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# Optimal Investment and Capital Structure with Stock Market Feedback*

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**Abstract**

This paper studies optimal investment and capital structure policies when firms learn from financial markets. We propose a tractable model of feedback with imperfect information aggregation and allow the firm to choose its capital structure and investment policy in a previous stage. Firms may benefit from committing to time-inconsistent investment strategies that feature more risk taking, which can be implemented by simple managerial compensation schemes. Issuing debt can increase market informativeness and firm value. Under the optimal capital structure, the time inconsistency of investment policies disappears. We derive empirical predictions regarding the relationship between market frictions, managerial compensation and capital structure.

**JEL classification:** G14, G30, D82.

**Keywords:** information aggregation, financial markets, feedback effect, investment, capital structure.

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# 1 Introduction

Financial markets have the potential to aggregate information that would otherwise be dispersed across many agents. If individuals believe a security is under-priced relative to its fundamentals, they have incentives to buy the security, pushing up the price and revealing the prevailing beliefs among market participants. As long as agents are correct on average, the collective information of all market participants may be a very precise signal – even if individually they have very imprecise information. Given financial markets' role of conveying information, a recent theoretical and empirical literature has emphasized how real decision markers, such as managers, use the information revealed by financial markets to guide real decisions – creating a *feedback effect*, as movements in secondary markets may not only reflect but also affect firms' cash flows through their impact on managerial decisions.

Due to market frictions, however, financial markets may fail to aggregate agents' dispersed information, therefore also failing to improve the efficiency of real decisions. This paper studies how firms' investment and funding decisions affect information aggregation in financial markets. We characterize optimal investment and capital structure policies when firms rely on information gleaned from financial markets.

We propose a tractable model of feedback in which trading in secondary financial markets is subject to frictions. A firm manager must choose whether or not to undertake a risky investment opportunity.[1] The profitability of investment depends on underlying fundamentals, about which the manager has imprecise information. The firm has its equity claims (stocks) traded in a secondary financial market, in which a continuum of speculators participate. By looking at trading activity, the manager tries to learn about the return of the firm's investment opportunity. Informed speculators receive private signals about fundamentals and may place orders to buy or sell the firm's shares upon paying some trading cost. When deciding whether or not (and how much) to trade, speculators must form expectations about the behavior of other speculators in the market, and about the managerial reaction to market activity. Trading costs make information aggregation not trivial.

The equilibrium in the financial market often features an inefficiently low level of informa-

[1]The risky opportunity may be, for instance, launching a new product, adopting a new production technology, expanding into a new market, a merger or acquisition of another firm, etc.

tion aggregation. The feedback effect may generate strategic complementarities in the trading behavior of speculators, potentially resulting in an coordination failure that completely shuts down market provision of information. We then study whether – and if yes, how – firms' investment and capital structure policies can potentiate learning from the market and increase firm value.

We first show that firms can benefit from committing to a particular investment policy. We characterize the optimal investment strategy of a firm that can commit to react to market activity in any arbitrary way (except for mild measurability restrictions). Under commitment, the firm chooses to invest in a broader set of states. The intuition is the following: If the firm is not expected to invest, speculators' information about the investment profitability is useless to forecast the value of the security, and hence there is no potential for trading profits. By investing more often, the firm boosts incentives to trade, increasing information aggregation. However, whatever information is obtained from the market is used in a less efficient manner by the firm ex post. The optimal investment policy strikes a balance between these two effects. In some cases, the investment policy under commitment removes strategic complementarities across speculators, eliminating a coordination failure equilibrium with no informed trading.

The optimal investment policy is often time inconsistent, in the sense that a decision maker seeking shareholder value maximization would like to deviate from it after observing market activity. However, we show that optimal investment can be easily implemented with a simple managerial compensation scheme that conditions the manager's payment on the firm's final cash flow only.[2]

Next, we study the firm's optimal capital structure problem. We allow the firm to issue debt in a previous stage as to maximize firm value, anticipating that the manager will maximize shareholder value in a subsequent stage. When setting its capital structure, the firm considers the impact it will have on market participants' incentives to trade and on the manager's incentives to invest. Debt affects information aggregation through two channels. First, raising debt has the direct effect of partially wiping off of the stock dividend the safe

[2]Specifically, the compensation scheme that implements the optimal investment policy protects the manager from downside risk and boosts the upside of her compensation as to induce more risk taking (as in option-based compensation schemes, for instance).

portion of the firm's returns, increasing potential trading profits for informed speculators. Second, higher debt indirectly boosts trading incentives since it increases managerial risk taking, making the information traders have about the investment profitability more valuable. The latter effect is only present if debt is risky.

The firm often achieves the first best with the optimal capital structure: By issuing an appropriate amount of safe debt, the firm may lead to maximum information aggregation without causing any distortion to the managerial use of information. This is the case when the firm has stronger investment opportunities available and there are less market frictions preventing informed trading. When the first best is not achievable, the firm may still benefit from issuing debt. In this case, the optimal capital structure involves the minimum amount of risky debt that leads to maximum information aggregation. The reason is that any marginal gain in informativeness induced by higher risky debt more than compensates the distortion to the ex-post use of information by the manager. Moreover, the optimal capital structure also often eliminates a coordination failure equilibrium with no market provision of information.

We also solve the problem of a firm that jointly determines its capital structure and commits to an investment policy before the observation of market activity. We show that the solution to this problem can be achieved with ex-post shareholder value maximization. That is, under an optimal capital structure, the time inconsistency problem in the firm's investment decision disappears. There is no longer scope for increasing firm value by making the manager deviate from shareholder value maximization (with particular compensation schemes, for instance).

We derive a number of empirical predictions. First, our model suggests that we should observe a positive correlation between firms' debt and common measures of informed trading in firms' stocks. The reason is two-fold: (i) debt boosts incentives for informed trading; and (ii) in markets where there is high potential for informed trading, firms have incentives to issue more debt. Second, in the presence of market frictions that increase trading costs, firms should in general issue more debt, as to counterbalance speculators' reduced incentives to trade. Third, firms with lower debt are expected to rely more on managerial contracts that induce more risk taking (for instance, option-based compensation). Fourth, we show that short-selling restrictions should induce firms to reduce debt. Such restrictions should also decrease firms' profitability and investment sensitivity to stock prices.

**Related literature.** Empirical studies have suggested that activity in financial markets affects real decisions in different contexts (Luo, 2005; Chen, Goldstein and Jiang, 2006; Bakke and Whited, 2010; Edmans, Goldstein and Jiang, 2012; Phillips and Sertsios, 2016; Edmans, Jayaraman and Schneemeier, 2017; Jayaraman and Wu, 2019). Motivated by this evidence, a growing body of theoretical work has been studying how information contained in market activity affects real decisions, what is often referred to as the *feedback effect.* Contributions on the topic include Dow and Gorton (1997), Goldstein and Guembel (2008), Bond and Eraslan (2010), Bond and Goldstein (2015), Edmans, Goldstein and Jiang (2015) and Goldstein and Yang (2019). This paper contributes to that literature by studying how firms' investment and funding decisions affect their ability to learn from financial markets.

Lin, Liu and Sun (2019) study managerial incentives in a model of stock market feedback. In their setting a firm manager learns from a single (large) informed speculator that decides on how much information to acquire, while in our setting there is a fringe of atomistic informed speculators, and the mechanisms at play work precisely through the strategic interactions between multiple speculators. We complement their work in several dimensions. First, we characterize firms' optimal investment policy from an ex ante perspective in a general way, and describe the compensation scheme that would implement it.[3] Second, we characterize firms' optimal capital structure and relate it to optimal managerial contracts and investment policies. We derive several novel empirical predictions in this regard.

Our baseline model is related to Dow, Goldstein and Guembel (2017). They study market provision of information in a model with feedback effects. Differently from this paper, their focus is on information acquisition – information aggregation is guaranteed in equilibrium once information has already been acquired.[4] Most importantly, we derive the ex-ante optimal investment and capital structure policies, and study how they are affected by frictions in the

[3] Lin, Liu and Sun (2019) study the optimal managerial contract in the class of compensation schemes that promise a fixed share of the firm's cash flows to the manager. In our setting, however, such sort of compensation schemes would often not allow the firm to implement its optimal investment strategy. Moreover, we show that under an optimal capital structure, there is no need to implement compensation schemes that deviate from shareholder value maximization.

[4] The decision to trade is made by an informed speculator, while the decision to acquire information is undertaken ex ante.

trading process.

As in Dow, Goldstein and Guembel (2017), there may be strategic complementarity or substitutability among speculators in our model. Strategic complementarities in trading are also present in Goldstein, Ozdenoren and Yuan (2013), Goldstein and Yang (2015) and Goldstein, Li and Yang (2013), for instance. We show that the mechanism through which firms' investment and capital structure choices increase firm value often goes through eliminating such strategic complementarities.

More broadly, the paper also relates to the extensive literature on how firm's capital structure affects firm value (Modigliani and Miller, 1958; Jensen and Meckling, 1976; Myers and Majluf, 1984; Harris and Raviv, 1990; Fulghieri and Lukin, 2001; and many others) as well as the market microstructure literature (Kyle, 1985; Glosten and Milgrom, 1985).

This remainder of the paper is organized as follows. Section 2 presents a baseline model of stock market feedback. Section 3 characterizes the equilibrium in the baseline model. Section 4 studies the ex-ante optimal investment policy. Section 5 analyzes the optimal capital structure. Section 6 summarizes the model's empirical predictions, and Section 7 concludes. All proofs are relegated to the appendix.

# 2 A model of feedback

Consider a firm whose equity claims are traded in a secondary financial market. A decision maker (manager) makes an investment decision after observing market activity, so there is potential for feedback from financial markets to managerial choices. In this section we present a standard model of feedback in which: (i) the manager chooses investment as to maximize shareholder value ex post (*after* observing market activity); and (ii) the firm is 100% equity-financed. In Section 4 we relax (i) by allowing the firm commit to an investment strategy ex ante (possibly different from ex-post shareholder value maximization). In Section 5 we relax (ii) by allowing the firm to issue debt claims, and study the optimal capital structure problem.

In the baseline model of this section, there are two dates. At date $t = 1$ (trading stage), speculators receive private signals about the firm's fundamentals and trade the firm's security.

At date $t = 2$ (investment stage), the manager takes an action that affects the firm's cash flow, after observing prices and quantities traded. The return of the firm's projects realize and dividends are paid to shareholders. All agents in the model are risk-neutral.

## 2.1 Firm value and investment decision

The value of the firm (cash flow) at the final period depends on a state variable $\theta \in \{\theta_L, \theta_H\}$ and on a managerial decision $a \in \{0, 1\}$, and is denoted by $v(\theta, a)$. Ex ante, each state $\theta$ is equally likely. The manager does not observe the realization of the state, but updates beliefs about it after observing activity in financial markets. If the manager chooses action 0, then the firm's cash flow is $v(\theta, 0) = V_0$. If action 1 is chosen, then firm value is state-contingent and is given by

$$v(\theta, 1) = \begin{cases} V_H & \text{if } \theta = \theta_H, \\ V_L & \text{if } \theta = \theta_L, \end{cases}$$

where $V_H > V_0 > V_L$. This implies that action 1 increases firm value only if the state is high ($\theta = \theta_H$). Action $a$ affects the asset side of the firm's balance sheet. One can interpret action $a = 0$ as choosing a safe project, or refraining from undertaking a risky investment opportunity. Action $a = 1$ represents investment in a risky project. This action could represent the adoption of a new technology, launching a new product, a merger or acquisition, etc.

## 2.2 Financial market

The financial market consists of a continuum of informed speculators indexed by $i \in [0, \alpha]$, where $\alpha > 0$, liquidity (noise) traders, and a competitive market maker. The quantity of outstanding equity claims (shares) is normalized to one. At date $t = 1$, each informed speculator can submit orders of up to one dollar to buy or (short) sell the firm's shares. Speculator $i$'s position in the stock market (in dollar amount) is denoted by $s_i \in [-1, 1]$, where the order size is limited due to wealth constraints, for instance.[5] Speculators incur a trading cost $\tau > 0$ per dollar traded in stocks. One can interpret this cost as the marginal

[5]The size of this limit is irrelevant: the results of the paper remain unchanged if speculators can buy or sell up to $K$ dollars, for any $K > 0$. What is important is that speculators cannot take unlimited positions.

operating cost incurred by the competitive market maker (which can be mapped, for instance, in the bid-ask spread or brokerage fees).

Before trading takes place, each speculator $i$ receives a private signal $m_i \in \{m_L, m_H\}$ about the state, where $\Pr(m_i = m_H|\theta = \theta_H) = \Pr(m_i = m_L|\theta = \theta_L) = \lambda > 0.5$. Conditional on the realization of the state, signals are independently distributed across speculators. The dividend payout to shareholders in the final period, $R(\theta, a)$, is equal to the equity value of the firm. Since the firm only has equity outstanding, $R(\theta, a) = v(\theta, a)$ (this will be relaxed in the next sections). Each speculator $i$ chooses an order $s_i \in [-1, 1]$ as to maximize her expected trading profit

$$s_i \mathbb{E}\left[\frac{R(\theta, a)}{P} - 1|m_i\right] - \tau|s_i|, \tag{1}$$

where $P$ denotes the stock price (to be determined endogenously). We define $R_H \equiv R(\theta_H, 1)$, $R_L \equiv R(\theta_L, 1)$, $R_0 \equiv R(\theta, 0)$, and $\Delta_R \equiv R_H - R_L$.

Besides the informed speculators, there are noise traders, who trade for exogenous reasons (to accommodate liquidity shocks, for instance). Noise traders submit an aggregate order of $\tilde{n}$ dollars for the firm's shares, where $\tilde{n}$ is drawn from a normal distribution with zero mean and variance $\sigma^2$. The total order flow in the stock market, in dollars, is then

$$\tilde{X} = \tilde{n} + \int_0^\alpha s_i di.$$

As in Kyle (1985), the market maker is competitive: after observing the total order flow, she meets the order at a price that equals the firm's expected cash flow.[6] That is, $P = \mathbb{E}\left[R(\theta, a)|\tilde{X}\right]$. Naturally, the market maker rationally anticipates the managerial reaction to market activity when setting the price.

Throughout the paper, we make the following parametric assumption:

**Assumption 1.** *Trading costs are sufficiently small,* $\tau < 2\lambda - 1$.

If Assumption 1 is violated, in all sections of the paper the equilibrium would trivially feature no informed trading and no information aggregation in financial markets. Table 1 summarizes the timing of events.

[6] As usual in the market microstructure literature, the market maker is able to take any long or short position on the asset to clear the market.

| $t = 1$ : Information and trade | $t = 2$ : Learning and investment |
|---|---|
| • State $\theta$ realizes<br>• Informed speculators receive signals<br>• Orders are placed<br>• Market maker sets price and meets orders | • Manager observes market activity and decides on investment<br>• Cash flow is realized<br>• Claim holders are paid |

Table 1: Timeline of events

## 2.3 Equilibrium definition

The equilibrium concept we adopt is Perfect Bayesian Equilibrium, which in our baseline model can be defined as follows.

**Definition 1.** An equilibrium consists of a price function $P(\tilde{X}) : \mathbb{R} \to \mathbb{R}_+$ ; an investment policy $a(\tilde{X}) : \mathbb{R} \to \{0, 1\}$; trading strategies $s_i(m_i) : \{m_L, m_H\} \to [-1, 1]$; beliefs $\mu(\tilde{X}) : \mathbb{R} \to [0, 1]$ for the market maker and the firm, and $\eta(m_i) : \{m_L, m_H\} \to [0, 1]$ for speculators (specifying the probabilities assigned to $\theta = \theta_H$), such that:

(i) Trading strategies for speculators maximize trading profits, given the price function, the investment policy of the manager, other traders' strategies and their private signals;

(ii) The investment policy of the manager maximizes shareholder value, given the information in the order flow $\tilde{X}$ and all other strategies;

(iii) The price function is such that the market maker breaks even in expectation, given the information in the order flow and all other strategies;

(iv) Beliefs $\mu(\tilde{X})$ and $\eta(m_i)$ are consistent with Bayes rule.

Notice that we condition the manager's investment decision only on aggregate orders and not on prices. This is without loss of generality, since once the aggregate order is observed, the manager obtains no additional information from the price.

# 3 Equilibrium in the baseline model

In this section we characterize the equilibrium in the baseline model of Section 2.

## 3.1 Beliefs

Speculators' beliefs are given by Bayes' rule: $\eta(m_H) = \lambda$ and $\eta(m_L) = 1-\lambda$. We now construct the manager's and market maker's beliefs. Fix strategies $\{s_i(m_H), s_i(m_L)\}_{i\in[0,1]}$ for speculators after observing high or low signals. Denote by $S_H \equiv \int_0^\alpha [\lambda s_i(m_H) + (1-\lambda)s_i(m_L)]\, di$ the aggregate order of informed speculators in the stock market when the state is high, and by $S_L \equiv \int_0^\alpha [\lambda s_i(m_L) + (1-\lambda)s_i(m_H)]\, di$ their aggregate order when the state is low. We then have that the aggregate order flow in the stock market is $\tilde{X} = \tilde{n} + S_H$ if $\theta = \theta_H$, and $\tilde{X} = \tilde{n} + S_L$ if $\theta = \theta_L$. Using Bayes rule, the posterior belief of the market maker (and the manager) when observing aggregate order $\tilde{X}$ can be written (with some abuse of notation) as:

$$\mu(X) = \frac{1}{1 + \exp\{-X\}}, \tag{2}$$

where

$$X = \frac{1}{2\sigma^2}(S_H - S_L)\left[\left(\tilde{X} - S_H\right) + \left(\tilde{X} - S_L\right)\right]. \tag{3}$$

Given speculators' strategies, $X$ is a sufficient statistic to update beliefs. Throughout the text we may refer to $X$ as 'aggregate order', since it provides a summary of market activity. To save on notation, we may hereafter write functions of $\tilde{X}$ as functions of $X$ instead – as we did for $\mu(\cdot)$ in (2).

