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Competing for Stock Market Feedback

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Abstract

We study how firms compete to attract informed trading when financial markets provide information to decision makers. Firms increase managerial risk taking to compete for market information, leading to a rat race in which firms overinvest in a (failed) attempt to increase their own stock informativeness. Efficiency gains of learning from the market may be eliminated: There is always an equilibrium where financial markets provide useful information, but are completely ignored by decision makers. Moreover, in any equilibrium firms react too little to market activity. Our results highlight that critically different outcomes arise when firms interact in integrated financial markets.

JEL classification: G14, G30, D82.

Keywords: information aggregation, financial markets, feedback effect, real efficiency.

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1 Introduction

Financial markets play a prominent role in providing information about economic fundamentals. As a result, decision makers are likely to rely on information gleaned from these markets to guide their actions, creating a two-way feedback between activity in financial markets and firms' cash flows (Bond, Edmans and Goldstein, 2012). Given that financial markets can be a useful source of information, firms may adjust their corporate policies in different margins as to affect incentives for market participants to produce information relevant to managerial decisions—a claim that has found empirical support in recent work (Foucault and Frésard, 2019; Lin, Liu and Sun, 2019). However, given that speculators have access to different securities and can choose to which ones to devote attention and resources, their incentives to trade a given firm's stock depend not only on the characteristics of that particular security, but also on the characteristics of all other securities available. In light of these observations, this paper studies how firms compete to attract informed trading to their stocks, and how this competition affects financial markets' potential to improve real efficiency.

We propose a model in which multiple firms have imperfect information about the profitability of their investment opportunities and have their stocks traded in an integrated financial market. Speculators have access to information about firms' fundamentals, and firms' corporate policies (managerial incentives) affect their learning and trading decisions. We show that firms compete to attract informed trading, and this competition may completely shut down the financial market's potential to improve real efficiency: Whenever two or more firms interact in an integrated financial market, there is always an equilibrium in which market activity has useful information to guide real decisions but is endogenously ignored by firms. Moreover, our competition channel always leads to an inefficiency: In any equilibrium, firms react too little to market information. This competition channel we uncover has empirical and normative implications for publicly listed firms.

Environment. In our model, each firm's controlling shareholders initially set up a managerial compensation scheme for a manager to whom they delegate a future investment decision. Firms and managers have imperfect information about firm-specific fundamentals affecting the profitability of their investment opportunities, so there is scope for learning from financial markets. The type of compensation contract offered to managers affects how each firm's investment will respond to market activity in a subsequent stage. For instance, a contract that incentivizes managerial risk taking makes the manager more prone to undertaking investment even if market activity suggests it is not so likely to be profitable.

After managerial contracts are set, a continuum of speculators make learning and trading decisions. Speculators are subject to wealth constraints and have limited attention, so they cannot learn about all securities and cannot trade unlimited amounts. Once a speculator chooses to learn about a firm's fundamental, she receives a noisy signal about it and chooses how much to buy or sell of the firm's stock. Her expected profits from trading the firm's security depends on the likehood of investment being undertaken by the firm, since information about investment profitability is useless in case the firm foregoes the investment opportunity. This gives rise to the following trade-off when firms set compensation contracts. By inducing more managerial risk taking, a firm boosts incentives for speculators to learn about and trade its security (which, all else equal, increases its stock price informativeness and leads to better decision making). But this comes at the cost of distorting the use the manager will make of information stemming from financial markets (which contributes to a reduction in real efficiency).

Key ingredients. Before detailing our results, we expand on some key features of our model. Our results rely on two basic ingredients. First, informed trading capacity is limited, meaning that more informed trading on a given stock means less informed trading on another stock. This arises naturally if speculators are subject to frictions such as wealth constraints, short-selling constraints or attention frictions. Second, firms can take measures that increase speculators' incentives to learn and trade, and they face a trade-off between incentivizing information production in financial markets and using this information efficiently. In our model, this measure is to set up a managerial compensation scheme that can lead to more risk taking. This is motivated by recent empirical evidence consistent with firms using managerial contracts as a margin to increase their stock price informativeness (Lin, Liu and Sun, 2019). This is the margin firms will use to compete to attract informed trading in our setting.

Notice that the use of the word 'competition' here does not refer to competition among firms in the product market. We abstract from any other strategic interactions among firms that are not driven by our mechanism—competition for information produced in secondary financial markets. This way, we isolate the effects of our mechanism and avoid any confounding effects unrelated to this informational channel.

Main results. Competition for market feedback creates a rat race among firms: To attract informed trading, firms have incentives to set compensation contracts that are more aggressive than that of their peers—in the sense of inducing more managerial risk taking. However, in equilibrium, they end up not reaping the benefits of this strategy. There are no effective gains in informativeness, and firms make worse use of information ex post, reacting too little to market activity. Our model has two types of equilibrium, to which we refer as the 'good' and the 'bad' equilibrium (although both are inefficient).

In the bad equilbrium, the efficiency gains that are usually thought to arise when firms can learn from financial markets are completely eliminated due to our competition channel. As a consequence of the aforementioned rat race, firms set compensation contracts that lead to an extreme level of managerial risk taking. Managers completely ignore information stemming from the market, even if that information is very accurate. Hence, in the presence of competition for information, *revelatory efficiency* may fail to translate into *gains in real efficiency*.[1] This result is in sharp contrast with what would happen in the absence of our competition channel. If markets were segmented or a single firm would try to learn

[1] 'Revelatory efficiency' refers to the extent to which market activity provides information useful to improve real efficiency. 'Gains in real efficiency' refer to the extent to which market activity *effectively* improves firm value. See also Bond, Edmans and Goldstein (2012).

from the market at a time, the unique equilibrium would feature no such inefficiency—firms would learn as much as possible and use information in the best possible way to guide real decisions. Competition for information among as little as two firms leads to the emergence of a dramatically different equilibrium outcome.

In the good equilibrium, managers rely on market activity to guide real decisions to some extent. However, equilibrium compensation contracts still lead to excessive risk taking, and managers under-react to market information. We derive comparative statics on different market parameters to shed further light on the empirical implications of our mechanism. In the good equilibrium, firms become more aggressive (more likely to undertake risky investments) when: (i) there are more sophisticated (potentially informed) speculators; (ii) speculators have access to better information; (iii) short-selling and wealth constraints are weaker. The idea is that the larger the market potential to aggregate information, the stronger firms' incentives to compete for that information. Hence, better availability of information leads to worse use of information in equilibrium. We also show that the number of publicly listed firms has a non-monotonic effect on equilibrium managerial risk taking and discuss the non-trivial consequences of financial market development for real efficiency.

An implication of our mechanism for empirical work is that a moderate investment-to-price sensitivity (or another measure of feedback effects) does not necessarily mean that firms do not consider financial markets a useful source of information. On the contrary, market information may be very valuable, and a moderate reaction to market activity may be a symptom of a strong competition motive.

One of the normative implications of our mechanism is that if all firms were unable to commit to certain investment policies (by delegating decisions to managers under a compensation scheme) they would all achieve higher firm value. This is what we call a *commitment paradox*: If firms could commit not to commit, they would all be better off. If a regulator could enforce that all firms offer contracts that align managers' and shareholders' incentives ex post, it would lead to the first best. Shareholder value maximization is then collectively optimal, but not individually optimal for firms, as an individual firm would always have incentives to distort managerial risk taking as to attract a larger share of informed trading if other firms maximized shareholder value.

Finally, we extend our main model and allow firms to endogenously decide whether or not to be publicly listed upon paying some underwriting cost. To better understand the inefficiencies arising from competition for information, we then consider what a social planner would do if it could control the number of publicly traded firms and compare it to the decentralized solution. We show that there is over-entry and that a Pigouvian tax on IPOs implements the social optimum.

Related literature. Empirical work has provided evidence consistent with firms learning from financial markets and using this information to guide real decisions (Luo, 2005; Chen, Goldstein and Jiang, 2006; Bakke and Whited, 2010; Edmans, Goldstein and Jiang, 2012; Edmans, Jayaraman and Schneemeier, 2017; Jayaraman and Wu, 2019). Recent theoretical work has studied this learning channel, which is often referred to as the *feedback effect.* Theoretical contributions on the topic include Dow and Gorton (1997), Goldstein and Guembel (2008), Bond and Eraslan (2010), Bond and Goldstein (2015) and Edmans, Goldstein and Jiang (2015).

The idea that firms actively try to shape how much they can learn from financial markets by using different tools has been explored in previous work, both theoretically and empirically. Examples include Goldstein and Yang, 2019 (information disclosure), Lin, Liu and Sun, 2019 (managerial compensation), Foucault and Frésard, 2012 (cross-listing) and Foucault and Frésard, 2019 (product differentiation).

Lin, Liu and Sun (2019) study, theoretically and empirically, how firms set up managerial compensation contracts to boost incentives for information production in financial markets.[2] By exploiting a randomized experiment, they find evidence consistent with that channel. In a similar spirit, Boleslavsky, Kelly and Taylor (2017) allow firms to commit to react to market

[2]Strobl (2014) also studies managerial contracts and incentives to trade in a model with a single firm, although there the manager is perfectly informed about fundamentals.

activity in certain ways as to increase informed trading, and Machado and Pereira (2020) also allow firms to adjust their capital structure for similar reasons. However, all those papers study settings with a single firm that do not compete to attract informed trading to their stock.

A few recent papers feature multiple firms in models with feedback effects. Schneemeier (2019) and Yang (2020) propose multi-firm models of feedback to study information disclosure. In Yang (2020), competitors in the product market choose disclosure policies as to affect how informative the financial market is. All firms learn from the same asset price (the price of a commodity future) and therefore there is no competition to attract informed trading. Schneemeier (2019) study information disclosure by two firms that learn from different asset prices.[3] As in our setting, speculators have limited attention, and firms benefit from attracting a larger share of informed trading. However, the information disclosure decisions analyzed in Schneemeier (2019) do not imply any trade-off between incentivizing information production in financial markets and using this information efficiently—which is a key driver of our results and arises naturally in our setting in which firms set managerial compensation schemes.

Another paper in which there are feedback effects and multiple firms is Foucault and Frésard (2019). In their setting, firms choose whether to differentiate their products from others. By choosing to produce a good similar to other publicly listed firms' products (conformity), firms can learn more from financial markets. However, in their setting firms do not need to compete to attract informed trading, since how much a speculator trades of a given stock does not affect how much she can trade of other stocks (as if markets were segmented and each firm had its own pool of speculators).[4] Hence, there is no competition

[3] In a different setting, Fishman and Hagerty (1989) also study disclosure policies similar to those in Schneemeier (2019) in a model with multiple firms. However, in Fishman and Hagerty (1989) financial markets can improve real efficiency for reasons unrelated to managerial learning, and the results are substantially different from those in Schneemeier (2019).

[4] This is also the case in the general equilibrium setting with multiple firms of Albagli, Hellwig and Tsyvinski (2018), although there the decision maker does not learn from the market. More specifically, in Albagli, Hellwig and Tsyvinski (2018) and Foucault and Frésard (2019) speculators can trade up to a given exogenous quantity of each firm's stock and buying less of a certain stock does not imply that the speculator can buy more of other stocks.

for market feedback.

More broadly, our paper also relates to the literature that studies limited attention in finance (see Veldkamp, 2011 for an overview) and the large literature on market frictions and limits to arbitrage (see Shleifer and Vishny, 1997 for a seminal contribution). From a technical perspective, our baseline model is related to Dow, Goldstein and Guembel (2017), although here we propose a multiple-firm setting where firms choose managerial contracts.

Outline. This paper is organized as follows. Section 2 presents the setting. Section 3 presents the equilibrium characterization and discusses our main results. Section 4 discusses normative implications and presents the extension with endogenous stock listing. Section 5 concludes. All proofs are presented in the appendix.

2 Environment

There are $J \geq 2$ ex-ante identical firms, and each firm has its stock traded in a secondary financial market. The economy has a continuum of mass $\overline{\alpha}$ of (sophisticated) speculators who have access to information about firms' fundamentals. Speculators must choose which firm to learn about, and then how much of the firm's stock to trade. Firms are indexed by $j \in \{1, ..., J\}$ and speculators are indexed by $i \in [0, \overline{\alpha}]$.

The financial market is composed of speculators, noise traders and a competitive market maker. The total order flow (in dollar amount) for stock j is denoted by Y_j, and $\boldsymbol{Y} = \{Y_1, ..., Y_J\}$ represents all market activity. Each firm j has a fundamental θ_j that can be low (L) or high (H). Managers and controlling shareholders have imperfect information about firms' fundamentals. At the initial stage, each firm's controlling shareholders offer a compensation contract to their manager, which is a function of the firm's final cash flow. Then, each speculator chooses which stock to learn about, and trading takes place in the financial market. Finally, the manager observes market activity and makes an investment decision whose profitability depends on the fundamental θ_j. The type of contract offered to the manager will effectively affect how firms' investment will react to market activity. The timeline is summarized below. In what follows we further detail the structure of the model, and in Section 2.3 we discuss our modeling choices.

