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ADJUSTED MONEY'S WORTH RATIOS IN LIFE ANNUITIES

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ABSTRACT

The Money's Worth Ratio (MWR) measures an annuity's actuarial fairness. It is calculated as the discounted present value of expected future payments divided by its cost. We argue that from the perspective of annuitants, this measure may overestimate the value-for-money obtained, since it does not adjust for liquidity or risk factors. Measuring these factors is challenging, requiring detailed knowledge of assets, liabilities, and of the stochastic processes followed by them. Using a multi-factor continuous-time model, we propose a simple solution for an adjusted MWR (AMWR), which does consider illiquidity and default risk. We implement this solution for the competitive Chilean annuity market, which offers MWRs above 1, finding that indeed these ratios are biased upward 7 percent on average. We also present estimates of default option values, asset insufficiency probabilities and implied credit spreads for each annuity provider.

KEYWORDS: Money's worth ratios, annuities, insurance companies, credit risk, liquidity premium, default probability, multi-factor continuous time models, Emerging Markets, Chile

JEL Classification: G22, G13, G28

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1. Introduction

Longevity is one of the most important risks faced upon retirement by pensioners, and life annuities are the best suited instruments to handle this risk. As explained in Brown, Mitchell, Poterba and Warshawsky (2001), there are many kinds of annuities, but they all share the longevity risk insurance attribute. Here we focus on fixed life annuities.

An important issue is to measure how good a deal a pensioner gets when buying a fixed annuity. In this context, Mitchell, Poterba, Warshawsky and Brown (1999) propose, define and estimate "Money's Worth Ratios" (MWR) for pension annuities in the US. MWRs are defined as the discounted present value of expected future annuity payments divided by the money (or premium) paid in advance for the annuity. Expected values are estimated using mortality tables and discounting takes place using the full term structure of risk-free interest rates. Higher money's worth ratios imply better deals for annuitants.

Many studies have used this methodology to determine the value-for-money that pensioners get around the world. For example, James and Song (2001) are surprised to find MWRs greater than one in many cases, among other reasons because there exist operational costs which must be paid by the pension annuity providers, whose present value should be added to that of annuity outflows (e.g. the same ratio from the perspective of the provider is larger). Fong, Mitchell and Koh (2010, 2011) evaluate MWRs of life annuities and discuss the impact of the government mandate to annuitize and its role as an annuity provider on the insurance market in Singapore. For Chile, the case study to which we apply our methodology below, because of detailed data availability, James, Martinez and Iglesias (2006) and Rocha, Morales and C. Thorburn (2008) find that MWRs are significantly larger than one.

We argue that Money's Worth Ratios of irrevocable life annuities should be adjusted to consider at least two factors: illiquidity and credit risk. Without these adjustments, measured MWRs are upward biased. Indeed, pension annuities tend to be completely illiquid liabilities from the perspective of annuity providers. Therefore, (ignoring longevity risk) a riskless annuity may be paid with certainty by investing in a portfolio of completely illiquid default-risk-free bonds. Liquidity spreads can be substantial, so the numerator of MWRs may be overstated simply because expected flows are discounted using interest rates which are "too low".[1]

Also, there is default risk, so future flows are not received with certainty. This also rather obviously reduces the present value of future payments (or the numerator of the MWRs).

Naturally, we are not the first to notice that credit risk is an important issue in life annuities. For instance, Babbel and Merrill (2007) model individual behavior under the possibility of default by the insurer issuing annuities, finding that even a little default risk can have a very large impact on annuity purchase decisions.

[1] See for example Pflueger and Viceira (2011) who find a liquidity risk Premium in TIPS of about 70 basis points in normal times. For a 10-year duration annuity this is an overstatement of 7%.

So it is important to model and measure default risk in this context, focusing on the supply side of annuities, which we do. It is perhaps due to practical difficulties that the default or credit risk adjustments have not been implemented in the annuities literature so far, since it requires detailed knowledge of the investment portfolio of the annuity provider, modeling the behavior of the asset classes they invest in and measuring economic leverage.

We model credit risk and illiquidity following two different literature strands: default risk and options associated with life insurance contracts.

Structural models of default start with Merton (1974), relating capital structure and the liabilities' credit risk. Default is triggered when the asset value falls below a certain threshold. Some articles determine this threshold endogenously, including Leland (1994), Leland and Toft (1996), Goldstein, Ju and Leland (2000) and Acharya and Carpenter (2002). These models are elegant when it comes to interpreting the equity holders' option to default, but are difficult to implement. Important practical differences in the case of annuity liabilities is that assets correspond to a portfolio of different kinds of investments and that liabilities are very long-term and completely illiquid.

The model is also related with the literature of embedded options in life insurance contracts, which begins with Brennan and Schwartz (1976) and Boyle and Schwartz (1977). Briys and de Varenne (1994), Miltersen and Persson (1998), Grosen and Jorgensen (2000) reconsider these articles to model life insurance contracts with associated savings that have guaranteed minimum rates of return. In addition, Brown and Walker (2009) use this conceptual framework to study the specific problem of maturity mismatching, typical of annuity insurance companies, which happens when assets mature before liabilities, allowing an analogy between reinvestment risk and prepayment risk.

In our case, the possible triggers for default are that either the market value of liabilities grows beyond the assets' (which happens when assets are shorter than liabilities and interest rates drop and/or when people on average live longer than expected) or when asset values fall faster than liabilities' (which may happen during a crisis, for example).

In order to determine the timing of the default option, which is crucial to determine its value, it is known since Merton (1974) that equityholders will postpone it as much as possible. We take an eclectic view in this aspect, and assume that the supervisory authority will check the annuity insurance company's equity at fixed time intervals. When equity turns negative, shareholders are required to invest more capital. If they don't, they have the option to walk away. The value of this option is paid by someone: the annuity holders, the State, or some combination thereof.

Another aspect that interests us is illiquidity. We conjecture that when negative equity is caused by a liquidity shock, implying a drop in the market values of relatively less liquid assets (such as during the 2007-2008 crisis; see Demyanyk and Hemert (2009), for example) it may not be optimal to require additional capital, since liquidity premia are mean-reverting. In any case, it may be difficult to disentangle liquidity risk from credit risk (Longstaff, Mithal et al. (2005)).

Here we propose a closed form solution for money's worth ratios which considers both, illiquidity and default risk. We call it the Adjusted Money's Worth Ratio (AMWR). This adjustment is done by taking as reference an illiquid default-risk-free replicating portfolio of bonds with payments identical to the expected annuities' and then adjust for default risk using option pricing theory. As expected, risk is summarized in the volatility of the annuity provider's equity, which depends on leverage, maturity mismatching, illiquidity mismatching and other assets' volatilities. These parameters themselves depend on the mean reversion in interest rates and in liquidity premia, among other factors.

We illustrate our results with a data-intensive application to the Chilean pension annuity industry. This market has been considered quite competitive even before the mandatory auction system (named SCOMP) was implemented in 2004. Walker (2006, 2009) documents a structural change in 2001 after which MWRs became larger than one.[2] James *et al* (2006) and Rocha *et al* (2006, 2008) find that the Chilean market seems to be among the most competitive in the world. The average MWR reported in Rocha is 1.064 for 2004, with an upward trend since 1999.[3] These high MWR levels may be interpreted as a puzzle, for the reasons discussed in James and Song (2001). However, our results indicate that the high MWRs for Chile indeed reflect lack of liquidity and credit risk adjustments. This may also explain the puzzle elsewhere.[4]

Finally, we discuss the regulatory incentives which have driven the Chilean market to this outcome.

2. A Multi-factor Model for Adjusted Money's Worth Ratios

This section first derives the Money's Worth Ratios adjusted for liquidity and credit risk, and then presents a multi-factor option pricing approach for life annuities. Finally, it also shows the linkage between the default probabilities of the annuity providers (i.e., insurance companies) and the parameters that determine the default option values.

a. Adjusted Money's Worth Ratios (AMWR)

The Money's Worth Ratios (MWR) of an annuity is a measure of actuarial fairness and is calculated as:

$$MWR_0 = \frac{G_0}{M_0} \tag{1}$$

[2] Walker op cit. shows that the annuity IRR that equates expected payments to the annuity premium became larger than the IRR of corresponding riskless bond (with identical payment structures), which is equivalent.

[3] For a more recent and comprehensive description of the annuity industry in Chile, see Mitchell and Ruiz (2011).

[4] As found in Doyle, Mitchell and Piggott (2004), adverse selection of annuities by long-lived individuals may also be an important issue from a supply side perspective, but this should bias the measured MWRs downward and not vice versa.

where G_0 is the discounted present value of expected future annuity payments and M_0 is the initial payment made by the annuitant. A standard practice is to discount the expected payments with the Treasury term structure of interest rates, therefore:

$$G_0 = \sum_{i=0}^{t} C[i] Z_0^L[i] \tag{2}$$

Here, $C[i]$ is the expected payment at time i, $Z_0^L[i]$ is the price of a zero-coupon liquid government bond maturing in i. The expected flow $C[i]$ considers the survival probability and also unexpected changes in the mortality tables, a risk that is uncorrelated with the interest rate risk.

We first argue that the expected flows should be discounted using the term structure of illiquid default-free bonds instead of using the Treasury yields. These are the correct discount rates, because an irrevocable life annuity is equivalent to a non-callable, non-puttable *illiquid* bond issued by an annuity provider.[5] Let D_0 be the present value of expected annuity flows using the term structure of illiquid default-free bonds:

$$D_0 = \sum_{i=0}^{t} C[i] Z_0^I[i] \tag{3}$$

where $Z_0^I[i]$ is the price of a zero-coupon default-risk-free illiquid zero coupon bond maturing in i. Note that $Z_0^I[i] \le Z_0^L[i]$ because illiquid bonds are discounted at higher rates than corresponding liquid bonds, therefore, $\frac{D_0}{G_0} \le 1$.

We also propose that an appropriate measure of the value-for-money of annuities should consider the possibility that the annuity insurance company (AIC) may default in the future. To do this in a simple way, we follow the structural model of Merton (1974). We assume that regulator defines an exogenous *regulatory horizon t* after which it will examine the market-value balance sheet of the annuity provider. For example, below we consider regulatory horizons of 1, 3 and 5 years. If the value of assets is below the liabilities', the AIC may invest more capital or decide to walk away, which implies bankruptcy. Ex-ante, the shareholder's equity has embedded the option to default, a value that is extracted from the annuity holders (and/or from the State, if there is a guarantee). For simplicity we assume that the annuity provider issues only one life annuity, hence, the liabilities of the insurance company are exactly that annuity.

The Money's Worth Ratio adjusting for both liquidity and credit risk is thus defined as:

[5] In the text we use the terms "annuity insurance company (AIC)" or "annuity provider" indistinctively.

$$AMWR_0[t] = \frac{B_0[t]}{M_0} \tag{4}$$

with

$$B_0[t] = D_0 - PV[Max[0, D_t - A_t]] \tag{5}$$

where $B_0[t]$ is the adjusted value of the annuity and $PV[Max[0, D_t - A_t]]$ corresponds to the value of the option to default owned by the AIC. Equation (5) shows that it is also the value extracted from the annuity holders (and/or from the State if guarantees exist). Note also that $\frac{B_0[t]}{D_0} \leq 1$ because the value of the option is non-negative. The following proposition formalizes both market-based adjustments considered for the MWR.

Proposition 1: *The Money Worth's Ratio adjusted for illiquidity and credit risk is:*

$$AMWR_0[t] = MWR_0 \cdot \frac{D_0}{G_0} \cdot \frac{B_0[t]}{D_0} \tag{6}$$

where the liquidity factor, $\frac{D_0}{G_0}$*, and the credit risk factor,* $\frac{B_0[t]}{D_0}$*, are both less than or equal to one. Therefore,*

$$AMWR_0[t] \leq MWR_0 \tag{7}$$

The proposition shows the sources of the overstatement of the MWR's. First, the liquidity factor adjusts the MWR for liquidity shocks in the market. During normal times this factor is close to one, however, if a liquidity shock occurs its value will decrease. Second, the credit risk factor adjusts the MWR for the option to default owned by the AIC. We know from the options literature that the value of this option is increasing in the leverage ratio, $\frac{D_0}{A_0}$, and in the volatility of the underlying equity, $D_t - A_t$, therefore, the credit risk factor is decreasing in these two variables. Finally, notice that if default of the AIC is certain in the horizon, then

$$AMWR_0[t] = MWR_0 \frac{A_0}{G_0} < MWR_0 \frac{D_0}{G_0} \tag{8}$$

b. Determinants of the AMWR

To quantify the determinants of the Adjusted Money's Worth Ratios (i.e., the liquidity and credit risk factors), we first need to consider a dynamic model of interest rates to obtain the default-free zero-coupon bonds $Z_0^L[i]$ and $Z_0^I[i]$, and the value of the option to default owned by the AIC shareholders.

We follow the affine term structure literature of Duffie and Kan (1996), Duffie, Pan and Singleton (2000), and Dai and Singleton (2000) and choose a Gaussian three-factor model for the illiquid default-free yields. The variables that determine the state of the fixed income market are: the short-term risk-free interest rate, r_t; the long-term risk-free interest rate, θ_t; and the short-term liquidity spread, s_t. These variables have the following dynamics:

$$dr_t = \kappa_r(\theta_t - r_t)\,dt + \sigma_r\,dW_{1,t}^Q \tag{9}$$

$$d\theta_t = \kappa_\theta(\bar{\theta} - \theta_t)\,dt + \sigma_{\theta 1}\,dW_{1,t}^Q + \sigma_{\theta 2}\,dW_{2,t}^Q \tag{10}$$

$$ds_t = \Big(\kappa_{\mathrm{sr}}(\theta_t - r_t) + \kappa_{\mathrm{s}\theta}(\bar{\theta} - \theta_t) + \kappa_s(\bar{s} - s_t)\Big)\,dt + \sigma_{\mathrm{s}1}\,dW_{1,t}^Q + \sigma_{\mathrm{s}2}\,dW_{2,t}^Q + \sigma_{\mathrm{s}3}\,dW_{3,t}^Q \tag{11}$$

where $W_{j,t}^Q$ are standard Brownian processes independent of each other. We define these processes under the risk-neutral measure *Q*, because it is convenient for valuation purposes. Later in the paper, we assume a constant risk-premia structure that will help us clarify if these risk factors are priced or not by the agents in the economy.

