

# The Economic Impact of Oil on Industry Portfolios\*

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## Abstract

We build an equilibrium model to disentangle industry-specific from business cycle effects of oil on stock returns. In our model oil is considered as an input factor for production and also as a macro variable. We estimate the model for 13 industries, including the oil industry. Our results suggest that the value of all non-oil industries decreases with an oil price shock. This result is explained by the effect of oil on the price-dividend ratios of the industries, in particular, by the significant negative effect of oil on their growth opportunities. The high persistence of the real oil price shocks makes these effects to be long-lived. The effect of oil on the current cash-flows is negative but small. This explains why the oil price shocks can produce such a significant effects on the US financial market despite the low US economy's oil intensity. The conditional expected portfolio returns decrease with the oil price because of the negative effect of oil on the market price of risk and interest rates. Moreover, industries with higher systematic risk have expected returns that are more affected by the oil price. We find that most of the systematic risk of the firms is explained by their output rather than by effect of oil on the cash-flows.

*Keywords:* Oil price, business cycle, asset pricing, time-varying risk premia, industry stock returns, conditional CAPM.

*JEL Classification:* G12, G17, Q43, E44, E32.

# 1 Introduction

There is a strong linkage between the oil price and the business cycle. For example, Hamilton (2008) documents that nine out of the last ten recessions in the United States were preceded by an increase in oil prices. Furthermore, a recent but meaningful fact, is that the business cycle plays a crucial role in determining the equity risk premia (e.g., Lettau and Ludvigson, 2001a; Cooper and Priestley, 2009). We combine both stylized facts in addition to the fact that oil is an important factor for production, to study the relationship between oil prices and stock returns. In particular, we propose a model with multiple transmission channels of oil that allows us to quantify the effect of each mechanism for different industry portfolios. Our model captures these effects in reduced form, for example, by assuming that the risk premium, interest rates and current and future cash flows depend on the oil price.

The effect of an oil shock on a firm may be decomposed into three categories depending on the level at which it works: business cycle, sectoral and individual levels. At the macro level, if oil is a good predictor for the business cycle, it should have some forecasting power for equity returns. Indeed, the empirical work of Casassus and Higuera (2011) tests this hypothesis and finds that the stock market excess returns are significantly affected by oil price changes. Bakshi, Panayotov, and Skoulakis (2011) finds a similar result using as a predictor the Baltic Dry Index, a shipping activity variable that is tied to economic activity and to energy prices. Also, a significant oil shock generally produces inflationary pressures, which generates shifts in the inflation expectations of the agents and contractionary monetary policies. Therefore, oil price increases could have an important impact on the financing cost of the firm (i.e., on the interest rates), which can cause significant reductions in the firm's growth opportunities and present value of its cash flows.

The sectoral effect of an oil shock operates through changes in the output demand and in the prices of raw materials and input factors, such as labor. These mechanisms vary across the different economic sectors. For example, firms in a durable sector experience important

reductions in their demands with an oil shock, whereas firms in other sectors may face even positive demand shocks (e.g., precious metals industry, as stated by Kilian and Park, 2009). Also, since oil is itself an input factor for most firms, potential substitution effects in the productive processes will cause the prices of other inputs to be affected.

At an individual level, if the impact of an oil shock depends on the firm's decisions, then the capacity to absorb these shocks becomes relevant. For example, oil price shocks can trigger firms's decisions related to energy efficiency, which have long run effects and can also influence the growth opportunities of the firms. Both the sectoral and individual effects determine the impact of oil on variables such as the firm's production costs, sales revenues, cash flows, dividends and growth opportunities. The interaction of these effects with the macroeconomic ones determines the final impact of oil on the firm's value. Finally, it is difficult to know a priori whether or not these oil price impacts on stock returns are purely systematic, i.e., priced in the economy. Some of the above-mentioned effects could impact the expected stock returns while others could simply be part of the idiosyncratic risk component.

We propose an equilibrium asset-pricing model that allows us to disentangle and quantify both industry-specific from business cycle impacts of the oil shocks on the stock price. The price of oil follows an exogenous stochastic process. For convenience, an exogenous pricing kernel is assumed to incorporate the macroeconomic effects of the oil price shocks in a simple way. In particular, we allow both the real risk-free interest rate and the market risk premium to be a function of the oil price. The industry-specific effects of the oil price acts through the current and future cash flows of the firm. Each industry is represented by a production technology that uses oil as an input factor. The effect of oil on the sales revenues depends on the oil elasticity of output (i.e., the oil intensity). To test the effect of oil on the growth opportunities of the firm we allow the output growth rate to depend on the oil price. The traditional supply-side transmission mechanism of the oil shocks is also considered by including the firm's oil expenditure in the production costs. Furthermore, a demand-side transmission channel of the oil shocks is included by assuming that the sales growth rate and the oil price

can be correlated.

Based on the intuition from the Gordon model (i.e., Gordon, 1959), we analyze the effect of oil of each industry portfolio by studying its effect on the current dividend, on the dividend growth rate and on the expected return of the portfolio. We also study the impact of oil on the price and price-dividend ratio and realized returns of the portfolios. To quantify these effects we present the oil price elasticities for each of these components. The relationship between oil prices and expected returns depends on: 1) the sensitivity of the real interest rate to the oil price, 2) the sensitivity of the market risk premium to the oil price and 3) quantity of systematic risk (i.e., the CAPM beta of the industry). The first two components affect all stocks in a similar way; however, the latter depends on industry-specific characteristics, such as the oil intensity of the firm's cash flows. The model provides closed-form expressions for the firm's optimal dividend and the dividend growth rate, however, a log-linearization is needed to obtain the price-dividend ratio of the portfolios.

We estimate the model with maximum likelihood using Kenneth French's industry portfolios returns from April 1983 to December 2010, along with market returns, crude oil prices and risk-free rates. According to our estimates, an oil price increase of 10% reduces on average the value of the non-oil industry portfolios by 1.8%, and increases the value of the oil industry portfolio by 1.5%. The result for the non-oil portfolios is explained because the average negative effect of a 10% rise of the oil price on the expected dividend growth rates (-1.1%) dominates the negative effect on the expected return (-0.8%). Consistent with the low oil intensities observed in the data, we find that the average effect of oil on the current dividend is negative but less economically significant (-0.4%). These results combined with the high-persistence of the oil shocks may explain why oil produces such a significant effect on the US financial market despite its low intensity in the economy.

With respect to the expected portfolio returns, our estimates suggest that both the real interest rate and the market risk premium are negatively affected by the oil price, therefore both macroeconomic transmission channels work in the same direction. This explains the

aggregate negative effect of oil on the conditional returns of the non-oil industries. We also find that industries with more systematic risk have a higher oil price elasticity of the expected returns. In our model the market beta of the industry is a linear combination of the oil market beta, the industry output market beta the market beta of a latent macro variable related to the interest rates. We find that oil increases the portfolio's systematic risk, because both the oil market beta and its weight in the portfolio's beta are negative. Conversely, for the oil industry this effect of oil act in an opposite directions because its weight for oil price's market beta is positive. With respect to the idiosyncratic effects of oil price shocks on the industry portfolio returns, we find that these are negative for most industries.

The rest structure of the paper is as follows. The following section reviews the related literature. Section 3 presents the partial equilibrium model and its basic implications. Section 4 proposes an approximate solution for our model and shows the main theoretical results and predictions of our model. Section 5 contains the empirical methodology and aspects related to the estimation of model. Section 6 presents the oil price elasticities and Section 7 analyzes the industry portfolio returns. Finally, Section 8 concludes.

## 2 Related literature

Our paper is motivated by a large literature that relates the oil price with both the business-cycle and the financial markets. The former became a mainstream topic with the empirical work of Hamilton (1983) that shows that the oil price changes strongly Granger-cause the GNP growth rate and the unemployment rate in the United States. He uses a bivariate vector autoregression approach (VAR) and Sims's (1980) six-variable VAR with quarterly data from 1948 to 1980, and finds that an oil price increase is followed by four successive quarters of lower real GNP growth rates.<sup>1</sup> The most common transmission channels of the oil shocks to the real economy are: inflation, terms of trade (Huntington, 2007) and the utilization rate of

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<sup>1</sup> Jones, Leiby, and Paik (2004) provide a recent survey of this literature.

capital (Finn, 2000). Subsequent studies note that this relationship weakens when data after 1980 is included, a period that coincides with the loss of market control by the OPEC (e.g., Mork, 1989; Lee, Ni, and Ratti, 1995; Hooker, 1996). This turned attention to possible non-linear relationships between the variables. Some economic mechanisms that can generate an asymmetric impact of an oil price shock are: the monetary policy (Ferderer, 1996; Bernanke, Gertler, and Watson, 1997; Hamilton and Herrera, 2004; Balke, Brown, and Yucel, 2002; Leduc and Sill, 2004), imperfect intersectoral mobility of factors (Lee and Ni, 2002; Lilien, 1982; Hamilton, 1988; Davis and Haltiwanger, 2001), irreversibility of investment (Bernanke, 1983; Dhawan and Jeske, 2008), salary rigidities (Lee, Ni, and Ratti, 1995) and interest rates (Balke, Brown, and Yucel, 2002). Kliesen (2008) obtains a significant economic impact of oil in an extended regression that includes the Chicago Fed National Activity Index (CFNAI), which is the first principal component of 85 monthly indicators of real economic activity.

Recently, Kilian (2009) proposes a novel methodology which allows to identify and disentangle demand from supply shocks in the global crude oil market. He finds that oil price shocks have been driven mainly by demand shocks (shocks to global aggregate demand and shocks to precautionary demand), rather than by oil supply shocks. Oil supply disruptions cause only temporary declines in real GDP and have a non-significant effect on prices. Also, positive aggregate demand shocks tend to raise prices and initially have a positive net effect on the economy; however they become recessionary after a while. Lastly, positive precautionary demand shocks lower real GDP and raise consumer prices. On the contrary, Kilian (2008) reports strong evidence that the exogenous supply shocks (although not all of them) did cause a significant impact on the GDP growth of G-7 countries. The empirical evidence strongly supports the existence of an economically and statistically significant linkage between oil price shocks and the business cycle.

Contrary to what happens on the macro side, the relationship between the oil shocks and the financial markets has received much less attention. Jones and Kaul (1996) find that the oil price shocks produce a significant and rational effect in the stock markets in the United States

and Canada, but evidence of overreaction is found for United Kingdom and Japan. Huang, Masulis, and Stoll (1996) test for Granger-causality from oil futures returns to stock returns and find a significant relation only for oil companies. Using a VAR model with oil price shocks and US stock returns, Sadorsky (1999) finds that oil shocks affect market returns, but not the other way around. On the theoretical side, Wei (2003) builds a general equilibrium model to analyze the impact of an oil price shock on the firm's value, which faces the problem of irreversible investment. Nevertheless, his model predicts a little impact of an oil shock and it is not able to explain the massive market decline in 1974. In a related contemporaneous work, Chiang, Hughen, and Sagi (2015) use crude oil futures and oil-related stock returns to estimate a reduced form model for crude oil prices. They show the oil factors extracted from oil-related data contain relevant information for explaining stock returns.

In a recent study, Driesprong, Jacobsen, and Maat (2008) find that oil price changes have a significant forecasting power for the stock market returns of developed countries, although this conclusion is not so strong for emerging markets. They argue that the origin of this predictability is caused by an initial underestimation of the impacts of oil price shocks by agents (underreaction to oil price changes is less pronounced in oil-related sectors), which then it is slowly corrected over time. Park and Ratti (2008) report evidence that oil price shocks have a significant negative impact on real stock returns of net importer countries. They find that positive oil price shocks are followed by short interest rate increases in almost all countries in the sample, including the US. Moreover, when linear oil price shocks are assumed, they find no evidence of asymmetry in the impacts of positive and negative oil price shocks. Kilian and Park (2009) apply the methodology of Kilian (2009) to evaluate the impact of the resulting oil shocks on US real stock returns. Their results show that supply, global aggregate demand and precautionary demand shocks account for 6%, 5% and 11% of the long run variation in real market stock returns, respectively. Furthermore, sectoral level evidence suggests that oil market shocks are transmitted to the economy as demand-side shocks rather than through production costs, as is commonly believed. Apergis and Miller (2009) employ a slightly different



methodology to do the same decomposition of oil shocks as in Kilian (2009), but for a sample of G-7 countries and Australia. In contrast to the findings in Kilian and Park (2009), they conclude that oil shocks have a negligible, although significant statistically, economic impact on the stock returns.

Most of the studies mentioned above use aggregate data, therefore can only account for the macroeconomic effects of the oil shocks on the financial markets. One of the few exceptions is Kilian and Park's (2009) who reveal that the sensitivities of the macro effects of oil vary considerably across industries. It is very important to use disaggregated data to better understand the relationship between the oil price and the financial market and to study how the macroeconomic and sectoral effects of oil interact to produce the response observed in the data.

### 3 The model

This section presents an asset-pricing model with an infinite-horizon manufacturing firm that is a price taker in the oil market and maximizes the present value of its cash flows. The model considers both an exogenous pricing kernel and an exogenous oil price process. These features allows us to test directly the macroeconomic and industry-specific relationships between the oil price and the firm discussed in the previous sections.

#### 3.1 The stock price

We consider a representative firm in a real economy that uses oil to produce an output good. Let the stock price of firm  $i$ ,  $P_t^i$ , be the present value of the cash flows generated by the firm:

$$P_t^i = \sup_{\{q_u^i \in \Psi\}} \mathbb{E}_t \left[ \int_t^\infty \frac{\Lambda_u}{\Lambda_t} D^i(q_u^i) du \right] \quad (1)$$

where  $\Lambda_u$  is the pricing kernel or stochastic discount factor and  $D^i(q_u^i)$  is the firm's cash flow that depends on the demand for oil for production purposes,  $q_u^i$ .

### 3.2 Pricing kernel dynamics

The stochastic discount factor follows the standard equilibrium dynamics:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t^f dt - \lambda_t dZ_{\Lambda,t} \quad (2)$$

where  $r_t^f$  is the real risk-free interest rate,  $\lambda_t$  is the market price of risk and  $Z_{\Lambda,t}$  is a standard Brownian motion that captures the systematic risk in the economy. The real interest rate is assumed to be a function of the real oil price,  $S_t$ , and a latent variable,  $y_t$ , that captures other macro effects not related to the oil price:

$$r_t^f = \alpha_0 + \alpha_s s_t + \alpha_\delta \delta_t + \alpha_y y_t \quad (3)$$

The dependence of the real interest rate on the oil price occurs because oil can affect both the inflation rate and the nominal interest rate. These effects can act in opposite directions if the monetary authority follows a contractionary policy that reacts to inflationary pressures caused by an increase in the oil price.<sup>2</sup>

We assume that the market price of risk is also an affine function of the state variables:

$$\lambda_t = \theta_0 + \theta_s s_t + \theta_\delta \delta_t + \theta_y y_t \quad (4)$$

Here  $\theta_s$  represents the effect of oil on the market price of risk, a transmission channel that is consistent with recent empirical studies that document that energy-related variables have a significant forecasting power for market excess returns.<sup>3</sup> Moreover, we expect a negative

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<sup>2</sup> See Bernanke, Gertler, and Watson (1997), Hamilton and Herrera (2004) and Leduc and Sill (2004) for a discussion about monetary policy and the oil price.

<sup>3</sup> See for example, Casassus and Higuera (2011) and Bakshi, Panayotov, and Skoulakis (2011).

$\theta_s$  consistent with the negative annual cumulative impact of oil price shocks on equity risk premium of 2.1% reported by Casassus and Higuera (2011). We also allow that other macro variables that affect the interest rates may also affect the market price of risk.

Our setting can be interpreted also as a conditional Capital Asset Pricing Model (CAPM) that considers the oil price and the latent variable  $y_t$  as conditioning variables (see Cochrane, 2005).<sup>4</sup>

### 3.3 State variables dynamics

We assume that the log real oil price follows a two-factor model:

$$ds_t = \kappa_s (\delta_t - s_t) dt + \sigma_s \left( \rho_s dZ_{\Lambda,t} + \sqrt{1 - \rho_s^2} dZ_{s,t} \right) \quad (5)$$

$$d\delta_t = \kappa_\delta (\bar{\delta} - \delta_t) dt + \sigma_\delta \left( \rho_\delta dZ_{\Lambda,t} + \sqrt{1 - \rho_\delta^2} dZ_{\delta,t} \right) \quad (6)$$

where  $\kappa_s > 0$  represents the speed of mean-reversion,  $\bar{\delta}$  is a parameter associated to the long-run mean of the real spot price,  $\sigma_s$  is the volatility of the oil price shocks and  $\rho_s$  represents the instantaneous correlation between the oil price returns and the pricing kernel. Therefore,  $\rho_s$  captures any possible systematic risk in the oil returns.  $\rho_\delta$  y  $Z_{\delta,t}$ .  $Z_{s,t}$  is a standard Brownian motion that represents the idiosyncratic shocks to the oil price and is independent of  $Z_{\Lambda,t}$ . The Brownian motion increments are correlated with  $dZ_{s,t}dZ_{\delta,t} = \rho_{s\delta}dt$ .

The latent variable is assumed to follow a standard Ornstein-Uhlenbeck process with zero unconditional mean and unit instantaneous standard deviation:

$$dy_t = -\kappa_y y_t dt + \left( \rho_y dZ_{\Lambda,t} + \sqrt{1 - \rho_y^2} dZ_{y,t} \right) \quad (7)$$

where  $\kappa_y > 0$  represents the speed of mean-reversion, and  $\rho_y$  is the instantaneous correlation between the shocks in  $y_t$  and the pricing kernel. The standard Brownian motion  $Z_{y,t}$  represents

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<sup>4</sup> Some related literature to this approach are Santos and Veronesi (2006) that develop a conditional CAPM on the labor income to consumption ratio, and Lettau and Ludvigson (2001b) that uses the consumption-wealth ratio (cay) as the conditioning variable.

the idiosyncratic shocks to  $y_t$  and is independent of the other Brownian motions in the model. The Brownian motion increments are correlated with  $dZ_{s,t}dZ_{y,t} = \rho_{sy}dt$ . The Brownian motion increments are correlated with  $dZ_{\delta,t}dZ_{y,t} = \rho_{\delta y}dt$ .

### 3.4 Firm's cash flows

We assume that the firm is fully funded by equity and that its cash flows take the following functional form:

$$D^i(X_t^i, q_t^i, S_t) = X_t^i(q_t^i)^{\gamma^i} - S_t q_t^i \quad \text{for } 0 < \gamma^i < 1 \quad (8)$$

where  $q_t^i$  is the demand for oil,  $\gamma^i$  is the oil intensity of the firm and  $X_t^i$  is an exogenous variable representing the non-oil determinants of the cash flows. The marginal cost of oil is the price of oil,  $S_t$ , and is the same for all firms. To concentrate on the effect of oil on the cash flows, we make the simplifying assumption that the firm only chooses its demand for oil. We also assume that the firm has a fully-flexible technology with respect to oil (i.e., there are no adjustment costs). The Cobb-Douglas form of the sales implies that the productivity of oil is higher for those firms with higher oil intensity,  $\gamma^i$ . Therefore, firms with higher  $\gamma^i$ 's demand more oil than those firms with lower  $\gamma^i$ 's.

In the rest of the paper we refer to the first component of the cash flows in equation (8) as the sales of the firm and to  $X_t^i$  as the output variable, however, it is clear that  $X_t^i$  captures not only the output price and the productivity of the firm, but also the effect of other inputs on the cash flows. Moreover, our model does not explicitly consider investment and a capital production factor, however, a time-varying expected growth rate of  $X_t^i$  could account for these issues in a reduced-form way.<sup>5</sup> Indeed, better investment opportunities will be represented by higher output growth rate, and viceversa. Also, if we assume that  $X_t^i$  changes only because of changes in the investment opportunities, then we will be able to disentangle the cash flows generated by current assets from those expected from future investments.

