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Do Estimated Taylor Rules Suffer from Weak Identification?

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Abstract

Over the last decade, applied researchers have estimated forward looking Taylor rules with interest rate smoothing via Nonlinear Least Squares. A common empirical finding for post-Volcker samples, based on asymptotic theory, is that the Federal Reserve adheres to the Taylor Principle. We explore the possibility of weak identification and spurious inference in estimated Taylor rule regressions with interest rate smoothing. We argue that the presence of smoothing subjects the parameters of interest to the Zero Information Limit Condition analyzed by Nelson and Startz (2007, *Journal of Econometrics*). We demonstrate that confidence intervals based on standard methods such as the delta method can have severe coverage problems when interest rate smoothing is persistent. We then demonstrate that alternative methodologies such as Fieller (1940, 1954), Krinsky and Robb (1986), and the Anderson-Rubin (1949) test have better finite sample coverage. We reconsider the results of four recent empirical studies and show that the evidence supporting the Taylor Principle can be reversed over half of the time.

KEYWORDS: Interest Rate Smoothing; Nonlinear Least Squares; Spurious Inference; Zero-Information-Limit-Condition.

JEL CLASIFICATIONS: C12, C22, E52.

1. Introduction

In a recent paper, Nelson and Startz (2007) document that a class of models, used widely in empirical work, suffer from weak identification. This occurs when a particular parameter, not necessarily the parameter of interest, approaches a critical point. They refer to this as the Zero Information Limit Condition (ZILC). These models include Nonlinear regression, IV with weak instruments, ARMA models, etc. They document that for models in which the ZILC holds, asymptotic theory is a poor approximation to the finite sample distribution of the parameter of interest. Standard errors of this parameter are underestimated, and the size of the *t*-test depends on the properties of the underlying DGP.

Over the last decade, applied researchers have estimated forward looking Taylor rules with interest rate smoothing via Nonlinear Least Squares (NLS) using real time data. This was a break from tradition when ex-post data were used, necessitating instrumental variables and estimation by GMM; *e.g.* Clarida, Galí and Gertler (2000). Using the delta method, which is an asymptotic approximation, a common finding from NLS estimation is that the confidence interval for the response of the Federal Reserve to changes in expected inflation lies entirely above unity, supporting the empirical conclusion that the central bank follows the Taylor Principle. We show in this paper that this nonlinear monetary policy regression model falls into the same framework as that discussed in Nelson and Startz (2007), suggesting that the parameter estimates, in particular the monetary response to changes in expected inflation in Taylor rules, may suffer from weak identification. In this case, confidence intervals based on the delta-method will not have correct finite sample coverage.

In this paper, we demonstrate that the parameters of interest in estimated Taylor rules are in fact subject to the ZILC. That is, the NLS estimators of the response to changes in expected inflation and the business cycle contain no information about the true parameter values as the degree of persistence of interest rate smoothing increases. Asymptotic theory is a poor approximation to the actual finite sample distribution of the parameter estimates, and the coverage probability of confidence intervals is too small for the empirically relevant range of interest rate smoothing and observed sample sizes.

We reconsider four recent empirical studies using methods which generate more accurate confidence intervals, and demonstrate that over one half of the results supporting the finding that the Taylor Principle holds in various post-Volcker subsamples is reversed.

2. A Nonlinear Taylor Rule Regression Model which is subject to the Zero Information Limit Condition

2.1 Nelson and Startz (2007 JoE)

Nelson and Startz (2007) consider the following nonlinear regression model:

$$y_t = \gamma(x_t + \beta z_t) + \varepsilon_t , \tag{2.1}$$

where β is the parameter of interest. This model is identified if $\gamma \neq 0$ and Nelson and Startz focus on the behavior of $\hat{\beta}_{NLS}$ as γ approaches the Zero Information Limit Condition (ZILC), in this case zero. They show that when the regression errors are Normal, γ controls the amount of information about β that is contained in the data for a given sample size. In particular, the asymptotic variance of $\hat{\beta}_{NLS}$ is proportional to γ^{-2}. They demonstrate that as γ approaches zero, information contained in $\hat{\beta}_{NLS}$ goes to zero, and its variance diverges. In the special case where x_t and z_t are standardized to have unit variance, and correlation ρ_{xz}, the asymptotic variance of $\hat{\beta}_{NLS}$ is given by the following expression:

$$\mathrm{var}(\hat{\beta}_{NLS}) = \left(\frac{\sigma^2}{T}\right)\left(\frac{1}{\gamma^2}\right)\frac{1+2\beta\rho_{xz}+\beta^2}{(1-\rho_{xz}^2)} . \tag{2.2}$$

In the limiting case that $\gamma = 0$ the variance of $\hat{\beta}$ becomes infinite. Nelson and Startz also show through a series of Monte Carlo simulations that the standard error of $\hat{\beta}_{NLS}$ is understated relative to the asymptotic formula. However, *t*-tests for β may be undersized or oversized, depending on the DGP, which for the model above boils down to correlation between x_t and z_t.

2.2A Simple Taylor Rule Model with Interest Rate Smoothing

In order to determine whether estimated Taylor rules suffer from weak identification, we extend the nonlinear model of Nelson and Startz to resemble a Taylor rule regression with interest rate smoothing. For simplicity, we consider the case in which the Fed focuses only on expected inflation while ignoring business cycle considerations. We can think of such a model as follows:

$$r_t = (1-\gamma) r_{t-1} + \gamma\beta z_t + \varepsilon_t \,. \tag{2.3}$$

In this case, the dependent variable is the Federal Funds Rate (FFR), z_t is expected inflation, β is the response to changes in expected inflation, and $(1-\gamma)$ is the degree of interest rate smoothing, which corresponds to ρ in our empirical parameterizations of Taylor rules. It is easily seen that as the degree of smoothing increases, γ approaches zero, and the Taylor rule coefficient of interest becomes weakly identified.

Some differences between the models are worth noting. γ appears twice on the right hand side of the regression and we have lagged r_t as a regressor, both due to smoothing. Unlike the model of Nelson and Startz, we do not need x_t to identify γ, so it has been dropped to keep the model tractable. Dropping x_t leaves only one right hand side correlation to consider; that between z_t and r_{t-1}.

