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INFORMATION ACQUISITION IN THE CONTEXT OF A CENTRALIZED SCHOOL ADMISSION SYSTEM

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Any errors or omissions are my own responsibility. Comments to: facorrea1@uc.cl

Abstract

In the context of a centralized school admission system—specifically a Deferred Acceptance- we study the process of information acquisition over the quality of a school. Under the assumption of homogeneous agents in everything except on their cost for acquiring information, we analyze the different equilibriums that can arise and how they differ from what a Social Planner does.

We present two versions of the model, one where the agents' valuations over the school are independent of each other, and a second where agents have common valuations over the school.

In the first model, we find both positive and negative externalities of the agents' information acquisition. The result depends on what the uninformed agents prefer to do. In the second one, the behavior of the uninformed agents' is endogenous to the model because of what we call the "curse of the uninformed", produced by those with small information acquisition costs taking advantage of that, and avoiding the school when it is of bad quality; giving those who do not acquire a greater probability of being selected in that case. There, in almost all cases, we found negative externalities from the acquisition of information.

This work suggests that, under certain conditions of the uninformed agents' behavior, when the preferences over schools are horizontally differentiated, there could be gains if a central authority implements policies to reduce the information's cost. However, if the agents' valuations are common, it is difficult to improve the aggregate welfare by making information cheaper. In almost all cases, it will make aggregate welfare fall. Making those with the highest cost worse off.

1. Introduction

This thesis focus is to model information acquisition effects in a centralized school admission system. The main motivation for this idea is the implementation since 2016 of the "Sistema de Admisión Escolar" (SAE) in Chile. Which is a Deferred Acceptance algorithm (DA) based on the one proposed by Gale and Shapley (1962), implemented for all schools that receive public funding[2].

This kind of matching algorithms are a useful tool to solve allocation problems where the regular market solution (i.e., using prices to equal demand and supply of seats at each school) is not feasible. The DA algorithm also has a series of properties that make it superior to other algorithms to solve this kind of problem. One of the main features is that reveal their true preferences is a dominant strategy for the agents (Pathak and Sonmez (2013)). This may look not that important, but Kapor, Neilson, and Zimmerman (2020) have proven that: "A switch to truthful reporting in the DA mechanism offers welfare improvements over the baseline given the belief errors we observe in the data," where the baseline was another kind of matching algorithm. These kinds of systems are efficient but have a problem. In reality, everything unfolds in a partial information world. Since the DA algorithms do not have prices to transmit information to users, these processes could require more effort and more information acquisition than an equivalent market one.

Going to the case of the publicly funded education system in Chile, a few empirical regularities motivate how we model the process. Eyzaguirre, Hernando, and Blanco (2018) show that: 1) the system has a significant excess of capacity[3] 2) for the demand side, they also show that few schools concentrate a significant amount of postulations, and 3) half of the participants did not apply to more than three schools. All this taking as given that in the existing system is almost free to add more options.

This thesis aims to understand the information acquisition process and its influence on the results of a DA algorithm in equilibrium. The performance of a DA mechanism is crucially influenced by the rankings submitted by students, which in turn depends on their incentives to acquire information. All this interacting with supply-side constraints, information constraints on the demand side, preferences of the agents over the acquisition of information, and believes or preferences over schools.

In this thesis, we analyze two models. In both, the center is the information acquisition decisions over a "special school" with unknown quality. With another school or "outside option" that has no restrictions in its capacity. In the first model, agents' valuations over the "special school" are independent of one to the other, while in the second, the valuations over the school are common. In other words, in the first one, all agents have the same probability of liking the school once they know its type, but the realization of the result is independent. In the second one, all have the same opinion about the special school's quality when its type is revealed to them.

Under the assumption of homogeneous agents in everything except on their cost for acquiring information, we analyze the different equilibriums that can arise and how they differ from what a Social Planner does. Specifically, we are interested in the proportion of agents that acquire information in each situation.

In the first model, we find positive and negative externalities of the agents' acquisition of information. They are positive when all uninformed agents are applying to the special school. To understand this, we

[2] This system has been implemented in stages, the first one was in 2016, and it included only one region. It has gradually included more regions until this year, which finally added the last one. Visit the official site for more information www.sistemadeadmisionescolar.cl/que-es/.

[3] The excess of capacity was of 47% for regions that participated in the 2017 edition.

can imagine the situation faced by one of the agents acquiring information. We know that she will apply only if the school is good for her (a situation independent of other agents opinions). If she finds it of bad quality, she will not apply, liberating space for other agents who may find the school good for them. There we can see the mechanism under which the positive externality operates.

The externalities are negative when no uninformed agent is willing to apply to the special school. If enough agents are acquiring (specifically, more than the proportion of seats available in the school), each new application will generate more congestion to the system. As in the previous situation, there will be seats liberated, but they will be in the outside option, a school that we model with infinite seats. Then, the acquisition of information does not improve the aggregate welfare in that case.

In the second model, the behavior of the uninformed agents is endogenous. This is because of what we call the "curse of the uninformed". This mechanism is produced by those agents with small information acquisition costs, which are always acquiring information, and with that, avoiding the special school when it is of bad quality. Then, those with a higher cost (and thus not acquiring information) have a greater probability of being selected in that case, almost auto-selecting themselves to the outside option or the special school when it is of bad quality. Because of that, in almost all cases, we found negative externalities from the acquisition of information. The two forces (congestion and the cost of the information) that the planner had in the first model are also present here, making the analysis more complicated.

However, the intuition is not that complicated once we understand that it is like a "zero-sum game" because the quantity of special seats is limited. In almost all cases, there is congestion when the special school is of good quality. Then, taking as given the congestion when it is of good quality, the agents with smaller information costs acquire to save themselves from the risk of being selected on the special school when it is of bad quality, or acquire to not go to the outside option. These are the same two reasons that made the agents acquire information in the first model. In the second, the channel that had the positive externality to function is closed. This makes the agents who do not acquire information worse off relative to the first model. Then, in almost all the possible cases, the Social Planner prefers to acquire less information than the equilibrium.

This work suggests that when the agents' preferences over schools are horizontally differentiated, and the uninformed agents are always applying to the school of interest, there could be gains if the central authority implements policies to reduce the information's cost. However, if the agents' valuations are common, it is difficult to improve the aggregate welfare by making information cheaper. In almost all cases, and because of the strong negative externalities, it will make the aggregate welfare fall.

1.1. *Literature Review*

As Caplin and Dean (2015) explain in their work, the standard theory of choice asserts that individuals act optimally given what they know. At least since Hayek (1945) and Stigler (1961), there has been a focus on optimizing knowledge itself, with decision makers trading off the cost of learning against improved decision quality.

The matching literature generally assumes agents know their preferences before entering a match. Some exceptions on this are the papers by Lo (2018), Lee & Schwarz (2017), and Chade, Lewis & Smith (2014), which we review next.

Lee and Schwarz (2017) generalize the one-to-one matching model from Gale and Shapley (1962) to allow for a stage of costly information acquisition. In the first stage, firms simultaneously choose a subset of workers to interview. In the second stage, firms and workers are matched one-to-one using a firm-proposing DA algorithm in which firms make offers to workers. This differs from the school matching mechanism because, in their model, the firms pay the information cost instead of the workers (as it will be in the school admission system). Either way, they propose a framework that combines incomplete information with a DA algorithm and presents the externalities created when workers go to multiple interviews, endogenizing in this way the probability that a worker accepts a job from a firm.

On the other hand, Lo (2018) studies market design's impact on costly information acquisition in a many-to-one school choice market. Following Azevedo and Leshno (2016), she models students in a tractable continuum matching market where every student knows a prior distribution for each school's utility. They must pay a cost to learn the actual utility realization, with independent realizations for each student. One significant result from Lo (2018) is that market outcomes are stable with respect to the acquired information and its cost. Another contribution to this work is that there can co-exist different stable outcomes under incomplete information. Taking this from Liu et al. (2014), who had already suggested a notion of stability with incomplete information. It is worth noting that this is not inconsistent with the fact that, under perfect information, there exists a unique stable outcome.

Finally, Lewis and Smith (2014) develop a decentralized Bayesian model of college admissions with two ranked colleges and two realistic match frictions: students find it costly to apply to college, and college evaluations of their applications are uncertain. The interesting part for this work is their finding that students face a non-trivial portfolio choice due to the costly applications. A problem that becomes trivial if the application's cost diminishes, as it happens in the DA applied in Chile. Implying that we do not need to worry about these kinds of frictions because the actual cost of adding one extra school is close to 0, reassuring us that the relevant cost is acquiring information about the schools' quality, and not other frictions.

2. Model with independent valuations

2.1. *Introduction:*

This thesis focuses on understanding the process of information acquisition and its effects on the equilibrium's arising under a DA algorithm. The first scenario we analyze is when agent valuations over schools are independent from each other.

In this model, the supply side has two schools: a special one with uncertain quality and limited seats and an outside option with a certain quality and unlimited seats. Valuations are independent, so what the others think about the special school's quality will not add new information for the uninformed agents. They already know with what probability the special school is good or bad for them. Then, they care only about the proportion of people acquiring and applying. Also, the costs of acquiring information are heterogeneous, so all agents face different costs.

What matters is if the uninformed agents prefer to apply or not to the special school. This defines when congestion arises, the optimum response from each agent, and the kind of externalities that arise.

The model has two kinds of equilibrium. The first kind is when uninformed agents always apply to the special school. In this situation, uninformed agents will acquire information to avoid being selected into a special school that they do not like. It will matter the expected value of the payoff received when they do not like the school.

The second kind is when uninformed agents do not apply to the special school. There, uninformed agents will acquire information to know if the special school is a good opportunity. It will matter the expected value of the payoff received when they like the school.

If we compare these two main kinds of equilibrium to a Social Planner, we will note that in both cases, congestion can arise in the special school. However, the role of information acquisition will be different in each situation.

If all the uninformed are applying, and an extra agent acquires information, we know that she will apply only if the school is good for her (a situation independent of other agents' opinions). If she finds it of bad quality, she will not apply, liberating space for other agents who may find the school good for them. There we can see the mechanism under which the externality operates, and we can say that information will have a positive externality to the system, helping get a more efficient allocation of the agents. Because the agents never internalize this extra benefit, the planner will always choose the same or more information acquisition than the equilibrium. This problem will be more serious when information is costlier and disappear when it is so cheap that all agents want to acquire.

In the second kind of equilibrium, the uninformed agents do not like to apply to the special school. If enough agents are acquiring (specifically, more than the proportion of seats available in the school), each new application will generate more congestion to the system. As in the previous situation, there will be seats liberated when they do not like it, but these seats will be in the outside option, a school that we model with unlimited capacity, thus blocking the mechanism of the positive externality. Because of that, the acquisition of information does not improve the aggregate welfare in this case.

We can say that the acquisition of information will have negative externalities in this second case. Each extra agent acquiring will increase congestion (or maintain it). A planner will make fewer people acquire information than the equilibrium when there is congestion. The problem will be more acute when information is cheap because more agents will acquire information, increasing the congestion. When it is more costly, fewer people will acquire at the equilibrium eliminating congestion, and then making the planner's solution converge to the equilibrium one.

The optimal policy recommendation will then depend on whether the agents apply or not to this special school when they do not acquire information.

The model has four parts: first, the general settings, where we introduce the notation and formalize the structure presented here, then the "Basic Trade-off" that presents the decision the agents have to take in a general way. Third, we solve the equilibrium, and lastly, we solve the problem of a "Social Planner" and compare its solution with the equilibrium. Also, all the proofs of the lemmas are available in the first Appendix.

2.2. *General setting and main assumptions:*

We assume that the supply side is composed of two schools: a "special one" with unknown quality and limited seats, and an "outside option", with certain quality and infinite seats available. Both schools have no preferences over students, also $\eta \in (0, 1)$ represents the limited capacity of the special one. This takes into account the fact that there exists excess capacity in the system, but that at the same time there are some schools with lots of congestion[4].

On the demand side, we assume a continuum of agents. For them, the outside option's utility is normalized to zero, and have a prior expectation that with probability p_1, the special school is good and gives them a utility $u_1 > 0$, and with probability p_2, is bad and gives them a utility of $u_2 < 0$. Since this are the only two possible situations, we can define $p = p_1$ and $p_2 = (1 - p)$. Then, the expected utility of being selected to the special school is:

$$pu_1 + (1 - p)u_2 \gtrless 0$$

The center of the model are the agents who have the opportunity of acquiring information about the special school at a cost $c = \lambda \cdot i$. Where λ is a positive number representing a common parameter for the level of the cost to all agents, and $i \in [0, 1]$ represents the position of the agent in the continuum[5]. This is the only difference between agents. The information reveals to them, with a 100% certainty, if they are going to like or not the special school, and apply to the special school only if they like it. Results present with probability p and $(1 - p)$, and independently for each agent. We will name $Q \in [0, 1]$ the fraction of agents that acquire information.

In this model, agents are atomistic. This is a very important assumption that drives the results. It implies that the agents take as given the probability of being selected on the special school, not worrying about the externality of their behavior over the system.

This model unfolds in the context of a centralized school admission system that uses a DA algorithm for matching the supply and demand for each school. Like a regular DA mechanism, people will always apply to their true preferences. With this clear, we can define:

Definition 1: "Pure-strategies Equilibrium". A pure-strategy equilibrium of this model is an information acquisition activity $\{a_i\}$, for every agent $i \in [0, 1]$, with $a_i \in \{0, 1\}$. And the property that no single agent can deviate unilaterally.

2.3. *Basic Trade-off:*

Every agent has to decide if she acquires information or not. To know that, she compares the expected utility of acquiring information with her opportunity cost. If she acquires, it needs to pay the cost λi, and with a probability p likes the school, obtaining a payoff of u_1. Then, we can define the utility if she acquires information as:

$$U_i (A_i \mid Q) = \mathbb{P}_{(Q)} p u_1 - \lambda i$$

Where $\mathbb{P}_Q$ represents the probability of being selected on the special school as a function of the proportion of agents that acquires information.

Each agent also needs to determine her opportunity cost (or her best response when she does not acquire information). We define this as the maximum between the utility obtained by applying uninformed to the special school ($\mathbb{P}_{(Q)}(pu_1 + (1 - p)u_2)$), and the utility of going to the outside option (0). Then, the utility

[4] This is based on how the system was applied in Chile. Technically, the schools have preferences given by a "Tómbola" or a random number generator that gives each student a number that, in the case of an excess of demand for one school, decides who is accepted and rejected.

[5] We also developed a similar base model, but with agents with homogeneous costs. This model arrives to similar conclusions, but with mixed strategies. Is available in the appendix 7.1.

if she does not acquire is equal to:

$$U_i\left(NA_i \mid Q\right) = max\left(\mathbb{P}_{(Q)}(pu_1 + (1-p)u_2), 0\right)$$

Then, an agent will acquire information if:

$$U_i\left(A_i \mid Q\right) \geqslant U_i\left(NA_i \mid Q\right)$$

Using the definitions, this can be expressed as:

$$\mathbb{P}_{(Q)}pu_1 - \lambda i \geq max\left(\mathbb{P}_{(Q)}(pu_1 + (1-p)u_2), 0\right) \tag{2.3.1}$$

$\mathbb{P}_{(Q)}$ represents the probability of being accepted to the special school. This probability depends on the agents that acquire and on the behavior of the uninformed. If they prefer to apply to the special school it will be equal to:

$$\mathbb{P}_{(Q)} = min\left(\frac{\eta}{1-(1-p)Q}, 1\right) \tag{2.3.2}$$

In the first case, the proportion of agents applying will be equal to all, minus those that acquire and did not like the school. Congestion arise if the proportion of seats available is smaller than the proportion of people that applies. If that does not happen or, there are more seats than applicants, the probability of being accepted is equal to one.

If the uninformed agents prefer to apply to the outside option instead of the special school, this probability changes to:

$$\mathbb{P}_{(Q)} = min\left(\frac{\eta}{pQ}, 1\right) \tag{2.3.3}$$

Where pQ represent the people actually applying to the special school. It will be less than one when the proportion of agents that acquires information and likes the special school is bigger than the proportion of seats.

Then, $\mathbb{P}_{(Q)}pu_1$ represents the utility the agent will have if she likes the special school and apply to it. If she does not like it, she applies to the outside option obtaining zero. λi represent the cost she pays as a function of her position on the continuum.

Because of that distinction in $\mathbb{P}_{(Q)}$, we analyze the different equilibriums in two cases. First, when the uninformed apply to the special school, there the information acquisition process produces a decrease in the congestion of the system. Each agent that gets informed will apply with probability $p < 1$ to the special school, assuring us that whenever $Q > 0$, the congestion will be less than that in the case of $Q = 0$.

The other case is when the uninformed do not apply to the special school. In this other case, we will see that the information acquisition process produces congestion in the system because people will start to apply to the special school with probability $p > 0$.

Is important to note that, since $\mathbb{P}_Q$ is always greater than zero, the actual condition that divides the two cases is a parametric one, given by $pu_1 + (1-p)u_2 \gtreqless 0$.

For each case, we will also present the results of a "Social Planner" who considers the externalities produced by the agents, and show how she differs from the equilibrium.

2.4. *Equilibrium:*

Here we solve the equilibrium for the two main cases discussed. We start with the one when all the uninformed wanted to apply to the special school, and then go to the one where they do not apply.

2.4.1. When the uninformed apply to the special school:

Assumption 1: To be in the case where the uninformed want to apply, we assume that $pu_1 + (1-p)u_2 > 0$.

Then $\mathbb{P}_{(Q)} = min(\eta/(1-(1-p)Q), 1)$. We can write down this version of the basic trade-off as:

$$min\left(\frac{\eta}{1-(1-p)Q}, 1\right) pu_1 - \lambda i \geq min\left(\frac{\eta}{1-(1-p)Q}, 1\right)(pu_1 + (1-p)u_2) \tag{2.4.1}$$

As we can note by the minimum, we have two main situations here. We know that if the probability of being accepted is less than one, there is congestion. Then:

$$\frac{\eta}{1-(1-p)Q} < 1$$

$$Q < \frac{1-\eta}{1-p}$$

Definition 2: We define

$$\tilde{Q} = \frac{1-\eta}{1-p}$$

With $Q \in [0, 1]$, as the maximum level of Q, consistent with a equilibrium with congestion (if $Q > \tilde{Q}$, there is no congestion). For $\eta \leq p$ this fraction is bigger or equal than one, and for $\eta > p$ this expression is smaller than one.

Based on this definition, we divide the cases we analyze in two zones: $\eta \leq p$, with exogenous congestion, and cases when $\eta > p$, with endogenous congestion.

1) Equilibrium with exogenous congestion:

Assumption 2: To be in the case with exogenous congestion, we assume $\eta \leq p$.

This implies $\tilde{Q} = \frac{1-\eta}{1-p} \geq 1$. The intuition is that the proportion of seats is always smaller than the proportion of people that finds the school of good quality. Then, even if all the agents acquire information (the situation with the smallest quantity of agents applying, given that the uninformed want to apply) the seats will not be enough to all the agents that like the school. With that, equation 2.4.1 is equal to:

$$\frac{\eta}{1-(1-p)Q} pu_1 - \lambda i \geq \frac{\eta}{1-(1-p)Q}(pu_1 + (1-p)u_2) \tag{2.4.2}$$

The prior expression can be written for the marginal agent as:

$$\frac{\eta}{1-(1-p)Q}(1-p)\bar{u}_2 - \lambda Q = 0 \tag{2.4.3}$$

Rearranging:

$$(1-p)\lambda Q^2 - \lambda Q + (1-p)\eta\bar{u}_2 = 0 \tag{2.4.4}$$

If this expression is smaller than zero, the marginal agent does not want to acquire information. If it is greater or equal to zero, the agent will want to do it. The intuition can be seen more easily on equation 2.4.3, replacing $Q = 0$. There the first agent will have a positive payoff that will make her acquire information.

Solving this quadratic equation for Q, we obtain two solutions:

$$\begin{aligned} Q^A &= \frac{\lambda - \sqrt{\lambda^2 - 4(1-p)^2\lambda\eta\bar{u}_2}}{2(1-p)\lambda} \\ Q^B &= \frac{\lambda + \sqrt{\lambda^2 - 4(1-p)^2\lambda\eta\bar{u}_2}}{2(1-p)\lambda} \end{aligned}$$

Lemma 1: If $\eta \leq p$ and $\lambda > \frac{(1-p)\eta\bar{u}_2}{p}$, then $Q = Q^A$ is the only solution.

We now check the solution when there is congestion, but $\lambda < \frac{(1-p)\eta\bar{u}_2}{p}$.

Lemma 2: If $\eta \leq p$ and $\lambda < \frac{(1-p)\eta\bar{u}_2}{p}$, $Q = 1$.

2. Equilibrium with endogenous congestion:

Assumption 3: To be in the case with endogenous congestion, we assume $\eta > p$.

This implies $\tilde{Q} < 1$. Then, if the Q of equilibrium is greater than $\tilde{Q}$, there will be no congestion. The intuition is that the proportion of seats is bigger than the proportion of of people that finds the school of good quality for them. Then, if all the agents acquire information, is impossible to have congestion.

Solving first the cases with congestion: the solution will be equal to the case with $\eta \leq p$, but restricted to $Q < \tilde{Q}$ instead of less than one. With that, equation 2.4.1 is equal to:

$$\frac{\eta}{1-(1-p)Q}pu_1 - \lambda i \geq \frac{\eta}{1-(1-p)Q}(pu_1 + (1-p)u_2)$$

That is the same as equation 2.4.2, but with the new restriction for Q. Then, we resume the solution for the case with congestion in one lemma:

Lemma 3: If $\eta > p$ and $\lambda > \frac{(1-p)^2\bar{u}_2}{1-\eta}$, then $Q = Q^A$ is the only solution.

Now solving the case without congestion: the solution will differ now, and the agents will be facing a different version of the basic trade-off, where the probability of being accepted in the special school is equal to one. With that, the equation 2.4.1 is equal to:

$$pu_1 - \lambda i \geq (pu_1 + (1-p)u_2) \tag{2.4.5}$$

$$i \leq \frac{(1-p)\bar{u}_2}{\lambda}$$

Then, the marginal agent acquiring information satisfies:

$$Q = \frac{(1-p)\bar{u}_2}{\lambda} \tag{2.4.6}$$

To be in this case, we need $Q \in [\tilde{Q}, 1]$. In the next two lemmas we resume where this is a valid solution for the equilibrium.

Lemma 4: If $\eta > p$ and $\lambda \in [(1-p)\bar{u}_2, \frac{(1-p)\bar{u}_2}{1-\eta}]$, then $Q = \frac{(1-p)\bar{u}_2}{\lambda}$.

