This section analyzes the steady state and the dynamics properties of the economy when firms choose the adoption target that maximizes the technology improvement.<sup>46</sup>

The problem faced by the R&D firm is presented in equations (18) to (20). The only variable that the R&D firm controls is the adoption technology target ( $A_i^T(t)$ ). As by assumption R&D firms in leading economies are not knowledge-constrained, they always

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<sup>44</sup>In these cases, policies that raise effective R&D investment ( $n(t)$ ) may help to transit to a better equilibrium. Note, however this strategy would imply to improve the R&D productivity above the leading economies' level.

<sup>45</sup>A low adoption capacity, however, produces a final adoption flow that could be lower than this frontier.

<sup>46</sup>In the early stages of development, the country could target less knowledge-intensive technologies if available. In fact, this strategy could help countries that are transiting to a low-growth equilibrium to increase its adoption capacity and elude this equilibrium. In this paper, we do not focus on this mechanism. We assume that the adoption knowledge intensity does not change across countries nor across time. (Mies, 2010, explores quantitatively this possibility).

choose the technology frontier as the adoption technology target.<sup>47</sup> Therefore, we focus on non-leading economies.

$$\max_{A_i^T(t)} \lambda \left( \frac{Kn(t)/A_i^T(t)}{Kn^*(t)/A_{\max}(t)} \right)^\gamma (A_i^T(t) - A_i(t)) \quad \text{st} \quad (18)$$

$$i : \quad A_i(t) < A_i^T(t) \leq A_{\max}(t) \quad (19)$$

$$ii : \quad \frac{Kn(t)/A_i^T(t)}{Kn^*(t)/A_{\max}(t)} \leq 1 \rightarrow A_i^T(t) \geq \frac{Kn(t)}{Kn^*(t)} A_{\max}(t) \quad (20)$$

The intuition for this problem is the following: The adoption technology target has two opposite effects on  $\bar{A}_i(t)$ . On one hand, a high target raises the potential technology achievable due to a higher technology gap, which induces the R&D firm to choose a target that makes the technology gap the largest possible. On the other hand, a higher target requires a larger stock of knowledge to master the new technology. For that reason, the R&D firm would prefer to choose a low target. Additionally, the firm faces two constraints: The first constraint (equation 19) states that the adoption target has to be higher than the current level of technology to provide a positive return (i.e.  $A_i^T(t) > A_i(t)$ ), but that this target can be at most the technology frontier ( $A_i^T(t) \leq A_{\max}(t)$ ). The second constraint (equation 20) reflects that knowledge constraint only binds when the effective stock of knowledge is lower than the corresponding stock of the leading economy. In other words, achieving a higher relative stock of knowledge (relative to  $Kn^*(t)/A_{\max}(t)$ ) does not provide any further benefit for the economy. Consequently, the R&D firm never chooses adoption targets that produces that  $Kn(t)/A_i^T(t)$  is over  $Kn^*(t)/A_{\max}(t)$ .

The first order condition for this problem yields the following expression:

$$\lambda \left( \frac{Kn(t)/A_i^T(t)}{Kn^*(t)/A_{\max}(t)} \right)^\gamma \left[ \frac{A_i^T(t)(1-\gamma) + \gamma A_i(t)}{A_i^T(t)} \right] \geq 0 \quad (21)$$

Whenever equation (21) holds with inequality, the optimal target is the technology frontier  $A_{\max}(t)$ . However, if equation (21) holds with equality, then the R&D firm will choose to target a lower technology. The general solution is given by

$$A_i^T(t) = \min [A_{\max}(t), \max (\gamma/(\gamma - 1)A_i(t), Kn(t)/Kn^*(t)A_{\max}(t))] \quad (22)$$

Equation (22) states that the R&D firm either chooses the technology frontier (when adoption is not knowledge intensive or when the economy approaches the leading country)

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<sup>47</sup>The leading economy maximizes  $\max_{A_i^T(t)} A_i(t) + (A^T(t) - A_i(t)) + A_i(t)s$ . *FOC* :  $1 > 0$ .

