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The macroeconomic consequences of raising the minimum wage: capital accumulation, employment and the wage distribution *

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Abstract

We study the quantitative impact of a rise in the minimum wage on macroeconomic outcomes such as employment, the stock of capital and the distribution of wages. Our modeling framework is the large-firm search and matching model. Our comparative statics are in line with previous empirical findings: a moderate increase in the minimum wage barely affects employment, while it compresses the wage distribution and generates positive spillovers on higher wages. The model also predicts an increase in the stock of capital. Next, we perform the policy experiment of introducing a 10 dollar minimum wage. Our results suggest large positive effects on capital (4.0%) and output (1.8%), with a decrease in employment by 2.8%. The introduction of a 9 dollar

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minimum wage would instead produce similar effects on capital accumulation without harming employment.

Keywords: Minimum wage; Search; Large firm; Employment; Capital; Wage distribution.

JEL codes: E24; J63; J68; L20.

1 Introduction

What are the macroeconomic consequences of raising the minimum wage? The literature has investigated extensively the effect on employment and the wage distribution. The consensus is that a moderate rise in the minimum wage does not lead to a significant decrease in employment (Card and Krueger, 1994), while, at the same time, it compresses the wage distribution, with positive spillovers on higher wages (Lee, 1999). However, less is known regarding its impact on capital accumulation and aggregate output.

In this paper, we study the quantitative effects of the minimum wage on several macroeconomic aggregates such as the stock of capital, the level of employment and output, as well as its impact on the whole distribution of wages in the economy. We address these questions by means of a model that generates impacts of minimum wage increases on employment and the wage distribution in line with the consensus predictions. Having established the plausibility of the model to explain these observed effects, we then use it to analyze the impact of the minimum wage on the stock of capital and aggregate output.

Our framework is the large-firm search and matching model with continuous wage renegotiation *à la* Stole and Zwiebel (1996a) and Stole and Zwiebel (1996b). Earlier contributions have documented that the strategic interactions between workers and the firm brought by bargaining distorts the decisions to hire labor and accumulate capital. In particular, the holdup problem gives the firm incentives to underinvest (Acemoglu and Shimer, 1999), while decreasing marginal returns to labor or the substitutability between labor types induce the firm to hire too many employees as compared to the first-best allocations (Smith, 1999; Cahuc et al., 2008). We analyze how changes in the minimum wage may exacerbate or limit these distortions.

We calibrate the model to the US economy. We match the distribution of wages across workers as well as the share of workers earning the minimum wage. These are important moments to match in order to derive precise comparative statics about the impact of a rise in the minimum wage on the levels of capital and employment and the distribution of wage earnings in the economy. We find that a moderate increase in the minimum wage—similar to changes analyzed in the empirical literature—barely affects the level of employment and gives incentives to invest more in capital. The increase in the minimum wage also compresses the distribution of wages in the economy and generates positive spillovers on higher wages, as has been illustrated in some empirical studies.

We also perform a policy analysis and ask how the introduction of a 10 dollar minimum wage would affect the current macroeconomic situation, as discussed in the public debate. This is a large policy change as about 18% of the employees in the data earns a wage equal or lower than ten dollars per hour, while about 6.6% of employees currently belong to the group of minimum-wage workers. The policy experiment predicts a positive increase for both capital (4.0%) and aggregate output (1.8%), but a negative effect on employment (-2.8%). Our analysis suggests that a more conservative increase in the minimum wage (i.e. $9 an hour) would generate a similar change in the capital stock without having a significant negative impact on employment.

The presence of a holdup problem on capital is an important feature of the model to generate the aforementioned comparative statics. In our model, the representative firm underinvests in capital in order to renegotiate lower wages with its workers. Because the minimum wage is a floor, an increase in the minimum wage alleviates the inefficiency. As a result of the increase in capital demand, employment is barely affected because capital and labor are complements in the production function. The increase in the capital stock also generates the observed spillovers on higher wages in the distribution of earnings since it also increases the marginal product of these workers.

Another mechanism, proper to Cahuc et al. (2008), helps generating these comparative statics. It is based on rent appropriation and occurs when the different types of labor considered in the production function are substitutes. Absent a minimum wage, the firm chooses to overemploy some labor types in order to renegotiate lower wages for other types: because labor types are substitutes, overemployment allows to reduce the other types' marginal products and so their wages. Unfortunately for the firm, this leads to an additional appropriation problem as the overemployed workers claim part of the decrease in the other types' wage rate when they negotiate with the firm. In a context where the wage of some workers is fixed by regulation, rent appropriation by those workers is no longer possible. Thus, overemploying these workers becomes a more attractive option for the firm to affect the wage of other types. Together with the effect on capital demand, this rent appropriation effect explains why a moderate rise in the minimum wage barely affects employment. It also provides an answer to the minimum wage paradox mentioned in Teulings (2000), who describes as a puzzle the weak response of employment to an increase in the minimum wage if the substitutability between labor types is estimated to be high. Additionally, the rent-appropriation effect explains part of the compression in the wage distribution as it helps to generate a larger mass of workers around the minimum wage peak.

Our analysis is closely related to the work of Smith (1999). This paper also studies the effect of a minimum wage in the context of a large-firm search model with firm entry and exit.[1] In his model, firms hire one type of labor and the production function displays decreasing returns to scale. Those assumptions generate two effects. First, firms tend to overemploy workers in order to decrease wages by decreasing their marginal product. Second, overemployment creates a negative externality that forces some firms out of the industry by increasing the labor-market search cost. As a result, there are too few firms in the economy and they are inefficiently too large. By introducing a minimum wage, it is possible to eliminate this inefficiency: firms stop overemploying, which increases firm entry and aggregate employment. However, the seminal study of Card and Krueger (1994) shows a positive effect of the minimum wage on firm size and no significant effect on the number of firms. Our results are consistent with Card and Kruger's findings.

Our paper is also related to the literature that discusses the effect of a minimum wage on capital accumulation. In a context with search frictions, Acemoglu and Shimer (1999) show that the holdup problem is avoided if the wage rate (as a function of the capital stock) is constant in the neighborhood of the efficient capital stock. In our model, introducing a minimum wage allows to fulfill this necessary condition. Acemoglu (2001) builds a model where firms open too few capital-intensive jobs because workers appropriate part of the return on capital. The introduction of a binding minimum wage helps correct for this externality and enhances the creation of capital-intensive jobs. However, an increase in the minimum wage always results in an increase in unemployment in his model, while the effect is marginally negative in the context of our model. Moreover, our model considers a richer set of strategic interactions between workers, the firm and capital. Similarly, Kaas and Madden (2008) illustrate the beneficial effects of a minimum wage for capital investment in the context of an oligopsonistic model, but they do not obtain a positive effect on employment.

The paper is organized as follows. Section 2 outlines the model, while Section 3 describes our calibration strategy. In Section 4 we analyze the quantitative performance of the model. In particular, we check that the model generates effects of a rise in the minimum wage in line with the available evidence for employment and the wage dis-

[1] Acemoglu and Hawkins (2014) extended the analysis in Smith (1999) by considering convex vacancy costs. This extension allows to generate the realistic feature of slow employment growth at the firm level. See also Ebell and Haefke (2009), Felbermayr and Prat (2011) and Janiak (2013). In a model without firm entry, Elsby and Michaels (2013) also match key properties of the cross-sectional distributions of employment and employment growth. However, none of these papers consider the impact of a minimum wage.

tribution, and we quantify the effects over capital and output. In addition, we analyze the potential macroeconomic effects of implementing a federal 10 dollar minimum wage in the US. Finally, we perform some robustness checks. Section 5 concludes.

2 The model

We consider an economy in steady state, where time is continuous and discounted at a rate r and agents are risk neutral. For notational simplicity, we suppress the time indices t when describing the economy and analyzing the equilibrium, while we denote by primes variables evaluated at time $(t + dt)$, where dt is an arbitrarily small interval of time.

2.1 A representative firm

Output is produced by a representative firm. The firm hires N types of workers in quantities n_i, with $i = 1, ..., N$, and owns capital in quantity k. Each labor type i is endowed with x_i efficiency units. We denote by $h \equiv \sum_{i=1}^{N} x_i n_i$ the total stock of efficiency units hired by the representative firm and $\mathbf{n} \equiv (n_1, n_2, ..., n_N)$ the vector of n_i. The production function $f(\mathbf{n}, k)$ is increasing and concave in each argument.

2.2 Labor

There are N labor markets corresponding to each type of labor. A worker of type i can only hold a job of type i. The total mass of workers of each type is $1/N$. Workers on each market can be either employed or unemployed. The presence of search and matching frictions explains the existence of unemployment on each market (in quantities u_i). Firms post vacancies at a flow cost cx_i in order to hire workers of type i. We denote by v_i the mass of posted vacancies by the representative firm on each labor market, while V_i is the aggregate mass of vacancies in the economy. In equilibrium, $v_i = V_i$, but the representative firm takes V_i as given while v_i is a control variable. Vacancies on market i are filled at a rate $q(\theta_i)$ that depends negatively on the labor market tightness $\theta_i \equiv \frac{V_i}{u_i}$, i.e. the vacancy-unemployment ratio. This rate is derived from a matching function $m(u_i, V_i)$ with constant returns to scale, increasing in both arguments, concave and satisfying the property $m(u_i, 0) = m(0, V_i) = 0$, implying that $q(\theta_i) = \frac{m(u_i, V_i)}{V_i} = m(\theta_i^{-1}, 1)$. Separations occur at an exogenous rate s.

2.3 Prices

Workers choose to earn either the minimum wage $\bar{w}$ or negotiate with the firm. There is also continuous wage renegotiation and workers can choose to earn the minimum wage at any time and at a zero cost.[2] If they choose to bargain, they obtain the negotiated wage $\breve{w}^i(\mathbf{n}, k)$. In this case, a division of the match value is proposed using a Nash-bargaining framework and, following Flinn (2006) and Flinn (2011), the minimum wage is then treated as a constraint on the bargaining outcome. The parameter $\beta \in (0, 1)$ refers to the bargaining power of workers.

It turns out that, for some values of the minimum wage, it may be the case that no equilibrium in pure strategy exists. This happens when the minimum wage has expansionary effects on labor demand.[3] For this reason, we consider a more general framework where workers can opt for mixed strategies when choosing between negotiating with the firm or earning the minimum wage. Because, in equilibrium, workers may be indifferent between earning the minimum wage or bargaining the wage with the firm, they randomize according to a mixed strategy, where the probability they bargain with the firm is χ_i. The probability χ_i is endogenous and determined through arbitrage.[4] We focus on symmetric equilibria, where all workers from a given labor category choose the same χ_i. We also denote by $\bar{n}_j = (1 - \chi_j)n_j$ and $\breve{n}_j = \chi_j n_j$ the mass of workers who choose to earn the minimum wage and negotiate their wage with the firm respectively.

Hence, the expected wage $w^i(\mathbf{n}, k)$ paid to a worker of type i is

$$w^i(\mathbf{n}, k) = \chi_i \breve{w}^i(\mathbf{n}, k) + (1 - \chi_i)\bar{w}. \tag{1}$$

Notice that our notation for wages explicitly emphasizes their dependence on the employment levels n_i and the capital stock k: the firm may choose a particular level of employment or capital before wages are negotiated in order to influence the outcome of the bargaining process *ex post*.[5] For example, Smith (1999) shows that when the production function is concave in each factor, the firm may choose to overemploy in

[2]See Hawkins (2015) for a model that considers commitment over wages.

[3]The intuition is the following: consider the case where a minimum wage is introduced, this minimum wage is marginally binding and it causes an increase in labor demand. Then, a contradiction would arise since workers would renegotiate higher wages because of the increase in labor demand, but this requires the minimum wage to be binding at the same time. Hence, no equilibrium in pure strategy would exist in this case, but an equilibrium in mixed strategy does.

[4]Section 2.8 gives further details on the determination of χ_i in equilibrium.