The short- and long-term interest rate factors are dedicated to match the Treasury term structure of interest rates. These factors follow mean-reverting processes towards a long-term mean $\bar{\theta}$. We assume that the speed of mean-reversions κ_r and κ_θ are positive. However, we validate these assumptions when we estimate the model. For the illiquid term structure we also consider the liquidity spread s_t. This variable also follows a mean-reverting process. In this case, the expected change in the spread depends on the shape of the Treasury term structure of interest rates through the parameters κ_{sr} and $\kappa_{\mathrm{s}\theta}$. For example, if $\kappa_{\mathrm{sr}} > 0$ then the spreads will tend to decrease when the short-rate is high. The only restriction that needs to be imposed is that $\kappa_s > 0$, a constraint that is also validated in the data.

Let the zero-coupon bonds be:

$$Z_0^L[t] = \mathbb{E}^Q\left[e^{-\int_0^t r_u\,du}\right] \quad \text{and} \quad Z_0^I[i] = \mathbb{E}^Q[e^{-\int_0^t (r_u+s_u)\,du}] \tag{12}$$

The affine term structure model presented in equations (9) to (11) provides closed-form solutions for these bonds, and therefore, for the annuities using both sets of discount rates (see equations (2) and (3)). In particular, the (log) price of the Treasury bond is linear in the short- and long-term interest rates, and the (log) price of the illiquid bond is linear in the interest rates and in the liquidity spread. These solutions allow us to get closed-form expressions for the liquidity adjustment multiplier for the AWMR. The following proposition shows this result.

Proposition 2: *Given the expected annuity payments and the affine term structure model in equations* (9) *to* (11)*, the liquidity factor has the following representation*

$$\frac{D_0}{G_0} = \frac{\sum_{i=0}^{t} C[i] Z_0^I[i]}{\sum_{i=0}^{t} C[i] Z_0^L[i]} \tag{13}$$

where the price of the liquid and illiquid zero-coupon bonds are given by

$$Z_0^L[t] = Exp\left[\alpha^L[t] + \beta_r^L[t]\, r_0 + \beta_\theta^L[t]\theta_0\right] \tag{14}$$

$$Z_0^I[t] = Exp\left[\alpha^I[t] + \beta_r^I[t]\, r_0 + \beta_\theta^I[t]\theta_0 + \beta_s^I[t]\, s_0\right] \tag{15}$$

respectively. Here, $\alpha^k[t]$ y $\beta_r^k[t]$ are deterministic functions of the maturity of the bond that depend on the parameters of the model.

Proof: See Appendix A.1.

To obtain the credit risk adjustment for the AMWR, we need to derive the dynamics of both assets and liabilities of the insurance company. Since the liabilities of the AIC are the life annuities, it is straightforward to obtain the dynamics of D_t by applying Ito's lemma to equation (3):

$$\frac{dD_t}{D_t} = (r_t + s_t)\, dt + \sigma_{D1}\, dW_{1,t}^Q + \sigma_{D2}\, dW_{2,t}^Q + \sigma_{D3}\, dW_{3,t}^Q \tag{16}$$

The expected liabilities' return under the risk-neutral measure is the illiquid interest rate, $r_t + s_t$, because of the illiquid nature of the life annuities. The parameters σ_{Dj} depend on the covariance and persistence of the state variables and on the expected annuity payments. In particular, the σ_{Dj}'s are represented by:

$$\sigma_{D1} = \frac{D_r}{D}\sigma_r + \frac{D_\theta}{D}\sigma_{\theta 1} + \frac{D_s}{D}\sigma_{s1}, \quad \sigma_{D2} = \frac{D_\theta}{D}\sigma_{\theta 2} + \frac{D_s}{D}\sigma_{s2} \quad \text{and} \quad \sigma_{D3} = \frac{D_s}{D}\sigma_{s3} \tag{17}$$

where the $\frac{D_x}{D}$ are weighted average β_x^I given by $\frac{D_x}{D} = \frac{\sum_{i=0}^{t} \beta_x^I[i] C[i] Z_0^I[i]}{\sum_{i=0}^{t} C[i] Z_0^I[i]}$ for $x = \{r_t, \theta_t, s_t\}$.

Note that the market value of liabilities is endogenous and can be spanned by the interest rates and liquidity spread state variables. The underlying assumption in equation (16) is that the AIC will keep annuities with the same duration and convexity over time. This is, as time goes by, the annuities are paid and new annuitants with a similar profile enter the company. This also implies that the volatility of the liabilities of the AIC is constant over time.

An annuity provider should invest most of its funds in fixed income securities, but it may also invest in other asset classes, such as, stocks, real estate and commodities. This means that the assets of the AICs are affected by risks which are unspanned by the term structures of interest rates. For this reason, we assume an exogenous process for the AICs' assets that has an extra source of uncertainty. For tractability, we assume that the expected return of the assets under the risk-neutral measure is also the illiquid risk-free rate because many of the assets of the AIC are illiquid. In particular, we consider the following Geometric Brownian Motion:

$$\frac{dA_t}{A_t} = (r_t + s_t)\, dt + \sigma_{A1}\, dW_{1,t}^{Q} + \sigma_{A2}\, dW_{2,t}^{Q} + \sigma_{A3}\, dW_{3,t}^{Q} + \sigma_{A4}\, dW_{4,t}^{Q} \tag{18}$$

Here, $W_{4,t}^{Q}$ is a standard Brownian process independant of the fixed income Brownian processes. It represents the uncertainty from the non-fixed income investments. The covariance parameters σ_{Aj} are estimated from an OLS time-series regression between the asset returns and the fixed income factors.

Using equations (16) and (18) we can obtain the value of the default option for any regulatory horizon t. Replacing the value of the option in equation (5) yields the following expression for the market-value of the liabilities, i.e., the adjusted value of the annuity:

$$B_0[t] = D_0 - \mathbb{E}^{Q}[e^{-\int_0^t (r_u + s_u)\, du} Max[0, D_t - A_t]] \tag{19}$$

Equation (19) shows that the value of the option to default corresponds to a European exchange option of the liabilities for the assets of the AIC, which matures at the regulatory horizon (see Margrabe, 1978). In the same way, we can obtain the equity market value of the insurance company that includes the option to default:

$$E_0[t] = A_0 - B_0[t] = \mathbb{E}^{Q}[e^{-\int_0^t (r_u + s_u)\, du} Max[A_t - D_t, 0]] \tag{20}$$

Note that the market value of the equity is greater than value of the equity without the option to default, i.e., $E_0[t] \geq A_0 - D_0$. Here, $A_0 - D_0$ also represents the equity value in the case that AIC shareholders are forced to invest more capital if at the regulatory horizon the company has insufficient assets to cover liabilities.

Our model provides a closed-form solution for the option value to default based on the Black-Scholes-Merton formula (see Black and Scholes, 1973, and Merton, 1973), and therefore, the credit risk adjustment factor and the market values of the equity and liabilities of the annuity provider can be obtained explicitly. The following proposition shows these results.

Proposition 3: *If the liabilities and assets of the AIC follow the dynamics in equations* (16) *and* (18)*, then the credit risk adjustment factor of the AMWR has the following solution:*

$$\frac{B_0[t]}{D_0} = N[d_2] + \frac{A_0}{D_0}(1 - N[d_1]) \tag{21}$$

where $N[d]$ *denotes the standard normal cumulative distribution function and* $d_2 = \frac{Log[\frac{A_0}{D_0}] - \frac{\sigma_K{}^2}{2}t}{\sigma_K\sqrt{t}}$*,* $d_1 = d_2 + \sigma_K\sqrt{t}$ *and* $\sigma_K = \sqrt{(\sigma_{A1} - \sigma_{D1})^2 + (\sigma_{A2} - \sigma_{D2})^2 + (\sigma_{A3} - \sigma_{D3})^2 + \sigma_{A4}{}^2}$*. Moreover, the market value of the equity considering the option to default is:*

$$E_0[t] = A_0 N[d_1] - D_0 N[d_2] \tag{22}$$

Proof: See Appendix A.2.

As in the Black-Scholes-Merton formula, $N[d_1]$ and $N[d_2]$ are adjusted probabilities of excercising the option. In our case, if $N[d_1] = N[d_2] = 0$, the firm will go banckrupt with probability 1 and $\frac{B_0[t]}{D_0} = \frac{A_0}{D_0}$ and $E_0[t] = 0$. If $N[d_1] = N[d_2] = 1$ the firm won't default under any event, therefore, $\frac{B_0[t]}{D_0} = 1$ and $E_0[t] = A_0 - D_0$.

c. Asset Insufficiency Probabilities

Our model allows us to obtain, for any regulatory horizon *t*, the probability that the value of assets is below the liabilities', which is therefore the asset insufficiency or default probability of the AIC. This probability is defined as:

$$Prob_0[t] = \mathbb{E}^P\left[\mathbb{I}_{\{A_t < D_t\}}\right] \tag{23}$$

where $\mathbb{I}_{\{.\}}$ is the indicator function such that $\mathbb{I}_{\{B\}} = \begin{cases} 1 \text{ if B is true} \\ 0 \text{ otherwise} \end{cases}$.

One difference with the pricing approach used above is that in this case we are interested in the distribution of the variables under the *true* probability measure *P*. For this reason we need to consider the systematic risk associated with each risk factor. We assume that the risk premia are constant, implying that both assets and liabilities also have constant risk premia. We denote these as λ_A and λ_D, respectively. This assumption is crucial to obtain a closed-form expression for the default probabilities in equation (23). The following proposition presents this result.

Proposition 3: *Assume that the liabilities and assets of the AIC have constant risk premium and that they are driven by equations* (16) *and* (18)*, respectively. Their dynamics under the true probability measure are:*

$$\frac{dA_t}{A_t} = (r_t + s_t + \lambda_A)\,dt + \sigma_{A1}\,dW_{1,t}^P + \sigma_{A2}\,dW_{2,t}^P + \sigma_{A3}\,dW_{3,t}^P + \sigma_{A4}\,dW_{4,t}^P \tag{24}$$

$$\frac{dD_t}{D_t} = (r_t + s_t + \lambda_D)\,dt + \sigma_{D1}\,dW_{1,t}^P + \sigma_{D2}\,dW_{2,t}^P + \sigma_{D3}\,dW_{3,t}^P \tag{25}$$

Moreover, the bankruptcy probability for the AIC at the regulatory horizon t, is given by:

$$Prob_0[t] = 1 - N[d_3]$$

with $d_3 = \frac{Log\left[\frac{A_0}{D_0}\right] + \left(\mu_K - \frac{1}{2}\sigma_K{}^2\right)t}{\sigma_K\sqrt{t}}$ *and* $\mu_K = \lambda_A - \lambda_D + \sigma_{D1}(\sigma_{D1} - \sigma_{A1}) + \sigma_{D2}(\sigma_{D2} - \sigma_{A2}) + \sigma_{D3}(\sigma_{D3} - \sigma_{A3})$.

Proof: See Appendix A.3.

It is important to notice that by assuming constant risk premia there is no predictability in these markets, because both assets and liabilities have permanent shocks. The predictability in stocks and bonds has been discussed widely in the financial literature with mixed results. As a robustness test, we estimated a version of the model considering that the risk premia are affine on the state variables, but we find none of these estimates to be significantly different from zero.

3. Data sources and input elaboration to estimate the default option values

The Annuity Insurance Companies' (AICs) regulator, the Superintendencia de Valores y Seguros (SVS) has given us access to detailed information regarding assets and liabilities for each AIC as of December 2008 and December 2009. Traded assets are valued ate market prices. In practice, we consider three sources for the valuation of these instruments: LVA Indices, a price provider for local fixed income; the price vector of the Superintendence of Pensions; and information of each AIC for instruments which are not local fixed income, such as mutual funds, stocks and a few international investments. It turns out that using the Superintendence of Pensions data or the LVA Indices data gives very similar results, so we adopt the former, since it's the official source used by the SVS.

There are two kinds of non-traded local fixed income instruments which are more difficult to price: mortgages and leasing contracts. Appendices B and C explain the methodologies used to estimate the market value of these instruments. In summary, to value mortgages, we empirically estimate the relationship between the interest rates of *new* mortgages in each month with a lagged reference interest rate and the characteristics of the individual mortgage and the debtor. Considering the regression results and the available data for the explanatory variables as of December 2008 and 2009 we estimate “market interest rates” for the mortgages, allowing us to estimate the market values of the mortgage portfolios. For leasing contracts, we assume that they are similar to large mortgages but with higher credit risk.

a. Present value of annuity liabilities assuming no default risk

Assuming initially no default risk, annuities are valued as the present value of the expected future payments discounted using a term-structure of interest rates with no default risk denominated in the same currency unit as the annuity liabilities (UF, the local inflation indexed unit of account).

The annuities’ expected cash flows are estimated using the most recently available mortality tables at the time of the study (2009 for the principals and 2006 for beneficiaries). We used the SVS official software (SEACSA) to estimate the aggregate expected annuity cash flows for each annuity provider.

The yield curve of local government bonds is likely to reflect the greater market liquidity that these bonds have, so these interest rates are “low” relative to less liquid but safe bonds. We have argued that given that irrevocable life annuities are completely illiquid, the replicating portfolio is composed of illiquid bonds, and that such a yield curve should be used to value them.

To take this illiquidity into account, here we use a local AAA-rated bond yield curve to estimate the present value of liabilities. It is true that AAA bonds have some credit risk, but the additional spread for credit risk is presumably small compared with the spread attributable to their lower liquidity. Still, for completeness, we also estimate the present value of annuity liabilities using government and local AA-rated yield curves.

The government bond yield curve was fitted using the Nelson & Siegel methodology, treating coupon bonds as portfolios of zero-coupons, using the last 5 trading days in 2008 and 2009. Figure 1 presents the fitted curves in each period.