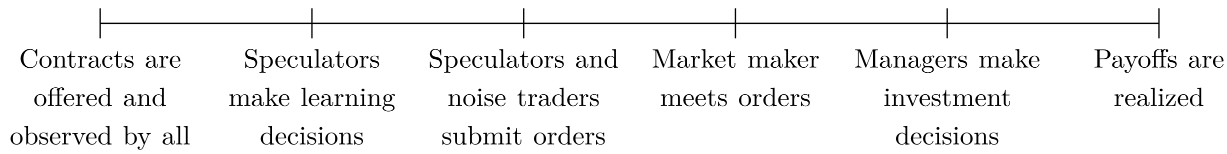

Figure 1: Timeline

2.1 Firms

The final cash flow of firm j depends on the firm's fundamental θ_j and on a managerial decision $a_j \in \{0, 1\}$, and is denoted by v_j. We refer to actions $a_j = 0$ and $a_j = 1$ as 'not investing' and 'investing', respectively. When investment is undertaken, the cash flow is given by

$$v_j = \begin{cases} V_H & \text{if } \theta_j = H, \\ 0 & \text{if } \theta_j = L. \end{cases}$$

If the manager does not invest, $v_j = V_0$, with $V_H > V_0 > 0$. Hence, investment increases firm value only in the high state.

Fundamentals are not observed by firms' controlling shareholders and managers, so there is potential for learning from market activity. We assume that firms' fundamentals $\{\theta_1, ..., \theta_J\}$ are mutually independent, and for every firm j each state is ex-ante equally likely. Hence, firms wish to learn about different objects and may therefore compete to attract informed trading to their own stock.

At the initial stage, controlling shareholders of each firm j offer the manager a compensation policy $w_j(v_j)$ that specifies the manager's payment as a function of the final cash flow. The compensation scheme must satisfy: (i) The feasibility constraint $w_j(v_j) \leq v_j$ for all v_j; (ii) The participation constraint $\mathbb{E}[w_j(v_j)] \geq \underline{u}$, where $\underline{u} > 0$ is the managers' common outside option. We assume that $\underline{u} < V_0$, which guarantees the participation constraint can always be satisfied.

Before deciding on the investment decision a_j, the manager of each firm j observes stock prices and total order flows in all stock markets, and updates her belief about the firm's state. Decision makers can learn useful information from financial markets to guide their investment decisions. Managers decide on investment as to maximize their expected compensation.

As usual in models of feedback, expectations about firms' investment affect speculators' incentives to trade. Hence, by offering different incentives for the manager to undertake the investment opportunity, firms can actively try to shape their informational environment. The dividend paid to shareholders at the final stage is $r_j \equiv v_j - w_j(v_j)$. Controlling shareholders choose $w_j(v_j)$ to maximize the expected firm value, $\mathcal{V}_j \equiv \mathbb{E}[r_j]$. Without loss of generality, we assume that controlling shareholders always offer contracts such that the participation constraint binds.[5] Expected firm value can then be written as:

$$\mathcal{V}_j = V_0 + \frac{1}{2}\Pr(a_j = 1|\theta_j = H)(V_H - V_0) - \frac{1}{2}\Pr(a_j = 1|\theta_j = L)V_0 - \underline{u}. \tag{1}$$

Note that, to maximize her expected compensation, each manager will undertake the investment opportunity whenever the probability she assigns to the high state is at least $\gamma_j \equiv w_j(V_0)/w_j(V_H)$.[6] The variable γ_j can be interpreted as the conservatism ratio chosen by controlling shareholders and captures how the firm's investment reacts to market activity. For example, a low γ_j means the contract offered induces the manager to undertake the investment opportunity even when the available information indicates that the chance of success is low. We say that a manager is a shareholder value maximizer if she is promised a fixed proportion of the firm's cash flow, i.e., $\gamma_j = \gamma^* \equiv V_0/V_H$. In that case her incentives are completely aligned with shareholders. For instance, if instead $\gamma_j < \gamma^*$, the manager may be

[5] As will become clear, learning and trading decisions are not affected by the amount of rents managers obtain. Therefore, controlling shareholders always choose not to leave any rents to managers in equilibrium.

[6] The feasibility constraint implies $w_j(0) = 0$, and hence the manager invests whenever $\Pr(\theta_j = H|\boldsymbol{Y})\, w_j(V_H) \geq w_j(V_0)$. As a tie-breaking convention, we assume the manager invests when indifferent.

willing to undertake the investment opportunity when shareholders would prefer not to do it ex post (after observing market activity).

Hereafter we often refer to the problem of controlling shareholders as that of choosing the manager's conservatism ratio γ_j, since this choice uniquely pins down the contract $w_j(\cdot)$.[7] Moreover, to ease the exposition and avoid technical complications, we assume that $V_H > 2V_0$, which implies that controlling shareholders would be willing to undertake the investment opportunity if they acted based solely on their prior.[8]

2.2 Financial market

Speculators. Before deciding on their trades and after observing managers' compensation contracts, sophisticated speculators make learning decisions. Speculators have limited attention and can only learn about the fundamentals of one firm. Specifically, if speculator i chooses to learn about firm j, she receives a signal $m_{i,j} \in \{m_L, m_H\}$ such that $\Pr(m_{i,j} = m_H | \theta_j = H) = \Pr(m_{i,j} = m_L | \theta_j = L) = \lambda > 0.5$. For every j, the signal $m_{i,j}$ is iid across speculators conditional on θ_j. We denote by $\ell_i(\boldsymbol{\gamma})$ the learning decision of investor i as a function of compensation contracts, as summarized by the conservatism ratios $\boldsymbol{\gamma} \equiv \{\gamma_1, ..., \gamma_J\}$. Once speculators observe their signals, they choose how much to trade of the security they learned about. Speculators are risk neutral and submit a market order to buy (or short-sell if negative) s_i dollars of that stock in order to maximize expected trading profits, which are given by:

$$\mathbb{E}\left[\frac{s_i}{p_j}(r_j - p_j)\middle| m_{i,j}\right],$$

where p_j denotes the price of the security of firm j (to be determined in equilibrium). Speculators hold no initial position on the stock of the firm and are wealth constrained. Each speculator initially has ω dollars, cannot borrow and face a margin requirement of ζ to

[7]Since the feasibility constraint implies $w_j(0) = 0$, controlling shareholders must choose only $w_j(V_H)$ and $w_j(V_0)$. Also, since the participation constraint must bind, a choice of γ_j pins down the pair $\{w_j(V_H), w_j(V_0)\}$.

[8]Under this assumption, there is a unique equilibrium in the trading stage subgame (on the equilibrium path), and this allows us not to have to take a stance on equilibrium selection.

short sell (that is, to short sell one dollar they need to provide $\zeta > 0$ dollars in guarantees in a margin account). This implies the following trading restriction: $s_i \in [-\omega/\zeta, \omega]$. The parameter ζ captures the strength of short-selling constraints and will be useful when deriving comparative statics.

Noise traders. Besides sophisticated speculators, there are noise traders that trade for exogenous motives. In particular, for each firm j noise traders submit an aggregate order of n_j dollars, where n_j is normally distributed with zero mean and variance $\sigma^2 > 0$, is iid across firms and independent of fundamentals.

Market maker. After firms set their compensation contracts and traders submit their orders, the market maker observes the aggregate order flows in all markets, $\boldsymbol{Y}$. The aggregate order flow for each stock j is given by the sum of sophisticated speculators' orders and noise traders' orders:

$$Y_j = \int_{\Omega_j} s_i di + n_j,$$

where $\Omega_j = \{i \in [0, \overline{\alpha}] : \ell_i = j\}$. For future reference, let α_j denote the mass of speculators that choose to learn about firm j (i.e., the mass of the set Ω_j). As in Kyle (1985), the market maker is competitive and liquidates the orders at the fair price:

$$p_j(\boldsymbol{Y}) = \mathbb{E}[r_j | \boldsymbol{Y}].$$

When setting the price the market maker rationally anticipates the managers' reaction to market activity.

2.3 Discussions of assumptions

We now discuss some of the modeling choices made.

The nature of competition. In our setting, firms' cash flows are not affected by other firms' fundamentals nor by other firms' investment decisions. The use of the word competition here, thus, does not refer to competition in the product market. We make the deliberate choice to abstract from competition in the product market to ensure that all the strategic interactions among firms are driven by our mechanism—competition for information produced in secondary financial markets. This way, we avoid any confounding effects unrelated to this informational channel.[9]

Managerial contracts as the margin of competition. In our model, firms try to shape their informational environment by offering certain managerial compensation schemes (that ultimately affect how investment reacts to market activity). We borrow this approach from Lin, Liu and Sun (2019). In their model, *a single firm* tries to increase its stock price informativeness by choosing a certain managerial contract, and they present empirical evidence supporting the claim that firms indeed explore this margin to affect the informational content of their stocks. We then take this as a natural margin for firms to compete for market feedback in our setting with multiple firms. There may be different margins through which firms compete to attract informed trading, and our insights apply more broadly. What is key for our results is that firms use some margin that implies a trade-off between incentivizing information production in financial markets and using this information efficiently.[10]

Limited attention. The assumption that speculators have limited attention is justified by the literature on rational inattention.[11] That speculators can only receive signals about one security is not essential, but greatly simplifies the exposition. What is key for our results is that sophisticated speculators face a trade-off: when trading more of stock x they have to reduce their trades in some stock y (informed trading is a scarce resource). Even if speculators had information about all securities, but still faced wealth constraints, such a trade-off would arise.

[9] The interplay between managerial compensation and product market competition has been studied in the industrial organization literature (see Fershtman and Judd, 1987, and Katz, 1991).

[10] For instance, firms could try to affect speculators' incentives to trade by distorting their capital structure (as in Machado and Pereira, 2020).

[11] See Sims (2003). For empirical evidence of limited attention in financial markets see Da, Engelberg and Gao (2011) and Sicherman et al. (2016).

Noise traders. We do not take a stance on what is the source of noise in the economy, and simply assume noise traders' orders are exogenous. This formulation implies that noise trading does not react to changes in managerial risk taking. This is justified if noise traders are not only uninformed about firms' fundamentals, but also about managerial incentives.

However, this assumption is not crucial for our results. In fact, accounting for noise traders' responses could amplify our competition mechanism. Suppose, for instance, that noise traders are a fringe of uninformed traders who choose securities to store value to accommodate liquidity shocks, and do so while trying to minimize expected trading losses. Given that the market maker always breaks even in expectation, any increase in expected trading profits for sophisticated speculators increases expected trading losses for noise traders. Hence, as a firm increases incentives for informed trading in its stock, it would also reduce incentives for noise trading, which could improve market informativeness for two reasons. Finally, given that we will focus on equilibria where firms play symmetric strategies, expected trading losses of noise traders are equal across securities in equilibrium, so they would have no incentives to switch securities.

Trading different securities. To ease notation, we imposed that speculators can only trade the security about which they learned. If we allowed speculators to trade any security, our results would be unchanged, since speculators would endogenously choose not to trade any other security. Speculators can never profit more in expectation by trading a security they have no information about than by trading the security about which they are informed.

2.4 Equilibrium definition

In our setting, a Perfect Bayesian Equilibrium (PBE) consists of price functions $p_j(\boldsymbol{Y})$; compensation contracts, as summarized by γ_j; trading strategies $s_i(m_{i,j})$; learning strategies $\ell_i(\boldsymbol{\gamma})$; investment strategies $I_j(\boldsymbol{Y})$; beliefs $\mu_j(\boldsymbol{Y})$ for the market maker and firm managers, and $\eta(m_{i,j})$ for speculators (specifying the probabilities assigned to $\theta_j = H$), such that:

- Trading and learning strategies for speculators maximize trading profits, given the price functions and all other agents' strategies;

- Compensation contracts maximize the ex-ante value of each firm, given the price functions and all other agents' strategies;
- Investment strategies maximize managers' expected compensation, given the price functions and all other agents' strategies;
- Price functions are such that the market maker breaks even in expectation;
- Beliefs $\mu_j(\boldsymbol{Y})$ and $\eta\,(m_{i,j})$ are consistent with Bayes rule.

Moreover, for tractability we restrict attention to Perfect Bayesian Equilibria in which firms offer the same compensation contract in equilibrium (symmetry). We say that a strategy profile is an equilibrium if it is a PBE that satisfies this additional requirement. To ease notation, and without loss of generality, we condition managers' beliefs and their investment decisions only on the total order flows and not on stock prices—once order flows are observed, prices convey no additional information to managers.

2.5 Benchmark case: Autarky

In order to better understand the effects of competition for market feedback, we will compare our setting to a benchmark model in which firms have their share of informed trading guaranteed. More precisely, to construct the benchmark we modify the main model by assuming that markets are segmented: an equal and exogenous mass of sophisticated speculators $\overline{\alpha}/J$ receive information about each security. We will refer to this benchmark setting as the case of 'autarky' or 'segmented markets'.

3 Characterization of equilibrium

We solve the model backwards. First, we solve for the market maker's beliefs, asset prices and the managers' decisions, taking as given managerial contracts (conservatism ratios), learning and trading decisions. Second, we characterize learning and trading decisions, taking as given the conservatism ratios. Finally, we solve for the equilibrium conservatism ratios.

