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AND COLLUSIVE BEHAVIOR**

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INDEX

ABSTRACT

1. INTRODUCTION	1
2. THE MODEL	6
2.1 Notation	6
2.2 Equilibria in unrelated markets	8
3. THE COMPETITIVE BUNDLING EQUILIBRIUM	9
4. COLLUSION	18
4.1 Collusive bundling	19
4.2 Sustaining a collusive agreement	22
4.3 Adding Bernheim&Whinston	27
5 EXTENSION: CONNECTING CITIES	28
6 CONCLUDING REMARKS	33
APPENDIX	34
REFERENCES	36

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Juan-Pablo Montero and Esperanza Johnson*

June 20, 2012

Abstract

We study the static and dynamic implications of non-linear pricing schemes (i.e., bundling) for otherwise unrelated products but for multimarket contact. Bundling is always present in competition but unlikely in a cartel agreement. Although it brings extra profits to the cartel —sometimes charging a premium rather than a discount for the bundle—, bundling makes deviation from the agreement far more attractive. Depending on the correlation of consumers' preferences, this deviation effect is either reinforced with milder punishments (for positive correlations) or partially offset with harsher punishments (for negative correlations). The deviation effect is so strong that it even dominates a zero-profit (pure-bundling) punishment.

Key words: multimarket contact, conglomerate merger, bundling, collusion

JEL classification: L13, L41

1 Introduction

More often we see firms in seemingly unrelated markets either merging or forming alliances which, among other things, allows them to bundle their products. Grocery stores in different countries, for example, offer their customers discount vouchers (fuelperks!) that can be redeemed when purchasing fuel at a particular gas station chain. In some cases the grocery store and the gas station engaged in this type of cross-market discounts are owned by the same conglomerate or parent company but in others they operate under

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a price alliance.¹ Similarly, consumers in some countries have now the option to buy their electricity and gas from the same supplier, possibly at a discount, or from two different suppliers;² likewise with regard to their mobile phone and cable television services.

In this paper we are interested in understanding the static and dynamic (i.e., the possibility of sustaining collusion) implications of having firms, either through conglomerate mergers or alliances, implementing non-linear pricing schemes for otherwise unrelated products. For example, we would like to know whether and when a proposal for a (second) conglomerate merger would make collusion less or more likely in the different markets the conglomerates will have contact.³ Likewise, we would like to know, or at least make a more informative guess, whether a substantial amount bundling (i.e., when we observe most consumers buying everything from either conglomerate because of the bundling discounts) is more likely the result of intensive competition or collusive behavior.⁴

There are two branches of the literature that are relevant to our study. One is the literature on multimarket contact and collusive behavior initiated with the intuition of Edwards (1955) and later formalized by Bernheim and Whinston (1990). The basic insight of the theory of Bernheim and Whinston (1990) is that two companies that encounter each other in more than one market can pool the incentive compatibility constraint for sustaining collusion of one market (e.g., mobile phone) with that of another market (e.g., cable-TV) into one multimarket incentive constraint. Put differently, this theory is one of transferring collusion discipline from markets in which collusion is easier to sustain to those in which is more difficult or simply not possible. The implication is that multimarket contact is irrelevant for markets that are identical in terms of sustaining collusion (e.g., same number of identical players in a repeated Bertrand game) and increasingly relevant as they differ in that regard.

The problem with the theory of Bernheim and Whinston (1990) in our context is that it assumes away any demand linkages that may exist across markets. This is a reasonable assumption for conglomerates that encounter each other in markets that are geographically distant,⁵ but not for conglomerates, as in the examples above, that face the same consumers in the different markets they serve. This takes us to the second literature relevant to our study, that of bundling in oligopoly. A conglomerate that supplies both phone and cable-TV to the same group of consumers can implement non-

¹See Goic et al (2011) and Gans and King (2006) for more details on these and other examples of cross-market discounts.

²See, e.g., the discussion in Granier and Podesta (2010).

³The anticompetitive effects of conglomerate mergers, as distinct from vertical and horizontal mergers, are more extensively discussed by Dave (2008).

⁴The model also sheds light on the question of whether mandating multiproduct firms à la carte pricing, i.e., forcing firms to unbundle their products, is beneficial to consumers. See Crawford and Yurukoglu (2012) find this to be case in the multichannel television market but for the increase in input costs. We take input costs to be fixed.

⁵A good example is the hotel industry (Fernandez and Marin, 1998).

linear pricing schemes (i.e., mixed or pure bundling) that single-suppliers cannot, which has implications for both static and dynamic competition.

There is now a vast literature on multiproduct non-linear pricing and bundling in the static context. The monopoly problem has been studied, among others, by Adams and Yellen (1976), Schmalensee (1984), McAfee et al. (1989), Armstrong (1996) and more recently by Chen and Riordan (2012). The central message of the more recent papers is that the optimality of bundling extends well beyond the original insight of Adams and Yellen (1976) that the correlation in consumers preferences must be negative for bundling to exist. But these papers are mainly concerned with establishing the conditions for the preferences of the consumers under which bundling, whether pure or mixed, dominates linear pricing (i.e., separate selling) and not with finding the profit-maximizing solution for the monopolist. As noted by Armstrong (1996), computing such solution is not trivial except in a few isolated examples. In principle, this complicates our analysis because we need to go through a similar computation exercise to obtain the optimal collusive agreement of a cartel and the optimal deviation from it. The (Salop) model we develop in Section 2 copes with these complications in that it provides closed-form solutions for both competitive and collusive behavior (including deviations) for the entire range of possible correlation in consumer preferences.

There is also a growing literature on the (static) oligopoly problem but here the results are dependent on to the underlying assumptions and model set-up (Stole, 2007). Some papers, e.g., Armstrong and Vickers (2001) and Thanassoulis (2007, section 3), adopt a one-stop shopping framework in that each consumer buys everything from the same supplier. Others assume that conglomerates can commit to either product compatibility (Matutes and Regibeau, 1992) or bundling discounts (Gans and King, 2006). In this paper we follow Armstrong and Vickers (2010), and also sections of Matutes and Regibeau (1992) and Thanassoulis (2007), in assuming that conglomerates set all prices simultaneously, including that of the bundle, and that each consumer is free to purchase all items from the same supplier or from different suppliers (and become a two-stop shopper).⁶ According to these papers —see, for example, Proposition 4 in Armstrong and Vickers (2010)—, it seems that the possibility of offering bundling discounts gives rise to a prisoners' dilemma for the conglomerates forcing them to price more aggressively than under linear pricing (or with single-product firms). The intuition is that bundling reduces product differentiation for at least some consumers which is what induces conglomerates

⁶We also assume that consumers have unit demands and there is complete market coverage. Armstrong and Vickers (2010) also cover elastic demands (see their Proposition 3 and note the connection to the unit demand case).

to compete more intensively.^{7,8}

The problem with these papers is that they focus on the specific case of zero correlation in consumers preferences. Except for Armstrong and Vickers (2010), which work with a general (and symmetric) density function of consumer preferences/locations over the unit square, the other papers simply assume that consumers are uniformly distributed over the unit square. But based on the monopoly works above and the emphasis they place on the correlation of valuations, this seems to be an important shortcoming. The only attempt we are aware of that relaxes the zero-correlation assumption in an oligopoly environment is Reisinger (2006). His model also builds upon the Salop model, but it is quite different from ours (among others, his model does not cover the entire range of correlations). Nevertheless, he finds, as we do, that the correlation plays a crucial role in dictating whether the price-discrimination effect of bundling—the only effect present in monopoly—dominates its business-stealing effect (i.e., making it easier for a conglomerate to steal consumers from the rival).

As explained in Section 3 of the paper, we find that bundling benefits conglomerates for positive correlations (Proposition 1) but it benefits consumers for negative correlations (Proposition 2). To provide some intuition start from the situation in which conglomerates are changing the equilibrium linear prices. When the correlation is highly positive each conglomerate enjoys a large fraction of "captive" consumers, i.e., consumers that strongly prefer all products from the same conglomerate. So, conglomerates can go after the two-stop shoppers with higher stand-alone prices⁹—while maintaining the original linear prices to the one-stop shoppers—without fear of losing them to the rival. This is the price-discrimination effect of bundling that allows conglomerates to extract extra surplus in equilibrium. This seems to be the prevailing effect in the recent work of Crawford and Yurukoglu (2012) for the multichannel television market that, provided input costs do not change, shows that consumers would benefit greatly if TV distributors are mandated to price à la carte.¹⁰

As the correlation drops, the fraction of captive consumers falls and a conglomerate finds it more attractive to bundle and offer both products at a lower price (lower than the sum of the linear prices) because this way it can attract a large number of consumers

⁷An example may help. Denote conglomerates by 1 and 2 and products by A and B . The marginal cost of producing either product is 1. Suppose consumers are of two types, I and II , and are in the same proportion. Let the pair $v_1^I = (7, 3)$ be type I 's valuations for conglomerate 1's products A and B , respectively. The other pairs are $v_2^I = (3, 7)$, $v_1^{II} = (3, 7)$, and $v_2^{II} = (7, 3)$. Note that consumers see absolutely no difference in the bundles, they value them equally at 10. The equilibrium price in the absence of conglomerates (or if conglomerates are forced to use linear pricing) is 7 and each conglomerate obtains a profit of $3 = \frac{1}{2}(7 - 1)$ from each item. Conversely, when bundling is available the equilibrium consists in a price for the bundle of 2 and conglomerates make zero profit.

⁸Note that in Chen's (1997) model, bundling plays the exact opposite role; it serves to soften competition by introducing product differentiation among otherwise single-product duopolists.

⁹The prevailing prices when purchasing from different suppliers.

¹⁰The problem, they explain, is that input costs may change substantially as a result of bilateral re-negotiations with input suppliers. In this paper we take input costs to be fixed.

that see the bundles not that different (the problem for the companies is that both think the same). This is the action of the business-stealing effect. Put it in more technical terms, when the correlation is positive a conglomerate’s optimal (global) response to linear pricing is to increase the stand-alone prices, which indirectly introduces a bundling discount for its one-stop shoppers, whereas when the correlation is negative the optimal (global) response is to directly lower the price to those buying the bundle. The strategic complementarity of prices explains the rest. In Section 5 (Extensions) we show that these results extend beyond the Salop model to the square unit with different distribution of consumers (in fact, several distribution of consumers in the square city can be represented as consumers placed in a collection of Salop cities).

In Section 4 we take these competitive bundling equilibrium results to the study of collusive behavior; in particular, we look at how the possibility of implementing bundling schemes, whether at the collusive, deviation, and/or punishment phase, makes it easier or more difficult for conglomerates to sustain collusion relative to individual firms in unrelated markets. The literature on collusion and bundling is more limited and there is nothing, as far as we know, like what we do in this paper. For example, Spector (2007) shows that a firm that is a monopolist in one market, say A , but an oligopolist in a second market B may decide to sell A bundled with B , in addition to B , only to facilitate collusion in market B . Indeed, such practice takes part of the demand—coming from consumers willing to buy both A and B —out of the reach of future deviators in market B . More recently, Dana and Fong (2012) find a similar mechanism operating in an intertemporal bundling model, which arises when customers contract for multi-period service arrangements with a single supplier (e.g., newspapers, magazines, etc). Intertemporal bundling only reduces the demand available at the period of deviation because any consumer that is approached that period anticipates the price war that initiates next period, and therefore, the prices available to him/her in later periods.

It is tempting to take the insights of these papers to our context and deduce that bundling also facilitates collusion by making it harder for a deviator to attract the rival’s one-stop shoppers. The exact opposite turns out to be true: conglomerates have unambiguously a much more difficult time in sustaining collusion than do firms in unrelated markets (or, equivalently, multiproduct firms that are forced to price à la carte).¹¹ Like in monopoly, bundling helps conglomerates to increase profits along the collusive path (Proposition 3)—sometimes charging a premium rather than a discount for the bundle. Although the amount of collusive bundling, measured by the fraction of one-stop shoppers, is substantially smaller than that observed under competitive bundling (Propositions 1 and 2), unless the correlation of preferences is perfectly positive in which case bundling becomes irrelevant for competition and for the cartel.

Despite the extra profits from collusive bundling, the problem for the cartel is that

¹¹Note that we can also view our consumers as being served by multi-period contracts. What is important is that the contracts do not vary with the shopping pattern.

the deviation from such collusive agreement can be far more attractive than the deviation from linear monopoly pricing (Proposition 5). The reason is simple. The introduction of a bundling discount creates consumers that are somehow indifferent about where to one-stop shop. In fact, if the correlation in consumers preferences is perfectly negative (see example in fn. 7), a conglomerate can serve the entire population of one-stop shoppers in the period of deviation by reducing the price of the bundle only slightly. One important policy implication from this collusion analysis is that observing a large fraction of one-stop shopping, together with bundling discounts, is more likely the result of competition than of collusive behavior.

One recommendation for the cartel then is to stick to linear (monopoly) prices. But even in cases where there is a threat of a zero-profit price war (perfectly negative correlation), collusion remains harder to sustain for the conglomerates (Proposition 4). The reason is again that bundling provides the deviator with an effective tool to steal consumers. This raises an obvious question in front of a proposal for a conglomerate merger: When is the anti-collusive effect of bundling likely to revert the pro-collusive effect of Bernheim and Whinston (1990)? Less likely as the markets involved in the merger (or price alliance) are more asymmetric from a B&W perspective. This is particularly true when the asymmetry is in product differentiation because bundling becomes irrelevant as one of the markets goes perfectly competitive.

We conclude in Section 6 with a discussion of how elements formally absent in the model can be incorporated and the implications they may have in the results; for example, when consumers may face a "shopping cost" if they purchase from firms that belong to different conglomerates. We also provide some ideas for future research taking advantage of the tractability of the model.

2 The model

2.1 Notation

Consider a circular (Salop) city of unit length and a unit mass of consumers uniformly distributed on its perimeter. There are two products, A and B , and two conglomerates/alliances, 1 and 2, each with two firms; one that produces good A at unit cost c_A and the other B at cost c_B . Each consumer desires at most one unit of each good and is characterized by the common reservation values, v_A and v_B , the common transport cost parameters, t_A and t_B , and her location in the city's perimeter $x = (x_{1A}, x_{1B}, x_{2A}, x_{2B})$, where $x_{ik} < 1/2$ is the distance of the consumer to the firm in conglomerate $i = 1, 2$ that produces good $k = A, B$.

