Pre-Distribution: Bargaining over Incentives with Endogenous Production

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Abstract

This article sets forth a model of multilateral negotiations with alternating offers and voting in which members bargain over the distribution of equity shares prior to making investment decisions that determine the surplus to distribute. We challenge the classical assumption embedded in the divide-the-dollar setting that the fund to distribute is exogenoulsy given. Bargaining over productive incentives is fundamentally a different problem that yields an allocation of the surplus which does not coincide with the exogenous fund case. The proposer faces a trade-off between rent-sharing and rent-generation subject to the constraint of receiving a majority vote. If he attempts to offer small equity shares to coalition partners, the size of the pie might shrink, rendering his large ownership percentage unfruitful. In equilibrium, the proposer forms a minimum winning coalition by offering two types of shares. Some members are incentivized to invest in the common project and they receive a share that yields a payoff greater than the ex ante value of the game, thus the proposer's rent-seeking incentives are mitigated by the production constraints. The remaining coalition partners are offered a share that *buys* their vote without inducing investment. We provide a new view on efficiency in bargaining games which relates to the approved incentive scheme, and not only the timing of the approval.

I. Introduction

It is quite common that people explicitly or implicitly agree to a rent-sharing scheme prior to engaging in mutually beneficial activities such as starting a business partnership, fighting a common enemy by forming a military coalition, or coauthoring a book. In many cases, the agreements that govern the relationship between two or more parties are the result of a negotiation process and the agreements that are reached determine simultaneously productive incentives and profit-sharing rules.[1] In this article, we set forth a game of *pre-distributive* multilateral bargaining in which economic agents determine a division of equity shares prior to engaging in productive activities. Our aim is to model a situation where all players are part of the bargaining process and production occurs after a profit-sharing agreement has been reached. We seek to analyze how the bargaining institutional variables such as the committee size, voting rule, and discount factor affect rent-seeking and rent-generating incentives.

While contracting theory has mostly been focused on how to provide the right incentives to create a surplus, bargaining theory has been mostly concerned with how to redistribute an existing surplus. In fact, the classical models of structured bargaining with alternating offers (Rubinstein (1982), Baron and Ferejohn (1989)) and the extensive generalizations that have followed[2], have assumed the existence of a fund which is to be divided via bargaining. This assumption might be appropriate in settings such as legislatures where committee members bargain over the distribution of a budget that has been generated by others, but it is certainly not suitable for the partnership context which we seek to study here. Here we show that

[1] One can think of situations in which there is no prior negotiation because a default rule is implicitly agreed upon. For example, in certain disciplines it is a common practice to list the names of coauthors in descending order of importance and those involved in the project know ex ante who is the lead researcher. In other disciplines, authors are listed in alphabetical order implying an equal split of the intellectual credit.

[2] For an extesive treatment of bilateral bargaining with alternating offers see Osborne and Rubinstein (1990). For a generalization of the Baron and Ferejohn (1989) model see Eraslan (2000). See Jackson and Moselle (2002) for a model that combines private and public goods in the multilateral bargaining setting and Yildirim (2007) for rent-seeking behavior in proposal rights. See Merlo and Wilson (1995) for a model of stochastic bargaining. For a brief review of experimental results regarding the Baron and Ferejohn game see Palfrey (2005) and Frechette, Kagel, and Morelli (2005).

bargaining over productive incentives is, indeed, a different problem because it poses a trade-off between productive and appropriative incentives which is absent in the classical setting. In our model, the two approaches coincide only for a restricted set of parameters in the limiting case when players are perfectly patient. We derive novel results regarding the distribution of the surplus and provide a new view on efficiency in bargaining games.

The model developed in this article employs the Baron and Ferejohn (1989; BF hereafter) game of multilateral bargaining as the negotiations protocol for establishing equity shares. In the original BF bargaining game[3], players negotiate with alternating offers and voting over the allocation of an exogenous fund.[4] The BF game is a cornerstone model of political economy and multilateral bargaining because it offers very clear predictions about the equilibrium bargaining outcome.[5] First, proposers disburse funds only to the smallest coalition of members required for approval of the proposal and those members included in the coalition are offered a share that exactly matches the continuation value of the game. Second, the proposer keeps a larger portion of the fund which is decreasing in the voting quota. Third, bargaining outcomes are efficient in the sense that proposals are accepted without delay.

In our model, players instead bargain over the distribution of equity shares which determine simultaneously productive incentives and a profit-sharing rule. Once an agreement is reached, partners proceed to an investment stage in order to determine the total fund. We consider a very simple production technology (linear and symmetric) because our aim is to describe the strategic interaction between the bargaining environment and the provision of productive incentives, and avoid the complications that might arise with a richer production

[3]Under the closed amendment rule, proposals on the floor are voted on without the possibility to modify them. Under the open rule, a proposal has to be seconded to take the floor for voting. If an amendment is proposed, then another member has to second the proposal. We focus on the closed rule because it has received much attention in the theoretical and experimental literature.

[4]In the BF game, a randomly selected member is called to propose a distribution of the common fund and then all members proceed to vote. If a majority of partners votes in favor, the game ends and payoffs are realized. If the voting quota is not met, the process repeats itself (and usually the fund is discounted).

[5]In this article we will focus on the stationary subgame perfect equilibrium refinement (SSPE) and refer to it as *equilibrium*.

context.

When bargaining over equity, the proposer faces a trade-off between rent-sharing and rent-generation. If he attempts to offer a small share of equity to coalition partners, the size of the pie might shrink, rendering his large ownership percentage unprofitable. In equilibrium, the proposer builds a coalition by offering two types of shares: Some members are incentivized to contribute (denoted hereafter as *productive members*) and others are simply offered a share that *buys* their vote without inducing contribution. Productive members receive a payoff that is greater than the continuation value of the game, while non-productive coalition partners receive a share that induces a payoff equal to the ex ante value of the game. Hence we are able to show that rent-extraction is mitigated by productive incentives. Redundant members (those whose vote is not required for approval) receive zero equity.

One application of our model can be found in the formation of international alliances as studied in the field of international relations. Several studies have seeked to answer why members of an alliance bear different burdens, especially in military alliances as measured by investments in military operations (see Olson and Zeckhauseer 1966, Thies 1987, Sandler 1993, Sandler and Hartley 2001). Some authors have argued that an alliance creates a public good and differences in valuations for the public good will generate differences in investments or expenditures between members. However, not all authors agree that alliances create pure public goods, and instead attribute the differences in burdens to the individual and private incentives that allies might have.[6] Our model takes the latter stance without imposing any asymmetry between potential allies. Instead, every member is ex ante equal. Ex post asymmetries resulting from the bargaining outcomes assign property rights over the benefits of the alliance which induce differences in investment levels or burdens.