With Normal errors, it can be shown that the Information matrix for $\hat{\beta}$ and $\hat{\gamma}$ is equal to:

$$I\left(\hat{\beta},\hat{\gamma}\right)=\begin{bmatrix} \dfrac{\gamma^2}{\sigma^2}Tm_{zz} & \left(-\dfrac{\gamma}{\sigma^2}Tm_{rz}+\dfrac{\gamma\beta}{\sigma^2}Tm_{zz}\right) \\ \left(-\dfrac{\gamma}{\sigma^2}Tm_{rz}+\dfrac{\gamma\beta}{\sigma^2}Tm_{zz}\right) & \left(\dfrac{1}{\sigma^2}Tm_{rr}+\dfrac{\beta^2}{\sigma^2}Tm_{zz}-\dfrac{2\beta}{\sigma^2}Tm_{rz}\right) \end{bmatrix}, \tag{2.4}$$

with determinant $\Delta = \dfrac{\gamma^2}{\sigma^4}T^2\left(m_{zz}m_{rr} - m_{rz}^2\right)$. m_{zz} is the 2nd sample moment of z_t, etc. This implies the following asymptotic variance for $\hat{\beta}_{NLS}$:

$$\text{var}(\hat{\beta}_{NLS}) = \left(\frac{\sigma^2}{T}\right)\left(\frac{1}{\gamma^2}\right)\frac{m_{rr} - 2\beta m_{rz} + \beta^2 m_{zz}}{(m_{rr}m_{zz} - m_{rz}^2)} \,. \tag{2.5}$$

This clearly demonstrates that as γ approaches zero, $\hat{\beta}_{NLS}$ contains no information about the true value of β, suggesting that estimated Taylor rules may be subject to spurious inference.

2.3A More General Taylor Rule Model with Interest Rate Smoothing

Most estimated Taylor rules consider some sort of business cycle activity in addition to expected inflation, typically the output gap. We extend equation (2.3) to allow for this as follows:

$$r_t = (1-\gamma)r_{t-1} + \gamma[\beta z_t + \omega\tilde{y}_t] + \varepsilon_t \,, \tag{2.6}$$

where $\tilde{y}_t$ is the deviation of output from its trend or potential, and ω is the change in the FFR when output deviates from its target. In this framework, it is possibly more transparent that as interest rate smoothing increases, the terms we care about in the squared bracket are being multiplied by a smaller number, suggesting that the ZILC may cause problems for inference on β and ω that is based on asymptotic theory.

3. A Review of the Literature on the Delta Method for the ratio of Coefficients

The problem of computing confidence intervals for ratios of parameters has a long tradition in economics, being particularly important within the literature estimating elasticities and/or long-run multipliers. In these setups, the well-known delta Wald-type method constitutes a general procedure to approximate the standard error of a nonlinear combination of estimates based on a first-order Taylor series expansion. However, this methodology is only valid asymptotically, provided that the transformation function is differentiable, with nonzero and bounded derivatives.

To fix ideas, consider the following first-order dynamic regression model:

$$y_t = \rho y_{t-1} + \theta_0 + \theta_1 x_t + \theta_2 z_t + \varepsilon_t \,, \tag{3.1}$$

where the long-run elasticity, or long-run multiplier depending on the context at hand, is defined as the ratio of the coefficient on a regressor to one minus the coefficient on the lagged dependent variable.

Within this framework, a $100 \cdot (1-\alpha)\%$ confidence interval for the ratio is given by the following expression:

$$DCI(\alpha) = \left[\frac{\hat{\theta}_j}{(1-\hat{\rho})} \pm z_{a/2} \left(\hat{G}' \hat{\Sigma} \hat{G} \right)^{1/2} \right]$$

$$\hat{G} = \left[\frac{1}{(1-\hat{\rho})}, \quad \frac{\hat{\theta}_j}{(1-\hat{\rho})^2} \right]'$$

where $z_{a/2}$ is the normal two-tailed α -level cutoff point and $\hat{\Sigma}$ is the estimated variance-covariance matrix associated with $(\hat{\theta}_j, \hat{\rho})$.

Notice that this transformation of the parameter vector becomes problematic (i.e., unbounded) as ρ approaches unity. In other words, this ratio is weakly identified over a subset of the parameter space, and as shown by Dufour (1997), standard procedures that are bounded by construction can have zero coverage probability. Therefore, this provides a clear connection between the ZILC of Nelson and Startz (2007) and inappropriateness of the delta-method that motivates our investigation.

A number of recent studies have compared the relative performance of alternative methods, such as the bootstrap procedure of Krinsky and Robb (1986) and the modification to Fieller's (1940,1954) original approach, and have concluded that the latter performs remarkably well in a variety of settings – see for example Hirschberg et al. (2008), Bolduc et al (2010), and Bernard et al. (2007). Like the delta method, Fieller's method also relies on asymptotic theory but confidence intervals are computed by inverting a test that does not require identifying the ratio, and are neither symmetric nor bounded.

We also consider the Anderson-Rubin (1949) test that has been predominately used in the weak instrument literature, but is also suitable for models that are weakly identified without endogenous regressors.

In the next section we describe the four alternative methodologies in detail, and in section 5 we conduct a series of Monte Carlo experiments to assess their relative performance within the context of a dynamic regression model that resembles a Taylor rule with interest rate smoothing.

4. Methods of Computing Confidence Intervals for the Ratio of Parameter Estimates

In this section, we discuss four methods of computing confidence intervals for the ratio of coefficients.

4.1 The Delta Method

The basic Taylor rule equation to be estimated is:

$$r_t = \rho r_{t-1} + (1-\rho)[\mu + \beta E_t \pi_{t+h} + \omega \hat{y}_t] + \varepsilon_t , \qquad (4.1)$$

where the Taylor Principle is satisfied if $\beta > 1$. There are two equivalent ways of testing a hypothesis about the value of β. One is to estimate Equation (4.1) by NLS, and compute:

$$\hat{\beta}_{NLS} \pm 1.96 asyse(\hat{\beta}_{NLS}) .$$

This can be done quite easily using canned software packages. Asymptotically, this confidence interval has 95% coverage, and a *t*-test based on the same output has asymptotic size of 5%. Equivalently, one can linearize Equation (4.1) as follows:

$$r_t = \rho r_{t-1} + \theta_0 + \theta_1 E_t \pi_{t+h} + \theta_2 \hat{y}_t + \varepsilon_t , \qquad (4.2)$$

where $\hat{\beta}_{OLS}$ can be computed as $\frac{\hat{\theta}_{1,OLS}}{1-\hat{\rho}_{OLS}}$. Since this is the ratio of OLS estimates, the delta method can be used to construct the asymptotic variance of $\hat{\beta}_{OLS}$. Both this method and NLS of Equation (4.1) give numerically equivalent results. Since $\beta = \frac{\theta_1}{1-\rho}$, the delta method is only valid when $\rho < 1$, which ensures that the derivatives of the transformation are bounded and continuously differentiable. We also note here that when $\rho = 1$, the ZILC holds.