The location of the firms in the city's perimeter is depicted in Figure 1. Firms selling the same product locate in front of one another (i.e., at a distance of $1/2$) and firms from the same conglomerate are $\theta \in [0, 1/2]$ miles apart, where θ is an exogenous parameter that captures the degree of correlation between the reservation values of the two goods

of either conglomerate, net of transportation (i.e., product differentiation) costs, across the population of consumers. We say that there is a positive correlation in conglomerate preferences when a consumer that buys product A from conglomerate i is more likely to buy product B from the same conglomerate. Thus, the case of perfectly positive correlation is when $\theta = 0$, i.e., when the two firms of a conglomerate (e.g., $1A$ and $1B$) are in the exact same location so that $x_{iA} = x_{iB}$ for all consumers; the case of perfectly negative correlation is when $\theta = 1/2$, i.e., when firms of different conglomerates (e.g., $1A$ and $2B$) are in the same location so that $x_{iA} = x_{-iB}$ for all consumers; and the case of no correlation is when $\theta = 1/4$. More generally, the one-to-one mapping from the distance θ to the correlation ρ is given by $\rho = 1 - 4\theta$.

*** figure 1 here or below ***

This correlation in preferences, which is crucial for competition, can also be seen in Figure 2 as we place our circular-city consumers in the square city of Armstrong and Vickers (2010); firms $1A$ and $1B$ are in the lower-left corner and $2A$ and $2B$ are in the upper-right corner and the distance between firms selling the same product is $1/2$ (as in the circular city). The perimeter of the rectangle indexed by θ_1 contains the unit mass of consumers uniformly distributed on the circular city $\theta = \theta_1$. Note that the fraction of consumers on either of the short sides of the rectangle is exactly equal to θ_1 , so when this fraction approaches zero ($\theta \rightarrow 0$), the correlation approaches 1 and when $\theta \rightarrow 1/2$, as in the second rectangle θ_2 in the figure, the correlation approaches -1 . Note also that our circular approach easily accommodates the case of consumers uniformly distributed over the square city, as in Matutes and Regibeau (1992), Thanassoulis (2007), Armstrong and Vickers' (2010) uniform example, etc., by simply considering a continuum of circular cities uniformly indexed by $\theta \in [0, 1/2]$. We come back to this connection in Section 5 (Extensions).

*** figure 2 here or below ***

We assume that v_A and v_B are sufficiently large that all consumers will purchase both products in equilibrium (a few additional restrictions on parameter values are introduced as we use them). Thus, a customer located at x that purchases A from conglomerate i and B from $-i$ gets utility

$$u(x) = v_A + v_B - p_{iA} - p_{-iB} - t_A x_{iA} - t_B x_{-iB}$$

and if she purchases both products from the same conglomerate $i = 1, 2$, she gets

$$u(x) = v_A + v_B - p_{iA} - p_{iB} + \gamma_i - t_A x_{iA} - t_B x_{iB}$$

where p_{ik} is the stand-alone price that conglomerate i charges for product k and $\gamma_i \geq 0$ is the bundle discount received by the one-stop shoppers that go to i (at times we will

use $p_{iAB} = p_{iA} + p_{iB} - \gamma_i$ to refer to the price that i charges for the bundle).¹² All prices are set simultaneously in each period. We focus on symmetric equilibria throughout.

2.2 Equilibria in unrelated markets

Before looking at non-linear pricing, it is useful to have the equilibrium solutions that would prevail in the absence of conglomerates or price alliances, as they would provide the relevant benchmarks. Since markets are totally unrelated, the preference of consumers for the two products, as captured by θ , is irrelevant for the equilibrium solution. The equilibrium of the one-shot game is well known (Tirole, 1988)

$$p_{ik} = p_{-ik} = p_k^u = c_k + \frac{t_k}{2} \quad (1)$$

for $k = A, B$.¹³ And since firms split the market, a firm's profit under the static Nash equilibrium (i.e., the "unrelated markets" equilibrium) is equal to $\pi_k^u = t_k/4$.

It is also well known that in a repeated interaction firms may be able to sustain better outcomes in equilibrium. Assuming that firms rely on grim trigger strategies to sustain collusion (i.e., deviations are punished with reversions to the static Nash equilibrium forever), the incentive compatibility constraint that must be satisfied for sustaining collusion in market $k = A, B$ is

$$\frac{\pi_k^c}{1 - \delta} \geq \pi_k^d + \frac{\delta \pi_k^u}{1 - \delta} \quad (2)$$

where δ is the discount factor common to all firms, $\pi_k^c > \pi_k^u$ is each firm's collusive profit and $\pi_k^d > \pi_k^c$ is the profit a firm earns in one period if it (optimally) deviates from the collusive agreement. Note that an optimal deviation here is to undercut the collusive price by $t_k/2$ and take the entire market.¹⁴

We know then from (2) that firms can sustain the monopoly outcome in (subgame-perfect) equilibrium, which is to charge $p_k^m = v_k - t_k/4$ in every period, if δ is above the critical level

$$\underline{\delta}_k = \frac{p_k^m - c_k - t_k}{2(p_k^m - c_k - t_k) + t_k/2} \quad (3)$$

Note that despite an increase in product differentiation leads to both a softer punishment and lower profits along the collusive path, it nevertheless makes it easier for firms to sustain monopoly profits in the sense that it expands the range of discount factors where

¹²As we discuss below, there will be cases in which conglomerates would like to charge a bundle premium, as opposed to a discount, to one-stop shoppers. This makes collusion more profitable but not necessarily easier to sustain. Yet, its implementation requires of perfect monitoring of sales which seems difficult in many cases. See McAfee et al. (1989) for a discussion on the latter.

¹³Recall that for this to be an equilibrium we need $t_k < v_k - c_k$ (Tirole, 1988).

¹⁴When the collusive price is the monopoly price ($v_k - t_k/4$), this optimal deviation requires $t_k < 4(v_k - c_k)/7$, which we assume to hold since v 's are large enough.

that can happen in equilibrium (i.e., lower $\underline{\delta}_k$). The reason is that a higher t makes deviation less attractive and this deviation effect dominates the other two effects. We will see a similar phenomenon when conglomerates try to sustain collusion in that bundling makes deviation from the collusive agreement very attractive.

The multimarket-contact result of Bernheim and Whinston (1990) can also be easily illustrated using (3). Suppose markets A and B differ in such a way that $\underline{\delta}_A > \delta > \underline{\delta}_B$. If now firms $1A$ and $1B$ merge to form conglomerate 1 and likewise $2A$ and $2B$ to form conglomerate 2, but stick to *linear prices* at all times, there will be cases in which the critical factor above which conglomerates can sustain monopoly profits in both markets, $\underline{\delta}^{BW}$, is below the firms' discount factor (i.e., $\underline{\delta}_B < \underline{\delta}^{BW} < \delta < \underline{\delta}_A$). If so, the multimarket contact generated by the mergers has allowed firms to transfer monopoly discipline from market B to market A . In this paper we are interested in a fundamentally different effect of multimarket contact on collusion —that of non-linear pricing— that will arise even if markets are identical from a Bernheim-Whinston's perspective ($\underline{\delta}_A = \underline{\delta}_B$). Obviously, the two effects are likely to be present but to keep the analysis clean (and simplify some of the proofs) we will impose symmetry in most places.

3 The competitive bundling equilibrium

In this section we will derive the static (symmetric) equilibrium when conglomerates may offer discounts to one-stop shoppers; we do so for a Salop city characterized by some exogenously given correlation in consumers' preferences $\theta \in [0, 1/2]$. In the Extensions we argue that the outcome of such equilibrium is not that different from the one that emerges in a supposedly more general square-city model that also allows for any correlation level.

Consider then the Salop-city θ of Figure 3. While conglomerates set three prices each (i.e., p_{iA} , p_{iB} and γ_i), consumers really care about four prices, the prices of each of the four choices they have: either buy both products from $i = 1, 2$ or A from i and B from $-i$. Given the vector of prices (p, γ) in the figure, conglomerate i 's demand for its product k is

$$q_{ik}(p, \gamma) = \frac{1}{2} + \frac{p_{-ik} - p_{ik}}{t_k} + \frac{\gamma_i}{2t_k} - \frac{\gamma_{-i}}{2t_k} \quad (4)$$

where $i = 1, 2$ and $k = A, B$. The first two terms are the familiar "Hotelling terms". The third term, $\gamma_i/2t_k$, corresponds to the additional demand for product k conglomerate i is able to attract with the bundling discount γ_i , that is, the two-stop shoppers that before the discount were buying product k from $-i$ and $-k$ from i . These two stop-shoppers are now willing to travel the extra distance $\gamma_i/2t_k$ (in the k -dimension) in order to pocket the discount γ_i . Similarly, the last term, $\gamma_{-i}/2t_k$, is the demand i losses to conglomerate $-i$ from the two-stop shoppers that now buy k (and $-k$) from $-i$.

*** figure 3 here or below ***

Expression (4) tells us only about the total demand for product k but not about how much is from one-stop shoppers and how much from two-stop shoppers, which is important for computing firms' profit. This division is easy to obtain when $p_{-ik} = p_{ik}$ for both $k = A, B$. Before any discount, the fraction of one-stop shoppers going to each conglomerate is $1/2 - \theta$; there are only one-stop shoppers in a city with perfectly positive correlation ($\theta = 0$), despite the absence of discounts, and none in a city with perfectly negative correlation ($\theta = 1/2$). Following the logic above, and given that there is a positive fraction of two-stop shoppers, the number of one-stop shoppers received by company i increases with its own discount (see figure 3)

$$\psi_i(\gamma_i, \gamma_{-i} | p_{-ik} = p_{ik}) = \frac{1}{2} - \theta + \frac{\gamma_i}{2t_A} + \frac{\gamma_i}{2t_B} \geq 0 \quad (5)$$

and, therefore, i 's profit is

$$\pi_i = \sum_{k=A,B} q_{ik}(\cdot)(p_{ik} - c_k) - \psi_i(\cdot)\gamma_i \quad (6)$$

where $q_{ik}(\cdot)$ is given by (4).

Since quantities in these Hotelling models depend on price differences (i.e., $p_{ik} - p_{-ik}$), we can impose symmetry in stand-alone prices and solve for the equilibrium discount directly using (6). Thus, maximizing π_i with respect to γ_i and letting $\gamma_i = \gamma_{-i} = \gamma$ yields

$$\gamma(\theta) = \frac{4}{3}\theta \frac{t_A t_B}{t_A + t_B} \quad (7)$$

Alternatively, the equilibrium bundling discount could have been obtained following Proposition 1 of Armstrong and Vickers (2010). Since in a symmetric equilibrium (i.e., $p_{ik} = p_{-ik} = p_k$ and $\gamma_i = \gamma_j = \gamma$) the fraction of two-stop shoppers is given by

$$\Phi(\gamma, \theta) = 1 - \psi_1 - \psi_2 = 2 \left(\theta - \frac{\gamma}{2t_A} - \frac{\gamma}{2t_B} \right) \geq 0 \quad (8)$$

the equilibrium discount for a Salop-city θ must satisfy the first-order condition

$$2\Phi(\gamma, \theta) + \gamma\Phi_\gamma(\gamma, \theta) = 0 \quad (9)$$

that has (7) as solution.

There are a couple of observations worth mentioning. First, that linear pricing ($\gamma = 0$) is an equilibrium only when there is perfectly positive correlation ($\theta = 0$). This is the only case in which there are only one-stop shoppers despite the zero discount. Otherwise, the discount is positive and increasing with θ , which is not surprising because the demand for the bundle increases with the fraction of two-stop shoppers that may eventually buy the bundle. Second, that the equilibrium discount increases as the correlation falls (i.e., moves towards -1) is in line with the monopoly results of Adam and Yellen (1976) and

McAfee et al. (1989). Yet, there are some caveats. One is that in McAfee et al (1989) it was sometimes optimal—for positive correlation levels—for the monopolist to charge a bundling premium, as opposed to a discount, to one-stop shoppers. While a premium is never part of the equilibrium in competition (see also Armstrong and Vickers (2010) for a discussion), it can be part of a collusive agreement, as we will see in the next section. Another caveat is that under monopoly the number of two-stop shoppers drops to zero as the correlation approaches -1 , which, according to (7) and (8), does not seem to be the case under duopoly competition.¹⁵ We cannot elaborate any further on this latter caveat without completing the characterization of the equilibrium.

Let us then obtain the stand-alone equilibrium prices p_A and p_B . Consider conglomerate 1's incentive to slightly reduce its stand-alone price p_A by ε , while keeping its discount unchanged at γ (so the price of its bundle is now $p_A + p_B - \gamma - \varepsilon$) and its stand-alone price for product B unchanged at p_B . This change in prices is captured by the dashed line in Figure 3, which corresponds to a horizontal movement of $\varepsilon/2t_A$. On the one hand, this price change has reported company 1 with inframarginal losses equal to $\varepsilon/2$ from one half of the market that was buying A before the price reduction. These losses are compensated with marginal gains from extra units of product A : $\varepsilon/2t_A$ units from consumers that before were buying the bundle from 2 and now buy product A from 1 and another $\varepsilon/2t_A$ units from consumers that before were buying A from 2 and B from 1 and now buy the bundle from 1.

In equilibrium, gains and losses must be equal

$$\frac{1}{2}\varepsilon = \frac{\varepsilon}{2t_A}(p_A - c_A) + \frac{\varepsilon}{2t_A}(p_A - \gamma - c_A)$$

which leads to the stand-alone equilibrium price for A

$$p_A = c_A + \frac{t_A}{2} + \frac{\gamma}{2} = p_A^u + \frac{\gamma}{2} \quad (10)$$

and, similarly, for B

$$p_B = c_B + \frac{t_B}{2} + \frac{\gamma}{2} = p_B^u + \frac{\gamma}{2} \quad (11)$$

where γ is given by (7).¹⁶ Note that the equilibrium price of the bundle is equal to the sum of the equilibrium prices in the case of unrelated markets, that is, $p_{AB} = p_{AB}^u \equiv p_A^u + p_B^u$.

Relative to the benchmark case in section 1, eq. (1), these results indicate that bundling has been quite beneficial for firms allowing them to extract extra surplus from two-stop shoppers, i.e., those that pay stand-alone prices. This result appears to be in sharp contrast with Proposition 4 in Armstrong and Vickers (2010) that says that

¹⁵If the equilibrium condition (9) is valid for all θ , then $\Phi(\gamma(\theta = 1/2), 1/2) = 1/3$. Shortly we will see however that it is not; indeed, the number of two-stop shoppers when $\theta = 1/2$ is zero.

¹⁶Below, after Proposition 1, we discuss the possibility of the (simultaneous) existence of a "trivial" pure-bundling equilibrium in which conglomerates charge very high prices for separate items.

bundling typically benefits all consumers because it engages conglomerates in fiercer competition for the one-stop shoppers, competition that spills over the two-stop shoppers.¹⁷ The next two propositions will show that there is no such contradiction.

