Price Rigidities and the Granular Origins of Aggregate Fluctuations*

Ernesto Pasten†, Raphael Schoenle‡, and Michael Weber§

This version: March 2017

Abstract

We study the aggregate implications of sectoral shocks in a multi-sector New-Keynesian model with intermediate inputs featuring sectoral heterogeneity in price stickiness, sector size, and input-output linkages. Heterogeneity in price stickiness by itself can generate aggregate fluctuations from idiosyncratic shocks but it also distorts the "granular" effect of large sectors on aggregate volatility as well as the "network" effect of central sectors in the production network. This distortion affects both the magnitude of GDP volatility due to sectoral shocks and the identity of sectors driving aggregate fluctuations. Price rigidity can even distort the sign of fluctuations. Importantly, sector sales are no longer a sufficient statistic for a sector's contribution to aggregate volatility, as in Hulten (1978). We calibrate a 348-sector version of the model to the BEA input-output tables and BLS micro pricing data and find: (i) sectoral heterogeneity of price rigidity alone generates sizable GDP volatility from sectoral shocks; (ii) sectoral heterogeneity of price rigidity amplifies both the "granular" and the "network" effects; and (iii) sectoral heterogeneity of price rigidity alters the identity and relative contributions of the most important sectors for aggregate fluctuations. Price rigidity generates "frictional" origins of aggregate fluctuations conceptually different from the granular and network effects.

JEL classification: E31, E32, O40

Keywords: Input-output linkages, sticky prices, idiosyncratic shocks

*Susanto Basu, Ben Bernanke, Francesco Bianchi, Saki Bigio, John Cochrane, Eduardo Engel, Yuriy Gorodnichenko, Gita Gopinath, Josh Hausman, Pete Klenow, Valerie Ramey, Harald Uhlig, and conference and seminar participants at the Central Bank of Chile, the NBER Monetary Economics meeting, UCLA, Stanford, UChile-Econ as well as the 2016 SED Meeting (Toulouse). Pasten thanks the support of the Universite de Toulouse Capitole during his stays in Toulouse. The contributions by Michael Weber to this paper have been prepared under the Lamfalussy Fellowship Program sponsored by the European Central Bank. Any views expressed are only those of the authors and do not necessarily represent the views of the ECB or the Eurosystem. We also thank Jose Miguel Alvarado, Will Cassidy, Stephen Lamb, and Matt Klepacz for excellent research assistance. The views expressed herein are those of the authors and do not necessarily represent the position of the Central Bank of Chile.

†Central Bank of Chile and Toulouse School of Economics. e-Mail: ernesto.pasten@tse-fr.eu

‡Brandeis University. e-Mail: schoenle@brandeis.edu.

§Booth School of Business, University of Chicago and NBER. e-Mail: michael.weber@chicagobooth.edu.

I Introduction

Identifying aggregate shocks that drive business cycles might be difficult (Cochrane (1994)). A recent literature advances the possibility shocks at the firm or sector level may be the origin of aggregate fluctuations. This view stands in contrast to the "diversification argument" of Lucas (1977), which conjectures that idiosyncratic shocks at a highly disaggregated level average out in the aggregate.[1] In contrast, Gabaix (2011) argues that the diversification argument does not readily apply when the firm size distribution follows a fat-tailed distribution, which is the empirically relevant case for the U.S. Intuitively, shocks to disproportionately large firms matter for aggregate fluctuations, the "granular" effect. In a similar vein, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) focus on sectoral shocks and show that input-output relationships across sectors can mute the diversification argument if measures of sector centrality follow a fat-tailed distribution, which is also the empirically relevant case for the U.S. They label this channel the "network" channel. Thus, either through a granular or network channel, microeconomic shocks to small numbers of firms or sectors may drive aggregate fluctuations, instead of aggregate shocks.[2] Both channels, however, operate under the assumption of perfectly flexible prices.

This paper studies whether and how nominal rigidities affect the importance of microeconomic shocks for aggregate fluctuations. To fix ideas, let us consider a multi-sector economy without linkages across sectors and consider a positive productivity shock to one sector. Marginal costs in this sector decrease and prices should fall in the absence of pricing frictions. But consider what happens if prices do not adjust. Demand for goods of the shocked sector remains unchanged, so production remains unchanged. Therefore, regardless of the size of the sector, the contribution of its shocks to aggregate fluctuations is zero except for some general equilibrium effects.[3] A similar logic applies to production networks. A price cut in one sector due to a positive productivity shock would propagate downstream by decreasing production costs, triggering price cuts in other sectors. But, if prices do not change in the shocked sector, marginal costs of downstream firms remain unchanged and there is no propagation regardless of the centrality of the shocked sector, except for small general equilibrium effects.

[1] Dupor (1999) takes a similar perspective as Lucas (1977) and implicitly anyone who models aggregate shocks driving aggregate fluctuations.

[2] A fast-growing literature has followed. Some recent examples are Acemoglu, Akcigit, and Kerr (2016); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017); Atalay (2015); Baqaee (2016); Bigio and La'O (2016); Caliendo, Parro, Rossi-Hansberg, and Sarte (2014); Carvalho and Gabaix (2013); Carvalho and Grassi (2015); Di Giovanni, Levchenko, and Méjean (2014); Di Giovanni, Levchenko, and Méjean (2016); and Foerster, Sarte, and Watson (2011).

[3] First, lower demand for inputs in the shocked sector decreases wages. Second, higher profits of firms in the shocked sector increase household income. However, these effects are small up to a first-order approximation.

In the data, prices are neither fully rigid nor fully flexible, and substantial heterogeneity of price rigidity across sectors in the U.S. exists (see Bils and Klenow (2004); Klenow and Kryvtsov (2008); Nakamura and Steinsson (2008)). How does the heterogeneity in nominal price rigidity interact with the granular effect of Gabaix (2011) and the network effect of Acemoglu et al. (2012) in affecting the ability of microeconomic shocks to generate sizable aggregate fluctuations? Do price rigidities distort the identity of sectors that are the origin of aggregate fluctuations? Can price rigidity create a "frictional" origin of aggregate fluctuations, conceptually different than the granular or the network origins already described in the literature?

We answer these questions in a multi-sector new-Keynesian model in which firms produce output using labor and intermediate inputs. Our model follows Basu (1995) and Carvalho and Lee (2011), but we make no simplifying assumptions on the steady-state distribution of sectoral value-added, input-output linkages, and the sectoral distribution of price-setting frictions which we model following Calvo (1983). Sectoral productivity shocks are the only source of variation in our model. We calibrate the model to the Input-Output tables of the Bureau of Economic Analysis (BEA) at the most disaggregated level and the micro data underlying the Producer Price Index (PPI) from the Bureau of Labor Statistics (BLS). After merging these two datasets, we end up with 348 sectors.

We first analytically study in a simplified version of our model the distortionary role of price rigidity on the granular and network origins of aggregate fluctuations. Up to a log-linear approximation, GDP is a linear combination of sectoral shocks, and the model nests Gabaix (2011) and Acemoglu et al. (2012) as special cases. When we abstract from intermediate inputs and price stickiness, we recover the granularity effect of Gabaix (2011): the ability of microeconomic shocks to generate aggregate fluctuations depends on the fat-tailedness of the sector size distribution which we measure by sector GDP.

Price stickiness introduces two new effects. First, it dampens the level of aggregate volatility originating from any shock, both sector-specific and aggregate. Second, there is a relative effect: The sectoral distribution of price rigidity distorts the relative importance of sectors for aggregate fluctuations. In particular, a sector is important when it is large, as in Gabaix (2011), and/or when its prices are much more flexible than most prices in the economy. Consider a scenario in which the sector size distribution is fat-tailed and size is negatively correlated with price rigidity; that is, larger sectors are more likely to have more flexible prices. In this case, shocks to large sectors become even more important for aggregate volatility than in a frictionless economy. In other words, the distribution of the multipliers mapping sectoral shocks into aggregate volatility is more fat-tailed than implied by the sector size distribution alone. The opposite holds if sector size is positively correlated with price rigidity. It is even possible that the diversification argument of Lucas (1977) gains bite due to sticky prices even though the conditions necessary for the granular channel hold in a frictionless economy. Therefore, price rigidity distorts the identity of the most important sectors for aggregate fluctuations as long as no monotone negative relationship between price rigidity and size exists. When idiosyncratic shocks drive aggregate fluctuations, price setting frictions may even distort the sign of the business cycle and not only generate inertia, as standard with aggregate shocks.

We reach similar results for the network effect of Acemoglu et al. (2012). With flexible prices, microeconomic shocks are more important for macroeconomic volatility as the distribution of sector centrality is more fat-tailed: large suppliers of intermediate inputs (first-order interconnection) and/or large suppliers to large suppliers of intermediate inputs (second-order interconnection) are important for aggregate volatility. With price stickiness, the most flexible sectors among large suppliers of intermediate inputs or the most flexible sectors among large suppliers to the most flexible large intermediate input suppliers are the most important for aggregate volatility. Thus, the multipliers of sectoral shocks to aggregate volatility may be more or less fat-tailed than the distribution of sector centrality. Heterogeneity in price rigidity invalidates the Hulten (1978) result which holds in Gabaix (2011) and Acemoglu et al. (2012): sector (or firm) total sales are no longer a sufficient statistic for the importance of GDP volatility.

In our quantitative analysis, we first confirm results from Gabaix (2011) and Acemoglu et al. (2012) for sector size and sector linkages and establish new facts for their interaction with price stickiness: (i) sectoral GDP follows a fat-tailed Pareto distribution; (ii) measures of centrality in the U.S. input-output tables follow a fat-tailed Pareto distribution; (iii) sectoral gross output (value added plus intermediate goods) follows a fat-tailed Pareto distribution;[4] (iv) the sectoral frequency of price changes in the U.S. follows a Pareto distribution, but is not fat-tailed; (iv) the correlation of the frequency of price changes with sectoral GDP in its upper tail is 6.73% and with measures of network centrality between 22.63% and 33.33%. As we discuss in Section III, however, measures of linear dependence between the frequency of price adjustments, sector size, and sector centrality do not suffice to inform us about the distortionary role of price rigidity for aggregate fluctuations originating from sectoral shocks.

In a series of experiments, we instead quantitatively show that price rigidity does indeed affect the importance of microeconomic shocks for aggregate fluctuations in a 348-sector economy. We focus our discussion on relative multipliers, that is, multipliers of sectoral productivity shocks on GDP volatility relative to the multiplier of aggregate productivity shocks

[4] This distribution is also remarkably similar to the distribution of firm total sales Gabaix (2011) reports. Gross output is conceptually the closest in our data to total sales.

on GDP volatility, because the effect of aggregate shocks is not invariant to the distribution of price rigidity, sectoral GDP and input-output linkages.

In our first experiment, we match sectoral GDP shares but assume equal input-output linkages across sectors. The relative multiplier of sectoral productivity shocks on GDP volatility increases from 11% when prices are flexible to 22.8% when price stickiness across sectors follows the empirical distribution. In the second experiment, we match input-output linkages to the U.S. data but assume equal sector sizes. Now, the relative multiplier increases from 8% with flexible prices to 11.5%. In a third experiment, differences in the frequency of price changes is the only source of heterogeneity across sectors. The relative multiplier of sectoral shocks is now 10.8%, twice as large compared to an economy with complete symmetry and equal price stickiness across sectors. Overall, when all three heterogeneities are present, the relative multiplier on GDP volatility is 24%, almost five times larger than in an economy with complete symmetry. The five-fold increase of the relative multiplier underscores the relevance of microeconomic shocks for aggregate fluctuations and shows that heterogeneity in sector size, input-output structure, and price stickiness are intricately linked and reinforce each other.

Price rigidity not only contributes to the importance of micro shocks driving aggregate volatility, but also has strong effects on distorting the identity and contribution of sectors driving aggregate fluctuations. For instance, the identity of the two most important sectors for aggregate volatility shifts from "Real Estate" and "Wholesale Trading" with flexible prices to "Petroleum Refineries" and "Oil and Gas Extraction" with sticky prices when we only consider network effects. When we also allow for sectoral heterogeneity in size, the two most important sectors with flexible and sticky prices are again the same, "Real Estate" and "Wholesale Trading." However, the relative contribution of the former is now cut in half relative to the flexible-price economy, and the importance of the latter doubles with sticky prices.

A. Literature review

At an abstract level, this paper shows not only the size or centrality of nodes in the network matter for the macro effect of micro shocks, but also the frictions that affect the capacity of nodes to propagate shocks. This point goes beyond sticky prices. We focus on sticky prices because prices are the central transmission mechanism of sectoral shocks in production networks. In addition, price stickiness is a measurable friction at a highly disaggregated level. The frictional origin of fluctuations also goes beyond production networks in a closed economy; it applies to all networks with heterogeneous effects of frictions across nodes, e.g., in international trade networks, financial networks, or social networks. Our work is thus related to an extensive literature that we do not attempt to summarize here.

Long and Plosser (1983) pioneer the microeconomic origin of aggregate fluctuations and Horvath (1998) and Horvath (2000) push this literature forward. Dupor (1999) argues microeconomic shocks matter only due to poor disaggregation. Gabaix (2011) invokes the firm size distribution and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) the sectoral network structure of the economy to document convincingly the importance of microeconomic shocks for macroeconomic fluctuations: under empirically plausible assumptions, microeconomic shocks do matter. Barrot and Sauvagnat (2016); Acemoglu, Akcigit, and Kerr (2016); and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016) provide empirical evidence for the importance of idiosyncratic shocks for aggregate fluctuations, and Carvalho (2014) synthesizes this literature.

The distortionary role of frictions (and price rigidity, in particular) is at the core of the business cycle literature that conceptualizes aggregate shocks as the driver of aggregate fluctuations, including the New-Keynesian literature. However, to the best of our knowledge, there is no parallel study of the distortionary role of frictions when aggregate fluctuations have microeconomic origins. That said, there are a few recent papers that include frictions in their analyses. Baqaee (2016) shows that entry and exit of firms coupled with CES preferences may amplify the aggregate effect of microeconomic shocks. Carvalho and Grassi (2015) study the effect of large firms in a quantitative business cycles model with entry and exit. Bigio and La'O (2016) study the aggregate effect of the tightening of financial frictions in a production network. Despite a different focus, we share our finding with Baqaee (2016) and Bigio and La'O (2016) that the Hulten theorem does not apply in economies with frictions.

Our model shares building blocks with previous work studying pricing frictions in production networks. Basu (1995) shows that frictions introduce misallocation resulting in nominal demand shocks looking like aggregate productivity shocks. Carvalho and Lee (2011) develop a new-Keynesian model in which firms' prices respond slowly to aggregate shocks and quickly to idiosyncratic shocks, rationalizing the findings in Boivin et al. (2009). We build on their work to answer different questions and relax assumptions regarding the production structure to quantitatively study the interactions of different heterogeneities.

Nakamura and Steinsson (2010); Midrigan (2011); and ?? (alv), among many others, endogenize price rigidity to study monetary non-neutrality in multi-sector menu cost models. Computational burden and calibration issues make such an approach infeasible in our highly disaggregated model. This is why we study the effect of disaggregation on monetary non-neutrality in a multi-sector Calvo model in a companion paper (Pasten, Schoenle, and Weber (2016)). Bouakez, Cardia, and Ruge-Murcia (2014) estimate a Calvo model with production networks using data for 30 sectors, and find heterogeneous responses of sectoral inflation to a monetary policy shock, but do not study the questions we pose in this paper.

