INSTITUTO DE ECONOMÍA

DOCUMENTO de TRABAJO

549

2020

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www.economia.puc.cl • ISSN (edición impresa) 0716-7334 • ISSN (edición electrónica) 0717-7593

Public transport and urban structure⋆

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August 31, 2020

Abstract

Public transport is central to commuting in most cities around the world. This paper studies the role of public transportation in shaping the urban structure. The main contribution of the paper is to propose a tractable model as a tool to study urban regulations and transport policies in the long-run. Using the classic monocentric city framework, we model public transport as a mode that can only be accessed by walking to a limited set of stops. By incorporating a discrete transport mode choice and income heterogeneity, the model remains simple yet can reproduce non-monotonous urban gradients observed in cities with public transport, and well-observed spatial patterns of sorting by income and use of public transport. For example, it can reproduce an inverted U-shape of transit usage along the city.

Keywords: Monocentric city model; public transport; mode choice; income groups sorting.

1. Introduction

Transportation and commuting are central to the theory of cities and urban structure. Early works from Alonso (1964), Mills (1967), and Muth (1969) developed the starting point of the modern urban economics literature in what is called the monocentric city model. In that model, production takes place at the Central Business District (CBD), where all jobs are located. The core of the model is that residents consume a numeraire good, housing, and must commute. As commuting costs increase with distance to the CBD, the differences

⋆We gratefully acknowledge financial support from the Instituto Sistemas Complejos de Ingeniería, ISCI (grant ANID PIA AFB180003), from the Center of Sustainable Urban Development, CEDEUS (grant ANID/FONDAP/15110020), and from FONDECYT grant 1191010. We also thank valuable comments from Kenzo Asahi, Jan Brueckner, Mogens Fosgerau, Sofia Franco, Juan Carlos Munoz, and participants of the 2017 ITEA conference, the 1st Workshop on Urban and Regional Economics (Bogota), and seminars at PUC and at the Markets, Organizations and Regulation (MORe) group of ISCI.

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in these costs along the city must be balanced by differences in the price of living space and consumption of housing (Brueckner, 1987). For this reason, land and rental prices, as well as population density, should decrease with distance to the CBD, and dwelling size should increase with distance to the CBD. This model has been extended in many directions, for example, to study amenities (Brueckner et al., 1999), local public goods (de Bartolome and Ross, 2003), and landscape preferences (Turner, 2005), among others. See Duranton and Puga (2015) for an extensive review.

Public transport is essential to commuting in many cities around the world. For example, the share of public transport trips is 28% in 25 of the European largest cities, while the share of private car trips was 33% in 2015 (EMTA, 2015).[1] The use of public transport is arguably more important in developing countries. The average share of trips made by public transport in 15 of the largest cities in Latin America was 43% in 2009, significantly higher than the share made by car, which was 28% (CAF, 2010). The public transport system's features and technology affects commuting costs and, thus, following the argument of its central importance, it should change the urban form.

This paper studies the role of public transport in shaping cities, through a novel use of the monocentric city model. We focus on three crucial aspects of the urban structure: (i) use of public and private transport according to location; (ii) spatial sorting of different income groups, and (iii) how housing price, land price, dwelling sizes, population density, and structural density (floor-to-area ratio) change with distance to the central business district. We show that the model provides significant predictory power regarding several (ir)regularities that are usually observed in cities with high use of public transport. When cities differ in commuting costs components, their structure and use of public transport will be different. For example, the discrete nature of stations in space allows us to obtain the inverted U-shape of public transport usage along with the city that has been identified in some metropolis. The main contribution of the paper is, therefore, to propose a tractable model that is a useful tool to address the efficiency and impact of transport policies in the long-run.

The literature about the role of public transport on the urban structure is scarce. The main contributions have focused on explaining the sorting of residents by income (LeRoy and Sonstelie, 1983; Glaeser et al., 2008; Su and DeSalvo, 2008). In particular, Glaeser et al. (2008) argue that "the primary reason for central city poverty is public transportation". As the interest of these papers is on sorting, the modeling of public transport is quite simple; indeed, public transport is simply modeled as a car that is slower and less expensive. The consequences of the simplification are that the patterns of car and public transport usage are segregated zones where either one or the other mode is dominant. Our modeling of

[1]These metropolitan areas include Amsterdam, Barcelona, Berlin, London, Madrid, and Paris, among others.

public transportation allows for more complex patterns that are relevant when studying transport taxes, emissions, vehicle-kilometer traveled, among other relevant outcomes.

On the other hand, the transport economics literature has modeled public transportation in a very detailed way to study its optimal level of service and the efficient pricing scheme. Furthermore, many studies have investigated the efficiency of policies such as subsidization, bus lanes, car congestion pricing, and combinations when there is interaction with other transport modes (for recent studies, see, e.g., Proost and Van Dender, 2008; Parry and Small, 2009; Kutzbach, 2009; and Basso and Silva, 2014). Nevertheless, this strand of the literature has adopted a short-run view by assuming that the housing market and location of households is exogenously fixed.[2] Our paper contributes to this literature by providing a framework to assess such transportation policies when public transportation and patterns of mode usage and income along with the city matter.

Our paper also contributes to the literature on the benefits of better access to transportation to consumers. There is ample empirical evidence that households value improved accessibility, and most of the studies have estimated the effect of closer proximity to a rail station on prices. Yet, the theory suggests that the average impact may mask significant heterogeneity concerning overall accessibility (or proximity to the CBD). Our theoretical model delivers price elasticities with respect to the distance to the station that change with distance to the CBD depending on structural parameters. Thus, it can shed light on how the effect of closer proximity changes along a transport corridor such as a rail line.

To further motivate and highlight the relevance of the model, we illustrate how property prices change after the announcement of a subway line in Santiago. Figure 1 shows the weighted average of housing prices for each segment of 100 meters, from the CBD to the last station of the subway line (Line 3). The solid grey and black lines are polynomial fits for housing prices along the corridor before and after the announcement of Line 3, respectively, and vertical dashed lines indicate the location of each announced station.[3] Before the announcement (solid grey line), the price gradient is non-monotonic but not statistically different from a downward-slope line. After the announcement (solid black line), we find a non-monotonic pattern with steeper peaks at the location of announced stations (vertical dashed lines). In this case, the 95% confidence interval cannot accommodate a straight line. We believe that this suggestive evidence provides support for our model as a useful tool for policy analysis, as it can reproduce those price patterns and underlying choices.

The discrete nature of the stops and the fact that people may walk downstream or upstream to access public transport drives the action and predictory power while remain-

[2]An exception is Brueckner (2005), who studies the effect of transport subsidies on the spatial expansion of cities, but also using a simplified transit system and without interaction with a different mode.

[3]The details of the data and methodology are in Appendix A.

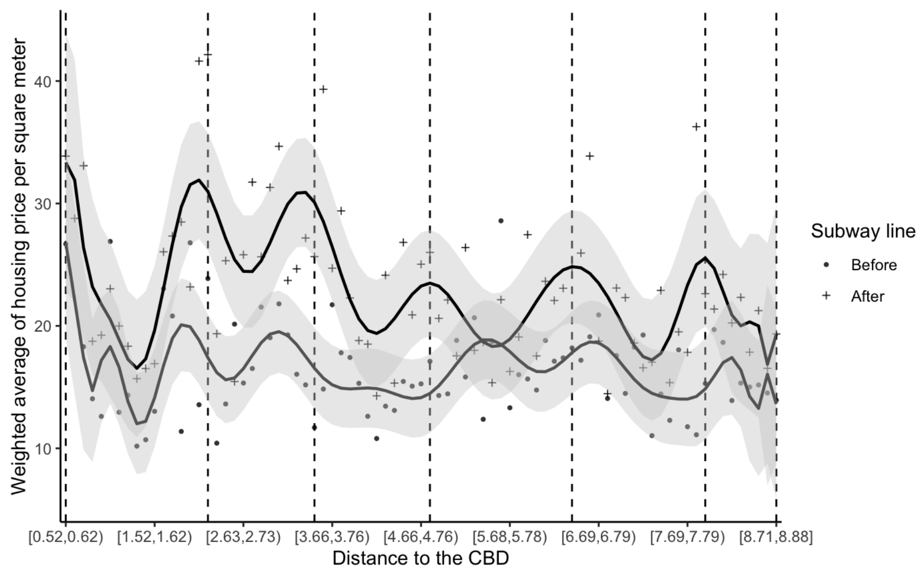


Figure 1: Housing prices before (grey line) and after (black line) the announcement of Line 3.

ing a rather simple model.[4] The non-monotonic commuting costs induce non-monotonic gradients with peaks in prices (as in Figure 1), population density, and structural density at stations, where dwelling sizes are smaller. These predictions have been suggested in the literature, for example, by Duranton and Puga (2015), and modeled in the context of complementary public transport modes, where people take buses to train stations but car is not available as an alternative mode (Kilani et al., 2010).

However, it is the combination of detailed modelling of public transport, a simple discrete choice model between car an transit, and income heterogeneity that makes the model stand out. Our model predicts that the use of cars can appear all along with the city and not only in long stretches of the city, where that mode dominates without any use of public transport, as currently available models predict. We also show that the presence of public transport can break the ordered sorting from the models without the need to have multiple modes of transportation. Our model with only public transportation and two income-groups has a large amount of mixing at the level of the distance between stops because, as the price gradients are non-monotonic due to the access cost to the stations, price bids can cross multiple times.[5]

The rest of the paper is organized as follows. Section 2 introduces the monocentric city model with public transport and characterizes the urban structure equilibrium. Section 3

[4]There is ample evidence showing that the walking time to access public transportation plays a vital role on its demand (see, e.g., Yáñez et al., 2010).

