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Bernardita Vial y Pilar Alcalde.

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Intermediary Commissions in a Regulated Market with Heterogeneous Customers

Pilar Alcalde* and Bernardita Vial†

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Abstract

Several studies predict that in markets with intermediation, changes in regulation intended to benefit consumers may have negative consequences for welfare when firms set commissions. We argue that, even if commissions are exogenous, transparent, and paid by customers, policies to diminish intermediaries' bias may have nontrivial effects on equilibrium outcomes. We provide evidence from a highly regulated retirement market by examining two subsequent policy changes that reduced the commission differential between products. Some of the patterns of the demand side are consistent with biased advice, but others are less intuitive. Our model helps understand these patterns and also predicts the firms' equilibrium reaction to commission levels. We show that, in many cases, prices move in the opposite direction to the intermediaries' bias: when intermediaries are cheaper and less biased, more customers follow their advice, making demand less elastic. This change induces firms to increase prices, producing nontrivial effects on welfare.

1 Introduction

In many industries, the complexity of products forces customers to rely on an intermediary's advice to make informed decisions. Many people seek financial advice before making investment decisions, choosing an insurance policy or planning their retirement, just as they seek medical guidance facing a health issue or legal counsel in the case of a legal problem. While expert advice may be convenient in most cases, the possibility that conflicts of interest may cause biased recommendations is a concern when the adviser's payoff depends on the alternative chosen by the customer.

In recent years, many countries have been discussing or implementing policies to minimize the adverse effects of biased advice or make potential conflicts of interest more transparent to consumers. Some of these policies include tightening regulation on commission-selling of financial products, mandating the disclosure of advisers' commissions, expanding fiduciary duty, or limiting contact between product providers and intermediaries. However, several studies predict that these changes in regulation will not always improve consumer welfare when firms set intermediaries' commissions (see Inderst and Ottaviani (2012a) and Robles-Garcia (2019)). In these studies, differences in commissions may be

*Universidad de Los Andes, Chile. palcalde@uandes.cl
†Pontificia Universidad Católica de Chile. bvial@uc.cl

useful to promote efficiency through the reaction of demand to supply-side incentives and differences in costs.

This cautionary tale can be extended to other settings as well. Even if commissions are fully transparent, exogenously set–so they are not related to cost differences– and paid directly by customers instead of firms, policies to diminish intermediaries' bias may have nontrivial effects on equilibrium prices and quantities. Our results indicate that it is not only consumers' lack of awareness that can have detrimental effects on welfare but also that trust in the intermediaries' expertise decreases the magnitude of the elasticity of demand, inducing firms to increase prices.

As motivating evidence, we look at a reform in a highly regulated retirement market. Consumers may choose between two types of intermediary –an independent adviser or a sales agent from an insurance company– and pay a commission to them after choosing one of two kinds of products– annuities and programmed withdrawals from their pension savings. These commissions are exogenous and set by the regulator. Initially, consumers only paid a commission if they purchased an annuity. Then, the regulator implemented two policy changes: first, a decrease in the commission for annuities for both types of intermediary and, second, the introduction of a new commission for programmed withdrawals, but only for advisers. The patterns we observe on the demand side are as expected if intermediaries react to the commission differential by changing their recommendations, and consumers follow advice even if it biased. First, the market share of annuities decreases among consumers who use independent advisers after both reforms, as the bias in favor of annuities declines. Moreover, consumers using sales agents select annuities less frequently only after the first reform, when their commissions were affected. Second, as intermediation using advisers gets more expensive after the second reform, the fraction of consumers who hire them decreases. However, other patterns in the data are less intuitive. The cost of intermediation decreases after the first reform, but the fraction of customers without intermediary increases. Additionally, sales agents become relatively less expensive after the second reform, but the fraction of customers using sales agents does not increase. Finally, the data suggest that after the second reform, the demand for annuities among customers hiring advisers becomes more elastic.

Our model helps rationalize these empirical patterns. We consider a continuum of consumers who must choose between two products, where the value of each one is partially observed and heterogeneous. Each customer has an *ex ante* preference for products, which is private information, but the suitability of each product depends on her unobserved type. Intermediaries care about their commissions, but also about the product's suitability for the customer due, for example, to genuine interest, reputation concerns, or fiduciary duties. The intermediary receives private information about the customer's type and sends a discrete message in the form of advice; hence, the customer may learn about her type from the recommendation, even if it is known to be biased. We show that consumers with intermediate *ex ante* valuations follow intermediaries' recommendations, while consumers with extreme valuations select their preferred products regardless of advice. The cutoffs for this decision depend on the price and commission levels. The effect of decreasing the commission for one product depends on how this affects each cutoff, changing the advice but also the likelihood of following it.

After a change in commissions, two effects arise: the cost of intermediation changes and intermediaries shift their recommendations. When the commission for programmed withdrawals increases, customers who find advisers too expensive may switch to sales agents if they prefer advice or may buy directly if they prefer programmed withdrawals regardless of recommendations. This cost-effect explains why the fraction of consumers without intermediation increases, and why their demand for annuities decreases, even though they never pay a commission. In addition to this cost-effect, the shift in recommendations affects their value in two manners. On the one hand, some customers with an *ex ante* preference for programmed withdrawals are willing to pay higher commissions for less biased advice. These consumers switch from sales agents to advisers, even though they are more expensive, explaining why the fraction of customers using sales agents remains almost constant after the second reform. On the other hand, some customers with an *ex ante* preference for annuities have a lower expected utility of following advice. This effect explains why the fraction of customers participating without intermediaries increases after the first reform, even though intermediaries are cheaper.

If independent advisers and sales agents were perfect substitutes, only consumers who follow recommendations would hire advisers after the second reform. But some customers may value intermediation beyond recommendations: for instance, advisers guide on related topics such as pension subsidies or taxes. Then, advisers and sales agents are not perfect substitutes for everyone, so several customers still prefer to hire advisers after the reform even if they do not follow their advice on product choice. This effect explains why the elasticity of the demand for annuities among customers using advisers increases after the second reform.

The model predicts that as commissions change, the elasticity of demand varies in a nontrivial way because consumers who trust intermediaries and follow advice are less sensitive to prices. We continue our theoretical analysis by including firms' equilibrium reaction to commission levels. We show that, in many cases, prices move in the opposite direction to the intermediaries' bias. When either the commission differential is lower–as under exogenous changes in commissions or after a reaction of firms under mandatory disclosure– or concern about product suitability for customers is greater–as under expansion of fiduciary duties–, intermediaries are less biased and more trusted, increasing the fraction of consumers who follow advice. This greater trust decreases the magnitude of the elasticity of demand and induces firms to raise prices. Therefore, there are nontrivial effects on welfare: consumers may benefit from hiring (cheaper) intermediaries or from following (less biased) advice, but the rise in prices hurts all consumers, especially those who participate in the market without intermediation. Our simulations show, counterintuitively, that if the regulator wants to reduce intermediation bias, in many cases it would be better to raise the lower commission, rather than reduce the higher commission, to avoid an increase in prices that is harmful to consumers.

In Inderst and Ottaviani (2012a), mandatory disclosures may reduce efficiency if firms are asymmetric and the market share of the cost-efficient firm is too small: commissions decrease after disclosure, and this reduction is more significant for the cost-efficient firm, further reducing its market share. In our model, in contrast, the potentially detrimental effect of a reduction in commissions is present even if the setting is symmetric. If firms are equally cost-efficient and recommendations are unbiased, our model predicts unambiguously that a marginal decrease in commissions decreases the magnitude of demand elasticity and increases prices in equilibrium. When recommendations are biased, if the bias decreases after the reduction in the commission (i.e., if the reduction applies to the high-commission product), this effect may be reinforced because intermediaries become more trustworthy. Then, customers rely on their advice more frequently, making demand less responsive to prices.

Various papers have documented biased advice in intermediation in different industries: general financial markets (Mullainathan et al., 2012), life insurance markets (Anagol et al., 2017), annuity markets (Bhattacharya et al., 2020), bond markets (Egan, forthcoming), mortgage markets (Robles-Garcia, 2019), housing markets (Hong, 2018; Barwick et al., 2017), medical advice (Chen et al., 2016; Dickstein, 2018), energy-efficient durable goods markets (Allcott and Sweeney, 2017), auto repair markets (Schneider, 2012) as well as in many laboratory experiments (Beyer et al., 2013; Cain et al., 2005; Chung and Harbaugh, 2018). Some of these papers focus on whether advice is biased toward high-commission products (Anagol et al., 2017; Mullainathan et al., 2012), or is responsive to commission level or structure (Beyer et al., 2013; Cain et al., 2005), or whether advice influences consumers (Bhattacharya et al., 2020; Chung and Harbaugh, 2018; Egan, forthcoming; Jin and Leslie, 2003). Our paper contributes to the empirical literature on intermediation, by documenting evidence of biased advice and the responses of customers to this bias in the retirement industry, and discussing the effect that different policies have on demand.

A smaller number of empirical papers focus on the effect of particular policies. Anagol et al. (2017),Chung and Harbaugh (2018) and Cain et al. (2005) study the impact of mandating disclosure of commissions or other incentives, finding detrimental or counterintuitive effects such as switching to the recommendation of other products or increasing the bias of the advice. This evidence is supported by some theoretical papers on the equilibrium effect of mandatory disclosure, which find that this policy may have adverse effects (Li and Madarász, 2008; Inderst and Ottaviani, 2012a). Regarding the extension of fiduciary duties, Bhattacharya et al. (2020) provide evidence of better customer-product fit while Finke and Langdon (2012) find no statistical differences in measures of business characteristics among states with stricter fiduciary standards. All these empirical papers refer to the relationship between intermediaries and customers but consider prices as given, which we include explicitly in our analysis. Robles-Garcia (2019) estimates a structural model that includes consumers, brokers, and firms for the mortgage market and predicts the equilibrium effects on welfare of restricting or reducing commission payments. We show that reducing intermediation bias changes the elasticity of demands, which in turn may affect equilibrium outcomes in unexpected ways under price competition.

Another discussion in the literature is whether intermediaries face a trade-off between selling high-commission products or adjusting to consumer beliefs. Mullainathan et al. (2012) and Anagol et al. (2017) report that advisers recommend products with higher fees more frequently, but also fail to de-bias their customers and often reinforce biases that are in their self-interest. Chung and Harbaugh (2018) find that the expert benefits by recommending an *a-priori* favored action even if the other action is better for him. This behavior may have several explanations. One is reputational concerns, as explored by Ottaviani and Sørensen (2006): in this scenario, experts' advice is biased towards the customer's prior belief. Another is that the intermediary pays a cost to make uninformed consumers understand complicated matters. Allcott and Sweeney (2017) and Calcagno and Monticone (2015) find evidence of intermediaries providing more information only to more interested or knowledgeable consumers, in two different settings. Also, Calcagno and Monticone (2015) find that a high degree of financial literacy increases the probability that an investor will consult an advisor, but reduces the likelihood of delegating the portfolio choice. We assume reputation concerns in our model, and also consider heterogeneity in consumers' valuations to capture how recommendations are received differently, depending on their prior preferences for products and advice.

By examining the effect decreasing intermediaries' bias on equilibrium prices and quantities, our paper also contributes to the theoretical literature on intermediation. Several authors have presented models where experts benefit from customers trusting their recommendations, but also have incentives to provide misleading advice for their gain. The paper most similar to ours is Inderst and Ottaviani (2012a). In this model, the role of commissions is to make the intermediary respond to supply-side incentives. We exclude this mechanism, making commissions exogenous and paid by the consumer, and still show that changes in commissions intended to benefit consumers may have unexpected consequences for welfare. Inderst and Ottaviani (2009) explore the agency relationship between the selling firm and its sales force, and in Inderst and Ottaviani (2012b), uses the models of Inderst and Ottaviani (2009) and Inderst and Ottaviani (2012a) to analyze the effect of different policies. Brown and Minor (2012) study the differences in incentives for misconduct in large branded firms as compared to independent sellers and experts with greater experience. Li and Madarász (2008) consider a cheap-talk model to study the quality of advice under mandatory disclosure or nondisclosure. Other papers have constructed models to address persuasion, strategic concealment or revelation of information (for instance Kamenica and Gentzkow (2011), Bourjade and Jullien (2011), Brocas and Carrillo (2007), Caplin and Leahy (2004) and Kartik et al. (2017)). This literature is related to the choice of the information structure, which we consider as given.

2 The role of intermediation in the Chilean market for pension products

Most of the literature, both theoretical and empirical, has focused on contexts where commissions are undisclosed and paid by firms to increase demand. We sidestep the issue of opacity by providing evidence from a market where commissions are fully transparent, exogenously set–so not related to cost differences– and paid directly by consumers instead of firms: the retirement market in Chile. In this highly regulated market, retirees must choose between two retirement products: an annuity or a programmed withdrawal of their pension savings, sold respectively by insurance companies and *Administradoras de Fondos de Pension*, AFP (retirement funds administrators).[1] Both types of firms are independent of each other and have different regulations. Even though most countries have low annuitization rates, this market is relatively large in Chile. In our data, 61.6% of retirees choose annuities: 38.6% choose immediate annuities, and 22.9% choose deferred annuities.

To begin drawing a pension, consumers must obtain a quote for the different firms' products through a centralized system and choose one of the quotes. This centralized system is known as SCOMP (*Sistema de Consultas y Ofertas de Montos de Pensión* in Spanish).[2] They can carry out this process independently (directly) or through an intermediary: either a sales agent from an insurance company or an independent adviser. Sales agents and independent advisers receive different commissions for the products chosen and, thus, have different incentives. Indeed, some policymakers believe that one explanation for Chile's high annuitization rate is the commission differential in favor of annuities. We analyze a reform of the pension system that introduced two subsequent changes that modified this differential.

Before the reform, both types of intermediaries received 2.5% of the retiree's retirement fund as a commission if the customer chose an annuity and no commission in the case of programmed withdrawal.[3] The reform first reduced the commission for annuities to 2%, with a cap of approximately US$2,500, starting in November 2008, and later introduced a commission of 1.2% for programmed withdrawal, although only for advisers, with a cap of approximately US$1,500, beginning in April 2009.

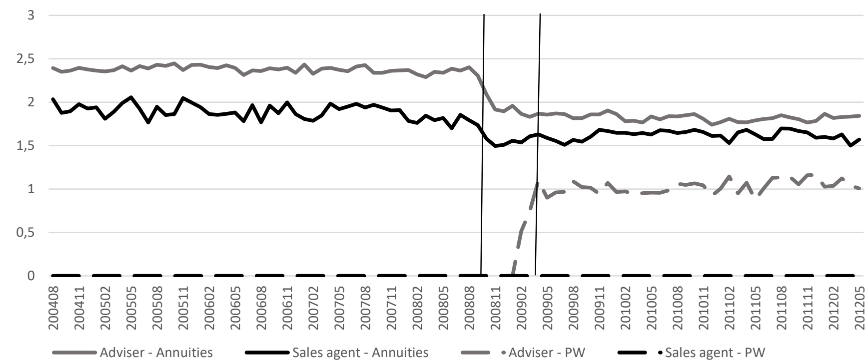
[1]Chile has a defined contribution pension system under which active workers are mandated to save in individual accounts administered by AFPs.

[2]For more information on the system or details of the data, see Alcalde and Vial (2019) and Larrain and Morales (2010).

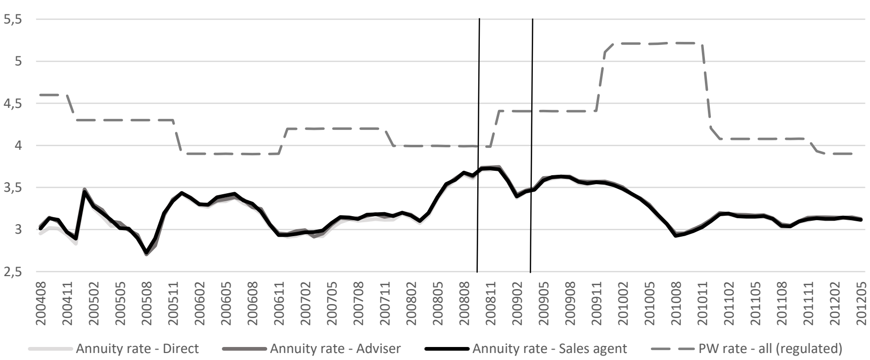
[3]The sales agent was paid only if the annuity is from his firm and the adviser in any event.

2.1 The data

The reform affected market shares for both intermediaries and retirement products. We show this effect using individual-level data from SCOMP. The dataset contains all quotes for all retirement processes between August 2004 and June 2012. We use a subsample of retirees with similar motives for choosing between the two products: those retiring at or after the standard retirement (that is, excluding early retirement), entering the system for the first time (excluding those switching from programmed withdrawal to annuity), quotes with no lump-sum withdrawal, and quotes above the minimum pension.[4]



(a) Intermediation commissions



(b) Annuity rates by intermediary

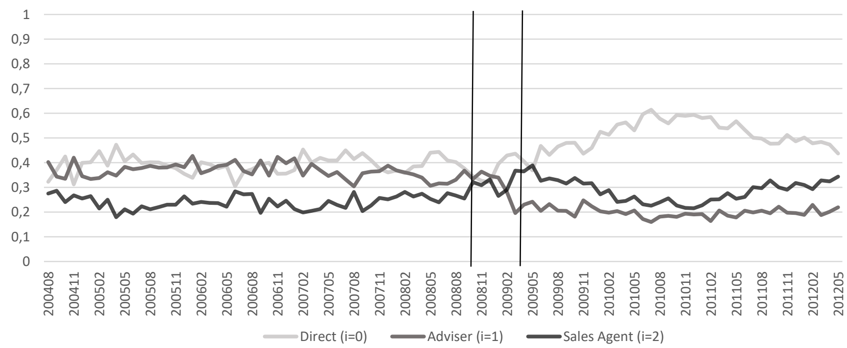
Figure 1: Prices of intermediaries and annuities.