Proposition 1 *Suppose $t_A = t_B = t$. For $\theta \leq \hat{\theta}$, there is a mixed-bundling symmetric equilibrium that is characterized by eqs. (7), (10) and (11), where*

$$\hat{\theta} = 4\frac{1}{2} - 3\sqrt{2} = 0.257$$

Proof. Appendix A covers some implications of having $t_A \neq t_B$. If companies are playing according to (7), (10) and (11), each obtains profit equal to

$$\pi_{\theta \leq \hat{\theta}}^b(\theta) = \frac{t}{2} + \frac{2t}{9}\theta^2$$

where the second term is the extra profit due to bundling. It is immediate that for $\theta = 1/2$, eqs. (7), (10) and (11) cannot constitute an equilibrium since a company can attract all the one-stop shoppers by slightly increasing the bundling discount. When $\theta = 1/2$ (and $t_A = t_B = t$), the one-stop shoppers, which amount to $1 - \Phi(\cdot) > 0$, are absolutely indifferent between equally-priced bundles (bundling has created homogenous products for these consumers). The same logic applies more generally for $\theta < 1/2$. Suppose that company i is considering a larger bundling discount $\tilde{\gamma} > \gamma(\theta)$ enough to attract $-i$'s one-stop shoppers (increasing the discount is cheaper than reducing the stand-alone price of either A or B). Given that $t_A = t_B = t$, to estimate $\tilde{\gamma}$ it suffices to focus on the decision of the one-stop shopper that locates halfway between firm $2A$ and $2B$ (see figure 1). Such consumer will switch bundles as long as the additional discount more than compensate the extra travel, that is

$$\tilde{\gamma} - \gamma(\theta) \geq (1 - \theta)t - \theta t = (1 - 2\theta)t \equiv \Delta \tag{12}$$

Note that as $\theta \rightarrow 1/2$ the extra discount approaches zero. The new discount $\tilde{\gamma}$ will attract not only one-stop shoppers but also some or all of the two-stop shoppers. It will attract them all if the most distant two-stop shopper (the one located next to firm $-ik$, see figure 1), decides to switch to the bundle, which happens if

$$p_A + p_B - \tilde{\gamma} + (1 - 2\theta + \theta)t \leq p_A + p_B + (1/2 - \theta)t \tag{13}$$

Thus, if we let $\tilde{\gamma} = \gamma(\theta) + \Delta$, then (13) becomes $\theta \leq 3/8$. Now, a company that deviates

¹⁷It strictly does if consumers are uniformly distributed over the square city; see uniform example in p. 39 and Matutes and Regibeau (1992).

and plays $\gamma(\theta) + \Delta$ when $\theta \leq 3/8$ earns

$$\tilde{\pi}(\theta) = 2 \left(\frac{t}{2} + \frac{\gamma(\theta)}{2} \right) - (\gamma(\theta) + \Delta)$$

from selling bundles to the entire market. It is easy to see that $\pi^b(\theta)|_{\theta \leq \hat{\theta}} \geq \tilde{\pi}(\theta) \iff \theta \leq \hat{\theta}$. ■

A main implication of Proposition 1 is that (mixed) bundling sometimes do benefit conglomerates (consistent with the finding of Crawford and Yurukoglu (2012) for the multichannel television market under constant input costs); it does when consumer preferences are positively correlated. Using a variation of the Salop-city model, but quite different from ours, Reisinger (2006) finds a similar result. The explanation is that for positive correlations or nearly so (i.e., $\theta < \hat{\theta}$) the price-discrimination effect of bundling dominates the business-stealing effect (it is a dominant strategy to bundle products). This result seems to go against the pro-competitive results of Matutes and Regibeau (1992) and Armstrong and Vickers (2010) for zero correlation environments. We will see next that it does not because implicit in Proposition 1 (and its proof) is that competition becomes more intense for values of $\theta > \hat{\theta}$.

But before we look into that, note that the game may simultaneously accept a pure bundling equilibrium which is to offer one price for the bundle —the price of the "unrelated-market package" (p_{AB}^u)— together with high enough stand-alone prices (e.g., $v_A + v_B - c_{-k}$) for each item k .¹⁸ No company can ever attract two-stop shoppers, so this is equivalent to restricting companies just to sell bundles. There are however good reasons why this "unrelated-package" equilibrium may not hold. We know from the proof of Proposition 1 that a conglomerate that undercuts the bundle price by $\Delta \equiv (1 - 2\theta)t$ is able to attract all the one-stop shoppers of its rival, increasing its profit from $t/2$, the profit under the unrelated package, to $2\theta t$. Thus, the unrelated package equilibrium cannot be such for $\theta > 1/4$. Another reason is that the mixed-bundling equilibrium in Proposition 1 Pareto dominates the unrelated-package one. Yet, a more practical reason is that there may exist consumers, albeit very few, that would like to buy just one of the products (Thanassoulis, 2007), i.e., that have a positive reservation value for one item and zero for the other. This would destroy the unrelated-package equilibrium even if the mass of such single-item consumers is infinitesimally small provided that they are happy to buy at the stand-alone prices of Proposition 1.¹⁹

We leave for later the intuition of why (competitive) bundling is good for firms when the correlation of preferences is positive but not when is negative, as it is established in the next proposition.

Proposition 2 *Suppose $t_A = t_B = t$. For $\theta > \hat{\theta}$, there is a pure-bundling symmetric*

¹⁸Strictly speaking conglomerates do not need to charge such high stand-alone prices, it suffices to charge the ones in Proposition 2.

¹⁹Or slightly higher but below the stand-alone prices in Proposition 2.

equilibrium in mixed strategies that is more competitive than the linear-pricing benchmark, i.e., eq. (1), and is characterized by (i) the price support $p_{AB} \in [\underline{p}_{AB}, \bar{p}_{AB}]$, where

$$\bar{p}_{AB} = \begin{cases} c_A + c_B + 2(1 - \sqrt{\theta})t & \text{if } \hat{\theta} < \theta \leq \tilde{\theta} \\ c_A + c_B + \frac{2}{1-\theta}\Delta & \text{if } \tilde{\theta} < \theta \leq \frac{1}{2} \end{cases} \quad (14)$$

and

$$\underline{p}_{AB} = \begin{cases} c_A + c_B + 2(1 + \theta - 2\sqrt{\theta})t & \text{if } \hat{\theta} < \theta \leq \tilde{\theta} \\ c_A + c_B + \frac{2\theta}{1-\theta}\Delta & \text{if } \tilde{\theta} < \theta \leq \frac{1}{2} \end{cases} \quad (15)$$

and (ii) the stand-alone prices

$$p_k \geq \begin{cases} c_k + (1 + \theta - \sqrt{\theta})t & \text{if } \hat{\theta} < \theta \leq \tilde{\theta} \\ c_k + \frac{2-3\theta}{1-\theta}t & \text{if } \tilde{\theta} < \theta \leq \frac{1}{2} \end{cases} \quad \text{for } k = A, B \quad (16)$$

where $\tilde{\theta} = \frac{1}{2}(3 - \sqrt{5}) = 0.382$.

Proof. The proof involves several steps (Appendix B covers some implications of having $t_A \neq t_B$). We first show that the pure-bundling equilibrium is more competitive than the linear benchmark. It is immediate since $\bar{p}_{AB} < p_A^u + p_B^u$ for all $\theta > \hat{\theta}$. Secondly, we establish, by contradiction, that the equilibrium must be in pure bundling.²⁰ If not, the bundling discount must be equal to (7) and the stand-alone prices to (10) and (11). But from Proposition 1 we know this is not possible when $\theta > \hat{\theta}$.

Third, and as part of the construction of the price support, we rule out two (pure-strategy) equilibrium candidates. One obvious candidate is the "unrelated package": $p_{AB}^u \equiv p_A^u + p_B^u$ (and stand-alone prices high enough to eliminate any two-stop shopper; we come back to this issue in the last step of the proof). We know this is an equilibrium only if conglomerates are restricted, for some reason, to marginal deviations from rival's prices. But as we discussed above, this equilibrium disappears when conglomerates are open to discrete deviations, not only marginal ones, and $\theta > 1/4$: a conglomerate that undercuts the bundle price by $\Delta \equiv (1 - 2\theta)t$ increases its profit from $t/2$ to $2\theta t$. A second equilibrium candidate then is one in which no conglomerate has incentives to deviate by undercutting by Δ , that is

$$\frac{1}{2}(p_{AB} - c_A - c_B) \geq p_{AB} - \Delta - c_A - c_B \quad (17)$$

Let $p_{AB}^L = c_A + c_B + 2\Delta$ denote the solution of the equality in (17). Neither can this be an equilibrium because now conglomerates have incentives to raise the price of the bundle to either $p_{AB}' = p_{AB}^L + \Delta$, and keep a fraction θ of the consumers (the increase

²⁰We have not come close to explore the possibility of an equilibrium in which conglomerates mix all three prices.

must be slightly less than Δ ; note also that $3\Delta\theta > 2\Delta/2$ when $\theta > \hat{\theta}$), or

$$p''_{AB} = \arg \max_p \left\{ (p - c_A - c_B) \left(\frac{1}{2} + \frac{p_{AB}^L - p}{2t} \right) \right\} = \frac{1}{2}(p_{AB}^L + t + c_A + c_B),$$

whichever is smaller. More generally, let us denote by

$$R(p_{AB}) = \min \left\{ p_{AB} + \Delta, \frac{1}{2}(p_{AB} + t + c_A + c_B) \right\}$$

i 's optimal upward response when $-i$ prices its bundle at p_{AB} and by

$$q(R(p_{AB}), p_{AB}) = \min \left\{ \theta, \frac{1}{2} + \frac{p_{AB} - R(p_{AB})}{2t} \right\}$$

i 's corresponding bundling demand.

We are now ready to construct the price support. Suppose conglomerate $-i$ offers its bundle for p_{-iAB} , where $p_{AB}^L < p_{-iAB} < p_{AB}^u$. We know from the previous step that is not optimal for i to play $p_{AB} = p_{-iAB}$ but rather a lower price equal to $p_{-iAB} - \Delta$ or a higher price equal to $R(p_{-iAB})$, whichever is more profitable. Therefore, there must exist a price $p_{-iAB} \in (p_{AB}^L, p_{AB}^u)$ for which either deviation is equally profitable for i , that is

$$\pi_i(p_{-iAB} - \Delta, p_{-iAB}) = \pi_i(R(p_{-iAB}), p_{-iAB}) \quad (18)$$

where $\pi_i(p_{-iAB} - \Delta, p_{-iAB}) = p_{-iAB} - \Delta - c_A - c_B$ and $\pi_i(R(p_{-iAB}), p_{-iAB}) = (R(p_{-iAB}) - c_A - c_B)q(R(p_{-iAB}), p_{-iAB})$. Let p_{AB}^* be the solution of (18); then, $\bar{p}_{AB} = R(p_{AB}^*)$. Since $R(p_{AB} < p_{AB}^*) < R(p_{AB}^*)$, no conglomerate will ever price above $R(p_{AB}^*)$ if it knows for certain that its rival's price is $R(p_{AB}^*)$ or lower. The same logic extends when i knows that $-i$ is, instead of setting a single given price, mixing prices in the support $p_{AB} \leq R(p_{AB}^*)$ according to some probability function $G(p_{AB})$. Note that at $\theta = \tilde{\theta}$, where $\tilde{\theta} + \sqrt{\tilde{\theta}} = 1$, $p_{AB}^* + \Delta = (p_{AB}^* + t + c_A + c_B)/2$, so below this level $\bar{p}_{AB} = (p_{AB}^* + t + c_A + c_B)/2$, otherwise $\bar{p}_{AB} = p_{AB}^* + \Delta$. These are the two terms in (14) that reflect the fact that p_{AB}^* changes its functional form when $\theta > \tilde{\theta}$. The lower limit of the support is obtained in a similar fashion. Since no conglomerate undercuts by Δ if it knows for certain that its rival's price is below p_{AB}^* —it rather reacts with $R(p_{AB}^*)$ —, no conglomerate will ever price below $p_{AB}^* - \Delta$; and this extends even if i knows that $-i$ is, instead of setting a single price, mixing prices in the support $p_{AB} \geq p_{AB}^* - \Delta$ according to some probability function $G(\cdot)$. Then, $\underline{p}_{AB} = p_{AB}^* - \Delta$. As with the upper limit, the two terms in (15) respond to the change in the determination of p_{AB}^* .

The proof concludes with establishing a (conservative) lower bound for the stand-alone prices for the bundling strategies described above to constitute an equilibrium. Consider for a minute a pure-strategy pure-bundling equilibrium in which both firms play $p_{AB} = c_A + c_B + z$ (z is the mark-up) and the (high) stand-alone p_k and p_{-k} . To just

"convert" a fraction 2ε of the most vulnerable one-stop shoppers (e.g., the ones located half way between $1A$ and $2B$; see figure 1) into two-stop shoppers that buy $-k$ from $-i$ and k from i , conglomerate i would need to reduce the stand alone price of product k from p_k to p'_k in an amount that satisfies

$$p'_k + p_{-k} + (1/2 - \theta)t = p_{AB} + (1/2 - \theta - \varepsilon)t + \theta t \quad (19)$$

Such a deviation is not profitable for i if the losses from its own one-stop shoppers switching to two-stop shopping are greater than the gains from serving the new two-stop shoppers, that is

$$\varepsilon(p_{AB} - c_A - c_B) \geq 2\varepsilon(p'_k - c_k) \quad (20)$$

Obtaining p'_k from (19), replacing it into (20) and letting $\varepsilon \rightarrow 0$ leads to

$$p_{-k} \geq c_{-k} + z/2 + \theta t$$

Finally, replacing z by the mark-ups in (14) yields the prices in (16). ■

The main implication of Proposition 2 is that competition is greatly intensified as we move away from positive or zero correlation environments; so much that it reduces to marginal-cost pricing when the correlation is perfectly negative (note how the support of prices collapses to $c_A + c_B$ as $\theta \rightarrow 1/2$).²¹ The reason for this is simple. The mere opportunity of bundling their products has taken conglomerates, by the way of competition, to create perfectly homogeneous products.²² And this is a prisoner's dilemma firms can not escape from (i.e., i 's optimal response when $-i$ is not bundling is to bundle). In other words, bundling has provided firms with an effective tool to steal consumers from the rival; too effective perhaps from the firms' perspective.

In a more technical matter, some readers may see the restriction placed upon the stand-alone prices too great to support the pure-bundling equilibrium in the proposition. It is not. On the one hand, the stand-alone prices in (16) discourage deviations even when the prices being played are the highest in the support. Thus, stand-alone prices can go somewhat below that without affecting the equilibrium outcome. On the other hand, the prices in (16) are only a bit higher than the stand-alone prices in Proposition 1 (even at $\hat{\theta}$ when they are highest) and, more decisively, smaller than the monopoly

²¹To know exactly how expected prices evolve as a function of $\theta > \hat{\theta}$ we would need to solve a second order differential equation for $G(p)$, the probability distribution function according to which both conglomerates choose the price of their bundles in equilibrium. For example (we let $c_k = 0$ and $t_k = 1$ for $k = A, B$)

$$G(p, \theta)|_{\theta=3/8} = \frac{d}{dp} \left(k_1 + \frac{2}{3}p + h(p) \left(k_2 + k_3 \int [h(p)(4p+1)(p-2)]^{-1} dp \right) \right)$$

where $h(p) = e^{-4p/3} (4p+1)^2 (p-2)$, $k_1 = 0.34$, $k_2 = 0.14$ and $k_3 = -2.66$.