[5]Other rationales for social mixing are two-dimensional heterogeneity of individuals such as income and commuting costs (Behrens, Combes, Duranton, Gobillon and Robert-Nicoud, 2014) and locations with two-dimensional heterogeneity, distance to the CBD and other exogenous feature such as amenities (Gaigné, Koster, Moizeau and Thisse, 2017).

extends the analysis by including the interaction between public and private transport on the urban structure and explores the implications of our model in the sorting of residents by income. Section 4 concludes.

2. Urban structure in the public transport city

We first describe the model in which public transport is the main transportation mode, to highlight the detailed modeling of stations and its implications. In the following Section, we introduce modal choice and income heterogeneity.

We assume all jobs are located at the CBD. Residents commute to their jobs (at the CBD) along a corridor, where walking and public transport are the only available commuting options. The public transport mode could be a bus rapid transit system, a tramway, or a subway line. In this sense, we consider a linear rather than a circular city. One way to interpret our city is represented in Figure 2. At both sides of the corridor, buildings vary on height, the number of apartments, and apartment size with distance to the CBD.

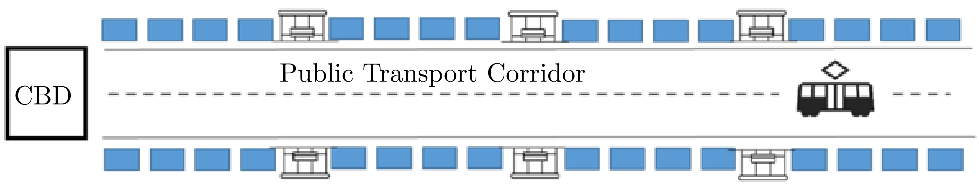


Figure 2: Linear city with public transport.

2.1. Housing demand side

Residents' preferences are represented by a standard strictly quasi-concave utility function $U(c, q)$, where c is the consumption of a composite non-housing good, and q is the consumption of housing (measured by floor space). In equilibrium, q will change with the distance to the CBD, which we denote by x. We assume that the price of the composite good is the same everywhere in the city, and is normalized to unity. The rental price per unit of housing floor space p can also vary with x.

Residents' income comes from two sources: a fixed amount E, which is unrelated to the hours of work, and $w \cdot H$, that comes from working H hours per day at a hourly wage of w. Thus, the budget constraint is $E + w \cdot H = c + pq + e$, where e is the public transport fare. The time constraint is $T = H + L + t$, where T is total available time in a day (discounting time for sleeping, eating, and so on), L is leisure time, and t is commuting time. Obtaining H from the time constraint and replacing it on the income constraint leads to $E + w(T - L - t) = c + pq + e$, which may then be re-organized as follows,

$$\underbrace{y}_{E+w(T-L)} = c + pq + \underbrace{\rho}_{e+wt} \tag{1}$$

Equation (1) highlights that commuting costs are the sum of the public transport trip fare and travel time costs. This sum is the generalized commuting cost and is represented by ρ.

The urban equilibrium is achieved when all individuals obtain the same utility level $\bar{U}$. Therefore, the equilibrium is impervious to the permutation of locations between any pair of residents. Obtaining c from equation (1) and replacing it into the utility function, individual utility maximization implies:

$$\max_{q} \quad V(y - pq - \rho, q) \tag{2}$$

where V is the indirect utility function. Consumers therefore face a tradeoff between housing and goods consumption, with substitution occurring through location decisions. Consumers' optimal choices can be characterized from the first-order condition of equation (2). Since residents choose q optimally conditional on prices, the first-order condition is:

$$\frac{\frac{\partial V(y-pq-\rho,q)}{\partial q}}{\frac{\partial V(y-pq-\rho,q)}{\partial c}} \quad = \quad p \tag{3}$$

Thus, the marginal rate of substitution between housing and non-housing goods must equal their price ratio. Next, since residents must reach the same utility at each location x, the spatial equilibrium condition which guarantees this requirement is (see, e.g., Duranton and Puga, 2015):

$$V(y - pq - \rho, q) = \bar{U} \tag{4}$$

The two conditions, individual optimality (3) and spatial equilibrium (4), allow for obtaining solutions for $p(x)$ and $q(x)$, which we denote as $\hat{p}(x, y, \rho, \bar{U})$ and $\hat{q}(x, y, \rho, \bar{U})$.

2.2. Housing supply side

Our housing supply model closely follows Brueckner (1987). Housing is produced with inputs of land l and capital K, according to the concave constant returns production function $H(K, l)$. This function gives floor space contained in a building. While the floor space is rented by residents at price p, land and capital are rented by producers at prices r and i, respectively. Therefore, the producer's profit is $pH(K, l) - iK - rl$. Using the constant returns property of H, Brueckner (1987) rewrites profit as $l[pH(K/l, 1) - iK/l - r]$. Denoting S the capital-land ratio K/l, the profit can be written as:

$$l[ph(S) - iS - r] \tag{5}$$

Where $h(S) \equiv H(S, 1)$ gives floor space per unit of land. S, then, becomes an index of the height of buildings, usually called *structural density*. It also follows that the marginal productivity of capital is positive $h'(S) > 0$ and also $h''(S) < 0$ because, for a fixed l, the use of additional capital will be less productive (e.g., thicker walls, deeper foundations). The producers maximize profit per square meters of land for a fixed l, choosing S. Using equation (5), the first-order condition that ensures firm profit maximization is:

$$ph'(S) = i \tag{6}$$

Moreover, the zero-profit condition, denoting long-run equilibrium is:

$$ph(S) - iS = r \tag{7}$$

The two conditions –firm optimality and long-run equilibrium– allow for obtaining solutions for $r(x, y, \rho, \bar{U})$ and $S(x, y, \rho, \bar{U})$. Again, if it was the case that $\rho = \tau x$, then it can be shown that r and S follow the same comparative statics as p when x, y and τ change (Brueckner, 1987). If we assume, without loss of generality, that each household contains one person, then the population density is given by:

$$D = \frac{h(S)}{q} \tag{8}$$

2.3. Public transport modelling and commuting costs

Up to this point, the model and assumptions are conventional, with the exception that the usual constant per-distance cost of commuting is replaced by the generalized cost of commuting, ρ. Indeed, one distinctive feature of this paper is the modeling of public transport, reflected in this commuting cost, which, besides the in-vehicle travel time, includes commuters' walking time to the station.

We consider that the public transport corridor has equally spaced stations at a distance d. The exogeneity of this spacing may be understood by minimum standard requirements, for example, that no one should walk more than one kilometer to a station. We assume that public transport is uncongested on the road, meaning that transit vehicles do not congest each other. This free-flow speed is denoted v. We further assume that residents do not experience waiting time, because public transport operates based on timetables.

To avoid the trivial solution in which everyone walks to the CBD, we assume that parameters are such that the indifferent resident between walking to the CBD and commuting by public transport is located between the CBD and the first station.[6] Denote $\bar{x}_0$ the location for which residents are indifferent between walking to the CBD or walking *in the opposite direction* to the first station (see Figure 3). Then $\bar{x}_0$ is obtained from equating

[6] People would always choose walking if either the public transport fare is prohibitively high or if the walking speed is equal to or higher than public transport speed.

commuting costs:

$$w \cdot \frac{\bar{x}_0}{\mu} = e_1 + w \cdot \frac{d}{v} + w \cdot \frac{d - \bar{x}_0}{\mu} \tag{9}$$

The left side of equation (9) is the generalized cost of walking from $\bar{x}_0$ to the CBD at speed μ, where the wage rate w is the value of time implied by our model. The right side of equation (9) represents the generalized cost of commuting from the first station located at $x = d$. The first term, e_1, is the public transport fare charged at station 1. The second term is the in-vehicle travel time between the first station and the CBD. The third term is the walking time from $\bar{x}_0$ to the first station located at d. From equation (9), we obtain:

$$\bar{x}_0 = \frac{d}{2} + \frac{\mu e_1}{2w} + \frac{d}{2}\frac{\mu}{v} \tag{10}$$

Note that $\bar{x}_0$ is always upstream (to the right) of $d/2$. The necessary condition on parameters needed for this to be interior (i.e. downstream of the station) is: $\frac{\mu e_1}{w(1-\mu/v)} < d$, where $0 < \frac{\mu}{v} < 1$. In other words, if the public transport system is not "good enough", both in terms of fare and speed, then people would prefer walking. Figure 3 shows the public transport line with the stops and the locations at which individuals are indifferent between which one to walk to.

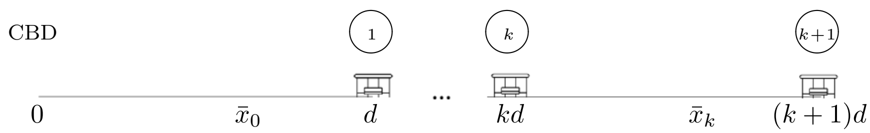


Figure 3: Public transport stations and indifference locations.

The equation that defines the location $\bar{x}_k$ of the indifferent commuter between station k and station $k+1$ is, again, obtained by equating generalized commuting costs:

$$e_k + w\left[\frac{kd}{v} + \frac{\bar{x}_k - kd}{\mu}\right] = e_{k+1} + w\left[\frac{(k+1)d}{v} + \frac{(k+1)d - \bar{x}_k}{\mu}\right] \tag{11}$$

The generalized commuting cost from $\bar{x}_k$, for $k \in \{1, ..., n-1\}$, by walking to station k (located at kd) and to station $k+1$ (located at $(k+1) \cdot d$) are on the left-hand and the right-hand side, respectively, of equation (11). Note that, in principle, e_k is different than e_{k+1}, meaning that the fare may be different at different stations. Of course, people to the right of the last station have no choice but to walk downstream to that station. From (11) we obtain:

$$\bar{x}_k = \left(dk + \frac{d}{2}\right) + \frac{\mu}{2w}(e_{k+1} - e_k) + \frac{d}{2}\frac{\mu}{v} \tag{12}$$

Having characterized the location of the indifferent commuters, we can study how the commuting costs ρ change along the city. This is important since commuting costs are pivotal in the choice of location, which, in turn, is the instrument that enables individuals to trade housing and consumption good. Consider people boarding at station $k > 1$. Everyone living in $[kd, \bar{x}_k]$, with $k > 1$, incur the same in-vehicle time to the CBD since they all board at station k. However, the walking time is increasing in that interval, indicating that total commuting cost is increasing with distance to the CBD in that interval. On the other hand, everyone living in $[\bar{x}_{k-1}, kd]$ also incur the same in-vehicle time since they all also board at station k, but the walking time is *decreasing* as people live closer to the station, indicating that total commuting cost is *decreasing* with x in this interval. For the interval between the CBD and the first station, people that are located downstream of $\bar{x}_0$ walk to the CBD.