## 3.2 Investment decision

After looking at market activity, the manager chooses whether or not to undertake the risky action $(a = 1)$. Action $a = 1$ is optimal whenever

$$\mu(X)\, R_H + [1 - \mu(X)]\, R_L \geq R_0.$$

Therefore, the manager uses a *cutoff strategy*: to invest if and only if $X \geq \overline{X}$, for some cutoff $\overline{X} \in [-\infty, \infty]$.[7] Whenever she observes a sufficiently large aggregate order $X$, she assigns a sufficiently high probability to the state being high, and consequently undertakes the risky project. Indifference implies that the equilibrium cutoff is[8]

$$\overline{X}^* \equiv \ln\left(\frac{V_0 - V_L}{V_H - V_0}\right). \tag{4}$$

Hereafter we often refer to the realizations of $X$ for which the manager invests by $A \subseteq \mathbb{R}$. Therefore, in equilibrium, $A = \left[\overline{X}^*, \infty\right)$. Also, we often refer to $\overline{X}^*$ given by (4) as the *ex-post optimal investment strategy*, since it maximizes firm value ex post.

## 3.3 Prices

The market maker observes aggregate orders in financial markets and sets prices as to reflect the expected value of the stock. We can then write:

$$P(X) = \begin{cases} \mu(X) R_H + [1 - \mu(X)] R_L & \text{if } X \in A, \\ R_0 & \text{otherwise.} \end{cases} \tag{5}$$

If the market maker observes orders such that $X \in A$, she anticipates that the manager will undertake the risky action, in which case the price reflects the market maker's beliefs about the state. If instead $X \notin A$, the market maker anticipates the manager will choose $a = 0$, and there is no uncertainty about dividends.

## 3.4 Trading stage

We define the variable $\kappa$, which will be key for the equilibrium characterization:

$$\kappa \equiv \frac{1}{2\sigma^2} (S_H - S_L)^2. \tag{6}$$

[7] We adopt the tie-breaking convention that the manager chooses $a = 1$ when indifferent. Notice that always investing or never investing can also be seen as cutoff strategies, as they may be represented on the extended real line by $\overline{X} = -\infty$ and $\overline{X} = \infty$, respectively.

[8] Recall that the firm has no debt, so we used $R(\theta, a) = v(\theta, a)$ to obtain (4).

One can think of $\kappa$ as a measure of how aggressively speculators trade on their information. To see that, consider that speculators place the same orders regardless of the signals they observe. In that case, we would have $\kappa = 0$. If speculators adopt very different trading strategies when optimistic or pessimistic about the state, this would be reflected in very different aggregate orders across high and low states, leading to a high value of $\kappa$. Moreover, $\kappa \to 0$ as the market becomes arbitrarily noisy ($\sigma \to \infty$), and $\kappa \to \infty$ if noise trading vanishes ($\sigma \to 0$) and $S_H \neq S_L$ (perfect information aggregation). Notice that $\kappa \in [0, \overline{\kappa}]$, where

$$\overline{\kappa} \equiv \frac{2\alpha^2 (2\lambda - 1)^2}{\sigma^2}. \tag{7}$$

If speculators' strategies when receiving good and bad signals are as different as possible – say, if everyone that receives a good (bad) signal buys (sell) as much as possible – we have that $\kappa = \overline{\kappa}$. We refer to $\kappa$ as a measure of *market informativeness*, and $\overline{\kappa}$ represents the maximum possible level of information aggregation.

### 3.4.1 Trading profits

The next result is critical for the equilibrium characterization. In particular, it shows that, from the perspective of an individual speculator, other speculators' strategies are summarized by $\kappa$. We denote the standard normal density and cumulative distribution functions by $\phi(\cdot)$ and $\Phi(\cdot)$, respectively.

**Lemma 1.** *Suppose the manager undertakes investment if and only if $X \in A$, where $A$ is any measurable subset of $\mathbb{R}$. Then, a speculator's trading profit disregarding trading costs only depends on other speculators' strategies through $\kappa$ and is given by:*

$$\Pi(s, m, \kappa) = s\left[2\eta(m) - 1\right] \int_A \left[\frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R}\right] \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du. \tag{8}$$

Since $\eta(m_H) = \lambda > 1/2 > 1 - \lambda = \eta(m_L)$, it follows from Lemma 1 that a speculator receiving a positive (negative) signal will necessarily buy (sell) or not trade the security (if transaction costs do not compensate). More precisely, a positively informed speculator chooses $s = 1$ if $\Pi(1, m_H, \kappa) > \tau$, $s = 0$ if $\Pi(1, m_H, \kappa) < \tau$, and is indifferent between any

$s \in [0,1]$ if $\Pi(1, m_H, \kappa) = \tau$. Analogously, a negatively informed speculator chooses $s = -1$ if $\Pi(-1, m_L, \kappa) > \tau$, $s = 0$ if $\Pi(-1, m_L, \kappa) < \tau$, and any $s \in [-1, 0]$ if $\Pi(-1, m_L, \kappa) = \tau$.

Moreover, the broader the set of states where the manager is expected to undertake investment, the higher the speculators' incentives to trade. This is captured by the integration limit $A$ in (8). As the price rule in (5) makes clear, speculators do not make any profit in the financial market in states where the firm does not undertake investment, since in those situations the information they have is not relevant to anticipate the firm's cash flow.

Importantly, Lemma 1 implies that each individual speculator best-responds to $\kappa$. It is useful to define the expected trading profit *per dollar* traded, disregarding trading costs and taking into account that optimistic traders never sell and pessimistic traders never buy. Trading profits per dollar spent in buying (selling) shares for a speculator receiving a good (bad) signal are given by:

$$\pi(\kappa) = (2\lambda - 1) \int_A \frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du. \tag{9}$$

### 3.4.2 Strategic effects

The next lemma highlights some properties of the trading profit function $\pi(\cdot)$ given a cutoff investment strategy. Notice that, if the manager plays according to a cutoff $\overline{X} \leq 0$, she would invest if no additional information were obtained from the market, and if $\overline{X} > 0$, she would not invest under the prior.[9]

**Lemma 2.** *Fix an investment cutoff $\overline{X}$, so that $A = \left[\overline{X}, \infty\right)$. Then, for any $R_H > R_L \geq 0$:*

1. *If $\overline{X} > 0$ (i.e., the manager would not invest under the prior), $\lim_{\kappa \to 0} \pi(\kappa) = 0$ and $\pi'(\kappa) > 0$ for low enough $\kappa$;*

2. *If $\overline{X} \leq 0$ (i.e., the manager would invest under the prior), $\pi(\kappa)$ is strictly decreasing.*

Lemma 2 implies that there can be both strategic complementarity and substitutability in speculators' strategies, as usual in models of feedback. There are two strategic effects at play: an *informational advantage effect*, and a *feedback effect*.

[9] Inspecting equations (3) and (6), one can verify that $\kappa = 0$ if and only if $S_H = S_L$, in which case $X$ is zero regardless of the realization of the noise traders' orders. That is, when market activity does not add any relevant information, $X = 0$ with probability one.

The informational advantage effect is a source of strategic substitutability across speculators. As others trade more on their information, $\kappa$ increases, market activity becomes more informative, and prices move closer to the real value of securities. This reduction in the informational advantage of informed speculators decreases potential trading profits on its intensive margin, and thus reduces incentives to trade.

The feedback effect operates through the managerial reaction to market activity. As $\kappa$ increases, the manager puts more weight on information stemming from the market, and this affects the probability that the risky action is undertaken. Hence, $\kappa$ also affects trading profits on the extensive margin: conditional on the manager choosing $a = 0$, expected profits are zero; conditional on $a = 1$, expected profits are positive and depend on the size of the informational advantage held by speculators.

The feedback effect may be a source of either strategic substitutability or complementarity across speculators. For instance, consider the case where the manager is ex-ante prone to undertaking the risky action ($\overline{X} \leq 0$) and suppose $\kappa = 0$ is expected (no information aggregation). If speculators were to trade more aggressively on information (an increase in $\kappa$), the manager would put some weight on information stemming from the market and would be more likely to reverse its prior and not invest. In this situation, a speculator has lower incentives to trade on information if she expects others to trade more aggressively. Hence, both the informational advantage and the feedback effect go in the same direction, explaining why $\pi(\kappa)$ is decreasing in that case.

Now consider the opposite case, where the manager is not prone to undertaking the risky action under the prior ($\overline{X} > 0$), and again, suppose $\kappa = 0$ is expected. An increase in $\kappa$ would make the manager rely on information gleaned from financial markets to some extent, and invest with some positive probability. In this case, speculators have more incentives to trade if others are trading on their information, and thus there are strategic complementarities. The latter situation may lead to a coordination problem among speculators: if a speculator expects others not to trade and the manager's prior is not favorable to investment, she has no incentives to trade either, and information provision by the market may be shut down altogether.

#### 3.4.3 Equilibrium in the trading stage

Proposition 1 characterizes equilibrium outcomes in the trading stage for a given investment cutoff $\overline{X}$.

**Proposition 1.** *Fix a cutoff investment rule* $\overline{X}$ *and let* $\underline{\tau} \equiv \pi(\overline{\kappa})$ *and* $\overline{\tau} \equiv \pi(0)$*. Then, for any* $R_H > R_L \geq 0$*:*

1. *If* $\overline{X} > 0$*, an equilibrium with* $\kappa^* = 0$ *exists;*

2. *If* $\overline{X} \leq 0$*, the equilibrium is (essentially) unique, with* $\kappa^* = 0$ *if* $\tau \geq \overline{\tau}$*,* $\kappa^* \in (0, \overline{\kappa})$ *if* $\tau \in (\underline{\tau}, \overline{\tau})$*, and* $\kappa^* = \overline{\kappa}$ *if* $\tau \leq \underline{\tau}$*.*[10]

The feedback effect explains why an equilibrium with zero information aggregation always exists when the manager is not willing to undertake the risky action under the prior ($\overline{X} > 0$). If speculators expect the manager not to invest, they have no incentives to trade. Then, the manager does not obtain any additional information and does not invest, confirming speculators' initial expectations.

Both the feedback effect and the informational advantage effect explain why the equilibrium is unique when the manager is willing to invest under the prior ($\overline{X} \leq 0$). For low $\kappa$, speculators know the manager gives high weight to her own prior and is likely to undertake the risky investment opportunity. Then, trading on information is profitable (investment is likely and the market maker learns little). As more speculators trade on their information (higher $\kappa$), the market aggregates information better, pushing prices closer to the real security value and reducing incentives to trade. As is often the case, strategic substitutabilities lead to a unique equilibrium.

Figure 1 illustrates these results. In panel 1a, consider $\kappa < \kappa^*$. For any such $\kappa$, each speculator individually would have incentives to trade as aggressively as possible (with $s_i(m_H) = 1$ and $s_i(m_L) = -1$). But this would lead the market to aggregate information much better, with $\kappa = \overline{\kappa}$, in which case expected trading profits would not compensate the trading cost $\tau$, and speculators would not be willing to trade ($s_i(m_H) = s_i(m_L) = 0$, leading

[10] An equilibrium with $\kappa^* \in (0, \overline{\kappa})$ can be either symmetric or asymmetric (for instance, a fraction of speculators may trade as much as possible and the rest not trade). We say the equilibrium is *essentially unique* in this case: there are multiple strategy profiles consistent with equilibrium, but market informativeness, stock prices and firm value are uniquely pinned down.

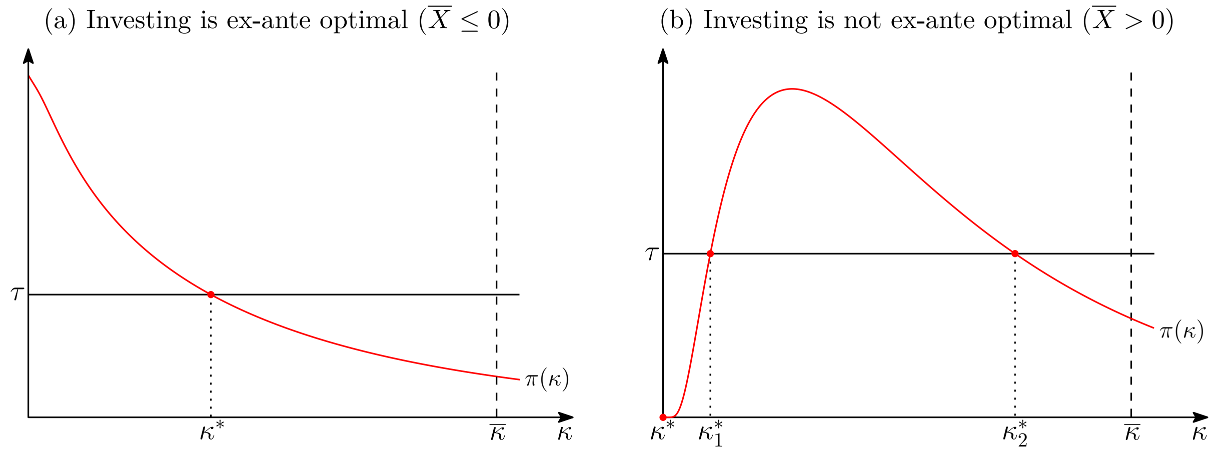


Figure 1: Equilibrium in the trading stage for a given $\overline{X}$. The figure illustrates the computation of equilibrium for different parameter cases.

to $\kappa = 0$). The equilibrium is reached at an intermediate level of $\kappa$ at which speculators are indifferent between any order size as long as $\kappa = \kappa^*$.

Panel 1b illustrates the result that $\pi(0) = 0$ when $\overline{X} > 0$. In that case, speculators do not have any incentives to trade on information if they expect others to refrain from trading. A coordination failure equilibrium exists ($\kappa^* = 0$), in which the market fails to aggregate information that would be useful to guide the managerial decision and increase firm value.

## 3.5 Efficiency and firm value

The next proposition shows that the more information is aggregated in financial markets, the larger the value of the firm in equilibrium.

**Proposition 2.** *The ex-ante firm value only depends on speculators' strategies through $\kappa$ and is strictly increasing in $\kappa$ under the ex-post optimal investment strategy $\overline{X}^*$. The maximal ex-ante firm value is achieved when $\kappa = \overline{\kappa}$ and the manager invests if and only if $X \geq \overline{X}^*$.*

The intuition is simple: the more aggressively speculators trade on their information, the more the manager learns from market activity, and the more likely she is to undertake efficient investment decisions. Since the manager maximizes shareholder value, and in the baseline model shareholder value equals firm value, more information can only lead to more efficient decisions. Naturally, the maximum possible ex-ante firm value is achieved when $\kappa = \overline{\kappa}$ and the manager makes the best possible use of this information. Proposition 2 justifies the following definition of a first best:

**Definition 2.** The first best (from the perspective of the firm) is achieved when $\kappa = \overline{\kappa}$ and the manager plays the cutoff strategy $\overline{X} = \overline{X}^*$.

In equilibrium, $\overline{X} = \overline{X}^*$, but as Proposition 1 makes clear, in many circumstances information aggregation is lower than its first-best level. In the next two sections of this paper we show how firm value can be improved by: (i) committing to certain investment policies (say, by providing appropriate managerial incentives); and (ii) issuing debt securities. The latter directly affects the *liability side* of the firm's balance sheet, and the former, the *asset side*.

# 4 Ex-ante optimal investment policy

We now modify the model of Section 2 by adding a previous date ($t = 0$) in which the firm commits to an investment strategy *before* observing market activity as to maximize expected firm value.

In particular, we allow the firm to choose any measurable subset $A \subseteq \mathbb{R}$ so that it invests if and only if $X \in A$, anticipating that its choice will affect the equilibrium played in the trading stage ($t = 1$). The manager at $t = 2$ then simply executes the chosen investment strategy (we discuss implementation through managerial contracts in Section 4.1.1). In other words, the firm commits to reacting to market activity in a certain way, possibly affecting how much information will be aggregated in the market. In short, what we do in this section is to compute the *ex-ante* optimal investment strategy (i.e., the one that maximizes ex-ante firm value).

Two technical highlights are in order. First, it is convenient to adopt the same change of variables as in the baseline model and state the firm's problem in terms of $X$ instead of the original aggregate order $\tilde{X}$. As discussed in Appendix A.2, the problem in which the firm conditions investment on $X$ is equivalent to the problem where it conditions investment on $\tilde{X}$. Second, to simplify the exposition, here we allow the firm to condition its investment on orders ($X$) but not on prices ($P$). This is without loss of generality. In Appendix A.3

we show that there is always an optimal investment strategy that does not condition on the price, even when we allow for it.

Depending on the investment strategy the firm chooses, there may be multiple equilibria. We take an approach similar to the adversarial/robust design in Inostroza and Pavan (2018), and assume that whenever there are multiple equilibria in the trading stage, the one with lowest information aggregation ($\kappa$) is selected. In equilibrium, those are the strategy profiles that yield lowest firm value, among all the equilibria of the trading stage.[11]

**Assumption 2.** *Whenever there are multiple equilibria in the trading stage, the one with lowest information aggregation is played.*

The problem of the firm is to choose a measurable set $A \subseteq \mathbb{R}$ to maximize expected firm value

$$\mathcal{V} = \frac{1}{2}\left[V_0 + \Pr\left(X \in A \middle| \theta = \theta_H\right)\left(V_H - V_0\right)\right] + \frac{1}{2}\left[V_0 - \Pr\left(X \in A \middle| \theta = \theta_L\right)\left(V_0 - V_L\right)\right], \quad (10)$$

subject to the restriction that those probabilities are consistent with speculators' equilibrium strategies for each $A$. Notice that if $A = \left[\overline{X}, \infty\right)$, the investment policy is a cutoff strategy, in which case we refer to it only by the cutoff $\overline{X}$. Recall that the ex-post optimal investment rule is a cutoff strategy, with $\overline{X}$ given by $\overline{X}^*$ in (4).

## 4.1 Equilibrium under commitment

When choosing its investment strategy under commitment, the firm potentially faces the following trade-off: By deviating from the ex-post optimal investment strategy, it makes worse use of the information obtained. On the other hand, it may lead to higher information aggregation. Market participants do not internalize the effect of their trading activity on information aggregation and firm value. To increase information aggregation, the firm needs to reward informed speculators when they trade more aggressively and reveal good or bad news. One way to reward speculators for their information provision is by investing in a broader range of states, since speculators only profit when investment is undertaken. The

[11] This selection criterion is not critical to the main results. The model yields qualitatively analogous predictions if we select any of the stable equilibria.

next proposition shows that the ex-ante optimal investment policy takes the form of a simple cutoff, possibly featuring investment in states where it is ex-post not optimal to invest.

**Proposition 3.** *There is always a cutoff strategy* $\overline{X}_c^*$ *that is ex-ante optimal. If there are gains from commitment, then* $\overline{X}_c^* < \overline{X}^*$.