²²Note that the bundles offered by the conglomerates are not entirely homogeneous when $t_A \neq t_B$, even for $\theta = 1/2$ (see Appendix B).

prices p_k^m .²³ The same readers may have noticed that the mixed-strategy equilibrium in Proposition 2 extends below $\hat{\theta}$ until $\theta = 1/4$, although it is Pareto dominated by the one in Proposition 1.

We now combine in Figure 4 the results of Propositions 1 and 2. The solid line provides a summary of how profits under competitive bundling, $\pi^b(\theta)$, evolve relative to the benchmark case, $\pi^u = \pi_k^u + \pi_{-k}^u$, as a function of the correlation of preferences $\rho = 1 - 4\theta$ (at the end of the section we refer to the dotted line). The evolution of π^b responds to the existence of two opposing effects that arise with bundling, namely, (i) a price-discrimination effect or the fact that bundling allows companies to better sort consumers, and (ii) a business-stealing effect or the fact that bundling makes it easier for a company to steal consumers from her rival when more homogeneous (i.e., bundled) products become available. When $\theta < \hat{\theta}$ the price-discrimination effect prevails; otherwise the business stealing effect does. They never cancel out, it is either one or the other.

*** figure 4 here or below ***

The intuition for this is that when the correlation is highly positive (small θ) each conglomerate enjoys a large fraction of "captive" consumers, i.e., consumers that strongly prefer *both* products from the same conglomerate. So, conglomerates can go after those few two-stop shoppers with higher (stand-alone) prices —while maintaining the original linear prices (p_k^u) to the one-stop shoppers— without fear of losing them to the rival. As the correlation falls (higher θ) and the fraction of captive consumers drops, a conglomerate finds it more attractive to bundle and offer both products at a lower price (lower than $p_k^u + p_{-k}^u$) because this would attract a large number of consumers (the problem for the companies is that both think the same). Put it differently, when θ is small a conglomerate's optimal response to linear pricing is to increase the stand-alone prices, which indirectly introduces a bundling discount,²⁴ whereas when θ is large the optimal response is to lower the price to those buying the bundle. The strategic complementarity of prices explains the rest.

Figure 4 also illustrates that bundling can have important implications for the sustainability of a collusive agreement by altering the pricing path that follows a deviation from the agreement. But before we go into that in the next section, it is important to emphasize that what we learn from Figure 4, with its implications for collusive behavior, is not specific to the Salop model. In Section 5 we extend the circular-city model to a situation in which consumers are all over the square city of Figure 2 and can be placed in

²³Obviously, the equilibrium in Proposition 2 would be somehow altered if there is a mass of single-item consumers, even if it is infinitesimally small, that are happy to buy at stand-alone prices lower than those in (16) or whatever the exact lower bound is. The equilibrium would probably be in mixed strategies for the three prices; but we have not given further thoughts to this.

²⁴It is not difficult to show that even when $\theta = 0$, i 's optimal response to $-i$ charging linear prices (p_k^u) is to introduce a discount; more precisely, to offer the bundle for $p_A^u + p_B^u$ and item k for $p_k^u + \gamma/2$, where $\gamma = t/2$.

a collection of cities that are ordered according to some distribution $f(\theta)$ in $\theta \in [0, 1/2]$. A full range of correlations in preferences can be produced varying $f(\theta)$ —for example, the uniform distribution of consumers over the square city is obtained with $f(\theta) = 2$ for all θ — with results that follow closely those in figure 4.

Another message from this latter exercise is that it is not innocuous the way in which correlations are produced. A simple example can help illustrate the point. Suppose that instead of letting θ vary, as we have done so far, we consider consumers located in two (extreme) cities, $\theta = 0$ and $\theta = 1/2$, and let the proportion of consumers allocated to each city, $(1 + \rho)/2$ and $(1 - \rho)/2$, respectively, vary with $\rho \in [1, -1]$. This way we can produce any aggregate correlation ρ we want. Not surprisingly, the equilibrium outcomes are quite different from those in Propositions 1 and 2 except when $\rho = 1$ and $\rho = -1$. As shown in Appendix C, the pure bundling (mixed-strategy) equilibrium moves gradually and monotonically from linear pricing ($\rho = 1$) to marginal-cost pricing ($\rho = -1$). Again, this is not surprising because allocating more consumers to city $\theta = 0$ can never restore the price-discrimination effect simply because we are adding consumers with perfectly positive correlations for which bundling is irrelevant. In other words, we are "liquefying" the pro-competitive (i.e., business-stealing) effect of bundling without really changing the form of the equilibrium.

We conclude the section with a remark on the second profit profile depicted in Figure 4 —the dotted line. It is drawn upon the analysis in Appendixes A and B and captures the evolution of (relative) bundling profits when $t_A \neq t_B$, and $(t_A + t_B)/2 = t$ so profits under linear pricing remain unchanged. One immediate implication is that the (pro-firm) equilibrium in Proposition 1 can extend well into negative correlations (even all the way if $\hat{\theta}' = 1/2$). The other is that the gap between bundling and benchmark profits closes with transport cost heterogeneity. Neither implication is really surprising after we understand that introducing such heterogeneity is equivalent to considering a more positive correlation in preferences. In fact, when one of the markets is perfectly competitive (e.g., $t_k = 0$), a consumer that buys product $-k$ from i is ready to buy k from the same conglomerate (or from the other conglomerate for that matter). Bundling becomes irrelevant when one of the markets is perfectly competitive or, which is the same, when there is a perfectly positive correlation.

4 Collusion

In the first part of the section we ask what is the optimal collusive agreement conglomerates can fashion, as if they were infinitely patient or $\delta \rightarrow 1$, and only then look at the issue of sustaining such an agreement or a less ambitious one. Throughout the section we maintain the assumption that $t_A = t_B = t$. But when we look at the sustainability of the agreement we make also explicit that $v_A = v_B = v$ and $c_A = c_B = c$ (actually, with full coverage it suffices to assume that $v_A - c_A = v_B - c_B$), so we can fully isolate the Bernheim and Whinston's effect of multimarket contact on collusion from that of

bundling. In the last part of the section we incorporate the Bernheim and Whinston's effect by relaxing these latter assumptions.

4.1 Collusive bundling

The question here is whether and how a cartel agreement of the two conglomerates can make use of bundling to raise profits above those obtained when charging the (linear) monopoly prices p_A^m and p_B^m . Figure 5 below provides a hint. Figure 5a considers a cartel charging monopoly prices to consumers living in city $\theta = 1/2$ (i.e., $\rho = -1$). There is essentially only two-stop shoppers: the half on the left of the city will buy A from 1 and B from 2 and the half on the right will buy A from 2 and B from 1. Only consumers exactly located halfway between $1A$ and $1B$ (and $2A$ and $2B$) are indifferent between shopping at either conglomerate or at different conglomerates; and these consumers are precisely the ones left with no surplus. All remaining consumers enjoy a surplus, particularly those located close to the firms.

*** figure 5 here or below ***

The cartel can get a hand on some of that surplus by introducing non-linear pricing: by raising stand-alone prices above monopoly levels while maintaining the monopoly prices for the one-stop shoppers (i.e., $p_{AB} = p_A^m + p_B^m$). As indicated by the dashed lines in the figure, this would reduce the number of two-stop shoppers and create a strictly positive mass of one-stop shoppers, all with zero surplus. Obviously, the cartel does not want to increase the stand-alone prices all the way as to eliminate all the two-stop shoppers (we come back shortly with the optimal increase).

Figure 5b considers the other extreme, that of a cartel charging monopoly prices to consumers living in city $\theta = 0$ (i.e., $\rho = 1$). Now, there are essentially no two-stop shoppers: consumers on the upper half of the city will buy everything from 1 and those on the lower half from 2. Only consumers exactly located halfway between $1B$ and $2B$ (and $1A$ and $2A$) are indifferent between shopping at either conglomerate or at different conglomerates; and these consumers are again the ones left with no surplus. As before, the cartel can get a hand on some the surplus enjoyed by most consumers by introducing non-linear pricing. But unlike in $\theta = 1/2$, here the cartel would like to charge a premium, as opposed to a discount, to the one-stop shoppers while maintaining the stand-alone prices at the monopoly levels. As indicated by the dashed lines in the figure and assuming that the cartel can perfectly monitor sales, this would reduce the number of one-stop shoppers and create a strictly positive mass of two-stop shoppers, all with zero surplus. Again, the cartel would not introduce a premium that would eliminate all the one-stop shoppers but something smaller. The exact amount is made precise in the following proposition.

Proposition 3 *A cartel always benefits from non-linear pricing, sometimes charging a*

premium (ω^c), as opposed to a discount (γ^c), to the one-stop shoppers. More precisely, the optimal non-linear pricing strategy for the cartel, provided it can be sustained and there is perfect monitoring of sales, is to offer the bundle for

$$p_{AB}^c = \begin{cases} p_A^m + p_B^m + \omega^c(\theta) & \text{if } 0 \leq \theta \leq \frac{1}{4} \\ p_A^m + p_B^m & \text{if } \frac{1}{4} \leq \theta \leq \frac{1}{2} \end{cases}$$

where $\omega^c(\theta) = (1 - 2\theta)t/4$, and to charge stand-alone prices

$$p_k^c = \begin{cases} p_k^m & \text{if } 0 \leq \theta \leq \frac{1}{4} \\ p_k^m + \frac{1}{2}\gamma^c(\theta) & \text{if } \frac{1}{4} \leq \theta \leq \frac{1}{2} \end{cases} \quad \text{for } k = A, B$$

where $\gamma^c(\theta) = \theta t/2$.

Proof. Let $\pi^m \equiv \frac{1}{2} \sum_k (p_k^m - c_k)$ be each cartel member's profit when charging monopoly prices. For any given city $\theta \in [0, 1/2]$, consider first the strategy of charging a premium $\omega > 0$ to one stop-shoppers while maintaining stand-alone prices at monopoly levels. The optimal premium is given by

$$\omega^c = \arg \max_{\omega} \{ \pi^m + \Psi(\omega, \theta) \omega / 2 \} \quad (21)$$

where $\Psi(\omega, \theta) = (1 - 2\theta - 2\omega/t) = 1 - \Phi(\cdot) \geq 0$ is the total number of one-stop shoppers (capital letters denote industry variables). Solving (21) gives the expression $\omega^c(\theta)$ in the proposition. Collusive profits under this bundling strategy are decreasing in θ from $\pi^m + t/16$, when $\theta = 0$, to π^m , when $\theta = 1/2$. Consider now the alternative strategy of increasing each stand-alone price by $\gamma/2$ while maintaining the price of the bundle at the monopoly level. The optimal discount is given by

$$\gamma^c = \arg \max_{\gamma} \{ \pi^m + \Phi(\gamma, \theta) \gamma / 2 \} \quad (22)$$

where $\Phi(\gamma, \theta) = 2(\theta - \gamma/t) \geq 0$ is the total number of two-stop shoppers. Solving (22) gives the expression $\gamma^c(\theta)$ in the proposition (note that the second-order conditions here and above hold). Collusive profits under this alternative bundling strategy are increasing in θ from π^m , when $\theta = 0$, to $\pi^m + t/16$, when $\theta = 1/2$. It is easy to check that at $\theta = 1/4$ either collusive-bundling strategy reports the same profits. It is also not difficult to check that the cartel cannot do better because an additional price increase would inevitably lead the cartel to give up full coverage, which is suboptimal when v 's are high, as assumed here. ■

There are several observations to make. An obvious one is that in our context it may seem unlikely that a cartel, or any conglomerate for that matter, can charge a premium to one-stop shoppers (perhaps not so much in the case of energy products —electricity and gas— since the point of delivery cannot be disguised so easily). But before we rule that out in the discussion below, it is interesting to notice that at $\theta = 1/4$ it is

equally profitable for the cartel to either charge the premium ω^c or offer the discount γ^c despite the different outcomes in terms of the number of one-stop shoppers vs. two-stop shoppers. This is because at $\theta = 1/4$ linear monopoly pricing (i.e., p_A^m and p_B^m) leaves all consumers with the same amount of total surplus regardless of whether they shop at one conglomerate or at both. Thus, the cartel must decide whether it goes after the surplus of the one-stop shoppers, with the premium, or that of the two-stop shoppers, with higher stand-alone prices. It cannot do both if it aims for full coverage.

Figure 6 may help (it also serves to elaborate further on the last sentence in the proof of the proposition). Figure 6a shows the case in which the cartel opts for the premium reducing the number of one-stop shoppers from those located between the conglomerates' firms (i.e., between 1A and 1B and between 2A and 2B) to those located between the dashed lines. Note the cartel cannot simultaneously increase stand-alone prices in an effort to capture some of the surplus of the two-stop shoppers. If it does that, consumers located sufficiently close to either plant 1B or 2B (1A or 2A) would not purchase product A (product B) from the cartel.

*** figure 6 here or below ***

Figure 6b shows the alternative case in which the cartels opts for the discount (i.e., increasing each stand-alone price by $\gamma/2$) adding more one-stop shoppers, as indicated by the dashed lines. Again, the cartel cannot simultaneously increase the price of the bundle above $p_A^m + p_B^m$ because if it does, consumers just to the left of point "x" in the figure will only purchase item A from the cartel. i.e., from plant 2A, which is $\gamma/2t$ miles away from point "x". Note that a consumer located exactly at "x" is indifferent between three choices, namely, (i) purchasing the bundle at $p_A^m + p_B^m$, (ii) purchasing A from 2A and B from 1B at prices $p_k^m + \gamma/2$, and (iii) purchasing A from 2A and B from a "neighbor friend" at v^B ; whatever the choice, his total surplus is $t/4 - \gamma > 0$. The reason why consumers right to the left of "x" do not get B from a friend is because they can get the bundle for $p_A^m + p_B^m$, but they will if the bundle becomes more expensive.