Note from equation (12) that, unless transit fares are heavily decreasing with distance, the commuting costs for people located exactly at public transport stations are increasing the farther away from the CBD they are. It is straightforward to conclude that commuting costs –including both in-vehicle and walking time– follow a sawtooth pattern, with an overall increasing trend.

To further characterize commuting costs, consider first the case where the public transport system has a flat fare, i.e., $e_k = e \quad \forall \quad k$. In this case, it follows from equation (12) that the distance between the indifferent commuter $\bar{x}_k$ and the downstream station is given by $\frac{d}{2} + \frac{d}{2}\frac{\mu}{v}$ for all $k \geq 1$. That is, all the indifferent commuters, except for $\bar{x_0}$ are located at the same location relative to their closest stations. Now, consider that the fare is not flat, but increasing with distance: $e_k \equiv e + \Delta \cdot k \quad \forall k > 0$. It follows that the distance from $\bar{x}_k$, for all $k \geq 1$, to the downstream station is now larger by an amount given by $\frac{\mu\Delta}{2w}$. Thus, in each interval the $\bar{x}_k$ when fares increase with distance will be to the left of the indifferent commuter under flat fares, in relative terms, if $\Delta < e$. Note that what changes are the positions of the indifferent commuters but the slope of both parts of a tooth, is always given by the walking speed μ.

With a distance-based fare, then, the sawtooth pattern has more asymmetric teeth, with a shorter decreasing section, overall resembling more the classical τx commuting cost. It is also simple to picture how a zone fare system affects commuting costs by changing the location of the indifferent commuter for sets of stations. Finally, note that changing the distance between stations would change the amplitude of the sawtooth pattern. With very close stations, the amplitude diminishes, moving towards the classic model. All these are shown in Figure 4.

2.4. Urban structure

In this section, we study how the presence of public transport shapes the internal structure of the city. That is, how housing and land prices, dwelling sizes, building heights, and population density change with distance to the CBD in equilibrium. The analysis shows that the model can capture some irregularities that are observed in cities with intensive use of public transport, and that a car-monocentric city model cannot.

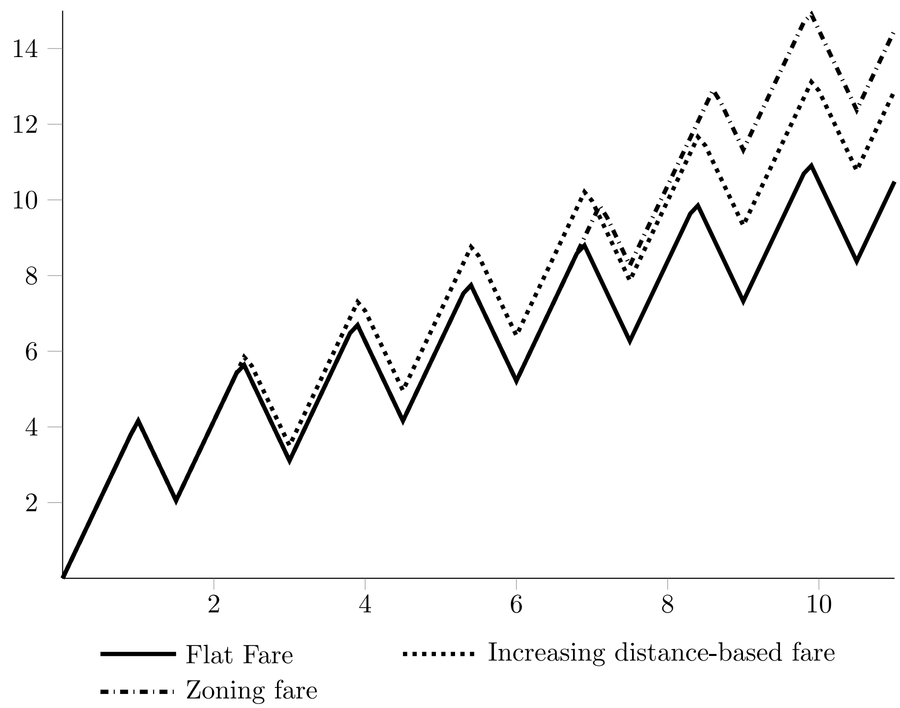


Figure 4: Commuting costs compared to flat fare for (b) increasing distance-based fare (c) zone-based fare.

First, we focus on describing how housing price (p), housing consumption (q), land price (r), structural density (S), and population density (D) change along the city. We also highlight the differences with the predictions of the traditional monocentric linear city model of Alonso-Mills-Muth. Totally differentiating the conditions obtained from the demand side (equation (3) and (4)) and from the supply side (equations (6) and (7)) with respect to x, ρ, y, and $\bar{U}$, allows for performing a complete analysis. Specifically, differentiating equation (4) with respect to x yields:

$$\frac{\partial V}{\partial c}\left(-q\frac{\partial p}{\partial x} - p\frac{\partial q}{\partial x} - \frac{\partial \rho}{\partial x} + \frac{\partial v}{\partial q}\frac{\partial q}{\partial x}\right) = 0 \tag{13}$$

Using that $\frac{\partial V}{\partial q} = p\frac{\partial V}{\partial c}$ from equation (3), and using equation (13) we obtain that $\frac{\partial p}{\partial x} = -\frac{1}{q}\frac{\partial \rho}{\partial x}$. That is, p follows the opposite trend than the commuting cost. From the analysis of commuting costs ρ above, we conclude that:

$$\text{sign}\left(\frac{\partial p}{\partial x}\right) = \begin{cases} < 0 & \text{when} \quad kd < \quad x \quad < \bar{x}_k \qquad \forall k \geq 0 \\ > 0 & \text{when} \quad \bar{x}_k < \quad x \quad < (k+1)d \quad \forall k \geq 0 \end{cases} \tag{14}$$

Based on Duranton and Puga (2015)'s notation, in this model equation (14) is analogous to the Alonso-Muth condition. The housing price decreases with distance to the CBD when the individual walks downstream to a station, as in $(kd, \bar{x}_k)$, and increases with distance to the CBD when she walks upstream, as in $(\bar{x}_k, (k+1)d)$. Thus, housing price shows peaks at public transport stations.

The difference with the classic model is that the commuting cost decreases when the resident walks upstream to the public transport station. That is, it follows a sawtooth pattern with an overall increasing trend. Consequently, the housing price will have the exact opposite pattern. In essence, in each interval between $\bar{x}_k$ and a station, there is a small classic monocentric city. Each station acts as a CBD, and the intuition remains: the change in the housing consumption cost with distance to the CBD has to be exactly offset by the change in commuting costs. Since in each of these small monocentric cities commuting costs are linear –given by the product of walking speed, distance, and the value of time–, it follows that $p(x)$ will be convex in each interval, as in Brueckner (1987). This pattern is represented in Figure 5. Furthermore, each station is a symmetry point, in that the monocentric city to the right and left of a station are identical, but with different directions. They develop until a $\bar{x}_k$ is reached; and since the $\bar{x}_k$ are to the right of the middle point between two stations, one obtains the overall decreasing pattern.

Dwelling size is also non-monotonic along the city, though its overall pattern is increasing, and it shows local peaks between stations at $\bar{x}_k$, i.e. the commuter's location that is indifferent between the upstream and downstream station (see Figure 5).[7]

Next, differentiating the zero-profit condition of the housing firms (7) with respect to x, and using equation (6), we obtain the gradient of land price as $\frac{\partial r}{\partial x} = h(S)\frac{\partial p}{\partial x}$. Recall that $h(S)$ gives the floor space per unit of land and is always positive. Therefore, the sign of the partial derivative of r with respect to x has the same sign as the sign of the partial derivative of p with respect to x. Just as in the traditional model, land price r changes follow housing price changes. Therefore, land price is overall decreasing, but with local peaks at stations.

Next, differentiating equation (6), with respect to x, yields $\frac{\partial S}{\partial x} = -\frac{\partial p}{\partial x}\frac{h'(S)}{p \cdot h''(S)}$. As the marginal productivity of capital $h'(S)$ is positive and increases at a decreasing rate, i.e., $h''(S) < 0$, the sign of the partial derivative of S with respect to x is the same as the sign of the partial derivative of p with respect to x. That is, the height of buildings is overall decreasing but with local peaks at stations. Finally, another important element of the urban structure is the population density. As $D = \frac{h(S)}{q}$ (equation (8)), it is straightforward to show that $\frac{\partial D}{\partial x}$ has the same sign as $\frac{\partial S}{\partial x}$. Therefore, the population density follows the same pattern as the structural density: it is overall decreasing with local peaks at stations.[8] All

[7]Differentiating the individual optimality condition (3) with respect to x and reordering we get $\frac{\partial q}{\partial x} = \frac{\partial p}{\partial x}\left[\frac{\partial\left(\frac{\partial V/\partial q}{\partial V/\partial c}\right)}{\partial q}\right]^{-1} = \frac{\partial p}{\partial x}\eta$ where $\eta \equiv \left[\frac{\partial\left(\frac{\partial V/\partial q}{\partial V/\partial c}\right)}{\partial q}\right]^{-1}$ is the slope of the indifference (constant utility) curve. We have already shown that the sign of $\frac{\partial p}{\partial x}$ is positive or negative, depending on the location. η is negative because indifference curves are convex as we assume that the utility function is strictly quasi-concave. Using (14) and the equation above, we obtain that housing consumption changes with distance to the CBD as follows: sign $\left(\frac{\partial q}{\partial x}\right) > 0$ when $kd < x < \bar{x}_k \forall k \geq 0$ and sign $\left(\frac{\partial q}{\partial x}\right) < 0$ when $\bar{x}_k < x < (k+1)d \forall k \geq 0$.

