Mandatory retirement savings in the presence of an informal labor market

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November 2020

MANDATORY RETIREMENT SAVINGS IN THE PRESENCE OF AN INFORMAL LABOR MARKET

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This paper shows how taking a job in the informal economy may help to circumvent the borrowing constraints activated by mandatory retirement savings. Agents with present bias may choose to work informally to avoid the mandatory contributions and increase their present consumption, even if they would be more productive in the formal economy. In this case, decreasing the contribution rate will actually increase their savings and welfare.

Keywords: mandatory savings, contribution rate, present bias, informal sector, borrowing constraints. JEL Codes: D14, E21, G11, H55.

1. INTRODUCTION

A key motivation for social security programs is the concern that individuals do not save enough for their retirement (Diamond (1977), Poterba (2014)). This undersaving is explained by present bias, a behavioral bias that leads to consuming too much in the present at the expense of future welfare (Feldstein (1985), Laibson (1998), İmrohoroğlu et al. (2003), Chetty (2015), Chu and Cheng (2019), Ericson and Laibson (2019)). As such, one might think that individuals should be mandated to contribute to their own retirement accounts (Summers (1989), Laibson (1998), Feldstein (2005), Cremer et al. (2008, 2009), Kaplow (2010), Andersen and Bhattacharya (2019), Moser and Olea de Souza e Silva (2019)). Nonetheless, this logic omits the existence of an informal sector, defined as economic activities where no taxes or social security contributions are enforced. Informal jobs tend to be low-productivity jobs (Amaral and Quintin (2006), Galiani and Weinschelbaum (2012), Ulyssea (2018, 2020)), but still represent a significant share of the labor force in

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emerging economies (Schneider and Enste (2013), La Porta and Shleifer (2014), Williams and Schneider (2016)). Individuals who are unaware of their present bias or simply cannot help themselves may take an informal job to avoid the mandatory contributions. In this context, what should be the appropriate policy to deal with the undersaving problem without missallocating the labor force into low-productivity activities?

In this paper, I focus on social security programs based on mandatory retirement savings: individuals working in the formal economy are forced to contribute a percentage of their wages into fully funded individual retirement accounts, mimicking the system implemented by Chile in 1981 and popularized across Latin America ever since. Some empirical works do indeed incorporate the informal sector on the study of social security reforms (e.g, Frölich et al. (2014), Joubert (2015), Becerra (2017, 2018), McKiernan (2020)), but none tries to tackle the design –starting from scratch– of the optimal social security program in the presence of an informal sector. This paper tries to fill this vacuum by looking at the optimal level of mandatory savings in the context of agents with present bias who may evade the mandatory contributions by working informally.

The results are driven by the interaction between the borrowing and saving decisions and the allocation of labor between the formal and the informal sectors. Mandatory savings are ineffectual unless a borrowing constraint prevents agents to turn future benefits into present consumption (Aaron (1966), Blanchard and Fisher (1989), Gale (1998), Andersen and Bhattacharya (2011)). Nonetheless, impatient agents with an active borrowing constraint will look for alternative ways to borrow, even if they are relatively expensive. I show how working in the informal sector is one of such ways. By working informally, individuals circumvent the borrowing constraint activated by the mandatory savings and increase their present consumption at the expense of their future consumption. As a consequence, decreasing the contribution rate will actually increase the savings and welfare of workers who would be more productive in the formal sector but are working informally to avoid the contributions.

The rationale behind the results is as follows. Imagine that an agent earns one dollar in the formal sector. However, the social planner tries to force her to save this dollar. The

agent faces the following dilemma: to earn this dollar in the formal economy or to earn less in the informal economy. The agent will choose to work informally if the informal earnings are above the present value of the gross return on the saved dollar. Such an event requires the agent to discount the cash flow at a rate higher than the net interest rate on savings. This happens when the agent would like to borrow to increase today's consumption but is not able to do so. Therefore, by trying to force the agent to save a dollar, the social planner may end up burning some cents.

In contradiction to conventional wisdom, a higher contribution rate not only reduces the future consumption of certain agents, but it may increase their present consumption as well. This implies that ignoring the existence of an informal economy may lead to the implementation of paternalistic social security programs that aim at helping agents to correct their present bias, but in the end may have the complete opposite effect.