[5]We assume that the firm acquires capital *before* wages are bargained, and that the capital stock of the firm cannot be adjusted while wage negotiation occurs. If capital could be freely adjusted, the holdup problem would not be present, see Cahuc and Wasmer (2001a) for a discussion.

order to reduce wages through a reduction in the marginal product of labor[6]. Cahuc and Wasmer (2001a) and Cahuc et al. (2008) show that the complementarity (substitutability) between different types of labor may induce the firm to underemploy (overemploy) one type of labor in order to reduce the wage of other workers.

Finally, the purchase of a unit of capital is priced one unit of final good and capital depreciates at a rate δ.

2.4 Value functions

The present-discounted value of profits of the representative firm is

$$\Pi(\mathbf{n},k) = \max_{\{v_1,\ldots,v_N,a\}} \frac{1}{1+rdt}\left(\left[f(\mathbf{n},k) - \sum_{j=1,\ldots,N}\left[w^j(\mathbf{n},k)n_j + v_j c x_j\right] - a\right]dt + \Pi(\mathbf{n}',k')\right), \tag{2}$$

subject to the constraints

$$\dot{n}_i = q(\theta_i)v_i - sn_i, \quad \forall i = 1,...,N \tag{3}$$

and

$$\dot{k} = a - \delta k, \tag{4}$$

where a denotes investment in physical capital and dt is an arbitrarily small interval of time. We specifically consider the case where dt tends to zero.

The value of being unemployed for an i-type worker follows a standard formulation and reads in steady state as

$$rU_i = bx_i + \theta_i q(\theta_i)\left[W_i - U_i\right], \tag{5}$$

with bx_i the flow utility of being unemployed, while the value of being employed follows

$$W_i = \max\{\breve{W}_i, \bar{W}_i\}, \tag{6}$$

where $\breve{W}_i$ is the value if a worker chooses to negotiate,

[6] We refer to a situation of *overemployment* when (in partial equilibrium) the firm hires a quantity of labor larger than the level that would prevail under a situation where the firm takes the stream of future wages as given. Similarly, when employment is below that level, we refer to a situation of *underemployment*.

$$r\breve{W}_i = \breve{w}^i(\mathbf{n}, k) + s\left[U_i - W_i\right], \tag{7}$$

and $\bar{W}_i$ is the value when a worker chooses to earn the minimum wage,

$$r\bar{W}_i = \bar{w} + s\left[U_i - W_i\right]. \tag{8}$$

2.5 Intra-firm Nash bargaining

In a context with mixed strategies, a worker randomizes between negotiating the wage with the firm with probability χ_i and earning the minimum wage with probability $(1-\chi_i)$. Consequently, for the purpose of the negotiation, χ_i is given, and only the mass of workers participating in the negotiation process $\breve{n}_i = \chi_i n_i$ is relevant. The remaining mass $\bar{n}_i = (1-\chi_i)n_i$ is taken as a given input in the production function, since their wage is exogenously determined and given by the minimum wage $\bar{w}$.

Denote by $f_i(\mathbf{n}, k)$ the partial derivative of the function f with respect to its i-th argument, i.e., $f_i(\mathbf{n}, k) = \frac{\partial f(\mathbf{n},k)}{\partial n_i}$. The wage rates under Nash bargaining $\breve{w}^i(\cdot)$ are determined following a standard Nash bargaining rule and are given by

$$\beta \Pi_i(\mathbf{n}, k) = (1-\beta)\left[\breve{W}_i - U_i\right], \quad \forall i = 1, ..., N, \tag{9}$$

subject to

$$\breve{w}^i \geq \bar{w}, \tag{10}$$

where $\breve{W}_i$ is defined in (7) and the firm's surplus $\Pi_i(\mathbf{n}, k)$ is calculated by applying the envelope theorem to (2):[7]

$$\Pi_i(\mathbf{n}, k) = \frac{f_i(\mathbf{n}, k) - \breve{w}^i(\mathbf{n}, k) - \sum_{j=1,\dots,N} \breve{w}_i^j(\mathbf{n}, k)\breve{n}_j}{r+s}, \quad \forall i = 1, ..., N.$$

The equation above describes the marginal value a negotiating worker brings to the firm. It is equal to the discounted sum of marginal profits, taking into account that hiring this marginal worker affects the wage of all negotiating workers (the last term in the equation above). Notice that, because a fraction $(1-\chi_j)$ of workers earn the minimum wage, the marginal worker only affects the wage of a share χ_j of workers. Hence, the lower χ_j, the more limited the ability of the firm to act strategically.

The Nash solution for the negotiated wage reads as

[7]Notice that, since $n_i \equiv \breve{n}_i + \bar{n}_i$, $f_i(\mathbf{n}, k) = \frac{\partial f(\mathbf{n},k)}{\partial \breve{n}_i}$ and $\Pi_i(\mathbf{n}, k) = \frac{\partial \Pi(\mathbf{n},k)}{\partial \breve{n}_i}$.

$$\breve{w}^i(\mathbf{n}, k) = \max \left\{ \beta f_i(\mathbf{n}, k) + (1-\beta) r U_i - \beta \sum_{j=1,...,N} \breve{w}_i^j(\mathbf{n}, k) \breve{n}_j, \bar{w} \right\}, \quad \forall i = 1, ..., N. \tag{11}$$

Apart from the constraint imposed by the minimum wage, the wage equation (11) differs from the standard equation from Pissarides (1985) through the term containing the derivative. Under Nash bargaining, workers can appropriate part of the decrease in wages of the other workers. This explains the difference between the wage equation (11) and the standard one. Notice that the lower χ_j is, the lower is the size of this additional term, because the set of wages that can be affected through intra-firm bargaining is smaller. This suggests that the minimum wage may affect the strategic behavior of the firm through a *rent-appropriation* effect.

Expression (11) states a system of nonlinear differential equations in $\breve{w}^i$. The following proposition characterizes the solution to this system.

Proposition 1. *The negotiated wage of an i-type worker is*

$$\breve{w}^i(\mathbf{n}, k) = \max \left\{ \beta \breve{\Omega}_i f_i(\mathbf{n}, k) + (1-\beta) r U_i, \bar{w} \right\}, \quad \forall i = 1, ..., N. \tag{12}$$

where

$$\breve{\Omega}_i = \frac{\int_0^1 f_i(\mathbf{n}\mathbf{M}(z), k) \breve{\varphi}(z) dz}{f_i(\mathbf{n}, k)}, \tag{13}$$

$$\breve{\varphi}(z) = \frac{1}{\beta} z^{\frac{1-\beta}{\beta}}$$

and $\mathbf{M}(z)$ *is a diagonal matrix where the* j*-th element of the diagonal is equal to* $\chi_j z + 1 - \chi_j$*.*

Proof. See Appendix A.1. □

Apart from the constraint imposed by the minimum wage, the wage equation (12) differs from the one-worker-per-firm wage equation by the presence of the overemployment factor $\breve{\Omega}_i$, as in Cahuc et al. (2008). We provide intuition for this term in Section 2.7.

2.6 First-order conditions of the firm

The first-order conditions for vacancy posting and capital investment are, respectively,

$$\frac{cx_i}{q(\theta_i)} = \frac{f_i(\mathbf{n},k) - w^i(\mathbf{n},k) - \sum_{j=1,...,N} w_i^j(\mathbf{n},k)n_j}{r+s}, \quad \forall i = 1, ..., N, \tag{14}$$

and

$$r + \delta = f_k(\mathbf{n},k) - \sum_{i=1,...,N} w_k^i(\mathbf{n},k)n_i \tag{15}$$

The vacancy-posting conditions (14) equate the expected search cost of hiring a worker of type i to the discounted sum of profits that the marginal worker brings to the firm after being hired. They differ from the condition of the standard model with one worker per firm (Pissarides, 1985) through two strategic effects. First, the employment level of group i may affect the wage of that group. Incentives to overemploy may appear when the production function is concave in n_i (Smith, 1999). Second, the employment level of group i may affect the wage of the other group $j \neq i$. Incentives to overemploy may appear when factors are substitutes and underemployment may result from complementarity between factors (Cahuc et al., 2008).

The capital investment condition (15) equates the opportunity cost of capital to the marginal income of capital. The latter differs from its neoclassical counterpart through the effect on wages: depending on the complementarity/substitutability of capital with labor, the representative firm may choose to underinvest/overinvest in order to reduce wages.

2.7 Distortion factors

Conditions (14) and (15) depend on the negotiated wages $\breve{w}^h$ and $\breve{w}^l$ through equation (1). The following proposition characterizes the vacancy-posting conditions and the capital investment condition considering the solution for negotiated wages given by expression (12).

Proposition 2. *The vacancy-posting conditions and the capital investment condition read respectively as*

$$(r+s)\frac{cx_i}{q(\theta_i)} = \Omega_i f_i(\mathbf{n},k) - w^i(\mathbf{n},k), \quad \forall i = 1, ..., N \tag{16}$$

and

$$r + \delta = \Omega_k f_k(\mathbf{n},k), \tag{17}$$

with the distortion factors defined as follows:

$$\Omega_i = \chi_i \breve{\Omega}_i + (1 - \chi_i)\bar{\Omega}_i, \quad \forall i = 1, ..., N \tag{18}$$

where $\breve{\Omega}_i$ *is defined as in* (13)*, and* $\bar{\Omega}_i$ *and* Ω_k *are*

$$\bar{\Omega}_i = \frac{\int_0^1 f_i\left(\mathbf{nM}(z), k\right) \bar{\varphi}(z) dz}{f_i(\mathbf{n}, k)}, \tag{19}$$

$$\Omega_k = \frac{\int_0^1 f_k\left(\mathbf{nM}(z), k\right) \bar{\varphi}(z) dz}{f_k(\mathbf{n}, k)}, \tag{20}$$

$$\bar{\varphi}(z) = \frac{1 - \beta}{\beta} z^{\frac{1-2\beta}{\beta}}.$$

Proof. See Appendix A.2. □

Notice the presence of the *distortion factors* $\Omega_i > 0$ and $\Omega_k > 0$ in equations (16) and (17), which we respectively call *overemployment* and *underinvestment factors*. Their presence is the outcome of the strategic interactions between workers and the representative firm in bargaining. When Ω_i for any $i = 1, ..., N$ takes a value larger than one, we refer to this situation as a situation of *overemployment*, in the sense that the firm employs a quantity of i-type workers larger than in the case where the firm considers future wages as given. *Underemployment* of factor i appears when the respective factor is lower than one. Similarly, the values of Ω_k illustrate how investment by the representative firm responds to contract incompleteness: we refer to *overinvestment* when $\Omega_k > 1$ and *underinvestment* when $\Omega_k < 1$.

To provide further intuition for the values of Ω_i, Ω_k, $\breve{\Omega}_i$ and $\bar{\Omega}_i$, we describe some special cases in Appendix B.

2.8 Equilibrium

In general equilibrium, the labor market tightness θ_i and the present-discounted value of being unemployed U_i are endogenous. The former is obtained in a standard way by equating the flows in and out of employment, leading to the Beveridge relations

$$n_i = \frac{\theta_i q(\theta_i)}{s + \theta_i q(\theta_i)} \tag{21}$$

while, by combining equations (5)-(8), one can show that the value of unemployment can be written as

$$rU_i = \frac{(r+s)bx_i + \theta_i q(\theta_i) w^i}{r+s+\theta_i q(\theta_i)}. \tag{22}$$

To close the model, we also need to determine the equilibrium values for the χ_i's. These have to be consistent with the optimizing behavior of workers: the choice by workers between negotiating with the firm or earning the minimum wage must be the most attractive one. Following Flinn (2011), denote by $\tilde{w}_i$ the negotiated wage for an i-type worker, ignoring the minimum wage constraint (10). We thus have the following equilibrium condition:[8]

Equilibrium condition 1. *Workers' wage strategy is optimal:*

- *The fraction* $\chi_i = 1$ *is an equilibrium if* $\tilde{w}_i > \bar{w}$.
- *The fraction* $\chi_i = 0$ *is an equilibrium if* $\tilde{w}_i < \bar{w}$.
- *The fraction* $\chi_i \in (0,1)$ *is an equilibrium if* $\tilde{w}_i = \bar{w}$.

for each $i = 1, ..., N$.