For the AAA and AA yield curves we tried to follow the same procedure, but the reduced number of transactions implied unreliable results. So instead we did the following: first, for each annuity provider (or AIC) we estimate the present value of annuity liabilities using the government yield curve. Then, for each AIC we estimate the Internal Rate of Return (IRR) which equates the present value of the expected flows with the present value using the entire government yield curve. Finally, we add to each AIC’s IRR the spread corresponding to AA and AAA bonds. Figure 2 presents the spreads for AA and AAA bonds over time. We take the spreads at the end of 2008 and 2009, which are presented in Table 1.

Table 2, Panel A, shows the present value of annuity liabilities using the methodology just described. Using the AAA curve we obtain an average IRR of 4.13% (measured in the indexed unit of account, UF) and a present value of USD 27,500 million, approximately.[6] The same calculation using the government bond yield curve and the AA curve, imply values of USD 29,100 million and USD 26,600 million, respectively.

Panel B of Table 2 shows the corresponding results for 2008. We clearly appreciate the effects of a "flight to quality". AAA and AA spreads were unusually high at the time.

b. Value of other liabilities

Annuity Insurance companies have liabilities other than annuities. We assume that the market value of such liabilities is equal to their book value. In addition, we assume that over time their market value behaves like a short-term government bond index.

c. Value of "operational liabilities"

We consider the concerns of James and Song (2001), that managing a portfolio of annuities has operational costs. To estimate the present value of such costs we consider how much pension funds charge in Chile for managing programmed withdrawals. At the time of this study the fee was 1.25% of each payment. If the cost is always a fixed fraction of the flow, then it also is a fixed fraction of the present value of the flow. So the present value of annuity liabilities is augmented by 1.25% to reflect these operational liabilities.[7]

d. Market-value balance sheets and mapping the components to market indices

Market-value balance sheets

As explained, results are very sensitive to the yield curve chosen to value annuity liabilities but not to the source used for pricing the assets, so we use the Superintendence of Pensions to value them and the *ad hoc* methodology described previously to value mortgages and leasing contracts.

Table 3 shows a summary of the market-value balance sheet for each AIC. We also value liabilities, as described, using government, AAA and AA yield curves. Total liabilities include "other" and "operational" in this case. We present estimates for December 2008 and 2009. AICs are ordered according to their leverage, using the AAA curve.

Considering the AAA reference curve, we observe that the system's average leverage is 28 times in 2009 and 71 times in 2008. In 2009 there are two companies with negative equity and nine are

[6] We use as reference 500 CLP / USD.
[7] We thank Jorge Matrangelo for this suggestion.

above the maximum leverage level allowed in the regulation.[8] In 2008 only two companies would have complied with the maximum allowed leverage, if we use the AAA curve to discount annuity liabilities.

Mapping the components to market indices

Table 4 shows how the different components of the asset-side of the balance sheets are mapped into market indices, for the case of one particular AIC.

The methodology used for valuing annuity liabilities was explained above as well as the value of the "operational liabilities". We map the "risk free annuity" to a "9-year plus" duration bond index ("LVAZCZ3A") whose components are rated AAA.[9] We checked the quality of an approximation assuming that this index behaves like a zero-coupon bond with maturity equal to its average Duration, finding that the approximation error is minimal, especially for the purposes of estimating variances and covariances. Therefore, in order to consider the differences in Duration between annuity providers, we took as reference the IRR of the index (LVAZCZ3A) and constructed a liability index for each AIC with the same average Duration of their annuities. (We call this index IRVAAAnn, where "nn" is the number assigned to the AIC). All of these indices' returns are perfectly correlated with each other and with the return of the reference index, but may have different variances and covariances because of the different Durations.

Given the behavior of assets and liabilities and the 2009 leverage ratio, the behavior of equity is residual.[10]

Other indicators

We also estimate complementary indicators, by asset class and the Duration of each. Table 5 presents the aggregate summary. Local fixed income has an average Duration of 8.5 years, an Internal Rate of Return of 4.6%. 41.8% of the assets are subject to prepayment risk. However, assuming that "other assets" have a 1-year Duration (with the exception of international fixed income, which is assumed long-term and currency hedged for these purposes) the average Duration lowers to 7.1 years.

Using the AAA curve, the average liability Duration is 9.6 years, including annuities and other liabilities. Given the relatively high average leverage (28.5 times using AAA), the equity's residual Duration is –63 years.

Regarding credit risk, 18.5% of the assets have AA or better credit risk rating and 56.7% has a rating of A or less.

[8] Which is 20 times. As will be explained later, this happens because AICs have neither been required to update the present value of their liabilities using recent interest rate levels nor to use the latest mortality tables to value older annuities.

[9] The actual index is constructed by LVA Indices. It represents a buy and hold investment strategy.

[10] Notice that Equity can be negative. This doesn't affect the option value calculations presented below.

4. Empirical Results

This section shows first the quasi-maximum likelihood method used to estimate the fixed income parameters of the model and the calibration of the parameters associated to each AIC portfolio. Then, it presents the liquidity and credit risk adjustment factors for each company. Finally, it shows the values of the options to default and their effect on annuities' implied credit spreads, and the asset insufficiency probabilities of each AIC.

a. QML estimation

To estimate the fixed income parameters of the model we use monthly data consisting on government and corporate AAA bonds between January 2002 and November 2010. In particular, we use data on bond portfolios provided by LVA Indices that are used to build their fixed income indices. For the government bonds we have 9 portfolios with average durations ranging from 0.82 to 12.13 years, while for the corporate AAA bonds we have four portfolios with average durations from 1.53 to 11.49 years.

Since the bond portfolios of LVA Indices are constructed with coupon paying bonds that we don't have in our data set, we build a fictitious bond for each index that matches the descriptors provided by LVA Indices (e.g., IRR, duration, convexity, average maturity). In particular, we create a bond that has the following fictitious payment structure,

$$Bond[IRR, g, T] = \frac{1}{IRR-g}\left(1 - \left(\frac{1+g}{1+IRR}\right)^{T}\right) \tag{26}$$

For each month we obtain an optimal T and g such that a linear function of the square difference of the duration, convexity and maturity between the data and the fictitious bond is minimized.

Once we have the fictitious bonds that replicate the data, we estimate the fixed income model using the maximum Likelihood methodology proposed by Chen and Scott (1993) and Pearson and Sun (1994). In our affine model, the state variables $\{r_t, \theta_t, s_t\}$ have a multivariate Gaussian distribution, which implies that the logarithm of the zero-coupon bonds and their IRR have a Normal distribution. However, in our case the IRR of the fictitious bonds have a different distribution, because these are coupon-paying bonds. Fisher and Gilles (1996) and Duffee and Stanton (2004) suggest that a reasonable way to estimate these models is with a quasi-maximum Likelihood methodology, where the density of the observed IRRs is approximated by a Gaussian distribution. Let $X_t = \{r_t, \theta_t, s_t\}$ and $\tilde{Y}_t$ be the observed IRR. In the model, the IRR of the bonds are a nonlinear function of the state variables, $Y_t = h(X_t)$. We assume that three IRRs are observed without error and back out the state variables $X_t = \mathrm{h}^{-1}(\tilde{Y}_t)$, numerically. The three error-free IRRs are: the government bonds with average durations of 0.82 and 7.86 years, and the AAA bond with an average duration of 1.53 years. From the first two IRRs we back out the short- and long-term interest rate state variables, while the liquidity spread is obtained from the AAA bond. The approximate conditional distribution of the IRR is:

$$f_Y\big(Y(t)|Y(t-1)\big) \approx abs(J_X) f_X\big(X(t)|X(t-1)\big) \tag{27}$$

where J_X is the approximate Jacobian transformation from $Y(t)$ to $X(t)$, i.e., $J_X = det(\mathrm{h}^{-1})$ which is also obtained numerically. For the quasi-maximum Likelihood estimator we calculate the first two conditional moments from $f_Y\big(Y(t)|Y(t-1)\big)$ and assume that the likelihood function is Normal.

Once we have the state variables for each period, $X(t)$, we obtain the remaining cross-section IRRs, which are 7 government portfolios and 3 AAA portfolios. We assume that these observations have measurement errors that follow AR (1) processes:

$$\tilde{Y}(t) = Y(t) + u(t) \quad \text{with} \quad u(t) = \rho\, u(t-1) + e(t) \tag{28}$$

where the errors $e(t)$ are jointly Normally distributed with zero mean and covariance matrix $\mathbb{E}[e\ e^T]$. The conditional likelihood function for measurement errors is

$$f_u\big(u(t)|u(t-1)\big) = f_e\big(e(t)\big) \tag{29}$$

The likelihood function for the panel data is the product of both the likelihood of the observed data without error and the likelihood of the measurement errors. More details about the estimation procedure can be found in the empirical appendices of Collin-Dufresne and Solnik (2001) and Casassus and Collin-Dufresne (2005).

Table 6 shows the QML estimates of the fixed income model. The estimates show that the three state variables follow stationary processes although with different degrees of mean reversion. The liquidity shocks are only temporary ($\kappa_s = 3.724$), while the shocks to the long-term rates are highly persistent ($\kappa_\theta = 0.065$). The long-term (real) interest rate is 5.4% and the long-term liquidity spread is 20 bp. The short- and long-term interest rates are negatively correlated (i.e., $\sigma_r\sigma_{\theta 1} < 0$) as are also the short-term rate and the liquidity spread (i.e., $\sigma_r\sigma_{s1} < 0$). Finally, the constant risk premia for each risk factor are significant at least at the 10% level.

To calibrate the parameters of the dynamics of AIC's assets, we study in detail the investment portfolios of the insurance companies. The appendices provide tables with the breakdown of the aggregate investments of the AICs.[11] As expected, corporate bonds dominate the portfolios, followed by subordinated bonds, Treasury bonds, mortgage loans, real estate and stocks.

Due to the large number of financial instruments included in these portfolios, it is crucial to group assets with similar characteristics (e.g. same asset class, same currency, and similar maturity, quality and liquidity). The objective is to determine the composition of the AIC portfolios at an aggregate level, but also to gather information about the dynamics of each of these asset groups. For the latter we need to map each asset class on one of the many indexes provided by LVA Indices, as explained in the previous section (see Table 4). We choose the portfolio composition of the AICs as of December 2009 to obtain the representative weights on each asset class. Once the portfolios have been mapped we assume that the companies keep the same composition over

[11] The portfolio details for each AIC are available upon request.

time. After obtaining the weights and considering the time-series of each one of the indexes, it is straightforward to obtain a proxy for the time-series of the assets of each AIC.

To calibrate the covariance coefficients of the assets in equation (18), we run an OLS regression for the realized asset returns using the risk-free interest rate and the other fixed-income factors as explanatory variables. The regression error corresponds to the uncorrelated factor of each company, $W_{4,t}^{Q}$.

b. Liquidity and credit risk factors

We argue that MWRs should be adjusted for liquidity and credit risk. Table 7 shows these factors for each annuity insurance company. These adjustment factors depend on the balance sheet and risks of each AIC and are independent of the type of policy, i.e., gender, early retirement, survivors, etc. This implies that the adjustment factors should be applied to all the annuity holders of the company.

The table shows that the factors vary considerably across companies. In particular, the liquidity factor ranges from 0.944 to 0.989 and is lower in general for companies that are more levered. The credit risk factors decrease with regulatory horizon because the option value increases with it. The cross-sectional variability of these factors also increases with the horizon and the ranges are 0.964-1.000, 0.947-0.997 and 0.934-0.992 for 1, 3 and 5 years, respectively. These factors are lower (i.e., credit risk is more important) for the most levered companies. Considering the two adjustment factors together brings down the multipliers to ranges of 0.917-0.989, 0.898-0.986 and 0.885-0.981 for 1, 3 and 5 years, respectively.

Overall, both adjustments considered here allow us to justify MWR up to 1.1, a figure that is even larger than the MWRs reported in Rocha, Morales and Thorburn (2008) (the average MWR in 2004 was 1.064). Indeed, if we adjust the MWRs from Rocha et. al. (2008) we get AWMRs below one.

Thus, liquidity and credit risk adjustments can potentially explain the high MWR offered in some particular markets.

c. Value of the default option

One of the objectives of this work is to value the life annuities considering that annuity providers can default. As explained before, the annuity is worth the discounted present value of expected future payments minus the value of the option to default held by the shareholders of the annuity insurance company.

Table 8 shows the leverage ratios and the values of the default options for each AIC, as a fraction of their equity as of December 2009 for different regulatory horizons. The second column shows the leverage ratio of each AIC considering the market value of the assets and the liabilities discounted with the illiquid AAA yields. The third to fifth columns present the default option values as a fraction of each AIC's equity (for regulatory horizons of 1, 3 and 5 years). The companies are ordered from high to low option/equity ratio, a ranking that almost coincides with the one using the leverage ratio. Notice that equity values include the default option value.

It is important to note that, conceptually, someone must bear the cost of the option to default: it could be the state, the annuitants or the shareholders in case they decide to invest more equity capital in the company to avoid bankruptcy. The aggregate value of this option is USD 460 million for a 1-year regulatory horizon, USD 1 billion for a 3-year horizon and USD 1.4 billion for a 5-year horizon. In absolute terms, the value of the option to default increases with the regulatory horizon. However, as a fraction of equity, the percentage might decrease with the horizon if the market value of equity without the option is negative, as is the case of the first two AICs in the table.

Since the regulatory leverage ratio for this industry is relatively high (20), the option value is a significant fraction of the total equity value, even for the companies that are near this threshold. However, the relevant question for the annuitants is how affected their life annuities may be because of the option to default. In other words, how important is the default option value for the AIC's liabilities. We find that in the aggregate, default options represent 1.7%, 3.6% and 5.1% of the riskless illiquid liability for regulatory horizons of 1, 3 and 5 years. These don't seem to be "large" numbers; nevertheless, they can imply IRRs for the liabilities significantly above the corresponding illiquid default-free yield.

Indeed, Table 9 shows the IRR and the spreads with respect to the AAA curve. As a reference, consider that in December 2009 the average spread of the AA curve over the AAA curve was 34 basis points (see Table 1). Table 9 shows that for the 1-year regulatory horizon (third column from right to left) the majority of annuities qualify as having implied spreads similar to a AA bond. However, when the regulatory horizon is increased, most companies increase their risk exponentially and for a 3-year horizon only 4 out of 16 companies have implied spreads lower than the AA spread. For a 5-year horizon, only 2 companies maintain low implied credit spreads.