The rest of this paper is organized as follows. Section 2 describes the model and its solution. Section 3 presents some analytical results based on comparative statics. Section 4 extends the analysis to a multi-agent setting. Section 5 discusses the empirical implications and suggests future paths of research.

2. THE MODEL

Consider a single agent who lives for two periods. She is endowed with one unit of time which she allocates between working in the formal sector and working in the informal sector. Leisure gives her no utility. Her earnings per unit of time are w^f in the formal sector and w^s in the informal sector, where $0 < w^s < w^f$. She is not able to work in her second period of life, hence her second-period consumption has to rely on savings. Her *experienced* lifetime utility is given by $u(c_1) + \beta u(c_2)$, where c_1 is the consumption in her first period of life and c_2 is the consumption in the second period. I assume that the instantaneous utility function u is 1) increasing, 2) strictly concave, 3) twice differentiable, and 4) satisfies $\lim_{c\to 0} u'(c) = \infty$. The parameter $\beta \in (0,1)$ is the discount factor the agent *should* be using when making her decisions. Nonetheless, she is too impatient for her own good and instead uses a discount factor $\delta\beta$, where $\delta \in (0,1)$ is the present-bias parameter.

The decisions she takes in her first period of life are made to maximize her *decision* lifetime utility, which is given by the function $u(c_1) + \delta\beta u(c_2)$.[1]

The agent chooses how much time $z \in [0, 1]$ to spend working in the formal economy and how much time $1-z$ to spend working in the informal economy. Therefore, her formal income y is given by $w^f z$ and her total income before any taxes or contributions is given by $w^s + \left(1 - \frac{w^s}{w^f}\right) y$. The agent is forced to save a fraction τ of her formal income, where $\tau \in [0, 1)$ represents the contribution rate. Her forced savings are invested in a riskless asset that yields a gross return $R > 0$, implying that the agent receives $R\tau y$ dollars in benefits in her second period of life. The agent may also voluntarily save x dollars at a gross return R, but she is subject to the borrowing constraint $x \geq 0$. Since voluntary and forced savings offer the same return, the borrowing constraint on voluntary savings is necessary to avoid the irrelevance of forced savings. In particular, if the agent could borrow at a rate R, any forced savings would be offset by the equivalent borrowing, implying that total savings would be the same as if there were no forced savings.[2]

The agent solves the following problem:

$$
\begin{aligned}
(\hat{c}_1, \hat{c}_2, \hat{y}, \hat{x}) \in \quad & \arg\max_{(c_1, c_2, y, x)} u(c_1) + \delta\beta u(c_2) \\
& \text{subject to:} \\
& c_1 = w^s + \left(1 - \tfrac{w^s}{w^f} - \tau\right) y - x \\
& c_2 = R\left(\tau y + x\right) \\
& y \leq w^f \\
& x \geq 0
\end{aligned}
\tag{1}
$$

First, consider the case where $1 - \frac{w^s}{w^f} - \tau \geq 0$, meaning that the contribution rate is not higher than the productivity gap between the formal and the informal sectors. In this case, it is optimal to work full-time in the formal economy. Second, consider the opposite case, where $1 - \frac{w^s}{w^f} - \tau < 0$. In this case, working in the informal sector increases

[1] For a recent discussion on present bias, including the adequacy of savings for retirement, see Ericson and Laibson (2019).

[2] See Aaron (1966) and Blanchard and Fisher (1989) for the theoretical formulation of this result and Gale (1998) and Chetty et al. (2014) for an empirical discussion on the offsetting of pension wealth.

today's take-home payment at the expense of future consumption. As such, working in the informal sector becomes an alternative way of borrowing. Still, the agent will not work in the informal sector as long as she has any voluntary savings left. Conversely, if she is working in the informal sector, it is because she ran out of voluntary savings. This is formally stated in Lemma 1:

LEMMA 1 *The solution to Problem (1) is unique and it has the following properties:*

i *If* $\hat{x} > 0$, *then* $\hat{y} = w^f$.

ii *If* $\hat{y} < w^f$, *then* $\hat{x} = 0$.

PROOF: It follows from the following first order conditions of Problem (1):

$$1 - \frac{\delta\beta u'(\hat{c}_2)}{u'(\hat{c}_1)} R \geq 0, \text{ with equality if } \hat{x} > 0. \tag{2}$$

$$\left(1 - \frac{w^s}{w^f}\right) - \left(1 - \frac{\delta\beta u'(\hat{c}_2)}{u'(\hat{c}_1)} R\right) \tau \geq 0, \text{ with equality if } \hat{y} < w^f. \tag{3}$$

Q.E.D.