Pre-distributive bargaining also provides a new perspective on the analysis of efficiency in bargaining games. In the redistributive bargaining context, efficiency has typically been

[6] See the seminal paper by Olson and Zeckhauser (1966) for the pure public goods nature of alliances. See Loehr (1973) and Sandler and Cauley (1975) for models challenging the Olson and Zeckhauser (1966) setting. See Sandler (1993) and Sandler and Hartley (2001) for comprehensive reviews of the literature on alliance formation and burden sharing.

measured by how costly it is to reach a decision, where the cost of delay is parametrized through the discount factor (shrinking of the total surplus) or players' impatience levels.[7] Both BF and Rubinstein (1982) predict efficient equilibria in the sense that agreements are reached in the first round of negotiations and the institutional variables such as the committee size and voting rule play no role. In the present setting, efficiency is not only about the timing of approval, but also about the agreement *per se* because total production will depend on the approved equity scheme. We find that efficiency, measured as total production, is inversely related to the voting requirement. With higher voting quotas, too much equity is diverted to *buying* votes and little is devoted to incentivize production, which implies that the unanimity voting rule results in the least efficient of outcomes. For two commonly used voting rules, unanimity and simple majority, efficiency is inversely related to the size of the committee. Thus, we provide a rationale for the existence of inefficiencies in committee decision-making processes which are not only due to the costs of bargaining (discounting), but also because of the distributional compromises that must be reached in order to implement a common project.

The equity feasibility constraint which states that the sum of ownership percentage shares must equal one, acts as a production technology constraint by setting a limit on the size of the fund that can be produced in equilibrium. Also, the fact that shares cannot be conditioned on productive decisions could be suppressing efficiency. For these reasons, we consider a variant of the model in which the proposer can condition compensations on members' contributions (we call this model contract bargaining). Here, the fully efficient outcome can be achieved regardless of the committee size, the voting rule, or the discount factor. In equilibrium certain members are offered a compensation that makes them slightly better off by contributing, but not enough to match the continuation value of the game. Hence, full efficiency does not entail unanimous approval.

The article proceeds as follows. Section 2 presents a literature review focused on multi-

[7] Rubinstein (1982) also considers a fixed cost of bargaining.

lateral bargaining games. Section 3 introduces the model. Section 4 solves the equilibrium and provides the comparative statics. Section 5 analyzes the setting in which partners can bargain over contracts contingent of investments. Section 6 concludes the paper. In the Appendix we present the main proofs.

II. Literature Review

The standard BF model under the closed amendment rule has been generalized in many directions and a comprehensive survey is provided in Eraslan and McLennan (2013). Eraslan (2002) considers the game in which players differ in their probability of being recognized as the proposer and hold different discount factors. It is shown that for a given parameter configuration, stationary subgame perfect equilibrium (SSPE) need not be unique but the ex ante values of the game are. Importantly, a player's ex ante value of the game is increasing in the probability of recognition and the discount factor. Eraslan and McLennan (2013) also show uniqueness of payoffs in a setting where the set of winning coalitions may vary for each proposer.

The BF game has also been studied in an environment in which the fund to distribute varies in time according to a stochastic process. Merlo and Wilson (1995) restrict attention to the unanimity voting rule and show that delay might arise in equilibrium because players are better off by waiting for a larger fund. Eraslan and Wilson (2002) consider any voting rule and show that when not every member's vote is required for approval, delay can also arise but it will be inefficient because agreements are reached too soon.[8]

In a closely related article, Baranski (2016) presents a game with the reverse timing: Players first engage in a production stage followed by a profit-sharing game of bargaining. Investments can be considered a sunk cost at the beginning of the bargaining stage, and if strategies are restricted to be history-independent, the resulting equilibrium prediction is

[8] See Eraslan and Merlo (2014) for a case where each proposer *brings* a total fund of different magnitude to be distributed (i.e. the proposer and fund stochastic processes are perfectly correlated).

that no one should contribute to the common fund. The reason is that the ex ante value of the bargaining game under the SSPE is equal to the average fund, which induces the same payoff structure as the linear public goods game.

Another application of the BF bargaining protocol to the firm context can be found in Britz, Herings, and Predtetchisnki (2013). In their setting, partners must agree unanimously over a future production plan for the firm in the midst of uncertainty. Differences in the risk attitudes of the committee members result in conflicts about which production plan to choose, but transfers that are payable prior to the realization of the state of nature are used to *grease the wheels* of the bargaining process. Their main result is that payoffs resulting from the equilibrium production plan and transfer scheme are equivalent to those specified by a generalized Nash bargaining solution where the relative bargaining power weights are given by the probability of being the proposer for each player.[9] Their model could be understood as a bargaining game of risk sharing, an aspect that is not present in the game of equity bargaining which we pose here.

The trade-off between productive and appropriative activities has been the object of study in political economy and organizational behavior. For example, Hirshleifer (1991), Skaperdas (1992), and Grossman and Kim (1995) develop models in which players can invest in *swords* as a means to appropriate others' production *or plowshares* that yield a given output. In these settings, too many swords are useless when there is little production, so that in equilibrium the marginal investment in appropriation activities yields the same return as the marginal investment in production. A similar trade-off is faced by the proposer in the game of equity bargaining studied here.

[9] Once they characterize the equilibrium in the payoff space, they show that there is a unique production plan and vector of transfers satisfying the Nash solution. This result holds in the limiting case where there is no discounting of future payoffs.

III. The Model

Let there be a committee with n players (odd) that are endowed with a wealth level normalized to 1. In stage 1, members bargain sequentially with alternating offers and voting on how to split the rents of a potential common fund that they will produce in stage 2. In each bargaining round (denoted by superscript t) within stage 1, a player is randomly called upon (with equal probability) to propose a scheme on how to divide a common fund. Denote by $\mathbf{s}^t = (s_1^t, ...s_n^t)$ a division of the fund such that $\sum s_j^t = 1$ and $s_j^t \in [0,1]$ for every player j. For each proposal on the floor, players vote to accept or reject and q votes are required for approval (including the proposer's vote).[10] In case of rejection, the proposal and voting rounds are repeated. Once an allocation is approved in round τ, players proceed to stage 2 in which they simultaneously choose a contribution $c_i \in [0,1]$. Each unit contributed is multiplied times $\alpha \in (1,q]$ and becomes part of the common fund.

Let h^t denote the history up to round t in the bargaining period, which includes the list of previous proposers and their proposals, as well as the voting record. We denote by H^t the set of all possible histories. A strategy for a proposer in round t is given by $s : H^t \to [0,1]^n$ and for voters it is given by $v : (s^t, H^t) \to \{Yes, No\}$.

A contribution strategy is a function $c_j : s^\tau \times H^\tau \to [0,1]$ where τ denotes the round in which the proposal is approved.[11] Player j's payoffs for a given profile of strategies $(\mathbf{s}^\tau, \mathbf{v}^\tau, \mathbf{c})$ are given by

$$u(\mathbf{s}^\tau, \mathbf{v}^\tau, \mathbf{c}) = \delta^{\tau-1}\left(s_j^\tau F - c_j + 1\right) \ , \tag{1}$$

where $\delta \in (0,1)$ is the discount factor and $F = \alpha \sum c_i(s^\tau, H^\tau)$ is the total profit of the committee.