4.2 Fieller

It is well known that Wald-type tests are not invariant to the formulation of the null hypothesis. An alternative formulation could be written as follows:

$$\theta_1 - \lambda_0(1-\rho) = 0,$$

where λ_0 is the value of the ratio of parameters under the null hypothesis. In order to compute a confidence interval, one could invert the corresponding *t*-statistic as follows:

$$\left| \frac{\hat{\theta}_1 - \lambda_0(1-\hat{\rho})}{asyv(\hat{\theta}_1) + 2(\lambda_0)asyc(\hat{\theta}_1, \hat{\rho}) + (\lambda_0)^2 asyv(\hat{\rho})} \right| \leq z_{\alpha/2},$$

which requires solving a quadratic inequality such that:

$$A(\lambda_0)^2 + 2B(\lambda_0) + C \leq 0,$$

where:

$$\begin{cases} A = (1-\hat{\rho}) - (z_{\alpha/2})^2 asyv(\hat{\rho}) \\ B = -\hat{\theta}_1(1-\hat{\rho}) - (z_{\alpha/2})^2 asyc(\hat{\theta}_1, \hat{\rho}) \\ C = \hat{\theta}_1^{\,2} - (z_{\alpha/2})^2 asyv(\hat{\theta}_1) \end{cases}$$

Following Bernard et al. (2007), it can be shown that if $A > 0$ the bounded solution is given by:

$$\left[\frac{-B-\sqrt{\Delta}}{A}, \frac{-B+\sqrt{\Delta}}{A} \right],$$

if and only if $\Delta \equiv B^2 - AC > 0$.

If the above condition is not met, the resulting $100 \cdot (1-\alpha)\%$ confidence interval will be unbounded since the interest rate smoothing parameter is close to unity. This was suggested by Dufour (1997) who pointed out that unbounded confidence intervals will occur often when the parameters of an econometric model are unidentified, or close to being so.

4.3 Krinsky and Robb

They propose a bootstrap procedure to compute a confidence interval for a ratio of parameters by sampling from their asymptotic distribution, then computing the ratio for each draw, and finally trimming the lower/upper 2.5% tails. The result is an approximation to an asymptotic 95% confidence interval, and this methodology has been widely used by empirical researchers in situations where the delta method is expected to fail.

4.4 Anderson-Rubin

The idea behind this approach is to test the null hypothesis indirectly, by testing the exclusion restrictions that are implied by it. In our setup, we start by imposing a null based on hypothesized values for all the parameters as follows:

$$r_t - \rho_0 r_{t-1} - (1-\rho_0)[\beta_0 E_t \pi_{t+h} + \omega_0 \ \hat{y}_t] = \varepsilon_{t,0} \ .$$

Note that the error term is observed under H_0. Therefore, the Anderson-Rubin test can be obtained as the usual F statistic on the exclusion of r_{t-1}, $E_t \pi_{t+h}$ and $\hat{y}_t$ in the auxiliary regression based on $\varepsilon_{t,0}$. We then invert this test to obtain a confidence set for the joint hypothesis, and then we project onto the axis for the response to the inflation coefficient. For this reason, in the empirical section we will refer to the Anderson-Rubin type procedure as projection-based confidence intervals.

5. Finite Sample Size of the Delta Method, Fieller, Krinsky and Robb, and AR

In this section, we compare the finite sample size of the t-statistic for the inflation response coefficient for the delta-method, Fieller's method, Krinsky and Robb, and Anderson and Rubin. In our simulations, we set the inflation coefficient to unity, so that the Taylor Principle fails to hold, and determine how often the estimated confidence interval contains the true value of unity. We generate artificial nominal interest rates based on parameter values from Orphanides (2004) using both *iid* and persistent inflation and output series. We then estimate the Taylor rule with interest rate smoothing by NLS.

We consider T = 25, 50, 100, 150, and 250. The first four sample sizes are empirically relevant, and we know of no empirical work with 250 observations. We consider the entire range for ρ, but will limit our discussion to values of ρ greater than 0.7 for two reasons. First, values of ρ less than 0.7 are far from the ZILC of unity, and as expected, all 4 methods work quite well in small samples that far away from the ZILC point. Second, values of ρ greater than 0.7 are empirically relevant aside from being close to the ZILC: the four empirical studies that we analyze have estimated values of ρ typically above 0.70 with two minor deviations from this, and as large as 0.95.

The delta-method rejection line is in blue, Fieller in red, and Krinksky and Robb in green, and Anderson Rubin in dashed grey. We plot the rejection frequencies only up to 0.25, so that their differences are more visually apparent, although we do note that for very small sample sizes, the delta-method often goes above 25%, while this is never true for the other three tests.

We begin with T = 25 and compare the rejection frequencies for both *iid* and persistent regressors in Figure 1. The top panel displays the results for non-persistent data and lower panel displays the results for persistent regressors. In both cases, the delta method is dominated by the other three methods. For *iid* data the delta method doubles its nominal size to 10% when the smoothing parameter is close to 0.85, whereas for persistent data it reaches 10% for $\rho = .80$. This distortion worsens when approaching the ZILC point of $\rho = 1$, reaching as high at 38% and 44%, respectively. The situation is much better for Fieller and Krinsky and Robb. Even for these small samples, these tests do not become doubly-sized until around $\rho = .95$, a smoothing coefficient which is higher than all of the empirical estimates we consider; although we know that there is a downward mean and median-bias in the NLS estimate of this coefficient.[1] It isn't until ρ approaches unity that the size distortion worsens, with

[1] We do not report the results here, but our simulations bear out our claim.

Krinsky and Robb never going over 22%. The Anderson-Rubin test appears overly conservative, with rejection frequencies close to 3%, up until 0.95 where actual size is restored. Beyond this point, size increases as a smoothing coefficient of unity is approached, but to a much smaller extent, 11%-13%, than the other three testing procedures.