The first order condition is always positive so that it is always optimal to choose the highest target possible.

or a lower target if adoption is a difficult task given the knowledge intensity of the activity, the sector's technology level, and the knowledge stock. For low knowledge intensities, precisely, for  $\gamma \leq [A_{\max}(t)/(A_{\max}(t) - A_i(t))]$ , the positive effect of a higher target on the technology gap dominates the adverse effect on the stock of knowledge and R&D firms target the technology frontier. Note that for all  $\gamma \leq 1$ , the optimal target is  $A_{\max}(t)$ . As the knowledge intensity increases, the effect on relative knowledge becomes more relevant. In this case, the target corresponds to the highest value among these two components:  $A_i^T(t) = \gamma/(\gamma - 1)A_i(t)$  and  $A_i^T(t) = Kn(t)/Kn^*(t)A_{\max}(t)$ . The technology target is increasing in the sector's technology and decreasing in the knowledge intensity  $\gamma$  (first component). Consequently, we can find sectors targeting different levels of technologies in an economy: Laggard sectors may be targeting less advanced technologies while leading sectors may be targeting the technology frontier.

The optimal target, however, also depends on the ratio  $Kn(t)/Kn^*(t)$ , the stock of knowledge between non-leading and leading economies (second component). If this ratio is large, then R&D firms will target a technology that is close to the frontier, even if the sector's technology is low. The reason is that a relatively large knowledge stock makes it unnecessary to sacrifice a large technology gap. In particular, if a country has attained the stock of knowledge of the leading economy, then all its R&D firms will target the technology frontier independently of the knowledge intensity  $\gamma$  and the technology level of the sector.

*Steady state.* There is no closed analytic solution for the aggregate variables; so we describe the equilibria through simulations. Since in this setup, the aggregate production function can be expressed as a neoclassical production function with constant returns to scale in physical capital and labor, a few parameters can adequately describe the equilibrium.<sup>48</sup> Table 1 presents the parameters used in the simulations.

We assign a value of 0.35 to the capital share of output  $\alpha$ <sup>49</sup>, 0.02 to the discount rate  $\rho$ <sup>50</sup>, and 0.03 to the depreciation rate  $\delta$ .<sup>51</sup> These values are widely used in neoclassical growth models. We choose a value of one for the inverse of the intertemporal rate of substitution.<sup>52</sup> We assign to the innovation rate  $s\mu$ , which is also the long run growth rate, the value of 2.2%, the average per capita US growth rate for the years 1960-2006.<sup>53</sup> The calibration of the R&D productivity  $\beta$ , and thus of the probability of success, is more problematic and such empirical measure is scarce. As this probability also reflects the rate of creative destruction, we obtained this value from Caballero and Jaffe (1993),

<sup>48</sup>The aggregate production function is described by the following Cobb-Douglas representation:  $Y(t) = [A(t)L]^{(1-\alpha)} K(t)^\alpha$ .

<sup>49</sup>See Gollin (2002).

<sup>50</sup>Gourinchas and Parker, 2002; Cagetti, 2003; and Laibson et al. (2007) estimate discount rates for the US in the range of 0.01 to 0.09. We choose 0.02 in order to obtain a plausible risk-free interest rate.

<sup>51</sup>Based on Musgrave (1992), who estimates depreciation rates with data for the US Manufacturing sector.

<sup>52</sup>This parameter typically is in the range of 1 to 3.5 (see Hall, 1998; Attanasio and Weber, 1993). As for the discount rate  $\rho$ , we choose  $\sigma = 1$ , in order to obtain a plausible risk-free interest rate. Changing this value, however, does not alter results.

<sup>53</sup>Calculation based on Maddison (2009). Averages for the years 1820-2006, 1900-2006 and 1960-2006 correspond to 1.84%, 2.08% and 2.21%, respectively. We choose the latter value as the US has shown a relatively constant GDP growth rate in this period.

who estimate an average rate of 3.6%.<sup>54</sup> With these parameters, we obtain the values of all endogenous variables. Implied steady states values are 4.5% for the risk-free interest rate, and 2.2 for the long-run growth rate of the high-growth equilibrium.

**Table 1: Simulations' parameters**

$\rho$	= 2.3%	$\sigma$	= 1.0
$\alpha$	= 0.35	$s\mu$	= 2.2%
$\delta$	= 0.03	$prob(success)$	= 3.6%

Figure 4 presents the results. The vertical axis measures the average steady-state technology level (productivity) of a country relative to the frontier. The two horizontal axes measure adoption barriers and the intensity of knowledge in adoption. The barriers parameter' varies between 0 and 1 (zero, corresponding to no barriers). We study knowledge intensities in the range of 0 and 2, which accounts for all relevant cases. This figure encompasses both steady-state technology levels and long-run growth rates. As the average steady-state technology is measured relative to the technology frontier, all values strictly positive ( $a_{ss} > 0$ ) imply that the technology level is growing at the growth rate of the technology frontier (high-growth equilibrium). In all these cases, differences in steady state values imply level differences. Relative steady-state technologies equal to zero ( $a_{ss} = 0$ ) imply that the economy is growing at a lower rate than the technology frontier in the long run. The average technology can still be growing in these economies (and technology levels are always positive), however measured relative to the technology frontier, technology levels are drifting away.