[10] Voting takes place sequentially and in a known preestablished order. This avoids trivial multiplicities of equilibria.

[11] If the proposal is never approved, then by definition $\mathbf{c} = \mathbf{0}$ and payoffs are 0 to everyone.

IV. Equilibrium

We restrict attention to stationary subgame perfect equilibria, meaning that bargaining strategies are history-independent. We assume that a member votes in favor if and only if she is offered a share that yields a payoff that is equal to or greater than the continuation value of the game.[12]

DEFINITION 1 A stationary equilibrium of the equity bargaining game, denoted by the stationary strategy profile $\boldsymbol{\sigma}^* := (\mathbf{s}^*, \mathbf{v}^*, \mathbf{c}^*)$ induces a vector of ex ante values of the game given by $\mathbf{V}^* = (V_1^*, ...V_n^*)$ such that $\boldsymbol{\sigma}^*$ and $\mathbf{V}^*$ satisfy:

1. For every player i we have that $u(\boldsymbol{\sigma}^*) \geq u(\tilde{\sigma}_i, \boldsymbol{\sigma}^*_{-i})$ for any other strategy $\tilde{\sigma}_i$;

2. s is only a function of the state of non-agreement, $v : \mathbf{s} \rightarrow \{Yes, No\}$, and $c : \mathbf{s} \rightarrow [0, 1]$;

3. $v = yes$ if and only if $u(\boldsymbol{\sigma}^*) \geq \delta V_i^*$;

4. If there are multiple profiles that satisfy (1)-(3), we select the one that yields the highest payoff to the proposer.

Note that condition (4) imposes a selection criteria in case there are multiple stationary equilibria.[13] In order to simplify our analysis we make two additional assumptions about the parameter space. First, we require that the discount factor is not *too small.*[14]

ASSUMPTION 1 The discount factor satisfies that $\frac{n}{n+\alpha-1} < \delta \leq 1$.

The second assumption states that productivity cannot be *too low.*

ASSUMPTION 2 The productivity parameter satisfies that $1 + \frac{\delta n}{n-\delta q+\delta} < \alpha \leq q$.

[12] This is a standard assumption in the literature.

[13] In Appendix B we show by example that there can exist other equilibria which are more efficient in terms of production but yield a lower payoff to the proposer. When we relax condition (4) in Definition 1 we are not able to find a closed-form solution, hence we provide a general characterization that a computer algorithm could easily solve. See Proposition 6.

[14] The importance of this assumption becomes clear in Lemma 2. It is a sufficient condition that ensures that all members in the proposer's coalition are offered a positive share, and rules out equilibria in which members receiving zero equity would also vote in favor.

The role of each assumption will become clear as we solve the game, however none of the restrictions are necessary for the existence of an equilibrium.

IV.A Stage 2: Investment Subgame

We start in stage two by characterizing the possible subgames after approval. For any approved distribution of shares in round τ of stage 1 ($\mathbf{s}^\tau$), a player's equilibrium strategy in stage 2 is given by

$$c_i^*(\mathbf{s}^\tau) = \begin{Bmatrix} 0 & \text{if} & \alpha s_i^\tau < 1 \\ 1 & \text{if} & \alpha s_i^\tau \geq 1 \end{Bmatrix} \tag{2}$$

which simply states that a player only finds it optimal to invest if and only if she has a positive return. It is straightforward to show that at most $\|\alpha\| \leq q$ players can be induced to contribute since the sum of shares must be equal to 1. From now on, we say a member is *incentivized* or *productive* if she is given a share such that $c_i^*(\mathbf{s}^\tau) = 1$.

IV.B Stage 1: Bargaining

When players are bargaining over shares, they are implicitly bargaining over the associated payoffs. Hence, we study the *implicit bargaining game* in the payoff space for which there are well-established results in the literature that will be invoked throughout the process.[15]

We denote the proposer's share by s_{Prop} and the share offered to incentivized members by s_{Cont}. We allow for the possibility that certain members are offered a positive share that does not induce contribution and denote such share by s_{Vote}.[16]

Let k denote the number of productive members excluding the proposer (those to whom s_{Cont} is offered) and let m denote the number of voters to whom s_{Vote} is offered. It follows that the fund is given by

[15] See Britz, Herings, and Predtetchinski (2013) for a similar approach.

[16] Although it seems we are imposing a solution structure in solving the game, all we are doing is assuming that if two members are incentivized, then the offered shares are equal. In order to meet the voting quota, the proposer might require the votes of other non-incentivized partners whom are offered s_{Vote}. Again, we do not impose how many members receive this share.

$$F = \alpha(k+1) \quad . \tag{3}$$

We can rewrite the equity constraint as

$$s_{\text{Prop}} + k s_{\text{Cont}} + m s_{\text{Vote}} = 1 \quad . \tag{4}$$

The ex ante value of the game can be defined as

$$V(m,k;q,n,\alpha) := \frac{1}{n} s_{\text{Prop}} F + \frac{k}{n} s_{\text{Cont}} F + \frac{m}{n}(s_{\text{Vote}} F + 1) + \left(1 - \frac{1+k+m}{n}\right) \tag{5}$$

and it is a weighted average of the payoffs that a player receives in each possible role that she might find herself in.[17] The last term, $1 - \frac{1+k+m}{n}$, denotes the expected payoff from not being assigned equity.[18] Using the feasibility constraint we obtain

$$V(m,k;q,n,\alpha) = \frac{F}{n} + 1 - \frac{1+k}{n} = \frac{(\alpha-1)(k+1)}{n} + 1 \quad . \tag{6}$$

Equation (6) has the nice interpretation that the ex ante value of the game, for a given k, is equal to the average fund net of contributions plus the endowment.

In order for the fund to be attainable, three production consistency conditions must be met:

$$1/\alpha \leq s_{\text{Prop}} \leq 1 \quad , \tag{7}$$

$$1/\alpha \leq s_{\text{Cont}} \leq 1 \quad , \tag{8}$$

$$0 \leq s_{\text{Vote}} < 1/\alpha \ . \tag{9}$$

[17] To calculate this, we are imposing symmetry (what player j offers i is what i offers j). We also assume that proposers randomize over whom to offer s_{Vote} and s_{Cont}. Hence, in k/n times a member is offered s_{Cont} and in m/n she is offerd s_{Vote}.

[18] Notice that $1 - \frac{1+k+m}{n}$ is the probability of being assigned $s_i = 0$ which implies $c_i^* = 0$. This results in a payoff of 1 (keeping the endowment).