Other recent applications of networks in different areas of macroeconomics and finance are Gofman (2011), who studies how intermediation in over-the-counter markets affects the efficiency of resource allocation; Di Maggio and Tahbaz-Salehi (2015), who study the fragility of the interbank market; Ozdagli and Weber (2016), who show empirically that input-output linkages are a key propagation channel of monetary policy shocks to the stock market; and Kelly, Lustig, and Van Nieuwerburgh (2013), who study the joined dynamics of the firm-size distribution and stock return volatilities. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) and Herskovic (2015) study the asset-pricing implications of production networks.

II Model

Our multi-sector model has households supplying labor and demanding goods for final consumption, firms with sticky prices producing different varieties with labor and intermediate inputs, and a monetary authority setting nominal interest rates according to a Taylor rule. Sectors are heterogeneous in three dimensions: their final goods production (which we interpret as value added or simply GDP), input-output linkages, and the frequency of price adjustment.

A. Households

The representative household solves

$$\max_{\{C_t, L_{kt}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{k=1}^{K} g_k \frac{L_{kt}^{1+\varphi}}{1+\varphi} dj \right),$$

subject to

$$\begin{aligned} \sum_{k=1}^{K} W_{kt} L_{kt} + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t &= P_t^c C_t \\ \sum_{k=1}^{K} L_{kt} &\leq 1, \end{aligned}$$

where C_t and P_t^c are aggregate consumption and aggregate prices, respectively. L_{kt} and W_{kt} are labor employed and wages are paid in sector $k = 1, ..., K$. Households own firms and receive net income, Π_{kt}, as dividends. Bonds, B_{t-1}, pay a nominal gross interest rate of I_{t-1}. Total labor supply is normalized to 1.

Households' demand of final goods, C_t and goods produced in sector k, C_{kt}, are

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{jkt}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}. \tag{2}$$

A continuum of goods indexed by $j \in [0,1]$ exists with total measure 1. Each good belongs to one of the K sectors in the economy. Mathematically, the set of goods is partitioned into K subsets $\{\Im_k\}_{k=1}^{K}$ with associated measures $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$.[5] We allow the elasticity of substitution across sectors η to differ from the elasticity of substitution within sectors θ.

The first key ingredient of our model is the vector of weights $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]$ in equation (1). These weights show up in households' sectoral demand:

$$C_{kt} = \omega_{ck} \left(\frac{P_{kt}}{P_t^c}\right)^{-\eta} C_t. \tag{3}$$

All prices are identical in steady state, so $\omega_{ck} \equiv \frac{C_k}{C}$, where variables without time subscript are steady-state quantities. In our economy, C_{kt} and C_t represent the sectoral and total production of final goods, that is, we interpret them as sectoral and total value-added. Hence, we can interpret Ω_c as the vector of steady-state sectoral GDP shares satisfying $\Omega_c' \iota = 1$ where ι denotes a column-vector of 1s. Away from the steady state, sectoral GDP shares depend on the gap between sectoral prices and the aggregate price index, P_t^c:

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{4}$$

We can interpret P_t^c as the GDP deflator. Households' demand for goods within a sector is given by

$$C_{jkt} = \frac{1}{n_k} \left(\frac{P_{jkt}}{P_{kt}}\right)^{-\theta} C_{kt} \text{ for } k = 1, ..., K. \tag{5}$$

Goods within a sector share sectoral GDP equally in steady state. Away from the steady state, the demand of goods within a sector is distorted by the gap between their price and the sectoral price, defined as

$$P_{kt} = \left[\frac{1}{n_k} \int_{\Im_k} P_{jkt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{6}$$

[5] The sectoral subindex is redundant, but it clarifies exposition.

The household first-order conditions determine labor supply and the Euler equation:

$$\frac{W_{kt}}{P_t^c} = g_k L_{kt}^{\varphi} C_t^{\sigma} \text{ for all } k, j, \tag{7}$$

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c} \right]. \tag{8}$$

We implicitly assume sectoral segmentation of the labor market, so labor supply in equation (7) holds for a sector-specific wage $\{W_{kt}\}_{k=1}^{K}$. We choose the parameters $\{g_k\}_{k=1}^{K}$ to ensure a symmetric steady state across all firms.

B. Firms

A continuum of monopolistically competitive firms exists in the economy operating in different sectors. We index firms by their sector, $k = 1, ...K$, and by $j \in [0, 1]$. The production function is

$$Y_{jkt} = e^{a_{kt}} L_{jkt}^{1-\delta} Z_{jkt}^{\delta}, \tag{9}$$

where a_{kt} is an i.i.d. productivity shock to sector k with $\mathbb{E}[a_{kt}] = 0$ and $\mathbb{V}[a_{kt}] = v^2$ for all k, L_{jkt} is labor, and Z_{jkt} is an aggregator of intermediate inputs:

$$Z_{jkt} \equiv \left[\sum_{k'=1}^{K} \omega_{kk'}^{\frac{1}{\eta}} Z_{jk}\left(k'\right)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \tag{10}$$

$Z_{kjt}(r)$ is the amount of goods firm j in sector k uses in period t as intermediate inputs from sector r.

The second key ingredient of our model is heterogeneity in aggregator weights $\{\omega_{kk'}\}_{k,k'}$. We denote these weights in matrix notation as Ω, satisfying $\Omega\iota = \iota$.

The demand of firm jk for goods produced in sector k' is given by

$$Z_{jkt}\left(k'\right) = \omega_{kk'} \left(\frac{P_{k't}}{P_t^k} \right)^{-\eta} Z_{jkt}. \tag{11}$$

We can interpret $\omega_{kk'}$ as the steady-state share of goods from sector k' in the intermediate input use of sector k. Away from the steady state, the gap between the price of goods in sector k' and the aggregate price relevant for a firm in sector k, P_t^k distorts input-output linkages:

$$P_t^k = \left[\sum_{k'=1}^{K} \omega_{kk'} P_{k't}^{1-\eta} \right]^{\frac{1}{1-\eta}} \text{ for } k = 1, ..., K. \tag{12}$$

P_t^k uses the sector-specific steady-state input-output linkages to aggregate sectoral prices.

The aggregator $Z_{jk}(k')$ gives the demand of firm jk for goods in sector k':

$$Z_{jk}(k') \equiv \left[n_{k'}^{-1/\theta} \int_{\Im_{k'}} Z_{jkt}(j', k')^{1-\frac{1}{\theta}} dj' \right]^{\frac{\theta}{\theta-1}}. \tag{13}$$

Firm jk's demand for an arbitrary good j' from sector k' is

$$Z_{jkt}(j', k') = \frac{1}{n_{k'}} \left(\frac{P_{j'k't}}{P_{k't}} \right)^{-\theta} Z_{jk}(k'). \tag{14}$$

In steady state, all firms within a sector share the intermediate input demand of other sectors equally. Away from the steady state, the gap between a firm's price and the price index of the sector it belongs to (see equation (6)) distorts the firm's share in the production of intermediate input.

Our economy has $K+1$ different aggregate prices, one for the household sector and K for each sector. In contrast, there are unique sectoral prices which the household sector and all sectors face.

The third key ingredient of our model is sectoral heterogeneity in price rigidity. Specifically, we model price rigidity à la Calvo with parameters $\{\alpha_k\}_{k=1}^K$ such that the pricing problem of firm jk is

$$\max_{P_{jkt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^\tau \left[P_{jkt} Y_{jkt+s} - MC_{kt+s} Y_{jkt+s} \right].$$

Marginal costs are $MC_{kt} = \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta} e^{-a_{kt}} W_{kt}^{1-\delta} \left(P_t^k \right)^\delta$ in reduced form after imposing the optimal mix of labor and intermediate inputs:

$$\delta W_{kt} L_{jkt} = (1-\delta) P_t^k Z_{jkt}, \tag{15}$$

and $Q_{t,t+s}$ is the stochastic discount factor between period t and $t+s$.

We assume the elasticities of substitution across and within sectors are the same for households and all firms. This assumption shuts down price discrimination among different customers, and firms choose a single price P_{kt}^*:

$$\sum_{\tau=0}^{\infty} Q_{t,t+\tau} \alpha_k^s Y_{jkt+\tau} \left[P_{kt}^* - \frac{\theta}{\theta-1} MC_{kt+\tau} \right] = 0, \tag{16}$$

where $Y_{jkt+\tau}$ is the total production of firm jk in period $t+\tau$.

We define idiosyncratic shocks $\{a_{kt}\}_{k=1}^{K}$ at the sectoral level, and it follows that the optimal price, P_{kt}^{*}, is the same for all firms in a given sector. Thus, aggregating among all prices within sectors yields

$$P_{kt} = \left[(1 - \alpha_k)\, P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{17}$$

C. Monetary policy, equilibrium conditions and definitions

The monetary authority set nominal interest rates according to a Taylor rule:

$$I_t = \frac{1}{\beta} \left(\frac{P_t^c}{P_{t-1}^c} \right)^{\phi_\pi} \left(\frac{C_t}{C} \right)^{\phi_y}. \tag{18}$$

Monetary policy reacts to GDP deflator inflation P_t^c and deviations from steady state total value-added C_t. We do not model monetary policy shocks.

Bonds are in zero net supply, $B_t = 0$, labor markets clear, and goods markets clear such that

$$Y_{jkt} = C_{jkt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{j'k't}\,(j,k)\, dj', \tag{19}$$

implying a wedge between gross output Y_t and GDP C_t.

III Theoretical Results in a Simplified Model

We can derive closed-form results for the importance of sectoral shocks for aggregate fluctuations in simplified version of our model. Given the focus of the paper, we study log-linear deviations from steady-state GDP. We report the steady-state solution and the full log-linear system solving for the equilibrium in the Online Appendix. All variables in lower cases denote log-linear deviations from steady state.

A. Simplifying Assumptions

We make the following simplifying assumptions:

(i) Households have log utility, that is, $\sigma = 1$, and linear disutility of labor, $\varphi = 0$. Thus,

$$w_{kt} = p_t^c + c_t;$$

that is, the labor market is integrated and nominal wages are proportional to nominal GDP.

(ii) Monetary policy targets constant nominal GDP, so

$$p_t^c + c_t = 0.$$

(iii) We replace Calvo price stickiness by a simple rule: all prices are flexible, but with probability λ_k a firm in sector k has to set its price before observing shocks. Thus,

$$P_{jkt} = \begin{cases} \mathbb{E}_{t-1}\left[P_{jkt}^*\right] & \text{with probability } \lambda_k, \\ P_{jkt}^* & \text{with probability } 1-\lambda_k, \end{cases}$$

where $\mathbb{E}_{t-1}$ is the expectation operator conditional on the $t-1$ information set.

Solution We show in the Online Appendix that under assumptions (i), (ii) and (iii), c_t is given by

$$c_t = \chi' a_t, \tag{20}$$

where $\chi \equiv (\mathbb{I}-\Lambda)\left[\mathbb{I}-\delta\Omega'(\mathbb{I}-\Lambda)\right]^{-1}\Omega_c$. Λ is a diagonal matrix with price rigidity probabilities $[\lambda_1, ..., \lambda_K]$ as diagonal, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is a vector of sectoral productivity shocks. Recall that Ω_c and Ω represent in steady state the sectoral consumption shares and intermediate input shares.

A linear combination of sectoral shocks describes the log-deviation of GDP from its steady state up to a first-order approximation. Thus, aggregate GDP volatility is

$$v_c = v\sqrt{\sum_{k=1}^{K}\chi_k^2} = \|\chi\|_2\, v, \tag{21}$$

because all sectoral shocks have the same volatility, that is, $\mathbb{V}[a_{kt}] = v^2$ for all k. $\|\chi\|_2$ denotes the Euclidean norm of χ.

Thus, χ is a vector of multipliers from the volatility of sectoral productivity shocks to GDP volatility. We will refer to these multipliers as *sectoral multipliers* in the following.

Below, we study the effect of heterogeneous price rigidity on the scale of aggregate volatility v_c in an economy with a given number of sectors K. We also investigate the effect on the rate of decay of v_c as the economy becomes increasingly more disaggregated, $K \to \infty$.

We use the following definition:

Definition 1 *A given random variable X follows a* ***power-law distribution with shape parameter*** *β when $\Pr(X > x) = (x/x_0)^{-\beta}$ for $x \geq x_0$ and $\beta > 0$.*

B. The granular effect and price rigidity

We now study the interaction of price rigidity with the granular effect of Gabaix (2011). The granular effect studies the role of the firm size distribution on the importance of microeconomic shocks as the origin of aggregate volatility. Gabaix (2011) measures firm size by total sales, which includes sales as final goods and as intermediate inputs. The setup of our model and data requirements have us study sectors instead of firms. However, this is only a nominal difference. We also shut down intermediate inputs, that is, $\delta = 0$, to disentangle the contribution of sector GDP to the size of sectors. Hence, sector size only depends on sectoral value added (sectoral GDP). With $\delta = 0$, our expressions mirror the ones in Gabaix (2011) in special cases. We study the effect of intermediate inputs, that is, the network effect of Acemoglu et al. (2012) below, and then study both channels jointly.

When $\delta = 0$,

$$\chi = \left(\mathbb{I} - \Lambda'\right) \Omega_c,$$

or, simply, $\chi_k = (1 - \lambda_k)\, \omega_{ck}$ for all k.

Recall that $\omega_{ck} = C_k / \sum_{k=1}^{K} C_k$. Hence, steady-state sectoral GDP shares fully determine sectoral multipliers only when prices are flexible. In general, sectoral multipliers also depend on the sectoral distribution of price stickiness. Sales are no longer a sufficient statistic for the importance of sectors for aggregate volatility breaking the Hulten (1978) result in the Gabaix (2011) framework.

The following lemma presents our first result for homogeneous price stickiness across sectors.

Lemma 1 *When $\delta = 0$ and $\lambda_k = \lambda$ for all k,*

$$v_c = \frac{(1-\lambda)\, v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(C_k\right) + \overline{C}_k^2},$$

where $\overline{C}_k$ and $\mathbb{V}(\cdot)$ are the sample mean and sample variance of $\{C_k\}_{k=1}^{K}$.[6]

This lemma follows from equation (21) when $\delta = 0$. As in Gabaix (2011), the volatility of GDP in an economy with K sectors depends on the cross-sectional dispersion of sector size, here measured by $\mathbb{V}(C_k)$. Price rigidity only has a scale effect on volatility depending on whether productivity shocks are sectoral or aggregate. The scale effect follows from equation (20): if $\delta = 0$, and all sectoral shocks are perfectly correlated, then $v_c = (1 - \lambda)\, v$.

The next proposition determines the rate of decay of v_c as the economy becomes increasingly

[6] We define $\mathbb{V}(X_k)$ of a sequence $\{X_k\}_{k=1}^{K}$ as $\mathbb{V}(X_k) \equiv \frac{1}{K}\sum_{k=1}^{K}\left(X_k - \overline{X}\right)^2$. The definition of the sample mean is standard.

more disaggregated, $K \to \infty$, in the presence of homogeneous price stickiness.

Proposition 1 (Granular effect) *If $\delta = 0$, $\lambda_k = \lambda$ for all k, and $\{C_k\}_{k=1}^K$ follows a power-law distribution with shape parameter $\beta_c \geq 1$, then*

$$v_c \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1 \end{cases}$$

where u_0 is a random variable independent of K and v.

