INSTITUTO DE ECONOMÍA

DOCUMENTO de TRABAJO

554
2020

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www.economia.puc.cl • ISSN (edición impresa) 0716-7334 • ISSN (edición electrónica) 0717-7593

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November 10, 2020

Abstract

Firms and other institutions frequently evaluate their employees. Some firms ask for their employees to complete self-evaluation reports (SERs). The consensus in the business literature is that SERs are not credible and should only be used as a developmental tool. I discuss when SERs are useful and when they are not for a firm that wishes to reward only its good workers: SERs are useful when the job requires multidimensional skills or when the employees have private information about the quality of their evaluators; they are not useful when the job description is unidimensional.

JEL classification: D82, H41

Keywords: mechanism design, evidence, communication.

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1 Introduction

In firms and in many other organizations, employees are evaluated by their supervisors. These evaluations are instrumental in determining the employee's salary and/or career progression. An increasily common practice is for firms to ask their employees to complete a self-evaluation report (SER), also referred to as self-appraisal, self-assessment or simply self-report in the business literature. In fact, this tendency has led to the emergence of a range of services which help workers fill out their SERs. However, the reliability of SERs has long been put into question; it is argued that employees are too lenient when self-evaluating compared to other more objective measures of performance (see, for example, the extensive review of Campbell and Lee, 1988). This perceived unreliability has led to the recommendation by several business guides to use SERs merely as a developmental tool for the employees. For example, in a 2011 article in the Harvard Business Review authored by Dick Grote and appropriately titled "Let's Abolish Self-Appraisal", the author relays the following interaction with a business manager: "One senior executive, describing his company's experience using a forced-ranking procedure to identify its A, B, and C performers, told me of the same problem (of unreliability): “The As are afraid they'll be considered Bs, the Bs are scared they'll be seen as Cs, and all the Cs are convinced that they're A players.”"

On the other hand, there are non-business related examples of evaluations which partially rely on self-evaluations. A clear example is that of criminal investigations. When there is doubt over a suspect's innocence, that suspect is often interrogated and asked a series of questions aimed at determining his guilt. Another example is the use of self-reporting of income for taxation purposes. In either of these examples, SERs make sense because the subject being evaluated is able to provide "evidence" that supports his claims.

At first glance, applying this sort of logic to the use of SERs in business appears problematic, because it seems unlikely that workers are able to provide hard evidence about their performance that is unavailable to the firm. SERs are mostly instruments that allow workers to argue for their worth; rather than presenting some physical evidence, they simply enunciate the reasons why they should be rewarded. In the economic theory language, SERs are cheap talk. Nevertheless, I argue in this article that even when SERs are merely cheap talk, they might still be useful. Specifically, I discuss when the use of SERs is a part of the "optimal" mechanism for the evaluator, who is assumed to want to reward more the better quality workers.

The main general results of the article are a sufficient condition for which SERs are not necessary for the evaluator to conduct an "optimal" evaluation and a sufficient condition for which they are. The result that SERs are sometimes necessary is based on the principle that the agent - the one being evaluated - might want to be evaluated differently depending on what he knows. To fix ideas, let me describe two simple examples; one where SERs are not likely to be necessary and another where they are.

Imagine that a telemarketing firm wants to evaluate its workers' performance over the course of the year in order to determine whether to fire them or retain them at the end of the year. The telemarketing firm only cares about the workers' ability to sell and observes each workers' yearly sales' record. In such a setting, the more able the worker is, the more likely he is to have a good sales' record. As a result, if, hypothetically, a good worker - a worker the evaluator would like to retain - was asked at the beginning of the year how he would like to be evaluated, he would say that he would prefer to be rewarded if his sales' record was good enough. Naturally, this is information the evaluator could deduce. As a result and as I prove in the article, in these circumstances, SERs are pointless.

Consider a slightly different example where a university evaluates its faculty members in order to determine whether to tenure them. The university cares about the faculty's ability to do research and to teach. Throughout the tenure track period, the university has access to the faculty's publications and to their teaching evaluations. Things are not as simple in this example as in the previous one because different good workers might prefer to be evaluated differently. For example, think of a faculty member who is not very good at teaching but is good enough at research for the university to want to tenure him. That faculty member would prefer to be evaluated based mostly on the value of his publications rather than on his teaching evaluations. By contrast, someone who is very good at teaching but not so good at research might prefer the opposite. When this happens, I show in the article that SERs must be a part of any optimal evaluation mechanism.

While in both examples, the workers being evaluated have private information that is relevant for the evaluator - their ability - the nature of their private information is different. Unlike the first example, in the second example there are different ways of being a good worker. Therefore, different good workers might prefer to be evaluated differently and that is what makes SERs useful. One can then conclude that SERs are not important for jobs that are one-dimensional, where there is only one way of doing the task required and the worker is either good or bad at it; but are important for jobs that are multidimensional, where not all good workers are the same. One such example that is discussed in the article is the much debated topic of how to evaluate teachers.

Teachers are supposed to teach a variety of skills to their students. Some of these skills are easier to measure than others: for example, the ability to perform simple arithmetic exercises or the ability to follow simple orders can be more easily measured than critical thinking. Additionally, it is natural for formal evaluations to put more weight on the metrics that are more reliable. The problem that has been identified is that this leads teachers to "teach to the test": teachers shift their resources from developing skills that are harder to measure (like critical thinking) to skills that are more easily measurable (like the ability to memorize and to follow simple rules, which can be measured more accurately through standardized tests).[1] I show that one can mitigate this problem by creating an evaluation mechanism that gives teachers the ability to choose how they want to be evaluated. In this way, those teachers who are better at teaching less measurable skills have an incentive to choose to put more weight into the (less reliable) signals which measure those skills rather than be forced to be evaluated mostly through their students' scores in standardized tests.

Finally, I also discuss another circumstance in which SERs are a part of any optimal evaluation mechanism which is when the subject being evaluated privately knows which of his many possible evaluators is better suited to evaluate him. This is something that happens frequently in academic work when authors are often asked to choose (either directly or indirectly) the journal (co-)editor they prefer to judge their work. SERs make sense because, once again, good workers might prefer to be evaluated differently; an author who writes a good mechanism design paper for example is likely to prefer to be evaluated by a mechanism design expert while an author who writes a good matching paper might prefer a matching expert instead.

The paper proceeds as follows. In section 2, I present the model; in section 3, I present the main general results; in section 4, I discuss the implications of the main results in terms of the determining how and whether to use SERs; in section 5, I discuss the related literature, while in section 6, I conclude.

[1] I discuss the debate over teaching evaluations in section 4.3.

2 Model

There is an agent (he) and an evaluator (she). The agent has a value $v \in [\underline{v}, \overline{v}]$ associated to him - his skill, ability, "fit" within the firm, etc. The evaluator wishes to reward the agent but only if his value v is larger than some (opportunity) cost $c \in (\underline{v}, \overline{v})$. Specifically, the evaluator chooses a probability $x \in [0, 1]$ to maximize $(v - c)\, x$. Cost c might include the cost of hiring the worker, the salary raise that might be given to the worker, the opportunity cost of not hiring a different worker of value c, etc. The worker simply wants to maximize the probability x that he is rewarded, regardless of his private information.

While c is commonly known, v is an unobservable random variable. However, both the agent and the evaluator observe signals correlated with v. In particular, the agent privately observes signal $s \in S$ while the evaluator privately observes signal $z \in Z$. I assume that both sets S and Z are finite and write $p(s, z)$ to denote the joint probability of vector (s, z). To facilitate the expositions of results, I also assume that $p(s, z) > 0$ for all $s \in S$ and $z \in Z$, so that every vector is possible (albeit possibly with a very small probability).

Finally, I make a more fundamental assumption that is meant to capture the idea that the agent has more information than the evaluator; that $E(v|s, z) = E(v|s)$ for all $s \in S$ and $z \in Z$. This means that knowing z is irrelevant (in terms of the conditional expectation) if one already knows s. Notice that this assumption is satisfied if the agent directly observes v, either because $s = v$ or because $s = (v, y)$ for some other random variable y that might be correlated with z. In the latter case, the agent knows something about the distribution of the signal of the expert in addition to knowing his own value.

Let

$$\overline{S} = \{s \in S : E(v|s) \geq c\}$$

and

$$\underline{S} = \{s \in S : E(v|s) < c\}.$$

Notice that if the evaluator was able to directly observe s, she would reward the agent $(x = 1)$ if $s \in \overline{S}$ and would not $(x = 0)$ if $s \in \underline{S}$. I sometimes refer to elements of $\overline{S}$ as "high" types and elements of $\underline{S}$ as "low" types. If either set is empty, the problem is trivial (the evaluator either blindly rewards the agent or she does not), so I assume that neither set is empty.