Unless the contribution rate is high enough to activate the borrowing constraint, the agent works full time in the formal sector. To show this, consider the marginal value of a formal dollar, as defined by the left-hand side of Inequality (3). The first term is the productivity gap. The second term is the wedge between benefits and contributions. If the borrowing constraint is not active, then the agent is willing to give up one dollar in the first period in exchange for R dollars in the second period, implying no wedge. In this case, the marginal value of a formal dollar equals the productivity gap and the agent works full-time in the formal sector. The threshold for the contribution rate that activates the borrowing constraint is equal to the saving rate in the absence of mandatory savings, as stated in the following lemma:

LEMMA 2 *Let* $\underset{\sim}{\tau}$ *be the voluntary savings rate in the absence of mandatory savings:*

$$\frac{\delta\beta u'(R\underset{\sim}{\tau} w^f)}{u'((1-\underset{\sim}{\tau})w^f)} R = 1.$$

If $\tau \leq \underset{\sim}{\tau}$*, then* $\frac{\delta\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R = 1$ *and* $\hat{y} = w^f$*.*

As long as the contribution rate is below the threshold $\underset{\sim}{\tau}$ that activates the borrowing constraint, any increase in mandatory savings is fully offset by an equal decrease in voluntary savings, leaving the consumption path unchanged. However, once the contribution rate is above $\underset{\sim}{\tau}$, the agent is not willing to give up one dollar in the first period in exchange for R dollars in the second period, generating a wedge between benefits and contributions. As a consequence, the agent may want to avoid some of the mandatory savings by working part-time in the informal economy. Whether the agent works informally or not depends on her productivity gap. If the gap is high enough, then the agent will still work full-time in the formal sector, despite the wedge between benefits and contributions. However, if the gap is low enough, then the agent will allocate some time to the informal sector, increasing her present consumption at the expense of her future consumption. This increases the discount factor up to the point where the marginal value of a formal dollar becomes zero. The following lemma formally characterizes this solution:

LEMMA 3 *Define* $\bar{t}$ *as the wedge between benefits and contributions if the agent works full-time in the formal economy and her borrowing constraint is active:*

$$\bar{t} := \left(1 - \frac{\delta\beta u'(R\tau w^f)}{u'\left((1-\tau)\, w^f\right)}R\right)\tau.$$

If $\tau \geq \underset{\sim}{\tau}$*, then the solution to Problem (1) is as follows:*

1. *If* $1 - \frac{w^s}{w^f} - \bar{t} \geq 0$*, then* $\hat{x} = 0$ *and* $\hat{y} = w^f$*.*
2. *If* $1 - \frac{w^s}{w^f} - \bar{t} < 0$*, then* $\hat{x} = 0$ *and* $\hat{y} < w^f$ *solves the following equation:*

$$\left(1 - \frac{w^s}{w^f}\right) - \left(1 - \frac{\delta\beta u'(R\tau\hat{y})}{u'\left(w^s + \left(1 - \frac{w^s}{w^f} - \tau\right)\hat{y}\right)}R\right)\tau = 0 \tag{4}$$

Note that if mandatory savings are of any consequence for welfare, they have to lead to a zero-savings corner solution:

REMARK *If* $\tau \geq \underset{\sim}{\tau}$*, then* $\hat{x} = 0$*.*[3]

[3]This feature is also stressed in the model of Andersen and Bhattacharya (2011).

3. RESULTS

Voluntary savings are exhausted well before the agent allocates any time to the informal economy. As soon as τ goes just above $\underset{\sim}{\tau}$, the borrowing constraint becomes active, but the agent will still tolerate some further forced savings before working any time in the informal economy. Nonetheless, if the agent has already started to work informally, an increase in the contribution rate will actually *decrease* her total savings. This happens because the rise in the contribution rate is not enough to offset the fall in formal income, as shown in the following proposition:

PROPOSITION 1 *Consider the elasticity of formal income $\hat{y}$ with respect to the contribution rate τ:*

$$\epsilon := \frac{d\hat{y}}{d\tau}\frac{\tau}{\hat{y}}.$$

If $\hat{y} < w^f$, then $\epsilon < -1$.

PROOF: See Appendix A.

