

# Risk premia at the ZLB: a macroeconomic interpretation

## preliminary\*

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### Abstract

Historically, high inflation is associated with low stock returns, leading investors to fear inflation. We document that this correlation changes after 2008, and positive signals about inflation are now associated with high stock returns. We interpret this as a change in the conditional covariance of economic activity and inflation. We then show how the zero lower bound (ZLB) on nominal interest rates can explain this change of covariance owing to the changing propagation mechanisms at the ZLB. This has important implications for asset prices since covariances determine risk premia. A fairly standard New Keynesian macroeconomic model generates positive term premia and inflation risk premia in normal times (far from the zero lower bound), but these premia fall as the economy becomes closer to the ZLB.

## 1 Introduction

The relation between inflation and economic activity is controversial, as illustrated by the widespread debate on the empirical relevance of the Phillips curve. The purpose of this paper is to use financial markets data to shed light on this relation, and to study the implications of this relation for asset pricing. In particular, we focus on the recent period in the United States when the zero lower bound (ZLB) constrained monetary policy. Standard macroeconomic models suggest that the response to aggregates shocks is different when the ZLB binds. Demand shocks may have little effect on inflation or economic activity if the ZLB does not bind because the central bank can offset demand fluctuations by changing the interest rate. But the same demand shocks may have large effects if the ZLB binds and the central bank cannot respond.<sup>1</sup> This change in the response to shocks affects the covariance of marginal

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<sup>1</sup>Similarly, supply shocks may have less positive effects on output if the ZLB does not bind, and larger negative effects on inflation, because monetary policy cannot accomodate them. As argued by Eggertsson (2012), a positive supply shocks may even be recessionary at the ZLB.

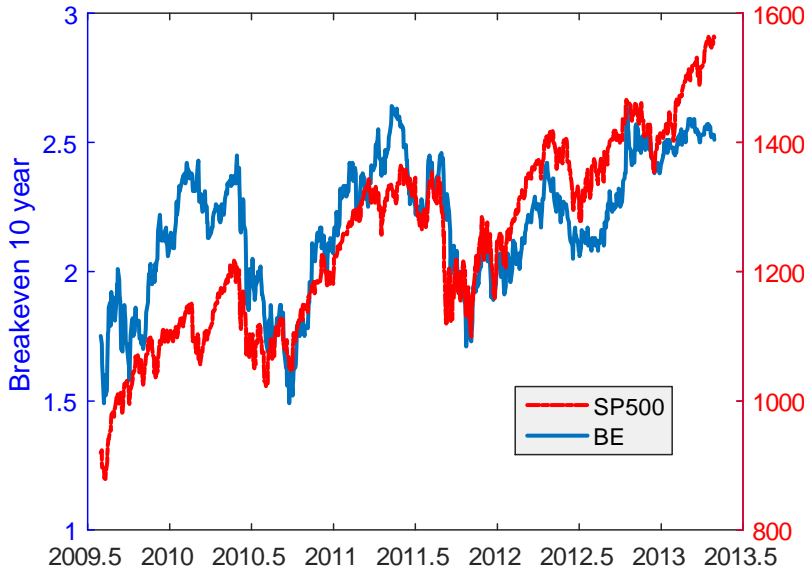


Figure 1: 10-year inflation breakeven (left-scale) and SP500 (right-scale): 2009:7:2013:5.

utility and inflation and consequently the inflation risk premium - the price investors are willing to pay to avoid bearing inflation risk.

The first contribution of the paper is to study the empirical relation between stock prices and inflation. We document that there has been a significant change in the response of stock prices to inflation in the United States after 2008. Historically, high inflation is associated with low stock returns, as documented in a long literature dating back at least to Fama and Schwert (1977)). But since 2008, stock prices appear to react positively to inflation. As a simple illustration, figure 1 depicts the strong correlation between stock prices and the 10-year breakeven (the difference between the yield of a 10 year nominal Treasury bond and a 10 year indexed Treasury bond) between 2009 and 2013. During this period, increases in stock prices - which typically reflected positive assessment of the economic recovery - were also associated with increases in inflation breakevens.

We demonstrate the change in the association of stock prices and inflation using three different sources: (i) correlations between the stock market and measures of inflation compensation (inflation breakevens, inflation swaps, or a portfolio of individual stocks constructed to mimic inflation); and (ii) the response of stock prices to inflation data releases during short windows; (iii) the response of monthly stock prices to monthly inflation. These facts are new to the best of our knowledge, and they are also consistent with the ZLB mechanism outlined above.

We interpret this change in correlation as reflecting a change in the conditional covariance of consumption growth (or more broadly economic activity) and inflation,  $Cov_t(\Delta c_{t+1}, \pi_{t+1})$ . This has important implications for the pricing of inflation risk, because (in a consumption based model) this pricing depends on the covariance of inflation and consumption growth. The change in the correlation suggests that the inflation risk premium is low or even negative, which is broadly consistent with other indirect

measures such as affine term structure models estimates.<sup>2</sup> More generally, in a world where supply shocks dominate, the covariance is strongly negative, investors fear inflation, and the risk premium for bearing inflation risk is positive. But in a world where demand shocks dominate (e.g. because of the ZLB), this covariance is positive, and the inflation risk premium may be negative. If the stock market reflects expectation of future output, and breakevens expectations of future inflation, then the covariance of stock prices and breakevens is a proxy for this covariance.<sup>3</sup>

The second contribution of our paper is to demonstrate theoretically how the covariance of economic activity and inflation *endogenously* changes depending on whether the ZLB binds. We solve a fairly standard New Keynesian dynamic stochastic general equilibrium (DSGE) model with high risk aversion, taking the ZLB into consideration, using nonlinear methods. As predicted by the simple intuition that demand (supply) shocks are amplified (weakened) at the ZLB, the model generates positive inflation and term premia in normal times (far from the ZLB), but lower inflation and term premia at the ZLB.

Overall, the paper connects the recent behavior of asset prices with a leading macroeconomic framework, and connects well-known recent observations about financial markets – nominal bonds appear to be good hedges – with macroeconomic theory: bonds are good hedges because demand shocks matter more at the ZLB. This implies lower term premia, hence the model help explains why long-term interest rates have remained so low since 2008.

Given the importance of long-term interest rates, which are a reference borrowing cost, understanding their movements is of primary interest for research and policy. The reduction in term and inflation premia also has some important “practical” consequences. First, there is a wide-ranging debate about the sources of the decline of interest rates, and to what extent this decline will last. Our model suggests that an upturn in the economy or in inflation may lead to a significant increase in interest rates because these risk premia change as the ZLB becomes less of a constraint. Second, economists and policymakers often use inflation compensation (i.e., inflation swaps or inflation breakevens) as a measure of expected inflation. It is well understood that inflation compensation may differ from expected inflation due to risk or liquidity premia; but the magnitude and even the sign of this adjustment are controversial. Our model argues that breakevens *underestimate* expected inflation when the economy operates close to the ZLB, but *overestimate* expected inflation when the economy is far from the ZLB. Third, our analysis is an indirect test of the widely-used ZLB New Keynesian macroeconomic model.

The paper is organized as follows. The rest of the introduction reviews briefly the related literature. Section 2 studies a simple example that illustrates how the covariance of macroeconomic variables affects the inflation risk premium and breakeven rates. Section 3 presents reduced form evidence that the link between the inflation and risky assets has changed since 2008. Section 4 introduces a stylized DSGE model. Section 5 provides some preliminary quantitative results. Section 6 concludes.

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<sup>2</sup>For some prominent models, see Kim and Wright (2005), Adrian, Crump and Moench (2013), D’Amico, Kim and Wei (2010), Ajello, Benzoni and Chyruk (2014).

<sup>3</sup>Obviously, this covariance is a sufficient statistic only in a very simple model (see Section 2). But the covariance of marginal utility - and hence economic activity - and inflation is important in a broad class of asset pricing models.

## 1.1 Related Literature

Our paper is related to several strands of literature. First, there is a large macro-asset pricing literature that attempts to explain the level and volatility of the term premium. This work typically uses endowment economies for tractability. Marshall (1992) is an early paper in this literature. Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2013) are recent studies that use the long-run risk framework. The underlying logic of how risk premia are determined is similar to our paper (and is discussed in section 2), but our contribution relative to these papers is to study the *sources* of the correlations between inflation and growth that are taken as primitives in these studies. David and Veronesi (2014) also study the changes in regimes with different correlations of inflation and asset prices.

Second, a subset of this literature consists of DSGE production models with nominal rigidities that attempt to replicate various features of asset prices. Key contributions include Rudebusch and Swanson (2008, 2012), Li and Palomino (2014), Christiano et al. (2010), Palomino (2012), and Swanson (2015). Especially close in spirit is the recent paper by Campbell, Pflueger and Viceira (2014) that emphasize structural breaks in monetary policy rules and how these affect asset prices and their correlations. Our contribution relative to all these papers is to introduce the ZLB and to focus on the recent changes since the Great Recession started. The contemporaneous study by Nakata and Tanaka (2016) is also closely related, with a fairly similar message but differences in empirical work and details of the model.

Third, our paper relates to the vast macro literature on the effects of the zero lower bound (ZLB). Seminal contributions include Krugman (1997) and Eggertsson and Woodford (2003). (2014) evaluates the pertinence of the ZLB mechanism, which remains disputed. In particular, our nonlinear method is related to the contributions of Fernandez-Villaverde et al. (2012), Ngo (2015) and Miao and Ngo (2015).

Finally, the broader question of the relation between stock prices and inflation has long a long history dating back at least to Fama and Schwert (1977) who showed that stocks appeared to be affected negatively by inflation, a result widely viewed as “puzzling” since stocks are claims to real assets. Modigliani and Cohn (1979) argued that investors suffered from money illusion. Boudoukh and Richardson (1993) and Campbell and Vuolteenaho (2004) revisited this issue. On the empirical side, Bernanke and Kuttner (2005) and Rigobon and Sack (2014) demonstrate that monetary policy surprises have a large effect on stock prices. Duarte (2013) also emphasizes the change in correlation and studies how inflation affects the cross-section of stock returns. Fleckenstein et al. (2014, 2015) study the pricing of TIPS and deflation while Ang, Bekaert and Wei (2008) and Hordahl and Tristani (2010) provide estimates of inflation premia. Gorodnichenko and Weber (2016) and Weber (2016) study the heterogeneity in price flexibility and demonstrate that it affects the responses of stocks to monetary policy shocks. Gali (2014) links monetary policy to asset pricing bubbles.

## 2 Why would the inflation risk premium be higher at the ZLB?

In order to provide some basic intuition, it is convenient to use a stripped-down, representative agent, endowment economy model. Suppose that the representative consumer has expected utility with constant

relative risk aversion:

$$E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

and that consumption growth and inflation are conditionally jointly log-normally distributed. Specifically, denote log consumption growth by  $\Delta c_{t+1} = \Delta \log C_{t+1}$  and log inflation by  $\pi_{t+1} = \Delta \log P_{t+1}$  (where  $P_t$  is the CPI) and assume that

$$\begin{pmatrix} \Delta c_{t+1} \\ \pi_{t+1} \end{pmatrix} \sim N \left( \begin{pmatrix} \mu_{c,t} \\ \mu_{p,t} \end{pmatrix}, \begin{pmatrix} \sigma_{c,t}^2 & \rho_{c,p,t} \\ \rho_{c,p,t} & \sigma_{p,t}^2 \end{pmatrix} \right),$$

Note that the conditional means, variances and covariances  $\mu_{c,t}, \mu_{p,t}, \sigma_{p,t}, \sigma_{c,t}, \rho_{c,p,t}$  can vary arbitrarily over time.

The critical parameter is  $\rho_{c,p,t}$ , which may be positive or negative, and measures the exposure of inflation to consumption growth risk:

$$\rho_{c,p,t} = Cov_t(\Delta c_{t+1}, \pi_{t+1}).$$

Intuitively, a positive  $\sigma_{p,t}$  corresponds to the case where “demand shocks” dominate: low consumption is associated with low inflation, while a negative  $\sigma_{p,t}$  corresponds to the case where “supply shocks” dominate: low consumption is associated with high inflation. This covariance determines the inflation risk premium, as we now show.