We know that if $\lambda < (1-p)\bar{u}_2$, the solution $Q = \frac{(1-p)\bar{u}_2}{\lambda}$ will give us a Q bigger than one, then it can not be possible. At the same time, because $\tilde{Q} < 1$, we know that this case will be without congestion, then:

Lemma 5: If $\eta > p$ and $\lambda < (1-p)\bar{u}_2$, then $Q = 1$.

With all this, we can write down two propositions that resume the possible equilibriums:

Proposition 1: If $\eta \leq p$ and $(pu_1 + (1-p)u_2) > 0$, the equilibrium, as a function of the cost parameter λ is given by:

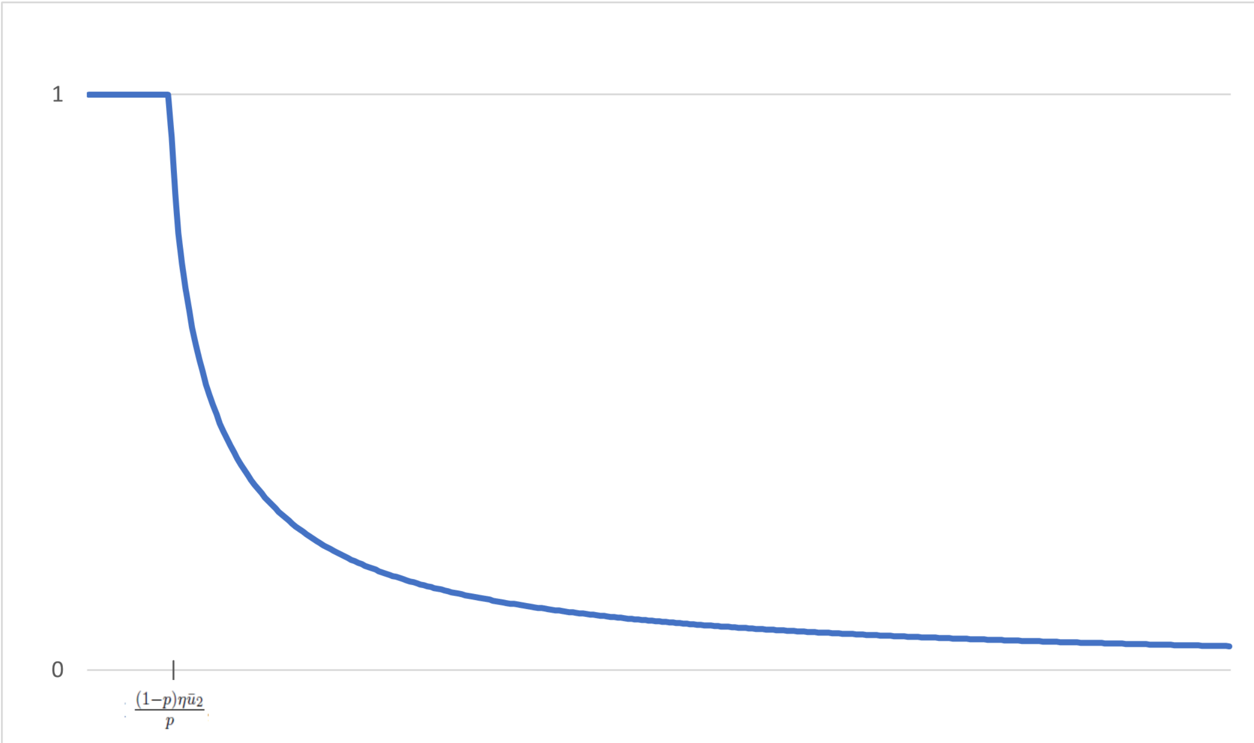
$$Q = \begin{cases} 1 & if \quad \lambda < \frac{(1-p)\eta\bar{u}_2}{p} \\ \frac{pu_1}{\lambda} & if \quad \lambda \geq \frac{(1-p)\eta\bar{u}_2}{p} \end{cases} \tag{2.4.7}$$

The results of this are in Figure 1. There the x-axis is for values of λ and the y-axis for values of Q. $\frac{(1-p)\eta\bar{u}_2}{p}$ represents the maximum λ compatible with an equilibrium with all agents acquiring information.

Proposition 2: If $\eta > p$ and $(pu_1 + (1-p)u_2) > 0$, the equilibrium, as a function of the cost parameter λ is given by:

$$Q = \begin{cases} 1 & if \quad \lambda < (1-p)\bar{u}_2 \\ \frac{(1-p)\bar{u}_2}{\lambda} & if \quad \lambda \in [(1-p)\bar{u}_2, \frac{(1-p)^2\bar{u}_2}{1-\eta}] \\ Q^A & if \quad \lambda \geq \frac{(1-p)^2\bar{u}_2}{1-\eta} \end{cases} \tag{2.4.8}$$

Figure 1



The results of this can be seen on Figure 2. The axis are the same as in the previous figure. $(1-p)\bar{u}_2$ represents the first value of λ where it is possible to be in a equilibrium with $Q = Q^{nc}$, and $\frac{(1-p)^2\bar{u}_2}{1-\eta}$ represents the last point compatible with a equilibrium in $Q = Q^{nc}$.

With all this, we can describe the utility of the agents in the form:

$$U_i = \begin{cases} \mathbb{P}_{(Q^*)} p u_1 - \lambda i & if \quad i \leq Q^* \\ \mathbb{P}_{(Q^*)}(p u_1 + (1-p) u_2) & if \quad i > Q^* \end{cases} \tag{2.4.9}$$

Where Q^* represents the proportion of agents that acquire information, given the parameters and $\mathbb{P}_{(Q^*)}$.

2.4.2. When the uninformed agents do not apply to the special school:

Assumption 4: To be in the case where the uninformed do not want to apply, we assume that $pu_1 + (1-p)u_2 \leq 0$.

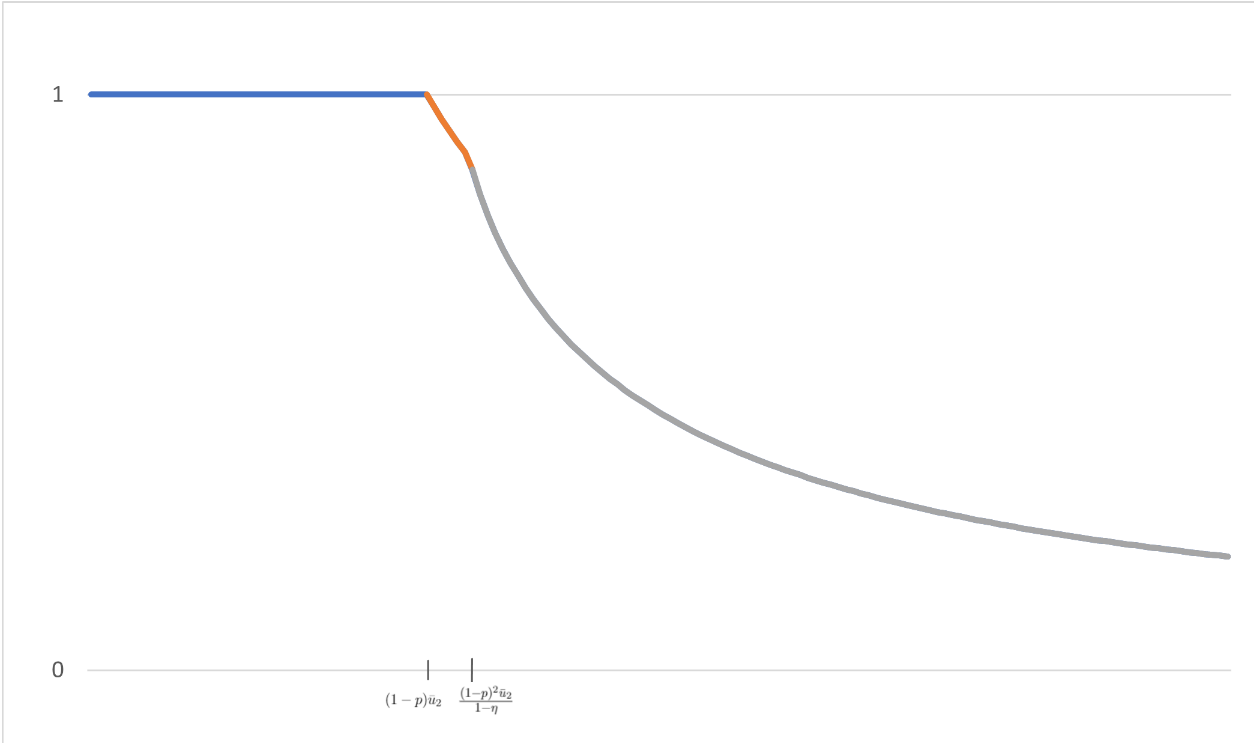
Then, $\mathbb{P}_{(Q)} = min(\eta/pQ, 1)$, and the basic trade-off for this case is:

$$min\left(\frac{\eta}{pQ}, 1\right) pu_1 - \lambda i \geq 0 \tag{2.4.10}$$

We know that if the probability of being accepted is less than one, there is congestion. Then:

$$\frac{\eta}{pQ} < 1$$

Figure 2



$$Q > \frac{\eta}{p}$$

Definition 3: We define

$$\hat{Q} = \frac{\eta}{p}$$

With $Q \in [0,1]$, as the minimum level of Q consistent with a equilibrium with congestion (if $Q < \hat{Q}$, there is no congestion). For $\eta > p$ this expression is bigger than one, and for $\eta \leq p$ is smaller or equal to one.

Based on definition 3, we analyze two zones: $\eta > p$ with exogenous congestion and $\eta \leq p$, with endogenous congestion.

1) Equilibrium with exogenous congestion:

Assumption 5: To be in the case with exogenous congestion, we assume $\eta > p$.

This implies $\hat{Q} > 1$. The intuition is that here the proportion of seats available is bigger than the proportion of agents that like the special school. Then, even if all the agents acquire information, there will be no congestion. With this, the equation 2.4.10 is equal to:

$$pu_1 - \lambda i \geq 0 \tag{2.4.11}$$

The marginal agent satisfies:

$$Q = \frac{pu_1}{\lambda} \tag{2.4.12}$$

The next lemmas characterize the equilibrium for the case with exogenous congestion:

Lemma 6: If $\eta > p$ and $\lambda > pu_1$, then $Q = \frac{pu_1}{\lambda}$.

Lemma 7: If $\eta > p$ and $\lambda \leq pu_1$, then the equilibrium is on $Q = 1$.

2) Equilibrium with endogenous congestion:

Assumption 6: To be in the case with endogenous congestion, we assume $\eta \leq p$.

This implies $\hat{Q} < 1$. Then, if the Q of equilibrium is greater than $\hat{Q}$, there will be congestion. If it is smaller than that, there is no congestion as in the previous case. The intuition for this case is that the proportion of seats is smaller than the proportion of agents that like the special school. Then, if all the agents acquire, there will be congestion.

Now, if $Q < \hat{Q}$, the problem is the same as in the zone with $\eta > p$. We define the solution for that case with the next lemma:

Lemma 8: If $\eta \leq p$ and $\lambda > \frac{p^2 u_1}{\eta}$, then the equilibrium is on $Q = \frac{pu_1}{\lambda}$.

If $Q \geq \hat{Q}$, we change to the zone with congestion. Then, equation 2.4.10 is:

$$\frac{\eta}{pQ} pu_1 - \lambda i \geq 0 \tag{2.4.13}$$

The marginal agent satisfies[6]:

$$Q = \sqrt{\frac{\eta u_1}{\lambda}} \tag{2.4.14}$$

We characterize when this solution is an equilibrium on the next two lemmas:

Lemma 9: If $\eta \leq p$, $\lambda \geq \eta u_1$ and $\lambda \leq \frac{p^2 u_1}{\eta}$, then $Q = \sqrt{\frac{\eta u_1}{\lambda}}$.

Lemma 10: If $\eta \leq p$ and $\lambda \leq \eta u_1$, $Q = 1$.

With all this, we can write down two propositions that resume the possible equilibriums of this model:

Proposition 3: If $\eta \leq p$ and $pu_1 + (1-p)u_2 < 0$, the equilibrium, as a function of the cost parameter λ, is given by:

$$Q = \begin{cases} 1 & if \quad \lambda < \eta u_1 \\ \sqrt{\frac{\eta u_1}{\lambda}} & if \quad \lambda \in [\eta u_1, \frac{p^2 u_1}{\eta}] \\ \frac{pu_1}{\lambda} & if \quad \lambda \geq \frac{p^2 u_1}{\eta} \end{cases} \tag{2.4.15}$$

[6] The expression has two solutions, but we use only the positive one, because $Q \in [0, 1]$.

Proposition 4: If $\eta > p$ and $pu_1 + (1-p)u_2 < 0$, the equilibrium, as a function of the cost parameter λ, is given by:

$$Q = \begin{cases} 1 & if \quad \lambda < pu_1 \\ \frac{pu_1}{\lambda} & if \quad \lambda \geq pu_1 \end{cases} \tag{2.4.16}$$

Then, when the uninformed agents do not apply to the special school, there are two conditions needed to have congestion: first, that a proportion of the agents, bigger than the amount of seats, likes the school, and second, that the cost of the acquisition of information is small enough. Then, a big proportion acquires information and makes congestion. In the other cases when one of this two necessary conditions is not met, there is no congestion and there is no point in worrying about externalities, since the problem changes to one where each agents is by their own. In other words, the seats available are not a binding restriction anymore.

2.5. *Social Planner:*

Her utility is the sum between the cost she needs to pay to make agents acquire information, the benefit obtained by the proportion Q of agents that acquire, and the benefits from those that do not acquire.

$$U_{sp}(Q) = -\int_0^Q \lambda i di + Q\mathbb{P}_{(Q)}[pu_1 - (pu_1 - (1-p)\bar{u}_2)\mathbb{1}_{\text{un. apply}}] + \mathbb{P}_{(Q)}[pu_1 - (1-p)\bar{u}_2]\mathbb{1}_{\text{un. apply}} \tag{2.5.1}$$

Where $\mathbb{1}_{\text{un. apply}}$ is a indicator function that takes the value of one when the uninformed agents apply to the special school and zero otherwise. And $\mathbb{P}_{(Q)}$ is defined in the same way as in the basic trade-off. As in the equilibrium, we are going to have to main cases, depending if the uninformed agents will like to apply to the special school or not. Also, inside each main case, we have two situations to analyze: when there is or there is not congestion.

2.5.1. When the uninformed apply to the special school:

Assumption 7: To be in the case when the uninformed agents apply, we assume $pu_1 + (1-p)u_2 > 0$.

As in the equilibrium, here we have two cases. We analyze the maximum value inside each case and then see what she will choose for every value of λ.

1) Exogenous congestion:

Assumption 8: To be in the case with exogenous congestion, we assume $\eta \leq p$.

Then, $\mathbb{P}_{(Q)} = \eta/(1 - Q(1-p))$, and the utility of the planner is defined by:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + Q\frac{\eta}{1 - Q(1-p)}[(1-p)\bar{u}_2] + \frac{\eta}{1 - Q(1-p)}[pu_1 - (1-p)\bar{u}_2] \tag{2.5.2}$$

Then, we have the next lemma:

Lemma 11: If $\eta < p$, for every level of λ the Social Planner chooses a level of Q^{sp} higher or equal than the Q of equilibrium.

Because of the complexity of the function and that we do not need the exact Q to understand the role of the externalities, we do not obtain an algebraic solution to the problem with congestion. Instead, we solve it numerically.

2) Endogenous congestion:

Assumption 9: To be in this case with endogenous congestion, we assume $\eta > p$.

Cases without congestion: $\mathbb{P}_{(Q)} = 1$ and Q belongs to the interval $[\tilde{Q}, 1]$. Then, the utility of the planner is defined by:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + Q[pu_1 - (pu_1 + (1-p)u_2)] + [pu_1 + (1-p)u_2]$$

Then, solving the equation for Q, we get the result presented in the next lemmas.

Lemma 12: If $\eta > p$ and $\lambda \in [(1-p)\bar{u}_2, \frac{(1-p)^2\bar{u}_2}{1-\eta}]$, $Q^{sp} = Q^{nc} = (1-p)\bar{u}_2/\lambda$.

Lemma 13: If $\eta > p$ and $\lambda < (1-p)\bar{u}_2$, then $Q^{sp} = 1$.

Because there is no congestion, this solution is the same that was obtained in the case when agents decided by their own.

Cases with congestion: $\mathbb{P}_{(Q)} = \eta/(1 - Q(1-p))$, and the utility of the planner is defined by equation 2.5.2. Then, the solution in this case resumed on the next lemma.

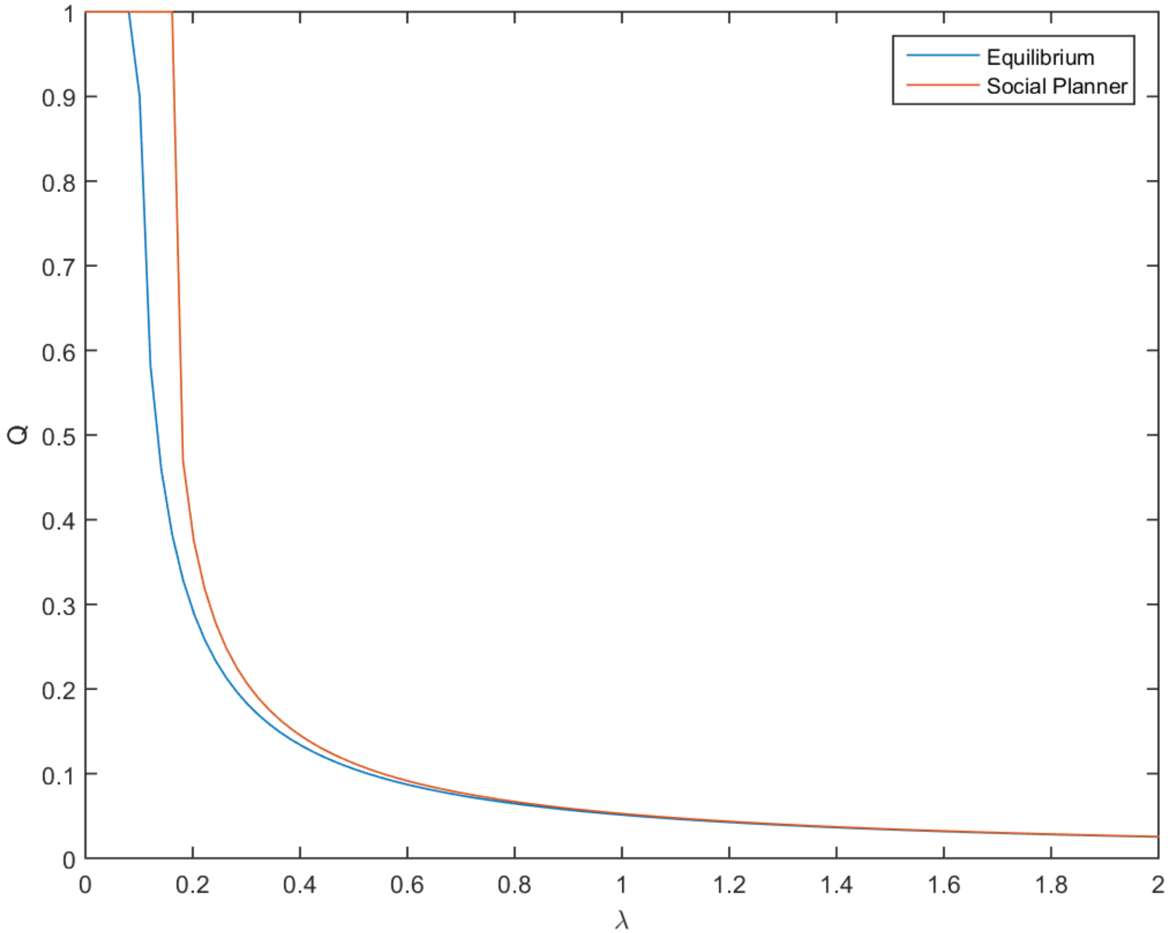
Lemma 14: If $\eta > p$ and $\lambda > \frac{(1-p)^2\bar{u}_2}{1-\eta}$, for every level of λ the Social Planner chooses a level of Q^{sp} higher than the Q of equilibrium.

We do not aggregate the results in propositions since the main result is a lower bound ($Q^{sp} \geq Q^{eq}$ for every λ).

Then, we directly compare the results of the equilibrium with the planner. We have two cases: $\eta \leq p$ and $\eta > p$. In the first one congestion is always exogenous, independent of the value of Q chosen. On the second one, there are zones without congestion for values of $Q > \tilde{Q}$.

An example of the results is on Figure 3 for the case with exogenous congestion. And on Figure 4 for the case with endogenous congestion.

Figure 3



In both figures we can see the results of the previous lemmas. Separating in both zones, we have that the Social Planner always acquires more (or the same if $Q = 1$) than the equilibrium when there is congestion. If there is no congestion, the Social Planner and the equilibrium coincide in their solution.