COROLLARY 1 *If $\hat{y} < w^f$, then $\frac{d\tau\hat{y}}{d\tau} < 0$.*

The effect of the contribution rate on present and future consumption might be the complete opposite of what a mechanical analysis that ignores the behavioral responses would suggest. Suppose that the contribution rate increases from τ to τ', as illustrated by Figure 1. If there is no opportunity to work in the informal sector and the borrowing constraint is active, then present consumption falls from $(1-\tau)w^f$ to $(1-\tau')w^f$ and future consumption increases from $R\tau w^f$ to $R\tau' w^f$. However, with an opportunity to work in the informal sector, agents working part-time in the informal sector respond to an increase in the contribution rate by evading even more contributions. Since mandatory savings are their only source of retirement income, their future consumption suffers. This is shown in Figure 1 as a fall on future consumption from $\hat{c}_2$ to $\hat{c}_2'$. Even more surprisingly, nothing rules out a positive effect of the contribution rate on present consumption. Mechanically, a higher contribution rate should decrease present consumption, given a fixed level of

formal income and an active borrowing constraint. However, the behavioral response of agents working part-time in the informal sector is to shift even more labor away from the formal sector. This helps them to increase their take-home payment, all of which is spent on present consumption. Their response not only helps them to offset the mechanical effect but it might completely overturn it, as in the example of Figure 1. In this example, present consumption *increases* from $\hat{c}_1$ to $\hat{c}_1'$. This is because the cost of present consumption in terms of future consumption actually *falls* from $R\tau\left(-\left(1-\frac{w^s}{w^f}-\tau\right)\right)^{-1}$ to $R\tau'\left(-\left(1-\frac{w^s}{w^f}-\tau'\right)\right)^{-1}$. The effect of the contribution rate on present and future consumption is formally stated in the following proposition:

PROPOSITION 2 *If* $\hat{y} < w^f$, *then* $\frac{d\hat{c}_2}{d\tau} = R\hat{y}(1+\epsilon) < 0$ *and* $\frac{d\hat{c}_1}{d\tau} = \frac{\hat{y}}{\tau}\left(\left(1-\frac{w^s}{w^f}-\tau\right)\epsilon-\tau\right)$.[4]

PROOF: It follows from Lemma 1 and Proposition 1. *Q.E.D.*

The effect of the contribution rate on welfare –as defined not by the decision utility but by the experienced utility– is not straightforward because of the ambiguous effect of the contribution rate on present consumption. Still, it can be shown that agents working part-time in the informal sector will be better off with a *lower* contribution rate:

PROPOSITION 3 *Define* $W := u(\hat{c}_1) + \beta u(\hat{c}_2)$ *as the indirect experienced lifetime utility. If* $\hat{y} < w^f$, *then* $\frac{dW}{d\tau} < 0$.

PROOF: See Appendix A.

4. MULTI-AGENT SETTING

A benevolent social planner would like to implement the contribution rate $\tilde{\tau}$ that maximizes the experienced lifetime utility if the option of working in the informal economy

[4] This is reminiscent to an overlooked result in Andersen and Bhattacharya (2019), where a contribution rate high enough can push middle-aged agents towards borrowing at a rate higher than the interest rate on savings. In this case, decreasing the contribution rate will increase their old-age consumption.