[8]Concavity of all these functions follow directly from the fact that each interval between the location of the indifferent commuter and the station is a classic monocentric city.

the relevant gradients are displayed in Figure 5, except for land price r which follows the same pattern as housing price p.

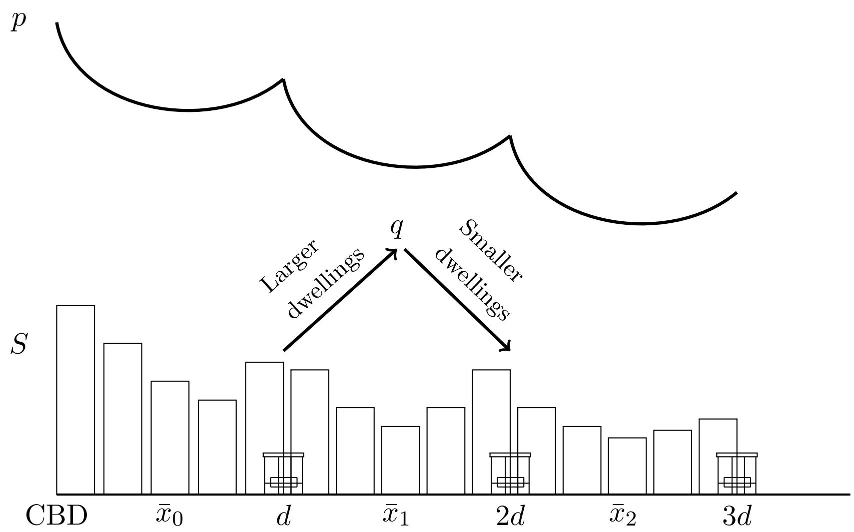


Figure 5: Housing price (p), housing consumption (q) and structural density (S) along the city.

The two conditions that characterize the equilibrium of the urban area: the urban land rent r equals the agricultural rent r_A at the city boundary $\bar{X}$ and the total urban population N has to fit inside the city. Both conditions are expressed in equations (15) and (16), respectively.

$$r(\bar{X}, y, \rho, u) = r_A \tag{15}$$

$$\int_0^{\bar{x}} D(x)dx = N \tag{16}$$

The presence of public transport may also give rise to suburbs that are disconnected from the city. That is, inhabitants that cluster around a transit station, but that has agricultural land on both sides. This is sometimes also known as leapfrog developments (see Duranton and Puga, 2015, for references). The intuition is displayed in Figure 6: as land price is not a monotonically decreasing function, it can be equal to the agricultural rent more than once as long as there is a public transport stop further away from the CBD from the first crossing point.[9]

We have studied how the presence of public transport affects the gradients predicted by the classic monocentric model. The overall trends are the same when moving away from the CBD: the housing price, land price, structural density, and population density

[9]Indeed, leapfrog development only occurs if that station is built. In work currently being developed, we show that it may well be the case that building the station, and thus inducing a disconnected suburb, is welfare-maximizing.

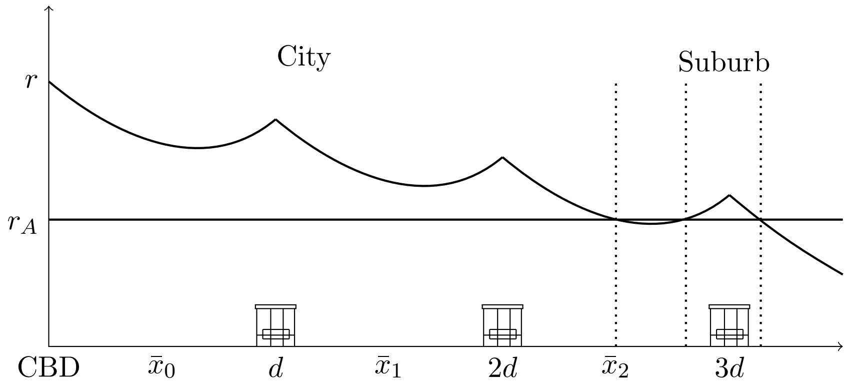


Figure 6: City boundary and the disconnected suburb.

decrease, while dwelling size increases. The critical difference is that these gradients are non-monotonic: our model predicts prices, population, and structural density peaks at stations, where dwelling sizes are smaller. In other words, around public transport stations, rental prices are higher, buildings are taller, and apartments are smaller. These predictions seem intuitive and representative of cities with intensive use of public transport. Figure 1 provides suggestive evidence for Santiago. Another example is Bowes and Ihlanfeldt (2001) who use data from Atlanta and find that "[...] properties that are between one and three miles from a station have a significantly higher value compared to those farther away". Other references that sustain the empirical finding that rental prices decrease as properties are farther away from subway stations, and which may lead to the pattern that our model delivers, are Landis et al. (1995); Knaap et al. (2001); Bae et al. (2003); Gibbons and Machin (2005); Armstrong and Rodriguez (2006); Hess and Almeida (2007); Liang and Cao (2007); Gu and Zheng (2010); Feng et al. (2011); Efthymiou and Antoniou (2013). These studies cover many different cities and countries. In terms of specific numbers, Knight and Sirmans (1996) reports that every 0.1 miles further from the metro station contributed to a 2.5% decrease in housing rent.

3. Mode choice and heterogeneous income

3.1. Private transport

We now add the car as a possible transport mode. We continue to abstract away from congestion externalities, and, for the time being, we keep the assumption that individuals are homogeneous. We further assume that everyone has access to commute by car.

If a resident commutes by car, the income constraint becomes $E+wW = c+pq+\gamma+\tau' x$, where γ represents a fixed (distance independent) cost, such as capital, insurance, or parking costs at the CBD. The time constraint is $T = H + L + t_c$, where t_c is the car travel time. As before, we obtain H from the time constraint and replace it on the income constraint.

Using that travel time t_c and location x are related through car speed, v_c, we obtain:

$$\underbrace{y}_{E+w(T-L)} = c + pq + \underbrace{\rho_c}_{\gamma+\left(\tau'+\frac{w}{v_c}\right)x}$$

The classic monocentric model would have $\gamma = 0$ and $\tau = \tau' + \frac{w}{v_c}$, such that commuting costs are τx, with τ capturing both operational and time costs.

We have now a discrete-continuous choice model where consumers have to choose where to locate, the dwelling size, consumption of the numeraire, and one of the three transport modes available: car, public transport or walking. At each location x, individuals choose the mode of transport with lower commuting costs, as this maximizes utility; the decision is individual and does not require any belief on what the rest of the people do since there are no externalities. Letting V be the utility associated to the transport mode with lowest cost at each location x, equilibrium conditions (3) and (4) above hold.

Identifying which mode of transport is used at different locations boils down to identifying where each mode has a lower commuting cost. In previous models (e.g. LeRoy and Sonstelie, 1983; Glaeser et al., 2008), as modes differ in both fixed and variable costs, this is reduced to find the intersection between two linear commuting costs. In our model, as public transport commuting costs are not monotonically increasing, there could be multiple crossing points. Mathematically, finding intersection points is simple to do, yet it is not very informative, so we do not linger on this. Instead, we rely on graphical analyses to illustrate the ensuing numerical simulations. Recall that public transport costs have the sawtooth shape discussed in Section 2.4 and displayed in Figure 4. What becomes relevant for our analyses here are the upper and lower *boundaries*, that is, the straight line connecting the lower points of the commuting cost function, which occur at stations, and the straight line connecting the peaks, which are located at $\bar{x}_k$ ($k \in \{1, ..., n\}$). It is simple to see that, in the case of a flat fare, the lower boundary line has an intercept equal to e and a slope given by $\frac{w}{v}$. The upper boundary line has an intercept given by $z = \bar{x}_0 \left(\frac{1}{\mu} - \frac{w}{v}\right) > e$ and the same slope $\frac{w}{v}$.

As discussed, car commuting costs are linear, with intercept γ and slope $\tau' + \frac{w}{v_c}$. There are several possible spatial arrangements of mode use, and the equilibrium pattern depends on the relative magnitude of both the intercepts (γ, e and z) and the slopes ($\tau' + \frac{w}{v_c}$, and $\frac{w}{v}$). It is reasonable to assume that cars go faster than public transport so that $v_c > v$; this pushes for the slope of the car commuting cost to be smaller. However, the cost per kilometer τ' has the opposite effect, as per-kilometer user expenses are higher for car travel than for transit. Consider a first case where $e < \gamma < z$ and suppose that the two opposite effects in the slope of the car commuting cost cancel out, so that $\tau' + \frac{w}{v_c} = \frac{w}{v}$. In this case, as shown in panel (a) of Figure 7, the prediction is that between consecutive stations, people closer to the stations would walk to the public transport system, while people in the center would use the car. This pattern does not change as x increases, so it is present in the entire city. Note that this result implies a decrease of the commuting costs compared with the case of only public transport, around what we denoted $\bar{x}_k$, implying that the previously predicted drop of rental and land prices, building height, and population density between stations would be softened. Consistently, dwelling sizes there would not increase as much between stations.