For simplicity, we will focus on one-period bonds. (One may think of the time period as being 10 years.) The real log stochastic discount factor is

$$\log M_{t+1} = \log \beta - \gamma \Delta c_{t+1},$$

and the nominal log stochastic discount factor is

$$\log M_{t+1}^{\$} = \log M_{t+1} - \pi_{t+1}.$$

Simple calculations show that the log real risk-free rate is

$$\begin{aligned} \log R_{t+1}^f &= -\log E_t(M_{t+1}), \\ &= -\log \beta + \gamma E_t(\Delta c_{t+1}) - \frac{\gamma^2}{2} Var_t(\Delta c_{t+1}). \end{aligned}$$

the familiar formula that decomposes the riskless rate into impatience, intertemporal substitution, and precautionary savings.

The log nominal risk-free rate is

$$\begin{aligned} \log R_{t+1}^{f,\$} &= -\log E_t(M_{t+1}^{\$}), \\ &= \log R_{t+1}^f + E_t(\pi_{t+1}) - \frac{1}{2} Var_t(\pi_{t+1}) - \gamma Cov_t(\pi_{t+1}, \Delta c_{t+1}). \end{aligned}$$

The breakeven rate is the difference in the yields of these two bonds, or in logs:

$$\begin{aligned} BE_t &= \log R_{t+1}^{\$} - \log R_{t+1} \\ &= E_t(\pi_{t+1}) - \frac{1}{2} Var_t(\pi_{t+1}) - \gamma Cov_t(\pi_{t+1}, \Delta c_{t+1}). \end{aligned}$$

This shows that the (log) breakeven rate is the sum of expected (log) inflation, a Jensen adjustment,<sup>4</sup> and a risk premium term which equals risk aversion  $\gamma$  multiplied by the covariance of consumption growth and inflation  $\rho_{c,p,t}$ . If investors are risk-neutral ( $\gamma = 0$ ), and neglecting the Jensen term, the breakeven measures perfectly expected (log) inflation. However, most macroeconomic models that replicate asset prices require high risk aversion, suggesting that the inflation risk premium component may be large.

Intuitively, if the covariance  $\rho_{c,p,t} < 0$ , supply shocks dominate, and breakevens overestimate inflation. Nominal bonds are risky assets, since their real payoff is low in states of the world where inflation is high, which on average coincide with low consumption growth and high marginal utility. Hence, agents require a premium to hold nominal bonds, so the nominal yield is higher than it would be under risk-neutrality. On the other hand, if  $\rho_{c,p,t} > 0$ , demand shocks dominate, inflation is a hedge, and breakevens underestimate inflation.

The covariance however does not depend solely on which kind of shocks are expected to dominate, but also on the propagation mechanisms at work. If the ZLB binds, demand shocks may be amplified, while supply shocks could have weak or even opposite effects on output than usual. This would affect the covariance even if the variances of the underlying fundamental shocks remain constant. To see this in more detail, suppose there are two fundamental shocks,  $\varepsilon_d$  and  $\varepsilon_s$ . Inflation goes up with “demand” shock, but down with “supply” shock. To a linear approximation, we can write

$$\Delta c_{t+1} = \lambda_{c,d}\varepsilon_{d,t+1} + \lambda_{c,s}\varepsilon_{s,t+1},$$

$$\pi_{t+1} = \lambda_{\pi,d}\varepsilon_{d,t+1} + \lambda_{\pi,s}\varepsilon_{s,t+1},$$

and as a result

$$Cov_t(\Delta c_{t+1}, \pi_{t+1}) = \underbrace{\lambda_{c,d}\lambda_{\pi,d}\sigma_d^2}_{>0} + \underbrace{\lambda_{c,s}\lambda_{\pi,s}\sigma_s^2}_{<0}$$

where  $\lambda_{c,d} > 0$ ,  $\lambda_{\pi,d} > 0$ ,  $\lambda_{c,s} > 0$  and  $\lambda_{\pi,s} < 0$  typically. At ZLB, both  $\lambda_{c,d}$  and  $\lambda_{\pi,d}$  increase, leading the covariance to increase and the inflation risk premium to fall. Moreover,  $\lambda_{c,s}$  falls and may even become negative, as the economy benefits less from positive supply shocks, while  $\lambda_{\pi,s}$  tends to become more negative. These forces conjure to make the covariance become more negative. We next turn to the data to see which case is more realistic - and we will argue that since 2009, the major movements in breakevens have been positively correlated with the stock market, which suggests that  $\rho_{c,p,t} > 0$ .<sup>5</sup>

### 3 Changes in the relation between stock prices and inflation

This section presents reduced-form evidence that asset markets now view inflation as a net positive for the economy. We first document changes in the correlation of inflation compensation (inflation breakevens or inflation swaps) with stock prices; we then review the response of asset prices to news about inflation and to actual inflation; and finally we construct from the cross-section of US stocks a portfolio that “mimics” news about inflation and document its behavior since 2005.

<sup>4</sup>The source of this term is that the real payoff of a nominal bond depends inversely on inflation. Consequently, higher uncertainty about inflation leads to higher expected payoffs. This term is typically small.

<sup>5</sup>Our approach is to use stock returns as a measure of news about the economy rather than consumption, which is notoriously difficult to measure. The small sample makes it attractive to rely on asset price measures.

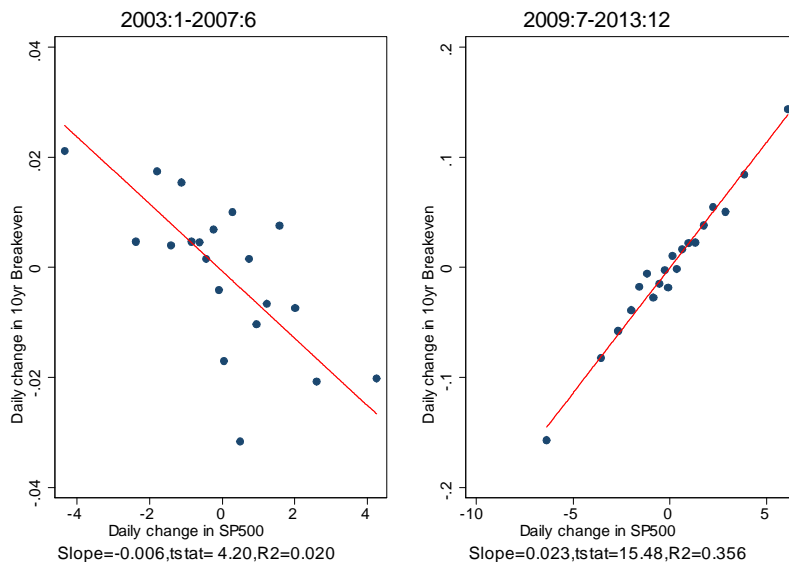


Figure 2: Scatter plot of daily changes in SP500 (x-axis) vs. daily changes in 10 year breakevens (y-axis) for two subsamples: before and after 2008, with regression lines superimposed.

### 3.1 Correlation of stock prices and inflation compensation

We start by illustrating how the correlation of breakeven inflations (the difference between the 10 year nominal and real (TIPS) yields) with stock prices changes after 2008. We focus on the period after mid-2009 because TIPS markets were disrupted during the peak of the financial crisis (see Fleckenstein et al. (2014)). Figure 2 plots the daily changes in SP500 vs. the daily changes in breakevens.<sup>6</sup> The left panel demonstrates that in the 2003-2007 sample, the correlation is essentially zero. The right panel shows that the correlation becomes very strong after 2009. A 1% increase in the SP500 is associated with an increase of 0.2bps before the crisis (t-stat: 1.5), but with an increase of 1.6bps after the crisis (t-stat: 11.7).  $\omega_{cnt}(\eta)$  1 reports these correlations for different maturities as well as correlations with nominal and real Treasury yields.

One might worry that nominal treasuries are “special” in terms of their liquidity. Inflation swaps provide an alternative measure of inflation compensation. Figure 4 shows that the results with one-day inflation swaps are very similar to those with breakevens: the slope is 1.4 instead of 1.6 post-crisis, and 0 instead of 0.2 pre-crisis.

Another potential concern is that the daily changes in prices reflect mostly market sentiment rather than hard news.<sup>7</sup> Figure 3 depicts the correlation between 20-day changes in SP500 vs. 20-day changes in 10 year breakeven. The change in the relation is still very striking between the two subsamples. The slope shifts from -0.8 to 2.6 between the two subsamples.

<sup>6</sup>We start the earliest sample in January 2003 because TIPS liquidity was limited before that date. Note also that in this and all the other scatter plots of the paper, we plot only 20 bins of the data to make the graphs easier to read. The regression statistics refer to the full sample.

<sup>7</sup>Indeed, there is evidence that the long-run response of the market to macro news is stronger than the short-run response; see XXX.

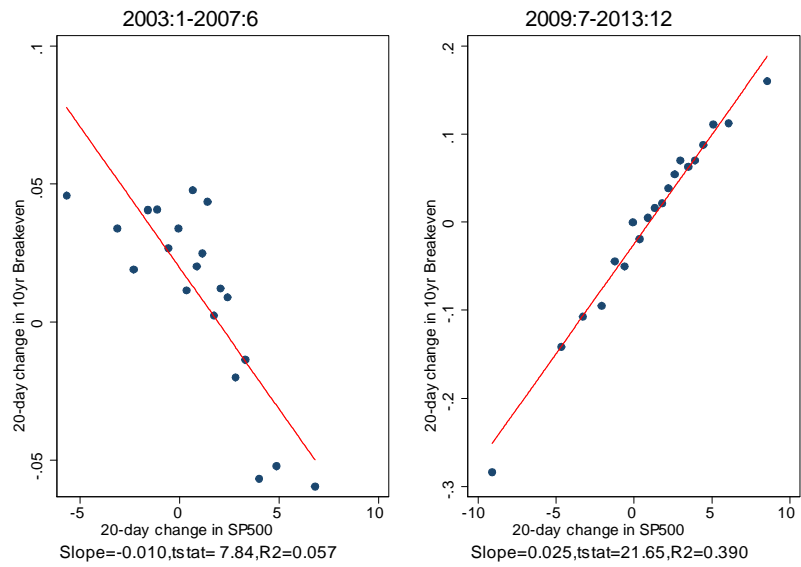


Figure 3: Scatter plot of 20-day changes in SP500 (x-axis) vs. 20-day changes in 10 year breakevens (y-axis) for two subsamples: before and after 2008, with regression lines superimposed.

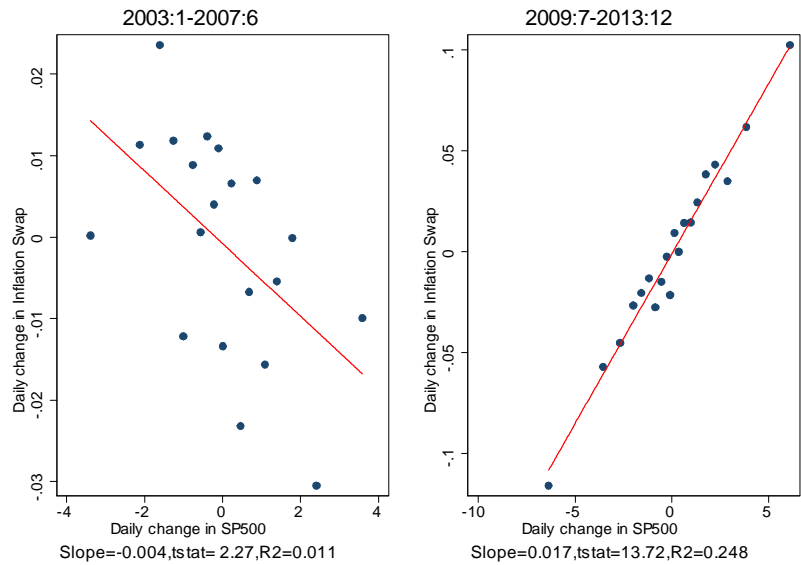


Figure 4: Scatter plot of daily changes in SP500 (x-axis) vs. daily changes in 10 year inflation swap (y-axis) for two subsamples: before and after 2008, with regression lines superimposed.