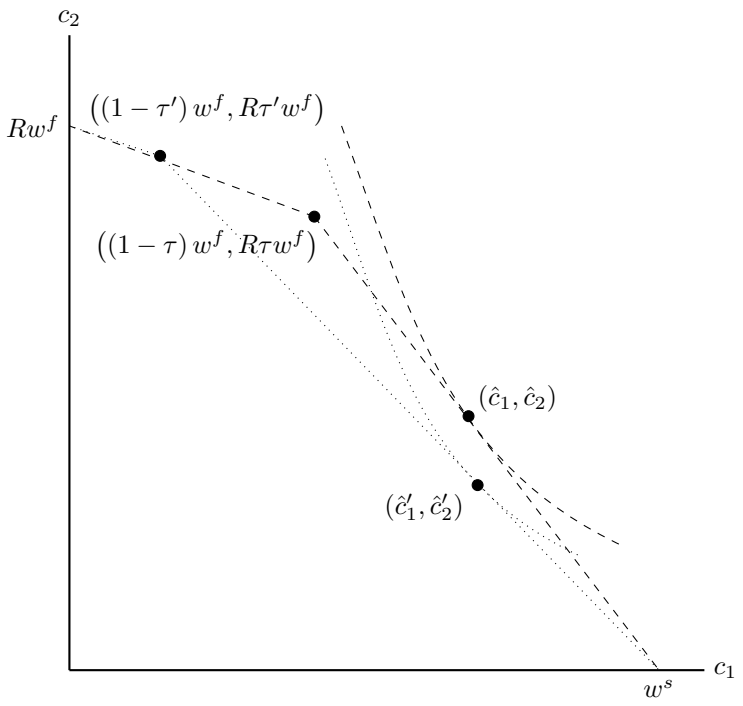

FIGURE 1.— Effect of the contribution rate on present and future consumption

were not available. This rate is a given by the following Euler equation:

$$\frac{\beta u'(R\tilde{\tau}w^f)}{u'((1-\tilde{\tau})w^f))}R = 1.$$

If the instantaneous utility u is isoelastic, then the contribution rate $\tilde{\tau}$ is independent of the formal wage w^f. Nonetheless, it follows from Lemma 3 that setting a uniform contribution rate $\tilde{\tau}$ will encourage those agents with a productivity gap below $(1-\delta)\tilde{\tau}$ to work informally. According to Proposition 3, these agents will be better off with a contribution rate lower than $\tilde{\tau}$. Therefore, the social planner would like to set the contribution rate according to the productivity gap.[5] Consider the case where there is a continuum of agents sorted according to their productivity gap in a closed interval $\mathcal{I} \subset \mathbb{R}$ that includes 0. Let w_i^f and w_i^s be the wages for type $i \in \mathcal{I}$ in the formal and the informal sectors, where w_i^f/w_i^s is increasing in i and $w_0^f = w_0^s$. The following corollary characterizes the optimal contribution rate for each type:

PROPOSITION 4 *Let τ_i^* be the contribution rate that maximizes the indirect experienced lifetime utility of type $i \in \mathcal{I}_+ \subset \mathcal{I}$.*

1. *If $1 - \frac{w_i^s}{w_i^f} - (1-\delta)\tilde{\tau} \geq 0$, then $\tau_i^* = \tilde{\tau}$.*
2. *If $1 - \frac{w_i^s}{w_i^f} - (1-\delta)\tilde{\tau} < 0$, then $\tau_i^* < \tilde{\tau}$ and τ_i^* satisfies the following equation:*

$$\left(1 - \frac{w_i^s}{w_i^f}\right) - \left(1 - \frac{\delta\beta u'\left(R\tau_i^* w_i^f\right)}{u'\left(\left(1-\tau_i^*\right)w_i^f\right)}R\right)\tau_i^* = 0 \tag{5}$$

PROOF: It follows from Lemma 3 and Proposition 3.

Unfortunately, the contribution rate schedule suggested by Proposition 4 is not implementable when the productivity gaps are private information. In this case, what is the best

[5] Sanchez Martin et al. (2014) show how a uniform contribution rate and a single investment portfolio may generate welfare losses due to a borrowing constraint that does not allow households to undo the default choices of a mandatory pension program. A similar point is made by Gomes et al. (2005), who show how the optimal contribution rate to tax-deferred saving accounts varies significantly across households with an uninsurable labor income.

the social planner can do without missallocating any labor? To answer this question, let $T(y)$ be the contributions to the individual retirement account when the formal earnings are y. In this setting, the *average* contribution rate $T(y)/y$ may differ from the *marginal* contribution rate $T'(y)$. The schedule T that achieves the highest experienced utility for each individual without missallocating any labor can be found by following these steps:

1. Find the maximum feasible contribution for type 0:

$$T(w_0^f) = \arg\max_x u(w_0^f - x) + \delta\beta u\left(Rx\right) \tag{6}$$

2. As long as $T(w_i^f) < \tilde{\tau} w_i^f$, set the marginal contribution for type $i > 0$ in such a way that:

$$\left(1 - \frac{w_i^s}{w_i^f}\right) - \left(1 - \frac{\delta\beta u'\left(RT(w_i^f)\right)}{u'\left(w_i^f - T(w_i^f)\right)} R\right) T'(w_i^f) = 0 \tag{7}$$

3. Once $T(w_\iota^f) = \tilde{\tau} w_\iota^f$ for a certain $\iota \in \mathcal{I}$, set $T(w_i^f) = \tilde{\tau} w_i^f$ for all $i \geq \iota$.[6]