Figures 2 and 3 depict size for T = 50 and 100, moderate but empirically relevant sample sizes. We see a similar pattern to a sample of 25 observations. The size distortion of the delta method is partially mitigated, as predicted by asymptotic theory, but it still reaches twice its nominal size around $\rho = .85$ and peaking around 18% or 30% at the ZILC point, depending on the sample size and persistence of the regressors, in that order. Fieller and Krinsky and Robb are much better behaved, only becoming oversized when ρ exceeds 0.95, although we do observe a slight divergence between these two tests for $\rho = .85$ and higher, with Krinsky and Robb being better sized in both settings. With respect to Anderson-Rubin, we continue to find that the test is conservatively sized, around 3%, for most of the range, with empirical size approaching nominal size fairly close to the ZILC point.

We finally consider T = 150 and 250, reported in Figures 4 and 5. These are interesting sample sizes, since one of our studies has 158 observations, and no empirical studies have 250. For the former, although the delta method continues to improve with the larger sample size, Krinsky and Robb is the clear winner, with a maximum size of around 7% when interest rate smoothing is high. Fieller is in the middle, and resembles the performance of the delta method when approaching $\rho = 1$. Anderson-Rubin continues to be conservative for most of the range of the smoothing parameter, although it achieves the most accurate empirical size closest to the ZILC point. When we increase the sample size to 250, Krinsky and Robb is again the clear winner, being almost correctly sized for the entire range of ρ. The Fieller test becoming oversized around the ZILC point continues, whereas Anderson and Rubin becomes almost entirely correctly sized as ρ approaches unity.

What emerges from this series of Monte Carlo simulations is the result that confidence intervals based on the delta method will be too narrow, given the false sense that they are informative, when in fact they are based on an oversized t-test. In contrast, the other three sets of confidence will necessarily be wider, as their actual size is closer to nominal size.

6. Confidence Intervals for Taylor Rule Coefficients with Better Finite Sample Coverage Properties

Given the simulation results in the previous Section, we now re-examine four empirical studies which estimate Taylor rules with interest rate smoothing and forward looking data, using nonlinear least squares, which again is equivalent to using the delta-method on the linearized version of the model. We compare the confidence intervals computed from the delta method, Fieller, Krinsky and Robb, and Anderson and Rubin.

6.1 Orphanides (2004 JMCB)

We start by re-examining Orphanides (2004). His was the first paper to estimate forward looking Taylor rules with real time data. This alleviated the endogeneity of the right hand side variables induced by using ex-post data, and eliminated the need for more complicated IV/GMM methods, allowing researchers to use NLS.

He estimates equations of the form[2]:

$$r_t = +\rho r_{t-1} + (1-\rho)[\mu + \beta E_t \pi_{t+h} + \omega \hat{y}_t] + \varepsilon_t , \qquad (6.1)$$

where r_t is the nominal Federal Funds rate, $E_t \pi_{t+h}$ is the forecast of inflation h horizons into the future, $\hat{y}_t$ is the estimated output gap, and ρ is the degree of interest rate smoothing.

Orphanides considers two samples, 1966:1-1979:2 and 1979:3-1995:4, corresponding the pre and post Volcker regimes. He also considers inflation forecasts from 1 to 4 quarters ahead. For these 8 regressions, the Taylor Principle is estimated to hold twice at the nominal 5% level, during the post Volcker regime for the 3 and 4 quarter ahead inflation forecasts. Of course, these findings of significance are based on the delta-method, which is only valid asymptotically, and the sample size in this case is

[2] In the published version of his paper, Orphanides uses AR(2) smoothing. We cannot replicate his results using his working paper data, but we will consider AR(2) smoothing in this paper with the same data.

quite small; $T = 66$. We consider whether this finding is robust to using methods which generate confidence intervals with more accurate coverage properties.

Table 1 reports the estimated interest smoothing coefficient and the response to expected inflation, along with the four sets of confidence intervals for each inflation forecast horizon. The estimated response to inflation is almost two for one. The first confidence interval column is the delta-method, which can be directly inferred from Orphanides' Table 1, by computing:

$$\hat{\beta} \pm 1.96 asyse(\hat{\beta}) .$$

The second column is Fieller, column 3 is Krinsky and Robb (KR), and the 4^{th} column is Anderson and Rubin.

For AR(1) smoothing, we see that while KR comes close to overturning Orphanides' result for a 3 period ahead inflation forecast, only Anderson and Rubin overturns the result for both inflation horizons. The published version of Orphanides' paper uses AR(2) smoothing. Using the data from his working paper, we also consider AR(2) smoothing. The results are reported in Table 2. For the 3 quarter ahead inflation forecast, all 4 confidence intervals have lower bounds less than unity, including the delta method, a result which is not reported in his paper. This completely reverses 1 of his 2 findings that the Taylor Principle held post-Volker. For the 4 quarter ahead inflation forecast, KR and Anderson-Rubin have a lower bound less than unity, consistent with an accommodative Fed.

6.2 Nikolsko-Rzhevskyy (2011 JMCB)

Nikolsko-Rzhevskyy (2011) estimates Taylor Rules with AR(1) interest rate smoothing from 1982:1-2007:1, 101 quarterly observations, for the "nowcast" of inflation, as well as for forecast horizons 1-6. His estimated inflation response coefficients are higher than those in Orphanides, ranging between 2 and almost 3. Based on the delta-method, for the seven regressions, he finds that the Taylor Principle holds for $h = 2 - 6$. In Table 3, we report his asymptotic confidence intervals for these horizons, as well as those based on Fieller, KR and Anderson-Rubin. For $h = 2 - 4$, all

3 alternatives to the delta method reverse the conclusion that $\beta > 1$. For $h = 5$, his finding only changes with Anderson-Rubin, while for $h = 6$ his conclusion is unchanged.

Nikolsko-Rzhevskyy also considers a shorter sample period, 1987:2 – 2007:1, containing 80 observations. He estimates a Taylor rule for the same inflation horizons with an output gap in one specification, plus output gap growth in the other. These are reported in Tables 4 and 5. For the non-output gap growth specification, he again finds that the delta method is consistent with the Taylor rule holding for $h = 3 - 6$. The Anderson Rubin test reverses these findings, although this is not the case for either Fieller or KR. For his results with output gap growth, all 4 methods are consistent with a Fed that fights expected inflation via raising the FFR.

We note here that the estimated interest rate smoothing coefficient is higher in all of Nikolsko-Rzhevskyy's estimated models than those of Orphanides. We also note that these estimates are higher in those specifications that allow for a response to output gap growth and so are the estimated responses to expected inflation. Thus, the results in Table 5 provide additional support to the idea that the Taylor Principle holds for post-Volker samples.