	SP500	G10	G5	GF5	TIPS10	TIPS5	TIPSF5	BE 10	BE 5	BE F5
<b>Panel A: Jan 2003 through May 2007</b>										
SP500	1.00	0.15	0.15	0.13	0.14	0.13	0.13	<b>0.05</b>	<b>0.06</b>	<b>0.02</b>
G10	0.15	1.00	0.96	0.95	0.85	0.79	0.75	0.50	0.40	0.38
G5	0.15	0.96	1.00	0.83	0.84	0.81	0.70	0.45	0.44	0.27
GF5	0.13	0.95	0.83	1.00	0.79	0.70	0.74	0.51	0.33	0.46
TIPS10	0.14	0.85	0.84	0.79	1.00	0.92	0.90	-0.03	0.00	-0.04
TIPS5	0.13	0.79	0.81	0.70	0.92	1.00	0.64	-0.00	-0.16	0.15
TIPSF5	0.13	0.75	0.70	0.74	0.90	0.64	1.00	-0.05	0.19	-0.25
Breakeven 10	0.05	0.50	0.45	0.51	-0.03	-0.00	-0.05	1.00	0.77	0.80
Breakeven 5	0.06	0.40	0.44	0.33	0.00	-0.16	0.19	0.77	1.00	0.22
Breakeven F5	0.02	0.38	0.27	0.46	-0.04	0.15	-0.25	0.80	0.22	1.00
<b>Panel B: June 2009 through Nov 2012</b>										
SP500	1.00	0.53	0.43	0.55	0.29	0.13	0.34	<b>0.46</b>	<b>0.40</b>	<b>0.37</b>
G10	0.53	1.00	0.92	0.95	0.77	0.58	0.72	0.61	0.52	0.50
G5	0.43	0.92	1.00	0.76	0.73	0.67	0.60	0.53	0.52	0.37
GF5	0.55	0.95	0.76	1.00	0.71	0.45	0.74	0.61	0.46	0.54
TIPS10	0.29	0.77	0.73	0.71	1.00	0.82	0.88	-0.04	0.00	-0.07
TIPS5	0.13	0.58	0.67	0.45	0.82	1.00	0.46	-0.11	-0.29	0.08
TIPSF5	0.34	0.72	0.60	0.74	0.88	0.46	1.00	0.03	0.24	-0.17
Breakeven 10	0.46	0.61	0.53	0.61	-0.04	-0.11	0.03	1.00	0.81	0.85
Breakeven 5	0.40	0.52	0.52	0.46	0.00	-0.29	0.24	0.81	1.00	0.38
Breakeven F5	0.37	0.50	0.37	0.54	-0.07	0.08	-0.17	0.85	0.38	1.00

Table 1: Correlation of the daily changes in the SP500, and in the changes in the of yields: 10 year, 5 year, and 5 year forward Treasuries, TIPS, and breakevens.

	SP500		5y Treasury		5y Inflation Swap		5 y Inflation Breakeven	
Employment	-0.002**	0.005**	0.053***	0.080***	0.021	0.031***	0.020***	0.027***
	(0.001)	(0.002)	(0.006)	(0.014)	(0.024)	(0.012)	(0.007)	(0.008)
CPIcore	<b>-2.218**</b>	<b>-0.435</b>	22.489***	-8.644	31.000**	20.763***	18.144***	18.178***
	(0.949)	(1.087)	(3.894)	(9.317)	(12.052)	(6.515)	(5.333)	(6.559)
PPIcore	<b>-0.628**</b>	<b>1.209**</b>	2.565	5.883**	3.920*	10.766***	2.212	8.484***
	(0.291)	(0.538)	(1.581)	(2.614)	(2.134)	(2.805)	(2.226)	(2.381)
Observations	11,697	1,704	11,697	1,704	1,042	1,703	1,731	1,703
R-squared	0.002	0.006	0.017	0.042	0.016	0.023	0.013	0.023
Before 2009	y	n	y	n	y	n	y	n
Since 2009	n	y	n	y	n	y	n	y

Robust (White) standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 2: Response of asset prices to surprise in macro announcements. Daily regression in samples before and after 2009.

### 3.2 Response to inflation releases

One concern is that inflation compensation might be reflecting other factors than expected inflation. This leads us to a second piece of evidence to study more directly the response of stock prices to inflation. We follow the event study approach and regress the daily return on the SP500 on the “surprise” on days of macro announcements, i.e. the difference between the data as released by the statistical agency and the median forecast made by economists (and collected by Action Economics /MMS).  $\omega_{cnt}(\eta)$  2 focuses on the CPI and PPI releases and adds the employment report (nonfarm payroll) for comparison. Interesting, column 1 shows that before 2009, the stock market had a significant *negative* response to both CPI and PPI (core) releases. Column 2 shows that the stock market has a weak and insignificant response to CPI core surprises, and a positive and significant response to PPI surprises, after 2009. In terms of magnitude, if CPI core inflation was one ‘tick’ (1/10th of a percent) higher than expected, stock prices on average fell 0.2% on that day before the crisis, but only an insignificant 0.04% after the crisis.

One simple way to understand this switch - and which is consistent with our ZLB argument - is that before 2009, an unexpected decrease in inflation led to the presumption that the Fed would cut rates, helping stocks. After 2009, the Fed is unable to respond. Note also that the responses of inflation breakeven or inflation swaps remain relatively unchanged suggests that the releases have a roughly similar informational content for inflation before and after 2009. The change in the response of stock prices to the employment report is, on the other hand, consistent with Boyd, Hu and Jagannathan (2005).

	1925-1939	1940-1959	1960-1984	1985-2002	1960-2007	2003-2007	2008-2015
$\beta$	1.534	-0.309	-1.625**	-1.716	-1.704***	-1.956***	1.510
	(1.065)	(0.376)	(0.700)	(1.527)	(0.546)	(0.722)	(1.388)
Obs	168	240	300	216	576	60	96
R2	0.013	0.003	0.021	0.007	0.019	0.098	0.020

White SE; \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 3: Response of the CRSP value-weighted return to inflation, for different subsamples. Monthly data.

### 3.3 Response to inflation

More directly, we can estimate the response of (nominal) stock returns to inflation, in the spirit of Fama and Schwert:

$$R_t = \alpha + \beta\pi_t + \varepsilon_t,$$

where  $\pi_t$  is CPI inflation,  $R_t$  is the CRSP total return.<sup>8</sup>  $\omega_{cnt}(\eta)$  3 confirms the results of Fama and Schwert, that is higher inflation is associated with lower nominal stock returns (rather than higher has one might expect under the Fisher hypothesis). This result varies significantly across periods, however. It holds in the 1960-2007 period, but not since 2008. Interestingly, the only other period where the coefficient was positive is the Great Depression - when short-term money market rates were also very low.

One might worry that this relation is driven by low frequency changes in inflation that are anticipated as opposed to unexpected “news” or “surprises” to inflation. We propose two ways of decomposing inflation into expected vs. unexpected components. First, following Fama and Schwert, we use the T-bill rate as a proxy for expected inflation - a good assumption if the real rate is fairly constant. Hence, we estimate

$$R_t = \alpha + \beta(\pi_t - Tbill_t) + \gamma Tbill_t + \varepsilon_t.$$

As a second method, we use as proxy for expected inflation the current inflation (year-over-year to smooth out the noise). This leads us to estimate

$$R_t = \alpha + \beta(\pi_t - \pi_t^E) + \gamma\pi_t^E + \varepsilon_t.$$

Last, we also report the results using core inflation rather than total inflation. All these results ( $\omega_{cnt}(\eta)$ s 4-6) point to a significant change in behavior post 2008.

<sup>8</sup>We use CRSP to extend the sample and include the Great Depression. Results are nearly identical if one uses SP500 where available.

	1925-1939	1940-1959	1960-1984	1985-2002	1960-2007	2003-2007	2008-2015
$\beta$	<b>1.533</b>	<b>-0.344</b>	<b>-1.681**</b>	<b>-2.405</b>	<b>-1.949***</b>	<b>-1.956***</b>	<b>2.186*</b>
	(1.073)	(0.388)	(0.822)	(1.572)	(0.589)	(0.733)	(1.228)
$\gamma$	1.489	-2.197	-1.539*	1.497	-1.197	-3.274	-32.845***
	(5.094)	(3.821)	(0.919)	(2.500)	(0.796)	(2.727)	(9.165)
Obs	168	240	300	216	576	60	96
R2	0.013	0.004	0.021	0.020	0.020	0.104	0.131
White SE; *** p<0.01, ** p<0.05, * p<0.1							

Table 4: Response of the CRSP value-weighted return to inflation minus expected inflation (proxied by the Tbill rate) and expected inflation, for different subsamples. Monthly data.

	1985-2002	1960-2007	2003-2007	2008-2015
$\beta$	<b>-2.608</b>	<b>-3.207***</b>	<b>-2.377**</b>	<b>2.087</b>
	(1.801)	(0.717)	(1.171)	(1.558)
$\gamma$	-2.466	-3.003***	-2.819**	1.156
	(1.714)	(0.676)	(1.241)	(1.519)
Obs	216	576	60	96
R2	0.010	0.030	0.107	0.127
White SE; *** p<0.01, ** p<0.05, * p<0.1				

Table 5: Response of the CRSP value-weighted return to inflation minus expected inflation (proxied by last month's year-over-year inflation) and expected inflation, for different subsamples. Monthly data.

	1985-2002	1960-2007	2003-2007	2008-2015
$\beta$	<b>-5.486</b>	<b>-2.659***</b>	<b>-6.675</b>	<b>2.966</b>
	(3.643)	(0.947)	(4.168)	(7.721)
$\gamma$	-4.763	-2.439***	-7.182*	0.723
	(3.430)	(0.900)	(4.199)	(7.251)
Obs	216	576	60	96
R2	0.017	0.013	0.067	0.042
White SE; *** p<0.01, ** p<0.05, * p<0.1				

Table 6: Response of the CRSP value-weighted return to core inflation minus expected core inflation (proxied by last month's year-over-year core inflation) and expected core inflation, for different subsamples. Monthly data.

### 3.4 An inflation-mimicking portfolio

Another place where we may find useful information about inflation is the cross-section of stocks. Some firms are naturally more sensitive to inflation, due to the nature of their assets, their business, their liabilities (debts, rents, pensions, etc.). We can create a long-short portfolio of stocks based on their inflation sensitivity. This allows tracking an asset that is a “inflation hedge” over a long period of time (the sample is longer than with TIPS or inflation swaps), using high-frequency data, and without the liquidity problems that TIPS or inflation swaps may have.

We implement this as follows. On the last day of each year, we sort the 500 stocks with largest market capitalization in CRSP by inflation sensitivity. The inflation sensitivity is estimated using the response of the stock to CPI announcements over the previous 3 years of data. Specifically, we run for each stock:

$$R_{it} = \alpha_i + \beta_i \text{NewsCPI}_t + \varepsilon_{it},$$

over the 36 (3 years times 12 months) days of CPI releases; here  $\text{NewsCPI}_t$  is the difference between actual CPI inflation and the forecast made by economists before the release.<sup>9</sup> As may be expected, we find that technology firms have typically low (or negative)  $\beta_i$  while commodity or energy firms and banks have positive  $\beta_i$ . (A list of the top and bottom 50 stocks by inflation sensitivity in 2011 is included in appendix.)

We then create an (equally-weighted) portfolio long the top quartile of inflation sensitivity and short the bottom quartile. This portfolio is effectively a “breakeven in the stock market”. We first document that this portfolio behaves similarly to actual breakevens. Figure 5 depicts the correlation of this portfolio with the SP500 before and after the crisis. We see that before the crisis, the correlation is strongly negative, but it becomes strongly positive after the crisis. This is similar to the results obtained with breakevens or inflation swaps. An alternative illustration of the same fact involves reporting the market beta of the inflation portfolio. This beta shoots up starting in 2008, see figure 6.

Finally, figure 7 shows the cumulated return on this long-short portfolio together with year-over-year total CPI and core inflation. The returns are high during the financial crisis - the strategy generates around +70% from 2007 through 2011 - then are low (-30% from 2011 through 2015). The returns broadly follow realized year-over-year inflation. In particular, it is perhaps not surprising that this strategy has a low return post 2011 - inflation was lower than expected during that period. The period of the financial crisis is more surprising since inflation fell while this portfolio did well. Perhaps the increase in value of the portfolio reflected a fear of inflation which never materialized. Alternatively, there may have been a repricing of risk whereby high inflation beta stocks, that were perceived as risky initially, became more attractive leading to a large increase in value.

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<sup>9</sup>We use core inflation. We obtained fairly similar results using total inflation, as well as PPI or core PPI inflation.