Let us look first at the evolution of commission rates and implicit prices. Figure 1a shows the average commission rate paid for intermediaries: f_A for annuities and f_{PW} for programmed withdrawals. During the whole period, average commissions were lower for sales agents than for advisers because agents only get paid if the annuity is from their firm while advisers get paid in any event. Sales agents were less affected by the reform than advisers, who experienced a more substantial decrease in the

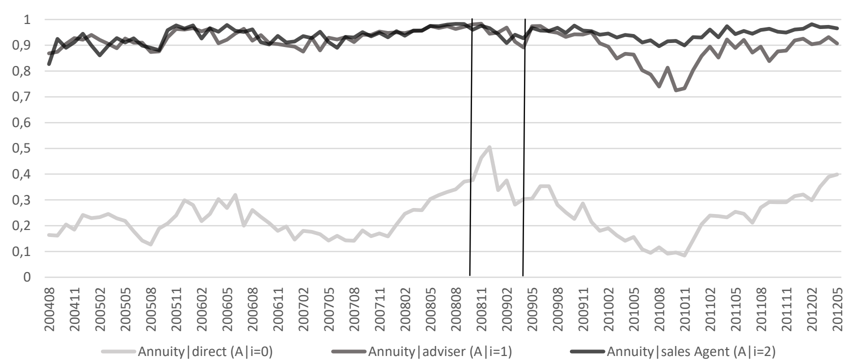
[4]We exclude these groups from our sample because some of them must necessarily choose one of the products, and to avoid composition problems: (a) retirees under early retirement choose programmed withdrawal more frequently than retirees under standard retirement, which may be the result of tax incentives, also the 2008 reform increased the requirements for early retirement, changing the composition of this group; (b) retirees already using programmed withdrawal are allowed to enter the system again but only to buy an annuity; (c) quotes that include a lump-sum withdrawal have the monthly payment fixed at a given level and compete by withdrawal amount and thus are different to regular quotes; also the 2008 reform increased the requirements for a lump-sum withdrawal, so we eliminate these quotes, not the individuals; (d) retirees with limited savings who only get quotes below the minimum pension are forced by law to choose programmed withdrawal.

average rate for annuities and started receiving a commission for programmed withdrawals. Note that, after the reform, average fees were lower than the limit of 2% and 1.2%, respectively, because of the cap. The figure shows a clear-cut reduction in f_A in October 2010, and an increase in f_{PW} in April 2009 for advisers.

We look at the discount rate of each product to analyze the evolution of prices: Figure 1b shows the evolution of the average annuity rate r_A (depending on firms' quotes)[5] and the programmed withdrawal rate r_{PW} (set by the corresponding regulator)[6] ; the two vertical lines show the implementation of the reform. Annuity rates fluctuated significantly over time: firms' offers vary according to costs (depending on market interest rates and the reserve and capital requirements mandated by the regulator) and presumably other market conditions. In particular, r_A fell considerably while r_{PW} increased after the reform (in 2008-2010, during the subprime crisis). By design of the system, firms ignore each retiree's intermediary; indeed, the figure shows practically no variation in the annuity rate between groups on average.



(a) Share of intermediaries



(b) Share of annuities by intermediary

Figure 2: Market shares of intermediaries and annuities.

[5]The *annuity rate* is defined as the internal rate of return on the annuity contract and is determined by the insurance company based on the retiree's fund size and demographic characteristics. Higher annuity rates imply that annuitants obtain higher expected payments in exchange for their premiums paid in advance. Rocha et al. (2008) find that the market for annuities in Chile shows high levels of annuity rates (relative to the risk-free rate), leading them to conclude that this market is very competitive.

[6]The *programmed withdrawal rate* is the discount rate of programmed withdrawal contracts and is determined by the regulator for each calendar year. The regulator set a single discount rate from 2004 to 2008, and a vector of rates for each future period of the contract from 2009 to 2014. We compute the single equivalent discount rate for 2009-2012, which we use as the programmed withdrawal rate in the figure.

Figure 2 describes the evolution of customer choices. Figure 2a shows the market shares for intermediaries: the proportion of retirees who completed the retirement process without an intermediary (direct, $i = 0$), with an adviser ($i = 1$), or with a sales agent ($i = 2$) in each month. The market shares were relatively constant before the reform. Still, after the reform, the fraction of retirees without intermediaries increased, and the proportion using intermediaries decreased, particularly for advisers. Figure 2b shows the market share of annuities by type of intermediary: before the reform, retirees using different intermediaries chose annuities in a similar proportion, close to 90%, and show almost no response to price changes. Instead, retirees without intermediaries varied their share of annuities over time, showing high responsiveness to price changes. After the reform, the annuity share for retirees with sales agents dropped slightly as the gap between rates increased, but the annuity share for retirees without intermediaries and with advisers dropped much more sharply.

2.2 Regression analysis

The patterns in Figure 2 may depend on the evolution of prices in the period, or may be subject to composition effects, as the gender, wealth, or age of retirees may have changed over time. Thus, we analyze the effect of the reform using a before-after specification for the probability of choosing an intermediary ($i = 0, 1, 2$) and selecting an annuity (both immediate and deferred annuities A, vs. programmed withdrawal PW) conditional on the type of intermediary, at the retiree level. We control for the changes in commissions using two dummy variables: one for the reduction in f_A between October 2008 and the end of the sample period, and other for the increase in f_{PW} between April 2009 and the end of the sample period. We use as regressors the prices of each product–average annuity and programmed withdrawal rates for the quotes received–and individual demographic characteristics–gender, age, fund size, beneficiary characteristics, and proxies for socioeconomic status. We use clustered errors by month in all specifications.

To check the robustness of the regression exercise, we gradually include additional regressors: variables for macro conditions such as interest and unemployment rates, variables that measure *ex ante* preferences for one of the retirement products, and fixed effects for geographic areas. These specifications with additional regressors are reported as different columns in the tables presented here. We also check the robustness of the results to changes in the specification of the policy variables[7] and changes in the estimation sample.[8] These robustness checks provide results parallel to those presented here; the tables with their results are not presented here but are available on request.

Tables 1 and 2 show the before-after estimation of the effect of the reform for all specifications on the intermediary and product choice respectively. The patterns in Figure 2a are robust to prices and composition effects. After both changes in commissions, the probability of participating without intermediary increased significantly, the probability of using advisers decreased significantly, and the probability of using sales agents remained almost constant.

Regarding product choice, after the decrease in f_A, the market share of annuities decreased among consumers using intermediaries, but this effect is significant in most specifications only for sales agents.

[7] For this, we run two checks. First, we redefine the dummy variable for the reduction in f_A to take value one between October 2008 and April 2009, to compare the effect of the increase in f_{PW} to the initial period. Second, we drop the dummy variable for the reduction in f_A and check only the effect of the increase in f_{PW} against the period up to April 2009.

[8] For this, we either drop the consumers who choose a deferred annuity, include the consumers who choose quotes with lump-sum withdrawal, and include the consumers who retire under early retirement. These groups of consumers were excluded from the sample to avoid composition problems, as Section 2.1 explains.

Table 1:
Before-after regression on the probability of choosing an intermediary

	(1)	(2)	(3)	(4)
Effect of a reduction in f_A:				
[1] Share of Direct: $\Pr(i=0)$	0.0523***	0.0896***	0.0982***	0.0998***
[2] Share of Advisers: $\Pr(i=1)$	-0.0467***	-0.0842***	-0.0932***	-0.1034***
[3] Share of Sales Agents: $\Pr(i=2)$	-0.0056	-0.0054	-0.005	0.0036
Effect of an increase in f_{PW}:				
[4] Share of Direct: $\Pr(i=0)$	0.0711***	0.0532**	0.0525**	0.0461*
[5] Share of Advisers: $\Pr(i=1)$	-0.0683***	-0.0491***	-0.0419***	-0.0384***
[6] Share of Sales Agents: $\Pr(i=2)$	-0.0028	-0.0041	-0.0106	-0.0077
Log-likelihood	-97793,9	-83779,1	-78926,6	-78337,5
Wald Chi^2	4993,3	7421,2	13334,8	138814,4
AIC	167759,7	167676,2	157985,2	156860,9

Notes: Regressions include 82,401 obs, 95 clusters by month; all include policy dummies with the following controls:
(1) includes prices (annuity and PW rates), demographics, beneficiary info, and other socioeconomic variables,
(2) adds variables for macro conditions: long-term interest rates and unemployment rate,
(3) adds variables that measure prior preference for one product,
(4) adds fixed effects for geographic area.
Coefficients are significant at $^{*}10\%$,$^{**}5\%$,$^{***}1\%$.

Table 2:
Before-after regression on the probability of choosing an annuity

	(1)	(2)	(3)	(4)
Effect of a reduction in f_A:				
[1] Annuity|Direct: $\Pr(A\|i=0)$				
Level of $\Pr(A\|i=0)$	0.0096	-0.0145	-0.0009	0.0014
Elasticity of $\Pr(A\|i=0)$ wrt. r_A	1.4513	1.6032	0.6597	0.5934
Elasticity of $\Pr(A\|i=0)$ wrt. r_{PW}	1.9051	1.0418	-0.8114	-0.8384
[2] Annuity|Adviser: $\Pr(A\|i=1)$				
Level of $\Pr(A\|i=1)$	0.0108	-0.0135	-0.0473**	-0.0445*
Elasticity of $\Pr(A\|i=1)$ wrt. r_A	-0.1485	-0.1827	-0.0862	-0.0973
Elasticity of $\Pr(A\|i=1)$ wrt. r_{PW}	-0.1427	-0.4185***	-0.3639***	-0.3614***
[3] Annuity|Sales Agent: $\Pr(A\|i=2)$				
Level of $\Pr(A\|i=2)$	-0.0435***	-0.0678***	-0.0824***	-0.079***
Elasticity of $\Pr(A\|i=2)$ wrt. r_A	0.4286***	0.3981***	0.4456**	0.4172**
Elasticity of $\Pr(A\|i=2)$ wrt. r_{PW}	0.1961*	-0.0982	-0.1041	-0.079
Effect of an increase in f_{PW}:				
[4] Annuity|Direct: $\Pr(A\|i=0)$				
Level of $\Pr(A\|i=0)$	-0.0265	-0.0167	-0.0482*	-0.0485*
Elasticity of $\Pr(A\|i=0)$ wrt. r_A	-0.695	-0.4928	-0.6766	-0.6289
Elasticity of $\Pr(A\|i=0)$ wrt. r_{PW}	-6.0679***	-5.083**	-3.1478*	-3.0025*
[5] Annuity|Adviser: $\Pr(A\|i=1)$				
Level of $\Pr(A\|i=1)$	-0.093***	-0.0832**	-0.0553**	-0.0543**
Elasticity of $\Pr(A\|i=1)$ wrt. r_A	1.1057***	1.2092***	0.8121***	0.817***
Elasticity of $\Pr(A\|i=1)$ wrt. r_{PW}	-0.6787***	-0.3442**	-0.5753***	-0.5103***
[6] Annuity|Sales Agent: $\Pr(A\|i=2)$				
Level of $\Pr(A\|i=2)$	0.0345**	0.0429***	0.0405**	0.0388**
Elasticity of $\Pr(A\|i=2)$ wrt. r_A	-0.155	-0.0715	-0.2726	-0.2483
Elasticity of $\Pr(A\|i=2)$ wrt. r_{PW}	-0.5802***	-0.2482	-0.3785***	-0.3708**
R^2	0,541	0,541	0,587	0,588
RootMSE	0,331	0,331	0,314	0,314
AIC	51618	51542,9	43006,8	42773,1

Notes: Regressions include 82,401 obs, 95 clusters by month, with the following variables:
(1) policy dummies, interacted with intermediary type and prices (annuity rate r_A, and PW rate r_{PW}),
(1) includes prices, demographics, beneficiary info, and other socioeconomic variables,
(2) adds variables for macro conditions: long-term interest rates and unemployment rate,
(3) adds variables that measure prior preference for one product,
(4) adds fixed effects for geographic area.
Coefficients are significant at *10%,** 5%,*** 1%.

After the increase in f_{PW}, the probability of choosing an annuity decreased for retirees using advisers and participating directly, and increased for retirees using sales agents, even though these two last groups are not directly affected by this policy. More importantly, the magnitude of the elasticity of demand for annuities with respect to r_A increased significantly for retirees using advisers, and the magnitude of the elasticity of demand for annuities with respect to r_{PW} increased significantly for all groups.

This evidence suggests that intermediaries' recommendations do respond to commission differentials and do affect the likelihood of choosing different products. Even though programmed withdrawals became more expensive after the reform, retirees using advisers selected them more frequently. But other results suggest more subtle effects of commissions on consumer behavior. First, Figure 2b shows that the reduction in the annuity share among customers with advisers was not uniform after the reform; the larger decline occurs when the relative price of annuities was unusually high (years 2009-2011), and then the effect declined considerably. This pattern is consistent with the increase in the demand elasticity described in Table 2: customers with advisers became more sensitive to changes in prices when f_{PW} increased. Second, the annuity share for retirees participating directly and using sales agents also changed, even though those consumers were not explicitly affected by the increase in f_{PW}. This pattern suggests a change in the composition of unobserved preferences in the group. Additionally, the share of consumers choosing sales agents did not increase significantly after the increase in f_{PW}, although sales agents became relatively cheaper than advisers. The model developed in Section 3 allows us to analyze these effects formally.

3 Theoretical framework

We consider a market with two firms, A and B, and a continuum of customers and intermediaries. Firm $n \in \{\text{A,B}\}$ provides product n at price $p_n \geq 0$. Each customer (*she*) must select a single product and may participate in the market without intermediation ($i = 0$) or through an intermediary (*he*). The index $i > 1$ denotes the type of the intermediary.[9,10]

The type of customer is $\theta \in \{A, B\}$. As in Inderst and Ottaviani (2012a) and Inderst and Ottaviani (2012b) we define product n as suitable for the customer if $n = \theta$. The customer's type θ is unobservable for everyone. We denote the prior probability of type A by q_0 and, for the sake of simplicity, we assume that $q_0 = 0.5$.

The intermediary may advise the customer in two respects, recommending one product (A or B) and providing guidance on the optimal use of the products. If the customer hires an intermediary i, she pays a commission $f_{ni} \geq 0$ to the intermediary when product n is selected. Commission levels are exogenous and publicly observed.

The intermediary may be concerned about the suitability of the product selected by the customer and, after meeting her, observes a private signal about her type θ and updates his prior belief to a posterior probability of type A, denoted by q. The recommendation is a straightforward message $m \in \{A, B\}$; $\Pr_i(m)$ is the probability that a type-i intermediary sends message m. If the intermediary

[9] As commissions are exogenous there is no role for competition among intermediaries in the model. Hence, we could also assume that there is a unique intermediary of each type who receives a signal for each customer and sends personalized messages to all of them accordingly.

[10] In Inderst and Ottaviani (2012 AER) the information conveyed through advice is necessary for trade. We relax the assumptions of the model and allow firms to sell directly to consumers without intermediation.

were concerned only about product suitability the probability of sending message m should depend only on his updated belief q. If instead, $\Pr_i(m)$ depends also on commission levels f_{ni}, we say that advice is biased. The customer is aware of the potential bias of the message, but may still want to hire an intermediary under certain circumstances.

Payoffs. The customer's valuation of the match is heterogeneous, and depends on her type θ (unknown) and on an idiosyncratic valuation x (private information). Her (gross) utility after buying product n without an intermediary is

$$V_n(\theta, x) \equiv \begin{cases} v + x \text{ if } n = \theta = A \\ v - x \text{ if } n = \theta = B \\ 0 \text{ if } n \neq \theta, \end{cases}$$

where $v > 0$ is the same for all customers. The idiosyncratic valuation x follows a distribution F symmetric around zero, with support $[-v, v]$ and density f. We assume that f is log-concave and that $f(-v) = f(v) = 0$.[11] The customer has an *ex ante* preference for product A if $x > 0$, and for product B otherwise.

If the customer hires an intermediary of type i, she updates her prior q_0 to $\Pr_i(A|m)$ after observing his message m. In addition to this recommendation, the intermediary may provide guidance on the optimal use of products A and B, improving the quality of the match for any customer type. For instance, independent advisers may provide information on subsidies, taxes, and annuity attributes. We assume for simplicity that when the customer hires an intermediary of type i her utility from product n is augmented by v_i, where $v_1 \in \{0, 1\}$ (private information) and $v_2 = 0$. In other words, the customer has an *ex ante* preference for receiving advice from an intermediary of type 1 if $v_1 = 1$.

The intermediary's valuation of the match reflects some degree of concern about the suitability of the product for the customer:

$$W_n(\theta) \equiv \begin{cases} w \text{ if } n = \theta \\ 0 \text{ if } n \neq \theta, \end{cases}$$

where $w > 0$. For the sake of simplicity we assume that w is the same for all intermediaries.

We denote by $i(x, v_i)$ the customer's intermediary-selection policy. The product-selection policy is $\Pr(n|x)$ when $i = 0$, and $\Pr_i(n|m, x)$ when $i > 0$. The vectors $\mathbf{p} = (p_A, p_B)$ and $\mathbf{f}_i = (f_{Ai}, f_{Bi})$ denote prices and commissions respectively.

The payoff of firm n if the customer purchases its product is simply the price p_n minus the unitary cost c_n (and zero otherwise), where the cost differential $\Delta c \equiv c_A - c_B$ is possibly different from zero.