Proof. See Online Appendix. ■

Proposition 1 revisits the central idea of the granular effect: when the size distribution of sectors is fat-tailed, given by $\beta_c < 2$, v_c converges to zero at a rate slower than the Central Limit Theorem implies, which is $K^{1/2}$. The rate of decay of v_c becomes slower as $\beta_c \to 1$. Intuitively, when the size distribution of sectors is fat-tailed, few sectors remain disproportionately large at any level of disaggregation. Gabaix (2011) documents that a power-law distribution with shape parameter close to 1 characterizes the upper tail of the empirical distribution of firm sizes. Below, we find the same result with sectoral data on value added. Thus, contrary to Dupor (1999), sectoral shocks can generate sizable aggregate fluctuations even if we study sectors at a highly disaggregated level. Homogeneous price rigidity plays no role for this result, except for the scale effect which we discuss in Lemma 1.

We now study the case of heterogeneous price rigidity across sectors.

Lemma 2 *When $\delta = 0$ and price rigidity is heterogeneous across sectors,*

$$v_c = \frac{v}{\overline{C}_k K^{1/2}} \sqrt{\mathbb{V}\left(\left(1-\lambda_k\right) C_k\right) + \left[\left(1-\overline{\lambda}\right)\overline{C}_k - \mathbb{COV}\left(\lambda_k, C_k\right)\right]^2},$$

where $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^K$ and $\mathbb{COV}(\cdot)$ is the sample covariance of $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$.[7]

For a fixed number of sectors, Lemma 2 states that the volatility of GDP depends on the sectoral dispersion of the convoluted variable $\{(1-\lambda_k) C_k\}_{k=1}^K$ as well as the covariance between sectoral price rigidity and sectoral GDP. This result holds independently of the dependence between price rigidity and sectoral GDP. The dependence between sectoral GDP and price rigidity is, however, important for the rate of decay of v_c as $K \to \infty$.

[7] We define $\mathbb{COV}(X_k, Q_k)$ of sequences $\{X_k\}_{k=1}^K$ and $\{Q_k\}_{k=1}^K$ as $\mathbb{COV}(X_k, Q_k) \equiv \frac{1}{K}\sum_{k=1}^K \left(X_k - \overline{X}\right)\left(Q_k - \overline{Q}\right)$.

Proposition 2 *If* $\delta = 0$, $\{\lambda_k\}_{k=1}^K$ *and* $\{C_k\}_{k=1}^K$ *are independently distributed, the distribution of* $\{\lambda_k\}_{k=1}^K$ *satisfies*

$$\Pr\left[1 - \lambda_k > y\right] = \frac{y^{-\beta_\lambda} - 1}{y_0^{-\beta_\lambda} - 1} \text{ for } y \in \left[y_0, 1\right], \beta_\lambda > 0,$$

and $\{C_k\}_{k=1}^K$ *follows a power-law distribution with shape parameter* $\beta_c \geq 1$, *then*

$$v_c \sim \begin{cases} \frac{u_0}{K^{\min\{1-1/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_0}{\log K} & \text{for } \beta_c = 1, \end{cases}$$

where u_0 *is a random variable independent of* K *and* v.

Proof. See Online Appendix. ■

Proposition 2 shows that price rigidity does not affect the rate of decay of v_c as $K \to \infty$ when $\{\lambda_k\}_{k=1}^K$ and $\{C_k\}_{k=1}^K$ are independent. The independence assumption and the lower bound in the support of the distribution of the frequency of price adjustment, λ_k, explain this result. If λ_k were unbounded below, $(1 - \lambda_k) C_k$ would follow a Pareto distribution with shape parameter equal to the minimum of the shape parameters of the distributions of C_k and $1 - \lambda_k$. But under the assumptions of Proposition 2, the convoluted variable $(1 - \lambda_k) C_k$, follows a Pareto distribution with shape parameter of the distribution of C_k.

Price rigidity is still economically important despite the irrelevance for the rate of convergence. Lemma 2 implies that price rigidity distorts the identity and the contribution of the most important sectors for the volatility of GDP. The distortion arising from price rigidity is central for policy makers who aim to identify the microeconomic origin of aggregate fluctuations, e.g., for stabilization purposes.

Proposition 2 assumes a specific functional form for the distribution of $\{\lambda_k\}_{k=1}^K$, because we cannot prove more general results. We show below that the distributional assumption characterizes the empirical marginal distribution of sectoral frequencies well. The distribution is Pareto with a theoretically bounded support (that is not binding in our sample of sectors).

We now move to the central result in this section.

Proposition 3 *Let* $\delta = 0$, *the distributions of* $\{\lambda_k\}_{k=1}^K$ *and* $\{C_k\}_{k=1}^K$ *are not independent such that the following relationships hold:*

$$\lambda_k = \max\left\{0, 1 - \phi C_k^{\mu}\right\} \text{ for some } \mu \in (-1, 1), \phi \in (0, x_0^{-\mu}), \tag{22}$$

and $\{C_k\}_{k=1}^K$ *follows a power-law distribution with shape parameter* $\beta_c \geq 1$.

If $\mu < 0$,

$$v_c \sim \begin{cases} \frac{u_1}{K^{\min\{1-(1+\mu)/\beta_c, 1/2\}}} v & \text{for } \beta_c > 1 \\ \frac{u_2}{K^{\min\{-\mu, 1/2\}} \log K} v & \text{for } \beta_c = 1. \end{cases} \tag{23}$$

If $\mu > 0$,

$$v_c \sim \begin{cases} \frac{u_2}{K^{\min\left\{1-\frac{1+\mathbf{1}\{K \leq K^*\}\mu}{\beta_c}, \frac{1}{2}\right\}}} v & \text{for } \beta_c > 1 \\ \frac{u_2}{K^{-\mathbf{1}\{K \leq K^*\}\mu} \log K} v & \text{for } \beta_c = 1, \end{cases} \tag{24}$$

for $K^* \equiv x_0^{-\beta_c} \phi^{-\beta_c/\mu}$.

Proof. See Online Appendix. ■

Proposition 3 studies the implications of the interaction between sector size and price rigidity on the rate of decay of GDP volatility, v_c, as the economy becomes more disaggregated. First, consider the case when $\mu < 0$, that is, when larger sectors have more rigid prices. When $\beta_c \in (\max\{1, 2(1+\mu)\}, 2)$, then v_c decays at rate $K^{1/2}$. In general, when $\beta_c \in [1, 2)$, a positive relationship between sectoral size and price stickiness slows down the rate of decay of v_c despite the bounded support of the price-stickiness distribution.

Consider the case when larger sectors have more flexible prices ($\mu > 0$). Equation (22) and the bounded support of the frequency of price adjustment generates a kink such that sectors with value added larger than $\phi^{-1/\mu}$ have perfectly flexible prices. This kink generates a kink in the rate of decay of aggregate volatility, v_c. If $\beta_c \in [1, 2)$, v_c decays at a rate faster than when sector size and price stickiness are independently distributed, as long as the number of sectors is weakly smaller than K^*, $K \leq K^*$. If the number of sectors is sufficiently large, price rigidity is irrelevant for the rate of decay of v_c, as in Proposition 2. Intuitively, sector size and price rigidity are independent for any sector with value added larger than $\phi^{-1/\mu}$. For $K > K^*$, there is a high enough probability that sectors with value added larger than $\phi^{-1/\mu}$ dominate the upper tail of the distribution of multipliers $\chi_k = (1 - \lambda_k) C_k$.

The central question now becomes what a sufficiently large number of sectors is empirically; that is, how large the threshold K^* is. We can answer this question within the context of Proposition 3. When $K > K^*$, a high density of sectors with fully flexible prices exists. In our calibration with 348 sectors, the finest level of disaggregation our data allows, there is not a single sector with fully flexible prices. Thus, when larger sectors tend to have more flexible prices, the price-setting frictions slow down the rate of decay of aggregate volatility v_c for any level of disaggregation with at most 348 sectors.

With no kink in the relationship between sectoral GDP and price stickiness for large sectors, price stickiness slows down the rate of decay of v_c for any level of disaggregation, just as in the case of $\mu < 0$.

For expositional convenience, we have assumed a deterministic relationship between sectoral GDP and price stickiness. However, if this relationship is stochastic, we trivially find that price rigidity distorts the identity of the most important sectors for GDP volatility—even if price rigidity is irrelevant for the rate of decay of GDP volatility,

The next corollary summarizes the results of this section.

Corollary 1 *In an economy in which sectors have heterogeneous sectoral GDP but no input-output linkages, sectoral heterogeneity of price rigidity distorts the magnitude of aggregate volatility generated by idiosyncratic sectoral shocks as well as the identity of sectors from which aggregate fluctuations originate.*

C. The Network Effect and Price Rigidity

We now develop how price rigidity affects the network effect of Acemoglu et al. (2012). We assume a positive intermediate input share, $\delta \in (0,1)$, but shut down the heterogeneity of sectoral GDP, that is, $\Omega_c = \frac{1}{K}\iota$. The vector of multipliers mapping sectoral shocks into aggregate volatility now solves

$$\chi = \frac{1}{K}(\mathbb{I} - \Lambda)\left[\mathbb{I} - \delta\Omega'(\mathbb{I} - \Lambda)\right]^{-1}\iota. \tag{25}$$

This expression nests the solution for the "influence vector" in Acemoglu et al. (2012) when prices are fully flexible, that is, $\lambda_k = 0$ for all $k = 1, ..., K$.[8]

In general, however, a non-trivial interaction between price rigidity and input-output linkages exists across sectors. To study this interaction, we follow Acemoglu et al. (2012) and use an approximation of the vector of multipliers truncating the effect of input-output linkages at second-order interconnections:

$$\chi \simeq \frac{1}{K}(\mathbb{I} - \Lambda)\left[\mathbb{I} + \delta\Omega'(\mathbb{I} - \Lambda) + \delta^2\left[\Omega'(\mathbb{I} - \Lambda)\right]^2\right]\iota.$$

Let us assume homogeneous price rigidity across sectors first.

Lemma 3 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$ and $\lambda_k = \lambda$ for all k, then*

$$v_c \geq \frac{(1-\lambda)\,v}{K^{1/2}}\sqrt{\kappa + \delta'^2\mathbb{V}(d_k) + 2\delta'^3\mathbb{COV}(d_k, q_k) + \delta'^4\mathbb{V}(q_k)}, \tag{26}$$

where $\kappa \equiv 1 + 2\delta' + 3\delta'^2 + 2\delta'^3 + \delta'^4$, $\delta' \equiv \delta(1-\lambda)$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample variance and

[8] The only difference here is $\chi'\iota = 1/(1-\delta)$, because Acemoglu et al. (2012) normalize the scale of shocks such that the sum of the influence vector equals 1.

covariance statistics across sectors and $\{d_k\}_{k=1}^K$ and $\{q_k\}_{k=1}^K$ are the *outdegrees* and *second-order outdegrees*, respectively, defined for all $k = 1, ..., K$ as

$$\begin{aligned} d_k &\equiv \sum_{k'=1}^{K} \omega_{k'k}, \\ q_k &\equiv \sum_{k'=1}^{K} d_{k'} \omega_{k'k}. \end{aligned}$$

Lemma 3 follows from equation (21), $d = \Omega'\iota$ and $q = \Omega'^2\iota$. We have an inequality, because the exact solution for the multipliers χ is strictly larger than the approximation. Acemoglu et al. (2012) coin the terms "outdegrees" and "second-order outdegrees" to measure the centrality of sectors in the production network. In particular, d_k is large when sector k is a large supplier of intermediate inputs. In turn, q_k is large when sector k is a large supplier of large suppliers of intermediate inputs.

Similarly to Lemma 1, homogeneous price rigidity across sectors only has a scale effect on aggregate volatility for a given level of disaggregation. Thus, as in Acemoglu et al. (2012), aggregate volatility from idiosyncratic shocks is higher if the production network is more asymmetric, that is, if a higher dispersion of outdegrees and second-order outdegrees exists across sectors.

The next proposition shows results for the rate of decay of v_c as $K \to \infty$ under the assumption of homogeneous price rigidity.

Proposition 4 (Network effect) *If $\delta \in (0, 1)$, $\lambda_k = \lambda$ for all k, $\Omega_c = \frac{1}{K}\iota$, the distribution of outdegrees $\{d_k\}$, second-order outdegrees $\{q_k\}$, and the product $\{d_k q_k\}$ follow power-law distributions with respective power parameters $\beta_d, \beta_q, \beta_z > 1$ such that $\beta_z \geq \frac{1}{2}\min\{\beta_d, \beta_q\}$, then*

$$v_c \geq \begin{cases} \frac{u_3}{K^{1/2}} v & \text{for } \min\{\beta_d, \beta_q\} \geq 2, \\ \frac{u_3}{K^{1-1/\min\{\beta_d, \beta_q\}}} v & \text{for } \min\{\beta_d, \beta_q\} \in (1, 2), \end{cases}$$

where u_3 is a random variable independent of K and v.

Proof. See Online Appendix. ■

Proposition 4 summarizes the network effect: the rate of decay of aggregate volatility depends on the distribution of measures of network centrality and their interaction. Thus, if some sectors are disproportionately central in the production network, sectoral idiosyncratic shocks have sizable effects on aggregate volatility even if sectors are defined at a highly disaggregated level. The fattest tail among the distributions of outdegrees and second-order outdegrees bounds the rate of decay of aggregate volatility, if the positive relation between outdegrees and second-order outdegrees is not too strong.

Acemoglu et al. (2012) document in the U.S. data that $\beta_d \approx 1.4$ and $\beta_q \approx 1.2$. We find slightly higher numbers in the data we use in our calibration.[9]

As before, homogeneous price rigidity across sectors only has a scale effect on GDP volatility. However, it has one implication worth noticing. Since $\beta_q < \beta_d$ in U.S. data, the distribution of second-order outdegrees contributes the most to the slow decay of v_c when K is large. Lemma 3 implies that this contribution is quantitatively less important as price rigidity increases because $\delta'/\delta = 1 - \lambda$.

Next, we turn to our results for the case of heterogeneous price rigidity across sectors.