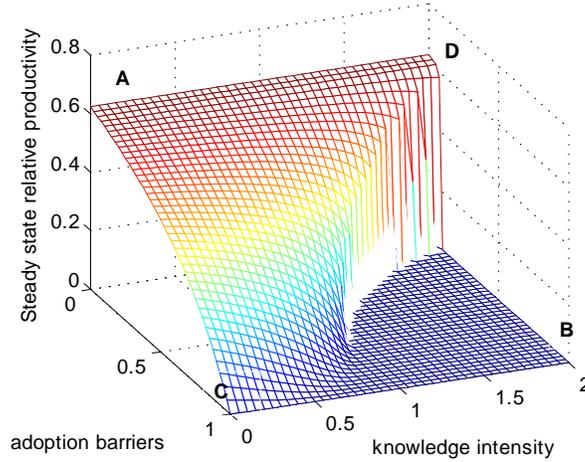
As discussed in previous sections, the steady-state relative technology level increases with lower adoption barriers and lower knowledge intensities. For a given R&D investment, point *A* is the highest relative steady-state technology level that can be observable. It reflects an R&D environment without barriers and an adoption production function that does not require knowledge (recall that the knowledge intensity parameter characterizes the adoption function and is given and equal for all countries). Point *B*, in contrast, reflects the opposite situation. At this point adoption barriers and adoption knowledge intensity are at their highest level. In such scenario, the economy shows the lowest growth and the lowest average technology.<sup>55</sup> In fact, point *B* characterizes an equilibrium of low growth. Point *C* denotes economies that have highest adoption barriers, but face an adoption activity that requires no knowledge. From this point, lowering adoption barriers produces increases in the steady state technology level (moving along the  $\overline{CA}$  axis). For low knowledge intensities, a high long-run equilibrium is achieved provided that some adoption is made. The higher the knowledge intensity (moving from point *C* to points *B* or *D*), the lower the adoption barriers that are coherent with high growth.

Targeting lower technologies allows the economy to achieve higher income levels and higher growth rates. The reason is that this strategy demands a lower knowledge stock

<sup>54</sup>This rate corresponds to the average estimated creative-destruction rate for the period 1965-1981 for 21 sectors in the US. The annual rate of creative destruction varies in the range of 2 to 7 percent over different periods.

<sup>55</sup>The figure shows only relative relations. Consequently, it is not possible to discriminate between growth rates and technology levels for countries that transit to a zero average relative technology.

Figure 4: Steady state average technology: Different knowledge intensities and adoption barriers for optimal adoption targets



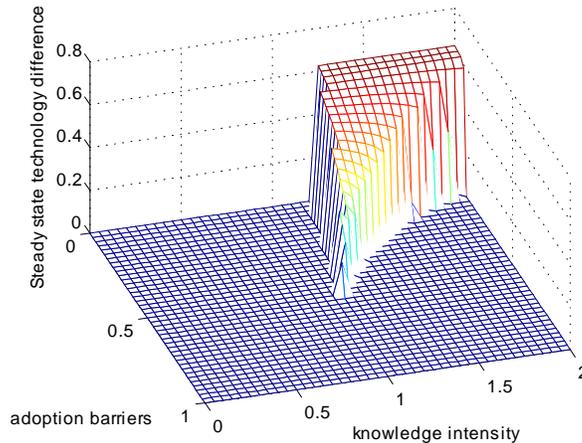
increasing thereby the expected technology improvement and the incentives to invest in R&D. Figure 5 shows the steady-state gains in relative average technology, when R&D firms choose the adoption target. All positive differences correspond to situations in which the economy achieves high growth by targeting a less-advanced technology, but would have reached low growth, had it targeted the technology frontier. These gains are decreasing in the adoption barriers. There are no gains for low adoption knowledge intensities ( $\gamma \leq 1$ ) since in these cases it is optimal to target the technology frontier. Relaxing the restriction of copying frontier technologies generates that resources are used in a more efficient way; this would indicate that too ambitious R&D policies may be misleading.

Turning now to the dynamics of the target choice, figure (6) shows the evolution of the ratio of the technology target (described in condition 22) relative to the technology frontier. When the target is the technology frontier, the ratio takes the value of one. Curves closer to the vertical axis correspond to less knowledge-intensive adoption activities. The horizontal axis measures time intervals.<sup>56</sup>

When the level of relative technology is low, R&D firms choose a low technology target. This target raises as the relative technology of the sector increases. Knowledge accumulation and technology improvement lead the R&D sector eventually to target the technology frontier. The more knowledge intensive is adoption, the longer R&D firms are targeting less advanced technologies and the lower is the adoption target for a given technology in use in sector  $i$ . If adoption barriers are relatively low, eventually R&D firms will target the technology frontier independently of the adoption knowledge intensity and

<sup>56</sup>The simulations were made for initial conditions that proxy the gap between the 20 poorest countries and the US as described in appendix C.