To illustrate this point further, Figure 3 shows the adjusted annuity IRR considering the different regulatory horizons. Note that for a 5-year horizon some AICs have IRRs that are higher than for the corresponding BBB bonds.

A final exercise is to estimate the AIC's default probabilities (see Table 10). Note that there is a close relationship with the previous results, i.e., companies whose default option value is larger, also have a higher default probability. An interesting difference is the effect of the expected return, because the probability of default might decrease if it is high. To reconcile this result with the previous ones, it is necessary to consider that, although the probability falls, the expected left-tail value in the probability distribution increases.

d. Discussion

We have found that using the AAA yield curve reference, leverage ratios for most annuity insurance companies are above the maximum allowed levels by the local regulation (which is 20 times). We also find that default options represent a significant fraction of total equity for most AICs, especially considering longer possible default horizons, even for the companies that would comply with the maximum leverage ratio. All of this is related with a significant probability of asset insufficiency at different horizons. Finally, in addition to the illiquidity adjustment factor that we propose for Money's Worth Ratios (which averages 0.95), the credit risk adjustment factor averages between 0.98 and 0.95, depending on the possible default horizon. For some companies these adjustment factors are significantly lower. Together, these factors justify observing measured MWRs as high as 1.1, which don't mean that annuitants get good deals in the Chilean market.

An interesting question is what incentives have brought the annuity industry to this point. In what follows we present a few facts that help explain these findings.

The SVS has regulated the annuity industry with two main tools: the matching rule (Norma de Calce or NDC, in Spanish) and the Asset Sufficiency Test (Test de Suficiencia de Activos or TSA, in Spanish).[12]

The NDC adds the future cash flows of all fixed income investments and classifies them in 10 successive maturity tranches. All inflation-indexed fixed income payments are considered for all the tranches except for the last 2 ones (starting in year 21), which exclude pre-payable assets' flows. These aggregate cash flows are compared with the aggregate expected liability flows (e.g. the annuities'). For the purposes of calculating the reserve requirements, if there are enough asset flows as to cover the liabilities' in any tranche, asset flows are discounted back to the present using government bond market interest rates; if not, the deficit is discounted back to the present using relatively "low" interest rates (3%, inflation-adjusted), increasing in this way the reserve requirement.

However, according to the NDC rule, market interest rates used for valuing annuities need not be updated. They correspond to the prevailing interest rates when the annuity was sold.[13] Therefore, the registered annuity liabilities do not take into account that interest rates have dropped systematically in about 3 percentage points since 2000, suggesting a sizeable underestimation of its value.

[12] Circular 1512 and NCG 209, respectively.

[13] The only exception is the weight given to the "low" interest rate when the level of cash flow mismatching changes.

On the other hand, there are instructions which indicate that the industry must use updated mortality tables only gradually.[14] Given that life expectancy has increased over time, here is another reason that explains why annuity liabilities are underestimated.

Finally, the asset sufficiency test (TSA) tries to determine whether under stress the present value of asset flows is enough to cover liabilities using a 3% (real) discount rate. Again, the TSA regulation considers all asset flows (as long as they are inflation-indexed) and requires subtracting a fraction of the payments of each instrument depending on its risk rating, to account for credit risk. For example, for (local) BBB-rated instruments, the factor is 3%. The 3% flow write-off in a 10-year Duration bond is equivalent to a spread of 30 basis points. This is several orders of magnitude smaller than the market spread, which is close to 250 basis points (see Figure 3). Similarly, a (local) BB-rated bond allows considering 93% of its flows, which is equivalent to a 70 basis point spread with respect to risk-free bonds in a 10-year Duration bond. The market spread for BB-rated bonds is much higher. Mortgages and leasing contract flows are excluded only if they are behind in their scheduled payments.

This succinct revision of the regulation allows us to conclude that it is not at all surprising to find that market-value leverage ratios are above their regulatory limit. Annuity insurance companies are not required to fully account for the effects of lower interest rate levels and of higher life expectancy. On the other hand, the existing regulations provide incentives for selling safer assets and buying riskier ones. Selling government or AAA bonds and buying BB-rated ones implies liberating reserves. This gives rise to "regulatory arbitrage",[15] making the default option more valuable.

Therefore, our results can be explained by a "perfect storm": high levels of competition in the annuity industry; drops in real interest rate levels; increases in life expectancy; and a regulation, which inadequately measures the leverage ratios and provides incentives for higher risk-taking. This leads previously reported Money's Worth Ratios to overstate the value-for-money that annuitants get in the Chilean market. There is no puzzle.

5. Conclusions

The Money's Worth Ratio (MWR) of an annuity is a measure of actuarial fairness. MWRs are estimated as the discounted present value of expected future annuity payments divided by the money (or premium) paid in advance for the annuity.

In this article we argue that Money's Worth Ratios (MWR) of irrevocable life annuities should be adjusted to consider at least two factors: illiquidity and credit risk. Without these adjustments,

[14] For example, rule NCG 178 of 2005 establishes that AICs have up to year 2015 to recognize the effects of the 2004 mortality table update. The latest update is 2009.

[15] For example, see Black (1995) or Bodie (1995), who analyzes stock investments by defined benefit pension plans.

measured MWRs are upward biased, in the sense that they do not reflect the value-for-money obtained by annuity buyers.

As intuitive as this may be, measuring these adjustment factors is not trivial, since it requires detailed knowledge of the nature of annuity (and other) liabilities, of the asset portfolios used to back the annuities and of the stochastic processes followed by assets and liabilities.

Here we propose a closed form solution for money's worth ratios which considers both, illiquidity and default risk in the context of a multi-factor continuous-time model. We call it the Adjusted Money's Worth Ratio (AMWR). The adjustments are done by taking as reference an illiquid default-risk-free replicating portfolio of bonds with payments identical to the expected annuities' and then adjusting for default risk using option pricing theory, in the spirit of Merton (1974).

We define a "regulatory horizon" after which the supervising authority will check whether the market value of assets suffices to cover liabilities'. If not, AIC equity holders must capitalize the company or default. So the default option is similar to an exchange option (Margrabe, 1978) by means of which equity holders can give the AIC's assets in exchange of the liabilities, with no further obligations.

As expected, risk is summarized in the volatility of the annuity provider's equity, which depends on leverage, maturity mismatching, illiquidity mismatching and other assets' volatilities. These parameters themselves depend on the mean-reversion in interest rates and in liquidity premia, among other factors.

We illustrate our results with a data-intensive application to the Chilean pension annuity industry, which is considered to be quite competitive. The average MWR reported in Rocha et al. (2008) is 1.064 for 2004, with an upward trend since 1999. High MWRs are often considered a puzzle.

We use a quasi-maximum likelihood method to estimate the fixed income parameters of the model and to calibrate the parameters associated with each annuity insurance company's (AIC) asset portfolio. This allows estimating the liquidity and credit-risk adjustment factors for each AIC, in addition to the default option values, the implied annuity credit spreads, and the asset insufficiency (or default) probabilities.

The aggregate value of the option to default is USD 460 million for a 1-year regulatory horizon, USD 1 billion for a 3-year horizon and USD 1.4 billion for a 5-year horizon. Since the regulatory leverage ratio for this industry is relatively high (20), for nearly all AICs, the option value is a significant fraction of the total equity value, even for the companies that are near this threshold.

Aggregate, default options represent 1.7%, 3.6% and 5.1% of the riskless illiquid liability for regulatory horizons of 1, 3 and 5 years, respectively, but with a wide cross-sectional dispersion. On average, these don't seem to be "large" numbers; nevertheless, they can imply IRRs for the liabilities significantly above the corresponding illiquid default-free yield. For the 1-year regulatory horizon most annuities qualify as having implied spreads similar to (local) AA bonds. However, when the regulatory horizon is extended, for most companies risk increases exponentially. For a 3-

year horizon only 4 out of 16 companies have implied spreads lower than the spread corresponding to AA bonds and for a 5-year horizon, only 2 companies have low implied credit spreads. In the latter case some AICs have IRRs that are higher than BBB bonds' (e.g. local junk). We also present asset insufficiency probabilities for each AIC at different horizons.

Finally, we discuss the regulatory incentives and the market forces which may have driven the Chilean annuity market to its current state. There are high levels of competition documented for this market, real interest rate levels have steadily fallen, life expectancy has increased, and regulation inadequately measures leverage and provides incentives for risk-taking. This leads previously reported Money's Worth Ratios to be overstated in the Chilean market, at least in the sense that they don't necessarily represent good deals for annuitants.

Our results thus indicate that the high MWRs for Chile indeed reflect lack of liquidity and credit risk adjustments. This may also help explain the high MWRs elsewhere.

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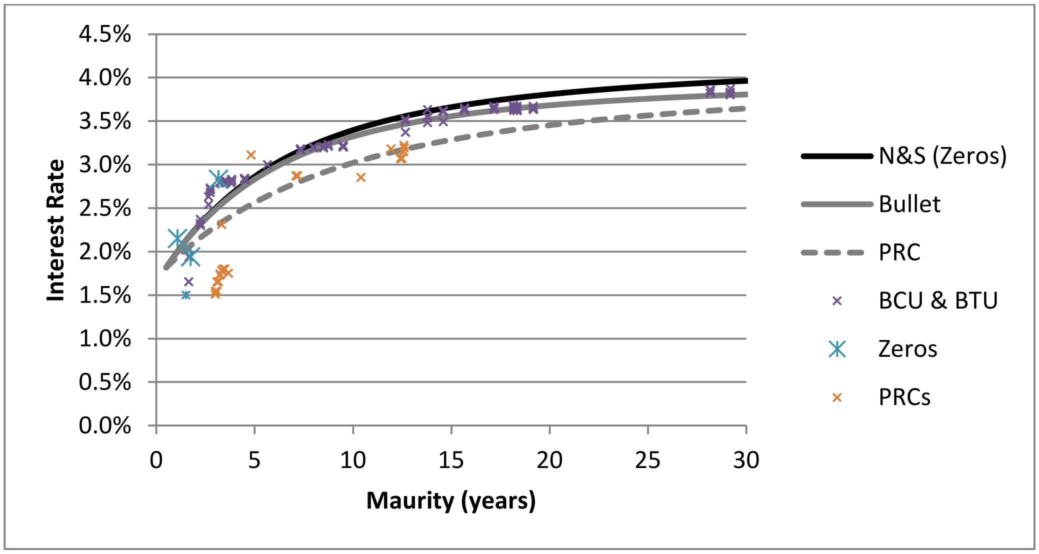
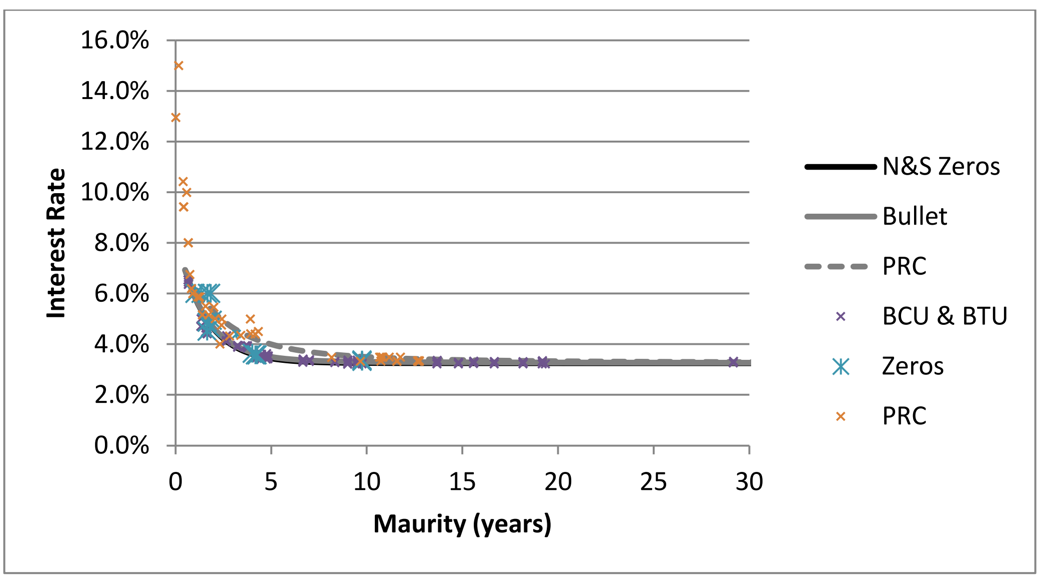
7. FIGURES AND TABLES