Lemma 4 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, and price rigidity is heterogeneous across sectors, then*

$$v_c \geq \frac{v}{K^{1/2}} \left[\begin{array}{c} \left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[\widetilde{\kappa} + \delta^2 \mathbb{V}\left(\widetilde{d}_k\right) + 2\delta'^3 \mathbb{COV}\left(\widetilde{d}_k, \widetilde{q}_k\right) + \delta'^4 \mathbb{V}\left(\widetilde{q}_k\right)\right] \\ -\left(\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2\right)\left[2\delta^2\left(1+\widetilde{\delta}+\widetilde{\delta}^2\right)\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) + \delta^4 \mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right)^2\right] \\ +\mathbb{COV}\left((1-\lambda_k)^2, \left(1+\delta\widetilde{d}_k + \delta^2\widetilde{q}_k\right)^2\right) \end{array} \right]^{\frac{1}{2}}, \tag{27}$$

where $\widetilde{\kappa} \equiv 1 + 2\widetilde{\delta} + 3\widetilde{\delta} + 2\widetilde{\delta} + \widetilde{\delta}$, $\widetilde{\delta} \equiv \delta\left(1 - \overline{\lambda}\right)$, $\overline{\lambda}$ is the sample mean of $\{\lambda_k\}_{k=1}^K$, $\mathbb{V}(\cdot)$ and $\mathbb{COV}(\cdot)$ are the sample variance and covariance statistics across sectors, and $\left\{\widetilde{d}_k\right\}_{k=1}^K$ and $\{\widetilde{q}_k\}_{k=1}^K$ are the **modified outdegrees** and **modified second-order outdegrees**, respectively, defined for all $k = 1, ..., K$ as

$$\begin{aligned} \widetilde{d}_k &\equiv \sum_{k'=1}^{K} (1-\lambda_{k'})\,\omega_{k'k}, \\ \widetilde{q}_k &\equiv \sum_{k'=1}^{K} (1-\lambda_{k'})\,\widetilde{d}_{k'}\omega_{k'k}. \end{aligned}$$

Lemma 4 follows from equation (21), $\widetilde{d} = \Omega'(\mathbb{I} - \Lambda)\,\iota$ and $\widetilde{q} = [\Omega'(\mathbb{I} - \Lambda)]^2\,\iota$ which we label the vectors of modified outdegrees and modified second-order outdegrees, respectively. These statistics measure centrality of sectors in the production network after adjusting nodes by their degree of price rigidity. In particular, $\widetilde{d}_k$ is high either when sector k is a large supplier of intermediate inputs and/or when it is a large supplier of the most flexible sectors. Similarly, $\widetilde{q}_k$ is large when sector k is a large supplier of the most flexible sectors, which are in turn large suppliers of the most flexible sectors.

[9] This is why we abstract from the case when $\min\{\beta_d, \beta_q\} = 1$ in Proposition 4.

The lower bound for v_c in Lemma 4 collapses to the one in Lemma 3 if price rigidity is homogeneous across sectors. The first line on the right-hand side of equation (27) is similar to the one of equation (27) in Lemma 3 with two differences. First, by Jensen's inequality,

$$\frac{1}{K}\sum_{k=1}^{K}(1-\lambda_k)^2 \geq \left(1-\overline{\lambda}\right)^2.$$

The muting effect of price rigidity on aggregate volatility is weaker if price rigidity is heterogeneous across sectors relative to an economy with $\lambda_k = \overline{\lambda}$ for all k.

Second, we now compute key statistics using modified outdegrees, that is, $\widetilde{d}$ and $\widetilde{q}$ instead of d and q. To see the implications, note

$$\begin{aligned}
\widetilde{d}_k &= \left(1-\overline{\lambda}\right) d_k - K\mathbb{COV}\left(\lambda_{k'}, \omega_{k'k}\right), \\
\widetilde{q}_k &= \left(1-\overline{\lambda}\right)^2 q_k - K\mathbb{COV}\left(\lambda_{k'}, \widetilde{d}_k\omega_{k'k}\right) - \left(1-\overline{\lambda}\right)\sum_{k'=1}^{K}\omega_{k'k}\mathbb{COV}\left(\lambda_s, \widetilde{d}_s\omega_{sk'}\right).
\end{aligned}$$

The dispersion of $\widetilde{d}$ is higher than the dispersion of $\left(1-\overline{\lambda}\right)d$ when $\mathbb{COV}\left(\lambda_{k'}, \omega_{k'k}\right)$ is more dispersed across sectors and when it is negatively correlated with d. In words, the dispersion of $\widetilde{d}$ is high when the intermediate input demand of the most flexible sectors is highly unequal across supplying sectors, and when large intermediate input supplying sectors are also large suppliers to flexible sectors. Similarly, the dispersion of $\widetilde{q}$ is higher than the dispersion of $\left(1-\overline{\lambda}\right)^2 q$ when $\mathbb{COV}\left(\lambda_{k'}, \widetilde{d}_k\omega_{k'k}\right)$ is more dispersed and is negatively correlated with q.

The second and the third line on the right-hand side of the lower bound for v_c in of equation (27) capture new effects. In particular, volatility of GDP is higher when $\mathbb{COV}\left(\lambda_k, \widetilde{d}_k\right) < 0$. That is, if sectors with high modified outdegree, $\widetilde{d}_k$, are the most flexible sectors (second line), and if Jensen's inequality effect is stronger (third line).

Analyzing the rate of decay of v_c as $K \to \infty$ is more complicated compared to a case with no intermediate inputs, $\delta = 0$.

Proposition 5 *If $\delta \in (0,1)$, $\Omega_c = \frac{1}{K}\iota$, price rigidity is heterogeneous across sectors, the distribution of modified outdegrees $\left\{\widetilde{d}_k\right\}$, modified second-order outdegrees $\{\widetilde{q}_k\}$ and the product $\left\{\widetilde{d}_k\widetilde{q}_k\right\}$ follow power-law distributions with respective power parameter $\widetilde{\beta}_d, \widetilde{\beta}_q, \widetilde{\beta}_z > 1$ such that $\widetilde{\beta}_z \geq \frac{1}{2}\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}$, then*

$$v_c \geq \begin{cases} \frac{u_4}{K^{1/2}}v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \geq 2, \\ \frac{u_4}{K^{1-1/\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\}}}v & \text{for } \min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} \in (1,2), \end{cases}$$

where u_4 is a random variable independent of K and v.

Proof. Identical to the proof of Proposition 4: see Online Appendix. ■

Proposition 5 resembles Proposition 3 in the context of production networks. If sectors with the most sticky (flexible) prices are also the most central in the price rigidity-adjusted production network such that $\min\left\{\widetilde{\beta}_d, \widetilde{\beta}_q\right\} > (<) \min\left\{\beta_d, \beta_q\right\}$, GDP volatility decays at a faster (slower) rate than when price rigidity is homogeneous across sectors or independent of network centrality. Also as before, regardless of the effect of price rigidity on the rate of decay of v_c as $K \to \infty$, price rigidity distorts the identity of the most important sectors driving GDP volatility originating from idiosyncratic shocks through the network effect.

The following corollary summarizes the findings of this section.

Corollary 2 *In an economy characterized as a production network, sectoral heterogeneity of price rigidity distorts the scale of aggregate volatility generated by idiosyncratic sectoral shocks as well as the identity of sectors from which aggregate fluctuations originate.*

The details of the analysis are different from Section III*B*., but the main message is the same. The inefficiency that price rigidity introduces dampens aggregate fluctuations, similar to an economy with aggregate shocks, but also changes the sectoral origin of aggregate fluctuations. Thus, when shocks are idiosyncratic, the inefficiency of price rigidity may even change the sign of cycles relative to a frictionless economy.

D. The Network Effect, the Granular Effect, and Price Rigidity

We now study the general case how price rigidity affects multipliers when both the granular and network effects are at work in order to gain some intuition for intermediate steps in the calibration. In the next subsection, we relax further modeling restrictions.

In the general case, the (transposed) vector of multiplier is

$$\chi' = (\mathbb{I} - \Lambda)\left[\mathbb{I} - \delta\left(\mathbb{I} - \Lambda\right)\Omega\right]^{-1}\Omega_c$$

which is identical to

$$\chi' = (\mathbb{I} - \Lambda)\left[\sum_{\tau=0}^{\infty}\left[\left(\mathbb{I} - \Lambda\right)\Omega\right]^{\tau}\right]\Omega_c. \tag{28}$$

When we truncate the middle term at second order, we have

$$\chi_k \geq (1 - \lambda_k)\left[\omega_{ck} + \delta\widehat{d}_k + \delta^2\widehat{q}_k\right]. \tag{29}$$

As before, ω_{ck} is the GDP share of sector k, $\widehat{d}_k$ is the "generalized outdegree" of sector k, and

$\widehat{q}_k$ is the "generalized second-order outdegree" of sector k

$$\begin{aligned}\widehat{d}_k &\equiv \sum_{k'=1}^{K} \omega_{ck'} \left(1-\lambda_{k'}\right) \omega_{k'k}, \\ \widehat{q}_k &= \sum_{k'=1}^{K} \widehat{d}_{k'} \left(1-\lambda_{k'}\right) \omega_{k'k}\end{aligned} \tag{30}$$

which combine the effect of heterogeneity of GDP shares, I/O linkages, and price rigidity across sectors.

As before, the multiplier is

$$\|\chi\|_2 = \sqrt{\sum_{k=1}^{K} \chi_k^2}.$$

$\sum_{k=1}^{K} \chi_k$ is the multiplier of an aggregate shock.

Observation 1: Assume input-output shares and sector sizes are homogeneous across sectors, that is, $\omega_{ck} = \omega_{kk'} = 1/K$ for all k, k'. It then follows from equation (30) that

$$\begin{aligned}\widehat{d}_k &= \frac{1}{K}\left(1-\overline{\lambda}\right), \\ \widehat{q}_k &= \frac{1}{K}\left(1-\overline{\lambda}\right)^2\end{aligned}$$

for $\overline{\lambda} \equiv \frac{1}{K}\sum_{k=1}^{K} \lambda_k$.

In this case, we can directly solve for the vector of multipliers

$$\chi_k = \frac{1-\lambda_k}{K\left(1-\delta\left(1-\overline{\lambda}\right)\right)}$$

Hence, the multiplier equals

$$\|\chi\|_2 = \frac{1}{K\left(1-\delta\left(1-\overline{\lambda}\right)\right)} \sqrt{\sum_{k=1}^{K} \left(1-\lambda_k\right)^2}$$

and the aggregate multiplier equals

$$\sum_{k=1}^{K} \chi_k = \frac{1-\overline{\lambda}}{1-\delta\left(1-\overline{\lambda}\right)}$$

Result 1: Multipliers increase in the dispersion of price stickiness across sectors analogous to discussion in Section III *B*.

Observation 2 Assume homogeneous input-output shares, that is, $\omega_{kk'} = 1/K$ but do not restrict sector size across sectors, while prices are frictionless.

Again $\widehat{d}_k$ and $\widehat{q}_k$ take the following simple form

$$\begin{aligned}\widehat{d}_k &= \frac{1}{K}\left(1-\overline{\overline{\lambda}}\right),\\ \widehat{q}_k &= \frac{1}{K}\left(1-\overline{\lambda}\right)\left(1-\overline{\overline{\lambda}}\right),\end{aligned}$$

where $\overline{\overline{\lambda}} \equiv \sum_{k=1}^{K} \omega_{ck}\lambda_k$.

We can again directly solve for the vector of multipliers

$$\chi_k = (1-\lambda_k)\left[\omega_{ck} + \frac{\delta\left(1-\overline{\overline{\lambda}}\right)/K}{1-\delta\left(1-\overline{\lambda}\right)}\right]. \tag{31}$$

Now there is an additional term within the bracket for finite K.

Assume prices are frictionsless, that is, $\lambda_k = 0$ for all k. Hence,

$$\chi_k = \omega_{ck} + \frac{\delta/K}{1-\delta}.$$

If $\delta = 0$, then $\chi_k = \omega_{ck}$ and we recover the granular effect of Section III *B*. However, when $\delta > 0$, an extra additive term exists. This term becomes negligible as $K \to \infty$. For finite K, this additive term reduces the dispersion of multipliers, χ. To see this, note the aggregate multiplier is $(1-\delta)^{-1}$, so it is not invariant to δ.

We can show

$$\frac{\sqrt{\sum_{k=1}^{K}\left(\omega_{ck} + \frac{\delta/K}{1-\delta}\right)^2}}{(1-\delta)^{-1}} \leq \sqrt{\sum_{k=1}^{K}\omega_{ck}^2}.$$

The left-hand size is the multiplier of sectoral shocks relative to aggregate shocks when $\delta \in (0,1)$ and the right-hand size is the same relative multiplier when $\delta = 0$. This inequality holds as long as $\sum_{k=1}^{K}\omega_{ck}^2 \geq 1/K$, which is ensured by Jensen's inequality (the average ω_{ck} is $1/K$).

Result 2: The multiplier is smaller in an economy with homogeneous input-output shares than in an economy without intermediate inputs when GDP shares are heterogeneous across sectors and prices are flexible.

Observation 3 With positive but homogeneous intermediate inputs across sectors, the additive term of χ_k in equation (31) is decreasing in the simple and weighted average of the heterogeneous price stickiness across sectors, $\overline{\overline{\lambda}}$ and $\overline{\lambda}$.

Result 3: For any distribution of GDP shares, $\{\omega_{ck}\}$, the relative multiplier of sectoral shocks is increasing in average price stickiness.

E. Relaxing Simplifying Assumptions

We now discuss the implications of further relaxing the simplifying modeling assumptions we made to derive the results in Sections III*B.* and III*C.*.

Non-linear Disutility of Labor When $\varphi > 0$, labor supply and demand jointly determine wages such that

$$w_{kt} = c_t + p_t^c + \varphi l_{kt}$$

becomes the log-linear counterpart of equation (7). Thus, with monetary policy targeting $c_t + p_t^c = 0$, it no longer holds that sectoral productivity shocks have no effect on sectoral wages.[10]

We now describe these effects one by one. First, the log-linear version of the production function implies that

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k \right).$$

Hence, conditioning on sectoral gross output, shocks in sector k have direct effects on labor demand in sector k and indirect effects on all other sectors to the extent the sector-specific aggregate price of intermediate inputs, $\{p_t^k\}_{k=1}^K$, changes (which depends on input-output linkages).

Second, aggregating demand for goods by households and firms implies that sectoral gross output depends on total gross output y_t and prices according to

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1-\psi) p_t^c - \psi \widetilde{p}_t \right] \right).$$

Hence, conditioning on total gross output, shocks in sector k affect sectoral gross output through the effects on the relative price between sectoral prices and the GDP deflator, p_t^c, and sectoral prices and the economy-wide aggregate price for intermediate goods, $\widetilde{p}_t$.

$\widetilde{p}_t$ is given by

$$\widetilde{p}_t = \sum_{k'=1}^{K} \zeta_{k'} p_{k't},$$

which uses steady-state shares of sectors, ζ_k, in the aggregate production of intermediate inputs

[10]The Online Appendix contains details of the derivations.

as weights,

$$\zeta_k \equiv \sum_{k'=1}^{K} n_{k'} \omega_{k'k}.$$

$\{n_k\}_{k=1}^{\infty}$ are the shares of sectors in aggregate gross output (which coincides with the measure of firms in each sector)

$$n_k = (1-\psi)\,\omega_{ck} + \psi \zeta_k \text{ for all } k = 1, ..., K.$$

$\psi \equiv \frac{Z}{Y}$ is the fraction of total gross output used as intermediate input in steady state.

Third, the response of total gross output y_t to the shocks depends on the response of value added, c_t, and production of intermediate inputs, z_t, according to

$$y_t = (1-\psi)\, c_t + \psi z_t,$$

such that z_t solves

$$z_t = (1+\Gamma_c)\, c_t + \Gamma_p \left(p_t^c - \widetilde{p}_t\right) - \Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't}.$$

$\Gamma_c \equiv \frac{(1-\delta)(\sigma+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}, \Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}$.

Thus, another channel through which sectoral productivity shocks affect labor demand is through their effects on the aggregate demand for intermediate inputs.