Figure 5: Steady state average technology gains from allowing R&D firms to choose the adoption target instead of targeting the technology frontier



of initial conditions (panel a of figure 6). If, however, knowledge intensity and adoption barriers are high, then the economy may not be able to copy the frontier technology even in the long run (panel b of figure 6).<sup>57</sup> In this case, R&D firms will permanently copy less-advanced technologies. Growth will be positive and higher than in the case in which the target is always the technology frontier, but growth will be lower than that of the leading economies.

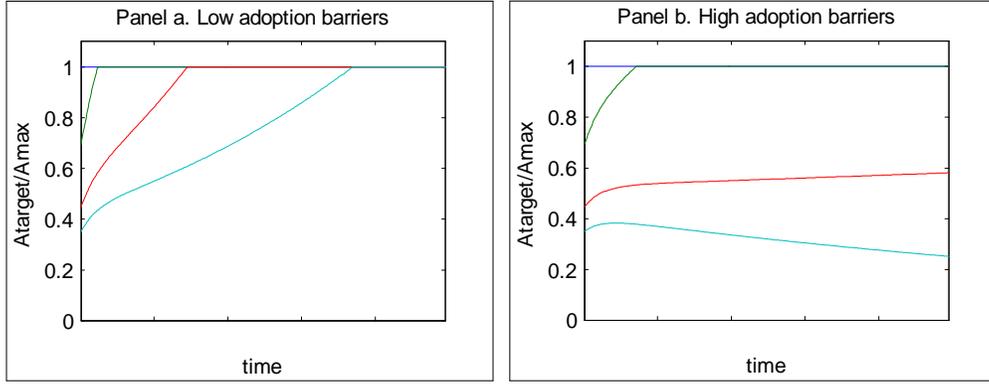
The knowledge intensity of the adoption activity determines the type of growth equilibria that the economy can reach. However, lowering barriers and improving the incentives to perform R&D are crucial to transit to a high growth equilibrium when low growth is a possibility. Economies may be targeting a lower adoption technology goal during the transition, but in the presence of high barriers, expected technology improvements can drastically fall, discouraging thereby R&D investment and reducing knowledge accumulation. In these cases, targeting lower technologies and improving the R&D environment may be the only way to elude the low growth equilibrium.

## 6 Concluding remarks

We develop a theoretical framework of endogenous adoption and innovation that is coherent with a wide set of income and growth trajectories. In particular, we discuss conditions that make economies more likely to be in a path of high growth (in which all economies share a common long-run growth rate) or in an environment in which growth traps are a possibility. The features of the model encompass, thereby, these two strands in the litera-

<sup>57</sup>Steady-state levels that tend to zero in figure 4.

Figure 6: Evolution of the optimal technology target ( $A_{target}$ ) relative to the technology frontier ( $A_{max}$ )



ture. The economic environment, the incentives to perform R&D, the adoption knowledge intensity, and the dynamics of the knowledge stock relative to the technology that R&D firms are aiming to implement define the type of path that the economy is following.

The stock of knowledge needed to absorb foreign technologies depends on the technology that the economy is targeting. Within this framework, two situations arise: If knowledge intensity in the adoption activity is low, then a low stock of relative knowledge does not affect decisively the adoption capacity of the economy. In this case, all countries achieve a high-growth equilibrium in the long run. However, differences in income levels and in the transition speed emerge if countries have different economic structures. If knowledge intensity in the adoption activity is high, then the economy may fall in a growth trap if its stage of development and relative knowledge are low. In this case, the adoption capacity can follow a decreasing path becoming an impediment for growth. Reducing adoption and innovation barriers may help to escape the low-growth equilibrium, as a better R&D environment can compensate the scarcity of knowledge in the early stages of development. However, the model shows that a complementary way to elude the low-growth equilibrium is to target technologies that are not at the frontier, as less advanced technologies require less knowledge and less R&D resources for being implemented. The model suggests that countries will adopt laggard technologies when they possess a low stock of knowledge and will target more advanced technologies as they develop. In such a scenario, the economy may improve its growth prospects and may eventually target frontier technologies and sustain high growth.