Conditions (7) - (9) guarantee that there are exactly $k+1$ productive members (including the proposer). Furthermore, we require that members to whom a positive share is offered find it optimal to vote in favor. Hence, the share offered must induce a payoff that is greater than or equal to the continuation value of the game. The voting consistency conditions are:

$$\begin{aligned} s_{\text{Prop}}F &\geq \delta V \quad , && (10) \\ s_{\text{Cont}}F &\geq \delta V \quad , && (11) \\ s_{\text{Vote}}F+1 &\geq \delta V \quad , && (12) \\ m+k+1 &\geq q \quad , && (13) \end{aligned}$$

where the last condition specifies that the proposer receives the necessary amount of votes.[19] We are now ready to present the maximization problem that a proposer faces:

$$\max_{\{k,m,s_{\text{Prop}},s_{\text{Cont}},s_{\text{Vote}}\}} s_{\text{Prop}} \cdot F \qquad (14)$$
$$\text{s.t. conditions (4) and (7)-(13)} \quad .$$

A few observations will help us rewrite the problem more concisely in terms of k only. Note that s_{Prop} is decreasing in m and that F is not dependent on m, so that the proposer will restrict the amount of s_{Vote} offers made to exactly meet the voting quota. Also, recall that $k \leq \|\alpha\| \leq q$. Hence, we have that $m = q-1-k$.

LEMMA 1 In equilibrium, $s_{\text{Cont}}(k) = \left\{ \begin{array}{ccc} 1/\alpha & \text{if} & k \geq \tilde{k} \\ \frac{\delta V}{F} & & \text{otherwise} \end{array} \right\}$ where $\tilde{k} := \frac{\delta n}{n-\delta(\alpha-1)} - 1$.

[19] Since we are working in the parameter space where $\alpha \leq q$ this implies that $k \leq q$.

PROOF. Conditions (8) and (11) imply that $s_{\text{Cont}} \geq \max\left\{1/\alpha, \frac{\delta V}{F}\right\}$ and we can show that

$$\begin{aligned} 1/\alpha &\geq \frac{\delta V}{F} \iff \\ k+1 &\geq \delta\left[\frac{(k+1)(\alpha+1)}{n}+1\right] \iff \\ k &\geq \frac{\delta n}{n-\delta(\alpha-1)} - 1 = \tilde{k}\ . \end{aligned}$$

Since the proposer's payoff decreases in s_{Cont}, in equilibrium it must be that constraints (8) and (11) bind. ∎

From condition (12) we solve for s_{Vote} to be

$$s_{\text{Vote}}(k) = \max\left\{\frac{\delta V - 1}{F}, 0\right\}$$

and again it is straightforward to verify that this constraint binds. In the following lemma we make use of Assumption 1 to determine $s_{\text{Vote}}(k)$.

LEMMA 2 If $\delta > \frac{n}{n+\alpha-1}$ then $\frac{\delta V-1}{F} > 0$ for all k.

PROOF. We have that $\delta > \frac{n}{n+\alpha-1} = \frac{1}{1+(\alpha-1)/n} \geq \frac{1}{1+(k+1)(\alpha-1)/n} \implies \delta\left[1+(k+1)(\alpha-1)/n\right] = \delta V > 1$ and the result follows. ∎

We are now able to write problem 14 as a function of the number of incentivized members (k):

$$\max_{k\in\{0,\ldots,\|\alpha\|\}} \Pi(k) := s_{\text{prop}} \cdot F = \left[1 - \frac{1}{\alpha} - k \cdot s_{\text{Cont}}(k) - (q-1-k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F(k)\ . \quad (15)$$

Temporarily, we drop the integrality requirement on k and solve the associated program (where $k \in \mathbb{R}$). In Appendix A (Lemma 4) we show that the optimal solution is given by

$$\bar{k} := \frac{1}{2}\frac{n(\alpha+\delta) - \delta q(\alpha-1)}{n-\delta(\alpha-1)} - 1 \quad (16)$$

and that $\bar{k} \in \left[\tilde{k} + \frac{1}{2}, \alpha\right]$, which sets us in the region where $s_{\text{Cont}}(k) = \frac{1}{\alpha}$. Since the objective function is a quadratic function in the region where the global maximum attains, the integer solution will be given by the closest integer to $\bar{k}$. We have that

$$k^* := \left\{ \begin{array}{c} \left\|\bar{k}\right\| \quad \text{if } \bar{k} - \left\|\bar{k}\right\| < 1/2 \\ \left\|\bar{k}\right\| + 1 \quad \text{otherwise} \end{array} \right\} \tag{17}$$

defines the optimal solution to problem (15).[20] In Corollary 4 in Appendix A we show that k^* is always feasible.

We summarize the equilibrium in the following proposition.

PROPOSITION 1 The equilibrium outcome of the equity bargaining game is as follows:

1. The proposer assigns $s^*_{\text{Cont}} = 1/\alpha$ to k^* members, $q - k^* - 1$ other members receive $s^*_{\text{Vote}} := s_{\text{Vote}}(k^*)$, and the proposer assigns herself $s^*_{\text{Prop}} := 1 - k^* s^*_{\text{Cont}} - (q - k^* - 1) s^*_{\text{Vote}}$. The remaining $n - q$ members receive a zero share.

2. All members offered s^*_{Cont} or s^*_{Vote} vote in favor.

3. The proposer and those who obtain a share s^*_{Cont} contribute all their endowment.

4. There is no delay in approval.

This equilibrium characterization presents two novel results in the multilateral bargaining literature. The first is that not every member of the coalition obtains the same payoffs. Note that non-productive coalition partners receive a share that yields exactly the continuation value of the game while productive members receive a higher share. This takes us to the second feature: productive incentives mitigate the proposer's rent-extraction capacities, hence the proposer cannot fully extract the rents of productive coalition members. In this

[20]It can be easily shown that $\bar{k} \geq \tilde{k}$ for any parameter values. Assumption 2 guarantees that $\bar{k} > \tilde{k} + \frac{1}{2}$ so that we can be sure that the closest integer to $\bar{k}$ yields the highest payoff.

sense, the payoffs from the equity bargaining game described above are not equivalent to the payoffs resulting from a standard BF.

We now relax Assumption 2 to show that there exist parameter values for which the standard BF game with an exogenous fund and the game of equity bargaining yield payoff equivalent outcomes.

As an example, let $\delta = 1$, $n = 5$, $q = 3$, and $\alpha = 2$ where one can verify that the the conditions in Assumption 2 do not hold. Note that, at most two members can be incentivized to produce because $s_{\text{Cont}} \geq \frac{1}{2}$, and if this were the case, the proposal would receive only two votes. Hence, it must be that only the proposer will produce, i.e. that $k^* = 0$. Here, every member of the coalition receives a share $s_{\text{Vote}} = \frac{1}{10}$ and the proposer's share is $s_{\text{Prop}} = \frac{8}{10}$ resulting in payoffs of $1 + \frac{1}{5}$ for coalition partners and $1 + \frac{3}{5}$ for the proposer. These are the same final payoffs that the standard BF game would predict if players were to be splitting a surplus of $\alpha - 1$ (production net of investments) and each player had an initial endowment of 1.

Proposition 2 (Baron and Ferejohn 1989 Proposition 2) Let $(k+1)(\alpha-1)$ be the total surplus to distribute and let every player be endowed with 1 unit of wealth prior to engaging in bargaining. Each member of the winning coalition earns a payoff of $1+\frac{\delta(k+1)(\alpha-1)}{n}$ and the proposer earns $1 + (k+1)(\alpha-1) - (q-1)\frac{\delta(k+1)(\alpha-1)}{n}$.