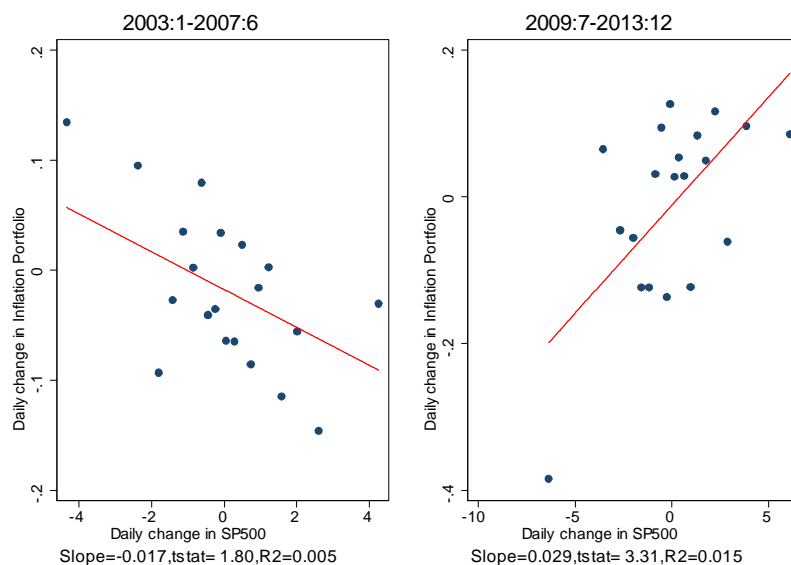


Figure 5: Scatter plot of daily changes in SP500 (x-axis) vs. daily changes in the inflation-mimicking portfolio (y-axis) for two subsamples: before and after 2008, with regression lines superimposed.

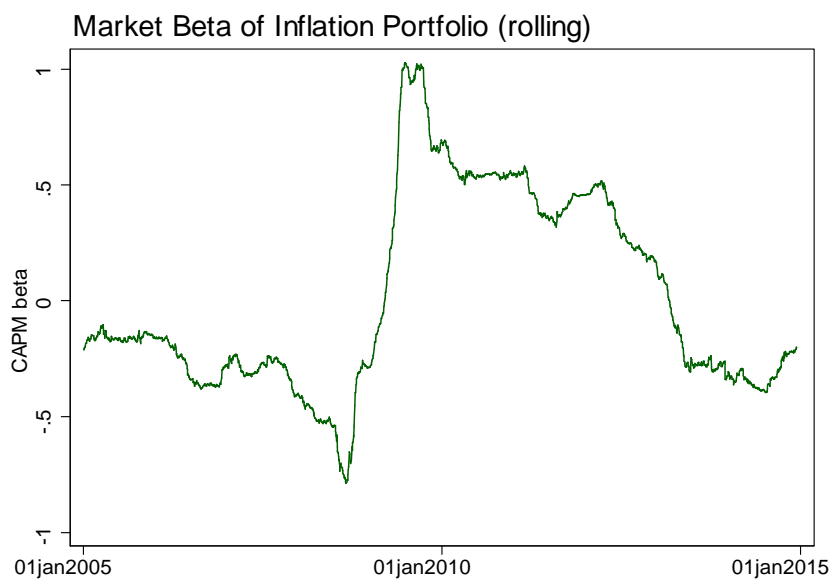


Figure 6: Rolling window (120 days) CAPM beta of the inflation-mimicking portfolio.

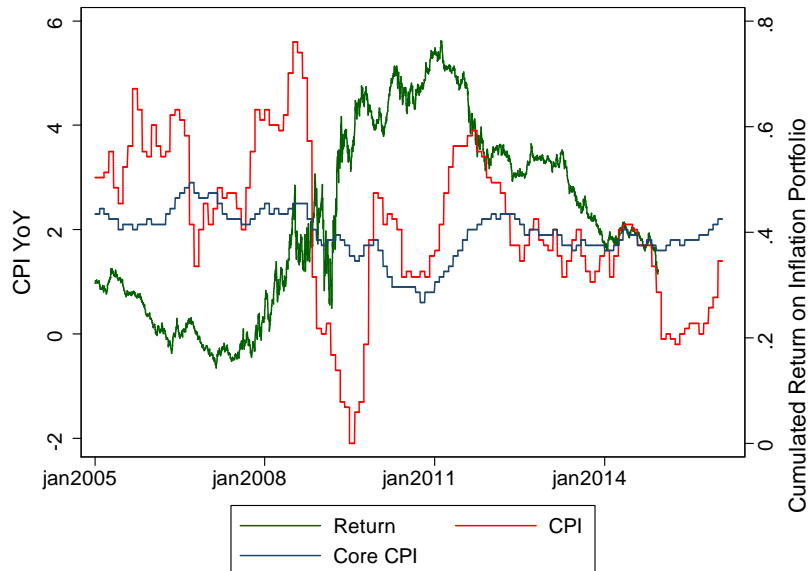


Figure 7: Cumulated return on the inflation-mimicking portfolio (long the top 25% inflation sensitivity stocks and short the bottom 25% inflation sensitivity stocks), together with year-over-year CPI and core CPI inflation.

## 4 Model

Our model follows closely Rudebusch and Swanson (2012, thereafter RS). The main difference is that we explicitly take into account the zero lower bound.<sup>10</sup> These authors themselves build closely on the standard New Keynesian model as outlined for instance in Gali (2012) and Woodford (2003). The main difference they introduce relative to the standard model is that they incorporate recursive preferences as in Epstein and Zin (1989) as well as different shocks.

### 4.1 Household

We follow Rudebusch and Swanson's version of Epstein and Zin (1989), except that we introduce a shock to the discount factor  $\beta_t$  :

$$V_t = (1 - \bar{\beta}) u(C_t, N_t) + \bar{\beta} \beta_t E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}} .$$

As RS emphasize, instantaneous utility may be negative so one may need to flip signs:

$$V_t = (1 - \bar{\beta}) u(C_t, N_t) - \bar{\beta} \beta_t E_t \left( (-V_{t+1})^{1-\alpha} \right)^{\frac{1}{1-\alpha}} .$$

The per period utility is assumed to be

$$u(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi N_t^{1+\nu}}{1+\nu} ,$$

where  $C_t$  is consumption,  $N_t$  is labor and  $Z_t$  is technology. The introduction of  $Z_t$  in the utility function is designed to ensure that employment remains stationary even with unit root technology. It may reflect

<sup>10</sup>We also use Rotemberg rather than Calvo pricing, chiefly to economize on state variables, and use different shocks, and a different monetary policy rule.

home production or changing tastes as the economy grows.

The labor supply equation is simply

$$w_t = \frac{u_2(C_t, N_t)}{u_1(C_t, N_t)} = C_t^\sigma \chi N_t^v. \quad (1)$$

The real stochastic discount factor is

$$M_{t,t+1} = \bar{\beta} \beta_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{V_{t+1}}{E_t(V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha},$$

and the nominal stochastic discount factor is

$$M_{t,t+1}^{\$} = \frac{M_{t,t+1}}{\Pi_{t+1}},$$

where  $\Pi_{t+1}$  is gross inflation  $P_{t+1}/P_t$ .

The first order condition links the nominal short-term interest rate to the nominal SDF:

$$1 = E_t \left[ Y_t^{\$, (1)} M_{t+1}^{\$} \right],$$

where  $Y_t^{\$, (1)}$  is the gross nominal yield on a 1-period asset.

## 4.2 Production and optimal price-setting

There is a number of identical monopolistically competitive firms, each of which operates a production function that is constant return to scale in labor:

$$Y_{it} = Z_t N_{it}. \quad (2)$$

Each firm faces a downward-sloping demand curve coming from the Dixit Stiglitz aggregator with elasticity of demand  $\varepsilon$ :

$$Y_{it} = Y_t \left( \frac{P_{it}}{P_t} \right)^{-\varepsilon}, \quad (3)$$

where  $P_t$  is the price aggregator

$$P_t = \left( \int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

We use the Rotemberg (1982) assumption of quadratic adjustment costs to changing prices.<sup>11</sup> Specifically, the cost of changing the price from  $P$  to  $P'$  is  $\frac{\phi}{2} Y \left( \frac{P'}{P} - \bar{\Pi} \right)^2$  where  $\phi$  captures the magnitude of the costs,  $Y$  are firm sales, and  $\bar{\Pi}$  is a parameter capturing “indexation”, i.e. it is costless to have an inflation of  $\bar{\Pi}$ .

Each period, firms set their price so as to maximize

$$E_t \sum_{k=0}^{\infty} M_{t,t+k}^{\$} \left( P_{it+k} Y_{it+k} - w_{t+k} N_{it+k} - \frac{\phi}{2} Y_{it+k} \left( \frac{P_{it+k}}{P_{it+k-1}} - \bar{\Pi} \right)^2 \right),$$

subject to the demand curve (3) and the production function (2).

<sup>11</sup>Miao and Ngo (2015) illustrate that some results (such as the size of the fiscal multiplier) may be affected by the price setting assumptions at the ZLB.



In equilibrium, all firms choose the same price, and given quadratic adjustment costs, they adjust their price each period. A standard derivation for the optimal price yields a nonlinear version of the forward-looking Phillips curve:

$$0 = \left(1 - \varepsilon + \varepsilon \left(\frac{w_t}{Z_t}\right) - \phi(\Pi_t - \bar{\Pi})\Pi_t\right) Y_t + \phi E_t (M_{t+1}(\Pi_{t+1} - \bar{\Pi})\Pi_{t+1} Y_{t+1}),$$

where  $M_{t+1}$  is the real stochastic discount factor.

The resource constraint reads

$$C_t = \left(1 - \frac{\phi}{2}(\Pi_t - \bar{\Pi})^2\right) Y_t, \quad (4)$$

since we need to subtract price adjustment costs from output. The definition of gross domestic product similarly takes into account that price adjustment is an intermediate input:

$$GDP_t = \left(1 - \frac{\phi}{2}(\Pi_t - \bar{\Pi})^2\right) Y_t = C_t.$$

### 4.3 Fundamental Shocks

We assume that:

- (1) The preference shock follows an AR(1) process:

$$\log \beta_t = \rho_\beta \log \beta_{t-1} + \varepsilon_{\beta,t},$$

with  $\varepsilon_{\beta,t}$  iid  $N(0, \sigma_\beta^2)$

- (2) The level of TFP follows an AR(1) process:

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_{z,t},$$

with  $\varepsilon_{z,t}$  iid  $N(0, \sigma_z^2)$ .<sup>12</sup>

### 4.4 Monetary Policy Rule

We assume that the central bank uses the following policy rule:

$$Y_t^{\mathbb{S},(1)} = \max \left\{ 1, Y^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{GDP_t}{GDP_t^*}\right)^{\phi_y} \beta_t^{\phi_\beta} \right\} \quad (5)$$

where  $GDP_t^*$  is the natural level of GDP, i.e. the one which would occur if prices were flexible (and in the case of our model economy, is simply  $Z_t$  times a constant). The max operator simply reflects the truncation implied by the ZLB.

In a somewhat unusual fashion, we allow the central bank to react directly to the preference shock  $\beta_t$ . We do this because it illustrates more starkly the effects of the ZLB. As long as the ZLB does not bind, the central bank is able to offset completely the effects of the preference shock. Once the ZLB binds however, this is impossible. Note that offsetting preference shocks is the optimal policy in this model, leading to perfect stabilization of inflation and output absent the ZLB - the ‘‘divine coincidence’’ of Blanchard and Gali (2005).

<sup>12</sup>We also explored shocks to the growth rate of TFP, i.e.  $\Delta \log Z_t = \rho_z \Delta \log Z_{t-1} + (1 - \rho_z)\mu_z + \varepsilon_{z,t}$ .

## 4.5 Asset prices: Bonds

In order to simplify the numerical computation of the model, we study the prices of geometric consols rather than zero-coupon bonds. A nominal geometric consol with parameter  $\lambda$  pays 1\$ next period, then  $\lambda$ \$ the period after, then  $\lambda^2$ \$, and so on. A real consol with parameter  $\lambda$  has the same payoffs, but in units of final goods rather than in \$.<sup>13</sup>

Hence the price for a (nominal or real) consol with parameter  $\lambda$  satisfies the recursion:

$$q_t^{i,\lambda} = E \left[ M_{t+1}^i \left( 1 + \lambda q_{t+1}^{i,\lambda} \right) \right] \quad (6a)$$

for  $i \in \{\$, \text{real}\}$ . The yield is defined as

$$Y_t^{i,\lambda} = \frac{1}{q_t^{i,\lambda}} + \lambda \quad (7)$$

where  $i \in \{\$, \text{real}\}$ .

We now calculate the risk-neutral price (and yield), i.e. the price (and yield) that would occur if agents were risk-neutral. The difference between the yield and the risk-neutral yield is a measure of term premium. (It coincides with the standard definition for zero-coupon bonds.)