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## 7 Appendix A: The disaggregate relations

This appendix presents the building blocks of the model that are not presented in the main sections of the paper. We start presenting the household's problem. Then, we focus on the firms' problem in the final and in the intermediate production sectors.

### 7.1 Households

There is a continuum of measure one of households that live infinitely. Households derive utility from the consumption of the final good only and supply inelastically their endowment of labor equal to one. There is no aggregate risk, markets are complete, and there is perfect access to foreign capitals.

Households have the option to save in an external asset that yields  $r_B t$ , in physical capital with return  $r(t)$ , and in stocks of R&D firms. All assets are denominated in consumption units, the numeraire of the economy. Even though households face a stochastic problem, an optimal path involves no aggregate risk. We present the problem in terms of this optimal path. As all agents are identical in terms of preferences and consumption choices, the path of aggregate consumption  $C(t)$  can be obtained from the maximization of problem stated in equation (23). As usual,  $\sigma$  denotes the coefficient of relative risk aversion,  $\rho$  the discount rate, and  $\delta$  the depreciation rate.  $B(t)$  and  $K(t)$  define the aggregate stocks of foreign bonds and physical capital, respectively.  $S(t)$  corresponds to the aggregate stock portfolio that renders  $\bar{r}_s t$  and  $\omega(t)$  to the salary. Households own all firms in the economy.  $R(t)$  corresponds to the consolidated residual benefits of these firms.

$$\text{Max } U(C(t)) = \text{Max} \int_t^\infty \frac{C(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \quad \text{subject to} \quad (23)$$

$$\begin{aligned} \dot{K}(t) + \dot{B}(t) + \dot{S}(t) &= (r(t) - \delta) K(t) + \\ &B(t) + \bar{r}_s(t) S(t) + R(t) + \omega(t)L - C(t) \end{aligned} \quad (24)$$

The Euler equation states that:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma} (r_B - \rho) \quad (25)$$

Equilibrium conditions for holdings of bonds, physical capital, and the stock portfolio imply that:

$$r(t) - \delta = \bar{r}_s(t) = r_B$$

## 7.2 Producers

This section follows the models presented in Aghion and Howitt (1998). The final goods sector is competitive and produces according to (1) presented in the main section of the paper. The intermediate goods sector comprises a monopoly that is producing the inputs and the R&D firm that is investing in R&D to contest this producer. Inputs differ in the productivity that they provide. Productivity of input  $x_i(t)$  is higher than the productivity of  $x_z(t)$  if  $A_i(t) > A_{zt}$ .

The monopoly in sector  $i$  produces input  $i$  with the following technology:  $x_i(t) = K_i(t)/A_i(t)$ . Physical capital requirements to produce one unit of  $x_i(t)$  depend on the technology level embedded in  $x_i(t)$ . The higher  $A_i(t)$ , the more physical capital is needed to embed this technology in  $x_i(t)$ .

Firm  $i$  sells inputs  $i$  to the final goods firm at the monopoly price  $p_i(t)$  and pays for the use of capital its competitive rental price  $r(t)$ . The firm chooses optimally the amount  $x_i(t) = L (\alpha^2/r(t))^{1/(1-\alpha)}$  which only depends on two aggregate variables: the rental price of capital and the total flow of labor.<sup>58</sup> Perfect access to foreign capital ensures that  $r(t) = r_B + \delta$  in equilibrium. Optimal  $x_i(t)$  is independent of the level of technology embedded in the input, because revenues and costs of producing  $x_i(t)$  are proportional to the level of technology. Therefore, every sector supplies an equal amount of inputs. Monopolists' profits, in contrast, depend on the technology embedded in input  $x_i(t)$  and correspond to  $A_i(t)\pi(t)$  as defined in equation (2).

## 7.3 The R&D Market

Every sector comprises an R&D firm that is trying to displace the incumbent monopoly producing the inputs. Displacement occurs only if the R&D firm accomplish a better technology to embed in input  $x_i(t)$ . If the R&D firm and the incumbent compete with the same technology, they compete Bertrand and each monopoly earns zero profits. Thus, the R&D firm only invests in R&D if it can improve the technology currently in use. We assume that every technology improvement is drastic, implying that if the R&D firm accomplishes a new technology, it becomes the only producer of the input and charges a monopoly price.<sup>59</sup> In this case, the R&D firm becomes the new monopoly and stops investing in R&D. Accordingly, the previous monopoly stops producing and starts engaging in R&D activities.

**R&D Investment.** We assume that investment in R&D only affects the probability of success.<sup>60</sup> The R&D firm chooses the amount  $I_i(t)$  to invest by considering the expected

<sup>58</sup>Optimal amount of  $x_i(t) = \arg \max_{x_i(t)} [p_i(t)x_i(t) - r(t)K_i(t)]$ .