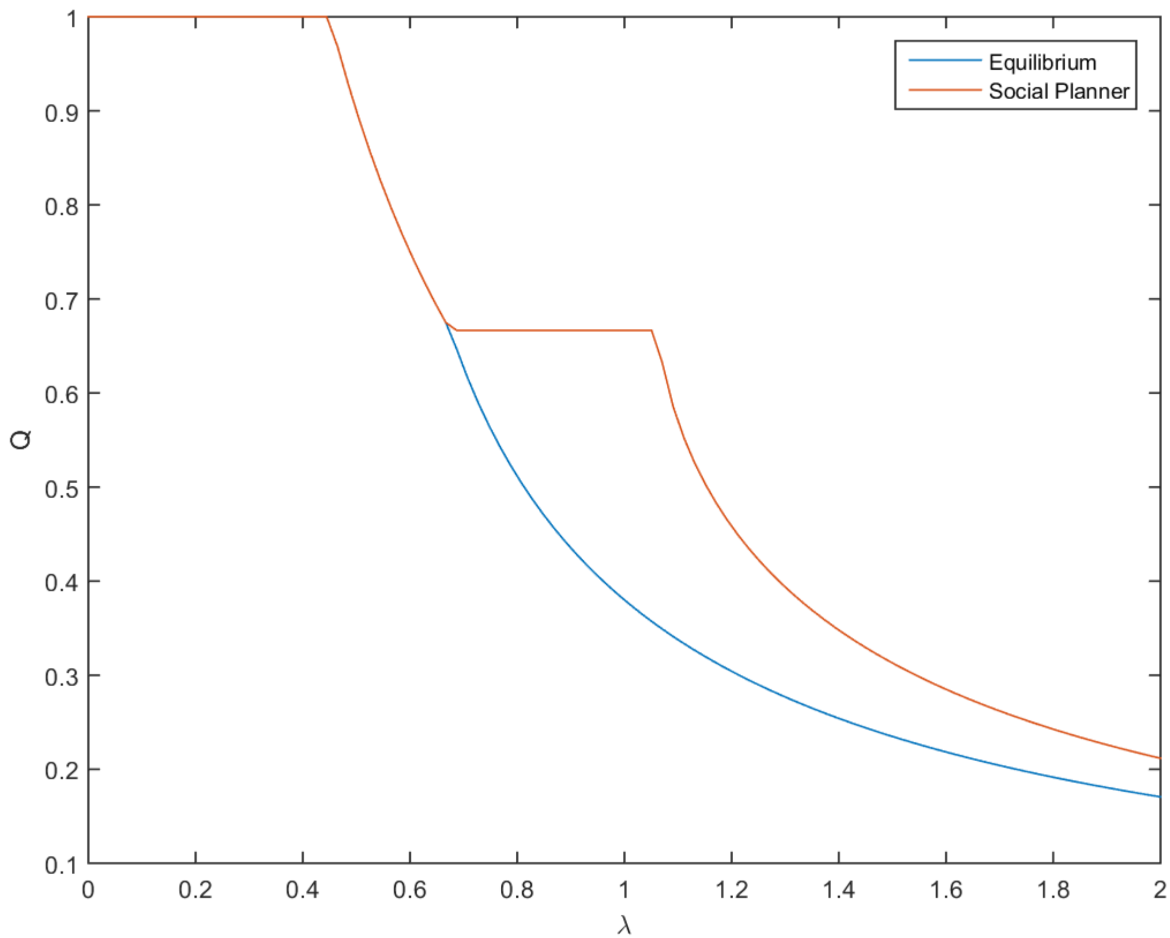
Separating if congestion is endogenous or exogenous, we have that: for the case with endogenous congestion, as soon as this starts, the Social Planner abruptly changes her behavior and choose $Q = \tilde{Q}$, maintaining congestion at zero (or the probability of being accepted at one) for the next values of λ. When the cost of the information is bigger than the benefit of the situation without congestion, she starts diminishing the proportion that acquires information, and thus tolerating a degree of congestion, but always maintaining the proportion over the one of equilibrium.

When congestion is exogenous, the planner chooses a bigger Q for every level of λ.

2.5.2. When the uninformed do not apply to the special school:

Assumption 10: To be in the case when the uninformed agents do not apply, we assume $pu_1 + (1-p)u_2 \leq 0$.

Figure 4



We proceed with the same structure as when the uninformed apply.

1) Exogenous congestion:

Assumption 11: To be in the case with exogenous congestion, we assume $\eta > p$.

Then, $\mathbb{P}_{(Q)} = 1$, and the utility of the planner is defined by:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + Q\,pu_1 \tag{2.5.3}$$

If we maximize the equation with respect to Q, we get the optimum Q for the planner. We summarize the results in the next lemmas:

Lemma 15: If $\eta > p$ and $\lambda > pu_1$, the planner chooses $Q = \frac{pu_1}{\lambda}$.

Lemma 16: If $\eta > p$ and $\lambda \leq pu_1$, the planner chooses $Q = 1$.

In this case, there is no congestion. Then, there are no externalities to take into account by the Social Planner, and these are the same solutions we obtain in the equilibrium with no congestion.

2) Endogenous congestion:

Assumption 12: To be in this case with endogenous congestion, we assume $\eta \leq p$.

Because of that $\mathbb{P}_{(Q)} = \eta/pQ$. Then, the utility of the planner is:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + \eta u_1 \tag{2.5.4}$$

We can note that the function is strictly decreasing in Q. With that we present the different cases of the results in the next lemmas:

Lemma 17: If $\eta \leq p$ and $\lambda > \frac{p^2 u_1}{\eta}$, the planner chooses $Q = \frac{pu_1}{\lambda}$.

Lemma 18: If $\eta \leq p$ and $\lambda \leq \frac{p^2 u_1}{\eta}$, the planner chooses $Q = \hat{Q}$.

We can summarize the solutions of the planner on two main propositions:

Proposition 5: If $pu_1 + (1-p)u_2 < 0$ and $\eta > p$, then the optimum acquisition of information of the Social Planner as a function of the cost parameter is:

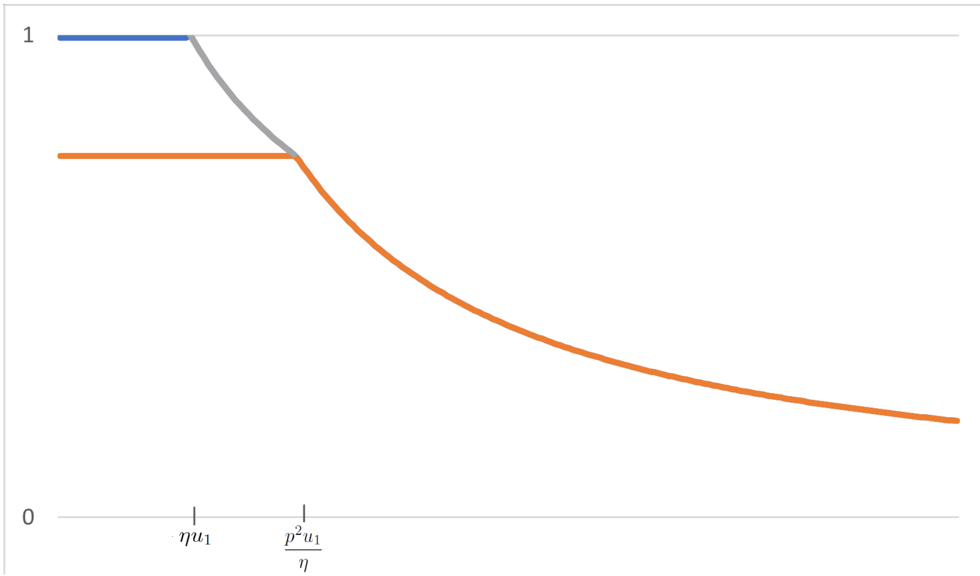
$$Q = \begin{cases} 1 & if \quad \lambda < pu_1 \\ \frac{pu_1}{\lambda} & if \quad \lambda \geq pu_1 \end{cases} \tag{2.5.5}$$

Proposition 6: If $pu_1 + (1-p)u_2 < 0$ and $\eta \leq p$, then the optimum acquisition of information of the Social Planner as a function of the cost parameter is:

$$Q = \begin{cases} \hat{Q} & if \quad \lambda < \frac{p^2 u_1}{\eta} \\ \frac{pu_1}{\lambda} & if \quad \lambda \geq \frac{p^2 u_1}{\eta} \end{cases} \tag{2.5.6}$$

We can note that both solutions are equal to the equilibrium when there is no congestion. Or, only when the equilibrium Q was defined in the interval $[\hat{Q}, 1]$ when $\eta \leq p$, the solution of the planner differed to the one of the equilibrium.

Figure 5

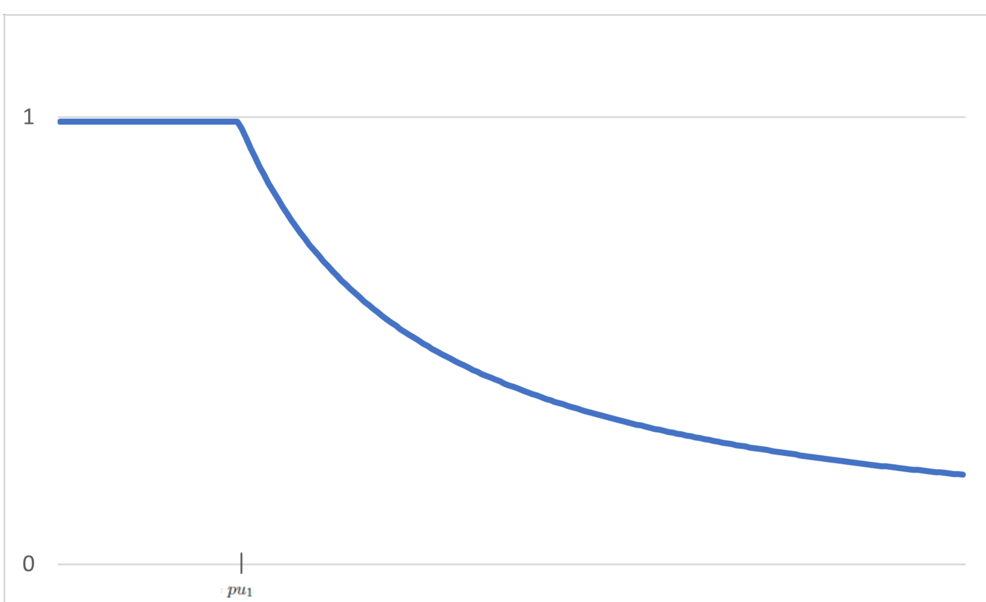


The comparison between the equilibrium and the planner when $\eta \leq p$ is on Figure 5. There, the x-axis is for λ and y-axis for Q. The blue line is for $Q = 1$ and gray line for the interior solution of the equilibrium. The planner is in the orange line. Both solutions overlap for values of $\lambda \geq \frac{p^2 u_1}{\eta}$.

Then, we can see that when information is cheap (or λ is low), an excess of agents acquires information relative to the Planner. This is because each agent takes as given the probability of being accepted to the special school, generating congestion in the system and a waste of resources relative to the centralized solution.

By internalizing the congestion, the Social Planner makes a fraction equal to $\hat{Q}$ acquire information, eliminating the negative externality caused by the excess of information acquisition. After that, when there is no congestion, both solutions are the same.

Figure 6



The Figure 6, following the same structure than the previous Figure, represents the equilibrium and the social planner for the cases where $\eta > p$ (both in the blue line). Since there is no congestion, both solutions coincide.

2.6. *Final Remarks of the model with independent valuations:*

After analyzing the two main variations inside this model, we can note that: when the uninformed agents apply ex-ante to the special school, they are acquiring information to avoid being selected in a school they do not like. Instead, when the uninformed agents prefer to not apply to the special school, agents are acquiring information to know if the special school is a good opportunity for them. Thus, in the first scenario, all the cutoffs are functions of u_2, while in the second one, they are functions of u_1.

Also, and as a consequence of the observation of the previous paragraph, when the uninformed apply to the special school, exist a positive externality from the acquisition of information, because it reduces congestion. Versus the situation where uninformed agents prefer to not apply, where the externality is negative. These two kinds of externalities are easy to see in the graphics; in the first case, the Social Planner acquires more or the same information than the agents by themselves. In contrast, for the second case, the Social Planner acquires less or the same amount of information.

The channels were these externalities generate is different. The positive externality generates from the diminution in applications to the special school that each extra agent acquiring information implies. On the other hand, the negative one generates because each extra agent (for $Q > \hat{Q}$) implies more congestion. When all uninformed agents apply and there is congestion, this last effect does not exist because congestion in the special school is independent of the agents' actions. When there is no congestion, the externalities disappear.

3. Model with common valuations:

3.1. *Introduction:*

In the previous model (and its extension on Appendix 7.1), we assumed that the beliefs were independent. This meant that if agent j knew the opinion of agent k (with $k \neq j$), this was not important to her in the decision of acquiring or not acquiring information about the special school. Now we remove the independence assumption and add to the model the possibility of correlations between the agents' preferences. If one agent finds the school of good quality, she will know that the rest of the agents will think the same. It is equivalent to think of the school as vertically differentiated. Every agent likes a "good" school, and every agent hates a "bad" school. There is no possibility of getting confused in the valuation when the agent acquires the information.

The basics are the same as in the previous model. The supply side has two schools, a special one with uncertain quality and limited seats, and an outside option with a certain quality and unlimited seats. On the demand side, the agents have ex-ante probabilities assigned to the two possible outcomes and heterogeneous costs.

In this model, we restrict the analysis to situations where the expected utility of applying to the special school is greater than zero. The reason is to show that, even when this happens, the preferences' correlation can make the uninformed not want to apply.

Because of that, the behavior of the uninformed agents is endogenous. To understand this, we can think of what a single agent does after she acquires information: if she finds the school is good, she applies, and if it is bad, she goes to the outside option. The uninformed agents know this. Also, they know the cost of acquiring information for her and the cost of the others. In other words, the uninformed agent does not know the school's quality but knows how many people are acquiring information and what the other uninformed are doing.

If we are on a base case, where no agent acquires information and all are applying, the probabilities of being selected are independent of the school's quality, just as in the previous model. However, if some agents start to acquire, they will apply only if the school is of good quality, then the probability of being selected when the school is of good quality is the same as before (all the agents are applying), but when the school is of bad quality, is bigger (fewer agents are applying). This problem gets worse when the proportion of people acquiring increases. At the maximum, they are facing a game with an expected utility smaller than zero, so they stop applying to the special school and go to the outside option. This new disadvantage produced by the externality of those with small information acquisition costs is what we call the "curse of the uninformed". This can also be seen as a form of self-selection, where the agents with small acquisition costs get the best schools.

This situation happens in all the cases we analyze. In some, because the expected payoff when the school is good is too big relative to the loss if it is of bad quality, the uninformed agents will apply. Nevertheless, it is worth noting that the "curse" will affect them, via making the probability of being selected in a bad quality school bigger than in the base case.

When we compare the equilibrium results with the Social Planner ones, they differ in a different way than in the previous model. Externalities are negative in almost all cases. Because of the "curse of the uninformed", there will be more incentives to apply for the marginal agent.

In some specific cases, the planner will acquire more or the same information. This is when the planner is acquiring information for a proportion of agents near the one that changes the behavior of the uninformed from not applying to applying or vice-versa.

The planner will want no congestion on the special school, just as in the previous model. Simultaneously, she will want to "save" the uninformed from applying when the school is of bad quality. These are two opposing forces. In some cases, it will choose a non-optimal level of congestion to protect the uninformed from applying. In other specific cases, it will choose no congestion and no uninformed applying. There is also a third (and simpler) force, that is, the cost of the information. When it goes up, it will force the planner to choose no agent acquiring, even though in the equilibrium, there is a proportion bigger than zero applying.

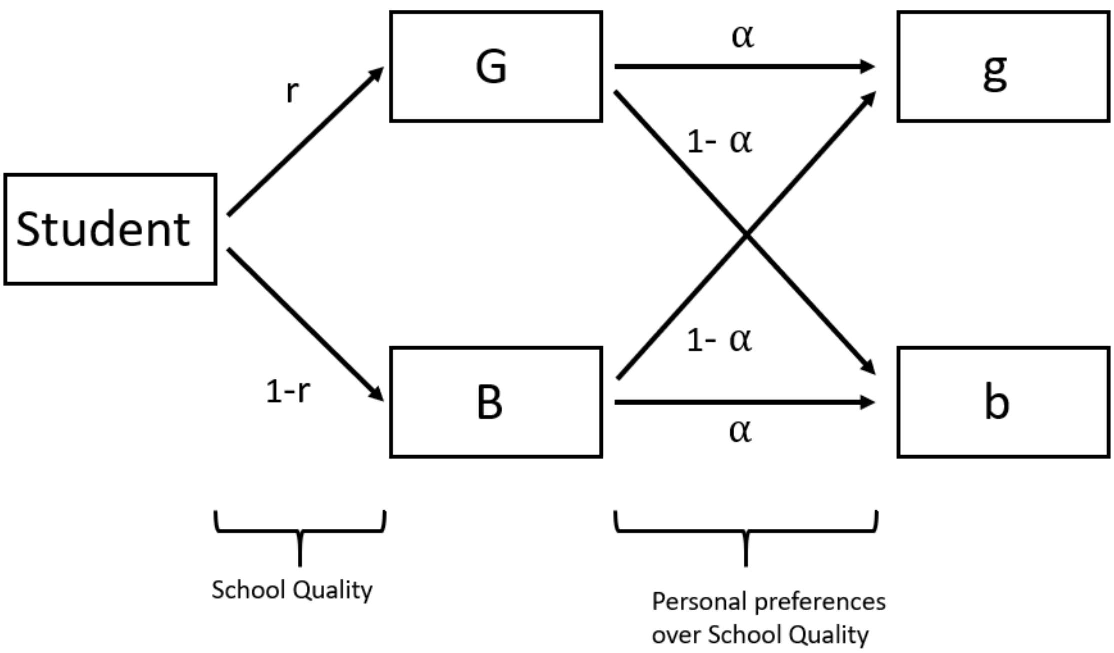
It is worth noting that it will choose no agent applying, even when the cost of the first ones is really small. That is because all agents have the same opinion about a school, then, if the first agent acquires, she will apply only when it is of good quality. If we add that all the uninformed are applying, this will reduce the congestion when the school is of bad quality, but since there are less seats that agents, there will be congestion in independent of the type of the school. Then, the aggregate well being will be the same. It will only matter the cost the agent incurred, even if it was low. And because of that, when the information acquisition is not enough to make the uninformed not apply, the planner will prefer to save the cost of the information, and avoid any kind of "curse" happening.

Then, the most important insights from this model: first, with homogeneous beliefs, the response of the uninformed is endogenous. Second, most of the time, the equilibrium will have too much information acquisition relative to the planner. This excess of information will result from the interaction of three forces: the externality of the congestion, the "curse of the uninformed," and the information's cost. However, in specific cases near the change of the uninformed agents' behavior, it will be optimum for the planner to acquire more information than the equilibrium.

3.2. *General settings:*

To remove the independence assumption and add the possibility of correlations between the agents' preferences, we start from the base model and introduce more structure to the problem. In specific, the assumed structure for the correlation in agents' preferences over the special school is displayed in Figure 7.

Figure 7



We assume that there exists a probability $r \in [0, 1]$ of the special school being of "Good quality" (G) and a probability of $1 - r$ of the school being of "Bad quality" (B).

After that comes the student's preferences over the special school's quality, with probability $\alpha \in [0.5, 1]$ if the school is Good, they also think that it is good for them (the same if it is Bad). With a probability $1 - \alpha$, if the school is Good, the agent will think it is bad for her[7].

In this scheme, the parameter α represents the correlation between the agents' preferences over the special school.

There, the probability that an agent ex-ante likes the special school is:

[7]The parameter α is defined in the interval $[0.5, 1]$ and not in $[0, 1]$ as r because if it is less than 0.5, there is no point in calling a college "of Good quality" if the majority of students think otherwise.

$$p = r\alpha + (1 - r)(1 - \alpha) = \alpha(2r - 1) + 1 - r \tag{3.2.1}$$

Where p is the probability that agents find the special school good, obtaining a payoff of u_1. It is important to note that, if the agents do not acquire information, they will know the exact value of the parameters r, α, but not the realization for the others or them of the special school's actual quality.

If we return to the previous example of two agents and assume $\alpha = 1$, if the agent j likes the special school, the agent k will also like it. This is where schools are vertically differentiated (one is always preferred over the other by every agent). In contrast, if we assume $\alpha = 0.5$, we will return to a situation where preferences are independent or a world where schools are "horizontally differentiated" between them.

This is the general framework. For simplicity, we add the assumption that $\alpha = 1$, so all the agents will have the same opinion over a certain school. The options are then vertically differentiated (one is always preferred over the other by every agent) and $p = r$. In contrast, if we had assumed $\alpha = 0.5$, we will return to a situation where preferences are independent or a world with "horizontal differentiation".

Also, over this general framework, we assume that the ex-ante utility of the special school is greater than zero.

$$pu_1 + (1 - p)u_2 > 0$$

This assumption is mainly to focus the attention on the cases where the behavior of the uninformed agents is endogenous to the model[8]. The cases when agents ex-ante do not want to apply will be reviewed during the development of the model, but only in the situations when the uninformed, as a result of the process of information acquisition, preferred to apply to the outside option.

3.3. *Basic trade-off:*

As in the first model, an agent will acquire if the utility of doing it is bigger than her opportunity cost. This cost will be the maximum between applying uninformed to the special school and going directly to the outside option, and will be the utility of not acquiring information. Then:

$$U_i\left(NA_i \mid Q\right) = max\left(\mathbb{P}_G\, pu_1 + \mathbb{P}_B(1 - p)u_2, 0\right)$$

$\mathbb{P}_G$ and $\mathbb{P}_B$ represent the probabilities of being accepted into the special school when it is of good or bad quality. Because the only difference between the agents is the cost at which they can acquire information, we know that all the uninformed are equal, so they will prefer the same option when confronted with this situation. With that, we can know the functional forms of the probabilities:

$$\mathbb{P}_G = \eta$$

Because if it is of good quality, all the agents will apply.

$$\mathbb{P}_B = min\left(\frac{\eta}{1 - Q}, 1\right)$$

[8]If $pu_1 + (1 - p)u_2 \leq 0$, the behavior of the uninformed will change. They will always prefer to apply to the outside option, independent of the value of Q. Is exogenous, as in the previous model.

If too few people acquire information, the uninformed applying to the special school will cause congestion even when it is bad.

With this, we define now the utility of acquiring information as:

$$U_i\left(A_i \mid Q\right) = \mathbb{P}_{(G|A)} p u_1 - \lambda i \geq$$

Where $\mathbb{P}_{(G|A)}$ represents the probability of being accepted in the special school conditional on acquiring information. After an agent acquires information, she knows the quality. Because of that, she will apply only if it is good. It will depend on the conduct of the uninformed agents the value of this probability. If the uninformed apply to the special school it will be:

$$\mathbb{P}_{(G|A)} = \eta$$

Because all the agents will be applying in this case. If the uninformed prefer to not apply to the special school, it will be:

$$\mathbb{P}_{(G|A)} = min\left(\frac{\eta}{Q}, 1\right)$$

It will be equal to the fraction whenever $Q > \eta$, and equal to one for smaller values of Q.

Then, an agent will acquire information if:

$$U_i\left(A_i \mid Q\right) \geqslant U_i\left(NA_i \mid Q\right)$$

In terms of the parameters of the model, this can be expressed as:

$$\mathbb{P}_{(G|A)} p u_1 - \lambda i \geq max\left(\mathbb{P}_G p u_1 + \mathbb{P}_B (1-p) u_2, 0\right) \tag{3.3.1}$$

The difference with the model of common valuations comes from the fact that now the uninformed agents' behavior is endogenous. In some cases, it will depend on Q what is the optimum choice for the uninformed, in other of the special school's expected payoffs. This is mainly because of the "curse of the uninformed". If too many agents are acquiring information, the uninformed agent will be selected with a very high probability when the school is of bad quality.

3.4. *Behavior of agents who do not acquire information:*

Because of the endogenous nature of the opportunity cost, we map the "reaction function" of the uninformed agents to different information acquisition levels.