We now turn to two additional cases, one where the slope of the car commuting cost is smaller than that of the lower boundary line, and one where it is larger. If speeds are fixed, which of the two cases arise depends on τ', which in turn is arguably mainly driven by gasoline prices. So, to fix ideas we could think of a North American case, where gasoline is relatively inexpensive, implying a small τ', and a European case, where gasoline is more expensive, meaning a larger τ'.[10]

For a small τ' –the North American case– it may then happen that $\tau' + \frac{w}{v_c} > \frac{w}{v}$. In this case, shown in panel (b) of Figure 7, there can exist a first zone where people either walk to the CBD or use only public transport, followed by a zone where people close to stations take the public transit and people in-between stations commute by car. The share of car use would be increasing as neighborhoods are farther away from the CBD, until a point in which people only commute by car. Note that in this case, there would be no disconnected suburbs. Panel (c) shows what happens when fuel is more expensive. In this case, the mix between car and public transport occurs close to the CBD, but people living farther away, in equilibrium, takes public transport.

One of the most compelling results is that the simple public transport/car model with stations and walking can provide a rationale for mixing in transport modes along the city, without needing to resort to different income groups. LeRoy and Sonstelie (1983), when considering two income groups and two modes, obtain up to four different zones but, if only one income group is considered, the city would be divided into two, one zone dominated by the car, and one zone by public transport. In reality, we observe smoother modal split changes as people live away from the CBD. Consider, for instance, what Glaeser et al. (2008) report for what they call *subway cities* in the US (Boston, Chicago, New York City, and Philadelphia). They show that public transport usage increases from around 30 to 40-50% in the first three miles, and then decreases to about 20% at the 10th mile. The inverted U-shape of the use of public transport mode is attributed to macro changes in the location of income groups, according to the model of LeRoy and Sonstelie. First, close to the CBD, rich people would use public transport; then, poor people would locate and use public transportation, and, finally, rich people would live in the outer part of the city and would commute by car. What we argue is that the inverted U-shape of public transport

[10] According to Statista (2018), as of the 2nd quarter of 2017, US gasoline prices were between half and a third of those in the UK, Germany, France, Sweden, Italy, Netherlands, and Norway.

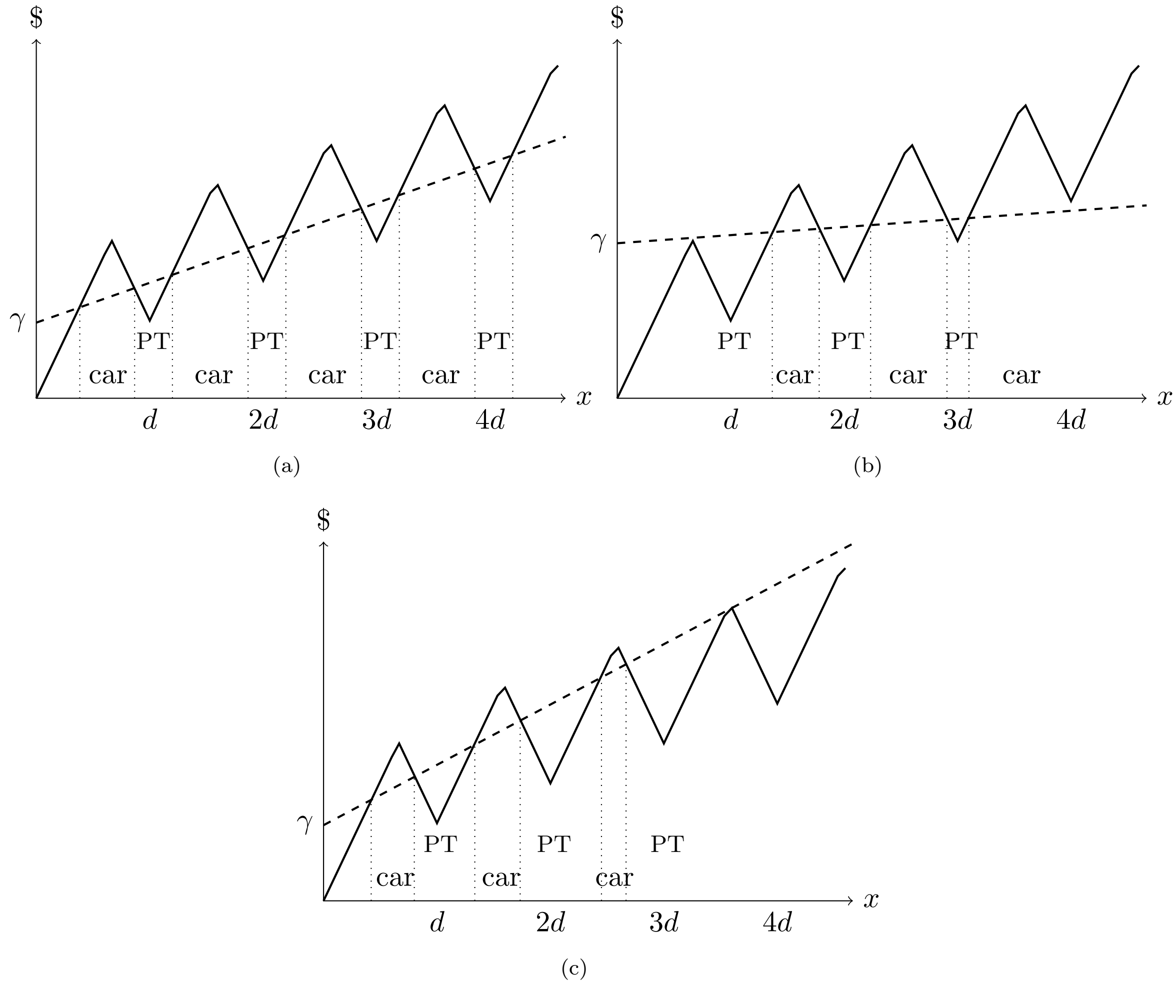


Figure 7: (a) spatial modal split with equal slopes, (b) low-gasoline price city, and (c) high-gasoline price city.PT refers to public transport.

usage in these subway cities may well be explained by the presence of public transport –and stations– alone, without necessarily incorporating different income groups, which add a second effect. Indeed, consider panel (b) in Figure 7 and suppose that we calculate the average public transit usage (recall that the density gradient is overall decreasing) between the CBD and the first station, and then between stations. It is intuitively clear that a U-shape for public transport ridership would be obtained.

To illustrate this better, we simulate the urban equilibrium structure of a city. The functional forms and parameters of the simulations come mainly from Wu and Plantinga (2003) and Bertaud and Brueckner (2005), which are based on US values. We discuss them briefly. The utility function chosen is a Cobb-Douglas $U(c,q) = c^{1-\alpha}q^{\alpha}$, which provides closed-form solutions for the key variables. We set α equal to 0.3, which is the average of the value used by Wu and Plantinga (2003), 0.5, and Bertaud and Brueckner (2005), 0.1. This implies that each household spends 30% of the income in housing. The hourly wage is US$ 16.86, which is the value used by Bertaud and Brueckner (2005). It is estimated using the income per household of the 2000 US census (US$ 42,151) and 2,000 work hours/year, which implies 5.48 work hours per day. Wu and Plantinga (2003) use an income of US$ 40,000 which is broadly consistent.

We also follow Bertaud and Brueckner (2005) in the production side and use a Cobb-Douglas production function normalizing the price of capital to unity. This gives a functional form for the housing output per unit of land $h(S) = g \cdot S^{\beta}$. We set g and β together with the agricultural land value r_A and the city population N to obtain a reasonable back-of-the-envelope estimate for the equilibrium number of dwellings per building in the city center as well as a reasonable length of the city. The distance between stops, d, is set at 1.5 km, and the public transport fare e is US$ 1. The fixed cost for a car trip is US$ 4, and the variable cost per km. is US$ 0.58. Walking speed is 4 km/hr, and the free-flow public transport speed is 24 km/hr. The parameter values are summarized in Table 1.

Parameter	**Value**	**Explanation**
α	0.3	Percentage of the income spent on housing.
y	US$40,000	Annual income per household
w	US$16.86	Hourly wage
g	1	Production function multiplier
β	0.75	Power of the production function
r_A	US$4,000	Agricultural land price
L	80,000 hab.	Population
d	1.5 km	Distance between public transport stops
e	US$ 1	Public transport (flat) fare

Table 1: Parameters values of the base case

Figure 8, panel (b), shows the public transport usage along the city of the numerical simulation. We obtain an inverted U-shape curve for public transport usage, with a companion flat income curve. The smoothness of the transit share curve comes from the mixing that occurs along the city, which is induced by the modeling of public transport stations and the associated walking costs.

A final point worth mentioning is that not only the price of fuel may lead to these different configurations. Indeed, for fixed fuel prices, various public transport technologies may involve different speeds (variable v), which would affect the trend of the sawtooth pattern, as opposed to that of the straight car line. Several transport policies may affect the slopes of the relevant curves that define the transport mode used at each location, and, therefore, may determine how much, and where, public transport is used. For instance, the pricing of parking, car ownership taxes, or transit subsidies will affect intercepts, while transit policies such as congestion pricing or dedicated bus lanes will affect speeds. We believe that this opens the door to analyzing the effects of these and other measures in terms of the spatial distribution of public transport usage and their welfare implications in a broader urban spatial context.

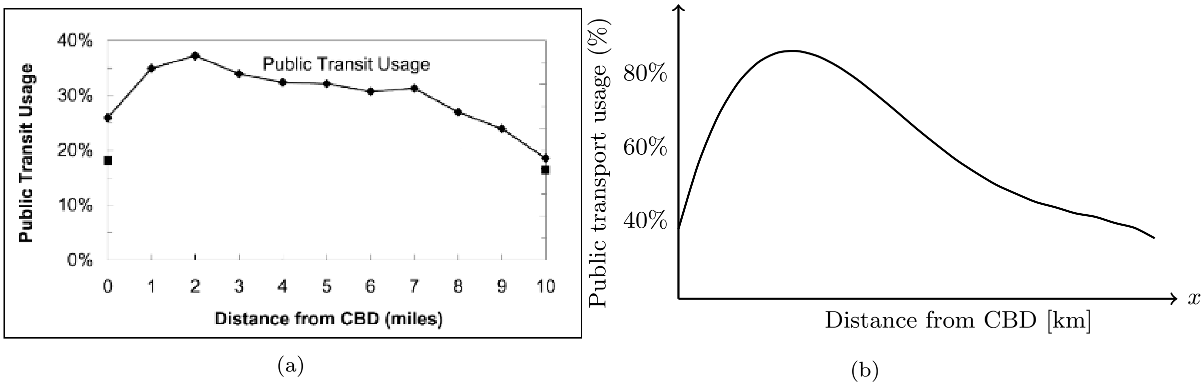


Figure 8: Public transit usage, and distance to CBD. (a) Adapted from Glaeser et al. (2008) and (b) our model with homogeneous individuals and two modes.