To sum up, equation (20) still gives the solution for c_t but the vector of multipliers χ is now

$$\chi \equiv (\mathbb{I}-\Lambda)\left[\gamma_1 \mathbb{I} + \gamma_2 \aleph \iota'\right]\left[\mathbb{I} - \varphi\left[\gamma_3 \iota \Omega_c' + \gamma_4 \iota \vartheta' - \gamma_5 \iota'\right](\mathbb{I}-\Lambda) - \gamma_6 \Omega' (\mathbb{I}-\Lambda)\right]^{-1} \Omega_c, \tag{32}$$

with $\gamma_1 \equiv \frac{1+\varphi}{1+\delta\varphi}$, $\gamma_2 \equiv \frac{\psi(1-\delta)\Gamma_a}{1+\delta\varphi}$, $\gamma_3 \equiv \frac{(1-\delta)[(1-\psi)\eta-1]}{1+\delta\varphi}$, $\gamma_4 \equiv \frac{\psi(1-\delta)(\eta-\Gamma_p)}{1+\delta\varphi}$, $\gamma_5 \equiv \frac{\gamma_2}{\Gamma_a}$, $\gamma_6 \equiv \delta\gamma_1$, $\aleph \equiv (n_1, ..., n_K)'$, and $\vartheta = (\zeta_1, ..., \zeta_K)'$.

Relative to the solution for χ in equation (25), multipliers take a richer functional form, capturing all three channels elastic labor demand introduces. However, we find quantitatively these channels have little importance.

Pricing Friction and Monetary Policy Rule. Calvo pricing frictions result in serial correlation in the response of prices even when shocks are i.i.d.:

$$p_{kt} = (1+\beta+\kappa_k)^{-1}\left[\kappa_k mc_{kt} + \beta \mathbb{E}\left[p_{kt+1}\right] + p_{kt-1}\right] \text{ for } k = 1, ..., K,$$

where $\kappa_k \equiv (1-\alpha_k)(1-\beta\alpha_k)/\alpha_k$.

A Taylor rule of the form

$$i_t = \phi_\pi^c \left(p_t^c - p_{t-1}^c\right) + \phi_c c_t$$

offsets some of the serial correlation which price rigidity introduces.

Serial correlation in price responses implies GDP is now given by

$$c_t = \sum_{\tau=0}^{\infty} \sum_{k=1}^{K} \rho_{k\tau} a_{kt-\tau}.$$

Hence, we have to redefine multipliers χ_k for $k = 1, ..., K$

$$\chi_k \equiv \sqrt{\sum_{\tau=0}^{\infty} \rho_{k\tau}^2}, \tag{33}$$

such that $v_c = \|\chi\|_2 \, v$ still holds.

In contrast to our simplified model, χ does not capture the effect of sectoral shocks on GDP c_t and aggregate volatility v_c. We thus adjust the definition of χ to simplify the comparison between our simplified model and our quantitative analysis below.

IV Data

This section describes the data we use to construct the input-output linkages, sectoral output, and the micro-pricing data we use to construct measures of price stickiness at the sectoral level.

A. Input-output Linkages and Sectoral Output

The Bureau of Economic Analysis (BEA) produces Input-Output Tables detailing the dollar flows between all producers and purchasers in the U.S. Producers include all industrial and service sectors, as well as household production. Purchasers include industrial sectors, households, and government entities. The BEA constructs the Input-Output Tables using Census data that are collected every five years. The BEA has published Input-Output tables every five years beginning in 1982 and ending with the most recent tables in 2012. The Input-Output tables are based on NAICS industry codes. Prior to 1997, the Input-Output Tables were based on SIC codes.

The Input-Output tables consist of two basic national-accounting tables: a "make" table and a "use" table. The make table shows the production of commodities by industry. Rows present industries, and columns present the commodities each industry produces. Looking across columns for a given row, we see all the commodities a given industry produces. The sum of the entries comprises industry output, gross GDP. Looking across rows for a given column, we see all industries producing a given commodity. The sum of the entries adds up the output of a commodity. The use table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities, and the columns show the industries and final users that utilize them. The sum of the entries in a row is the output of that commodity. The columns document the products each industry uses as inputs and the three components of value added: compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The sum of the entries in a column adds up to industry output.

We utilize the Input-Output tables for 2002 to create an industry network of trade flows. The BEA defines industries at two levels of aggregation: detailed and summary accounts. We use the detailed levels of aggregation to create industry-by-industry trade flows, also to report robustness results using the summary table. The BEA data also serve to construct our measure of sectoral GDP, Ω_C.

The BEA provides concordance tables between NAICS codes and Input-Output industry codes. We follow the BEA's Input-Output classifications with minor modifications to create our industry classifications. We account for duplicates when NAICS codes are not as detailed as Input-Output codes. In some cases, an identical set of NAICS codes defines different Input-Output industry codes. We aggregate industries with overlapping NAICS codes to remove duplicates.

We combine the make and use tables to construct an industry-by-industry matrix which details how much of an industry's inputs other industries produce. We use the make table ($MAKE$) to determine the share of each commodity C that each industry k produces. We define the market share ($SHARE$) of industry k's production of commodity C as

$$SHARE = MAKE \odot (\mathbb{I} - MAKE)^{-1}_{k,k'} .$$

We multiply the share and use tables (USE) to calculate the dollar amount that industry k' sells to industry k. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix,

$$REVSHARE = SHARE \times USE.$$

We then use the revenue share matrix to calculate the percentage of industry k' inputs purchased from industry k and label the resulting matrix $SUPPSHARE$:

$$SUPPSHARE = REVSHARE \odot \left((\mathbb{I} - MAKE)^{-1}_{k,k'} \right)'. \tag{34}$$

The input-share matrix in this equation is an industry-by-industry matrix and therefore consistently maps into our model.[11] The BEA also provides a direct-requirement. This table is a commodity-by-industry matrix, and the mapping to our theoretical model is therefore less straightforward. A commodity-by-commodity direct-requirements table would be an alternative to our approach of modeling input-output relations, but is not readily available. We report calibration results using direct requirements in the appendix for comparison with the literature (see, e.g., Acemoglu et al. (2012)).

B. Frequencies of Price Adjustments

We use the confidential microdata underlying the producer price data (PPI) from the BLS to calculate the frequency of price adjustment at the industry level.[12] The PPI measures changes in prices from the perspective of producers, and tracks prices of all goods-producing industries, such as mining, manufacturing, and gas and electricity, as well as the service sector. The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service sector output. Our sample ranges from 2005 to 2011.

The BLS applies a three-stage procedure to determine the sample of goods. First, to construct the universe of all establishments in the U.S., the BLS compiles a list of all firms filing with the Unemployment Insurance system. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or the number of employees. The BLS collects prices from about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments. Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price adjustment at the goods level, FPA, as the ratio of the number of price changes to the number of sample months. For example, if an observed price

[11] Ozdagli and Weber (2016) follow a similar approach.

[12] The data have been used before in Nakamura and Steinsson (2008); Goldberg and Hellerstein (2011); Bhattarai and Schoenle (2014); Gorodnichenko and Weber (2016); Gilchrist, Schoenle, Sim, and Zakrajšek (2016); Weber (2015); and D'Acunto, Liu, Pflueger, and Weber (2016), among others.

path is \$10 for two months and then \$15 for another three months, one price change occurs during five months, and the frequency is 1/5. We aggregate goods-based frequencies to the BEA industry level.

The overall mean monthly frequency of price adjustment is 22.15%, which implies an average duration, $-1/\log(1 - FPA)$, of 3.99 months. Substantial heterogeneity is present in the frequency across sectors, ranging from as low as 4.01% for the semiconductor manufacturing sector (duration of 24.43 months) to 93.75% for dairy production (duration of 0.36 months).

V Calibration

We calibrate the steady-state input-output linkages of our model Ω to the U.S. input-share matrix in 2002. The same data also serve to calibrate sectoral size, Ω_C. The Calvo parameters match the frequency of price adjustments between 2005 and 2011, using the micro data underlying the PPI from the BLS. After we merge the input-output and the frequency of price adjustment data, we end up with 348 sectors.

The detailed input-output table has 407 unique sectors in 2002. We lose sectors for three reasons. First, some sectors produce almost exclusively final goods, so there are not enough observations of such goods in the PPI data to compute frequency. Second, the goods some sectors produce do not trade in a formal market, so there are no prices to record. Examples of missing sectors are (with I/O industry codes in parentheses) "video tape and disc rentals" (532230), "bowling centers" (713950), "military armored vehicle, tank, and tank component manufacturing" (336992), and "religious organizations" (813100).

We show results for several calibrations of our model. **MODEL1** has linear disutility of labor, $\varphi = 0$, and monetary policy targeting constant nominal GDP. This is the closest parametrization of our full-blown New-Keynesian model to the simplified model we study in Section III with the modeling of the pricing friction as the only difference.[13]

MODEL2 is an intermediate case in which $\varphi = 0$, and monetary policy that follows the Taylor rule we specified in Section II with parameters $\phi_c = 0.33/12 = 0.0275$ and $\phi_\pi = 1.34$.

In **MODEL3**, monetary policy follows this same Taylor rule, but we set the inverse-Frisch elasticity to $\varphi = 2$.

These calibrations are at a monthly frequency, so the discount factor is $\beta = 0.9975$ (implying an annual risk-free interest rate of about 3%). We set the labor share $1-\delta$ to 0.5 and the elasticity

[13] We also calibrate the simplified model of Section III as MODEL1, interpreting the frequencies of price adjustments as the probability a sector can adjust prices after the shock. Results are very close to the results for MODEL1 we discuss below.

of substitution across sectors to $\eta = 2$ and within sectors to $\theta = 6$.

VI Quantitative Results

A. Power Laws

Guided by our theoretical results in Section III, we first report the shape parameters of the power law distribution for sector sizes, outdegrees, and the frequencies of price adjustments following Gabaix and Ibragimov (2011). To do so, we compute the OLS estimator of the empirical log-counter cumulative distribution on the log sequence of the variables using the data in the upper 20% tail.

First, we find the shape parameter of sectoral GDP is 0.8859 (st dev 0.1497) for our calibration sample of 348 sectors matched to PPI micro data. This estimate contrasts with a shape parameter of 1.021 (st dev 0.1604) when we repeat the procedure for all 407 sectors. The exclusion of some sectors suggest a stronger granular effect if prices are frictionless. The shape parameter of sectoral GDP is significantly different from 0 and 2 in both cases, and not significantly different from 1.

As a robustness check, we report some results below for a sample of 345 sectors, which excludes the three sectors with the largest GDP share: "Retail trade" (4A0000), "Real Estate" (531000) and "Wholesale trade" (420000). The shape parameter of GDP in this sample is 1.006 (st dev 0.1713). We also compute this shape paramter for sectoral gross output (value added plus intermediate inputs). To do so, we combine data for both sectoral GDP and input linkages. The estimated parameter is 1.0603 (st dev 0.1792). This estimate is similar to the shape parameter of firm sales in the U.S. economy of 1.054 (see Gabaix (2011)).[14]

Second, the shape parameter of outdegrees for our calibration sample is 1.5676 (st dev 0.2797).[15] Our estimate is significantly different from 0 and 2. We estimate a shape parameter of 1.2834 (st dev 0.2017) for second-order outdegrees.[16]

Third, in terms of price rigidity, the shape parameter of the sectoral distribution of frequency of price changes is 2.5773 (st dev 0.4050); that is, the distribution of the frequency of price adjustment is not fat-tailed.

[14] Gross output is conceptually the closest counterpart to sectoral total sales in Gabaix (2011).

[15] Acemoglu et al. (2012) report an estimate of 1.4559 (st dev 0.2461) for a sample of 416 sectors. In addition to having a slightly different sample, they also treat the input-output linkages in a different way. Our shape parameters for outdegrees are not statistically different from theirs at the 5% significance level.

[16] Acemoglu et al. (2012) report an estimate of 1.3019 (st dev 0.2201).

B. Correlations

In our simplified model of Section III, we derived several results for the rate of decay of aggregate volatility from idiosyncratic shocks for increasingly more disaggregated economies which are a function of the dependence structure between the frequency of price adjustment, the sector size, and the input-output structure.

Motivated by these results, we report some relevant reduced-form correlations. The correlations between the frequency of price adjustment with sectoral GDP $\{C_k\}_{k=1}^{K}$, outdegrees d, and second-order outdegrees q are 5.1%, 18.8%, and 22.2% for the whole sample and 6.7%, 22.6%, and 33.3% in the 20% upper tail. The correlation between the degree of price stickiness and sectoral GDP is rather low, but we will see below that an interesting interaction between these variables still exists.

In fact, although the correlation between sectoral price stickiness and measures of sector centrality is substantially higher than with sectoral GDP, our simple model suggests that correlations do not fully capture the intricate interaction between pricing frictions and the input-output structure of the economy.

C. Multipliers

We now show economically important amplification effects of price stickiness exist, despite some of these low reduced-form correlations and the absence of fat tails in the distribution of the frequency of price changes.

Table 1 reports multipliers, $\|\chi\|$, for different experiments that map the volatility of sectoral productivity shocks into aggregate GDP volatility. We formally define the multiplier, χ, in equation (33). We report multipliers in levels but also relative to the multiplier that maps aggregate productivity shocks into aggregate GDP volatility (we will sometimes refer to the latter as "aggregate multiplier"). Price rigidity has a mechanical effect on aggregate volatility, dampening volatility originating from idiosyncratic, but also aggregate shocks. The relative multiplier controls for the general dampening effect of price rigidity on aggregate volatility.

C.1 Multipliers: Flexible Prices

We start in Panel A with MODEL1, which corresponds to the simplified model of Section III except for the modeling of the pricing friction; that is, it features Calvo price stickiness, a constant nominal GDP target in the monetary policy rule, and linear disutility of labor. Column (1) assumes flexible prices to isolate the quantitative strength of the pure granular effect due to the empirical distribution of sectoral GDP, the pure network effect due to the empirical input-output structure of the U.S. economy, and their joint effect.

We start with an economy in which all sectors are homogeneous, that is, when they have equal size and uniform input-output linkages. As the model in Section III suggests, the multiplier then equals $K^{-1/2}$ for K=348 and it is 5.36% of the aggregate multiplier, which equals 1. The multiplier is 0.1994 when GDP shares Ω_C are calibrated to U.S. data, but intermediate input use is shut down, $\delta = 0$. This isolates the granular effect from the previous uniform network effect. GDP volatility increases by a factor of 4 with sectoral heterogeneity in size relative to uniform GDP shares across sectors, showing a strong granular effect from idiosyncratic shocks for aggregate volatility.

Intermediate inputs ($\delta = 0.5$) with homogeneous steady-state input-output linkages, Ω, mute the strength of the granular channel of idiosyncratic shocks. As line (3) shows, the relative multiplier is now 11% rather than almost 20%.

In line (4), we focus on the fully heterogeneous network channel for aggregate fluctuations; that is, we impose equal GDP shares across sectors but calibrate Ω to the actual, heterogeneous U.S. input-output tables. The multiplier is now 0.0795. The network channel increases the multiplier by 50% relative to the multiplier in an economy with homogeneous steady-state input-output structure (0.0536), but the network channel is substantially smaller than the granular channel for aggregate fluctuations.

The last lines study granular and network channels jointly. The multiplier is now 17.45%, indicating the potential of idiosyncratic shocks to be a major driving force behind aggregate fluctuations.

C.2 Multipliers: Homogeneous Sticky Prices

We next allow for rigid prices in column (2) of Table 1 but impose homogeneous price stickiness across sectors. Specifically, we calibrate the sectoral Calvo parameter to the average frequency of price adjustment in the U.S. for all sectors.