Now, if one rules out monitoring of sales in Proposition 3, we still have that

Lemma 1 *The optimal bundling strategy for a cartel of two conglomerates, provided it can be sustained and there is no monitoring of sales, is to offer the bundle for $p_{AB}^c = p_A^m + p_B^m$ and to charge stand-alone prices $p_k^m + \gamma^c(\theta)/2$ for $k = A, B$, where $\gamma^c(\theta) = \theta t/2$ and $\theta \in [0, 1/2]$.*

The proof follows directly from that of Proposition 3. The cartel always finds it optimal to implement a non-linear pricing strategy except when there is perfectly positive correlation. As long as linear (monopoly) pricing creates two-shoppers, the cartel will have incentives to price discriminate them with higher prices.²⁵

²⁵ And so will a monopolist. In fact, if we replace conglomerate 2 by a fringe of firms supplying at

It is also interesting to notice that the amount of collusive bundling in Lemma 1, measured by the fraction of one-stop shoppers, is strictly smaller than that of competitive bundling regardless of consumers' preferences. This can be readily seen from (8) and the fact that the competitive-bundling discount, whether in pure or mixed bundling, is greater than the collusive-bundling discount. Or more clearly from what happens when there is perfectly negative correlation (see figure 5a). Competitive bundling results in pure bundling (100% of one-stop shoppers) while (optimal) collusive-bundling in a perfectly balanced mixed bundling with 50% of one-stop shoppers and 50% of two-stop shoppers. Therefore, and without even entering into the issue of sustainability, these results give us some answer to a question postulated in the introduction: a large fraction of one-stop shopping, together with bundling discounts,²⁶ is more likely the result of competition than of collusive behavior. We will see next that the latter answer gets only reinforced when we look at the sustainability of the collusive agreement.

4.2 Sustaining a collusive agreement

There are two questions that come to mind when studying the sustainability of collusion. In connection with Bernheim and Whinston (1990), the first question is whether the formation of conglomerates (or price alliances) makes it easier, or more difficult, for firms to sustain some given level of collusive profits, in particular, the monopoly profits that result from charging p_A^m and p_B^m . And the second and related question is how the sustainability of collusion changes as the collusive agreement of the two conglomerates moves from a linear pricing scheme (e.g., p_A^m and p_B^m) to the non-linear pricing schemes of Proposition 3. The answer to either question is not immediately obvious because of the changes in the profit levels that enter in the incentive compatibility condition that must be satisfied to sustain collusion in equilibrium.

We take both questions in order. As standard in the literature, we focus on changes in the critical discount factor above which a collusive agreement can be sustained in equilibrium. In that sense, we say that a lower critical discount factor makes it easier (or more likely) for the cartel to sustain the collusive agreement. Recall that in this part we are explicitly assuming, besides $t_A = t_B = t$, that $v_A = v_B = v$ and $c_A = c_B = c$.

Proposition 4 *The formation of conglomerates (or price alliances) makes it more difficult for firms to sustain linear monopoly prices, i.e., $p_A^m = p_B^m = p^m$, regardless of consumers' preferences.*

marginal cost, conglomerate 1's optimal pricing strategy is to offer item $k = A, B$ for $c_k + t/4 + \gamma/2$ and a discount for the bundle equal to $\gamma = t\theta$ for $\theta \in [0, \check{\theta} < 1/2]$, where $\check{\theta}$ is the point where the monopolist gives up the one-stop shoppers. The latter does not need to happen if the monopolist has a sufficient cost advantage. Obviously, the monopolist can do better for low values of θ if he can introduce a bundling premium.

²⁶There is also the possibility that the large fraction of one-stop shopping is the result of a highly positive correlation of preferences together with zero discounts.

Proof. We need to demonstrate that

$$\underline{\delta}^b(\theta) \equiv \frac{\pi^d(\theta) - \pi^m}{\pi^d(\theta) - \pi^b(\theta)} > \frac{\pi_s^d - \pi_s^m}{\pi_s^d - \pi_s^u} \equiv \underline{\delta}_s \quad (23)$$

holds for all $\theta \in [0, 1/2]$. Note that subscript "s" denotes profit at the single market level that can be either A or B since they are the same for collusion purposes (i.e., $\underline{\delta}_A = \underline{\delta}_B$), π^d is the profit a conglomerate obtains when optimally deviates —possibly using non-linear prices— from the agreement, $\pi^m = 2\pi_s^m = 2p^m - 2c$ is the profit each conglomerate obtains from charging monopoly prices, π^b is the profit in the competitive bundling equilibrium as dictated by either Proposition 1 or 2 (note that the absence of a subscript denotes profit at the conglomerate level). Since we know that the only terms in (23) that change with θ are π^d and π^b and that a conglomerate can at least replicate, and typically improve upon, single-market deviations (i.e., $\pi^d \geq 2\pi_s^d$), we only need to pay attention to the case when π^b is lowest, that is, $\pi^b(\theta)|_{\theta=1/2} = 0$. Recall from (3) that in the case of a single (unrelated) market the optimal deviation is to undercut the monopoly price by $t/2$ and take the entire market. In the case of multi-markets the optimal deviation is also to undercut the price of the bundle by $t/2$ and take the entire market as well (note that the same undercut applies for any $\theta \geq 1/4$). This is because when $\theta = 1/2$ each conglomerate is already selling one item to each consumer. Thus, we have that

$$\underline{\delta}^b(\theta)|_{\theta=1/2} > \underline{\delta}_s \iff \frac{2p^m - 2c - t/2 - \frac{1}{2}(2p^m - 2c)}{2p^m - 2c - t/2 - 0} > \frac{p^m - c - t/2 - \frac{1}{2}(p^m - c)}{p^m - c - t/2 - t/4} \quad (24)$$

$$\iff \frac{p^m - c - t/2}{2(p^m - c - t/2) + t/2} > \frac{p^m - c - t}{2(p^m - c - t) + t/2} \quad (25)$$

$$\iff p^m - c - t < p^m - c - t/2 \quad (26)$$

■

The proposition shows that the possibility of selling bundles makes the deviation from a collusive agreement so attractive that it fully offsets the opposite effect of facing the harshest possible punishment. Note that despite the proposition has been derived for linear monopoly prices it holds through for any (linear) collusive prices.

The results of Proposition 4 serve as a natural introduction to the second question advanced above. Can the cartel improve the sustainability of collusion by increasing profits along the collusive path, that is, by moving to the collusive-bundling agreement of Proposition 3? The answer is again not obvious. Collusive bundling not only results in higher profits along the collusive path, which facilitates collusion, but also it can reduce product differentiation —particularly evident in Figure 5a where the one-stop shoppers are absolutely indifferent where to shop—, which hinders collusion because deviation from the agreement becomes even more attractive. Note however that when collusive bundling takes the form of a premium, as opposed to a discount, it is not longer evident

that the deviation is more attractive since there will be fewer one-stop shoppers that under linear pricing. One may see the introduction of a premium as an increase in product differentiation. The proposition below presents results for two extreme cities $\theta = 0$ ($\rho = 1$) and $\theta = 1/2$ ($\rho = -1$). These results illustrate cleanly the mechanism at play and so they provide the basis for the discussion that follows covering the other cases.

Proposition 5 *In cities with either perfectly positive correlation ($\theta = 0$) or perfectly negative correlation ($\theta = 1/2$), it is more difficult for the two conglomerates to sustain the collusive-optima bundling agreement of Proposition 3 than linear monopoly prices.*

Proof. Consider first the case of a collusive-bundling agreement in city $\theta = 1/2$: $p_{AB} = 2p^m$ and $p_s = p^m + \gamma/2$ for $s = A$ or B . Note that the bundling discount γ does not need to be equal to γ^c for the result to hold. The critical discount factor for such collusive agreement is given by

$$\underline{\delta}^b(\gamma, \theta) = \frac{\pi^d(\gamma, \theta) - \pi^c(\gamma, \theta)}{\pi^d(\gamma, \theta) - \pi^b(\theta)}$$

where $\pi^c(\gamma, \theta) = \pi^m + \frac{1}{2}\Phi(\gamma; \theta)\gamma$ are per-period profits along the collusive-bundling path and $\pi^d(\gamma, \theta)$ are (optimal) deviation profits in the period of deviation for a given bundling discount γ . From the proof of Proposition 3 we have that $\pi^c(\gamma, \theta) = \pi^m + \theta\gamma - \gamma^2/t > \pi^m$. To obtain the optimal deviation, note first that when $\theta \rightarrow 1/2$ the most difficult consumers to attract for conglomerate i with a reduction in the price of the bundle are the two-stop shoppers. Since a two-stop shopper that decides to buy the bundle (from either conglomerate for that matter) enjoys the bundle discount γ , the reduction in the price of the bundle just enough to attract the "most distant" two-stop shopper is equal to $t/2 - \gamma$. Thus, deviation profits can be written as $\pi^d(\gamma, \theta) = \pi^d(\theta) + \gamma$, where $\pi^d(\theta)$ are profits when optimally deviating from linear monopoly prices (see above). Therefore,

$$\underline{\delta}^b(\gamma, \theta) > \underline{\delta}^b(\theta) \iff \frac{\pi^d(\theta) - \pi^m + \gamma - \theta\gamma + \gamma^2/t}{\pi^d(\theta) - \pi^b(\theta) + \gamma} > \frac{\pi^d(\theta) - \pi^m}{\pi^d(\theta) - \pi^b(\theta)} \quad (27)$$

$$\iff 1 - \theta + \frac{\gamma}{t} > \frac{\pi^d(\theta) - \pi^m}{\pi^d(\theta) - \pi^b(\theta)} \quad (28)$$

The left-hand side of (28) is greater than $1/2$ for $\theta = 1/2$ and $\gamma > 0$; hence, it suffices to show that the right-hand side of (28) is smaller than $1/2$ for all $t > 0$, which is the case as indicated by the term on the left-hand side of (25).

Consider now the case of a collusive-bundling agreement in city $\theta = 0$: $p_{AB} = 2p^m + \omega$ and $p_s = p^m$ for $s = A$ or B (again ω does not need to be equal to ω^c for the result to hold). Proceeding as before, we have from Proposition 3 that collusive profits are given by $\pi^c(\omega, \theta) = \pi^m + \frac{1}{2}\Psi(\omega, \theta)\omega = \pi^m + (\frac{1}{2} - \theta)\omega - \omega^2/t > \pi^m$. To obtain the optimal deviation $\pi^d(\omega, \theta)$, note first that when $\theta \rightarrow 0$ the most difficult consumers for

conglomerate i to attract with a reduction in the price of the bundle are $-i$'s one-stop shoppers. As seen in Proposition 1, it is required to reduce the price of the bundle by $\Delta \equiv (1 - 2\theta)t$. Since everyone now ends up paying the premium, deviation profits become $\pi^d(\omega, \theta) = \pi^d(\theta) + \omega$. Therefore,

$$\underline{\delta}^b(\omega, \theta) > \underline{\delta}^b(\theta) \iff \frac{\pi^d(\theta) - \pi^m + \omega - (\frac{1}{2} - \theta)\omega + \omega^2/t}{\pi^d(\theta) - \pi^b(\theta) + \omega} > \frac{\pi^d(\theta) - \pi^m}{\pi^d(\theta) - \pi^b(\theta)} \quad (29)$$

$$\iff \frac{1}{2} + \theta + \frac{\omega}{t} > \frac{\pi^d(\theta) - \pi^m}{\pi^d(\theta) - \pi^b(\theta)} \quad (30)$$

As above, the left-hand side of (30) is greater than $1/2$ for $\theta = 0$ and $\omega > 0$, which concludes the proof. ■

The two cases presented in the proposition share the exact same outcome: the additional deviation incentives created by collusive-bundling are far greater than the extra profit. The reason for this, however, is different. In the case of $\theta = 1/2$ the deviation incentives are large because the bundling discount γ has made it easier for i to steal consumers from $-i$; in fact, $-i$'s one-stop shoppers can be attracted at almost no cost. Conversely, in the case of $\theta = 0$ the deviation incentives are large not because it has become easier for i to steal consumers (actually it remains equally difficult) but because the bundling premium ω has made it increasingly attractive to attend more consumers.

Understanding these two polar cases gives a hint on how things change as we move away from them. Consider first a departure from $\theta = 0$ (towards lower correlations). There will be a city $\theta^\omega < 1/4$ for which it will be equally difficult for deviator i to attract $-i$'s one stop-shoppers, with a reduction in the price of the bundle of $\Delta = (1 - 2\theta)t$, and the most distant of the two-stop shoppers, with a reduction of $t/2 + \omega$ (recall that such two-stop shopper must be compensated not only for the extra travel, $t/2$, but also for the premium, ω). Solving $(1 - 2\theta)t = t/2 + \omega$ we obtain (note that $\theta^\omega = 1/4$ for $\omega = 0$)

$$\theta^\omega = \frac{1}{4} - \frac{\omega}{2t}$$

City θ^ω is plot in Figure 7a that, according to Proposition 3, reduces to $\theta^\omega = 1/6$ when $\omega = \omega^c(\theta)$. More importantly, for cities $\theta \in (0, \theta^\omega]$ expression (30) in Proposition 5 continues holding because the extra deviation incentives brought by the premium remains the same, that is, $\pi^d(\omega, \theta) = \pi^d(\theta) + \omega$. However, for cities $\theta \in (\theta^\omega, 1/4]$, these extra incentives start falling because the price reduction is not longer $(1 - 2\theta)t$ but $t/2 + \omega$, and at $\theta = 1/4$ these extra incentives completely disappear, that is, $\pi^d(\omega, \theta)|_{\theta=1/4} = \pi^d(\theta)|_{\theta=1/4} = 2p^m - 2c - t/2$. Thus, there exists a city $\theta^* \in (\theta^\omega, 1/4)$ for which $\underline{\delta}^b(\omega, \theta) = \underline{\delta}^b(\theta)$. Between θ^* and $1/4$, the premium makes it easier to sustain collusion, i.e., the extra collusive-profit effect of the premium dominates its deviation effect.

*** figure 7 here or below ***

Likewise, consider a departure from $\theta = 1/2$ (towards higher correlations). There will be a city $\theta^\gamma > 1/4$ for which it will be equally difficult for deviator i to attract $-i$'s one stop-shoppers, with a reduction in the price of the bundle of $\Delta = (1 - 2\theta)t$, and the most distant of the two-stop shoppers, with a reduction of $t/2 - \gamma$. Solving we obtain (note that $\theta^\gamma = 1/4$ for $\gamma = 0$)

$$\theta^\gamma = \frac{1}{4} + \frac{\gamma}{2t}$$

City θ^γ is plot in Figure 7b that, according to Proposition 3, reduces to $\theta^\gamma = 1/3$ when $\gamma = \gamma^c(\theta)$. Again, for cities $\theta \in [\theta^\gamma, 1/2)$ expression (28) in Proposition 5 continues holding because the extra deviation incentives brought forward by the discount remains the same, that is, $\pi^d(\gamma, \theta) = \pi^d(\theta) + \gamma$. However, for cities $\theta \in [1/4, \theta^\gamma)$, these extra incentives start falling because the price reduction is not longer $t/2 - \gamma$ but $(1 - 2\theta)t$, and at $\theta = 1/4$ these extra incentives completely disappear. Again, there exists a city $\theta^{**} \in (1/4, \theta^\gamma)$ for which $\underline{\delta}^b(\gamma, \theta) = \underline{\delta}^b(\theta)$ and between $1/4$ and θ^{**} the introduction of a bundling discount makes it easier for the cartel to sustain collusion.