We know that the uninformed agents have only two options: to apply or not to the special school. They will apply if the utility of doing so is greater than applying to the outside option (zero). Also, we know that all agents, independent of their position in the continuum, have the same payoffs for this. Then, all are equal in this, and a generic uninformed applies to the special school if:

$$\eta p u_1 + \min\left(\frac{\eta}{1-Q}, 1\right)(1-p) u_2 \geq 0 \tag{3.4.1}$$

The equation is from the opportunity cost in the previous sub-section. Then, we have two cases: congestion and no congestion when the school is of bad quality. In the scenario without congestion, an agent applies if:

$$\eta p u_1 + (1-p)u_2 \geq 0, \quad \text{if } \frac{\eta}{1-Q} > 1$$

Solving for η:

$$\eta \geq \frac{(1-p)\bar{u}_2}{pu_1}$$

Definition 4: We define

$$\eta^* = \frac{(1-p)\bar{u}_2}{pu_1} \tag{3.4.2}$$

As the minimum level of seats available on the special school consistent with all uninformed agents applying to that school, and conditional on the special school having no congestion when it is of bad quality.

In the second one, now solving for Q:

$$\eta p u_1 + \frac{\eta}{1-Q}(1-p)u_2 \geq 0, \quad \text{if } \frac{\eta}{1-Q} \leq 1$$

$$Q \leq 1 - \frac{(1-p)\bar{u}_2}{pu_1}$$

Definition 5: We define

$$Q^* = 1 - \frac{(1-p)\bar{u}_2}{pu_1} \tag{3.4.3}$$

As the level of information acquisition that makes the uninformed agents indifferent between applying or not to the special school when there is congestion in it independent of its quality.

Then, and since congestion when it is of bad quality is endogenous to the equilibrium (and latter to the decision of the planner), we can separate the space of possible parameters on two cases: $\eta < \eta^*$ and $\eta \geq \eta^*$. We resume the results on one lemma for each situation:

Lemma 19: if $\eta < \eta^*$, the uninformed agents apply to the special school only if $Q < Q^*$.

Lemma 20: if $\eta \geq \eta^*$, the uninformed agents apply to the special school.

Then, an important parametric condition is if $\eta \gtrless \eta^*$. Having clear the behavior of the uninformed agents, we can go to solve the equilibrium and the problem of the Social Planner.

3.5. *Equilibrium:*

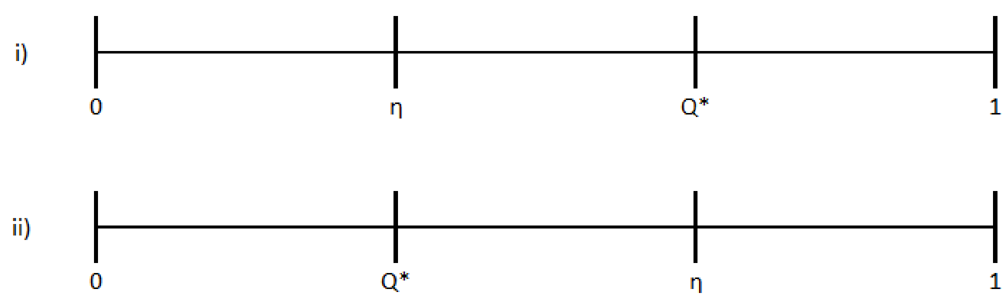
In this section we solve the equilibrium for different parametric conditions, that we can divide in two great cases: one when $\eta \geq \eta^*$ and the other when $\eta < \eta^*$.

1) $\eta < \eta^*$: From lemma 19, we know that if $Q < Q^*$ the uninformed agents will apply to the special school. In that situation, because Q^* is smaller than $1-\eta$ and all uninformed agents apply, we have

congestion independent of the type of the school[9]. If $Q > Q^*$, we are going two have two sub-cases that will depend on the parameters' values. One were $Q \geq \eta$ and other were $Q < \eta$. It will be important to analyze because when the uninformed agents do not apply to the special school and $Q > \eta$, we have congestion if the special school is of good quality[10]. Moreover, for values of Q smaller than η, but bigger than Q^*, we do not have congestion. That case exists only if $Q^* < \eta$, a situation that does not always happen. In Figure 8 are the two possible parametric cases that can arise.

We continue now solving each possible case separately. First the cases when $Q \geq max(Q^*, \eta)$, then when $Q \in [Q^*, \eta]$ and last when $Q < Q^*$. After that, we have all the possible equilibrium cases covered, so we obtain two propositions, following the structure of Figure 8.

Figure 8



Now, checking the different tranches of Q that can exist in this two cases:

Assumption 13: We assume $Q \geq max(Q^*, \eta)$.

Because $Q > Q^*$, the uninformed do not apply to the special school. Also, because $Q > \eta$, there will be congestion when the special school is of good quality. An agent acquires if:

$$\frac{\eta}{Q} p u_1 - \lambda i \geq 0$$

The marginal agent satisfies:

$$\frac{\eta}{Q} p u_1 = \lambda Q$$

$$Q = \sqrt{\frac{\eta p u_1}{\lambda}} \tag{3.5.1}$$

We resume where this solution is the equilibrium in the next lemmas:

Lemma 21: If $\eta < \eta^*$, $Q^* > \eta$ and $\lambda \in \left[\eta p u_1, \frac{\eta p u_1}{Q^{*2}}\right]$, then $Q = \sqrt{\frac{\eta p u_1}{\lambda}}$.

[9] When the special school is of bad quality, the probability of being accepted is: $\mathbb{P}_B = min\left(\frac{\eta}{1-Q}, 1\right)$. Then, there is congestion if the fraction is less than one: $\frac{\eta}{1-Q} < 1$. Expression that is equivalent to: $Q < 1 - \eta$.

[10] When the uninformed agents do not apply to the special school, no agent apply to the special school when it is of bad quality.

Lemma 22: If $\eta < \eta^*$, $Q^* \leq \eta$ and $\lambda \in \left[\eta p u_1, \frac{p u_1}{\eta^2}\right]$, then $Q = \sqrt{\frac{\eta p u_1}{\lambda}}$.

Lemma 23: If $\eta < \eta^*$ and $\lambda < \eta p u_1$, then $Q = 1$.

Assumption 14: We assume $Q \in [Q^*, \eta]$.

This case is only possible when $Q^* < \eta$. Here, as in the previous one, we know from lemma 19 that the uninformed agents do not apply to the special school. Also, $Q < \eta$, so there is no congestion when the school is of good quality. Then, an agent acquires information if:

$$p u_1 - \lambda i \geq 0$$

Then, the marginal agent:

$$p u_1 - \lambda Q = 0$$
$$Q = \frac{p u_1}{\lambda} \tag{3.5.2}$$

In the next lemma we specify the range of parameters where this is a solution to the equilibrium.

Lemma 24: If $\eta < \eta^*$, $Q^* < \eta$ and $\lambda \in \left[\frac{p u_1}{\eta}, \frac{p u_1}{Q^*}\right]$, then $Q = \frac{p u_1}{\lambda}$.

Assumption 15: We assume $Q < Q^*$.

It is the third possible case when $\eta < \eta^*$. From lemma 19 we know the uninformed agents will apply to the special school. Also, because $Q^* < 1 - \eta$, there will be congestion when the school is of bad quality. Then, an agent acquires information if:

$$\eta p u_1 - \lambda i \geq \eta p u_1 + \frac{\eta}{1 - Q}(1 - p) u_2$$

The marginal agent acquiring information:

$$\frac{\eta}{1 - Q}\bar{u}_2 - \lambda Q = 0 \tag{3.5.3}$$

Rearranging terms, we can write it down like:

$$\lambda Q^2 - \lambda Q + \eta(1 - p)\bar{u}_2 = 0$$

Equation that has two solutions:

$$Q^C = \frac{\lambda - \sqrt{\lambda^2 - 4(1 - p)\lambda\eta\bar{u}_2}}{2\lambda}$$

$$Q^D = \frac{\lambda + \sqrt{\lambda^2 - 4(1-p)\lambda\eta\bar{u}_2}}{2\lambda}$$

In the next lemma we define the optimum solution and the values of λ consistent with it.

Lemma 25: If $\eta < \eta^*$ and $\lambda > \frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}$, then $Q = Q^C$.

Because of the change in the behavior of the uninformed agents', if we compare the thresholds between the different cases, we can note that for $\eta < Q^*$ and $\eta \geq Q^*$ we have zones with multiple equilibriums. We characterize them in the next lemmas.

Lemma 26: If $\eta < \eta^*$, $Q^* > \eta$ and $\lambda \in \left[\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}, \frac{\eta p u_1}{Q^{*2}}\right]$, there are multiple equilibriums, with $Q = Q^C$ or $Q = \sqrt{\frac{\eta p u_1}{\lambda}}$.

For the cases when $Q^* \leq \eta$ we are going to have two sub-cases, depending on the different values of η. We check them separately:

Lemma 27: If $\eta < \eta^*$, $Q^* \leq \eta$, $\eta > \sqrt{Q^*}$ and $\lambda \in \left[\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}, \frac{p u_1}{Q^*}\right]$, there are multiple equilibriums, with $Q = Q^C$ or $Q = \frac{p u_1}{\lambda}$.

Lemma 28: If $\eta < \eta^*$, $Q^* \leq \eta$, $\eta \leq \sqrt{Q^*}$ and $\lambda \in \left[\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}, \frac{p u_1}{Q^*}\right]$, there are multiple equilibriums for Q. These are:

If $\lambda \in \left[\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}, \frac{p u_1}{\eta}\right]$:

$$Q = Q^C \quad \text{or} \quad \sqrt{\frac{\eta p u_1}{\lambda}} \tag{3.5.4}$$

If $\lambda \in \left[\frac{p u_1}{\eta}, \frac{p u_1}{Q^*}\right]$:

$$Q = Q^C \quad \text{or} \quad \frac{p u_1}{\lambda} \tag{3.5.5}$$

With this, we can write down the propositions that summarize the results for the three possible cases:

Proposition 7: If $\eta < \eta^*$ and $Q^* \geq \eta$ the equilibrium as a function of the acquisition cost parameter is:

$$Q = \begin{cases} 1 & if \;\; \lambda \leq \eta p u_1 \\ \sqrt{\frac{\eta p u_1}{\lambda}} & if \;\; \lambda \in \left[\eta p u_1, \frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}\right] \\ Q^C \;\; or \;\; \sqrt{\frac{\eta p u_1}{\lambda}} & if \;\; \lambda \in \left[\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}, \frac{\eta p u_1}{Q^{*2}}\right] \\ Q^C & if \;\; \lambda > \frac{\eta p u_1}{Q^{*2}} \end{cases} \tag{3.5.6}$$

Proposition 8: If $\eta \in [\sqrt{Q^*}, \eta^*]$ the equilibrium, as a function of the acquisition cost parameter, is:

$$Q = \begin{cases} 1 & if \ \ \lambda \leq \eta p u_1 \\ \sqrt{\dfrac{\eta p u_1}{\lambda}} & if \ \ \lambda \in \left[\eta p u_1, \dfrac{p u_1}{\eta}\right] \\ \dfrac{p u_1}{\lambda} & if \ \ \lambda \in \left[\dfrac{p u_1}{\eta}, \dfrac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}\right] \\ Q^C \ \ or \ \ \dfrac{p u_1}{\lambda} & if \ \ \lambda \in \left[\dfrac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}, \dfrac{p u_1}{Q^*}\right] \\ Q^C & if \ \ \lambda > \dfrac{p u_1}{Q^*} \end{cases} \tag{3.5.7}$$

Proposition 9: If $\eta \in [Q^*, min(\eta^*, \sqrt{Q^*})]$ the equilibrium, as a function of the acquisition cost parameter, is:

$$Q = \begin{cases} 1 & if \ \ \lambda \leq \eta p u_1 \\ \sqrt{\dfrac{\eta p u_1}{\lambda}} & if \ \ \lambda \in \left[\eta p u_1, \dfrac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}\right] \\ Q^C \ \ or \ \ \sqrt{\dfrac{\eta p u_1}{\lambda}} & if \ \ \lambda \in \left[\dfrac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}, \dfrac{p u_1}{\eta}\right] \\ Q^C \ \ or \ \ \dfrac{p u_1}{\lambda} & if \ \ \lambda \in \left[\dfrac{p u_1}{\eta}, \dfrac{p u_1}{Q^*}\right] \\ Q^C & if \ \ \lambda > \dfrac{p u_1}{Q^*} \end{cases} \tag{3.5.8}$$

Here the cases with multiple equilibriums arise because of the discontinuous change produced when the uninformed start (or stop) to apply to the special school.

An example of the different results under each proposition is displayed on Figure 9. There, the x-axis is for values of λ and the y-axis for values of Q. We can see there, on the lines of different colors, how the multiple equilibriums arise at different values of λ for each case. The only difference between the three graphics is in the value of η, that has to be changed to fit the restrictions.

2) When $\eta > \eta^*$: from lemma 20, we know that in this case, independent of the value of Q, the uninformed agents will always apply to the special school. We have two sub-cases, depending on if there is congestion when the special school is of bad quality or not.

Assumption 16: We assume $Q > 1 - \eta$.

In this sub-case we do not have congestion when the school is of bad quality. Then, an agent acquires information if:

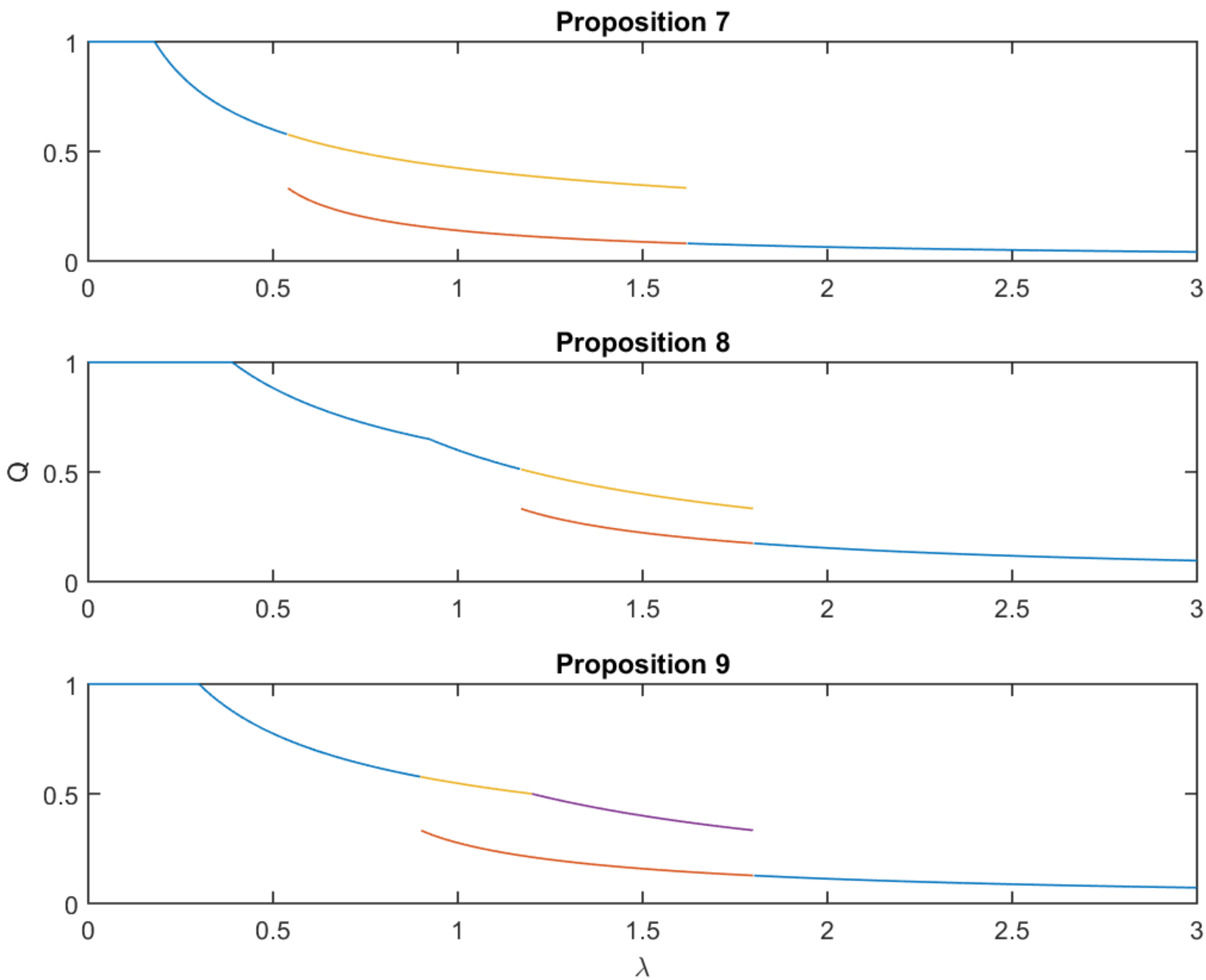
$$\eta p u_1 - \lambda i \geq \eta p u_1 + (1-p) u_2$$

The marginal agent accomplishes:

$$(1-p)\bar{u}_2 = \lambda Q$$

$$Q = \frac{(1-p)\bar{u}_2}{\lambda} \tag{3.5.9}$$

Figure 9



In the next two lemmas we summarize when there is a equilibrium with $Q > 1 - \eta$.

Lemma 29: If $\eta > \eta^*$ and $\lambda \in \left[(1-p)\bar{u}_2, \frac{(1-p)\bar{u}_2}{1-\eta}\right]$, then $Q = \frac{(1-p)\bar{u}_2}{\lambda}$.

Lemma 30: If $\eta > \eta^*$ and $\lambda < (1-p)\bar{u}_2$, then $Q = 1$.

Assumption 17: We assume $Q \leq 1 - \eta$.

In this case, there is congestion even when the special school is of bad quality. Then, an agent acquires if:

$$\eta p u_1 - \lambda i \geq \eta p u_1 + \frac{\eta}{1-Q}(1-p)u_2$$

The marginal agent accomplishes:

$$\frac{\eta}{1-Q}(1-p)\bar{u}_2 - \lambda Q = 0$$

Here we arrive to the same as when $\eta < \eta^*$ and $Q < Q^*$. Then, we know that the solution is:

$$Q^C = \frac{\lambda - \sqrt{\lambda^2 - 4(1-p)\lambda\eta\bar{u}_2}}{2\lambda}$$

The cases under which this solution represents the Q of equilibrium, as a function of λ, are defined on the next lemma.

Lemma 31: If $\eta \geq \eta^*$ and $\lambda > \frac{(1-p)\bar{u}_2}{1-\eta}$, then $Q = Q^C$.

We can summarize the results for this case in the next proposition:

Proposition 10: If $\eta \geq \eta^*$, the equilibrium as a function of the acquisition of information cost parameter, is:

$$Q = \begin{cases} 1 & if \ \ \lambda \leq (1-p)\bar{u}_2 \\ \dfrac{(1-p)\bar{u}_2}{\lambda} & if \ \ \lambda \in \left[(1-p)\bar{u}_2, \dfrac{(1-p)\bar{u}_2}{1-\eta}\right] \\ Q^C & if \ \ \dfrac{(1-p)\bar{u}_2}{1-\eta} \end{cases} \tag{3.5.10}$$

There are no multiple equilibriums on Proposition 10 because the uninformed agents apply to the special school independent of the Q. It is the same intuition as in the first model.

An example of this proposition is available on Figure 10. There, the three kinds of equilibrium in pure strategies are in different colors.

3.6. *Social Planner:*

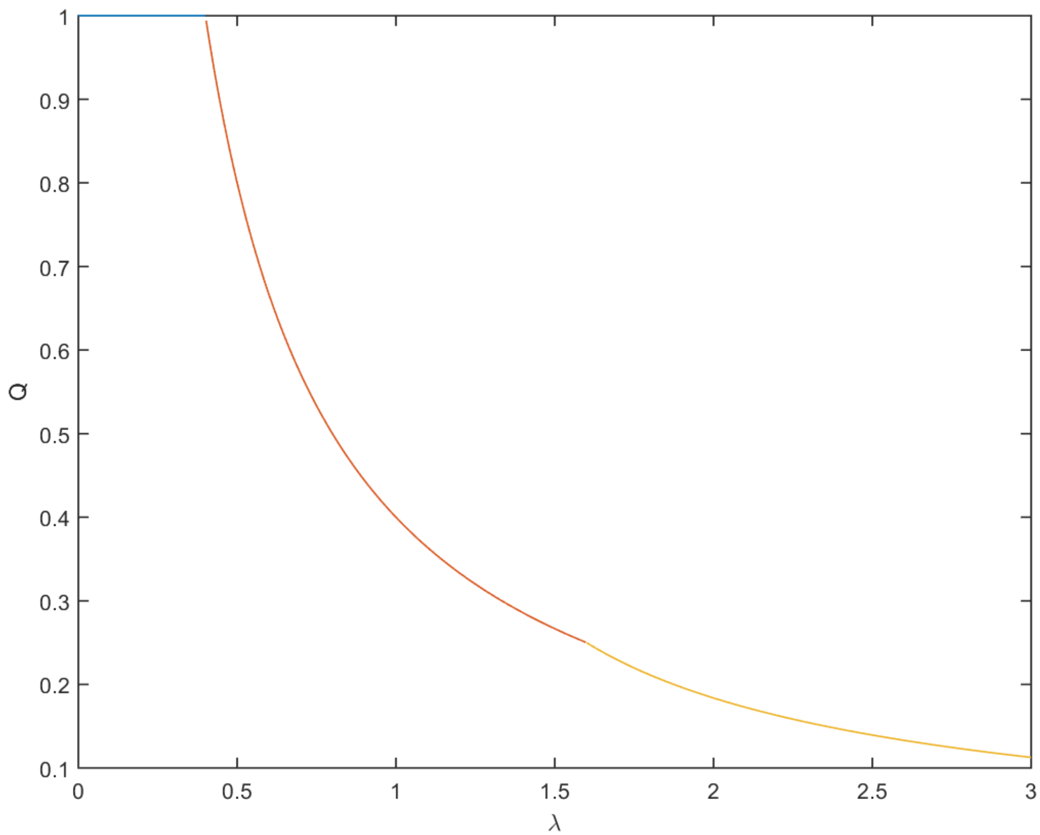
In this section, we analyze the problem from the perspective of a planner. We show first her utility in general, then for the different cases, and lastly, we find the planner's optimum responses given the parameters.

