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ABSTRACT

This paper shows that exponential discounting may have an advancing effect on the timing of investment, not captured by sensitivity analysis carried out for the complete range of instantaneous discount rates implicit in declining discounting.

JEL: Classification: D90, H43.

Key words: declining discounting, exponential discounting, hyperbolic discounting, investment timing, project evaluation.

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THE EFFECT OF A CONSTANT OR A DECLINING DISCOUNT RATE ON OPTIMAL INVESTMENT TIMING

The importance of the discount rate on project evaluation is well established. A low or high discount rate makes a typical investment project highly profitable or unprofitable. In addition, the discount rate generally has an effect on the optimal timing of investment.

This paper will show, through a simple numerical example, that the shape of the discount function also has an effect on optimal investment timing. Specifically, it will show that exponential discounting may have an advancing effect on the timing of investment, not captured by sensitivity analysis carried out for the complete range of instantaneous discount rates implicit in declining discounting.

Until recently, and ever since Samuelson proposed the discounted utility model back in 1937, using a constant rate of discount was the norm among economists. Lately, however, many studies have argued in favor of declining discount rates based on different rationales.

One such rationale is hyperbolic discounting (Cropper and Laibson, 1999), term that is applied to the manifestation of increasing impatience as time horizons get shorter, and which is generally associated with time inconsistencies. As Strotz (1955-1956) showed more than forty

years ago, time-inconsistent preferences arise whenever non-exponential discounting is assumed at the individual level.

Another rationale leading to declining rates of discount is based on the well-known formula (Dasgupta, 2001, p. 181)

$$(1) \quad r_t = \mathbf{d} + \mathbf{h}(C_t)(dC_t / dt) / C_t$$

where r_t is the interest rate at time t , using aggregate consumption as numeraire; δ is the rate of pure time preference, or social rate of discount using utility or well-being as numeraire; $\eta(C_t)$ is the elasticity of marginal well-being with respect to consumption and $(dC_t/dt)/C_t$ is the percentage rate of change of C_t . Assuming a constant elasticity of marginal well-being, if consumption growth is expected to decline in the future, then the interest rate would accordingly decline over time. A decline in consumption growth in the future would be consistent with what Barro and Sala-i-Martin (1995, p. 28) call conditional convergence: an economy grows faster the further it is from its own steady-state value. The decline in the instantaneous discount rate refers, in this case, to the equilibrium interest rate in the economy, and could arise with or without exponential discounting at the individual level.

Newell and Pizer (2000) provide yet another rationale for declining rates. They show that when the future path of the interest rate is uncertain but highly correlated over time, then the distant future should be discounted at lower rates than suggested by the current rate.

Still another rationale for declining discount rates is provided by Weitzman (2001), who argues that "... even if every individual believes in a constant discount rate, the wide spread of opinion on what it should be makes the effective social discount rate decline significantly over time." He based this result on a survey of 2,160 economists, where he asked the appropriate rate to be used for evaluating environmental projects over a long time horizon.

Assuming the answers to follow a gamma probability distribution, he shows that the discount function, $A(t)$, is equal to

$$(2) \quad A(t) = \frac{1}{(1 + tS^2 / m)^{m^2 / s^2}}$$

and that the instantaneous rate of discount, $R(t)$, is equal to

$$(3) \quad R(t) = -\frac{A'}{A} = \frac{m}{1 + tS^2 / m}$$

where m and s^2 are the mean and variance of the distribution. He finds a sample mean and a standard deviation of 4 and 3 percent per annum respectively.

The function $A(t)$ is a generalized hyperbolic discount function and exhibits time inconsistency. If every person sampled were asked again, some years later, the rate of discount to be used, the answers, to be consistent, can be expected to be the same. This would result in the same mean and the same variance. The function $R(t)$ would start again at m . Hence, the inconsistency.¹

While the above list of rationales is undoubtedly far from complete, and the interested reader is well advised to look at the critical review on time discounting and time preference by Frederick et. al. (2002), it shows how the profession is leaning towards, or at least accepting, the idea of declining discount rates.