Timeline. The timeline of the game is as follows:

- firms set prices p_A and p_B
- each customer observes commissions $\mathbf{f}_i$ (exogenous) and prices $\mathbf{p}$ and chooses whether to hire an intermediary ($i > 0$) or not ($i = 0$)
- if hired, each intermediary observes the private signal about the type of customer and sends a message (recommendation) $m \in \{A, B\}$

[11] Hence, the distribution is unimodal, with $F(-a) = 1 - F(a)$, $f(-a) = f(a)$ and $f'(-a) = f'(a)$.

- each customer with $i > 0$ observes the message m and updates her belief
- each customer chooses a product $n \in \{$A,B$\}$

Equilibrium. We analyze the informative, Perfect Bayesian Equilibrium of the game. An equilibrium is a set of strategies and beliefs such that:

- given intermediary and customer strategies, p_n is optimal for each firm $n \in \{A, B\}$
- given customer strategies, the intermediaries' message policy $m_i(q)$ is optimal for any posterior belief q for $i = 1, 2$,
- the customer's intermediary-selection policy $i(x, v_i)$ is optimal for any (x, v_i), and the product-selection policies $\Pr_i(n|m, x)$ and $\Pr(n|x)$ are optimal for any feasible message m and (x, v_i) (according to the posterior belief $\Pr_i(\theta|m)$)
- q and $\Pr_i(\theta|m)$ are consistent beliefs.

In Section 4 we analyze a setting that can be closely mapped to the Chilean market for pension products described in Section 2. We consider two types of intermediaries: independent advisers ($i = 1$) and sales agents ($i = 2$), in addition to the outside option ($i = 0$). As we control for annuity rates in the empirical analysis, we assume that prices p_A and p_B are exogenous in this section. In other words, we analyze the proper subgame beginning at the second stage (after prices are determined).

In Section 5, by contrast, we set aside the specific characteristics of the pension annuity market and explicitly model competition with intermediation in a more general setting, analyzing the game starting at the first stage. To focus the analysis on the competition between firms rather than on the choice of different types of intermediation we simplify the model in this section considering only one type of intermediary (i.e., $i \in \{0, 1\}$) and assuming that $v_i = 0$.

Belief updating and intermediary strategy. The intermediary updates his prior q_0 to a posterior belief q upon observation of a private signal, where this posterior belief has distribution G. We assume that the density g is log-concave, with $G(0.5) = 0.5$. If advice is followed in equilibrium, the intermediary recommends product A if the expected utility of this message, w_{Ai}, is larger than for message B, w_{Bi}, where

$$w_{ni}(q) = qW_n(A) + (1 - q)W_n(B) + \lambda f_{ni},$$

where $\lambda \in \{0, 1\}$ captures the intermediary's concern about his own monetary payoff. Then, $w_{Ai}(q) > w_{Bi}(q)$ happens when $q > q_i^*$, where

$$q_i^* = \frac{1}{2}\left(1 - \lambda\frac{\Delta f_i}{w}\right), \tag{1}$$

and $\Delta f_i \equiv (f_{Ai} - f_{Bi})$ denotes the commission differential for a type-i intermediary.[12] Then, the probability that the intermediary sends message $m = A$ is $\Pr_i(A) = 1 - G(q_i^*)$. Note that q_i^* depends on Δf_i–and advice is biased–if $\lambda = 1$.

The customer's posterior belief $\Pr_i(A|m)$ under an informative equilibrium corresponds to:

$$\Pr_i(A|m) = \begin{cases} E(q|q > q_i^*) \equiv \overline{q}_i \text{ if } m = A, \\ E(q|q \leq q_i^*) \equiv \underline{q}_i \text{ if } m = B, \end{cases} \tag{2}$$

[12]The commission differential must satisfy $\lambda\Delta f_i \in [-w, w]$, otherwise the intermediary always sends the same message regardless of the signal, and advice is not informative. Hence, we restrict attention to values of Δf_i and w such that this condition always holds.

where $\underline{q}_i \leq q_0 \leq \overline{q}_i$, and consistency of beliefs implies that

$$(1 - G(q_i^*))\overline{q}_i + G(q_i^*)\underline{q}_i = q_0. \tag{3}$$

If the message was uninformative, the customer's posterior probability would be equal to her prior probability; i.e., $\Pr_i(A|m) = q_0$.

Customers' strategies. Without intermediation the customer chooses product A if the expected utility with this product, u_{A0}, is larger than for product B, u_{B0}, where

$$u_{A0}(x|p_A) = q_0(v + x) - p_A, \tag{4}$$

$$u_{B0}(x|p_B) = (1 - q_0)(v - x) - p_B. \tag{5}$$

As $q_0 = 0.5$, the equilibrium product-selection policy when $i = 0$ is

$$\Pr(A|x) = \begin{cases} 1 \text{ if } x > \Delta p, \\ 0 \text{ if } x \leq \Delta p. \end{cases}$$

In other words, without additional information, the customer's choice depends only on her *ex ante* preference for the products and the price differential.

With intermediation the customer can still decide whether to follow the intermediary's recommendation or not. The customer's expected utility with product n after hiring an intermediary of type i and observing message m is given by

$$u_{ni}(x, v_i|m, p_n, f_{ni}) = \Pr_i(A|m)V_n(A, x) + (1 - \Pr_i(A|m))V_n(B, x) + v_i - p_n - f_{ni}, \tag{6}$$

Conditional on the intermediary type i and the message m, the equilibrium product-selection policy when $i > 0$ is

$$\Pr_i(A|m, x) = \begin{cases} 1 \text{ if } m = A \text{ and } x > \Delta p + \Delta f + v(1 - 2\overline{q}_i), \\ 0 \text{ if } m = A \text{ and } x \leq \Delta p + \Delta f + v(1 - 2\overline{q}_i), \\ 1 \text{ if } m = B \text{ and } x \geq \Delta p + \Delta f + v(1 - 2\underline{q}_i), \\ 0 \text{ if } m = B \text{ and } x < \Delta p + \Delta f + v(1 - 2\underline{q}_i) \end{cases}$$

Because $\overline{q}_i > \underline{q}_i$, this policy implies that individuals with extreme valuations for the product will choose their *ex ante* preferred products (i.e., $n = A$ if x is large, and $n = B$ is x is small) for any recommendation. Individuals with intermediate valuations for the product will follow the recommendation.

Regarding the selection of an intermediary, we can compare between choosing $i = 0$ and a given $i > 0$. A customer with intermediary i and an extreme value of x buys product n after any message; her expected utility u_{ni} can be written using equations (3) and (6) as

$$u_{ni}(x, v_i|p_n, f_{ni}) = u_{n0}(x|p_n) + v_i - f_{ni}, \tag{7}$$

If the customer will not follow recommendation, she prefers to hire intermediary i rather than no intermediation at all ($i = 0$, obtaining u_{n0}) if the benefit of receiving advice is greater than the cost of intermediation, i.e., if $v_i > f_{ni}$.

A customer with intermediary i and an intermediate value of x who follows the recommendation buys product $n = m$; her expected utility u_{Mi} can be written using again equations (3) and (6) as

$$u_{Mi}(x, v_i|\mathbf{p}, \mathbf{f}_i) = v_i + v(q_0 + G(q_i^*)(1 - 2\underline{q}_i)) + x(q_0 - G(q_i^*)) - \mathrm{E}(p|q_i^*) - \mathrm{E}(f_i|q_i^*), \tag{8}$$

where $\mathrm{E}(p|q_i^*)$ denotes the expected payment to the firms and $\mathrm{E}(f_i|q_i^*)$ the expected payment to the intermediary. We denote by $\overline{x}_i$ the cutoff that solves $u_{A0}(\overline{x}_i|p_A) = u_{Mi}(\overline{x}_i|\mathbf{p},\mathbf{f}_i) - v_i$, and by $\underline{x}_i$ the cutoff that solves $u_{B0}(\underline{x}_i|p_B) = u_{Mi}(\underline{x}_i|\mathbf{p},\mathbf{f}_i) - v_i$:

$$\begin{aligned} \overline{x}_i &= \Delta p + (1 - G(q_i^*))\kappa_i & (9) \\ \underline{x}_i &= \Delta p - G(q_i^*)\kappa_i, & (10) \end{aligned}$$

where κ_i is defined as

$$\kappa_i \equiv \frac{vG(q_i^*)(1 - 2\underline{q}_i) - E(f_i|q_i^*)}{G(q_i^*)(1 - G(q_i^*))}. \tag{11}$$

In addition we define

$$\begin{aligned} \overline{\mu}_i &= \frac{1}{G(q_i^*)} \min\{v_i, f_{Ai}\}, \\ \underline{\mu}_i &= \frac{1}{1 - G(q_i^*)} \min\{v_i, f_{Bi}\}, \end{aligned}$$

where $u_{Mi}(x, v_i|\mathbf{p},\mathbf{f}_i) > \max_{n\in\{A,B\}} \max\{u_{n0}(x|p_A), u_{ni}(x, v_i|p_n, f_{ni})\}$ when $x \in [\underline{x}_i - \underline{\mu}_i, \overline{x}_i + \overline{\mu}_i]$.

Then, when we consider an intermediary i the customer's preference is summarized as follows: if $x \in [\underline{x}_i - \underline{\mu}_i, \overline{x}_i + \overline{\mu}_i]$ the customer prefers to hire intermediary i and follow his recommendation rather than choosing $i = 0$ or hiring i disregarding his message. If $x > \overline{x}_i + \overline{\mu}_i$, the customer prefers to select product A in any case (either with $i = 0$ if $v_i < f_{Ai}$ or hiring intermediary i ignoring his recommendation if $v_i \geq f_{Ai}$). Similarly, if $x < \underline{x}_i - \underline{\mu}_i$, the customer prefers to select product B (either with $i = 0$ if $v_i < f_{Bi}$ or hiring intermediary i disregarding his message if $v_i \geq f_{Bi}$). The demand for product n for customers with intermediary i thus depends on the commission differential: Δf_i affects the intermediary's policy through q_i^*, which in turn affects the fraction of customers following recommendations, and also the fraction $G(q_i^*)$ among them who select B. If $v_i \geq f_{ni}$, the demand for product n for customers with intermediary i also depends on the level of the commission f_{ni} and on prices p_A and p_B. Indeed, f_{ni} affects the fraction of customers choosing intermediaries, while p_A and p_B affect the choice of products for customers disregarding their recommendations.

To compare $i = 1$ and $i = 2$ it is necessary to describe the commissions. In the special case where commissions are the same for both intermediaries (i.e., when $\mathbf{f}_1 = \mathbf{f}_2$) the cutoff $q_1^* = q_2^*$ is unique and intermediaries are equally biased. Therefore, we can supress the index i everywhere, and the choice between types depends only on the customer's preference for receiving advice, v_i. Indeed, the customer prefers a type-1 intermediary if $v_1 = 1$, and is indifferent otherwise. Then, the maximum utility attainable following an intermediary's advice is

$$\begin{aligned} u_M(x, v_1|\mathbf{p},\mathbf{f}) &= \max\{u_{M1}(x, v_1|\mathbf{p},\mathbf{f}), u_{M2}(x|\mathbf{p},\mathbf{f})\} \\ &= v_1 + v(q_0 + G(q^*)(1 - 2\underline{q})) + x(q_0 - G(q^*)) - E(p|q^*) - E(f|q^*). \end{aligned} \tag{12}$$

Customers' policies can thus be defined by $\overline{\mu} = \max\{\overline{\mu}_1, \overline{\mu}_2\}$ and $\underline{\mu} = \max\{\underline{\mu}_1, \underline{\mu}_2\}$. Lemma 1 summarizes the equilibrium customers' policies in this case:

Lemma 1. [Customer policies, Symmetric case] *In any informative equilibrium, if $\mathbf{f}_1 = \mathbf{f}_2$, customers with $x \in [\underline{x} - \underline{\mu}, \overline{x} + \overline{\mu}]$ choose $i > 0$ and follow their intermediary's (personalized) recommendations. Customers with $x > \overline{x} + \overline{\mu}$ always choose product A, and those with $x < \underline{x} - \underline{\mu}$ always choose product B; these customers choose $i > 0$ only if $v_i \geq f_n$. Customers with $i > 0$ choose $i = 1$ if $v_1 = 1$ and are indifferent between $i = 1$ and $i = 2$ otherwise.*

See the Appendix for proofs. We focus in what follows on the set of parameter values such that an informative equilibrium with intermediation may exist.

In the general case, a change in the commission f_{ni} affects the expected utility u_{Mi} through two different channels: it changes the cost of intermediation (direct effect) and may also change the distribution of messages $G(q_i^*)$, affecting the benefit of hiring an intermediary (indirect effect, through q_i^*):

$$\frac{\partial u_{Mi}(x, v_i|\mathbf{p}, \mathbf{f}_i)}{\partial f_{ni}} = -\frac{\partial E(f_i|q_i^*)}{\partial f_{ni}} + g(q_i^*)\left(\Delta f_i\left(1+\frac{v}{w}\right) + (\Delta p - x)\right)\frac{\partial q_i^*}{\partial f_{ni}}. \tag{13}$$

Lemma 2 shows how the effect of a change in f_n on the expected utility with and without intermediation translates to the effect on $\underline{x}$ and $\overline{x}$ when $\mathbf{f}_1 = \mathbf{f}_2$.

Lemma 2. [Cutoffs, symmetric case] *If $\mathbf{f}_1 = \mathbf{f}_2$, the effect of a change in f_n on $\underline{x}$ and $\overline{x}$ is*

$$\frac{\partial \underline{x}}{\partial f_n} = \frac{1}{1-G(q^*)}\frac{\partial E(f|q^*)}{\partial f_n} + \frac{\partial \underline{x}}{\partial q^*}\frac{\partial q^*}{\partial f_n}$$

and

$$\frac{\partial \overline{x}}{\partial f_n} = -\frac{1}{G(q^*)}\frac{\partial E(f|q^*)}{\partial f_n} + \frac{\partial \overline{x}}{\partial q^*}\frac{\partial q^*}{\partial f_n},$$

where

$$\frac{\partial \underline{x}}{\partial q^*} = -\frac{g(q^*)}{1-G(q^*)}\left(\Delta f\left(1+\frac{v}{w}\right) + \kappa G(q^*)\right),$$

and

$$\frac{\partial \overline{x}}{\partial q^*} = \frac{g(q^*)}{G(q^*)}\left(\Delta f\left(1+\frac{v}{w}\right) - \kappa(1-G(q^*))\right).$$

Note that $\underline{\mu} = \overline{\mu} = 0$ when $v_1 = 0$ (as in Section 5). In other words, in this simplified setting, customers' policies are defined only by $\underline{x}$ and $\overline{x}$, and they hire intermediaries only when they are willing to follow their recommendations. We further analyze the equilibrium and its comparative statics in the settings of Sections 4 and 5 below.

4 Product selection with two types of intermediation

In this section, we consider a setting to map the Chilean market for pension products described in Section 2. We consider two types of intermediaries who receive the same commission $f_A > 0$ if the customer selects product A and zero otherwise. Therefore, $u_{B0} = u_{B2} \leq u_{B1}$, and $u_{B0} < u_{B1}$ if and only if $v_1 > 0$. We then analyze the effect of a reduction in f_A for all intermediaries ($i = 1, 2$) and an increase in f_B for independent advisers ($i = 1$). We compare the theoretical predictions of our model with the empirical patterns observed in the data described in Section 2.

The Chilean data provide supporting evidence for the assumption that $v_1 = 0$ for some customers and $v_1 > 0$ for others. First, if $v_1 > 0$ and $f_B = 0$, participating without intermediaries is a strictly dominated strategy: one can benefit from hiring an adviser without paying a commission. Then, the fraction of customers selecting B without intermediaries when $f_B = 0$ is only consistent with $v_1 = 0$ for some retirees. Second, if $v_1 = 0$ and $f_B > 0$, hiring an adviser is a strictly dominated strategy for customers not willing to follow advice: they pay a commission without getting any benefit. Then, only customers willing to follow advice would hire advisers and their demand would be perfectly inelastic.

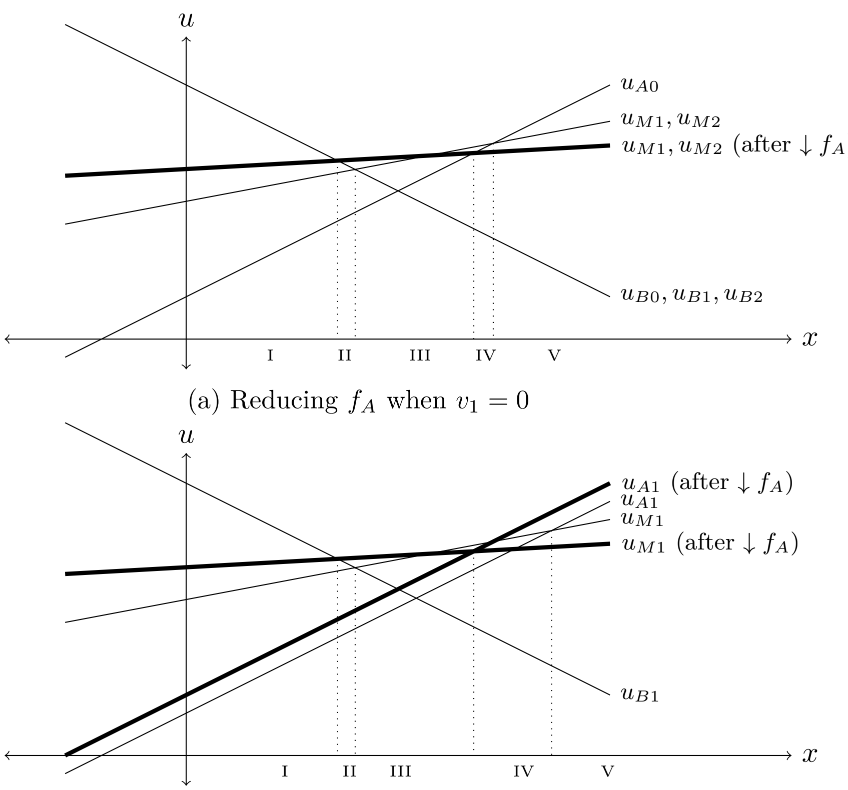
Then, the responsiveness to prices of the demand of consumers using advisers when $f_B > 0$ is only consistent with $v_1 > 0$ for some retirees–for them, sales agents and advisers are not perfect substitutes. In the analysis that follows, we show how the reforms affect both groups of customers.