Comparing across columns (1) and (2), price rigidity reduces the level of aggregate volatility that sectoral shocks generate by an order of magnitude, just as our model in Section III predicts. However, sticky prices also tend to dampen aggregate volatility due to aggregate shocks in general. Hence, we focus our discussion on relative multipliers. Multipliers relative to an aggregate productivity shock are similar to the case with flexible prices in column (1), but with two exceptions: (i) introducing homogeneous input-output linkages offsets the granular channel less than under flexible prices (compare rows (2) and (3) across columns (1) and (2)); (ii) the granular effect in row (4) becomes slightly weaker (going from 7.95% to 6.07%). We expect these results based on our analysis in Section III. The pricing friction more strongly mitigates the network effect of second-order outdegrees than of outdegrees. Since the distribution of second-order outdegrees is more fat-tailed than outdegrees, even a homogeneous price friction reduces the quantitative strength of the network effect.

C.3 Multipliers: Heterogeneous Sticky Prices

Column (3) of Table 1 presents the main results of this section. The calibration captures the empirical sectors' size distribution, the actual input-output structures of the U.S. economy at the most granular level, as well as detailed, heterogeneous output price stickiness across sectors, and allows us to analysis the relevance of idiosyncratic shocks for aggregate fluctuations.

The calibration empirically confirms our theoretical predictions of Section III. First, across rows, we see heterogeneous price rigidity increases the *level* of aggregate fluctuations originating from idiosyncratic shocks by at least 45% and up to 100% relative to the case of equal price stickiness in column (2).

Second, heterogeneity in price stickiness alone increases the *relative* multiplier of sectoral shocks on GDP volatility: the relative multiplier goes from 5.36% to 10.77% in a calibration with equal GDP shares across sectors and homogeneous input-output linkages (see row (1)). Heterogeneous price stickiness thus increases the relative multiplier by more than the network effect, which generates a relative multiplier of 7.95% and 6.07% depending on whether prices are flexible or homogeneously sticky across sectors (see row (4) in columns (1) and (2)). Thus, heterogeneous price rigidity creates a "frictional" channel of aggregate volatility independent of the "granular" or "network" channels in the literature.

Third, the interaction between the granular effect of heterogeneous sector sizes and the frictional channel is quite strong despite a low correlation between sectoral size and the frequency of price changes. Without intermediate inputs, $\delta = 0$, the relative multiplier is 24.87% instead of 19.94% when prices rigidity is equal for all sectors (see rows 2 in columns (2) and (3)). If $\delta = 0.5$ and input-output linkages are equal for all sectors, the relative multiplier is 22.77%, whereas it is only 11% in an economy with flexible price and 16.4% in an economy with equal price stickiness across sectors (see row 3).

Fourth, there is also a strong interaction between the network channel of aggregate fluctuations and the frictional channel: the relative multiplier is 11.52% with sticky prices calibrated to the U.S. economy, whereas it is only 7.95% in a flexible-price economy and 6.07% in an economy with equal price stickiness (compare row (4) across columns).

Overall, when our model matches the sectoral size distribution, the input-output linkages of the U.S. economy, and the distribution of price rigidity across sectors, the relative multiplier that maps sectoral productivity shocks into aggregate volatility equals almost a quarter of the multiplier of an aggregate productivity shock, which is an almost 40% increase compared to a relative multiplier of 17.45% in a frictionless economy (last row).

C.4 Multipliers: Alternative Model Specifications

Panels B and C of Table 1 report similar results for two alternative model specifications. MODEL2 assumes a standard Taylor rule instead of a monetary policy targeting constant nominal GDP, whereas MODEL3 additionally drops the assumption of linear disutility of labor. The level of the multipliers differ from MODEL1 in Panel A but the relative multipliers are almost identical across different calibrations.

Table 2 reports multipliers in levels and relative to the aggregate multiplier for the same cases, but studies only the impact effect of sectoral shocks on GDP. Multipliers differ only slightly relative to the ones in reported in Table 1, suggesting the Calvo assumption introduces only a small degree of persistence relative to the simple specification of price rigidity we study in the simplified model of Section III.

Table 3 excludes the three sectors with the largest GDP shares but is otherwise identical to Table 1: "Retail trade," "Real Estate," and "Wholesale trade." The exclusion of these sectors mutes the quantitative strength of the granular effect somewhat, but it is still quantitatively large. Overall, we confirm all the key results of Table 1 when we introduce price rigidity and allow for heterogeneous price stickiness across sectors.

D. Distorted Idiosyncratic Origin of Fluctuations

Table 4 shows introducing heterogeneity in the frequency of price adjustment across sectors changes the identity and relative contribution of the five most important sectors for the multiplier for different calibrations of MODEL1. Relative contributions sum to 1 and the entries in Table 4 tell us directly the fraction of the multiplier coming from the reported sectors.[17] Overall, sectoral heterogeneity of price rigidity distorts the identity and the relative contribution of the most important sectors for aggregate fluctuations originating from sectoral productivity shocks.

In column (1), we calibrate Calvo parameters to the sectoral frequency of price changes in the U.S., but impose equal GDP shares across sectors, and input-output linkages are homogeneous. The five most important sectors are the five sectors with most flexible prices, which are mostly farming products: "Dairy cattle and milk production" (112120), "All other crop

[17] Results are similar for MODEL2 or MODEL3.

farming" (1119B0), "Cattle ranching and farming" (1121A0), "Primary smelting and refining of copper" (331411), "Poultry and egg production" (112300). The relative contributions to the overall multiplier range from 6.05% to 4.59%. If all sectors were perfectly identical, the contribution of any sector would be 0.29% (348^{-1}).

Columns (2) and (3) in Table 4 allow us to study how price rigidity affects the granular channel of idiosyncratic shocks. Column (2) assumes flexible prices but steady-state sectoral GDP shares that match GDP shares in the data. Column (3) also matches the sectoral frequency of price changes. The three most important sectors are the same in both calibrations, but their relative contributions change substantially: with flexible prices, the relative contributions of "Retail trade" (4A0000), "Real estate" (531000), and "Wholesale trade" (420000) are 22.58%, 21.3%, and 18.42%, respectively; once we allow for rigid prices, the contribution of "Wholesale trade" (420000) doubles from 18.42% to 36.13%, whereas the contribution of "Retail trade" (4A0000) is now smaller by 50%.

The relative contribution to the granular multiplier of the fourth most important sector, "Monetary authorities and depository credit intermediation" (52A000), increases from 4.92% when prices are flexible to 10.93% with sticky prices. The fifth most important sector, instead, changes its identity: when prices are flexible, it is "Offices of physicians, dentists, and other health practitioners" (621A00) with a contribution of 3.6%, whereas it becomes "Telecommunications" (517000) with a contribution of 8.47% when prices are sticky.

Columns (4) and (5) of Table 4 expose the distortion price rigidity introduces on the network effect of aggregate fluctuations. Column (4) assumes flexible prices and steady-state input-output linkages calibrated to the U.S. input-output tables, whereas column (5) also matches sectoral frequencies of price adjustment. Now the identity of the five most important sectors changes completely: with flexible prices, the most important sectors is "Wholesale trade" (420000), with a contribution of 25.22%, followed by "Real estate" (531000), "Monetary authorities and depository credit intermediation" (52A000), "Electric power generation, transmission, and distribution" (221100), and "Petroleum refineries" (324110), with relative contributions of 9.44%, 3.59%, 3.42%, and 2.87%.

Once we allow for sticky prices, the contributions of the five most important sectors range between 6.65% to 5.5% which in descending order are "Petroleum refineries" (324110), "Electric power generation, transmission, and distribution" (221100), "Cattle ranching and farming" (1121A0), "All other crop farming" (1119B0) and "Primary smelting and refining of copper" (331411). In short, energy sectors become the most important followed by farming sectors, once we allow for price rigidity.

Finally, we compare columns (6) and (7) in Table 4 to see how the introduction of heterogeneous price stickiness across sectors changes the importance and identity of sectors for the multiplier when both the granular and network channels are at work.

The relative contributions of the five most important sectors with flexible prices are: "Real estate" (531000), "Wholesale trade" (420000), "Monetary authorities and depository credit intermediation" (52A000), "Retail trade" (4A0000) and "Telecommunications" (517000). With flexible prices, "Real estate" on its own contributes a third to the overall multiplier. When we turn on sticky prices, the contribution, instead, drops to below 20%. For "Wholesale trade," we see exactly the opposite changes in relative importance across calibrations. These effects mirror the changes we observe in columns (2) and (3) when we study the granular channel in isolation but the changes are more pronounced. The identity of the other three most important sectors remains unchanged but their relative importance changes slightly.

The evidence on the changing identity and relative importance of sectors for aggregate fluctuations originating from sectoral shocks we observe in Table 4 underlines the importance to study granular, network, and frictional channels in combination. A central bank which aims to stabilize sectoral prices of "big" or "central" sectors might make systematic policy mistakes if it does not take into account the "frictional" origin of aggregate fluctuations.

VII Concluding Remarks

This paper studies the "frictional origin" of aggregate fluctuations originating from sectoral shocks when nominal output prices are rigid. We do so theoretically and quantitatively in the context of a multi-sector New-Keynesian model with intermediate inputs. We calibrate a 348-sector model to the most granular input-output linkages and sector sizes from the Bureau of Economic Analysis and the sectoral frequency of price adjustments from the Bureau of Labor Statistics.

We show analytically in a simplified model that the aggregate propagation of a sectoral productivity shock depends on sectors' size (measured by its GDP), its centrality in the production network, and the distribution of price rigidity across sectors. In particular, a shock to a sector has stronger aggregate effects when it hits a large/central sector with highly flexible prices that sells to large/central sectors with flexible prices. We derive conditions under which the interaction between the frictional, granular, and network sources may amplify the scale of aggregate fluctuations from microeconomic shocks. We also show theoretically that pricing friction changes the identity and relative contribution of the most important sectors driving aggregate fluctuations. Thus, price rigidity not only generates aggregate inertia, as is standard when shocks are aggregate, but may also distort the sign of aggregate fluctuations given the idiosyncratic nature of microeconomic shocks.

Quantitatively, the pricing friction alone creates sizable effects of microeconomic shocks on GDP volatility. Thus, there is a "frictional" origin of aggregate fluctuations that is conceptually different from the granular or network mechanisms already described in the literature.

Overall, price rigidity generates a frictional origin of aggregate fluctuations, it amplifies the granular and network channels of idiosyncratic shocks, and it changes the identity and relative importance of sectors for aggregate fluctuations originating from sectoral shocks. A central bank which aims to stabilize sectoral prices of "big" or "central" sectors might make a systematic policy mistake if it does not take into account the "frictional" origin of aggregate fluctuations.

References

Acemoglu, D., U. Akcigit, and W. Kerr (2016). Networks and the macroeconomy: An empirical exploration. *NBER Macroannual 30*(1), 273–335.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The network origins of aggregate fluctuations. *Econometrica 80*(5), 1977–2016.

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2017). Microeconomic origins of macroeconomic tail risks. *The American Economic Review 107*(1), 54–108.

Atalay, E. (2015). How important are sectoral shocks? *Unpublished Manuscript, University of Wisconsin*.

Baqaee, D. R. (2016). Cascading failures in production networks. *Unpublished Manuscript, LSE*.

Barrot, J.-N. and J. Sauvagnat (2016). Input specificity and the propagation of idiosyncratic shocks in production networks. *The Quarterly Journal of Economics 131*(3), 1543–1592.

Basu, S. (1995). Intermediate goods and business cycles: Implications for productivity and welfare. *The American Economic Review 85*(3), 512–531.

Bhattarai, S. and R. Schoenle (2014). Multiproduct firms and price-setting: Theory and evidence from U.S. producer prices. *Journal of Monetary Economics 66*, 178–192.

Bigio, S. and J. La'O (2016). Financial frictions in production networks. Technical report, National Bureau of Economic Research.

Bils, M. and P. J. Klenow (2004). Some evidence on the importance of sticky prices. *Journal of Political Economy 112*(5), 947–985.

Boivin, J., M. P. Giannoni, and I. Mihov (2009). Sticky prices and monetary policy: Evidence from disaggregated U.S. data. *The American Economic Review 99*(1), 350–384.

Bouakez, H., E. Cardia, and F. Ruge-Murcia (2014). Sectoral price rigidity and aggregate dynamics. *European Economic Review 65*, 1–22.

Caliendo, L., F. Parro, E. Rossi-Hansberg, and P.-D. Sarte (2014). The impact of regional and sectoral productivity changes on the U.S. economy. Technical report, National Bureau of Economic Research.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Carvalho, C. and J. W. Lee (2011). Sectoral price facts in a sticky-price model. *Unpublished Manuscript, PUC-Rio*.

Carvalho, V. and X. Gabaix (2013). The great diversification and its undoing. *The American Economic Review 103*(5), 1697–1727.

Carvalho, V. M. (2014). From micro to macro via production networks. *The Journal of Economic Perspectives 28*(4), 23–47.

Carvalho, V. M. and B. Grassi (2015). Large firm dynamics and the business cycle. *Unpublished Manuscript, University of Cambridge*.

Carvalho, V. M., M. Nirei, Y. U. Saito, and A. Tahbaz-Salehi (2016). Supply chain disruptions: Evidence from the great east Japan earthquake. *Unpublished Manuscript*.

Cochrane, J. H. (1994). Shocks. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 41, pp. 295–364. Elsevier.

D'Acunto, F., R. Liu, C. E. Pflueger, and M. Weber (2016). Flexible prices and leverage. *Unpublished Manuscript, University of Chicago*.

Di Giovanni, J., A. A. Levchenko, and I. Méjean (2014). Firms, destinations, and aggregate fluctuations. *Econometrica 82*(4), 1303–1340.

Di Giovanni, J., A. A. Levchenko, and I. Méjean (2016). The micro origins of international business cycle comovement. Technical report, National Bureau of Economic Research.

Di Maggio, M. and A. Tahbaz-Salehi (2015). Collateral shortages and intermediation networks. *Unpublished Manuscript*.

Dupor, B. (1999). Aggregation and irrelevance in multi-sector models. *Journal of Monetary Economics 43*(2), 391–409.

Durrett, R. (2013). *Probability: theory and examples* (4th ed.). Cambridge; New York: Cambridge University Press.

Foerster, A. T., P.-D. G. Sarte, and M. W. Watson (2011). Sectoral versus aggregate shocks: A structural factor analysis of industrial production. *Journal of Political Economy 119*(1), 1–38.

Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica 79*(3), 733–772.

Gabaix, X. and R. Ibragimov (2011). Rank-1/2: A simple way to improve the OLS estimation of tail exponents. *Journal of Business & Economic Statistics 29*(1), 24–39.

Gilchrist, S., R. Schoenle, J. Sim, and E. Zakrajšek (2016). Inflation dynamics during the financial crisis. *American Economic Review (forthcoming)*.

Gofman, M. (2011). A network-based analysis of over-the-counter markets. *Unpublished Manuscript*.

Goldberg, P. P. and R. Hellerstein (2011). How rigid are producer prices? *FRB of New York Staff Report*, 1–55.

Gorodnichenko, Y. and M. Weber (2016). Are sticky prices costly? Evidence from the stock market. *American Economic Review 106*(1), 165–199.

Herskovic, B. (2015). Networks in production: Asset pricing implications. *Unpublished Manuscript, UCLA*.

Herskovic, B., B. T. Kelly, H. Lustig, and S. Van Nieuwerburgh (2016). The common factor in idiosyncratic volatility: Quantitative asset pricing implications. *Journal of Financial Economics 119*(2), 249–283.

Horvath, M. (1998). Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics 1*(4), 781–808.