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<sup>5</sup> This occurs because the output growth rate is directly related to the growth rates of the sales and cash flows of the firm.

The output variable is governed by the following geometric Brownian process:

$$\frac{dX_t^i}{X_t^i} = (\mu_0^i + \mu_s^i s_t + \mu_\delta^i \delta_t + \mu_y^i y_t) dt + \sigma_X^i \left( \rho_X^i dZ_{\Lambda,t} + \sqrt{1 - \rho_X^i{}^2} dZ_{X,t}^i \right) \quad (9)$$

We assume that the drift of the output growth rate is affine in the log oil price, allowing the growth opportunities to depend on the oil price. If  $\mu_s^i < 0$  then the growth opportunities of the firm decrease with the oil price. The parameter  $\sigma_X^i$  is the volatility of the output growth rate, and  $\rho_X^i$  and  $\rho_{X_s}^i$  represent the correlations of this variable with the pricing kernel and the oil price shocks, respectively. In particular,  $\rho_X^i$  determines how the systematic shocks are transmitted to the firm's sales, so firms that are more sensitive to the business cycle will have a higher correlation (e.g., firms whose output corresponds to durable consumption goods sector). This variable can be also interpreted as a measure of demand cyclicality since it quantifies the comovement between the output growth and the aggregate economic growth. The correlation  $\rho_{X_s}^i$  has two sources: one due to the systematic risk factor ( $dZ_{\Lambda,t}$ ) and another one due to the idiosyncratic oil price shocks ( $dZ_{s,t}$ ). The inclusion of both  $\mu_s^i$  and  $\rho_{X_s}^i$  is based on the sectoral evidence from Kilian and Park (2009) that suggests that oil shocks propagate through the demand for the firm's output by consumers rather than through its production costs. Finally,  $Z_{X,t}^i$  is a standard Brownian motion that represents the idiosyncratic shocks to  $X_t^i$ . The Brownian motion increments are correlated with  $dZ_{s,t} dZ_{X,t}^i = \rho_{sX}^i dt$ . The Brownian motion increments are correlated with  $dZ_{\delta,t} dZ_{X,t}^i = \rho_{\delta X}^i dt$ . The Brownian motion increments are correlated with  $dZ_{y,t} dZ_{X,t}^i = \rho_{yX}^i dt$ .

Although the final impact of oil in the cash flows must be evaluated considering that the firm reacts by changing its demand for oil, it is important to highlight that oil impacts both sales and production costs of the firm. Furthermore, today's oil price impacts current cash flows and also expected future cash flows through the persistence of the oil shocks and the growth opportunities represented by the expected output growth rate. A third mechanism that related oil to the cash flows is that an unexpected shock to the oil price could also generate and unexpected change in the output growth rate, and therefore, an unexpected change in the

future cash flow of the firm.

### 3.5 Firm's equilibrium

The representative firm optimally demands oil in every period to maximize its market value in equation (1), subject to the dynamics of the state variables that drive the macroeconomic variables, its production costs and its sales revenue in equations (4) to (9). It is important to note that the stock price is not a function of  $\Lambda_t$ , because in the discount process the relevant information is contained in the discount factor  $\frac{\Lambda_u}{\Lambda_t}$  which, given its geometric nature, is independent on the level of  $\Lambda_t$ .

It is straightforward to verify that the value function satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \max_{\{q_t^i\}} \left\{ D^i(X_t^i, q_t^i, e^{s_t}) \Lambda_t dt + \mathbb{E}_t[d(\Lambda_t P^i(s_t, \delta_t, y_t, X_t^i))] \right\} \quad (10)$$

subject to

$$\lim_{T \rightarrow \infty} \mathbb{E}[\Lambda_T P^i(s_T, \delta_T, y_T, X_T^i)] = 0 \quad (11)$$

where the transversality condition in equation (11) rules out any possible asset-price bubble. The solution of this optimization problem is considered in the next section.

## 4 Solution and theoretical results

This section shows the solution of the firm's optimization problem. First we obtain the optimal demand for oil and the cash flows of the firm. Then, we exploit a property of our model to express the resulting partial differential equation (PDE) as a function of the price-dividend ratio. This new PDE is approximated and solved using the log-linear expansion for dividend-

price ratio developed by Campbell and Viceira (2002). Finally, we show some theoretical results and the main predictions of our model.

## 4.1 Optimal demand for oil

The representative firm chooses the optimal demand for oil  $q_t^i$  based on the rule that is presented in the following proposition.

**Proposition 1** *The optimal demand for oil and cash flows of the firm are given by:*

$$q_t^{i*} = \left( \frac{\gamma^i X_t^i}{S_t} \right)^{\frac{1}{1-\gamma^i}} \quad (12)$$

$$D_t^{i*} = D^i(X_t^i, S_t) = \left( \frac{(\gamma^i)^{\gamma^i} X_t^i}{S_t^{\gamma^i}} \right)^{\frac{1}{1-\gamma^i}} (1 - \gamma^i) = \frac{1 - \gamma^i}{\gamma^i} q_t^{i*} S_t \quad (13)$$

Moreover, the optimal growth rate of the cash flows is

$$\frac{dD_t^{i*}}{D_t^{i*}} = \left( \frac{\gamma^i (s_t - \delta_t) \kappa_s + \mu_0^i + \mu_s^i s_t + \mu_\delta^i \delta_t + \mu_y^i y_t}{1 - \gamma^i} + \zeta^i \right) dt + \zeta_\Lambda^i dZ_{\Lambda,t} + \zeta_s^i dZ_{s,t} + \zeta_X^i dZ_{X,t} \quad (14)$$

where  $s_t = \log(S_t)$  and

$$\zeta^i = \frac{\gamma^i (\gamma^i \sigma_s^2 - 2\sigma_s \sigma_X^i \rho_s \rho_X^i + \sigma_X^{i2})}{2(1 - \gamma^i)^2}, \quad \zeta_\Lambda^i = \frac{\sigma_X^i \rho_X^i - \gamma^i \sigma_s \rho_s}{1 - \gamma^i},$$

$$\zeta_s^i = -\frac{\gamma^i \sigma_s \sqrt{1 - \rho_s^2}}{1 - \gamma^i}, \quad \zeta_X^i = \frac{\sigma_X^i \sqrt{1 - \rho_X^{i2}}}{1 - \gamma^i} \quad (15)$$

**Proof** See Appendix A.1.  $\square$

The proposition shows that the optimal demand for oil,  $q_t^{i*}$ , is increasing in the output variable  $X_t^i$  and decreasing in the oil price  $S_t$ . That is, a shock to  $X_t^i$  increases the marginal benefit of oil and, therefore, the demand for oil. In contrast, the firm reduces its oil demand when the

marginal cost of oil increases. Moreover, ceteris paribus, firms with higher  $\gamma^i$  will demand more oil because this input is more productive.

Equation (13) shows that the optimal cash flow is also increasing in  $X_t^i$  and decreasing in  $S_t$ . The oil price elasticity of the cash flow, i.e., the percentage change in the current cash flow caused by a percentage change in oil price is given by:

$$E_{D,S}^i = \frac{\partial \log(D_t^{i*})}{\partial s_t} = -\frac{\gamma^i}{1 - \gamma^i} \quad (16)$$

This elasticity is always negative and depends only on the oil intensity. As expected, the cash flows of industries with higher oil intensities will be more affected by an increase in the oil price.

To understand the effect of oil on the growth opportunities of the firm we can derive from equation (14) the oil price elasticity of the cash-flows growth rate. This elasticity measures the percentage change in the annualized expected cash flows growth rate due to a percentage change in the oil price and is given by:

$$E_{g,S}^i = \frac{\partial \frac{1}{dt} \mathbb{E}_t \left[ \frac{dD_t^{i*}}{D_t^{i*}} \right]}{\partial s_t} = \frac{\gamma^i \kappa_s + \mu_s^i}{1 - \gamma^i} \quad (17)$$

This equation confirms that the sensitivity of the growth opportunities to the oil price,  $\mu_s^i$ , is a key determinant of the cash flows growth rate of the firm. Also, the speed of mean reversion of the oil price,  $\kappa_s$ , has a positive effect on the growth rate, because the higher this parameter, the less persistent is the oil shock and, therefore, the oil price will decrease faster towards its long-term mean increasing net periods cash flow. This effect is amplified by the oil intensity in the cash flows,  $\gamma^i$ .

## 4.2 Price-Dividend ratio

We assume that the all cash flows are paid as dividends to the shareholders, therefore, equation (13) also represents the dividends of the firm. Let the price-dividend ratio of the stock



be:

$$H_t^i = H^i(s_t, \delta_t, y_t) = \frac{P^i(s_t, \delta_t, y_t, X_t^i)}{D^i(X_t^i, e^{st})} \quad (18)$$

Note that in our model, the shocks to the output variable are permanent (see equation (9)), therefore, they affect the current and future dividends in the same proportion, implying that the price of the stock is also amplified by the magnitude of the shock. This means that even if the price and the dividend are functions of  $X_t^i$ , the price-dividend ratio will be unaffected by this variable.<sup>6</sup>

By replacing in equation (10) both the stock price  $P_t^i$  from the equation above and the optimal dividend from equation (13), and doing some algebra yields the following PDE for the price-dividend ratio  $H_t^i$ :

$$\begin{aligned} 0 = & \frac{1}{H_t^i} - r_t^f - \frac{\gamma^i}{1 - \gamma^i} (\kappa_s (\delta_t - s_t) - \lambda_t \sigma_s \rho_s) + \frac{1}{1 - \gamma^i} (\mu_0^i + \mu_s^i s_t + \mu_\delta^i \delta_t + \mu_y^i y_t - \lambda_t \sigma_X^i \rho_X^i) \\ & + \frac{1}{2} \frac{\gamma^{i2}}{(1 - \gamma^i)^2} \sigma_s^2 - \frac{\gamma^i}{(1 - \gamma^i)^2} \sigma_s \sigma_X^i \rho_s \rho_X^i + \frac{1}{2} \frac{\gamma^i}{(1 - \gamma^i)^2} \sigma_X^i{}^2 + \frac{H_{s,t}^i}{H_t^i} \left( \kappa_s (\delta_t - s_t) - \lambda_t \sigma_s \rho_s - \frac{\gamma^i}{1 - \gamma^i} \sigma_s^2 \right. \\ & \left. + \frac{1}{1 - \gamma^i} \sigma_s \sigma_X^i \rho_s \rho_X^i \right) + \frac{H_{\delta,t}^i}{H_t^i} \left( \kappa_\delta (\delta - \delta_t) - \lambda_t \sigma_\delta \rho_\delta - \frac{\gamma^i}{1 - \gamma^i} \sigma_s \sigma_\delta \left( \rho_s \rho_\delta + \rho_{s\delta} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_\delta^2} \right) \right) \\ & + \frac{1}{1 - \gamma^i} \sigma_\delta \sigma_X^i \rho_\delta \rho_X^i - \frac{H_{y,t}^i}{H_t^i} \left( \kappa_y y_t + \lambda_t \rho_y + \frac{\gamma^i}{1 - \gamma^i} \sigma_s \left( \rho_s \rho_y + \rho_{sy} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_y^2} \right) \right. \\ & \left. - \frac{1}{1 - \gamma^i} \sigma_X^i \rho_y \rho_X^i \right) + \frac{1}{2} \frac{H_{ss,t}^i}{H_t^i} \sigma_s^2 + \frac{H_{s\delta,t}^i}{H_t^i} \sigma_s \sigma_\delta \left( \rho_s \rho_\delta + \rho_{s\delta} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_\delta^2} \right) \\ & + \frac{H_{sy,t}^i}{H_t^i} \sigma_s \left( \rho_s \rho_y + \rho_{sy} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_y^2} \right) + \frac{1}{2} \frac{H_{\delta\delta,t}^i}{H_t^i} \sigma_\delta^2 + \frac{H_{\delta y,t}^i}{H_t^i} \sigma_\delta \left( \rho_\delta \rho_y + \rho_{\delta y} \sqrt{1 - \rho_\delta^2} \sqrt{1 - \rho_y^2} \right) \\ & + \frac{1}{2} \frac{H_{yy,t}^i}{H_t^i} \end{aligned} \quad (19)$$

To our knowledge, there is no exact solution for equation (19), however, Campbell and Viceira (2002) obtain an approximated solution for a similar equation using a simple log-linearization technique.<sup>7</sup> Their approach is based on the discrete-time approximation of the

<sup>6</sup> The price-dividend ratio is independent of the level of the output variable, but it is affected by the parameters that determine the dynamics of  $X_t^i$ .

<sup>7</sup> In equation (5.20) of Campbell and Viceira (2002)  $H_t^i$  is the wealth-consumption ratio and it is a function of the real interest rate. In their case, they have a non-homogeneous ODE whose associated homogeneous equation belongs to the degenerate hypergeometric equation family. Polyanin and Zaitsev (1995) shows an exact solution for that equation, but it is too complex to allow for any economic intuition. We would have the same type of

price-dividend ratio of Campbell and Shiller (1988). We follow this approach and pursue log-linear approximation around the unconditional mean of the logarithm of price-dividend ratio,  $\mathbb{E}[p_t^i - d_t^{i*}]$ . This approach gives the following result:

$$\frac{1}{H_t^i} = \frac{1}{\exp(p_t^i - d_t^{i*})} \approx h_0^i - h_1^i \log(H_t^i) \quad (20)$$

where  $p_t^i$  is the logarithm of  $P_t^i$ ,  $d_t^{i*}$  is the logarithm of  $D_t^{i*}$ ,  $h_0^i = h_1^i(1 - \log(h_1^i))$  and  $h_1^i = \exp(-\mathbb{E}[p_t^i - d_t^{i*}])$ . After replacing the first term of equation (19), i.e.  $(H_t^i)^{-1}$ , with its approximation shown in equation (20), we find that the new approximated HJB equation have a close form solution that is presented in the following proposition.

**Proposition 2** *The approximated price-dividend ratio of the firm is an exponential affine function given by*

$$H^i(s_t, \delta_t, y_t) = \exp(a^i + b^i s_t + c^i \delta_t + d^i y_t) \quad (21)$$

where  $a^i$ ,  $b^i$ ,  $c^i$  and  $d^i$  are constant coefficients that depend on the parameters of the model and are presented in the appendix.

**Proof** See Appendix A.2.  $\square$

The price-dividend ratio varies with the state variables  $s_t$ ,  $\delta_t$  and  $y_t$ . Also, the associated constant coefficients depends on the parameters of the dynamics of the state variables, the interest rate, the market price of risk, the firm's cash flows and the parameter of log-linear approximation  $h_1^i$ . Moreover, from equation (21) we can derive the oil price elasticity of the price-dividend ratio of the stock:

$$E_{H,S}^i = \frac{\partial \log(H_t^i)}{\partial s_t} = b^i \quad (22)$$

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solution if the latent variable  $y_t$  were constant.—revisar—

That is, the parameter  $b^i$  measures the percentage change in price-dividend ratio due to a percentage change in oil price, holding everything else constant.

### 4.3 Stock returns

Using equation (18) along with the results in Propositions 1 and 2 we are able to derive an expression for the stock price, which can be used to obtain the instantaneous stock return. Let  $G_t^i$  be the gain process for an investor who has a long position on the stock. The return for the investor is:

$$\frac{dG_t^i}{G_t^i} = \frac{dP_t^i}{P_t^i} + \frac{D_t^{i*} dt}{P_t^i} \quad (23)$$

The following proposition shows the expression for the stock return.

**Proposition 3** *The conditional stock return is given by:*

$$\frac{dG_t^i}{G_t^i} = (r_t^f + \eta_\Lambda^i \lambda_t) dt + \eta_\Lambda^i dZ_{\Lambda,t} + \eta_s^i dZ_{s,t} + \eta_\delta^i dZ_{\delta,t} + \eta_y^i dZ_{y,t} + \eta_X^i dZ_{X,t} \quad (24)$$

where,

$$\eta_\Lambda^i = \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1 - \gamma^i} \sigma_X^i \rho_X^i \quad (25)$$

$$\eta_s^i = \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \sqrt{1 - \rho_s^2} \quad (26)$$

$$\eta_\delta^i = c^i \sigma_\delta \sqrt{1 - \rho_\delta^2} \quad (27)$$

$$\eta_y^i = d^i \sqrt{1 - \rho_y^2} \quad (28)$$

$$\eta_X^i = \frac{1}{1 - \gamma^i} \sigma_X^i \sqrt{1 - \rho_X^i{}^2} \quad (29)$$

**Proof** See Appendix A.3.  $\square$

The proposition shows that the stock return has two components: a conditional expected return and the weighted sum of the shocks considered in the model. The conditional expected return

of stock  $i$  is given by:

$$\mathbb{E}_t \left[ \frac{dG_t^i}{G_t^i} \right] = (r_t^f + \eta_\Lambda^i \lambda_t) dt = (\alpha_0 + \eta_\Lambda^i \theta_0 + (\alpha_s + \eta_\Lambda^i \theta_s) s_t + (\alpha_\delta + \eta_\Lambda^i \theta_\delta) \delta_t + (\alpha_y + \eta_\Lambda^i \theta_y) y_t) dt \quad (30)$$

The expected stock return is the sum of the instantaneous risk-free rate and a time-varying risk premium that is given by the (negative) covariance between the pricing kernel and the stock return. Also, the quantity of risk  $\eta_\Lambda^i$  is constant, therefore, the conditional expected stock return is an affine function of the state variables  $s_t$  and  $y_t$  through the varying interest rate and market price of risk. From the equation above we can derive the oil price elasticity of the expected stock return, which measures the percentage change in the expected stock return due to a percentage change in the oil price:

$$\begin{aligned} E_{r,S}^i &= \frac{\partial \frac{1}{dt} \mathbb{E}_t \left[ \frac{dG_t^i}{G_t^i} \right]}{\partial s_t} = \alpha_s + \eta_\Lambda^i \theta_s \\ &= \alpha_s + \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1 - \gamma^i} \sigma_X^i \rho_X^i \right) \theta_s \quad (31) \end{aligned}$$

This equation shows that the response of the expected stock return to oil price changes varies across the firms. It also shows that the effect of  $\theta_s$  on the expected stock returns is twofold. It has a direct impact through the market price of risk and an indirect effect through the parameters  $b^i$  and  $c^i$  that affect the quantity of risk  $\eta_\Lambda^i$ . The impact of an oil shock is increasing in  $\sigma_X^i$ , i.e., firms that face more volatile output are more risky and more vulnerable to oil price changes. Also, firms with higher cyclicality,  $\rho_X^i$ , tend to be more negatively affected by oil price increases because an oil shock is a bad signal about the future economic growth. A classical example of this type of firms are those who produce durable consumption goods. Finally, we expect to find that the impact of an oil price change is an increasing function of the oil intensity, i.e., firms with higher  $\gamma^i$ 's demand more oil, hence, oil is more important in their cash flows. Consequently, these firms are more affected by the oil shocks.