4.1 Reducing f_A for both types of intermediary

If we assume that $f_A > 0$ and $f_B = 0$, when f_A decreases for all intermediaries, the expected utility of hiring an intermediary changes. Indeed, the expected utility of hiring an intermediary $i > 0$ but choosing product A regardless of his recommendation, u_{Ai}, increases; see equation (7). In addition, the expected utility of following advice, u_{Mi}, also changes; in this case equation (13) becomes

$$\frac{\partial u_{Mi}(x, v_i|\mathbf{p}, \mathbf{f})}{\partial f_A} = -(1 - G(q^*)) - g(q^*)\frac{f_A}{2w}\left(1 + \frac{v}{w}\right) - g(q^*)\left(\frac{\Delta p - x}{2w}\right). \tag{14}$$

Then, u_{Mi} increases for low values of x after a reduction in f_A: the expected cost of intermediation decreases, and the bias is reduced, and both effects increase the expected utility of following recommendations. But u_{Mi} may decrease for high values of x. The ambiguity comes from the third term in equation (14): as the probability of recommending B increases, the option of following advice worsens for customers with an *ex ante* preference for product A (i.e., with $x > \Delta p$). Meanwhile, u_{A0} and u_{Bi} for $i \geq 0$ remain constant. This implies that the fraction of customers selecting B regardless of recommendations decreases, while the fraction of them choosing A regardless of advice may increase. Indeed, Lemma 2 shows that $\underline{x}$ decreases after a reduction in f_A, but the effect on $\overline{x}$ is ambiguous.



(b) Reducing f_A when $v_1 > f_A > 0$

Figure 3: Effect of a reduction in f_A on expected utilities

Figure 3 summarizes the effect of a reduction in f_A on expected utilities for two different cases, assuming that u_{M1} increases for low values and decreases for large values of x. Figure 3a describes the effect of the reform when $v_1 = 0$, where we can identify five groups of consumers that are affected differently. Consumers with the lowest values of x (group I) select product B regardless of recommendations before and after the reform, and are indifferent between $i \geq 0$. Consumers in group II switch from selecting product B to following recommendations of intermediaries, while those in group III follow recommendations before and after the reform. Groups II and III are indifferent between hiring $i > 0$. Consumers in group IV cease to hire intermediaries after the reform and prefer product A without any intermediary. Finally, consumers with the highest values of x (group V) select product A without any intermediary before and after the reform. The effect of group IV explains the increase in the fraction of customers participating without intermediaries (findings [1] and [2] in Table 1). Additionally, the fraction of customers with sales agents who selected annuities decreased, which is consistent with a shift in recommendation in group III (finding [3] in Table 2).

On the other hand, Figure 3b shows how the reduction in f_A affects different groups of consumers with $v_1 > 0$, all of them hiring advisers before and after the reform. In this case, consumers in group II switch from selecting product B to following the adviser's recommendation. On the other hand, consumers in group IV switch from following recommendations to selecting product A regardless of advice. The combination of the effects of groups II and IV with the shift in advisers' recommendations explains why the fraction of customers with advisers who select annuities barely decreases (finding [2] in Table 2).

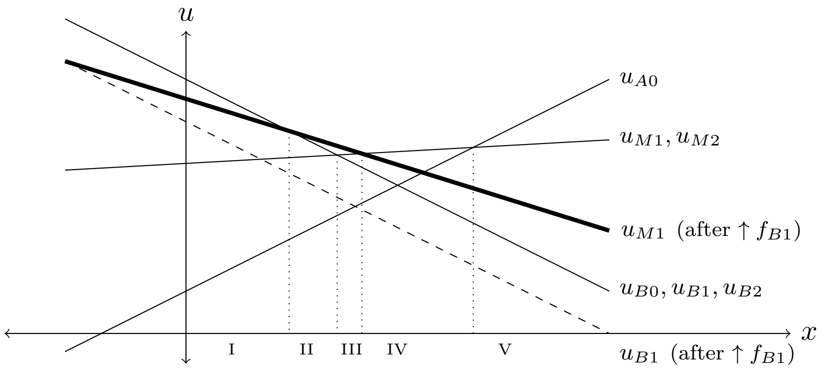
4.2 Increasing f_{B1}, only for intermediaries of type 1

When f_{B1} increases, the bias in favor of annuities reduces and advisers' recommendations change. This effect on recommendations explains the reduction in the market share of annuities only among consumers using advisers (finding [5] in Table 2). Beyond this direct effect for advisers, the expected utility with an adviser is affected. Indeed, the expected utility of hiring a type-1 intermediary but choosing product B regardless of the recommendation, u_{B1}, decreases. In addition, the expected utility of following advice, u_{Mi}, also changes and Equation (13) becomes

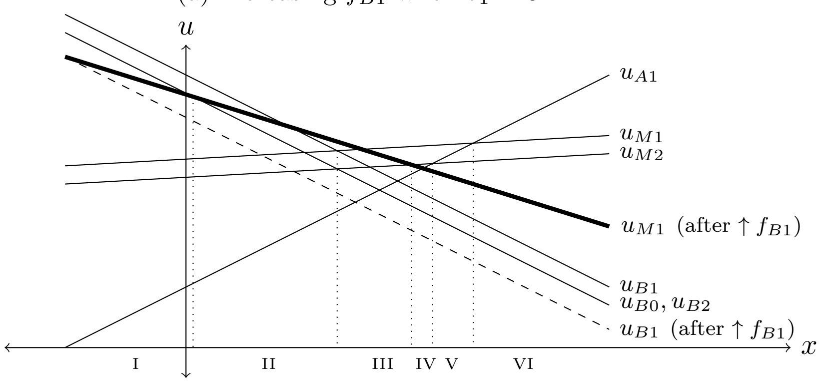
$$\frac{\partial u_{M1}(x|\mathbf{p}, \mathbf{f}_1)}{\partial f_{B1}} = -G(q_1^*) + g(q_1^*)\frac{\Delta f_1}{2w}\left(1 + \frac{v}{w}\right) + g(q_1^*)\left(\frac{\Delta p - x}{2w}\right). \tag{15}$$

The first term in equation (15) is negative since the expected cost of intermediation $E(f_1|q_1^*)$ increases. The second term is positive because $\Delta f_1 > 0$ and the bias is reduced, increasing the expected utility of following recommendations. The third term is positive if and only if $x < \Delta p$.

Figure 4 summarizes the effect of a change in f_{B1} on expected utilities when we assume that u_{M1} increases for low values and decreases for large values of x. Figure 4a describes the effect of the reform when $v_1 = 0$, where we can identify five groups of consumers that are affected differently. Consumers in group I select product B regardless of recommendations; they were indifferent between $i \geq 0$, but they cease to hire advisers when f_{B1} increases. This effect explains the rise in the fraction of customers without intermediation and the reduction in the fraction of customers hiring advisers (findings [4] and [5] in Table 1). Consumers in group II switch from selecting B to following advisers' recommendations. Consumers in groups III and IV were indifferent between following recommendations of sales agents or advisers before the reform; those in group III switch to advisers looking for less-biased advice, while consumers in group IV switch to sales agents looking for less-expensive advice. The combined effects of groups II, III and IV explains why the fraction of customers using sales agents does not change when f_{B1} increases (finding [6] in Table 1), and why the composition of consumers hiring sales agents changes, affecting the fraction who select annuities (finding [6] in Table 2). Finally, consumers with the highest values of x (group V) select product A without any intermediary before and after the reform. The combined effects of groups I and V may explain the change in the demand for annuities among consumers without intermediaries (finding [4] in Table 2): within the pool of consumers without intermediation, the fraction with a strong preference for annuities (high values of x) decreases.



(a) Increasing f_{B1} when $v_1 = 0$



(b) Increasing f_{B1} when $v_1 > f_A > 0$

Figure 4: Effect of an increase in f_{B1} on expected utilities

Figure 4b shows how the increase in f_{B1} affects different groups of consumers when $v_1 > 0$. All these groups were hiring advisers before the reform. Consumers in group I always select product B regardless of recommendations; they switch to sales agents or not intermediation. This effect reinforces the effect for group I in Figure 4a, which explains finding [4] in Tables 1 and 2: the fraction of consumers without intermediation increases and the composition of this pool also changes in favor of selecting product B. Consumers in group II switch from always selecting product B to following advice. Consumers in group III do not change their behavior and always follow advice. In contrast, consumers in groups IV and V used to follow advice, but they switch to sales agents and to choose A regardless of advice, respectively. Finally, consumers in group VI do not change their behavior and always choose A regardless of advice. The combined effect of groups II to VI generates an ambiguous effect on the size and composition of the group of consumers hiring advisers. Still, we argue that group V should be more relevant than group II to explain finding [5] in Table 2. We find that after the increase in f_{B1}, the demand for annuities among customers with advisers became more elastic. Thus it is necessary that $v_1 > 0$ for some consumers and that the fraction of consumers who do not follow advice increases among advisers.[13] Indeed, if $v_1 = 0$ for all consumers, then after the reform, only

[13]The reform also changed independent advisers from insurance brokers to pension agents, introducing certification

those consumers willing to follow recommendations would choose to hire advisers, and the demand for annuities within this group would be completely inelastic to prices. Formally, this effect depends on the joint distribution of x and v_1, because it affects the mass of customers near the cutoffs $\underline{x} - \underline{\mu}$ and $\overline{x} + \overline{\mu}$. This issue is a central piece of the analysis in the next section, where we analyze competition among firms.

5 Price competition with intermediation

We continue our theoretical analysis by including firms' equilibrium reaction to commission levels and changes in demand elasticity. For this, we return to the baseline model from Section 3 and extend it to explicitly model firm competition, by introducing a first stage where each firm n selects its price p_n. We then use this model to analyze the equilibrium effects of two different policy changes designed to reduce the bias of intermediaries' recommendations.

For tractability, we consider only one type of intermediation. Thus, we assume that $v_1 = 0$–advice involves only a recommendation regarding product selection– and suppress index i where it was used. The intermediary selection policy is $i(x)$, an indicator that takes value 1 when the customer chooses to hire an intermediary and 0 otherwise.

If $v_1 = 0$, Lemma 1 implies that customers with $x \in [\underline{x}, \overline{x}]$ hire intermediaries and follow their recommendations. Customers with $x > \overline{x}$ always choose product A, and those with $x < \underline{x}$ always choose product B. As intermediation is costly, customers hire an intermediary only when willing to follow his recommendation. In other words, the cutoffs $\underline{x}$ and $\overline{x}$ define both the product and intermediary selection policies. Then, aggregate demand for product n, denoted by D_n, is simply described by

$$D_{\mathrm{B}} = (F(\overline{x}) - F(\underline{x}))G(q^*) + F(\underline{x}),$$

and $D_{\mathrm{A}} = 1 - D_{\mathrm{B}}$.

Let $H(x) \equiv F(x+\kappa)G(q^*) + F(x)(1-G(q^*))$. Then, $D_B = H(\underline{x})$. Note that D_{A} and D_{B} depend on the price differential Δp (not on price levels) through $\underline{x}$, and also on the commission levels f_A and f_B through q^* and κ. As $\frac{\partial \underline{x}}{\partial p_A} = -\frac{\partial \underline{x}}{\partial p_B} = 1$ it follows that[14]

$$\frac{\partial D_n}{\partial p_n} = -\frac{\partial D_n}{\partial p_{n'}} = -H'(\underline{x}) \leq 0.$$

Firms' strategies. The expected payoff of firm's n is

$$\Pi_n(p_n) = (p_n - c_n)D_n.$$

Firm n's best response $p_n^*(p_{n'})$ is obtained from the maximization of $\Pi_n(p_n)$ given $p_{n'}$. It satisfies[15]

$$p_n^*(p_{n'})\left(1 + \frac{1}{\eta_n}\right) = c_n, \tag{16}$$

requirements for the latter. As a consequence, v_1 or w could have increased, which reinforces the effects just described.

[14] While

$$\frac{\partial^2 D_n}{\partial {p_n}^2} = -\frac{\partial^2 D_{n'}}{\partial {p_{n'}}^2} = -\frac{\partial^2 D_n}{\partial p_n \partial p_{n'}} = \frac{\partial^2 D_{n'}}{\partial p_n \partial p_{n'}},$$

for any $n \in \{A, B\}$, with $n' \neq n$.

[15] Note that the expected profit of firm n is $p_n - c_n$ when $\Pr(n) = 1$ (while its profit is zero when $\Pr(n) = 0$). Hence, the unrestricted maximization of Π_A is equivalent to maximizing Π_A subject to $p_A \in [p_B - \Delta v - \Delta f, p_B + \Delta v - \Delta f]$ (the

where $\eta_n = -H'(\underline{x})\frac{p_n}{D_n}$ is the price-elasticity of demand for product n. Then, the slope of the best-response function can be obtained by applying the implicit function theorem in equation (16). The second-order condition of the firm's maximization problem implies that the slope of the best-response function is less than one for both firms.

Lemma 3. [Best-response functions] *In any interior solution of the firm's problem the slope of the best-response function is less than one. If the price-elasticity of demand for product n is increasing in $p_{n'}$, then the best-response function is increasing; i.e., $\frac{\partial p_n^*}{\partial p_{n'}} \in (0,1)$.*

5.1 Characterization of an informative equilibrium.

Under Lemma 3, if a Nash equilibrium of the pricing game with two active firms exists, it is unique. Moreover, at least one of the best-response functions has a positive slope at the equilibrium price levels p_A^* and p_B^*. As the equilibrium of the pricing game is unique, the informative equilibrium is also unique.

Proposition 1. [Informative equilibrium] *If an informative equilibrium with two active firms exists, it is unique. In this equilibrium firms set prices (p_A^*, p_B^*) in a first stage. Customers with idiosyncratic valuations x in the interval $[\underline{x}, \overline{x}]$ hire intermediaries and follow their recommendations. Customers with extreme idiosyncratic valuations x do not hire any intermediary; they choose A if $x > \Delta p$, and B otherwise. If hired, each intermediary observes a private signal about the customer's type and recommends product A whenever his posterior belief q is above q^*. A necessary condition for the existence of an informative equilibrium is that $\overline{x} > \underline{x}$; i.e., that $\kappa > 0$.*

In the analysis below we focus on parameter values that sustain an informative equilibrium with two active firms. In this case, equation (16) must be satisfied for both firms simultaneously and the price differential Δp and the cost differential Δc must satisfy

$$\Delta p - \Delta c + \left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) = 0, \tag{17}$$

where the demand elasticities depend on Δp (price differential, not level) and commissions f_A and f_B (levels). Moreover,

$$\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) = \frac{2H(\underline{x}) - 1}{H'(\underline{x})},$$

which is increasing in Δp.[16]

Note that $\Delta p = \Delta c$ only if $H(\underline{x}) = 0.5$. This can happen, for example, if $G(q^*) = 0.5$: if $\Delta p = 0$, it follows that $\overline{x} = -\underline{x} = 0.5\kappa$ and $H(\underline{x}) = 0.5$. From this point, we obtain the following result:

Lemma 4. [Symmetric case] *If $\Delta f = 0$ (which implies that $G(q^*) = 0.5$), then:*

- *$\Delta p = 0$ and $D_A = D_B = 0.5$ in equilibrium if $\Delta c = 0$.*

optimum is always attained within that set). If $c_A > p_B + \Delta v - \Delta f$, then $p_A^* = p_B + \Delta v - \Delta f$ is indeed a best response from firm A; otherwise, $p_A^* \in [p_B - \Delta v - \Delta f, p_B + \Delta v - \Delta f)$. Similarly, the unrestricted maximization of Π_B is equivalent to maximizing Π_B subject to $p_B \in [p_A - \Delta v + \Delta f, p_A + \Delta v + \Delta f]$, and $p_B^* = p_A + \Delta v + \Delta f$ is a best response from firm B only if $c_B > p_A + \Delta v + \Delta f$. Then, the first-order condition for an interior solution is: $D_n + (p_n - c_n)\frac{\partial D_n}{\partial p_n} = 0$.

[16] The proof can be found in Appendix A.4.

- $0 < \Delta p < \Delta c$ *and* $D_B > D_A$ *in equilibrium if* $\Delta c > 0$.
- $0 > \Delta p > \Delta c$ *and* $D_B < D_A$ *in equilibrium if* $\Delta c < 0$.

Applying the implicit function theorem to equations (16) and (17) we can extend the results of Lemma 4 to a more general setting with $\Delta f \neq 0$ in the following sense: If Δc increases the price differential Δp also increases in equilibrium, and therefore the demand for product B increases.

In general, an increase in any variable ψ generates a direct effect on D_B and an indirect effect through equilibrium prices:

$$\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}\psi} = \frac{\partial H(\underline{x})}{\partial \psi} + H'(\underline{x})\frac{\mathrm{d}\Delta p}{\mathrm{d}\psi}, \tag{18}$$

where $\frac{\partial H(\underline{x})}{\partial \psi}$ corresponds to the effect of a change in ψ on D_B, holding prices constant, and $\frac{\mathrm{d}\Delta p}{\mathrm{d}\psi}$ denotes its effect on the price differential in equilibrium.