This yields the following definition of equilibrium:

Definition 1. *A steady-state general equilibrium is a set of employment levels* n_i*, a capital stock* k*, a set of fractions* χ_i *of workers who earn the negotiated wage, negotiated wage rates* $\breve{w}_i$ *and labor market tightness* θ_i *such that the first-order conditions* (14) *and* (15)*, the wage equations* (12)*, the value of unemployment* (22)*, the Beveridge relations* (21) *and Equilibrium condition 1 are satisfied, given a minimum wage* $\bar{w}$.

Equilibrium condition 1 suggests the possibility of multiple equilibria. In the quantitative exercises of Section 4, we focus only on situations where the equilibrium is unique.[9]

2.9 Discussion on the impact of the minimum wage

In the next sections we numerically analyze the impact of the minimum wage on capital and employment, as well as on the wage distribution. We consider the case of a production function with constant returns to scale in h and k. In this case, capital and effective labor are complements, implying that the overemployment factors Ω_i take values larger than 1 in equilibrium and the underinvestment factor Ω_k takes a value

[8] In order to obtain the equilibrium allocations and prices, we follow the standard approach and check the Kuhn-Tucker conditions for when the minimum wage constraint binds or not. See Appendix E for further details on the numerical algorithm used.

[9] See Bauducco and Janiak (forthcoming) for an example with multiple equilibria.

lower than 1. The former is due to the concavity of the production function in h: the firm overemploys labor in order to renegotiate lower wages by decreasing the marginal products of labor. The latter is due to the complementarity between capital and labor and the presence of a holdup problem: the firm can decrease wages and marginal products of labor by underinvesting in capital.

In this context, the minimum wage may have different effects on employment and capital. First, it may alleviate the holdup problem. Indeed, when the minimum wage is binding, the firm can no longer reduce the wage of minimum-wage workers by underinvesting, as the minimum wage is a floor on wages. As a result, capital demand increases and employment may also increase because of the complementarity between capital and labor. We call this effect the *capital demand* effect.

A second mechanism, which we call the *rent appropriation* effect, may also imply an increase in employment because the different types of labor considered in the production function are substitutes. Cahuc et al. (2008) have shown that, absent a minimum wage, the firm chooses to overemploy one type of labor in order to reduce the wage of the other types: because the equilibrium wage depends on the marginal product of labor under Nash bargaining, overemployment allows to reduce the other type's wage by decreasing its marginal product. Unfortunately for the firm, this leads to an additional appropriation problem as the overemployed workers claim part of the decrease in the other types' wage rates when they negotiate with the firm. In a context where the wage of some types of workers is fixed by regulation, rent appropriation by those workers is no longer possible. Thus, overemploying these workers becomes a more attractive option for the firm in order to affect the wage of the other types. Third, the minimum wage may have standard adverse effects on employment: labor demand decreases because labor is too costly. In this case, the demand for capital may also decrease because of the complementarity between capital and labor.

The minimum wage may also have an impact on the wage distribution. Evidently, it directly affects the wage of minimum-wage workers, but it may also have spillover effects on the wage of other worker categories. This is due to the capital demand effect, as shown in equation (12): the increase in capital demand increases the negotiated wage of all workers by increasing the marginal product of labor thanks to the complementarity between capital and labor.

Compression of the wage distribution may also happen for two reasons. First, it may occur as a result of the minimum wage increasing faster than wages of the rest of the distribution. Second, the rent-appropriation effect may also play a role as it increases the demand for minimum-wage workers, generating a larger mass of workers around the minimum-wage peak. Notice that the capital demand effect, however, tends to play against compression because of the aforementioned spillover effects.

Table 1: Calibration: parameter values

Parameter	Value	Description	Target	Source
μ	0.578	Wage distribution scale	Min. wage normalized to 1	CPS MORG
x_{min}	0.189	Wage distribution truncation	6.6% Min. wage workers	CPS MORG
σ	0.825	Wage dist. dispersion	Max. likelihood estimation	CPS MORG
η	0.5	Matching func. elasticity	Standard	Pissarides (2009)
β	0.5	Bargaining power	Standard	Pissarides (2009)
r	0.004	Discount rate	Interest rate	Pissarides (2009)
m_0	0.921	Matching function scale	Job finding probability	JOLTS and BLS data
s	0.035	Job separation rate	Job separation probability	JOLTS and BLS data
c	6.622	Vacancy cost	Aggregate tightness	JOLTS and BLS data
b	1.424	Flow value of unemp.	40% of marginal product	Shimer (2005)
δ	0.004	Capital depreciation	5% annual deprec.	Cooley and Prescott (1995)
α	0.537	Production func. elasticity	Labor share	Cooley and Prescott (1995)
A	0.333	TFP	Fixed	Fixed

3 Calibration

Our calibration strategy embeds the identification of the distribution of efficiency units across workers within a standard calibration of the search and matching model. We target moments of the US economy for the year 2015 and consider that a unit interval of time represents a month.[10]

We consider Cobb-Douglas specifications for both the matching functions and the production function:

$$f(\mathbf{n}, k) = A \left(\sum_{i=1}^{N} x_i n_i \right)^{1-\alpha} k^{\alpha},$$

$$m(V_i, u_i) = m_0 V_i^{\eta} u_i^{1-\eta}.$$

We choose the number of labor markets $N = 100$ so as to accurately fit the wage distribution. This is an important object to match in order to generate realistic comparative statics of an increase in the minimum wage. The distribution of efficiency units x_i across worker categories is identified using data on wages from the Merged

[10] It might be argued that, for the comparative statics displayed in Section 4.1, we should consider a longer time period for the calibration as the economy may still have been on its recovery path in 2015. Two elements motivate our choice of time period for the calibration: firstly, unemployment had already reached its pre-crisis level in 2015 with a 5.3% rate. Secondly, by calibrating the model to 2015, our analysis is better suited to perform the policy experiment in Section 4.3.

Outgoing Rotation Groups of the Current Population Survey. Following Autor et al. (2016), we consider individuals aged 18-64 and exclude the self employed as well as workers from the District of Columbia. We compute hourly wages as weekly earnings divided by weekly hours worked. Wages are then normalized by the minimum wage.[11] Finally, we consider that individuals reporting wages below the minimum earn the minimum. As a result, 6.6% of the employed labor force are minimum wage workers in our interpretation of the data.

Since there is no one-to-one match between wages and the stock of efficiency units held by workers subject to the minimum wage, we cannot identify the x_i's for these workers using earnings data alone. In order to overcome this issue, we choose to assume a parametric form of the distribution of efficiency units to be able to identify the x_i's of minimum-wage workers. To this end, we consider a truncated log-normal distribution, as the log-normal distribution is known to fit the earnings distribution well.[12] We denote by μ and σ^2 the mean and variance of the non-truncated distribution, while x_{min} is the level of the truncation, i.e. there is no mass of workers with $x_i < x_{min}$. The parameters $\mu = 0.578$ and $x_{min} = 0.219$ are calibrated such that the minimum wage takes value 1 in the calibrated economy and there is about 6.6% of minimum-wage workers. The resulting share of minimum-wage workers is 6.3% in the calibrated economy and the first 6 categories of workers are subject to the minimum wage.[13] We estimate the dispersion parameter $\sigma = 0.825$ of the distribution with maximum likelihood. Notice that this value is larger than the standard deviation of log wages in the data (0.649) because we actually are fitting a truncated distribution. Figure 1 compares the cumulative distribution function of wages in the data with the calibrated economy. Although this distribution does not deliver a great fit of the right tail, it replicates reasonably well the low end of the earnings distribution:[14] the estimated 10-50 ratio, a measure of dispersion commonly used in the literature, is -0.708 (in logs),

[11] An issue with the US economy is that minimum wages differ by state because workers can earn the state minimum wage when it is above the federal one. The National Conference of State Legislatures documents the minimum wage levels across states. They vary from $7.25 per hour (the federal minimum wage) in states like Texas or Georgia to $10.00 per hour in California or Massachusetts. Table 2 in the Appendix lists the values taken by the minimum wage in each state.

[12] We use a truncated distribution because workers with too low levels of x_i are not employed in our model, and therefore are not observed in the data on labor earnings.

[13] The share of minimum-wage workers is larger than 6% in the calibrated economy because labor demand for minimum-wage workers is larger for this group of workers through the rent-appropriation and capital demand effects discussed in Section 2.9.

[14] The fact that the log-normal distribution does not fit very well the right tail of earnings distribution is common. Because we think that a good fit of the left part of the distribution is required for the comparative statics in Section 4.1 and the policy experiment in Section 4.3, we prefer to consider this specification instead of other popular forms such as the Pareto distribution, which typically fit right tails better.

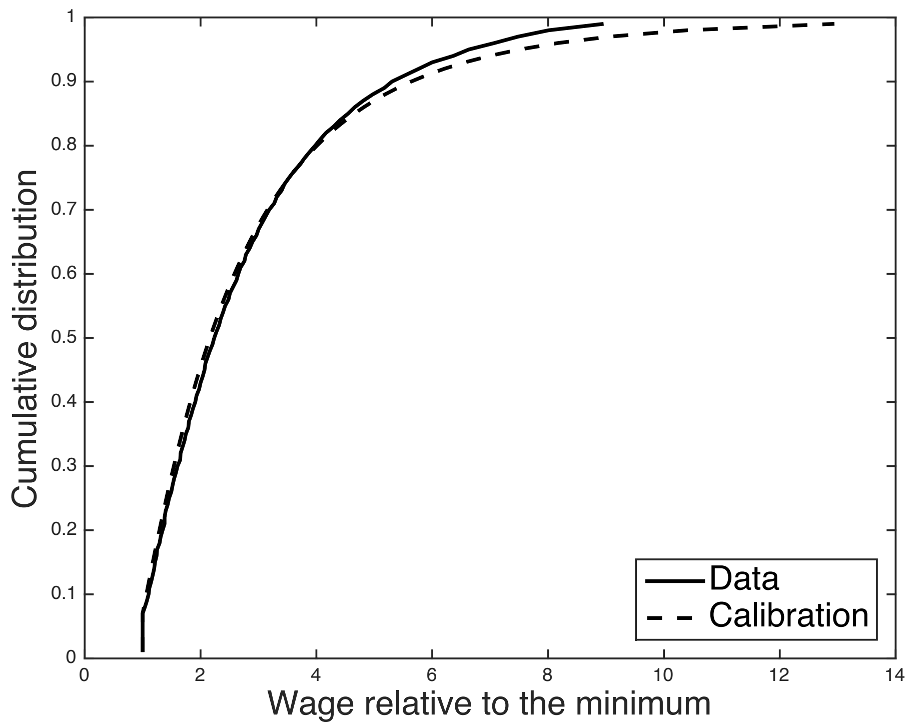

Figure 1: Cumulative distribution function of wages: model versus data

which is in line with its empirical counterpart (−0.706) and the evidence provided in Autor et al. (2016). Another relevant characteristic the distribution replicates is the following: the share of workers earning $10 or less is 18.1% in the data, while the share of workers earning a wage lower or equal than 1.24 in our calibrated economy—which we argue in Section 4.3 is the counterpart of a $10 wage— is 17.8%.