Proposition 3 Let $k^* \in \left\{0, ..., \left\|\tilde{k}\right\|\right\}$ be the optimal solution to problem (14) and $\delta = 1$. Then, the equilibrium payoffs of the game correspond to payoffs of the Baron and Ferejohn (1989) game in which players are initially endowed with 1 unit of wealth and the surplus to distribute is given by $(k^* + 1)(\alpha - 1)$.

The proof relies on the fact that $s_{\text{Cont}} = \frac{\delta V}{F}$ for $k \leq \tilde{k}$ and that non-productive coalition partners are offered a share $s_{\text{Vote}} = \frac{\delta V - 1}{F}$. These conditions imply that all coalition partners (except the proposer) earn a payoff that is equal to the continuation value of the game. It follows that $q-1$ coalition partners (except the proposer) earn $\delta V = \delta\left[\frac{(k+1)(\alpha-1)}{n} + 1\right]$ which is equal to $1 + \frac{\delta(k+1)(\alpha-1)}{n}$ (the payoff of a coalition member in the BF game) if and only if $\delta = 1$.

IV.C Comparative Statics

We are now ready to present various comparative statics results regarding the equilibrium size of the fund. We will focus on the equilibrium region where $s_{\text{Cont}} = 1/\alpha$ and k^* is given by (17).

Corollary 1 In equilibrium, the following results hold about $F(k^*)$:

1. it is weakly decreasing in q;

2. it is weakly increasing in n;

3. it is weakly decreasing in δ if $n < (q-\alpha)(\alpha-1)$ and weakly increasing if $n \geq (q-\alpha)(\alpha-1)$;

4. it is increasing in α.

Results (1), (2) and (4) of Corollary 1 are straightforward to verify by computing $\partial\bar{k}/\partial q$, $\partial\bar{k}/\partial n$, and $\partial\bar{k}/\partial\alpha$.[21] The first result implies that unanimity is the worst rule from an efficiency standpoint. When more votes are required for approval, the proposer must weigh the benefits of adding an extra productive or voting member and satisfy the feasibility constraint. It turns out that as q increases, adding an extra productive member is too expensive in terms of equity and the overall effect is a reduction in the fund.

The positive relationship between the equilibrium fund and the committee size is quite surprising. To see why this is the case, notice that when q is fixed, adding more members to the committee does not alter the voting constraint and that as n gets larger, the ex ante value of the game becomes smaller. This implies that the share offered to a non-productive

[21] The comparative statics in (1)-(3) specify weakly monotonic relations is due to the fact that a change in the variable in question might not be enough to induce a discrete change in k^*. We omit the proof because these are simple arithmetic calculations.

voter decreases and it can reach a point where the proposer has enough available equity to upgrade a non-productive coalition partner's share so that she becomes productive.

The relationship between the optimal fund and the discount factor is guided by more subtle dynamics. In equilibrium, as δ increases, the share offered to a non-productive coalition member (s_{Vote}) increases (for a fixed k), while s_{Cont} is constant. This means that the proposer must give up own equity if he wants to sustain the same level of production. Alternatively, he can take one of two paths: (1) sacrifice a productive member and replace her by a voting member or (2) replace a voting member by a productive member.[22] Summarizing, the proposer must weigh the payoffs from maintaining the level of output by sacrificing own equity, increasing output and sacrificing equity, and reducing the fund and augmenting own equity.[23] The optimal choice will depend on the parameter configuration, which we proceed to explain.

Fixing q and n, the inequality $n > (q-\alpha)(\alpha-1)$ is more likely to hold when either α is large (close to q) or small (close to 1), which set us in the region where the fund increases with δ. A large α makes s_{Cont} relatively cheap in terms of equity, thus making it more attractive to enhance a non-productive partner's share to productive levels. When α is small, replacing a voting member by a productive member induces a big loss of equity to the proposer but this loss is less than proportional to the percentage increase in production. Note that for a small α, the total fund is small as well. Thus, adding a productive member generates a large proportional increase in the fund.

For intermediate values of α, adding a productive member is no longer as cheap as it is for large α, nor does it induce a large proportional change in the size of the fund as it happens for small values of α. Hence, as δ increases, the proposer finds it optimal to replace a productive member by a voting member in this region, a decision that entails a reduction in the total fund.

[22] Notice that the replacements take place in order to meet the voting quota.

[23] When sacrificing a productive member, the proposer is able to increase his equity because $s_{\text{Vote}} < s_{\text{Cont}}$.

EXAMPLE 1 Let $n = 15$. For $q = 11$ and $\alpha = 11$ we have that $k^* = 6$ if $\delta = 1$ and $k^* = 5$ if $\delta = 0.5$. Hence, the fund decreases as δ decreases. The opposite effect happens when $n = 21$, $q = 18$ and $\alpha = 15$ because we have that $k^* = 5$ if $\delta = 1$ and $k^* = 6$ when $\delta = 0.5$.

We now examine the effect of committee size on the total fund for the simple majority and unanimity voting rules.[24]

COROLLARY 2 Larger committees yield (weakly) lower efficiency under the unanimity and simple majority voting rules.

A proof can be found in Appendix A. Recall that in Corollary (1) we had fixed the voting quota and considered a change in the size of the committee. Here, the voting quota is pegged to the size of the committee. Therefore, increasing the committee size entails that more votes need to be *bought* in order to obtain approval. Although non-productive coalition partners become cheaper, the decrease in equity that must be disbursed to them is surpassed by the increase in votes that must be *bought*.

V. Bargaining over Contracts

Previously, we considered a setting in which member's compensations could not be conditioned on contribution levels. We now relax this assumption and model a situation in which the proposer can offer members a contract of the form $f_i(c_i) = a_i c_i$ where a_i specifies the compensation per unit contributed by player i. A proposal in period t is denoted by $\mathbf{a}^t = (a_2^t, ...a_n^t)$ where the i^{th} entry is player i's contract. Without loss of generality we identify the proposer with the player index 1. In order to avoid unnecessary theoretical complications, we simply define the proposer as the residual claimant, i.e. his payoff is defined as the amount remaining after paying out contracts based on contributions.

At a terminal bargaining node τ in which the approved proposal is $\mathbf{a}^\tau$, a player's optimal

[24] Note that for the unanimity rule we have that $q = n$ and for the simple majority rule $q = (n + 1)/2$.

contribution strategy is given by

$$c_i^*(\mathbf{a}^\tau) = \left\{ \begin{array}{lll} 0 & \text{if} & a_i^\tau < 1 \\ 1 & \text{if} & a_i^\tau \geq 1 \end{array} \right\}. \tag{18}$$

The total fund is given by $F(k_c) = \alpha(1 + k_c)$ where k_c is the number of productive members excluding the proposer.