Proposition 3 also shows that the different shocks that influence the stock return come from

the relationships between the state variables and the two components of the stock price: the price-dividend ratio and the dividends or cash flows (i.e.,  $P_t^i = H_t^i D_t^{i*}$ ). These components have both systematic and non-systematic risks. For example, the impact of the systematic shock  $dZ_{\Lambda,t}$  on the stock return, which is the only component of the risk that is priced by the agents, comes from the correlations between the systematic risks of the state variables with  $H_t^i$  and  $D_t^{i*}$ . Indeed, in the first term of  $\eta_{\Lambda}^i$  in equation (25),  $b^i$  and  $-\frac{\gamma^i}{1-\gamma^i}$  measure the impact of the systematic oil shocks on the dynamics of the price-dividend ratio and the cash flows, respectively. The same interpretation applies to the other components of the systematic term  $\eta_{\Lambda}^i$ .

The remaining shocks account for the non-systematic risk of the stock return. These risks are not priced, therefore, they don't affect the expected stock return. The idiosyncratic oil shock,  $dZ_{s,t}$ , affects both the price-dividend ratio and the dividend, and its associated coefficient ( $\eta_s^i$ ) captures the oil-related idiosyncratic risk of the stock return. The first term of  $\eta_s^i$  in equation (26) shows the direct impact of the idiosyncratic oil shock on  $H_t^i$  and  $D_t^{i*}$ , while the second term measures its impact through the output growth rate which can also be correlated with the idiosyncratic oil shock. This last term shows that the demand-side effect of oil on the cash-flows also impacts the stock returns. Finally, the other idiosyncratic shocks  $dZ_{y,t}$  and  $dZ_{X,t}^i$  affect the stock return through their impact on the dynamics of the price-dividend ratio and cash flows, respectively.<sup>8</sup>

#### 4.4 Decomposition of the systematic risk

In the traditional CAPM framework the market portfolio is efficient and is perfectly correlated with the pricing kernel.<sup>9</sup> Therefore, we assume that the instantaneous return on the market

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<sup>8</sup> Note from equations (13) and (21) that the macro variable  $y_t$  affect the price-dividend ratio, but not the cash flows. Conversely, the output variable affect the cash flows of the firm, but not its price-dividend ratio.

<sup>9</sup> To be more precise, the correlation between the market returns and the changes in the pricing kernel is -1.

portfolio is defined as:

$$\frac{dG_t^m}{G_t^m} = (r_t^f + \sigma_m \lambda_t) dt + \sigma_m dZ_{\Lambda,t} \quad (32)$$

where  $\sigma_m$  is the volatility of the market portfolio return. Using equation (32), it is straightforward to show that equation (24) can be expressed in a standard CAPM form:

$$\frac{dG_t^i}{G_t^i} = (r_t^f + \beta^i \lambda_t^m) dt + \eta_{\Lambda}^i dZ_{\Lambda,t} + \eta_s^i dZ_{s,t} + \eta_{\delta}^i dZ_{\delta,t} + \eta_y^i dZ_{y,t} + \eta_X^i dZ_{X,t} \quad (33)$$

where  $\lambda_t^m = \sigma_m \lambda_t$  is the excess return on the of market portfolio. The CAPM beta of the stock  $i$  with respect to market portfolio is given by:

$$\beta^i = \frac{\text{Cov}_t \left( \frac{dG_t^i}{G_t^i}, \frac{dG_t^m}{G_t^m} \right)}{\text{Var}_t \left( \frac{dG_t^m}{G_t^m} \right)} = \frac{\eta_{\Lambda}^i}{\sigma_m} \quad (34)$$

The market betas are constant, because the instantaneous stock returns in the model are Gaussian. The beta of a stock can be expressed as a linear combination of the oil market beta  $\beta_s$ , the latent variable beta  $\beta_y$ , and the output variable market beta  $\beta_X^i$ :

$$\begin{aligned} \beta^i &= \frac{1}{\sigma_m} \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_{\delta} \rho_{\delta} + d^i \rho_y + \frac{1}{1 - \gamma^i} \sigma_X^i \rho_X^i \right) \\ &= \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \beta_s + c^i \beta_{\delta} + d^i \beta_y + \frac{1}{1 - \gamma^i} \beta_X^i \end{aligned} \quad (35)$$

Equation (35) shows that the systematic risk of the stock comes from several sources. The market betas of the macroeconomic state variables  $s_t$ ,  $\delta_t$  and  $y_t$  constitute common sources of systematic risk for all stocks. However, the weights of these betas depend on the firm's idiosyncratic characteristics, therefore, the systematic shocks of the macroeconomic variables affect each firm differently. The stock's market beta also depends on the output variable market beta, which affects the expected stock return through the systematic risk of the cash flows.

## 4.5 Time-varying expected stock return and predictability

Changes in the oil price produce macroeconomic effects which are reflected in the real interest rate and in the market price of risk. These systematic effects affect all industries and their magnitude depend on the persistence of the oil shocks.<sup>10</sup> After an oil shock, the conditional expected portfolio returns change for all the industries. As time goes by, the interest rate, market price of risk and, therefore, the conditional expected returns go back gradually to their values before oil price shock. Because the conditional expected return of a portfolio is affine on the log spot price and the latent variable  $y_t$ , it inherits the mean-reverting behavior of these variables. The following proposition formalizes this result and shows that the conditional expected stock return and the log oil price follow a multivariate mean-reverting Ornstein-Uhlenbeck process:

**Proposition 4** *The join process followed by the annualized conditional expected stock return, the log spot oil price, and the log long-run oil price is*

$$dJ_t^i = C^i(\mu + \xi(C^i)^{-1}J_t^i)dt + C^i\Sigma_J dZ_{J,t} \quad (36)$$

where

$$J_t^i = \begin{pmatrix} \bar{r}_t^i - (\alpha_0 + \eta_\Lambda^i \theta_0) \\ s_t \\ \delta_t \end{pmatrix}, \quad \bar{r}_t^i = \frac{1}{dt} \mathbb{E}_t \left[ \frac{dG_t^i}{G_t^i} \right]$$

$$C^i = \begin{pmatrix} \alpha_s + \eta_\Lambda^i \theta_s & \alpha_\delta + \eta_\Lambda^i \theta_\delta & \alpha_y + \eta_\Lambda^i \theta_y \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mu = \begin{pmatrix} 0 \\ \kappa_\delta \bar{\delta} \\ 0 \end{pmatrix}, \quad \xi = \begin{pmatrix} -\kappa_s & \kappa_s & 0 \\ 0 & -\kappa_\delta & 0 \\ 0 & 0 & -\kappa_y \end{pmatrix}$$

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<sup>10</sup> Recall that the log real oil price follows a mean-reverting process and its long-run mean is  $\bar{\delta} - \frac{\sigma_s^2}{2\kappa_s}$ .

$$\Sigma_J = \begin{pmatrix} \sigma_s \rho_s & \sigma_s \sqrt{1 - \rho_s^2} & 0 & 0 \\ \sigma_\delta \rho_\delta & 0 & \sigma_\delta \sqrt{1 - \rho_\delta^2} & 0 \\ \rho_y & 0 & 0 & \sqrt{1 - \rho_y^2} \end{pmatrix} \quad \text{and} \quad dZ_{J,t} = \begin{pmatrix} dZ_{\Lambda,t} \\ dZ_{s,t} \\ dZ_{\delta,t} \\ dZ_{y,t} \end{pmatrix} \quad (37)$$

**Proof** See Appendix A.4.  $\square$

Equation (36) is equivalent to the bivariate equation (7.1.28) of Campbell, Lo, and MacKinlay (1996), but in continuous time. Consistently with their analysis, the positive serial correlation in expected returns appears in realized stock returns. Therefore, a positive oil shock has two simultaneous effects: it changes future expected returns (see equation (36)) and it directly affects realized stock returns (see equation (24)). The sign of the first effect depends on the sign of  $\alpha_s + \eta_\Lambda^i \theta_s$ , while the second one inherits the sign of  $\eta_s^i$ . The sign and the magnitude of the autocorrelation in realized stock returns, if there exists, will depend on the signs and forces of these two effects caused by oil price shocks. The proposition also verifies that the realized stock returns are predictable by the log oil price according to equation (24).

## 5 Data and empirical implementation

We use four datasets for the estimation of the model: stock market returns, industry portfolio returns, crude oil prices and risk-free rates. These series are deflated using the Consumer Price Index (CPI) for all urban consumers from the US Bureau of Labor Statistics. We have 311 monthly observations from September 1989 to September 2015 for each one of the variables in the datasets. Our proxy for the market portfolio is the value weighted CRSP index (CRSP-VW), from which we obtained the log-returns. Crude oil prices are proxied by the closest-to-maturity crude oil futures. These series are from the New York Mercantile Exchange (NYMEX) and were



obtained from the US Energy Information Administration (EIA) website. For the nominal risk-free rate, we use the one-month Treasury bill rate (from Ibbotson Associates) that is available from Kenneth French’s web page.

Stock returns consist of monthly log-returns on the 17 industry sorted portfolios from Kenneth French’s web site.<sup>11</sup> These portfolios are formed on the basis of four-digit SIC (Standard Industrial Classification) code. We use industry portfolios for the following reasons. First, we prefer to use portfolios of firms because individual stock returns are too noisy. Second, if we use individual securities the estimation technique would become extremely demanding in computational terms. Third, the usage of portfolios grouped by a different criterion such as size or b/m is useless, because our model is built to capture industry-specific characteristics. In other words, only in the industry portfolios the idiosyncratic shocks that survive are of similar nature.

Table 1 presents a brief summary statistics of the time series used in our study: the monthly real log-returns on the CRSP-VW ( $r_t^m$ ), the logarithm of the real price of crude oil one-month maturity futures ( $s_t$ ), the one-month real Treasury bill rate ( $r_t^f$ ) and the monthly real log-returns on the industry portfolios ( $r_t^P$ ), where  $r_t^P$  is a vector of size 13. The table describes the summary statistics for the macroeconomic variables (Panel A) and for the industry portfolio returns (Panel B). In our sample, the average month market excess return is 0.46% and the average real oil price is approx \$20 dollars per barrel at April 1983 prices. Panel B shows that the portfolio with highest average return is the food industry, while the one with the lowest average return is the steel industry. Also, the machinery portfolio is the industry whose returns are more correlated with the market portfolio returns, with a coefficient of 0.87, while the oil industry has the lowest correlation with the market portfolio.

Finally, to correctly identify the technological parameter  $\gamma^i$  from data we require that the aggregation is done using firms whose production functions are reasonable alike. We give more

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<sup>11</sup> We are aware that our model does not include both debt and taxes while return data available for us are being affected by these factors. However, we have no reliable information that allows us to correct these effects in the data.

details about the calibration of  $\gamma^i$  in the next section.

## 5.1 Calibrating the oil intensity $\gamma^i$

The oil intensity of each industry,  $\gamma^i$ , is a critical parameter for the predictions of our model, therefore, it needs to be estimated as accurate as possible. Fortunately, given the Cobb-Douglas assumption for the sales of the firm it is straightforward to calibrate  $\gamma^i$  from the data. Indeed, using equation (12) we observe that  $\gamma^i$  is the ratio of oil expenditure to sales revenue (hereafter, OESR ratio):

$$\gamma^i = \frac{S_t q_t^{i*}}{X_t^i (q_t^{i*})^{\gamma^i}} \quad (38)$$

We calibrate this parameter using data from the Manufacturing Energy Consumption Survey (MECS) of the EIA.<sup>12</sup> In particular, we calculate the OESR ratio for all the surveyed industries using Tables 1.2, 3.2, 6.1 and 7.2 that are available for 1994, 1998, 2002, 2006 and 2010. Oil expenditure in million dollars is proxied by the total energy expenditure, which is obtained multiplying the Consumption of First Use Energy for All Purposes (Table 1.2) and the Average Prices of Purchased Energy Sources (Table 7.2). On the other hand, sales revenue in billion dollars is computed dividing the Energy Consumption as a Fuel (Table 3.2) by the Ratio of Fuel Consumption to Value of Shipments (Table 6.1). Then, we obtain the OESR ratio for each industry code.

A direct match between industry portfolios and OESR ratios is only available for 1994. For this year, the MECS Tables were built using SIC codes, so for each industry code we assign a portfolio tag matching this code with the corresponding range of SIC codes from the portfolio definition (industry definitions file). For example, SIC code 2011 (Meat Packing Plants) is assigned to “Food” portfolio, because the range of SIC codes 2010-2019 (Meat products) belong to this portfolio. After each industry code in the MECS Tables was associated to a portfolio

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<sup>12</sup> <http://www.eia.gov/emeu/mecs/mecs2006/2006tables.html>

tag, the parameter  $\gamma^i$  of each portfolio is computed as a simple average of the OESR ratios with the same portfolio tag.

For the other years the MECS Tables were built using NAICS (North American Industry Classification System) codes. We use the correspondence tables between both classification systems, provided by US Census Bureau, for matching the portfolios with the OESR ratios.<sup>13</sup> For the data from 1998 we use the “1997 NAICS to 1987 SIC” concordances, while for the other years we use the “2002 NAICS to 1987 SIC” concordances. The correspondence tables provide full compatibility between six-digits NAICS codes and four-digits SIC codes, however, industry codes in the MECS tables include NAICS codes with three to six digits. To face this problem we first assign the OESR ratios of NAICS codes with three/four/five digits to all NAICS codes starting with those three/four/five digits in the correspondence table. Then, when a NAICS code in the correspondence table has associated more than one OESR ratio, we kept the ratio assignation for the NAICS code with more digits. For example, the NAICS code 311221 (Wet Corn Milling) is included in the MECS tables, therefore it has a direct OESR ratio, while it also shares with others the ratios of the NAICS codes 3112 (Grain and Oilseed Milling) and 311 (Food). In this case prevails the six-digits code assignation. However, for the NAICS code 311222 (Soybean Processing), which is not in the MECS tables, we assign the OESR ratio belonging to NAICS code 3112. After this procedure, we do the portfolio matching and calculate the OESR using the SIC codes in the correspondence table in the same way as for the year 1994.

We use the procedure described above to calculate the OESR of 9 portfolios for 1994 and 13 portfolios for the other years. We drop the portfolio “Other” because it is not representative of any particular industry. Our estimate of  $\gamma^i$  is given by the average of the time series. The result of this calibration exercise is shown in Table 2. In general, as expected, the OESR ratios are low and only exceed the 10% for the oil and chemicals portfolios.

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<sup>13</sup> <http://www.census.gov/eos/www/naics/concordances/concordances.html>

## 5.2 Model estimation

We estimate a discretized version of the model by Maximum Likelihood (ML). The likelihood function is obtained in closed-form due to the log-linear approximation and the Gaussian nature of the equations that estimate. The empirical model consists of the following relationships: market returns, industry portfolio returns, log oil futures prices and risk-free interest rates. All these variables can be expressed as linear functions of the following vector of latent state variables:

$$V_t := \begin{pmatrix} s_t \\ \delta_t \\ y_t \end{pmatrix} \quad (39)$$

The natural candidates to back-out the variables  $s_t$  and  $\delta_t$  are (log) oil futures prices, while for  $y_t$  it is used the interest rate because of its affine structure on  $V_t$  (see equation (3)). Therefore, we can use the following linear transformation to connect the observable with the unobservable variables:

$$\begin{pmatrix} f_t^1 \\ f_t^{15} \\ r_t^f \end{pmatrix} = \begin{pmatrix} A_0^1 \\ A_0^{15} \\ \alpha_0 \end{pmatrix} + \begin{pmatrix} A_s^1 & A_\delta^1 & A_y^1 \\ A_s^{15} & A_\delta^{15} & A_y^{15} \\ \alpha_s & \alpha_\delta & \alpha_y \end{pmatrix} \begin{pmatrix} s_t \\ \delta_t \\ y_t \end{pmatrix}$$

$$W_t = A + B V_t \quad (40)$$

where the parameters of the oil futures prices,  $A_i^\tau$ , are derived in Appendix B.

Following Chen and Scott (1993), Pearson and Sun (1994) and Fisher and Gilles (1996), and assuming that the variables in  $W_t$  are observed without error, we are able to pin down  $V_t$ . We also include the realized returns from the market and  $P$  industry portfolios, which enhance

$W_t$  to a vector of  $P + 4$  observable random variables:

$$Y_t = \begin{pmatrix} r_t^m \\ W_t \\ r_t^P \end{pmatrix} \quad (41)$$

Here,  $r_t^m$  and  $r_t^P$  are the one-month return for the market and industry portfolios, respectively. Finally, recall that the solution of the model requires a log-linear approximation of dividend-price ratio, therefore, a value for the parameter  $h_1^i$  in equation (20) needs to be calibrated (see also equations (A9)-(A10)). We calibrate  $h_1^i$  using the methodology of Ang and Bekaert (2007) with the log price-dividend ratio from CRSP-VW. In our sample, the average value of the log price-dividend ratio is 3.777, implying a calibrated value of  $h_1^i = 0.023$  (portfolios). The details of the estimation strategy are given in Appendix C.

### 5.3 MLE Estimates

Table 3 shows the maximum likelihood estimates and significance of the macroeconomic parameters (Panel A) and the portfolio parameters (Panel B).

Panel A of Table 3 shows that the impact of the real oil price on the real risk-free interest rate is negative ( $\alpha_s = -0.007$ ) and significant at the 10% level (t-stat =  $-1.75$ ). This result confirms the significant effect of the oil price shocks in the real economy and can be understood as an underreaction of the monetary authority with respect to the oil shocks. Indeed, it is likely that due to inflationary pressures caused by an oil price increase the monetary authority increases the nominal short-term interest rate. However, a negative effect of an oil shock in the real interest rate, suggest that the inflation rate increases even more than the nominal interest rate. Also, the significant estimate of  $\alpha_y = 0.162$  reveals that, as expected, there are other economic forces driving the real interest rate. The estimate of the market price of risk

parameter  $\theta_s$  is also negative ( $-0.445$ ) and highly significant. The fact that an increase in the oil price causes a reduction in the aggregate risk premia confirms that positive oil price shocks precede economic recessions. Furthermore, the negative and highly significant estimate of  $\theta_y$  evidences the existence of factors that have at the same time a positive effect in the real interest rate and a negative effect on market price of risk.

The mean reversion parameters  $\kappa_s$  and  $\kappa_y$ , are both positive and statistically significant. However, the different magnitudes suggest that oil shocks are much more persistent than the shocks to the latent variable  $y_t$ . Also, the estimate of the correlation parameter  $\rho_s$  is negative but insignificant, meaning that the oil price is mainly driven by idiosyncratic shocks. This evidence suggests that the oil price is an exogenous variable for US economy, which is consistent with the assumption that this variable is governed by an exogenous process.<sup>14</sup> On the other hand, changes in the latent variable  $y_t$  are strongly positively correlated with the pricing kernel, which captures the negative correlation observed in data between stock market return and risk-free rate (see Table 1 Panel A) and also the negative correlation between current consumption and interest rates.

The results from Panel B of Table 3 motivate the following general observations. First, none of the estimates of  $\rho_{X_s}^i$  is significant implying that oil and output shocks are not correlated for any industry. Second, there are no counter-cyclical industries in our sample because all industries have output growth rates that are negatively correlated with the pricing kernel (i.e.,  $\rho_{X_s}^i > 0$  for all industries). Third, our results are in line with those reported by Kilian and Park (2009), because they strongly support the hypothesis of the demand-side transmission channel of oil price changes, i.e., oil price affects negatively the output growth rate of all non-oil industries in our sample ( $\mu_s^i < 0$ ).