An allocation is denoted by $h : S \times Z \rightarrow [0, 1]$ and is incentive compatible (IC) if

$$E\left(h\left(s,z\right)|s\right) \geq E\left(h\left(s',z\right)|s\right)$$

for all $s' \in S$ and $s \in Z$. Notice that this definition of incentive compatibility assumes that the agent reports his type - his signal s - *before* knowing the realization of the expert's signal z. That is how incentives are given to the agent; the agent might have different beliefs about z depending on the realization of s.

The article studies optimal mechanisms; mechanisms that the evaluator wants to implement anticipating that agents will play a Bayes-Nash equilibrium of the evaluator's choosing. I assume that the evaluator is able to implement the mechanism that she chooses before z is realized (or before the agent knows its realization). Therefore, in the example of the introduction, the evaluation mechanism of a faculty member is determined at the beginning of the tenure clock, before the faculty member knows how well he will publish and what his teaching evaluations will look like. By the revelation principle (Myerson, 1979), it is enough to focus on direct revelation mechanisms, i.e., on IC allocations. An optimal IC allocation is an allocation h that maxmizes the evaluator's expected payoff, which is given by

$$W(h) \equiv E\left(\left(v-c\right)h\left(s,z\right)\right),$$

among all IC allocations.

3 General results

Before stating the results, it is necessary to first introduce some concepts. Let Γ denote the set of all possible orderings of elements of Z. For example, if $Z = \{a, b, c\}$, then

$$\Gamma = \left\{\langle a,b,c\rangle, \langle a,c,b\rangle, \langle b,a,c\rangle, \langle b,c,a\rangle, \langle c,a,b\rangle, \langle c,b,a\rangle\right\}.$$

Let n denote the number of elements of set Z and define set $\widehat{S} \subseteq \overline{S}$ to be the set of elements $s \in \overline{S}$ for which there exists $\gamma = \langle z_1, ..., z_n\rangle \in \Gamma$ such that, for all $s' \in \underline{S}$, $\frac{p(z_i|s)}{p(z_i|s')}$ is weakly increasing with i for all $i = 1, ..., n$. I say that (s, z) satisfies *partial monotonicity* (PM) if $\widehat{S} = \overline{S}$ and *complete monotonicity* (CM) if there is a single $\gamma = \langle z_1, ..., z_n\rangle \in \Gamma$ such that, for all $s \in \overline{S}$ and $s' \in \underline{S}$, $\frac{p(z_i|s)}{p(z_i|s')}$ is weakly increasing with i for all $i = 1, ..., n$.

In words, if a high type s belongs to set $\widehat{S}$ it is because one can reorder Z in such a way that the likelihood ratio $\frac{p(\cdot|s)}{p(\cdot|s')}$ is weakly increasing for any low type s'. If it is possible to do this for all high types, there is PM. CM implies PM but also requires that the reordering of Z is the same for all high types. Below, I present an example that illustrates these concepts.

Example 1 *Assume that* $S = \{s_1, s_2, s_3, s_4\}$ *and that*

$$E(v|s_4) > E(v|s_3) > c > E(v|s_2) > E(v|s_1)$$

so that

$$\overline{S} = \{s_3, s_4\} \text{ and } \underline{S} = \{s_1, s_2\}.$$

Finally, assume that $Z = \{l, m, h\}$.

In the following table on the left I represent the probability of each $z \in Z$ *given* $s \in S$. *On the right, I represent the likelihood ratio* $\frac{p(z|\overline{s})}{p(z|\underline{s})}$ *for all* $z \in Z$ *and any pair* $(\overline{s}, \underline{s})$ *such that* $\overline{s} \in \overline{S}$ *and* $\underline{s} \in \underline{S}$.

$p(z\|s)$	l	m	h
s_4	0.25	0.25	0.5
s_3	0.2	0.4	0.4
s_2	0.5	0.25	0.25
s_1	0.4	0.4	0.2

→

$\frac{p(z\|\overline{s})}{p(z\|\underline{s})}$	l	m	h
$\frac{s_4}{s_2}$	0.5	1	2
$\frac{s_4}{s_1}$	0.625	0.625	2.5
$\frac{s_3}{s_2}$	0.4	1.6	1.6
$\frac{s_3}{s_1}$	0.5	1	2

To see whether s_4 *belongs to set* $\widehat{S}$ *one has to verify if it is possible to reorder* Z *in such a way that both* $\frac{p(\cdot|s_4)}{p(\cdot|s_2)}$ *and* $\frac{p(\cdot|s_4)}{p(\cdot|s_1)}$ *are weakly increasing. A close look at the right hand side of the table is enough to see that order* $\gamma = \langle l, m, h \rangle$ *works, so* $s_4 \in \widehat{S}$. *Indeed, using the same order* $\gamma = \langle l, m, h \rangle$ *one can see that both* $\frac{p(\cdot|s_3)}{p(\cdot|s_2)}$ *and* $\frac{p(\cdot|s_3)}{p(\cdot|s_1)}$ *are also weakly increasing. As a result, not only does* s_3 *also belong to* $\widehat{S}$, *so that there is PM; it is also the case that the same order works for all types in set* $\widehat{S}$, *which means that there is CM.*

Suppose instead that we replace the previous table by the following:

$p(z\|s)$	l	m	h
s_4	0.25	0.25	0.5
s_3	0.25	0.5	0.25
s_2	0.5	0.25	0.25
s_1	0.4	0.4	0.2

→

$\frac{p(z\|\overline{s})}{p(z\|\underline{s})}$	l	m	h
$\frac{s_4}{s_2}$	0.5	1	2
$\frac{s_4}{s_1}$	0.625	0.625	2.5
$\frac{s_3}{s_2}$	0.5	2	1
$\frac{s_3}{s_1}$	0.625	1.25	1.25

Notice that $\widehat{S} = \overline{S} = \{s_3, s_4\}$ *as before but the difference is that the order that allows* s_4 *to join set* $\widehat{S}$ *is not the same order that allows* s_3 *to join the same set. In particular, for the likelihood ratios of* s_4 *to be weakly increasing, the order used must be* $\gamma = \langle l, m, h\rangle$ *but for* s_3 *it must be* $\gamma = \langle l, h, m\rangle$*. Therefore, in this case, we have PM but not CM.*

Finally, it is also possible to have neither as the following table illustrates:

$p(z\|s)$	l	m	h
s_4	0.25	0.25	0.5
s_3	0.4	0.35	0.25
s_2	0.5	0.25	0.25
s_1	0.4	0.4	0.2

$\rightarrow$

$\frac{p(z\|\overline{s})}{p(z\|\underline{s})}$	l	m	h
$\frac{s_4}{s_2}$	0.5	1	2
$\frac{s_4}{s_1}$	0.625	0.625	2.5
$\frac{s_3}{s_2}$	0.8	1.4	1
$\frac{s_3}{s_1}$	1	0.875	1.25

In this case, it follows that $s_4 \in \widehat{S}$ *(using order* $\gamma = \langle l, m, h\rangle$*) but there is no order that allows* s_3 *to join set* $\widehat{S}$*; for* $\frac{p(\cdot|s_3)}{p(\cdot|s_2)}$ *to be weakly increasing, the order would have to be* $\gamma = \langle l, h, m\rangle$*, while for* $\frac{p(\cdot|s_3)}{p(\cdot|s_1)}$ *to be weakly increasing, the order would have to be* $\gamma = \langle m, l, h\rangle$*. Therefore, because* $\widehat{S} \subset \overline{S}$*, we have neither CM nor PM.*

3.1 When are SERs not necessary?

The following proposition partially characterizes an optimal IC allocation.

Proposition 1 *Let* $\underline{s}$ *be any arbitrary element of* $\underline{S}$*. There is an optimal IC allocation* $\widehat{h}$ *such that i) for each* $s \in \widehat{S}$*, there is* $\alpha(s) \in \mathbb{R}_+$ *and* $\beta(s) \in [0,1]$ *such that*

$$\widehat{h}(s,z) = \begin{cases} 1 \text{ if } \frac{p(z|s)}{p(z|\underline{s})} > \alpha(s) \\ \beta(s) \text{ if } \frac{p(z|s)}{p(z|\underline{s})} = \alpha(s) \\ 0 \text{ if } \frac{p(z|s)}{p(z|\underline{s})} < \alpha(s) \end{cases}$$

and ii) for each $s \in \underline{S}$*,* $\widehat{h}(s,\cdot) = \widehat{h}(\omega(s),\cdot)$*, where*

$$\omega(s) \in \arg\max_{s' \in \overline{S}} E\left(\widehat{h}(s',z)\,|s\right).$$

Proof. See appendix. ■

Proposition 1 describes the lotteries over Z assigned to the agent when $s \in \widehat{S}$ (part i) and when $s \in \underline{S}$ (part ii). Let us start by part i) and assume that $s \in \widehat{S}$. That means that there is some order $\gamma^s \in \Gamma$ for which the likelihood ratio between type s and any other type $s' \in \underline{S}$ is weakly increasing with respect to that order. The proposition states that, in that case, the agent is rewarded if and only if z is high enough according to order γ^s.