Figure 1. **TERM STRUCTURE OF INTEREST RATES FOR UF-DENOMINATED GOVERNMENT BONDS**

A. 2009

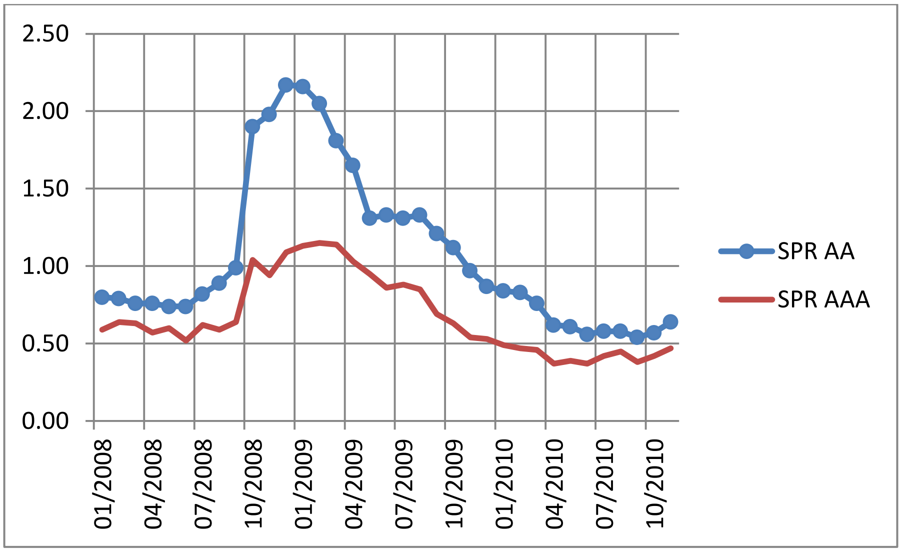


B. 2008



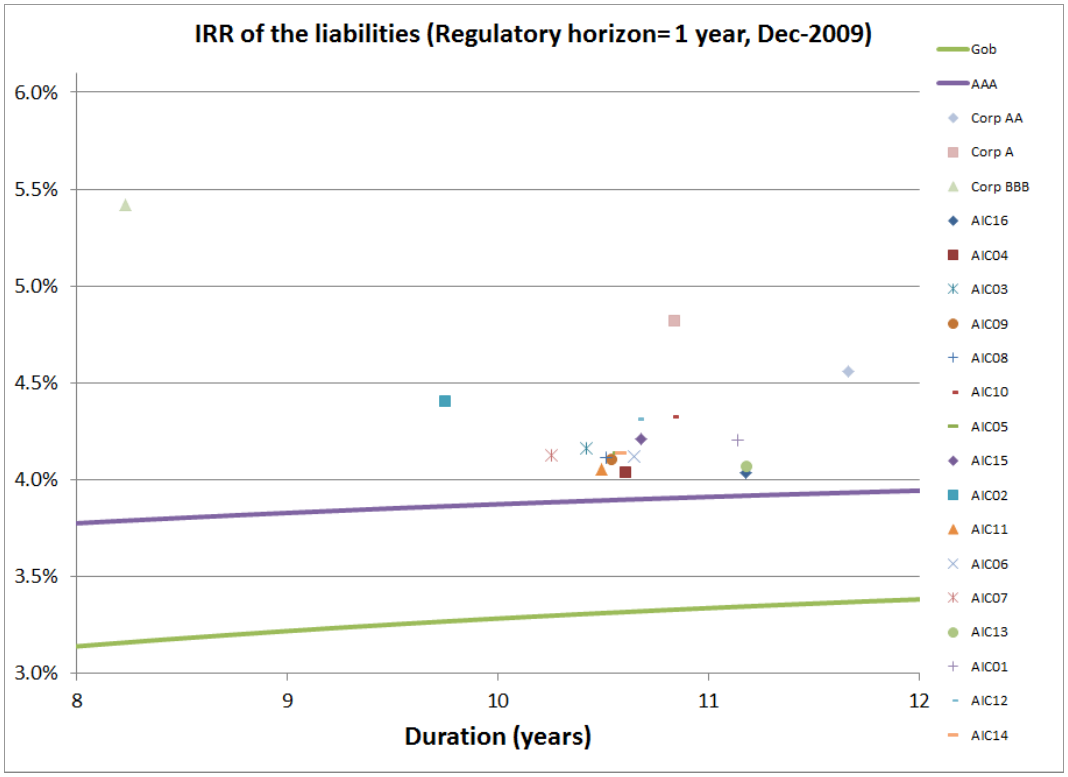
Note: Estimation based on the last 5 trading days of each year, excluding instruments with maturity of less than 1 year. PRCs are annuity-like, BTU and BCU are Bullet bonds.

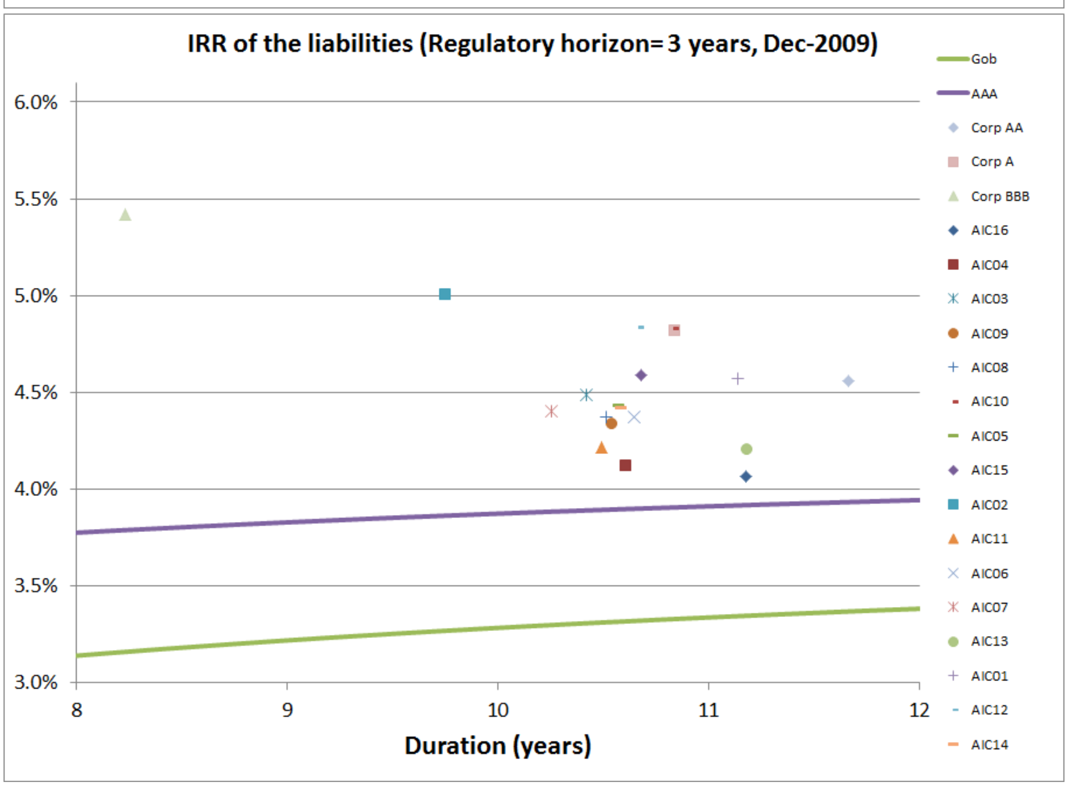
Figure 2. SPREADS FOR AAA AND AA BONDS (%)

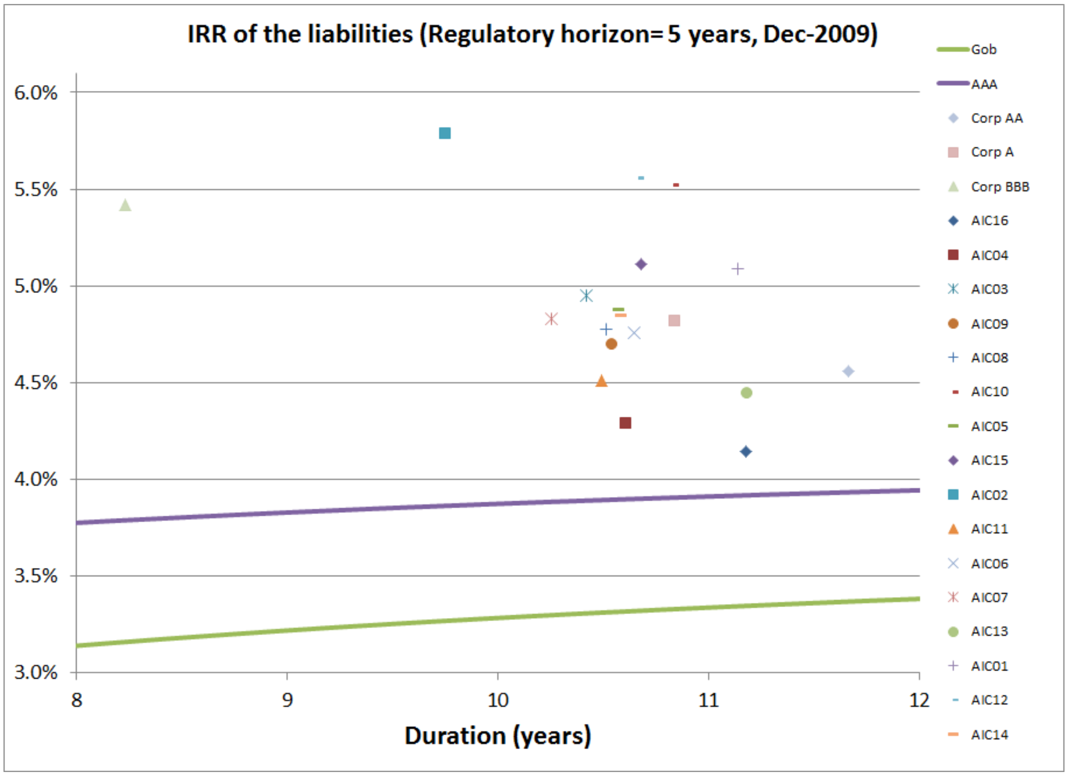


Source: Estimated here based on LVA Indices denominated in UF with Duration 9 and more, for Government, AAA and AA bonds.

Figure 3. ESTIMATED IRR OF THE LIABILITIES CONSIDERING THE CREDIT RISK ADJUSTMENT







IRR of the liabilities (Regulatory horizon= 5 years, Dec-2009)
6.0%
5.5%
5.0%
4.5%
4.0%
3.5%
3.0%
8
9
10
11
12
Duration (years)
Gob
AAA
Corp AA
Corp A
Corp BBB
AIC16
AIC04
AIC03
AIC09
AIC08
AIC10
AIC05
AIC15
AIC02
AIC11
AIC06
AIC07
AIC13
AIC01
AIC12
AIC14

Table 1. SPREADS (%)

	AAA	AA
December 2008	1.09	2.17
December 2009	0.53	0.87

Source: See Figure 2

Table 2. PRESENT VALUE (PV) OF ANNUITY LIABILITIES[16]

A. Year 2009

AIC	Size	Dur	IRR Gov.	IRR AAA	IRR AA	PV Gov. (Mn USD)	PV AAA (Mn USD)	PV AA (Mn USD)	Ratio Gov./AAA	Ratio AAA/AA
AIC01	LARGE	11.27	3.62%	4.15%	4.49%				1.06	1.037
AIC02	MED	9.49	3.53%	4.06%	4.40%				1.05	1.031
AIC03	LARGE	10.55	3.59%	4.12%	4.46%				1.056	1.034
AIC04	SMALL	10.74	3.60%	4.13%	4.47%				1.057	1.035
AIC05	MED	10.71	3.59%	4.12%	4.46%				1.057	1.035
AIC06	LARGE	10.78	3.59%	4.12%	4.46%				1.057	1.035
AIC07	SMALL	10.38	3.57%	4.10%	4.44%				1.055	1.034
AIC08	LARGE	10.64	3.59%	4.12%	4.46%				1.057	1.035
AIC09	MED	10.67	3.59%	4.12%	4.46%				1.057	1.035
AIC10	LARGE	10.97	3.61%	4.14%	4.48%				1.058	1.036
AIC11	SMALL	10.62	3.59%	4.12%	4.46%				1.056	1.035
AIC12	SMALL	10.8	3.60%	4.13%	4.47%				1.058	1.035
AIC13	MED	11.31	3.62%	4.15%	4.49%				1.06	1.037
AIC14	MED	10.72	3.59%	4.12%	4.46%				1.057	1.035
AIC15	SMALL	10.81	3.60%	4.13%	4.47%				1.058	1.035
AIC16	SMALL	11.31	3.62%	4.15%	4.49%				1.06	1.037
SYSTEM		**10.77**	**3.60%**	**4.13%**	**4.47%**	**29,088**	**27,510**	**26,575**	**1.057**	**1.035**

[16] IDs have been assigned randomly.

B. Year 2008

AIC	Size	Dur	IRR Gov.	IRR AAA	IRR AA	PV Gov. (Mn USD)	PV AAA (Mn USD)	PV AA (Mn USD)	Ratio Gov./AAA	Ratio AAA/AA
AIC16	SMALL	12.37	3.30%	4.39%	5.47%				1.132	1.12
AIC04	SMALL	11.73	3.31%	4.40%	5.48%				1.124	1.112
AIC03	LARGE	11.38	3.31%	4.40%	5.48%				1.121	1.11
AIC09	MED	11.51	3.31%	4.40%	5.48%				1.122	1.111
AIC08	LARGE	11.46	3.31%	4.40%	5.48%				1.122	1.11
AIC10	LARGE	11.91	3.31%	4.40%	5.48%				1.127	1.115
AIC05	MED	11.45	3.31%	4.40%	5.48%				1.122	1.111
AIC15	SMALL	11.67	3.31%	4.40%	5.48%				1.124	1.113
AIC02	MED	10.15	3.33%	4.42%	5.50%				1.107	1.098
AIC11	SMALL	11.53	3.31%	4.40%	5.48%				1.123	1.112
AIC06	LARGE	11.6	3.31%	4.40%	5.48%				1.123	1.112
AIC07	SMALL	11.19	3.31%	4.40%	5.48%				1.119	1.109
AIC13	MED	12.22	3.30%	4.39%	5.47%				1.13	1.118
AIC01	LARGE	12.19	3.30%	4.39%	5.47%				1.13	1.117
AIC12	SMALL	11.73	3.31%	4.40%	5.48%				1.125	1.113
AIC14	MED	11.62	3.31%	4.40%	5.48%				1.124	1.112
SYSTEM		**11.62**	**3.31%**	**4.40%**	**5.48%**	**29,949**	**26,656**	**23,970**	**1.124**	**1.112**

Table 3. Market Value Balance Sheet and Leverage

A. Year 2009

AIC	Size	Liabilities			Debt / Equity		
		AA (% Assets)	AAA (% Assets)	Gov. (% Assets)	AA (Times)	AAA (Times)	Gob (Times)
AIC02	MED	100.0	102.2	106.0	9117	-46	-18
AIC10	LARGE	97.0	100.4	106.2	32	-251	-17
AIC12	SMALL	96.3	99.6	105.1	26	245	-20
AIC15	SMALL	95.5	98.4	103.2	21	61	-32
AIC01	LARGE	94.3	97.8	103.6	17	44	-28
AIC03	LARGE	94.4	97.3	102.2	17	36	-47
AIC14	MED	93.5	96.3	101.1	14	22	-92
AIC07	SMALL	93.1	96.0	101.0	13	24	-99
AIC05	MED	92.7	95.7	100.8	13	22	-125
AIC06	LARGE	92.7	95.7	100.6	13	22	-159
AIC09	MED	92.8	95.4	99.8	13	21	487
AIC08	LARGE	92.7	95.4	100.0	13	21	7822
AIC13	MED	89.6	92.4	97.3	9	12	36
AIC04	SMALL	89.5	91.0	93.6	8	10	15
AIC11	SMALL	87.0	90.1	95.1	7	9	20
AIC16	SMALL	83.3	83.8	84.7	5	5	6
SYSTEM		**93.7**	**96.6**	**101.5**	**15**	**28**	**-67**