Nikolsko-Rzhevskyy (2011) uses Green book data through 2001:4, which was what was available at the time of his writing. With this in mind, we also estimate his model using Green book data only. This of course shortens the sample. The results are reported in Tables 6 – 8. The estimated inflation coefficients here are between 1.8 and 2.5. For the sample starting in 1982:1, based on the delta method, the Taylor Principle only holds for $h = 5 - 6$, but these results are overturned by Fieller, KR and Anderson-Rubin for $h = 5$ and KR and Anderson-Rubin for $h = 6$. For the samples starting in 1987:2, 2 of the 4 specfications with no response to output gap growth are now also overturned by KR and all by Anderson-Rubin. When a response to the output gap growth is allowed all horizons are consistent with a Fed that raises the FFR more than point-for-point with expected inflation.

6.3.1 Coibion and Gorodnichenko (2011 AER)

Coibion and Gorodnichenko (2011) estimate 6 Taylor rule regressions, using data up through 1979, and data after 1982. The data are at the FOMC frequency. Two specifications have significant estimates of β being larger than unity. The first is a forward looking Taylor rule with a 2 quarter ahead inflation forecast, forecasted output gap, and forecasted output growth as regressors. The second has a 2 quarter ahead inflation forecast, and the nowcast for output gap and output growth as regressors, called a mixed Taylor rule. Both specifications use AR(2) smoothing.

In Table 9 the estimated inflation coefficients are around 2.5. We report confidence intervals for each of the 2 specifications. For both the forward looking and mixed specifications, all the confidence intervals are consistent with the Taylor Principle holding, supporting Coibion and Gorodnichenko's finding that the Taylor Principle was followed post 1982.

6.3.2 Coibion and Gorodnichenko (2012 AEJ, Macro)

Our final empirical result estimates a Taylor rule with AR(1) smoothing, with and without output growth. The results are reported in Table 10. The estimated inflation coefficients are approximately 2. For both regressions, the other 3 confidence intervals do not overturn the delta method result that the Taylor rule held post 1987.

7. Conclusions

In this paper we demonstrate that forward looking Taylor rules with interest rate smoothing estimated by NLS are subject to a variety of problems related to the difference in the predictions based on asymptotic theory and the actual finite sample distributions of the parameters of interest. We use methods to construct confidence intervals for the Fed's change in its nominal interest rate target to changes in expected inflation that have better finite sample properties than the delta-method.

We reconsider 4 empirical studies which estimate Taylor Rule regressions via NLS, and find that about half of the failures to reject based on the delta-method are not robust to other testing methods which have better finite sample size properties. Specifically, many examples of apparently informative confidence intervals based on the delta-method widen when better procedures are used to become much less informative, in that they do not rule out that the Taylor Principle did not hold for a variety of samples and sample sizes.

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Figure 1 - Actual Size of a Nominal 5% Test, T=25