The parameter influencing the flow value of being unemployed b is another parameter that is known to affect the quantitative impact of labor-market policies. This has been discussed for instance in Costain and Reiter (2008), who show that small-surplus calibrations—as in Hagedorn and Manovskii (2008)—generate an unrealistic impact of changes in unemployment insurance. We rely on Shimer (2005) and require the flow value of being unemployed to be equal to 40% of the marginal product of labor, implying $b = 1.424$ in our calibration. A higher value of b would arguably generate changes in the minimum wage that are less employment-friendly. Hall and Milgrom (2008) for instance consider a b equal to 71% of the marginal product of labor.[15] We show in

[15]We do not analyze the business-cycle properties of the calibrated economy here. It is well-known that these values of b generate volatilities of unemployment and vacancies that are too low as compared to the

Section 4.5 that a calibration based on Hall and Milgrom (2008) does not produce an impact of the minimum wage significantly different from our strategy that relies on Shimer (2005). Similarly, we show that lower values of b predict a lower impact of the minimum wage on employment.

The identification of the scale parameter of the matching function, the job separation rate and the flow cost of posting a vacancy is done by targeting labor market transition probabilities and the aggregate labor market tightness. By using data from the JOLTS and the BLS, we establish a job finding probability of 65.3% and a job separation probability of 3.5%. These two values imply $m_0 = 0.921$ and $s = 0.035$. We target an aggregate labor-market tightness $\frac{\sum_{i=1}^{N} V_i}{\sum_{i=1}^{N} u_i} = 0.67$, yielding $c = 6.622$.

The technological parameters δ and α are calibrated using standard moments of the RBC literature. Following Cooley and Prescott (1995), $\delta = 0.0043$ produces a 5% annual capital depreciation. The value of $\alpha = 0.537$ targets a 60% labor share as also considered in Cooley and Prescott (1995). This value of the elasticity of output with respect to capital is in line with estimates of the IO literature, as in e.g. Doraszelski and Jaumandreu (2013), who show that the estimated coefficients of a Cobb-Douglas production function of capital and labor are similar. Without loss of generality, we fix $A = 1/3$.

Finally, we fix the rest of the parameters equal to common values used in the literature (Pissarides, 2009) such as the elasticity of the matching function ($\eta = 0.5$), the bargaining power of workers ($\beta = 0.5$) and the discount rate ($r = 0.004$).

4 Quantitative results

4.1 Comparative statics

In this section, we use the calibrated model economy in order to analyze the quantitative implications of increasing the minimum wage. Figure 2 shows the behavior of capital and aggregate employment for values of the minimum wage that range from 1 to 1.35. As wages are normalized to the current level of the US minimum wage,[16] the value of 1 reflects the benchmark scenario, while 1.35 corresponds to a 35% increase in the overall US minimum wage, which amounts roughly to setting the federal minimum

data. Nevertheless, one could obtain volatilities more in line with the data by introducing fixed matching costs as in Pissarides (2009).

[16]More precisely, because the minimum wage may vary from on state to the other, wages are normalized by the average level of the minimum wage across US states.

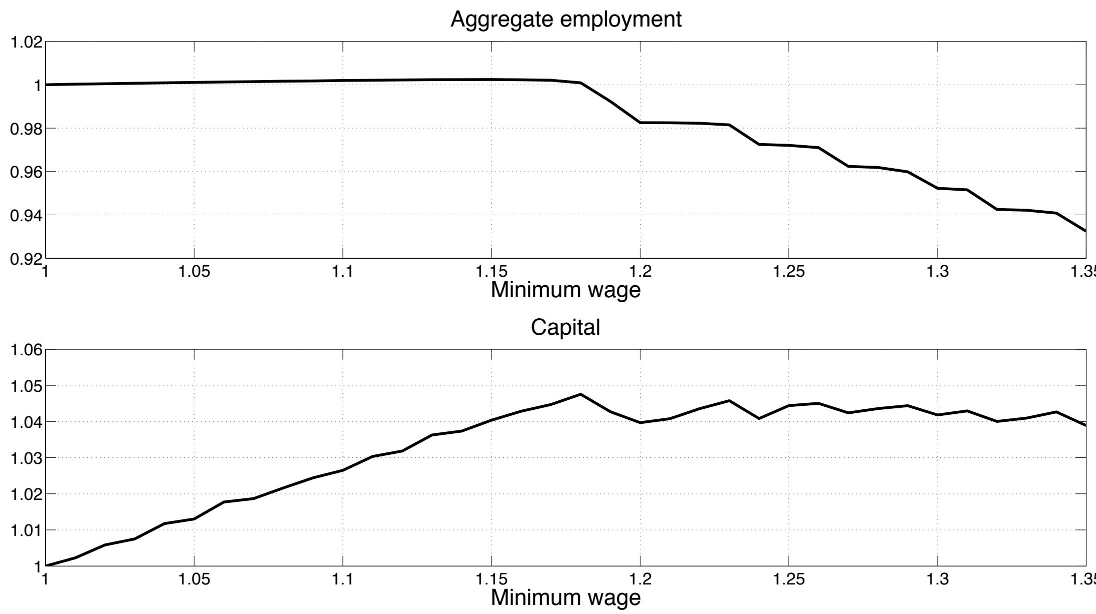

Figure 2: Aggregate employment and capital as a function of the minimum wage

wage equal to \$11.25.[17] The aggregate stocks of capital and employment are both normalized to 1 in the benchmark calibrated economy. The sharp oscillations on Figure 2 are due to the discretization of the labor force into $N = 100$ markets and occur when the minimum wage starts binding for some labor types or when the representative firm chooses not to hire workers from a specific labor type anymore.

As the minimum wage increases, aggregate employment initially (marginally) increases and, after a certain value of the minimum wage, starts to decline. Similarly, the capital stock in the economy increases for a moderate rise in the minimum wage, while it displays a downward trend for more substantial changes in this variable. This behavior of aggregate allocations is governed by the rent appropriation and capital demand effects discussed in Section 2.9. Firstly, as the minimum wage increases, the values taken by the χ_i's of minimum-wage workers decrease and more labor types are included among the employees earning the minimum wage, alleviating the holdup problem on capital investment and encouraging firms to invest more. This explains the upward trend in capital for values of the minimum wage between 1 and 1.17. Since capital and labor are complements, the increase in capital leads to an increase in the

[17] The values taken by the χ_i's indicate how binding a minimum wage might be. With a minimum wage equal to one, the first six employment categories with the lowest productivity earn the minimum wage. The five least productive categories face $\chi_i = 0$. For the sixth most productive category, $\chi_6 = 0.812$.

marginal productivity of labor and, as a consequence, to an increase in aggregate employment. This is the *capital-demand* effect at work. Secondly, because workers who earn the minimum wage do not claim part of the change in the wage of other workers, the representative firm chooses to over employ these workers by more in order to decrease the wage of those who negotiate their wages. This is the *rent-appropriation* effect. Both the capital demand and the rent appropriation effects explain the positive effect on aggregate employment exerted by moderate increases in the minimum wage.

For larger increases of the minimum wage (beyond 18%), while the capital demand and rent appropriation effects still play a role in the determination of aggregate allocations, they are not strong enough to guarantee an increase in labor and capital. For relatively high values of the minimum wage, the marginal productivity of the least productive categories of employment might be too low when compared with the minimum wage, in which case the representative firm ceases to hire these workers. This is the reason behind the decline in aggregate employment shown in Figure 2. Due to the complementarily of capital and labor, capital also decreases.

4.2 Minimum wage elasticities

In order to evaluate the empirical plausibility of our results, we check whether the changes in aggregate labor predicted by our model are in line with those found in the empirical literature on the minimum wage. The seminal paper of Card and Krueger (1994) reports slightly positive effects on employment for a change in New Jersey's minimum wage from \$4.25 to \$5.05. This represents an increase by 18.8%, but because the original \$4.25 minimum wage was not actually binding for most of the establishments in their database, the change only represented an average increase by 11%. Our calibrated model predicts a 0.21% increase in aggregate employment following such an increase in the effective wage floor. This mild positive effect is quantitatively in line with the findings of Card and Krueger (1994) (see table 5 in the paper), and is qualitatively similar to other findings in the literature for similar changes in the effective minimum wage (see for example Stewart (2004) and Card (1992)).

For large changes in the minimum wage, the model predicts sizable negative effects on aggregate labor. This is to be expected, as workers with low productivities cease to be employed when the floor wage becomes large. Our results are quantitatively in line with some empirical estimates of the effects of the minimum wage on vulnerable population groups. For example, Machin et al. (2003) report an elasticity of employment of $-0.15/-0.4$ for the introduction of a minimum wage higher than the wage perceived by 30% of employees previous to the reform. Our numerical exercise yields an elasticity of -0.19 following a 35% increase in the minimum wage.

4.3 Policy experiment: a 10 dollar minimum wage

The recent political discussion on minimum wages in the US has centered around increasing the federal minimum wage to \$10 an hour, from the current value of \$7.25 an hour. The idea, originally introduced by members of the Democrat party and defended by former US President Barack Obama, has been revitalized by President Donald Trump when he showed his support to the policy measure during his presidential campaign.

A \$10 minimum wage would impact each US state differently, depending on the specific level of the state minimum wage. To obtain an estimate of the nation-wide increase in the minimum wage, we compute the increase in the minimum wage by state implied by the policy measure, taking into account that the effective wage floor in each state is the maximum between the state and the federal minimum wage. We compute the average increase in the effective state minimum wage, weighted by the labor force of each state. This yields an increase in the overall minimum wage of approximately 24%.

In light of our model, a 24% increase in the minimum wage would imply a 4.08% increase in the capital stock and a 2.75% decrease in labor. In turn, these changes in production factors would translate into a 1.8% increase in output. Arguably, the macroeconomic implications of the proposed policy measure are large, but they stem from a substantial increase in the wage floor: prior to the reform, the share of workers that earn a wage equal or lower than 1.24 is 18.1% in the data and 17.8% in the calibrated economy. Then, the suggested increase in the minimum wage would potentially affect around one fifth of the labor force, a very large figure compared to the 6.3% mass of workers earning the minimum wage in the benchmark calibrated economy. According to our model, the percentage of workers earning the minimum wage after the reform is 14.9%. The difference between the ex-ante and ex-post figures reflect two effects from our model: first, as discussed in the previous section, for large increases in the minimum wage some low productivity workers stop being hired, as their marginal productivities are too low compared to the wage floor. Second, as discussed in Section 2.9, there are spillover effects of the minimum wage on workers with higher productivities which might then also see their wages increased by the reform.

Given the negative effects on employment that the \$10 minimum wage reform would have, we next ask the following question: within the context of our model, what would be the necessary change in the minimum wage in order to maximize the increase in employment? It turns out that a 15% increase in the minimum wage (an increase of \$1.75, or a \$9 minimum wage) would produce a 0.24% increase in employment, the highest possible given our benchmark calibration. In this context, capital and output

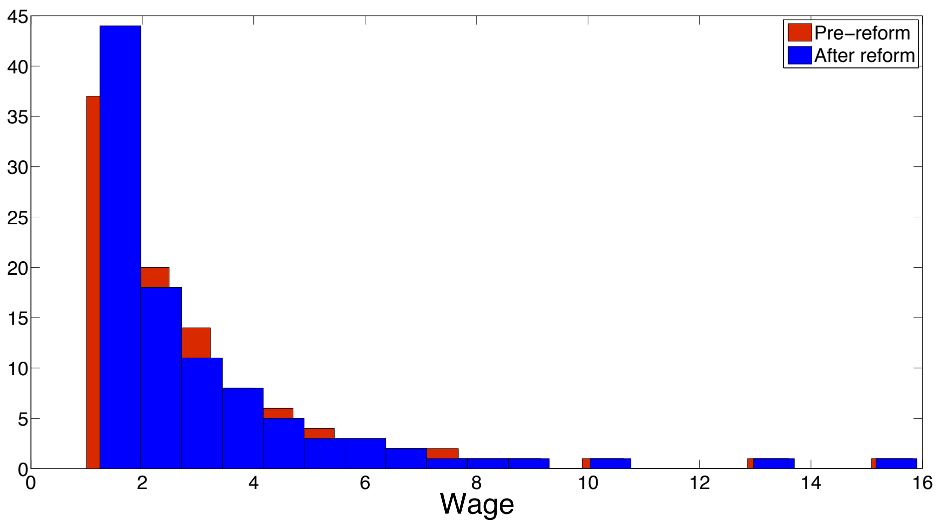

Figure 3: Histograms of the wage distribution pre- and post-reform

would increase 4.04% and 2.19%, respectively.