PROPOSITION 4 In equilibrium, $n - q$ members receive $a_i = 1$; $q - 1$ members receive $a_i = \max\{\delta\alpha, 1\}$ and total production is $F(k_c^*) = \alpha n$. A member votes in favor if and only if $a_i \geq \delta\alpha$. There is no delay in approval.

A sketch of the proof is provided and the details can be found in Appendix A. Note that the smallest a that induces contribution is $a = 1$. For each possible number of productive members, it always pays to incentivize an additional partner because the proposer appropriates a portion of the generated rents. Hence, everyone will obtain a contract in which a is at least 1. In order to meet the voting quota, the proposer must offer a contract such that $q - 1$ members will vote in favor. The minimum winning coalition receives a contract that yields a payoff equal to the continuation value of the game.

Note that when partners are very impatient (such that $\delta\alpha < 1$) every member receives the same contract ($a_i = 1$) and the proposer appropriates all the production net of investments. For comparison with the equity bargaining model, Corollary 3 contains the comparative statics.

COROLLARY 3 In equilibrium, the following results hold about $F(k_c^*)$:

1. it is constant in q and increasing in n;

2. it is constant in δ;

3. it is increasing in α.

The equivalence of payoff distribution between the contract bargaining game and the standard BF redistributive game holds for a broader range of parameters than for the equity bargaining game but it still a necessary condition that $\delta = 1$. It straightforward to note that all coalition partners (those who vote in favor) in the contract bargaining game receive the same payoff which matches the continuation value of the standard BF game only in the limiting case without discounting.

Proposition 5 The equilibrium payoffs of the contract bargaining game correspond to payoffs of the Baron and Ferejohn (1989) game in which players are initially endowed with 1 unit of wealth and the surplus to distribute is given by $(k_c^* + 1)(\alpha - 1)$ if and only if $\delta = 1$.

VI. Conclusion

We have shown that negotiating the division of a surplus does not yield the same equilibrium payoffs as the game in which players negotiate the incentives to generate a surplus. Until now, the bargaining literature has not made a distinction between the redistributive and pre-distributive approaches, yet we show that these are fundamentally different problems. Equivalence between the two approaches occurs only in a parameter region where productivity is not too high and there is no discounting.

Appropriation incentives on behalf of the agenda setter (proposer) predicted by the standard divide-the-dollar model can be mitigated if he faces the need to provide productive incentives because having too much equity is useless when the rents to appropriate are small. In equilibrium, the proposer offers a share that exactly induces certain members to produce, while other coalition partners are given shares large enough to buy their vote without incentivizing them to produce. Notably, productive members earn a payoff that is higher than the continuation value of the game, thus not every coalition partner obtains the same payoff and maximum rent extraction by the proposer is not possible.

We also confirm a common intuition that committees with higher voting requirements are less productive[25]. In our model, this trade-off takes place because much of the available equity must be used for buying votes instead of fostering investments, which evidences how the bargaining process can take a toll at efficiency.

It is straightforward to see that, under an equal distribution of shares, players would be essentially facing a linear public goods game in which it is a dominant strategy not to invest and attempt to appropriate a portion others' production.[26] We have shown that reassigning equity shares through pre-distributive bargaining can help circumvent the tragedy of the commons since any equilibrium entails positive production. However, the amount of contributors is almost never socially optimal because of the proposer's rent-extracting incentives and the equity feasibility constraint (i.e. the fact that shares must sum to 1).

Although the bargaining protocol and the production technology are quite stylized and abstract from relevant factors at play in *real* business partnerships or other organizational forms, it has been well documented that certain firms opt for a *retrospective* profit redistribution mechanism (or *a posteriori* as in Baranski (Forthcoming)) while others implement a *prospective* (or *a priori*) profit redistribution scheme as in the current model. Our theoretical results support that claim that prospective mechanisms are better from an efficiency standpoint in the context of multilateral negotiations. However, the experiments in Baranski (Forthcoming) revealed that ex post bargaining can be used to reward high contributors and punish low contributors, thus inducing almost fully efficient outcomes. It remains an open experimental question of whether or not pre-distributive bargaining will also enhance efficiency beyond the theoretical predictions and outperform the efficiency levels observed in the investment games with ex post bargaining.

We also considered a variant of the model in which the proposer can negotiate contracts that specify payments contingent on individual investments. Here, the fully efficient outcome can be achieved under any voting rule and committee size. However, appropriation incentives

[25] This idea is best captured in the quote attributed to the 19^{th} century American inventor Charles Kettering: "If you want to kill any idea in the world, get a committee working on it".

[26] See Ledyard (1995) for a comprehensive review of the model, its variants, and experimental results.

are still present and only a minimum winning coalition of partners are offered a contract that induces them to vote in favor.

Pre-distributive bargaining has been widely neglected in the literature where the focus has been on the "divide-the-dollar" paradigm. We have shown that negotiating incentives poses strategic dynamics that are absent in the classical setting with an exogenously given fund. Future research should focus on varying the production technology or bargaining protocol to better understand the trade-off between rent-generation and rent-extraction in the multilateral bargaining context, a setting that closely resembles many economic and political activities.

References

Baranski, Andrzej. "Voluntary Contributions and Collective Redistribution". *American Economic Journal: Microeconomics* (Forthcoming).

Baron, David P., and John A. Ferejohn. "Bargaining in legislatures." *American political science review* 83, no. 04 (1989): 1181-1206.

Britz, Volker, P. Jean-Jacques Herings, and Arkadi Predtetchinski. "A bargaining theory of the firm." *Economic Theory* 54, no. 1 (2013): 45-75.

Buchanan, James M., and Gordon Tullock. *The Calculus of Consent.* Vol. 3. Ann Arbor: University of Michigan Press, 1962.

Eraslan, Hülya. "Uniqueness of stationary equilibrium payoffs in the Baron–Ferejohn model." *Journal of Economic Theory* 103, no. 1 (2002): 11-30.

Eraslan, Hulya, and Antonio Merlo. "Some Unpleasant Bargaining Arithmetic?." (2014).

Frechette, Guillaume, John H. Kagel, and Massimo Morelli. "Behavioral identification in coalitional bargaining: An experimental analysis of demand bargaining and alternating offers." *Econometrica* 73, no. 6 (2005): 1893-1937.

Grossman, Herschel I., and Minseong Kim. "Swords or plowshares? A theory of the security of claims to property." *Journal of Political Economy* (1995): 1275-1288.

Hirshleifer, Jack. "The Paradox of Power." *Economics and Politics* 3 (1991): 177-200.

Jackson, Matthew O., and Boaz Moselle. "Coalition and party formation in a legislative voting game." *Journal of Economic Theory* 103, no. 1 (2002): 49-87.

Krishna, Vijay, and Roberto Serrano. "Multilateral bargaining." *The Review of Economic Studies* 63, no. 1 (1996): 61-80.

Ledyard, John. "Public Goods: A Survey of Experimental Research" in John H. Kagel and Alvin E. Roth, eds., *Handbook of experimental economics.* Princeton, NJ: Princeton University Press, 1995, pp. 111-194.