3.1 Equilibrium prices and managerial decisions

Beliefs. Here we present the managers' and market maker's posterior beliefs (after observing market activity). Fix managerial contracts, and learning and trading strategies. We define sophisticated speculators' aggregate order for stock j when the state θ_j is high and low as $S_{j,H} \equiv \int_{\Omega_j} [\lambda s_i(m_H) + (1-\lambda) s_i(m_L)]\, di$ and $S_{j,L} \equiv \int_{\Omega_j} [\lambda s_i(m_L) + (1-\lambda) s_i(m_H)]\, di$, respectively. That means that $Y_j = S_{j,H} + n_j$ if the fundamental of firm j is high, and $Y_j = S_{j,L} + n_j$ if it is low. By Bayes rule, the probability that the manager of firm j and the market maker assign to $\theta_j = H$ can be written (with some abuse of notation) as:

$$\mu_j(X_j) = \frac{1}{1 + \exp(-X_j)}, \tag{2}$$

where

$$X_j \equiv \frac{1}{2\sigma^2}(S_{j,H} - S_{j,L})\left[(Y_j - S_{j,H}) + (Y_j - S_{j,L})\right]. \tag{3}$$

Given speculators' strategies, X_j summarizes all the information about firm j contained in market activity. Hence, we often refer to X_j as the 'aggregate order' for stock j (instead of Y_j). Also, to keep notation to a minimum, we hereafter often refer to functions of $\boldsymbol{Y}$ as functions of X_j, whenever a variable only depends on $\boldsymbol{Y}$ through X_j—as we did in equation (2).

Managerial decision. As already discussed, the manager of firm j invests if and only if $\mu_j(X_j) \geq \gamma_j$. Using (2), this condition can be rewritten as $X_j \geq \overline{X}_j$, where:

$$\overline{X}_j \equiv \ln\left(\frac{\gamma_j}{1-\gamma_j}\right).$$

That is, the manager invests whenever the aggregate order for stock j is larger than $\overline{X}_j$. The cutoff $\overline{X}_j$ is an increasing function of the conservatism ratio γ_j, so $\overline{X}_j$ itself is also a measure of managerial conservatism. The larger $\overline{X}_j$, the larger the aggregate order required for the manager to be confident enough about the firm's fundamentals to invest. We also define the shareholder value maximizing investment cutoff as $\overline{X}^* \equiv \ln\left(\frac{\gamma^*}{1-\gamma^*}\right)$.[12]

Prices. After observing aggregate orders, the competitive market maker sets the price of each security equal to its expected dividend, anticipating managerial decisions. Hence, stock prices are given by:

$$p_j\left(X_j\right) = \begin{cases} \mu_j(X_j)R_H & \text{if } X_j \geq \overline{X}_j, \\ R_0 & \text{otherwise,} \end{cases} \tag{4}$$

where $R_H \equiv V_H - w_j(V_H)$ and $R_0 \equiv V_0 - w_j(V_0)$.

3.2 Trading and learning decisions

For each firm j we define the following measure of market informativeness:

$$\kappa_j = \frac{1}{2\sigma^2}\left(S_{j,H} - S_{j,L}\right)^2.$$

If aggregate orders from sophisticated speculators are the same in both states ($S_{j,H} = S_{j,L}$), then $\kappa_j = 0$. The larger the difference in aggregate orders across states, the larger κ_j. The variable κ_j is the relevant measure of market informativeness for stock j in the following sense. Consider any arbitrary binary decision problem in which the final payoff depends on θ_j and the decision maker observes market activity before making a decision. Then, the expected utility of the decision maker is increasing in κ_j (see Appendix A.12 for a formal statement and proof). Lemma 1 characterizes trading strategies once learning decisions have been made.

[12]Recall that firm j's compensation contract uniquely pins down γ_j, so the investment cutoff $\overline{X}_j$ reflects that compensation contract. Also, recall that γ^* summarizes the contract that leads the manager to maximize shareholder value ex post (after market activity has been observed).

Lemma 1 (Equilibrium in the trading stage)**.** *Fix any* $\overline{X}_j \leq 0$.[13] *In equilibrium, speculators buy (sell) as much as possible upon receiving good (bad) news about a firm* j*:* $s_i(m_H) = \omega$ *and* $s_i(m_L) = -\omega/\zeta$ *for all* $i \in \Omega_j$.

Trading decisions are as expected: once an agent has learned about a given security, she buys on good news and sells on bad news. Lemma 1 implies that, in equilibrium,

$$\kappa_j = c\alpha_j^2, \quad \text{where} \quad c \equiv \frac{\omega^2 (1+\zeta)^2 (2\lambda - 1)^2}{2\zeta^2 \sigma^2}$$

and α_j is the result of speculators' learning decisions in the previous stage. The parameter c captures all the exogenous factors affecting market informativeness for security j: c is increasing in speculators' wealth ω and the precision of their signals λ, and decreasing in the short-selling margin requirement ζ and the amount of noise trading σ. The endogenous component of market informativeness is captured by α_j, the mass of speculators that choose to learn about stock j.

Before we turn to the learning decision of speculators, we need to compute their expected profits for trading each security j in the subsequent round. We denote by $\Phi(\cdot)$ and $\phi(\cdot)$ the standard normal CDF and PDF, respectively.

Lemma 2 (Trading profits)**.** *Fix any finite* $\overline{X}_j \leq 0$*. Under the optimal trading strategy, the expected trading profit per dollar traded in stock* j *is given by*

$$\pi\left(\kappa_j; \overline{X}_j\right) = (2\lambda - 1)\left[1 - \Phi\left(\frac{\overline{X}_j + \kappa_j}{\sqrt{2\kappa_j}}\right)\right] \tag{5}$$

for any $\kappa_j > 0$*, which is strictly decreasing in* $\overline{X}_j$ *and* κ_j.

Note that trading profits only depend on other speculators' strategies through $\kappa_j = c\alpha_j^2$. To understand why trading profits are decreasing in $\overline{X}_j$, notice that speculators only have information about the profitability of the investment opportunity. If the firm is unlikely

[13] Compensation policies that imply $\overline{X}_j > 0$ are off the equilibrium path, so we refrain from discussing that case in the main text.

to undertake investment ($\overline{X}_j$ is large), there is little potential to convert this informational advantage into profits.

Lemma 2 also shows that, if the manager of firm j plays an investment cutoff $\overline{X}_j \leq 0$, learning decisions are strategic substitutes: the more speculators are expected to learn about (and trade) security j, the less a speculator wants to do so.[14] For intuition, let $\overline{X}_j \leq 0$ and consider a limiting case where there is no informed trading on firm j's stock ($\kappa_j = X_j = 0$). Then, firm j's manager invests with probability one, following her prior. As more speculators trade stock j ($\kappa_j > 0$), the manager starts to put weight on market information, and may refrain from investing if a low order realizes, which reduces expected trading profits. These properties of the profit function will be useful for the remainder of the equilibrium characterization.

The equilibrium condition for learning is straightforward: If an agent chooses to learn about security j, then it must be that $\pi\left(c\alpha_j^2; \overline{X}_j\right) \geq \pi\left(c\alpha_{j'}^2; \overline{X}_{j'}\right)$, for every $j' \neq j$. It follows from this condition and Lemma 2 that, if all firms have a common investment cutoff $\overline{X} \leq 0$ (due to the choice of a common compensation contract), then their stocks have an equal share of informed trading, $\alpha_j = \overline{\alpha}/J$ for all j.[15]

3.3 Equilibrium contracts

Finally, we can solve for firms' choice of managerial compensation contracts and derive the main results of the paper. Note that the equilibrium in the learning, trading and investment subgames only depend on compensation contracts through the implied investment cutoffs $\overline{X}_j$ (which have a one-to-one map to the chosen conservatism ratios γ_j). Hence, we describe a firm's choice in terms of the investment cutoff directly, instead of γ_j.

In order to characterize the equilibrium investment cutoffs $\overline{X}_1, ..., \overline{X}_J$, we first need to describe how ex-ante firm value depends on firms' decisions and on speculators' choices. Firm value, as was the case for trading profits, only depends on speculators' strategies through

[14] When $\overline{X}_j > 0$ (which is off the equilibrium path), learning decisions are not strategic substitutes globally.
[15] See Lemma A.3 in the appendix for a proof.

market informativeness κ_j, as shown in the next lemma.

Lemma 3. *For any $\kappa_j > 0$, the ex-ante value of firm j can be written as*

$$\mathcal{V}_j(\kappa_j, \overline{X}_j) = \frac{1}{2}\left[V_H - \Phi\left(\frac{\overline{X}_j - \kappa_j}{\sqrt{2\kappa_j}}\right)(V_H - V_0)\right] + \frac{1}{2}\Phi\left(\frac{\overline{X}_j + \kappa_j}{\sqrt{2\kappa_j}}\right)V_0 - \underline{u}. \tag{6}$$

Notice that, although for a given κ_j the value of firm j does not depend on other firms' compensation policies, in equilibrium it will depend on those policies through their effect on κ_j (that is, through their effect on speculators' learning decisions).

3.3.1 Equilibrium contracts in the benchmark case

We now characterize the equilibrium investment cutoffs in our autarky benchmark, in which markets are segmented and therefore firms do not compete to attract informed trading.

Proposition 1 (Benchmark)**.** *Under autarky, firms choose the cutoff that maximizes shareholder value ex post, $\overline{X}_j = \overline{X}^*$ for every j. Market informativeness for each security is $\kappa_j = c\left(\overline{\alpha}/J\right)^2$.*

By choosing the contract that maximizes shareholder value ex post, firms can guarantee the maximum possible value of market informativeness with segmented markets. Hence, not surprisingly, that is what they choose. The only reason to use managerial incentives as to commit to some investment policy that is not optimal ex post is to try to boost market informativeness.

3.3.2 Equilibrium contracts with competition for market feedback

We now characterize the equilibrium cutoffs $\overline{X}_1, ..., \overline{X}_J$ in the main model. Next proposition shows our first main result, namely that competition for market feedback may completely shut down stock markets' potential to improve real efficiency.

Proposition 2 (Bad equilibrium)**.** *When firms compete for market feedback, there is always an equilibrium in which firms do no react to market information: $\overline{X}_j = -\infty$ for every j.*

There is always an equilibrium in which all managers act solely based on their prior, regardless of how informative financial markets are about the profitability of investment. In particular, a possible equilibrium outcome is that market activity is as informative as possible about the fundamentals of firm j, but its manager does not react to market activity anyway. Hence, *revelatory efficiency* (the extent to which market activity provides information useful to improve real efficiency) fails to translate into *gains in real efficiency* (the extent to which market activity *effectively* improves firm value).[16]

The stark result of Proposition 2 is the consequence of a rat race. Suppose that initially all firms choose a finite cutoff $\overline{X} \leq \overline{X}^*$, and thus react to information stemming from financial markets. Firm j may have incentives to deviate to a lower cutoff (conservatism ratio) as to get a larger portion of informed trading: By being more prone to invest, firm j boosts trading profits for its own stock and attracts a larger fraction of speculators. This deviation may pay off despite the implied distortion on the managerial use of information ex post. But other firms may then have incentives to undercut firm j's conservatism ratio. Through this process in which one firm tries to be more aggressive than the other, they end up choosing very unresponsive investment strategies that prevent them from reaping the benefits of informed trading.

It is an interesting exercise to compare our setting to an analogous model with a single firm (the typical case in the feedback literature). If we had only one firm, it is easy to see that the unique equilibrium in the model would be as the one of the autarky case (Proposition 1): The firm would learn as much as possible from financial markets and would use this information as efficiently as possible to increase firm value ($\overline{X}_1 = \overline{X}^*$ and $\kappa_1 = c\overline{\alpha}^2$). Once we introduce a second firm in the model, a dramatically different equilibrium arises in which, due the competition for market feedback, neither firm reacts to market activity and there are no efficiency gains whatsoever from this informational channel.

The equilibrium of Proposition 2 is not the unique equilibrium in our model. There exists

[16]For a discussion of the concept of revelatory efficiency, see Bond, Edmans and Goldstein (2012).

another, less extreme equilibrium in which firms still react to market activity. However, they react less than they would under autarky, and expected firm value is lower. The next proposition describes this other equilibrium.

Proposition 3 (Good equilibrium)**.** *When firms compete for market feedback, ex-ante firm value is always lower than under autarky. There exists only one equilibrium with* $\overline{X} \neq -\infty$ *(that is, one additional equilibrium besides that of Proposition 2). In such equilibrium, firms choose the cutoff* $\overline{X} < \overline{X}^*$ *that satisfies*

$$\frac{J}{J-1}\left[1 - J\frac{\overline{X}}{c\overline{\alpha}^2}\right] = \frac{e^{\overline{X}^*} + e^{\overline{X}}}{e^{\overline{X}^*} - e^{\overline{X}}}, \tag{7}$$

and market informativeness for each security is $\kappa_j = c(\overline{\alpha}/J)^2$*.*

Proposition 3 highlights that, even in a less extreme equilibrium where managers still react to market activity, an inefficiency arises: Firms offer managerial contracts that induce overinvestment to attract informed trading, but end up with the same level of informativeness as they would in autarky. Because of competition for market feedback, the equilibrium features a distortion in all firms' risk taking behavior ($\overline{X}$ is strictly smaller than $\overline{X}^*$).