B. Year 2008

AIC	Size	Liabilities			Debt / Equity		
		AA (% Assets)	AAA (% Assets)	Gov. (% Assets)	AA (Times)	AAA (Times)	Gob (Times)
AIC14	MED	101.6	111.4	123.4	-64	-10	-5
AIC02	MED	98.9	106.5	115.8	90	-16	-7
AIC08	LARGE	93.0	101.9	112.7	13	-54	-9
AIC11	SMALL	91.9	101.1	112.5	11	-88	-9
AIC03	LARGE	92.4	101.1	111.8	12	-91	-10
AIC15	SMALL	91.9	100.7	111.5	11	-141	-10
AIC06	LARGE	90.4	99.6	110.7	9	223	-10
AIC05	MED	89.7	99.0	110.4	9	99	-11
AIC07	SMALL	89.1	98.5	110.0	8	66	-11
AIC01	LARGE	87.3	97.1	109.3	7	34	-12
AIC04	SMALL	91.1	96.5	103.1	10	27	-33
AIC09	MED	88.3	96.4	106.3	8	27	-17
AIC12	SMALL	86.7	96.2	107.8	7	25	-14
AIC16	SMALL	94.6	96.1	97.9	18	25	47
AIC10	LARGE	87.0	95.1	105.1	7	19	-21
AIC13	MED	86.7	94.7	104.5	7	18	-23
SYSTEM		**90.0**	**98.6**	**109.2**	**9**	**71**	**-12**

Table 4. MAPPING OF AN AIC'S PORTFOLIO (DEC. 2009)

Assets	Percentage	Index
Bank Bonds	1.12%	LVAZ0105
Corp. Bonds	23.42%	
AAA/0-2	0.00%	LVAZCS3A
AAA/3-5	0.29%	LVAZCM3A
AAA/6-8	0.57%	LVAZCL3A
AAA/9+	3.19%	LVAZCZ3A
AA/0-2	0.10%	LVAZCS2A
AA/3-5	0.43%	LVAZCM2A
AA/6-8	1.91%	LVAZCL2A
AA/9+	1.82%	LVAZCZ2A
A/0-2	0.06%	LVAZCS1A
A/3-5	2.41%	LVAZCM1A
A/6-8	3.94%	LVAZCL1A
A/9+	4.04%	LVAZCZ1A
BBB (or less)	4.67%	LVAXC3BM
Gov. Bonds	15.53%	
1-	2.61%	LVAXG1
2	1.36%	LVAXG2
3	2.17%	LVAXG3
4	2.45%	LVAXG4
5	1.66%	LVAXG5
6	1.20%	LVAXG6
7	1.01%	LVAXG7
8	0.62%	LVAXG8
9+	2.45%	LVAXG9
Securitized Bonds	2.21%	LVAZ0102
Subordinate Bonds	4.69%	LVAZ0108
Leasings	11.68%	Leasings
Mortgage Bonds	9.92%	
1-	0.02%	LVAXH1
2	0.42%	LVAXH2
3	0.43%	LVAXH3
4	1.02%	LVAXH4
5	1.15%	LVAXH5
6	1.66%	LVAXH6
7	0.83%	LVAXH7
8	1.39%	LVAXH8
9+	2.99%	LVAXH9
Mortgages	12.99%	Mutuos
Other Fixed Income	5.09%	Average portfolio
Local Equity	3.96%	MSCI Chile
Mutual Funds	1.25%	
Type 1 + 2	1.25%	LVAXG1
Type 3	0.00%	LVAXC10
Type 4,5,6 and 8	0.00%	MSCI Chile
Real Estate	0.38%	MSCI Chile
International Fixed Income	0.54%	
AA	0.00%	LVAXCAA
A	0.00%	LVAXCA
BBB (or less)	0.54%	LVAXC3BM
Renta Variable Internacional	4.18%	MSCI
Derivatives	-0.05%	Int'l Fixed Income
Other Assets	3.10%	LVAXC1
Subtotal	94.91%	
Total Assets	100.00%	
Liabilities		
Short-term	13.35%	LVAXG1
Operational	1.07%	IRVAAA_AICXX
Annuities	85.58%	IRVAAA_AICXX
Total liabilities	100.00%	

Table 5. MARKET-VALUE BALANCE SHEET (SYSTEM, 2009)

Assets	Amt. (Mn USD)	% Assets	Duration	Avg. IRR	% Prepayable	R_e
Bank Bonds	827	2.5%	8.5	4.5	13.3%	
Copr. Bonds	10073	31.0%	9.3	4.6	53.5%	
Gov. Bonds	2734	8.4%	7.7	3.8	0.0%	
Secur. Bonds	451	1.4%	5.9	5.7	87.2%	
Subord. Bonds	2989	9.2%	10.2	4.6	2.4%	
Leasings	2463	7.6%	7.3	4.6	27.4%	
Mortgage Bonds	1958	6.0%	6.0	4.8	5.9%	
Mortgages	3834	11.8%	8.4	4.9	100.0%	
Other Investments	1104	3.4%	1.0			
Local Equity	1099	3.4%	1.0			
Mutual Fund Shares	428	1.3%	1.0			
Real Estate	1440	4.4%	1.0			
Internat. Fixed Income	1195	3.7%	8.3			
Internat. Equity	658	2.0%	1.0			
Derivatives	180	0.6%	1.0			
Other Assets	1060	3.3%	1.0			
TOTAL ASSETS	32490	100.0%	7.1	0.0		
SUB TOTAL Local Fixed Inc.	25328	78.0%	8.5	4.6	41.8%	

Liabilities	Amt. (Mn USD)	% Assets	Duration	Avg. IRR	Debt / Equity
Annuities_Gov	29088	89.5%	10.8	3.6%	
Annuities_AAA	27510	84.7%	10.7	4.1%	
Annuities_AA	26575	81.8%	10.1	4.5%	
Operational_Gov	364	1.1%	10.8	-	
Operational_AAA	344	1.1%	10.7	-	
Operational_AA	332	1.0%	10.1	-	
Other	3534	10.9%	1	-	
Total Liabilities_Gov	32985	101.5%	9.7		-67
Total Liabilities_AAA	31388	96.6%	9.6		28
Total Liabilities_AA	30441	93.7%	9.0		15
Equity					
Equity_Gov.	-495	-1.5%	179		
Equity_AAA	1102	3.4%	-63		
Equity_AA	2049	6.3%	-21		

Assets	Amt. (Mn USD)	% Assets
Government	2734	8.4%
AAA	3266	10.1%
AA	8076	24.9%
A	4549	14.0%
BBB	234	0.7%
BBB_m	100	0.3%
No rating	71	0.2%
Leasings	2463	7.6%
Mortgages	3834	11.8%
Other Investments	1104	3.4%
Local Equity	1099	3.4%
Mutual Fund Shares	428	1.3%
Real Estate	1440	4.4%
Internat. Fixed Income	1195	3.7%
Internat. Equity	658	2.0%
Derivatives	180	0.6%
Other Assets	1060	3.3%
TOTAL ASSETS	32490	100.0%

Table 6. QUASI-MAXIMUM LIKELIHOOD ESTIMATES USING GOVERNMENT AND CORPORATE AAA BONDS DATA BETWEEN JANUARY 2002 AND NOVEMBER 2010

Mean-reversion parameters	Estimates	T stats
κ_r	0.741	7.1
κ_θ	0.065	3.8
κ_{sr}	0.357	0.9
$\kappa_{\mathrm{s}\theta}$	0.418	2.1
κ_s	3.724	4.7

Long-term parameters	Estimates	T stats
$\bar{\theta}$	0.054	8.7
$\bar{s}$	0.002	1.1

Covariance parameters	Estimates	T stats
σ_r	0.031	12.7
$\sigma_{\theta 1}$	-0.005	3.3
$\sigma_{\theta 2}$	0.013	9.8
σ_{s1}	-0.020	1.7
σ_{s2}	-0.008	0.8
σ_{s3}	0.092	5.3

Risk premium parameters	Estimates	T stats
λ_1	-0.666	2.2
λ_2	-0.328	1.7
λ_3	2.209	3.7

Table 7. LIQUIDITY AND CREDIT RISK FACTORS FOR THE AICS (DECEMBER 2009)

AIC	Liquidity factor	Credit risk factor			Liquidity and credit risk factors		
	$\frac{D_0}{G_0}$	$\frac{B_0[1]}{D_0}$	$\frac{B_0[3]}{D_0}$	$\frac{B_0[5]}{D_0}$	$\frac{D_0}{G_0} \cdot \frac{B_0[1]}{D_0}$	$\frac{D_0}{G_0} \cdot \frac{B_0[3]}{D_0}$	$\frac{D_0}{G_0} \cdot \frac{B_0[5]}{D_0}$
AIC02	0.964	0.964	0.947	0.935	0.929	0.913	0.901
AIC10	0.945	0.970	0.950	0.936	0.917	0.898	0.885
AIC12	0.948	0.971	0.949	0.934	0.920	0.899	0.885
AIC15	0.953	0.981	0.963	0.950	0.935	0.918	0.906
AIC01	0.944	0.982	0.962	0.948	0.927	0.908	0.895
AIC03	0.952	0.986	0.968	0.956	0.939	0.922	0.910
AIC14	0.953	0.989	0.971	0.959	0.942	0.925	0.913
AIC05	0.950	0.989	0.970	0.957	0.940	0.922	0.910
AIC07	0.949	0.990	0.973	0.960	0.940	0.924	0.911
AIC08	0.951	0.991	0.974	0.962	0.943	0.927	0.915
AIC06	0.956	0.991	0.974	0.962	0.947	0.931	0.920
AIC09	0.954	0.992	0.977	0.965	0.946	0.932	0.921
AIC11	0.950	0.997	0.984	0.971	0.947	0.934	0.922
AIC13	0.972	0.997	0.985	0.975	0.969	0.958	0.948
AIC04	0.947	0.999	0.991	0.983	0.946	0.939	0.931
AIC16	0.989	1.000	0.997	0.992	0.989	0.986	0.981

Table 8. VALUE OF THE DEFAULT OPTION FOR REGULATORY HORIZON OF 1, 3 AND 5 YEARS AS A FRACTION OF EQUITY (DECEMBER DE 2009)

AIC	Leverage (Liability/Equity)	Regulatory horizon = 1 year % option	Regulatory horizon = 3 years % option	Regulatory horizon = 5 years % option
AIC02	-45.8	249	169	150
AIC10	-250.8	115	109	107
AIC12	245.5	88	93	94
AIC15	61.4	53	70	76
AIC01	43.8	44	62	69
AIC03	36.4	34	54	62
AIC14	26.4	23	43	52
AIC05	22.4	20	40	49
AIC07	24.2	20	40	49
AIC06	22.0	17	36	45
AIC08	20.8	16	35	44
AIC09	20.7	14	33	42
AIC13	12.2	4	15	24
AIC11	9.1	3	13	21
AIC04	10.1	1	8	15
AIC16	5.2	0	1	4

Table 9. VALUE OF THE ANNUITY MINUS THE OPTION TO DEFAULT FOR REGULATORY HORIZON OF 1, 3 AND 5 YEARS AS A FRACTION OF ANNUITY, IRR AND SPREADS (DECEMBER DE 2009)

AIC	IRR $B_0[1]$ (annual %)	IRR $B_0[3]$ (annual %)	IRR $B_0[5]$ (annual %)	$Spread$ $B_0[1]$ (bp, annual)	$Spread$ $B_0[3]$ (bp, annual)	$Spread$ $B_0[5]$ (bp, annual)
AIC02	4.4	5.0	5.8	40	100	178
AIC10	4.3	4.8	5.5	29	80	149
AIC12	4.3	4.8	5.6	28	81	153
AIC15	4.2	4.6	5.1	18	56	109
AIC01	4.2	4.6	5.1	17	53	105
AIC03	4.2	4.5	5.0	14	46	93
AIC14	4.1	4.4	4.8	11	40	82
AIC05	4.1	4.4	4.9	11	41	85
AIC07	4.1	4.4	4.8	10	39	81
AIC08	4.1	4.4	4.8	9	35	75
AIC06	4.1	4.4	4.8	9	34	73
AIC09	4.1	4.3	4.7	8	31	67
AIC11	4.1	4.2	4.5	3	19	49
AIC13	4.1	4.2	4.4	3	17	41
AIC04	4.0	4.1	4.3	1	10	26
AIC16	4.0	4.1	4.1	0	3	10

Table 10. ASSET INSUFFICIENCY PROBABILITIES FOR REGULATORY HORIZONS OF 1, 3 AND 5 YEARS

Annuity Insurance Company	Prob[1]	Prob[3]	Prob[5]
AIC02	0.432	0.237	0.151
AIC12	0.336	0.251	0.197
AIC10	0.333	0.209	0.144
AIC15	0.211	0.135	0.089
AIC01	0.206	0.148	0.105
AIC03	0.160	0.111	0.074
AIC05	0.139	0.126	0.098
AIC14	0.131	0.102	0.071
AIC07	0.122	0.099	0.071
AIC08	0.112	0.102	0.077
AIC06	0.104	0.085	0.059
AIC09	0.083	0.065	0.043
AIC11	0.038	0.070	0.068
AIC13	0.029	0.035	0.025
AIC04	0.010	0.018	0.013
AIC16	0.000	0.002	0.001

8. APPENDICES

APPENDIX A – PROOF OF PROPOSITIONS

A.1 Proof of Proposition 2

Let $Z_0[t;\lambda]$ be a generic bond defined as:

$$Z_0[t;\lambda] = \mathbb{E}_0^Q\left[e^{-\int_0^t (r_u+\lambda\, s_u)\,du}\right] \tag{A1}$$

This definition implies that the present value of $Z[t;\lambda]$ is a Q-Martingale under the risk-neutral measure Q (see Duffie, Pan and Singleton (2000) and Dai and Singleton (2000)), therefore:

$$\mathbb{E}_v^Q\left[d\left(e^{-\int_0^v (r_u+\lambda\, s_u)\,du} Z_v[t;\lambda]\right)\right] = 0 \tag{A2}$$

Using Ito's lemma, we obtain a partial differential equation (PDE) for the price of the generic bond:

$$\begin{aligned}\frac{\partial Z}{\partial t} + \kappa_r(\theta - r)Z_r + \kappa_\theta(\bar{\theta} - \theta)Z_\theta + \left(\kappa_{sr}(\theta - r) + \kappa_{s\theta}(\bar{\theta} - \theta) + \kappa_s(\bar{s} - s)\right)Z_s + \frac{1}{2}\sigma_r^2 Z_{rr} + \frac{1}{2}(\sigma_{\theta 1}^2 + \sigma_{\theta 2}^2)Z_{\theta\theta} \\ + \frac{1}{2}(\sigma_{s1}^2 + \sigma_{s2}^2 + \sigma_{s3}^2)Z_{ss} + \sigma_r\sigma_{\theta 1}Z_{r\theta} + \sigma_r\sigma_{s1}Z_{rs} + (\sigma_{\theta 1}\sigma_{s1} + \sigma_{\theta 2}\sigma_{s2})Z_{\theta s} - (r + \lambda\, s)Z = 0\end{aligned} \tag{A3}$$

with boundary condition $Z_t[t,\lambda] = 1$. In this equation, $Z_x = \frac{\partial Z}{\partial x}$ and $Z_{xy} = \frac{\partial^2 Z}{\partial x \partial y}$.