Therefore, when looking for an ex-ante optimal investment strategy, we can restrict attention to cutoff strategies $\overline{X}_c^*$ such that $\overline{X}_c^* \leq \overline{X}^*$. The next proposition characterizes the ex-ante optimal investment strategy when the firm would be willing to invest under its prior ($\overline{X}^* \leq 0$). In Appendix A we define the bounds $\tilde{\tau}_1$, $\tilde{\tau}_2$, and $\tilde{\tau}_3$ to which we refer in the next propositions.

**Proposition 4.** *Suppose investment is profitable under the prior* ($\overline{X}^* \leq 0$)*. Then, for* $\tau \leq \tilde{\tau}_2$ *and* $\tau \geq \tilde{\tau}_3$*, there is no gain in commitment* ($\overline{X}_c^* = \overline{X}^*$ *is ex-ante optimal*)*. For* $\tau \in (\tilde{\tau}_2, \tilde{\tau}_3)$*, the ex-ante optimal cutoff is some* $\overline{X}_c^* < \overline{X}^*$.

Proposition 4 states that, if $\overline{X}^* \leq 0$ and the firm can commit to an investment strategy ex ante, it commits to a lower cutoff $\overline{X}_c^*$ if trading costs are in an intermediate range. For very low or high trading costs, there is no gain in deviating from the ex-post optimal investment strategy. The intuition is the following: If trading costs are sufficiently large, speculators would optimally choose not to trade regardless of their beliefs about other speculators' behavior, so distorting the use of information ex post to try to improve information aggregation does not pay off. If trading costs are sufficiently low, speculators trade on information as much as possible if the firm plays the ex-post optimal cutoff $\overline{X}^*$(the first best is achieved). Hence, committing to a different cutoff does not pay off either, since maximum information aggregation is already achieved without the need to distort the ex-post use of information.

If instead trading costs are in an intermediate range, the firm optimally commits to a lower investment cutoff. This means that projects with *ex-post* negative NPV – i.e., projects appraised as having negative NPV *after* the observation of market activity – will be undertaken with positive probability. If the firm would set $\overline{X}_c^* = \overline{X}^*$, this would lead to an equilibrium with interior $\kappa^*$, in which there would be some margin to improve information aggregation ex ante by investing in a broader set of states (with $X$ to the left of $\overline{X}^*$). Moreover, at $\overline{X}_c^* = \overline{X}^*$ the marginal cost of distorting the ex-post use of information is zero. That is, if the manager is making the very best use of information ex post – only investing for $X \geq \overline{X}^*$ – committing to a marginally lower investment cutoff ex ante compensates, due to the improvement in information aggregation (even though information is somewhat misused). Figure 2a illustrates Proposition 4 for different cases of parameters.

The next proposition details the optimal investment rule under commitment when risky investment would not be undertaken under the prior. In what follows, define:

$$\tilde{\kappa} \equiv 2\left[\Phi^{-1}\left(\frac{V_0 - V_L}{V_H - V_L}\right)\right]^2. \tag{11}$$

**Proposition 5.** *Suppose investment is not profitable under the prior ($\overline{X}^* > 0$). Then:*

1. *If $\overline{\kappa} \leq \tilde{\kappa}$ and/or $\tau \geq \tilde{\tau}_3$, there is no gain in commitment: $\overline{X}_c^* = \overline{X}^*$ is ex-ante optimal;*

2. *If $\overline{\kappa} > \tilde{\kappa}$ and $\tau \leq \tilde{\tau}_1$, the ex-ante optimal cutoff is $\overline{X}_c^* = 0$ and it leads to maximum informativeness ($\kappa^* = \overline{\kappa}$);*

3. *If $\overline{\kappa} > \tilde{\kappa}$ and $\tau \in (\tilde{\tau}_1, \tilde{\tau}_3)$, whenever there are gains in commitment, the ex-ante optimal cutoff is some $\overline{X}_c^* \leq 0$.*

Note that whenever commitment to an investment strategy different from $\overline{X}^*$ is optimal, the cutoff $\overline{X}_c^*$ is such that the coordination failure among speculators is eliminated, that is, $\overline{X}_c^* \leq 0$. This is a necessary condition for commitment to pay off, since any $\overline{X}_c^* > 0$ would lead to an equilibrium in the trading stage with no information being aggregated in the market ($\kappa^* = 0$). Committing to an investment policy can only increase firm value if it eliminates strategic complementarities across speculators.

The intuition behind Proposition 5 is somewhat different from Proposition 4. First, regardless of trading costs, if the maximum possible level of information aggregation is too low, it is never optimal to set a lower cutoff and distort the use of information ex post. If $\overline{\kappa}$ is sufficiently large, then whether or not it pays off to commit to an investment rule that leads to ex-post sub-optimal investment depends on trading costs. For very low trading costs, the firm commits to a cutoff that imposes the minimum distortion while leading to maximum information aggregation, $\overline{X}_c^* = 0$. This result contrasts with the case with $\overline{X}^* \leq 0$: there, for very low $\tau$ there was no reason to commit to a different investment rule. The key

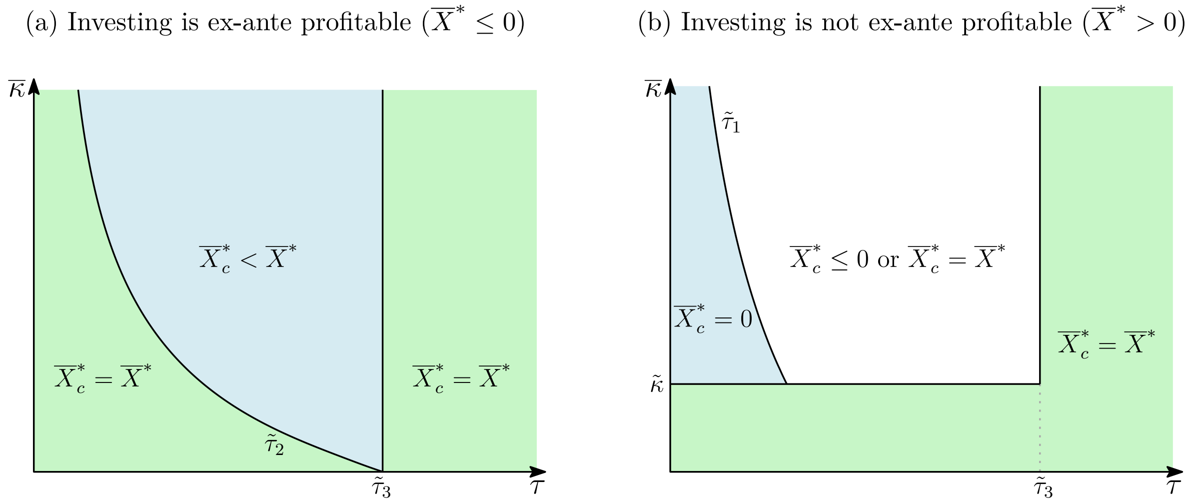


Figure 2: Optimal investment rule under commitment. The figure illustrates how the ex-ante optimal investment policy depends on $\tau$ and $\overline{\kappa}$, for a given $\lambda$.

difference here is that under no commitment there is always no trade in secondary markets, and commitment allows the firm to eliminate the coordination failure equilibrium and causes a discrete jump in information aggregation and firm value.

For intermediate values of the trading cost, maximum aggregation is not achieved with $\overline{X}^*_c = 0$, but it may still pay off to set $\overline{X}^*_c$ sufficiently low as to achieve $\kappa^* = \overline{\kappa}$. Whether the optimum solution under commitment features a distortion relative to the ex-post optimal cutoff or not depends on parameters.

Finally, as in Proposition 4, if trading costs are very large and speculators are not willing to trade on information regardless of their beliefs about their peers' behavior, then simply playing according to $\overline{X}^*$ is optimal, since there is no scope for improving market informativeness. Figure 2b summarizes the results in Proposition 5.

### 4.1.1 Implementation: managerial compensation

Notice that, whenever $\overline{X}^*_c < \overline{X}^*$, the ex-ante optimal investment policy is time inconsistent in the sense that a decision maker seeking shareholder value maximization would like to deviate from it after observing market activity. We now comment on how the solution under commitment can be easily implemented with a managerial compensation scheme that conditions the manager's payment on the firm's realized cash flow at the final period.

Denote the manager's compensation by $w(v)$. For simplicity, we consider a manager that has an outside option $\overline{U} \approx 0$ (her required compensation is arbitrarily low relative to the scale of the firm's projects). Define $w_H \equiv w(V_H)$, $w_L \equiv w(V_L)$, and $w_0 \equiv w(V_0)$, and consider $w_H \geq w_0 \geq w_L$. Given such a compensation scheme, the manager at $t = 2$ will choose $a = 1$ if and only if

$$X \geq \overline{X}_w = \ln\left(\frac{w_0 - w_L}{w_H - w_0}\right). \tag{12}$$

Note that any cutoff $\overline{X}_w \leq \overline{X}^*$ can be achieved by setting $w(\cdot)$ appropriately at $t = 0$. Hence, the firm can implement its ex-ante optimal investment policy $\overline{X}_c^*$ with such simple contracts. The cutoff $\overline{X}_w$ is a function of the ratio between the potential downside and upside in the manager's compensation scheme. Therefore, since $\overline{X}_c^* \leq \overline{X}^*$, to implement its commitment solution the firm would set a managerial contract that always incentivizes the manager to be weakly more risk-tolerant than shareholders would be (ex post), with a compensation scheme that offers a relatively lower downside risk (higher upside) than the company's assets, with $\frac{w_0 - w_L}{w_H - w_0} \leq \frac{V_0 - V_L}{V_H - V_L}$.

Moreover, note that promising a fixed proportion of the firm's cash flow to the manager, i.e., $w(v) = \beta v$, is often sub-optimal in this setting. Such a compensation scheme can only lead to the ex-post optimal investment cutoff $\overline{X} = \overline{X}^*$, while implementing a cutoff $\overline{X}_c^* < \overline{X}^*$ often leads to larger firm value, as shown in Propositions 4 and 5.

# 5 Optimal capital structure

In this section, we modify the model of Section 2 by assuming that the firm issues debt and equity securities in a previous stage ($t = 0$) as to maximize its value. The firm anticipates that its security issuance will affect trading behavior and managerial decisions (at $t = 2$, the manager maximizes shareholder value as in the baseline model). For simplicity of exposition, in the main text assume that equity is publicly traded while debt is not. In Appendix A.4 we show that the optimal capital structure presented here is actually optimal when both securities are (potentially) publicly traded. We conclude this section analyzing the joint decision of capital structure and ex-ante investment policy (Section 5.4).

We start analyzing how debt affects the equilibrium at $t = 1$ (trading stage) and $t = 2$

(investment stage), and then solve for the optimal choice of debt by the firm at $t = 0$. Suppose the firm has outstanding debt in the amount of $D$ dollars. Speculators are protected by limited liability and therefore stock dividends in the final period are

$$R(\theta, a) = \max\{v(\theta, a) - D, 0\}. \tag{13}$$

## 5.1 Debt and direct incentives to trade

We first study the direct effect of debt on speculator's trading profits, taking as given the manager's investment policy. Recall that, if investment is undertaken if and only if $X \in A$, for a given $\kappa$ we have that trading profits per dollar traded are given by (9). Using the expression for dividends in (13), trading profits become

$$\pi(\kappa) = \begin{cases} (2\lambda - 1)\int_A \frac{\mu(u)(V_H - V_L)}{V_L - D + \mu(u)(V_H - V_L)} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du & \text{if } D < V_L, \\ (2\lambda - 1)\int_A \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du & \text{if } D \in [V_L, V_H), \\ 0 & \text{if } D \geq V_H. \end{cases} \tag{14}$$

If $D \geq V_H$, we have that $R_H = P = 0$ (i.e., debt is so high that the stock is worthless). There is no potential for trading profits, and any arbitrarily small trading cost would eliminate any informed trading in stock markets.

For $D < V_L$, a marginal increase in $D$ has a direct positive effect on $\pi(\cdot)$. The reason is the following: Every additional dollar in debt means a dollar less in returns to shareholders regardless of the state (the distance between $R_H$ and $R_L$ does not change), and the expected profit per share traded is not affected. However, the stock price falls, and hence a speculator can purchase more shares with a dollar, making higher trading profits in total.

For $D \in [V_L, V_H)$, direct incentives to trade are not affected by marginal changes in $D$. The reason is that, since the return in the low state is always zero for $D$ in that range and the return in the high state is decreasing, expected profits per share conditional on investment are decreasing. But as before, a larger $D$ also implies lower stock price, so more shares can be purchased with a dollar, and these two effects exactly cancel out.

Notice that the direct incentives to trade stocks are maximized when $D \in [V_L, V_H)$. The intuition is that, in this case, the whole portion of the firm's cash flow that is certain (namely $V_L$) is wiped off of dividends and stock prices. Therefore, speculators do not have to 'waste' part of their limited resources on a safe bet. Every dollar spent by informed speculators in this security is then converted into expected informational profits.[12]

## 5.2 Debt and managerial decisions

We now turn to the effect of debt on the managerial decision. Since the manager maximizes shareholder value, the risky action will be undertaken whenever $\mu(X) R_H + [1 - \mu(X)] R_L \geq R_0$. Using the expression for dividends in (13), it is easy to see that if $D \geq V_0$, the risky action is always undertaken. If $D \in (V_L, V_0)$, investment is undertaken if and only if

$$X \geq \ln\left(\frac{V_0 - D}{V_H - V_0}\right) \equiv X^*(D). \tag{15}$$

If $D \leq V_L$, investment is undertaken if and only if

$$X \geq \ln\left(\frac{V_0 - V_L}{V_H - V_0}\right) = \overline{X}^*.$$

We then have that safe debt ($D \leq V_L$) does not distort managerial incentives in relation to the ex-post optimal decision. Risky debt ($D > V_L$), on the other hand, induces the manager to take more risks, as usual in the corporate finance literature (Jensen and Meckling, 1976). Importantly, in this setting, more risk taking by the manager implies a broader set $A$ where investment takes place, boosting incentives to trade and potentially increasing information aggregation. However, this distortion implies that whatever information is obtained from the market is used less efficiently ex post. The next section characterizes the optimal capital structure in face of these effects.

[12] If $D < V_L$, dividends are always at least $V_L - D$, and hence this certain component of the dividend is fully priced in by the market maker. In this case, buying a stock is equivalent to buying a portfolio consisting of a risk-free asset that pays $V_L - D$ plus a risky asset. All potential trading profits emerge from the risky portion of this portfolio.

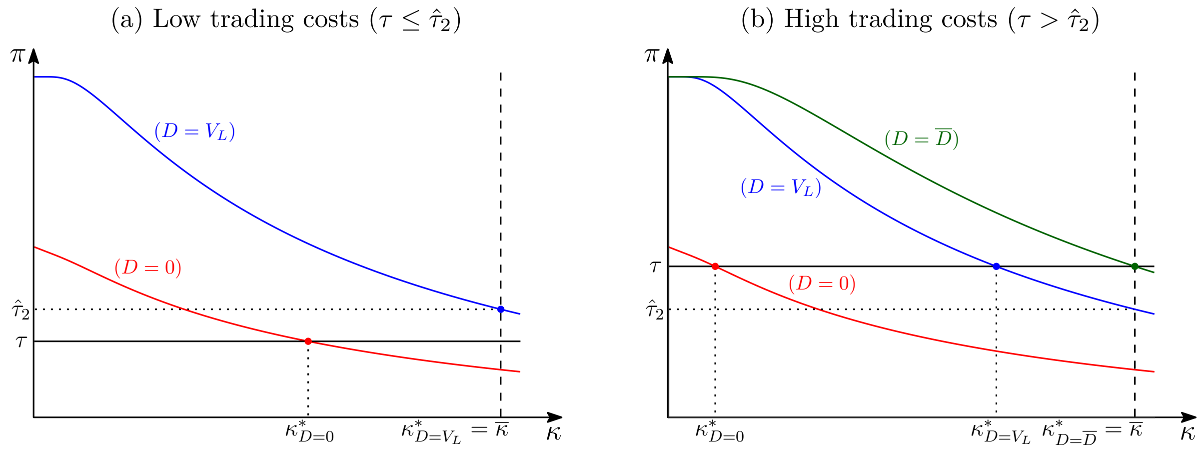


Figure 3: Optimal debt when $\overline{X}^* \leq 0$. For low $\tau$ (left panel), setting $D = V_L$ leads to the first best. For high $\tau$ (right panel), $D = V_L$ leads to an interior $\kappa^*$, and the firm optimally issues risky debt $\overline{D}$ as to achieve $\kappa^* = \overline{\kappa}$.

## 5.3 Optimal debt under feedback effects

The next two propositions characterize the firm's optimal choice of debt at $t = 0$. In Appendix A we define the variables $\hat{\tau}_1$, $\hat{\tau}_2$, $\hat{\tau}_3$, $\underline{D}$, $\overline{D}$ and $\hat{D}$, to which we refer in this section.

**Proposition 6.** *Suppose* $\overline{X}^* \leq 0$. *Then:*

1. *If* $\tau \leq \hat{\tau}_2$, *the first best can be achieved with safe debt: any* $D \in [\max(0, \underline{D}), V_L]$ *is optimal and it leads to* $\kappa^* = \overline{\kappa}$;

2. *If* $\tau > \hat{\tau}_2$, *the optimal capital structure involves risky debt:* $D = \overline{D} > V_L$ *is optimal and it leads to* $\kappa^* = \overline{\kappa}$.

Proposition 6 implies that, whenever investment is profitable under the prior, maximum information aggregation is always achieved under the optimal capital structure. If trading costs are low enough, the firm can achieve the first-best outcome by issuing an appropriate amount of safe debt. In particular, for very low $\tau$, the first best would be achieved even if $D = 0$, while for $\tau$ closer to $\hat{\tau}_2$ the firm must issue enough debt ($D \geq \underline{D}$) as to partially remove the safe portion of the firm's cash flow from the price of the stock, enhancing incentives to trade. As long as debt is safe ($D \leq V_L$), managerial incentives to take risk are not affected, and all the information extracted from market activity is efficiently used. Note that, in particular, setting $D = V_L$ is always optimal for $\tau \leq \hat{\tau}_2$.