Why cannot shareholder value maximization by all firms be sustained in equilibrium? Suppose that initially all firms choose the (ex-post) shareholder value maximizing cutoff $\overline{X}^*$. Then, any firm j necessarily has incentives to deviate to a lower cutoff. This deviation has to pay off despite the implied distortion on the managerial use of information ex post. This is so because when the use of information is the very best ($\overline{X}_j = \overline{X}^*$), the marginal cost of distorting the managerial decision is zero—an envelope theorem reasoning—while the benefits in terms of market informativeness is always positive. Figure 2 illustrates firms' incentives to deviate from shareholder value maximization with $J = 2$.[17]

[17]The reader might wonder whether the two equilibria presented survive a selection criterion based on some stability concept. We have numerically verified that both equilibria are stable, for all parameters considered, in the following sense: In a neighborhood of each equilibrium, the iteration of firms' best-responses converges to that equilibrium.

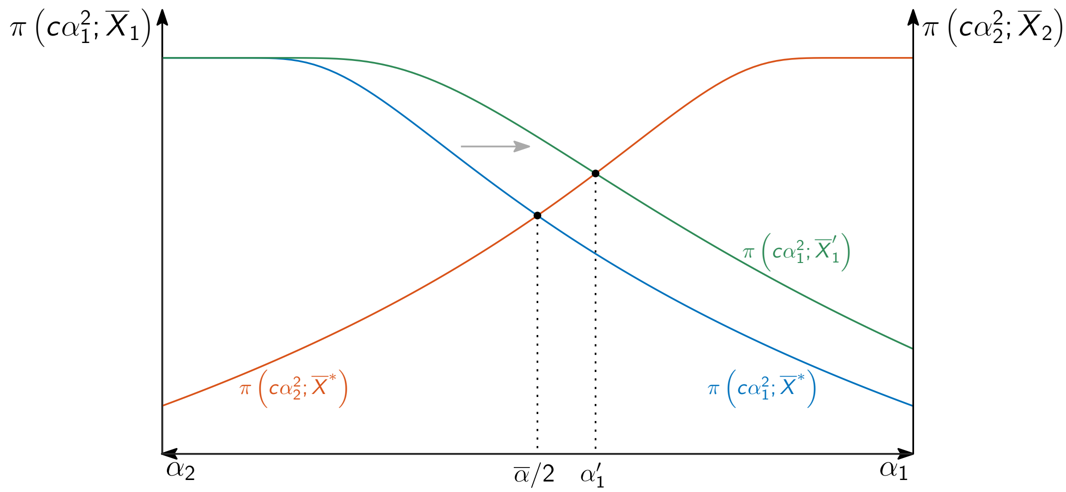

Figure 2: Deviation from shareholder value maximization. The length of the horizontal axis is $\overline{\alpha}$; α_1 runs from left to right, and α_2 runs from right to left. The figure shows trading profits for firms 1 and 2 as a function of α_1 and α_2, respectively, and for different investment cutoffs. If initially both firms choose $\overline{X}^*$, each gets half of total informed trading. However, the best response of firm 1 would be to deviate to $\overline{X}_1' < \overline{X}^*$, which would increase its share of informed trading to α_1'.

3.4 Comparative statics

In this section we conduct comparative statics to study how market parameters affect managerial conservatism and consequently firms' use of market information. Naturally, we focus on the good equilibrium (Proposition 3) in this section, since the managerial contract is independent of parameters in the bad equilibrium (Proposition 2).

Proposition 4. *Consider the good equilibrium (Proposition 3). Managerial conservatism (as measured by $\overline{X}$) is decreasing in the mass of sophisticated speculators $\overline{\alpha}$, their wealth ω and the precision of their signals λ, and is increasing in the strength of short-selling constraints ζ and the amount of noise trading σ.*

The intuition is the following. The larger the market potential to aggregate information, the stronger the firms' incentives to compete for market feedback by distorting managerial incentives. Therefore, better availability of information ends up leading to worse use of information by managers in equilibrium.

We now analyze the effect on managerial conservatism of a change in the number of firms that have stocks publicly traded.

Proposition 5. *Consider the good equilibrium (Proposition 3). For large enough $\overline{\alpha}$, managerial conservatism is decreasing (increasing) in the number of firms for low (high) J.*

To understand why the effect of J on managerial conservatism may be non-monotonic, notice that an increase in J has two effects: a *competition effect* and an *information supply* effect. First, an increase in J makes competition for market feedback fiercer, as there are more firms trying to attract informed trading. This increases incentives for firms to become more aggressive in their compensation contracts (decreasing $\overline{X}$). Second, it reduces the potential for the market to aggregate information for a given firm, as the number of informed traders per firm is smaller. The latter effect on the information supply per firm makes an increase in J reduce incentives for firms to distort the managerial decision (increasing $\overline{X}$). As J becomes large relative to the measure of speculators $\overline{\alpha}$, the second effect dominates.

In fact, if the measure of speculators increases in the same proportion when the number of publicly traded firms increases (so that $\overline{\alpha}/J$ remains constant), only the competition effect is present. In this case, the equilibrium cutoff $\overline{X}$ decreases globally in J. The next proposition states this result formally. Figure 3 illustrates the results of Propositions 5 and 6.

Proposition 6. *Consider the good equilibrium (Proposition 3). If J and $\overline{\alpha}$ increase in the same proportion, managerial conservatism decreases.*

In our model, $\overline{\alpha}$ and J capture two dimensions of financial market development: the participation of sophisticated speculators in financial markets and the number of publicly traded firms. Proposition 6 predicts that as these two dimensions advance hand-in-hand, firms take more excessive risk. Also, it comes as a corollary of this result that a proportional increase in the number of traded firms and the number of speculators is not neutral for expected firm value: it reduces the expected value of each *individual* publicly traded firm, due to the amplification of the competition motive. In the next section, we analyze the effects of our mechanism for efficiency from an aggregate perspective.

(a) Effect of J on $\overline{X}$ (b) Effect of J on $\overline{X}$ via competition effect

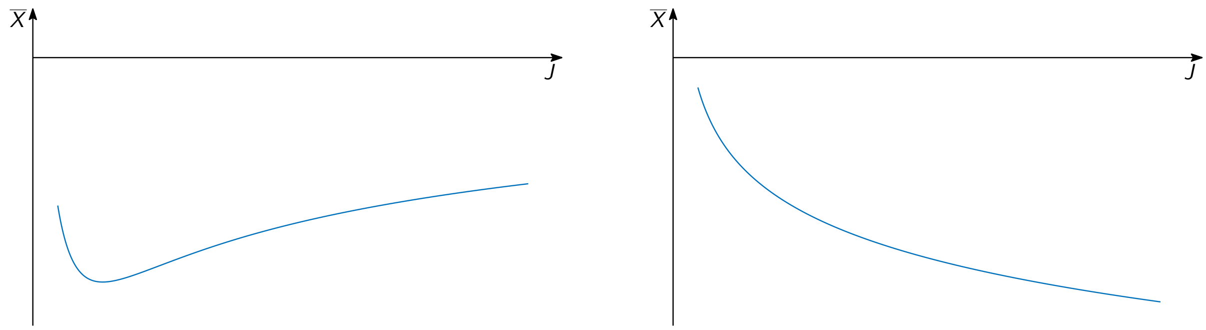

Figure 3: Managerial conservatism and number of firms. The left panel shows how managerial conservatism changes as J increases. The right panel does the same exercise, but increasing $\overline{\alpha}$ as J increases as to keep $\overline{\alpha}/J$ constant.

4 Implications for real efficiency

In this section we present some normative implications of our mechanism. First, we discuss how firms' ability to commit to certain investment policies collectively reduces real efficiency. Then, we extend our main model to incorporate endogenous listing of stocks, which allows us to study the inefficiencies that arise in firms' decisions to go public.

4.1 A commitment paradox and implications for regulation

By offering managers a certain compensation contract at the initial stage, firms can effectively commit to particular investment policies that are time-inconsistent. Ex post, after market information has been revealed, shareholders would always prefer to enforce a shareholder value maximizing strategy—investing if and only if $X_j \geq \overline{X}^*$. The reason why firms commit to time-inconsistent investment policies is to try to improve their informational environment, but as Propositions 2 and 3 make clear, this turns out to be very ineffective. In fact, this sort of commitment power reduces firm value: If firms could sign a contract and collectively *commit not to commit*, they would all achieve higher firm value. This is what we call a *commitment paradox*.

The commitment paradox implies that regulations that actively limit managerial risk taking in publicly listed firms may improve real efficiency, defined here as the total expected value of all firms. For instance, consider a social planner that enforces shareholder value maximization through some regulation, so $\overline{X}_j = \overline{X}^*$ for every j. Such a regulation would implement the autarky equilibrium and eliminate all the inefficiencies arising from firms' competition for market feedback. The next corollary summarizes that discussion.

Corollary 1 (Commitment paradox)**.** *If firms were unable to commit to investment policies, real efficiency would increase.*

The fact that firms can fully delegate investment decisions to managers and set up compensation schemes ex ante reduces real efficiency. This result contrasts with other models of feedback in which deviating from shareholder value maximization can increase firm value, such as Lin, Liu and Sun (2019) and Dow, Goldstein and Guembel (2017). In those papers, commitment to investment strategies increases information production in financial markets, and the results rely on the presence of information acquisition costs or trading costs. However, unlike here, in those papers firms do not compete for market feedback. We have then shown that once one considers the strategic interactions of multiple firms in integrated financial markets, a force in favor of ex-post shareholder value maximization arises.[18]

4.2 IPOs and real efficiency

In this section, our goal is to understand the inefficiencies that arise in firms' decision to go public and how those inefficiencies can be fixed.

We extend our model by adding a previous stage in which ex-ante identical firms can decide whether to have their stocks publicly traded upon paying an underwriting fee $\psi > 0$. The total number of firms in the economy is denoted by $\tilde{J} \geq 2$. The number of firms that go public is still denoted by J, but is now endogenous. Naturally, firms that are not listed have nothing to learn from financial markets and simply invest according to their prior. We

[18] If we introduced information acquisition or trading costs, our results would hold as long as those costs are sufficiently low.

assume that underwriting costs are not prohibitively high: $\psi < \mathcal{V}_j\left(c\overline{\alpha}^2, \overline{X}^*\right) - (V_H/2 - \underline{u})$, which ensures that going public is not a strictly dominated choice.

To understand the inefficiencies that arise in firms' decision to go public, we also consider the problem of a social planner that chooses J as to maximize total welfare $\mathcal{W}$, which is defined as the expected total wealth in the economy:

$$\mathcal{W} = \sum_{j=1}^{J} \mathbb{E}\left[\mathcal{V}_j\right] + \sum_{j=J+1}^{\tilde{J}} \left(\frac{V_H}{2} - \underline{u}\right) - J\psi.$$

Since firms are ex-ante identical, without loss of generality we assume the planner assigns the firms with lowest indexes to go public. After the social planner chooses J, there are no further interventions, and a decentralized equilibrium is played.

It is straightforward to see that if $J = 1$ the unique equilibrium in the subsequent stages has all speculators trading the firm's security and the firm maximizing shareholder value ex post ($\overline{X}_1 = \overline{X}^*$). If $J \geq 2$, we know from Section 3 that there are two equilibria in the subsequent stages. We assume in this section that the good equilibrium of Proposition 3 is played in that case.[19] Next proposition compares the social optimum to the decentralized equilibrium.

Proposition 7. *Let J^* be the socially optimal number of publicly listed firms and J^{eq} be the number of publicly listed firms in a decentralized equilibrium. Then, $J^* \leq J^{eq}$ and whenever $J^* < J^{eq}$ the social optimum can be implemented with a Pigouvian tax on IPOs.*

In the social optimum, the planner strikes a balance between granting access to market information for a large number of firms and preventing too much distortion in their managerial choices due to the competition effect.

In a decentralized equilibrium, firms do not internalize how their decision to go public affects all other publicly listed firms. When an additional firm goes public, it affects the value

[19] If the bad equilibrium were to be played, it is trivial that the optimal J would be one. Since in the bad equilibrium competition drives the value of publicly listed firms to the value of private firms, having $J \geq 2$ would only mean incurring additional underwriting costs without any benefit.

of other listed firms $\mathcal{V}_j\left(\kappa_j, \overline{X}_j\right)$ in two ways. First, there is a negative externality through κ_j: an additional firm reduces the amount of information each firm obtains from the market. Second, there is an externality through $\overline{X}_j$ that can be positive or negative, depending on whether a larger J increases or reduces managerial conservatism in equilibrium (Proposition 5). However, it turns out that the overall externality is always negative: one additional listed firm always reduces the value of the existing public firms.