Intuitively, individuals who earn the same in the formal and the informal sectors should be forced to save what they would have done voluntarily. This helps to set the mandatory savings of individuals more productive formally as high as possible. Still, marginal contributions are set in such a way that no individual more productive formally works anytime in the informal sector. Naturally, the average contribution $T(w_i^f)/w_i^f$ should never be above the average contribution rate $\tilde{\tau}$ that would be optimal in the absence of an informal sector. The schedule T ends up partitioning $\mathcal{I}$ into three subsets:

i The set of types $i < 0$ who are more productive informally and so contribute nothing.

ii The set of types $i \geq 0$ with a productivity gap not high enough to implement an average contribution rate $\tilde{\tau}$.

[6] Let g_i^f and g_i^s be the growth rates for wages in the formal and the informal sector, respectively. For $i > 0$ such that $T(w_i^f)/w_i^f \leq \tilde{\tau}$, incentive compatibility requires the following second order condition:

$$-u'(w_i^f - T(w_i^f))\left(g_i^f - g_i^s\right) - u''(w_i^f - T(w_i^f))\left(1 - \frac{w_i^s}{w_i^f} - T'(w_i^f)\right) < 0.$$

This inequality is guaranteed by the assumption that w_i^f/w_i^s is increasing in i.

iii The set of types $i \geq 0$ with a productivity gap high enough to implement an average contribution rate $\tilde{\tau}$.

5. DISCUSSION

The effect on savings and welfare of a uniform contribution rate will depend on the worker's comparative advantage between the formal and the informal sectors. Mandatory savings are inconsequential for workers who are more productive in the informal sector, since they will work full-time in the informal sector no matter the contribution rate. For workers with a comparative advantage in the formal sector high enough to be working full-time in the formal sector, a marginal increase in the contribution rate may in turn increase their welfare. However, in the case of individuals who are more productive in the formal sector but with a comparative advantage low enough to encourage them to evade their contributions by working informally, *decreasing* the contribution rate will actually *increase* their savings and welfare. As estimated by Doligalski and Rojas (2019), the productivity gap between the formal and the informal sectors increases with the productivity in the formal sector. Therefore, if the poorest workers tend to be more productive in the informal sector, a uniform rate of mandatory savings will be inconsequential for the bottom segment of the income distribution, beneficial for those at the top, but detrimental for those individuals who fall in between.

Some simple calculations suggest that a uniform contribution rate may indeed generate welfare losses for a significant share of the working population of at least some of the emerging economies that have implemented this policy. Consider the case of Colombia, where about half the working population works in the informal sector and the contribution rate is currently 16%.[7] Suppose that $R = 6.94$, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\sigma = 2.35$, $\beta = 0.279$ and

[7]The institutional details of the Colombian social security system are in fact notoriously more complicated, but they do not seem to contradict the result of the numerical exercise. Among other things: 1) Individuals may opt for a parallel pay-as-yo-go public system, but this is only convenient for the top 5% of earners (Montenegro et al. (2018)). 2) Out of the 16% contribution, only 11.5 percentage points (pp) go to the individual retirement account. The rest correspond to fees and insurances (3pp) and a transfer to a public fund designed to guarantee a minimum pension benefit (1.5pp). Nevertheless, access to this

$\delta = 0.482$.[8] Under these parameters, only workers with a formal wage 8.3% higher than their informal wage will work full time in the formal sector. Workers with a productivity gap below 8.3% but still more productive in the formal sector represent 12.5% of the working population, according to Table I. These workers, who under the aforementioned parameters have incentives to work in the informal sector, would be better off with a lower contribution rate.[9] However, the implementation of a lower contribution rate for this set of agents requires decreasing the contribution rate for agents with a productivity gap higher than 8.3%. The contribution schedule that implements no misallocation of labor is presented in Figure 2.[10]

Proposing to decrease the contribution rate of low-earning workers may raise the eyebrows of staunch defenders of forced savings, especially in the context of an aging population. However, this policy proposal by no means implies that a lower contribution rate will lead to enough retirement savings. Instead, it suggests that the labor income of individ-

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minimum pension subsidy is attached to contributing for a number of years which the bottom half rarely reach when they retire (Villar and Forero (2018)).