For higher trading costs, the first best cannot be achieved. In this case, the firm is willing to increase debt beyond the range where there is no distortion to managerial incentives. That is, under the optimal debt, $\overline{X}$ will be strictly smaller than $\overline{X}^*$, meaning that the manager will not make the ex-post efficient use of information (see equation (15)). It turns out that increasing risky debt always pays off while there is scope for increasing information aggregation, despite the managerial distortion. The firm then issues the minimum amount of risky debt that yields maximum information aggregation ($\overline{D}$). Figure 3 illustrates the results in Proposition 6.

**Proposition 7.** *Suppose $\overline{X}^* > 0$. Then, positive levels of information aggregation are only achieved with risky debt. Moreover:*

1. *For $\overline{\kappa} \geq \tilde{\kappa}$ and $\tau \leq \hat{\tau}_3$, the firm issues the minimum amount of risky debt that leads to $\kappa^* = \overline{\kappa}$: the optimal $D$ is $\hat{D}$ if $\tau \leq \hat{\tau}_1$ and $\overline{D} > \hat{D}$ if $\tau \in (\hat{\tau}_1, \hat{\tau}_3]$;*

2. *For $\overline{\kappa} < \tilde{\kappa}$ or $\tau > \hat{\tau}_3$, any $D < \hat{D}$ is optimal and leads to $\kappa^* = 0$.*

When investment is not optimal under the prior, the optimal capital structure can never lead to the first best, as opposed to the case with $\overline{X}^* \leq 0$. Still, the firm might increase its value by issuing risky debt optimally at $t = 0$, at least when the potential maximum level of informativeness is high enough ($\overline{\kappa} \geq \tilde{\kappa}$) and trading costs are low enough ($\tau \leq \hat{\tau}_3$). Recall that $\overline{\kappa}$ is larger the higher the precision of informed speculators' signals ($\lambda$), the larger the number of informed speculators ($\alpha$), and the smaller the amount of noise traders ($\sigma$) – see equation (7).

If $\overline{\kappa} \geq \tilde{\kappa}$ and trading costs are sufficiently low ($\tau \leq \hat{\tau}_1$), the firm issues the precise amount of debt that implements a cutoff $\overline{X} = 0$ (namely $\hat{D}$), so that the economy switches from a coordination failure equilibrium (with $\kappa^* = 0$) to an equilibrium with maximum information aggregation. Even though the information aggregated in the market is not used in the most efficient manner ex post (since $\overline{X} = 0 < \overline{X}^*$), given that the information revealed is of good enough quality, firm value is maximized.

If $\overline{\kappa} \geq \tilde{\kappa}$ but trading costs are higher ($\tau \in (\hat{\tau}_1, \hat{\tau}_3]$), by choosing the level of debt that implements $\overline{X} = 0$, the firm can no longer achieve the maximum level of information aggregation, even though it would eliminate the coordination failure equilibrium and lead to a positive $\kappa$ in equilibrium. In this case, it pays off to further increase $D$ to $\overline{D} > \hat{D}$ as to reach

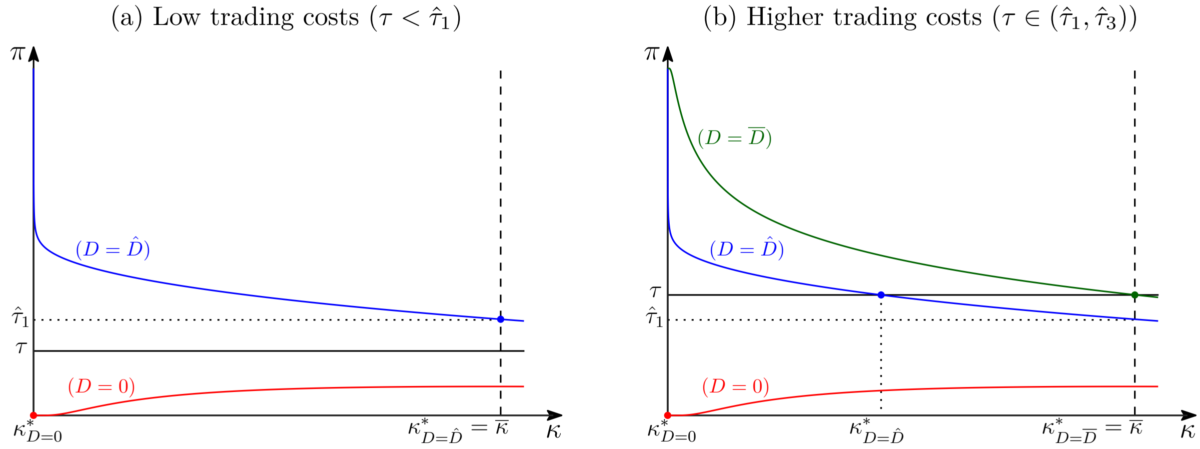


Figure 4: Optimal debt when $\overline{X}^* > 0$ and $\overline{\kappa} > \tilde{\kappa}$. For low $\tau$ (left panel), setting $D = \hat{D}$ eliminates the coordination failure and leads to $\kappa^* = \overline{\kappa}$. For higher $\tau$ (right panel), $D = \hat{D}$ leads to an interior $\kappa^*$, and the firm optimally issues risky debt $\overline{D}$ as to achieve $\kappa^* = \overline{\kappa}$.

maximum information aggregation. Despite higher debt further distorting the managerial decision rule ex post, the gain in informativeness always more than compensates the losses of potentially misusing the information obtained. Figure 4 illustrates some of the results in Proposition 7 when there is high potential for information aggregation.

Finally, if $\overline{\kappa}$ is too low ($\overline{\kappa} < \tilde{\kappa}$) or trading costs are too high ($\tau > \hat{\tau}_3$), the firm opts for a low enough level of debt so that the market continues not to aggregate any information and the firm manager continues to act according to her prior only (foregoing the risky investment opportunity). If debt was too high ($D \geq \hat{D}$), the distortion in the managerial decision rule (with excessive risk taking) would not compensate the improvement in information aggregation, since the potential for information production is low. This is the situation described in the last part of Proposition 7. To sum up, if the investment opportunity is not ex-ante optimal and there is low potential for information aggregation in secondary markets, the firm is better off limiting its debt so that the manager is not overly incentivized to react to market activity, potentially undertaking bad projects with high probability.

Figure 5 summarizes the results in Propositions 6 and 7 for different sets of parameters. The left panel illustrates the case with ex-ante profitable investment opportunities ($\overline{X}^* \leq 0$). For low trading costs the first best is achieved when the firm issues the right amount of safe debt. For a given $\overline{\kappa}$, as $\tau$ increases the firm optimally issues risky debt as to achieve maximum

(a) Investing is ex-ante profitable ($\overline{X}^* \leq 0$) (b) Investing is not ex-ante profitable ($\overline{X}^* > 0$)

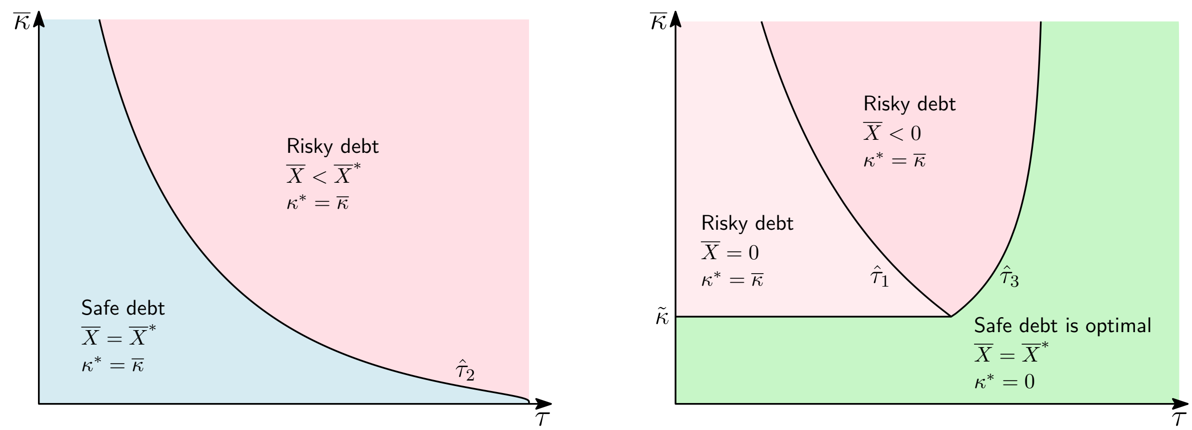


Figure 5: Optimal capital structure. The figure illustrates how the optimal capital structure – and the implied equilibrium values of $\kappa$ and $\overline{X}$ – depend on $\tau$ and $\overline{\kappa}$, for a given $\lambda$.

information aggregation. The right panel illustrates the case with $\overline{X}^* > 0$. In this case the firm can never trigger information aggregation with safe debt. If the potential to aggregate information is low ($\overline{\kappa} < \tilde{\kappa}$) or trading costs are high ($\tau > \hat{\tau}_3$), the firm has no incentives to issue debt, as raising enough debt to leave the coordination failure ($\kappa^* = 0$) would be too costly in terms of managerial distortion. If there is enough potential to aggregate information ($\overline{\kappa} > \tilde{\kappa}$) the firm finds it profitable to issue risky debt as to achieve $\kappa^* = \overline{\kappa}$ (as long as $\tau$ is low enough).

To shed further light on those results, we discuss next the effect of debt on investment probabilities.

#### 5.3.1 Risky debt and the probability of investment

If follows from the previous discussion that issuing safe debt never reduces firm value: it has a positive direct effect on speculators' incentives to trade, and it does not distort the use of market information by the manager ex post. Moreover, whenever the firm issues risky debt as to achieve a positive level of information aggregation, it has incentives to further increase risky debt until it reaches *maximum* information aggregation – despite the distortion in the use of information. To help understand this result, the next proposition analyzes the effects of risky debt on the probabilities of investment in each state.

**Proposition 8.** *Suppose the firm issues an amount of risky debt $D > V_L$ that leads to an equilibrium informativeness $\kappa^* \in (0, \overline{\kappa})$. Then, a marginal increase in $D$:*

1. *Does not affect the probability of investment being undertaken in the bad state,* $\Pr(a = 1|\theta = \theta_L) = \frac{\tau}{2\lambda - 1}$*;*

2. *Increases the probability of investment being undertaken in the good state.*

A marginal increase in risky debt increases incentives for the manager to undertake risk, which for a given level of market informativeness $\kappa > 0$ would lead to a higher probability of investment being undertaken in the bad state. However, precisely because the manager is more willing to undertake risk, speculators have larger incentives to trade and market informativeness increases, which contributes to a *reduction* in the probability of a bad investment being undertaken. In fact, these two effects cancel out: raising risky debt does not effectively lead to more investment in bad states once we take into account the gains in market informativeness.

Moreover, both the increase in market informativeness and the increase in incentives for the manager to take risk explain why higher risky debt raises the probability of investment in the good state.

In sum, a marginal increase in risky debt keeps unchanged the probability of investment taking place when it is not (ex-post) optimal for the firm, and decreases the probability of good investment opportunities being missed. This is why firm value increases as the firm raises risky debt – up to the point where informativeness reaches its maximum, $\kappa^* = \overline{\kappa}$.

## 5.4 Debt and investment policy

Throughout this section, when deriving the optimal capital structure we have assumed that the firm manager maximizes shareholder value. We now allow the firm to jointly determine its optimal capital structure and investment policy, potentially committing to an investment policy that is not consistent with (ex-post) shareholder value maximization. Formally, we assume that, at $t = 0$, the firm can choose $D \in \mathbb{R}_+$ and a measurable set $A \subseteq \mathbb{R}$ such that it invests if and only if $X \in A$. The next proposition discusses the solution to this more general problem.

**Proposition 9.** *Consider the problem where the firm can choose its capital structure and investment policy at $t = 0$. Then, the optimum can always be achieved with an investment policy consistent with shareholder value maximization ex post.*

In other words, when the firm can choose its level of debt optimally, there is no reason for the firm to deviate from shareholder value maximization (for instance, by designing certain managerial incentives as was the case in Section 4). The intuition for this result is the following. First, the firm can often achieve the first best by simply issuing enough safe debt, boosting incentives to trade and keeping the managerial decision unaffected. If the first best is not achievable and it is optimal for the firm to induce more risk taking by the manager, this can be implemented by issuing an appropriate amount of risky debt – since raising debt beyond $V_L$ does not hamper incentives to trade, but incentivizes investment in more states.

Another implication of our results is that, as long as trading costs are not too high, the firm can often achieve the first best by issuing the right amount of debt (Proposition 4). This result contrasts with the case of commitment in Section 4: there, either the first best would be achieved even without commitment, or it would not be achieved.

# 6 Empirical predictions

We now derive some comparative statics that are useful to discuss some of the model's empirical implications.

**Proposition 10.** *The smallest and largest optimal levels of debt are increasing in the mass of informed traders $\alpha$, and increasing in trading costs $\tau$ for sufficiently low $\tau$.*[13]

**Informed trading.** Our model predicts that larger debt increases speculators' incentives to trade stocks, leading to higher informed trading in equilibrium. Therefore, we should observe a positive correlation between firms' debt and measures of informed trading in their

[13] We refer to the smallest and largest optima since for some parameters there are multiple optimal values of $D$. Another way to state the result is that the optimal $D$ is increasing in those paramenters in the strong set order sense (see Topkis, 1978, or Vives, 1999). Also, if trading costs are very large and $\overline{X}^* > 0$, the level of debt needed to guarantee information provision by the market is too high and the firm gives up exploring this channel (see also Figure 5b).

stocks – such as probability of informed trading (PIN) and price nonsynchronicity (see Chen, Goldstein and Jiang, 2006).

Moreover, we can interpret markets with large $\alpha$ as markets in which many traders possess information that is relevant for decisions makers (so there is a large potential for stock market feedback, $\overline{\kappa}$ is large).[14] Higher market sophistication ($\alpha$) makes firms more willing to issue debt as to better explore the market provision of information. Hence, in markets where there are more informed traders, we should expect firms to issue more debt. This is another reason why we should observe a positive correlation between firms' debt and measures of informed trading, and our model suggests the causality goes both ways.

**Market frictions.** In markets with higher trading costs, informed speculators have less incentives to trade, which all else equal tends to reduce market informativeness. For moderate trading costs, an increase in trading costs then incentivizes firms to issue more debt as to counterbalance speculators' reduced incentives to trade and fully explore the market potential to aggregate information. In general, we should observe a positive relationship between trading costs in secondary markets and firms' debt. This relationship should be more pronounced in sectors with higher potential for information transmission from financial markets (larger $\overline{\kappa}$).

**Managerial compensation.** From Proposition 9, under the optimal capital structure firms have no incentives to induce the manager to depart from shareholder value maximization. However, if there are other factors that lead a firm to choose debt below the optimum predicted by our model, then such firm should rely more on incentive schemes that encourage risk taking. Hence, we should expect firms with lower debt to rely more on compensation schemes that protect managers from downside risk, inducing more risk taking (for instance, option-based compensation). Moreover, our results suggest that this relationship is stronger when market sophistication or market frictions are large. These predictions complement the analysis of managerial compensation under feedback effects in Lin, Liu and Sun (2019).

[14]For instance, we should expect $\alpha$ to be larger in industries where demand-side information (such as consumers' tastes) is important to forecast investment profitability, and $\alpha$ to be lower in industries where profitability is mostly determined by technological features mostly known by insiders.

**Short-selling restrictions.** Our model has implications for the effect of short-selling restrictions on market informativeness and firms' capital structure. In the model, short-selling constraints are isomorphic to a decrease in the maximum potential informativeness $\overline{\kappa}$, since those constraints prevent pessimistic investors with no initial position on the stock from trading. (For a formal treatment of this matter, see Appendix A.5). Therefore, short-selling restrictions have the same effect as a decrease in the mass of informed traders $\alpha$.

We should then expect firms to decrease their level of debt following a short-selling ban. Lin, Liu and Sun (2019) study empirically the effect of a removal of short-selling restrictions on managerial compensation, exploring a channel of learning from financial markets. Our results suggests that, in the presence of such feedback effects, a removal of short-selling restrictions could also impact firm's funding decisions (leading to more debt).

Additionally, removing short-selling constraints should increase firms' investment sensitivity to stock prices and profitability, and the more so for firms that increase debt following the removal. Those firms should be able to take more advantage of the higher potential for learning from the market.

# 7 Final remarks

This paper proposes a tractable model of stock market feedback and studies optimal investment and capital structure policies. Firms may benefit from committing to an investment policy that features more risk taking, as to strike a balance between boosting information aggregation in secondary markets and making good use of information ex post. The ex-ante optimal investment policy is in general time inconsistent, but can be implemented with simple managerial compensation schemes.

The optimal capital structure often leads to the first best. When the firm has strong investment opportunities available and market frictions are not large, by issuing enough safe debt the firm leads to maximum information aggregation without distorting the managerial use of information. When the first best is not achievable but there is still high potential for information provision in financial markets, the optimal capital structure involves the minimum amount of risky debt that leads to maximum information aggregation. Under an optimal capital structure, the time inconsistency in investment policies disappears: the firm's optimal investment policy ex ante is consistent with ex-post shareholder value maximization.

Our empirical predictions shed light on the relationship between firms' capital structure, the probability of informed trading, market frictions, and managerial contracts. Our theory can help guide future empirical work on the real effects of financial markets.