Horvath, M. (2000). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics 45*(1), 69–106.

Hulten, C. R. (1978). Growth accounting with intermediate inputs. *The Review of Economic Studies 45*(3), 511–518.

Kelly, B., H. Lustig, and S. Van Nieuwerburgh (2013). Firm volatility in granular networks. Technical report, National Bureau of Economic Research.

Klenow, P. J. and O. Kryvtsov (2008). State-dependent or time-dependent pricing: does it matter for recent us inflation? *The Quarterly Journal of Economics 123*(3), 863–904.

Long, J. B. and C. Plosser (1983). Real business cycles. *The Journal of Political Economy 91*(1), 39–69.

Lucas, R. E. (1977). Understanding business cycles. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 5, pp. 7–29. Elsevier.

Midrigan, V. (2011). Menu costs, multiproduct firms, and aggregate fluctuations. *Econometrica 79*(4), 1139–1180.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics 123*(4), 1415–1464.

Nakamura, E. and J. Steinsson (2010). Monetary non-neutrality in a multisector menu cost model. *Quarterly Journal of Economics 125*(3), 961–1013.

Ozdagli, A. and M. Weber (2016). Monetary policy through production networks: Evidence from the stock market. *Unpublished Manuscript, University of Chicago*.

Pasten, E., R. Schoenle, and M. Weber (2016). Production networks and the propagation of monetary policy shocks. *Unpublished manuscript*.

Weber, M. (2015). Nominal rigidities and asset pricing. *Unpublished manuscript, University of Chicago Booth School of Business*.

Table 1: **Multipliers of Sectoral Shocks into Aggregate Volatility**

This Table reports multipliers of sectoral productivity shocks on GDP volatility, $\|\chi\|$, with relative multipliers, $\|\chi\|$, in parenthesis. The latter are relative to the multiplier of an aggregate productivity shock on GDP volatility.

			Flexible Prices		Homogeneous Calvo		Heterogeneous Calvo	
			(1)		(2)		(3)	
					Panel A: MODEL1			
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0028	(5.36%)	0.0052	(10.77%)
(2)	het Ω_c	δ =0	0.1994	(19.94%)	0.0167	(19.94%)	0.0249	(24.87%)
(3)	het Ω_c	hom Ω	0.1100	(11.00%)	0.0085	(16.40%)	0.0126	(22.77%)
(4)	hom Ω_c	het Ω	0.0795	(7.95%)	0.0032	(6.07%)	0.0063	(11.52%)
(5)	het Ω_c	het Ω	0.1745	(17.45%)	0.0098	(18.86%)	0.0143	(23.95%)
					Panel B: MODEL2			
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0050	(5.36%)	0.0060	(7.54%)
(2)	het Ω_c	δ =0	0.1994	(19.94%)	0.0343	(19.94%)	0.0432	(25.96%)
(3)	het Ω_c	hom Ω	0.1100	(11.00%)	0.0138	(14.81%)	0.0177	(18.66%)
(4)	hom Ω_c	het Ω	0.0795	(7.95%)	0.0059	(6.38%)	0.0077	(8.62%)
(5)	het Ω_c	het Ω	0.1745	(17.45%)	0.0171	(18.40%)	0.0219	(21.45%)
					Panel C: MODEL3			
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0048	(5.36%)	0.0076	(8.26%)
(2)	het Ω_c	δ =0	0.1994	(19.94%)	0.0401	(19.94%)	0.0517	(25.78%)
(3)	het Ω_c	hom Ω	0.1135	(11.00%)	0.0137	(15.40%)	0.0196	(18.72%)
(4)	hom Ω_c	het Ω	0.0775	(7.95%)	0.0056	(6.23%)	0.0093	(9.09%)
(5)	het Ω_c	het Ω	0.1755	(17.45%)	0.0166	(18.55%)	0.0234	(21.10%)

Table 2: **Multipliers of Sectoral Shocks into Aggregate Volatility: Impact Response**

This Table reports the impact multipliers of sectoral productivity shocks on GDP volatility, $\|\chi\|$, with relative multipliers, $\|\chi\|$, in parentheses. The latter are relative to the multiplier of an aggregate productivity shock on GDP volatility.

			Flexible Prices		Homogeneous Calvo		Heterogeneous Calvo	
			(1)		(2)		(3)	
					Panel A: MODEL1			
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0015	(5.36%)	0.0048	(11.71%)
(2)	het Ω_c	δ =0	0.1994	(19.94%)	0.0107	(19.94%)	0.0188	(23.53%)
(3)	het Ω_c	hom Ω	0.1100	(11.00%)	0.0054	(18.58%)	0.0095	(22.40%)
(4)	hom Ω_c	het Ω	0.0795	(7.95%)	0.0016	(5.43%)	0.0056	(12.37%)
(5)	het Ω_c	het Ω	0.1745	(17.45%)	0.0056	(19.51%)	0.0102	(22.94%)
					Panel B: MODEL2			
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0033	(5.36%)	0.0055	(7.87%)
(2)	het Ω_c	δ =0	0.1994	(19.94%)	0.0265	(19.94%)	0.0377	(25.58%)
(3)	het Ω_c	hom Ω	0.1135	(11.35%)	0.0098	(16.01%)	0.0147	(18.58%)
(4)	hom Ω_c	het Ω	0.0775	(7.75%)	0.0036	(5.91%)	0.0068	(8.77%)
(5)	het Ω_c	het Ω	0.1755	(17.55%)	0.0114	(18.71%)	0.0175	(21.00%)
					Panel C: MODEL3			
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0048	(5.36%)	0.0059	(7.72%)
(2)	het Ω_c	δ =0	0.1994	(19.94%)	0.0401	(19.94%)	0.0456	(25.50%)
(3)	het Ω_c	hom Ω	0.1135	(11.35%)	0.0136	(15.28%)	0.0167	(18.44%)
(4)	hom Ω_c	het Ω	0.0775	(7.75%)	0.0055	(6.12%)	0.0075	(8.64%)
(5)	het Ω_c	het Ω	0.1755	(17.55%)	0.0165	(18.51%)	0.0205	(20.99%)

Table 3: **Multipliers of Sectoral Shocks into Aggregate Volatility (excl largest sectors)**

This Table reports multipliers of sectoral productivity shocks on GDP volatility, $\|\chi\|$*, with relative multipliers,* $\|\chi\|$*, in parentheses. The latter are relative to the multiplier of an aggregate productivity shock on GDP volatility. We exclude the three sectors with the largest GDP shares: Retail trade (4A0000), Real Estate (531000) and Wholesale trade (420000). We adjust all numbers by a factor* $\left(\frac{345}{348}\right)^{1/2}$.

			Flexible Prices		Homogeneous Stickiness		Heterogeneous Calvo	
			(1)		(2)		(3)	
			Panel A: MODEL1					
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0024	(5.36%)	0.0052	(10.82%)
(2)	het Ω_c	δ =0	0.1496	(14.96%)	0.0108	(14.96%)	0.0204	(21.61%)
(3)	het Ω_c	hom Ω	0.088	(8.80%)	0.0056	(12.52%)	0.0103	(20.05%)
(4)	hom Ω_c	het Ω	0.0702	(7.02%)	0.0026	(5.76%)	0.0064	(11.69%)
(5)	het Ω_c	het Ω	0.1384	(13.84%)	0.0064	(14.42%)	0.0121	(21.47%)
			Panel B: MODEL2					
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0043	(5.36%)	0.006	(7.57%)
(2)	het Ω_c	δ =0	0.1496	(14.96%)	0.0224	(14.96%)	0.0313	(21.68%)
(3)	het Ω_c	hom Ω	0.088	(8.80%)	0.0091	(11.42%)	0.0134	(15.40%)
(4)	hom Ω_c	het Ω	0.0702	(7.02%)	0.0048	(5.95%)	0.0076	(8.49%)
(5)	het Ω_c	het Ω	0.1386	(13.84%)	0.0114	(14.19%)	0.0169	(17.94%)
			Panel C: MODEL3					
(1)	hom Ω_c	hom Ω	0.0536	(5.36%)	0.0039	(5.36%)	0.0076	(8.26%)
(2)	het Ω_c	δ =0	0.1496	(14.96%)	0.0251	(14.96%)	0.039	(22.14%)
(3)	het Ω_c	hom Ω	0.0904	(9.04%)	0.0086	(11.86%)	0.0156	(15.78%)
(4)	hom Ω_c	het Ω	0.0686	(6.86%)	0.0043	(5.86%)	0.0094	(9.08%)
(5)	het Ω_c	het Ω	0.1386	(13.86%)	0.0104	(14.27%)	0.019	(17.89%)

Table 4: **Contribution of Sectors to Multiplier of Sectoral Shocks on GDP Volatility**

This table reports the contribution of five most important sectors to the multiplier of sectoral shocks on GDP volatility for MODEL1 and the identity of sectors in parentheses. The different columns represent calibrations which match the frequency of price adjustments (λ), the distribution of consumption shares (Ω_c), or the actual input-output matrix (Ω). 112120: Dairy cattle and milk production; 1119B0: All other crop farming; 1121A0: Cattle ranching and farming; 331411: Primary smelting and refining of copper; 112300: Poultry and egg production; 4A0000: Retail trade; 531000: Real estate; 420000: Wholesale trade; 52A000: Monetary authorities and depository credit intermediation; 621A00: Offices of physicians, dentists, and other health practitioners; 517000: Telecommunications; 221100: Electric power generation, transmission, and distribution; 324110: Petroleum refineries; 211000: Oil and gas extraction; 331411: Primary smelting and refining of copper.

λ (1)	Ω_c (2)	λ, Ω_c (3)	Ω (4)	λ, Ω (5)	Ω_c, Ω (6)	λ, Ω_c, Ω (7)
6.05% (112120)	22.58% (4A0000)	36.13% (420000)	25.22% (420000)	6.65% (324110)	33.92% (531000)	32.76% (420000)
5.13% (1119B0)	21.30% (531000)	17.31% (531000)	9.44% (531000)	6.54% (211000)	16.71% (420000)	19.32% (531000)
5.11% (1121A0)	18.42% (420000)	11.86% (4A0000)	3.59% (52A000)	5.92% (1121A0)	10.27% (4A0000)	12.09% (52A000)
5.04% (331411)	4.92% (52A000)	10.93% (52A000)	3.42% (221100)	5.62% (1119B0)	8.13% (52A000)	9.56% (4A0000)
4.59% (112300)	3.60% (621A00)	8.47% (517000)	2.87% (324110)	5.50% (331411)	5.85% (517000)	9.04% (517000)

Online Appendix:
The Frictional Origin of Aggregate Fluctuations

Ernesto Pasten, Raphael Schoenle, and Michael Weber

Not for Publication

I Steady-State Solution and Log-linear System

A. Steady-State Solution

Without loss of generality, set $a_k = 0$. We show below conditions for the existence of a symmetric steady state across firms in which

$$W_k = W, Y_{jk} = Y,\ L_{jk} = L,\ Z_{jk} = Z, P_{jk} = P \text{ for all } j, k.$$

Symmetry in prices across all firms implies

$$P^c = P^k = P_k = P$$

such that, from equations (1), (2), and (10) in the main body of the paper and (13),

$$\begin{aligned} C_k &= \omega_{ck} C, \\ C_{jk} &= \frac{1}{n_k} C_k, \\ Z_{jk}\left(k'\right) &= \omega_{kk'} Z, \\ Z_{jk}\left(j', k'\right) &= \frac{1}{n_{k'}} Z_{jk}\left(k'\right). \end{aligned}$$

The vector $\Omega_c \equiv [\omega_{c1}, ..., \omega_{cK}]'$ represents steady-state sectoral shares in value-added C, $\Omega = \{\omega_{kk'}\}_{k,k'=1}^{K}$ is the matrix of input-output linkages across sectors, and $\aleph \equiv [n_1, ..., n_K]'$ is the vector of steady-state sectoral shares in gross output Y.

It also holds that

$$\begin{aligned} C &= \sum_{k=1}^{K} \int_{\Im_k} C_{jk} dj, \\ Z_{jk} &= \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}\left(j', k'\right) dj' = Z. \end{aligned}$$

From Walras' law in equation (19) and symmetry across firms, it holds

$$Y = C + Z. \tag{A.1}$$

Walras' law and results above yield, for all j, k:

$$\begin{aligned} Y_{jk} &= C_{jk} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{jk}\left(j', k'\right) dj' \\ Y &= \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left(\sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right) Z, \end{aligned}$$

so $\aleph$ satisfies

$$\begin{aligned} n_k &= (1-\psi)\,\omega_{ck} + \quad \psi \sum_{k'=1}^{K} n_{k'} \omega_{k'k}, \\ \aleph &= (1-\psi) \left[I - \psi \Omega' \right]^{-1} \Omega_c, \end{aligned}$$

for $\psi \equiv \frac{Z}{Y}$. Note by construction $\aleph' \iota = 1$, which must hold given the total measure of firms is 1.

Steady-state labor supply from equation (7) is

$$\frac{W_k}{P} = g_k L_k^{\varphi} C^{\sigma}.$$

In a symmetric steady state, $L_k = n_k L$, so this steady state exists if $g_k = n_k^{-\varphi}$ such that $W_k = W$ for all k. Thus, steady-state labor supply is given by

$$\frac{W}{P} = L^{\varphi} C^{\sigma}. \tag{A.2}$$

Households' budget constraint, firms' profits, production function, efficiency of production (from equation (15)) and optimal prices in steady state respectively are

$$CP = WL + \Pi \tag{A.3}$$

$$\Pi = PY - WL - PZ \tag{A.4}$$

$$Y = L^{1-\delta} Z^{\delta} \tag{A.5}$$

$$\delta WL = (1-\delta)\, PZ \tag{A.6}$$

$$sP = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{A.7}$$

for $\xi \equiv \frac{1}{1-\delta} \left(\frac{\delta}{1-\delta} \right)^{-\delta}$.

Equation (A.7) solves

$$\frac{W}{P} = \left(\frac{\theta - 1}{\theta \xi} \right)^{\frac{1}{1-\delta}}. \tag{A.8}$$

This latter result together with equations (A.5), (A.6), and (A.7) solve

$$\frac{\Pi}{P} = \frac{1}{\theta} Y.$$

Plugging this result in equation (A.4) and using equation (A.1) yields

$$\begin{aligned} C &= \left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]Y \\ Z &= \delta\left(\frac{\theta-1}{\theta}\right)Y, \end{aligned} \tag{A.9}$$

such that $\psi \equiv \delta\left(\frac{\theta-1}{\theta}\right)$.

This result and equation (A.7) gives

$$L = \left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-\frac{\delta}{1-\delta}} Y,$$

where Y solves from this latter result, equations (A.2), (A.9) and (A.8):

$$Y = \left(\frac{\theta-1}{\theta\xi}\right)^{\frac{1}{(1-\delta)(\sigma+\varphi)}} \left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{\frac{\delta\varphi}{(1-\delta)(\sigma+\varphi)}} \left[1-\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-\frac{\sigma}{\sigma+\varphi}}.$$

B. Log-linear System

B.1 Aggregation

Aggregate and sectoral consumption (interpreted as value-added) given by equations (1) and (2) are

$$\begin{aligned} c_t &= \sum_{k=1}^{K} \omega_{ck} c_{kt}, \\ c_{kt} &= \frac{1}{n_k}\int_{\Im_k} c_{jkt}dj. \end{aligned} \tag{A.10}$$

Aggregate and sectoral production of intermediate inputs are given by

$$\begin{aligned} z_t &= \sum_{k=1}^{K} n_k z_{kt}, \\ z_{kt} &= \frac{1}{n_k}\int_{\Im_k} z_{jkt}dj, \end{aligned} \tag{A.11}$$

where (10) and (13) imply that $z_{jk} = \sum_{r=1}^{K} \omega_{kr} z_{jk}(r)$ and $z_{jk}(r) = \frac{1}{n_r}\int_{\Im_r} z_{jk}(j', r)\, dj'$.