Her utility can be written down on two sections:

$$Q \geq \eta: \quad U_{sp}(Q) = \frac{-\lambda Q^2}{2} + p\eta u_1 + (1-p)\min\left(\frac{\eta}{1-Q}, 1\right)(1-Q)u_2 \mathbb{1}_{\text{un. apply}} \tag{3.6.1}$$

$$Q < \eta: \quad U_{sp}(Q) = \frac{-\lambda Q^2}{2} + p\left[Qu_1 + (\eta - Q)u_1 \mathbb{1}_{\text{un. apply}}\right] + (1-p)\min\left(\frac{\eta}{1-Q}, 1\right)u_2 \mathbb{1}_{\text{un. apply}} \tag{3.6.2}$$

Figure 10



Where $\mathbb{1}_{\text{un. apply}}$ is an indicator function that takes the value of one when the uninformed agents apply to the special school and zero otherwise. The difference between $Q \geq \eta$ and $Q < \eta$ is that in the first one, there is always congestion when the school is of good quality, and in the second, there is no congestion in that situation. It is important to note that there is always congestion when all the uninformed are applying, and the school is of good quality.

Also, and as in the equilibrium, the uninformed agents' behavior is endogenous to the Q selected by the planner.

3.6.1. Utility function:

From lemmas 19 and 20, we know the uninformed agents' behavior to different levels of Q. With that, we analyze the utility function of the planner under two main parametric constraints. These constraints are if the proportion of seats available is smaller or bigger than η^*. Here we characterize the different forms that the utility function takes under each restriction.

1) If $\eta < \eta^*$: as in the equilibrium when $\eta < \eta^*$, the utility of the Social Planner has three sections. We resume them in the next lemmas:

Lemma 32: If $\eta < \eta^*$ and $Q \geq max\,(Q^*, \eta)$, the utility function of the Social Planner is equal to: $-\frac{\lambda Q^2}{2} + \eta p u_1$

Lemma 33: If $\eta < \eta^*$ and $Q \in [Q^*, \eta]$, the utility function of the Social Planner is equal to: $-\frac{\lambda Q^2}{2} + p u_1 Q$.

Lemma 34: If $\eta < \eta^*$ and $Q < Q^*$, the utility of the Social Planner is equal to: $-\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2)$.

2) When $\eta \geq \eta^*$: because here the uninformed apply to the special school independent of Q, there are only two possible situations in this case. We characterize them in the next two lemmas.

Lemma 35: If $\eta > \eta^*$ and $Q > 1-\eta$, the utility of the Social Planner is equal to: $-\frac{\lambda Q^2}{2} + p\eta u_1 + (1-p)(1-Q)u_2$.

Lemma 36: If $Q \leq 1-\eta$, the utility of the Social Planner is equal to: $-\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2)$.

3.6.2. Optimal acquisition of information:

Taken as given the planner's utility for each sub-case, we obtain the level of Q and compare their utility between cases to get the optimal response under the case of $\eta < \eta^*$ and $\eta \geq \eta^*$.

1) $\eta < \eta^*$: as in the point one of the equilibrium we have two possible cases depending on the value of the parameters. Both situations are displayed on Figure 8. We analyze the different sections that compose this two cases:

Assumption 18: We assume $Q^* \geq \eta$.

Using lemmas 32 and 34, we know that under Assumption 18, the utility function of the Social Planner has two zones:

$$U_{sp}(Q) = \begin{cases} -\frac{\lambda Q^2}{2} + p\eta u_1 & if \quad Q \geq Q^* \\ -\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2) & if \quad Q < Q^* \text{ and } pu_1 + (1-p)u_2 > 0 \end{cases} \tag{3.6.3}$$

The Social Planner maximizes in each zone. The next two lemmas show the optimum value of Q for each one with its corespondent utility.

Lemma 37: If $\eta < \eta^*$, $Q^* > \eta$ and $Q \geq Q^*$, the local optimum Q for the Social Planner is $Q = Q^*$, and the utility she receives is: $-\frac{\lambda Q^{*2}}{2} + p\eta u_1$.

Lemma 38: If $\eta < \eta^*$ and $Q < Q^*$, the local optimum Q for the Social Planner is $Q = 0$, and the utility she receives is: $\eta(pu_1 + (1-p)u_2)$.

Then, the Planner compares this two utilities and chooses the biggest one. Checking when the utility of $Q = Q^*$ is higher:

$$U_{sp}(0) \leq U_{sp}(Q^*)$$

Replacing:

$$\eta(pu_1 + (1-p)u_2) \leq -\frac{\lambda Q^{*2}}{2} + p\eta u_1$$

Then:

$$\lambda \leq \frac{2\eta(1-p)\bar{u}_2}{(Q^*)^2}$$

Definition 6: We define

$$\tilde{\lambda} = \frac{2\eta(1-p)\bar{u}_2}{Q^{*2}} \tag{3.6.4}$$

As the level of λ that leaves the Social Planner indifferent between the solution with $Q = 0$ and the one with $Q = Q^*$.

If $\lambda < \tilde{\lambda}$, information is relatively cheaper, then the planner chooses an equilibrium where a fraction Q^* acquires information and the uninformed do not apply to the special school. There is congestion only if the school is of good quality[11]. In contrast, if $\lambda \geq \tilde{\lambda}$, information is more expensive, then the planner chooses $Q = 0$ and makes all the uninformed agents apply. When that happens, there is always congestion, independent of the actual quality of the school.

With this, we summarize the response of the planner for this case in the next proposition:

Proposition 11: If $\eta < \eta^*$ and $Q^* > \eta$, the Social Planner chooses:

$$Q = \begin{cases} Q^* & if \quad \lambda \leq \tilde{\lambda} \\ 0 & if \quad \lambda > \tilde{\lambda} \end{cases} \tag{3.6.5}$$

An example of the utility the planner receives, in this case, is in Figure 11. The x-axis is for values of Q and the y-axis for values of $U_{sp}(Q)$. Both zones of the utility are decreasing on Q, but because of $\lambda < \tilde{\lambda}$, the maximum is $Q = Q*$. With all the uninformed agents not applying to the special school, and enough people acquiring to have congestion on it (because of $Q^* > \eta$). It is worth noting that this is the minimum amount of people acquiring compatible with a solution where the uninformed agents do not apply. The planner is then willing to sacrifice some congestion (make more people acquire than the number of seats available) in exchange for the uninformed to do not start applying.

If she diminishes the acquisition of information by a small and positive quantity "ϵ", all the uninformed will start to apply, leaving her with a cost $\epsilon\lambda$ smaller, but with a much smaller benefit, because if the school is of good quality it was already full, but previously it was empty when it was of bad quality.

[11] All this subject to $\tilde{\lambda}$ being smaller or equal to the maximum value of λ where the solution with $Q = Q^*$ is defined. If it is bigger than that, the new cutoff is the maximum value of λ where the solution with $Q = Q^*$ is defined.

Now, since there are $1 - Q^* + \epsilon$ uninformed agents applying, there will be agents selected in the bad case too, generating an expected dis-utility of the change.

Figure 11

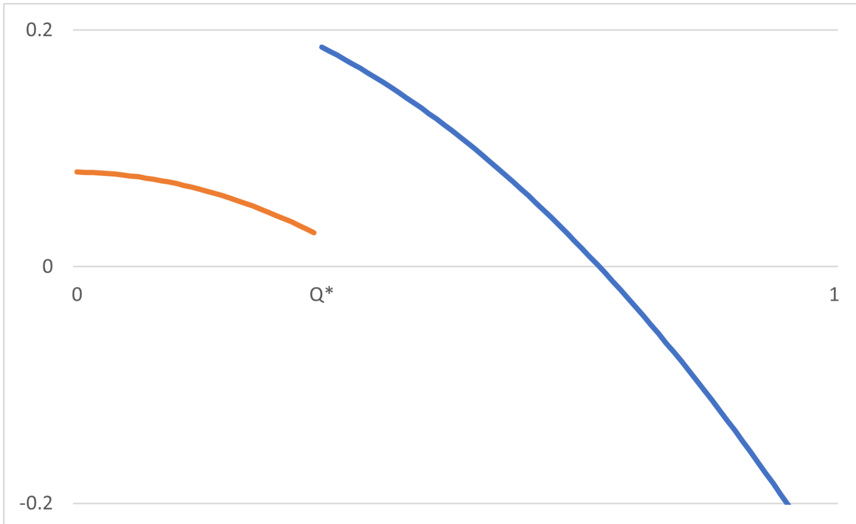
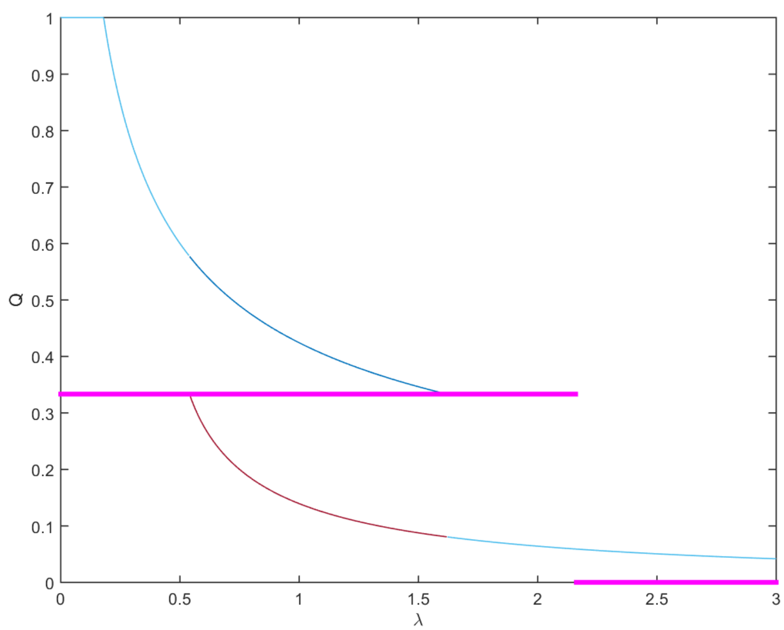


Figure 12



Comparison with the equilibrium solution:

When $\eta < \eta^*$, we know that the uninformed agents' behavior is endogenous and depends on the level of Q. Thus, it is not as straightforward as in the model with independent valuations, the comparison between the equilibrium and what the planner prefers to do.

Then, we characterize and comment the behavior of the planner relative to the equilibrium in the next two lemmas.

Lemma 39: If $\eta < \eta^*$, $\eta \leq Q^*$ and $\eta^* \geq 0.5$, the Social Planner makes the uninformed agents not apply to the special school for bigger values of λ than the equilibrium.

Remembering $\eta^* = (1-p)\bar{u}_2/pu_1$, we know that when the expected loss of applying uninformed ($(1-p)\bar{u}_2$) divided by the expected benefit (pu_1) is greater than 0.5, the planner will prefer to make the uninformed not apply for values of λ bigger than the ones found on the equilibrium.

On Figure 12 there is a comparison between the Social Planner for this case. There the x-axis is for λ and the x-axis for Q. The pink line represents the solution of the planner. We can see there three interesting phenomena:

i. The planner chooses a higher value of λ to stop making the uninformed not apply to the special school. When the expected loss/win ratio that is η^* is high enough, the planner will prefer to avoid people participating in that lottery as much as possible, even though she and the agents are not risk-averse.

ii. When the planner is making the uninformed apply, she chooses $Q = 0$. This situation is because, in this model, all agents have the same valuations. To understand this, we can imagine a situation where no agent is acquiring information, and all the uninformed apply. If one agent decides to acquire, it will pay the cost and apply only if the school is of good quality. However, since there is congestion independent of the school's condition, the aggregate benefit will be the same, but the cost will be $i\lambda$ bigger.

   The planner prefers $Q = 0$ when the information is costlier than the benefit of the situation with a fraction Q^* acquiring and no uninformed applying. In contrast with the equilibrium. There the agents with low costs will be acquiring, and those with higher ones applying uninformed, but with higher probabilities of entering the special school when it is bad, because of the "curse of the uninformed".

iii. When λ is smaller than the cutoff between the zone with the uninformed not applying and multiple equilibriums, the planner prefers to minimize congestion, conditional on being in the zone where the uninformed agents do not apply. In other words, she minimizes congestion, conditional on the uninformed agents are not applying to the special school.

   For bigger , the planner enters the zone where there are multiple equilibriums. Because of that, we are not sure if she it is choosing more or less information acquisition than the equilibrium. We are only sure that the planner prefers that the uninformed do not apply to the special school for bigger values of λ than the equilibrium.

Lemma 40: If $\eta < \eta^*$, $\eta < Q^*$ and $\eta^* < 0.5$ the Social Planner makes the uninformed agents not apply to the special school for smaller values of λ than the equilibrium.

In this situation, the intuition is similar, the main difference is that here the value of the expected loss, relative to the expected utility (η^*) is smaller. This means that it is not as bad as before to go uninformed to the special school when $Q = 0$. Because of that, the planner prefers to change the behavior of the agents at a smaller cost than before.

ii) $Q^* < \eta$: taken as given the parameters and using lemmas 32, 33, and 34 we know that the utility of the Social Planner has three zones:

$$U_{sp}(Q) = \begin{cases} -\frac{\lambda Q^2}{2} + p\eta u_1 & if \quad Q \geq \eta \\ -\frac{\lambda Q^2}{2} + pu_1 Q & if \quad Q \in [Q^*, \eta] \\ -\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2) & if \quad Q < Q^* \end{cases} \tag{3.6.6}$$

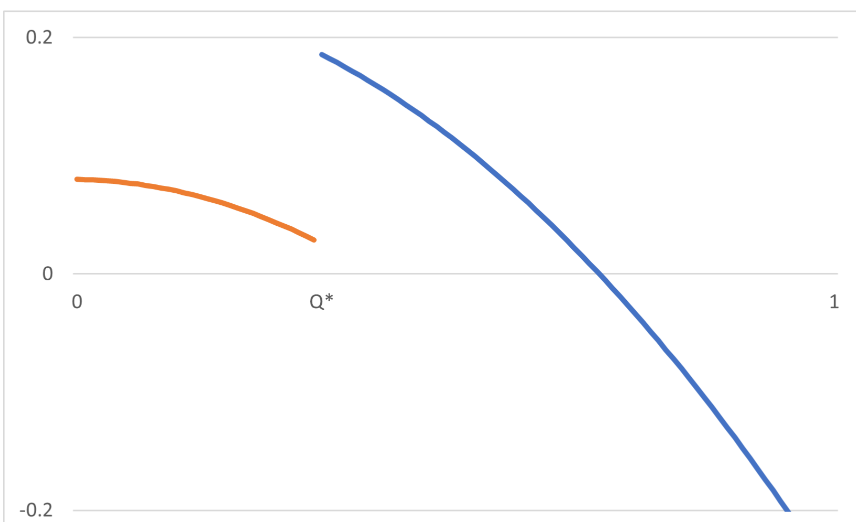
As in the previous scenario, the Social Planner maximizes inside each zone. We use two lemmas to show the optimum value of Q for each situation, with its corresponding utility.

Lemma 41: If $\eta < \eta^*$, $Q^* < \eta$, $\lambda \in [\frac{pu_1}{\eta}, \frac{pu_1}{Q^*}]$ and $Q \in [Q^*, \eta]$, the local optimum for the Social Planner is $Q = \frac{pu_1}{\lambda}$, and the utility she receives is equal to: $\frac{p^2u_1^2}{2\lambda}$.

Lemma 42: If $\eta < \eta^*$, $Q^* < \eta$, $\lambda < \frac{pu_1}{\eta}$ and $Q \geq \eta$, the local optimum Q for the Social Planner is $Q = \eta$, and the utility she receives is equal to: $-\frac{\lambda\eta^2}{2} + p\eta u_1$.

Lemma 43: If $\eta < \eta^*$ and $Q < Q^*$, the local optimum Q for the Social Planner is $Q = 0$, and the utility she receives is equal to: $\eta(pu_1 + (1-p)u_2)$.

Figure 13



With the optimum Q for each segment and the utilities, we check for what values of λ the planner prefers the uninformed agents applying or not.

First, we can note that the cases on lemmas 41 and 42 are defined for certain values of λ. If λ is outside that range, these are not feasible. The maximum value where they exist is $\lambda = \frac{pu_1}{Q^*}$. If λ is any bigger than that, the only feasible case is the one on lemma 43, where $Q = 0$.

For values of λ smaller than $\frac{pu_1}{Q^*}$, we have two more cases when the planner has to choose. We analyze first the case presented on lemma 41. There, all the uninformed agents stop applying to the special school and $Q < \eta$. We compare it to the utility of the planner with $Q = 0$.

Then, if the planner has to choose between $Q = 0$, and $Q = pu_1/\lambda$, she chooses the latter if:

$$U_{sp}(0) \leq U_{sp}(\frac{pu_1}{\lambda})$$

$$\eta(pu_1 + (1-p)u_2) \leq \frac{p^2u_1^2}{2\lambda}$$

$$\lambda \leq \frac{p^2u_1^2}{2\eta(pu_1 + (1-p)u_2)}$$

Definition 7: We define

$$\tilde{\lambda} = \frac{p^2u_1^2}{2\eta(pu_1 + (1-p)u_2)} \tag{3.6.7}$$

As the level of λ that leaves the Social Planner indifferent between the solution with $Q = 0$ and the one with $Q = pu_1/\lambda$.

Then, for values $\lambda < \tilde{\lambda}$, the Social Planner prefers acquiring information for some agents[12].

In the next two lemmas we clarify in what ranges of the cost parameter λ, the planner prefers a solution with agents acquiring over the one with $Q = 0$.

Lemma 44: If $\eta < \eta^*$, $Q^* < \eta$, $\lambda \in [\frac{pu_1}{\eta}, \frac{pu_1}{Q^*}]$ and $\eta \leq 0.5$, $U_{sp}(0) \leq U_{sp}(\frac{pu_1}{\lambda})$.

Lemma 45: If $\eta < \eta^*$, $Q^* < \eta$, $\lambda \in [\frac{pu_1}{\eta}, \tilde{\lambda}]$ and $\eta > 0.5$, $U_{sp}(0) \leq U_{sp}(\frac{pu_1}{\lambda})$.

Lemma 46: If $\eta < \eta^*$, $\eta > Q^*$ and $\lambda < \frac{pu_1}{\eta}$, $U_{sp}(0) \leq U_{sp}(\eta)$.

As we show on lemmas 44 and 45, the optimum depend if η is smaller or greater than 0.5. Because of that, we describe the optimum election for the planner, as a function of λ, on two propositions:

Proposition 12: If $\eta \in [Q^*, 0.5]$, the Social Planner chooses:

$$Q = \begin{cases} \eta & if \quad \lambda < \frac{pu_1}{\eta} \\ \frac{pu_1}{\lambda} & if \quad \lambda \in \left[\frac{pu_1}{\eta}, \frac{pu_1}{Q^*}\right] \\ 0 & if \quad \lambda > \frac{pu_1}{Q^*} \end{cases} \tag{3.6.8}$$

[12] All of this assuming that $pu_1 + (1-p)u_2 > 0$ and that $\tilde{\lambda}$ is smaller than the maximum value where the solution with $Q = pu_1/\lambda$ is defined, that is: $\lambda = \frac{pu_1}{Q^*}$

Proposition 13: If $\eta \in [max(Q^*, 0.5), \eta^*]$, the Social Planner chooses:

$$Q = \begin{cases} \eta & if \quad \lambda < \frac{pu_1}{\eta} \\ \frac{pu_1}{\lambda} & if \quad \lambda \in \left[\frac{pu_1}{\eta}, \tilde{\lambda}\right] \\ 0 & if \quad \lambda > \tilde{\lambda} \end{cases} \tag{3.6.9}$$

Figure 14

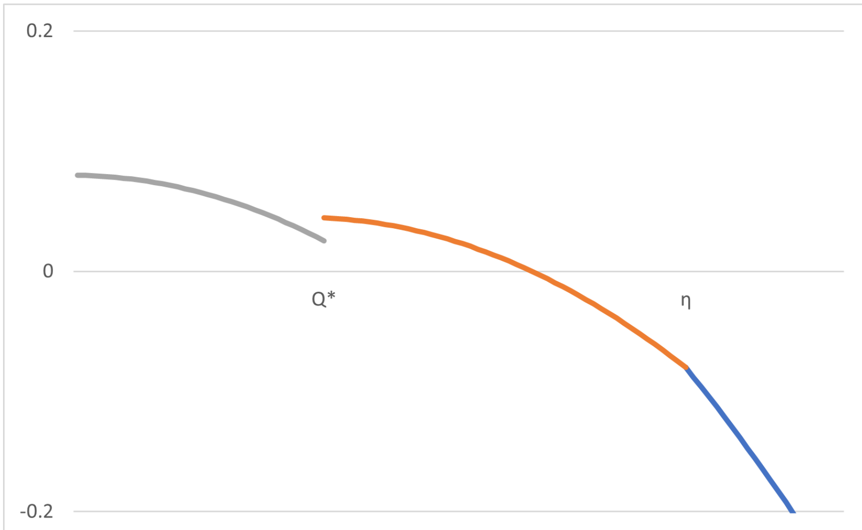
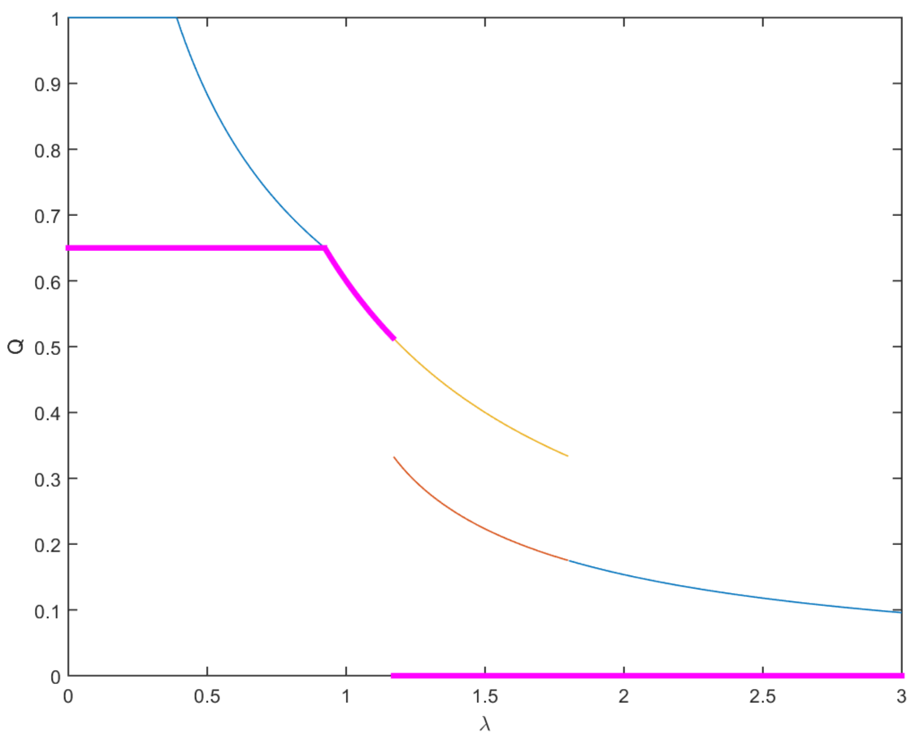


Figure 15



With this two propositions we cover the whole range of values where η is defined in this case ($\eta < \eta^*$

and $Q^* < \eta$, or $\eta \in [Q^*, \eta^*]$).