The intuition is as follows: recall that the evaluator simply wishes to reward the agent as much as possible when $s \in \overline{S}$ and as little as possible when $s \in \underline{S}$. By rewarding a high type like s according to order γ^s she ensures that low types who deviate to reporting s are less likely to get rewarded, because, by construction, the higher one is in order γ^s the more likely it is that the agent's signal is s and the less likely it is some $s' \in \underline{S}$.

When $s \in \underline{S}$, the proposition simply states that the agent will receive a lottery that is equal to the lottery he would be assigned if his type was $\omega(s)$ - a high type. This then implies that the number of lotteries assigned to the agent is not larger than the number of high types.

The argument is fairly straightforward: notice that, whenever $s \in \underline{S}$, the agent must be indifferent to mimicking a high type; if not, the evaluator could gain by reducing his rewards. But then, if the agent is indifferent between reporting $s \in \underline{S}$ and some other $\omega(s) \in \overline{S}$ whenever s is realized, the evaluator is also indifferent about what he decides to do, so she may as well just assign the agent the lottery assigned to type $\omega(s) \in \overline{S}$.

Proposition 1 is particularly useful when there is at least PM, because, in that case, it fully characterizes the optimal IC allocation. Indeed, I will use proposition 1 in the following sections to characterize optimal IC allocations. From a more practical point of view, the main implication of proposition 1 is the corollary below, which determines a sufficient condition for the irrelevance of SERs.

Let allocation h^* be such that

$$h^*(s,z) = \begin{cases} 1 \text{ if } E(v|z) \geq c \\ 0 \text{ if } E(v|z) < c \end{cases}$$

for all $(s,z) \in S \times Z$. Notice that allocation h^* is an optimal allocation when SERs are not allowed, because, of all allocations which do not depend on the agent's signal s, h^* is the evaluator's preferred allocation. Whenever allocation h^* is an optimal IC allocation, I write that SERs are not necessary.

Corollary 1 *If (s,z) satisfies CM, then SERs are not necessary.*

The intuition for the result is as follows: SERs are only necessary when different types have different things they would like to communicate to the evaluator. By the nature of the problem, the evaluator is only interested in indulging the high types. If there is CM, all high types prefer to be "evaluated in the same way", which negates the need for SERs.

Being "evaluated in the same way" has a specific meaning. Divide set Z into two disjoint sets Z^1 and Z^0 such that the agent is rewarded ($x = 1$) if $z \in Z^1$ and is not rewarded if $z \in Z^0$. Consider the following thought experiment: ask the agent of type $\overline{s} \in \overline{S}$ to determine sets Z^1 and Z^0 with the constraint that some type $\underline{s} \in \underline{S}$ must have an expected payoff of (at most) $\overline{u} \in [0, 1]$. The property of CM implies that, for any $\underline{s} \in \underline{S}$ and any $\overline{u} \in [0, 1]$, all high types answer the same pair (Z^1, Z^0). It is in this sense that every high type wants to be evaluated in the same way.

Below, I provide the proof of the result which follows almost directly from proposition 1.

Proof. If (s, z) satisfies CM, there is a single order $\gamma \in \Gamma$ such that $\frac{p(\cdot|\overline{s})}{p(\cdot|\underline{s})}$ is weakly increasing with respect to that order for all $\overline{s} \in \overline{S}$ and $\underline{s} \in \underline{S}$. Proposition 1 implies that the optimal IC allocation is such that the rewards of any type in set $\overline{S}$ are placed at the top of that order until reaching some threshold. The fact that said allocation is IC implies that the thresholds of all types in $\overline{S}$ have to be the same. This, together with part ii) of proposition 1 means that every type in set S receives the same lottery over Z. As a result, the optimal allocation of proposition 1 is not strictly preferred to allocation h^*, making the latter an optimal IC allocation. ■

3.2 When are SERs necessary?

As example 1 illustrates, the model need not satisfy CM. In that case, corollary 1 does not apply, so it might be that SERs matter. In this section, I present a sufficient condition for SERs to be necessary. The approach is to start by considering allocation h^* - the optimal allocation among those which do not require SERs - and find some deviation that is both incentive compatible and leaves the evaluator better off.

Let

$$Z^* = \{z \in Z : E(v|z) \geq c\}$$

and notice that

$$h^*(s, z) = \begin{cases} 1 \text{ if } z \in Z^* \\ 0 \text{ if } z \notin Z^* \end{cases}.$$

Proposition 2 *SERs are necessary if there is* $\overline{s} \in \overline{S}$ *and a pair* (z', z'') *such that* $z' \in Z^*$, $z'' \notin Z^*$ *and*

$$\frac{p(z''|\overline{s})}{p(z''|\underline{s})} > \frac{p(z'|\overline{s})}{p(z'|\underline{s})}$$

for all $\underline{s} \in \underline{S}$.

Proof. See appendix. ■

The deviation that proposition 2 explores is one where a single high type $\overline{s} \in \overline{S}$ receives a different lottery over Z than what he would have gotten with allocation h^*; every other type receives the same lottery. Under the conditions stated, it is possible to improve type $\overline{s}$ without increasing the incentives of low types to deviate by rewarding him (with some probability) when $z = z''$ instead of rewarding him as frequently when $z = z'$.

Proposition 2 works as an algorithm to check whether SERs are necessary. It requires that one looks for high types such that there are two signal realizations z' and z'' with the stated properties. That process is even simpler when focusing on high types that belong to set $\widehat{S}$. Let $\Gamma^s \subseteq \Gamma$ denote the set of orderings γ of Z for which $\frac{p(\cdot|s)}{p(\cdot|\underline{s})}$ is weakly increasing in γ for all $\underline{s} \in \underline{S}$ and notice that Γ^s is not empty whenever $s \in \widehat{S}$.

Corollary 2 *SERs are necessary if there is* $\widehat{s} \in \widehat{S}$ *such that* $h^*(\widehat{s}, \cdot)$ *is not weakly increasing with respect to any* $\gamma \in \Gamma^{\widehat{s}}$.

Proof. The statement implies that there is some $z' \in Z^*$ and $z'' \notin Z^*$ such that

$$\frac{p(z''|\widehat{s})}{p(z''|s)} > \frac{p(z'|\widehat{s})}{p(z'|s)}$$

for all $s \in \underline{S}$, so the result direct follows from proposition 2. ■

In words, corollary 2 states that for SERs not to be necessary, it must be that in the optimal mechanism without SERs, every type s in set $\widehat{S}$ is rewarded according to some order $\gamma \in \Gamma^s$, i.e., according to the order which allows type s to belong to set $\widehat{S}$. Therefore, in the event that there is PM ($\widehat{S} = \overline{S}$) but no CM, SERs are typically necessary.[2]

[2]Specifically, SERs are necessary unless for all $s \in \overline{S}$ there is some $\gamma \in \Gamma^s$ for which all elements of Z^* come before all elements outside of Z^*.

The following example illustrates how to use these results to determine whether SERs are necessary.

Example 2 *Assume that $v \in \{0.1, 0.6, 0.9\}$ where each realization is equally likely; $s = v$; $c = 0.5$ and $z \in \{0, 1\}$ where*

s	0.1	0.6	0.9
$\Pr\{z = 1 \vert s\}$	0.5	0.1	0.9

.

Notice that there is PM but no CM because $\Gamma^{0.6} = \{\langle 1, 0\rangle\}$ while $\Gamma^{0.9} = \{\langle 0, 1\rangle\}$.

If no SERs are used, the optimal mechanism for the evaluator is to reward the agent iff $z = 1$ (because $E(v|z = 1) > c > E(v|z = 0)$). To apply the corolary, one checks to see whether the lottery assigned by the evaluator is not weakly increasing for all elements of either set $\Gamma^{0.6}$ or $\Gamma^{0.9}$. It is clear that $h^*(s, \cdot)$ is decreasing with respect to order $\langle 1, 0\rangle$, so it follows that, in this example, SERs are necessary.

One can use proposition 1 to characterize the optimal IC allocation and a mechanism that implements it. The optimal mechanism is as follows: allow the agent to choose between evaluation A and evaluation B. If the agent chooses evaluation A, then $x = z$ while if he chooses evaluation B, then $x = 1 - z$. In the Bayes-Nash equilibrium, the agent chooses evaluation A if $v = 0.9$, evaluation B if $v = 0.6$ and is indifferent between the two if $v = 0.1$.