<sup>59</sup>Innovation or adoption is drastic, if the previous incumbent cannot produce and make nonnegative profits when the current one is charging the monopoly price (see references in Aghion and Howitt, 1998).

<sup>60</sup>Assuming that R&D investment only affects the probability of success and not the technology improvement simplifies the discussion of the mechanisms and makes the model more tractable. The cost

profits  $W_i(t)$  that it will get if it is successful and the expenses in R&D, which affect the probability of success.

The present value of a successful R&D firm  $W_i(t)$  is given by equation (26) which corresponds to the profits of the monopoly for as long as it remains producing. According to equation (2), time  $t$ 's profits are given by the term  $\bar{A}_i(t)\pi(t)$ , where  $\bar{A}_i(t)$  is the level of technology that a successful R&D firm achieves. This technology level is constant for the whole period in which the firm remains as the monopoly. The firm discounts its flows at the cost of capital  $rc(t)$  and the displacement rate  $\phi(t)$ . The displacement rate corresponds to the probability that the rival firm obtains an improved technology in the future.

$$W_i(t) = \bar{A}_i(t)w_i(t) = \int_t^\infty \bar{A}_i(t)\pi(t)e^{-\int_t^z (rc_s + \phi_s) ds} dz \quad (26)$$

The probability of success is defined as  $p_i(t) \equiv n_i(t)\beta$ . Recall that  $n_i(t) \equiv I_i(t)/\bar{A}_i(t)$ . Although risk is idiosyncratic; there is no aggregate risk and firms maximize the expected net benefit from R&D as in equation (27).

$$\max_{I_i(t)} (I_i(t)/\bar{A}_i(t)\beta) W_i(t) - I_i(t) \quad (27)$$

$$FOC : \beta w_i(t) = 1 \quad (28)$$

From the following equilibrium condition, we get the optimal R&D investment. The equilibrium condition is obtained by deriving equation (26) with respect to time. As there is no aggregate risk, in equilibrium the cost of capital is equal to the risk-free rate  $r_B$ .

$$\pi(t)/w(t) + \dot{w}_i/w_i + \dot{\bar{A}}_i(t)/\bar{A}_i(t) - \phi(t) = r_B \quad (29)$$

The LHS of equation (29) corresponds to the expected instantaneous return of the R&D firm (profits of the monopolist in time  $t$  plus the change in the value of the firm minus the probability of being displaced) which is equal to the risk-free rate.<sup>61</sup> Combining the equilibrium condition with the former result, we obtain optimal  $n_i(t)$  and, consequently,

is that the framework does not allow to analyze how the technology improvement in a particular sector is affected by the resources invested. However, resources invested in every sector affect the average technology improvement of the economy (section 4 discusses implications for aggregate relations).

<sup>61</sup>Deriving equation (28) with respect to time, we get  $\dot{w}_i(t) = 0$ . The incumbent does not invest in R&D to improve its own technology, so that  $\dot{\bar{A}}_i(t)/\bar{A}_i(t) = 0$ . As the incumbent does not face any cost advantage for investing in R&D, it does not invest to improve its current technology. For a given amount of R&D invested by the incumbent and the contestant, successful R&D would leave the incumbent with a technology  $\bar{A}'_i$  and incremental profits  $W(\bar{A}'_i) - W(\bar{A}_i)$  which are strictly less than the incremental profits  $W(\bar{A}'_i)$  obtained by the contestant. As a result, the former would find no financing in equilibrium or, alternatively, it would prefer to invest in another R&D firm.

the research level  $I_i(t)$  presented in equations (30) and (31), respectively.<sup>62</sup>

$$n_i(t) = n(t) = (\pi(t)\beta - r_B) / \beta \quad (30)$$

$$I_i(t) = \bar{A}_i(t)n(t) \quad (31)$$

## Appendix B. The dynamic system

### B1. The leading economy

The following equations correspond to the dynamic system of a leading economy with a unique equilibrium described by equations (14) and (13). As  $n(t)$  is constant in equilibrium, we omit the time subscript for this variable.

$$\begin{bmatrix} \dot{a}(t) \\ \dot{kn}(t) \end{bmatrix} = \begin{bmatrix} -n\beta + n\beta s - g & 0 \\ -n + ns & -g \end{bmatrix} \begin{bmatrix} a(t) \\ kn(t) \end{bmatrix} + \begin{bmatrix} n\beta \\ -n \end{bmatrix}$$

This system has two negative eigenvalues,  $n\beta(s-1) - g$  and  $-g$ , implying a stable equilibrium.