The PDE above is linear in the state variables which suggests that functional form of the price of the generic bond is:

$$Z_v[t;\lambda] = Exp[\alpha[t - v;\lambda] + \beta_r[t - v;\lambda]r_v + \beta_\theta[t - v;\lambda]\,\theta_v + \beta_s[t - v;\lambda]\, s_v] \tag{A4}$$

Replacing this equation in the PDE and grouping terms, yields the following expressions for functions $\alpha[\tau;\lambda]$ and $\beta_x[\tau;\lambda]$:

$$\beta_s[\tau;\lambda] = -\lambda\frac{1 - e^{-\tau\kappa_s}}{\kappa_s} \tag{A5}$$

$$\beta_r[\tau;\lambda] = -\left(1 + \lambda\frac{\kappa_{sr}}{\kappa_r - \kappa_s}\right)\frac{1 - e^{-\tau\kappa_r}}{\kappa_r} + \lambda\frac{\kappa_{sr}}{\kappa_r - \kappa_s}\frac{1 - e^{-\tau\kappa_s}}{\kappa_s} \tag{A6}$$

$$\beta_\theta[\tau;\lambda] = \frac{\kappa_r}{\kappa_r-\kappa_\theta}\left(1+\lambda\frac{\kappa_{sr}}{\kappa_r-\kappa_s}\right)\frac{1-e^{-\tau\kappa_r}}{\kappa_r} \\ -\left(\frac{\kappa_r}{\kappa_r-\kappa_\theta}+\lambda\left(\frac{\kappa_{s\theta}}{\kappa_\theta-\kappa_s}+\frac{\kappa_{sr}\kappa_\theta}{(\kappa_r-\kappa_\theta)(\kappa_\theta-\kappa_s)}\right)\right)\frac{1-e^{-\tau\kappa_\theta}}{\kappa_\theta} \\ +\lambda\left(\frac{\kappa_{s\theta}}{\kappa_\theta-\kappa_s}+\frac{\kappa_s\kappa_{sr}}{(\kappa_r-\kappa_s)(\kappa_\theta-\kappa_s)}\right)\frac{1-e^{-\tau\kappa_s}}{\kappa_s} \tag{A7}$$

$$\alpha[\tau;\lambda] = -\int_0^\tau\Big(\kappa_\theta\bar{\theta}\beta_\theta[u;\lambda]+(\kappa_{s\theta}\bar{\theta}+\kappa_s\bar{s})\beta_s[u;\lambda]+\frac{1}{2}\sigma_r^2\beta_r[u;\lambda]^2+\frac{1}{2}(\sigma_{\theta 1}^2 \\ +\sigma_{\theta 2}^2)\beta_\theta[u;\lambda]^2+\frac{1}{2}(\sigma_{s1}^2+\sigma_{s2}^2+\sigma_{s3}^2)\beta_s[u;\lambda]^2+\sigma_r\sigma_{\theta 1}\beta_r[u;\lambda]\beta_\theta[u;\lambda] \\ +\sigma_r\sigma_{s1}\beta_r[u;\lambda]\beta_s[u;\lambda]+(\sigma_{\theta 1}\sigma_{s1}+\sigma_{\theta 2}\sigma_{s2})\beta_\theta[u;\lambda]\beta_s[u;\lambda]\Big)\,du \tag{A8}$$

Finally, to obtain the value of the liquid and illiquid bond we just need to recognize that $\alpha^L[t]=\alpha[\tau;0]$, $\beta_r^L[t]=\beta_r[\tau;0]$, $\beta_\theta^L[t]=\beta_\theta[\tau;0]$, $\alpha^I[t]=\alpha[\tau;1]$, $\beta_r^I[t]=\beta_r[\tau;1]$, $\beta_\theta^I[t]=\beta_\theta[\tau;1]$ y $\beta_s^I[t]=\beta_s[\tau;1]$.

A.2 Proof of Proposition 3

To obtain the credit risk factor it is easier to first calculate the market value of the equity with the option pricing approach. Using that K_t is the inverse of the levarage ratio (i.e., $K_t=\frac{A_t}{D_t}$) and defining $Z_t=\frac{D_t e^{-\int_0^t(r_u+s_u)\,du}}{D_0}$, we can write the equity of the AIC as:

$$E_0[t]=D_0\mathbb{E}^Q\big[Z_t Max[K_t-1,0]\big] \tag{A9}$$

Note that $Z_0=1$, $Z_t>0$ and Z_t is a Q-Martingale:

$$\frac{dZ_t}{Z_t}=\frac{d\left(D_t e^{-\int_0^t(r_u+s_u)\,du}\right)}{D_t e^{-\int_0^t(r_u+s_u)\,du}}=\frac{dD_t}{D_t}-(r_t+s_t)\,dt=\sigma_{D1}\,dW_{1,t}^Q+\sigma_{D2}\,dW_{2,t}^Q+\sigma_{D3}\,dW_{3,t}^Q \tag{A10}$$

therefore, it is a valid change of measure. Using Girsanov's Theorem (see Duffie, 2001, and Shreve, 2004), we obtain that:

$$E_0[t]=D_0\mathbb{E}^R\big[Max[K_t-1,0]\big] \tag{A11}$$

where the new probability measure R is defined by:

$$dW_{1,t}^{R} = -\sigma_{D1}\, dt + dW_{1,t}^{Q}, \quad dW_{2,t}^{R} = -\sigma_{D2}\, dt + dW_{2,t}^{Q},$$

$$dW_{3,t}^{R} = -\sigma_{D3}\, dt + dW_{3,t}^{Q} \quad \text{and} \quad dW_{4,t}^{R} = dW_{4,t}^{Q} \tag{A12}$$

To solve for the expectation under *R*, we first need to determine the dynamics of the inverse leverage ratio under this probability measure:

$$dK_t = (\sigma_{A1} - \sigma_{D1})\, dW_{1,t}^{R} + (\sigma_{A2} - \sigma_{D2})\, dW_{2,t}^{R} + (\sigma_{A3} - \sigma_{D3})\, dW_{3,t}^{R} + \sigma_{A4}\, dW_{4,t}^{R} \tag{A13}$$

or

$$dK_t = \sigma_K\, dW_{K,t}^{R} \tag{A14}$$

with $\sigma_K = \sqrt{(\sigma_{A1} - \sigma_{D1})^2 + (\sigma_{A2} - \sigma_{D2})^2 + (\sigma_{A3} - \sigma_{D3})^2 + \sigma_{A4}{}^2}$.

By analogy to the Black-Scholes-Merton formula, we can obtain that:

$$E_0[t] = D_0(K_0 N[d_1] - N[d_2]) \tag{A15}$$

with $d_2 = \frac{Log[K_0] - \frac{\sigma_K{}^2}{2} t}{\sigma_K \sqrt{t}}$ and $d_1 = d_2 + \sigma_K \sqrt{t}$. To obtain the credit risk adjustment factor we replace the equity on equation (20) and obtain $B_0[t]$.

A.3 Proof of Proposition 4

The probability that the inverse of the leverage ratio is less than a given value x at time t is defined as:

$$Prob_0[t, x] = \mathbb{E}^P\left[\mathbb{1}_{\{K_t < x\}}\right] = \mathbb{E}^P\left[\mathbb{1}_{\{Log[K_t] < Log[x]\}}\right] \tag{A16}$$

Using Ito's lemma and the dynamics of the inverse leverage from the previous proposition, we obtain the following stochastic differential equation (SDE):

$$dLog[K_t] = dLog\left[\frac{A_t}{D_t}\right] = \left(\mu_K - \frac{1}{2}\sigma_K{}^2\right) dt + \sigma_K\, dW_{K,t}^{P} \tag{A17}$$

with $\mu_K = \lambda_A - \lambda_D + \sigma_{D1}(\sigma_{D1} - \sigma_{A1}) + \sigma_{D2}(\sigma_{D2} - \sigma_{A2}) + \sigma_{D3}(\sigma_{D3} - \sigma_{A3})$.

Solving the SDE for $\mathrm{Log}[K_t]$ and replacing this variable in the probability above yields:

$$\begin{aligned} Prob_0[t,x] &= \mathbb{E}^P\left[\mathbb{1}_{\left\{Log[K_0]+\left(\mu_K-\frac{1}{2}\sigma_K{}^2\right)t+\sigma_K W_{K,t}^P<Log[x]\right\}}\right] \\ &= \mathbb{E}^P\left[\mathbb{1}_{\left\{\frac{Log\left[\frac{K_0}{x}\right]+\left(\mu_K-\frac{1}{2}\sigma_K{}^2\right)t}{\sigma_K\sqrt{t}}<-\frac{W_{K,t}^P}{\sqrt{t}}\right\}}\right] \end{aligned} \tag{A18}$$

where $-\frac{W_{K,t}^P}{\sqrt{t}}$ is a standard Normal random variable. The probability of default is $\mathrm{Prob}_0[t,1]$.

APPENDIX B – ESTIMATION OF THE MARKET VALUE OF MORTGAGES AND GENERATION OF A CONSISTENT TIME SERIES

Table 5 shows that about 12 percent of AIC assets are invested in mortgages. In order to assign a market value to them, we try to establish a statistical relationship between the effective interest rates charged for new mortgages each month with market interest rates, in addition to loan term, amount lent and other control variables. The database consists of every mortgage operation intermediated by agencies supervised by the SVS, therefore excluding mortgages originated by bank-related agencies. In any case, given that this is a competitive market, we expect our sample to be representative.

Given the significant prepayment levels observed between 200 and 2005 and also given that our purpose is to estimate the mortgage portfolio's market value ant the end of 2009 (and 2008) the sample used covers the period 2007-2009.

The market reference interest rates correspond to those of traded mortgage bonds (Letras Hipotecarias) originated and backed by a State-owned bank called BancoEstado. These instruments have experienced moderate prepayment as compared with other mortgage bonds. The corresponding interest rates are lagged one period (REF_TM1) in order to consider the delays between deal closings and registration. To "synchronize" mortgage interest rates and market rates, the relevant question is what interest rates are being set in the recent deals at a given point in time, which will be published as the mortgage rates in the future. Given a statistical relationship with lagged market interest rates, plus the effects of term, amount lent, collateral, payments to income and other effects, we can use the latest market interest rate as an input in the equations which determine the relevant interest rates to value the mortgages.

Estimation procedure

We considered several estimation procedures to finally settle for the following relatively simple formulation:

(1)

$$\mathrm{LOG(EFF\_INTRATE)} = a_1\mathrm{LOG(REF\_TM1)} + a_2\mathrm{TERM} + a_3\mathrm{TERM}^{\wedge}2 + a_4\mathrm{D1\_AMNT} + a_5\mathrm{D2\_\ AMNT} + a_6\mathrm{D3\_\ AMNT} + a_7(\mathrm{D4\_\ AMNT} + \mathrm{D5\_\ AMNT}) + a_8\ \mathrm{DEBT\_COLLAT} + a_9\mathrm{PROP\_INCOME}$$

We use logarithms because the distribution of interest rates has a long right tail which is empirically similar to a log-Normal distribution. This formulation also allows for a non-linear relationship between interest rate levels and term to maturity (measured in years) allowing for curvature in the relevant segment. We expect interest rates to be a concave function of the term to maturity.

Given extreme values in the mortgage amounts, and the effect that these extreme values have on the other estimated parameters, we use dummies representing segments:

D1 = 1 if amount <= 500 UF

D2 = 1 if 500 < amount <= 1000

D3 = 1 if 1000 < amount <= 5000

D4 = 1 if 5000 < amount <= 10000

D5 = 1 if 10000 < amount <= 30000

D6 = 1 if amount > 30000

In the latest sample period there are no loans greater than 30,000 UF and in equation 1 we add together D4 and D5.

DEBT_COLLAT corresponds to the Loan/Collateral ratio when the mortgage is issued. PROP_INCOME corresponds to the payment/income ratio. In both cases we expect a positive relationship with the mortgage interest rate.

There are two kinds of mortgages, named “fines generales” (general purposes) and “vivienda” (housing), but there were no significant differences. In addition, we searched for differences between mortgage loans to individuals and to legal entities. We present our results separately.

Table B.I, Panel A, presents our results considering only individuals. We obtain all the expected signs. Mortgage interest rates are nearly proportional to the reference rate, with a (n) (exponential) coefficient not significantly different from 1. Mortgage loan rates are increasing and concave in term to maturity ($a_2>0$ and $a_3<0$).