The oil industry is the one with both the lowest output volatility ( $\sigma_X^i = 0.145$ ) and the least cyclicity in the sample (i.e., the lowest  $\rho_X^i$ ). This industry is also the one with the least affected

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<sup>14</sup> A causality relationship from the US consumption to the oil price would be consistent with a negative correlation between oil price changes and pricing kernel (i.e.,  $\rho_s > 0$ ), because a drop in consumption would cause simultaneously an decrease in the oil price and an increase in the pricing kernel.

growth opportunities by crude oil price (i.e. it has the lowest  $|\mu_s^i|$  and it is insignificant). This occurs because for the oil industry, oil has two effect on the growth opportunities that net each other. A higher oil price is a negative macroeconomic signal for the growth opportunities of all industries, however, for this specific industry it also means a higher output price in the future. On the other side, the most volatile is the steel industry ( $\sigma_X^i = 0.276$ ) and the most cyclical is the machinery industry ( $\rho_X^i = 0.826$ ), evidence that is consistent with the high volatility and cyclicity of US investment growth. Finally, the machinery industry is also the one whose expected output growth is most sensitive to the oil price ( $\mu_s^i = -0.145$ ). That is, its growth opportunities are the most affected by an oil price increase. Again this evidence gives strong support to the demand-side effects of oil price, because oil intensity in this industry barely reaches 0.8%.

It is worth noting that our results are in line with those studies supporting the predictive power of oil for future economic growth in US economy. An oil price increase today will be followed by slower economic activity, so more (less) cyclical industries should see their growth opportunities more (less) negatively affected. Consequently, the correlation between the estimates of  $\rho_X^i$  and  $\mu_s^i$  for all industries is  $-0.88$ .

## 6 Oil price elasticities and industry portfolios

We use the intuition from the Gordon model and study the effect of oil on the stock prices by analyzing its effect on the current cash flows or dividends, on the expected growth rates and on the discount rates. Our model allows us to decompose the oil impacts into short- and long-term effects. Indeed, the current dividend accounts for the short-term effects of oil, while the price-dividend ratio of the stock accounts for the long-term effects.

## 6.1 Current dividend and oil price

Table 4 presents the oil price elasticity of the current cash flows or dividends (see column  $E_{D,S}^i$ ). The table shows that these elasticities are negative and small for all industries, something expected because they depend only on the oil intensities, which are generally low. An oil price increase of 10% reduces the dividends of most industries by less than 1%, except for the oil and chemical industries which have a higher, but still minor effect (-1.9% and -1.4%, respectively). The average effect of oil on the current dividend is negative and economically insignificant (only -0.4%). Thus, the short-term and most obvious effect of oil on the stock price is very small, as highlighted by some macroeconomic studies (e.g. Barsky and Kilian, 2004). To see the overall effect of oil we must consider the impact on the future discount rates and growth opportunities of the industries. The next section covers this topic.

## 6.2 Price-dividend ratio and oil price

Column  $E_{H,S}^i$  of Table 4 presents a very different picture for the long-term effect of oil. The oil price elasticities of the price-dividend ratio varies across industries, but are negative and economically significant for most industries. For example, an oil price increase of 10% causes a reduction of 2.4% in dividend-price ratio of the construction portfolio and a decrease of 2.6% in the machinery portfolio. For the chemical industry this elasticity is zero, while for the oil industry it is positive and significant ( $E_{H,S}^i = 0.34$ ).

What drives this significant effect of oil in the price-dividend ratios? The last two columns of table (i.e., columns  $E_{g,S}^i$  and  $E_{r,S}^i$ ) present the oil price elasticities of the dividend growth rates and expected portfolio returns and shed some light to answer this question. The table shows that the oil price affects negatively the dividend growth rates and discount rates of all industries, however, the impact on the dividend growth rates dominates explaining the negative effect of oil on the dividend-price ratios. For the non-oil portfolios the average negative effect



of a 10% rise in the oil price on the expected dividend growth rates is -1.1%, while the average negative effect on the expected return is -0.8%. For the oil industry the price-dividend ratio is increasing in the oil price because its dividend growth rate has the lowest sensitivity to oil price. Furthermore, we can infer from the table that the price-dividend ratio of the chemical industry is not sensitive to the oil price, because the dividend growth rate and the discount rate effects cancel out. However, in contrast to the prediction of the original Gordon model, the elasticity of the price-dividend ratio can not be fully explained by the elasticities of the dividend growth rate and discount rate. In our model the dividend growth rates and the expected returns vary over time and their dynamics are driven by the persistence of both the oil price and the macro variable. These effects that are considered in the stock price but not directly in the instantaneous elasticities shown in Table 4.

A key determinant in the elasticity of the price-dividend ratio is the sensitivity of the output growth rate to the oil price,  $\mu_s^i$ . In fact, in our sample the correlation between  $b^i$  and  $\mu_s^i$  is 0.88, implying that industries whose growth opportunities are more affected by oil prices, have also the higher elasticity of the price-dividend ratio.<sup>15</sup> Another important parameter is the volatility of the output of each industry. In general, the higher the volatility, the higher the sensitivity of price-dividend ratio of that industry to the oil price. In this case, the mechanism works through the effect of oil in the discount rate. Industries with higher output volatilities have expected returns that are more sensitive to the oil price. On the other hand, the oil intensity and the demand cyclicalities play a minor role in the long-term effect of oil.

Finally, the total impact of oil on the market value of an industry portfolio can be calculated from the effects of oil on both the price-dividend ratio and the dividends of the portfolio. Table 4 also shows the oil price elasticity of the market value of the portfolio which is labeled as  $E_{P,S}^i$ . In general, the effect of oil on the value of the portfolios are negative and economically significant for all non-oil industries. Indeed, an oil price increase of 10% reduces on average the value of the non-oil industry portfolios by 1.8%. Oil has the highest effect on the construction portfolio

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<sup>15</sup> Recall from equation (22) that the oil price elasticity of the price-dividend ratio is  $b^i$ .

that decreases its value by 3%. The only industry that increases its value with an oil shock is the oil industry. The value of this portfolio grows by 1.5% after a rise of 10% in oil price.

### 6.3 Out-of-sample test for the market price-dividend ratio

This section shows that the oil price elasticities of the price-dividend ratio that our model predicts for the market portfolio is consistent with the one observed in the data. Indeed, a direct implication of equations (21) and (40) is that log price-dividend ratio ( $H_t^i$ ) can be expressed as an affine function of  $f_t^1$ ,  $f_t^{15}$  and  $r_t^f$ , that is:

$$\begin{aligned}
h_t^i = \log(H_t^i) &= a^i + \begin{bmatrix} b^i & c^i & d^i \end{bmatrix} V_t \\
&= a^i + \begin{bmatrix} b^i & c^i & d^i \end{bmatrix} B^{-1} (W_t - A) \\
&= \left( a^i - \begin{bmatrix} b^i & c^i & d^i \end{bmatrix} B^{-1} A \right) + \begin{bmatrix} b^i & c^i & d^i \end{bmatrix} B^{-1} W_t \\
&= \hat{a}^i + \hat{b}^i f_t^1 + \hat{c}^i f_t^{15} + \hat{d}^i r_t^f
\end{aligned} \tag{42}$$

We can use this equation to conduct an out-of-sample test of our model. Unfortunately, a time series of the latent variable  $y_t$  cannot be obtained without using our parameter estimates, so we prefer to discard this variable to avoid doubts about our out-of-sample comparison. We run an OLS time-series regression for the market log price-dividend ratio on the log oil price, and compare the estimates with those predicted by our model. We use the time series of the log price-dividend ratio on the CRSP-VW Index described in Section 5.2.<sup>16</sup>

The results of this test are shown in Table 5 and suggest that our model correctly captures the empirical relationship between the aggregate price-dividend ratio and the oil price. Note that the market price-dividend ratio series were not included in the estimation of our model,

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<sup>16</sup> We calibrate our model for an average industry, which is defined by the average values of  $\gamma^i$ ,  $\sigma_X^i$ ,  $\rho_X^i$  and  $\mu_s^i$  in our sample. The parameter  $\mu_0^i$  is needed for calibrating the parameter  $a^i$ , but it is not identifiable from our empirical technique because it is not present in the stock returns. Therefore, we use that  $\log(h_1^i) = -a^i[\mu_0^i] - b^i(\bar{\delta} - \sigma_s^2/2\kappa_s)$  to pin down this parameter.

therefore, this is a purely out-of-sample result. Consistent with the theoretical prediction, the OLS estimates evidence that  $s_t$  has a negative and significant impact on  $h_t^i$ . Moreover, the magnitudes of the OLS estimate of  $b^i$  (-0.131) and the one predicted for the average industry (-0.122) are similar. The overall fit of the time-series regression, measured by the statistic  $R^2$ , is 2.3% which is reasonable for a study based on monthly frequency data.

## 7 Oil price and industry portfolio returns

In this section we study the effect of the oil price on both the expected returns and the realized returns of the industry portfolios. Our estimates suggest that the oil shocks are mostly idiosyncratic ( $\rho_s \sim 0$ ), therefore, we will assume that the oil price changes because of a change in  $dZ_{s,t}$ .

### 7.1 Conditional expected returns and oil prices

We first analyze the business cycle effects of oil on the conditional expected real stock return through the real interest rate and the market price of risk. The negative signs and significance of  $\alpha_s$  and  $\theta_s$  in Table 3 suggest that both business-cycle channels work in the same direction. An increase of one standard deviation in the log real oil price ( $\sigma_s = 0.289$ ) produces a decrease of 0.2% in the risk-free rate and a decrease of 0.129 in the market price of risk. Figures 1 and 2 show the effect of these variables on the expected stock returns.<sup>17</sup> It is clear from the pictures that both channels are responsible for the negative relationship between the conditional expected stock return and the oil price, however, the stock returns are more sensible to  $\theta_s$  than to  $\alpha_s$ . Indeed, the plots confirm that the most influential macroeconomic transmission channel of oil on the conditional expected stock return works through the market price of risk.

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<sup>17</sup> The default parameters used are those reported in Table 3 Panel A. For the latent variable we assume that  $y_t = \lim_{T \rightarrow \infty} E_t[y_T] = 0$ . For comparison purposes the figures show also the expected returns when  $\alpha_s$  and  $\theta_s$  are zero and when they have the same magnitude as the estimated ones but with the opposite sign.

Nevertheless, the overall effect of oil on the expected portfolio returns depend on  $\eta_{\Lambda}^i$  which is an industry-specific parameter (see equation (31)). Table 6 shows that the estimates of  $\eta_{\Lambda}^i$  are positive for the 13 industry portfolios, therefore, the relationship between the oil price and the conditional expected stock returns is unambiguously negative. This implies that industries that carry a higher systematic risk are those whose expected return are more affected by the oil price. The most sensitive industry to the oil price is the steel industry ( $\eta_{\Lambda}^i = 0.231$ ) and the less sensitive is the oil industry ( $\eta_{\Lambda}^i = 0.107$ ). Figure 3 shows the relationship between oil and the expected returns for these industries and also for the average industry. The parameter  $\eta_{\Lambda}^i$  is responsible of the different slopes in the plot. As explained before,  $\eta_{\Lambda}^i$  and, therefore, the elasticity of the expected return, are highly sensitive to the volatility and cyclicity of the industry's output.<sup>18</sup> The steel and oil industries confirm this prediction, because the former has the highest volatility ( $\sigma_X^i = 0.276$ ) and is cyclical ( $\rho_X^i = 0.787$ ), while the latter has the lowest volatility and cyclicity ( $\sigma_X^i = 0.145$  and  $\rho_X^i = 0.630$ ).

In addition, figures 4 and 5 show the positive effect of  $\sigma_X^i$  and  $\rho_X^i$  on the oil elasticity of the expected return.<sup>19</sup> Indeed, firms that face more volatile output are more risky and more vulnerable to oil price changes. Also, firms with higher cyclicity  $\rho_X^i$ , tend to be more negatively affected by oil price shock because this is a bad signal about the future economic growth. A classical example of this type of firms are those who produce durable consumption goods. Figures 6 and 7 show that the oil intensity,  $\gamma^i$ , and the growth opportunities parameter,  $\mu_s^i$ , have only a minor effect on the relation between the oil price and the expected stock return. Firms with higher  $\gamma^i$ 's demand more oil, hence, oil is more important in their cash flows and they are more affected by the oil shocks. Also, firms whose growth opportunities are more negatively affected by oil, have a more negative oil elasticity of expected returns, but this effect is economically insignificant. Yet, we would expect the growth opportunities parameter to have a significant effect for longer-horizon stock returns. Overall, Figures 4 to 7 suggest that

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<sup>18</sup> This result comes from the effect of the covariance term  $\sigma_X^i \rho_X^i$  on the expected stock returns.

<sup>19</sup> The industry-specific parameters are obtained as an average of the estimates in Panel B of Table 3, while the comparative values of these parameters are based on minimum and maximum values in this table. Again, for the latent variable we assume that  $y_t = \lim_{T \rightarrow \infty} E_t[y_T] = 0$ .

the output volatility,  $\sigma_X^i$ , is the most important industry-specific parameter in the relationship between the conditional expected stock return and the oil price.

## 7.2 The CAPM and oil prices

We can also understand the systematic effects of oil by studying the CAPM beta of each portfolio (see equations (34) and (35)). The beta of an industry is equal to  $\eta_\Lambda^i$  divided by the standard deviation of market portfolio return ( $\sigma_m$ ). Equation (35) shows that the industry portfolio's beta is a weighted average of the market betas of the state variables. Table 7 presents the beta decomposition for the different industries. The main result is that the betas of the industries are mostly explained by the output betas,  $\beta_X^i$  (see the last two columns of the table). This is consistent with the fact that the volatility and cyclicity of the output are by far the most important factors in determining the systematic risk of the portfolios.<sup>20</sup> Another interesting result is that for all non-oil industries the weights on the oil beta are also negative. These negative weights arise from the negative impact of oil on the price-dividend ratio (with weight  $b^i < 0$ ) and current dividend (with weight  $-\frac{\gamma^i}{1-\gamma^i} < 0$ ). The negative beta of oil ( $\beta_s = -0.087$ ) suggests that the aggregate effect of oil on the systematic risks of the industries are positive, however, it explains a very little fraction (less than 2%) of the industries betas. Finally, in line with our previous results, the steel and oil industries have the highest and lowest systematic risks, respectively. The large difference in the  $\beta_X^i$  (1.343 versus 0.563) explains the differences in the systematic risk of both industry portfolios (1.430 versus 0.663).

A fact that deserves special attention is the positive weight for  $\beta_s$  that is present in the market beta of the oil portfolio. This result is consistent with the fact that the oil industry is the only one that increases its value with an oil price shock.<sup>21</sup> The oil industry is the only industry whose growth opportunities are not significantly impacted by the oil price and,

<sup>20</sup> Indeed, by definition the industry-specific parameter  $\beta_X^i$  is increasing in  $\sigma_X^i$  and  $\rho_X^i$ , since  $\beta_X^i = \frac{\sigma_X^i \rho_X^i}{\sigma_m}$ .

<sup>21</sup> Probably, this is a consequence of the higher flexibility of this industry to transfer oil price increases to the future sales prices, along with a low price-demand elasticity.

therefore the effect of lower discount rates dominates the decrease in the future dividends. Interestingly, the negative beta of oil implies that the oil industry has a lower systematic risk, although this has only a minor impact on the total systematic risk of the portfolio. This result is important for CAPM-based portfolio management, because it suggests that to reduce the systematic risk of portfolio is more efficient to take long positions in crude oil futures than to invest in oil-related stocks. In summary, our estimates suggest that the partial effect of oil on the conditional expected returns of industry portfolios depends critically on the output betas ( $\beta_X^i$ ).

### 7.3 Idiosyncratic effects of oil price shocks and stock returns

The idiosyncratic risk of a stock is important to understand the dynamics of the stock returns. In our oil-based model the idiosyncratic oil shocks have also a direct impact on the realized industry portfolio returns and its weight depends on the parameter  $\eta_s^i$  (see equation (24)).

Table ?? shows the decomposition of  $\eta_s^i$  in two sources, one associated to the oil price and the other to the output variable, because an idiosyncratic oil shock affects the dynamics of these two variables. The first effect is negative for all non-oil industries because an idiosyncratic shock decreases the market value of those portfolios. This effect is positive for the oil industry. The second effect is always positive and is driven by the positive correlation between the oil and the output variable of each industry, although this last effect is not statistically significant because of depends on parameters  $\rho_{X_s}^i$  and  $\rho_s$  (see Table 3). The overall effect an idiosyncratic oil shock in the realized returns varies across industries and can be positive or negative depending on the strength of the two components discussed above.

### 7.4 Dynamic effects of oil price shocks on stock returns

Bringing all together, an idiosyncratic oil price shocks cause an immediate impact on the realized stock return. This effect can be positive or negative depending on the industry. This increase

in the oil price produces a negative effect on the expected returns of all industries through both the interest rate and the market price of risk. These effects are long-live because of the high persistence in real oil price. Gradually, the oil effect will disappear and macro variable will increase towards their long-run means.

Figure 8 shows the impact over time of an idiosyncratic oil shock on the conditional expected portfolio returns of the oil industry, the steel industry and the average industry. We simulate an increase of one standard deviation in log real oil price ( $\sigma_s = 0.289$ ) in period  $t = 0$ , assuming that both state variables are at their unconditional means before the shock ( $t = -1$ ), that is  $s_{-1} = 3.140$  and  $Y_{-1} = 0$ . The figure shows that initially the conditional expected return on all industries fall and, as expected from Section 7.2, the most impacted industry is steel industry. The conditional expected return decreases by 3.2%, 2.3% and 1.6% for the steel, average and oil industries, respectively.

Driesprong, Jacobsen, and Maat (2008) report a significant forecasting power of the lagged log return on oil price for stock returns using monthly data. They find a negative relationship between the conditional expected stock returns and the crude oil returns. In their industry portfolio analysis they comment the following: “effect is weaker in the sectors in which the effect of an oil price change is easiest to assess” and “Predictability tends to be strong, however, in the non-oil-related sectors”. Our model reproduces their results, however it provides an alternative explanation for the same empirical facts. In our model, the oil price effects on the oil industry are weaker than on the other industries, because of its low output market beta, while in the non-oil-related sectors the impact of oil price changes is larger since these industries have more volatile and cyclical sales incomes.

As can be seen from Figure 8, after an oil shock, the mean-reversion in log oil price comes into play and increase the real interest rate and the market price of risk over time. The conditional expected portfolio returns increase at longer horizons and the speed depends on persistence of the oil shocks. The mean-reverting process produces an U-shaped pattern in expected stock returns, with these last ones declining until the 13th month and rising later. This dynamics

is consistent with the empirical evidence reported by Casassus and Higuera (2011), who show that the partial effect of the fourth quarterly lag of oil futures returns has a highly significant impact on the conditional expected stock market return. Finally, it is worth noting that from equation (36) we see that the future expected stock returns are increasing in  $s_t$ , however the speed of mean-reversion parameter associated to this variable is very small, which is also evident from Figure 8.

## 8 Conclusions

In this paper we present a partial equilibrium model that allows us to disentangle the multiple effects of the oil price on the industry portfolios. Our model takes into account that oil shocks are transmitted through business-cycle and industry-specific channels. Macroeconomic channels have been widely documented previously and are associated to the risk-free interest rate and the market price of risk. The less studied industry-specific channels can be decomposed into those affecting the current (i.e., oil intensity and oil demand) and future cash-flows (i.e., growth opportunities).

We derive the asset pricing implications of the model and study the oil price elasticities of the current dividend, of the dividend growth rate and of the expected return of the portfolio. We find that on average, the non-oil industries decrease their value by 1.8% after an oil price increase of 10%. This effect is dominated by the negative effect of oil on the growth opportunities of the industries. Oil has also a minor negative effect on the current dividend of the firms, which is consistent with the low intensity of oil in the economy.