[8]The gross interest R corresponds to the compounding for 45 years of an annual net interest rate of 4.4%, which has been the real return in the last 5 years for the best performing retirement portfolio in Colombia (Parra et al., 2020). The discount factor β and the inverse of the elasticity of intertemporal substitution σ are taken from Granda and Hamann (2015). The parameter δ corresponds to the average present-bias in Moser and Olea de Souza e Silva (2019).

[9]An important caveat applies to this statement, though: workers who would be more productive in the formal economy but will not reach the legal minimum wage –7.5% of the working population, according to Table I– would like to work at least some time in the formal sector. However, they are actually being excluded altogether from the formal labor market, due to the legal impossibility of contributing to social security when earnings are below the legal minimum wage. Decreasing the contribution rate will help them as long as the minimum wage is quoted after the employer's contributions and the cut is on the employer's contribution rate. In the context of Problem (1), there would be the additional constraint given by $(1-\tau_f)y$ not being lower than the minimum wage, where τ_f is the employer's contribution rate, $\tau = \tau_f + \tau_w$ and τ_w is the worker's contribution rate. In Colombia, $\tau_f = 12\%$.

[10]As long as the average contribution rate is below $\tilde{\tau}$, the contribution for the type j next to i in Table I is found by solving the equation $\left(1 - \frac{w_j^s}{w_j^f}\right) - \left(1 - \frac{\delta\beta u'\left(RT(w_j^f)\right)}{u'\left(w_j^f - T(w_j^f)\right)}R\right)\frac{T(w_j^f)-T(w_i^f)}{w_j^f - w_i^f} = 0$, which is the discrete version of Equation (7).

Productivity gap	w^s *	w^f *	Cumulative	Marginal
-0.87%	0.844	0.837	25.55%	2.41%
0.86%	0.869	0.877	28.01%	2.46%
2.50%	0.895	0.918	30.50%	2.49%
4.06%	0.922	0.961	33.00%	2.50%
5.55%	0.951	1.007	35.50%	2.50%
6.96%	0.981	1.054	37.99%	2.49%
8.31%	1.012	1.104	40.47%	2.48%
9.60%	1.045	1.156	42.92%	2.46%
10.84%	1.080	1.211	45.35%	2.44%
12.02%	1.116	1.268	47.78%	2.42%
13.15%	1.153	1.328	50.19%	2.41%
14.23%	1.193	1.391	52.59%	2.40%
15.27%	1.234	1.456	54.98%	2.39%

*Legal minimum wages (PPP $5868 2013 US dollars per year).

Source: Adapted from Doligalski and Rojas (2019).

TABLE I

Zoom on profiles of wages and productivity gaps.