The intuition for this is as follows. The externality through $\overline{X}_j$ is positive if an increase in J leads to an increase in managerial conservatism. Remember from Section 3.4 that an increase in J has two effects on managerial conservatism: a competition effect (which pushes conservatism downwards) and a information supply effect (which pushes conservatism upwards). Hence, the externality through $\overline{X}_j$ is positive if the information supply effect dominates, which happens if information is very valuable for firms. But then it is also the case that the negative externality through κ_j is large. In short, the positive externality through $\overline{X}_j$ can only exist when the negative externality through κ_j is large, which is the reason why the combined externality is always negative.

Each firm compares the private benefit they get by accessing market information with the underwriting cost, without internalizing the negative externality on others. Therefore, in the decentralized equilibrium it is often the case that too many firms decide to go public, and a tax on IPOs can correct the inefficiency.[20]

As a final remark, note that even if one assumes that any increase in the number of listed firms J leads to a proportional increase in the mass of speculators $\overline{\alpha}$ (as in Proposition 6), the results in Proposition 7 would remain unchanged. The reason is that increases in J would keep market informativeness for each firm (κ_j) constant and reduce managerial conservatism ($\overline{X}_j$) in equilibrium, so the negative externality of IPOs would still be present.

[20]Due to the discreteness of J, J^* and J^{eq} could coincide for some parameters despite the negative externality.

5 Concluding remarks

This paper studies how firms compete to attract informed trading when secondary financial markets provide useful information to decisions makers. From a substantive point of view, the paper emphasizes that competition for market feedback leads to a rat race that induces excessive managerial risk taking and under-reaction to market activity, potentially eliminating all the gains in real efficiency that learning from financial markets could generate. From a methodological perspective, the paper contributes to the feedback literature by highlighting the importance of understanding the strategic interaction of firms in integrated financial markets, since it can lead to critically different equilibrium outcomes.

References

Albagli, E., Hellwig, C. and Tsyvinski, A. (2018): Imperfect financial markets and shareholder incentives in partial and general equilibrium. *Toulouse School of Economics (TSE) Working Paper*.

Bakke, T.E. and Whited, T.M. (2010): Which firms follow the market? An analysis of corporate investment decisions. *Review of Financial Studies*, 23, 5, 1941–1980.

Boleslavsky, R., Kelly, D. and Taylor, C. (2017): Selloffs, bailouts, and feedback: Can asset markets inform policy? *Journal of Economic Theory*, 169, 294–343.

Bond, P., Edmans, A. and Goldstein, I. (2012): The real effects of financial markets. *Annu. Rev. Financ. Econ.*, 4, 1, 339–360.

Bond, P. and Eraslan, H. (2010): Information-based trade. *Journal of Economic Theory*, 145, 5, 1675–1703.

Bond, P. and Goldstein, I. (2015): Government intervention and information aggregation by prices. *Journal of Finance*, 70, 6, 2777–2812.

CHEN, Q., GOLDSTEIN, I. and JIANG, W. (2006): Price informativeness and investment sensitivity to stock price. *Review of Financial Studies*, 20, 3, 619–650.

DA, Z., ENGELBERG, J. and GAO, P. (2011): In search of attention. *Journal of Finance*, 66, 5, 1461–1499.

DOW, J., GOLDSTEIN, I. and GUEMBEL, A. (2017): Incentives for information production in markets where prices affect real investment. *Journal of the European Economic Association*, 15, 4, 877–909.

DOW, J. and GORTON, G. (1997): Stock market efficiency and economic efficiency: Is there a connection? *Journal of Finance*, 52, 3, 1087–1129.

EDMANS, A., GOLDSTEIN, I. and JIANG, W. (2012): The real effects of financial markets: The impact of prices on takeovers. *Journal of Finance*, 67, 3, 933–971.

EDMANS, A., GOLDSTEIN, I. and JIANG, W. (2015): Feedback effects, asymmetric trading, and the limits to arbitrage. *American Economic Review*, 105, 12, 3766–3797.

EDMANS, A., JAYARAMAN, S. and SCHNEEMEIER, J. (2017): The source of information in prices and investment-price sensitivity. *Journal of Financial Economics*, 126, 1, 74–96.

FERSHTMAN, C. and JUDD, K.L. (1987): Equilibrium incentives in oligopoly. *American Economic Review*, 927–940.

FISHMAN, M.J. and HAGERTY, K.M. (1989): Disclosure decisions by firms and the competition for price efficiency. *Journal of Finance*, 44, 3, 633–646.

FOUCAULT, T. and FRÉSARD, L. (2012): Cross-listing, investment sensitivity to stock price, and the learning hypothesis. *Review of Financial Studies*, 25, 11, 3305–3350.

FOUCAULT, T. and FRÉSARD, L. (2019): Corporate strategy, conformism, and the stock market. *Review of Financial Studies*, 32, 3, 905–950.

GOLDSTEIN, I. and GUEMBEL, A. (2008): Manipulation and the allocational role of prices. *Review of Economic Studies*, 75, 1, 133–164.

Goldstein, I. and Yang, L. (2019): Good disclosure, bad disclosure. *Journal of Financial Economics*, 131, 1, 118–138.

Jayaraman, S. and Wu, J.S. (2019): Should I stay or should I grow? Using voluntary disclosure to elicit market feedback. *Review of Financial Studies, Forthcoming.*

Katz, M.L. (1991): Game-playing agents: Unobservable contracts as precommitments. *The RAND Journal of Economics*, 307–328.

Kyle, A. (1985): Continuous auctions and insider trading. *Econometrica*, 53, 6, 1315–1336.

Lin, T.C., Liu, Q. and Sun, B. (2019): Contractual managerial incentives with stock price feedback. *American Economic Review*, 109, 7, 2446–68.

Luo, Y. (2005): Do insiders learn from outsiders? Evidence from mergers and acquisitions. *Journal of Finance*, 60, 4, 1951–1982.

Machado, C. and Pereira, A.E. (2020): Optimal investment and capital structure with stock market feedback.

Schneemeier, J. (2019): Optimal disclosure and fight for attention.

Shleifer, A. and Vishny, R.W. (1997): The limits of arbitrage. *Journal of finance*, 52, 1, 35–55.

Sicherman, N., Loewenstein, G., Seppi, D.J. and Utkus, S.P. (2016): Financial attention. *Review of Financial Studies*, 29, 4, 863–897.

Sims, C.A. (2003): Implications of rational inattention. *Journal of Monetary Economics*, 50, 3, 665–690.

Strobl, G. (2014): Stock-based managerial compensation, price informativeness, and the incentive to overinvest. *Journal of Corporate Finance*, 29, 594–606.

Veldkamp, L.L. (2011): *Information choice in macroeconomics and finance.* Princeton University Press.

Yang, L. (2020): Disclosure, competition and learning from asset prices. *Rotman School of Management Working Paper*, 3095970.

A Appendix

For convenience, we start by proving two auxiliary results. The next lemma nests Lemma 2.

Lemma A.1. *For a speculator receiving a good (bad) signal about stock j, the expected trading profit per dollar bought (sold) of that stock is given by (5) for any $\kappa_j > 0$, and for $\kappa_j = 0$ it is given by*

$$\pi\left(0; \overline{X}_j\right) = \begin{cases} 2\lambda - 1 & \text{if } \overline{X}_j \leq 0, \\ 0 & \text{if } \overline{X}_j > 0. \end{cases} \tag{A.1}$$

$\pi\left(\kappa_j; \overline{X}_j\right)$ is strictly decreasing in $\overline{X}_j$ for $\kappa_j > 0$, and strictly decreasing in κ_j for any finite $\overline{X}_j \leq 0$.

Proof. From (3), note that conditional on $\theta_j = H$, $X_j = \kappa_j + \frac{S_{j,H} - S_{j,L}}{\sigma^2} n$ and is then normally distributed with mean κ_j and variance $2\kappa_j$. Conditional on $\theta = L$, $X_j = -\kappa_j + \frac{(S_{j,H} - S_{j,L})}{\sigma^2} n$ and is normally distributed with mean $-\kappa_j$ and variance $2\kappa_j$. Suppose agent i has chosen to learn about stock j, i.e., $\ell_i(\boldsymbol{\gamma}) = j$. Define $R_L = 0$ as the dividend in the low state. For a given realization of $\theta_j \in \{L, H\}$ and X_j, the realized trading profit of placing an order of s_i dollars for stock j is $\frac{s_i}{p_j(X_j)} \left[R_{\theta_j} - p_j(X_j)\right]$ if $X_j \geq \overline{X}_j$ and zero otherwise (recall that $p_j(X_j) = R_0$ if $X_j < \overline{X}_j$, so the price is equal to the realized dividend). Using equation (4) and the distribution of X_j, the expected trading profit from placing an order of s_i dollars for stock j is

$$s_i \left\{ \eta(m_{i,j}) \int_{\overline{X}_j}^{\infty} \frac{[1 - \mu_j(u)]}{\mu_j(u)} \frac{1}{\sqrt{2\kappa_j}} \phi\left(\frac{u - \kappa_j}{\sqrt{2\kappa_j}}\right) du - [1 - \eta(m_{i,j})] \int_{\overline{X}_j}^{\infty} \frac{1}{\sqrt{2\kappa_j}} \phi\left(\frac{u + \kappa_j}{\sqrt{2\kappa_j}}\right) du \right\}$$

for any $\kappa_j > 0$. Using the fact that $(1 - \mu_j(u)) \phi\left(\frac{u - \kappa_j}{\sqrt{2\kappa_j}}\right) = \mu_j(u) \phi\left(\frac{u + \kappa_j}{\sqrt{2\kappa_j}}\right)$, the expression above can be rewritten as

$$s_i \left[2\eta(m_{i,j}) - 1\right] \left[1 - \Phi\left(\frac{\overline{X}_j + \kappa_j}{\sqrt{2\kappa_j}}\right)\right]. \tag{A.2}$$

Notice that expected trading profits only depend on other speculators' strategies through κ_j. Bayes rule implies $\eta(m_H) = \lambda > 1/2$ and $\eta(m_L) = 1 - \lambda < 1/2$, and therefore dividing the expression above by $|s_i|$ yields the expression in (5), proving the first statement.

Now consider $\kappa_j = 0$. In that case $X_j = 0$ with probability one, and $\mu_j(0) = 1/2$. If $\overline{X}_j \leq 0$, the firm invests with probability one, the stock price is $p_j(0) = R_H/2$ and trading profits are given by

$$\eta(m_{i,j}) \frac{s_i}{p_j(0)} \left[R_H - p_j(0)\right] + \left[1 - \eta(m_{i,j})\right] \frac{s_i}{p_j(0)} \left[0 - p_j(0)\right] = s_i \left[2\eta(m_{i,j}) - 1\right], \tag{A.3}$$

which equals $s_i(2\lambda-1)$ for a speculator receiving a good signal and $-s_i(2\lambda-1)$ for a speculator receiving a bad signal. If instead $\overline{X}_j > 0$, the firm does not invest, prices are equal to the dividend R_0, and trading profits are always zero. Hence, the expected trading profit per dollar bought (sold) of stock j following a good (bad) signal is given by (A.1) for $\kappa_j = 0$, proving the second statement.

That $\pi\left(\kappa_j;\overline{X}_j\right)$ is strictly decreasing in $\overline{X}_j$ for $\kappa_j > 0$ is straightforward. Finally,

$$\frac{\partial \pi\left(\kappa_j;\overline{X}_j\right)}{\partial \kappa_j} = (2\lambda-1)\phi\left(\frac{\overline{X}_j+\kappa_j}{\sqrt{2\kappa_j}}\right)\frac{\overline{X}_j-\kappa_j}{(2\kappa_j)^{3/2}}, \tag{A.4}$$

which is strictly negative for $-\infty < \overline{X}_j \leq 0$ and $\kappa_j > 0$. Moreover, for any $\kappa_j > 0$, $\pi\left(\kappa_j;\overline{X}_j\right) \leq 2\lambda - 1 = \pi\left(0;\overline{X}_j\right)$, which concludes the proof. □

Lemma A.2. *Let firm value $\mathcal{V}_j(\kappa_j,\overline{X}_j)$ for a given $\overline{X}_j$ and $\kappa_j > 0$ be as given by (6). Then,*

$$sign\left(\frac{\partial \mathcal{V}_j(\kappa_j,\overline{X}_j)}{\partial \kappa_j}\right) = sign\left(\overline{X}_j\left(e^{\overline{X}_j}-e^{\overline{X}^*}\right)+\kappa_j\left(e^{\overline{X}_j}+e^{\overline{X}^*}\right)\right), \tag{A.5}$$

$$sign\left(\frac{\partial \mathcal{V}_j(\kappa_j,\overline{X}_j)}{\partial \overline{X}_j}\right) = sign\left(\overline{X}^*-\overline{X}_j\right). \tag{A.6}$$

Proof. The proof is analogous to Lemma A.2 in Machado and Pereira (2020) for the case of a single firm. Taking partial derivatives of (6), we have:

$$\frac{\partial \mathcal{V}_j}{\partial \kappa_j} = \frac{1}{2}\phi\left(\frac{\overline{X}_j-\kappa_j}{\sqrt{2\kappa_j}}\right)(V_H-V_0)\left(\frac{\overline{X}_j+\kappa_j}{2^{3/2}\kappa_j^{3/2}}\right) - \frac{1}{2}\phi\left(\frac{\overline{X}_j+\kappa_j}{\sqrt{2\kappa_j}}\right)V_0\left(\frac{\overline{X}_j-\kappa_j}{2^{3/2}\kappa_j^{3/2}}\right). \tag{A.7}$$