We also study the effect of oil on expected and realized stock returns. The macroeconomic effect of oil on the market price of risk is negative and highly significant, implying a negative linkage between oil and expected returns. We also find that firms whose expected returns are more sensitive to the oil price are those who have a higher systematic risk. However, we find



that most of the systematic risk of the firms is explained by the systematic risk of their output rather than by effect of oil. The effect of oil on the realized stock returns can have either sign, but depend on the effect of oil on the value of the portfolio and its correlation with the output of the industry.

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# Appendix

## A Proofs

This appendix contains the proofs of Propositions 1 to 4.

### A.1 Proof of Proposition 1

To obtain the result of Proposition 1, first equation (8) is substituted into the HJB equation (10). Then, we apply Itô's Lemma to the second term in the HJB equation and take conditional expectations:

$$\begin{aligned}
0 = \max_{\{q_t^i\}} & \left\{ X_t^i (q_t^i)^{\gamma^i} - S_t q_t^i - P_t^i r_t^f + P_{s,t}^i (\kappa_s (\delta_t - s_t) - \lambda_t \sigma_s \rho_s) + P_{\delta,t}^i (\kappa_\delta (\bar{\delta} - \delta_t) - \lambda_t \sigma_\delta \rho_\delta) \right. \\
& - P_{y,t}^i (\kappa_y y_t + \lambda_t \rho_y) + P_{X,t}^i X_t^i (\mu_0^i + \mu_s^i s_t + \mu_\delta^i \delta_t + \mu_y^i y_t - \lambda_t \sigma_X^i \rho_X^i) + \frac{1}{2} P_{ss,t}^i \sigma_s^2 \\
& + P_{s\delta,t}^i \sigma_s \sigma_\delta \left( \rho_s \rho_\delta + \rho_{s\delta} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_\delta^2} \right) + P_{sy,t}^i \sigma_s \left( \rho_s \rho_y + \rho_{sy} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_y^2} \right) \\
& + P_{sX,t}^i X_t^i \sigma_s \sigma_X^i \rho_s \rho_X^i + \frac{1}{2} P_{\delta\delta,t}^i \sigma_\delta^2 + P_{\delta y,t}^i \sigma_\delta \left( \rho_\delta \rho_y + \rho_{\delta y} \sqrt{1 - \rho_\delta^2} \sqrt{1 - \rho_y^2} \right) \\
& \left. + P_{\delta X,t}^i X_t^i \sigma_\delta \sigma_X^i \rho_\delta \rho_X^i + \frac{1}{2} P_{yy,t}^i + P_{yX,t}^i X_t^i \sigma_y \rho_y \rho_X^i + \frac{1}{2} P_{XX,t}^i X_t^i \sigma_X^i \sigma_X^i \right\} \quad (A1)
\end{aligned}$$

subject to

$$\lim_{T \rightarrow \infty} \mathbb{E}_t [\Lambda_T P^i(s_T, \delta_T, y_T, X_T^i)] = 0$$

The first result of Proposition 1 arises from the first order condition of this optimization problem, while the second one from substituting the optimal oil demand in equation (8).

Expressing result in equation (13) as:

$$D_t^{i*} = \exp \left( \frac{1}{1 - \gamma^i} \log(X_t^i) - \frac{\gamma^i}{1 - \gamma^i} s_t \right) (\gamma^i)^{\frac{\gamma^i}{1 - \gamma^i}} (1 - \gamma^i) \quad (A2)$$

Finally, dividend's growth rate results from applying Itô's lemma to equation (A2).

### A.2 Proof of Proposition 2

Substituting (20) in equation (19) produces:

$$\begin{aligned}
0 = & h_0^i - h_1^i \log(H_t^i) - r_t^f - \frac{\gamma^i}{1 - \gamma^i} (\kappa_s (\delta_t - s_t) - \lambda_t \sigma_s \rho_s) + \frac{1}{1 - \gamma^i} (\mu_0^i + \mu_s^i s_t + \mu_\delta^i \delta_t + \mu_y^i y_t - \lambda_t \sigma_X^i \rho_X^i) \\
& + \frac{1}{2} \frac{\gamma^{i2}}{(1 - \gamma^i)^2} \sigma_s^2 - \frac{\gamma^i}{(1 - \gamma^i)^2} \sigma_s \sigma_X^i \rho_s \rho_X^i + \frac{1}{2} \frac{\gamma^i}{(1 - \gamma^i)^2} \sigma_X^i \sigma_X^i + \frac{H_{s,t}^i}{H_t^i} \left( \kappa_s (\delta_t - s_t) - \lambda_t \sigma_s \rho_s - \frac{\gamma^i}{1 - \gamma^i} \sigma_s^2 \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{1-\gamma^i} \sigma_s \sigma_X^i \rho_s \rho_X^i + \frac{H_{\delta,t}^i}{H_t^i} \left( \kappa_\delta (\bar{\delta} - \delta_t) - \lambda_t \sigma_\delta \rho_\delta - \frac{\gamma^i}{1-\gamma^i} \sigma_s \sigma_\delta \left( \rho_s \rho_\delta + \rho_{s\delta} \sqrt{1-\rho_s^2} \sqrt{1-\rho_\delta^2} \right) \right) \\
& + \frac{1}{1-\gamma^i} \sigma_\delta \sigma_X^i \rho_\delta \rho_X^i - \frac{H_{y,t}^i}{H_t^i} \left( \kappa_y y_t + \lambda_t \rho_y + \frac{\gamma^i}{1-\gamma^i} \sigma_s \left( \rho_s \rho_y + \rho_{sy} \sqrt{1-\rho_s^2} \sqrt{1-\rho_y^2} \right) \right) \\
& - \frac{1}{1-\gamma^i} \sigma_X^i \rho_y \rho_X^i + \frac{1}{2} \frac{H_{ss,t}^i}{H_t^i} \sigma_s^2 + \frac{H_{s\delta,t}^i}{H_t^i} \sigma_s \sigma_\delta \left( \rho_s \rho_\delta + \rho_{s\delta} \sqrt{1-\rho_s^2} \sqrt{1-\rho_\delta^2} \right) \\
& + \frac{H_{sy,t}^i}{H_t^i} \sigma_s \left( \rho_s \rho_y + \rho_{sy} \sqrt{1-\rho_s^2} \sqrt{1-\rho_y^2} \right) + \frac{1}{2} \frac{H_{\delta\delta,t}^i}{H_t^i} \sigma_\delta^2 + \frac{H_{\delta y,t}^i}{H_t^i} \sigma_\delta \left( \rho_\delta \rho_y + \rho_{\delta y} \sqrt{1-\rho_\delta^2} \sqrt{1-\rho_y^2} \right) \\
& + \frac{1}{2} \frac{H_{yy,t}^i}{H_t^i}
\end{aligned} \tag{A3}$$

We guess the following exponential affine solution for the price-dividend ratio:

$$H^i(s_t, \delta_t, y_t) = \exp(a^i + b^i s_t + c^i \delta_t + d^i y_t) \tag{A4}$$

and replace it in equation (A3). After substituting  $r_t^f$  and  $\lambda_t$  from equations (3) and (4), and collecting terms that are linear on  $s_t$  and  $y_t$ , we obtain the following system of linear equations for the unknown constants  $a^i$ ,  $b^i$  and  $c^i$ :

$$\begin{aligned}
0 = & h_0^i - a^i h_1^i - \alpha_0 + c^i \kappa_\delta \bar{\delta} + \frac{1}{1-\gamma^i} \mu_0^i + \frac{1}{2} \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right)^2 \sigma_s^2 + \frac{1}{2} \frac{\gamma^i}{(1-\gamma^i)^2} \sigma_X^i{}^2 + \frac{1}{1-\gamma^i} d^i \sigma_X^i \rho_y \rho_X^i \\
& + \frac{1}{1-\gamma^i} \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \sigma_X^i \rho_s \rho_X^i + \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \left( \rho_s \rho_\delta + \rho_{s\delta} \sqrt{1-\rho_s^2} \sqrt{1-\rho_\delta^2} \right) c^i \sigma_s \sigma_\delta + \frac{1}{2} d^i{}^2 \\
& + \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \left( \rho_s \rho_y + \rho_{sy} \sqrt{1-\rho_s^2} \sqrt{1-\rho_y^2} \right) d^i \sigma_s + \left( \rho_\delta \rho_y + \rho_{\delta y} \sqrt{1-\rho_\delta^2} \sqrt{1-\rho_y^2} \right) c^i d^i \sigma_\delta \\
& + \frac{1}{2} c^i{}^2 \sigma_\delta^2 + \frac{1}{1-\gamma^i} c^i \sigma_\delta \sigma_X^i \rho_\delta \rho_X^i - \theta_0 \left( \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1-\gamma^i} \sigma_X^i \rho_X^i \right) \tag{A5}
\end{aligned}$$

$$\begin{aligned}
0 = & -b^i h_1^i - \alpha_s + \frac{1}{1-\gamma^i} \mu_s^i - \theta_s \left( \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1-\gamma^i} \sigma_X^i \rho_X^i \right) \\
& - \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \kappa_s \tag{A6}
\end{aligned}$$

$$\begin{aligned}
0 = & -c^i h_1^i - \alpha_\delta + \frac{1}{1-\gamma^i} \mu_\delta^i - c^i \kappa_\delta - \theta_\delta \left( \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1-\gamma^i} \sigma_X^i \rho_X^i \right) \\
& + \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \kappa_s \tag{A7}
\end{aligned}$$

$$0 = -d^i h_1^i - \alpha_y + \frac{1}{1-\gamma^i} \mu_y^i - d^i \kappa_y - \theta_y \left( \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1-\gamma^i} \sigma_X^i \rho_X^i \right) \tag{A8}$$

From equations (A6), (A7) and (A8) we obtain  $b^i$ ,  $c^i$  and  $d^i$ :

$$\begin{aligned}
b^i = & \left( \theta_s ((h_1^i + \kappa_\delta) ((1-\gamma^i) \alpha_y \rho_y - \mu_y^i \rho_y - (h_1^i + \kappa_y) (\sigma_X^i \rho_X^i - \gamma^i \sigma_s \rho_s)) + (h_1^i + \kappa_y) (\gamma^i \kappa_s - \mu_\delta^i) \right. \\
& \left. + (1-\gamma^i) \alpha_\delta) \sigma_\delta \rho_\delta + (\mu_s^i + \gamma^i \kappa_s - (1-\gamma^i) \alpha_s) ((h_1^i + \kappa_\delta) (h_1^i + \kappa_y + \rho_y \theta_y) + (h_1^i + \kappa_y) \sigma_\delta \rho_\delta \theta_\delta) \right) / \\
& \left( (1-\gamma^i) ((h_1^i + \kappa_\delta) ((h_1^i + \kappa_s) (h_1^i + \kappa_y + \rho_y \theta_y) + (h_1^i + \kappa_y) \sigma_s \rho_s \theta_s) + (h_1^i \theta_\delta + (\theta_s + \theta_\delta) \kappa_s) (h_1^i \right.
\end{aligned}$$

$$+\kappa_y)\sigma_\delta\rho_\delta)) \quad (\text{A9})$$

$$\begin{aligned} c^i &= \left( \theta_\delta((h_1^i + \kappa_s)((1 - \gamma^i)\alpha_y\rho_y - \mu_y^i\rho_y - (h_1^i + \kappa_y)\sigma_X^i\rho_X^i) + (h_1^i + \kappa_y)(\gamma^i h_1^i + (1 - \gamma^i)\alpha_s - \mu_s^i)\sigma_s\rho_s) \right. \\ &\quad + (\mu_\delta^i - (1 - \gamma^i)\alpha_\delta)((h_1^i + \kappa_s)(h_1^i + \kappa_y + \rho_y\theta_y) + (h_1^i + \kappa_y)\sigma_s\rho_s\theta_s) - \kappa_s((\gamma^i h_1^i + (1 - \gamma^i)\alpha_s \\ &\quad - \mu_s^i)(h_1^i + \kappa_y + \rho_y\theta_y) + \theta_s((h_1^i + \kappa_y)\sigma_X^i\rho_X^i + (\mu_y^i - (1 - \gamma^i)\alpha_y)\rho_y)) \left. \right) / \left( (1 - \gamma^i)((h_1^i + \kappa_\delta)((h_1^i \right. \\ &\quad + \kappa_s)(h_1^i + \kappa_y + \rho_y\theta_y) + (h_1^i + \kappa_y)\sigma_s\rho_s\theta_s) + (h_1^i\theta_\delta + (\theta_s + \theta_\delta)\kappa_s)(h_1^i + \kappa_y)\sigma_\delta\rho_\delta) \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} d^i &= \left( \theta_y((h_1^i + \kappa_s)((1 - \gamma^i)\alpha_\delta - \mu_\delta^i)\sigma_\delta\rho_\delta - (h_1^i + \kappa_\delta)\sigma_X^i\rho_X^i) + ((h_1^i + \kappa_\delta)\sigma_s\rho_s + \kappa_s\sigma_\delta\rho_\delta)(\gamma^i h_1^i - \mu_s^i \right. \\ &\quad + (1 - \gamma^i)\alpha_s) + (\mu_y^i - (1 - \gamma^i)\alpha_y)((h_1^i + \kappa_\delta)(h_1^i + \kappa_s + \sigma_s\rho_s\theta_s) + ((h_1^i + \kappa_s)\theta_\delta + \kappa_s\theta_s)\sigma_\delta\rho_\delta) \left. \right) / \\ &\quad \left( (1 - \gamma^i)((h_1^i + \kappa_\delta)((h_1^i + \kappa_s)(h_1^i + \kappa_y + \rho_y\theta_y) + (h_1^i + \kappa_y)\sigma_s\rho_s\theta_s) + (h_1^i\theta_\delta + (\theta_s + \theta_\delta)\kappa_s)(h_1^i \right. \\ &\quad + \kappa_y)\sigma_\delta\rho_\delta) \end{aligned} \quad (\text{A11})$$

while  $a^i$  is obtained from equation (A5) after replacing the solution in equations (A9), (A10) and (A11). This completes the proof of Proposition 2.

### A.3 Proof of Proposition 3

We obtain the price of the stock from equations (18), (13) and (21):

$$\begin{aligned} P_t^i &= H^i(s_t, \delta_t, y_t)D^i(X_t^i, e^{s_t}) \\ &= \exp(a^i + b^i s_t + c^i \delta_t + d^i y_t) X_t^{i \frac{1}{1-\gamma^i}} \exp\left(-\frac{\gamma^i}{1-\gamma^i} s_t\right) (\gamma^i)^{\frac{\gamma^i}{1-\gamma^i}} (1-\gamma^i) \\ &= \exp\left(a^i + \left(b^i - \frac{\gamma^i}{1-\gamma^i}\right) s_t + c^i \delta_t + d^i y_t + \frac{1}{1-\gamma^i} \log(X_t^i)\right) (\gamma^i)^{\frac{\gamma^i}{1-\gamma^i}} (1-\gamma^i) \end{aligned} \quad (\text{A12})$$

Applying Itô's Lemma and dividing by  $P_t^i$  yields:

$$\begin{aligned} \frac{dP_t^i}{P_t^i} &= \left( \frac{1}{1-\gamma^i} \mu_0^i + \frac{1}{2} \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right)^2 \sigma_s^2 + \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) (\rho_s \rho_y + \rho_{sy} \sqrt{1-\rho_s^2} \sqrt{1-\rho_y^2}) d^i \sigma_s \right. \\ &\quad + \frac{1}{1-\gamma^i} \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \sigma_X^i \rho_s \rho_X^i + \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) (\rho_s \rho_\delta + \rho_{s\delta} \sqrt{1-\rho_s^2} \sqrt{1-\rho_\delta^2}) c^i \sigma_s \sigma_\delta \\ &\quad + (\rho_\delta \rho_y + \rho_{\delta y} \sqrt{1-\rho_\delta^2} \sqrt{1-\rho_y^2}) c^i d^i \sigma_\delta + \frac{1}{1-\gamma^i} d^i \sigma_X^i \rho_y \rho_X^i + \frac{1}{1-\gamma^i} c^i \sigma_\delta \sigma_X^i \rho_\delta \rho_X^i \\ &\quad + \frac{1}{2} \frac{\gamma^i}{(1-\gamma^i)^2} \sigma_X^i{}^2 + c^i \kappa_\delta \bar{\delta} + \frac{1}{2} c^i{}^2 \sigma_\delta^2 + \frac{1}{2} d^i{}^2 + \left( \frac{1}{1-\gamma^i} \mu_s^i - \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \kappa_s \right) s_t \\ &\quad + \left( \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \kappa_s - c^i \kappa_\delta + \frac{1}{1-\gamma^i} \mu_\delta^i \right) \delta_t + \left( \frac{1}{1-\gamma^i} \mu_y^i - d^i \kappa_y \right) y_t \right) dt \\ &\quad + \left( \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1-\gamma^i} \sigma_X^i \rho_X^i \right) dZ_{\Lambda,t} + c^i \sigma_\delta \sqrt{1-\rho_\delta^2} dZ_{\delta,t} \\ &\quad + \left( b^i - \frac{\gamma^i}{1-\gamma^i} \right) \sigma_s \sqrt{1-\rho_s^2} dZ_{s,t} + d^i \sqrt{1-\rho_y^2} dZ_{y,t} + \frac{1}{1-\gamma^i} \sigma_X^i \sqrt{1-\rho_X^i{}^2} dZ_{X,t} \end{aligned} \quad (\text{A13})$$

Also, from equation (20) the approximated dividend-price ratio is given by:

$$\begin{aligned}
\frac{D_t^{i*}}{P_t^i} &= h_0^i - h_1^i \log(H_t^i) = h_0^i - h_1^i (a^i + b^i s_t + c^i \delta_t + d^i y_t) \\
&= \alpha_0 - \frac{1}{2} \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right)^2 \sigma_s^2 - \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \left( \rho_s \rho_y + \rho_{sy} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_y^2} \right) d^i \sigma_s \\
&\quad - \frac{1}{2} \frac{\gamma^i}{(1 - \gamma^i)^2} \sigma_X^2 - c^i \kappa_\delta \bar{\delta} + \theta_0 \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1 - \gamma^i} \sigma_X^i \rho_X^i \right) \\
&\quad - \frac{1}{1 - \gamma^i} c^i \sigma_\delta \sigma_X^i \rho_\delta \rho_X^i - \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \left( \rho_s \rho_\delta + \rho_{s\delta} \sqrt{1 - \rho_s^2} \sqrt{1 - \rho_\delta^2} \right) c^i \sigma_s \sigma_\delta \\
&\quad - \frac{1}{1 - \gamma^i} \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \sigma_X^i \rho_s \rho_X^i - \left( \rho_\delta \rho_y + \rho_{\delta y} \sqrt{1 - \rho_\delta^2} \sqrt{1 - \rho_y^2} \right) c^i d^i \sigma_\delta - \frac{1}{2} d^{i2} \\
&\quad - \frac{1}{2} c^{i2} \sigma_\delta^2 - \frac{1}{1 - \gamma^i} d^i \sigma_X^i \rho_y \rho_X^i - \frac{1}{1 - \gamma^i} \mu_0^i - b^i h_1^i s_t - c^i h_1^i \delta_t - d^i h_1^i y_t
\end{aligned} \tag{A14}$$

where the term  $a^i h_1^i$  is substituted from equation (A5).