The risk-neutral price for a (nominal or real) consol with parameter  $\lambda$  satisfies:

$$q_t^{i,\lambda,RN} = E_t \left[ \left( 1 + \lambda q_{t+1}^{i,\lambda,RN} \right) E_t \left[ M_{t+1}^i \right] \right], \quad (8)$$

and we can define the risk-neutral yield similar to equation 7.

The holding period returns on consols is given by the standard formula:

$$R_{t+1}^{i,\lambda} = \frac{1 + \lambda q_{t+1}^{i,\lambda}}{q_t^{i,\lambda}} \quad (9)$$

where  $i \in \{\$, \text{real}\}$ .

We define the term premium as the difference between the log yield and the log risk-neutral yield:

$$TP_t^{i,\lambda} = y_t^{i,\lambda} - y_t^{i,\lambda,RN} \quad (10)$$

where  $i \in \{\$, \text{real}\}$ . Note that  $TP_t^{i,0} = 0$  by construction.

We define the *inflation term premium* as the difference between the nominal term premium and the real term premium:

$$ITP_t^\lambda = TP_t^{\$, \lambda} - TP_t^{\text{real}, \lambda}. \quad (11)$$

The slope of the yield curve is the difference between the log yield of a  $\lambda$ -consol and the log yield of a 0-consol:

$$SL_t^{i,\lambda} = y_t^{i,\lambda} - y_t^{i,0} \quad (12)$$

where  $i \in \{\$, \text{real}\}$ .

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<sup>13</sup>For our simulations, we calculate these yields and prices for three values of  $\lambda$ . These values are chosen so that  $\lambda_1 = 0$ , and  $\lambda_2$  and  $\lambda_3$  are such that these consols have the durations of actual 5 and 10 year bonds.

## 4.6 Asset prices: Breakevens and Inflation premia

*Inflation breakevens* are the difference between the log nominal yield and the log real yield

$$BE_t^\lambda = y_t^{s,\lambda} - y_t^\lambda. \quad (13)$$

We define expected log inflation (over the lifetime of a consol) recursively as:

$$ELI_t^\lambda = (1 - \lambda)E_t(\log \Pi_{t+1}) + \lambda E_t(ELI_{t+1}^\lambda).$$

For  $\lambda = 0$ , this is simply the expected inflation next period, and for  $\lambda \rightarrow 1$ , this is the long-run average inflation in the future.

Last, the *inflation risk premium* is the difference between breakevens and expected inflation:

$$IRP_t^\lambda = BE_t^\lambda - ELI_t^\lambda.$$

Inflation risk premia are closely related to inflation term premia.<sup>14</sup>

## 4.7 Asset Prices: Stocks

Following Abel (1999), we define a stock as an asset with payoff  $D_t = C_t^\xi$ , where  $\xi > 1$  reflects leverage.

The real stock price satisfies the recursion

$$P_t^s = E_t [M_{t+1} (P_{t+1}^s + D_{t+1})],$$

so that if we define the P/D ratio as  $q_t^s = \frac{P_t^s}{D_t^s}$ , we have the following recursion for the P-D ratio:

$$q_t^s = E_t \left[ M_{t+1} (q_{t+1}^s + 1) \frac{D_{t+1}}{D_t} \right],$$

and the realized return on equity from  $t$  to  $t + 1$  is

$$R_{t+1}^e = \frac{P_{t+1}^s + D_{t+1}}{P_t^s} = \frac{q_{t+1}^s + 1}{q_t^s} \frac{D_{t+1}}{D_t}.$$

## 5 Quantitative Results

This section studies the quantitative implications of the model presented in the previous section. These results are preliminary - we are not yet at the stage where a “best-fitting” calibration can be presented. Rather, the goal for now is more to illustrate “comparative statics” - how some effects vary depending on whether the economy is close to the ZLB.

Due to the presence of the ZLB, we need to solve carefully the model using nonlinear methods. This is especially important because asset prices can be highly sensitive to nonlinearities. We use projection methods with cubic spline that build on the methods used in Ngo (2015) a Miao and Ngo (2015), and Fernandez-Villaverde et al. (2014).

<sup>14</sup>The appendix discusses the relation between the two concepts in the more common case of zero-coupon bonds.

Parameter	Description and source	Value
$\beta$	Subjective discount factor, RS (AEJ 2012), Woodford (2003)	0.99
$\alpha$	Curvature with respect to next period value	-145
$\chi_c$	IES is 0.15, RS (AEJ 2012)	6
$v$	Frisch labor supply elasticity is 0.4, RS (AEJ 2012)	2.5
$\chi$	Normalized to 1, e.g. Fernandez-Villaverde et al (JEDC 2015)	1
$\varepsilon$	Gross markup is 1.15, e.g. Fernandez-Villaverde et al (JEDC 2015)	7.66
$\phi_\pi$	Weight on inflation in the Taylor rule, Swanson (2015)	2
$\phi_y$	Weight on output in the Taylor rule , 0.75 in Swanson (2015)	0.9
$\phi_\beta$	Weight on preference shock in Taylor rule	-1
$\phi$	Adjustment cost, corresponding to the Calvo parameter of 0.85	238
$\rho_z$	Persistence of technology shock, 0.95 in SR12, 0 in Swanson (2015)	0.9
$\rho_\beta$	Persistence of preference shock	0.8
$\sigma_z$	unconditional std. dev. of the technology shock	0.012
$\sigma_\beta$	unconditional std. dev. of the preference shock	0.007

Table 7: Parameter values used and sources.

## 5.1 Parameter choices

Table 7 presents the parameters that we use together with the source of the value we use. Most of these parameters are standard in the New Keynesian literature. The level of risk aversion is large, to generate more plausible levels of risk premia. The level of price rigidity  $\phi$  is also somewhat higher than usual. The effective risk aversion is 208 (see Swanson (2013) for an explanation). Following Rudebusch and Swanson (RS, 2012), we consider small values of the IES of consumption and the Frisch elasticity of labor supply as they help generate a high average term premium, consistent with the data (far from the ZLB).

## 5.2 Response to shocks far from the ZLB

This section discusses the effects of technology and demand shocks when the economy is far from the ZLB.<sup>15</sup>

### 5.2.1 TFP shocks

As shown in the policy functions plotted in figure 8 or in the impulse response plotted in figure 9, an increase in productivity leads to higher consumption (and hence lower marginal utility) and higher stock prices. The latter reflects that stock price equals a present discounted value of dividends, which are proportional to consumption and hence increase with the productivity shock. Higher productivity also leads to lower inflation - as is typical with the New Keynesian model, since rigid prices prevent a

<sup>15</sup>Throughout the paper, we use “demand” and “preference” shock labels interchangeably.

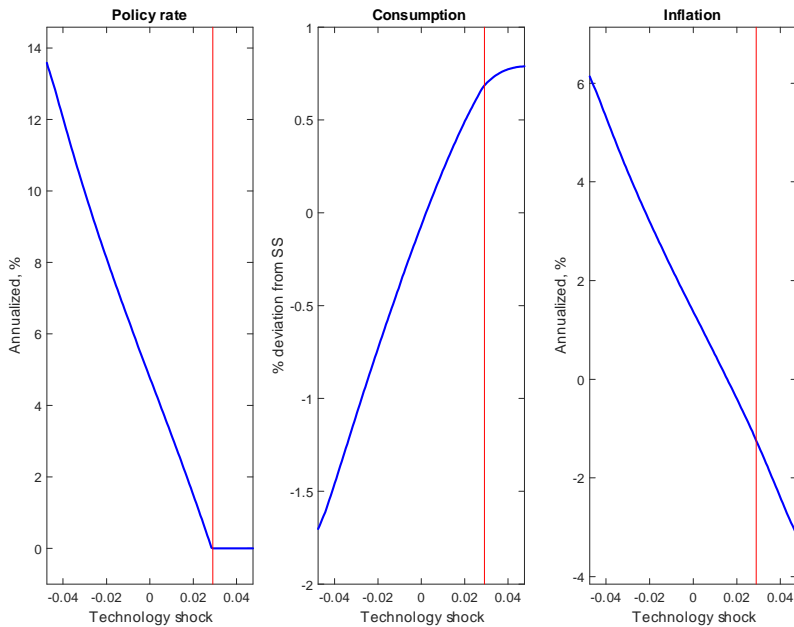


Figure 8: Policy functions - TFP shock.

full expansion of output. As a result, the covariance of consumption growth and inflation is negative, generating a positive inflation risk premium.

The effect of productivity on long-term interest rates depends both on the monetary policy rule and, mostly, on the process for productivity. Given that the later is mean-reverting, interest rates tend to go up when the level of productivity is low, since agents rationally expect higher real consumption growth in the future. This explains why in figure 10 the real rate falls with productivity. The breakeven rate reflects inflation expectations which are lower when productivity is high, since productivity is persistent. The nominal yield reflects both the real yield and inflation expectations, and hence is more strongly downward-sloping than the real yield. Overall, the real return on holding long-term bonds (nominal or real) is high if productivity goes up, given that yields fall with such a shock. Hence, long-term bonds are risky and require a positive risk premium, creating an upward-sloping yield curve.<sup>16</sup>

### 5.2.2 Demand shocks

Given the assumed monetary policy rule, the effect of demand shocks is small outside the ZLB. Hence, as shown for policy functions in the figure 11 and in the impulse response 12, the demand shock has only mild effects on consumption and inflation unless the ZLB binds. There is a large effect on stock prices since interest rates fall persistently with the demand shock. The effect on nominal and real long-term interest rates comes directly from the preference shock which is fully accommodated by the central bank. Given that consumption has little response to this shock, the term premium is not affected much by these bond responses. As a result, overall when the economy operates far from the ZLB, bond premia

<sup>16</sup>When the technology shock is a random walk, the real yield curve is approximately flat since expected growth is approximately constant (not exactly, owing to the effects of monetary policy on real growth). Swanson (2015) proposes a model that fits the yield curve with random walk shocks by considering a slightly different Taylor rule.

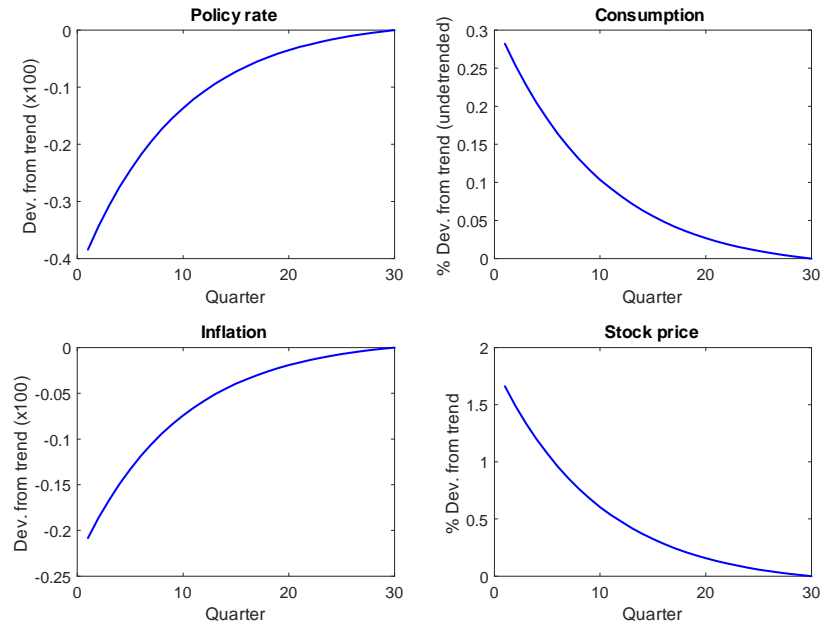


Figure 9: Impulse response to a 1% productivity shock, far from the ZLB.