An example of Proposition 13 is on Figure 14. The x-axis is for Q and the y-axis for $U_{sp}(Q)$. There $\lambda > \tilde{\lambda}$, so the maximum is on $Q = 0$. We can note the discontinuity on Q^*, produced by the change in the uninformed agents' behavior. Also, we can note the change of the utility in η. For values of Q bigger than that, congestion starts when the special school is of good quality. It is easy to note how congestion negatively affects the utility, so the planner never chooses a $Q > \eta$, or in other words, she never chooses a solution with congestion in this case.

Comparison with the equilibrium solution:

This case is similar to the previous one, but with some differences. Because $\eta \geq Q^*$, there will be a zone where the planner is making agents acquire information, the uninformed are not applying, and there is no congestion when the school is of good quality. The planner has one less restriction, so here she does not have the zone with congestion when the special school is of good quality to make the uninformed not apply, as in the case with $\eta < Q^*$.

From the previous model, we know that if there is no congestion in the equilibrium, the planner will want the same solution as the equilibrium; this is a result we see in this case for both propositions and certain values of λ. We review these situations in detail ahead.

The next two lemmas helps us understand the properties of this solution:

Lemma 47: If $\eta < \eta^*$, $Q^* < \eta$ and $\eta \leq 0.5$, the Social Planner makes the uninformed not apply to the special school until the same λ that the equilibrium does.

Lemma 48: If $\eta < \eta^*$, $Q^* < \eta$ and $\eta > 0.5$, the Social Planner makes the uninformed not apply to the special school until a smaller level of λ that the equilibrium does.

On Figure 17 there is a comparison between the equilibrium found on proposition 8 with the solution of the planner found on proposition 17. There the x-axis represents λ and the y-axis values of Q. We can see clearly the result of lemma 48 there. If lemma 47 was graphed in the figure, the pink line when $Q = \frac{pu_1}{\lambda}$ will continue until the same point of the equilibrium (in the yellow line in the figure) and the solutions with $Q = 0$ will start at the level of λ that the first trench of the solution ends.

As we commented at the beginning of this comparison, the intuition is similar to the previous case. However, the possibility of having no congestion on the special school when agents do not acquire, lifts a binding restriction for the planner, making her change the behavior as the optimum Q is approaching from above to Q^*. In this sense, this case is an unrestricted form of the previous.

The forces interacting are the same, but without the restriction that made the planner acquire more in a specific case. Then, the planner always acquires less or the same level of information as the equilibrium.

2) $\eta \geq \eta^*$: from lemma 20, we know that in this case, all the uninformed agents apply to the special school, independent of the Q. When the special school is good, the payoff is so big that even when

the probability of being selected when the school is of bad quality is equal to one (or the "curse of the uninformed" is as big as it can be), they prefer to apply. Thus, we always have congestion when the special school is of good quality because all the agents will apply in this case.

The only threshold that changes the planner's utility function is when Q is bigger than $1-\eta$. Over that point, the proportion $(1-Q)$ of uninformed agents applying to the special school when it is of bad quality is smaller than η, so all the uninformed will be selected in it.

Using the lemmas 35 and 36, we can describe the Social Planner's utility with this two zones:

$$U_{sp}(Q) = \begin{cases} -\frac{\lambda Q^2}{2} + p\eta u_1 + (1-p)(1-Q)u_2 & if \quad Q > 1-\eta \\ -\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2) & if \quad Q \leq 1-\eta \end{cases} \tag{3.6.10}$$

As in the previous case, the Social Planner maximizes her utility in each zone. We now have two lemmas that shows us the optimum value of Q for each one:

Lemma 49: If $\eta > \eta^*$, $\lambda \in \left[(1-p)\bar{u}_2, \frac{(1-p)\bar{u}_2}{1-\eta}\right]$ and $Q > 1-\eta$, the optimum of the Social Planner is $Q = \frac{(1-p)\bar{u}_2}{\lambda}$, and the utility she receives is equal to: $p\eta u_1 + (1-p)u_2 - \frac{(1-p)^2 u_2}{2\lambda}$.

Lemma 50: If $\eta > \eta^*$, $\lambda < (1-p)\bar{u}_2$ and $Q = 1$, the local optimum of the Social Planner is $Q = 1$, and the utility she receives is equal to $-\frac{\lambda}{2} + p\eta u_1$.

Lemma 51: If $\eta > \eta^*$ and $Q \leq 1-\eta$, the local optimum of the Social Planner is $Q = 0$, and the utility she receives is equal to: $\eta(pu_1 + (1-p)u_2)$.

Knowing each zone's utility, the planner can now compare them to pick the best one.

First, and as in the previous cases, we can note that if $\lambda > \frac{(1-p)\bar{u}_2}{1-\eta}$, the only case that exists is where $Q = 0$. If we continue for values of λ smaller than $\frac{(1-p)\bar{u}_2}{1-\eta}$, there will be two more cases defined, then we need to check the optimum for the planner in each situation.

We compare the utility the planner obtains in each case, and for what values of λ she prefers the case with information acquisition over the other:

$$U_{sp}(0) \leq U_{sp}\left(\frac{(1-p)\bar{u}_2}{\lambda}\right)$$

$$\eta(pu_1 + (1-p)u_2) \leq p\eta u_1 + (1-p)u_2 - \frac{(1-p)^2 u_2}{2\lambda}$$

$$\lambda \leq \frac{(1-p)\bar{u}_2}{2(1-\eta)}$$

Definition 8: We define

$$\lambda^{**} = \frac{(1-p)\bar{u}_2}{2(1-\eta)} \tag{3.6.11}$$

As the level of λ that leaves the Social Planner indifferent between $Q = 0$ and $Q = \frac{(1-p)\bar{u}_2}{\lambda}$.

If $\lambda < \lambda^{**}$ (information is cheap), the planner prefers a solution with $Q = \frac{(1-p)\bar{u}_2}{1-\eta}$. If it is bigger than the cutoff, she will prefer the one where $Q = 0$.

Now, we check if λ^{**} belongs to the interval where the solution $Q = \frac{(1-p)\bar{u}_2}{\lambda}$ is defined. We test first if λ^{**} is smaller than the maximum level of λ were it is defined:

$$\frac{(1-p)\bar{u}_2}{1-\eta} \leq \lambda^{**}$$

$$\frac{(1-p)\bar{u}_2}{1-\eta} \leq \frac{(1-p)\bar{u}_2}{2(1-\eta)}$$

Expression that is always false. Then, we check if λ^{**} is bigger than the minimum value where the case with $Q = \frac{(1-p)\bar{u}_2}{1-\eta}$ exists:

$$(1-p)\bar{u}_2 \leq \frac{(1-p)\bar{u}_2}{2(1-\eta)}$$

$$\eta \geq 0.5 \tag{3.6.12}$$

If η is big enough, the case with $Q = \frac{(1-p)\bar{u}_2}{1-\eta}$ will be the chosen by the planner when $\lambda \in [(1-p)\bar{u}_2, \lambda^{**}]$. If $\eta < 0.5$, for every $\lambda \in [(1-p)\bar{u}_2, \infty[$, the optimum will be with $Q = 0$. Because is the only case defined.

Last, we compare the utility of the planner when $Q = 1$ to the case when $Q = 0$, to see when she prefers one over the other:

$$U_{sp}(0) \leq U_{sp}(1)$$

$$\eta(pu_1 + (1-p)u_2) \leq -\frac{\lambda}{2} + p\eta u_1$$

$$\lambda \leq 2\eta(1-p)\bar{u}_2$$

Definition 9: We define

$$\lambda^{***} = 2\eta(1-p)\bar{u}_2 \tag{3.6.13}$$

As the level of λ that leaves the Social Planner indifferent between $Q = 0$ and $Q = 1$.

If $\lambda^{***} \leq (1-p)\bar{u}_2$, $Q = 1$ will give to the Social Planner more utility than $Q = 0$. Then, we compare λ^{***} with $\lambda = (1-p)\bar{u}_2$, or the maximum level of λ where the solution with $Q = 1$ is feasible:

$$\lambda^{***} \leq (1-p)\bar{u}_2$$

$$2\eta(1-p)\bar{u}_2 \leq (1-p)\bar{u}_2$$

Expression that is true only if $\eta \leq 0.5$. With all this, we can describe the solution to this case via two propositions:

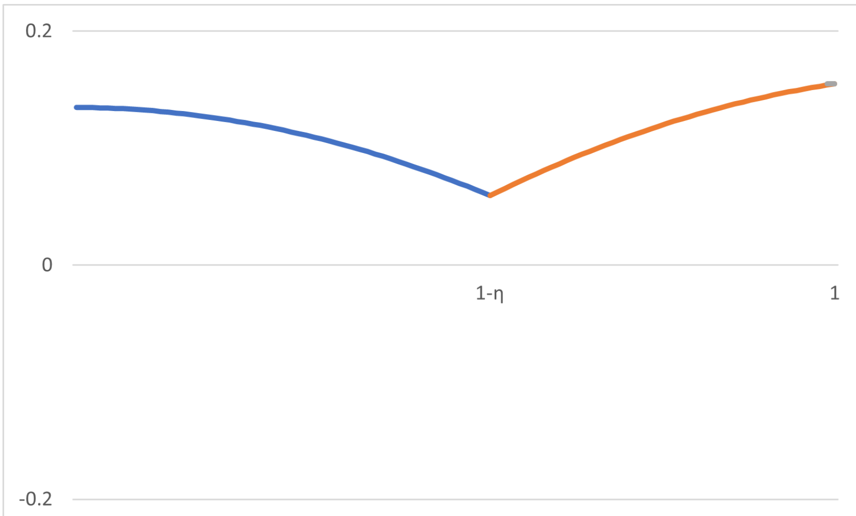
Proposition 14: If $\eta > \eta^*$ and $\eta \leq 0.5$.

$$Q = \begin{cases} 1 & if \quad \lambda \leq \lambda^{***} \\ 0 & if \quad \lambda > \lambda^{***} \end{cases} \tag{3.6.14}$$

Proposition 15: If $\eta > \eta^*$ and $\eta > 0.5$.

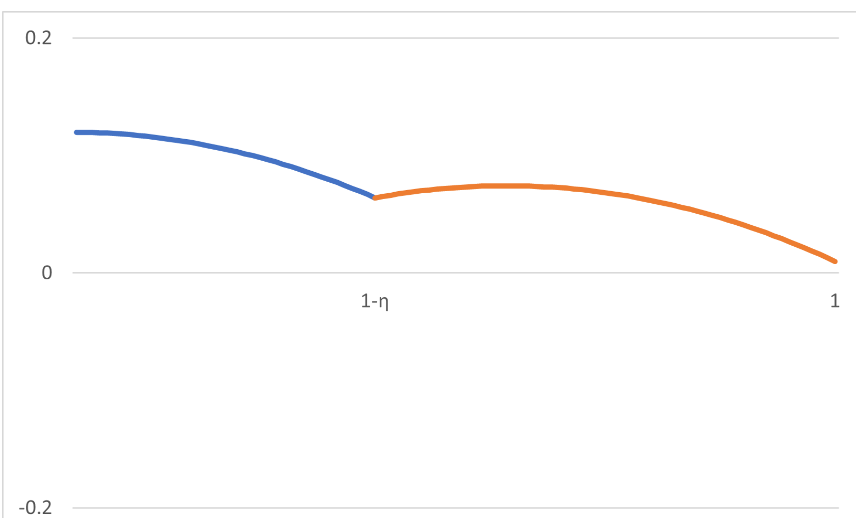
$$Q = \begin{cases} 1 & if \quad \lambda \leq (1-p)\bar{u}_2 \\ \frac{(1-p)\bar{u}_2}{\lambda} & if \quad \lambda \in [(1-p)\bar{u}_2, \lambda^{**}] \\ 0 & if \quad \lambda > \lambda^{**} \end{cases} \tag{3.6.15}$$

Figure 16



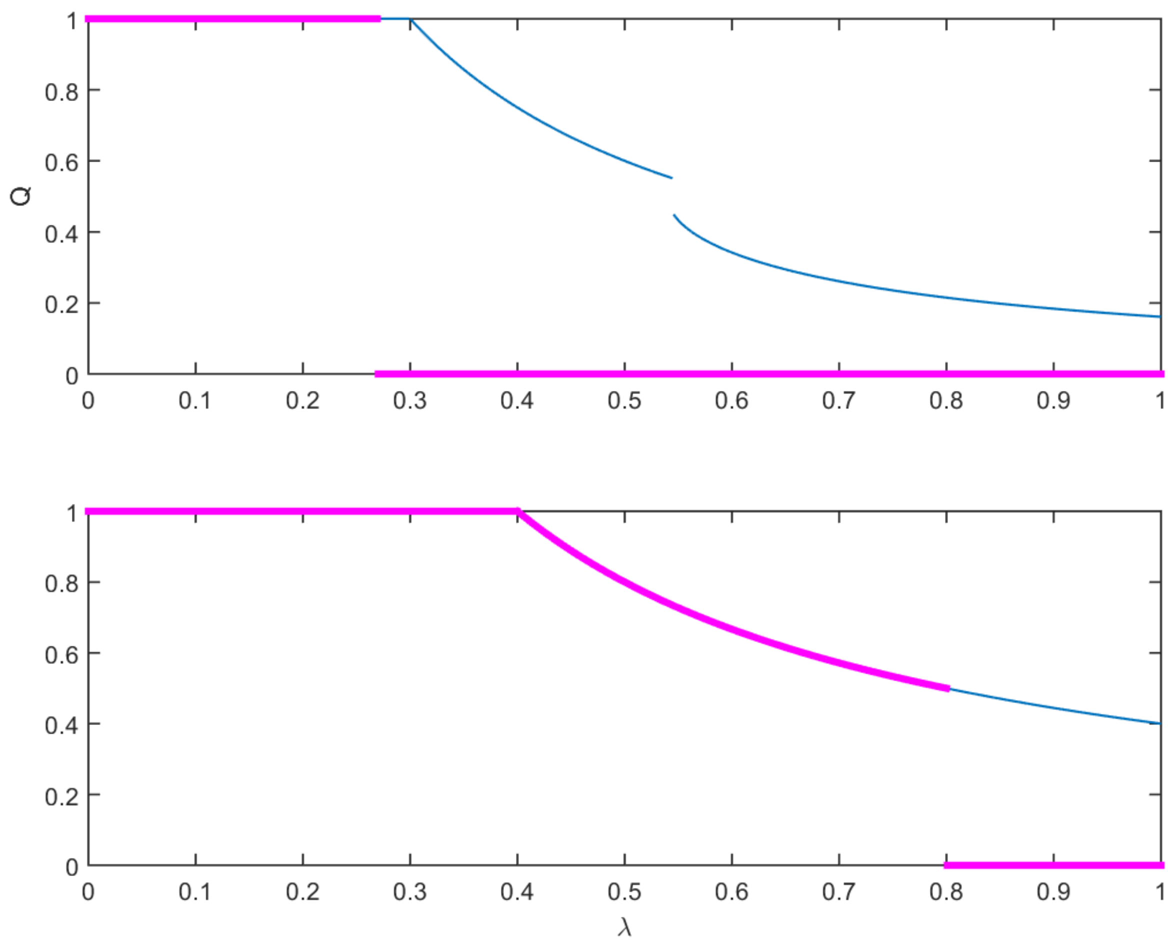
An example of proposition 14 is on Figure 16. There x-axis is for Q and the y-axis for $U_{sp}(Q)$. There $\lambda \leq \lambda^{**}$, so the maximum is on $Q = 1$. Also, we can note that in this case, there is no discontinuity. That is because there is no reaction from the uninformed agents to the choices of the Social Planner. They will always apply to the special school when $\eta > \eta^*$.

Figure 17



An example of proposition 15 is on Figure 17. There, $\eta < 0.5$ and a level of $\lambda > \lambda^{***}$. Because of that, the maximum is on $Q = 0$. Also, we can note that the main difference with the previous figure is that the orange zone has an interior maximum. This is because η is smaller than 0.5.

Figure 18



We can note that, in this case with $\eta \geq \eta^*$, the cutoffs depend on u_2. Here the uninformed agents will apply to the special school independent of the Q, so the motivation for acquiring information is to save themselves from the event where the special school is of bad quality. This is a finding that is shared with the cases when the uninformed apply always in the models when agents have independent preferences.

Comparison with the equilibrium:

As we discussed before on this same model, for values of $\eta \geq \eta^*$, the behavior of the uninformed agents is exogenous. For this values, they always apply to the special school.

Because of this, the equilibrium and the problem of the planner are simpler than in the cases when $\eta < \eta^*$. We characterize the problem in the next lemma:

Lemma 52: If $\eta \geq \eta^*$, the Social Planner acquires always less or the same amount of information than the equilibrium for any level of λ.

Until now, we have reviewed common properties for both propositions. Now we focus on their differences. The basic difference between both propositions is that fewer seats are available in the first one than in the second one. To understand how this affects the planner's decision, we analyze the solution of proposition 15 and then the one from proposition 14.

In proposition 15, at the highest λ that the planner acquires information, the Q she chooses is equal to $2(1-\eta)$. With that proportion of people acquiring information, there is are $1-2(1-\eta) = 2\eta - 1 > 0$ uninformed agents. What is interesting is that this proportion is always smaller than η. Then, there is no congestion when the special school is of bad quality. At the same time, there is always congestion when the school is of good quality. Then, the planner is minimizing the number of agents going to the special school if it is of bad quality, taking as given the uninformed agents' behavior.

Because the optimum Q for the planner decreases with λ, we know that for every $\lambda < \lambda^{**}$, she is minimizing the number of agents suffering the "curse of the uninformed", until she reaches $Q = 1$, when no agent has to go to the school if it is of bad quality.

The difference between the planner and the equilibrium under proposition 15 comes when λ is bigger than λ^{**}. There, the planner chooses $Q = 0$. There are two forces behind this, first is that the cost of the information is higher than before. The second is that the gain in utility given by an extra agent acquiring is equal to zero because there will be congestion when the school is of bad quality. If one agent acquires and scapes from that situation, there will be others replacing it. The private benefit of acquiring is big, but the social benefit is equal to zero because the planner values all agents as equals.

With this, we know that the planner only tries to avoid the "curse of the uninformed" for the greatest proportion of agents. When information is cheap she avoid it by making a big proportion acquire, so the minimum amount of agents goes to the special school if it is bad. If information is more expensive, she makes no agent acquire, so all have the same probability of being accepted, independent of the quality. The planner takes this always since the expected payoff is greater than zero and there is no risk aversion.

Then, when we are in the first case (with $\eta < 0.5$), the proportion of people needed to acquire information, so there is no congestion when the school is of bad quality, must be greater than in the case with $\eta > 0.5$. Then, the cost of that strategy is higher at each level of λ, making the planner prefer $Q = 0$ at smaller levels of λ than on proposition 15.

On Figure 18 is a comparison between the equilibrium and the two solutions of the planner for values of $\eta > \eta^*$. The x-axis is for values of λ and y-axis for values of Q, and the planner is on the pink line. The first figure is proposition 14 and the second proposition 15.

3.7. *Final Remarks of the model with common valuations:*

After analyzing the model as whole, we can note that the new force that we called: "the curse of the uninformed" changes the behavior of the agents, even though we assume $pu_1 + (1-p)u_2 > 0$, making the behavior of the uninformed agents endogenous to the model.

Moreover, we can affirm that even though the kind of "self-selection" that the "curse" represents, does not change the behavior of the uninformed in all the cases, it always changes the agents' utility. The uninformed gain less in expected value than in a situation without it. Furthermore, those with smaller acquisition costs can gain rents because of their condition, always having better payoffs than those preferred by a Social Planner. In other words, for them, the private benefit of acquiring information is bigger than the social benefit.

The forces driving the planner's behavior are three in this case: the "curse", the externalities as in the first model, and the cost of the information. The first force is an externality too, but we separate it because it acts in a less obvious way than the main effect.

When the cost is high enough, the information's cost always prevails, making the planner choose $Q = 0$. Nevertheless, that situation is always with a smaller Q than the equilibrium; there, the agents with small acquisition costs are always acquiring, generating a loss in the aggregate well being as we explained before.

The last important result from this model is that the planner will prefer to acquire less information than the equilibrium in almost all cases. The exception to that is in a few situations where the uninformed agents' behavior is changing between applying or not. The reason is that, in this model, the mechanism that made possible, in the first model, the positive externality when all agents applied to the special school is closed. The channel was that those who acquire left more seats available when they did not like the school, but now since all have the same opinion about it, there is a negative externality in that situation.

4. Further discussion:

In this work, we tested how in a school admission system selecting applicants under a DA mechanism, the equilibrium changed with two different assumptions over the agents' preferences. Specifically, if the preferences were independent or common when there is one special school of interest, showing that there were positive and negative externalities arising from the process when there are independent preferences. A situation that was different when valuations over the special school were common, externalities were negative in almost all situations.

This work falls short in two directions first, how things could change if the correlation on the preferences is not perfect. Second, how these findings interact with situations when there are more options available for acquiring information. We think that the second one is the mayor pitfall, and it is an interesting topic to continue researching.

Despite the limitations, this work gives plenty of insight into the discussion of policies aiming to help those agents with higher information acquisition costs. Moreover, it shows how important it is to understand if the preferences are vertically or horizontally differentiated. It may sound counterintuitive, but in some cases making information scarcer can help the aggregate welfare.