Replacing (A13) and (A14) in equation (23) yields:

$$\begin{aligned}
\frac{dG_t^i}{G_t^i} &= \left( \alpha_0 + \theta_0 \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1 - \gamma^i} \sigma_X^i \rho_X^i \right) + \left( \frac{1}{1 - \gamma^i} \mu_s^i \right. \right. \\
&\quad \left. \left. - \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \kappa_s - b^i h_1^i \right) s_t + \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \kappa_s - c^i \kappa_\delta + \frac{1}{1 - \gamma^i} \mu_\delta^i - c^i h_1^i \right) \delta_t \right. \\
&\quad \left. + \left( \frac{1}{1 - \gamma^i} \mu_y^i - d^i \kappa_y - d^i h_1^i \right) y_t \right) dt + \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \sqrt{1 - \rho_s^2} dZ_{s,t} \\
&\quad + \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{1}{1 - \gamma^i} \sigma_X^i \rho_X^i \right) dZ_{\Lambda,t} + d^i \sqrt{1 - \rho_y^2} dZ_{y,t} \\
&\quad + c^i \sigma_\delta \sqrt{1 - \rho_\delta^2} dZ_{\delta,t} + \frac{1}{1 - \gamma^i} \sigma_X^i \sqrt{1 - \rho_X^2} dZ_{X,t}
\end{aligned} \tag{A15}$$

Then, using equations (A9), (A10) and (A11) it is easy to see that:

$$\frac{\mu_s^i}{1 - \gamma^i} - \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \kappa_s - b^i h_1^i = \alpha_s + \theta_s \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{\sigma_X^i \rho_X^i}{1 - \gamma^i} \right) \tag{A16}$$

$$\left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \kappa_s - c^i \kappa_\delta + \frac{\mu_\delta^i}{1 - \gamma^i} - c^i h_1^i = \alpha_\delta + \theta_\delta \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{\sigma_X^i \rho_X^i}{1 - \gamma^i} \right) \tag{A17}$$

$$\frac{\mu_y^i}{1 - \gamma^i} - d^i \kappa_y - d^i h_1^i = \alpha_y + \theta_y \left( \left( b^i - \frac{\gamma^i}{1 - \gamma^i} \right) \sigma_s \rho_s + c^i \sigma_\delta \rho_\delta + d^i \rho_y + \frac{\sigma_X^i \rho_X^i}{1 - \gamma^i} \right) \tag{A18}$$

Finally, replacing (A16), (A17) and (A18) in equation (A15) and using the definitions of  $r_t^f$  and  $\lambda_t$  in equations (3) and (4), gives the result in Proposition 3.



## A.4 Proof of Proposition 4

Taking the conditional expectation to equation (24) and annualizing the return gives:

$$\begin{aligned}
\bar{r}_t^i &= r_t^f + \eta_\Lambda^i \lambda_t \\
&= \alpha_0 + \alpha_s s_t + \alpha_\delta \delta_t + \alpha_y y_t + \eta_\Lambda^i (\theta_0 + \theta_s s_t + \theta_\delta \delta_t + \theta_y y_t) \\
&= (\alpha_0 + \eta_\Lambda^i \theta_0) + [\alpha_s + \eta_\Lambda^i \theta_s \quad \alpha_\delta + \eta_\Lambda^i \theta_\delta \quad \alpha_y + \eta_\Lambda^i \theta_y] \begin{pmatrix} s_t \\ \delta_t \\ y_t \end{pmatrix}
\end{aligned} \tag{A19}$$

$$\bar{r}_t^i = (\alpha_0 + \eta_\Lambda^i \theta_0) + [\alpha_s + \eta_\Lambda^i \theta_s \quad \alpha_\delta + \eta_\Lambda^i \theta_\delta \quad \alpha_y + \eta_\Lambda^i \theta_y] V_t \tag{A20}$$

Let's define the following trivariate vector:

$$\begin{aligned}
J_t^i &= \begin{pmatrix} \bar{r}_t^i - (\alpha_0 + \eta_\Lambda^i \theta_0) \\ s_t \\ \delta_t \end{pmatrix} \\
&= \begin{pmatrix} \alpha_s + \eta_\Lambda^i \theta_s & \alpha_\delta + \eta_\Lambda^i \theta_\delta & \alpha_y + \eta_\Lambda^i \theta_y \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} V_t \\
J_t^i &= C^i V_t
\end{aligned} \tag{A21}$$

Applying Itô's Lemma:

$$\begin{aligned}
dJ_t^i &= C^i dV_t \\
&= C^i ((\mu + \xi V_t) dt + \Sigma_J dZ_{J,t})
\end{aligned} \tag{A22}$$

So, substituting by  $V_t = (C^i)^{-1} J_t^i$  and reordering terms gives the result in Proposition 4.

## B Futures valuation

In matricial terms, the dynamics of the state variables is:

$$dV_t = \begin{pmatrix} ds_t \\ d\delta_t \\ dy_t \end{pmatrix} = \kappa_V (\bar{V} - V_t) dt + \Sigma_V dZ_t \tag{B1}$$

where,

$$\kappa_V = \begin{pmatrix} \kappa_s & -\kappa_s & 0 \\ 0 & \kappa_\delta & 0 \\ 0 & 0 & \kappa_y \end{pmatrix} \tag{B2}$$

$$\bar{V} = \begin{pmatrix} \bar{\delta} \\ \bar{\delta} \\ 0 \end{pmatrix} \tag{B3}$$

$$\Sigma_V = \begin{pmatrix} \sigma_s \rho_s & \sigma_s \sqrt{1 - \rho_s^2} & 0 & 0 & 0_{P'} \\ \sigma_\delta \rho_\delta & 0 & \sigma_\delta \sqrt{1 - \rho_\delta^2} & 0 & 0_{P'} \\ \rho_y & 0 & 0 & \sqrt{1 - \rho_y^2} & 0_{P'} \end{pmatrix} \quad (\text{B4})$$

$$dZ_t = \begin{pmatrix} dZ_{\Lambda,t} \\ dZ_{s,t} \\ dZ_{\delta,t} \\ dZ_{y,t} \\ dZ_{X,t} \end{pmatrix} \quad (\text{B5})$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0_{P'} \\ 0 & 1 & \rho_{s\delta} & \rho_{sy} & 0_{P'} \\ 0 & \rho_{s\delta} & 1 & \rho_{\delta y} & 0_{P'} \\ 0 & \rho_{sy} & \rho_{\delta y} & 1 & 0_{P'} \\ 0_P & 0_P & 0_P & 0_P & I_P \end{pmatrix} \quad (\text{B6})$$

$$dZ_t dZ_t' = M dt \quad (\text{B7})$$

and  $P$  is the number of industry portfolios.

Under the historical measure, the dynamics of the futures price with maturity  $\tau = T - t$ , denoted by  $F_t^\tau = F(\tau, V_t)$ , satisfies:

$$E_t \left( \frac{d(\Lambda_t F_t^\tau)}{\Lambda_t F_t^\tau} + r_t^f dt \right) = 0 \quad (\text{B8})$$

Applying Ito's lemma and taking conditional expectation, the following PDE is obtained:

$$\frac{1}{F_t^\tau} \left( -\frac{\partial F_t^\tau}{\partial \tau} + (\bar{V} - V_t)' \kappa_V' \frac{\partial F_t^\tau}{\partial V_t} + \frac{1}{2} \text{tr} \left( \frac{\partial^2 F_t^\tau}{\partial V_t \partial V_t'} \Sigma_V M \Sigma_V' \right) - \Sigma_{V1}' \frac{\partial F_t^\tau}{\partial V_t} (\theta_0 + \theta_V' V_t) \right) = 0 \quad (\text{B9})$$

where,  $\Sigma_{V1}$  is the first column of the matrix  $\Sigma_V$  and  $\theta_V = [\theta_s \ \theta_\delta \ \theta_y]'$ .

Using the following guess for the futures price

$$F_t^\tau = \exp(A_0^\tau + A_V^\tau' V_t) \quad (\text{B10})$$

it results in the next differential equation system:

$$\begin{aligned} \frac{dA_0^\tau}{d\tau} &= \bar{V}' \kappa_V' A_V^\tau + \frac{1}{2} A_V^\tau' \Sigma_V M \Sigma_V' A_V^\tau - \Sigma_{V1}' A_V^\tau \theta_0 \\ \frac{dA_V^\tau}{d\tau} &= -\kappa_V' A_V^\tau - \theta_V A_V^\tau' \Sigma_{V1} \end{aligned} \quad (\text{B11})$$

For the estimation of the model, the following discretized version of this system is used:

$$\begin{aligned} A_0^{\tau+\Delta\tau} &= A_0^\tau + \left( \bar{V}' \kappa_V' A_V^\tau + \frac{1}{2} A_V^\tau' \Sigma_V M \Sigma_V' A_V^\tau - \Sigma_{V1}' A_V^\tau \theta_0 \right) \Delta\tau \\ A_V^{\tau+\Delta\tau} &= A_V^\tau - (\kappa_V' A_V^\tau + \theta_V A_V^\tau' \Sigma_{V1}) \Delta\tau \end{aligned} \quad (\text{B12})$$

with the following initial conditions:

$$\begin{pmatrix} A_0^0 \\ A_s^0 \\ A_\delta^0 \\ A_y^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (\text{B13})$$

## C MLE estimation

This appendix contains all the details about the estimation of the model.

### C.1 State variables

Discretizing the equations (32), (5), (6) and (7), the following discrete time dynamics of the state variables is obtained:

$$X_{t+\Delta t} = \begin{pmatrix} r_{t+\Delta t}^m \\ s_{t+\Delta t} \\ \delta_{t+\Delta t} \\ y_{t+\Delta t} \end{pmatrix} = \mu_X + \Phi_X X_t + \Sigma_X \Delta Z_{t+\Delta t}^X \quad (\text{C1})$$

where,

$$\mu_X = \begin{pmatrix} (\alpha_0 + \sigma_m \theta_0) \Delta t \\ 0 \\ \kappa_\delta \bar{\delta} \Delta t \\ 0 \end{pmatrix} \quad (\text{C2})$$

$$\Phi_X = \begin{pmatrix} 0 & (\alpha_s + \sigma_m \theta_s) \Delta t & (\alpha_\delta + \sigma_m \theta_\delta) \Delta t & (\alpha_y + \sigma_m \theta_y) \Delta t \\ 0 & 1 - \kappa_s \Delta t & \kappa_s \Delta t & 0 \\ 0 & 0 & 1 - \kappa_\delta \Delta t & 0 \\ 0 & 0 & 0 & 1 - \kappa_y \Delta t \end{pmatrix} \quad (\text{C3})$$

$$\Sigma_X = \begin{pmatrix} \sigma_m & 0 & 0 & 0 \\ \sigma_s \rho_s & \sigma_s \sqrt{1 - \rho_s^2} & 0 & 0 \\ \sigma_\delta \rho_\delta & 0 & \sigma_\delta \sqrt{1 - \rho_\delta^2} & 0 \\ \rho_y & 0 & 0 & \sqrt{1 - \rho_y^2} \end{pmatrix} \quad (\text{C4})$$

$$\Delta Z_{t+\Delta t}^X = \begin{pmatrix} \Delta Z_{\Lambda, t+\Delta t} \\ \Delta Z_{s, t+\Delta t} \\ \Delta Z_{\delta, t+\Delta t} \\ \Delta Z_{y, t+\Delta t} \end{pmatrix} \quad (\text{C5})$$

It is important to note that  $r_t^m$  is included because systematic risk is unspanned by both oil market and bond market, and we are assuming complete markets at the level of the whole economy.

To back out the latent variables in the model, the following equation is used:

$$Y_t^X = \begin{pmatrix} r_t^m \\ f_t^1 \\ f_t^{15} \\ r_t^f \end{pmatrix} = A_X + B_X X_t \quad (\text{C6})$$

where,

$$A_X = \begin{pmatrix} 0 \\ A_0^1 \\ A_0^{15} \\ \alpha_0 \end{pmatrix} \quad (\text{C7})$$

$$B_X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A_s^1 & A_\delta^1 & A_y^1 \\ 0 & A_s^{15} & A_\delta^{15} & A_y^{15} \\ 0 & \alpha_s & \alpha_\delta & \alpha_y \end{pmatrix} \quad (\text{C8})$$

It is important to note that stock returns (included  $r_t^m$ ) are returns from  $t - \Delta t$  to  $t$ , while  $r_t^f$  is the risk-free return between  $t$  and  $t + \Delta t$ .

Substituting equation (C1) in equation (C6):

$$\begin{aligned} Y_{t+\Delta t}^X &= A_X + B_X (\mu_X + \Phi_X X_t + \Sigma_X \Delta Z_{t+\Delta t}^X) \\ &= (A_X + B_X \mu_X) + B_X \Phi_X X_t + B_X \Sigma_X \Delta Z_{t+\Delta t}^X \\ &= (A_X + B_X \mu_X) + B_X \Phi_X B_X^{-1} (Y_t^X - A_X) + B_X \Sigma_X \Delta Z_{t+\Delta t}^X \\ &= ((I_4 - B_X \Phi_X B_X^{-1}) A_X + B_X \mu_X) + B_X \Phi_X B_X^{-1} Y_t^X + \Sigma_Y^X \Delta Z_{t+\Delta t}^X \end{aligned} \quad (\text{C9})$$

where  $\Sigma_Y^X = B_X \Sigma_X$ .

## C.2 Oil futures prices observed with error

It is assumed that the 3-month, 6-month, 12-month and 18-month crude oil futures prices are observed with error, that is:

$$Y_t^\varepsilon = \begin{pmatrix} f_t^3 \\ f_t^6 \\ f_t^{12} \\ f_t^{18} \end{pmatrix} = A_\varepsilon + B_\varepsilon X_t + \varepsilon_t \quad (\text{C10})$$

where,  $X_t = B_X^{-1} (Y_t^X - A_X)$  and

$$A_\varepsilon = \begin{pmatrix} A_0^3 \\ A_0^6 \\ A_0^{12} \\ A_0^{18} \end{pmatrix} \quad (\text{C11})$$

$$B_\varepsilon = \begin{pmatrix} 0 & A_s^3 & A_\delta^3 & A_y^3 \\ 0 & A_s^6 & A_\delta^6 & A_y^6 \\ 0 & A_s^{12} & A_\delta^{12} & A_y^{12} \\ 0 & A_s^{18} & A_\delta^{18} & A_y^{18} \end{pmatrix} \quad (\text{C12})$$

Following to Casassus and Collin-Dufresne (2005), the measurement errors are supposed to follow an AR(1) process given by:

$$\varepsilon_{t+\Delta t} = \rho_\varepsilon \varepsilon_t + \nu_{t+\Delta t} \quad (\text{C13})$$

where  $\nu_t \sim MVN(0_R, \Omega)$ ,  $\Omega = E(\nu_t \nu_t')$ ,  $R = 4$  and  $\rho_\varepsilon$  is the autoregressive parameter, which is assumed to be the same for all maturities.

### C.3 Industry portfolio returns

The equation of the returns on  $P$  industry portfolios is:

$$\begin{aligned} Y_{t+\Delta t}^P = r_{t+\Delta t}^P &= A_P + B_P X_t + \Sigma_Y^P \Delta Z_{t+\Delta t} \\ &= A_P + B_P B_X^{-1} (Y_t^X - A_X) + \Sigma_Y^P \Delta Z_{t+\Delta t} \\ &= (A_P - B_P B_X^{-1} A_X) + B_P B_X^{-1} Y_t^X + \Sigma_Y^P \Delta Z_{t+\Delta t} \end{aligned} \quad (\text{C14})$$

where,  $Y_{t+\Delta t}^P$  is a column vector of size  $P$  ( $P = 13$ ) and

$$A_P = ( (\alpha_0 + \eta_\Lambda \theta_0) \Delta t ) \quad (\text{C15})$$

$$B_P = ( 0_P \quad (\alpha_s + \eta_\Lambda \theta_s) \Delta t \quad (\alpha_\delta + \eta_\Lambda \theta_\delta) \Delta t \quad (\alpha_y + \eta_\Lambda \theta_y) \Delta t ) \quad (\text{C16})$$

$$\Sigma_Y^P = ( \eta_\Lambda \quad \eta_s \quad \eta_\delta \quad \eta_y \quad \text{diag}(\eta_X) ) \quad (\text{C17})$$

$$\Delta Z_{t+\Delta t} = \begin{pmatrix} \Delta Z_{\Lambda, t+\Delta t} \\ \Delta Z_{s, t+\Delta t} \\ \Delta Z_{\delta, t+\Delta t} \\ \Delta Z_{y, t+\Delta t} \\ \Delta Z_{X, t+\Delta t} \end{pmatrix} \quad (\text{C18})$$

### C.4 Industry portfolio price-dividend ratios observed with error

It is assumed that the log price-dividend ratios of the industry portfolios are observed with error. Accordingly with equation (21), we have:

$$Y_t^e = \log(H_t) = A_e + B_e X_t + e_t \quad (\text{C19})$$

where,

$$A_e = \begin{pmatrix} a^1 \\ \vdots \\ a^P \end{pmatrix} \quad (\text{C20})$$

$$B_e = \begin{pmatrix} 0 & b^1 & c^1 & d^1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & b^P & c^P & d^P \end{pmatrix} \quad (\text{C21})$$

It is assumed that the measurement errors follow the AR(1) process given by:

$$e_{t+\Delta t} = \rho_e e_t + \omega_{t+\Delta t} \quad (\text{C22})$$

where  $\omega_t \sim MVN(0_P, \Pi)$ ,  $\Pi = E(\omega_t \omega_t')$  and  $\rho_e$  is the autoregressive parameter, which is assumed to be the same for all industry portfolios.

## C.5 Log-likelihood function

Putting together equations (C9) and (C14) gives the equation of all observable variables:

$$Y_{t+\Delta t} = \begin{pmatrix} Y_{t+\Delta t}^X \\ Y_{t+\Delta t}^P \end{pmatrix} = A + BY_t + \Psi \Delta Z_{t+\Delta t} \quad (\text{C23})$$

where,

$$A = \begin{pmatrix} (I_4 - B_X \Phi_X B_X^{-1}) A_X + B_X \mu_X \\ A_P - B_P B_X^{-1} A_X \end{pmatrix} \quad (\text{C24})$$

$$B = \begin{pmatrix} B_X \Phi_X B_X^{-1} & 0_{4P} \\ B_P B_X^{-1} & 0_{PP} \end{pmatrix} \quad (\text{C25})$$

$$\Psi = \begin{pmatrix} \Sigma_Y^X & 0_{4P} \\ \Sigma_Y^P \end{pmatrix} \quad (\text{C26})$$

Then,  $Y_{t+\Delta t}$  follows the conditional distribution shown below:

$$Y_{t+\Delta t} | Y_t \sim N(A + BY_t, \Psi M \Psi' \Delta t) \quad (\text{C27})$$

So, the log-likelihood function of the all observable variables is

$$\begin{aligned} L_Y &= \sum_{t=t_0}^{T-\Delta t} \log(f_Y(Y_{t+\Delta t} | Y_t)) \\ &= \sum_{t=t_0}^{T-\Delta t} -\frac{P+4}{2} \log(2\pi) - \frac{P+4}{2} \log(\Delta t) - \frac{1}{2} \log(\det(\Psi M \Psi')) \\ &\quad - \frac{1}{2\Delta t} (Y_{t+\Delta t} - A - BY_t)' (\Psi M \Psi')^{-1} (Y_{t+\Delta t} - A - BY_t) \end{aligned} \quad (\text{C28})$$

On the other hand, the log-likelihood function of the measurement errors of the log oil futures prices is

$$\begin{aligned} L_\varepsilon &= \sum_{t=t_0}^{T-\Delta t} \log(f_\varepsilon(\varepsilon_{t+\Delta t} | \varepsilon_t)) \\ &= \sum_{t=t_0}^{T-\Delta t} -\frac{4}{2} \log(2\pi) - \frac{1}{2} \log(\det(\Omega)) - \frac{1}{2} (\varepsilon_{t+\Delta t} - \rho_e \varepsilon_t)' \Omega^{-1} (\varepsilon_{t+\Delta t} - \rho_e \varepsilon_t) \end{aligned} \quad (\text{C29})$$

In the same way, the log-likelihood function of the measurement errors of the log industry portfolio

price-dividend ratios is

$$\begin{aligned}
L_e &= \sum_{t=t_0}^{T-\Delta t} \log(f_e(e_{t+\Delta t}|e_t)) \\
&= \sum_{t=t_0}^{T-\Delta t} -\frac{P}{2} \log(2\pi) - \frac{1}{2} \log(\det(\Pi)) - \frac{1}{2} (e_{t+\Delta t} - \rho_e e_t)' \Pi^{-1} (e_{t+\Delta t} - \rho_e e_t) \quad (\text{C30})
\end{aligned}$$

Finally, the log-likelihood function to maximize is:

$$L = L_Y + L_\varepsilon + L_e \quad (\text{C31})$$

Table 1: **Statistical summary, 1989M09-2015M12**

All variables are expressed in real terms and sampled at a monthly frequency. Panel A shows the statistics for macroeconomic variables.  $r_t^m$  is the monthly return on the CRSP-VW,  $f_t^1$  is the logarithm of the crude oil 1-month maturity futures,  $f_t^{15}$  is the logarithm of the crude oil 15-month maturity futures,  $r_t^f$  is the annualized continuously compounded Treasury bill rate. Panel B reports the statistics for the monthly returns on the 13 industry portfolios in Table 2.