4 Discussion

In this section, I discuss some of the implications of the general results presented in the previous section.

4.1 Connection to moral hazard

To know whether or not firms should use SERs, one first has to take a step back and understand why it is that firms want to evaluate workers in the first place. There are two main reasons: moral hazard and selection.

The moral hazard explanation comes from Holmstrom (1979) and is well known. Firms benefit from the effort $e \in \mathbb{R}_+$ their workers exert but effort is unobservable. Instead, the firm only has access to the worker's performance $z \in \mathbb{R}$, which works as a signal of effort. A classical result from Holmstrom (1979) is that if $\frac{p(z|e)}{p(z|e')}$ is increasing with z for any $e > e'$, i.e., if z has the monotone likelihood ratio property (MLRP), then the optimal contract offered by the firm is such that the salary paid to the worker is increasing in the worker's performance z. Evaluations are then necessary to determine z, which in turn determines the agent's compensation. In a moral hazard model, the role of SERs is quite clear; they are not necessary because the agent has no private information at the time he signs the contract with the firm.

A different explanation is that firms conduct evaluations because they want to identify the workers with more quality and treat them better in order to retain their services (Fuchs, 2015). In a sense, effort is a trait instead of a choice and is a mix of talent, skill and fit with the firm. By contrast to moral hazard motivations for conducting evaluations, in a model such as this, SERs could potentially make sense for the firm, because it is natural that workers are better informed than firms (presumably, they would know their skill level better). A natural assumption is that z has the MLRP just like in Holmstrom (1979). Refine the model and assume that $s = v\,(= e)$, $Z \subset \mathbb{R}$ and that the distribution of z conditional on s has the MLRP.

Proposition 3 *Under the conditions described, SERs are not necessary.*

Proof. By definition, (s, z) satisfies CM. Therefore, one can directly apply corolary 1. ■

The MLRP assumption is not particularly farfetched and has been used for the last 40 years in the moral hazard models that have followed Holsmstrom (1979), so this result seems to suggest that SERs are not that important for firms. That turns out not to be the case as I argue in the next few sections.

4.2 Multidimensional jobs

There are many times where a firm cares about multiple skills that the worker might have. A fitting example is that of a university which evaluates a faculty member and cares about his research skill but also his teaching skill. Formally, assume that $v = v_1 + v_2$ where v_1 represents reseach ability while v_2 represents teaching ability. To keep matters simple, assume that $v_i \in \{\underline{v}, \overline{v}\}$ and let the probability that $v_i = \overline{v}$ be denoted by $q_i \in (0, 1)$ for $i = 1, 2$. Assume that $c \in (2\underline{v}, \underline{v} + \overline{v})$ and that the agent knows both his research ability and his teaching ability, i.e., $s = (v_1, v_2)$. This means that $\underline{S} = \{(\underline{v}, \underline{v})\}$ while $\overline{S} = \{(\overline{v}, \underline{v}), (\underline{v}, \overline{v}), (\overline{v}, \overline{v})\}$. Finally, assume that $Z_i \subset \mathbb{R}$ for $i = 1, 2$, where the distribution of each z_i depends only on v_i and is denoted by $\overline{p}_i(z_i)$ when $v_i = \overline{v}$ and by $\underline{p}_i(z_i)$ when $v_i = \underline{v}$; z_1 might correspond to the number of publications while z_2 might represent teaching evaluations. As in the previous section, assume that $\frac{\overline{p}_i(\cdot)}{\underline{p}_i(\cdot)}$ is increasing for $i = 1, 2$ so that a high realization of z_i indicates that v_i is more likely to be $\overline{v}$.

Proposition 4 *SERs are necessary as long as* $1 < |Z^*| < n - 1$.

Proof. See appendix. ■

The previous proposition states that, in general, SERs are necessary. The reason is that, even though each z_i has the MLRP, vector z does not. To see why that is, compare the following two high types: type $(\overline{v}, \underline{v})$ who is good at research and bad at teaching, and type $(\underline{v}, \overline{v})$ who is good at teaching but bad at research. These two types have very different preferences regarding how they would prefer to be evaluated; the former would prefer to be evaluated mostly based on z_1, while the latter would prefer the focus to be on z_2. As a result, corollary 1 does not apply; there is something to be communicated from the agent to the evaluator.

This model is particularly helpful in that there is partial monotonicity (as is always the case when there is a single low type). As a result, one can use proposition 1 to characterize the optimal IC allocation and compare it to allocation h^* and verify that the former is better as long as Z^* has more than one element and less than $n-1$, which happens in general. The following proposition describes the mechanism which induces the optimal IC allocation characterized (in general) in proposition 1.

Proposition 5 *There is an optimal mechanism as follows: the agent chooses one of three options: A, B and C. If he chooses A, he is rewarded if z_1 is large enough; if he chooses B, he is rewarded if z_2 is large enough; if he chooses C, he is rewarded if $\chi(z_1, z_2)$ is large enough, where $\chi : Z_1 \times Z_2 \to \mathbb{R}$ is strictly increasing in both arguments. In the Bayes-Nash equilibrium, type $(\overline{v}, \underline{v})$ picks option A, type $(\underline{v}, \overline{v})$ picks option B while type $(\underline{v}, \underline{v})$ picks option C.*

Proof. See appendix. ■

This example demonstrates how SERs might help firms evaluate their workers when their value is multidimensional. SERs allow the firm to let the worker choose how he wants to be evaluated. This allows workers to signal where their relative strength is, which enables the firm to tailor the evaluation to those strengths; in the example, those who (believe they) are better at research ex-ante prefer to be evaluated based on their publications while those who (believe they) are better at teaching prefer to be evaluated based on their teaching evaluations.

4.3 Teachers' evaluations

The evaluation of school teachers has long been a topic of discussion. Historically, in most countries, public school teachers have been paid according to their education and experience. This practice has been criticized and there is a growing movement for linking teachers' salaries to their performance, just like what happens with most other workers. The hope is that, in this way, i) teachers will devote more time and effort to performing better and ii) teachers who are unable to perform at high standards will be replaced by others who are more capable.

The counterargument is that teachers' performance is harder to measure than the performance of other workers; more accurately, *some* aspects of a teacher's performance are hard to measure. For example, while the ability to teach memorization or simple mathematical algorithms can be ascertained more accurately through the standardized testing of the students, the ability to teach critical thinking is harder to measure in the sense that any test devised to measure it would not be very accurate. This asymmetry between the accuracy with which different skills can be measured may lead to the excessive allocation of time and effort by teachers towards developing the skills that are more easily measurable, i.e., teachers might "teach to the test".[3]

Holmstrom and Milgrom (1991) model this problem as a multi-task moral hazard problem, where the agent chooses how much effort to exert on different tasks, which generate (imperfect) signals of performance. They find that it might make sense to pay fixed (or close to fixed) wages to workers "even when good, objective output measures are available and agents are highly responsive to incentive pay." The argument is that since the agent is able to shift (too many) resources towards tasks which can be more easily measurable, the principal (evaluator) has no better alternative than to simply refrain from evaluating the agent, so that this undesirable rearrangement of resources does not happen.

[3] See Pogdursky and Springer (2007) or Lavy (2007) for a more detailed discussion over this debate.

As mentioned above, moral hazard motives are not the only motives which lead firms and institutions to evaluate their employees. In particular, it is argued that school teachers should be evaluated not only to induce them to exert sufficient effort but also to select the better able ones. In that sense, and as is shown in the previous section, SERs do help the evaluator and, as I argue below, might mitigate this "teaching to the test" effect which has concerned policy makers.

Let us consider the model of the previous section and think of v_1 as the ability to teach memorization and v_2 as the ability to teach critical thinking. Make the following additional assumptions: v_i is i.i.d., $Z_1 = Z_2$ and ratios $\frac{\overline{p}_1(\cdot)}{\overline{p}_2(\cdot)}$, $\frac{\underline{p}_2(\cdot)}{\underline{p}_1(\cdot)}$ and $\frac{\frac{\overline{p}_1(\cdot)}{\underline{p}_1(\cdot)} - \frac{\overline{p}_2(\cdot)}{\underline{p}_2(\cdot)}}{\frac{\overline{p}_1(\cdot)}{\underline{p}_1(\cdot)} \frac{\overline{p}_2(\cdot)}{\underline{p}_2(\cdot)}}$ are all increasing. These conditions ensure that z_1, the test that is used to measure whether the student has learned how to memorize, is more accurate than z_2, the test that is used to measure critical thinking.