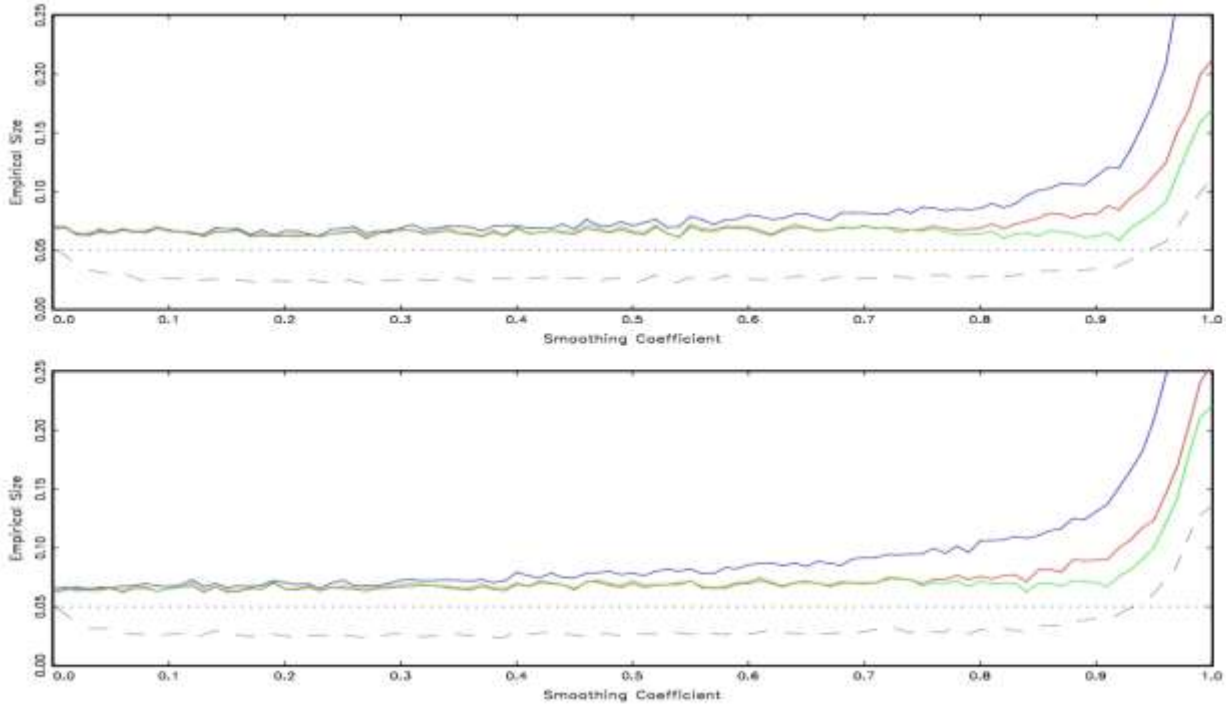

Figure 2 - Actual Size of a Nominal 5% Test, T=50

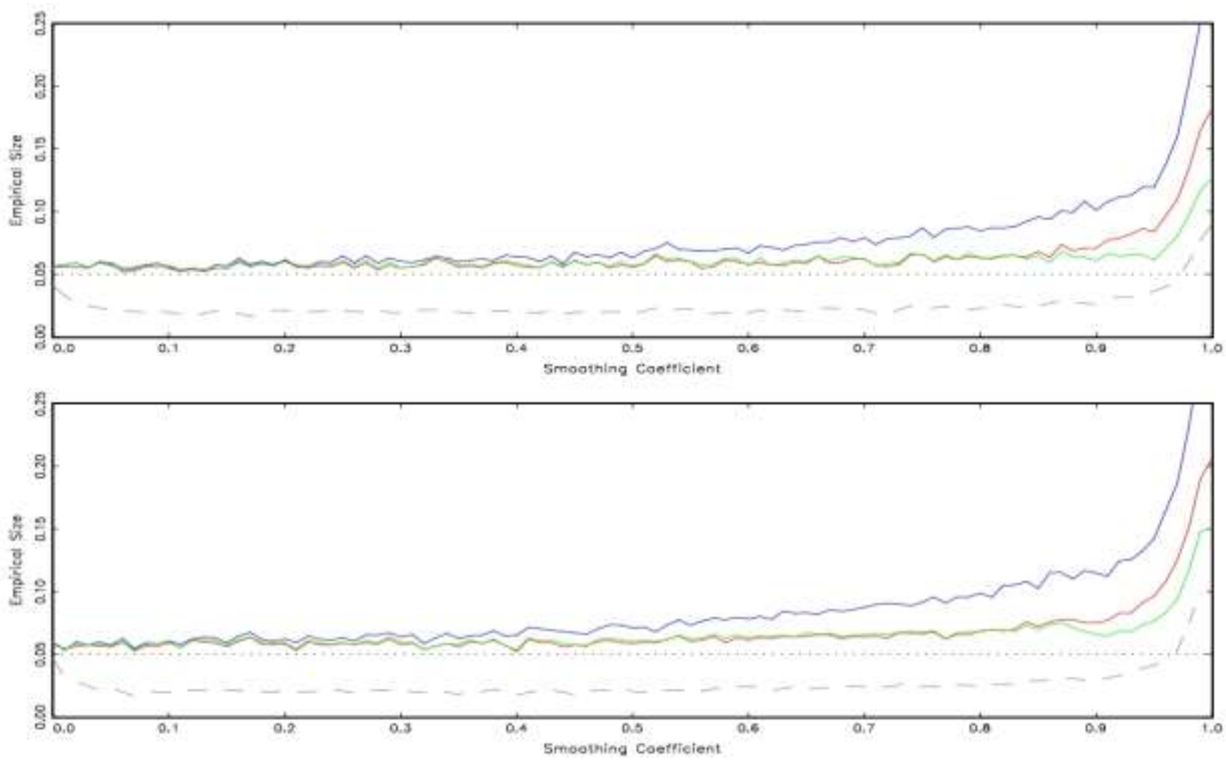

Figure 3 - Actual Size of a Nominal 5% Test, T=100

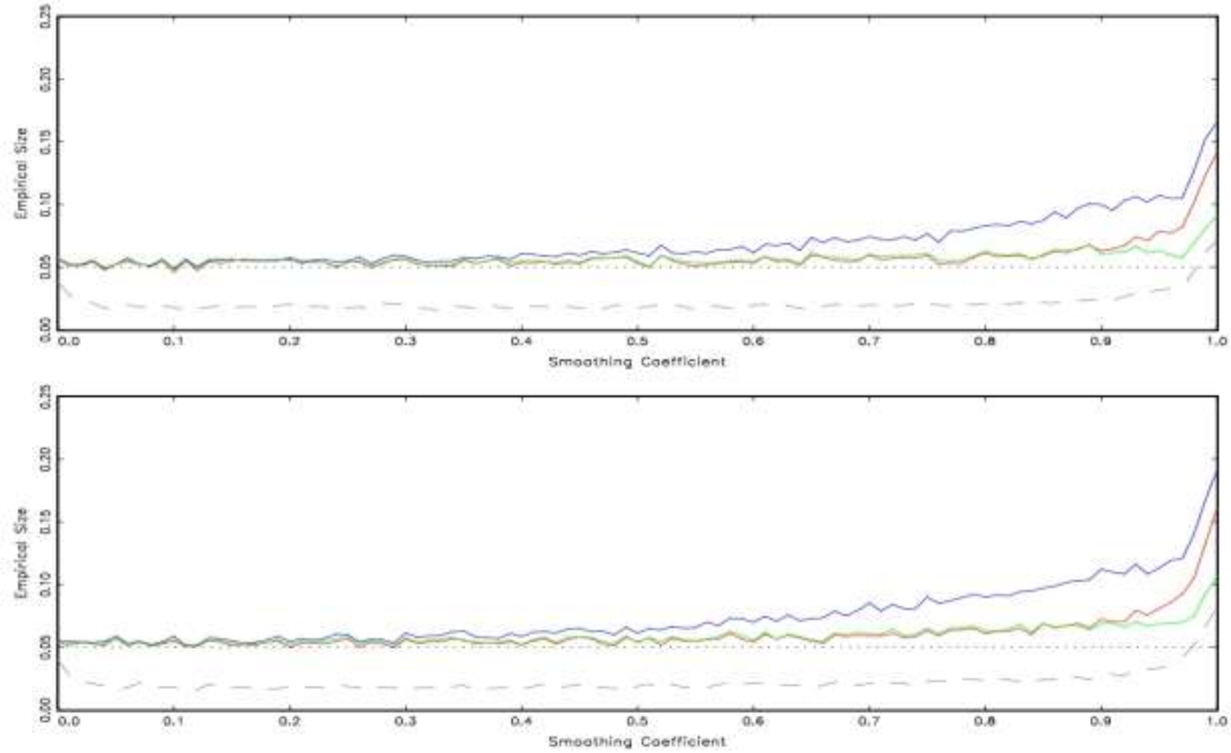

Figure 4 - Actual Size of a Nominal 5% Test, T=150

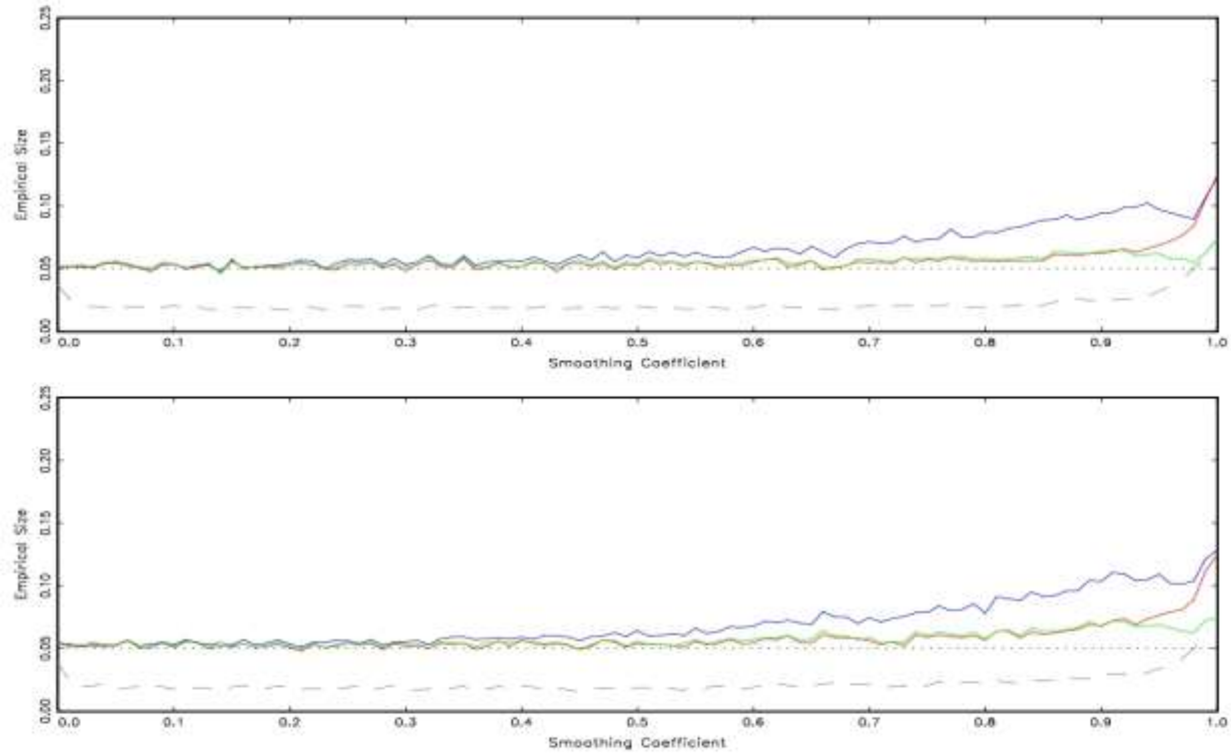

Figure 5 - Actual Size of a Nominal 5% Test, T=250

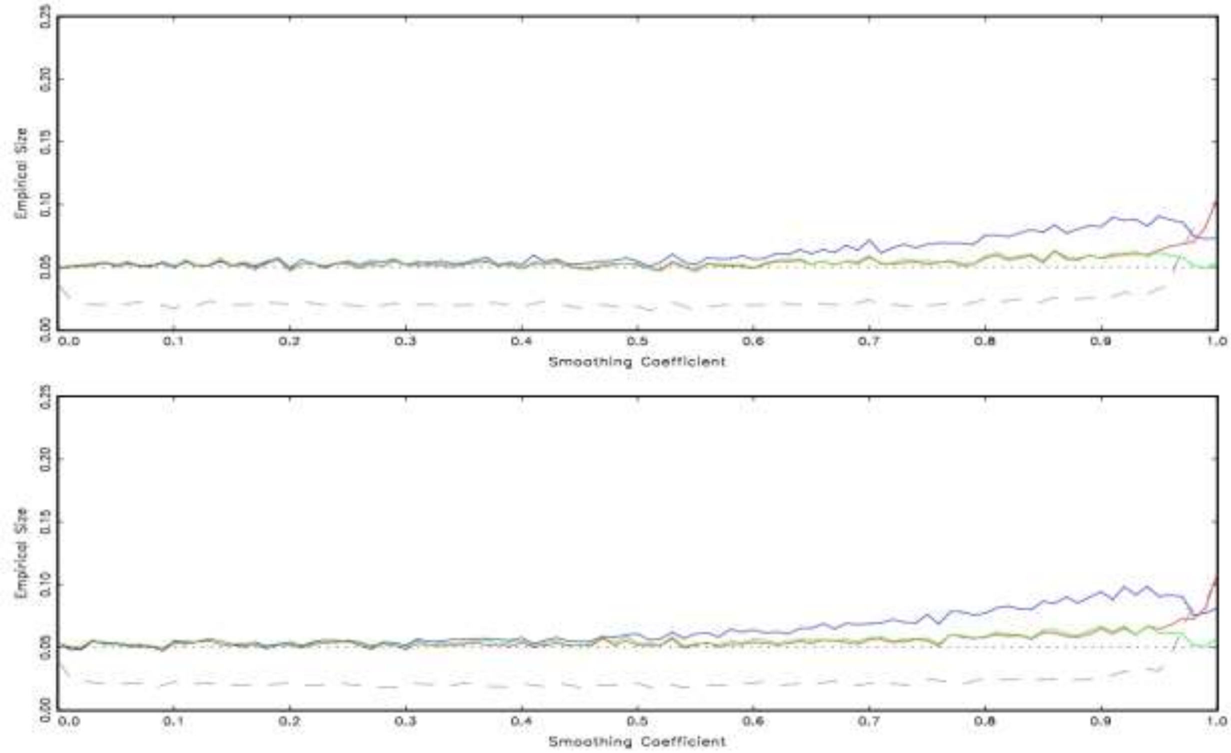

Table 1 - Orphanides (2004) / 95% Confidence Intervals

Sample: 1979:3-1995:4 (T = 66) - AR(1) smoothing

Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 3$	0.76	1.89	[1.09, 2.69]	[1.06, 2.85]	[1.01, 2.81]	$[\mathbf{0.79}, \mathbf{3.25}]$
$h = 4$	0.74	1.95	[1.18, 2.73]	[1.16, 2.86]	[1.12, 2.83]	$[\mathbf{0.91}, \mathbf{3.17}]$

Note: Newey-West HAC standard errors.

Table 2 - Orphanides (2004) / 95% Confidence Intervals

Sample: 1979:3-1995:4 (T = 66) - AR(2) smoothing

Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 3$	0.76	1.85	$[\mathbf{0.94}, \mathbf{2.76}]$	$[\mathbf{0.92}, \mathbf{2.92}]$	$[\mathbf{0.85}, \mathbf{2.88}]$	$[\mathbf{0.42}, \mathbf{3.43}]$
$h = 4$	0.74	1.91	[1.03, 2.79]	[1.03, 2.94]	$[\mathbf{0.96}, \mathbf{2.91}]$	$[\mathbf{0.38}, \mathbf{3.45}]$

Note: Newey-West HAC standard errors.