4.4 Distributional analysis

As discussed in Section 2.9, an increase in the minimum wage impacts the distribution of wages. In this section we analyze this impact and show that it is in line with the available evidence from empirical studies on the distributional effects of changes in the minimum wage legislation.

To this end, we consider the $10 minimum wage reform studied in the previous section and analyze the effects on the wage distribution of such policy reform. Figure 3 plots the histogram of the wage distribution prior to the reform (in red) and after the reform has taken place (in blue). It is immediate to see that there are spillover effects from the increase in the minimum wage, as after the reform the whole distribution appears shifted to the right. As mentioned in Section 2.9, spillovers arise as a consequence of the increase in the demand for capital which, in turn, is due to the holdup problem being alleviated when a higher minimum wage implies that a larger fraction of workers does not negotiate wages.[18]

To corroborate that, following the minimum wage reform, there is compression of the wage distribution, we follow much of the literature and compute the 10-50

[18] Autor et al. (2016) acknowledge that these spillovers are *"... a potentially important and little understood effect of minimum wage laws..."*. While Autor et al. (2016) provide a statistical explanation for the presence of spillovers, our model is able to generate and rationalize spillovers from an economic perspective.

ratio for wages before and after the reform. As reported in Section 3, the 10-50 ratio prior the reform is -0.708 (in logs) in our calibrated economy, whereas following the reform, this ratio is -0.595 (in logs). This result confirms that the increase in the minimum wage compresses the distribution of wages, at least at the left tail. Moreover, this compression is quantitatively in line with some of the estimates available in the literature: the elasticity of the 10-50 ratio with respect the minimum wage is 0.469 in the model, a value close to what Lee (1999) finds for the US. As discussed in Section 2.9, compression of the wage distribution is due to the rent-appropriation effect along with partial spillovers to wages of non-minimum-wage workers.

4.5 Some robustness analysis

4.5.1 Flow value of unemployment

Our benchmark calibration considers a flow value of being unemployed equal to 40% of the marginal product of labor, in line with Shimer (2005). As this parameter is traditionally an important determinant of the quantitative effects of labor market policies, in this section we investigate to what extent our results depend on the specific value assigned to it.

First, we consider the case in which bx_i is unemployment insurance, and b is calibrated such that the total expenditure on unemployment insurance over GDP is equal to 0.4%, which is the corresponding figure for public unemployment spending for US in 2013. This calibration implies a value of $b = 0.5210$, which is substantially lower than the value in the benchmark case. Figure 4 shows the behavior of capital and aggregate employment as a function of the minimum wage.

It is clear from Figure 4 that aggregate employment and capital display similar trajectories as in the benchmark calibration. In this case, following a 24% increase in the minimum wage, capital increases 4.57%, aggregate employment falls by 1.96% and output increases by 2.14%.

Next, we set the flow value of being unemployed equal to 71% of the marginal product of labor, a value based on Hall and Milgrom (2008), which yields $b = 2.827$. Figure 5 shows aggregate employment and capital as a function of the minimum wage in this case.

Again, the trajectories of aggregate employment and capital are similar to the ones depicted in Figure 2 for the benchmark economy. In this case, the effects of a 24% increase in the minimum wage on employment, capital and output are -2.72%, 4.39% and 2.02%, respectively. These values are very similar to the ones reported in Section 4.3 for the benchmark case.

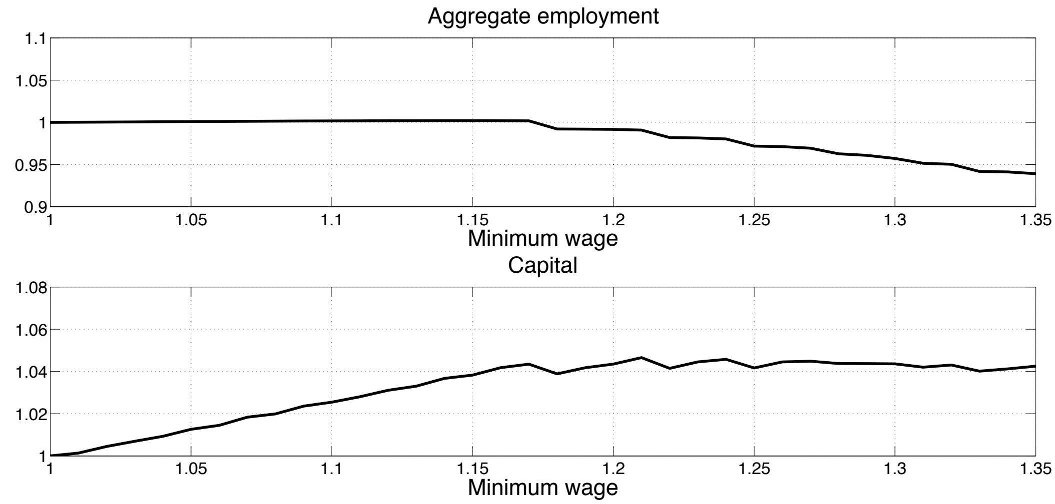

Figure 4: Aggregate employment and capital as a function of the minimum wage - Low b

From the analysis in the section we can conclude that the flow value of being unemployed is not a crucial parameter to generate the quantitative results discussed in previous sections.

4.5.2 Fraction of workers subject to the minimum wage

In the benchmark specification, we calibrate the initial fraction of workers subject to the minimum wage to match its counterpart in the data. In this section, we show that the quantitate results discussed in Section 4 depend crucially on this fraction being tightly matched. It is thus important to calibrate properly the share of workers subject to the minimum wage as we did in Section 3, as well as identifying the dispersion of productivities among the workers of this category.

To this end, we perform the following exercise: we calibrate the economy using the implied wage distribution estimated by maximum likelihood, but assuming that all categories of workers perceiving a wage equal to the minimum wage share the same productivity parameter, and that the minimum wage is marginally binding for all these categories, meaning that $\chi_i = 1 \; \forall i = 1, ..., N$. Therefore, the fraction of workers subject to the minimum wage in this specification is zero.

Figure 6 shows aggregate employment and capital as a function of the minimum wage for this case. As can be readily seen on the graph, the effects on capital are much larger than in the case in which a 6.3% of workers initially are subject to the minimum wage. This is the case because, when the minimum wage increases, the fraction of workers that earn the minimum wage rapidly grows from 0 to around 6%. Then, the

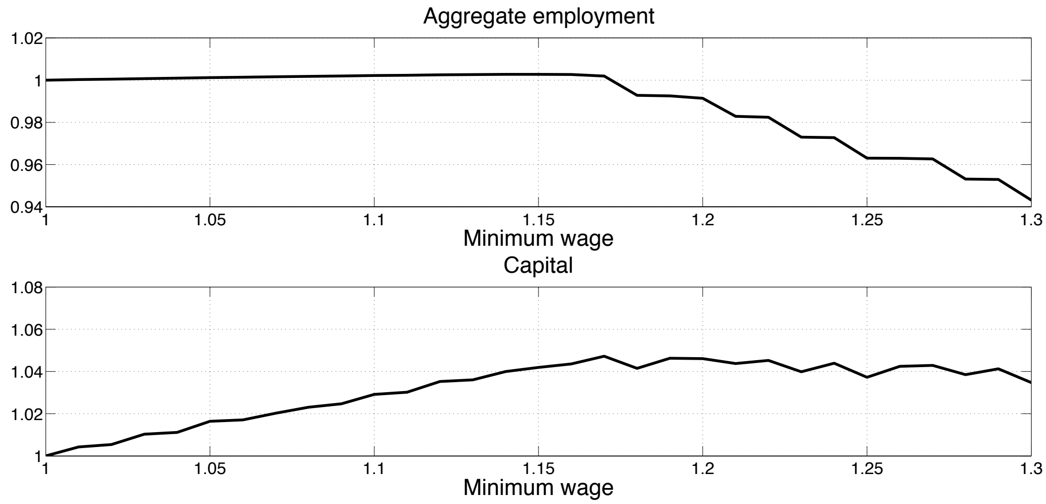

Figure 5: Aggregate employment and capital as a function of the minimum wage - High b

capital demand effect kicks in in full force, and capital increases substantially. This explains the initial convex trajectory of capital. Notice that this initial increase in the capital stock explains 5% of the overall increase in capital due to a rise in the minimum wage. The effects of a 24% increase in the minimum wage on employment, capital and output are 0.67%, 10.14% and 5.54%, respectively. These values are quite different to the ones reported in Section 4.3 for the benchmark case.

5 Conclusion

Consensus evidence suggests that a moderate increase in the minimum wage does not lead to a decrease in employment, but it compresses the wage distribution and it generates positive spillovers on higher wages. We provide a quantitative analysis of the impact of an increase in the minimum wage on macroeconomic outcomes in the large-firm search and matching model. In the context of our model, a moderate increase in the minimum wage generates a weak (positive) effect on employment, a large impact on capital accumulation and comparative statics on the wage distribution in line with the available evidence.

Two effects explain these comparative statics. First, the minimum wage fosters the demand for capital by alleviating a holdup problem that leads to underinvestment. The increase in capital also generates a positive spillover on higher wages. We call this a *capital demand* effect. Second, since labor types are substitutes, the firm strategically overemploys minimum wage workers to exert a downward pressure over higher wages.

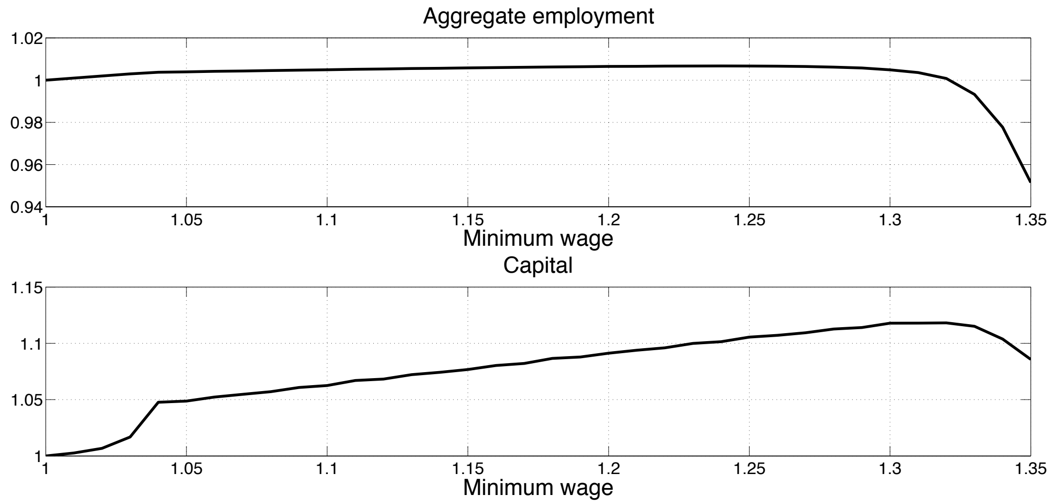

Figure 6: Aggregate employment and capital as a function of the minimum wage - 0% of workers subject to the minimum wage initially

We call this a *rent appropriation* effect. This effect also helps to generate compression of the wage distribution.

Next, we perform a policy experiment and analyze the quantitative consequences of introducing a 10 dollar minimum wage in the economy. The predictions of our model are a positive increase for both capital (4.0%) and aggregate output (1.8%), but a negative effect on employment (-2.8%). Our analysis suggests that a more conservative increase in the minimum wage (i.e. $9 an hour) would generate a similar change in the capital stock without harming employment.