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# A Appendix

We define here some variables to which we refer in the main text:

$$\tilde{\tau}_1 \equiv (2\lambda - 1) \int_0^{\infty} \frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \frac{1}{\sqrt{2\overline{\kappa}}} \phi\left(\frac{u + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right) du, \tag{A.1}$$

$$\tilde{\tau}_2 \equiv (2\lambda - 1) \int_{\overline{X}^*}^{\infty} \frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \frac{1}{\sqrt{2\overline{\kappa}}} \phi\left(\frac{u + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right) du, \quad \tilde{\tau}_3 \equiv (2\lambda - 1)\frac{\Delta_R}{R_L + R_H}, \tag{A.2}$$

$$\hat{\tau}_1 \equiv (2\lambda - 1)\left[1 - \Phi\left(\sqrt{\frac{\overline{\kappa}}{2}}\right)\right], \quad \hat{\tau}_2 \equiv (2\lambda - 1)\left[1 - \Phi\left(\frac{\overline{X}^* + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right)\right], \quad \hat{D} \equiv 2V_0 - V_H, \tag{A.3}$$

$$\overline{D} = \overline{D}(\tau) \equiv V_0 - (V_H - V_0)\, e^{\Omega(\tau)}, \quad \text{where} \quad \Omega(\tau) \equiv \sqrt{2\overline{\kappa}}\Phi^{-1}\left(1 - \frac{\tau}{2\lambda - 1}\right) - \overline{\kappa}. \tag{A.4}$$

Let $\hat{\tau}_3$ be defined implicitly by

$$1 - \Phi\left(\frac{X^*\left(\overline{D}\left(\hat{\tau}_3\right)\right) - \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right) = \frac{V_0 - V_L}{V_H - V_0}\left[1 - \Phi\left(\frac{X^*\left(\overline{D}\left(\hat{\tau}_3\right)\right) + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right)\right], \tag{A.5}$$

and $\underline{D}$ by

$$\tau = (2\lambda - 1) \int_{\overline{X}^*}^{\infty} \frac{\mu(u)\left(V_H - V_L\right)}{V_L - \underline{D} + \mu(u)\left(V_H - V_L\right)} \frac{1}{\sqrt{2\overline{\kappa}}} \phi\left(\frac{u + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right) du. \tag{A.6}$$

We now present and prove two auxiliary lemmas that will be used in subsequent proofs.

**Lemma A.1.** *Let $\pi(\kappa)$ be as given by (9). Then,*

$$\lim_{\kappa\to 0}\pi(\kappa) = \begin{cases} \frac{\Delta_R}{R_L+R_H}(2\lambda-1) & \textit{if } 0\in A, \\ 0 & \textit{if } 0\notin A. \end{cases}$$

*Proof.* Using (9), we can write:

$$\pi(\kappa) = (2\lambda-1)\, e^{-\frac{\kappa}{4}}\int_A \frac{\mu(u)\Delta_R}{R_L+\mu(u)\Delta_R} e^{-\frac{u}{2}}\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u}{\sqrt{2\kappa}}\right)du.$$

Letting $\delta(u)$ be the Dirac delta function, we have that

$$\lim_{\kappa\to 0} e^{-\frac{\kappa}{4}}\int_A \frac{\mu(u)\Delta_R}{R_L+\mu(u)\Delta_R} e^{-\frac{u}{2}}\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u}{\sqrt{2\kappa}}\right)du = \int_A \frac{\mu(u)\Delta_R}{R_L+\mu(u)\Delta_R} e^{-\frac{u}{2}}\delta(u)du,$$

which equals $\frac{\mu(0)\Delta_R}{R_L+\mu(0)\Delta_R}$ if $0\in A$ and 0 otherwise. Using the fact that $\mu(0)=1/2$ yields the desired result. □

**Lemma A.2.** *Suppose the firm plays a cutoff strategy $\overline{X}$ and let $\mathcal{V}(\overline{X},\kappa)$ denote the ex-ante firm value for a given $\overline{X}$ and $\kappa>0$. Then,*

$$\textit{sign}\left(\frac{\partial\mathcal{V}(\overline{X},\kappa)}{\partial\kappa}\right) = \textit{sign}\left(\overline{X}\left(e^{\overline{X}}-e^{\overline{X}^*}\right)+\kappa\left(e^{\overline{X}}+e^{\overline{X}^*}\right)\right), \tag{A.7}$$

$$\textit{sign}\left(\frac{\partial\mathcal{V}(\overline{X},\kappa)}{\partial\overline{X}}\right) = \textit{sign}\left(\overline{X}^*-\overline{X}\right). \tag{A.8}$$

*Proof.* If the manager adopts a cutoff investment strategy $\overline{X}$, we can write the ex-ante value of the firm as:

$$\mathcal{V} = \frac{1}{2}\left[V_H-\Phi\left(\frac{\overline{X}-\kappa}{\sqrt{2\kappa}}\right)(V_H-V_0)\right]+\frac{1}{2}\left[V_L+\Phi\left(\frac{\overline{X}+\kappa}{\sqrt{2\kappa}}\right)(V_0-V_L)\right], \tag{A.9}$$

and therefore:

$$\frac{\partial\mathcal{V}}{\partial\kappa} = \frac{1}{2}\phi\left(\frac{\overline{X}-\kappa}{\sqrt{2\kappa}}\right)(V_H-V_0)\left(\frac{\overline{X}+\kappa}{2^{3/2}\kappa^{3/2}}\right)-\frac{1}{2}\phi\left(\frac{\overline{X}+\kappa}{\sqrt{2\kappa}}\right)(V_0-V_L)\left(\frac{\overline{X}-\kappa}{2^{3/2}\kappa^{3/2}}\right), \tag{A.10}$$

which has the same sign as

$$\phi\left(\frac{\overline{X}-\kappa}{\sqrt{2\kappa}}\right)(V_H-V_0)\left(\overline{X}+\kappa\right)-\phi\left(\frac{\overline{X}+\kappa}{\sqrt{2\kappa}}\right)(V_0-V_L)\left(\overline{X}-\kappa\right).$$

Since $\phi\left(\frac{\overline{X}-\kappa}{\sqrt{2\kappa}}\right)/\phi\left(\frac{\overline{X}+\kappa}{\sqrt{2\kappa}}\right) = e^{\overline{X}}$ and $\frac{V_0-V_L}{V_H-V_0} = e^{\overline{X}^*}$, the expression above has the same sign as $\overline{X}\left(e^{\overline{X}}-e^{\overline{X}^*}\right)+\kappa\left(e^{\overline{X}}+e^{\overline{X}^*}\right)$. Moreover, we have that

$$\frac{\partial\mathcal{V}}{\partial\overline{X}} = -\frac{1}{2\sqrt{2\kappa}}(V_H-V_0)\,\phi\left(\frac{\overline{X}-\kappa}{\sqrt{2\kappa}}\right)+\frac{1}{2\sqrt{2\kappa}}(V_0-V_L)\,\phi\left(\frac{\overline{X}+\kappa}{\sqrt{2\kappa}}\right). \tag{A.11}$$

One can verify that the expression above has the same sign as $(\overline{X}^* - \overline{X})$. $\square$

## A.1 Proofs

### A.1.1 Proof of Lemma 1

Define:

$$n = \frac{1}{\sigma^2}(S_H - S_L)\tilde{n}. \tag{A.12}$$

Notice that $n \sim N(0, 2\kappa)$. Using equations (3) and (A.12), we can write:

$$X = \begin{cases} n + \kappa & \text{if } \theta = \theta_H, \\ n - \kappa & \text{if } \theta = \theta_L. \end{cases} \tag{A.13}$$

Let $N_H(\kappa, A) = \{n \in \mathbb{R} : (n + \kappa) \in A\}$ and $N_L(\kappa, A) = \{n \in \mathbb{R} : (n - \kappa) \in A\}$. For a speculator with beliefs $\eta = \eta(m)$, the expected profit from trading $s$ dollars, ignoring transaction costs, is:

$$\Pi = s\left\{\eta \int_{N_H(\kappa,A)} \left[\frac{R_H}{P(n+\kappa)} - 1\right] \frac{1}{\sqrt{2\kappa}}\phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn + (1-\eta)\int_{N_L(\kappa,A)} \left[\frac{R_L}{P(n-\kappa)} - 1\right] \frac{1}{\sqrt{2\kappa}}\phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn\right\},$$

Using the price function (5) and applying the changes of variable $u = n + \kappa$ and $u = n - \kappa$, for the first and second integral, respectively:

$$\Pi = s\left\{\eta \int_A \left[\frac{(1-\mu(u))\,\Delta_R}{R_L + \mu(u)\,\Delta_R}\right] \frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u-\kappa}{\sqrt{2\kappa}}\right) du - (1-\eta)\int_A \left[\frac{\mu(u)\,\Delta_R}{R_L + \mu(u)\,\Delta_R}\right] \frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du\right\}.$$

Using the fact that $(1-\mu(u))\,\phi\left(\frac{u-\kappa}{\sqrt{2\kappa}}\right) = \mu(u)\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right)$, we can write $\Pi$ as in (8). $\square$

### A.1.2 Proof of Lemma 2

The first statement in the first part of Lemma 2 is a special case of Lemma A.1, with $A = \left[\overline{X}, \infty\right)$. That $\pi(\kappa)$ is increasing for low $\kappa$ when $\overline{X} > 0$ follows from the fact that $\lim_{\kappa\to 0}\pi(\kappa) = 0$ and $\pi(\kappa) > 0$ for $\kappa > 0$ and $\Delta_R > 0$.

We now prove Lemma 2, part 2. Consider $\overline{X} \le 0$ and $R_H > R_L \ge 0$. Using Lemma 1, we know that, conditional on $\kappa$, the specific strategies played by speculators do not matter to compute speculator's payoffs. We can then consider, without loss of generality, the following aggregate orders conditional on the state: $S_H = a\beta$, $S_L = -c\beta$, where $\beta \ge 0$, $a = \frac{R_H}{R_L + R_H}$ and $c = \frac{R_L}{R_L + R_H}$. Note that there is a one to one mapping from $\beta$ to $\kappa$ (the aggregate strategies considered imply $\kappa = \beta^2/2\sigma^2$). Bayesian updating implies:

$$\mu\left(\tilde{X}\right) = \Pr\left(\theta = \theta_H|\tilde{X}\right) = \frac{\Pr\left(\tilde{X}|\theta = \theta_H\right)\Pr(\theta = \theta_H)}{\Pr\left(\tilde{X}\right)},$$

which becomes

$$\mu\left(\tilde{X}\right)=\frac{\phi\left(\frac{\tilde{X}-S_H}{\sigma}\right)}{\phi\left(\frac{\tilde{X}-S_H}{\sigma}\right)+\phi\left(\frac{\tilde{X}-S_L}{\sigma}\right)}=\frac{\phi\left(\frac{\tilde{X}-a\beta}{\sigma}\right)}{\phi\left(\frac{\tilde{X}-a\beta}{\sigma}\right)+\phi\left(\frac{\tilde{X}+c\beta}{\sigma}\right)}.$$

We can write beliefs for a given $\tilde{n}$ in states $\theta_H$ and $\theta_L$, respectively, as:

$$\mu_H\left(\tilde{n}\right)=\frac{\phi\left(\frac{\tilde{n}}{\sigma}\right)}{\phi\left(\frac{\tilde{n}}{\sigma}\right)+\phi\left(\frac{\tilde{n}+\beta}{\sigma}\right)},\quad \text{and}\quad \mu_L\left(\tilde{n}\right)=\frac{\phi\left(\frac{\tilde{n}-\beta}{\sigma}\right)}{\phi\left(\frac{\tilde{n}-\beta}{\sigma}\right)+\phi\left(\frac{\tilde{n}}{\sigma}\right)}. \tag{A.14}$$

The manager invests whenever $X\geq\overline{X}$. Since $X=\frac{1}{2\sigma^2}\beta\left[2\tilde{X}-\beta\frac{\Delta_R}{R_L+R_H}\right]$ given the strategies assumed, she invests whenever

$$\tilde{X}\geq\frac{2\sigma^2\overline{X}}{\beta}+\beta\frac{\Delta_R}{R_L+R_H}\equiv\hat{X}\left(\beta\right).$$

Let $\tilde{\pi}(\beta)$ denote the expected trading profits for a speculator buying one dollar in stocks for a given belief $\eta$. Given that $\kappa=\beta^2/2\sigma^2$, to prove the second part of Lemma 2 it suffices to show that $\tilde{\pi}(\beta)$ is decreasing in $\beta$ for $\eta>0.5$ (and thus, given Lemma 1, trading profits from selling when $\eta<0.5$ are decreasing in $\beta$). If the state is high, the manager invests whenever $\tilde{n}\geq\hat{X}(\beta)-a\beta$. If the state is low, she invests whenever $\tilde{n}\geq\hat{X}(\beta)+c\beta$. We can thus write:

$$\tilde{\pi}(\beta)=\eta\int_{\hat{X}(\beta)-a\beta}^{\infty}\left[\frac{R_H}{R_L+\mu_H\left(\tilde{n}\right)\Delta_R}-1\right]\frac{1}{\sigma}\phi\left(\frac{\tilde{n}}{\sigma}\right)d\tilde{n}+(1-\eta)\int_{\hat{X}(\beta)+c\beta}^{\infty}\left[\frac{R_L}{R_L+\mu_L\left(\tilde{n}\right)\Delta_R}-1\right]\frac{1}{\sigma}\phi\left(\frac{\tilde{n}}{\sigma}\right)d\tilde{n}.$$

Using (A.14) and performing changes of variables such that $u=\tilde{n}+a\beta$ in the first integral and $u=\tilde{n}-c\beta$ in the second, we have:

$$\tilde{\pi}(\beta)=(2\eta-1)\frac{1}{\sigma}\Delta_R\int_{\hat{X}(\beta)}^{\infty}M(u,\beta)du,\quad\text{where}\quad M(u,\beta)=\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)\phi\left(\frac{u-a\beta}{\sigma}\right)}{R_L\phi\left(\frac{u+c\beta}{\sigma}\right)+R_H\phi\left(\frac{u-a\beta}{\sigma}\right)}.$$

Using the Leibniz rule, we have that

$$\frac{\partial\tilde{\pi}}{\partial\beta}=(2\eta-1)\frac{1}{\sigma}\Delta_R\left[\int_{\hat{X}(\beta)}^{\infty}\frac{\partial M\left(u,\beta\right)}{\partial\beta}du-\frac{d\hat{X}(\beta)}{d\beta}M(\hat{X}(\beta),\beta)\right].$$

Since for $\overline{X}\leq 0$, $\frac{d\hat{X}(\beta)}{d\beta}>0$, it suffices to show that $\frac{\partial M(u,\beta)}{\partial\beta}\leq 0$. $\frac{\partial M(u,\beta)}{\partial\beta}$ has the same sign as:

$$aR_L\left(u-a\beta\right)\phi\left(\frac{u+c\beta}{\sigma}\right)-cR_H\left(u+c\beta\right)\phi\left(\frac{u-a\beta}{\sigma}\right),$$

which is zero if $R_L=0$, since in this case $c=0$. Consider now $R_L>0$. It remains to show that the expression above is smaller than or equal to zero. Given that $aR_L = cR_H$, this is the case whenever

$$(u - a\beta)\,\phi\left(\frac{u + c\beta}{\sigma}\right) \leq (u + c\beta)\,\phi\left(\frac{u - a\beta}{\sigma}\right). \tag{A.15}$$

If $u - a\beta = 0$, $u + c\beta > 0$ and (A.15) is trivially satisfied. There are then two cases to be considered: $u - a\beta < 0$ or $u - a\beta > 0$. First, consider $u - a\beta > 0$. Inequality (A.15) can be written as

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} \leq \frac{(u + c\beta)}{(u - a\beta)}$$

which holds since

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} < 1 \quad \text{and} \quad \frac{(u + c\beta)}{(u - a\beta)} > 1.$$

Now, consider $u - a\beta < 0$. We need to show that

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} \geq \frac{(u + c\beta)}{(u - a\beta)}.$$

If $u + c\beta \geq 0$, the inequality above is satisfied, since the $LHS > 0$ and the $RHS \leq 0$. If instead $u + c\beta < 0$,

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} > 1 \quad \text{and} \quad \frac{(u + c\beta)}{(u - a\beta)} \in (0, 1)\,.$$

We have then established that function $M(u, \beta)$ is decreasing in $\beta$. Hence, for $\overline{X} \leq 0$, $\pi(\kappa)$ is strictly decreasing in $\kappa$. □

### A.1.3 Proof of Proposition 1

Consider $\pi(\kappa)$ as given by (9), with $A = \left[\overline{X}, \infty\right)$. Suppose $\overline{X} > 0$. Since, from Lemma 2, $\lim_{\kappa\to 0}\pi(\kappa) = 0 < \tau$, if all other speculators are expected not to trade, it is a best response for each speculator not to trade, and hence there always exists an equilibrium with $\kappa^* = 0$.