Table 3 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1982:1 – 2007:1 (T = 101) - AR(1) smoothing

Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 2$	0.85	2.44	$[1.20, 3.69]$	$\mathbf{[0.72, 4.40]}$	$\mathbf{[0.79, 4.23]}$	$\mathbf{[0.0^{*}, 5.89]}$
$h = 3$	0.85	2.58	$[1.19, 3.96]$	$\mathbf{[0.98, 4.43]}$	$\mathbf{[1.00, 4.32]}$	$\mathbf{[0.36, 5.43]}$
$h = 4$	0.84	2.65	$[1.34, 3.96]$	$\mathbf{[0.96, 4.35]}$	$\mathbf{[0.98, 4.25]}$	$\mathbf{[0.0^{*}, 5.41]}$
$h = 5$	0.82	2.84	$[1.69, 3.99]$	$[1.39, 4.34]$	$[1.38, 4.26]$	$\mathbf{[0.70, 5.05]}$
$h = 6$	0.83	2.72	$[1.50, 3.94]$	$[1.43, 4.24]$	$[1.39, 4.16]$	$[1.01, 4.92]$

Note: Newey-West HAC standard errors.

Table 4 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1987:2 – 2007:1 (T = 80) - AR(1) smoothing

Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 3$	0.82	2.06	$[1.07, 3.05]$	$[1.22, 3.27]$	$[1.08, 3.21]$	$\mathbf{[0.92, 3.60]}$
$h = 4$	0.80	2.10	$[1.15, 3.04]$	$[1.08, 3.13]$	$[1.05, 3.09]$	$\mathbf{[0.75, 3.49]}$
$h = 5$	0.80	2.08	$[1.18, 3.00]$	$[1.14, 3.12]$	$[1.08, 3.09]$	$\mathbf{[0.83, 3.49]}$
$h = 6$	0.80	2.16	$[1.20, 3.12]$	$[1.21, 3.27]$	$[1.15, 3.23]$	$\mathbf{[0.98, 3.67]}$

Note: Newey-West HAC standard errors.

Table 5 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1987:2 – 2007:1 (T = 80) - AR(1) smoothing

Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 3$	0.88	2.50	$[1.20, 3.80]$	$[1.60, 5.54]$	$[1.56, 5.14]$	$[1.44, 6.0^{*}]$
$h = 4$	0.87	2.49	$[1.39, 3.59]$	$[1.64, 4.63]$	$[1.57, 4.56]$	$[1.45, 6.0^{*}]$
$h = 5$	0.87	2.50	$[1.41, 3.58]$	$[1.65, 4.59]$	$[1.57, 4.53]$	$[1.45, 6.0^{*}]$
$h = 6$	0.87	2.56	$[1.39, 3.73]$	$[1.63, 4.69]$	$[1.55, 4.65]$	$[1.42, 6.0^{*}]$

Note: Newey-West HAC standard errors, with a response to output gap growth.

Table 6 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1982:1 – 2001:4 (T = 80) - AR(1) smoothing						
Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 5$	0.78	2.51	$[1.23, 3.78]$	$[\mathbf{0.90}, \mathbf{4.85}]$	$[\mathbf{0.84}, \mathbf{4.56}]$	$[\mathbf{0.0}^{*}, \mathbf{6.0}^{*}]$
$h = 6$	0.79	2.39	$[1.02, 3.76]$	$[1.01, 4.57]$	$[\mathbf{0.93}, \mathbf{4.53}]$	$[\mathbf{0.50}, \mathbf{6.0}^{*}]$

Note: Newey-West HAC standard errors.

Table 7 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1987:2 – 2001:4 (T = 59) - AR(1) smoothing						
Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 3$	0.68	1.79	$[1.03, 2.55]$	$[1.04, 2.87]$	$[\mathbf{0.97}, \mathbf{2.84}]$	$[\mathbf{0.77}, \mathbf{3.51}]$
$h = 4$	0.65	1.80	$[1.10, 2.49]$	$[1.01, 2.56]$	$[\mathbf{1.00}, \mathbf{2.53}]$	$[\mathbf{0.72}, \mathbf{2.85}]$
$h = 5$	0.67	1.79	$[1.08, 2.49]$	$[1.08, 2.71]$	$[1.02, 2.67]$	$[\mathbf{0.83}, \mathbf{3.14}]$
$h = 6$	0.67	1.82	$[1.09, 2.55]$	$[1.16, 2.84]$	$[1.12, 2.82]$	$[\mathbf{0.98}, \mathbf{3.31}]$

Note: Newey-West HAC standard errors.

Table 8 - Nikolsko-Rzhevskyy (2011) / 95% Confidence Intervals

Sample: 1987:2 – 2001:4 (T = 59) - AR(1) smoothing						
Horizon	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
$h = 3$	0.76	2.02	$[1.41, 2.63]$	$[1.57, 3.31]$	$[1.55, 3.23]$	$[1.47, 3.92]$
$h = 4$	0.74	2.00	$[1.50, 2.50]$	$[1.59, 2.82]$	$[1.55, 2.82]$	$[1.48, 3.40]$
$h = 5$	0.74	2.00	$[1.48, 2.52]$	$[1.58, 2.91]$	$[1.54, 2.90]$	$[1.47, 3.58]$
$h = 6$	0.76	2.04	$[1.43, 2.64]$	$[1.56, 3.11]$	$[1.51, 3.09]$	$[1.45, 3.83]$

Note: Newey-West HAC standard errors.

Table 9 – Coibion and Gorodnichenko (2011) / 95% Confidence Intervals

Sample: post 1982 (T = 158) - AR(2) smoothing						
Specification	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
Forward	0.94	2.54	$[1.35, 3.72]$	$[1.29, 4.99]$	$[1.24, 4.65]$	$[1.59, 3.05]$
Mixed	0.92	2.20	$[1.41, 2.99]$	$[1.47, 3.38]$	$[1.43, 3.37]$	$[1.32, 2.80]$

Note: Newey-West HAC standard errors.

Table 10 – Coibion and Gorodnichenko (2012) / 95% Confidence Intervals

Sample: 1987:4 – 2006:4 (T = 77) - AR(1) smoothing						
Specification	$\hat{\rho}$	$\hat{\beta}$	*Delta Method*	*Fieller*	*Krinsky–Robb*	*Anderson-Rubin*
g_y	0.83	2.29	$[1.71, 2.86]$	$[1.77, 2.99]$	$[1.73, 2.97]$	$[1.66, 3.11]$
$no\ g_y$	0.69	1.78	$[1.36, 2.19]$	$[1.32, 2.17]$	$[1.32, 2.18]$	$[1.19, 2.27]$

Note: Newey-West HAC standard errors.