Using the facts that $\phi\left(\frac{\overline{X}_j-\kappa_j}{\sqrt{2\kappa_j}}\right)/\phi\left(\frac{\overline{X}_j+\kappa_j}{\sqrt{2\kappa_j}}\right) = e^{\overline{X}_j}$ and $\frac{V_0}{V_H-V_0} = e^{\overline{X}^*}$, we get that the expression above has the same sign as $\overline{X}_j\left(e^{\overline{X}_j}-e^{\overline{X}^*}\right)+\kappa_j\left(e^{\overline{X}_j}+e^{\overline{X}^*}\right)$. Moreover, we have that

$$\frac{\partial \mathcal{V}_j}{\partial \overline{X}_j} = -\frac{1}{2\sqrt{2\kappa_j}}(V_H-V_0)\phi\left(\frac{\overline{X}_j-\kappa_j}{\sqrt{2\kappa_j}}\right) + \frac{1}{2\sqrt{2\kappa_j}}V_0\phi\left(\frac{\overline{X}_j+\kappa_j}{\sqrt{2\kappa_j}}\right), \tag{A.8}$$

which, one can verify, has the same sign as $(\overline{X}^*-\overline{X}_j)$. □

A.1 Proof of Lemma 1

Fix $\overline{X}_j \leq 0$. Recall that expected trading profits are given by (A.2) for $\kappa_j > 0$ and by (A.3) for $\kappa_j = 0$. Since $\eta(m_H) = \lambda > 1/2$ and $\eta(m_L) = 1-\lambda < 1/2$, a speculator that learns about stock j wants to increase (respectively, decrease) s_i as much as possible upon receiving signal m_H (respectively, m_L). Hence, $s_i(m_H) = w$, $s_i(m_L) = -w/\zeta$. □

A.2 Proof of Lemma 2

Lemma 2 is a special case of Lemma A.1 above. □

A.3 Proof of Lemma 3

As already shown in the first paragraph of the proof of Lemma A.1, conditional on $\theta_j = H$, X_j is normally distributed with mean κ_j and variance $2\kappa_j$, and conditional $\theta_j = L$, it is normally distributed with mean $-\kappa_j$ and variance $2\kappa_j$. Therefore, $\Pr(a_j = 1|\theta_j = H) = \Pr\left(X_j > \overline{X}_j|\theta_j = H\right) = 1 - \Phi\left(\frac{\overline{X}_j - \kappa_j}{\sqrt{2\kappa_j}}\right)$ and $\Pr(a_j = 1|\theta_j = L) = \Pr\left(X_j > \overline{X}_j|\theta_j = L\right) = 1 - \Phi\left(\frac{\overline{X}_j + \kappa_j}{\sqrt{2\kappa_j}}\right)$. Substituting those probabilities in (1) yields the desired result. □

A.4 Proof of Proposition 1

From Lemma A.1, for any $\overline{X}_j \leq 0$ trading profits are strictly positive. Hence, if a firm chooses any cutoff $\overline{X}_j \leq 0$, speculators trade as much as possible, and $\kappa_j = c\left(\overline{\alpha}/J\right)^2$ (as in Lemma 1). By Lemma A.2, equation (A.6), we can then conclude that $\overline{X}_j = \overline{X}^*$ yields larger firm value than any other $\overline{X}_j \leq 0$. Now suppose firm j chooses some cutoff $\overline{X}'_j > 0$ that leads to some $\kappa'_j \leq c\left(\overline{\alpha}/J\right)^2$. By Lemma A.2 we have that $\mathcal{V}_j\left(\kappa'_j, \overline{X}'_j\right) < \mathcal{V}_j\left(\kappa'_j, \overline{X}^*\right) \leq \mathcal{V}_j\left(c\left(\overline{\alpha}/J\right)^2, \overline{X}^*\right)$. Hence, the firm chooses $\overline{X}_j = \overline{X}^*$. □

A.5 Proof of Proposition 2

Recall that, by Lemma A.1, trading profits per dollar are strictly decreasing in $\overline{X}_j$ for any $\kappa_j > 0$. Also, if a firm plays $\overline{X}_j = -\infty$, trading profits are $\pi\left(\kappa_j; \overline{X}_j\right) = 2\lambda - 1$ for all κ_j, which is the maximum possible trading profit. Suppose all firms play $\overline{X}_j = -\infty$, in which case their expected value is $V_H/2 - \underline{u}$. Wlog, consider a deviation by firm $j = 1$ to any other cutoff $\overline{X}'_1$. Since, $\pi\left(\kappa_1; \overline{X}'_1\right) < 2\lambda - 1$ for every $\kappa_1 > 0$, such deviation would lead to $\kappa_1 = \alpha_1 = 0$ in equilibrium, in which case the expected value of firm $j = 1$ would be, again, $V_H/2 - \underline{u}$. Hence, no firm has incentives to deviate, and there always exists an equilibrium with $\overline{X}_j = -\infty$ for all j. □

A.6 Proof of Proposition 3

We start by proving two auxiliary lemmas.

Lemma A.3. *Suppose that $\overline{X}_j = \overline{X}_{j'} = \overline{X} \leq 0$ for some $j \neq j'$, with $\overline{X} \neq -\infty$. Then, the equilibrium in the learning stage must have $\alpha_j = \alpha_{j'}$.*

Proof. Let $\overline{X}_j = \overline{X}_{j'} = \overline{X} \leq 0$ for some $j \neq j'$, with $\overline{X} \neq -\infty$. Suppose by contradiction that the equilibrium in the learning stage has $\alpha_{j'} > \alpha_j$ (and hence $\kappa_{j'} > \kappa_j$). Since $\pi\left(\kappa; \overline{X}\right)$ is strictly decreasing in κ, speculators in Ω_j would have incentives to deviate and learn about firm j' instead, so it cannot be an equilibrium. It must then be that $\alpha_{j'} = \alpha_j$ in the learning stage equilibrium. $\square$

Lemma A.4. *Suppose that in equilibrium $\overline{X}_j = \overline{X}$ for all j. Then, it must be that $\overline{X} < \overline{X}^*$.*

Proof. First, notice from (A.4) that $\pi\left(\kappa_j; \overline{X}_j\right)$ is strictly increasing in κ_j for $\kappa_j < \overline{X}_j$ and strictly decreasing for $\kappa_j > \overline{X}_j$. Also, note that the possible equilibrium values of κ are the interval $[0, \overline{\kappa}]$, where $\overline{\kappa} \equiv c\overline{\alpha}^2$. Suppose that in equilibrium $\overline{X}_j = \overline{X}$ for all j.

Case i) Suppose $\overline{X} \geq \overline{\kappa}$. Notice that $\pi\left(\kappa, \overline{X}\right)$ is strictly increasing for all $\kappa \in [0, \overline{\kappa}]$ and that $\frac{\partial \pi\left(\kappa; \overline{X}_j\right)}{\partial \overline{X}_j} < 0$ for any $\kappa > 0$ (Lemma A.1). Wlog, suppose firm $j = 1$ deviates to $\overline{X}_1 = \overline{X}^* < 0 < \overline{X}$. Since $\pi\left(\kappa; \overline{X}_1\right)$ becomes strictly decreasing in κ for all $\kappa \in [0, \overline{\kappa}]$, and $\pi\left(\overline{\kappa}; \overline{X}_1\right) > \pi\left(\overline{\kappa}; \overline{X}\right)$, we have that $\pi\left(\kappa; \overline{X}_1\right) > \pi\left(\kappa'; \overline{X}\right)$ for all $\kappa, \kappa' \in [0, \overline{\kappa}]$. Hence, the unique equilibrium in the learning stage following the deviation features $\kappa_1 = \overline{\kappa}$. Lemma A.2 then implies that $\mathcal{V}_1\left(\overline{\kappa}, \overline{X}_1\right) > \mathcal{V}_1\left(\overline{\kappa}, \overline{X}\right) \geq \mathcal{V}_1\left(\kappa, \overline{X}\right)$ for any $\kappa \leq \overline{\kappa}$, and therefore that deviation is profitable. Hence, all firms playing $\overline{X} \geq \overline{\kappa}$ cannot be an equilibrium.

Case ii) Suppose $0 < \overline{X} < \overline{\kappa}$. From (A.4), notice that $\pi\left(\kappa_j; \overline{X}\right)$ is single peaked in κ_j, with its maximum achieved when $\kappa_j = \overline{X}$. Suppose first that in equilibrium $\max_j \kappa_j \leq \overline{X}$. Wlog, assume that firm $j = 1$ deviates to $\overline{X}_1 = \overline{X}^* < 0$. Following the deviation, it must be that $\kappa_1 > \overline{X}$ in the trading stage. To see this, suppose by contradiction that $\kappa_1 \leq \overline{X}$ after the deviation. Using (A.4) and the fact that $\pi\left(\kappa_j, \overline{X}_j\right)$ is strictly decreasing in $\overline{X}_j$, we have that $\pi\left(\kappa_1; \overline{X}^*\right) \geq \pi\left(\overline{X}; \overline{X}^*\right) > \pi\left(\overline{X}; \overline{X}\right) \geq \pi\left(\kappa; \overline{X}\right)$ for all $\kappa \in [0, \overline{\kappa}]$. But then all speculators would choose to learn about the stock of firm 1, and after that deviation $\kappa_1 = \overline{\kappa} > \overline{X}$, a contradiction. Hence, following the deviation to $\overline{X}_1 = \overline{X}^*$, $\kappa_1 > \overline{X}$. Finally, notice that the deviation must be profitable: from Lemma A.2, $\mathcal{V}_1\left(\overline{X}^*, \kappa_1\right) > \mathcal{V}_1\left(\overline{X}^*, \kappa\right) > \mathcal{V}_1\left(\overline{X}, \kappa\right)$ for any $\kappa < \kappa_1$.

Now suppose that in equilibrium $\max_j \kappa_j > \overline{X}$. Once again, we will show that there exists j' such that $\kappa_{j'}$ increases following a deviation to $\overline{X}_{j'} = \overline{X}^*$. If initially the equilibrium has $\kappa_{j'} < \overline{X}$ for some j', firm j' clearly has incentives to deviate to $\overline{X}^*$ (once again, since $\pi\left(\kappa; \overline{X}^*\right) > \pi\left(\kappa'; \overline{X}\right)$ for every $\kappa < \overline{X}$ and $\kappa' \in [0, \overline{\kappa}]$ the firm would end up with a larger κ after that deviation). Suppose then that, instead, the initial equilibrium has $\kappa_j > \overline{X}$ for all j. Since $\pi\left(\kappa; \overline{X}\right)$ is strictly positive and strictly decreasing in κ for $\kappa > \overline{X}$ it must be that κ_j is the same for all j and $\kappa_j = c\left(\frac{\overline{\alpha}}{J}\right)^2 \equiv \kappa^\dagger$. If firm $j = 1$ deviates to $\overline{X}^*$, κ_1 must increase and the deviation must be profitable. To see this, suppose by contradiction that $\kappa_1 \leq \kappa^\dagger$ following the deviation. Then, $\pi\left(\kappa_1; \overline{X}^*\right) \geq \pi\left(\kappa^\dagger; \overline{X}^*\right) > \pi\left(\kappa^\dagger; \overline{X}\right) \geq \pi\left(\kappa; \overline{X}\right)$ for all $\kappa \geq \kappa^\dagger$, which implies $\kappa_j < \kappa^\dagger$ for all $j \neq 1$ after the deviation. But since, after the deviation trading profits are strictly positive for the security of firm 1 regardless of κ, every speculator must still trade as much as possible following the deviation, which leads to a contradiction: it cannot be that $\kappa_1 \leq \kappa^\dagger$ and that κ_j decreases for all $j \neq 1$ after the deviation. We have then shown that κ_1 must increase following the deviation to $\overline{X}^*$, which is then a profitable deviation. Hence, all firms playing $\overline{X} \in (0, \overline{\kappa})$ cannot be an equilibrium.

Case iii) Suppose $\overline{X}^* < \overline{X} \leq 0$. In this case $\pi\left(\kappa; \overline{X}\right)$ is strictly decreasing for all $\kappa \in [0, \overline{\kappa}]$, and by Lemma A.3, $\alpha_j = \frac{\overline{\alpha}}{J}$ for all j. Hence, since $\pi\left(\kappa; \overline{X}\right) > 0$ for all κ, $\kappa_j = c\left(\frac{\overline{\alpha}}{J}\right)^2 \equiv \kappa^\dagger$ for all j. Suppose firm $j = 1$ deviates to $\overline{X}^*$. Since $\pi\left(\kappa, \overline{X}^*\right) > \pi\left(\kappa, \overline{X}\right) > 0$ for every κ, this deviation must increase κ_1 and is therefore profitable (Lemma A.2 implies $\mathcal{V}_1\left(\kappa_1, \overline{X}^*\right) > \mathcal{V}_1\left(\kappa^\dagger, \overline{X}^*\right) > \mathcal{V}_1\left(\kappa^\dagger, \overline{X}\right)$ for $\kappa_1 > \kappa^\dagger$). Hence, all firms playing $\overline{X} \in \left(\overline{X}^*, 0\right]$ cannot be an equilibrium.