We also find that (monotonically), larger loans have lower interest rates, since $a_4 > a_5 > a_6 > a_7$.

Finally, we find that the variables associated with credit quality also turn out to be significant and have the correct sign.

Comparing these results to those obtained when we also include legal entities. The number of observations increases in 345 (from 21175 to 21520). The goodness of fit marginally worsens but the coefficients keep their orders of magnitude. This justifies using a single equation to determine the interest rate levels used for valuing mortgages for individuals and legal entities.

Out of sample checks

To check the estimation’s robustness, we verified the out-of-sample explanatory power of the equation presented in Table I.A using the rest of the explanatory variables, for the period January-October 2010. This conditional prediction is compared with the observed mortgage interest rates. Table II presents these results. The out-of-sample performance is significant (with an out-of-sample R^2 between 57% and 59% for 3,000 observations), but we cannot discard a systematic bias in the conditional prediction. The fitted line of observed interest rates against predicted rates should have a slope equal to one and a constant equal to zero. The constant is significantly positive and the slope statistically significantly smaller than one.

Therefore, even though our results may be precise enough to value about 12 percent of total assets, it may be necessary to check in the future the causes for these possible biases.

Valuation

To price the mortgages we use the results presented in Table B.II. We use the characteristics of each mortgage in each portfolio, with the following adjustments:

MARKET REFERENCE RATE: is the interest rate at the end of the month when the mortgages are valued (recall that the equation uses a lagged market interest rate reference).

TERM: residual term of the loan.

AMOUNT: amount outstanding according to the loan's interest rate (which we call the par value).

DEBT/COLLATERAL: fraction of the total loan outstanding as a proportion of the initial loan, multiplied by the initial Debt/Collateral ratio when the loan was originated.

PAYMENT/INCOME: we use the same ratio reported when the loan was originated.

PREPAYMENT: to consider the possible effect of future debt prepayments, we used market information for mortgage bonds (which are prepayable). We could establish that 90% of these bonds are never traded above 110% of its par value. So we set an upper limit of 110% of par value for each mortgage.

DEFAULT: we follow the same criteria of the SVS.

MORTGAGES ORIGINATED BY BANKS: in this case we only have the characteristic of the loan and not of the debtor. We don't have the information regarding collateral or the debtor's income. We assume that these characteristics are equal o the median of the other mortgages for which we do have information.

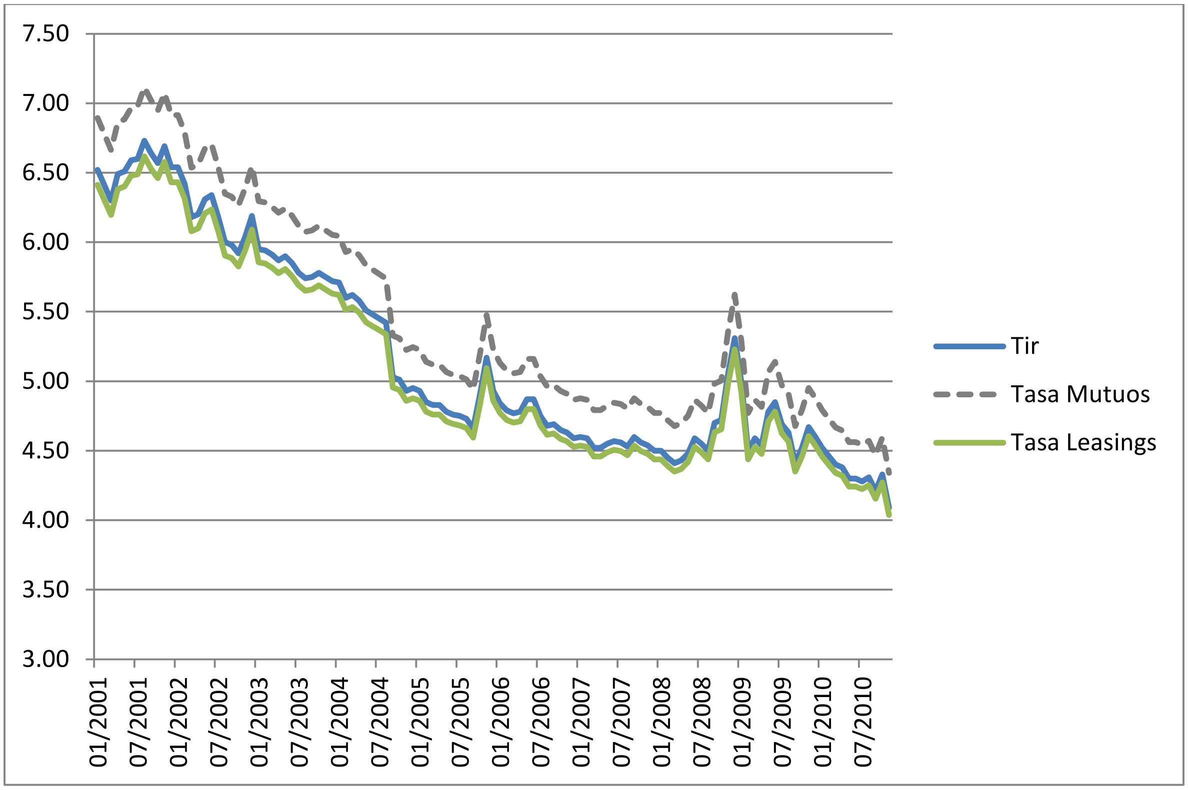
Generation of a consistent time series

To estimate the mortgages' variances and covariances with other asset classes, we need a "market interest rate" for these instruments. For this purpose we took the results presented in Table B.II below, replacing in the equation the median Debt/Collateral and Income/payment ratios, assume that it corresponds to a 20-year mortgage. Results are illustrated in Figure B.1.

Para estimar las varianzas de los mutuos y sus covarianzas con otras clases de activos, es necesario obtener una tasa de interés "de mercado" para éstos sincronizada con las del resto. Para obtener una serie de tiempo para los mutuos se tomaron los resultados del Cuadro II, la tasa de interés de referencia (sin rezago) y un mutuo a 20 años, con relación deuda/garantía y dividendo/renta igual a la mediana de los mutuos otorgados. El resultado es una serie de tasas de interés casi paralelo por encima de la tasa de interés de referencia (véase gráfico AI.2.1). Con esta serie de tasas de interés se construye un índice de riqueza y se pueden estimar los retornos mensuales.

Figure B.1

Estimated Time-Series for Mortgages (Mutuos) and Leasing Contracts Interest Rates



B.I In-Sample results: 2007-2009

A. Only individuals

Dependent Variable: LOG(EFF_INTRATE)
Method: Least Squares
Included observations: 21175
White heteroskedasticity-consistent standard errors & covariance
LOG(EFF_INTRATE) = a_1LOG(REF_TM1)+ a_2TERM+ a_3TERM^2+ a_4D1_AMNT+ a_5D2_AMNT+ a_6D3_AMNT + a_7(D4_AMNT+D5_AMNT)+ a_8 DEBT_COLLAT + a_9PROP_INCOME

	Coefficient	Std. Error	t-Statistic	Prob.
a_1	1.011417	0.027957	36.17775	0.0000
a_2	0.008119	0.000953	8.517036	0.0000
a_3	-0.000155	2.13E-05	-7.280410	0.0000
a_4	0.211163	0.043485	4.855975	0.0000
a_5	0.034044	0.043459	0.783351	0.4334
a_6	-0.118302	0.043454	-2.722468	0.0065
a_7	-0.188317	0.043486	-4.330565	0.0000
a_8	0.068714	0.006049	11.36025	0.0000
a_9	0.050486	0.011487	4.395056	0.0000

R-squared	0.461156	Mean dependent var	1.619949
Adjusted R-squared	0.460952	S.D. dependent var	0.146642
S.E. of regression	0.107664	Akaike info criterion	-1.619177
Sum squared resid	245.3467	Schwarz criterion	-1.615794
Log likelihood	17152.04	Hannan-Quinn criter.	-1.618073
Durbin-Watson stat	1.301536		

B. Individuals and legal entities

Dependent Variable: LOG(EFF_INTRATE)
Method: Least Squares
Included observations: 21520
White heteroskedasticity-consistent standard errors & covariance
LOG(EFF_INTRATE) = a_1LOG(REF_TM1)+ a_2TERM+
a_3TERM^2+ a_4D1_AMNT+ a_5D2_AMNT+ a_6D3_AMNT
+ a_7(D4_AMNT+D5_AMNT)+ a_8DEBT_COLLAT
+ a_9PROP_INCOME

	Coefficient	Std. Error	t-Statistic	Prob.
a_1	1.016002	0.014733	68.95888	0.0000
a_2	0.008714	0.000926	9.412435	0.0000
a_3	-0.000170	2.08E-05	-8.204085	0.0000
a_4	0.202246	0.021517	9.399383	0.0000
a_5	0.026314	0.021159	1.243619	0.2137
a_6	-0.125676	0.021053	-5.969491	0.0000
a_7	-0.185854	0.021117	-8.801201	0.0000
a_8	0.063829	0.006066	10.52293	0.0000
a_9	0.044113	0.011254	3.919748	0.0001

R-squared	0.452731	Mean dependent var	1.619663
Adjusted R-squared	0.452528	S.D. dependent var	0.146352
S.E. of regression	0.108288	Akaike info criterion	-1.607627
Sum squared resid	252.2442	Schwarz criterion	-1.604291
Log likelihood	17307.06	Hannan-Quinn criter.	-1.606539
Durbin-Watson stat	1.302125		

TABLE B.II Out-of-Sample Results (2010)

A. Only individuals

Dependent Variable: LOG(EFF_INTRATE)
Method: Least Squares
Sample: 83837 86821 IF NATURAL=1
Included observations: 2934

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(EFF_INTRATE_F)	0.954625	0.014799	64.50622	0.0000
C	0.125844	0.023671	5.316465	0.0000

R-squared	0.586638	Mean dependent var	1.648832
Adjusted R-squared	0.586497	S.D. dependent var	0.142752
S.E. of regression	0.091796	Akaike info criterion	-1.937822
Sum squared resid	24.70633	Schwarz criterion	-1.933743
Log likelihood	2844.785	Hannan-Quinn criter.	-1.936353
F-statistic	4161.053	Durbin-Watson stat	1.552178
Prob(F-statistic)	0.000000		

B. Individuals and legal entities

Dependent Variable: LOG(EFF_INTRATE)
Method: Least Squares
Sample (adjusted): 83837 86821
Included observations: 2985 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(EFF_INTRATE_F)	0.947208	0.015115	62.66649	0.0000
C	0.137952	0.024172	5.707121	0.0000

R-squared	0.568312	Mean dependent var	1.648922
Adjusted R-squared	0.568168	S.D. dependent var	0.142261
S.E. of regression	0.093485	Akaike info criterion	-1.901355
Sum squared resid	26.06995	Schwarz criterion	-1.897334
Log likelihood	2839.772	Hannan-Quinn criter.	-1.899908
F-statistic	3927.089	Durbin-Watson stat	1.587885
Prob(F-statistic)	0.000000		

APPENDIX C – VALUATION OF LEASING CONTRACTS[17]

As shown in Table 5, about 8% of total assets are invested in leasing contracts (LC). It is not evident what reference interest rates should be used in this case. Sin we found no significant differences between mortgages oriented to individuals and to legal entities, a reasonable alternative may be to use the equivalent mortgage interest rates. However, we must consider the following:

Collateral: Mortgages are issued with a debtor equity of 20 to 25%. This need not be the case in LCs.

Payment risk: the source of payments in the case of mortgages is the debtors labor income. In the case of LCs, it corresponds to enterprise risk.

In any case, it is not clear that human capital is riskier than firm risk. Still, we use similar rates for LCs than for contracts, but in order to consider possible differences in terms of collateral we looked at information about mortgages which is presented in Tables C.I.

A reasonable upper limit for debt/collateral ratio in the case of mortgages is 80%, since it represents between 96% and 99% of all cases.

Looking at the ratio Payment/Income, we observe that for nearly the entire sample the ratio is less than 30%. We thus take these parameters (80% and 30%, respectively) to determine the corresponding interest rate. This implies that, except for the credit size, LCs should have higher interest rates than mortgages. But in Figure B.1 we observe that LCs are larger implying that their interest rates are lower.

[17] We thank the suggestions of Jorge Mastrangelo in this point.

TABLE C.I

A. COLLATERAL

Descriptive Statistics for DEBT_COLLATERAL
Categorized by values of DEBT_COLLATERAL
Sample: 1 86821
Included observations: 86821

DEBT_COLLATERAL	Mean	Std. Dev.	Obs.
[0, 0.1)	0.076753	0.027528	673
[0.1, 0.2)	0.147204	0.030652	568
[0.2, 0.3)	0.259014	0.027833	1161
[0.3, 0.4)	0.355916	0.028208	2563
[0.4, 0.5)	0.458330	0.029076	5424
[0.5, 0.6)	0.554969	0.029231	9964
[0.6, 0.7)	0.656475	0.029108	18646
[0.7, 0.8)	0.762947	0.030460	44522
[0.8, 0.9)	0.800691	0.005939	3288
[0.9, 1)	0.954910	0.030243	10
[1, 1.1)	1.000000	0.000000	2
All	0.670537	0.141381	86821

B. PAYMENT / INCOME

Descriptive Statistics for PROP_INCOME
Categorized by values of PROP_INCOME
Sample: 1 86821
Included observations: 86821

PROP_INCOME	Mean	Std. Dev.	Obs.
[0, 0.1)	0.068965	0.023540	7738
[0.1, 0.2)	0.157422	0.027316	39714
[0.2, 0.3)	0.232671	0.019780	37650
[0.3, 0.4)	0.334881	0.027072	1194
[0.4, 0.5)	0.438773	0.027875	312
[0.5, 0.6)	0.542800	0.032144	115
[0.6, 0.7)	0.646873	0.031841	45
[0.7, 0.8)	0.744392	0.027235	24
[0.8, 0.9)	0.851121	0.025799	19
[0.9, 1)	0.929810	0.018369	10
All	0.186788	0.064604	86821