A Appendix: proofs

A.1 Proof of Proposition 1

Denote by $\breve{\mathbf{n}} \equiv (\breve{n}_1, ..., \breve{n}_N,)$ the vector of $\breve{n}_i$'s and $\bar{\mathbf{n}} \equiv (\bar{n}_1, ..., \bar{n}_N,)$ the vector of $\bar{n}_i$'s. Define $\tilde{f}(\breve{\mathbf{n}}, \bar{\mathbf{n}}, k) \equiv f(\breve{\mathbf{n}} + \bar{\mathbf{n}}, k)$. Given that $f_{\breve{n}_i}(\breve{\mathbf{n}} + \bar{\mathbf{n}}, k) = \tilde{f}_{\breve{n}_i}(\breve{\mathbf{n}}, \bar{\mathbf{n}}, k)$, the system of differential equations to be solved can be rewritten as

$$\begin{cases} \breve{w}^1 = \max\left\{\beta \tilde{f}_{\breve{n}_1} + (1-\beta) r U_h - \beta \sum_{j=1,...,N} \breve{w}^j_{\breve{n}_1} \breve{n}_j, \bar{w}\right\} \\ \vdots \\ \breve{w}^N = \max\left\{\beta \tilde{f}_{\breve{n}_N} + (1-\beta) r U_h - \beta \sum_{j=1,...,N} \breve{w}^j_{\breve{n}_N} \breve{n}_j, \bar{w}\right\}. \end{cases}$$

The solution to this system is given in Cahuc et al. (2008), who use spherical coordinates to solve for it. This leads to the wage expression

$$\breve{w}^i = \max\left\{\beta \breve{\Omega}_i \tilde{f}_{\breve{n}_i} + (1-\beta) r U_i, \bar{w}\right\}, \tag{23}$$

where

$$\breve{\Omega}_i = \frac{\int_0^1 \frac{1}{\beta} z^{\frac{1-\beta}{\beta}} \frac{\partial \tilde{f}(z\breve{\mathbf{n}}, \bar{\mathbf{n}}, k)}{\partial (z\breve{n}_i)} dz}{\frac{\partial \tilde{f}(\breve{\mathbf{n}}, \bar{\mathbf{n}}, k)}{\partial (\breve{n}_i)}}. \tag{24}$$

Given the definition of $\tilde{f}$ we introduced above, the distortion factor in equation (24) can be rewritten as

$$\breve{\Omega}_i = \frac{\int_0^1 \frac{1}{\beta} z^{\frac{1-\beta}{\beta}} \frac{\partial f(z\breve{\mathbf{n}}, \bar{\mathbf{n}}, k)}{\partial (z\breve{n}_i)} dz}{\frac{\partial f(\breve{\mathbf{n}}, \bar{\mathbf{n}}, k)}{\partial (\breve{n}_i)}},$$

implying the more compact formulation in equation (13).

A.2 Proof of Proposition 2

A.2.1 Underinvestment factor

To obtain equation (17), first notice that, by using equation (1) and the definition of $\tilde{f}$ in Appendix A.1, equation (15) can be rewritten as

$$r + \delta = \tilde{f}_k(\mathbf{n}, k) - \sum_{j=1,...,N} \breve{w}^j_k(\mathbf{n}, k) \breve{n}_j \tag{25}$$

Then, by deriving (23) with respect to k, we have that

$$\breve{w}^j_k(\mathbf{n}, k) = \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial^2 \tilde{f}(z\breve{\mathbf{n}}, \bar{\mathbf{n}}, k)}{\partial (z\breve{n}_j) \partial k} dz, \quad \forall j = 1, ..., N.$$

We integrate by parts $\sum_{j=1,...,N} \breve{w}^j_k(\mathbf{n}, k) \breve{n}_j$ as

$$\sum_{j=1,\dots,N} \breve{w}_k^j(\mathbf{n},k)\breve{n}_j = \tilde{f}_k(\breve{\mathbf{n}},\bar{\mathbf{n}},k) - \frac{1-\beta}{\beta}\int_0^1 z^{\frac{1-2\beta}{\beta}}\tilde{f}_k(z\breve{\mathbf{n}},\bar{\mathbf{n}},k)dz.$$

Plugging this expression in condition (25) yields

$$r+\delta = \int_0^1 \frac{1-\beta}{\beta} z^{\frac{1-2\beta}{\beta}}\tilde{f}_k(z\breve{\mathbf{n}},\bar{\mathbf{n}},k)dz.$$

Finally, by using the definition of $\tilde{f}$ in Appendix A.1, the equation above can be rewritten as equation (17) with the respective expression for Ω_k.

A.2.2 Overemployment factors

To obtain equation (16), notice first that condition (14) can be rewritten as

$$\frac{cx_i}{q(\theta_i)} = \frac{f_i(\mathbf{n},k) - w^i(\mathbf{n},k) - \sum_{j=1,\dots,N}\breve{w}_i^j(\mathbf{n},k)\breve{n}_j}{r+s}, \quad \forall i \in \{h,l\}, \tag{26}$$

given equation (1).

Then, derive (12) with respect to n_i:

$$\breve{w}_i^j(\mathbf{n},k) = \int_0^1 (\chi_i z + 1 - \chi_i) z^{\frac{1-\beta}{\beta}} f_{ji}\left(\mathbf{n}\mathbf{M}(z),k\right)dz$$

and calculate $\sum_{j=1,\dots,N}\breve{w}_i^j(\mathbf{n},k)\breve{n}_j$:

$$\sum_{j=1,\dots,N}\breve{w}_i^j(\mathbf{n},k)\breve{n}_j = \sum_{j=1,\dots,N}\int_0^1 (\chi_i z + 1 - \chi_i) z^{\frac{1}{\beta}-1} f_{ji}\left(\mathbf{n}\mathbf{M}(z),k\right)\chi_j n_j dz$$

$$\begin{aligned}
&\sum_{j=1,\dots,N}\breve{w}_{n_i}^j(\mathbf{n},k)\breve{n}_j = \\
&\quad \chi_i \int_0^1 z^{\frac{1}{\beta}} \sum_{j=1,\dots,N} f_{ji}\left(\mathbf{n}\mathbf{M}(z),k\right)\chi_j n_j dz \\
&\quad +(1-\chi_i)\int_0^1 z^{\frac{1}{\beta}-1}\sum_{j=1,\dots,N} f_{ji}\left(\mathbf{n}\mathbf{M}(z),k\right)\chi_j n_j dz
\end{aligned}$$

By integrating by parts the two integrals in the equation above, we obtain

$$
\begin{aligned}
&\sum_{j=1,\dots,N} \breve{w}^{j}_{n_i}(\mathbf{n},k)\breve{n}_j = \\
&\qquad \chi_i f_i(\mathbf{n},k) - \chi_i \int_0^1 \frac{1}{\beta} z^{\frac{1}{\beta}-1} f_i\left(\mathbf{n}\mathbf{M}(z),k\right) dz \\
&\qquad +(1-\chi_i) f_i(\mathbf{n},k) - (1-\chi_i) \int_0^1 \frac{1-\beta}{\beta} z^{\frac{1}{\beta}-2} f_i\left(\mathbf{n}\mathbf{M}(z),k\right) dz
\end{aligned}
$$

By plugging this expression in condition (26), we obtain (16) and the respective expressions for Ω_i and $\bar{\Omega}_i$. This completes the proof.

B Distortion factors: special cases

B.1 No minimum wage earner

First, notice that, when $\chi_i = 1$, for all $i = 1, ..., N$, Ω_i, $\breve{\Omega}_i$ and Ω_k take the same form as in Cahuc et al. (2008) as all elements in the diagonal of the $\mathbf{M}(z)$ matrix are equal to z:

$$\Omega_i = \breve{\Omega}_i = \frac{\int_0^1 f_i\left(\mathbf{n}z, k\right) \breve{\varphi}(z)dz}{f_i\left(\mathbf{n}, k\right)},$$

$$\Omega_k = \frac{\int_0^1 f_k\left(\mathbf{n}z, k\right) \bar{\varphi}(z)dz}{f_k(\mathbf{n}, k)},$$

while, from (18), the value of $\bar{\Omega}_i$ becomes irrelevant.

Ω_i is the ratio of two elements: its denominator is the marginal product of labor, while its numerator is a weighted average of the infra-marginal products, where the weights in the integral are given by the density $\breve{\varphi}(z)$, with $\int_0^1 \breve{\varphi}(z)dz = 1$. Notice that $\Omega_i = 1$ when $\breve{\varphi}(z)$ has a mass point around $z = 1$ and $\breve{\varphi}(z) = 0$ for all $z < 1$, since the numerator is equal to the denominator in this case. Ω_i is also equal to one when the marginal product of i-type labor is independent of the n_j's ($\forall j = 1, ..., N$). For other values of $\breve{\varphi}(z)$ and with a non-linear production function, Ω_i may differ from one.

Three effects may drive the value of the distortion factors away from one.[19] First, the concavity in n_i of the production function tends to increase their value: the more concave the production function is, the larger are the incentives for the firm to overemploy i-type workers in order to reduce their wage. Second, the substitutability (complementarity) with j-type workers tends to increase (decrease) the value of Ω_i: overemployment (underemployment) allows to decrease the wage of j-type workers by decreasing their marginal product. Third, the shape of the density $\breve{\varphi}(z)$ also affects the values of Ω_i by weighting the different infra-marginal products of labor at a different intensity. Specifically, when the bargaining power of workers is large, the representative firm has more incentives to reduce wages.

Moreover, under Nash bargaining the negotiated wage is a function of the capital stock. Because workers do not share the cost of *ex ante* investments in the absence of binding wage contracts, this leads to underinvestment (overinvestment) when capital is complementary (substitutable) to labor: the representative firm anticipates that investing more (less) in physical capital amounts to bargaining to a higher wage.

B.2 One labor type

Consider the case where χ_i may differ from 1, but we only have one labor type. In this case, a first difference with respect to Cahuc et al. (2008) appears: the overemployment factor Ω_i becomes an average of the overemployment factor for negotiating workers $\breve{\Omega}_i$

[19] See Cahuc et al. (2008) and Cahuc and Wasmer (2001a) for more details.

and the overemployment factor for minimum wage workers $\bar{\Omega}_i$, as shown in equation (18).

With the change of variable $x = \chi z + 1 - \chi$, these distortion factors can be interpreted more easily:

$$\Omega_k = \frac{\int_0^1 f_k\left(nx, k\right)\bar{\psi}(x)dx}{f_k},$$

$$\breve{\Omega} = \frac{\int_0^1 f_1\left(nx, k\right)\breve{\psi}(x)dx}{f_1\left(n, k\right)} \quad \text{and} \quad \bar{\Omega} = \frac{\int_0^1 f_1\left(nx, k\right)\bar{\psi}(x)dx}{f_1\left(n, k\right)},$$

with

$$\breve{\psi}(x) = \begin{cases} 0 & \text{if } x < 1 - \chi \\ \frac{(x-1+\chi)^{\frac{1-\beta}{\beta}}}{\beta\chi^{1/\beta}} & \text{if } x \geq 1 - \chi \end{cases}$$

and

$$\bar{\psi}(x) = \begin{cases} 0 & \text{if } x < 1 - \chi \\ \frac{(1-\beta)(x-1+\chi)^{\frac{1-\beta}{\beta}}}{\beta\chi^{\frac{1-\beta}{\beta}}} & \text{if } x \geq 1 - \chi \end{cases}$$

where the subscripts i are not considered since we only have one labor type. Notice that the densities $\breve{\psi}(x)$ and $\bar{\psi}(x)$ fulfill the property $\int_0^1 \breve{\psi}(x)dx = \int_0^1 \bar{\psi}(x)dx = 1$.