Now, suppose $\overline{X} \leq 0$. From Lemma A.1, $\lim_{\kappa\to 0}\pi(\kappa)$ is strictly positive, and from Lemma 2, $\pi(\kappa)$ is strictly decreasing in $\kappa$. The best response to some $\kappa$ from a speculator is: to trade as much as possible ($s_i(m_H) = 1$, $s_i(m_L) = -1$) if $\pi(\kappa) > \tau$; not to trade if $\pi(\kappa) < \tau$; and to place any orders $s_i(m_H) \in [0, 1]$ and $s_i(m_L) \in [-1, 0]$ if $\pi(\kappa) = \tau$. If $\tau \geq \overline{\tau} = \pi(0)$, $\tau > \pi(\kappa)$ for any $\kappa > 0$, and hence speculators' best response to any $\kappa > 0$ is not to trade, so the unique equilibrium features $\kappa^* = 0$. If $\tau \leq \underline{\tau} = \pi(\overline{\kappa})$, $\pi(\kappa) > \tau$ for any $\kappa < \overline{\kappa}$, and hence speculators' best response to any $\kappa \in [0, \overline{\kappa})$ is to trade as much as possible, that is, $s_i(m_H) = 1$ and $s_i(m_L) = -1$, so the unique equilibrium features $\kappa^* = \overline{\kappa}$. Finally, consider $\tau \in (\underline{\tau}, \overline{\tau})$. By continuity, there exists $\kappa' \in (0, \overline{\kappa})$ satisfying $\pi(\kappa') = \tau$. For any $\kappa < \kappa'$, speculators would like to trade as much as possible, leading to $\kappa = \overline{\kappa} > \kappa'$, but for every $\kappa > \kappa'$, speculators would like not to trade, leading to $\kappa = 0$. The equilibrium is reached when $\kappa^* = \kappa'$, and then any $s_i(m_H) \in [0,1]$ and $s_i(m_L) \in [-1,0]$ are best responses. Any strategy profile for speculators leading to $\kappa = \kappa'$ is consistent with equilibrium. The equilibrium is essentially unique in the sense that the level of informativeness $\kappa'$ and firm value are uniquely determined. □

### A.1.4 Proof of Proposition 2

Suppose the firm invests if, and only if, $X \in A$. Then, firm value is given by equation (10). By equations (A.12) and (A.13), the distribution of $X$ conditional on a given state only depends on $\kappa$, which concludes the proof of the first statement. It follows from Lemma A.2 that under the ex-post optimal strategy $\overline{X} = \overline{X}^*$, $\frac{\partial \mathcal{V}}{\partial \kappa^*} > 0$. Lemma A.2 also implies that, for a given $\kappa$, the ex-ante optimal investment strategy is the ex-post optimal cutoff $\overline{X}^*$. Since firm value is strictly increasing in $\kappa$ under that cutoff strategy, the firm value can never be larger than when $\kappa = \overline{\kappa}^*$ and $\overline{X} = \overline{X}^*$. □

### A.1.5 Proof of Proposition 3

Let $A \subseteq \mathbb{R}$ be a measurable set such that the firm invests if, and only if, $X \in A$. By inspection of (8), one can easily verify that an equilibrium exists for any $A$. Let $N_H(\kappa, A) = \{n \in \mathbb{R} : (n+\kappa) \in A\}$ and $N_L(\kappa, A) = \{n \in \mathbb{R} : (n-\kappa) \in A\}$. Using (A.13), the firm payoff for a given $\kappa$ and $A$ can be written as

$$\mathcal{V} = V_0 + \frac{1}{2}\Pr\left((n+\kappa) \in A\right)(V_H - V_0) - \frac{1}{2}\Pr\left((n-\kappa) \in A\right)(V_0 - V_L),$$

where

$$\Pr\left((n+\kappa) \in A\right) = \int_{N_H(\kappa,A)} \frac{1}{\sqrt{2\kappa}}\phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn \quad \text{and} \quad \Pr\left((n-\kappa) \in A\right) = \int_{N_L(\kappa,A)} \frac{1}{\sqrt{2\kappa}}\phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn.$$

Applying the change of variables $u = n+\kappa$ in the first probability above and $u = n-\kappa$ in the second one, we have

$$\mathcal{V} = V_0 + \frac{1}{2}\int_A (V_H - V_0)\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u-\kappa}{\sqrt{2\kappa}}\right) du - \frac{1}{2}\int_A (V_0 - V_L)\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du,$$

which using the fact that $\phi\left(\frac{u-\kappa}{\sqrt{2\kappa}}\right)/\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) = e^u$ becomes

$$\mathcal{V}(A,\kappa) = V_0 + \frac{1}{2}\int_A J(u)\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du, \tag{A.16}$$

where $J(u) \equiv e^u (V_H - V_0) - (V_0 - V_L)$. Notice that $J(\cdot)$ is strictly increasing. Moreover, from (9), $\pi(\cdot)$ can be written as

$$\pi(\kappa;A)=(2\lambda-1)\int_A Q(u)\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right)du,$$

where $Q(u)\equiv\frac{\mu(u)\Delta_R}{R_L+\mu(u)\Delta_R}$. (For convenience, we condition the profit function on $A$ throughout the proof). Notice that $Q(\cdot)$ is increasing in $u$.

The problem of the firm is to choose a measurable set $A\subseteq\mathbb{R}$ and $\kappa\in[0,\overline{\kappa}]$ to maximize (A.16) subject to $\kappa=f(A)$, where $f(A)$ denotes the lowest value of $\kappa$ consistent with equilibrium in the trading stage for a given $A$. Instead of showing that there is a cutoff strategy that is optimal in the original problem, we do so in a relaxed problem in which the firm chooses $\kappa$ and $A$ to maximize (A.16) subject to $\kappa\le f(A)$ (step 1). We then show that in the solution of this relaxed problem $\kappa=f(A)$, implying that a cutoff strategy is also optimal in the original problem (step 2).

**Step 1.** We first show that, in the relaxed problem, there is always a cutoff strategy that is optimal. Suppose first that there exists a non-cutoff strategy $(A^*,\kappa^*)$ that is optimal and leads to $f(A^*)=0$, so $\kappa^*=0$. This implies that $X=0$ with probability one. Then, there exists a cutoff strategy $\overline{X}$ that is optimal: if $0\in A^*$, any $\overline{X}\le 0$ is optimal; if $0\notin A^*$, any $\overline{X}>0$ is optimal.

Now, suppose by contradiction that there exists a non-cutoff strategy $(A^*,\kappa^*)$ that is optimal and is such that: (i) $f(A^*)>0$; (ii) there exists bounded sets $B_1$ and $B_2$ with strictly positive measure satisfying $B_1\subseteq A^*$, $B_2\subseteq(A^*)^c$, and $X_2>X_1$ for every $X_1\in B_1$ and $X_2\in B_2$. Consider a deviation to $(A',\kappa^*)$ such that $A'=(A^*\setminus C_1)\cup C_2$, where $C_1$ and $C_2$ are measurable subsets of $B_1$ and $B_2$, respectively, with

$$\int_{C_1}\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right)du=\int_{C_2}\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right)du.$$

Such deviation increases firm value since $J(u)$ is strictly increasing. It remains to check that such deviation is feasible, meaning that $\kappa^*\le f(A')$. It then suffices to verify that $f(A')\ge f(A^*)$. To see that, first notice that $\pi(\kappa;A')\ge\pi(\kappa;A^*)$ for every $\kappa$, since $Q(u)$ is increasing. Next, notice that since $f(A^*)>0$, it must be that $\pi(\kappa;A^*)>\tau$ for all $\kappa<f(A^*)$ (or else $f(A^*)$ would not be the smallest $\kappa$ consistent with the equilibrium in the trading stage). It must then be that $f(A')\ge f(A^*)$, and hence $A^*$ is not optimal. Therefore, there must be a cutoff strategy that is optimal in the relaxed problem.

**Step 2.** Denote by $(A^*,\kappa^*)$ a cutoff strategy that is optimal in the relaxed problem, where $X^*$ is the cutoff associated to that strategy (i.e., $A^*=[X^*,\infty)$). We now show that $\kappa^*=f(A^*)$. If $f(A^*)=0$, this is trivially satisfied. If $X^*>0$, from Lemma A.1, $f(A^*)=0$, and hence $\kappa^*=f(A^*)$. Therefore, in the remaining of the proof we consider $X^*\le 0$ and $f(A^*)>0$.

If $X^*=\overline{X}^*$, firm value is strictly increasing in $\kappa$ (by Lemma (A.2)), and hence the largest $\kappa$ possible is chosen, $\kappa^*=f(A^*)$. That statement is also true if $X^*<\overline{X}^*$.

Now suppose $X^*>\overline{X}^*$ and $0<\kappa^*<f(A^*)$. In that case, for a given $\kappa$, the firm could increase

its value by decreasing $X^*$ (see Lemma A.2) and keeping the same $\kappa$ – one can check that such deviation is feasible, since trading profits would increase for every $\kappa$. Therefore there can be no optimum with $X^* > \overline{X}^*$ and $\kappa^* \in (0, f(A^*))$. Finally, assume $X^* > \overline{X}^*$ and $\kappa^* = 0 < f(A^*)$. Then, the firm could deviate and achieve the same payoff by choosing $X^* = \overline{X}^*$ and the same $\kappa$. But then, the firm can increase its payoff by increasing $\kappa^*$ when $X^* = \overline{X}^*$ (which is feasible since trading profits increase for every $\kappa$ as $X^*$ approaches $\overline{X}^*$).

Therefore, we have shown that there is always a cutoff strategy that is optimal in the relaxed problem, and that the constraint $\kappa^* \leq f(A^*)$ is binding in that optimum. This implies that in the original problem, a cutoff strategy is also optimal. The second statement of Proposition 3 follows from Propositions 4 and 5 (note that we only use the first statement of Proposition 3 to prove those propositions). □

### A.1.6 Proof of Proposition 4

Consider $\overline{X}^* \leq 0$. Denote by $\mathcal{V}\left(\overline{X}, \kappa\right)$ the ex-ante value of the firm for some cutoff $\overline{X}$ and some $\kappa$, as given by equation (A.9). Denote by $\kappa^*\left(\overline{X}\right)$ the equilibrium level of informativeness when $\overline{X}$ is played. Throughout the proof, we use the results in Lemma A.2, which imply, in particular, that $\frac{\partial \mathcal{V}}{\partial \kappa} > 0$ for all $\kappa > 0$ if $\overline{X} \leq \overline{X}^*$, and if $\overline{X} > \overline{X}^*$, $\frac{\partial \mathcal{V}}{\partial \kappa} < 0$ for low values of $\kappa$.

We first show that if commitment increases firm value, the ex-ante optimal cutoff is some $\overline{X}_c^* < \overline{X}^*$. First note that if the firm commits to a cutoff $\overline{X}' > 0$, $\kappa^*\left(\overline{X}'\right) = 0$ (given Assumption 2), and thus the manager keeps her prior and always chooses $a = 0$. Hence

$$\mathcal{V}\left(\overline{X}', \kappa^*\left(\overline{X}'\right)\right) = V_0 \leq \frac{V_H + V_L}{2} = \mathcal{V}\left(\overline{X}^*, 0\right) \leq \mathcal{V}\left(\overline{X}^*, \kappa^*\left(\overline{X}^*\right)\right),$$

and therefore any cutoff $\overline{X}' > 0$ is weakly dominated by the ex-post optimal one, $\overline{X}^*$.

Now, suppose that the firm commits to some $\hat{X} \in \left(\overline{X}^*, 0\right]$. From Proposition 1, for high enough $\tau$, $\kappa^*\left(\hat{X}\right) = \kappa^*\left(\overline{X}^*\right) = 0$, hence $\mathcal{V}\left(\hat{X}, \kappa^*\left(\hat{X}\right)\right) = \mathcal{V}\left(\overline{X}^*, \kappa^*\left(\overline{X}^*\right)\right)$ and $\overline{X}^*$ weakly dominates $\hat{X}$. Consider then a low enough $\tau$ such that $\kappa^*\left(\hat{X}\right) > 0$. Suppose $\kappa^*\left(\hat{X}\right)$ is such that $\left.\frac{\partial \mathcal{V}}{\partial \kappa}\right|_{\kappa = \kappa^*(\hat{X})} \leq 0$. Then $\frac{\partial \mathcal{V}}{\partial \kappa} < 0$ for all $\kappa \in \left(0, \kappa^*\left(\hat{X}\right)\right)$, and hence

$$\mathcal{V}\left(\hat{X}, \kappa^*\left(\hat{X}\right)\right) < \mathcal{V}\left(\hat{X}, 0\right) = \mathcal{V}\left(\overline{X}^*, 0\right) \leq \mathcal{V}\left(\overline{X}^*, \kappa^*\left(\overline{X}^*\right)\right),$$

so $\overline{X}^*$strictly dominates $\hat{X}$. Hence, $\hat{X} \in \left(\overline{X}^*, 0\right]$ is not ex-ante optimal if $\left.\frac{\partial \mathcal{V}}{\partial \kappa}\right|_{\kappa = \kappa^*(\hat{X})} \leq 0$. Now suppose that $\left.\frac{\partial \mathcal{V}}{\partial \kappa}\right|_{\kappa = \kappa^*(\hat{X})} > 0$, which implies $\frac{\partial \mathcal{V}}{\partial \kappa} > 0 \; \forall \kappa > \kappa^*\left(\hat{X}\right)$. We could then increase $\mathcal{V}$ with a marginal decrease in $\hat{X}$: it directly increases $\mathcal{V}$ for a given $\kappa$ since $\frac{\partial \mathcal{V}}{\partial \overline{X}} < 0$ for $\hat{X} > \overline{X}^*$, and it would shift $\pi(\cdot)$ upwards, increasing the equilibrium $\kappa^*$. Hence, $\hat{X}$ cannot be ex-ante optimal. We have then shown that no cutoff in $\left(\overline{X}^*, 0\right]$ dominates $\overline{X}^*$. Hence, if commitment increases firm value, it must be that $\overline{X}_c^* < \overline{X}^*$.

Now note that the effect of a marginal change in $\overline{X}$ on the ex-ante firm value is given by:

$$\frac{d\mathcal{V}}{d\overline{X}} = \frac{\partial \mathcal{V}}{\partial \overline{X}} + \frac{\partial \mathcal{V}}{\partial \kappa^*}\frac{\partial \kappa^*}{\partial \overline{X}}.$$

Consider $\tau \leq \tilde{\tau}_2$ (where $\tilde{\tau}_2$ is the value of $\pi(\overline{\kappa})$ for $\overline{X} = \overline{X}^*$, as defined in (A.2)). From Proposition 1, the unique equilibrium features $\kappa^* = \overline{\kappa}$ for any $\overline{X} \leq \overline{X}^*$. Committing to some $\overline{X} < \overline{X}^*$ would reduce firm value since $\partial\mathcal{V}/\partial\overline{X} > 0$ for $\overline{X} < \overline{X}^*$ and $\partial\kappa^*/\partial\overline{X} = 0$ (since $\kappa^*$ is already at its maximum). Hence, $\overline{X}_c^* = \overline{X}^*$ is ex-ante optimal. Now consider $\tau \in (\tilde{\tau}_2, \tilde{\tau}_3)$, where $\tilde{\tau}_3$ is the value of $\pi(0)$ for any $\overline{X} \leq 0$, as given in (A.2) (its computation follows from Lemma A.1). Given Proposition 1, at $\overline{X} = \overline{X}^*$, $\kappa^*\left(\overline{X}^*\right) \in (0, \overline{\kappa})$. Hence, at $\overline{X} = \overline{X}^*$ decreasing $\overline{X}$ increases firm value, since $\frac{\partial \mathcal{V}}{\partial \overline{X}} = 0$, $\frac{\partial \mathcal{V}}{\partial \kappa} > 0$, and $\frac{\partial \kappa^*}{\partial \overline{X}} < 0$ (since $\pi(\cdot)$ is decreasing). We must then have that the optimal cutoff is some $\overline{X}_c^* < \overline{X}^*$. Finally, if $\tau \geq \tilde{\tau}_3$, at $\overline{X} = \overline{X}^*$ speculators have no incentives to trade, $\kappa^*\left(\overline{X}^*\right) = 0$, and $\mathcal{V} = \frac{V_L + V_H}{2}$. There is no reduction in $\overline{X}$ that could lead to an equilibrium with interior $\kappa$, since $\pi(0) = \tilde{\tau}_3 \leq \tau$ for all $\overline{X} \leq 0$ (from Lemma A.1) and $\pi(\cdot)$ is strictly decreasing in $\kappa$ . Therefore, for $\tau \geq \tilde{\tau}_3$, there is no cutoff that dominates $\overline{X}^*$. □

### A.1.7 Proof of Proposition 5

Consider $\overline{X}^* > 0$. Denote by $\mathcal{V}\left(\overline{X}, \kappa\right)$ the ex-ante value of the firm for some cutoff $\overline{X}$ and some $\kappa$, as given by equation (A.9), and denote by $\kappa^*\left(\overline{X}\right)$ the equilibrium level of $\kappa$ when $\overline{X}$ is played. From Proposition 1 (and given Assumption 2), $\kappa^*\left(\overline{X}^*\right) = 0$. Commitment to any $\overline{X}' > 0$ does not affect firm value, since $\kappa^*\left(\overline{X}'\right) = 0$ and hence $\mathcal{V}\left(\overline{X}', \kappa^*\left(\overline{X}'\right)\right) = \mathcal{V}\left(\overline{X}^*, \kappa^*\left(\overline{X}^*\right)\right) = V_0$.

Now, suppose the firm commits to some $\overline{X}' \leq 0$. Notice that under that threshold strategy $\pi(\kappa)$ is decreasing (Lemma 2). Using Lemma A.2, if $\kappa^*\left(\overline{X}'\right) > 0$, a marginal decrease in $\overline{X}'$ has two effects: (i) it directly reduces $\mathcal{V}$ since $\partial\mathcal{V}/\partial\overline{X} > 0$, and (ii) it increases $\kappa^*$, indirectly increasing $\mathcal{V}$ since $\partial V/\partial\kappa > 0$.

Consider $\tau \leq \tilde{\tau}_1$ (where $\tilde{\tau}_1$ is the value of $\pi(\overline{\kappa})$ when $\overline{X} = 0$, as given in (A.1)). Proposition 1 implies that for any $\overline{X} \leq 0$ the unique equilibrium features $\kappa^* = \overline{\kappa}$, so it is never optimal to commit to any $\overline{X} < 0$, since $\partial\mathcal{V}/\partial\overline{X} > 0$ for $\overline{X} < \overline{X}^*$. If the firm commits to the cutoff $\overline{X}' = 0$, ex-ante firm value is

$$\mathcal{V}(0, \kappa^*(0)) = \mathcal{V}(0, \overline{\kappa}) = \frac{1}{2}\left[V_H - \Phi\left(-\sqrt{\frac{\overline{\kappa}}{2}}\right)(V_H - V_0)\right] + \frac{1}{2}\left[V_L + \Phi\left(\sqrt{\frac{\overline{\kappa}}{2}}\right)(V_0 - V_L)\right], \quad \text{(A.17)}$$

which using the fact that $\Phi\left(-\sqrt{\frac{\overline{\kappa}}{2}}\right) = 1 - \Phi\left(\sqrt{\frac{\overline{\kappa}}{2}}\right)$, becomes

$$\mathcal{V}(0, \overline{\kappa}) = \frac{V_L + V_0}{2} + \frac{V_H - V_L}{2}\Phi\left(\sqrt{\frac{\overline{\kappa}}{2}}\right).$$

Under the ex-post efficient cutoff $\overline{X}^* > 0$, $\kappa^*\left(\overline{X}^*\right) = 0$ and $\mathcal{V} = V_0$. Hence, committing to $\overline{X}' = 0$ dominates whenever $\mathcal{V}(0, \overline{\kappa}) > V_0$, which is equivalent to

$$\Phi\left(\sqrt{\frac{\overline{\kappa}}{2}}\right) > \frac{V_0 - V_L}{V_H - V_L} \in (0.5, 1).$$

The inequality above is equivalent to $\overline{\kappa} > \tilde{\kappa} = 2\left[\Phi^{-1}\left(\frac{V_0 - V_L}{V_H - V_L}\right)\right]^2$. For $\overline{\kappa} = \tilde{\kappa}$, the commitment to $\overline{X}' = 0$ leads to the same expected firm value as under $\overline{X}^*$, and for $\overline{\kappa} < \tilde{\kappa}$, the ex-post efficient cutoff $\overline{X}^*$ dominates. Hence, for $\tau \le \tilde{\tau}_1$, if $\overline{\kappa} > \tilde{\kappa}$, the ex-ante optimal cutoff is $\overline{X}_c^* = 0$, and if $\overline{\kappa} \le \tilde{\kappa}$, there is no gain in commitment ($\overline{X}_c^* = \overline{X}^*$ is optimal).