3.2. Adding heterogeneity of income

We now extend the analysis to include income heterogeneity. We adopt the simplest approach, namely considering two income groups. Suppose first that only public transport (and walking) is available. The income groups, denoted by superscripts h for high-income and l for low-income differ in the wage rate and in the utility function. As nothing else changes, the individual optimality condition in eq. (3) and the spatial equilibrium condition in eq. (4) apply to each group:

$$\frac{\frac{\partial V^i(y^i-pq-\rho^i,q)}{\partial q}}{\frac{\partial V^i(y^i-pq-\rho^i,q)}{\partial c}} = p \qquad\qquad V^i(y^i-pq-\rho^i,q)=\bar{U}^i$$

This system of equations, just as derived in Section 2, allows for obtaining the solutions for rental prices and housing consumption for each group as a function of the parameters of the model. As the income and utility function are different, the price of housing and the housing consumption will, in general, be different at each location. To distinguish them we use superscripts so that $p^i(x)$ represents the willingness to pay for housing by the group $i \in \{l, h\}$ at location x. As previous authors have assumed (see, e.g., Wheaton, 1976) land is allocated to the consumers willing to pay more, i.e. those in the group with higher $p^i(x)$. In other words, $p^i(x)$ are the bids for land by consumers.

As housing is essential, both groups consume housing, and therefore, there must be at least one location in which low-income residents outbid the high-income residents, and the reverse must also hold. Suppose there is a point $\hat{x}$ in which both bids are the same, therefore $p^l(\hat{x}) = p^h(\hat{x})$ holds. Using the Alonso-Muth condition (see equation (14)) the slope of the bid rent for each group is given by:

$$\frac{\partial p^i}{\partial x} = -\frac{1}{q^i}\frac{\partial \rho^i}{\partial x}$$

As housing is a normal good, $q^h(\hat{x}) > q^l(\hat{x})$ holds because prices are the same. Therefore, if ρ were the same for both income groups, as is the case in the more classical version of the monocentric city, the price gradient would be steeper for the low-income group, and they would be located nearer to the CBD with the high-income residents in the suburbs. This has been pointed out as early as Alonso (1964). In our model, however, ρ is different for the two income groups for two reasons. First, the wage rate is different so that the opportunity cost of time is different. Second, the location in which residents are indifferent between walking upstream or downstream, $\bar{x}_k$, are generally different.[11] This implies that at some locations the commuting cost and thus the bid rent gradient for low- and high-income groups have different signs. Therefore, due to the sawtooth pattern and the difference in the critical points, the bid rent curves can cross multiple times. This implies that mixing of income groups may happen several times along the city, inducing mixed neighborhoods. In contrast, close to the CBD and the city edge, one income group will eventually prevail. We illustrate the possible multiple crossing in Figure 9.

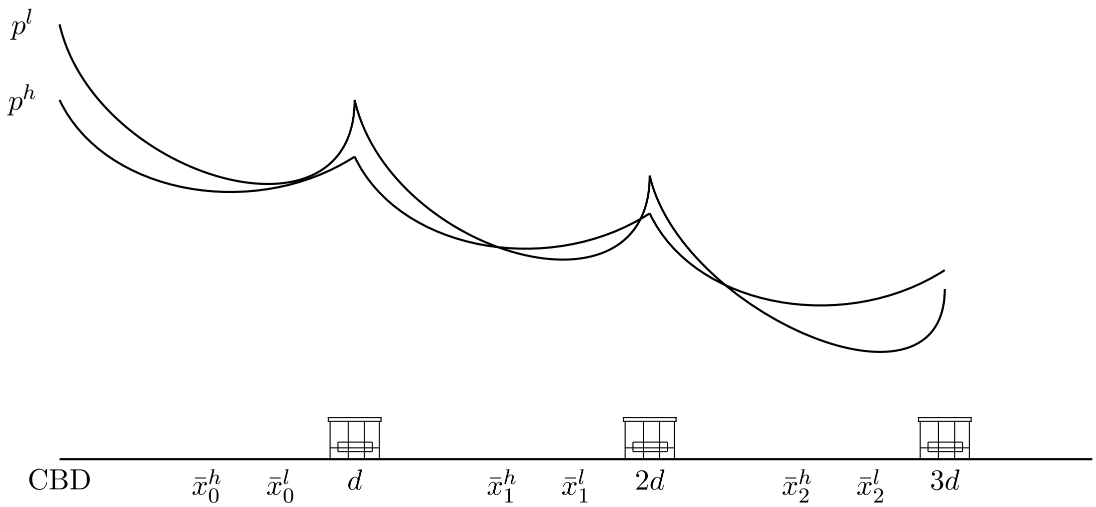


Figure 9: Example of the possible multiple crossing of the price of housing for low-income individuals (p^l) and high-income individuals (p^h).

We illustrate the multiple crossing with a simulation based on the parameters of the numerical simulation of Section 3.1. The differences are: (i) the population of each group is half of the total population (40,000 inhabitants each group); (ii) the high-income group earns 25% more than the base case population (wage rate equals to US$ 21.1) and spends 25% of the income on housing ($\alpha^h = 0.25$); (iii) the low-income group earns 25% less than base case population (wage rate equals to US$ 12.6) and spends 35% of the income on housing ($\alpha^l = 0.35$). Figure 10 shows the equilibrium price of rent in the city and the sorting of income groups.

Figure 10 shows that in the first 5.2km, there are only people from the high-income

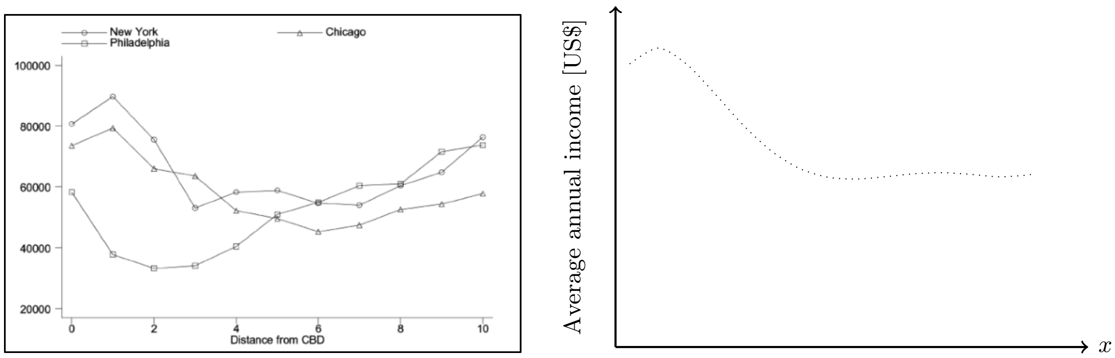
[11] From equation (10) it is straightforward to see that $\bar{x}_k$ is always different for the two groups $\forall k > 0$.

Figure 10: Rental price of housing in US$ per sq.ft. per year and sorting of income groups in the public transport city.

group, and beyond the 7.7km, only people from the low-income group. Thus, the numerical simulation has a low-income suburb in which people commute by public transportation and a high-income inner city in which residents walk to the CBD in the first kilometer and commute by public transport after that. In between, there is plenty of mixing: there are two high-income neighborhoods around the stations located at 6km and 7.5km. In between, there are low-income neighborhoods of less than a km long.

Mixing of two income groups, as opposed to having just two regions, one for the poor and one for the rich has been obtained before, but always at somewhat macro scales. For instance, if amenities or open spaces are present, then the mixing of income groups may also occur (Gaigné et al., 2017). LeRoy and Sonstelie (1983) find that mixing is possible with up to four regions along the city if the two income groups have two transport modes available. But the level of mixing we obtain, at a smaller scale, is novel, it does not require a second transport mode, and it provides a much smoother transition of the average income of neighborhoods as distance increases from the CBD. Consider for instance Figure 11, panel (a), from Glaeser et al. (2008) which shows smooth transitions of income levels in American cities. With only two income groups, if we average income between stations, we obtain Figure 11, panel (b), which roughly resembles the data reported for Chicago.

The existence of public transportation as a substitute for private transportation has been argued to be the reason why the poor centralize in the United States by LeRoy and Sonstelie (1983) and Glaeser et al. (2008). This would precisely be the cause for the inverted U-shape of income in Figure 11, panel 11a. In the simulation we present, the city center is dominated by the rich before the mixing zone (as in Chicago). Still, other parameters do place the poor at the center, followed by a mixing zone.



(a) (b)

Figure 11: Income and distance from the CBD. (a) Glaeser et al. (2008) and (b) this model.

3.3. Full model and simulations

When substitution between car and public transport is added to income heterogeneity, one obtains a model that remains simple, yet flexible enough to keep track of the non-monotonous nature of urban gradients in cities with public transport. At the same time, the model allows for reproducing well-observed spatial patterns of sorting by income and use of public transport.

To show this, we now combine the previous simulations in Section 3 and 3.2. The simulation is reasonable in that back-of-the-envelope calculations inspired in the linear city of Figure 1 and assuming three buildings per 100m on either side of the line lead to sensible outcomes. In the first 200m of the city, there are 1950 dwellings in 12 buildings, endogenously giving 163 dwellings per building. The same calculation for the first kilometer gives an average of 128 dwellings per building. Using a reference of 12 dwellings per floor, we obtain buildings of an average height of 14 floors in the first 200m and 11 floors in the first km. The same calculation for the kilometer furthest away from the CBD yields buildings of just one floor. Furthermore, the percentage of individuals that end up walking to the CBD instead of taking public transport or traveling by car is 9.4%. We believe that these are reasonable numbers for a metro or rapid transit line.