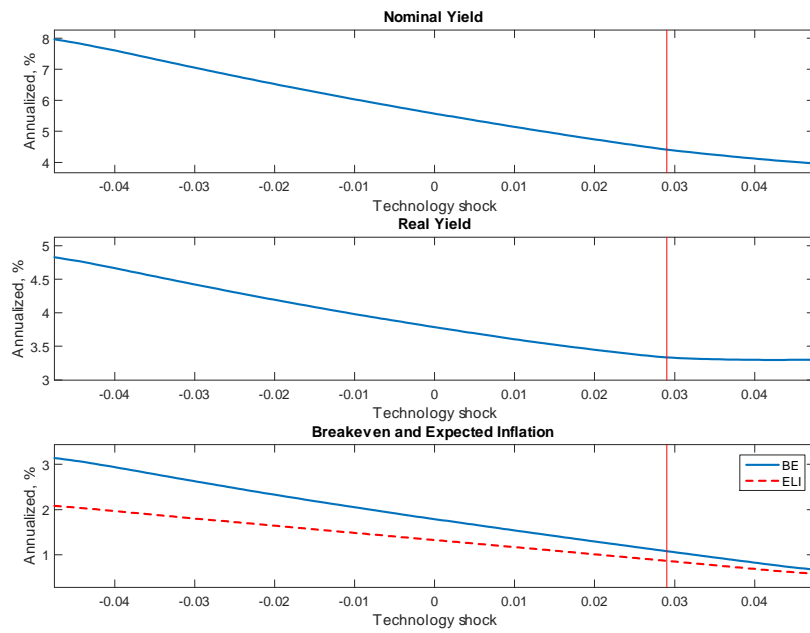


Figure 10: Policy function of long-term nominal and real yield and long-term breakeven and expected inflation, as a function of the TFP shock.

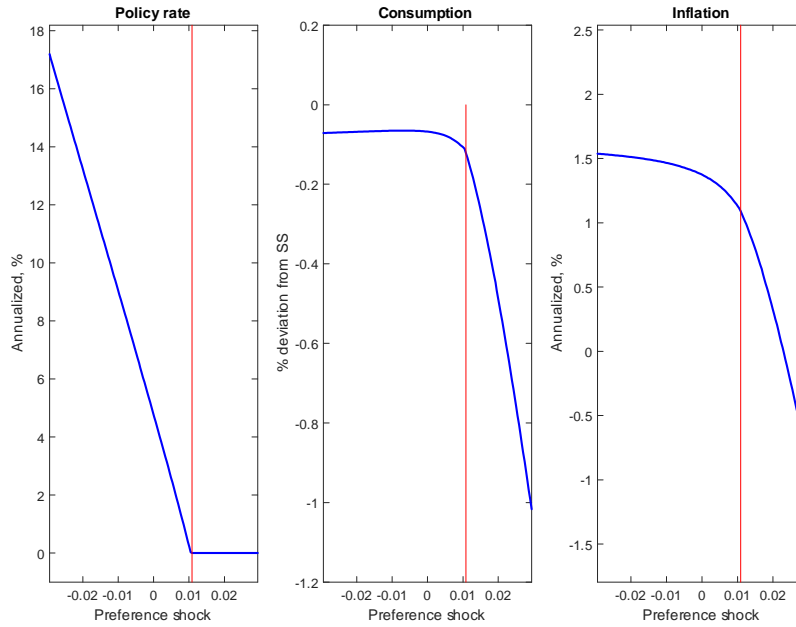


Figure 11: Policy function for the interest rate, consumption and inflation as a function of the preference shock.

reflect the effect of productivity shocks.

### 5.3 Response to shocks at the Zero Lower Bound

We again discuss separately the effect of demand and productivity shocks, but now focus on the case when the ZLB binds.

#### 5.3.1 Demand shocks

Once the ZLB binds, the response to shocks changes significantly. As shown in the policy function figure 11, once the ZLB binds, negative demand shocks lead to a significant decline of output and inflation, since monetary policy is unable to respond.<sup>17</sup> To illustrate the difference of response to a preference shock when the ZLB binds vs. not, we calculate an impulse response function (IRF) when the economy is at the ZLB and compare it to the steady-state IRF (i.e., the one shown in the previous section).<sup>18</sup> The blue line corresponds to the IRF shown above (effect of a demand shock if the economy starts

<sup>17</sup>Implicitly, we assume that fiscal policy is not used to offset this shock. Moreover, we abstract from so-called “unconventional” policies such as forward guidance or LSAP. We plan to explore these in the future. Note, however, that it is usually believed that unconventional policies are less efficient, more uncertain, and politically more risky, leading central banks to be more reluctant to pursue them (see Evans et al. (2015) for a discussion).

<sup>18</sup>The details of the calculations are as follows. We calculate the difference between two paths: (i) a path with a large shock to beta that brings the economy at the ZLB, and (ii) a path with the same shock, plus 1%. The difference gives us the effect of a 1% shock at the ZLB. We replicate the same calculation but instead of having a shock to beta that makes the ZLB bind, we just have a zero shock. The figure below plots these two differences (and obviously, the second one is the standard IRF discussed in the previous section). Note that we refer to IRF but do not employ the strict definition as the effect of a shock on the conditional expectation of a future variable,  $E_t y_{t+k} - E_{t-1} y_{t+k}$ . Rather, these are example paths when the economy starts in the nonstochastic steady-state and the shock sequence follow a deterministic process corresponding to the AR(1) shock. (There is a difference because the model is nonlinear.)

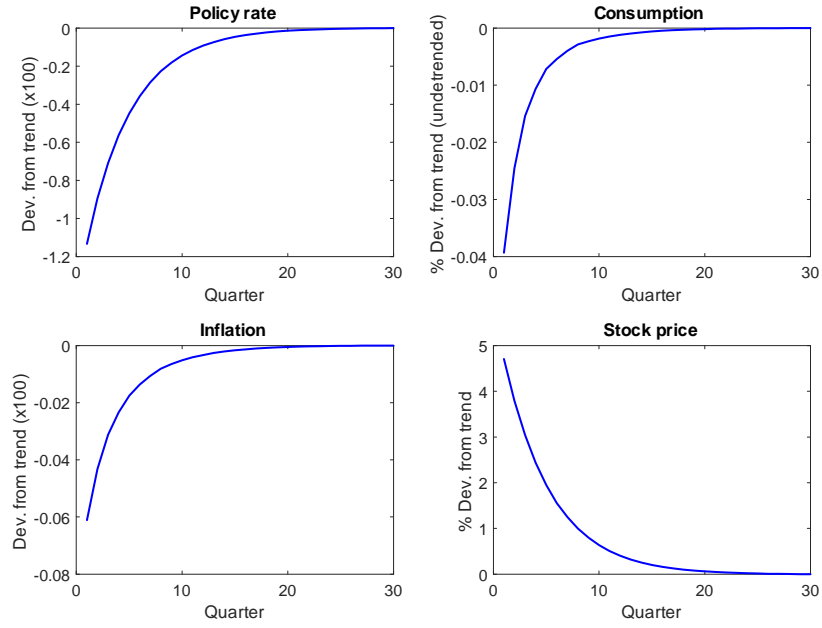


Figure 12: Impulse response to a 1% demand shock, far from the ZLB.

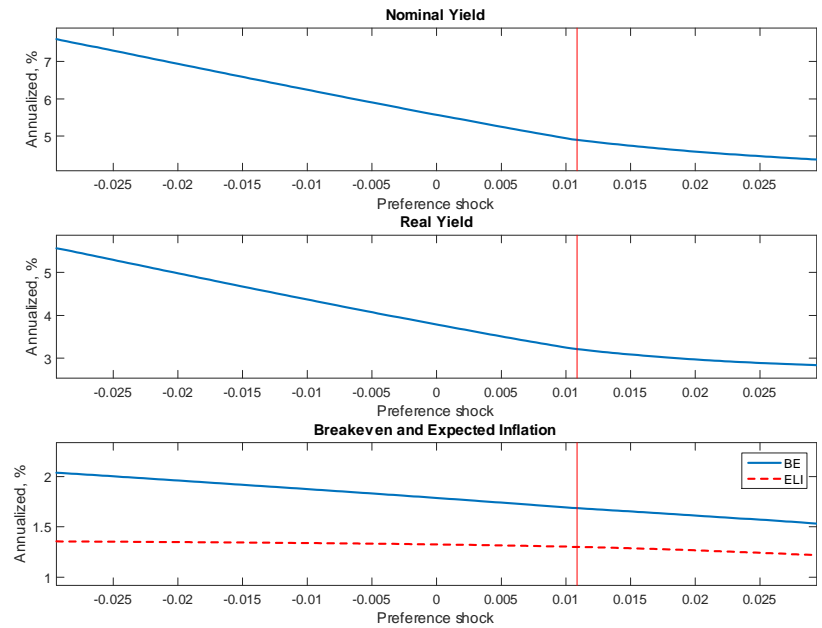


Figure 13: Policy function of long-term nominal and real yield and long-term breakeven and expected inflation, as a function of the preference shock.



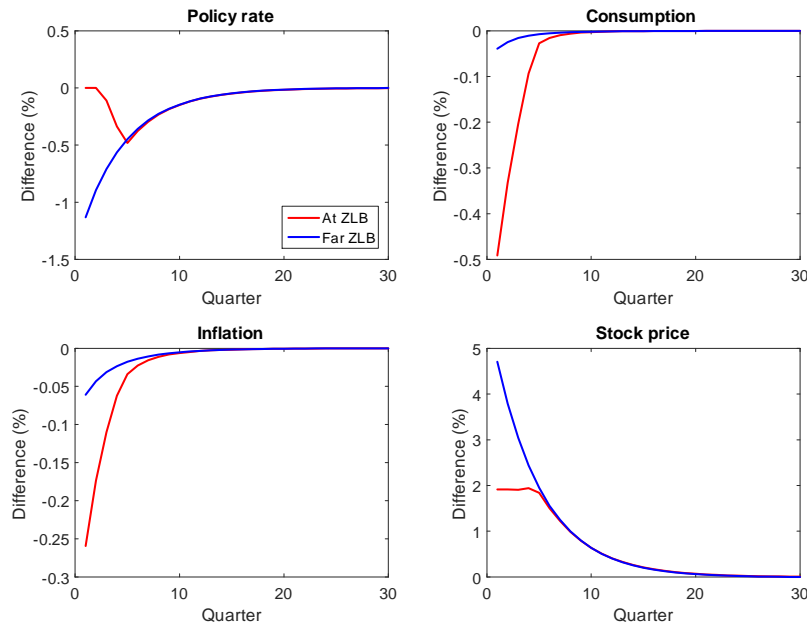


Figure 14: Impulse response to a 1% preference shock when the economy is at the ZLB vs. in steady-state.

far from the ZLB). The red line demonstrates that the economy responds very differently to the same shock if it starts at the ZLB. Specifically, the interest rate cannot respond near the ZLB. This leads consumption and inflation to drop much more significantly. Clearly, the covariance of consumption and inflation implied by this shock is much larger at the ZLB. As a result, stock prices do not rise as much as in the far from ZLB case since dividends (assumed to be proportional to consumption) fall.

### 5.3.2 Productivity shocks

We now demonstrate how the ZLB affects the propagation of productivity shocks. Figure 15 displays the effect of a 1% productivity shock when the economy is at the ZLB and when it is off the ZLB. In normal times, higher TFP leads to higher consumption, lower inflation, and a lower interest rate. However, when the ZLB binds, consumption rises less, and inflation falls much more significantly. (Consumption may even fall on impact, depending on parameter values.) The overall effect on the covariance of consumption and inflation is overall uncertain, but it tends to be less negative (i.e. the covariance increases) for most parameter values. Stock prices also tend to increase less at the ZLB as they mimic the path of consumption.<sup>19</sup>

## 5.4 ZLB and risk premia

The key result of this paper is that the covariance of consumption and inflation changes as the economy operates close to the ZLB. To illustrate this, we calculate the conditional covariance of consumption

<sup>19</sup>There is a large debate in the macroeconomics literature debating the empirical relevance of these model dynamics (e.g. Wieland (2014)). However, it is important to note that for the purpose of this paper, we do not actually require that consumption falls with positive productivity shocks. It is enough that consumption increases less, and inflation decreases more, to affect the key covariance of consumption and inflation.