Proposition 6 *In the optimal IC allocation which does not require SERs (which is h^*), the agent's expected payoff is larger when $v = (\overline{v}, \underline{v})$ than when $v = (\underline{v}, \overline{v})$.*

Proof. See appendix. ■

The previous proposition confirms that teachers who are better at teaching the more measurable skills are better evaluated (on average) than those who are better at teaching less measurable skills. The intution is that it is easier for the evaluator to reward type $(\overline{v}, \underline{v})$ without rewarding type $(\underline{v}, \underline{v})$ than it is to reward type $(\underline{v}, \overline{v})$ without rewarding $(\underline{v}, \underline{v})$. As a result, it is not only to be expected that teachers start neglecting those less measurable skills and focus on the more measurable ones but also that the teachers who refuse to do it simply get replaced by those who do. The contribution of this article to this literature is that it shows that SERs might help mitigate this unintended consequence of evaluating teachers.

Notice that without SERs, the evaluator faces a trade-off: she would like to increase the expected payoff of teachers of type $(\underline{v}, \overline{v})$ in order to reduce their incentive to "teach to the test" but that is not possible without, at the same time, either making types $(\overline{v}, \overline{v})$ or $(\overline{v}, \underline{v})$ worse off or making type $(\underline{v}, \underline{v})$ better off. Consider the following mechanism which uses SERs.

At the beginning of the evaluation period, the teacher is asked to choose how to he wants to evaluated in his SER. If he wants to, he can be evaluated as before, where he is rewarded if and only if $E(v|z) \geq c$. However, he also has a second option, which is to be rewarded according only to signal z_2; in particular, the agent would be rewarded if $z_2 > \widehat{z}_2$ and would not to be rewarded if $z_2 < \widehat{z}_2$ for some $\widehat{z}_2$. In this way, the teacher may choose to exclude the supposedly more accurate signals of performance and be evaluated only on the signals of performance that fit him best ex-ante.

Proposition 7 *If $1 < |Z^*| < n-1$ (i.e., if h^* is not optimal), then there is $\widehat{z}_2$ such that the agent strictly prefers the alternative evaluation if and only $v = (\underline{v}, \overline{v})$.*

Proof. See appendix. ■

The new mechanism diminishes some of the negative effects of performance-based pay because it reduces the incentives to "teach to the test" without sacrificing the evaluation. By allowing teachers who are better at teaching less measurable skills like critical thinking to opt out of being evaluated through standardized testing, one is able to increase their expected reward without rewarding more those whose teaching ability is low.

4.4 Self-selected evaluators

One circumstance in which the person being evaluated would like to be evaluated differently based on what he knows is when that person is an expert. By the very definition of what it means to be an expert, their performance is hard to be evaluated by those who are not experts themselves. In that sense, experts have private information about the identity of those possible evaluators who are more familiar with their work. As a result, SERs might allow the expert (the agent) to select which evaluator should evaluate him; a practice that is already employed in academia, for example, where several academic journals allow authors to select their editor/co-editor and/or referees.

To formalize this idea, I refine the model as follows. Assume that $s = (v, y)$, where $y \in \{1, 2\}$ is a random variable that is independent of v, and that $z = (z_1, z_2) \in Z_1 \times Z_2$, where the distribution of each z_i is as follows:

$$p(z_i|v, y) = \begin{cases} f^i(z_i|v) & \text{if } y = i \\ f^0(z_i) & \text{if } y \neq i \end{cases}$$

for all $v \in [\underline{v}, \overline{v}]$, $y \in \{1, 2\}$ and $z_i \in Z_i$. Expression $p(z_i|v, y)$ represents the probability of each $z_i \in Z_i$ given $s = (v, y)$ and I assume that $f^i(\cdot|v)$ has the MLRP for $i = 1, 2$.

In words, z_i is only correlated with v if $y = i$. If that does not happen, i.e., if $y \neq i$, then z_i is independent of the value of the agent. If we think of an academic submission of an article, z_1 and z_2 are referee reports submitted by different referees; one of which is an expert on the subject of the article, while the other one is not. Crucially, while the author (the agent) knows which of the referees is which, the editor of the journal (the evaluator) does not.

What would happen if SERs were not used? As I have shown in the previous sections, the author would be rewarded if and only if $E(v|z_1, z_2) \geq c$. This, however, is highly undesirable because both referee reports are being used even though one of them is uncorrelated with the agent's value; the uninformative signal is making the information that the editor receives less accurate. From the point of view of the editor, it would be preferable to use only the report of the referee who is informed. Under some conditions, SERs can accomplish that.

Consider the following alternative mechanism. When submitting his article to the journal, the author selects (in his SER) which of the two referees he prefers; he is then rewarded if and only if that referee believes his value is above cost c. Formally, the author starts by selecting $i \in \{1, 2\}$ and is then rewarded if and only if $E(v|z_i, i) \geq c$.

Proposition 8 *The alternative mechanism induces an optimal IC allocation if*

$$\sum_{z_i \in Z_i} f^i(z_i|\underline{v}) \mathbf{1}\{E(v|z_i, i) \geq c\} \geq \sum_{z_j \in Z_j} f^0(z_j) \mathbf{1}\{E(v|z_j, j) \geq c\}$$

for $i = 1, 2$ *and* $j \neq i$.

Proof. See appendix. ■

If the condition of proposition 8 holds, it is incentive compatible for the agent who observes signal y to select referee y regardless of his value; in that sense, the mechanism induces "truthful reporting". Whenever that happens, the alternative mechanism is optimal; the intuition is that even though the agent does not report v directly (because of corollary 1), he reports everything else (the y in this case), which helps the evaluator make as good of a decision as possible.

5 Related Literature

There are two main theories that explain why firms evaluate their workers: moral hazard and selection. As described in the text, it might be that firms need to give incentives for workers to exert effort. In order to do that, firms must condition the workers' rewards on the available signals of effort, which are a result of an evaluation. If, however, evaluations are motivated (only) by moral hazard problems, SERS are pointless because the worker has no private information at the time of the writing of the contract.

By contrast, as was demonstrated in the text, if the firm is motivated by selection, it might benefit from SERs, depending on how they are used.[4] As mentioned in the introduction, SERs are commonly perceived as unreliable signals of performance. Part of the reason why that is has a lot to do with the informal way in which SERs are used; in general, the firm simply asks the worker for his SER without being clear on what the implications of those SERs will be. This interaction resembles the cheap talk model of Crawford and Sobel (1982), with the exception that the worker (the sender) has type independent preferences. Indeed, in a model such as that, there would be no information transmitted in any (perfect Bayesian) equilibrium.

In the literature, it has been argued that the model of Crawford and Sobel (1982) might exaggerate the difficulties in obtaining informative equilibria, because it restricts the private information of the sender to one-dimensional signals; see, for example, Chakraborty and Harbaugh (2007, 2010, 2014), Levy and Razin (2007) and Che, Dessein and Kartik (2013). To see why the multidimensionality helps, think of the example in Chakraborty and Harbaugh (2007) about an advisor who recommends students on the academic job market. If the advisor only has one student, he will be unable to convey the true ability of the student to the market, because he will be tempted to exaggerate. If, however, the advisor has two students that he cares equally about, he is at least able to compare them by sending messages of the sort "Candidate A is better than candidate B". The advisor has enough incentives to want to report truthfully, because he is indifferent; in one case, he helps candidate A, in the other he helps candidate B. In general, the sender trades off one dimension for the other in each of the messages.

I make an apparently similar argument: I argue that the conditions of corollary 1 are

[4] What is more likely is that most firms evaluate for both reasons. Indeed, there are several papers which combine moral hazard motives and selection; e.g., Laffont and Tirole (1987,1988), Levin (2003), MacLeod (2003) or Gerardi and Maestri (2020).

more likely to hold when the worker's signal is unidimensional, while the conditions of corollary 2 are more likely to hold when it is multidimensional. In that sense, it follows that SERs are more likely to be useful in the latter case. Nevertheless, the two arguments are quite different. If I was to use a cheap talk model a la Crawford and Sobel (1982), SERs would not be beneficial regardless of how many dimensions the worker's signal has. The reason is that the firm's only concern is with a single dimension of the agent's private information; his expected valuation.

The two main differences between my work and Crawford and Sobel (1982) are that the receiver is informed (i.e., observes her own private signal) and is able to commit. The literature on cheap talk with an informed receiver (where the receiver cannot commit) is mixed with respect to the impact of the receiver's private information on the information transmitted by the sender; while Lai (2014), Chen (2012), de Barreda (2013), Ishida and Shimizu (2016) show that a better informed receiver induces the sender to share less information, Watson (1996) and Ishida and Shimizu (2019) show that the opposite can happen. Watson (1996) is particularly related to my work because the sender is also assumed to have type independent preferences. The author finds that the correlation between both players' signals might lead to informative communication; indeed, the optimal IC allocation described in section 4.4 can be implemented by such a cheap talk model. Moreover, the author finds conditions under which there are equilibria where the sender sends *all* of his private information to the receiver. The main difference to my work is that I assume that firms can commit to an allocation rule, which expands the set of allocations that can be implemented. In that sense, this article complements Watson (1996) by showing general conditions under which *some* information is sent by the sender to the receiver when the latter is able to commit.