**Panel A: Macroeconomic variables**

	$r_t^m$	$f_t^1$	$f_t^{15}$	$r_t^f$
Average	0.006	3.249	3.224	0.005
Standard deviation	0.043	0.513	0.530	0.043

Correlation matrix				
	$r_t^m$	$f_t^1$	$f_t^{15}$	$r_t^f$
$r_t^m$	1.000			
$f_t^1$	-0.047	1.000		
$f_t^{15}$	-0.037	0.975	1.000	
$r_t^f$	-0.098	-0.299	-0.282	1.000

**Panel B: Portfolio returns**

Portfolio	Average	Standard deviation	Correlation with $r_t^m$
Food	0.008	0.040	0.618
Oil	0.007	0.053	0.577
Clths	0.008	0.061	0.726
Durbl	0.005	0.056	0.811
Chems	0.007	0.057	0.776
Cnsum	0.009	0.042	0.626
Cnstr	0.008	0.058	0.820
Steel	0.004	0.082	0.804
FabPr	0.007	0.055	0.782
Machn	0.008	0.071	0.865
Cars	0.006	0.067	0.733
Trans	0.007	0.049	0.800
Rtail	0.007	0.048	0.775



Table 2: **Calibration of the parameter  $\gamma^i$**

The table shows the estimates of the oil intensity  $\gamma^i$  for 13 out of the 17 French's industry portfolio. The portfolio "Other" was discarded because it is not representative of any particular industry. This calibration was done with information from the Manufacturing Energy Consumption Survey (MECS) of the EIA. The last column has the average of the time series.

Name	Portfolio	Years					Average
	Description	1994	1998	2002	2006	2010	
Food	Food	0.045	0.013	0.018	0.025	0.019	0.024
Oil	Oil and Petroleum Products	0.149	0.217	0.153	0.105	0.061	0.137
Clths	Textiles, Apparel & Footware	0.021	0.013	0.016	0.017	0.018	0.017
Durbl	Consumer Durables	0.016	0.014	0.013	0.014	0.015	0.015
Chems	Chemicals	0.125	0.103	0.127	0.142	0.064	0.112
Cnsum	Drugs, Soap, Prfums, Tobacco		0.038	0.033	0.038	0.026	0.034
Cnstr	Construction and Construction Materials	0.108	0.035	0.033	0.041	0.042	0.052
Steel	Steel Works Etc	0.074	0.060	0.054	0.044	0.043	0.055
FabPr	Fabricated Products		0.016	0.017	0.017	0.014	0.016
Machn	Machinery and Business Equipment	0.005	0.008	0.009	0.010	0.009	0.008
Cars	Automobiles	0.006	0.006	0.006	0.007	0.007	0.006
Trans	Transportation		0.007	0.007	0.007	0.007	0.007
Rtail	Retail Stores		0.011	0.014	0.017	0.016	0.015
Total		9	13	13	13	13	13

Table 3: **Parameter estimates, 1989M09-2015M12**

Maximum likelihood estimates of the model parameters. Panel A reports the estimates for the global parameters, while Panel B presents the estimates for the parameters of each portfolio.

**Panel A: Global parameters**

Parameter	Estimate	t-stat
$\sigma_m$	0.149	25.22
$\alpha_0$	0.045	2.61
$\alpha_s$	0.078	3.94
$\alpha_\delta$	-0.092	-4.89
$\alpha_y$	0.132	23.04
$\theta_0$	1.352	2.38
$\theta_s$	-0.381	-2.00
$\theta_\delta$	0.114	0.95
$\theta_y$	0.020	0.89
$\bar{\delta}$	3.267	11.14
$\kappa_s$	1.638	22.70
$\kappa_\delta$	0.037	3.67
$\kappa_y$	7.338	14.10
$\sigma_s$	0.349	25.38
$\sigma_\delta$	0.186	25.55
$\rho_s$	0.099	2.03
$\rho_\delta$	0.259	5.33
$\rho_y$	-0.155	-3.05
$\rho_{s\delta}$	0.726	25.16
$\rho_{sy}$	-0.548	-11.70
$\rho_{\delta y}$	-0.471	-10.33
$\rho_\epsilon$	0.701	28.31
Log-likelihood	15141.283	

Panel B: Portfolio parameters

Portfolio	$\mu_s^i$		$\mu_\delta^i$		$\mu_y^i$		$\sigma_X^i$		$\rho_X^i$	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
Food	-0.034	-2.39	-0.002	-0.66	0.147	2.92	0.135	24.80	0.634	18.54
Oil	0.139	3.11	-0.154	-3.45	0.022	0.74	0.132	24.73	0.577	15.01
Clths	-0.181	-3.09	0.130	2.22	0.047	0.92	0.206	24.69	0.734	27.84
Durbl	-0.022	-0.72	-0.033	-1.12	0.102	1.92	0.192	24.78	0.810	41.35
Chem	-0.069	-1.73	0.035	0.97	0.071	1.42	0.167	24.74	0.767	32.72
Cnsum	-0.040	-2.57	0.002	0.68	0.124	2.38	0.143	24.80	0.641	19.05
Cnstr	-0.169	-3.42	0.118	2.55	0.075	1.50	0.188	24.76	0.826	45.53
Steel	0.094	1.49	-0.151	-2.51	0.077	1.19	0.251	24.73	0.800	39.01
FabPr	-0.068	-1.92	0.022	0.78	0.103	1.94	0.182	24.77	0.776	34.24
Machn	0.102	1.75	-0.175	-3.20	0.167	2.74	0.246	24.82	0.868	62.19
Cars	-0.128	-2.10	0.075	1.28	0.128	1.79	0.227	24.70	0.729	27.25
Trans	-0.016	-0.52	-0.033	-1.06	0.155	3.16	0.170	24.77	0.802	39.59
Rtail	-0.108	-2.63	0.061	1.49	0.194	3.98	0.162	24.73	0.804	39.85

Table 4: **Oil price elasticities**

$E_{D,S}^i$  is the oil price-dividend elasticity (equation (16)),  $E_{H,S}^i$  is the oil price-price-dividend ratio elasticity (equation (22)),  $E_{P,S}^i$  is the oil price-value elasticity ( $E_{D,S}^i + E_{H,S}^i$ ),  $E_{g,S}^i$  is the oil price-expected dividend growth elasticity (equation (17)) and  $E_{r,S}^i$  is the oil price-expected stock return elasticity (equation (31)).

Portfolio	$E_{D,S}$	$E_{H,S}$	$E_{P,S}$	$E_{g,S}$	$E_{r,S}$
Food	-0.02	-0.02	-0.05	0.01	0.05
Oil	-0.16	0.23	0.07	0.42	0.04
Clths	-0.02	-0.11	-0.12	-0.16	0.02
Durbl	-0.01	-0.01	-0.02	0.00	0.02
Chems	-0.13	0.07	-0.06	0.13	0.02
Cnsum	-0.03	-0.02	-0.05	0.02	0.04
Cnstr	-0.05	-0.06	-0.12	-0.09	0.02
Steel	-0.06	0.12	0.06	0.19	-0.01
FabPr	-0.02	-0.04	-0.06	-0.04	0.02
Machn	-0.01	0.07	0.06	0.12	0.00
Cars	-0.01	-0.08	-0.09	-0.12	0.01
Trans	-0.01	-0.02	-0.03	0.00	0.03
Rtail	-0.01	-0.07	-0.08	-0.09	0.03

Table 5: **Out-of-sample test for log price-dividend ratio, 1989M09-2015M12**

Relationship between log price-dividend ratio and log oil price, given that the latent variable  $y_t$  is at its unconditional mean. Column “Model” reports equation (42), given  $y_t = 0$ , calibrated for an average industry, which is defined by the average values of  $\gamma^i$ ,  $\sigma_X^i$ ,  $\rho_X^i$  and  $\mu_s^i$  in our sample. Parameter  $\mu_0^i$  was calculated using that  $\log(h_1^i) = -a^i(\mu_0^i) - b^i \times (\bar{s} - \sigma_s^2/2\kappa)$ . Column “OLS” presents the estimation of the same relationship by ordinary least squares assuming that  $H_t^i$  is observed with measurement errors, which we assume to be iid normal. Dependent variable is the log price-dividend ratio on the CRSP-VW Index.

	Model	OLS
Constant	4.161	4.155 ( 30.17 )
$s_t$	-0.122	-0.131 ( -2.78 )
$R^2$		0.023

Table 6: **Sensitivities of industry portfolio returns with respect to the different shocks**

The table shows the estimates of the  $\eta$  parameters in equation (24).

Portfolio	$\eta_{\Lambda}^i$	$\eta_s^i$	$\eta_{\delta}^i$	$\eta_y^i$	$\eta_X^i$
Food	0.086	-0.017	-0.001	0.002	0.107
Oil	0.108	0.025	0.057	-0.015	0.125
Clths	0.154	-0.043	0.010	-0.012	0.142
Durbl	0.157	-0.009	0.000	-0.004	0.114
Chems	0.155	-0.021	0.042	-0.007	0.121
Cnsum	0.092	-0.018	-0.003	-0.001	0.113
Cnstr	0.165	-0.041	0.015	-0.007	0.112
Steel	0.225	0.022	0.033	-0.007	0.159
FabPr	0.148	-0.019	0.021	-0.004	0.116
Machn	0.213	0.022	-0.014	0.004	0.123
Cars	0.169	-0.030	0.020	-0.001	0.156
Trans	0.137	-0.009	0.002	0.003	0.102
Rtail	0.128	-0.029	-0.001	0.008	0.098

Table 7: **Systematic risk decomposition**

The table shows decomposition of the industry portfolios market beta in the market betas of the state variables according to equation (35).  $\beta_s = 0.231$ ,  $\beta_\delta = 0.324$  and  $\beta_y = -1.036$ .

Portfolio	$b^i - \frac{\gamma^i}{1-\gamma^i}$	$c^i$	$d^i$	$\frac{1}{1-\gamma^i}$	$\beta_X^i$	$\left(b^i - \frac{\gamma^i}{1-\gamma^i}\right) \beta_s$	$c^i \beta_\delta$	$d^i \beta_y$	$\frac{1}{1-\gamma^i} \beta_X^i$	$\beta^i$
Food	-0.049	-0.003	0.002	1.025	0.576	-0.011	-0.001	-0.002	0.590	0.575
Oil	0.072	0.314	-0.015	1.159	0.509	0.017	0.102	0.015	0.590	0.723
Clths	-0.124	0.055	-0.012	1.017	1.016	-0.029	0.018	0.012	1.033	1.034
Durbl	-0.025	0.001	-0.004	1.015	1.041	-0.006	0.000	0.004	1.056	1.055
Chems	-0.060	0.231	-0.007	1.126	0.862	-0.014	0.075	0.008	0.971	1.039
Cnsum	-0.051	-0.017	-0.001	1.035	0.615	-0.012	-0.006	0.001	0.636	0.619
Cnstr	-0.118	0.084	-0.008	1.055	1.039	-0.027	0.027	0.008	1.096	1.104
Steel	0.063	0.183	-0.007	1.058	1.347	0.015	0.059	0.008	1.425	1.507
FabPr	-0.056	0.115	-0.004	1.016	0.946	-0.013	0.037	0.004	0.961	0.990
Machn	0.064	-0.080	0.004	1.008	1.434	0.015	-0.026	-0.004	1.446	1.430
Cars	-0.086	0.110	-0.001	1.006	1.109	-0.020	0.035	0.001	1.115	1.132
Trans	-0.026	0.012	0.003	1.007	0.916	-0.006	0.004	-0.003	0.922	0.917
Rtail	-0.085	-0.004	0.008	1.015	0.874	-0.020	-0.001	-0.009	0.887	0.857

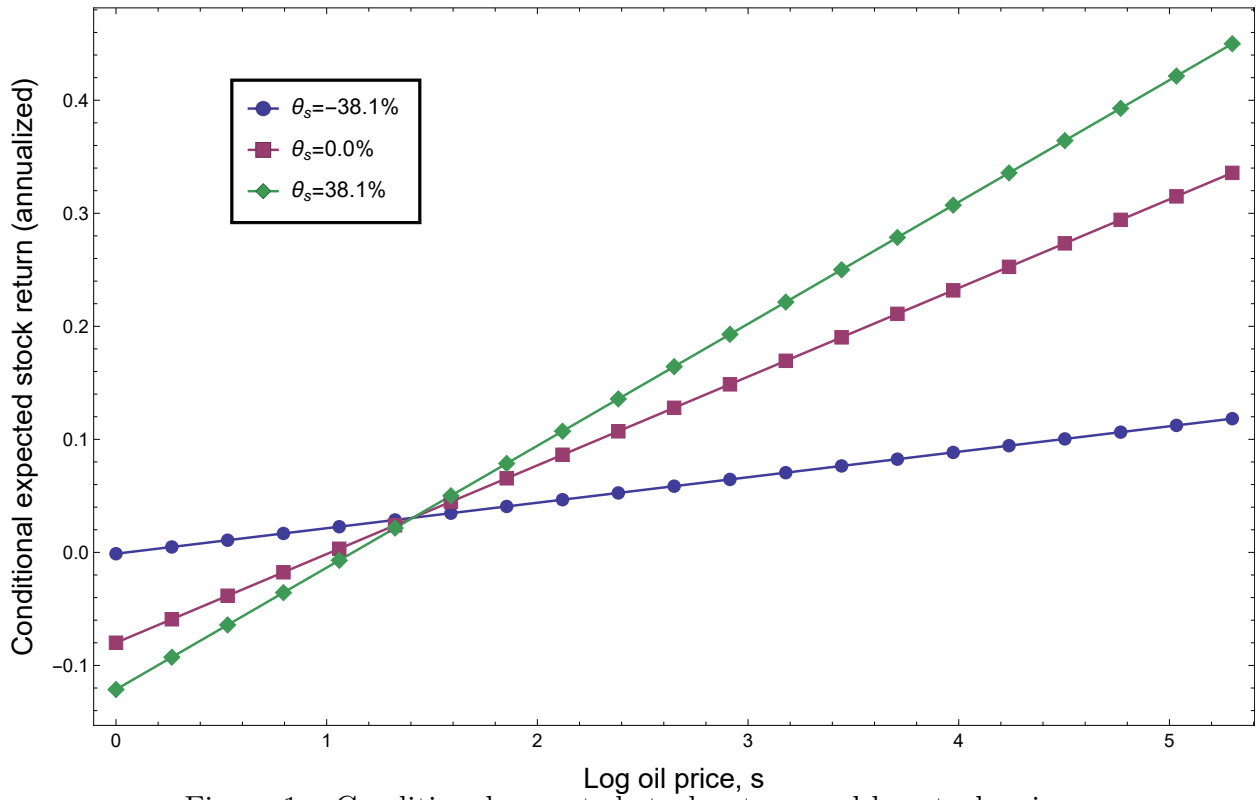


Figure 1: Conditional expected stock return and log stock price.

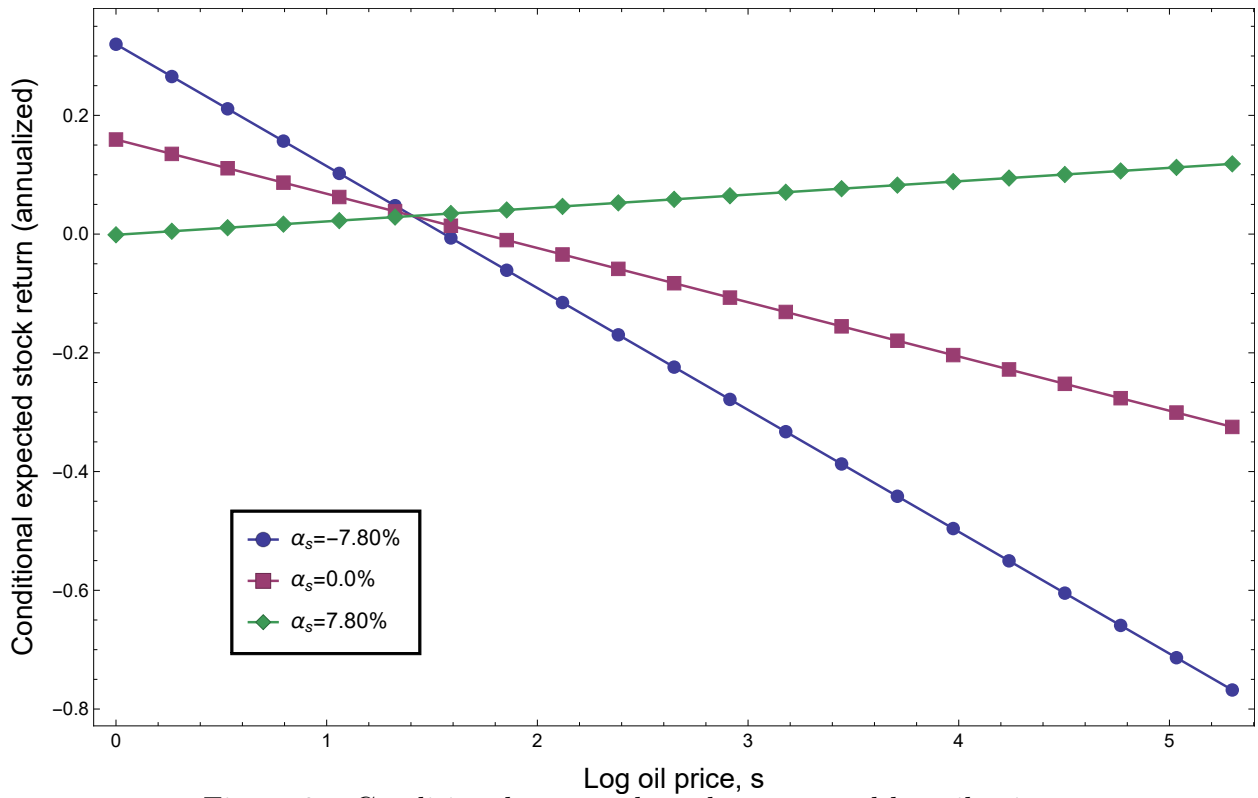


Figure 2: Conditional expected stock return and log oil price.