### B2. The non-leading economy

This subsection analyzes the conditions for the equilibrium point  $(a, kn) = (0, 0)$  to be locally stable. The dynamic system is given by equations (12) and (10). Note, that for the case of  $\gamma < 1$ , any positive value of  $kn$  moves the system away from the origin. As for the cases of  $\gamma = 1$  and  $\gamma > 1$ , we linearize the dynamic system around this steady state obtaining:

$\begin{bmatrix} \dot{a} \\ \dot{kn} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} a - a_{ss} \\ kn - kn_{ss} \end{bmatrix}$ , where  $a_1, a_2, a_3, a_4$  correspond to the following expressions (time subscripts are omitted):

$$a_1 = n\beta \left[ s\mu \left( \frac{kn}{kn^*} \right)^\varepsilon - \lambda \left( \frac{kn}{kn^*} \right)^\gamma \right] - g$$

$$a_2 = \frac{n\beta}{kn^*} \left[ \lambda\gamma \left( \frac{kn}{kn^*} \right)^{\gamma-1} (1-a) + a s\mu \varepsilon \left( \frac{kn}{kn^*} \right)^{\varepsilon-1} \right]$$

$$a_3 = n \left[ 1 - \lambda \left( \frac{kn}{kn^*} \right)^\gamma + s\mu \left( \frac{kn}{kn^*} \right)^\varepsilon \right]$$

---

<sup>62</sup>Interior solutions and probability bounded in the range  $[0, 1]$  require  $r_B(t)/\pi(t) \leq \beta \leq (1 + r_B(t))/\pi(t)$ .

$$a_4 = \frac{n}{kn^*} \left[ \lambda \gamma \left( \frac{kn}{kn^*} \right)^{\gamma-1} (1-a) + as\mu\varepsilon \left( \frac{kn}{kn^*} \right)^{\varepsilon-1} \right] - g$$

**Case 1:**  $\gamma = 1$ .

$$\begin{bmatrix} \dot{a} \\ \dot{kn} \end{bmatrix} = \begin{bmatrix} -g & \frac{n\beta\lambda}{kn^*} \\ n & \frac{n\lambda}{kn^*} - g \end{bmatrix} \begin{bmatrix} a - a_{ss} \\ kn - kn_{ss} \end{bmatrix}$$

The eigenvalues are:

$$r_1, r_2 = \frac{1}{2} \frac{n\lambda}{kn^*} - g \pm \sqrt{\left( \frac{n\lambda}{kn^*} - 2g \right)^2 + \frac{4}{kn^*} (-kn^*g^2 + \lambda gn + \beta\lambda n^2)}$$

If  $\lambda - 2g \frac{kn^*}{n} \equiv v_1 > 0$ , there is at least one positive eigenvalue. In this case, the origin is not a stable equilibrium. However, if  $v_1$  is negative, then the equilibrium point  $(a, kn) = (0, 0)$  is stable provided that  $\lambda < \frac{gkn_{ss}^*}{n} \left[ \frac{g}{n\beta+g} \right]$ . This condition corresponds to equation (17) in the main section of the paper. Condition (17) also satisfies that  $v_1 < 0$ .

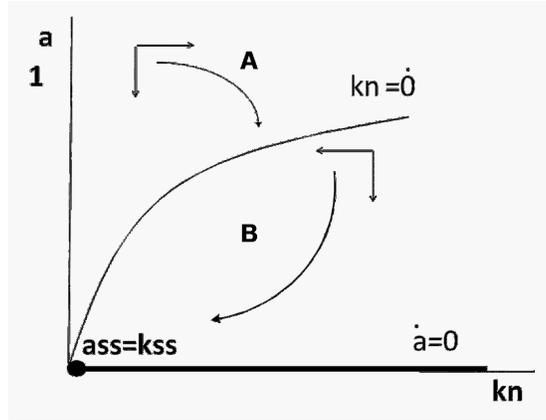
**Case 2:**  $\gamma > 1$ . In this case, the eigenvalues at  $(a, kn) = (0, 0)$  are negative and both equal to  $-g$ . Thus, the origin is a locally stable equilibrium. In this case, however, there are other equilibria. Initial conditions determine the equilibrium to which the economy transits.