The effect of a change in ψ on Δp can be obtained by applying the implicit function theorem to equation (17). Indeed, the change in the fraction of customers hiring an intermediary affects the elasticity of demand according to

$$\frac{\partial \eta_n}{\partial \psi} = -\left(\frac{p_n}{D_n}\frac{\partial H'(\underline{x})}{\partial \psi} + \frac{\eta_n}{D_n}\frac{\partial D_n}{\partial \psi}\right). \tag{19}$$

If the magnitude of the price-elasticity of demand for n increases, then the firm's best-response function decreases pointwise. Lemma 5 describes the effect of a change in a variable ψ on equilibrium prices.

Lemma 5. [Effect on prices] *The total effect of the change of any variable ψ on prices is*

$$\frac{\mathrm{d}p_A}{\mathrm{d}\psi} = M(\underline{x})\left(\left(\frac{H''(\underline{x})}{H'(\underline{x})} - H'(\underline{x})\right)\frac{\partial H(\underline{x})}{\partial \psi} - (2 - H(\underline{x}))\frac{\partial H'(\underline{x})}{\partial \psi}\right),$$

and

$$\frac{\mathrm{d}p_B}{\mathrm{d}\psi} = M(\underline{x})\left(\left(H'(\underline{x}) + \frac{H''(\underline{x})}{H'(\underline{x})}\right)\frac{\partial H(\underline{x})}{\partial \psi} - (1 + H(\underline{x}))\frac{\partial H'(\underline{x})}{\partial \psi}\right),$$

where $M(\underline{x}) = \left[3H'(\underline{x})^2 + (1 - 2H(\underline{x}))H''(\underline{x})\right]^{-1}$*, with* $H'(\underline{x}) > 0$ *and* $M(\underline{x}) > 0$.

Then, the total effect of a change of ψ on Δp is

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}\psi} = -M(\underline{x})\left(2H'(\underline{x})\frac{\partial H(\underline{x})}{\partial \psi} + (1 - 2H(\underline{x}))\frac{\partial H'(\underline{x})}{\partial \psi}\right),$$

5.2 Reducing the bias: Changing q^*

In what follows, we analyze the effect of two different classes of policy changes designed to reduce intermediation bias: policies that increase the concern for product suitability w, and policies that reduce the commission differential Δf. Without loss of generality, if $\Delta f > 0$, then $q^* < 0.5$ and both a reduction in Δf and an increase in w will raise q^*. Intermediaries will be less biased and more trusted, increasing the fraction of consumers who follow advice. In the model, this is the single mechanism for policies that increase the concern for product suitability. Examples of such policies include the expansion of fiduciary duties or requiring better qualifications for intermediaries.

However, policies that change commissions also include a cost-effect: even if q^* remains constant, the fraction of customers hiring an intermediary–and thus demand– will change as the expected cost of intermediation changes. Therefore, we separate the discussion of these policies. We begin describing the effect of a change in q^* on equilibrium prices and demands, which is common to all policies considered. This analysis allows us to examine the impact on the equilibrium of changes that increase w, i.e., that only affect the demand through q^*. Next, we analyze the effect of policies that affect commissions $\mathbf{f}$, considering the cost-effect (i.e., the impact of a change in $\mathbf{f}$ holding q^* constant), and the overall effect ($\mathbf{f}$ affects the cost of intermediation and also recommendations).

When $\Delta f > 0$ and q^* increase, Lemma 2 implies that the effect on $\overline{x}$ is ambiguous, but $\underline{x}$ decreases, and thus the fraction of customers selecting B without intermediation decreases. Using this result, Lemma 6 describes the direct effect of a change in q^* on demand for product B.

Lemma 6. [Direct effect of q^*] *The direct effect (i.e., holding prices constant) of a change in q^* on demand for product B is*

$$\frac{\partial H(\underline{x})}{\partial q^*} = g(q^*)(F(\overline{x}) - F(\underline{x})) \\ + g(q^*)\left((f(\overline{x}) - f(\underline{x}))\Delta f\left(1 + \frac{v}{w}\right) - \kappa(f(\overline{x})(1 - G(q^*)) + f(\underline{x})G(q^*))\right). \quad (20)$$

In the symmetric case with $\Delta f = \Delta c = 0$, Lemmas 4 and 5 imply that the price differential changes according to $\frac{\mathrm{d}\Delta p}{\mathrm{d}q^*} = -\frac{2}{3H'(\underline{x})}\frac{\partial H(\underline{x})}{\partial q^*}$. Then, equation (18) implies that the total effect on demand preserves the sign of the direct effect above: $\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}q^*} = \frac{1}{3}\frac{\partial H(\underline{x})}{\partial q^*}$. Lemma 7 describes the total effect of a change in q^* in the general case.

Lemma 7. [Total effect of q^*] *The total effect of the change of q^* on D_B is*

$$\frac{\mathrm{d}D_B}{\mathrm{d}q^*} = M(\underline{x})\left(\left(H'(\underline{x})^2 + (1 - 2H(\underline{x}))H''(\underline{x})\right)\frac{\partial H(\underline{x})}{\partial q^*} - (1 - 2H(\underline{x}))\frac{\partial H'(\underline{x})}{\partial q^*}H'(\underline{x})\right),$$

where $M(\underline{x}) = \left[3H'(\underline{x})^2 + (1 - 2H(\underline{x}))H''(\underline{x})\right]^{-1} > 0$.

5.3 Increasing concern for suitability

Policies that increase the concern for product suitability enter the model only through an increase in w that affects q^* and recommendations. Examples of these policies are the expansion of fiduciary duties, which mandate that intermediaries provide advice in the best interest of their clients, or an increase in the qualifications required for intermediaries, using certifications, exams, or mandatory training. The change in w affects intermediaries' recommendations and customers' belief updating only if $\Delta f \neq 0$ and $\lambda = 1$. In this case, an increase in w always reduces bias: q^* increases if $\Delta f > 0$ and $q^* < 0.5$, while q^* decreases if $\Delta f < 0$ and $q^* > 0.5$.

The total effect on D_{B} includes the direct effect $\frac{\partial H(\overline{x})}{\partial q^*}$ (described in Lemma 6) and the indirect effect through prices $H'(\overline{x})\frac{\mathrm{d}\Delta p}{\mathrm{d}q^*}$ (described in Lemma 5). As $\frac{\partial q^*}{\partial w} = \frac{\lambda \Delta f}{2w^2}$, the total effect of a change in w on D_B is

$$\frac{\mathrm{d}D_B}{\mathrm{d}w} = \frac{\mathrm{d}D_B}{\mathrm{d}q^*}\frac{\lambda \Delta f}{2w^2},$$

where $\frac{\mathrm{d}D_B}{\mathrm{d}q^*}$ is described in Lemma 7.

If $\Delta f \neq 0$, an increase in w makes intermediaries less biased, and the fraction of customers with intermediaries increases. Because customers who hire intermediaries base their decisions only on advice, the magnitude of the elasticity of demand may decrease, and prices may increase in equilibrium.

Figure 5 shows a simulation of the effect of increasing w for 243,972 combinations of parameters and considering symmetric Beta distributions for F and G.[17] The first column shows the results for combinations with $\Delta f = \Delta c = 0$, where we observe that the effect is null, as expected. The second and third columns show the results for $\Delta f > 0$ and $\Delta f < 0$ respectively, for different levels of Δc. The simulations show that the fraction of customers hiring an intermediary $(F(\overline{x}) - F(\underline{x}))$ always increases, but the demand for product B, $H(\underline{x})$, may increase or decrease. The increase in intermediation decreases the magnitude of the elasticity of demand. Prices p_A and p_B also increase in all cases, but the change in the price differential depends on the initial level of commissions: Δp decreases if $\Delta f > 0$, as $Pr(m = B)$ increases, and increases otherwise.

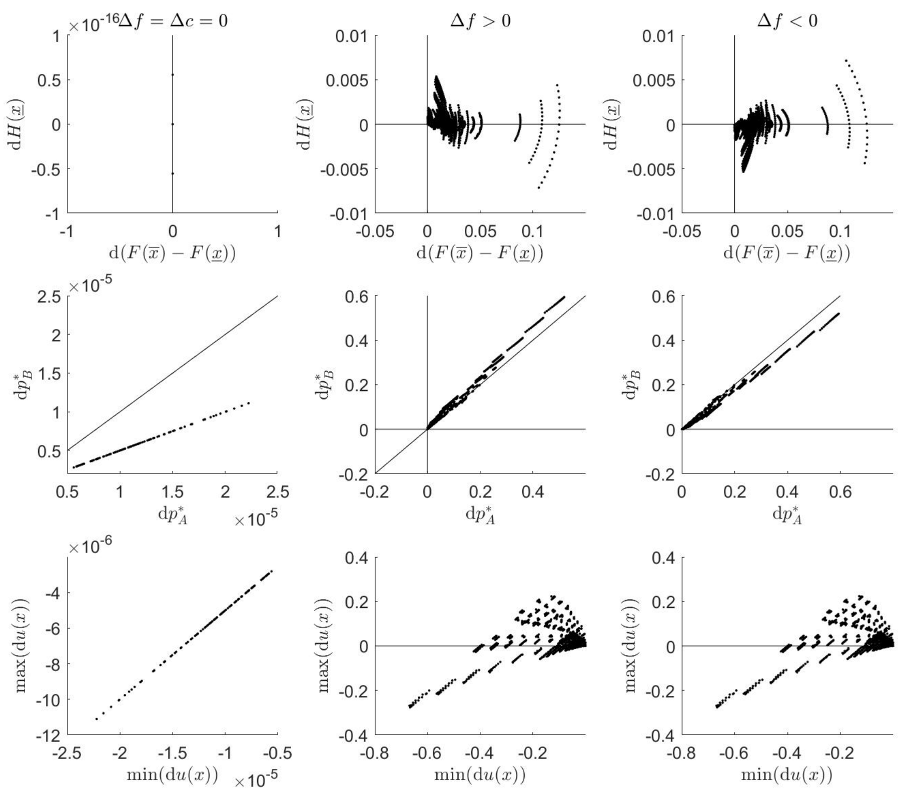


Figure 5: Effect of an increase in w

Note: Utility parameters $v \in [8, 20]$, $w \in [2, 16]$; symmetric beta distributions $F(a, a)$, $a \in [2, 4]$, and $G(b, b)$, $b \in [1, 3]$; firm costs $c_A, c_B \in [0, 4]$, and commissions $f_A, f_B \in [0.5, 4]$.

The effect on welfare is ambiguous. Customers without intermediaries are worse off when prices

[17] The simulations include different combinations of parameters in the following range of values: utility parameters $v \in [8, 20]$, $w \in [2, 16]$; symmetric beta distributions $F(a, a)$, $a \in [2, 4]$, and $G(b, b)$, $b \in [1, 3]$; firm costs $c_A, c_B \in [0, 4]$; commissions $f_A, f_B \in [0.5, 4]$. For a detailed description of the results, see Table 3 in Appendix A.11. The magnitude of the change in w is 0.5.

increase ($\min(\mathrm{d}u) < 0$ always), but for customers that hire intermediaries, the effect depends on the magnitude of the price increase relative to the gain from less biased advice. In some cases, some customers are better off (i.e., $\max(\mathrm{d}u) > 0$), while in some cases, all customers are worse off (i.e., $\max(\mathrm{d}u) < 0$). This result is relevant to the literature that discusses the effect of extending fiduciary duties: similar to Bhattacharya et al. (2020), we predict that this extension would generate a better customer-product fit, but at the risk of increasing prices and hurting some consumers.

5.4 Changing commissions

Policies that reduce the commission differential affect the demand through two channels. Indeed, holding prices constant, the direct effect of a change in f_n on demand for product B is

$$\frac{\partial H(\underline{x})}{\partial f_n} = \frac{\partial E(f|q^*)}{\partial f_n}\left(f(\underline{x}) - f(\overline{x})\right) + \frac{\partial H(\underline{x})}{\partial q^*}\frac{\partial q^*}{\partial f_n}. \tag{21}$$

The first term is the cost-effect: holding q^* constant, the change in the expected cost of intermediation affects the fraction of customers hiring an intermediary. The second term is the intermediation bias that accounts for the effect of the change in advice, that we previously discussed in Lemma 6.

Thus, we first consider the role of the cost-effect, and then analyze the overall effect that includes both mechanisms. To isolate the cost-effect, we need q^* to remain constant. The comparative statics is the same as if we assume unbiased advice (i.e., if $\lambda = 0$).

Cost effect (q^* constant, same as under unbiased advice)

With q^* constant, Lemma 2 implies that $\underline{x}$ increases and $\overline{x}$ decreases when f_n increases. The higher expected cost of intermediation decreases the fraction of customers hiring an intermediary (holding prices constant), for both $n = A, B$. This effect has an ambiguous impact on demand, depending on whether the customers ceasing to use intermediation will choose A or B. The direct cost-effect (first term in equation (21)) is positive when f_n increases if $f(\underline{x}) > f(\underline{x} + \kappa)$.

The change in the fraction of customers hiring intermediaries also affects the elasticity of demand. Equation (19) implies that the effect of a change in f_n on the elasticity depends on its effect on $H'(\underline{x})$, which changes according to

$$\frac{\partial H'(\underline{x})}{\partial f_n} = \frac{\partial E(f|q^*)}{\partial f_n}(f'(\underline{x}) - f'(\overline{x})).$$

Then, firms' best-response functions are also affected, and equilibrium prices change according to Lemma 5. The total cost effect is described in the following lemma.

Lemma 8. [Total cost-effect] *If q^* is constant (as under unbiased advice), the cost effect of the change of f_n on D_B is*

$$\left.\frac{\mathrm{d}D_B}{\mathrm{d}f_n}\right|_{q^*} = \frac{\partial E(f|q^*)}{\partial f_n}M(\underline{x})\Big((f(\underline{x}) - f(\overline{x}))H'(\underline{x})^2 + (2H(\underline{x}) - 1)(f'(\underline{x})f(\overline{x}) - f(\underline{x})f'(\overline{x}))\Big), \tag{22}$$

where $\overline{x} = \underline{x} + \kappa$ and $M(\underline{x}) = \left[3H'(\underline{x})^2 + (1 - 2H(\underline{x}))H''(\underline{x})\right]^{-1} > 0$.

In the symmetric case with $\Delta f = \Delta c = 0$, Lemmas 4, 5 and 8 imply that $\frac{\mathrm{d}\Delta p}{\mathrm{d}f_n}\Big|_{q^*} = \frac{\mathrm{d}D_\mathrm{B}}{\mathrm{d}f_n}\Big|_{q^*} = 0$, so the effect of changing the commission differential depends uniquely on the advice channel.[18] Intuitively, an increase in f_n reduces the fraction of customers using intermediaries, but this change does not affect demands because both products were equally costly as $\Delta f = \Delta p = \Delta c = 0$. If $\Delta c \neq 0$, a reduction in the fraction of customers using advisers does affect the demand, as $\Delta p \neq 0$ in this case and customers ceasing to use intermediation care about price differentials. Indeed, log-concavity of f allows us to characterize the cost effect. The following lemma describes the cost effect when $\Delta f = 0$.

Lemma 9. [Total cost-effect, symmetric case] *If $\Delta f = 0$, log-concavity of f implies that*

- $\frac{\mathrm{d}\Delta p}{\mathrm{d}f_n}\Big|_{q^*} = 0$ *and* $\frac{\mathrm{d}D_B}{\mathrm{d}f_n}\Big|_{q^*} = 0$ *if* $\Delta c = 0$.
- $\frac{\mathrm{d}D_B}{\mathrm{d}f_n}\Big|_{q^*} > 0$ *if* $\Delta c > 0$ *and* $\frac{\mathrm{d}D_B}{\mathrm{d}f_n}\Big|_{q^*} < 0$ *if* $\Delta c < 0$.

This result can be extended in the following sense: when Δp is high (i.e., when $\underline{x}$ is high), an increase in commissions increases demand for product B. Indeed, since f is log-concave, then $\frac{f(x)}{f(x+\kappa)}$ is a monotonically increasing function of x with domain $[-v, v]$ and range $[0, \infty)$ (see An (1998)). Then, $f'(\underline{x})f(\underline{x}+\kappa) > f(\underline{x})f'(\underline{x}+\kappa)$. As H is aso increasing, it follows that there is some x^* such that $H(\underline{x}) > 0.5$ and $f(\underline{x}) > f(\underline{x}+\kappa)$ for all $\underline{x} > x^*$. Lemma 8 implies that demand for B increases for all $\underline{x} > x^*$. We summarize this result in the following lemma.

Lemma 10. [Total cost-effect, general case] *If f is log-concave, there exists a level $x^* \in (-v, v)$ such that $\frac{\mathrm{d}D_B}{\mathrm{d}f_n}\Big|_{q^*} > 0$ for all $\underline{x} \geq x^*$. In other words, demand for product B shows a positive cost-effect when either commission f_A or f_B increases if $\underline{x}$ is sufficiently high.*

Note that even though the cost of product B increases when f_B increases, D_B increases if $\underline{x} > x^*$: the customers ceasing to use intermediation only care about Δp; then, the demand for product B increases if product B is sufficiently cheap.

Lemmas 9 and 10 above describe the effect of a change in f_n only on quantities and the price differential. Prices levels change according to demand elasticities and firms' best-response functions. In the symmetric case with $\Delta f = \Delta c = 0$ discussed in Lemma 4, $\frac{\partial H'(\underline{x})}{\partial f_n} > 0$ because $f'(\underline{x}) = -f'(\overline{x}) > 0$. Then, Lemma 9 and equation (19) imply that $\frac{\partial \eta_{n'}}{\partial f_n}\Big|_{q^*} \leq 0$. Intuitively, the fraction of customers hiring intermediaries increases when f_n decreases, decreasing the magnitude of demand elasticity. In turn, both p_A and p_B increase in equilibrium. Similarly, when f_n increases, the effects are opposite and equilibrium prices decrease. This intuition also applies in more general settings, as shown below.