This additional analysis shows that there can be cases in which moving to the more profitable agreement of Proposition 3 (or Lemma 1) can help the cartel not only with higher profits but also with more room for sustaining the agreement. It should be clear, however, that even in such cases conglomerates have a harder time in sustaining collusion than do individual firms in unrelated markets.

Lemma 2 *Sustaining the collusive agreement of Proposition 3 [or Lemma 1] in an intermediate city $\theta \in (\theta^*, \theta^{**})$ [or in city $\theta < \theta^{**}$] for the conglomerates is more difficult than sustaining monopoly prices for firms in unrelated markets.*

Proof. Regardless of whether the cartel follows Proposition 3 or Lemma 1, we only need to focus on those cases that exhibit tougher punishments, i.e., in cities $\theta \in (\hat{\theta}, \theta^{**})$, and highest profits along the collusive path, i.e., $\gamma = \gamma^c(\theta)$. If the Lemma holds then it holds everywhere. Hence, we just need to demonstrate that

$$\underline{\delta}^b(\gamma, \theta) \equiv \frac{\pi^d(\gamma, \theta) - \pi^c(\gamma, \theta)}{\pi^d(\gamma, \theta) - \pi^b(\theta)} > \frac{\pi_s^d - \pi_s^m}{\pi_s^d - \pi_s^u} \equiv \underline{\delta}_s \quad (31)$$

for $\theta \in (\hat{\theta}, \theta^{**})$ and $\gamma = \gamma^c(\theta)$. An expression for $\pi^c(\gamma, \theta)$ is found in either Proposition 3 or in the proof Proposition 5: $\pi^c(\gamma, \theta) = \pi^m - \theta\gamma + \gamma^2/t$. Using $\pi^m = 2\pi_s^m$ and the definition of $\gamma^c(\theta)$ we obtain $\pi^c(\gamma^c(\theta), \theta) = 2\pi_s^m + \theta^2 t/4$. An expression for $\pi^d(\gamma, \theta)$ can be constructed upon the discussion following Proposition 5. For $\theta \in (1/4, \theta^\omega)$, $\pi^d(\gamma, \theta) = \pi^m - (1 - 2\theta)t$ and $\pi^d = \pi^m - t/2$, so $\pi^d(\gamma, \theta) = \pi^d(\theta) + (2\theta - 1/2)t$. But when $\theta > 1/4$, $\pi^d = 2\pi_s^d + t/2$; hence, $\pi^d(\gamma, \theta) = 2\pi_s^d + 2\theta t$. As for $\pi^b(\theta)$, we take a conservative value by assuming that both conglomerates play the lower limit of the price support in Proposition 2, which yields the lower bound $\underline{\pi}^b = 2\pi_s^u - (2\sqrt{\theta} - \theta - 1/2)t < 2\pi_s^u$. Since

we know from (25) that $\underline{\delta}_s < 1/2$ for all $t > 0$, the problem in (31) can be re-written as

$$\begin{aligned} \underline{\delta}^b(\gamma, \theta) &> \underline{\delta}_s \iff \frac{2\theta t - \theta^2 t/4}{2\theta t + (2\sqrt{\theta} - \theta - 1/2)t} > \frac{1}{2} \\ &\iff 1 + 6\theta - 4\sqrt{\theta} - \theta^2 > 0 \end{aligned}$$

which holds easily for any $\theta \in [0, 1/2]$. ■

The previous lemma can be seen as an extension of Proposition 4; together they imply that the formation of conglomerates (or prices alliances) makes it unambiguously more difficult for firms to sustain collusion. This is one of the main messages of the paper (and the fact that a collusive agreement will make little or not use of bundling discounts when the sustainability of the agreement is an issue, except perhaps when $\theta < \theta^{**}$, but even then the discounts will be way below those under competitive bundling). This result however hinges on the assumption that the only multimarket contact effect that comes with conglomerate formation is that of bundling. In the next section we incorporate a second effect that works in the opposite direction, the so-called Bernheim&Whinston (B&W) effect.

4.3 Adding Bernheim&Whinston

Now that we have isolated and understood the effect of bundling on collusion we are ready to add the B&W effect. Our intention here is not to provide a comprehensive analysis but simply to illustrate situations in which both effects are present —the anti-competitive B&W effect and the pro-competitive bundling effect—,²⁷ and explain what to make of them, for example, in a conglomerate merger proposal. We do this by revisiting Proposition 4 for the case in which $v_A \neq v_B$ and/or $c_A \neq c_B$, so that $\underline{\delta}_A \neq \underline{\delta}_B$. Note that if $v_k - c_k > v_{-k} - c_{-k}$, then $\underline{\delta}_k > \underline{\delta}_{-k}$.

More specifically, suppose that markets A and B differ in such a way that $\underline{\delta}_B < \underline{\delta}_A$. Consider now the formation of the two conglomerates that for some reason are restricted to charge linear prices at all times. Pooling the incentive compatibility constraints yields the critical factor above which the (restricted) conglomerates can sustain monopoly prices in both markets

$$\underline{\delta}^{BW} = \frac{\frac{1}{2} \sum_k (p_k^m - c_k) - t}{\sum_k (p_k^m - c_k) - 2t + t/2}$$

where $\underline{\delta}_B < \underline{\delta}^{BW} < \underline{\delta}_A$. If it turns out that $\underline{\delta}_B < \underline{\delta}^{BW} < \delta < \underline{\delta}_A$, where δ is the actual factor used by firms to discount future profits, we say that multimarket contract has made it possible for the conglomerates to sustain monopoly profits in both markets in equilibrium. This is essentially how the B&W effect operates, by transferring monopoly discipline from market B to A .

²⁷Perhaps we should talk about the pro-collusive B&W effect and the anti-collusive bundling effect.

Suppose now that conglomerates are free to charge non-linear prices, specifically, along the punishment path and in the deviation stage. The critical factor above which the conglomerates now can sustain (linear) monopoly prices in both markets is (we focus on $\theta = 1/2$ because it is the "least favorable" case for bundling; see the proof of Proposition 4)

$$\underline{\delta}^b(\theta)|_{\theta=1/2} = \frac{\frac{1}{2} \sum_k (p_k^m - c_k) - t/2}{\sum_k (p_k^m - c_k) - t + t/2}$$

It is not difficult to see that $\underline{\delta}^b > \underline{\delta}^{BW}$. One is inclined to interpret this result as that the anti-collusive bundling effect always dominates the pro-collusive B&W effect, at least for this sort of market asymmetries. That would be wrong for two reasons. On the technical side it is rather obvious that $\underline{\delta}^b > \underline{\delta}^{BW}$ after having seen Proposition 4. The B&W effect (i.e., the pooling of the IC constraints) is embedded in the construction of $\underline{\delta}^b$. On the policy side, such interpretation fails to take into account the role played by market asymmetries. The following lemma can help.

Lemma 3 $\underline{\delta}^b(\theta)|_{\theta=1/2} < \underline{\delta}_A \iff (p_A^m - c_A) - (p_B^m - c_B) > t$.

The proof is straightforward and hence omitted. A merger or price alliance that connects two markets that are collusion-alike (i.e., $\underline{\delta}_A \approx \underline{\delta}_B$) should be less of a concern, from a collusion perspective, to antitrust authorities because the bundling effect is the only present. It can very well happen that with the merger it would not longer be possible to sustain monopoly prices at all, i.e., $\underline{\delta}_B \approx \underline{\delta}^{BW} \approx \underline{\delta}_A < \delta < \underline{\delta}^b$. The concern should rise, however, if the merger connects two markets that are quite different (i.e., $\underline{\delta}_A \gg \underline{\delta}_B$). As indicated in Lemma 3, it may well happen now that with such a merger it would be possible for the firms to sustain monopoly prices in both markets, i.e., $\underline{\delta}_B < \underline{\delta}^{BW} < \underline{\delta}^b < \delta < \underline{\delta}_A$. In this case the bundling effect plays a minor role, although still positive, relative to the B&W effect.

5 Extension: Connecting cities

The idea here is to show that the competitive bundling equilibrium characterized in Section 3 extends to more general settings. The few consumers in the square (Hotelling) city of figure 2 come from just considering two circular (Salop) cities, θ_1 and θ_2 . To place more consumers on that same square city, consider then a continuum of Salop cities ordered according to the density function $f(\theta)$ over $\theta \in [0, 1/2]$. Note that if $f(\theta)$ is the uniform distribution consumers would be uniformly distributed over the square city—each point in the square city would be crossed twice by 45° diagonals.

To obtain the competitive bundling equilibrium in the Hotelling city we proceed much as in Section 3. Given some bundling discount γ , the total number of two-stop shoppers

in a symmetric equilibrium is (throughout the section we maintain that $t_A = t_B = t$)

$$\Phi(\gamma) = \int_0^{1/2} \max\{0, \Phi(\gamma, \theta)\} f(\theta) d\theta = \int_{\gamma/t}^{1/2} 2(\theta - \gamma/t) f(\theta) d\theta$$

where $\max\{0, \Phi(\gamma, \theta)\}$ is the non-negative number of two-stop shoppers in Salop city θ and $\Phi(\gamma, \theta)$ is given by (8). Note that two-stop shoppers only exist in cities $\theta > \gamma/t$.

The equilibrium discount, which can be conveniently obtained using the equilibrium condition (9), solves

$$\int_{\gamma/t}^{1/2} (2\theta - 3\gamma/t) f(\theta) d\theta = 0 \quad (32)$$

that for the uniform distribution (i.e., $f(\theta) = 2$) reduces to²⁸

$$\gamma = \frac{t}{4} \quad (33)$$

To obtain the equilibrium stand-alone prices in the Hotelling city, let again conglomerate 1 reduce the price of item A (and of the bundle) by ε . The loss of doing so is the left-hand side of the equation below (recall that 1 is serving half of the consumers in equilibrium)

$$\int_0^{1/2} \varepsilon \frac{1}{2} f(\theta) d\theta = \int_0^{\gamma/t} \frac{\varepsilon}{2t} (p_A + p_B - \gamma - c_A - c_B) f(\theta) d\theta \quad (34)$$

$$+ \int_{\gamma/t}^{1/2} \frac{\varepsilon}{2t} (p_A - c_A) f(\theta) d\theta + \int_{\gamma/t}^{1/2} \frac{\varepsilon}{2t} (p_A - \gamma - c_A) f(\theta) d\theta \quad (35)$$

$$+ \int_{\xi}^{1/2} (p_A + p_B - \gamma - c_A - c_B) (1/2 - \theta + \gamma/t) f(\theta) d\theta \quad (36)$$

where $\xi = 1/2 - \varepsilon/2t$ is the threshold-city above which all the one-stop shoppers in cities $\theta \in [\xi, 1/2]$ buy 1's bundle (this should become clear below).

In equilibrium, the loss of such slight price reduction must be equal to the marginal gain, which can be divided in three parts (two more than in the equivalent analysis of section 3). The first part, line (34), comes from $\varepsilon/2t$ units from consumers that before were buying 2's bundle and now buy 1's bundle. Examples of these consumers are captured by the short and thick bars in city Salop θ_1 in Figure 8 (recall that item A 's price reduction is captured by a horizontal movement of $\varepsilon/2t$, as indicated by the dashed line in the figure). The second part, line (35), comes from both $\varepsilon/2t$ units from consumers that before were buying 2's bundle and now buy product A from 1 and $\varepsilon/2t$ units from consumers that before were buying A from 2 and B from 1 and now buy the bundle from 1. Examples of these consumers are captured by the short and thick bars in city θ_2 . Finally, the gains in line (36) correspond to that large fraction (i.e., $\psi_2 = 1/2 - \theta + \gamma/t$)

²⁸Note also that our equilibrium figure is half the one in Armstrong and Vickers' (2010) uniform example because their city is twice as long.

of one-stop shoppers living in cities above ξ , for example in θ_3 in the figure, that switch from 2's bundle to 1's bundle (note that city ξ is the one that just touches the dashed line). Examples of these consumers are captured by the long and thick bar in city θ_3 .

*** figure 8 here or below ***

As we let $\varepsilon \rightarrow 0$, the term in (36), which can also be written as $(p_A + p_B - \gamma - c_A - c_B)[1 - F(1/2 - \varepsilon/2t)](\varepsilon/2t + \gamma/t)$, reduces to

$$(p_A + p_B - \gamma - c_A - c_B) \frac{\varepsilon}{2t} f(1/2) \frac{\gamma}{t}$$

hence, the equilibrium condition (34)-(36) becomes

$$t = (2p_A - 2c_A - \gamma)[1 - F(\gamma/t)] + (p_A + p_B - c_A - c_B - \gamma) \left[F(\gamma/t) + f(1/2) \frac{\gamma}{t} \right]$$

A similar equation is obtained when we let 1 reduce instead the price of product B by $\varepsilon \rightarrow 0$. Using both equations we arrive at the equilibrium stand-alone prices

$$p_k = c_k + \frac{t}{2} \left(1 + f(1/2) \frac{\gamma}{t} \right)^{-1} + \frac{\gamma}{2} \quad (37)$$

that for the uniform distribution reduce to (recall that $\gamma = t/4$)

$$p_k = c_k + \frac{t}{3} + \frac{\gamma}{2} = c_k + \frac{11}{24}t < c_k + \frac{1}{2}t \equiv p_k^u \quad (38)$$

for $k = A, B$.

We have replicated the pro-competitive result of Matutes and Regibeau (1992) and Armstrong and Vickers (2010), among others, for the "uniform" square city.²⁹ Both one- and two-stop shoppers benefit from bundling with lower prices. And unlike the pro-competitive result of Proposition 2, here the equilibrium is in mixed bundling. But what is most striking about all this however is that a small perturbation in the distribution of consumers can destroy this equilibrium, not much the pro-competitive part, as we will see shortly, but the mixed-bundling part. In fact, if $f(1/2) \rightarrow 0$, eqs. (32) and (37) suggest that while the new "equilibrium" bundling discount remains virtually the same, the "equilibrium" stand-alone prices jump abruptly to the (pro-firm) levels of Proposition 1, that is, $p_k = c_k + t/2 + \gamma/2 > p_k^u$.

There are two questions here. The first is how the equilibrium can be so sensitive to removing some, not even all, of the consumers living in the northwest-southeast diagonal, which in terms of number consumers represents a negligible fraction of the total. The answer lies in the same eq. (37). Conglomerates compete fiercely for consumers located exactly in that diagonal because even a slightest reduction in prices reports a non-trivial

²⁹They have also documented that these equilibrium strategies are global best responses to each other.

movement of consumers in that diagonal. That Bertrand-type force increases with $f(1/2)$ and leads to marginal cost pricing (for the bundle) as $f(1/2) \rightarrow \infty$. The second question is whether imposing $f(1/2) = 0$ can effectively take the equilibrium to the "pro-firm" form of Proposition 1. The answer is not. As we also elaborate further below, the equilibrium remains pro-competitive, in that that prices do not go above p_k^u , but abandons its mixed-bundling form for a pure-bundling one very much like in Proposition 2.