This paper examines, through a simple numerical example, the advancing or postponing effect of assuming a constant or a decreasing rate

¹ In this case, the inconsistency is the result of what Weitzman calls the irreducible uncertainty about discount rates among the different “experts” in the profession. It is the result of aggregation of individually consistent or exponential discount factors. In this sense, the problem is analogous to the intransitivity in collective preferences present even when individual preferences are transitive. On the other hand, this result can be interpreted as the consequence of incomplete capital markets where the different opinions on the rate of discount cannot be arbitrated.

of discount on the optimal timing of a project. Gamma and constant discounting will be used to show that the optimal timing of a project depends not only on the level of the discount rate but also on its slope with respect to time. Specifically, the example shows that using a constant rate of discount may have an advancing effect on the timing of investment, not captured by sensitivity analysis carried out for the complete range of instantaneous discount rates implicit in gamma discounting.

To abstract from and avoid the problem of inconsistency present in gamma discounting, the gamma discount function in equation (2), which gives rise to the instantaneous rate of discount, $R(t)$, in equation (3), will be interpreted as representing adequately the evolution of interest rates in the economy. This means that instantaneous market rates would be expected to decline in the future, consistent, at least qualitatively, with the hypothesis of conditional convergence as described above. Alternatively, one can think of equations (2) and (3) as intertemporally inconsistent, but where precommitment forces actions to follow irrevocably the original decision plan.

Assume a project with initial investment costs declining, due to expected technological change or market conditions, at an annual rate of 3 percent starting at 10 dollars in year zero. Assume benefits in year t to be

equal to $F(t) = 1$, $s \leq t \leq s+25$. Finally, the discount function is $A(t)$, so that the present value of net benefits is:

$$(4) \quad VP(S) = -10e^{-0.03s} A(s) + \int_s^{s+25} A(t) dt$$

If gamma discounting is assumed, then equation (4) can be written, after the necessary calculations which include the substitution of equation (2) for $A(t)$, as

$$(5) \quad VP(s) = \frac{-10e^{-0.03s}}{(1+0.0225s)^{\frac{16}{9}}} + \frac{400(1+0.0225s)^{\frac{-7}{9}}}{7} - \frac{400(1+0.0225(s+25))^{\frac{-7}{9}}}{7}$$

where m and s are equal to 4 and 3 percent per annum respectively, following Weitzman (2001).

The optimal value of s is $s^* = 8.19$ years. Since the project lasts for 25 years, this means that the project ends at $t = 33.19$.

The instantaneous discount rate at time t is

$$(6) \quad R(t) = -\frac{A'(t)}{A(t)} = \frac{m}{1+tS^2 / m} = \frac{0.04}{1+0.0225t}$$

This means that for $0 \leq t \leq 33.19$, which is the full time horizon of the project, the instantaneous discount rate will be in the range $0.0229 \leq R(t) \leq 0.04$.

For a constant discount rate λ , equation (4) would be

$$(7) \quad VP(s) = -10e^{-(0.03+I)s} + \int_s^{s+25} e^{-It} dt$$

After the necessary calculations, it can be shown that the optimal timing of investment is

$$(8) \quad s^* = \frac{\ln\left[\frac{10(0.03+I)}{(1-e^{-25I})}\right]}{0.03}$$

To evaluate the effect of a constant rate of discount on the optimal timing of investment, values between the two extreme instantaneous discount rates derived above, namely 0.0229 and 0.04, were used.

For $I = 0.04$, $s^* = 3.4$ years. For $I = 0.0229$, $s^* = 6.45$ years. These results mean that if a gamma discount function is the appropriate way to discount future flows, then no constant value, in the complete range of values, gives the optimal starting time $s^* = 8.19$ years. The optimal starting time of the project is advanced in all cases.

The main conclusion to be extracted from this simple example is that, beyond the specific functional forms and numbers used, the practical

use of a constant rate of discount may have an important advancing effect on the timing of projects, even when sensitivity analysis is performed.²

The opposite is also true. If a constant rate is to be used, then using a decreasing rate of discount may have an undesired postponing effect on the timing of projects.

While sensitivity analysis on a constant rate, performed for the full range of instantaneous discount rates implicit in a declining discount schedule, is sufficient to determine a range where the net present value of a typical project lies, it is not sufficient to determine an analogous range for the optimal timing of investment.

² This result does not require gamma discounting but only a decreasing discount function. On the other hand, even with gamma discounting, the advancing effect for the full range of values is not always true.

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