Merlo, Antonio, and Charles Wilson. "A stochastic model of sequential bargaining with complete information." *Econometrica: Journal of the Econometric Society* (1995): 371-399.

Olson, Mancur, and Richard Zeckhauser. "An economic theory of alliances." *The Review of Economics and Statistics* (1966): 266-279.

Osborne, Martin J., and Ariel Rubinstein. *Bargaining and Markets.* Vol. 34. San Diego: Academic press, 1990.

Palfrey, Thomas R. "Laboratory experiments in political economy." *Annual Review of Political Science* 12 (2009): 379-388.

Rubinstein, Ariel. "Perfect equilibrium in a bargaining model." *Econometrica: Journal of the Econometric Society* (1982): 97-109.

Sandler, Todd. "The Economic Theory of Alliances A Survey." *Journal of conflict resolution* 37.3 (1993): 446-483.

Sandler, Todd, and Jon Cauley. "On the economic theory of alliances." *Journal of conflict resolution* 19.2 (1975): 330-348.

Sandler, Todd, and Keith Hartley. "Economics of alliances: The lessons for collective action." *Journal of economic literature* 39.3 (2001): 869-896.

Skaperdas, Stergios. "Cooperation, conflict, and power in the absence of property rights." *The American Economic Review* (1992): 720-739.

Thies, Wallace J. "Alliances and Collective Goods A Reappraisal." *Journal of Conflict Resolution* 31.2 (1987): 298-332.

Yildirim, Huseyin. "Proposal power and majority rule in multilateral bargaining with costly recognition." *Journal of Economic Theory* 136, no. 1 (2007): 167-196.

Appendix A. Proofs

Positive production in equilibrium

LEMMA 3 In equilibrium, at least one member will produce.

PROOF. Suppose that $\mathbf{s}$ is an equilibrium allocation of shares such that $\alpha_i s_i < 1$ for all i. Then, the equilibrium payoffs of the game are 1 for each player. A proposer can unilaterally deviate by giving ϵ to $q-1$ members where ϵ is small and keeping $1-q\epsilon > 1/\alpha$. The proposal will be approved and it yields a higher payoff to some members because the proposer is incentivized to produce. ■

Derivation of the Optimal k.

LEMMA 4 For $k \in [0, \|\alpha\| - 1] \subset \mathbb{R}$ the function

$$\Pi(k) := \begin{cases} \left[1 - \frac{1}{\alpha} - k \cdot \frac{1}{\alpha} - (q-1-k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F(k) & \text{if} \quad k \in [\tilde{k}, \|\alpha\|] \\ \left[1 - \frac{1}{\alpha} - k \cdot \frac{\delta V}{F(k)} - (q-1-k)\left(\frac{\delta V - 1}{F(k)}\right)\right] \cdot F(k) & \text{if} \quad k \in [0, \tilde{k}] \end{cases} \tag{19}$$

has the following properties:

1. It is linear and increasing in $k \in [0, \tilde{k}]$.

2. It is quadratic in $k \in [\tilde{k}, \|\alpha\|]$with a maximum at $\bar{k}$.

3. It is continuous at $k = \tilde{k}$.

4. $\tilde{k} \leq \bar{k}$ always.

Proof. The proofs for (1)-(3) are simple arithmetic computations. For (4) we have that

$$\begin{aligned} \tilde{k} &\leq \bar{k} \iff \\ \frac{\delta n}{n - \delta(\alpha - 1)} - 1 &\leq \frac{1}{2}\frac{n(\alpha + \delta) - \delta q(\alpha - 1)}{n - \delta(\alpha - 1)} - 1 \iff \\ \delta q(\alpha - 1) &\leq n(\alpha - \delta) \end{aligned}$$

and the last inequality holds because $(\alpha - 1) \leq (\alpha - \delta)$ and $\delta q \leq n$. ■

Optimal k is within bounds

Corollary 4 $k^* \in \{\left\|\tilde{k}\right\| + 1, ..., \|\alpha\| - 1\}$.

Proof. It is straightforward to show that $\tilde{k} + \frac{1}{2} \leq \bar{k}$ if and only if $q \leq \frac{n(\alpha - \delta - 1)}{\delta(\alpha - 1)} + 1$ (Assumption 2). This is a sufficient condition for $\left\|\tilde{k}\right\| + 1 \leq k^*$ to hold. Recall that, due to the equity feasibility constraint, at most $\|\alpha\|$ members (including the proposer) can be incentivized to produce. Thus, we require that $k^* \leq \|\alpha\| - 1$, and it suffices to show that $\bar{k} + \frac{1}{2} \leq \alpha - 1$ which we proceed to demonstrate. By Corollary 1 $\bar{k}$ can be increasing or decreasing in δ, thus we examine the lowest and highest possible values of $\bar{k}$. We define $\bar{k}_1 := \bar{k}\big|_{\delta=1}$ and $\bar{k}_2 := \bar{k}\big|_{\delta=\frac{n}{n+\alpha-1}}$. Now we compute

$$\begin{aligned} g_1 &: = \alpha - 1 - \left(\bar{k}_1 + \frac{1}{2}\right) = \frac{(\alpha - 1)(2n + 1 - 2\alpha)}{2(n - \alpha + 1)} \\ g_2 &: = \alpha - 1 - \left(\bar{k}_2 + \frac{1}{2}\right) = \frac{(\alpha - 1)(2n + 1 - \alpha)}{2n} \end{aligned}$$

and it is clear that $g_1 > 0$ and $g_2 > 0$ because $1 < \alpha \leq n$. Thus we have concluded that $\left\|\bar{k}\right\| + 1 \leq \|\alpha\| - 1$ which guarantees that k^* is feasible. ■

Proof of Corollary 2

Let $\bar{k}^M$ and $\bar{k}^U$ denote $\bar{k}$ given by (16) under the simple majority and unanimity rules. For the majority rule we are evaluating at $q = (n+1)/2$ and for the unanimity at $q = n$. We have that

$$\frac{\partial \bar{k}^M}{\partial n} = -\frac{\delta(\alpha - 1)[\alpha(2-\delta) - 3(1-\delta)]}{4(n + \delta - \alpha\delta)^2}$$

and this equation is positive only for $\alpha \in \left(1, \frac{3(1-\delta)}{2-\delta}\right)$. One can see that the upper bound $\frac{3(1-\delta)}{2-\delta}$ reaches its maximum at $\delta = 0$. This means that the largest α for which increasing the committee size generates an increase in $\bar{k}^M$ is for $\alpha = 3/2$. However, note that in this region there can be at most one member producing (the proposer) because $\|\alpha\| = 1$. We conclude that the fund will not increase in n.

Similarly for the unanimity rule we have that

$$\frac{\partial \bar{k}^M}{\partial n} = -\frac{\delta(\alpha - 1)[\alpha(1-\delta) + (1-2\delta)]}{2(n + \delta - \alpha\delta)^2}$$

and this equation is positive only for $\alpha \in \left(\frac{1-2\delta}{1-\delta}, 1\right)$. It follows that the total fund (weakly) decreases as n increases for any $\alpha \geq 1$.