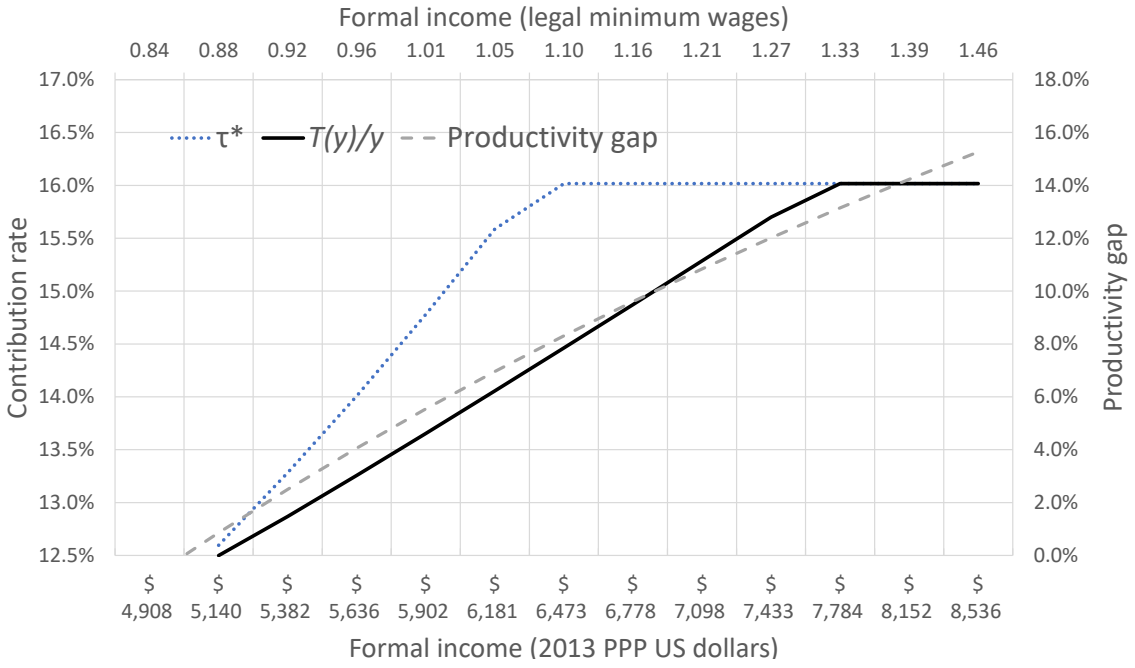

FIGURE 2.— Average contribution rates

uals prone to informality is not the appropriate source of funding for retirement savings. An alternative is the transfer of labor income from the top of the income distribution to the retirement funds of the bottom, since the opportunity cost of moving to the informal sector is comparatively higher for top earners. Still, the design of such a policy would require solving a screening problem across the working population, since formal income is self-reported and the productivity gap is not observable for social planners.[11] Furthermore, the screening problem will have to account for the elasticity of the labor supply, which adds another margin of reaction against mandatory savings.[12] The ambiguous effect of the contribution rate on present consumption hints towards consumption taxes as another alternative source for social security funding, as it is currently discussed in Chile.

[11]See Amador et al. (2006), Yu (2018), Moser and Olea de Souza e Silva (2019) and Farhi and Gabaix (2020) for examples on optimal paternalistic policies under private information.

[12]See Lockwood (2020) for an analysis of optimal taxation in a framework with no informality but where labor supply is distorted by present bias.

Overall, future research should look for better mechanisms to finance retirement savings in the presence of an informal labor market.

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APPENDIX A: PROOFS

PROOF OF PROPOSITION 1

Equation (4) implies that:

$$\epsilon = \frac{(u'(\hat{c}_1) - \delta\beta R u'(\hat{c}_2))\tau + u''(\hat{c}_1)\left(1 - \frac{w^s}{w^f} - \tau\right)\tau y - \delta\beta u''(\hat{c}_2)(R\tau)^2 y}{u''(\hat{c}_1)\left(1 - \frac{w^s}{w^f} - \tau\right)^2 y + \delta\beta u''(\hat{c}_2)(R\tau)^2 y} \tag{8}$$

Suppose that $\epsilon \geq -1$. If this were the case, then Equations (4) and (8) imply that:

$$\frac{-u''(\hat{c}_1)}{u'(\hat{c}_1)}\left(1 - \frac{w^s}{w^f} - \tau\right) y \geq 1, \tag{9}$$

which contradicts $1 - \frac{w^s}{w^f} - \tau < 0$.

Q.E.D.

PROOF OF PROPOSITION 3

Beware that $u(c_1)+\beta u(c_2)$ is *not* the agent's objective function, so the envelope theorem does not apply. From the definition of W it follows that:

$$\frac{dW}{d\tau}\frac{\tau}{y}\frac{1}{u'(\hat{c}_1)} = \left(1-\frac{w^s}{w^f}-\left(1-\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R\right)\tau\right)\epsilon-\left(1-\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R\right)\tau \tag{10}$$

First, consider the case where $\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R \leq 1$. Since $1-\frac{w^s}{w^f}-\left(1-\delta\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R\right)\tau = 0$, then $1-\frac{w^s}{w^f}-\left(1-\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R\right)\tau > 0$, implying that $\frac{dW}{d\tau} < 0$. Second, consider the case where $\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R > 1$. Introducing $1-\frac{w^s}{w^f}-\left(1-\delta\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R\right)\tau = 0$ into Equation (10) implies that:

$$\frac{dW}{d\tau}\frac{\tau}{y}\frac{\delta}{u'(\hat{c}_1)} = -\left(1-\frac{w^s}{w^f}-(1-\delta)\tau\right)(1+\epsilon)+\left(1-\frac{w^s}{w^f}\right)\delta\epsilon. \tag{11}$$

Since $1-\frac{w^s}{w^f}-\left(1-\frac{\delta\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R\right)\tau = 0$ and $\frac{\beta u'(\hat{c}_2)}{u'(\hat{c}_1)}R > 1$, then $1-\frac{w^s}{w^f}-(1-\delta)\tau < 0$. This last inequality and Proposition 1 imply that $\frac{dW}{d\tau} < 0$.

Q.E.D.