We illustrate the results of the simulation in Figure 12. This figure shows the rental (per unit of floor space) gradient.

As discussed in the previous section, this last gradient matches what a large body of empirical literature has found: that rental prices decrease as units are farther away from the public transport station. However, one can observe that close to the CBD, the peaking effect of stations is strong, but it weakens as the car becomes more prevalent among people living in the range of 5 to 15 km from the CBD. Eventually, the use of cars by lower-income people takes over completely (which also means that no disconnected suburb occurs for this simulation).

In Figure 13, we show the distance elasticity of rental price, calculated as the change in price 0.1 miles away from a station. Values range from 1% to 3%, with a non-monotonic

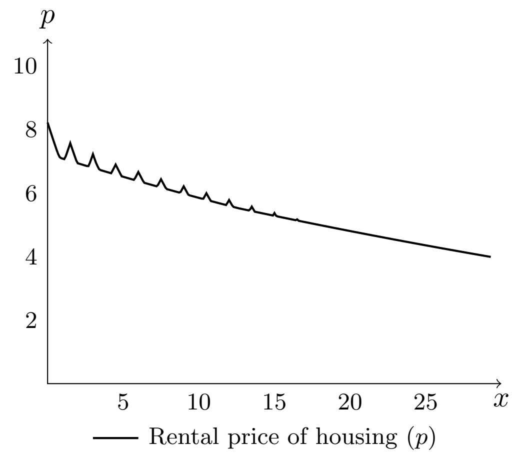


Figure 12: Numerical simulation of a city. Base case. Rental price per unit of floor space.

spatial pattern, although decreasing as the station is closer to the CDB between kilometers 4 and 15. These figures are roughly consistent with the 2.5% found for Washington DC by Knight and Sirmans (1996). Moreover, Gu and Guo (2008) and Gu and Zheng (2010) found that the distance elasticity is smaller closer to the CBD.

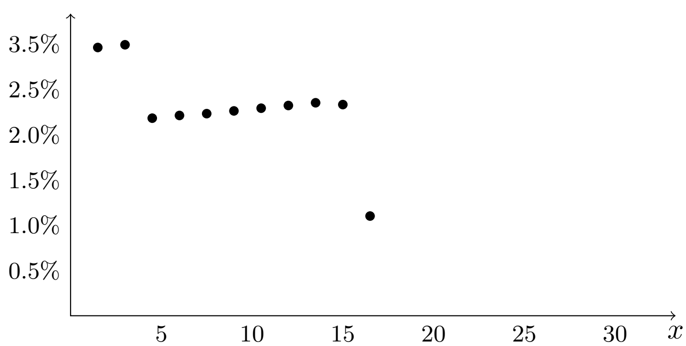
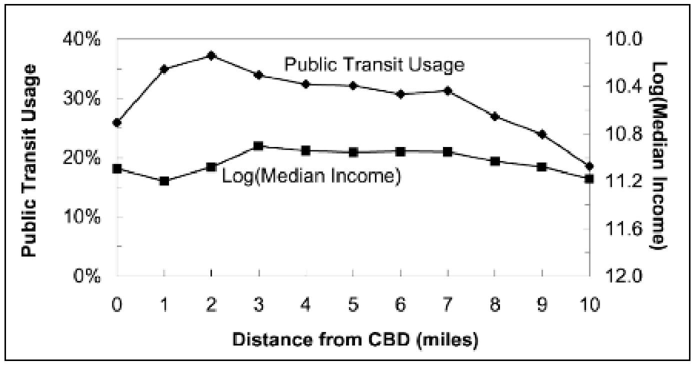


Figure 13: Distance elasticity of rental price.

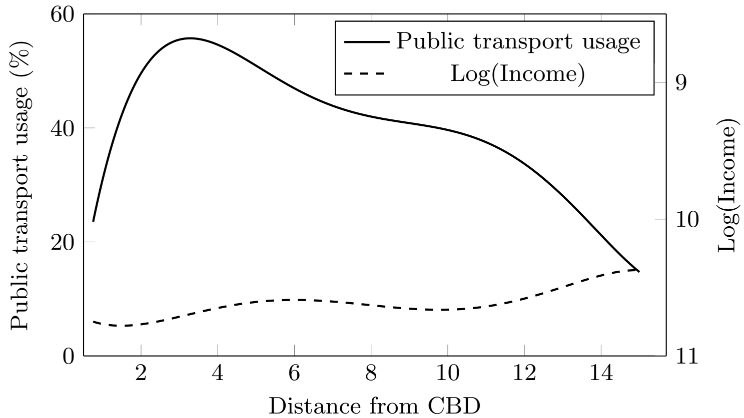
On the other hand, income mixing is strong, especially in the 12 kilometers closest to the CBD. Transport usage and income sorting along the city is shown in Figure 14. In this case, both the U-shape of transport usage and the U-and-then-inverse-U-shape for income that Glaeser et al. (2008) report as representative of some American cities are reproduced.

4. Conclusions

In this paper, we have studied the role of public transport in shaping urban structure. We extend the analysis of public transportation in the monocentric city model by explicitly modeling that it can be accessed through a limited set of stations. This gives rise to non-monotonic gradients for all the essential variables. In particular, our theoretical model shows that around public transport stations rental prices are higher, buildings are taller, and apartments are smaller, as it is observed in reality. This simple model can also explain



(a)



(b)

Figure 14: Public transit usage, income, and distance to CBD. (a) Glaeser et al. (2008) and (b) this model.

the presence of disconnected suburbs (leapfrog development) and to reproduce observed patterns of modal and income mixing along the city.

We argue that this model is useful to address the efficiency and impact of public transport policies as it captures many of the urban (ir)regularities. In particular, it helps evaluate the implications of changes in transportation systems or technology, pricing schemes, and taxes, among others, on relevant variables such as vehicle-kilometers traveled emissions and congestion. The main reason why this model is appropriate and improves upon others is that it can predict and capture the critical essential features in a simple way. For example, there is evidence that car ownership and fuel consumption decreases in the catchment area of a new subway station and that the effect is heterogeneous to the distance to the city center (Zhang et al., 2017). Such long-run effects and their heterogeneity are captured in our model.

The analysis can be extended in various directions. Arguably the most natural extension is to analyze different distance-based public transport pricing schemes. There is a current debate about whether distance-based fares should be abandoned or embraced. Yet, we are not aware of a study that addresses this question with the endogenous location of households. Another avenue is incorporating private and public transport externalities and study the efficiency of urban transport pricing policies as well as investment in infrastructure. Car congestion and public transport boarding delays are logical candidates. A different avenue for future research is to consider the dynamics of the cities by considering durable housing and redevelopment. This would make the study closer to a medium-run intra-city analysis.

Appendix A. Background, data and methodology for Figure 1

Appendix A.1. Santiago and its public transit system

Santiago is the capital and the most populated city of Chile, with almost 7 million inhabitants. Santiago has two features that make it an interesting case study in the context of this paper.

First, Santiago's jobs are mainly concentrated in a spine-shaped area at the geographic center of the city (what we will refer to as the CBD), as we show in Figure A.15. Through main avenues, the CBD connects to residential sectors on the outskirts. Overall, these avenues shape the majority of surface transportation in the city. That is, through these routes, most of the inhabitants commute radially to the CBD either by car or by public transit. Although this is far from perfect, this feature resembles the mechanics of the monocentric city model presented in Section 2.

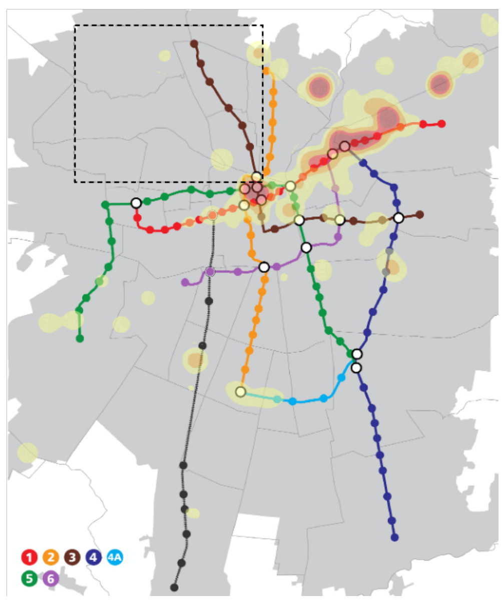


Figure A.15: Job locations area and the current subway network

Second, Santiago has expanded its rail-transport network significantly in the last ten years, opening two new subway lines and one suburban train. One segment of the subway expansion parallels one of the radial main avenues of the city. This segment belongs to the recently inaugurated Line 3 (see Figure A.15) and connects the CBD with the suburbs in the north. Line 3 was announced in July 2011, and its construction took eight years until its inauguration in February 2019. In this paper, we analyze the north segment of Line 3, which has eight stations and is 8.5 km long.

In summary, Santiago is a city with commuting patterns that may be approximated as monocentric, which, through its subway expansion on a radial route, provides an opportunity to evaluate the model's predictions empirically.

Appendix A.2. Data sources and methodology

To analyze the effect of Line 3 on the spatial structure of its radial route, we use data that includes: (i) the location of the subway line and its stations, and (ii) the location of dwellings and their characteristics as price.

The geographic location of Line 3 and its stations are public information.[12] This data is accessible either through the Metropolitan Public Transit Authority's website or even through Google Maps. For each subway line, there are records of both the date of announcement and the date of opening (month and year). The announcement of a new subway line generally involves details on the route, but also on the location of the stations, which was the case of Line 3 in July 2011.