But before we proceed with the characterization of the equilibrium for different correlation levels, we compute next the pure bundling (symmetric) equilibrium in the Hotelling city (below we explain when it is actually valid). If p_{AB} is the equilibrium price of the bundle, a conglomerate that reduces its price to $p'_{AB} = p_{AB} - \varepsilon$ obtains

$$\pi(p'_{AB}, p_{AB}) = (p'_{AB} - c_A - c_B) \int_0^{1/2} q(p'_{AB}, p_{AB}, \theta) f(\theta) d\theta$$

where

$$q(p'_{AB}, p_{AB}, \theta) = \begin{cases} 1 & \text{if } \theta \geq \frac{1}{2} - \frac{p_{AB} - p'_{AB}}{2t} \\ \frac{1}{2} + \frac{1}{2t}(p_{AB} - p'_{AB}) & \text{if } \theta < \frac{1}{2} - \frac{p_{AB} - p'_{AB}}{2t} \end{cases}$$

Since in equilibrium we must have

$$\left. \frac{\partial \pi}{\partial \varepsilon} \right|_{\varepsilon=0} = - \left[1 - \int_0^{1/2} \frac{1}{2} f(\theta) d\theta \right] + (p_{AB} - c_A - c_B) \left[\int_0^{1/2} \frac{1}{2t} f(\theta) d\theta + \frac{1}{4t} f(1/2) \right] = 0,$$

rearranging terms gives the equilibrium price

$$p_{AB} = c_A + c_B + t \left[1 + \frac{1}{2} f(1/2) \right]^{-1} \quad (39)$$

that for the uniform distribution reduces to

$$p_{AB} = c_A + c_B + \frac{t}{2}$$

Not surprisingly, this equilibrium is far more competitive than the mixed-bundling equilibrium described in (33) and (38). Note again the dramatic effect that removing some consumers in the northwest-southeast diagonal (i.e., making $f(1/2) = 0$) has on the equilibrium outcome.

We now complete the characterization of the equilibrium by considering different correlation scenarios using variations in $f(\cdot)$. For tractability, we keep $f(\cdot)$ uniform and let instead the support of the distribution vary. Consider first then the case in which $f(\theta) = 1/\eta$ over $\theta \in [0, \eta < 1/2]$, so we fully cover the range of positive correlations by gradually adding circular cities from $\rho = 1$, when $\eta = 0$, to $\rho \rightarrow 0$, when $\eta \rightarrow 1/2$. Proceeding as above —except for line (36)— it is possible to establish

Lemma 4 *Consider a Hotelling city $\theta \sim U[0, \eta]$, where $\eta \in (0, 1/2)$. For cities where $\eta \leq$*

$\hat{\eta} = 0.437$, there exists a (symmetric) mixed-bundling competitive equilibrium characterized by the bundling discount $\gamma = \eta t/2$ and the stand-alone prices $p_k = c_k + t/2 + \gamma/2 > p_k^u$ for $k = A, B$. And for cities where $\eta \in (\hat{\eta}, 1/2)$, there exists a (symmetric) pure-bundling competitive equilibrium in mixed strategies.

The first part of the lemma is a mirror of Proposition 1, so we omit its proof and much of the intuition behind it. The threshold $\hat{\eta}$ is obtained the same way as $\hat{\theta}$ and represents the same: the point at which the mixed-bundling strategies in the lemma are not longer optimal global responses. But unlike in Proposition 1, here it is not optimal to respond taking the entire market. The second part of the lemma requires some explanation. As shown in the Appendix, the pure-strategy pure-bundling equilibrium in (39), i.e., $p_{AB} = c_A + c_B + t = p_{AB}^u$, is only valid for $\eta \leq \tilde{\eta} = 0.422 < \hat{\eta}$; above that a conglomerate responds undercutting the price of the bundle by $t(1 - 2\sqrt{\eta/3})$. It should be clear then that above $\hat{\eta}$ the equilibrium must be in mixed strategies (similar to the one in Proposition 2) and, more importantly, that is more competitive than the linear-pricing benchmark.

Consider now the case of non-positive correlations, i.e., $f(\theta) = 2/(1 - 2\mu)$ over $\theta \in [\mu \geq 0, 1/2]$, by gradually deleting cities from $\rho = 0$, when $\mu = 0$, to $\rho \rightarrow -1$, when $\mu \rightarrow 1/2$.

Lemma 5 *For a Hotelling city $\theta \sim U[\mu, 1/2]$, where $\mu \in [0, 1/2)$, there exists a (symmetric) mixed-bundling competitive equilibrium characterized by the stand-alone prices (37), with $f(1/2) = 2/(1 - 2\mu)$, and the bundling discount*

$$\gamma = \begin{cases} \frac{1}{4}t & \text{if } \mu \leq 1/4 \\ \left(\frac{1}{6} + \frac{\mu}{3}\right)t & \text{if } \mu > 1/4 \end{cases} \quad (40)$$

There also exists a (symmetric) pure-bundling competitive equilibrium given by (39), with $f(1/2) = 2/(1 - 2\mu)$, and sufficiently high stand-alone prices.

The proof is omitted because follows the derivations above (note though that the form of the bundling discount in (40) changes at $\mu = 1/4$ because all Salop cities $\theta \in (1/4, 1/2)$ contain a positive fraction of two stop-shoppers given the equilibrium discount). There are three observations. The first is that the (pro-competitive) mixed-bundling equilibrium found for $\rho = 0$ extends all the way to $\rho \rightarrow -1$. At this point, $f(1/2) \rightarrow \infty$ in (37), so only two-stop shoppers end up paying above costs.³⁰ The second is that the pure-bundling equilibrium also approaches marginal cost pricing (for the bundle) as $\rho \rightarrow -1$.

The third observation, which is somehow implicit in the lemma, is that neither of the pure strategy equilibria in the lemma survives the removal of some consumers in the northwest-southeast diagonal, i.e., neither survives in a Hotelling city $\theta \sim U[\mu, \bar{\theta}]$, where $\bar{\theta} < 1/2$. In such a case the (symmetric) equilibrium takes a pure-bundling form and is in mixed strategies (very much like in Proposition 2); although it converges to the

³⁰Note from (40) that $\lim_{\mu \rightarrow 1/2} \gamma = t/3$ and, hence, $\lim_{\mu \rightarrow 1/2} \Phi(\gamma) = 1/3$.

pure-bundling equilibrium in the lemma as $\bar{\theta} \rightarrow 1/2$. In that sense we can say that this (pure strategy) pure-bundling equilibrium is robust to perturbations.³¹ Conversely, the mixed-bundling equilibrium in the lemma is not; it is only restored when $\bar{\theta} = 1/2$.

This last observation suggests that the existence of such pro-competitive mixed-bundling equilibrium is less likely than one might have thought.³² What is unambiguous is that there is a point (again, around zero correlation) in which the competitive bundling equilibrium in either of its forms starts delivering lower prices than the linear benchmark and more so as we move towards negative correlations. And what is particularly relevant for our collusion analysis is that this description follows closely what we already learned from the Salop model of Section 3 and that was summarized in Figure 4.

6 Concluding remarks

We have developed a Salop model of competition to study the static and dynamic (i.e., the possibility of sustaining collusion) implications of having conglomerates (or multiproduct firms) implementing bundling schemes for otherwise unrelated markets or products. The model is tractable enough to consider the entire range of correlations in consumer preferences. The main message that emerges from the analysis is that while conglomerates can sometimes—for positive correlations—benefit from bundling in a static context, they have a much more difficult time in sustaining collusion than do single-product firms. These results come with important policy implications; for example, that mandating à la carte pricing or preventing a conglomerate merger is not necessarily beneficial for consumers or that observing a large fraction of one-stop shopping, together with bundling discounts, is more likely the result of competition than of collusive behavior.

The model readily accommodates to different extensions. One is the introduction of market asymmetries, for example, in monopoly mark-ups or in the level of product differentiation. Market asymmetries enhance the Bernheim and Whinston's (1990) multi-market contact effect over the anti-collusive effect of bundling by increasing the collusive discipline that can be transferred from one market—the one where collusion is easier to sustain—to the other. This is particularly true when the asymmetry is in product differentiation because bundling becomes irrelevant as one of the markets goes perfectly competitive.

Another extension is to let consumers face, in addition to product differentiation, a "shopping cost" when they purchase from firms that belong to different conglomerates (e.g., they would need to pay two separate bills rather than one, visit two stores rather

³¹For the same reason that the equilibrium outcome in the standard Hotelling (i.e., linear) city is robust to removing a few consumers in the middle of the city.

³²It surely becomes more likely as t_A departs from t_B since that would introduce some product differentiation in the bundle even for consumers living in the northwest-southeast diagonal. As shown in Appendix A, note also that $t_A \neq t_B$ extends the range in which the (pro-firm) bundling equilibrium of Proposition 1 is valid.

than one, deal with two customer services, etc). In terms of sorting consumers, the role of this shopping cost is not that different from that of a bundling discount. But since it is largely exogenous to firms, there is little firms can do to suppress its pro-competitive (static/dynamic) effect. Competitive pure bundling would arrive sooner than otherwise (i.e., lower $\hat{\theta}$ in Proposition 1)³³ and collusion would become even more difficult to sustain for the conglomerates.

The model can also handle higher levels of product differentiation in all markets (i.e., higher t 's) so that the optimal deviation from a collusive agreement (or the optimal global response to a given set of prices for that matter) is not longer to take the entire market but a fraction of it. It can be shown that in such cases the conglomerate that deviates will utilize all instruments at hand, namely, the price of the bundle and that of the separate items. While the optimal deviation will always entails a reduction in the price of the bundle, the adjustment in stand-alone prices can go either way. With these additional instruments, a deviation from a collusive agreement can only become more attractive, so none of the conclusions above change.

This model tractability should permit, in future research, its extension in other relevant directions as well. One is to allow companies to have some influence on consumers' preferences/locations, and hence on competition, either directly by changing product attributes or indirectly by choosing who to merge with. Another, in the spirit of Crawford and Yurukoglu (2012), is to let conglomerate costs be endogenous to the outcome of bilateral negotiations with input suppliers.

7 Appendix

A. Proposition 1 for different transport costs

Let $t_A = t + \tau$ and $t_B = t - \tau$, so profits under linear pricing remain unchanged. It will become clear shortly that τ must be strictly smaller than t . From (7), (10) and (11) we obtain, respectively, equilibrium expressions for $\gamma(\theta, \tau) = 2\theta t/3 - 2\theta\tau/3$, $p_A(\tau) = c_A + t/2 + \gamma(\theta, \tau)/2 + \tau/2$ and $p_B(\tau) = c_B + t/2 + \gamma(\theta, \tau)/2 - \tau/2$. Furthermore, $\pi^b(\theta, \tau) = t/2 + 2\theta^2 t/9 - 2\theta^2 \tau^2/9t$. It is evident from these expressions that asymmetries in product differentiation make bundling less profitable (actually, we do not need any of this to see that bundling becomes irrelevant when one of the markets is perfectly competitive). Following the proof of Proposition 1, consider now a (higher) bundling discount enough to attract all the rival's one-stop shoppers, which is

$$\tilde{\gamma}(\tau) = \gamma(\theta, \tau) + (1 - 2\theta) \min\{t_A, t_B\} + \frac{1}{2}|t_A - t_B| = \gamma(\theta, \tau) + (1 - 2\theta)(t - \tau) + \tau \quad (41)$$

³³For a sufficiently large shopping cost, $\hat{\theta}$ may not even exist in that competitive bundling takes the form of pure bundling throughout; although in pure strategies in the range of positive correlations, i.e., $\theta \leq 1/4$, with $p_{AB} = p_A^u + p_B^u$.

This new discount also happens to attract all the rival's two-stop shoppers if

$$\theta \leq \frac{3t^2 + 3t\tau}{8t^2 + 6t\tau - 6\tau^2}$$

Since the latter always holds in the relevant range of parameter values, (41) is a profitable deviation as long as

$$p_A(\tau) + p_B(\tau) - \tilde{\gamma}(\tau) - c_A - c_B > \pi^b(\theta, \tau)$$

or

$$\theta > \hat{\theta}'(\tau) \equiv \frac{3}{2(t^2 - \tau^2)} \left(3t^2 - 3t\tau - t\sqrt{2(4t - 5\tau)(t - \tau)} \right) \quad (42)$$

Eq. (42) contains two important results that are easy to check: $\partial\hat{\theta}'/\partial\tau > 0$ for all τ in the relevant range, i.e., for all $\hat{\theta}' \leq 1/2$, and (ii) $\hat{\theta}'(\tau)|_{\tau=0.456t} = 1/2$. The latter implies that the "pro-firm" result identified in Proposition 1 can extend throughout in the presence of enough heterogeneity.

B. Proposition 2 for different transport costs

Rather than looking at how $t_A \neq t_B$ affects the equilibrium for any value of θ , we focus on $\theta = 1/2$ where we have an equilibrium in pure strategies. We can refer to figure 5a for help with the analysis. Suppose again $t_A > t_B$. If p_{AB} is the equilibrium price of the bundle, consumers on the west side of the city will buy the bundle from conglomerate 1 and those on the east side will buy it from 2. Note that a price reduction of $\frac{1}{2}(t_A - t_B)$ allows a conglomerate to take the entire market. Consider instead a smaller price reduction $\varepsilon < \frac{1}{2}(t_A - t_B)$. Since the additional demand of doing so is $\varepsilon/(t_A - t_B) < 1/2$, the "gain" from such deviation is

$$\pi(p_{AB} - \varepsilon, p_{AB}) = (p_{AB} - \varepsilon - c_A - c_B) \left(\frac{1}{2} + \frac{\varepsilon}{t_A - t_B} \right)$$

Solving the first-order condition $\partial\pi/\partial\varepsilon = 0$ for $\varepsilon = 0$ yields the equilibrium price

$$p_{AB} = c_A + c_B + \frac{1}{2}(t_A - t_B)$$

It is not difficult to show that this effect of product-differentiation heterogeneity on competition extends also to the mixed strategy equilibria in the proposition, provided they exist (see Appendix A).