Figure 7 shows a simulation of the effect of a change in f_n, holding q^* constant (cost effect) in our simulations.[19] Figure 7a shows the effect of a reduction in f_A: the fraction of customers hiring an intermediary ($F(\overline{x}) - F(\underline{x})$) increases and prices p_A and p_B also increase in all cases. Similarly, Figure 7b shows the effect of an increase in f_B; in this case ($F(\overline{x}) - F(\underline{x})$) always decreases and prices p_A and p_B decrease in almost all cases. The effect of these changes on demands depends on Δf and Δc. When $\Delta f = \Delta c = 0$, D_B and Δp remain constant, as Lemma 9 predicts.

[18] Note that $H(\underline{x}) = 0.5$ and $f(\underline{x}) = f(\overline{x}) = f(0.5\kappa)$ in this case.

[19] For details, see Table 3 in Appendix A.11. The magnitude of the change in f_n is 0.5 in the simulations.

Our simulations indicate that, in a vast majority of cases, commissions and prices change in the opposite direction, meaning that the welfare effect is ambiguous. Customers without intermediaries are unambiguously worse off when prices increase and they benefit when prices decrease. For customers with intermediaries, the effect depends on the magnitude of the price change. When f_A decreases the expected cost $E(p|q^*)+E(f|q^*)$ increases and utility decreases for all levels of x in a majority of cases. Similarly, when f_B increases, the expected cost $E(p|q^*) + E(f|q^*)$ decreases and utility increases for all levels of x in more than 40% of the cases.

Overall effect (q^* changes, biased advice)

We can now analyze the overall effect of a change in f_n, bringing together the effect of q^* described in section 5.2, and the cost effect described above. The cost effect is positive if Δp is sufficiently high while the bias effect can be positive or negative. If Δf decreases intermediaries recommend product B more frequently, increasing demand for it. However, as customers realize that the advice is biased, the fraction of them who follow it also changes when f_n changes and this may even reduce demand for product B.

The overall effect of the change of f_n on prices is therefore

$$\frac{\mathrm{d}p_{n'}}{\mathrm{d}f_n} = \left.\frac{\mathrm{d}p_{n'}}{\mathrm{d}f_n}\right|_{q^*} + \frac{\mathrm{d}p_{n'}}{\mathrm{d}q^*}\frac{\partial q^*}{\partial f_n},$$

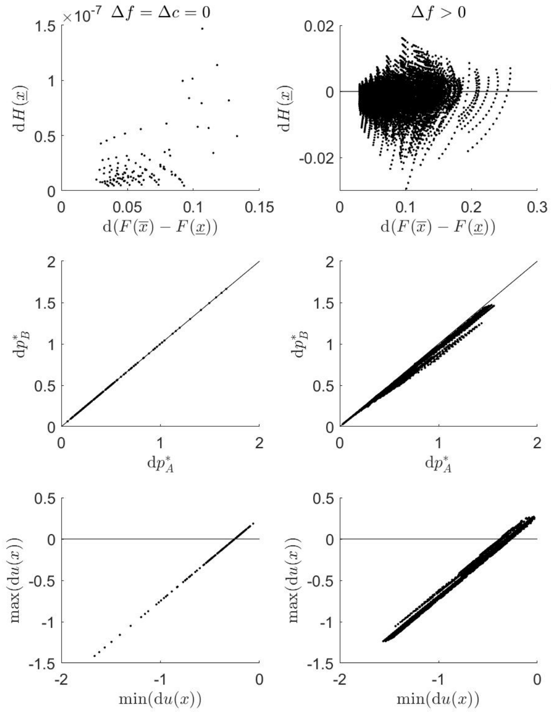
and the overall effect of the change of f_n on D_{B} under biased advice is

$$\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}f_n} = \left.\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}f_n}\right|_{q^*} + \frac{\mathrm{d}D_B}{\mathrm{d}q^*}\frac{\partial q^*}{\partial f_n}.$$

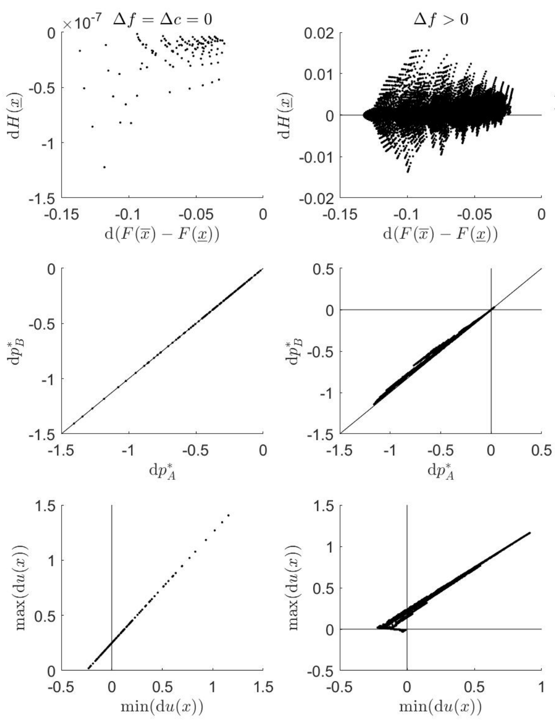
In the symmetric case with $\Delta f = \Delta c = 0$, Lemma 9 implies that $\left.\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}f_n}\right|_{q^*} = 0$. Then, the analysis in section 5.2 implies that $\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}f_n} = \frac{1}{3}\frac{\partial H(x)}{\partial q^*}\frac{\partial q^*}{\partial f_n}$. As before, the total effect on demand preserves the sign of the partial effect holding prices constant. In a general case, the overall effect adds the total effect of q^* described in Lemma 7 and the total cost-effect described in Lemma 10.

Figure 9 shows the overall effect of a change in f_n in our simulations, taking into account that q^* increases when f_A decreases or f_B increases.[20] Figure 9a shows the effect of a reduction in f_A. As the bias is reduced when $\Delta f > 0$, the bias effect reinforces the cost effect, which was positive, and the fraction of customers hiring intermediaries increases, with prices p_A and p_B still increasing in all cases. On the other hand, when $\Delta f < 0$, the bias effect is in the opposite direction to the cost effect. As the bias increases in this case, the fraction of customers hiring intermediaries decreases in some cases, thus increasing the magnitude of the elasticity of demand. Prices p_A and p_B decrease when this bias effect dominates and increase otherwise. Even though the bias is reduced when $\Delta f > 0$, the welfare effect is ambiguous: customers without intermediaries are unambiguously worse off because prices increase, but some customers hiring an intermediary may benefit because f_A decreases and the bias is reduced. The figure shows that some customers are indeed better off when $\Delta f > 0$. However, the simulations show that the expected cost $E(p|q^*)+E(f|q^*)$ increases in a majority of cases and the welfare effect is negative for all levels of x in almost 40% of the cases (and is negative for some levels of x in all cases).

[20] For details, see Table 4 in Appendix A.11.



(a) Effect of a reduction in f_A, q^* constant



(b) Effect of an increase in f_B, q^* constant

Figure 7: Simulations for changes in f_A and f_B, q^* constant

Note: Utility parameters $v \in [8, 20]$, $w \in [2, 16]$; symmetric beta distributions $F(a, a)$, $a \in [2, 4]$, and $G(b, b)$, $b \in [1, 3]$; firm costs $c_A, c_B \in [0, 4]$, and commissions $f_A, f_B \in [0.5, 4]$.

Similarly, Figure 9b shows the effect of an increase in f_B. When $\Delta f > 0$, the bias is reduced and the bias effect is in the opposite direction to the cost effect–which was negative– and the fraction of customers hiring an intermediary increases in some cases. On the other hand, when $\Delta f < 0$, the bias effect reinforces the cost effect but the simulations nonetheless show that prices decrease in the vast majority of cases for all levels of Δf. Customers without intermediaries benefit when prices decrease, but those who hire intermediaries may be worse off because f_B increases and also because the bias increases when $\Delta f < 0$. The welfare effect is negative for all levels of x in a small fraction of cases, mostly when $\Delta f < 0$ and the bias increases. Instead, utility increases for all levels of x in a fraction of cases that ranges from 20% to 35%.

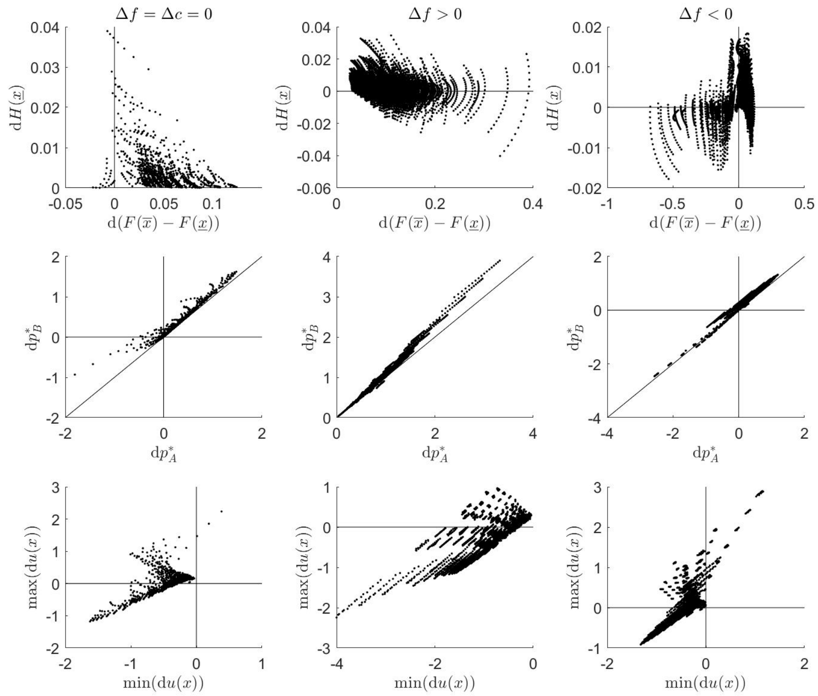
The welfare analysis highlights the relevance of taking into account firms' responses when evaluating a policy intended to reduce intermediation bias. Indeed, if the fraction of customers hiring an intermediary increases when the bias is reduced and the magnitude of the price elasticities of demand is reduced, prices increase. While the reduction in bias may benefit customers hiring an intermediary, the rise in prices hurts those without intermediaries. Even though our simulations are merely illustrative, they reveal an interesting and probably unexpected result: when $\Delta f > 0$, increasing f_B seems to be a better policy option from customers' standpoint than reducing f_A since, although both policies reduce the bias, prices tend to decrease under the former and increase under the latter.

6 Final Remarks

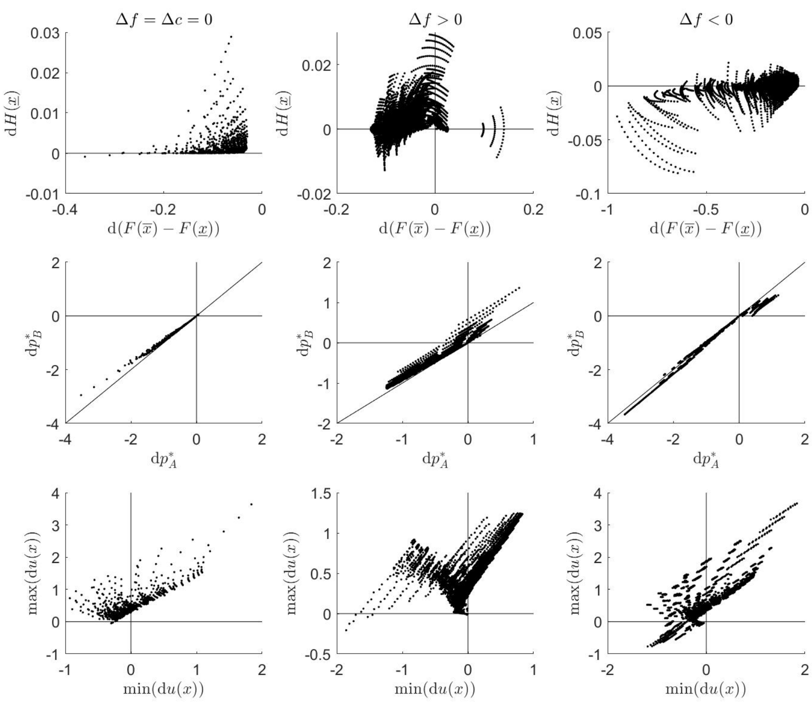
The model shows that customers' choices may be affected by the intermediary's advice in different –and possibly contradictory–ways. Indeed, a decrease in the commission differential affects the intermediary's recommendation, reducing the bias in favor of the high-commission product and modifying customers' product choice. A second and more subtle effect involves the selection of intermediary: when advice is less biased, intermediation becomes more attractive. This effect could explain why the fraction of customers hiring sales agents did not increase when the commission charged by financial advisers increased in the Chilean pension products market.

When we model price competition with intermediation explicitly, another dimension comes to notice: the magnitude of demand elasticities may decrease when the fraction of customers hiring intermediaries increases. Then, a reduction in intermediation bias–either due to a decrease in the commission differential or the increase in the concern for product suitability–may have the unintended effect of increasing prices in equilibrium. While customers using intermediaries may still benefit from the policy change, customers participating in the market without intermediaries will be unambiguously worse off in this case.

Price effects should be taken into account when evaluating policies to reduce the bias of intermediaries' recommendations. In particular, commission differentials could be decreased by lowering the highest commission or raising the lowest one. Excluding price effects, the former policy change dominates from the customers' standpoint: both the cost of intermediation and the bias of the recommendation are reduced, thus benefiting customers. However, prices tend to increase when the commission decreases because more customers hire intermediaries, and demand becomes less elastic. This price effect unambiguously hurts customers without intermediaries. If instead the lowest commission is raised, prices may decrease, thus reducing the cost. In this case, the second policy change may dominate from the customers' standpoint: even though the cost of intermediation increases, prices decrease, and the bias of the recommendation is reduced, thus benefiting customers.



(a) Effect of a reduction in f_A, overall effect



(b) Effect of an increase in f_B, overall effect

Figure 9: Simulations for changes in f_A and f_B, overall effect

Note: Utility parameters $v \in [8, 20]$, $w \in [2, 16]$; symmetric beta distributions $F(a, a)$, $a \in [2, 4]$, and $G(b, b)$, $b \in [1, 3]$; firm costs $c_A, c_B \in [0, 4]$, and commissions $f_A, f_B \in [0.5, 4]$.

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Appendix

A.1. Proof of Lemma 1

Proof. The expected utility without intermediation is

$$u_{A0}(x|p_A) = v_l + q_0(v + x) - p_A,$$
$$u_{B0}(x|p_B) = v_l + (1 - q_0)(v - x) - p_B,$$

while the expected utility with product n with an intermediary of type i ignoring his advice is

$$u_{ni}(x,y|p_n,f_{ni}) = u_{n0}(x|p_n) + v_i - f_{ni}$$

and the maximum utility attainable following an intermediary's advice is

$$u_M(x,v_1|\mathbf{p},\mathbf{f}) = v_1 + v(q_0 + G(q^*)(1-2\underline{q})) + x(q_0 - G(q^*)) - E(p|q^*) - E(f|q^*).$$

Recall that $\overline{x} = \Delta p + (1-G(q^*))\kappa$ and $\underline{x} = \Delta p - G(q^*)\kappa$, where

$$\kappa \equiv \frac{vG(q^*)(1-2\underline{q}) - E(f|q_i^*)}{G(q^*)(1-G(q^*))}$$

and

$$\overline{\mu} = \max\{\frac{1}{G(q^*)}\min\{v_1,f_{A1}\}, \frac{1}{G(q^*)}\min\{v_2,f_{A2}\}\},$$

while

$$\underline{\mu} = \max\{\frac{1}{1-G(q^*)}\min\{v_1,f_{B1}\}, \frac{1}{1-G(q^*)}\min\{v_2,f_{B2}\}\}.$$

It follows that $\max\{u_{A0}(x|p_A), u_{A1}(x,v_1|p_A,f_A), u_{A2}(x,v_2|p_A,f_A)\} > u_m(x,y|\mathbf{p},\mathbf{f})$ if $x > \overline{x}+\overline{\mu}$. Therefore, the customer chooses A ignoring recommendations in this case.