Data on the dwellings' location and their characteristics is available from the Real Estate Registry of Santiago de Chile. We use this administrative data geocoded by TOCTOC.[13] This dataset contains all real estate transactions from 2007 to 2017 and their corresponding geographic location. In particular, the available variables are sale price, dwelling size, type (house or apartment), condition, age, and quality of construction material.

Note that the years available in our dataset corresponds to years before and after the announcement of Line 3 (July 2011), but they do not extend until its inauguration (February 2019). Therefore, observing changes in dwelling sizes and buildings' height is difficult since the real estate developers usually take longer to respond. With this in mind, we analyze changes in housing prices since they internalize transportation infrastructure announcements faster.

The subway line affects all surrounding dwellings within an influence area. For this analysis, we include all houses up to one kilometer away from their nearest point on the subway line. Since the model predicts the effect on housing prices (y-axis) from the CBD to the last station along the subway line (x-axis), we need to turn spatial data into two dimensions. Hence, we set consecutive bins of 100 meters long from the CBD to the last station and calculate the average of housing prices based on their distance to the subway line within each segment. In this calculation, housing prices closer to the subway line take a higher weight, following a quantic function. Finally, we fit a polynomial of degree 20 to

[12]The geographic coordinates of these locations are in latitude and longitude format, which can be displayed and analyzed on any GIS software.

[13]TOCTOC is a private company that provides real estate services. They provided the data for research purposes only.

the sequence of average prices. This function is flexible and does not impose any structure based on the expected prediction.

Figure 1 shows the weighted average of housing prices for each segment of 100 meters, from the CBD to the last station of Line 3. The solid grey and black lines are polynomial fits for housing prices along the subway line before and after the announcement of Line 3, respectively. Vertical dashed lines indicate the location of each announced station. Before the announcement (solid grey line), we find a non-monotonic pattern with more frequent but less steep peaks that decrease slowly along the city. One possible explanation for this finding can be the presence of bus stops. Bus stops are usually less spaced between each other and lead to a more attenuated effect on housing prices. After the announcement (solid black line), we find a non-monotonic pattern with steeper peaks, especially near the CBD. Notice that these peaks appear at the location of announced stations (vertical dashed lines) without imposing any structure on the polynomial. This finding provides empirical evidence for one of the main predictions of the model.

We believe that this evidence provides support for the model as a useful tool to evaluate policies by calibrating the model to fit observations and then simulating counterfactual scenarios through parameter modifications. The model also can accommodate more features, such as durable housing, and provide additional insights or be used to evaluate transportation policies or housing development regulations (e.g., transit fare schemes, value capture tools, etc.).

References

Alonso, William, *Location and land use*, Harvard University Press Cambridge, MA, 1964.

Armstrong, Robert J. and Daniel A. Rodriguez, "An evaluation of the accessibility benefits of commuter rail in eastern Massachusetts using spatial hedonic price functions," *Transportation*, 2006, *33* (1), 21–43.

Bae, Chang-Hee Christine, Myung-Jin Jun, and Hyeon Park, "The impact of Seoul's subway Line 5 on residential property values," *Transport policy*, 2003, *10* (2), 85–94.

Basso, Leonardo J. and Hugo E. Silva, "Efficiency and substitutability of transit subsidies and other urban transport policies," *American Economic Journal: Economic Policy*, 2014, *6* (4), 1–33.

Behrens, Kristian, Pierre-Philippe Combes, Gilles Duranton, Laurent Gobillon, and Frédéric Robert-Nicoud, "Household sorting in the city," *Manuscript in progress, University of Pennsylvania, Wharton School*, 2014.

Bertaud, Alain and Jan K. Brueckner, "Analyzing building-height restrictions: predicted impacts and welfare costs," *Regional Science and Urban Economics*, 2005, *35* (2), 109–125.

Bowes, David R. and Keith R. Ihlanfeldt, "Identifying the impacts of rail transit stations on residential property values," *Journal of Urban Economics*, 2001, *50* (1), 1–25.

Brueckner, Jan K., "The structure of urban equilibria: A unified treatment of the Muth-Mills model," in "Handbook of Regional and Urban Economics," Vol. 2, Elsevier, 1987, pp. 821–845.

Brueckner, Jan K, "Transport subsidies, system choice, and urban sprawl," *Regional Science and Urban Economics*, 2005, *35* (6), 715–733.

Brueckner, Jan K., Jacques-François Thisse, and Yves Zenou, "Why is central Paris rich and downtown Detroit poor?: An amenity-based theory," *European Economic Review*, 1999, *43* (1), 91–107.

CAF, *Observatorio de Movilidad Urbana para América Latina* Corporación Andina de Fomento 2010.

de Bartolome, Charles A.M. and Stephen L. Ross, "Equilibria with local governments and commuting: income sorting vs income mixing," *Journal of Urban Economics*, 2003, *54* (1), 1–20.

Duranton, Gilles and Diego Puga, "Urban land use," in "Handbook of Regional and Urban Economics," Vol. 5, Elsevier, 2015, pp. 467–560.

Efthymiou, D. and C. Antoniou, "How do transport infrastructure and policies affect house prices and rents? Evidence from Athens, Greece," *Transportation Research Part A: Policy and Practice*, 2013, *52*, 1 – 22.

EMTA, *Barometer 2015 of public transport in the European metropolitan areas.* 2015.

Feng, Changchun, Weixuan Li, and Fanfan Zhao, "Influence of rail transit on nearby commodity housing prices: A case study of Beijing Subway Line Five," *Acta Geographica Sinica*, 2011, *66* (8), 1055–1062.

Gaigné, Carl, Hans Koster, Fabien Moizeau, and Jacques-François Thisse, "Amenities and the Social Structure of Cities," *CEPR Discussion Paper No. DP11958. Available at SSRN: https://ssrn.com/abstract=2954479*, 2017.

Gibbons, Stephen and Stephen Machin, "Valuing rail access using transport innovations," *Journal of Urban Economics*, 2005, *57* (1), 148 – 169.

Glaeser, Edward L., Matthew E. Kahn, and Jordan Rappaport, "Why do the poor live in cities? The role of public transportation," *Journal of urban Economics*, 2008, *63* (1), 1–24.

Gu, Yi-Zhen and Rui Guo, "The impacts of the rail transit on property values: empirical study in Batong line of Beijing," *Economic Geography*, 2008, *3*, 016.

Gu, Yizhen and Siqi Zheng, "The impacts of rail transit on property values and land development intensity: The case of No. 13 Line in Beijing," *Acta Geographica Sinica*, 2010, *2* (010).

Hess, Daniel Baldwin and Tangerine Maria Almeida, "Impact of proximity to light rail rapid transit on station-area property values in Buffalo, New York," *Urban studies*, 2007, *44* (5-6), 1041–1068.

Kilani, Moez, Fabien Leurent, and André De Palma, "Monocentric city with discrete transit stations," *Transportation research record*, 2010, *2144* (1), 36–43.

Knaap, Gerrit J., Chengr Ding, and Lewis D. Hopkins, "Do plans matter? The effects of light rail plans on land values in station areas," *Journal of Planning Education and Research*, 2001, *21* (1), 32–39.

Knight, John R. and C.F. Sirmans, "Depreciation, maintenance, and housing prices," *Journal of housing economics*, 1996, *5* (4), 369–389.

Kutzbach, Mark J., "Motorization in developing countries: Causes, consequences, and effectiveness of policy options," *Journal of Urban Economics*, 2009, *65* (2), 154–166.

Landis, John, Subhrajit Guhathakurta, William Huang, Ming Zhang, and Bruce Fukuji, "Rail transit investments, real estate values, and land use change: a comparative analysis of five California rail transit systems," 1995.

LeRoy, Stephen F. and Jon Sonstelie, "Paradise lost and regained: Transportation innovation, income, and residential location," *Journal of Urban Economics*, 1983, *13* (1), 67–89.

Liang, Qi and Hua Cao, "Property prices and bank lending in China," *Journal of Asian Economics*, 2007, *18* (1), 63–75.

Mills, Edwin S., "An aggregative model of resource allocation in a metropolitan area," *The American Economic Review*, 1967, *57* (2), 197–210.

Muth, Richard, "Cities and housing: The spatial patterns of urban residential land use," *University of Chicago, Chicago*, 1969, *4*, 114–123.

Parry, Ian W.H. and Kenneth A. Small, "Should urban transit subsidies be reduced?," *American Economic Review*, 2009, *99* (3), 700–724.

Proost, Stef and Kurt Van Dender, "Optimal urban transport pricing in the presence of congestion, economies of density and costly public funds," *Transportation Research Part A: Policy and Practice*, 2008, *42* (9), 1220–1230.

Statista, "Gas prices around the world Q2 2017," Technical Report, https://www.statista.com/statistics/221368/gas-prices-around-the-world/ 2018.

Su, Qing and Joseph S. DeSalvo, "The effect of transportation subsidies on urban sprawl," *Journal of Regional Science*, 2008, *48* (3), 567–594.

Turner, Matthew A., "Landscape preferences and patterns of residential development," *Journal of Urban Economics*, 2005, *57* (1), 19–54.

Wheaton, William C., "On the optimal distribution of income among cities," *Journal of Urban economics*, 1976, *3* (1), 31–44.

Wu, JunJie and Andrew J. Plantinga, "The influence of public open space on urban spatial structure," *Journal of Environmental Economics and Management*, 2003, *46* (2), 288–309.

Yáñez, María F, Sebastián Raveau, and Juan de Dios Ortúzar, "Inclusion of latent variables in mixed logit models: modelling and forecasting," *Transportation Research Part A: Policy and Practice*, 2010, *44* (9), 744–753.

Zhang, Yingjie, Siqi Zheng, Cong Sun, and Rui Wang, “Does subway proximity discourage automobility? Evidence from Beijing,” *Transportation Research Part D: Transport and Environment*, 2017, *52*, 506 – 517. Land use and transportation in China.