### B3. Full adoption barriers: The case of $\lambda = 0$

Figure (7) presents the phase diagram for the case of  $\lambda = 0$ . Function  $\dot{a}(t) = 0$  coincides with the horizontal axis. Pair of points  $(a, kn)$  above the equation  $\dot{kn}(t) = 0$  produce paths with increasing knowledge. A higher relative technology implies larger R&D investment and higher knowledge growth relative to the obsolescence rate (see equation 10). On the other hand, points  $(a, kn)$  above the equation  $\dot{a}(t) = 0$  produce decreasing relative technology paths. A higher relative technology reduces the potential technological improvement producing a fall in its relative level (see equation 10). An analogous argument follows for pair of points  $(a, kn)$  below both curves.

A minimum amount of adoption is necessary to achieve and maintain a high-growth rate in the long run. Successful innovation maintains at most the sector's productivity in line with the technology frontier. However, as successes do not occur in every sector, average relative technology inevitably falls. As innovation improvements are not enough to maintain the relative position of the economy, relative technology falls from every starting pair of points  $(a, kn)$ . The stock of knowledge can increase for some periods if average productivity provides a sufficient innovation base to stimulate R&D (region A).

Figure 7:  $\lambda = 0$



However, this stock eventually starts to fall.

### C. Simulation exercises

The following exercises complement the results in the main sections of the paper, whenever analytical solutions are not possible. The simulations are based on the parameters presented table 1, section 5. Sources and definitions are described in section 5.

#### Simulations' parameters

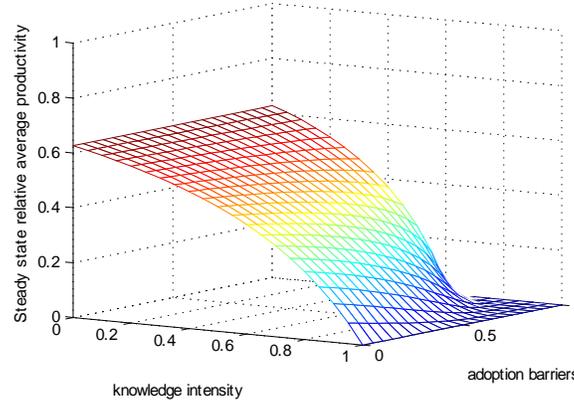
$\rho$	= 2.3%	$\sigma$	= 1
$\alpha$	= 0.35	$s\mu$	= 2.2%
$\delta$	= 0.03	$prob(success)$	= 3.6%

#### Knowledge intensity and adoption efficiency relations

The following figures present steady state relative technology levels for different pairs of knowledge intensities ( $\gamma$ ) and adoption barriers ( $\lambda$ ). In the figure, the no-barriers case corresponds to a value of  $\lambda = 0$ . Figure 8 presents the high-growth case. In this situation, the economy always achieves a positive relative technology steady state. As discussed in the paper, different knowledge intensities and adoption barriers have only level effects.

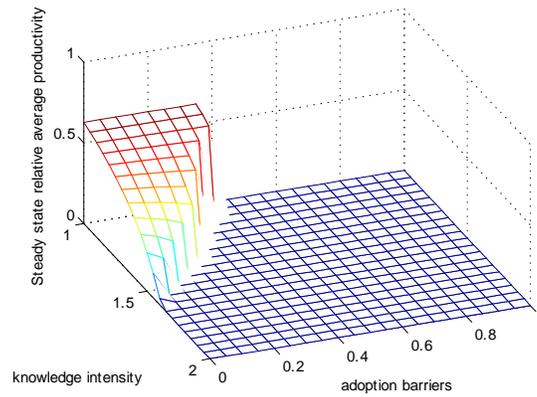
Next, we present the case in which the economy can reach a low-growth rate in the long run. As these cases can depend on initial conditions, we analyze two different starting points for non-leading economies. Highly poor initial conditions were proxied by the relation of the 20 poorest countries (in terms of per capita GDP) relative to the per capita US GDP in 2008. This measure accounts for a difference in per capita income of

Figure 8: Low knowledge intensities and adoption barriers



ca. 63 times. The better initial condition considers the world average per capita GDP in relation to the per capita US GDP. The income difference falls to 4.5 times if income-weighted average are used and to 3.7 if simple average are taken (source: The CIA World Factbook, 2008, PPP per capita GDP). Figure 9 presents the results for the basic case considering the less favorable average.

Figure 9: High knowledge intensities and adoption barriers



### Effects of initial conditions

Figure 10 presents the gains in steady state average productivity of having the initial conditions of the world average group instead of to the initial conditions of the poorest group

(both groups as defined in the previous paragraph). All positive differences correspond to situations in which the economy achieves high growth by having average initial conditions, but would have reached low growth, had it the conditions of the poorest countries.

Figure 10: High knowledge intensities: Effect of initial conditions (differences in steady-state productivity values)