Proof of Proposition 4

We will solve the model where the proposer's contract is defined as a transfer irrespective of his contribution. Without loss of generality we assign the role of player 1 to the proposer. Thus, we have that the proposer's payoff is given by $\sum_{i=1}^{n} \alpha_i c_i - \sum_{i=2}^{n} a_i c_i$. This simplifies the analysis greatly because it guarantees that the partnership is always solvent to honor the contracts offered. Define $\Lambda := \{i > 1 \text{ s.t. } a_i \geq \delta V\}$, which represents the set of those who will vote in favor and $\Omega := \{i > 1 \text{ s.t. } a_i \geq 1\}$ represents the set of incentivized members besides the proposer. Clearly, $\Lambda \subset \Omega$ because $\delta V > 1$ (Assumption 1). As before, we are characterizing equilibria in which the proposer produces. Let $k_c := |\Omega|$ and $F(k_c) = \alpha(k_c+1)$ denote the size of the fund.

The voting approval constraint is given by

$$|\Lambda| \geq q - 1 \tag{20}$$

and the proposer's maximization problem is given by

$$\max_{k_c, \{a_i\}_{i=2}^{n}} F(k_c) - \sum_{i \in \Omega} a_i \quad \text{s.t. (20)} \quad . \tag{21}$$

In the problem above we have imposed equilibrium behavior in the subgame in the sense that $c_i = 1$ if $i \in \Omega$.

It is clear that condition (20) binds because the proposer's payoff decreases in a_i. Hence, we have that there will be $q-1$ members receiving $\bar{a}_i(k_c) = \max\left\{\delta\left[\frac{(\alpha-1)(k_c+1)}{n} + 1\right], 1\right\}$. It is useful to note that $F(k_c) - \sum_{i \in \Omega} a_i$ is increasing in k_c as long as $a_i \leq \alpha$ (i.e. the contract offers a compensation lower than the member's productivity). It follows that $k_c^* = n - 1$. Given that the proposer already has the necessary votes and that his payoff decreases in a_i, he chooses $a_i = 1$ for the remaining $n - q$ partners.

In equilibrium $\bar{a}_i^* := a_i(k_c^*) = \max\{\delta\alpha, 1\}$ and the proposer's payoff if given by $\alpha n - (q-1)\max\{\delta\alpha, 1\} - (n-q-1)$. It is straightforward to show that his payoff increases in n and decreases in q (constant in q when $\delta\alpha < 1$).

Appendix B. Other Stationary Equilibria

Consider the following example with a five person committee. Let $\alpha = q = 3$ and $\delta = 1$. Plugging these values into (17) we obtain that $k^* = 1$. This implies that $F(k^*) = 6$, $s_{\text{Prop}} = 8/15$, $s_{\text{Vote}} = 2/15$, and $s_{\text{cont}} = 1/3$, and the ex ante value of the game given by $V = 9/5$. The proposer's payoff is $6 \times 8/15 = 48/15$.

Now I show that $k = 2$, s_{cont} =1/3, and $s_{\text{Prop}} = 1/3$ is a stationary equilibrium as well, but it yields a lower payoff to the proposer. When $k = 2$, the fund is equal to 9 and the stationary value of the game is $V = 11/5$. The payoff to each member of the coalition, including the proposer, is 3. Note that if the proposer deviates and assigns himself a larger share, one productive member must be sacrificed because of the binding equity constraint. We focus on the most profitable deviation when the non-productive coalition member is taken against the continuation value, then he must be offered $s_{\text{vote}} = 1/5$. Recall that a single deviation does not change the continuation value of the game due to stationarity. The proposer keeps 7/15 shares and obtains a payoff of $42/15 < 3$. This implies that the deviation was not profitable. In the equilibrium characterized by $k^* = 1$ the proposer earns a payoff of $48/15 > 3$.

However, if we consider a committee with $n = 7$, $q = \alpha = 4$ then it is not true that maximum efficiency can be attained in equilibrium. The reason is that if $k = 3$, s_{cont} =1/4,and $s_{\text{Prop}} = 1/4$, is part of a stationary equilibrium, the proposer can deviate to $s_{\text{Prop}} = 5/14$, $k = 2$, and $s_{\text{Vote}} = 1/7$ and receive a higher payoff while still obtaining the required votes.

We are not able to provide a closed-form solution to the problem of finding the most efficient stationary equilibrium but in the following proposition we provide a characterization of equilibrium strategies.

Proposition 6 Let $\hat{\boldsymbol{\sigma}} = (\hat{\boldsymbol{\sigma}}_i, \hat{\boldsymbol{\sigma}}_{-i})$ be a stationary strategy profile where $\hat{k}$ members are incentivized and the stationary values of the game are given by $\hat{V} := V(\hat{k})$. Then $\hat{\boldsymbol{\sigma}}$ is the most efficient equilibrium if the equilibrium strategies satisfy following conditions:

1. $s_{\text{Prop}}(\hat{k},\hat{V}) = 1 - \hat{k}\cdot s_{\text{Cont}}(\hat{k},\hat{V}) - m s_{\text{Vote}}(\hat{k},\hat{V})$, $s_{\text{Cont}}(\hat{k},\hat{V}) = \max\left\{\frac{1}{\alpha}, \frac{\delta\hat{V}}{\hat{k}}\right\}$, $s_{\text{Vote}}(\hat{k},\hat{V}) = \frac{\delta\hat{V}-1}{F(\hat{k})}$.

2. A member votes in favor if and only if $s_{\text{Cont}}F(\hat{k}) \geq \delta\hat{V}$, $s_{\text{Vote}}(\hat{k},\hat{V})F(\hat{k}) + 1 \geq \delta\hat{V}$, or $s_{\text{Prop}}(\hat{k},\hat{V})F(\hat{k}) \geq \delta\hat{V}$ and the proposal receives q votes ($q = m + \hat{k} + 1$).

3. Given $\hat{\boldsymbol{\sigma}}_{-i}$, $\hat{k} = \arg\max_{k\in\{0,\ldots,\|\alpha\|-1\}} s_{\text{Prop}}(k,\hat{V})F(k)$ subject to conditions (1) and (2) above.

4. $\hat{k} = \arg\max_{k\in\{0,\ldots,\|\alpha\|-1\}} F(k)$ subject to conditions (1), (2), and (3) above.

Proof. Conditions (1) and (2) state that no resources should be wasted, otherwise, this could result in lower efficiency or the proposer could improve his position (for a given $\hat{V}$) without failing to obtain the majority vote. Hence, productive members are offered the smallest share that induces contribution and non-productive coalition partners are offered a share that yields exactly the continuation value. Condition (3) states that the proposer cannot deviate to any other equity scheme and earn a higher profit while still obtaining the majority vote. Finally, condition (4) states that we choose the highest amount of productive members for which the previous conditions hold. ■

Existence is guaranteed because the equilibrium characterized in Proposition 1 satisfies (1)-(3), thus we know that $\hat{k} \in [k^*, ..., \|\alpha\|]$.