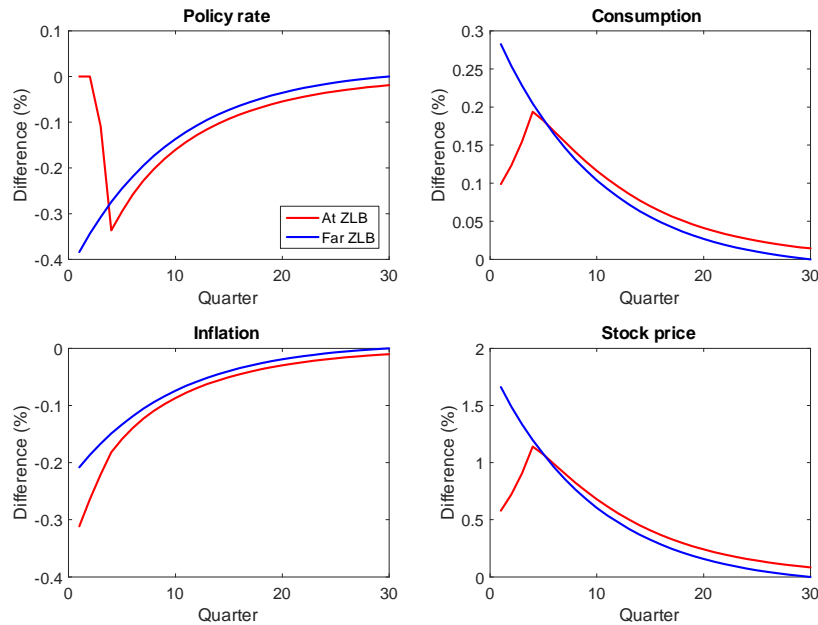


Figure 15: Impulse response to a 1% productivity shock when the economy is at the ZLB vs. in steady-state.

growth and inflation and plot it in figure 16 against the current value of the state variables (TFP and preference shocks; note that the ZLB binds when the economy is in the Southwest quadrant). We see that in normal times, the covariance is negative, but it rises substantially when the economy operates close to the ZLB.<sup>20</sup>

Figure 17 depicts the inflation risk and term premia for a 10-year equivalent consol, and shows that it is positive in normal times, but becomes smaller when the economy is close to the ZLB. This reflects the large change in the conditional covariance of consumption and inflation together with the high risk aversion. The nominal and real term premium also tend to fall as the economy becomes closer to the ZLB.

To understand the change in nominal and real bond premia, recall that the TFP shock generates a positive bond premium while the preference shock generates a negative one. At the ZLB, consumption reacts much more to the preference shock, which tends to increase the preference-shock induced risk premium. Inversely, consumption becomes less sensitive to TFP (as seen in the policy functions), which reduces the term premium from the TFP shock. On top of that, inflation becomes more procyclical as discussed above. Overall, these effects tend to reduce bond premia.

## 5.5 Simulated moments

We now simulate the model assuming that the economy is driven by both TFP shocks and preference shocks. Table 8 reports the moments both in full sample and in a sample “close” to the ZLB as well as “far” from the ZLB. A key point from this table is that the inflation risk premium goes from 52bps in the sample “far” from ZLB to 32bps in the sample “close”. This implies that a significant change in the

<sup>20</sup>We verify that the covariance of stock returns and breakevens, or stock returns and inflation, also tends to rise when the economy becomes closer to the ZLB. This validates our empirical strategy.

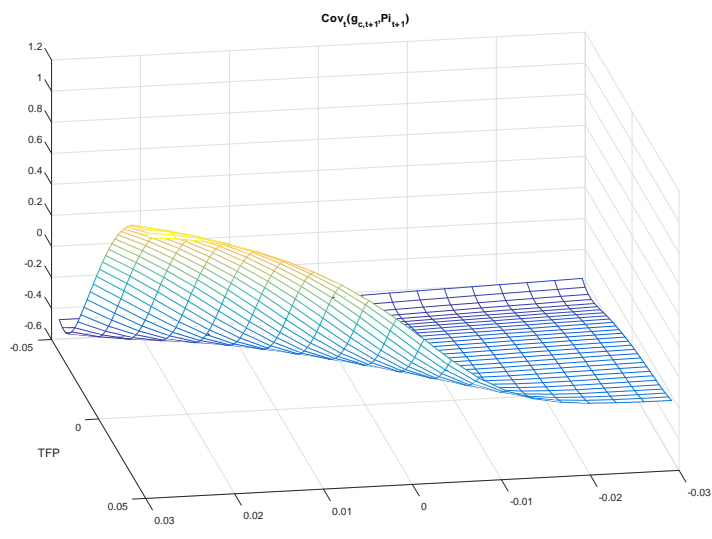


Figure 16: Conditional covariance of consumption growth and inflation next period, given the current state variables (TFP and preference shocks).

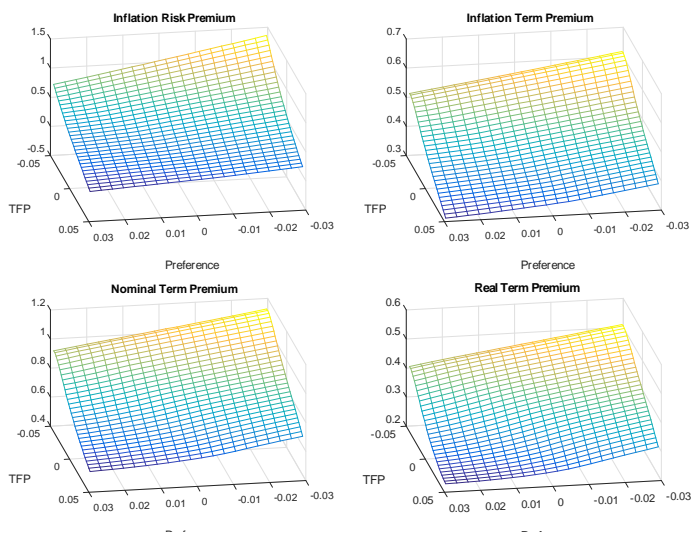


Figure 17: Inflation risk premium, inflation term premium, nominal term premium, and real term premium, as a function of the current state variables (TFP and preference shocks).

	Average		Conditional on $i < 1\%$		Conditional on $i > 3\%$	
	mean	sd	mean	sd	mean	sd
$\Delta \log Y$	0.00	0.84	0.14	1.03	-0.04	0.76
$\Delta \log N$	0.00	1.33	-0.24	1.18	0.07	1.38
$\pi$	1.34	1.10	0.16	0.93	1.71	0.95
$y^{s(1)}$	4.91	3.38	0.17	0.30	6.67	2.53
$y^{s(40)}$	5.63	0.70	4.68	0.23	5.97	0.54
$y^{(1)}$	3.51	2.89	-0.21	0.82	4.94	2.28
$y^{(40)}$	3.82	0.45	3.20	0.10	4.05	0.35
$BE^{(40)}$	1.81	0.31	1.48	0.22	1.92	0.28
inflation $RP$	0.47	0.13	0.32	0.06	0.52	0.11
nominal term premium	0.80	0.07	0.72	0.04	0.84	0.05
real term premium	0.37	0.04	0.32	0.02	0.38	0.03

Table 8: Simulated moment. Columns 1 and 2 give the mean and standard deviation. Columns 3-6 give the mean and standard deviation by subsamples ("close" to ZLB vs. "far" from ZLB).

decline of breakeven from the first sample to the second is not driven by a decline in expected inflation. As the breakeven declines from 1.92% to 1.48%, or 44bps, we see that 20bps correspond to risk premia and "only" 24bps to expectations. We also see in this table that, consistent with the figures above, the nominal and real term premia are lower when the economy operates close to the ZLB - nominal term premia fall by 12bps, and real term premia by 6bps. Hence, the inflation term premium also falls by 6(=12-6) bps. These magnitudes are fairly modest for our current calibration and model.

## 5.6 Effect of high risk aversion macroeconomic dynamics

Our model is a standard New Keynesian model with the ZLB, but with high risk aversion. How do these "nonstandard" preferences affect the responses of consumption and inflation, which have been studied extensively in the New Keynesian literature in models with low risk aversion?<sup>21</sup> In our current calibration, there is a small but significant effect of risk aversion on macro dynamics. The logic is as follows. When the economy hits the ZLB, macro volatility rises because the effect of preference shocks on consumption and inflation becomes larger. This higher volatility in turn leads to higher precautionary savings which reinforce the recession. This effect is stronger with high risk aversion. As a result, we observe that inflation and consumption fall more when the economy becomes closer to the ZLB in the case of high risk aversion, than in the case of low risk aversion. Figures 18 and 19 depict the response to preference shocks when the economy is far/close to the ZLB. We see that quantities fall by a larger amount in the model with high risk aversion.

<sup>21</sup>As in Eggertsson and Woodford (2003) for instance.

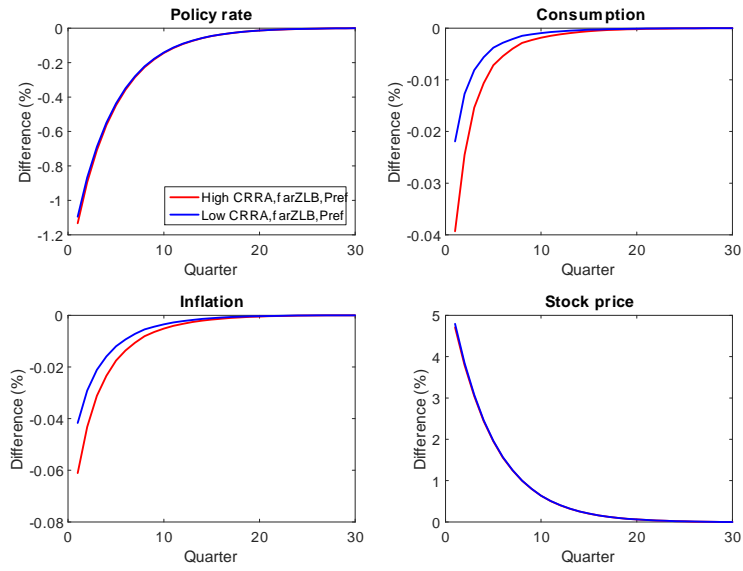


Figure 18: Comparison of responses to a preference shock for high and low risk aversion.

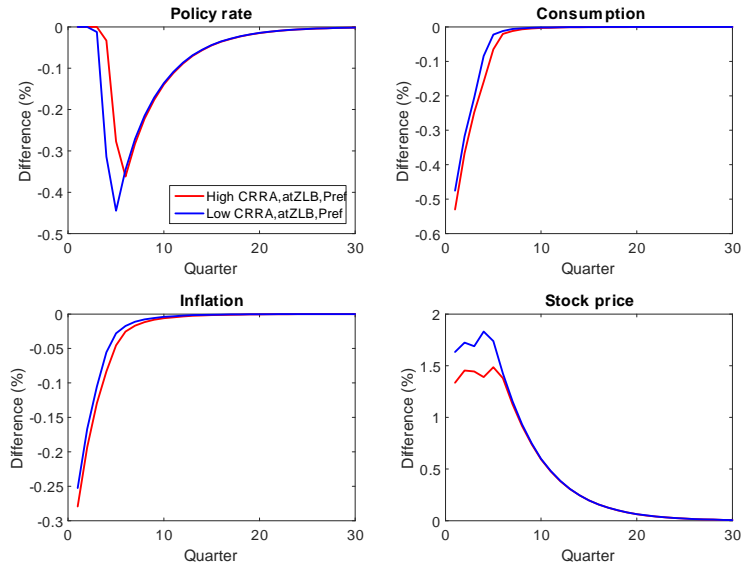


Figure 19: Comparison of responses to a preference shock when the economy is at the ZLB, for high and low risk aversion.

## 5.7 Role of Monetary Policy Rule

Our specification is somewhat nonstandard in that we allow the central bank to observe the preference shock  $\beta_t$  and react to offset it one-for-one (which is optimal if the ZLB does not bind). The main reason we do this is to obtain clearer results - the effect of demand shocks is very small far from the ZLB - but we also want to avoid a negative bond premium driven by the preference shock (for the reasons explained above).

It is true, however, that monetary policy could do even better. First, monetary policy could also react to the productivity shock in a more efficient way - and, absent the ZLB, could stabilize inflation (and the output gap) perfectly. Second, monetary policy could anticipate the possibility that the ZLB might bind, leading to sharper declines of interest rates close to the ZLB (Adam and Billi (2006)).

## 6 Conclusion

Financial markets data suggest that inflation, while it is typically associated with bad economic outcomes, became associated with good outcomes post 2008. A simple New Keynesian model that incorporates the zero lower bound can rationalize this. Demand shocks have much larger effects on inflation and consumption at the ZLB than off the ZLB, when monetary policy can largely offset them. The comovement of inflation with output changes considerably and tends to reduce inflation and term premia.

We plan to extend our study in a couple of directions. First, we will bring international evidence to bear on this question. Second, we plan to study the implications of different monetary policy rules. Finally, while the ZLB may be the most natural explanation of our empirical findings, alternative stories deserve to be explored quantitatively. For instance, it is plausible that higher inflation was perceived as beneficial because it facilitates household deleveraging.