The distortion factors have a structure similar to the one in Cahuc et al. (2008) with the difference that the share of negotiating workers χ has an effect on the density that appears in the integral. In particular, the lower the value of χ is, the more concentrated are the $\breve{\psi}(x)$ and $\bar{\psi}(x)$ distributions around $x = 1$. Figure 7 depicts examples of the $\breve{\psi}(x)$ density for $\beta = \frac{1}{2}$ and several values of χ.

The effect of χ on the densities suggests that the firm's strategic behavior gets more limited as the share of negotiating workers decreases. For example, when capital and labor are complements, the firm chooses to underinvest (i.e. $\Omega_k < 1$); but, as χ decreases and the distribution $\bar{\psi}(x)$ gets more concentrated around $x = 1$, underinvestment becomes weaker, generating an increase in the demand for capital: the lower the fraction of workers that negotiate their wage with the firm is, the lower are the effects that the wage negotiation exerts over the investment decision. This implies that $\Omega_k \to 1$ when $\chi_i \to 0$ for $i \in \{h, l\}$.[20]

There may also be situations where the firm's strategic behavior actually gets exacerbated. To understand when this may happen, first notice that the $\breve{\psi}(x)$ density

[20]Notice also that, if $\chi_i \to 0\ \forall\ i \in \{h, l\}$, $\breve{\Omega}_i \to 1$ and $\bar{\Omega}_i \to 1\ \forall\ i \in \{h, l\}$, as in this case the firm cannot strategically influence any wage rate.

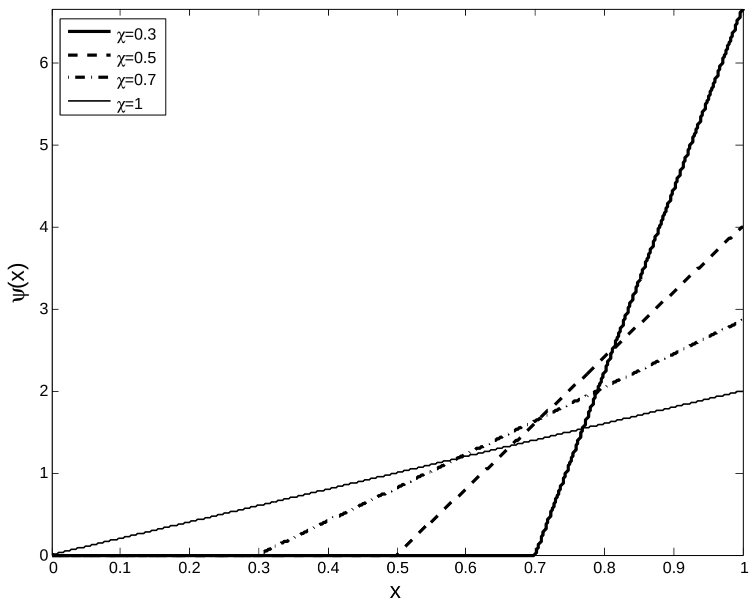

Figure 7: The effect of χ on the $\breve{\psi}(x)$ density, examples with $\beta = 1/2$

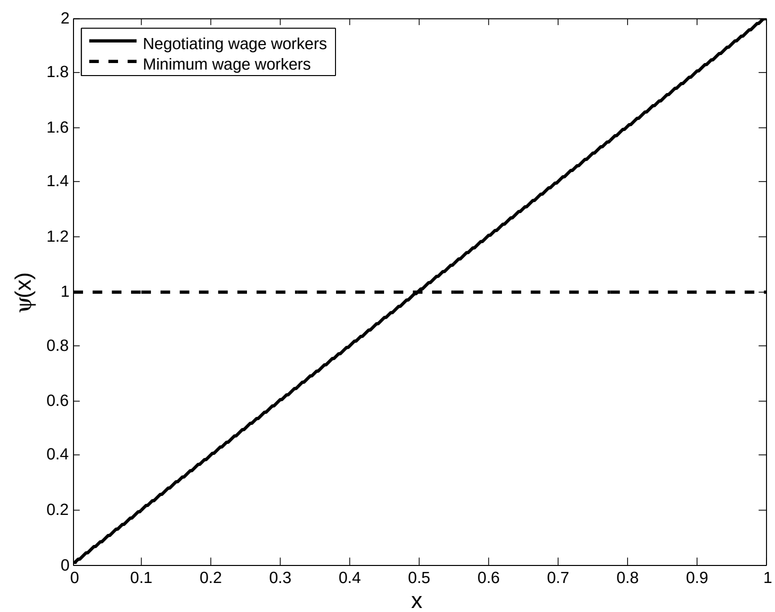

Figure 8: Comparing the $\breve{\psi}(x)$ and $\bar{\psi}(x)$ densities, example with $\beta = 1/2$ and $\chi = 1$

is more concentrated around $x = 1$ than $\bar{\psi}(x)$. Figure 8 illustrates this fact with an example: it compares the $\breve{\psi}(x)$ density (solid line) with $\bar{\psi}(x)$ (dashed line) in the case where $\beta = \frac{1}{2}$. This suggests that, *ceteris paribus*, overemployment (underemployment) should be stronger in the case of minimum wage workers than in the case of negotiating workers. The intuition for this is *rent appropriation*: negotiating workers claim part of the change in the wage of other workers resulting from intra-firm bargaining, while minimum wage workers do not. As a consequence, overemployment (or underemployment) may become stronger as χ decreases because rent appropriation by workers is more limited. This can be observed in equation (18), where Ω depends more on $\bar{\Omega}$ than $\breve{\Omega}$ as χ decreases.

B.3 General case

In the general case with two labor types and χ_i that may differ from one, the distortion factors are written as in (13), (18), (19) and (20). All the comparative statics and interpretations in Sections B.1 and B.2 still hold with the additional ingredient that the share of negotiating workers may interact with the fact that the two labor types are substitutes or complements.

Consider the example of two labor types which are substitutable. In this case, the firm has incentives to overemploy them. If χ_1 decreases, the firm would have less incentives to overemploy type 2 workers because it would influence the wage of a lower fraction of type 1 workers. In the case of overemployment of type 1 workers, the effect is ambiguous. On the one hand, overemployment may be more limited as in the case of type 2 workers. On the other hand, lower rent appropriation by type 1 workers may actually enhance the firm's strategic behavior, leading to higher overemployment.

C Unique versus multiple equilibria: some examples

The possibility of multiple equilibria is confirmed by Figure 9, which illustrates several examples of the determination of χ with one type of labor. Each panel in the figure compares the value of the minimum wage with $\tilde{w}$ as a function of χ to pin down possible equilibria.

The upper left panel illustrates the case with one labor type, no capital and a linear production function. In this case, because $\bar{w} > \tilde{w}$, nobody negotiates with the firm and the equilibrium value of χ is zero.

The two upper right panels illustrate the situation with decreasing returns to scale and no capital. The production function is $f(n) = n^{\alpha}$, with $\alpha \in (0, 1)$. The first example is characterized by multiple equilibria (two equilibria are in pure strategy and one in mixed strategy), while the second example shows one equilibrium in pure strategy. Intuitively, multiple equilibria arise in the former example due to a general-equilibrium effect. When agents coordinate to a higher χ and a larger proportion of agents negotiate, the representative firm chooses to overhire to reduce their wage because there are decreasing returns to labor. However, the firm does not consider the effect on labor-market tightness in general equilibrium: overemployment pushes the tightness upwards, which actually increases wages above the minimum wage. When the agents coordinate to a lower value of χ, the opposite happens.

Finally, the bottom panels show the determination of χ for three values of the minimum wage when the production function is $f(n, k) = n^{\alpha}k^{1-\alpha}$. There is uniqueness in these cases. When the minimum wage is too low, χ equals one. When it starts to bind, the unique equilibrium is characterized by mixed strategies. For higher values of the minimum wage, χ is equal to zero.

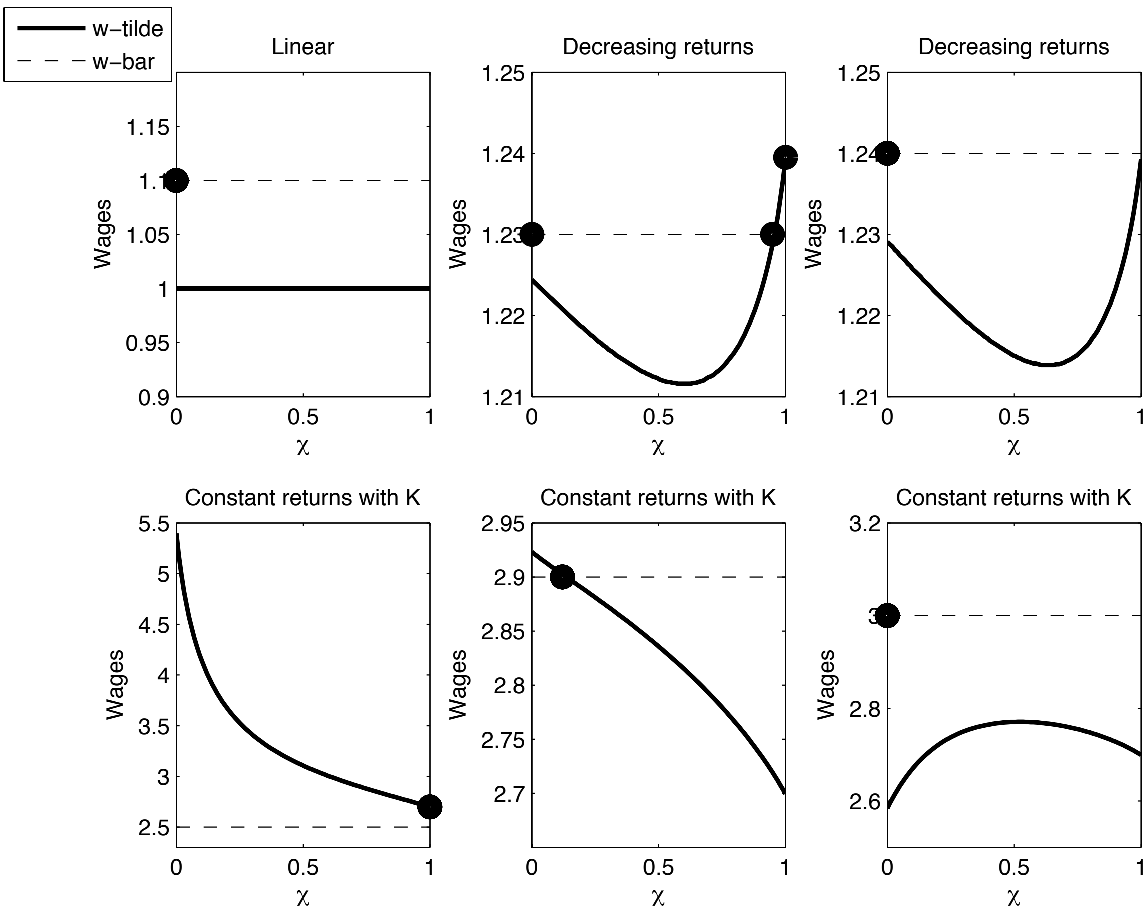

Figure 9: Determination of χ: several examples

Notes: the upper left panel illustrates the case with one labor type, no capital and a linear production function. The production function for the two upper right panels is $f(n) = n^{\alpha}$, with $\alpha \in (0, 1)$. The bottom panel show the determination of χ for three values of the minimum wage when the production function is $f(n, k) = n^{\alpha} k^{1-\alpha}$.

D Welfare

D.1 Properties

Our model is characterized by two types of inefficiencies. First, congestion externalities are not necessarily internalized by the Nash bargaining rule as in Hosios (1990). Second, appropriability distorts employment and capital decisions, as in Grout (1984) and Cahuc et al. (2008) among others. Both inefficiencies may partly compensate each other; for instance, the social losses of a large bargaining power, which leads to too few vacancies in the standard model with one worker per firm, may be reduced by an overemploying representative firm.