5. References:

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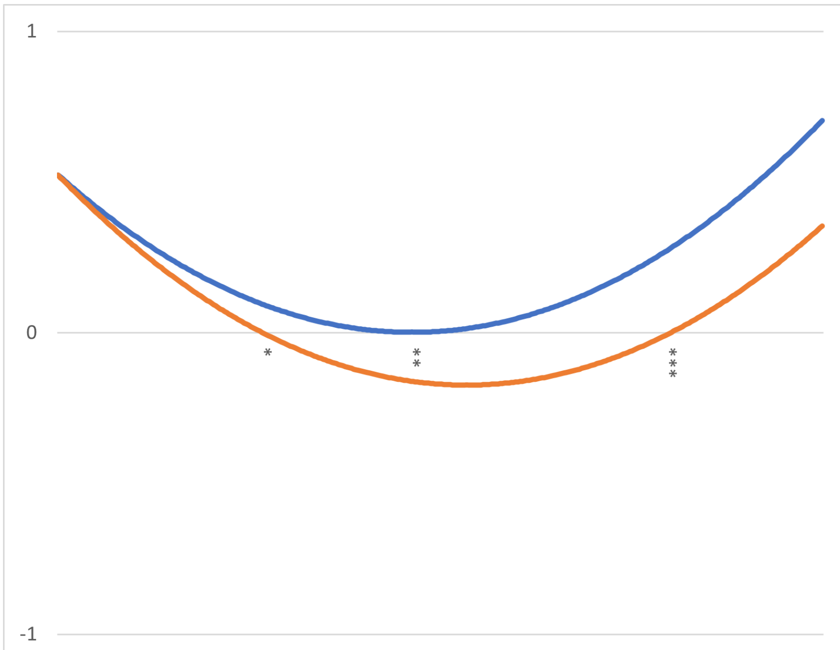
6. Appendix: Proofs

Proof lemmas 1 and 3: We have to prove two parts. First we show that Q^B is unstable, so it can be a real solution. Then we show that if λ is greater than the expression of the respective lemma, Q^A is the valid solution to the problem.

First part: if $Q^B \in \mathbb{R}$ and is less than one, Q^B is always unstable.

Plotting the equation 2.4.4 as a function of Q, we get the Figure 19. There, a solution to the equation is represented by the function crossing the x axis.

Figure 19



If we have a small random disturbance in the proportion of agents acquiring information around Q^B (noted as the point (***) in the Figure 19), from Q^* to Q^{rd}, we have that, if $Q^{rd} < Q^*$ the new marginal agent will have the equation 2.4.4 smaller than zero. This implies that the new marginal lost utility by acquiring information, making the agent not acquire. This will happen to every new marginal agent until $Q = Q^*$ of Q^A (represented by the point (*) in the same Figure). If $Q^{rd} > Q^*$, the new marginal agent will have the equation 2.4.4 greater than zero, then she will want to acquire information, and the same with the next new marginal agent until $Q = 1$. Thus, this second solution is not stable because, for every Q near Q^*, the system does not converge back to it.

Second part: we now prove that for $\lambda > min\left(\frac{(1-p)^2\bar{u}_2}{1-\eta}, \frac{(1-p)\eta\bar{u}_2}{p}\right)$ Q^A is as solution to the problem. We prove that the values inside the minimum are the values consistent with $Q < min(\tilde{Q}, 1)]$. Also that Q^A is always greater than zero and that for the range of feasible values of λ, this solution always exists.

Then, for Q^C to be a real solution to the problem, we need that: $(\lambda^2 - 4(1-p)^2\lambda\eta\bar{u}_2) \geq 0$ and that it belong to the interval $[0, 1]$.

For the determinant to be greater than zero:

$$\lambda^2 \geq 4(1-p)^2\lambda\eta\bar{u}_2$$

$$\lambda \geq 4(1-p)^2 \eta \bar{u}_2 \tag{6.0.1}$$

Defining with this:

$$\lambda^{min} = 4(1-p)^2 \eta \bar{u}_2 \tag{6.0.2}$$

As the minimum value of λ needed for the solution to be a real number. In Figure 19, the blue line represents the quadratic equation when $\lambda = \lambda^{min}$.

Now, we check if Q^A is greater than zero, when is smaller than $\tilde{Q}$ (lemma 3) and when it is smaller than 1 (lemma 1).

1) Greater than zero:

$$\frac{\lambda - \sqrt{\lambda^2 - 4(1-p)^2 \eta \bar{u}_2}}{2(1-p)\lambda} \geq 0$$

Solving, we obtain:

$$4(1-p)^2 \bar{u}_2 \eta \geq 0$$

Expression that is always true.

2) Less than $\tilde{Q}$:

$$\frac{\lambda - \sqrt{\lambda^2 - 4(1-p)^2 \eta \bar{u}_2}}{2(1-p)\lambda} \leq \frac{1-\eta}{1-p}$$

Solving, we obtain:

$$\lambda \geq \frac{(1-p)^2 \bar{u}_2}{1-\eta}$$

We can note that the minimum level of λ compatible with this solution is the same as $\tilde{\lambda}$, or the maximum level of cost compatible with an equilibrium without congestion.

$$\tilde{\lambda} = \frac{(1-p)^2 \bar{u}_2}{1-\eta}$$

Where $\tilde{\lambda}$ is always bigger than λ^{min}. Because of that, Q^A always belongs to the Real Numbers when $\eta > p$[13]

3) Less than one (lemma 1):

$$\frac{\lambda - \sqrt{\lambda^2 - 4(1-p)^2 \lambda \eta \bar{u}_2}}{2(1-p)\lambda} \leq 1$$

Solving, we obtain:

$$\lambda \geq \frac{(1-p)\eta \bar{u}_2}{p} \tag{6.0.3}$$

Defining:

$$\tilde{\lambda} = \frac{(1-p)\eta \bar{u}_2}{p} \tag{6.0.4}$$

[13] The proof of $\tilde{\lambda} \geq \lambda^{min}$ is:

$$\frac{(1-p)^2 \bar{u}_2}{1-\eta} \geq 4(1-p)^2 \eta \bar{u}_2$$

$$1 \geq 4(1-\eta)\eta$$

As the minimum value of lambda compatible with Q^A been less or equal to one. Now, we check if $\tilde{\lambda} \geq \lambda^{min}$:

$$\frac{(1-p)\eta \bar{u}_2}{p} \geq 4(1-p)^2 \eta \bar{u}_2$$

Simplifying:

$$1 \geq 4(1-p)p$$

Expression that is always true.

Then, we have proved the two parts of this lemma. ■

Proof lemma 2: We know from the proof of the lemma 3 that, when $\lambda < \frac{(1-p)\eta \bar{u}_2}{p}$, Q^A is no longer useful, because it will suggests values of Q bigger than 1. Then, we get first some intuition from what the agents do and finally prove the lemma.

Suppose that only the first agent of the continuum ($i = 0$) is acquiring information. Then, we have that equation 2.4.4 is equal to:

$$(1-p)\eta \bar{u}_2 > 0$$

Which, as it is easy to see, is an expression greater than zero. Because there are no costs and the benefit is always greater than zero, the first agent always wants to acquire information. Every time an agent has the equation 2.4.4 greater than zero, we know that she will want to acquire information.

The second part of this proof comes from Figure **??** and Q^A. As we check before, every time that $\lambda \leq \frac{(1-p)\eta \bar{u}_2}{p}$, Q^A will be greater than one, or in terms of the Figure, (*) > 1. Then, for all the agents of the continuum, the curve will pass over zero.

If we take together that if equation 2.4.4 is greater than zero agents in that point want to acquire, and that when $\lambda < \frac{(1-p)\eta \bar{u}_2}{p}$ the equation is always greater than zero for every agent of the continuum, we have proven that for any $\lambda < \frac{(1-p)\eta \bar{u}_2}{p}$, all agents want to acquire information. ■

Proof lemma 4: If $\eta > p$ we know that $\tilde{Q} < 1$. Then, there will exist values of Q between $\tilde{Q}$ and one were this problem exists. We already have the algebraic form of the solution:

$$Q^{nc} = \frac{(1-p)\bar{u}_2}{\lambda}$$

Then, we have to check when this solution belongs to the interval:

i) $Q^{nc} \geq \tilde{Q}$:

$$\frac{(1-p)\bar{u}_2}{\lambda} \geq \frac{1-\eta}{1-p}$$

$$\lambda \leq \frac{(1-p)^2 \bar{u}_2}{1-\eta}$$

Then, we can define:

$$\tilde{\lambda} = \frac{(1-p)^2 \bar{u}_2}{1-\eta}$$

As the maximum level of λ compatible with a equilibrium without congestion, or the level of λ compatible with $Q = \tilde{Q}$.

ii) $Q^{nc} \leq 1$:

$$\frac{(1-p)\bar{u}_2}{\lambda} \leq 1$$

Then, we can define:

$$\lambda^1 = (1-p)\bar{u}_2$$

As the value of λ compatible with $Q^{nc} = 1$.

Then, the lemma is proved. ■

Proof lemma 5: If $\eta > p$ and $\lambda < (1-p)\bar{u}_2$, the solution $Q = Q^{nc}$ will give us a Q bigger than one. But at the same time is impossible to have congestion since $\tilde{Q} < 1$. Then, the only plausible solution is to chose $Q = 1$. ■

Proof lemma 6: We know that the solution in this case for $Q \in [0, 1]$, then, we check the superior limit, since the first one is common to the previous lemma. Then $Q^{nc} \leq 1$:

$$\frac{pu_1}{\lambda} \leq 1$$

$$\lambda > pu_1$$

Then, the lemma is proved. ■

Proof lemma 7: If $\eta \geq p$ there is no congestion. It is equivalent to a situation where each agent solves its problem by their own. Then, every time that an agent faces:

$$i \leq \frac{pu_1}{\lambda}$$

She will acquire information. Is easy to see that we can find a λ small enough that even the agent with $i = 1$ (or the one with the highest cost) will want to acquire information. Now, looking for that λ:

$$1 \leq \frac{pu_1}{\lambda}$$

$$\lambda \leq pu_1 = \tilde{\lambda}$$

There, it is proved that for every value of λ smaller than $\tilde{\lambda}$, all agents will acquire information. ■

Proof lemma 8: We know that if $\eta < p$ the solution in this case for $Q \in [0, \hat{Q}]$, then, we check both limits:

1) Greater or equal to zero: is easy to note that this is always true, since $\lambda, p, u_1 > 0$.

2) $Q^{nc} \leq Q^{min}$:

$$\frac{pu_1}{\lambda} \leq \frac{\eta}{p}$$

$$\lambda \geq \frac{p^2 u_1}{\eta}$$

Then, the lemma is proved. ■

Proof lemma 9: We know that $Q \in [\hat{Q}, 1]$ to be in the case of a interior solution with congestion. Then, we prove that for $\lambda \geq \eta u_1$ and $\lambda \leq \frac{p^2 u_1}{\eta}$, the equilibrium is equal to $Q = \sqrt{\frac{\eta u_1}{\lambda}}$. The prove has two parts. A first one where see for what values the solution is smaller than one, and a second one when we check for what values of λ it is bigger than $\hat{Q}$.

1) Smaller than one:

$$\sqrt{\frac{\eta u_1}{\lambda}} \leq 1$$

$$\eta u_1 \leq \lambda$$

2) Bigger than $\hat{Q}$:

$$\sqrt{\frac{\eta u_1}{\lambda}} \geq \frac{\eta}{p}$$

$$\lambda \leq \frac{p^2 u_1}{\eta}$$

Then, the lemma is proved. ■

Proof lemma 10: With $\eta < p$. To prove this, we have to check for what range of parameters the last agent of the continuum (or the one with the biggest cost) has a utility of acquiring information greater or equal to zero:

$$1 \leq \frac{\eta}{pQ\lambda} p u_1$$

The reason is that, if she acquires, it is also convenient for everyone else to do that. Then we replace $Q = 1$ and solve:

$$\lambda \geq \eta u_1 \tag{6.0.5}$$

Then, if the proportion of seats multiplied by u_1 is greater than the biggest cost an agent has to pay, all agents acquire information. ■

Proof lemma 11: We test this on various part. First, for the zone where $Q = 1$ (or $Q = \tilde{Q}$ when $\eta > p$). Then for the zone of the interior solution.

Zone where $Q = 1$: If the utility of the Social Planner is convex, she always chooses $Q = 1$.

When a function is convex, the maximum is always on one side. For this case is either on $Q = 0$ or in $Q = 1$. Given the way we modeled the cost, the first agent always has $i = 0$, and the benefit of acquiring information is always greater or equal to zero. This agent always wants to acquire information. Then, $Q = 0$ is not a feasible solution.

Because of that, every time that the function is convex, the only possible solution is $Q = 1$. If $\eta > p$, the only difference is that now the interval for the solution is defined between zero and $\tilde{Q}$, instead of between zero and one.

Now, we prove that for small values of λ, the solution is convex and for λ bigger than the threshold, is always concave.

For this we use the second derivative (with respect to Q) to check the convexity of the function. This is:

$$\begin{aligned}\frac{\partial^2 U_{sp}(Q)}{\partial Q \partial Q} = -\lambda + \frac{\eta(1-p)^2\bar{u}_2}{(1-Q(1-p))^2} + \left[\frac{\eta(1-p)}{(1-Q(1-p))^2} + \frac{Q\eta(1-p)^2 2}{(1-Q(1-p))^3}\right](1-p)\bar{u}_2 \\ + \frac{\eta(1-p)^2 \cdot 2}{(1-Q(1-p))^3} \cdot [pu_1 - (1-p)\bar{u}_2]\end{aligned} \tag{6.0.6}$$

Because the restrictions we impose on the parameters ($u_1 \geq 0, \bar{u}_2 \leq 0, \eta \in (0,1), p \in (0,1), pu_1 - (1-p)\bar{u}_2 \geq 0$ and $\eta \leq p$) $1 - Q(1-p) \geq 0 \ \forall \ Q$, and the sign of the expression will depend on the parameter λ to determine if its greater than zero or not, with the second, third and fourth terms being always bigger than zero. Then, the threshold that makes the second derivative equal to zero is λ.

For a λ small enough, the function is convex, with its maximum being in $Q = 1$. If the parameter is sufficiently high to make the second derivative less than zero, the utility function becomes concave, and the maximum will be some $Q \in (0,1)$.

To prove that the function is always convex and then concave (or only concave), we need to prove that the second derivative is always increasing on Q. To do that, we derive the numerator (because this determines the changes of sign) of the second derivative with respect to Q, obtaining the expression:

$$3\lambda(1-p)\left(\left(p^2 - 2p + 1\right)Q^2 + (2p-2)Q + 1\right) \geq 0 \tag{6.0.7}$$

Which is always greater than zero. Then we know that the second derivative is always increasing on Q. And then, that the solution is first convex and then concave or always concave.

If $\eta > p$, the only difference is that now the interval for the solution is defined between zero and $\tilde{Q}$, instead of between zero and one.

Now, we continue to prove the final part that the Q of the planner is bigger or equal to the one of equilibrium.

We are going to prove this on two steps, first that the Social Planner chooses $Q = 1$ until a bigger λ than the equilibrium, and then the same, but for every interior solution, and also for $\eta \leq p$ and $\eta > p$.

If we are in the case when $\eta \leq p$: For the first part, we are going to derive the utility of the Social Planner with respect to Q, then evaluate it on $\lambda = \tilde{\lambda}(= \frac{(1-p)\eta\bar{u}_2}{p})$[14], and then on $Q = 1$. Then, if the derivative of the Social Planner evaluated on that point is greater than zero, it means that she will choose to continue having $Q = 1$ for bigger values of λ. Now, the first derivative with respect to Q is:

$$-\lambda Q + \frac{\eta(1-p)\bar{u}_2}{1-(1-p)Q} + \frac{(1-p)\eta((1-p)\bar{u}_2 Q + ((1-p)\bar{u}_2 + pu_1)}{(1-Q(1-p))^2} = 0 \tag{6.0.8}$$

Then, we evaluate it on $\lambda = \tilde{\lambda} = (1-p)\eta\bar{u}_2/p$ and $Q = 1$ and see if this expression is greater than zero:

$$\frac{(1-p)\eta((2-p)\bar{u}_2 + pu_1)}{p^2} - \frac{(1-p)\eta\bar{u}_2}{p} \geq 0$$

Simplifying we obtain:

$$pu_1 \geq -2(1-p)\bar{u}_2$$

Expression that is always true. With this, we have proven the first part. For the second one, we are

[14]Using the fact that this λ represents the cutoff when the equilibrium goes from having $Q = 1$ to the interior solution.

comparing the interior solutions. For the Social Planner, these are given by equaling the first derivative to zero, and it is the same as the one on the previous part of the proof. We can note that this expression has some terms in common with the equation **??**, which was:

$$-\lambda Q + \frac{\eta(1-p)\bar{u}_2}{1-(1-p)Q} = 0$$

The Q of equilibrium is chosen from this equation. If we compare it with the equation 6.0.8, we can see that the first derivative of the Social Planner the first two terms are equal, but then it has another one that is always positive. Then, for a given level of λ, the Social Planner will always have a bigger derivative. Then, choosing a bigger Q.

If we are in the case when $\eta > p$: the argument is the same as in the previous case, where congestion was exogenous. The difference is that now instead of being equal to one, the solution, is equal to $\tilde{Q}$.

To prove that $Q = \tilde{Q}$ is the solution, and not any other bigger proportion, we can check what happens if she chooses a $Q > \tilde{Q}$. If that happens, she will move to the case without congestion, were the solution is given by $Q = (1-p)\bar{u}_2/\lambda$. But, since $\lambda \geq \tilde{\lambda}$, she will always choose a $Q < \tilde{Q}$. Then, whenever $\lambda \geq \tilde{\lambda}$, and the utility is convex, the optimum for the Social Planner will be $Q = \tilde{Q}$. ■

Proof lemma 12: If $\eta > p$ and we are in the case when the uninformed want to apply to the special school, the utility function of the planner is:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + Q[pu_1 - (pu_1 + (1-p)u_2)] + [pu_1 + (1-p)u_2]$$

To know where is its maximum point we check its convexity with the second derivative:

$$\frac{\partial^2 U_{sp}(Q)}{\partial Q \partial Q} = -\lambda \tag{6.0.9}$$

This is always less than zero, meaning that the function is strictly concave and that we can find the Q by deriving it once with respect to Q and then equating to zero the expression:

$$-\lambda Q + (1-p)\bar{u}_2 = 0$$

Solving for Q:

$$Q = \frac{(1-p)\bar{u}_2}{\lambda} \tag{6.0.10}$$

As it is the same solution for the case without congestion and $Q < 1$ showed on a previous lemma for the equilibrium, the values of λ under which it is defined (make the solution less than one and bigger than $\tilde{Q}$) are the same ones. Then the lemma is proved. ■

Proof lemma 13: Is the same as in lemma number five. ■

Proof lemma 14: Is the same as in lemma 11, but instead of starting from 1, it is for levels of Q smaller than $\tilde{Q}$. ■

Proof lemma 15: With $\eta \geq p$ the argument is similar to the previous lemma, but instead of $Q < \hat{Q}$, now $Q < 1$. The solution is the same, since the utility does not depend on the relation between η and p when there is no congestion, so we directly prove that:

$$\frac{pu_1}{\lambda} < 1$$

And get:

$$\lambda > pu_1$$

Then the lemma is proved. ■

Proof lemma 16: With $\eta \geq p$ we know that is the case with exogenous congestion. We have already proven that the utility function of the planner is always concave.

Then, we go to the first derivative with respect to Q of the utility of the planner:

$$\frac{\partial U_{sp}(Q)}{\partial Q} = -\lambda Q + pu_1$$

Is easy to note that for values of λ smaller than pu_1, it will be needed a level of Q greater than 1. With that we know that the interior solution is not valid for those values of λ.

Then, for values of $\lambda < pu_1$ we will be in a corner solution, where $Q = 1$. Then it is proved. ■

Proof lemma 17: We know from the equilibrium that when $pu_1 + (1-p)u_2 < 0$ and $\eta < p$, there is a zone of $Q \in [0, \hat{Q}]$ without congestion. In other words, congestion arises when a proportion higher than $\hat{Q}$ acquires information.

With that, we maximize the utility function of the planer:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + Q\, pu_1$$

We analyze the convexity of the function by checking its second derivative with respect to Q:

$$\frac{\partial^2 U_{sp}(Q)}{\partial Q \partial Q} = -\lambda \tag{6.0.11}$$

Signaling that the function is strictly concave, thus the maximum is a interior solution and we can obtain it by deriving the function and equating to zero:

$$-\lambda Q + pu_1 = 0$$

Solving for Q:

$$Q = \frac{pu_1}{\lambda} \tag{6.0.12}$$

Expression that is smaller than $\hat{Q}$ whenever λ is bigger than $\frac{p^2 u_1}{\eta}$, as we proved previously for the equilibrium. ■

Proof lemma 18: With $\eta < p$ we know there are cases where congestion can arise, as we saw on the equilibrium. Now, going to the case of the Social Planner, if we maximize her utility function with respect to Q subject to $Q > \hat{Q}$. We get that the planner chooses $Q = \hat{Q}$, since its utility depends only in its cost of Q, so she will prefer the minimum possible value of Q.

To be in the case where the planner actually is restricted to $Q \geq \hat{Q}$ we need that no other solution is plausible in that parametric space.

We already proved on the results without congestion, that when $\eta < p$ and $\lambda > \frac{p^2 u_1}{\eta}$, the planner has another solution, she chooses $Q = pu_1/\lambda$. The maximum value of Q in that case is $Q = \hat{Q}$ when $\lambda = \frac{p^2 u_1}{\eta}$.

For values of $\lambda < \frac{p^2 u_1}{\eta}$ the utility of the planner on the previous case changes to the one we analyzed in this section with congestion. Then, we have that for $\lambda \in [0, \frac{p^2 u_1}{\eta}]$ the solution will be $Q = \eta/p = \hat{Q}$. Then, the lemma is proved. ■

Proof lemma 19 : We have two possibly different situations that we check. First, if $Q \geq 1 - \eta$ and then if $Q < 1 - \eta$.

Checking first the case when $\eta < \eta^* = \frac{(1-p)\bar{u}_2}{pu_1}$ and $Q \geq 1 - \eta$. There the uninformed agents apply to the special school if:

$$p\eta u_1 + (1 - p)u_2 \geq 0$$

Expression that is equivalent to:

$$\eta \geq \frac{(1 - p)\bar{u}_2}{pu_1}$$

Which is the contrary to the first assumption needed to be in this case. Then, uninformed agents will not apply in this situation.

Continuing with the second situation, $Q < 1 - \eta$, the uninformed agents apply to the special school if:

$$p\eta u_1 + (1 - p)\frac{\eta}{1 - Q}u_2 \geq 0$$

Expression that is equivalent to:

$$Q \leq 1 - \frac{(1 - p)\bar{u}_2}{pu_1} = Q^*$$

Where Q^* is always smaller than $1 - \eta$:

$$Q^* < 1 - \eta$$

$$1 - \frac{(1 - p)\bar{u}_2}{pu_1} < 1 - \eta$$

$$\frac{(1 - p)\bar{u}_2}{pu_1} > \eta$$

Which is the assumption that we made to be in this case.