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## 8 Appendix

### 8.1 Detrended system

We denote with a  $\tilde{\cdot}$  the variables detrended by  $Z_t$ . That is,

$$\begin{aligned}\tilde{C}_t &= \frac{C_t}{Z_t}, \\ \tilde{Y}_t &= \frac{Y_t}{Z_t},\end{aligned}$$

and so on. The system of equations to solve is hence:

(1) Taylor rule

$$Y_t^{\$, (1)} = \max \left\{ 1, Y^* \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left( \frac{\widetilde{GDP}_t}{GDP^*} \right)^{\phi_y} \beta_t^{\phi_\beta} \right\} \quad (14)$$

(2) Resource constraint:

$$\tilde{C}_t = \left( 1 - \frac{\phi}{2} (\Pi_t - \bar{\Pi})^2 \right) \tilde{Y}_t \quad (15)$$

(3) Production Function

$$\tilde{Y}_t = N_t \quad (16)$$

(4) Phillips curve:

$$0 = (1 - \varepsilon + \varepsilon \tilde{w}_t - \phi(\Pi_t - \Pi)\Pi_t) \tilde{Y}_t + \phi E_t \left[ e^{\Delta \log Z_{t+1}} M_{t+1}^{\$} (\Pi_{t+1} - \Pi) \Pi_{t+1} \tilde{Y}_{t+1} \right] \quad (17)$$

(5) Euler equation

$$1 = E_t \left[ Y_t^{\$, (1)} M_{t+1}^{\$} \right] \quad (18)$$

(6) Labor supply

$$\tilde{w}_t = \chi N_t^\nu \tilde{C}_t^\sigma \quad (19)$$

(7) Utility - in this case we define

$$\tilde{V}_t = \frac{V_t}{Z_t^{1-\sigma}}$$

and we now have:

$$\tilde{V}_t = (1 - \bar{\beta}) \left( \frac{\tilde{C}_t^{1-\sigma}}{1-\sigma} - \frac{\chi N_t^{1+\nu}}{1+\nu} \right) + \bar{\beta} \beta_t E_t \left( \tilde{V}_{t+1}^{1-\alpha} e^{(1-\alpha)(1-\sigma)\Delta \log Z_{t+1}} \right)^{\frac{1}{1-\alpha}} \quad (20)$$

(8) Real SDF:

$$\begin{aligned}M_{t,t+1} &= \bar{\beta} \beta_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \\ &= \bar{\beta} \beta_t \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} e^{-\sigma \Delta \log Z_{t+1}} \left( \frac{\tilde{V}_{t+1} e^{(1-\sigma)\Delta \log Z_{t+1}}}{E_t \left( \tilde{V}_{t+1}^{1-\alpha} e^{(1-\alpha)(1-\sigma)\Delta \log Z_{t+1}} \right)^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \\ &= \bar{\beta} \beta_t \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\sigma} e^{-(\sigma+\alpha(1-\sigma))\Delta \log Z_{t+1}} \left( \frac{\tilde{V}_{t+1}}{E_t \left( \tilde{V}_{t+1}^{1-\alpha} e^{(1-\alpha)(1-\sigma)\Delta \log Z_{t+1}} \right)^{\frac{1}{1-\alpha}}} \right)^{-\alpha}\end{aligned} \quad (21)$$

(9) Nominal SDF:

$$M_{t,t+1}^{\$} = \frac{M_{t,t+1}}{\Pi_{t+1}} \quad (22)$$

(10) Definition of potential as measured by the Central Bank:

$$\widetilde{YPOT}_t = e^{\phi_{ybar}(\mu_z - \Delta \log Z_t)} \widetilde{YPOT}_{t-1}^{\phi_{ybar}} \widetilde{C}_t^{1-\phi_{ybar}} \quad (23)$$

The state variables are  $YPOT_t$ ,  $\Delta \log Z_t$ , and  $\log \beta_t$ . This is a system of 12 equations (once we add the shock law of motion) in 12 unknowns:

$$Y_t^{\$, (1)}; \widetilde{YPOT}_t; \widetilde{Y}_t; \widetilde{C}_t; N_t; M_{t+1}, M_{t+1}^{\$}, \widetilde{V}_t; \widetilde{w}_t, \Delta \log Z_t \log \beta_t.$$

## 8.2 Calculation of asset prices

Recall that the stock is an asset with payoff  $D_t = C_t^{\xi}$ , and that the real stock price satisfies the recursion

$$P_t^s = E_t [M_{t+1} (P_{t+1}^s + D_{t+1})],$$

so that if we define the P/D ratio as  $q_t^s = \frac{P_t^s}{D_t^s}$ , then we need to solve the recursion for the P-D ratio:

$$q_t^s = E_t \left[ M_{t+1} (q_{t+1}^s + 1) \frac{D_{t+1}}{D_t} \right],$$

and the return on equity from  $t$  to  $t+1$  is

$$R_{t+1}^e = \frac{P_{t+1}^s + D_{t+1}}{P_t^s} = \frac{q_{t+1}^s + 1}{q_t^s} \frac{D_{t+1}}{D_t}.$$

We can define detrended dividend as

$$\widetilde{D}_t = \frac{D_t}{Z_t^{\xi}},$$

and hence

$$\widetilde{D}_t = \widetilde{C}_t^{\xi},$$

and hence we can solve for the P-D ratio  $q_t^s$  (which is stationary, so no need for detrending) using the recursion

$$q_t^s = E_t \left[ M_{t+1} (q_{t+1}^s + 1) \frac{\widetilde{D}_{t+1}}{\widetilde{D}_t} e^{\xi \Delta \log Z_{t+1}} \right].$$

We could also define the detrended price

$$\widetilde{P}_t^s = \frac{P_t^s}{Z_t^{\xi}},$$

and use the recursion

$$\widetilde{P}_t^s = E_t \left[ M_{t+1} (\widetilde{P}_{t+1}^s + \widetilde{D}_{t+1}) \right] e^{\xi \Delta \log Z_{t+1}}.$$

All the quantities defined for bonds do not require detrending since interest rates and inflation are stationary.

### 8.3 Definition of asset price moments using zero-coupon bonds

In the paper we define asset pricing object (e.g., the term premium) using our “geometric consol” assets. For clarity and completeness, this section defines the same object for the more standard zero-coupons assets. Let  $P_t^{(n)}$  the price of a zero coupon real bond (in real terms). We have

$$P_t^{(n)} = E_t \left( M_{t+1} P_{t+1}^{(n-1)} \right)$$

and  $P_t^{(0)} = 1$ , where  $M_{t+1}$  is the real SDF. We define the yield as

$$\frac{1}{\left(1 + Y_t^{(n)}\right)^n} = P_t^{(n)}$$

In log, let  $p_t^{(n)} = \log P_t^{(n)}$  and  $y_t^{(n)} = \log \left(1 + Y_t^{(n)}\right)$ , then

$$y_t^{(n)} = -\frac{1}{n} p_t^{(n)}.$$

The holding period return of a bond of maturity  $n$  is defined as

$$R_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}$$

We can have the same exact relationships with nominal yields. We denote them with a \$. Of course we need to use the nominal SDF, and the bond price is now the \$ price of a nominal bond.

The breakeven is defined as the difference of log yields:

$$BE_t^{(n)} = y_t^{\$(n)} - y_t^{(n)},$$

and the inflation risk premium is defined as the difference between breakevens and expected log inflation:

$$IRP_t^{(n)} = BE_t^{(n)} - ELI_t^{(n)}$$

where, denoting  $Q$  the price level:

$$ELI_t^{(n)} = E_t \left( \log \frac{Q_{t+n}}{Q_t} \right) = E_t \sum_{k=1}^n \pi_{t+k}$$

where  $\pi_{t+1} = \log \left( \frac{Q_{t+1}}{Q_t} \right)$  is log inflation.

The term premium is defined as the difference between the log yield of a  $n$ -period bond and the expected short rate over  $n$  periods, for  $n \geq 2$ :

$$TP_t^{(n)} = y_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t y_{t+k}^{(1)}.$$

We can use this definition for nominal or for real term premia.

The inflation term premium is defined as be the difference between the nominal and real term premium. This is close, but not exactly equal, to the inflation risk premium. To see this, note that

$$\begin{aligned} ITP_t^{(n)} &= TP_t^{\$(n)} - TP_t^{(n)}, \\ &= y_t^{\$(n)} - y_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left( y_{t+k}^{\$(1)} - y_{t+k}^{(1)} \right), \\ &= BE_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left( BE_{t+k}^{(1)} \right), \end{aligned}$$

Company Name	rank of (-m	beta_CPIc
SEAGATE TECHNOLOGY PLC	489	-13.8341
INTUITIVE SURGICAL INC	201	-8.00282
FIRSTENERGY CORP	200	-7.42508
WILLIAMS PARTNERS L P	208	-7.41301
ANALOG DEVICES INC	349	-7.27096
A E S CORP	394	-6.71653
ECOLAB INC	209	-5.73729
WATSON PHARMACEUTICALS INC	470	-5.69494
AGILENT TECHNOLOGIES INC	305	-5.50695
ABBOTT LABORATORIES	29	-5.42261
RESEARCH IN MOTION LTD	480	-4.79886
GOODRICH CORP	234	-4.78433
ALTERA CORP	311	-4.77769
TIFFANY & CO NEW	428	-4.68851
COMCAST CORP NEW	60	-4.58358
ACCENTURE PLC IRELAND	95	-4.53146
K L A TENCOR CORP	447	-4.49506
MOODYS CORP	487	-4.48298
GENERAL MILLS INC	141	-4.42412
XILINX INC	431	-4.3596
TELLUS CORP	451	-4.31558
MAXIM INTEGRATED PRODUCTS INC	475	-4.31163
CONAGRA INC	339	-4.25522
TEXAS INSTRUMENTS INC	100	-4.17465
KRAFT FOODS INC	39	-4.17177
COMCAST CORP NEW	261	-4.14002

and since  $y_{t+k}^{s(1)} - y_{t+k}^{(1)} = BE_{t+k}^{(1)} = IRP_{t+k}^{(1)} + ELI_{t+k}^{(1)}$ , we have

$$ITP_t^{(n)} = BE_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left( IRP_{t+k}^{(1)} + ELI_{t+k}^{(1)} \right)$$

and since  $ELI_t^{(n)} = E_t \sum_{k=1}^n \pi_{t+k} = E_t \sum_{k=1}^n ELI_{t+k}^{(1)}$ , we have

$$\begin{aligned} ITP_t^{(n)} &= BE_t^{(n)} - ELI_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left( IRP_{t+k}^{(1)} \right) \\ &= IRP_t^{(n)} - \frac{1}{n} \sum_{k=0}^{n-1} E_t \left( IRP_{t+k}^{(1)} \right). \end{aligned}$$

In the case where the inflation risk premium is constant, then  $ITP_t^{(n)} = IRP_t^{(n)} - IRP_t^{(1)}$ . Generally, since the inflation risk premium is thought to be small at short horizons  $IRP_t^{(1)} \simeq 0$ , and the difference between the inflation term premium and the inflation risk premium ought to be small.

## 8.4 Numerical method

Our numerical method follows Miao and Ngo (2015) and Fernandez-Villaverde et al. (2012). We use projection methods with cubic splines. (Details to be added.)

## 8.5 List of stocks

Lowest inflation betas as of 2011 Highest inflation betas as of 2011

POTASH CORP SASKATCHEWAN INC	89	6.362555
BANK OF AMERICA CORP	48	6.399535
YAMANA GOLD INC	334	6.720925
WELLS FARGO & CO NEW	14	6.783916
MARKET VECTORS E T F TRUST	405	6.808509
RED HAT INC	457	6.953241
JOY GLOBAL INC	461	7.054629
M & T BANK CORP	374	7.301363
NEWMONT MINING CORP	120	7.326789
HARLEY DAVIDSON INC	399	7.361314
AMERICAN EXPRESS CO	49	7.511855
MOSAIC COMPANY NEW	237	7.756494
A F L A C INC	181	7.965894
SILVER WHEATON CORP	358	8.104337
KEYCORP NEW	492	8.391482
ELDORADO GOLD CORP NEW	476	8.662218
C N H GLOBAL N V	419	8.955379
GOLDCORP INC NEW	88	9.136707
KINROSS GOLD CORP	292	9.315031
CAPITAL ONE FINANCIAL CORP	190	10.27686
INTERNATIONAL PAPER CO	293	10.57276
SUNTRUST BANKS INC	379	11.58631
AMERICAN INTERNATIONAL GROUP INC	68	12.5351