We now illustrate these ideas in the context of the constrained social planner problem.[21] The value function characterizing the social planner's solving problem is

$$\mathcal{V}(\mathbf{n},k) = \max_{\{v_1,...,v_N,a\}} \tfrac{1}{1+rdt} \left(\left[f(\mathbf{n},k) + b\left(1 - \textstyle\sum_{j=1,...,N} n_j\right) - \textstyle\sum_{j=1,...,N} v_j c - a \right] dt + \mathcal{V}(\mathbf{n}',k') \right), \tag{27}$$

subject to the constraints (3), (4), $\theta_i = \frac{v_i}{1/N_i - n_i}$ $\forall i = 1,...,N$.

We show in Appendix D.2.1 that the first-order conditions for a maximum are, in steady state,

$$\frac{c}{q(\theta_i)} = \frac{(f_i(\mathbf{n},k) - b)(1 - \eta(\theta_i))}{r + s - \theta_i^2 q'(\theta_i)}, \quad \forall i \in \{h,l\}, \tag{28}$$

and

$$r + \delta = f_k(\mathbf{n},k), \tag{29}$$

where $\eta(\theta_i) \equiv -\frac{\theta_i q'(\theta_i)}{q(\theta_i)}$.

These optimality conditions can be compared to the vacancy-posting and capital-investment conditions of the representative firm in the context of the steady-state equilibrium given in Definition 1. This allows us to establish the following result on the efficiency of the equilibrium:

Proposition 3. *The constrained-efficient allocations are a set of employment levels n_i, a capital stock k and labor-market tightness θ_i for all $i = 1,...,N$ such that the optimality conditions (28) and (29) and the Beveridge relations (21) are satisfied.*

Hence, a steady-state equilibrium is efficient if, $\forall i = 1,...,N$,

[21] See also Smith (1999) and Cahuc and Wasmer (2001b).

$$\beta = \frac{\breve{\Omega}_i f_i(\mathbf{n}, k) - b - \Delta_i (f_i(\mathbf{n}, k) - b)(r+s)}{\breve{\Omega}_i f_i(\mathbf{n}, k) - b + \Delta_i (f_i(\mathbf{n}, k) - b)\theta_i q(\theta_i)} \qquad \textit{for } 0 < \chi_i \leq 1, \tag{30}$$

$$\bar{w} = \bar{\Omega}_i f_i(\mathbf{n}, k) - \Delta_i (r+s)(f_i(\mathbf{n}, k) - b) \qquad \textit{for } 0 \leq \chi_i < 1, \tag{31}$$

$$\Omega_k = 1 \qquad \textit{for } 0 < \chi_i < 1, \tag{32}$$

where $\Delta_i = \frac{1-\eta(\theta_i)}{r+s+\theta_i q(\theta_i)\eta(\theta_i)}$.

Proof. See Appendix D.2.2. □

Condition (32) is a standard condition for an efficient capital allocation, while condition (30) is an augmented Hosios-Pissarides condition, with (31) being its counterpart in presence of a binding minimum wage. It is easy to show that both reduce to the standard condition of a model with one worker per firm when $\breve{\Omega}_i = \bar{\Omega}_i = 1$, with $\beta = \eta(\theta_i)$ when $\chi_i > 0$ and $\bar{w} = f_i(\mathbf{n}, k) - \Delta_i(r+s)(f_i(\mathbf{n}, k) - b)$ when $\chi_i = 0$.[22] When $\breve{\Omega}_i > 1$, a value for β larger than $\eta(\theta_i)$ is required in order to compensate for overemployment by the representative firm. The opposite occurs when $\breve{\Omega}_i < 1$.

A minimum wage can fulfill one of the efficiency conditions given in Proposition 3. For example, condition (31) may be satisfied when the decentralized equilibrium wage is inefficiently too low absent a minimum wage legislation. This happens when the representative firm has incentives to overemploy ($\breve{\Omega}_i > 1$) or when the share it obtains under wage bargaining is too high, generating congestion on the vacancy side. Similarly, condition (32) is satisfied when the minimum wage is binding for both labor groups. In this case, intrafirm bargaining cannot take place, which alleviates the holdup problem.

However, these efficiency conditions are rarely satisfied together. Moreover, the fulfilment of a subset of them does not necessarily produce an improvement in welfare. It may be the case that reaching optimality on one market leads to augmented inefficiencies on another market. For example, implementing (31) in the case of some workers may induce the overemployment factor for other workers to deviate even more from its social optimum.

D.2 Proof of Proposition 3

D.2.1 Constrained-efficient allocations

We show first how to obtain equations (28) and (29). Define $p(\theta) = \theta q(\theta)$. The first-order conditions of the program in (27) are

[22]It can be shown that, if $\bar{\Omega}_i = 1$ and $\chi_i = 0$, then

$$\bar{w} = \eta(\theta_i) f_i(\mathbf{n}, k) + (1 - \eta(\theta_i))b + \eta(\theta_i)\theta_i c.$$

$$c = \mathcal{V}_{n_i}(\mathbf{n}, k)p'(\theta_i), \quad \forall i = 1, ..., N,$$

and

$$\mathcal{V}_k(\mathbf{n}, k) = 1.$$

By applying the envelope theorem, we get

$$(r + s - \theta_i^2 q'(\theta_i))\mathcal{V}_{n_i}(\mathbf{n}, k) = f_i(\mathbf{n}, k) - b, \quad \forall i = 1, ..., N,$$

and

$$(r + \delta)\mathcal{V}_k(\mathbf{n}, k) = f_k(\mathbf{n}, k).$$

Plugging these two equations in the first-order conditions yields equation (29) and

$$\frac{c}{p'(\theta_i)} = \frac{f_i(\mathbf{n}, k) - b}{r + s - \theta_i^2 q'(\theta_i)}, \quad \forall i = 1, ..., N.$$

Notice that $p'(\theta_i) = q(\theta_i)(1 - \eta(\theta_i))$. Hence,

$$c \left(r + s - \theta_i^2 q'(\theta_i)\right) = \left(f_i(\mathbf{n}, k) - b\right) q(\theta_i)(1 - \eta(\theta_i)), \quad \forall i = 1, ..., N.$$

By rearranging this equation, one can obtain equation (28).

D.2.2 Efficiency of the decentralized equilibrium

Equations (30)-(32) are obtained by comparing (28) and (29) with the first-order conditions in the case of the decentralized equilibrium. From equations (1), (12), (16) and (18), the vacancy creation condition can be rewritten as

$$(r + s)\frac{c}{q(\theta_i)} = \chi_i(1 - \beta)\left(\breve{\Omega}_i f_i(\mathbf{n}, k) - rU_i\right) + (1 - \chi_i)\left(\bar{\Omega}_i f_i(\mathbf{n}, k) - \bar{w}\right), \forall i = 1, ..., N \tag{33}$$

while, as a reminder, the capital investment condition is

$$r + \delta = \Omega_k f_k(\mathbf{n}, k). \tag{34}$$

Notice that, from equations (1), (12) and (22), and imposing the equilibrium condition by which $\breve{w}^i = \bar{w}$ when $0 < \chi_i < 1$, we can write

$$rU_i = \frac{(r + s)b + \beta\theta_i q(\theta_i)\breve{\Omega}_i f_i(\mathbf{n}, k)}{r + s + \beta\theta_i q(\theta_i)} \quad \forall\ 0 < \chi_i \leq 1. \tag{35}$$

Plugging equation (35) in (33), we obtain

$$\frac{c}{q(\theta_i)} = \chi_i(1-\beta)\frac{\breve{\Omega}_i f_i(\mathbf{n},k) - b}{r+s+\beta\theta_i q(\theta_i)} + (1-\chi_i)\frac{\bar{\Omega}_i f_i(\mathbf{n},k) - \bar{w}}{r+s}. \tag{36}$$

Equation (28) can be rewritten as

$$\frac{c}{q(\theta_i)} = \chi_i(1-\eta(\theta_i))\frac{f_i(\mathbf{n},k) - b}{r+s+\theta_i q(\theta_i)\eta(\theta_i)} + (1-\chi_i)(1-\eta(\theta_i))\frac{f_i(\mathbf{n},k) - b}{r+s+\theta_i q(\theta_i)\eta(\theta_i)}. \tag{37}$$

Comparing (36) to (37) we obtain conditions (30) and (31). Similarly, by comparing (34) with (29), we obtain condition (32). This completes the proof.

E Appendix: numerical algorithm

We briefly describe here the numerical algorithm used to calibrate the model and to perform the exercises described in Section 4.

E.1 Calibration

1. As discussed in Section 3, we set as targets of the calibration the job finding probability, aggregate market tightness, the flow value of unemployment as a fraction of marginal product, and the labor share, and the N percentiles of the estimated wage distribution.
2. We assign values to parameters and provide initial guesses for parameters c, α, b, m_0 and the vector of productivities $\mathbf{x}$.
3. Given these parameter values, we solve the model according to the algorithm described in the following section.
4. Given equilibrium allocations, we compute the model counterparts to the calibration targets.
5. We iterate on 2-4 until the moments generated by the model are sufficiently close to the calibration targets defined in 1.
6. After obtaining values for the parameters, we check ex-post that x_{min} used in the estimation of the parametric wage distribution is such that the fraction of workers subject to the minimum wage in the model is close to its counterpart in the data.

E.2 Numerical solution of the model

1. We set a value for the minimum wage $\bar{w}$ and assign functional forms and values to the parameters of the model.
2. We assume the minimum wage $\bar{w}$ is not binding for any category. Therefore, $\chi_i = 1\ \forall i = 1, ..., N$. We solve for the allocations θ_i, n_i and $k\ \forall i = 1, ..., N$ by solving the system of equations given by (16), (17) and (21). Distortion factors are numerically calculated using the Gauss-Legendre algorithm.
3. For every $i = 1, ..., N$ we compare the negotiated wage $\tilde{w}_i$ with the minimum wage $\bar{w}$. If $\tilde{w}_i > \bar{w}$, then $\chi_i = 1$ is effectively an equilibrium. Otherwise, we set $\chi_i = 0$ and solve the system of equations (16), (17) and (21). Distortion factors are again numerically calculated using the Gauss-Legendre algorithm.
4. For every i for which we set $\chi_i = 0$, we compare the negotiated wage $\tilde{w}_i$ with the minimum wage $\bar{w}$. If $\tilde{w}_i < \bar{w}$, then $\chi_i = 0$ is effectively an equilibrium. Otherwise, we look for the allocations and $\chi_i \in (0, 1)$ that solve the system of equations (16), (17) and (21) and guarantee that $\tilde{w}_i = \bar{w}$. Distortion factors are again numerically calculated using the Gauss-Legendre algorithm.

5. We check that the equilibrium is unique, i.e., only one of the previous situations arises.

F Appendix: tables

Table 2: Minimum wages by US State

State	Minimum wage (in $US)	State	Minimum wage (in $US)
Alabama	7.25	Montana	7.25
Alaska	9.75	Nebraska	9
Arizona	8.05	Nevada	7.25
Arkansas	8	New Hampshire	7.25
California	10	New Jersey	8.38
Colorado	8.31	New Mexico	7.5
Connecticut	9.6	New York	9
Delaware	8.25	North Carolina	7.25
Florida	8.05	North Dakota	7.25
Georgia	7.25	Ohio	7.25
Hawaii	8.5	Oklahoma	7.25
Idaho	7.25	Oregon	9.25
Illinois	8.25	Pennsylvania	7.25
Indiana	7.25	Rhode Island	9.6
Iowa	7.25	South Carolina	7.25
Kansas	7.25	South Dakota	8.55
Kentucky	7.25	Tennessee	7.25
Louisiana	7.25	Texas	7.25
Maine	7.5	Utah	7.25
Maryland	8.25	Vermont	9.6
Massachusetts	10	Virginia	7.25
Michigan	8.5	Washington	9.47
Minnesota	7.25	West Virginia	8.75
Mississipi	7.25	Wisconsin	7.25
Missouri	7.65	Wyoming	7.25

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