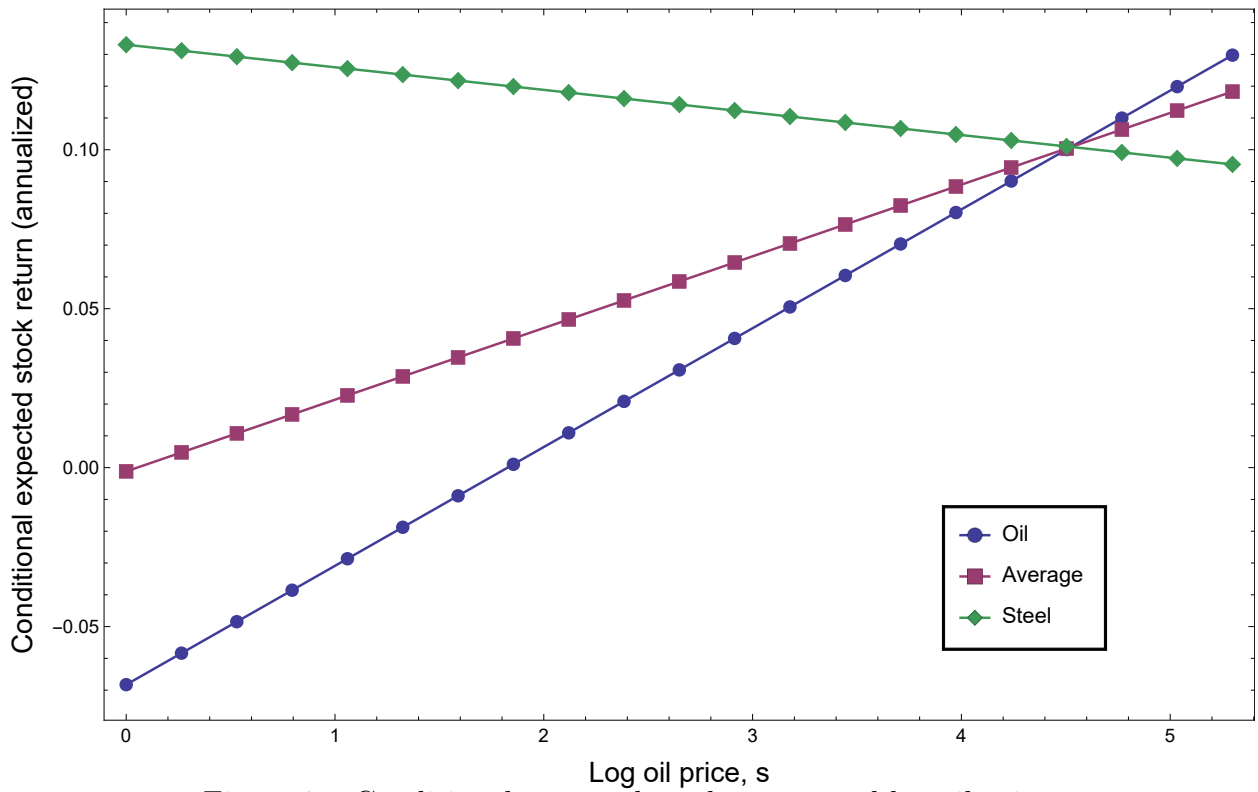


Figure 3: Conditional expected stock return and log oil price.

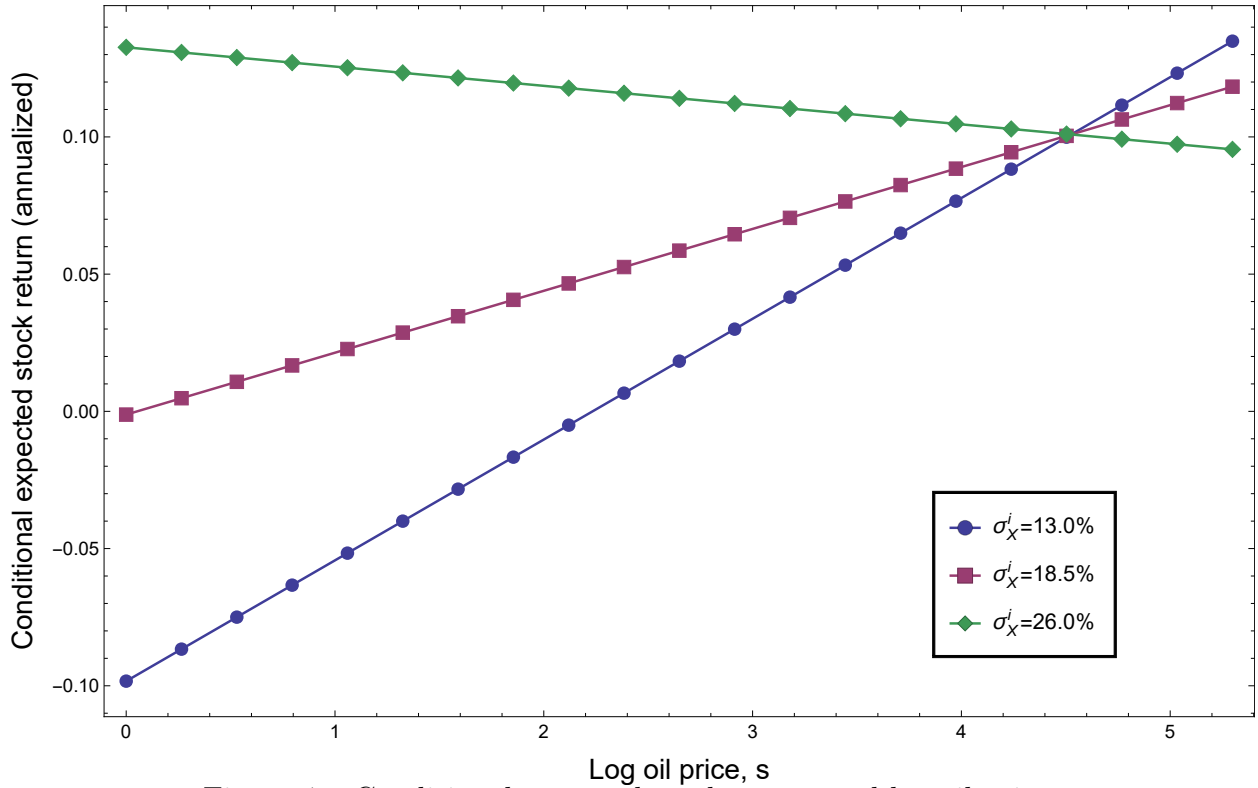


Figure 4: Conditional expected stock return and log oil price.

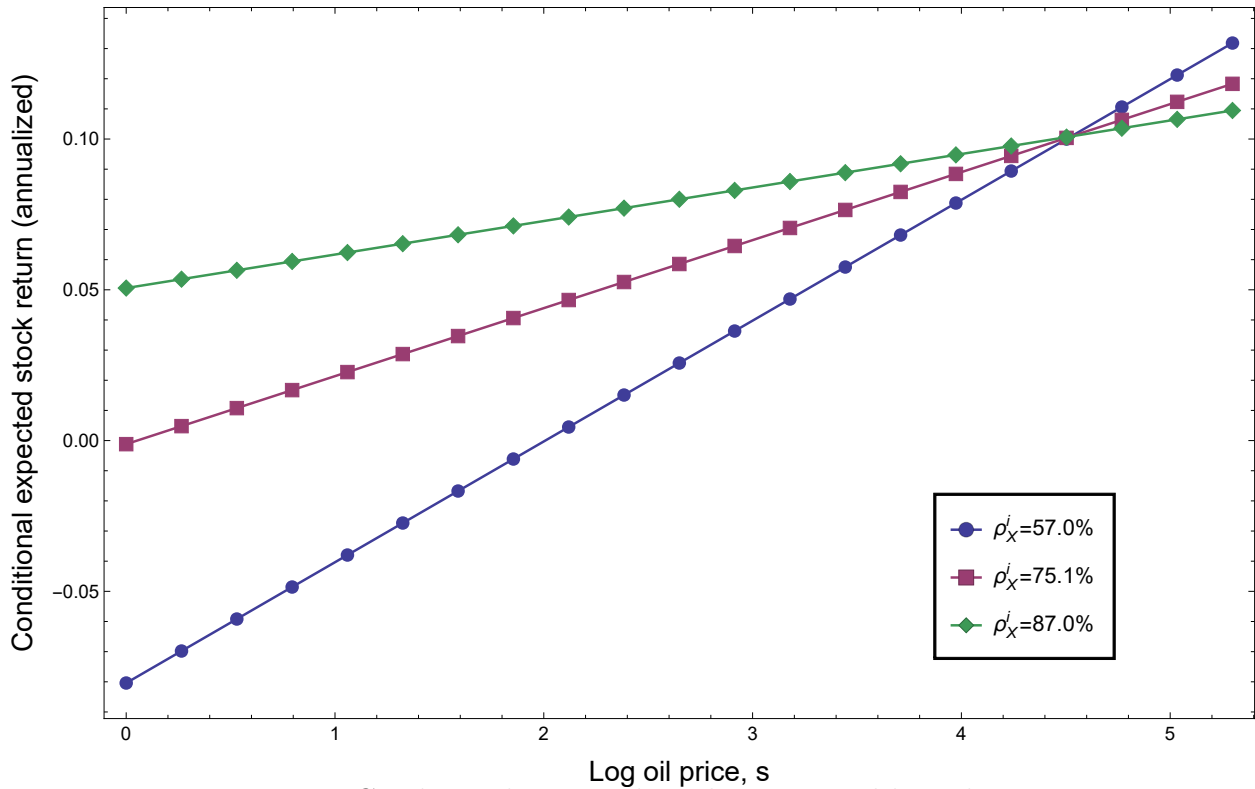


Figure 5: Conditional expected stock return and log oil price.

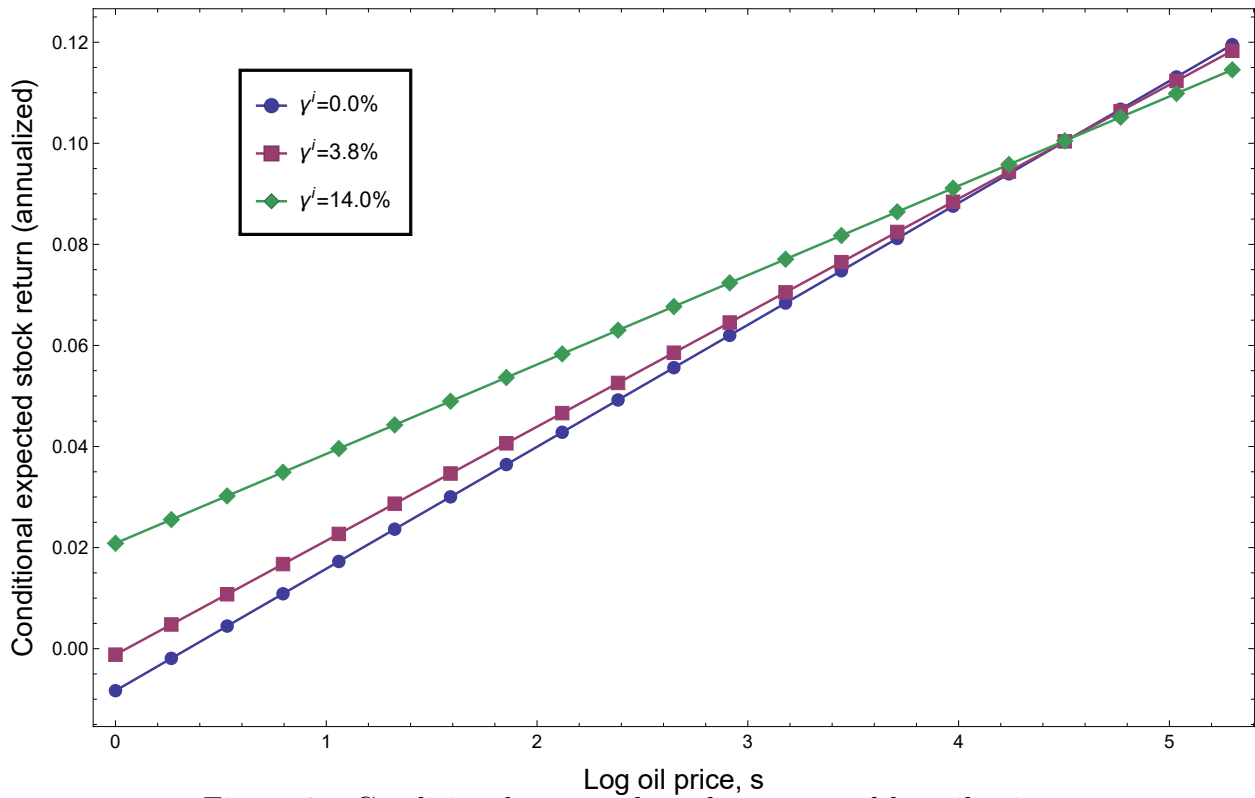


Figure 6: Conditional expected stock return and log oil price.

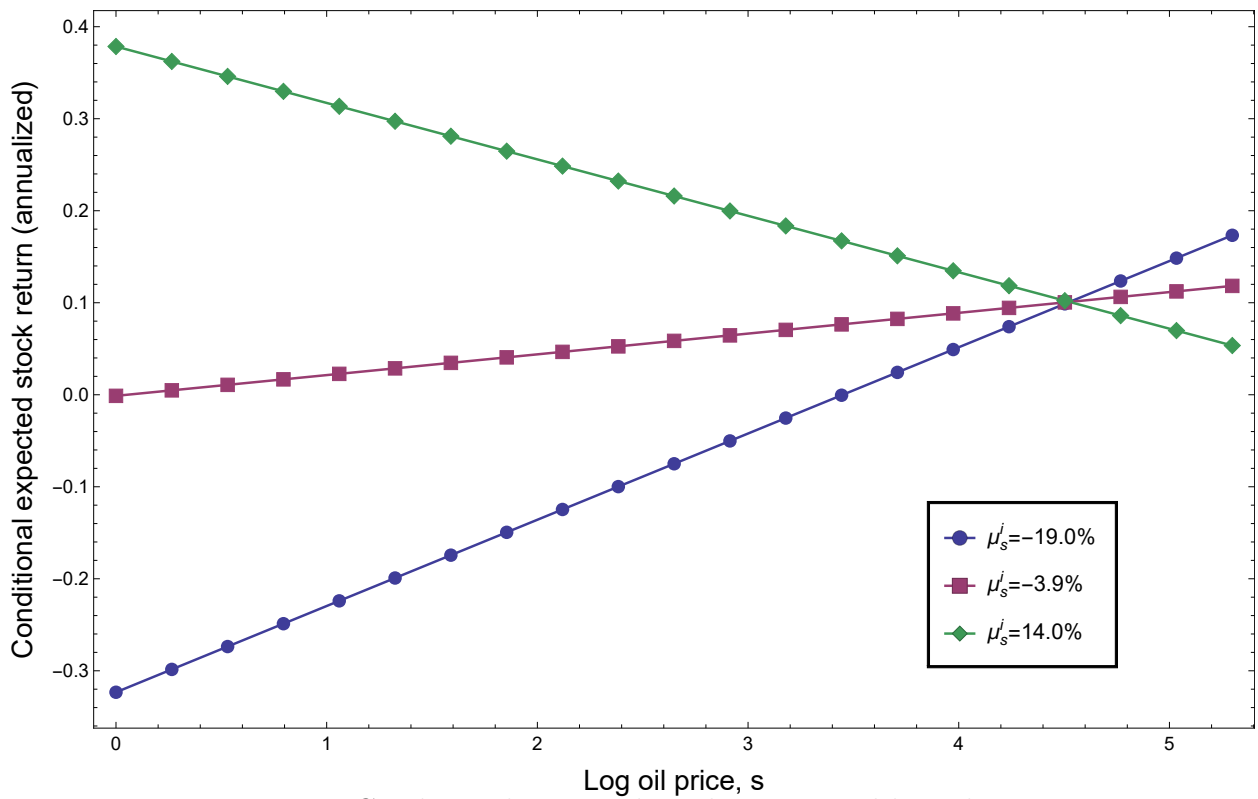


Figure 7: Conditional expected stock return and log oil price.

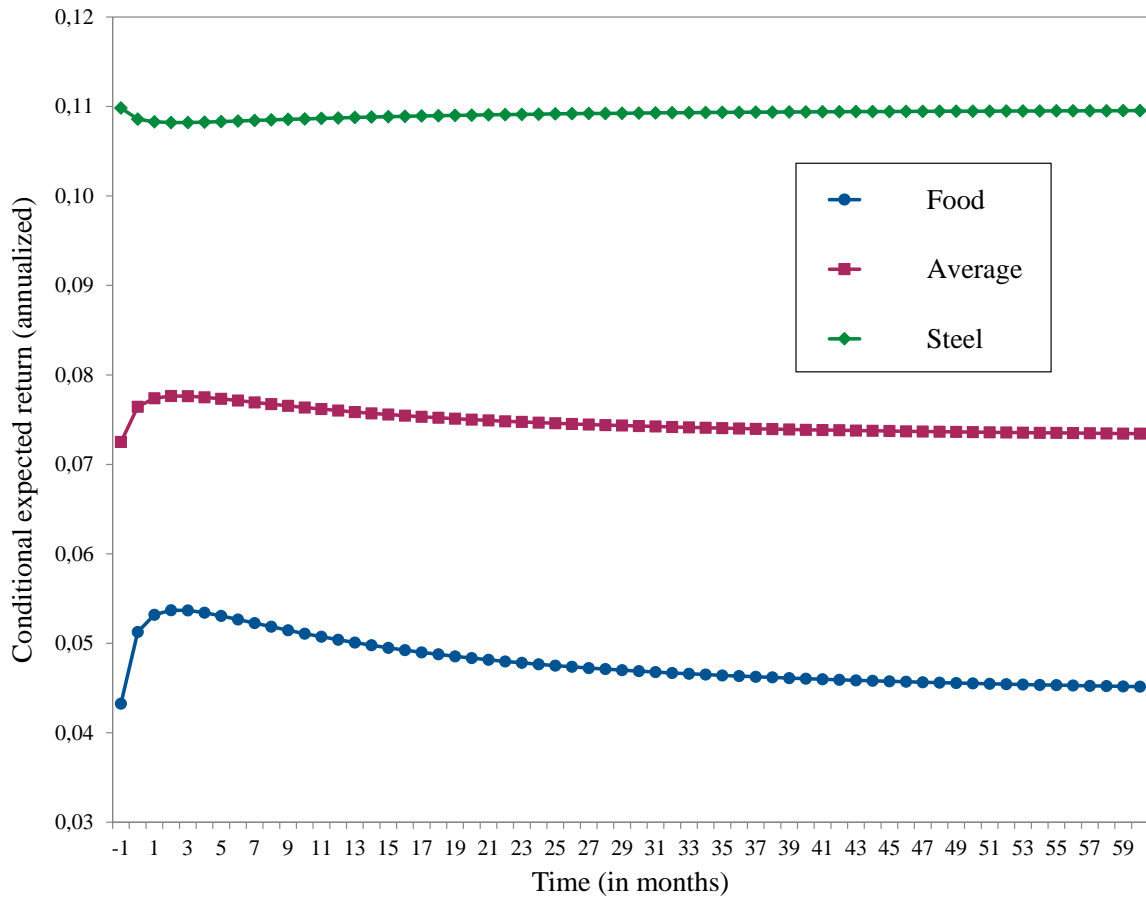


Figure 8: Conditional Expected return and log oil price shock at different horizons.