Similarly, $\max\{u_{B0}(x|p_B), u_{B1}(x,v_1|p_B,f_B), u_{B2}(x,v2|p_B,f_B)\} > u_m(x,y|\mathbf{p},\mathbf{f})$ if $x < \underline{x}-\underline{\mu}$. Therefore, the customer chooses B ignoring recommendations. □

A.2. Proof of Lemma 2

Proof. The effect of f_n on $\underline{x}$ and $\overline{x}$ is

$$\frac{\partial \underline{x}}{\partial f_n} = \frac{1}{1-G(q^*)}\frac{\partial E(f)}{\partial f_n} - \left(g(q^*)\kappa + G(q^*)\frac{\partial \kappa}{\partial q^*}\right)\frac{\partial q^*}{\partial f_n},$$

and

$$\frac{\partial \overline{x}}{\partial f_n} = -\frac{1}{G(q^*)}\frac{\partial E(f)}{\partial f_n} - \left(g(q^*)\kappa - (1-G(q^*))\frac{\partial \kappa}{\partial q^*}\right)\frac{\partial q^*}{\partial f_n},$$

where

$$\frac{\partial \kappa}{\partial q^*} = \frac{g(q^*)}{(1-G(q^*))G(q^*)}\left(\Delta f\left(1+\frac{v}{w}\right) - \kappa(1-2G(q^*))\right).$$

Then,

$$\frac{\partial \underline{x}}{\partial f_n} = \frac{1}{1-G(q^*)}\frac{\partial E(f)}{\partial f_n} - \frac{g(q^*)}{1-G(q^*)}\left((1-G(q^*))\kappa + \Delta f\left(1+\frac{v}{w}\right) - \kappa(1-2G(q^*))\right)\frac{\partial q^*}{\partial f_n},$$

and we conclude

$$\frac{\partial \underline{x}}{\partial f_n} = \frac{1}{1-G(q^*)}\frac{\partial E(f)}{\partial f_n} - \frac{g(q^*)}{1-G(q^*)}\left(G(q^*)\kappa + \Delta f\left(1+\frac{v}{w}\right)\right)\frac{\partial q^*}{\partial f_n}$$

and similarly,

$$\frac{\partial \overline{x}}{\partial f_n} = -\frac{1}{G(q^*)}\frac{\partial E(f)}{\partial f_n} - \frac{g(q^*)}{G(q^*)}\left(\kappa(1-G(q^*)) - \Delta f\left(1+\frac{v}{w}\right)\right)\frac{\partial q^*}{\partial f_n},$$

As $\frac{\partial q^*}{\partial f_A} = -\frac{1}{2w}$ and $\frac{\partial q^*}{\partial f_B} = \frac{1}{2w}$, we conclude that $\frac{\partial \underline{x}}{\partial f_A} > 0$ if $\Delta f \geq 0$ and $\frac{\partial \overline{x}}{\partial f_B} < 0$ if $\Delta f \leq 0$ (while the remaining effects are ambiguous). □

A.3. Proof of Lemma 3

Proof. The first and second-order conditions for an interior solution of the firm's problem are

$$D_n + (p_n - c_n)\frac{\partial D_n}{\partial p_n} = 0,$$

$$2\frac{\partial D_n}{\partial p_n} + (p_n - c_n)\frac{\partial^2 D_n}{\partial {p_n}^2} < 0.$$

It follows that firm-n's best response $p_n^*(p_{n'})$ satisfies

$$p_n^* = c_n - \frac{D_n}{\partial D_n / \partial p_n}. \tag{23}$$

Using the first-order condition, the second-order condition can be written as

$$2\left(\frac{\partial D_n}{\partial p_n}\right)^2 - D_n \frac{\partial^2 D_n}{\partial {p_n}^2} > 0.$$

Note that p_n^* is the fixed point of the function $B_n(p_n) \equiv c_n - \frac{D_n}{\partial D_n / \partial p_n}$ defined by the RHS of equation (23). The slope of B_n is

$$\frac{\partial B_n}{\partial p_n} = -1 + D_n \frac{\partial^2 D_n / \partial p_n^2}{\left(\partial D_n / \partial p_n\right)^2},$$

which is less than one at the fixed point if and only if the second order condition is satisfied in this critical point.

Using $\frac{\partial D_n}{\partial p_{n'}} = -\frac{\partial D_n}{\partial p_n}$, $\frac{\partial^2 D_n}{\partial p_n \partial p_{n'}} = -\frac{\partial^2 D_n}{\partial {p_n}^2}$, the slope of the best response function can be written as

$$\frac{\partial p_n^*}{\partial p_{n'}} = \frac{\left(\frac{\partial D_n}{\partial p_n}\right)^2 - D_n \frac{\partial^2 D_n}{\partial {p_n}^2}}{2\left(\frac{\partial D_n}{\partial p_n}\right)^2 - D_n \frac{\partial^2 D_n}{\partial {p_n}^2}}.$$

The second order condition for the firm's problem implies that the denominator is positive. If the slope of the best response function for firm n is negative, then $\frac{\partial^2 D_n}{\partial {p_n}^2} > 0$. But as $\frac{\partial^2 D_n}{\partial {p_n}^2} = -\frac{\partial^2 D_{n'}}{\partial {p_{n'}}^2}$, then the best response function for firm n' must be positive in this case. $\square$

A.4. Proof of $\frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial \Delta p} > 0$

Lemma 11.

$$\frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial \Delta p} > 0$$

in any informative equilibrium with two active firms.

Proof. Note first that

$$\frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial \Delta p} = \frac{2(H'(\underline{x}))^2 - (2H(\underline{x}) - 1)H''(\underline{x})}{(H'(\underline{x}))^2}.$$

Recall (see proof of Lemma 3) that the slope of the best response functions can be written as

$$\frac{\partial p_A^*}{\partial p_B} = \frac{H'(\underline{x})^2 + (1 - H(\underline{x}))H''(\underline{x})}{2H'(\underline{x})^2 + (1 - H(\underline{x}))H''(\underline{x})},$$
$$\frac{\partial p_B^*}{\partial p_A} = \frac{H'(\underline{x})^2 - H(\underline{x})H''(\underline{x})}{2H'(\underline{x})^2 - H(\underline{x})H''(\underline{x})}.$$

The second order conditions for the firm's A and B are respectively

$$-2H'(\underline{x})^2 - (1 - H(\underline{x}))H''(\underline{x}) < 0,$$
$$-2H'(\underline{x})^2 + H(\underline{x})H''(\underline{x}) < 0.$$

Then, the denominator of $\frac{\partial p_n^*}{\partial p_{n'}}$ is always positive.

If both best-response functions have positive slope, then the numerator of $\frac{\partial p_n^*}{\partial p_{n'}}$ is also positive for both firms. Adding $H'(\underline{x})^2 + (1 - H(\underline{x}))H''(\underline{x})$ and $H'(\underline{x})^2 - H(\underline{x})H''(\underline{x})$ and rearranging we obtain

$$2(H'(\underline{x}))^2 - (2H(\underline{x}) - 1)H''(\underline{x}) > 0,$$

as required, and we can conclude that $\frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial \Delta p} > 0$ in this case.

If the slope of the best-response function for firm A is negative, then $H''(\underline{x}) < 0$, implying that $-H(\underline{x})H''(\underline{x}) > 0$. And the second order contion for firm A implies that $2H'(\underline{x})^2 + (1 - H(\underline{x}))H''(\underline{x}) > 0$. Adding both inequalities we obtain the required condition.

Similarly, if the slope of the best-response function for firm B is negative, then $H''(\underline{x}) > 0$, implying that $(1 - H(\underline{x}))H''(\underline{x}) > 0$, while the second-order condition for firm B implies that $2H'(\underline{x})^2 - H(\underline{x})H''(\underline{x}) > 0$. Adding both inequalities, we again obtain the required condition.

We conclude then that $\frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial \Delta p} > 0$ in any possible case. □

A.5. Proof of Lemma 5

The effect of the change of any variable ψ on prices is

$$\frac{\mathrm{d}p_n}{\mathrm{d}\psi} = \frac{\frac{\partial p_n^*}{\partial p_{n'}}\frac{\partial p_{n'}^*}{\partial \psi} + \frac{\partial p_n^*}{\partial \psi}}{1 - \frac{\partial p_n^*}{\partial p_{n'}}\frac{\partial p_{n'}^*}{\partial p_n}}.$$

Replacing and rearranging we obtain

$$\frac{\mathrm{d}p_A}{\mathrm{d}\psi} = \frac{1}{3H'(\underline{x})^2 + (1 - 2H(\underline{x}))H''(\underline{x})}\left(\left(\frac{H''(\underline{x})}{H'(\underline{x})} - H'(\underline{x})\right)\frac{\partial H(\underline{x})}{\partial \psi} - (2 - H(\underline{x}))\frac{\partial H'(\underline{x})}{\partial \psi}\right),$$

and

$$\frac{\mathrm{d}p_B}{\mathrm{d}\psi} = \frac{1}{3H'(\underline{x})^2 + (1 - 2H(\underline{x}))H''(\underline{x})}\left(\left(H'(\underline{x}) + \frac{H''(\underline{x})}{H'(\underline{x})}\right)\frac{\partial H(\underline{x})}{\partial \psi} - (1 + H(\underline{x}))\frac{\partial H'(\underline{x})}{\partial \psi}\right).$$

Then, the effect of a change of ψ on Δp is

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}\psi} = -\frac{1}{3H'(\underline{x})^2 + (1 - 2H(\underline{x}))H''(\underline{x})}\left(2H'(\underline{x})\frac{\partial H(\underline{x})}{\partial \psi} + (1 - 2H(\underline{x}))\frac{\partial H'(\underline{x})}{\partial \psi}\right).$$

A.5. Proof of Lemma 4

Proof. We already know that $\underline{x} = -0.5\kappa$ and $H(-0.5\kappa) = 0.5$ when $\Delta p = 0$.

If $\Delta c = 0$, then $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) = 0$ when $\Delta p = \Delta c = 0$. Then, equation (17) is satisfied with $\Delta p = \Delta c$, and $D_\mathrm{A} = D_\mathrm{B} = 0.5$ in equilibrium.

If $\Delta c > 0$, then $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) = 0$ when $\Delta p = 0$, and $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) > 0$ when $\Delta p = \Delta c$. Then, equation (17) is not satisfied with $\Delta p = \Delta c$. As $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)$ is increasing in Δp, the equilibrium can only be attained with $0 < \Delta p < \Delta c$. This in turn implies that $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) = \frac{2H(\underline{x})-1}{H'(\underline{x})} > 0$ in equilibrium, and therefore $D_B = H(\underline{x}) > 0.5$.

Similarly, if $\Delta c < 0$, then then $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) = 0$ when $\Delta p = 0$ when $\Delta p = 0$ and $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) < 0$ when $\Delta p = \Delta c$. It follows that equation (17) is not satisfied with $\Delta p = \Delta c$. As $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)$ is increasing in Δp, the equilibrium can only be attained with $0 > \Delta p > \Delta c$. This in turn implies that $\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right) = \frac{2H(\underline{x})-1}{H'(\underline{x})} < 0$ in equilibrium, and therefore $D_B = H(\underline{x}) < 0.5$. □

A.6. Proof of Lemma 7

Proof. Holding prices constant, the effect of a change in q^* on D_B is

$$\frac{\partial D_B}{\partial q^*} = g(q^*)\left((f(\overline{x}) - f(\underline{x}))\Delta f\left(1 + \frac{v}{w}\right) - \kappa\left(f(\overline{x})(1 - G(q^*)) + f(\underline{x})G(q^*)\right) + F(\overline{x}) - F(\underline{x})\right).$$

Additionally,

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}q^*} = -\frac{\frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial q^*}}{1 + \frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial \Delta p}},$$

where

$$\frac{\partial\left(\frac{p_n}{\eta_n}\right)}{\partial q^*} = \frac{\frac{\partial D_n}{\partial q^*}}{\frac{\partial D_n}{\partial p_n}} - \frac{D_n}{\left(\frac{\partial D_n}{\partial p_n}\right)^2}\frac{\partial^2 D_n}{\partial p_n \partial q^*}$$

and therefore

$$\begin{aligned}
\frac{\partial\left(\frac{p_A}{\eta_A} - \frac{p_B}{\eta_B}\right)}{\partial q^*} &= \frac{\frac{\partial D_A}{\partial q^*} - \frac{\partial D_B}{\partial q^*}}{-H'(\underline{x})} - \frac{1}{H'(\underline{x})^2}\left(D_A\frac{\partial^2 D_A}{\partial p_A \partial q^*} - D_B\frac{\partial^2 D_B}{\partial p_B \partial q^*}\right) \\
&= 2\frac{g(q^*)}{H'(\underline{x})}\left((f(\overline{x}) - f(\underline{x}))\Delta f\left(1 + \frac{v}{w}\right) - \kappa\left(f(\overline{x})(1 - G(q^*)) + f(\underline{x})G(q^*)\right) + F(\overline{x}) - F(\underline{x})\right) \\
&+ (1-2H(\underline{x}))\frac{g(q^*)}{H'(\underline{x})^2}\left((f'(\overline{x}) - f'(\underline{x}))\Delta f\left(1 + \frac{v}{w}\right) - \kappa\left(f'(\overline{x})(1 - G(q^*)) + f'(\underline{x})G(q^*)\right) + f(\overline{x}) - f(\underline{x})\right).
\end{aligned}$$

It follows that

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}q^*} = -2g(q^*)\frac{H'(\underline{x})\left((f(\overline{x})-f(\underline{x}))\Delta f\left(1+\frac{v}{w}\right)-\kappa\left(f(\overline{x})(1-G(q^*))+f(\underline{x})G(q^*)\right)+F(\overline{x})-F(\underline{x})\right)}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}$$
$$-(1-2H(\underline{x}))g(q^*)\frac{\left((f'(\overline{x})-f'(\underline{x}))\Delta f\left(1+\frac{v}{w}\right)-\kappa\left(f'(\overline{x})(1-G(q^*))+f'(\underline{x})G(q^*)\right)+f(\overline{x})-f(\underline{x})\right)}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})},$$

and

$$\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}q^*} = g(q^*)\left((f(\overline{x})-f(\underline{x}))\Delta f\left(1+\frac{v}{w}\right)-\kappa\left(f(\overline{x})(1-G(q^*))+f(\underline{x})G(q^*)\right)+F(\overline{x})-F(\underline{x})\right)$$
$$-2g(q^*)\frac{H'(\underline{x})^2\left((f(\overline{x})-f(\underline{x}))\Delta f\left(1+\frac{v}{w}\right)-\kappa\left(f(\overline{x})(1-G(q^*))+f(\underline{x})G(q^*)\right)+F(\overline{x})-F(\underline{x})\right)}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}$$
$$-(1-2H(\underline{x}))g(q^*)\frac{\left((f'(\overline{x})-f'(\underline{x}))\Delta f\left(1+\frac{v}{w}\right)-\kappa\left(f'(\overline{x})(1-G(q^*))+f'(\underline{x})G(q^*)\right)+f(\overline{x})-f(\underline{x})\right)H'(\underline{x})}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}.$$

Rearranging,

$$\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}q^*} = g(q^*)\frac{H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}\left((f(\overline{x})-f(\underline{x}))\Delta f\left(1+\frac{v}{w}\right)-\kappa\left(f(\overline{x})(1-G(q^*))+f(\underline{x})G(q^*)\right)\right)$$
$$+g(q^*)\frac{H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}\left(F(\overline{x})-F(\underline{x})\right)$$
$$-(1-2H(\underline{x}))g(q^*)\frac{\left((f'(\overline{x})-f'(\underline{x}))\Delta f\left(1+\frac{v}{w}\right)-\kappa\left(f'(\overline{x})(1-G(q^*))+f'(\underline{x})G(q^*)\right)+f(\overline{x})-f(\underline{x})\right)H'(\underline{x})}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}.$$

□

A.7. Proof of Lemma 8

Proof. Under unbiased advice f_n does not affect the intermediaries' recommendation (either because $\lambda = 0$ or because $\frac{\partial \Pr(m_A)}{\partial q^*} = 0$). Hence, $\frac{\partial D_n}{\partial q^*} = \frac{\partial \eta_n}{\partial q^*} = 0$, and f_n affect firms' best response functions only through its direct effect on customers' choices:

$$\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}f_n} = \frac{\partial E(f)}{\partial f_n}\left(f(\underline{x})-f(\overline{x})\right)+\frac{\partial D_{\mathrm{B}}}{\partial \underline{x}}\frac{\mathrm{d}\Delta p}{\mathrm{d}f_n},$$

where

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}f_n} = -\frac{\frac{\partial\left(\frac{p_A}{\eta_A}-\frac{p_B}{\eta_B}\right)}{\partial f_n}}{1+\frac{\partial\left(\frac{p_A}{\eta_A}-\frac{p_B}{\eta_B}\right)}{\partial \Delta p}}.$$

But

$$\frac{\partial\left(\frac{p_A}{\eta_A}\right)}{\partial f_n} = \frac{\partial E(f)}{\partial f_n}\frac{\left(H'(\underline{x})(f(\underline{x})-f(\overline{x}))-D_A(f'(\overline{x})-f'(\underline{x}))\right)}{H'(\underline{x})^2}$$

and

$$\frac{\partial\left(\frac{p_B}{\eta_B}\right)}{\partial f_n} = -\frac{\partial E(f)}{\partial f_n}\frac{\left(H'(\underline{x})(f(\underline{x})-f(\overline{x}))+D_B(f'(\overline{x})-f'(\underline{x}))\right)}{H'(\underline{x})^2}.$$

It follows that

$$\frac{\partial\left(\frac{p_A}{\eta_A}-\frac{p_B}{\eta_B}\right)}{\partial f_n}=\frac{\partial E(f)}{\partial f_n}\frac{(2H'(\underline{x})(f(\underline{x})-f(\overline{x}))+(2H(\underline{x})-1)(f'(\overline{x})-f'(\underline{x})))}{H'(\underline{x})^2},$$

and therefore the effect on Δp is

$$\frac{\mathrm{d}\Delta p}{\mathrm{d}f_n}=-\frac{\partial E(f)}{\partial f_n}\frac{(2H'(\underline{x})(f(\underline{x})-f(\overline{x}))+(2H(\underline{x})-1)(f'(\overline{x})-f'(\underline{x})))}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})}. \tag{24}$$

The effect of f_n on D_B is therefore

$$\frac{\mathrm{d}D_\mathrm{B}}{\mathrm{d}f_n}=\frac{\partial E(f)}{\partial f_n}\left(f(\underline{x})-f(\overline{x})\right)+\frac{\partial D_\mathrm{B}}{\partial \underline{x}}\frac{\mathrm{d}\Delta p}{\mathrm{d}f_n}. \tag{25}$$

$$\frac{\mathrm{d}D_\mathrm{B}}{\mathrm{d}f_n}=\frac{\partial E(f)}{\partial f_n}\frac{(f(\underline{x})-f(\overline{x}))H'(\underline{x})^2+(2H(\underline{x})-1)(f'(\underline{x})f(\overline{x})-f(\underline{x})f'(\overline{x}))}{3H'(\underline{x})^2-(2H(\underline{x})-1)H''(\underline{x})} \tag{26}$$

where $\overline{x}=\underline{x}+\kappa$ and where the denominator is positive. □

A.8. Proof of Lemma 9

Proof. In the symmetric case with $\Delta f=\Delta c=0$ discussed in Lemma 4, $H(\underline{x})=0.5$ and $f(\underline{x})=f(\overline{x})=f(0.5\kappa)$ in equilibrium. Lemma 5 and Lemma 8 imply that $\frac{\mathrm{d}\Delta p}{\mathrm{d}f_n}\Big|_{q^*}=\frac{\mathrm{d}D_\mathrm{B}}{\mathrm{d}f_n}\Big|_{q^*}=0$ in this case.