Case iv) Suppose $\overline{X} = \overline{X}^*$. Suppose that firm $j = 1$, deviates to a slightly lower cutoff $\overline{X}_1 < \overline{X}^*$. Notice that, from Lemma A.3, after the deviation we have that $\alpha_j = \frac{\overline{\alpha}-\alpha_1}{J-1}$ for all $j \neq 1$ in the learning stage equilibrium. Given the necessary condition for equilibrium in the learning stage presented in Section 3.2, the equilibrium in the learning stage following the deviation requires

$$\pi\left(c\alpha_1^2; \overline{X}_1\right) = \pi\left(c\left(\frac{\overline{\alpha}-\alpha_1}{J-1}\right)^2; \overline{X}^*\right) \Leftrightarrow \frac{\overline{X}_1 + c\alpha_1^2}{\sqrt{2c\alpha_1^2}} = \frac{\overline{X}^* + c\left(\frac{\overline{\alpha}-\alpha_1}{J-1}\right)^2}{\sqrt{2c\left(\frac{\overline{\alpha}-\alpha_1}{J-1}\right)^2}}.$$

Using the implicit function theorem,

$$\frac{\partial \alpha_1}{\partial \overline{X}_1} = -\frac{\alpha_1}{c\alpha_1^2 \frac{J}{J-1} - \overline{X}_1 - \overline{X}^* (J-1) \frac{\alpha_1^2}{(\overline{\alpha}-\alpha_1)^2}}.$$

At $\overline{X}_1 = \overline{X}^*$, $\alpha_1 = \frac{\overline{\alpha}}{J}$ and the derivative above becomes

$$\left.\frac{\partial \alpha_1}{\partial \overline{X}_1}\right|_{\overline{X}_1 = \overline{X}^*} = -\frac{(J-1)\,\overline{\alpha}}{c\overline{\alpha}^2 - J^2\overline{X}^*}.$$

We can compute the effect of a deviation by firm $j = 1$ on its value. Using (6), we have that

$$\begin{aligned}\frac{d\mathcal{V}_1}{d\overline{X}_1} = &-\frac{1}{2\sqrt{2\kappa_1}}(V_H - V_0)\,\phi\left(\frac{\overline{X}_1 - \kappa_1}{\sqrt{2\kappa_1}}\right) + \frac{1}{2\sqrt{2\kappa_1}} V_0 \phi\left(\frac{\overline{X}_1 + \kappa_1}{\sqrt{2\kappa_1}}\right) \\ &+ c\alpha_1\left[\phi\left(\frac{\overline{X}_1 - \kappa_1}{\sqrt{2\kappa_1}}\right)(V_H - V_0)\left(\frac{\overline{X}_1 + \kappa_1}{2^{3/2}\kappa_1^{3/2}}\right) - \phi\left(\frac{\overline{X}_1 + \kappa_1}{\sqrt{2\kappa_1}}\right) V_0\left(\frac{\overline{X}_1 - \kappa_1}{2^{3/2}\kappa_1^{3/2}}\right)\right]\frac{\partial \alpha_1}{\partial \overline{X}_1}, \quad \text{(A.9)}\end{aligned}$$

which evaluated at $\overline{X}_1 = \overline{X}^*$ has the same sign as $\left.\frac{\partial \alpha_1}{\partial \overline{X}_1}\right|_{\overline{X}_1 = \overline{X}^*}$, which is strictly negative. Therefore, if $\overline{X}_j = \overline{X}^*$ for all j, firm 1 would benefit from a marginal reduction in $\overline{X}_1$, so all firms playing $\overline{X}^*$ cannot be an equilibrium. □

We are now ready to prove Proposition 3. From Lemma A.4, if there exists a symmetric equilibrium where $\overline{X}_j = \overline{X} \neq -\infty$ for all j, then we must have $\overline{X} < \overline{X}^*$. Therefore, suppose all firms play according to some cutoff $\overline{X} < \overline{X}^*$. Suppose firm $j = 1$ makes a small deviation to some cutoff $\overline{X}_1 < 0$. Once again, the equilibrium in the learning and trading stages requires that $\alpha_j = \frac{\overline{\alpha}-\alpha_1}{J-1}$ for all $j \neq 1$, $\kappa_j = c\alpha_j^2$ for all j, and

$$\pi\left(c\alpha_1^2; \overline{X}_1\right) = \pi\left(c\left(\frac{\overline{\alpha} - \alpha_1}{J-1}\right)^2; \overline{X}\right) \Leftrightarrow \frac{\overline{X}_1 + c\alpha_1^2}{\sqrt{2c\alpha_1^2}} = \frac{\overline{X} + c\left(\frac{\overline{\alpha}-\alpha_1}{J-1}\right)^2}{\sqrt{2c\left(\frac{\overline{\alpha}-\alpha_1}{J-1}\right)^2}}.$$

Using the implicit function theorem,

$$\frac{\partial \alpha_1}{\partial \overline{X}_1} = -\frac{\alpha_1}{c\alpha_1^2 \frac{J}{J-1} - \overline{X}_1 - \overline{X}\left(J-1\right)\frac{\alpha_1^2}{\left(\overline{\alpha}-\alpha_1\right)^2}}. \tag{A.10}$$

Using equation (A.9) and $\kappa_1 = c\alpha_1^2$, we have that $\frac{d\mathcal{V}_1}{d\overline{X}_1}$ has the same sign as

$$e^{\overline{X}^*} - e^{\overline{X}_1} + \frac{\partial \alpha_1}{\partial \overline{X}_1}\left[-\frac{\overline{X}_1}{\alpha_1}\left(e^{\overline{X}^*} - e^{\overline{X}_1}\right) + c\alpha_1\left(e^{\overline{X}^*} + e^{\overline{X}_1}\right)\right]. \tag{A.11}$$

A necessary condition for $\overline{X}_j = \overline{X}$ for all j to be an equilibrium is that $\frac{d\mathcal{V}_1}{d\overline{X}_1}$ is zero when evaluated at $\overline{X}_1 = \overline{X}$ and $\alpha_1 = \overline{\alpha}/J$, so that a local deviation is not profitable. Plugging (A.10) into (A.11), evaluating the latter at $\overline{X}_1 = \overline{X}$ and $\alpha_1 = \overline{\alpha}/J$, and equating it to zero yield the equation in (7). Notice that the right-hand side of (7) (RHS) has the following properties for $\overline{X} \in \left(-\infty, \overline{X}^*\right)$: (i) it is strictly increasing in $\overline{X}$; (ii) $\lim_{\overline{X}\to\overline{X}^{*-}} RHS = \infty$; (iii) $\lim_{\overline{X}\to-\infty} RHS = 1$. The left-hand side of (7) (LHS) is strictly decreasing in $\overline{X}$, and $\lim_{\overline{X}\to-\infty} LHS = \infty$. This implies that there is a unique solution to (7) with $\overline{X} < \overline{X}^*$, which is then the only equilibrium candidate with $\overline{X} \neq -\infty$. Denote that candidate solution hereafter by $\hat{X}$.

It remains to show that there is no global deviation that is profitable when all firms play the cutoff $\hat{X}$. Let $f\left(\overline{X}_1\right)$ be the value of firm 1 when it chooses to play a cutoff $\overline{X}_1$ and all other firms play according to $\hat{X}$. We already know that $\hat{X}$ is a critical point of f. We now show that it is the only critical point in the range $\left(-\infty, \overline{X}^*\right]$. Using (A.10) with $\overline{X} = \hat{X}$ and (A.11), we have that for $\overline{X}_1 \in \left(-\infty, \overline{X}^*\right]$, $f'\left(\overline{X}_1\right) = 0$ if and only if

$$\frac{\left(J-1\right)\hat{X}}{c} = \left(\overline{\alpha} - \alpha_1\right)^2\left[\frac{J}{J-1} - \left(\frac{e^{\overline{X}^*} + e^{\overline{X}_1}}{e^{\overline{X}^*} - e^{\overline{X}_1}}\right)\right].$$

The LHS of the expression above is a negative constant. We claim that the RHS is decreasing in $\overline{X}_1$ whenever it is negative. To see that, first notice that the term in brackets is strictly decreasing in $\overline{X}_1$. Also, whenever the RHS is negative, it must be that $\left(\overline{\alpha} - \alpha_1\right)^2 > 0$ and the term in brackets is negative. Finally, $\left(\overline{\alpha} - \alpha_1\right)^2$ is increasing in $\overline{X}_1$ (see (A.10) with $\overline{X} = \hat{X}$), which proves our claim. Moreover, $\lim_{\overline{X}_1\to-\infty} RHS \geq 0$ and $\lim_{\overline{X}_1\to\overline{X}^{*-}} RHS = -\infty$ (since $\lim_{\overline{X}_1\to\overline{X}^{*-}} \left(\overline{\alpha} - \alpha_1\right)^2 > 0$), which implies that LHS and RHS cross only once in the range $\left(-\infty, \overline{X}^*\right]$. Therefore, there is a unique critical point of f in that range, which is $\hat{X}$.

Now we show that $\hat{X}$ is a global maximum in the interval $\left(-\infty, \overline{X}^*\right]$. First, once again using

(A.10) with $\overline{X} = \hat{X}$ and (A.11), one can verify that $f'\left(\overline{X}^*\right) < 0$, which together with the fact that $\hat{X}$ is the unique critical point in $\left(-\infty, \overline{X}^*\right]$, implies that $f\left(\hat{X}\right) > f\left(\overline{X}^*\right)$. Second, assume that $\overline{X}_1 = -\infty$ (firm 1 always invests), which leads to some $\kappa_1 = \kappa'$. In that case, firm value is

$$\lim_{\overline{X}_1 \to -\infty} f\left(\overline{X}_1\right) = \lim_{\overline{X}_1 \to -\infty} \mathcal{V}_1\left(\kappa', \overline{X}_1\right) = \lim_{\overline{X}_1 \to -\infty} \mathcal{V}_1\left(0, \overline{X}_1\right) = \mathcal{V}_1\left(0, \hat{X}\right) < \mathcal{V}_1\left(c\left(\overline{\alpha}/J\right)^2, \hat{X}\right) = f\left(\hat{X}\right).$$

The inequality follows from Lemma A.2, the second equality follows from the fact that κ does not affect firm value if the firm always invests, and the third equality follows from the fact that if $\kappa = 0$ the firm always invests for any negative cutoff. Therefore, we have shown that $\hat{X}$ is a global maximum of f in $\left(-\infty, \overline{X}^*\right]$.

To conclude the proof that $\hat{X}$ is a global maximum of f, it suffices to verify that $f\left(\overline{X}_1\right) > f\left(\hat{X}\right)$ for $\overline{X}_1 > \overline{X}^*$. Let κ' and κ'' be the equilibrium values of κ_1 when all other firms play $\hat{X}$, and firm 1 plays according to $\overline{X}^*$and $\overline{X}_1 > \overline{X}^*$, respectively. Note that $\kappa' \geq \kappa''$. Then,

$$f\left(\overline{X}_1\right) = \mathcal{V}_1\left(\kappa'', \overline{X}_1\right) < \mathcal{V}_1\left(\kappa'', \overline{X}^*\right) \leq \mathcal{V}_1\left(\kappa', \overline{X}^*\right) = f\left(\overline{X}^*\right) < f\left(\hat{X}\right),$$

where the first two inequalities follows from Lemma A.2. Hence, $\hat{X}$ is the unique symmetric equilibrium with $\overline{X} \neq -\infty$.