C. Correlations using two extreme cities

It should be clear that the equilibrium must be in mixed strategies since the slightest reduction in the price of the bundle is enough to take all the one-stop shoppers in city

$\theta = 1/2$. Let $\alpha = (1 - \rho)/2$ be the fraction of consumers living in that city and $1 - \alpha$ the fraction in city $\theta = 0$. To characterize the equilibrium for any $\alpha \in (0, 1)$, we first find the price of the bundle p_{AB} that makes a conglomerate indifferent between lowering the price of it in $\varepsilon \rightarrow 0$ or increasing it according to some optimal response, $R(p_{AB}) > p_{AB}$, when the other conglomerate is charging p_{AB} , that is

$$\pi(p_{AB} - \varepsilon, p_{AB}) = \pi(R(p_{AB}), p_{AB}) \quad (43)$$

where $\pi(p_{AB} - \varepsilon, p_{AB}) = \alpha(p_{AB} - c_A - c_B) + (1 - \alpha)(p_{AB} - c_A - c_B)^{\frac{1}{2}}$ and $\pi(R(p_{AB}), p_{AB}) = (1 - \alpha)(R(p_{AB}) - c_A - c_B)[\frac{1}{2} - (R(p_{AB}) - p_{AB})/2t]$. Since a price increase gives up all consumers in city $\theta = 1/2$, we have the standard Hotelling best response for city $\theta = 0$: $R(p_{AB}) = (p_{AB} + t + c_A + c_B)/2$. Replacing all these terms into (43) and solving yields

$$p_{AB}^*(\alpha) = c_A + c_B + \frac{t}{1 - \alpha} \left(1 + 3\alpha - \sqrt{8\alpha(1 + \alpha)} \right)$$

Following the logic in the proof of Proposition 2, we are ready to construct the price support of the mixed strategy equilibrium: $\underline{p}_{AB}(\alpha) = p_{AB}^*(\alpha)$ and $\bar{p}_{AB}(\alpha) = R(p_{AB}^*(\alpha))$. Conglomerate i will never price below $\underline{p}_{AB}(\alpha)$ when $-i$ is pricing at or above that level (even if randomly). Similarly, i will never price above $\bar{p}_{AB}(\alpha)$ when $-i$ is pricing at or below that level. It remains to establish that the price support is always below the linear benchmark. Note first that $p_{AB}^*(\alpha)|_{\alpha \rightarrow 0} = c_A + c_B + t$, so $\underline{p}_{AB}(0) = \bar{p}_{AB}(0) = c_A + c_B + t = p_{AB}^u$; and second that $p_{AB}^*(\alpha)$ is decreasing in α ($\lim_{\alpha \rightarrow 1} p_{AB}^*(\alpha) = c_A + c_B$), and so are $\underline{p}_{AB}(\alpha)$ and $\bar{p}_{AB}(\alpha)$. Note also that, unlike in Proposition 2 when $\theta \rightarrow 1/2$, the price support $(\underline{p}_{AB}, \bar{p}_{AB})$ does not collapse to $c_A + c_B$ as $\alpha \rightarrow 1$ but to $(c_A + c_B, c_A + c_B + t/2)$. This is because the mass of consumers in city $\theta = 0$ is not exactly zero, unless $\alpha = 1$.

D. Pure-bundling equilibrium threshold

In a Hotelling city $\theta \sim U[0, \eta]$, where $\eta \in (0, 1/2)$, a conglomerate that prices its bundle below the "equilibrium" level p_{AB} at $p_{AB} - \epsilon$ obtains

$$\pi(p_{AB} - \epsilon, p_{AB}) = (p_{AB} - \epsilon - c_A - c_B) \left[1 - \int_0^{1/2 - \epsilon/2t} \left(\frac{1}{2} - \frac{\epsilon}{2t} \right) \frac{1}{\eta} d\theta \right]$$

The optimal price reduction ϵ^* solves the first-order condition $\partial\pi(\cdot)/\partial\epsilon = 0$ that together with $p_{AB} = c_A + c_B + t$ yields $\epsilon^* = t(1 - 2\sqrt{\eta/3})$ and $\pi(p_{AB} - \epsilon^*, p_{AB}) = \frac{4}{3}t\sqrt{\eta/3}$ (note that $\pi(\epsilon^*)$ is increasing in η). Thus, solving $\pi(p_{AB} - \epsilon^*, p_{AB}) = \pi^b(p_{AB}, p_{AB}) = t/2$ for η gives $\tilde{\eta} = 27/64$.

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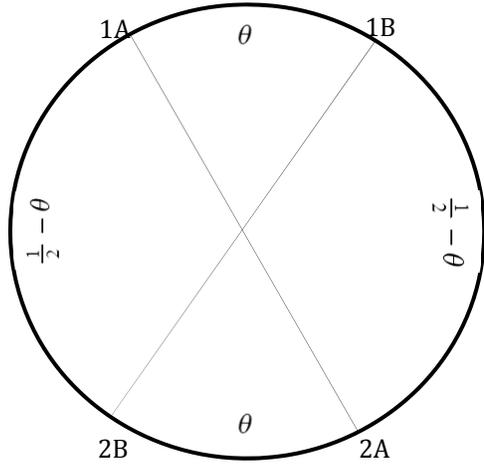


Figure 1. Location of firms in the Salop model

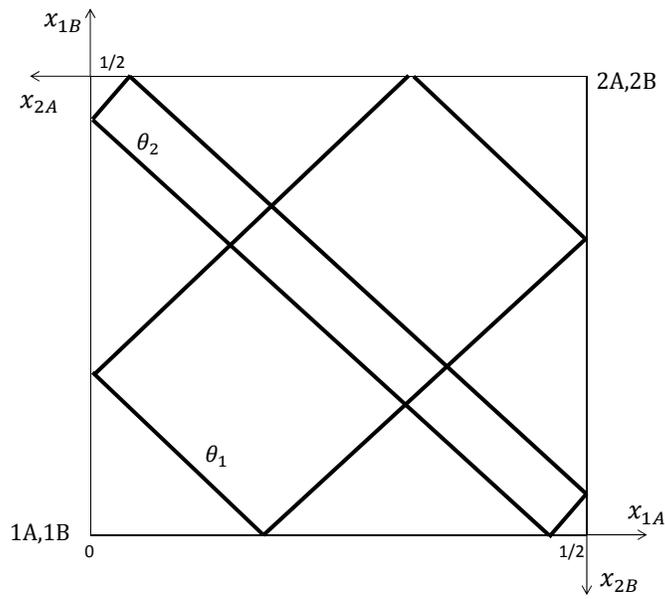


Figure 2. Salop cities in the square city

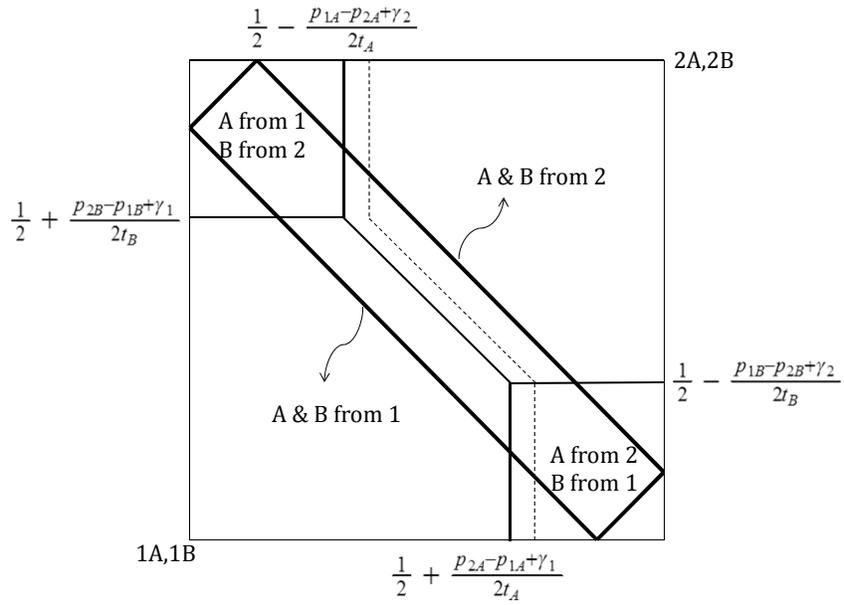


Figure 3. Demand of one and two-stop shoppers

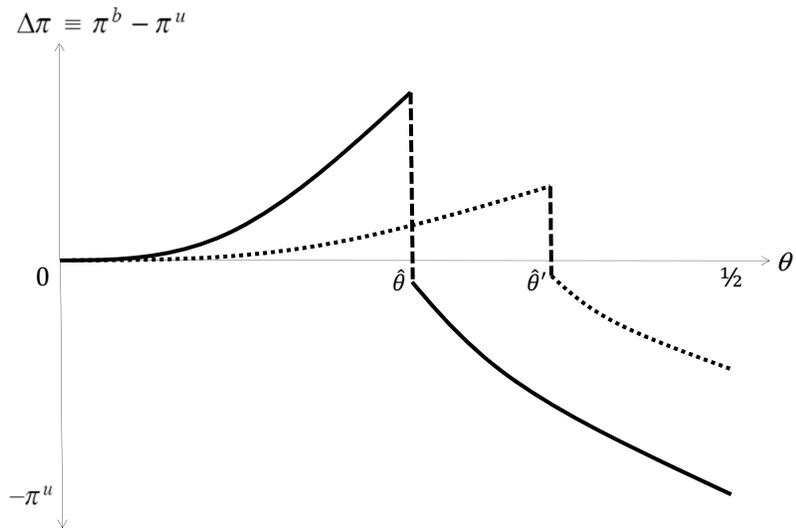


Figure 4. Evolution of profits under competitive bundling

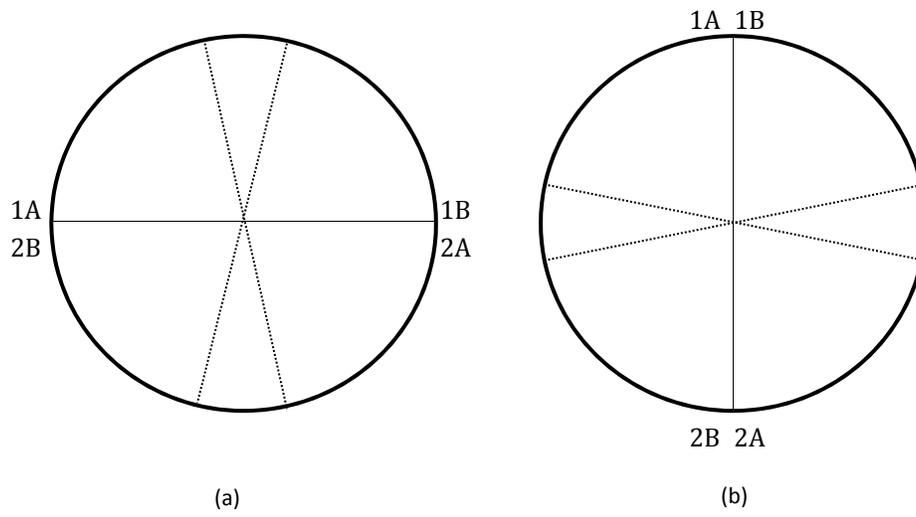


Figure 5. Collusive bundling agreement for two cities

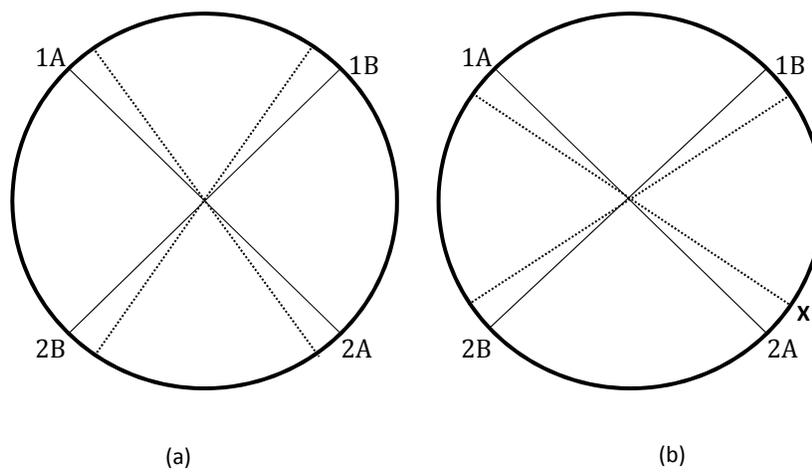


Figure 6. Equally profitable collusive agreements

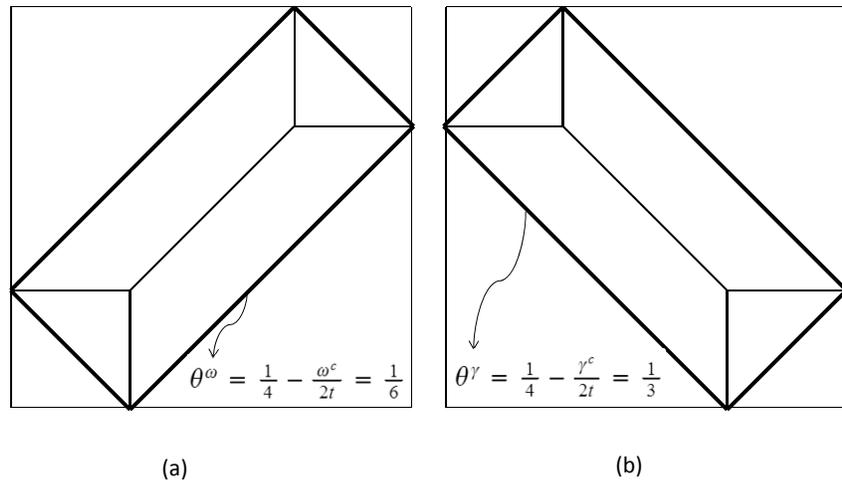


Figure 7: Deviation incentives from the cartel agreement

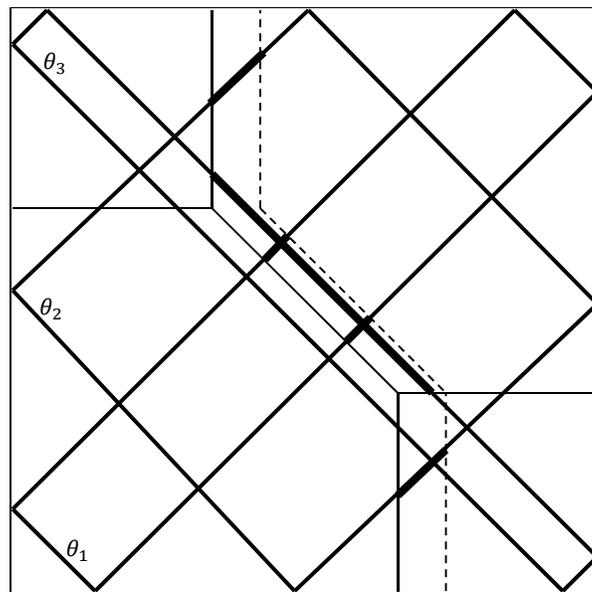


Figure 8. Connecting Salop and Hotelling cities