Now consider $\overline{\kappa} \le \tilde{\kappa}$ and $\tau > \tilde{\tau}_1$. Committing to a cutoff $\overline{X}' \le 0$ leads to firm value

$$\mathcal{V}\left(\overline{X}', \kappa^*\left(\overline{X}'\right)\right) \le \mathcal{V}\left(\overline{X}', \overline{\kappa}\right) \le \mathcal{V}(0, \overline{\kappa}) \le V_0 = \mathcal{V}\left(\overline{X}^*, 0\right),$$

where the first inequality holds since $\partial\mathcal{V}/\partial\kappa > 0$, the second holds since $\partial\mathcal{V}/\partial\overline{X} > 0$, and the third inequality is due to $\overline{\kappa} \le \tilde{\kappa}$. Hence there is no gain in commitment: $\overline{X}_c^* = \overline{X}^*$ is ex-ante optimal.

Next, consider $\overline{\kappa} > \tilde{\kappa}$ and $\tau \in (\tilde{\tau}_1, \tilde{\tau}_3)$ (where $\tilde{\tau}_3$ is the value of $\pi(0)$ when $\overline{X} \le 0$, as given in (A.2)). In this case, the ex-ante optimal cutoff is $\overline{X}_c^* = \overline{X}^*$ if $V_0 > \max_{\overline{X} \le 0} \mathcal{V}\left(\overline{X}, \kappa^*\left(\overline{X}\right)\right)$, and $\overline{X}_c^* = \arg\max_{\overline{X} \le 0} \mathcal{V}\left(\overline{X}, \kappa^*\left(\overline{X}\right)\right)$ otherwise.[15] Finally, if $\tau \ge \tilde{\tau}_3$, since $\kappa^*(\overline{X}) = 0$ for any $\overline{X}$, there is no gain in committing to a cutoff different from $\overline{X}^*$. □

### A.1.8 Proof of Proposition 6

Suppose $\overline{X}^* \le 0$. If $D \le V_L$, as follows from the discussion in Section 5.2, investment is undertaken if and only if $X \ge \overline{X}^*$. Also, from Proposition 1, part 2, we have that the unique equilibrium in this case features $\kappa^* = \overline{\kappa}$ for $\tau \le \pi(\overline{\kappa})$, which for $\overline{X} = \overline{X}^*$ and for a given $D \le V_L$, becomes

$$\tau \le (2\lambda - 1) \int_{\overline{X}^*}^{\infty} \frac{\mu(u)(V_H - V_L)}{V_L - D + \mu(u)(V_H - V_L)} \frac{1}{\sqrt{2\overline{\kappa}}} \phi\left(\frac{u + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right) du.$$

Since the RHS of the equation above is increasing in $D$ and assumes value $\hat{\tau}_2$ at $D = V_L$, the interval of values of $D$ satisfying the equation above is non-empty for any $\tau \le \hat{\tau}_2$, and is given by $[\max(0, \underline{D}), V_L]$, where $\underline{D}$ is given by (A.6). The proof of Proposition 6, part 1, is then concluded with the observation that, if $\tau \le \hat{\tau}_2$, for any $D \in (V_L, V_H)$ the same level of information aggregation is achieved ($\kappa^* = \overline{\kappa}$) and the investment rule makes less efficient use of information, leading to

[15] The sets of parameters for which each of those possibilities happen are non-empty. To see that, note that as $\tau \to \tilde{\tau}_1^+$ and $\overline{\kappa} \to \infty$, $\mathcal{V}(0, \kappa^*(0))$ approaches $\lim_{\overline{\kappa} \to \infty} \mathcal{V}(0, \overline{\kappa}) = \frac{V_0 + V_H}{2} > V_0$, so $\overline{X}^*$ is dominated by committing to $\overline{X}' = 0$. Also, for any $\overline{X} \le 0$, $\mathcal{V}\left(\overline{X}, \kappa^*\left(\overline{X}\right)\right) \le \mathcal{V}\left(0, \kappa^*\left(\overline{X}\right)\right)$ and as $\tau \to \tilde{\tau}_3^-$, $\kappa^*\left(\overline{X}\right) \to 0$ and $\mathcal{V}\left(0, \kappa^*\left(\overline{X}\right)\right)$ approaches $\frac{V_L + V_H}{2} < V_0$, so the ex-post optimal cutoff dominates any $\overline{X} \le 0$.

strictly lower firm value. Formally, note that

$$\frac{d\mathcal{V}}{dD} = \frac{\partial \overline{X}}{\partial D}\left[\frac{\partial \mathcal{V}}{\partial \overline{X}} + \frac{\partial \mathcal{V}}{\partial \kappa^*}\frac{\partial \kappa^*}{\partial \overline{X}}\right]. \tag{A.18}$$

For $D \geq V_0$ , $\overline{X} = -\infty$ (the manager always invests), $\frac{\partial \overline{X}}{\partial D} = 0$, and hence $\frac{d\mathcal{V}}{dD} = 0$. For $D \in (V_L, V_0)$, $\overline{X} = X^*(D)$ (as given by (15)) and hence $\frac{\partial \overline{X}}{\partial D} = -\frac{1}{V_0 - D}$. The other partial derivatives are as given in Lemma A.2. For $\tau \leq \hat{\tau}_2$, any $D \in (V_L, V_0)$ leads to $\kappa^* = \overline{\kappa}$, and hence $\frac{\partial \kappa^*}{\partial \overline{X}} = 0$. Also, since for $D \in (V_L, V_0)$ we have that $\frac{\partial \overline{X}}{\partial D} < 0$ and $\frac{\partial \mathcal{V}}{\partial \overline{X}} > 0$ (given that $X^*(D) < \overline{X}^*$), $\frac{d\mathcal{V}}{dD} < 0$ and $D > V_L$ cannot be optimal.

We now prove the second part of Proposition 6. From Lemma A.1:

$$\lim_{\kappa \to 0} \pi(\kappa) = \begin{cases} (2\lambda - 1)\frac{V_H - V_L}{V_L + V_H - 2D} & D < V_L, \\ 2\lambda - 1 & D \in [V_L, V_H), \end{cases} \tag{A.19}$$

so the maximum value of $\lim_{\kappa \to 0} \pi(\kappa)$ is achieved when $D \in [V_L, V_H)$. Given that $\pi(\kappa)$ is strictly decreasing, if $\tau \geq 2\lambda - 1$, trading profits are always smaller than trading costs for any positive $\kappa$ and for any $D$, and the unique equilibrium has $\kappa^* = 0$. This is the uninteresting case ruled out by Assumption 1. Consider then $\tau \in (\hat{\tau}_2, 2\lambda - 1)$. Inspection of (14) shows that, for $D < V_L$ , $\pi(\kappa)$ is increasing in $D$ for any $\kappa$. Hence, $D < V_L$ is never optimal, since increasing $D$ to $V_L$ would increase $\kappa^*$ without distorting the investment rule (as $\overline{X} = \overline{X}^*$ for all $D \leq V_L$). For $D \in [V_L, V_0)$, if the equilibrium value of informativeness $\kappa^*$ is smaller than $\overline{\kappa}$, it satisfies:

$$\pi(\kappa^*) = (2\lambda - 1)\left[1 - \Phi\left(\frac{\overline{X} + \kappa^*}{\sqrt{2\kappa^*}}\right)\right] = \tau,$$

and by the implicit function theorem, $\frac{\partial \kappa^*}{\partial \overline{X}} = -\frac{2\kappa^*}{(\kappa^* - \overline{X})}$. Also, in equilibrium $\overline{X} = X^*(D)$, and hence $\frac{\partial \overline{X}}{\partial D} = -\frac{1}{V_0 - D}$. $\frac{\partial \mathcal{V}}{\partial \kappa}$ and $\frac{\partial \mathcal{V}}{\partial \overline{X}}$ are given by (A.10) and (A.11). After some algebra, one can verify that (A.18) becomes:

$$\frac{d\mathcal{V}}{dD} = \frac{\kappa^*}{\kappa^* - X^*(D)}\frac{1}{\sqrt{2\kappa^*}}\phi\left(\frac{X^*(D) + \kappa^*}{\sqrt{2\kappa^*}}\right), \tag{A.20}$$

for any $D \in \left[V_L, \overline{D}\right)$, where $\overline{D}$ is the level of $D$ that leads to $\kappa^* = \overline{\kappa}$. That is, $\overline{D}$ satisfies

$$\tau = (2\lambda - 1)\left[1 - \Phi\left(\frac{X^*\left(\overline{D}\right) + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right)\right],$$

which solving for $\overline{D}$ yields the expression in (A.4). (A.20) is larger than zero since $X^*(D) \leq 0$ for $D \in \left[V_L, \overline{D}\right)$. Since $\frac{d\mathcal{V}}{dD} > 0$ for all $D < \overline{D}$ and increasing $D$ beyond $\overline{D}$ decreases firm value ($\frac{\partial \mathcal{V}}{\partial \overline{X}}\frac{\partial \overline{X}}{\partial D} < 0$ and $\frac{\partial \kappa^*}{\partial \overline{X}} = 0$), the firm chooses $D^* = \overline{D}$ for all $\tau \in (\hat{\tau}_2, 2\lambda - 1)$. □

### A.1.9 Proof of Proposition 7

Consider $\overline{X}^* > 0$. For any $D < V_L$, in equilibrium $\overline{X} = \overline{X}^* > 0$. For $D \in [V_L, V_0)$, $\overline{X} = X^*(D)$, and for $D \geq V_0$ investment is always undertaken. Since $X^*(D)$ is decreasing, for any $D < \hat{D} \equiv 2V_0 - V_H$, $\overline{X} > 0$, and from Proposition 1, $\kappa^* = 0$. Hence, $\mathcal{V} = V_0$ for $D < \hat{D}$. For $D \geq \hat{D}$, $\overline{X} = X^*(D) \leq 0$, and from Proposition 1 the equilibrium features $\kappa^* > 0$ – since for $D \geq \hat{D} > V_L$, $\overline{\tau} = \pi(0) = 2\lambda - 1 > \tau$ (see Assumption 1 and (A.19)).

Suppose $\tau \leq \hat{\tau}_1$ (defined in (A.3)). In this case, setting $D \geq \hat{D}$ leads to $\kappa^* = \overline{\kappa}$, since $\pi(\kappa)$ is decreasing for $X^*(D) \leq 0$, and

$$\pi(\overline{\kappa}) = (2\lambda - 1)\left[1 - \Phi\left(\frac{X^*(D) + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right)\right] \geq (2\lambda - 1)\left[1 - \Phi\left(\frac{\overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right)\right] = \hat{\tau}_1 \geq \tau.$$

Since for any positive $\kappa$ we have $\partial V/\partial\overline{X} > 0$ for $\overline{X} < \overline{X}^*$, $D > \hat{D}$ cannot be optimal (it distorts the ex-post use of information without further increasing $\kappa$). Since $X^*(\hat{D}) = 0$, setting $D = \hat{D}$ yields:

$$\mathcal{V} = \frac{V_L + V_0}{2} + \frac{V_H - V_L}{2}\Phi\left(\sqrt{\frac{\overline{\kappa}}{2}}\right), \tag{A.21}$$

which, one can verify, is larger than $V_0$ if and only if $\overline{\kappa} \geq \tilde{\kappa} = 2\left[\Phi^{-1}\left(\frac{V_0 - V_L}{V_H - V_L}\right)\right]^2$. Therefore, for $\tau \leq \hat{\tau}_1$ and $\overline{\kappa} \geq \tilde{\kappa}$, it is optimal to set $D = \hat{D}$. If $\tau \leq \hat{\tau}_1$ and instead $\overline{\kappa} < \tilde{\kappa}$, any $D \in \left[0, \hat{D}\right)$ is optimal as it yields $V_0$.

Now, notice that if $\overline{\kappa} < \tilde{\kappa}$ and $\tau > \hat{\tau}_1$, the optimal capital structure is also $D \in \left[0, \hat{D}\right)$: any $D \geq \hat{D}$ would lead to an equilibrium with $\kappa^* > 0$, but the ex ante firm value would be strictly smaller than the one in (A.21), which is itself smaller than $V_0$. Hence, the optimal $D$ is $D^* \in \left[0, \hat{D}\right)$ for $\overline{\kappa} < \tilde{\kappa}$.

In the remainder of the proof, consider $\overline{\kappa} \geq \tilde{\kappa}$ and $\tau > \hat{\tau}_1$. In this case, optimal debt is either some $D^* \in \left[\hat{D}, V_0\right)$ that leads to $\kappa^* > 0$, or any $D \in \left[0, \hat{D}\right)$, leading to $\kappa^* = 0$ and $\mathcal{V} = V_0$. Recall that $\overline{D}$, as defined in (A.6), is the minimum $D$ that leads to $\kappa^* = \overline{\kappa}$. For $D \in \left[\hat{D}, \overline{D}\right)$, $\overline{X} = X^*(D) \leq 0$ and $\partial\mathcal{V}/\partial D$ (which is as given in (A.20)) is strictly positive. For $D > \overline{D}$, using (A.18), $\partial\mathcal{V}/\partial D < 0$ (since, as implied by Lemma A.2, $\frac{\partial\mathcal{V}}{\partial\overline{X}}\frac{\partial\overline{X}}{\partial D} < 0$ and $\frac{\partial\kappa^*}{\partial\overline{X}} = 0$, as $\kappa^* = \overline{\kappa}$). The candidates for optimal debt are then $D < \hat{D}$, in which case $\mathcal{V} = V_0$, and $D = \overline{D}$, in which case

$$\mathcal{V} = \frac{1}{2}\left[V_H - \Phi\left(\frac{X^*\left(\overline{D}\right) - \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right)(V_H - V_0)\right] + \frac{1}{2}\left[V_L + \Phi\left(\frac{X^*\left(\overline{D}\right) + \overline{\kappa}}{\sqrt{2\overline{\kappa}}}\right)(V_0 - V_L)\right]. \tag{A.22}$$

As $\tau \to \hat{\tau}_1^+$, $\overline{D} \to \hat{D}$, $\overline{X}^*\left(\overline{D}\right) \to 0$ and $\mathcal{V}$ approaches (A.21), which is larger than $V_0$ since $\overline{\kappa} \geq \tilde{\kappa}$, so $D^* = \overline{D}$. Now consider the limit where $\tau \to (2\lambda - 1)^-$. In this case, $\overline{D} \to V_0$ and $X^*(\overline{D}) \to -\infty$, and $\mathcal{V}$ approaches $\frac{V_H + V_L}{2} < V_0$, so it is optimal to set any $D \in \left[0, \hat{D}\right)$. Since $\overline{D}$ is increasing in $\tau$,

$X^*(\cdot)$ is decreasing, and $\partial V/\partial\overline{X}$ is increasing for $\overline{X} < \overline{X}^*$, firm value in (A.22) is decreasing in $\tau$. Hence, there exists $\hat{\tau}_3 \in (\hat{\tau}_1, 2\lambda - 1)$ such that the optimal $D$ is $\overline{D}$ for $\tau \leq \hat{\tau}_3$, and any $D < \hat{D}$ is optimal otherwise. Writing $\overline{D}$ as a function of $\tau$ (see (A.4)), $\hat{\tau}_3$ is the value of $\tau$ that equates (A.22) to $V_0$, as given by (A.5). □

#### A.1.10 Proof of Proposition 8

Given an investment cutoff $\overline{X}$, using (A.13) we have that

$$\Pr(a = 1|\theta = \theta_L) = 1 - \Phi\left(\frac{\overline{X} + \kappa^*}{\sqrt{2\kappa^*}}\right) \quad \text{and} \quad \Pr(a = 1|\theta = \theta_H) = 1 - \Phi\left(\frac{\overline{X} - \kappa^*}{\sqrt{2\kappa^*}}\right).$$

Consider some $D \in \left(V_L, \overline{D}\right)$ that leads to an equilibrium with interior $\kappa^*$. In equilibrium, $\overline{X} = X^*(D) \leq 0$ (or else $\kappa^*$ would be zero) and $\kappa^*$ satisfies

$$(2\lambda - 1)\left[1 - \Phi\left(\frac{\overline{X} + \kappa^*}{\sqrt{2\kappa^*}}\right)\right] = \tau,$$

which automatically implies that $\Pr(a = 1|\theta = \theta_L) = 1 - \Phi\left(\frac{\overline{X}+\kappa^*}{\sqrt{2\kappa^*}}\right) = \frac{\tau}{2\lambda-1}$ in equilibrium. Moreover,

$$\frac{d\Pr(a = 1|\theta = \theta_H)}{dD} = \frac{\partial\overline{X}}{\partial D}\left[\frac{\partial\Pr(a = 1|\theta = \theta_H)}{\partial\overline{X}} + \frac{\partial\Pr(a = 1|\theta = \theta_H)}{\partial\kappa^*}\frac{\partial\kappa^*}{\partial\overline{X}}\right].$$

Computing all the partial derivatives above and rearranging yield:

$$\frac{d\Pr(a = 1|\theta = \theta_H)}{dD} = \frac{1}{V_0 - D}\frac{1}{\sqrt{2\kappa^*}}\phi\left(\frac{\overline{X} - \kappa^*}{\sqrt{2\kappa^*}}\right)\left[1 + \frac{\kappa^* + \overline{X}}{\kappa^* - \overline{X}}\right],$$

which is positive since $\overline{X} = X^*(D) \leq 0$. □

#### A.1.11 Proof of Proposition 9

First notice that the proofs of Propositions 3, 4 and 5 were written for any arbitrary $R_H \geq R_L \geq 0$. Hence, for any given level of debt, there is a cutoff investment strategy $\overline{X}_c \leq \overline{X}^*$ that is ex-ante optimal for the firm. Therefore, we hereafter restrict attention to the problem of choosing $D$ and a cutoff $\overline{X}_c \leq \overline{X}^*$ at $t = 0$. Define

$$X^p(D) = \begin{cases} \overline{X}^* & \text{if } D \leq V_L, \\ X^*(D) & \text{if } D \in (V_L, V_0), \\ -\infty & \text{if } D \geq V_0, \end{cases}$$

which is the investment strategy that maximizes shareholder value ex post for a given level of debt (see Section 5.2).