Sectoral and aggregate prices are given from equations (4), (6), and (12),

$$\begin{aligned} p_{kt} &= \int_{\Im_k} p_{jk}dj \text{ for } k = 1, ..., K \\ p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt}, \\ p_t^k &= \sum_{k'=1}^{K} \omega_{kk'} p_{k't}. \end{aligned}$$

Aggregation of labor is

$$
\begin{aligned}
l_t &= \sum_{k=1}^{K} l_{kt}, \\
l_{kt} &= \int_{\Im_k} l_{jkt} dj.
\end{aligned} \tag{A.12}
$$

B.2 Demands

Households' demand for sectors and goods in equations (3) and (5) for all $k = 1, ..., K$ become

$$
\begin{aligned}
c_{kt} - c_t &= \eta \left(p_t^c - p_{kt}\right), \\
c_{jkt} - c_{kt} &= \theta \left(p_{kt} - p_{jkt}\right).
\end{aligned} \tag{A.13}
$$

In turn, firm jk's demand for sectors and goods in equation (11) and (14) for all $k, r = 1, ..., K$,

$$
\begin{aligned}
z_{jkt}\left(k'\right) - z_{jkt} &= \eta \left(p_t^k - p_{k't}\right), \\
z_{jkt}\left(j', k'\right) - z_{jkt}\left(k'\right) &= \theta \left(p_{k't} - p_{j'k't}\right).
\end{aligned} \tag{A.14}
$$

Firms' gross output satisfies Walras' law,

$$
y_{jkt} = (1-\psi) c_{jkt} + \psi \sum_{k'=1}^{K} \int_{\Im_{k'}} z_{j'k't}(j, k) \, dj'. \tag{A.15}
$$

Total gross output follows from the aggregation of equations (19),

$$
y_t = (1-\psi) c_t + \psi z_t. \tag{A.16}
$$

B.3 IS and labor supply

The household Euler equation in equation (8) becomes

$$
c_t = \mathbb{E}_t\left[c_{t+1}\right] - \sigma^{-1}\left\{i_t - \left(\mathbb{E}_t\left[p_{t+1}^c\right] - p_t\right)\right\}.
$$

The labor supply condition in equation (7) is

$$
w_{kt} - p_t^c = \varphi l_{kt} + \sigma c_t. \tag{A.17}
$$

B.4 Firms

Production function:

$$
y_{jkt} = a_{kt} + (1-\delta) l_{jkt} + \delta z_{jkt} \tag{A.18}
$$

Efficiency condition:

$$
w_{kt} - p_t^k = z_{jkt} - l_{jkt} \tag{A.19}
$$

Marginal costs:

$$
mc_{kt} = (1-\delta) w_{kt} + \delta p_t^k - a_{kt} \tag{A.20}
$$

Optimal reset price:

$$p_{kt}^{*} = (1 - \alpha_k \beta)\, mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[p_{kt+1}^{*} \right]$$

Sectoral prices:

$$p_{kt} = (1 - \alpha_k)\, p_{kt}^{*} + \alpha_k p_{kt-1}$$

B.5 Taylor rule:

$$i_t = \phi_\pi \left(p_t^c - p_{t-1}^c \right) + \phi_c c_t$$

II Solution of Key Equations in Section III

A. Solution of Equation (25)

Setting $\sigma = 1$ and $\varphi = 0$ in equation (A.17) yields

$$w_{kt} = c_t + p_t^c = 0,$$

where the equality follows from the assumed monetary policy rule, so equation (A.20) becomes

$$mc_{kt} = \delta p_t^k - a_{kt}.$$

Here, sectoral prices for all $k = 1, ..., K$ are governed by

$$\begin{aligned} p_{kt} &= (1 - \lambda_k)\, mc_{kt} \\ &= \delta\, (1 - \lambda_k)\, p_t^k - (1 - \lambda_k)\, a_{kt}, \end{aligned}$$

which in matrix form solves

$$p_t = - \left[\mathbb{I} - \delta\, (\mathbb{I} - \Lambda)\, \Omega\right]^{-1} (\mathbb{I} - \Lambda)\, a_t.$$

$p_t \equiv [p_{1t}, ..., p_{Kt}]'$ is the vector of sectoral prices, Λ is a diagonal matrix with the vector $[\lambda_1, ..., \lambda_K]'$ in its diagonal, Ω is the matrix of input-output linkages, and $a_t \equiv [a_{1t}, ..., a_{Kt}]'$ is the vector of realizations of sectoral technology shocks.

The monetary policy rule implies $c_t = -p_t^c$, so

$$c_t = \left(\mathbb{I} - \Lambda'\right) \left[\mathbb{I} - \delta \left(\mathbb{I} - \Lambda'\right) \Omega'\right]^{-1} \Omega_c' a_t.$$

Solution of Equation (32)

Setting $\sigma = 1$ and $\varphi > 0$ in (A.17) yields

$$w_{kt} = \varphi l_{kt}^s + c_t + p_t^c = \varphi l_{kt}^d,$$

which follows from the assumed monetary policy rule.

Labor demand is obtained from the production function in equation (A.18), the efficiency condition for production in equation (A.19), and the aggregation of labor in equation (A.12):

$$l_{kt}^d = y_{kt} - a_{kt} - \delta \left(w_{kt} - p_t^k\right).$$

y_{kt} follows from equations (A.10), (A.11), (A.13), (A.14), and (A.15):

$$y_{kt} = y_t - \eta \left(p_{kt} - \left[(1 - \psi)\, p_t^c + \psi \sum_{k=1}^{K} n_k p_t^k\right]\right),$$

where

$$\widetilde{p}_t \equiv \sum_{k=1}^{K} n_k p_t^k = \sum_{k=1}^{K} \zeta_k p_{kt},$$

with $\zeta_k \equiv \sum_{k'=1}^{K} n_{k'} \omega_{k'k}$.

To solve for y_t, we use equations (A.11), (A.12), (A.16) and $y_t = \sum_{k=1}^{K} \int_{\Im_k} y_{jkt} dj$ to get

$$y_t = c_t + \psi \left[\Gamma_c c_t - \Gamma_p \left(\widetilde{p}_t - p_t^c \right) - \Gamma_a \sum_{k=1}^{K} n_k a_{kt} \right],$$

where $\Gamma_c \equiv \frac{(1-\delta)(1+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}$, $\Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}$, $\Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}$.

Putting together all these equations, sectoral wages solve

$$w_{kt} = \frac{\varphi}{1+\delta\varphi} \left[\begin{array}{c} (1+\psi\Gamma_c) c_t - a_{kt} - \psi\Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't} \\ \left[(1-\psi)\eta + \psi\Gamma_p \right] p_t^c + \psi (\eta - \Gamma_p) \widetilde{p}_t + \delta p_t^k - \eta p_{kt} \end{array} \right].$$

With this expression, the solution to equation (32) follows the same steps as the solution to equation (25).

III Proofs

Most proofs below are modifications of the arguments in Gabaix (2011), Proposition 2, which rely heavily on the Levy's Theorem (as in Theorem 3.7.2 in Durrett (2013) on p. 138).

Theorem 5 (Levy's Theorem) *Suppose $X_1, ..., X_K$ are i.i.d. with a distribution that satisfies*
(i) $\lim_{x\to\infty} \Pr[X_1 > x] / \Pr[|X_1| > x] = \theta \in (0,1)$
(ii) $\Pr[|X_1| > x] = x^{-\zeta} L(x)$ *with* $\zeta < 2$ *and* $L(x)$ *satisfies* $\lim_{x\to\infty} L(tx)/L(x) = 1$.
Let $S_K = \sum_{k=1}^{K} X_k$,

$$a_K = \inf\{x : \Pr[|X_1| > x] \le 1/K\} \text{ and } b_K = K\mathbb{E}\left[X_1 \mathbf{1}_{|X_1|\le a_K}\right] \tag{A.21}$$

As $K \to \infty$, $(S_K - b_K)/a_K \xrightarrow{d} u$ *where* u *has a nondegenerated distribution.*

A. Proof of Proposition 1

When $\delta = 0$ and $\lambda_k = \lambda$ for all k,

$$\|\chi\|_2 = \frac{1-\lambda}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K}\sum_{k=1}^{K} C_k^2}.$$

Given the power-law distribution of C_k, the first and second moments of C_k exist when $\beta_c > 2$, so

$$K^{1/2}\|\chi\|_2 \longrightarrow \frac{\sqrt{\mathbb{E}\left[C_k^2\right]}}{\mathbb{E}[C_k]}.$$

In contrast, when $\beta_c \in (1,2)$, only the first moment exists. In such cases, by the Levy's theorem,

$$K^{-2/\beta_c}\sum_{k=1}^{K} C_k^2 \xrightarrow{d} u_0^2,$$

where u_0^2 is a random variable following a Levy's distribution with exponent $\beta_c/2$ since $\Pr\left[C_k^2 > x\right] = x_0^{\beta} x^{-\beta_c/2}$.

Thus,

$$K^{1-1/\beta_c}\|\chi\|_2 \xrightarrow{d} \frac{u_0}{\mathbb{E}[C_k]}.$$

When $\beta_c = 1$, the first and second moments of C_k do not exist. For the first moment, by Levy's theorem,

$$\left(\overline{C}_k - \log K\right) \xrightarrow{d} g,$$

where g is a random variable following a Levy distribution.

Since the second moment is equivalent to the result above,

$$(\log K)\|\chi\|_2 \xrightarrow{d} u'.$$

B. Proof of Proposition 2

Let λ_k and C_k be two independent random variables distributed as specified in the Proposition, the counter-cumulative distribution of $z_k = (1-\lambda_k) C_k$ is given by

$$f_Z(z) = \int_z^{z/y_0} f_{C_k}(z/y) f_{1-\lambda_k}(y)\, dy,$$

which follows as Pareto distribution with shape parameter β_c. The proof of the Proposition then follows the proof of Proposition 1 for

$$\|\chi\|_2 = \frac{1}{K^{1/2}\overline{C}_k}\sqrt{\frac{1}{K} z_k^2}. \tag{A.22}$$

C. Proof of Proposition 3

As specified in the proposition, λ_k and C_k are related through $Z_k = (1-\lambda_k) C_k = \phi C_k^{1+\mu}$. When $\mu < 0$, Z_k is distributed Pareto with shape parameter $\beta_c/(1+\mu)$. Proceeding similarly to the proof of Proposition 1, when $\beta_c > \max\{1, 2(1+\mu)\}$, both $\mathbb{E}\left[Z_k^2\right]$ and $\mathbb{E}[C_k]$ exist, so $v_c \sim v/K^{1/2}$. When $\beta_c \in (1, \max\{1, 2(1+\mu)\})$, $\mathbb{E}[C_k]$ exist but $\mathbb{E}\left[Z_k^2\right]$ does not.

Applying Levy's theorem,

$$K^{-2(1+\mu)/\beta_c} \sum_{k=1}^{K} Z_k^2 \xrightarrow{d} u^2.$$

Thus, $v_c \sim \frac{u_1}{K^{1-(1+\mu)/\beta}} v$.

When $\beta_c = 1$, the last result also holds. But now $\mathbb{E}[C_k]$ does not exist. As in Proposition 1, $\left(\frac{1}{K}\sum_{k=1}^{K} C_k - \log K\right) \xrightarrow{d} g$. Thus, if $\mu \in [-1/2, 0]$, $v_c \sim \frac{u_2}{K^{-\mu}\log K} v$, whereas if $\mu \in (-1, -1/2)$, $v_c \sim \frac{u_2}{K^{1/2}\log K} v$.

The proposition for $\mu < 0$ is then obtained by rearranging terms.

When $\mu > 0$, Z_k is distributed piece-wise Pareto such that

$$\Pr[Z_k \geq z] = \begin{cases} x_0^{\beta_c} z^{-\beta_c} & \text{for } z > \phi^{-2/\mu} \\ z_0^{-\beta_c/(1+\mu)} z^{-\beta_c/(1+\mu)} & \text{for } z \in \left[z_0^2, \phi^{-2/\mu}\right]. \end{cases}$$

We now follow the same steps as in the proof of Proposition 1. When $\beta_c > 2(1+\mu)$, $\mathbb{E}\left[Z_k^2\right]$ and $\mathbb{E}[C_k]$ exist, so $v_c \sim v/K^{1/2}$. When $\beta_c \in (1, 2(1+\mu))$, $\mathbb{E}[C_k]$ exists but $\mathbb{E}\left[Z_k^2\right]$ does not. Applying Levy's theorem,

$$\frac{1}{a_K}\sum_{k=1}^{K} Z_k^2 \xrightarrow{d} u^2,$$

where

$$a_K = \begin{cases} x_0^2 K^{2/\beta_c} & \text{for } K > K^* \\ z_0^2 K^{2(1+\mu)/\beta_c} & \text{for } K \leq K^* \end{cases}$$

for $K^* \equiv x_0^{-\beta_c}\phi^{-\beta_c/\mu}$. Thus, $v_c \sim \frac{u_1}{K^{1-\frac{1+\mathbb{1}\{K\leq K^*\}\mu}{\beta_c}}} v$ for some random variable u_1.

When $\beta_c = 1$, $\left(\frac{1}{K}\sum_{k=1}^{K} C_k - \log K\right) \xrightarrow{d} g$, so now $v_c \sim \frac{u_2}{K^{-\mathbb{1}\{K\leq K^*\}\mu}\log K} v$ for some random variable u_2, completing the proof.

D. Proof of Proposition 4

When $\delta \in (0,1)$, $\lambda_k = \lambda$ for all k, and $\Omega_c = \frac{1}{K}\iota$, we know from equation XX that

$$\begin{aligned} \|\chi\|_2 &\geq \frac{1-\lambda}{K}\sqrt{\sum_{k=1}^{K}\left[1+\delta' d_k + \delta'^2 q_k\right]^2} \\ &\geq (1-\lambda)\sqrt{\frac{1+2\delta'+2\delta'^2}{K} + \frac{\delta'^2}{K^2}\sum_{k=1}^{K}\left[d_k^2 + 2\delta' d_k q_k + \delta'^2 q_k^2\right]}. \end{aligned}$$

Following the same argument as in Proposition 2,

$$\begin{aligned} K^{-2/\beta_d}\sum_{k=1}^{K} d_k^2 &\longrightarrow u_d^2, \\ K^{-2/\beta_q}\sum_{k=1}^{K} q_k^2 &\longrightarrow u_q^2, \\ K^{-1/\beta_z}\sum_{k=1}^{K} d_k q_k &\longrightarrow u_z^2, \end{aligned}$$

where u_d^2, u_q^2 and u_z^2 are random variables. Thus, if $\beta_z \geq 2\min\{\beta_d, \beta_q\}$,

$$v_c \geq \frac{u_3}{K^{1-1/\min\{\beta_d,\beta_q\}}} v$$

where u_3^2 is a random variable.