There has also been some work which combines the three main features of my model: that the receiver is informed and has commitment power and that the sender's preferences are type independent. Some of this literature has already identified a few reasons why self-reporting might be a part of the optimal mechanism. In Silva (2019), I study a model with multiple agents with correlated types. I find that self-reports are a crucial part of the optimal mechanism because of the information externalities contained in each individual report.[5] In Siegel and Strulovici (2020), plea bargaining (the act of confessing one's guilt in exchange for a reduced sentence) is a part of the optimal mechanism as a way to reduce risk for the players. In order to negate the arguments made in these articles, I consider a model with a single agent and assume

[5] Baliga and Sjostrom (2001), Marx and Squintani (2009) and Tamada and Tsai (2014) also study the use of SERs when there are multiple agents, albeit with very different models.

that both players are risk neutral.

As mentioned in the introduction, the common rationale for the use of SERs is the possibility that the sender has of providing "hard evidence". Green and Laffont (1986) studies a principal-agent model with hard evidence, which has since been refined by Bull and Watson (2007), Deneckere and Severinov (2008) and Strausz (2017). Models of hard evidence have been used to study the information disclosure of firms (Dye, 1986), their price policy (Sher and Vohra, 2015) or political lobbying (Cotton, 2012) among many other topics. Hard evidence is also a key component of the literature on costly verification, where the truthfulness of the sender's messages might be ascertained at a cost; see, for example, Baron and Besanko (1984), Ben-Porath, Dekel and Lipman (2014) or Mylovanov and Zapechelnyuk (2017).

Finally, this article is complementary of Deb and Stewart (2018), where the authors study how a firm should determine the tasks its workers perform in order to best determine their ability. They focus on tasks with binary outcomes (pass/fail tasks) and realize that firms might not want to always select more informative tasks. They then find a sufficient condition called group monotonicity for more informative tasks to be preferred; if all high types are better than all low types in tasks A and B, then the firm should use task A if it is more informative than task B. Even though group monotonicity is somewhat similar to the condition of corolary 1, it neither implies it nor is it implied by it, because it serves a different purpose. The two models differ in that i) in Deb and Stewart (2018), a worker is able to fail the test with certainty if he so chooses, while, in my model, that is not allowed, and ii) I assume that the firm is able to commit to any menu of contracts it offers the worker. Indeed, in my model, it can be shown that the firm prefers to select the more informative signals if given that opportunity regardless of whether group monotonicity holds or not.

6 Conclusion

SERs are used by many firms and institutions. Despite this, the overall consensus in the business literature is that SERs should not be used to determine a worker's compensation; they should merely be used as a learning tool for both the firm and the worker. This reluctance in using SERs comes from their perceived unreliability, as, empirically, workers typically exaggerate their accomplishments. This is to be expected, however, only if firms use SERs in a naive manner, as they typically do. If the firm simply uses SERs as a way to ask its workers whether they did a good job or not, to then reward only those who report affirmatively, it is no wonder that the majority of workers exaggerates. The fact that this happens does not suggest that SERs should not be used; it simply means that using them in this naive way is pointless. In order to determine whether there is any point to using SERs in determining a worker's compensation one has to consider the worker's incentives, i.e., one has to focus on incentive compatible mechanisms which use SERs. That is the approach that I follow in this article.

I discuss when it is possible to use SERs in the design of superior evaluation mechanisms that determine a worker's compensation and when it is not possible, regardless of how one designs the SERs. When they are useful, SERs allow the worker to communicate to the firm how he wants to be evaluated. In that sense, SERs are less about workers telling their employers how good a job they have done and they are more about firms giving power to the workers to choose between different possible evaluation criteria. Therefore, SERs are not likely to be necessary when the workers perform unidimensional tasks, where there is only way of doing things well, because all workers the firm would like to reward prefer to be evaluated in the same way. By contrast, when workers have multidimensional skills or have better information about which evaluator is more knowledgeable, SERs do make sense if properly designed.

7 Appendix

7.1 Proof of proposition 1

Consider a relaxed program R where the only incentive constraints considered are as follows:

$$E\left(h\left(s,z\right)|s\right) \geq E\left(h\left(s',z\right)|s\right)$$

for all $s \in \underline{S}$ and $s' \in \overline{S}$, i.e., low types are prevented from mimicking high types. Notice that a solution h' to R always exists because R is a linear program with finite variables that are chosen from a compact set. The proof uses h' to construct allocation $\widehat{h}$.

For all $s \in \widehat{S}$,

$$\widehat{h}\left(s,z\right) = \begin{cases} 1 \text{ if } \frac{p(z|s)}{p(z|\underline{s})} > \alpha\left(s\right) \\ \beta\left(s\right) \text{ if } \frac{p(z|s)}{p(z|\underline{s})} = \alpha\left(s\right) \\ 0 \text{ if } \frac{p(z|s)}{p(z|\underline{s})} < \alpha\left(s\right) \end{cases},$$

for all $z \in Z$, where $\alpha(s) \in \mathbb{R}_+$ and $\beta(s) \in [0,1]$ are such that

$$E\left(\widehat{h}(s,z)|s\right) = E\left(h'(s,z)|s\right).$$

Let γ^s be such that $\frac{p(\cdot|s)}{p(\cdot|s')}$ is weakly increasing with respect to order γ^s for all $s' \in \underline{S}$. Lottery $\widehat{h}(s,\cdot)$ places the rewards at the top of order γ^s until it reaches the same expected payoff for type s as lottery $h'(s,\cdot)$. (Notice that $\alpha(s)$ and $\beta(s)$ exist because $E(h'(s,z)|s) \in [0,1]$.)

For all $s \in \overline{S}$ such that $s \notin \widehat{S}$, let $\widehat{h}(s,\cdot) = h'(s,\cdot)$, while, for all $s \in \underline{S}$, $\widehat{h}(s,z) = h'(\omega(s),z)$, where $\omega(s)$ is defined in the statement of the result.

By construction, $\widehat{h}$ satisfies all incentive constraints considered by R, because low types can only choose from the set of lotteries over Z that are assigned to high types. It is also the case that

$$E\left(h'(s,z)|s\right) \geq E\left(h'(s',z)|s\right) \geq E\left(\widehat{h}(s',z)|s\right) \tag{1}$$

for all $s \in \underline{S}$ and $s' \in \overline{S}$. The first inequality follows trivially because h' satisfies the incentive constraints of R. As for the second inequality, it follows directly if $s' \notin \widehat{S}$, because $h'(s',\cdot) = \widehat{h}(s',\cdot)$. If, however, $s' \in \widehat{S}$, the statement is not direct and is proven below.

Claim 1 *$E(h'(s',z)|s) \geq E\left(\widehat{h}(s',z)|s\right)$ for all $s \in \underline{S}$ and $s' \in \widehat{S}$.*

Proof. Fix any $s \in \underline{S}$ and $s' \in \widehat{S}$. It is sufficient to show that $\widehat{h}(s',\cdot)$ is a solution of the following problem:

$$\min_{f:Z\to[0,1]} E(f(z)|s) \text{ s.t. } E(f(z)|s') = E(h'(s',z)|s').$$

The corresponding Lagrangean is given by

$$\mathcal{L} = -E(f(z)|s) + \lambda\left(E(f(z)|s') - E(h'(s',z)|s')\right) + \sum_{z\in Z}\left(\overline{\mu}(z)f(z) + \underline{\mu}(z)(1-f(z))\right).$$

where $\lambda \in \mathbb{R}$, $\overline{\mu}(z) \geq 0$ for all $z \in Z$ and $\underline{\mu}(z) \geq 0$ for all $z \in Z$. Notice that

$$\frac{d\mathcal{L}}{df(z)} = -p(z|s) + \lambda p(z|s') + \overline{\mu}(z) - \underline{\mu}(z).$$

Let $z^* \in Z$ be such that
$$\frac{p(z^*|s')}{p(z^*|\underline{s})} = \alpha(s')$$
and let
$$\lambda = \frac{p(z^*|s)}{p(z^*|s')},$$
$$\overline{\mu}(z) = \begin{cases} 0 \text{ if } \frac{p(z|s')}{p(z|\underline{s})} \geq \alpha(s) \\ p(z|s) - \lambda p(z|s') \text{ if } \frac{p(z|s')}{p(z|\underline{s})} < \alpha(s) \end{cases}$$
and
$$\underline{\mu}(z) = \begin{cases} -p(z|s) + \lambda p(z|s') \text{ if } \frac{p(z|s')}{p(z|\underline{s})} \geq \alpha(s) \\ 0 \text{ if } \frac{p(z|s')}{p(z|\underline{s})} < \alpha(s) \end{cases}$$
for all $z \in Z$. It follows that
$$\frac{d\mathcal{L}}{df(z)} = 0$$
and that
$$\lambda\left(E\left(\widehat{h}(s',z)|s'\right) - E(h'(s',z)|s')\right) = 0,$$
$$\overline{\mu}(z)\widehat{h}(s',z) = 0$$
and
$$\underline{\mu}(z)\left(1 - \widehat{h}(s',z)\right) = 0$$
for all $z \in Z$. Given that this program is linear, it then follows that $\widehat{h}(s',\cdot)$ solves it. ■

Condition (1) implies that allocation $\widehat{h}$ is also a solution of R (because, by construction, the high types' expected payoff does not change from allocation h' to allocation $\widehat{h}$). The proof is completed by showing that $\widehat{h}$ is IC. There are two types of deviations one needs to rule out; low types mimicking other low types and high types mimicking other types. The former deviations are ruled out by construction of $\widehat{h}$, because all low types choose from the same set of lotteries over Z. I show below that high types do not have an incentive to deviate either.