If $\Delta c>0$, by contrast, $H(\underline{x})>0.5$ and $\Delta p>0$ in equilibrium; then, log-concavity of f implies that $f(\underline{x})>f(\overline{x})$. Therefore $\frac{\mathrm{d}D_\mathrm{B}}{\mathrm{d}f_n}\Big|_{q^*}>0$ in this case. Similarly, $f(\underline{x})<f(\overline{x})$ and, therefore, $\frac{\mathrm{d}D_\mathrm{B}}{\mathrm{d}f_n}\Big|_{q^*}<0$ if $\Delta c<0$.

Indeed, as f is log-concave, we know that for all $x_1<x_2$ and for all $\delta\geq 0$,

$$f(x_1+\delta)f(x_2)\geq f(x_2+\delta)f(x_1),$$

and for all $\alpha\geq 0$,

$$f(x_1-\alpha)f(x_2)\leq f(x_2-\alpha)f(x_1)$$

(see Lemma 1 in An (1998)).

Let $x_1=-G(q^*)\kappa$ and $x_2=(1-G(q^*))\kappa$. As $\kappa>0$, then $x_1<x_2$. Moreover, if $G(q^*)=0.5$, then $f(x_1)=f(x_2)$.

Assume that $\Delta p>0$, and let $\delta=\Delta p$. Then, we conclude that $f(x_1+\delta)\geq f(x_2+\delta)$ if $G(q^*)=0.5$. In other words, $f(\underline{x})\geq f(\overline{x})$.

Similarly, if $\Delta p<0$, we can define $\alpha=-\Delta p$. In this case, we conclude that $f(x_1-\alpha)\leq f(x_2-\alpha)$ if $G(q^*)=0.5$, and therefore $f(\underline{x})\leq f(\overline{x})$.

Using Lemma 4 and Lemma 8 we obtain the desired results. □

A.9. Proof of Lemma 10

Proof. Since f is log-concave, then $r(x) \equiv \frac{f(x)}{f(x+\kappa)}$ is a monotonically increasing function from $[-\alpha, \alpha]$ to $[0, \infty)$. Therefore, $f'(\underline{x})f(\underline{x}+\kappa) > f(\underline{x})f'(\underline{x}+\kappa)$. Moreover, as $H(\underline{x})$ is increasing and as $f(\underline{x}) = f(\underline{x}+\kappa)$ for some $\underline{x}$ in $[-\alpha, \alpha]$, then there is some $\underline{x}^*$ such that $\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}f_n} > 0$ for all $\underline{x} \geq \underline{x}^*$. $\square$

A.10. Simulations

Table 3: Simulations for a change in f_n, q^* constant (cost effect)

		Range				Frequency §		
		min	max	min %	max %	% negative	% zero	% positive
Reducing f_A, q^* constant								
$\Delta f =$	$d(F(\overline{x}) - F(\underline{x}))$	0.0266	0.133	3.29%	99.22%	0%	0%	100%
$\Delta c = 0$	dH_0	-8.73e-07	1e-06	0%	0%	0%	100%	0%
	dp_A^*	0.0637	1.6636	0.68%	20.05%	0%	0%	100%
	dp_B^*	0.0637	1.6636	0.68%	20.05%	0%	0%	100%
	$dE(p+f\|q^*)$	-0.1863	1.4136	-2.93%	14.46%	28.68%	0%	71.32%
	$du(x)^\dagger$					71.32%	0%	0%
$\Delta f > 0$	$d(F(\overline{x}) - F(\underline{x}))$	0.0267	0.2596	3.81%	4962.19%	0%	0%	100%
	dH_0	-0.0192	0.0173	-3.01%	4.8%	33.4%	45.11%	21.49%
	dp_A^*	0.03	1.5602	0.32%	17.4%	0%	0%	100%
	dp_B^*	0.0298	1.4702	0.32%	16.47%	0%	0%	100%
	$dE(p+f\|q^*)$	-0.2693	1.2353	-4.54%	9.65%	41.86%	0.26%	57.88%
	$du(x)^\dagger$					58.14%	0%	0%
$\Delta f < 0$	$d(F(\overline{x}) - F(\underline{x}))$	0.0166	0.1292	2.4%	3619.5%	0%	0%	100%
	dH_0	-0.0153	0.0155	-2.43%	4.18%	17.84%	55.09%	27.07%
	dp_A^*	0.0154	1.3395	0.16%	14.93%	0%	0%	100%
	dp_B^*	0.0155	1.3618	0.17%	15.1%	0%	0%	100%
	$dE(p+f\|q^*)$	-0.1948	1.1088	-3.2%	8.79%	43.63%	0.52%	55.85%
	$du(x)^\dagger$					56.37%	0%	0%
Increasing f_B, q^* constant								
$\Delta f =$	$d(F(\overline{x}) - F(\underline{x}))$	-0.1362	-0.0289	-100%	-3.58%	100%	0%	0%
$\Delta c = 0$	dH_0	2.83e-07	3.31e-06	0%	0%	0%	100%	0%
	dp_A^*	-1.4055	-0.0209	-13.62%	-0.22%	100%	0%	0%
	dp_B^*	-1.4055	-0.0209	-13.62%	-0.22%	100%	0%	0%
	$dE(p+f\|q^*)$	-1.1555	0.2291	-8.54%	4.4%	60.47%	0%	39.53%
	$du(x)^\dagger$					0%	0%	60.47%
$\Delta f > 0$	$d(F(\overline{x}) - F(\underline{x}))$	-0.1321	-0.0186	-3619.59%	-2.68%	100%	0%	0%
	dH_0	-0.0137	0.0139	-3.71%	2.21%	14.72%	63.49%	21.79%
	dp_A^*	-1.1629	0.0299	-10.5%	0.81%	97.58%	0%	2.42%
	dp_B^*	-1.1446	0.0299	-10.4%	0.82%	97.58%	0%	2.42%
	$dE(p+f\|q^*)$	-0.9119	0.2408	-5.96%	4.29%	44.18%	0.56%	55.25%
	$du(x)^\dagger$					2.42%	0%	44.75%
$\Delta f < 0$	$d(F(\overline{x}) - F(\underline{x}))$	-0.2713	-0.0317	-4963.21%	-4.14%	100%	0%	0%
	dH_0	-0.0164	0.015	-4.52%	2.36%	25.89%	56.9%	17.21%
	dp_A^*	-1.2359	0.0622	-11.04%	1.69%	95.45%	0.3%	4.24%
	dp_B^*	-1.3094	0.0617	-11.43%	1.66%	95.48%	0.28%	4.24%
	$dE(p+f\|q^*)$	-0.9938	0.3719	-6.3%	8.16%	42.82%	0.22%	56.96%
	$du(x)^\dagger$					4.52%	0%	43.04%

Notes: Utility parameters $v \in [8, 20]$, $w \in [2, 16]$; symmetric beta distributions $F(a, a)$, $a \in [2, 4]$, and $G(b, b)$, $b \in [1, 3]$; firm costs $c_A, c_B \in [0, 4]$, and commissions $f_A, f_B \in [0.5, 4]$.

§ The frequency columns represent for how many combinations of parameters, the change in the corresponding variable is considered negative ($\Delta < -9e{-}04$), zero ($-9e{-}04 < \Delta < 9e{-}04$), or positive ($\Delta > 9e{-}04$).

† Similarly, for the change in utility, the frequency columns represent for how many combinations of parameters, utility weakly decreases $\forall x \in [-v, v]$ ($du(x) < 9e{-}04 \forall x$, "% negative"), utility is constant $\forall x \in [-v, v]$ ($-9e{-}04 < du(x) < 9e{-}04 \forall x$, "% zero"), or utility weakly increases $\forall x \in [-v, v]$ ($du(x) > -9e{-}04 \forall x$, "% positive"). These frequencies do not total 100%: the difference represents combinations of parameters with mixed results, that is, utility strictly decreases for some x and strictly increases for others.

Table 4: Simulations for a change in f_n, q^* changes (overall effect)

		Range				Frequency §		
		min	max	min %	max %	% negative	% zero	% positive
Reducing f_A, overall effect (q^* changes)								
$\Delta f =$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	-0.029	0.1243	-16.04%	92.64%	3.59%	0.29%	96.12%
$\Delta c = 0$	$\mathrm{d}H_0$	9.87e-06	0.0131	0%	2.62%	0%	52.71%	47.29%
	$\mathrm{d}p_A^*$	-1.8056	1.4787	-10.02%	18.7%	10.08%	0%	89.92%
	$\mathrm{d}p_B^*$	-0.9312	1.6216	-5.17%	19.41%	7.27%	0.29%	92.44%
	$\mathrm{d}E(p+f\|q^*)$	-1.4466	1.3102	-7.81%	13.68%	41.28%	0.39%	58.33%
	$\mathrm{d}u(x)^\dagger$					35.17%	0%	0.29%
$\Delta f > 0$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	0.0245	0.3925	3.15%	5732.46%	0%	0%	100%
	$\mathrm{d}H_0$	-0.0316	0.0291	-4.73%	8.79%	23%	39.26%	37.75%
	$\mathrm{d}p_A^*$	0.0404	3.3183	0.43%	24.55%	0%	0%	100%
	$\mathrm{d}p_B^*$	0.0402	3.8959	0.43%	26.27%	0%	0%	100%
	$\mathrm{d}E(p+f\|q^*)$	-0.4656	3.0557	-5.38%	18.18%	37.07%	0.26%	62.67%
	$\mathrm{d}u(x)^\dagger$					39.5%	0%	0%
$\Delta f < 0$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	-0.6734	0.1187	-617.14%	2475.33%	11.18%	0.5%	88.32%
	$\mathrm{d}H_0$	-0.0137	0.0144	-3.43%	3.88%	10.14%	55.31%	34.55%
	$\mathrm{d}p_A^*$	-2.5656	1.1878	-17.52%	13.18%	13.51%	0.17%	86.32%
	$\mathrm{d}p_B^*$	-2.4576	1.3219	-16.16%	13.76%	12.1%	0.08%	87.82%
	$\mathrm{d}E(p+f\|q^*)$	-2.6373	1.0132	-15.65%	7.73%	60.95%	0.51%	38.54%
	$\mathrm{d}u(x)^\dagger$					23.05%	0%	1.02%
Increasing f_B, overall effect (q^* changes)								
$\Delta f =$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	-0.3601	-0.0305	-265.9%	-3.78%	100%	0%	0%
$\Delta c = 0$	$\mathrm{d}H_0$	-0.0003	0.0097	-0.06%	1.93%	0%	76.74%	23.26%
	$\mathrm{d}p_A^*$	-3.5193	0.0547	-20.8%	1.48%	99.71%	0%	0.29%
	$\mathrm{d}p_B^*$	-2.9473	0.0504	-19.69%	1.36%	99.71%	0%	0.29%
	$\mathrm{d}E(p+f\|q^*)$	-2.8493	0.414	-15.38%	8.81%	63.95%	0.19%	35.85%
	$\mathrm{d}u(x)^\dagger$					0.29%	0%	35.47%
$\Delta f > 0$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	-0.131	0.1398	-2752.39%	79.33%	96.3%	0.27%	3.43%
	$\mathrm{d}H_0$	-0.0135	0.0139	-3.65%	2.26%	9.71%	58.55%	31.74%
	$\mathrm{d}p_A^*$	-1.2451	0.7799	-10.5%	5%	93.69%	0.1%	6.21%
	$\mathrm{d}p_B^*$	-1.1415	1.3599	-10.29%	8.8%	92.68%	0.03%	7.29%
	$\mathrm{d}E(p+f\|q^*)$	-0.9266	1.0185	-5.89%	6.79%	38.09%	0.42%	61.5%
	$\mathrm{d}u(x)^\dagger$					1.23%	0%	20.7%
$\Delta f < 0$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	-0.9086	-0.0326	-6218.76%	-4.21%	100%	0%	0%
	$\mathrm{d}H_0$	-0.0332	0.0291	-8.72%	5.6%	27.14%	47.73%	25.13%
	$\mathrm{d}p_A^*$	-3.4786	1.1872	-22.53%	22.12%	91.92%	0.2%	7.88%
	$\mathrm{d}p_B^*$	-3.6651	0.7678	-23.05%	13.31%	92.03%	0.1%	7.87%
	$\mathrm{d}E(p+f\|q^*)$	-3.2732	1.544	-18.46%	20.43%	47.44%	0.26%	52.3%
	$\mathrm{d}u(x)^\dagger$					7.97%	0%	27.01%

Notes: Utility parameters $v \in [8, 20]$, $w \in [2, 16]$; symmetric beta distributions $F(a, a)$, $a \in [2, 4]$, and $G(b, b)$, $b \in [1, 3]$; firm costs $c_A, c_B \in [0, 4]$, and commissions $f_A, f_B \in [0.5, 4]$.

§ The frequency columns represent for how many combinations of parameters, the change in the corresponding variable is considered negative ($\Delta < -9\mathrm{e}{-04}$), zero ($-9\mathrm{e}{-04} < \Delta < 9\mathrm{e}{-04}$), or positive ($\Delta > 9\mathrm{e}{-04}$).

† Similarly, for the change in utility, the frequency columns represent for how many combinations of parameters, utility weakly decreases $\forall x \in [-v, v]$ ($\mathrm{d}u(x) < 9\mathrm{e}{-04} \forall x$, "% negative"), utility is constant $\forall x \in [-v, v]$ ($-9\mathrm{e}{-04} < \mathrm{d}u(x) < 9\mathrm{e}{-04} \forall x$, "% zero"), or utility weakly increases $\forall x \in [-v, v]$ ($\mathrm{d}u(x) > -9\mathrm{e}{-04} \forall x$, "% positive"). These frequencies do not total 100%: the difference represents combinations of parameters with mixed results, that is, utility strictly decreases for some x and strictly increases for others.

Table 5: Simulations for a change in w

		Range				Frequency §		
		min	max	min %	max %	% negative	% zero	% positive
Increasing w								
$\Delta f =$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	2.4e-14	3.06e-12	0%	0%	0%	100%	0%
$\Delta c = 0$	$\mathrm{d}H_0$	8.9e-08	1.1e-06	0%	0%	0%	100%	0%
	$\mathrm{d}p^*_A$	5.62e-06	2.22e-05	0%	0%	0%	100%	0%
	$\mathrm{d}p^*_B$	2.81e-06	1.11e-05	0%	0%	0%	100%	0%
	$\mathrm{d}E(p + f\|q^*)$	4.21e-06	1.67e-05	0%	0%	0%	100%	0%
	$\mathrm{d}u(x)^\dagger$					0%	100%	0%
$\Delta f > 0$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	-0.0002	0.1255	-0.03%	150.46%	0%	54.17%	45.83%
	$\mathrm{d}H_0$	-0.0058	0.0053	-0.86%	1.6%	0.41%	98.51%	1.07%
	$\mathrm{d}p^*_A$	-0.0011	0.5191	-0.01%	3.74%	0.005%	21.76%	78.24%
	$\mathrm{d}p^*_B$	4.53e-05	0.5939	0%	3.92%	0%	15.85%	84.15%
	$\mathrm{d}E(p + f\|q^*)$	-0.0604	0.4978	-0.7%	3.17%	15.5%	27.31%	57.19%
	$\mathrm{d}u(x)^\dagger$					5.82%	1.86%	7.44%
$\Delta f < 0$	$\mathrm{d}(F(\overline{x}) - F(\underline{x}))$	-0.0002	0.1255	-0.03%	150.46%	0%	54.17%	45.83%
	$\mathrm{d}H_0$	-0.0053	0.0058	-0.79%	1.75%	1.07%	98.51%	0.41%
	$\mathrm{d}p^*_A$	3.29e-05	0.5939	0%	3.92%	0%	15.84%	84.16%
	$\mathrm{d}p^*_B$	-0.0011	0.5191	-0.01%	3.74%	0.005%	21.91%	78.09%
	$\mathrm{d}E(p + f\|q^*)$	-0.0604	0.4978	-0.7%	3.17%	15.5%	27.3%	57.21%
	$\mathrm{d}u(x)^\dagger$					5.83%	1.86%	7.45%

Notes: Utility parameters $v \in [8, 20]$, $w \in [2, 16]$; symmetric beta distributions $F(a, a)$, $a \in [2, 4]$, and $G(b, b)$, $b \in [1, 3]$; firm costs $c_A, c_B \in [0, 4]$, and commissions $f_A, f_B \in [0.5, 4]$.

§ The frequency columns represent for how many combinations of parameters, the change in the corresponding variable is considered negative ($\Delta < -9\text{e}{-04}$), zero ($-9\text{e}{-04} < \Delta < 9\text{e}{-04}$), or positive ($\Delta > 9\text{e}{-04}$).

† Similarly, for the change in utility, the frequency columns represent for how many combinations of parameters, utility weakly decreases $\forall x \in [-v, v]$ ($\mathrm{d}u(x) < 9\text{e}{-04} \forall x$, "% negative"), utility is constant $\forall x \in [-v, v]$ ($-9\text{e}{-04} < \mathrm{d}u(x) < 9\text{e}{-04} \forall x$, "% zero"), or utility weakly increases $\forall x \in [-v, v]$ ($\mathrm{d}u(x) > -9\text{e}{-04} \forall x$, "% positive"). These frequencies do not total 100%: the difference represents combinations of parameters with mixed results, that is, utility strictly decreases for some x and strictly increases for others.