Suppose that $\left(\overline{X}_c^*, D^*\right)$ is a solution to the problem considered. First, assume $\overline{X}_c^* > 0$. Then, it must be that $\overline{X}^* > 0$. In equilibrium $\kappa^* = 0$, and hence $\left(\overline{X}^*, 0\right)$ is also optimal and it is consistent with ex-post shareholder value maximization. Now, assume $\overline{X}_c^* \leq 0$. In this case, firm value is increasing in $\kappa$ (Lemma A.2). Recall that, from (14), for any $\kappa$, $\pi(\kappa)$ achieves its maximum for any $D \in [V_L, V_H)$. Hence, setting $D = V_L$ and keeping the cutoff $\overline{X}_c^*$ cannot decrease $\kappa$ and therefore cannot decrease firm value, so $\left(\overline{X}_c^*, V_L\right)$ is also optimal. Now notice that there exists $D' \in [V_L, V_0)$ that leads to $X^p(D') = \overline{X}_c^*$ (recall that $\overline{X}_c^* \leq \overline{X}^*$). Therefore, setting $(X^p(D'), D')$ is optimal since it leads to the same cutoff and the same $\kappa^*$ as $\left(\overline{X}_c^*, V_L\right)$ (as for a given cutoff, $\pi(\kappa)$ is independent of $D$ for $D \in [V_L, V_0)$). We have then shown that the optimum can always be achieved with ex-post shareholder value maximization. □

### A.1.12 Proof of Proposition 10

**Case 1.** We start looking at the case in which $\overline{X}^* \leq 0$. Suppose first that $\tau \leq \hat{\tau}_2$ and hence the firm initially chooses $D^* \in [\max(0, \underline{D}), V_L]$ (see Proposition 6). Note that $\underline{D}$ is strictly increasing in $\tau$ (see (A.6)). Therefore, following an increase in $\tau$ to some $\tau' \leq \hat{\tau}_2$, the optimal values of $D$ are in an interval $[\max(0, \underline{D}'), V_L]$, where $\underline{D}' > \underline{D}$. Moreover, following an increase in $\tau$ to some $\tau'' > \hat{\tau}_2$, the optimal level of debt is given by $\overline{D}(\tau'') > V_L$. Now suppose that initially $\tau > \hat{\tau}_2$. Any increase in $\tau$ increases the optimal level of debt since $\overline{D}$ is strictly increasing in $\tau$ (see (A.4)). We have then shown that, for $\overline{X}^* \leq 0$, the largest and smallest optimal debt are always increasing in $\tau$.

Now consider the effect of an increase in $\alpha$, which affects the optimal debt only through $\overline{\kappa}$. Suppose that initially $\tau \leq \hat{\tau}_2$. Notice that $\hat{\tau}_2$ in (A.3) is decreasing in $\overline{\kappa}$. Also, it follows from Lemma 2 that the RHS of (A.6) is decreasing in $\overline{\kappa}$, and hence $\underline{D}$ is increasing in $\overline{\kappa}$. Therefore, an increase in $\alpha$ (and hence in $\overline{\kappa}$) that reduces $\hat{\tau}_2$ to some $\hat{\tau}_2' \geq \tau$ changes the set of optimal levels of debt from $[\max(0, \underline{D}), V_L]$ to some $[\max(0, \underline{D}'), V_L]$ with $\underline{D}' > \underline{D}$. An increase in $\alpha$ that, instead, reduces $\hat{\tau}_2$ to some $\hat{\tau}_2'' < \tau$ leads to an optimal debt $\overline{D} > V_L$. Finally, if initially $\tau > \hat{\tau}_2$, any increase in $\alpha$ increases the optimal level of debt since $\overline{D}$ is increasing in $\overline{\kappa}$ for every $\tau > \hat{\tau}_2$, as can be verified using (A.4) and the definition of $\hat{\tau}_2$.

**Case 2.** We now consider the case where $\overline{X}^* > 0$. The comparative statics with respect to $\tau$ hold for any values of $\tau$ in the range $(0, \hat{\tau}_3]$. The result follows from Proposition 7, the fact that $\overline{D}(\tau)$ is strictly increasing in $\tau$, and that $\hat{D}$ is independent of $\tau$.

Now consider the effect of an increase in $\alpha$ (which increases $\overline{\kappa}$, as before). From Proposition 7, either the firm chooses some $D < \hat{D}$, leading to an equilibrium with $\kappa^* = 0$ (and firm value $V_0$), or it chooses the minimum level of debt that triggers $\kappa^* = \overline{\kappa}$, which is either $\hat{D}$ (if $\tau \leq \hat{\tau}_1$) or $\overline{D} > \hat{D}$ (if $\tau > \hat{\tau}_1$). Notice that the minimum level of debt needed to trigger an equilibrium with $\kappa^* = \overline{\kappa}$ is (weakly) increasing in $\overline{\kappa}$, given that the trading profit function $\pi(\kappa)$ is decreasing for any $D \geq \hat{D}$. (This follows from Lemma 2 and the fact that $X^*(D) \leq 0$ for $D \geq \hat{D}$). Hence, to prove that the optimal value of $D$ never falls following an increasing in $\overline{\kappa}$ we just need to show that, if the firm is initially choosing $\hat{D}$ or $\overline{D}$, it is never optimal to choose $D < \hat{D}$ following an increase in $\overline{\kappa}$.

Denote by $\nu(D, \kappa^*)$ the ex-ante value of the firm when it chooses debt $D$ and equilibrium informativeness is $\kappa^*$. Note that, if $D = \hat{D}$, $\nu(D, \kappa^*)$ is given by (A.21) with $\overline{\kappa}$ replaced by $\kappa^*$. First consider that, for an initial set of parameters, the optimal debt is $\hat{D}$. Note that $\nu\left(\hat{D}, \kappa^*\right)$ is increasing in $\kappa^*$. Therefore, if $\overline{\kappa}$ increases to some $\overline{\kappa}'$ and the minimum $D$ that leads to $\kappa^* = \overline{\kappa}'$ is still $\hat{D}$, then $\hat{D}$ must still be the optimum since $\nu\left(\hat{D}, \overline{\kappa}'\right) > \nu\left(\hat{D}, \overline{\kappa}\right) > V_0$. Now suppose the firm initially chooses $\hat{D}$ and, following an increase from $\overline{\kappa}$ to some $\overline{\kappa}''$, the minimum $D$ leading to $\kappa^* = \overline{\kappa}''$ is $\overline{D}'' > \hat{D}$. If after the increase in $\overline{\kappa}$ the firm would still choose $D = \hat{D}$, equilibrium informativeness would be some $\check{\kappa} \in (\overline{\kappa}, \overline{\kappa}'')$, since $\pi(\kappa)$ is decreasing. Hence, it cannot be optimal to choose $D < \hat{D}$ since $\nu\left(\hat{D}, \check{\kappa}\right) > \nu\left(\hat{D}, \overline{\kappa}\right) > V_0$. In fact, from Proposition 7, the optimal debt would be $\overline{D}''$ in that case.

Finally, suppose that for an initial set of parameters the optimal debt is $\overline{D}$, leading to $\kappa^* = \overline{\kappa}$. If $\overline{\kappa}$ goes up to some $\overline{\kappa}'$, the level of debt that leads to maximum informativeness is higher (denote it by $\overline{D}'$). Notice that if the firm kept its level of debt unchanged, firm value would still be $\nu\left(\overline{D}, \overline{\kappa}\right)$. Hence, the firm cannot be willing to choose any $D < \hat{D}$, since $\nu\left(\overline{D}, \overline{\kappa}\right) > V_0$. In fact, from Proposition 7, we know that in this case the firm would increase debt to $\overline{D}'$. □

## A.2 Change of variables in the commitment problem

Suppose the firm can commit to an investment strategy such that it invests if and only if $\tilde{X} \in \tilde{A}$, where $\tilde{A}$ is a measurable subset of the real line. Given its choice of $\tilde{A}$, the firm anticipates aggregates orders conditional on the state, $S_H$ and $S_L$, that are consistent with the equilibrium with the lowest $\kappa$. If $S_H \neq S_L$, then there is a one-to-one continuous map from $X$ to $\tilde{X}$, and hence there exists a measurable subset $A$ such that $X \in A \Leftrightarrow \tilde{X} \in \tilde{A}$. If $S_H = S_L$, then $\kappa = 0$ and $X = 0$ for every $\tilde{X}$. Then, the firm obtains no information from the market, so it must be that either (i) not investing regardless of market activity ($\tilde{A} = \emptyset$) is optimal or (ii) always investing ($\tilde{A} = \mathbb{R}$) is optimal. Therefore, the firm can attain the same payoff as the optimal $\tilde{A}$ by investing iff $X \in A$, where $A$ is such that it does not includes 0 (if (i) is optimal) or it includes 0 (if (ii) is optimal).

## A.3 Ex-ante investment and prices

In this section, for convenience, we write the price setting rule $P$ and beliefs $\mu$ as the original functions of $\tilde{X}$ (see Section 2). Under the most general formulation, the problem of the firm is to choose the strategies of all agents (manager, informed traders and market maker) subject to those being consistent with equilibrium in the trading stage. More precisely, the firm chooses a measurable subset $Z \subseteq \mathbb{R}^2$, trading strategies for informed speculators, and a price-setting rule $P(\tilde{X})$ such that: (i) the manager invests iff $\left(\tilde{X}, P(\tilde{X})\right) \in Z$; (ii) speculators' strategies are consistent with the equilibrium with the lowest $\kappa$ in the trading stage for a given $Z$ and $P(\cdot)$; (iii) the market maker breaks even in expectation for every $\tilde{X}$. We call this the general ex-ante investment problem. In Section 4 we claimed that the solution presented there would also be a solution to this more general problem, which we now prove.

**Lemma A.3.** *There is an ex-ante investment strategy that only conditions investment on $X$ that is a solution to the general ex-ante investment problem.*

*Proof.* For the market maker to break even, the price-setting rule must satisfy

$$P(\tilde{X}) = \begin{cases} R_L + \mu\left(\tilde{X}\right)\Delta_R & \text{if } \left(\tilde{X}, P\left(\tilde{X}\right)\right) \in Z, \\ R_0 & \text{otherwise.} \end{cases} \tag{A.23}$$

Suppose that at the optimum $Z = Z^*$. Since, given $Z^*$, $P(\tilde{X})$ is uniquely pinned down for each $\tilde{X}$, the manager invests if and only if $\tilde{X} \in \tilde{A}$, where $\tilde{A} = \{y : (y, P(y)) \in Z^*\}$. Therefore, the firm could achieve the same outcome by choosing a set $\tilde{A}$ such that the manager invests iff $\tilde{X} \in \tilde{A}$ and the same speculator's strategies, with the market maker setting prices according to $P(\tilde{X}) = R_L + \mu\left(\tilde{X}\right)\Delta_R$ if $\tilde{X} \in \tilde{A}$ and $P(\tilde{X}) = R_0$ otherwise. □

## A.4 Extension: publicly traded debt

In this section we adapt our main model to allow for two different security markets. Suppose the firm raises debt by issuing corporate bonds that are later traded in secondary financial markets. Speculators then have two distinct venues to trade on their information: they may speculate on the expected return to shareholders, but also on the default risk of the firm.

We adjust our setting in the following way. For a given level of debt $D$, let $R_B(\theta, a) \equiv \min\{D, v(\theta, a)\}$ denote the return to a bond holder in the final period, as a function of the state and the investment decision. Each speculator can trade $s_i$ dollars in stocks and $b_i$ dollars in bonds, subject to the resources constraint that $|s_i| + |b_i| \leq 1$. Moreover, noise traders place a random order $\tilde{n}_B$ (in dollars) for bonds, which is normally distributed with zero mean and variance $\sigma_B^2$ (independent from noise traders' orders in the stock market). For each dollar traded in bonds, speculators pay a trading cost $\tau_B$. For simplicity, let $\sigma_B = \sigma$ and $\tau_B = \tau$ – i.e., there is the same amount of noise and the same trading costs in both markets. After receiving signal $m_i$, speculator $i$ solves the following problem:

$$\max_{s_i, b_i} s_i \mathbb{E}\left[\frac{R(\theta, a)}{P} - 1 \middle| m_i\right] + b_i \mathbb{E}\left[\frac{R_B(\theta, a)}{P_B} - 1 \middle| m_i\right] - \tau\left(|s_i| + |b_i|\right)$$

$$\text{s.t.} \quad |s_i| + |b_i| \leq 1.$$

The aggregate order in the bond market is $\tilde{X}_B = \tilde{n}_B + \int_0^\alpha b_i di$. For a given strategy profile for informed speculators $\{s_i(m_H), s_i(m_L), b_i(m_H), b_i(m_L)\}_{i\in[0,1]}$, their aggregate order for stocks is as previously defined, and their aggregate order for bonds is $B_H = \int_0^\alpha [\lambda b_i(m_H) + (1-\lambda) b_i(m_L)] di$ in the high state and $B_L = \alpha [\lambda b_i(m_L) + (1-\lambda) b_i(m_H)]$ in the low state. We can then redefine variables $X$ and $\kappa$ as:

$$X \equiv \frac{1}{\sigma^2}\left[-\frac{1}{2}\left(S_H^2 - S_L^2\right) - \frac{1}{2}\left(B_H^2 - B_L^2\right) + (S_H - S_L)\tilde{X} + (B_H - B_L)\tilde{X}_B\right],$$

$$\kappa = \frac{1}{2\sigma^2}(S_H - S_L)^2 + \frac{1}{2\sigma^2}(B_H - B_L)^2. \tag{A.24}$$

Importantly, $X$ is a sufficient statistic for activity in both markets, and as before, speculators' trading profits only depend on other speculators' strategies through $\kappa$. Beliefs after observing activity in both markets ($\tilde{X}$ and $\tilde{X}_B$) are as given by (2). Lemma 1 continues to hold, and trading profits per dollar traded in the stock market are $\pi(\kappa)$, as given by (9).[16]

Bonds are priced in an analogous way to stocks in financial markets. After observing $\tilde{X}$ and $\tilde{X}_B$, or analogously, $X$, the market maker sets $P_B(X) = \mathbb{E}[R_B(\theta, a)|X]$. Trading profits in the bond market are then computed in an equivalent manner as profits in the stock market, only replacing $R_L$ by $R_L^B \equiv R_B(\theta_L, 1)$ and $\Delta_R$ by $\Delta_R^B \equiv R_H^B - R_L^B$, with $R_H^B \equiv R_B(\theta_H, 1)$, in equation (9). For a given $\kappa$ and set $A$ such that investment is undertaken for $X \in A$, we can write the trading profits per dollar in the stock and bond markets, respectively, as $\pi(\kappa) = \int_A G^S(u)\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right)du$ and $\pi^B(\kappa) = \int_A G^B(u)\frac{1}{\sqrt{2\kappa}}\phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right)du$, where

$$G^S(u) = \frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \quad \text{and} \quad G^B(u) = \frac{\mu(u)\Delta_R^B}{R_L^B + \mu(u)\Delta_R^B}.$$

Notice that if $D \leq V_L$, $R_L^B = R_H^B = D$, and $\Delta_R^B = 0$. If $D \in (V_L, V_H)$, $R_L^B = V_L$, $R_H^B = D$, so $\Delta_R^B = D - V_L$. Finally, if $D \geq V_H$, bond holders become the residual claimants of the company (as if bonds become stocks). We then have:

$$G^S(u) = \begin{cases} \frac{\mu(u)(V_H - V_L)}{V_L - D + \mu(u)(V_H - V_L)} & \text{for } D \leq V_L, \\ 1 & \text{for } D \in (V_L, V_H), \\ 0 & \text{for } D \geq V_H, \end{cases} \quad \text{and} \quad G^B(u) = \begin{cases} 0 & \text{for } D \leq V_L, \\ \frac{\mu(u)(D - V_L)}{V_L + \mu(u)(D - V_L)} & \text{for } D \in (V_L, V_H), \\ \frac{\mu(u)(V_H - V_L)}{V_L + \mu(u)(V_H - V_L)} & \text{for } D \geq V_H. \end{cases}$$

It follows from the expressions above that, for any set $A$ and for any $\kappa$, trading profits per dollar in the stock market are always strictly larger than trading profits for bonds – except when debt is so

[16] The proof is omitted since it is identical to the proof of Lemma 1 if one uses the new definitions of $X$, $\kappa$ and redefines $n$ as $n \equiv \frac{1}{\sigma^2}(S_H - S_L)\tilde{n} + \frac{1}{\sigma_B^2}(B_H - B_L)\tilde{n}_B$.

high that bonds become stock (such values of debt would never be optimal for the firm).[17] Therefore, the secondary market for bonds in this setting is endogenously illiquid: speculators always choose $b_i(m_H) = b_i(m_L) = 0$. The results presented in Section 5 remain unchanged when we allow bonds to be (potentially) publicly traded.

## A.5 Extension: Short-selling restrictions

Suppose that short-selling is not allowed and informed speculators have no initial position on the stock. In this case, speculators cannot submit negative orders, i.e., $s_i \in [0, 1]$. The model with short-selling restrictions can be solved in an analogous manner to the main model. By equation (8), it is easy to see that negatively informed traders will not trade in equilibrium ($s_i(m_L) = 0$ for every $i$). For positively informed speculators, trading profits per dollar bought are given by (9). The maximum possible level of informativeness (reached when $s_i(m_H) = 1$ for every $i$) is then $\overline{\kappa}^{ss} \equiv \frac{\alpha^2(2\lambda-1)^2}{2\sigma^2} = \frac{\overline{\kappa}}{4}$. All results previously derived apply, only replacing $\overline{\kappa}$ by $\overline{\kappa}^{ss}$. Hence, imposing short-selling restrictions has an effect equivalent to reducing $\overline{\kappa}$.

[17]Recall that setting any $D \geq V_0$ leads the manager to play a cutoff $\overline{X} = -\infty$. Hence, when bonds are publicly traded, if $D \geq V_H$ the manager always invests (she does not react to market activity). If $\overline{X}^* > 0$, this is clearly not optimal, as setting $D = 0$ leads to $\overline{X} = \overline{X}^*$, $\kappa = 0$, and achieves a strictly larger firm value. If $\overline{X}^* \leq 0$, firm value when $D \geq V_H$ is the same as in a situation with $\kappa = 0$ and $\overline{X} = \overline{X}^*$ (in which the firm always invests), but since $\frac{\partial \mathcal{V}}{\partial \kappa}\big|_{\overline{X}=\overline{X}^*} > 0$ (see (A.7)), setting $D = V_L$ is strictly better, as it leads to $\overline{X} = \overline{X}^*$ and $\kappa > 0$.