Claim 2 *For any solution h'' of the relaxed program R,*
$$E(h''(s,z)|s) \geq E(h''(s',z)|s)$$
for all $s' \in S$ and $s \in \overline{S}$.

Proof. Suppose not. Then, there is some $s \in \overline{S}$ and some $s' \in S$ such that

$$E\left(h''\left(s,z\right)|s\right) < E\left(h''\left(s',z\right)|s\right).$$

Let allocation h''' to be such that $h'''\left(s'',\cdot\right) = h''\left(s'',\cdot\right)$ for all $s'' \in S$ except when $s'' = s$. In that case, let $h'''\left(s,\cdot\right) = h''\left(s',\cdot\right)$. Allocation h''' satisfies all the incentive constraints considered by R (because h'' is a solution of R) and is such that $W\left(h'''\right) > W\left(h''\right)$, which contradicts the optimality of h''. ■

7.2 Proof of proposition 2

Let

$$r = \max\left\{\frac{p\left(z''|s\right)}{p\left(z'|s\right)} : s \in \underline{S}\right\}.$$

Consider the following lottery $g : Z \to [0,1]$, where $g\left(z\right) = h^*\left(s,z\right)$ for all $z \neq z', z''$, $g\left(z'\right) = 1-\varepsilon$ for some $\varepsilon \in \left(0, \min\left\{r,1\right\}\right)$ and $g\left(z''\right) = \frac{\varepsilon}{r}$. Finally, consider allocation h' as follows:

$$h'\left(s,\cdot\right) = \begin{cases} h^*\left(s,\cdot\right) \text{ if } \frac{p(z''|s)}{p(z'|s)} \leq r \\ g\left(\cdot\right) \text{ if } \frac{p(z''|s)}{p(z'|s)} > r \end{cases}.$$

Notice that

$$E\left(h^*\left(s,z\right)|s\right) \geq E\left(g\left(z\right)|s\right) \Leftrightarrow \frac{p\left(z''|s\right)}{p\left(z'|s\right)} \leq r$$

so allocation h' is trivially incentive compatible. It is also the case that $W\left(h'\right) > W\left(h^*\right)$ because $\frac{p(z''|\overline{s})}{p(z'|\overline{s})} > r$.

7.3 Proof of proposition 4

Notice that

$$E\left(v|z\right) = E\left(v_1|z_1\right) + E\left(v_2|z_2\right)$$

and that $E\left(v_i|z_i\right)$ is increasing with z_i for $i = 1,2$ because of the MLRP of z_i. Let $\overline{z}_i = \max\left\{z_i \in Z_i\right\}$, $\underline{z}_i = \min\left\{z_i \in Z_i\right\}$ and $\widetilde{z}_i = \max\left\{z_i \in Z_i \backslash \overline{z}_i\right\}$. Given that $\left|S^*\right| > 1$, it follows that $\left(\overline{z}_1, \overline{z}_2\right) \in Z^*$ and either $\left(\overline{z}_1, \widetilde{z}_2\right) \in Z^*$ or $\left(\widetilde{z}_1, \overline{z}_2\right) \in Z^*$. Without loss of generality, assume the former. If SERs are not necessary, it must be that the conditions of proposition 2 are not satisfied. Therefore, it must be that $\left(z_1, \overline{z}_2\right) \in Z^*$ for all $z_1 \in Z_1$, because

$$\frac{p\left(z_1|\underline{v}\right)p\left(\overline{z}_2|\overline{v}\right)}{p\left(z_1|\underline{v}\right)p\left(\overline{z}_2|\underline{v}\right)} > \frac{p\left(\overline{z}_1|\underline{v}\right)p\left(\widetilde{z}_2|\overline{v}\right)}{p\left(\overline{z}_1|\underline{v}\right)p\left(\widetilde{z}_2|\underline{v}\right)}.$$

In particular, this means that $(\underline{z}_1, \overline{z}_2) \in Z^*$. But then, by the same argument, it follows that $(z_1, z_2) \in Z^*$ for all $z_2 \in Z_2$ and $z_1 > \underline{z}_1$, because

$$\frac{p(z_1|\overline{v})\, p(z_2|\underline{v})}{p(z_1|\underline{v})\, p(z_2|\underline{v})} > \frac{p(\underline{z}_1|\overline{v})\, p(\overline{z}_2|\underline{v})}{p(\underline{z}_1|\underline{v})\, p(\overline{z}_2|\underline{v})}.$$

In particular, this means that $(z_1, \underline{z}_2) \in Z^*$ for all $z_1 > \underline{z}_1$. Therefore, the only pair that might not belong to set Z^* is pair $(\underline{z}_1, \underline{z}_2)$, which means that $|S^*| \geq n - 1$, which is a contradiction.

7.4 Proof of proposition 5

Notice that $\overline{S} = \widehat{S}$ and that

$$\frac{p(z|\,(\overline{v}, \underline{v}))}{p(z|\,(\underline{v}, \underline{v}))} = \frac{\overline{p}_1(z_1)}{\underline{p}_1(z_1)}$$

is increasing with z_1 and independent of z_2,

$$\frac{p(z|\,(\underline{v}, \overline{v}))}{p(z|\,(\underline{v}, \underline{v}))} = \frac{\overline{p}_2(z_2)}{\underline{p}_2(z_2)}$$

is increasing with z_2 and independent of z_1, and

$$\frac{p(z|\,(\overline{v}, \overline{v}))}{p(z|\,(\underline{v}, \underline{v}))} = \frac{\overline{p}_1(z_1)\, \overline{p}_2(z_2)}{\underline{p}_1(z_1)\, \underline{p}_2(z_2)} \equiv \chi(z_1, z_2).$$

is increasing in both z_1 and z_2. Therefore, in the optimal IC allocation characterized in proposition 1, there is $\widehat{z}_1 \in [0, 1]$, $\widehat{z}_2 \in [0, 1]$ and $\alpha \in \mathbb{R}_+$ such that i) type $(\overline{v}, \underline{v})$ is rewarded $(x = 1)$ if $z_1 > \widehat{z}_1$ and is not rewarded if $z_1 < \widehat{z}_1$, ii) type $(\underline{v}, \overline{v})$ is rewarded $(x = 1)$ if $z_2 > \widehat{z}_2$ and is not rewarded if $z_2 < \widehat{z}_2$; type $(\overline{v}, \overline{v})$ is rewarded $(x = 1)$ if $\chi(z_1, z_2) > \alpha$ and is not rewarded if $\chi(z_1, z_2) < \alpha$.

7.5 Proof of proposition 6

Let q denote the probability that $v_i = \overline{v}$. For convenience, define $\varphi_i(z_i) \equiv \frac{\overline{p}_i(z_i)}{\underline{p}_i(z_i)}$ for $i = 1, 2$ and notice that $\varphi_i(\cdot)$, $\frac{\varphi_1(\cdot)}{\varphi_2(\cdot)}$ and $\frac{\varphi_1(\cdot) - \varphi_2(\cdot)}{\varphi_1(\cdot)\varphi_2(\cdot)}$ are all increasing, which also implies that $\varphi_1(\cdot) - \varphi_2(\cdot)$ is increasing.