Finally, using Lemma A.2, $\lim_{\overline{X} \to -\infty} \mathcal{V}_j\left(\kappa, \overline{X}\right) = \mathcal{V}_j\left(0, \hat{X}\right) < \mathcal{V}_j\left(c\left(\overline{\alpha}/J\right)^2, \hat{X}\right) < \mathcal{V}_j\left(c\left(\overline{\alpha}/J\right)^2, \overline{X}^*\right)$ for any κ, which implies that in both the good ($\overline{X} = \hat{X}$) and the bad equilibrium ($\overline{X} = -\infty$) firm value is lower than in autarky. □

A.7 Proof of Proposition 4

Let $\overline{X}$ be the unique solution to (7) with $\overline{X} < \overline{X}^* < 0$. An increase in c or $\overline{\alpha}$ shifts down the LHS of (7), while it does not affect its RHS. Given that the RHS is strictly increasing in $\overline{X}$ in for $\overline{X} < \overline{X}^*$ and the LHS is strictly decreasing in $\overline{X}$, the cutoff $\overline{X}$ is strictly decreasing in c and $\overline{\alpha}$. The proof is then concluded by the observation that $\frac{dc}{d\omega}, \frac{dc}{d\lambda} > 0$ and $\frac{dc}{d\zeta}, \frac{dc}{d\sigma} < 0$. □

A.8 Proof of Proposition 5

Let $\overline{X}$ be the unique solution to (7) with $\overline{X} < \overline{X}^* < 0$. Define

$$G\left(\overline{X}, J\right) = 1 - J\frac{\overline{X}}{c\overline{\alpha}^2} - \frac{(J-1)}{J}\left(\frac{e^{\overline{X}^*} + e^{\overline{X}}}{e^{\overline{X}^*} - e^{\overline{X}}}\right).$$

Using that in equilibrium $G\left(\overline{X}, J\right) = 0$, by the implicit function theorem, $\frac{d\overline{X}}{dJ} = -\frac{\partial G/\partial J}{\partial G/\partial \overline{X}}$. Also,

$$\frac{\partial G}{\partial J} = -\left[\frac{J(J-2)\overline{X} + c\overline{\alpha}^2}{J(J-1)c\overline{\alpha}^2}\right] \tag{A.12}$$

and

$$\frac{\partial G}{\partial \overline{X}} = -\frac{J}{c\overline{\alpha}^2} - \frac{(J-1)}{J}\frac{2e^{\overline{X}+\overline{X}^*}}{\left(e^{\overline{X}^*} - e^{\overline{X}}\right)^2} < 0.$$

Hence, $\frac{d\overline{X}}{dJ}$ has the same sign as $\frac{\partial G}{\partial J}$ in (A.12), so $\frac{d\overline{X}}{dJ} > 0$ if and only if

$$J(J-2)\overline{X} < -c\overline{\alpha}^2. \tag{A.13}$$

For $J = 2$, the inequality above is always violated. Using the equilibrium condition in (7), we have that for $J > 2$ the inequality above is equivalent to

$$\frac{e^{\overline{X}^*} + e^{\overline{X}}}{e^{\overline{X}^*} - e^{\overline{X}}} > \frac{J}{J-2}. \tag{A.14}$$

We can verify how this condition is affected as $\overline{\alpha}$ varies, for a given J. Deriving the left-hand side of (A.14) with respect to $\overline{\alpha}$, we have:

$$\frac{dLHS}{d\overline{\alpha}} = \frac{2e^{\overline{X}^*+\overline{X}}}{\left(e^{\overline{X}^*} - e^{\overline{X}}\right)^2}\frac{\partial \overline{X}}{\partial \overline{\alpha}} < 0,$$

since $\frac{\partial \overline{X}}{\partial \overline{\alpha}} < 0$ (see Proposition 4), so the LHS of (A.14) is strictly decreasing in $\overline{\alpha}$. Moreover, from the equilibrium condition in (7), $\lim_{\overline{\alpha}\to 0}\overline{X} = \overline{X}^*$, which implies $\lim_{\overline{\alpha}\to 0} LHS = \infty$; and $\lim_{\overline{\alpha}\to\infty}\overline{X} = \overline{X}'$, where $\overline{X}'$ satisfies $\frac{e^{\overline{X}^*}+e^{\overline{X}'}}{e^{\overline{X}^*}-e^{\overline{X}'}} = \frac{J}{J-1}$, which implies $\lim_{\overline{\alpha}\to\infty} LHS = \frac{J}{J-1} < \frac{J}{J-2}$. Fix any $\overline{J} > 2$. For each $J \in \left[2, \overline{J}\right]$, there exists an $\underline{\alpha}_J$ such that the inequality in (A.14) is reversed for $\overline{\alpha} > \underline{\alpha}_J$. Define $\tilde{\alpha} \equiv \sup\{\underline{\alpha}_J\}_{J\in[2,\overline{J}]}$. We then have that for $\overline{\alpha} > \tilde{\alpha}$, $\frac{d\overline{X}}{dJ} < 0$ for all $J \in \left[2, \overline{J}\right]$.

Now fix $\overline{\alpha} > \tilde{\alpha}$. Notice that as J becomes arbitrarily large, the LHS of (A.13) goes to $-\infty$ (as $\overline{X}$ is bounded from above by $\overline{X}^* < 0$), while the RHS of (A.13) does not change. Hence, for J large enough, $\frac{d\overline{X}}{dJ} > 0$. □

A.9 Proof of Corollary 1

The result is a direct consequence of Propositions 1 and 3. □

A.10 Proof of Proposition 6

Suppose $\overline{\alpha} = J\Theta$, where $\Theta > 0$ is a constant. Then, the equilibrium condition (7) of Proposition 3 can be written as

$$\frac{J}{J-1}\left[1 - \frac{\overline{X}}{Jc\Theta^2}\right] = \frac{e^{\overline{X}^*} + e^{\overline{X}}}{e^{\overline{X}^*} - e^{\overline{X}}}.$$

The LHS of the expression above is a strictly decreasing linear function of $\overline{X}$, while the RHS is strictly increasing in $\overline{X}$ for $\overline{X} < \overline{X}^*$ and is such that $\lim_{\overline{X}\to-\infty} RHS = 1$ and $\lim_{\overline{X}\to\overline{X}^{*-}} RHS = \infty$. Since for $\overline{X} < \overline{X}^* < 0$ the LHS is strictly decreasing in J, we have that an increase in J reduces the equilibrium investment cutoff $\overline{X}$. $\square$

A.11 Proof of Proposition 7

Let $\mathcal{V}(\kappa, \overline{X})$ denote the value of a listed firm as a function of the equilibrium κ and $\overline{X}$. In this proof, for a given $J \geq 1$ we let $\kappa = \kappa(J) = c\,(\overline{\alpha}/J)^2$ and let $\overline{X} = \overline{X}(J)$ be the cutoff $\overline{X} < \overline{X}^*$ that satisfies (7) for $J > 1$, and $\overline{X} = \overline{X}(J) = \overline{X}^*$ for $J = 1$. The total wealth in the economy is then given by

$$\mathcal{W}(J) = J\left[\mathcal{V}\left(\kappa(J), \overline{X}(J)\right) - \psi\right] + \left(\tilde{J} - J\right)\left(\frac{V_H}{2} - \underline{u}\right)$$

for $J \geq 1$, and $\mathcal{W}(0) = \tilde{J}\,(V_H/2 - \underline{u})$. Consider $J \geq 1$ and let $g(J) = \mathcal{V}\left(\kappa(J), \overline{X}(J)\right)$. Note that:

$$g'(J) = \frac{\partial \mathcal{V}}{\partial \kappa}\kappa'(J) + \frac{\partial \mathcal{V}}{\partial \overline{X}}\overline{X}'(J).$$

$\overline{X}'(J)$ is obtained by applying the implicit function theorem to (7). The other derivatives are given by (A.7), (A.8), and $\kappa'(J) = \frac{-2\kappa}{J}$. Plugging those in the expression above, and using $\overline{\alpha} = J\sqrt{\kappa/c}$ we get, after some algebra, that $g'(J)$ has the same sign as

$$-\frac{\left(e^{\overline{X}^*} - e^{\overline{X}}\right)^2}{2\kappa e^{\overline{X}+\overline{X}^*}(J-1) + \left(e^{\overline{X}^*} - e^{\overline{X}}\right)^2} - 1 < 0,$$

so $g(J)$ is strictly decreasing.

First consider the case where $J^* > 1$. The optimality of J^* implies $\mathcal{W}(J^*) \geq \mathcal{W}(J^* - 1)$, which is equivalent to

$$g(J^*) - \psi - (V_H/2 - \underline{u}) \geq (J^* - 1)\left[g(J^* - 1) - g(J^*)\right].$$

Since $g(J)$ is strictly decreasing, $g(J^* - 1) > g(J^*)$, and hence $g(J^*) - \psi - (V_H/2 - \underline{u}) > 0$. But then $g(J) - \psi - (V_H/2 - \underline{u}) > 0$ for all $J \leq J^*$, which implies that there can be no decentralized equilibrium with $J^{eq} < J^*$ (if there were $J < J^*$ listed firms, a firm not listed would strictly prefer to go public). Now notice that given our assumption on ψ, $g(1) - \psi - (V_H/2 - \underline{u}) > 0$, so $J^{eq} \geq 1$. Hence if $J^* = 1$, we also have $J^{eq} \geq J^*$. We have then shown that $J^{eq} \geq J^*$ always.

We now prove that, whenever $J^{eq} > J^*$, a Pigouvian tax implements the social optimum J^* in a decentralized equilibrium. Given that $\psi < g(1) - (V_H/2 - \underline{u})$, the planner always lists at least one firm, so hereafter assume $J^{eq} > J^* \geq 1$. Suppose the planner taxes in τ dollars every firm that goes public. Then, J^* is a decentralized equilibrium after the tax if and only if the following no-entry and no-exit conditions hold:

$$g\left(J^{*}+1\right)-(\psi+\tau)\leq V_H/2-\underline{u},$$

$$g\left(J^{*}\right)-(\psi+\tau)\geq V_H/2-\underline{u}.$$

Rearranging, those equations become $\tau \geq g\left(J^{*}+1\right)-\psi-\left(V_H/2-\underline{u}\right)\equiv\underline{\tau}$ and $\tau\leq g\left(J^{*}\right)-\psi-\left(V_H/2-\underline{u}\right)\equiv\overline{\tau}$. Notice that $\overline{\tau}>\underline{\tau}$ and $\underline{\tau}\geq g\left(J^{eq}\right)-\psi-\left(V_H/2-\underline{u}\right)$, where we used that $g(J)$ is strictly decreasing and $J^{eq}\geq J^{*}+1$. Finally, since J^{eq} is a decentralized equilibrium with no taxes, it must be that $g\left(J^{eq}\right)-\psi-\left(V_H/2-\underline{u}\right)\geq 0$, so $\underline{\tau}\geq 0$. Hence, any tax $\tau\in[\underline{\tau},\overline{\tau}]$ implements the social optimum. □

A.12 Informativeness measure

Here we state formally and prove the claim we made in the text about κ_j being the relevant measure of market informativeness for stock j in our setting. Naturally, we present the result for non-trivial binary decisions problems, i.e., when the optimal choice differs across states under perfect information. If this is not the case, then the expected utility does not vary with κ_j and therefore is only weakly but not strictly increasing in κ_j.

Lemma A.5. *Consider the problem of a decision maker who maximizes expected utility and must choose an action $z\in\{A,B\}$. Her utility depends on z and on the fundamental of firm j (θ_j), and is given by a function $u(z,\theta_j)$, with $u(A,H)>u(B,H)$ and $u(A,L)<u(B,L)$ (so that her problem does not have a trivial solution that involves always choosing the same action). The decision maker has a prior that $\theta_j=H$ with probability $\xi\in(0,1)$ and before making her decision she observes market activity $\boldsymbol{Y}$. Then, the decision maker's expected utility is strictly increasing in κ_j.*

Proof. Since fundamentals are independent across firms, Bayes Rule implies that the decision maker's posterior probability of $\theta_j=H$ is given by

$$\Pr\left(\theta_j=H|\boldsymbol{Y}\right)=\frac{1}{1+\left(\frac{1-\xi}{\xi}\right)\exp\left\{-X_j\right\}}.$$

Hence, she strictly prefers action A whenever $X_j>\ln\left(\frac{1-\xi}{\xi}\frac{u(B,L)-u(A,L)}{u(A,H)-u(B,H)}\right)\equiv X^{\ddagger}$. If $X_j<X^{\ddagger}$ she chooses B and is indifferent when $X_j=X^{\ddagger}$. Following the same steps in Lemma 3 and its proof, for any $\kappa_j>0$ the expected utility under the optimal choice is given by:

$$\mathcal{U}=\xi\left[u(A,H)-\Phi\left(\frac{X^{\ddagger}-\kappa_j}{\sqrt{2\kappa_j}}\right)\Delta_H\right]+(1-\xi)\left[u(A,L)+\Phi\left(\frac{X^{\ddagger}+\kappa_j}{\sqrt{2\kappa_j}}\right)\Delta_L\right],$$

where $\Delta_H = u(A, H) - u(B, H)$ and $\Delta_L = u(B, L) - u(A, L)$. Now note that:

$$\frac{d\mathcal{U}}{d\kappa_j} = \xi \Delta_H \phi \left(\frac{X^{\ddagger} - \kappa_j}{\sqrt{2\kappa_j}} \right) \left(\frac{X^{\ddagger} + \kappa_j}{2^{3/2}\kappa_j^{3/2}} \right) - (1 - \xi)\, \Delta_L \phi \left(\frac{X^{\ddagger} + \kappa_j}{\sqrt{2\kappa_j}} \right) \left(\frac{X^{\ddagger} - \kappa_j}{2^{3/2}\kappa_j^{3/2}} \right).$$

Using that $\phi \left(\frac{X^{\ddagger} - \kappa_j}{\sqrt{2\kappa_j}} \right) / \phi \left(\frac{X^{\ddagger} + \kappa_j}{\sqrt{2\kappa_j}} \right) = e^{X^{\ddagger}} = \frac{1-\xi}{\xi} \frac{\Delta_L}{\Delta_H}$, one can verify after some algebra that the expression above has the same sign as κ_j. Hence, $\mathcal{U}$ is strictly increasing in κ_j. □