Claim 3 *For all $a > b$, $E(v|a, b) > E(v|b, a)$.*

Proof. Fix any $a, b \in Z_i$ such that $a > b$. Define

$$k_1 \equiv q^2 \varphi_1(a) \varphi_2(b)$$

and

$$k_2 \equiv q(1-q)(\varphi_1(a) + \varphi_2(b)).$$

Likewise define

$$k_1' \equiv q^2 \varphi_1(b) \varphi_2(a)$$

and

$$k_2' \equiv q(1-q)(\varphi_1(b) + \varphi_2(a)).$$

Notice that $k_1 > k_1'$, because $\frac{\varphi_1(\cdot)}{\varphi_2(\cdot)}$ is increasing, $k_2 > k_2'$, because $\varphi_1(\cdot) - \varphi_2(\cdot)$ is increasing, and $\frac{k_1}{k_1'} > \frac{k_2}{k_2'}$, because $\frac{\varphi_1(\cdot)-\varphi_2(\cdot)}{\varphi_1(\cdot)\varphi_2(\cdot)}$ is increasing. By defining

$$k_3 \equiv (1-q_1)(1-q_2)$$

we have that

$$E(v|a,b) = \frac{k_1 2\overline{v} + k_2(\overline{v} + \underline{v}) + k_3 \underline{v}}{k_1 + k_2 + k_3}$$

and that

$$E(v|b,a) = \frac{k_1' 2\overline{v} + k_2'(\overline{v} + \underline{v}) + k_3 \underline{v}}{k_1' + k_2' + k_3}.$$

Let

$$k_4 = (k_1 + k_2 + k_3)(k_1' + k_2' + k_3) > 0.$$

and notice that

$$\begin{aligned}
(E(v|a,b) - E(v|b,a)) k_4 &= (k_1 k_2' + k_1 k_3 - k_2 k_1' - k_3 k_1') 2\overline{v} + \\
&\quad (k_2 k_1' + k_2 k_3 - k_1 k_2' - k_3 k_2')(\overline{v} + \underline{v}) + \\
&\quad (k_3 k_1' + k_3 k_2' - k_1 k_3 - k_2 k_3) 2\underline{v} \\
&\geq (k_1 k_2' - k_2 k_1')(\overline{v} - \underline{v}) \\
&> 0
\end{aligned}$$

■

Define

$$\varrho(z_1, z_2) = \mathbf{1}\{E(v|z_1, z_2) \geq c\}$$

for all $(z_1, z_2) \in [0,1]^2$ and recall that $\varrho(\cdot,\cdot)$ is weakly increasing in both arguments. Let

$$U_1 \equiv \sum_{a\in Z_i}\sum_{b\in Z_i} \overline{p}_1(a)\,\underline{p}_2(b)\,\varrho(a,b)$$

and

$$U_2 \equiv \sum_{a\in Z_i}\sum_{b\in Z_i} \underline{p}_1(a)\,\overline{p}_2(b)\,\varrho(a,b)$$

represent the expected payoff of types $(\overline{v},\underline{v})$ and $(\underline{v},\overline{v})$ respectively under allocation h^*. The proof is completed by showing that $U_1 \geq U_2$.

Claim 4 $U_1 \geq U_2$.

Proof. Notice that

$$\begin{aligned} U_1 &\geq \left(\sum_{a\in Z_i} \overline{p}_2(a)\frac{\overline{p}_1(a)}{\overline{p}_2(a)}\right)\left(\sum_{a\in Z_i}\sum_{b\in Z_i}\overline{p}_2(a)\,\underline{p}_2(b)\,\varrho(a,b)\right) \\ &= \sum_{a\in Z_i}\sum_{b\in Z_i}\overline{p}_2(a)\,\underline{p}_2(b)\,\varrho(a,b) \\ &\geq \left(\sum_{b\in Z_i}\underline{p}_1(b)\frac{\underline{p}_2(b)}{\underline{p}_1(b)}\right)\left(\sum_{a\in Z_i}\sum_{b\in Z_i}\underline{p}_1(b)\,\overline{p}_2(a)\,\varrho(a,b)\right) \\ &= \sum_{a\in Z_i}\sum_{b\in Z_i}\overline{p}_2(a)\,\underline{p}_1(b)\,\varrho(a,b) \\ &\equiv U_1', \end{aligned}$$

where the first inequality follows because $\frac{\overline{p}_1(\cdot)}{\overline{p}_2(\cdot)}$ and $\varrho(\cdot,b)$ are increasing, while the second inequality follows because $\frac{\underline{p}_2(\cdot)}{\underline{p}_1(\cdot)}$ and $\varrho(a,\cdot)$ are increasing.

Finally, notice that

$$U_1' = \sum_{a\in Z_i}\sum_{b\in Z_i, b\leq a}\left(\overline{p}_2(a)\,\underline{p}_1(b)\,g(a,b) + \overline{p}_2(b)\,\underline{p}_1(a)\,\varrho(b,a)\right)$$

while

$$U_2 = \sum_{a\in Z_i}\sum_{b\in Z_i, b\leq a}\left(\overline{p}_2(a)\,\underline{p}_1(b)\,g(b,a) + \overline{p}_2(b)\,\underline{p}_1(a)\,\varrho(a,b)\right).$$

Therefore, it follows that

$$U_1' - U_2 = \sum_{a\in Z_i}\sum_{b\in Z_i, b\leq a}\left(\overline{p}_2(a)\,\underline{p}_1(b) - \overline{p}_2(b)\,\underline{p}_1(a)\right)\left(\varrho(a,b) - \varrho(b,a)\right) \geq 0$$

because $\varrho(a,b) \geq \varrho(b,a)$ for all $a \geq b$ and because $\frac{\overline{p}_2(\cdot)}{\underline{p}_1(\cdot)}$ is increasing. ■

7.6 Proof of proposition 7

Let $g \equiv h^*(s,\cdot)$ for any $s \in S$ and consider lottery $\widehat{g} : Z \to [0,1]$, where

$$\widehat{g}(z) = \begin{cases} 1 \text{ if } z_2 > \widehat{z}_2 \\ \widehat{\tau} \text{ if } z_2 = \widehat{z}_2 \\ 0 \text{ if } z_2 < \widehat{z}_2 \end{cases},$$

where $\widehat{\tau} \in [0,1]$ and $\widehat{z}_2$ are such that type $(\underline{v}, \overline{v})$ is indifferent between lotteries g and g'. By claim 1 (on the proof of proposition 1), it follows that the $E(\widehat{g}(z)|v) \leq E(g(z)|v)$ for all v. The fact that $1 < |Z^*| < n$ implies that $E(\widehat{g}(z)|v) < E(g(z)|v)$ for all $v \neq (\underline{v}, \overline{v})$. This means that there is some $\widehat{\tau}' \in [0,1]$ and $\widehat{z}'_2 \in Z_i$ such that lottery $E(\widehat{g}'(z)|(\underline{v}, \overline{v})) > E(g(z)|(\underline{v}, \overline{v}))$ and $E(\widehat{g}'(z)|v) < E(g(z)|v)$ for all $v \neq (\underline{v}, \overline{v})$, where

$$\widehat{g}'(z) = \begin{cases} 1 \text{ if } z_2 > \widehat{z}'_2 \\ \widehat{\tau} \text{ if } z_2 = \widehat{z}'_2 \\ 0 \text{ if } z_2 < \widehat{z}'_2 \end{cases}.$$

7.7 Proof of proposition 8

Consider a relaxed problem where the evaluator chooses an allocation h to maximize $E(v - c|s,z)$ subject to the following incentive constraints only:

$$E(h(v',i)|v',y) \geq E(h(v'',i)|v',y)$$

for all $v', v'' \in [\underline{v}, \overline{v}]$. In words, I do not allow the agent to misreport over y. By corollary 1 and because $f^i(\cdot|v)$ has the MLRP, it follows that a solution to this relaxed problem is allocation $\widehat{h}$, where

$$\widehat{h}((v,y),(z_1,z_2)) = \begin{cases} 1 \text{ if } E(v|z_y,y) \geq c \\ 0 \text{ if } E(v|z_y,y) < c \end{cases}.$$

Notice that

$$E\left(\widehat{h}((v',i),(z_1,z_2))|v'',i\right) \geq E\left(\widehat{h}((\underline{v},i),(z_1,z_2))|\underline{v},i\right)$$

for all $v', v'' \in [\underline{v}, \overline{v}]$ and $i = 1, 2$, because $E(v|z_i, i)$ is increasing with z_i and $f^i(\cdot|v)$ has the MLRP, while

$$E\left(\widehat{h}\left((v', i), (z_1, z_2)\right)|v'', j\right) = E\left(\widehat{h}\left((\underline{v}, i), (z_1, z_2)\right)|\underline{v}, j\right)$$

for all $v', v'' \in [\underline{v}, \overline{v}]$ and $i, j = 1, 2$. As a a result, allocation $\widehat{h}$ is incentive compatible provided

$$E\left(\widehat{h}\left((\underline{v}, i), (z_1, z_2)\right)|\underline{v}, i\right) \geq E\left(\widehat{h}\left((\underline{v}, j), (z_1, z_2)\right)|\underline{v}, i\right)$$

for $i, j = 1, 2$, which is the condition of the statement.

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