Then, if $Q < Q^*$ the uninformed agents will apply, and if $Q \geq 1 - \eta$, will not. ■

Proof lemma 20: As in the previous lemma, we test the two possible outcomes an agent can get if she decides to apply to the special school uninformed. First, if $\eta \geq \eta^*$ and $Q \geq 1 - \eta$, the uninformed agents apply to the special school if:

$$p\eta u_1 + (1 - p)u_2 \geq 0$$

Expression that is equivalent to:

$$\eta \geq \frac{(1-p)\bar{u}_2}{pu_1}$$

Which is the first assumption needed to be in this case. Then, uninformed agents will apply in this situation.

We have proven the first part. Now, if $Q < 1-\eta$, the uninformed apply to the special school if:

$$p\eta u_1 + (1-p)\frac{\eta}{1-Q}u_2 \geq 0$$

Expression that is equivalent to:

$$Q \leq 1 - \frac{(1-p)\bar{u}_2}{pu_1} = Q^*$$

Where Q^* is always bigger than $1-\eta$:

$$Q^* > 1-\eta$$

$$1 - \frac{(1-p)\bar{u}_2}{pu_1} > 1-\eta$$

$$\frac{(1-p)\bar{u}_2}{pu_1} < \eta$$

Which is the assumption that we made to be in this case. Then, the lemma is proved for every possible value of Q. ■

Proof lemmas 21 and 22: If $\eta < \eta^*$, $Q = \sqrt{\frac{\eta pu_1}{\lambda}}$ and we want to be on $Q \geq max(Q^*, \eta)$ we need to test for what values of the cost of information acquisition we are in the desired interval. We check this in two cases:

1. When the solution is less than one:

   $$\sqrt{\frac{\eta pu_1}{\lambda}} < 1$$

   Then,

   $$\lambda > \eta pu_1$$

2. When $Q^* > \eta$:

   $$\sqrt{\frac{\eta pu_1}{\lambda}} > Q^*$$

   Then,

   $$\lambda < \frac{\eta pu_1}{Q^{*2}}$$

3. When $Q^* < \eta$:

   $$\sqrt{\frac{\eta pu_1}{\lambda}} > \eta$$

   $$\lambda < \frac{pu_1}{\eta}$$

We can note that, when $Q^* < \eta$, the case will be defined between $[\eta, 1]$. Then, the maximum λ where this is a defined solution is $\lambda = \frac{pu_1}{\eta}$, expression that will be smaller than $\frac{\eta pu_1}{Q^{*2}}$. And the opposite will be true when $Q^* \geq \eta$. Then, we can define the interval of valid solutions for this case as:

$$\lambda \in \left[\eta p u_1, max\left(\frac{pu_1}{\eta}, \frac{pu_1}{Q^{*2}}\right)\right]$$

Then, the lemma is proved. ■

Proof lemma 23: We have proved on the previous lemma that when $\eta < \eta^*$ and $Q > max(Q^*, \eta)$ the optimum $Q = \sqrt{\frac{\eta p u_1}{\lambda}}$. And prove also that when $\lambda = \eta p u_1$ the optimum Q it was equal to 1, then for values of λ smaller than that, the solution suggests levels of Q bigger than one.

We know that for $Q = 1$ the uninformed agents do not apply to the special school. The agent i acquires information if:

$$\frac{\eta}{1} p u_1 - \lambda i \geq 0$$

If we go to the agent with the biggest cost: $i = 1$, then this is:

$$\eta p u_1 \geq \lambda$$

Then, we have proven that when $\lambda < \eta p u_1$ and $\eta < \eta^*$, all the agents acquire information. ■

Proof lemma 24: If $\eta < \eta^*$,

i) $Q^* \geq \eta$, $Q = \frac{pu_1}{\lambda}$ and we want that $Q \in [Q^*, \eta]$, we need to check for what values of λ the solution belongs to the desired interval.

   1. When the solution is greater than Q^*:
$$\frac{pu_1}{\lambda} \geq Q^*$$
$$\frac{pu_1}{Q^*} \geq \lambda$$
   2. When the solution is smaller than η:
$$\frac{pu_1}{\lambda} \geq \eta$$
$$\frac{pu_1}{\eta} \geq \lambda$$

   Then we have that λ must belong to $\left[\frac{pu_1}{\eta}, \frac{pu_1}{Q^*}\right]$ for Q to belong into the interval needed for the case to exist.

■

Proof lemma 25: For $\eta < \eta^*$ and $Q < Q^*$, we first show that the second solution is always unstable, and then that the first one is valid for this case only for values of $\lambda > \frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}$.

The first part of the prove is the same as the one of we shown proving lemma number three. About the second part of the proof, we start from:

$$\frac{\lambda - \sqrt{\lambda^2 - 4(1-p)\lambda\eta\bar{u}_2}}{2\lambda} < Q^*$$

And after rearranging and simplifying terms, we get to:

$$\lambda > \frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}$$

Then, the lemma is proved. ■

Proof lemma 26: If $\eta < \eta^*$, $Q^* > \eta$ we have already proven the limits of both solutions that we say can occur for a same interval. To prove the lemma, we show here that the intervals of both solutions intersect. Then:

$$\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)} \leq \frac{\eta p u_1}{Q^{*2}}$$

$$\frac{(1-p)\bar{u}_2}{pu_1} \geq 0$$

Expression that is always true. ■

Proof lemma 27: If $\eta \in [max(Q^*, \sqrt{Q^*}), \eta^*]$ we have already proven the limits of both solutions. To prove the lemma, we show here that the intervals of both solutions intersect. Then:

$$\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)} \leq \frac{pu_1}{Q^*}$$

$$\eta \leq 1$$

Expression that is always true. Also, if we compare $\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}$ with $\frac{pu_1}{\lambda}$:

$$\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)} > \frac{pu_1}{\lambda}$$

$$\eta > \sqrt{Q^*}$$

Then we get the extra condition for η. Also, we can note that because $Q^* < 1$, $Q^* < \sqrt{Q^*}$. ■

Proof lemma 28: If $\eta \in [Q^*, min(\eta^*, \sqrt{Q^*})]$ we have already proven the limits of both solutions. To prove the lemma, we show here that the intervals of the solutions intersect.

In lemma 27 we have already proven that Q^C and $\frac{pu_1}{\lambda}$ intersect. Here we prove that $\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}$ is smaller than pu_1/η, so the multiple equilibriums are possible when $Q =$ also. And that $\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}$ is bigger than $pu_1\eta$. Then:

$$\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)} \leq \frac{pu_1}{\eta}$$

$$\eta \leq \sqrt{Q^*}$$

Expression that is true in some cases. Also, if we compare $\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)}$ with $\eta p u_1$:

$$\frac{\eta(1-p)\bar{u}_2}{Q^*(1-Q^*)} > \eta p u_1$$

$$\eta^* > 0$$

Expression that is always true. ■

Proof lemma 29: For $\eta > \eta^*$ and $Q > 1-\eta$, we need that the solution belongs to the interval $[1-\eta, 1]$. Then we show under which values of λ it does it:

1. The solution is greater than $1-\eta$:
$$\frac{(1-p)\bar{u}_2}{\lambda} \geq 1-\eta$$
$$\frac{(1-p)\bar{u}_2}{1-\eta} \geq \lambda$$
2. The solution is smaller than one:
$$\frac{(1-p)\bar{u}_2}{\lambda} \leq 1$$
$$(1-p)\bar{u}_2 \leq \lambda$$

Then, if $\lambda \in \left[(1-p)\bar{u}_2, \frac{(1-p)\bar{u}_2}{1-\eta}\right]$, this is a solution to the equilibrium. ■

Proof lemma 30: For $\eta \geq \eta^*$ and $Q = 1$. We know that uninformed agents apply always in this case. Then, we check for what values of λ the agent with biggest acquisition cost will find it profitable to acquire information:

$$\eta p u_1 - \lambda \geq \eta p u_1 + (1-p)u_2$$

We get:

$$\lambda \leq (1-p)\bar{u}_2$$

Then the lemma is proved. ■

Proof lemma 31: Is the same as in the case when $\eta < \eta^*$ and $Q < Q^*$, but restricted to $Q < 1-\eta$ instead. The first part of the proof is the same as in lemma 25, so we continue to see the interval where it is defined this solution:

$$\frac{\lambda - \sqrt{\lambda^2 - 4(1-p)\lambda\eta\bar{u}_2}}{2\lambda} < 1-\eta$$
$$\lambda > \frac{(1-p)\bar{u}_2}{1-\eta}$$

Then, the lemma is proved. ■

Proof lemma 32: If $\eta < \eta^*$ and $Q \geq max\,(Q^*, \eta)$. From lemma 19 we know that when $Q \geq Q^*$, the uninformed will not apply.

Knowing that, we go to equation 3.6.1 and replace $\mathbb{1}_{\text{un. apply}} = 0$. Thus, obtaining:

$$U_{sp} = -\frac{\lambda Q^2}{2} + p\eta u_1$$

We do not need to check U_{sp} when $Q < \eta$, because to be in this case we need $Q \geq max\,(Q^*, \eta)$. Then, the lemma is proved. ■.

Proof lemma 33: We know that $\eta < \eta^*$ and $Q \in [Q^*, \eta]$. From lemma 19 we know that in this case, if $Q > Q^*$ no uninformed agent will apply to the special school. Then, we can go to equation 3.6.2 (because $Q \leq \eta$ always in this case) and replace $\mathbb{1}_{\text{un. apply}} = 0$, obtaining:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + pu_1 Q$$

Then, the lemma is proved. ■

Proof lemma 34: We know that $\eta < \eta^*$ and $Q < Q^*$. Also, from lemma 19, we know that in this case, all the uninformed agents will want to apply to the special school. Then, we go to equation 3.6.2 and replace: $\mathbb{1}_{\text{un. apply}} = 1$ and use $\eta/(1-Q)$ as the probability of being selected when the school is of bad quality (because here this fraction is smaller than one), and obtain:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2)$$

Then, the lemma is proved. ■

Proof lemma 35: If $\eta \geq \eta^*$ and $Q > 1 - \eta$, from lemma 20, we know that all the uninformed agents will want to acquire information. Then, we can replace $\mathbb{1}_{\text{un. apply}} = 1$ in the utility function (independent of Q being greater or equal than η). Also, and because Q is big enough, we replace the probability of being accepted on the special school when this is of bad quality by one.

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + p\eta u_1 + (1-p)(1-Q)u_2$$

Then the lemma is proved. ■

Proof lemma 36: If $Q \leq 1 - \eta$, we know from lemma 20 that all the uninformed agents will want to acquire information. Then, replacing $\mathbb{1}_{\text{un. apply}} = 1$ in the utility function (independent of Q being greater or equal than η) and because Q is big enough, the probability of being accepted on the special school when this is of bad quality is less than one. And get:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2)$$

Then, the lemma is proved. ■

Proof lemma 37: From lemma 32 we know that if $\eta < \eta^*$ and $Q \geq Q^*$, then $U_{sp}(Q) = -\frac{\lambda Q^2}{2} + p\eta u_1$. A function that is always decreasing on Q because only the cost depends on it. Then, in a world without restrictions, the Social Planner will choose $Q = 0$. We have a cutoff in Q^*. If $Q < Q^*$ the uninformed start to apply to the special school. Then, she always chooses the minimum feasible value which is $Q = Q^*$.

The utility is obtained by replacing $Q = Q^*$ on the one from lemma 32. ■

Proof lemma 38: From lemma 34 we know that if $\eta < \eta^*$ and $Q < Q^*$, then $U_{sp}(Q) = -\frac{\lambda Q^2}{2} + \eta(pu_1 +$

$(1-p)u_2)$. A function that is always decreasing on Q because only the cost depends on it. Then, the Social Planner will want to minimize the cost, choosing the minimum value of Q possible. In this case, it is equal to zero. The utility is obtained by replacing $Q = 0$ on the one obtained from the lemma 34. ■

Proof lemma 39: Taken as given $\eta < \eta^*$ and $\eta \leq Q^*$, because are the necessary conditions to be in this case. The equivalent to this situation on the equilibrium, is the solution proposed at proposition 7. From there, we know that the optimal Q is decreasing on λ (not in a strict way). Then, a higher λ is related to a smaller or equal level of Q.

We know that whenever $Q \geq Q^*$, the uninformed are not applying to the special school. And also, that when $\lambda \leq \tilde{\lambda}$, the planner is making a proportion equal to Q^* acquire information.

Then, to prove the lemma, we need to show that $\tilde{\lambda}$ is bigger than the maximum level of λ under which the equilibrium could be (because of the multiple equilibriums):

$$\tilde{\lambda} \geq \frac{\eta p u_1}{Q^{*2}}$$

We get:

$$\eta^* \geq 0.5$$

Then, the lemma is proved. ■

Proof lemma 40: Is the same as in lemma 39, but because here we assume $\eta^* < 0.5$, the last stage is false. ■

Proof lemma 41: From lemma 33 we know that if $\eta < \eta^*$ and $Q \in [Q^*, \eta]$. Then, the utility of the Social Planner is equal to:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + pu_1 Q$$

To analyze where is the maximum with respect to Q we check its concavity by its second derivative:

$$\frac{\partial^2 U_{sp}(Q)}{\partial^2 Q} = -\lambda$$

An expression that is always smaller than zero. Then, the utility function is always concave, and we can obtain its maximum by simply deriving with respect to Q and equalize to zero:

$$-\lambda Q + pu_1 = 0$$

Then:

$$Q = \frac{pu_1}{\lambda}$$

But, this Q must belong to the interval $[Q^*, \eta]$, to check that we see for what values of λ the optimum Q will belong to the interval were it is defined:

$$1) \quad \frac{pu_1}{\lambda} \leq \eta; \text{ then: } \quad \lambda \geq \frac{pu_1}{\eta}$$

$$2) \quad \frac{pu_1}{\lambda} \geq Q^*; \text{ then:} \quad \lambda \leq \frac{pu_1}{Q^*}$$

We need that $\lambda \in [\frac{pu_1}{\eta}, \frac{pu_1}{Q^*}]$.

Lastly, we obtain the utility by replacing this expression on the planner's utility for this case. ■

Proof lemma 42: In this situation the function decreases with Q. Then, the planner chooses the minimum possible value.

From lemma 41, we know that the Social Planner will be in the case without congestion when the special school is of good quality and $\lambda \in [\frac{pu_1}{\eta}, \frac{pu_1}{Q^*}]$. Then, if $\lambda < \frac{pu_1}{\eta}$, the planner will be in a situation when she wants a bigger Q, but the congestion starts in the special school. With that, her utility changes to $U_{sp}(Q) = -\frac{\lambda\eta^2}{2} + p\eta u_1$. The threshold between this two cases is $\lambda = \frac{pu_1}{\eta}$. There, both utilities are equal. If λ is smaller than that, the Social Planner will always choose $Q = \eta$. ■

Proof lemma 43: Is the same as the one showed on lemma 38, because this case is independent of η. ■

Proof lemma 44: $U_{sp}(0) \leq U_{sp}(\frac{pu_1}{\lambda})$ is true for $\lambda < \tilde{\lambda}$. But, if $\tilde{\lambda} > \frac{pu_1}{Q^*}$, this is not a value of λ that belongs to the interval $\lambda \in [\frac{pu_1}{\eta}, \frac{pu_1}{Q^*}]$ were $U_{sp}(\frac{pu_1}{\lambda})$ is defined. So it is not possible that $U_{sp}(0) \leq U_{sp}(\frac{pu_1}{\lambda})$ in that case.

Then, we check when $\frac{pu_1}{Q^*} < \tilde{\lambda}$:

$$\frac{pu_1}{Q^*} < \frac{p^2u_1^2}{2\eta(pu_1 + (1-p)u_2)}$$
$$\frac{1}{Q^*} < \frac{1}{2\eta Q^*}$$
$$\eta < 0.5$$

This is possible only if η is small enough. Because of this, if $\eta \leq 0.5$, the threshold between when the planner prefers one situation over the other must be $\lambda = \frac{pu_1}{Q^*}$ and not $\tilde{\lambda}$. ■

Proof lemma 45: Is the same as in lemma 44, but because in this case $\eta > 0.5$, $\tilde{\lambda}$ is always the threshold.

Also, check if $\lambda = \frac{pu_1}{\eta} > \tilde{\lambda}$:

$$\frac{pu_1}{\eta} < \frac{p^2u_1^2}{2\eta(pu_1 + (1-p)u_2)}$$
$$0.5 < \frac{(1-p)\bar{u}_2}{pu_1}$$

Expression that is always true when $\eta > 0.5$. ■

Now we go to see what is the optimum choice for the planner when $\lambda < \frac{pu_1}{\eta}$. From lemmas 42 and 43 we know there are two other cases defined. One where all the uninformed apply and $Q = 0$ and other that the uninformed do not apply and $Q = \eta$. She prefers the latter over the former if:

$$U_{sp}(0) \leq U_{sp}(\eta)$$

$$\eta(pu_1 + (1-p)u_2) \leq -\frac{\lambda\eta^2}{2} + p\eta u_1$$

$$\lambda \leq \frac{2(1-p)\bar{u}_2}{\eta} = \lambda^* \tag{6.0.13}$$

Where we can define λ^* as the cutoff between the two cases. If $\lambda < \lambda^*$, the planner will prefer that a fraction η acquires information. Now we have a lemma to show us where is the exact cutoff between both cases.

Proof lemma 46: $U_{sp}(0) \leq U_{sp}(\eta)$ is true for every $\lambda < \lambda^*$. But, $U_{sp}(\eta)$ is defined only if $\lambda < \frac{pu_1}{\eta}$. Then we check if $\frac{pu_1}{\eta}$ is smaller than λ^*:

$$\frac{pu_1}{\eta} \leq \frac{2(1-p)\bar{u}_2}{\eta}$$

$$0.5 \leq \frac{(1-p)\bar{u}_2}{pu_1}$$

$$0.5 \leq \eta^* \tag{6.0.14}$$

Expression that is always true in this case. To see this, we can start by noting that in this case:

$$Q^* < \eta < \eta^*$$

Using the definitions of $Q^* = 1 - \eta^*$:

$$1 - \eta^* < \eta^*$$

$$0.5 \leq \eta^* \tag{6.0.15}$$

That is the same that we found on 6.0.14. ■

Proof lemma 47: With $\eta \in [Q^*, 0.5]$ and $\eta < \eta^*$, we know that the planner will acquire information until $\lambda = \frac{pu_1}{Q^*}$.

In the equilibrium there are two possible solutions for these case, which are presented on propositions 8 and 9. For the purpose of this proof, both solutions are equivalent, because we are interested on the maximum value of λ where an equilibrium with the uninformed not applying can exist. That value is $\lambda\frac{pu_1}{Q^*}$ on both cases, that is the same value we showed before. Then the lemma is proved. ■

Proof lemma 48: With $\eta \in [max(Q^*, 0.5), \eta^*]$, we know that the planner acquires information until $\lambda = \tilde{\lambda}$. From lemma 45 we know that in this case $\frac{pu_1}{\eta} < \tilde{\lambda} < \frac{pu_1}{Q^*}$.

Then, the lemma is proved. ■

Proof lemma 49: Taken as given $\eta > \eta^*$ and $Q > 1 - \eta$, we know from lemma 35 that:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + p\eta u_1 + (1-p)(1-Q)u_2$$

If we derive twice with respect to Q we get:

$$\frac{\partial^2 U_{sp}(Q)}{\partial^2 Q} = -\lambda$$

An expression that is always negative. Hence, the utility function in this zone is convex, and we can find its optimum deriving and equating the equation to zero:

$$-\lambda Q - (1-p)u_2 = 0$$

$$Q = \frac{(1-p)\bar{u}_2}{\lambda} \tag{6.0.16}$$

And get the expression of the lemma for the optimum Q in this zone. Then, the utility is obtained by replacing $Q = \frac{(1-p)\bar{u}_2}{\lambda}$.

Then, we test for what values of λ this solution belongs to the interval $Q \in [1-\eta, 1]$:

1. Smaller than one:

$$\frac{(1-p)\bar{u}_2}{\lambda} \leq 1$$

$$\lambda \geq (1-p)\bar{u}_2$$

2. Bigger than $1-\eta$:

$$\frac{(1-p)\bar{u}_2}{\lambda} \geq 1-\eta$$

$$\lambda \leq \frac{(1-p)\bar{u}_2}{1-\eta}$$

There we get that $\lambda \in \left[(1-p)\bar{u}_2, \frac{(1-p)\bar{u}_2}{1-\eta}\right]$. ■

Proof lemma 50: This is a sub-case of the previous lemma, because when $\lambda < (1-p)\bar{u}_2$ the previous solution starts to gives us $Q > 1$, which is not feasible in the context of the model. Then, we know that for every value of λ smaller than this, we have another zone where $Q = 1$ always. ■

Proof lemma 51: If $\eta > \eta^*$ and $Q \leq 1-\eta$, by lemma 36, we know that the utility of the planner is:

$$U_{sp}(Q) = -\frac{\lambda Q^2}{2} + \eta(pu_1 + (1-p)u_2)$$

We can see there that only the cost depends on Q. Then the Q that maximizes this function is the minimum possible, which is $Q = 0$. We obtain the utility by replacing $Q = 0$ into the one obtained previously. ■

Proof lemma 52: With $\eta \geq \eta^*$, to prove this lemma we compare the solutions of the planner to the one of equilibrium for every value of λ in both propositions. Then:

1. If $\eta \leq 0.5$, or proposition 14 is the valid one. We know that λ^{***} is less than $(1-p)\bar{u}_2$. For that reason, we know that the planner chooses $Q = 1$ until a smaller level than the equilibrium. Because for values of $\lambda > \lambda^{***}$ the planner chooses $Q = 0$, and the equilibrium is for values smaller than one, but bigger than zero, the lemma is proved in this section.

2. If $\eta > 0.5$, or proposition 15 is the valid one. We know that until $\lambda = \lambda^{**}$, the planner and the equilibrium do the same. The difference comes for $\lambda > \lambda^{**}$, because there the planner choose $Q = 0$, and the equilibrium chooses values of Q always bigger than zero.

Then, the lemma is proved. ■

7. Appendix: Model with homogeneous costs and independent values

7.1. *Agents with homogeneous information acquisition costs:*

This is a different version of the base model. The difference is that here all agents face the same cost for the acquisition of information about the special school. As we will see, even under this assumption of homogeneity, different kinds of equilibriums can result.

Also, equilibriums are different from before because we have equilibriums in pure and mixed strategies. Then, we define:

Definition A 1: "Symmetric Equilibrium". A symmetric equilibrium of this model is an information acquisition activity $Q = 0$ or $Q = 1$, equal for every agent, and applications according to their beliefs, taking as given the capacity η of the special school and the DA algorithm.

The mixed strategies are equilibriums were this it is not accomplished. Meaning that only a fraction of the agents acquires information. Even though they all have the same expected payoff.

We also analyze the social Planner's reactions and show how her solutions differ from those founded in the equilibrium. The conclusions reached in this version of the model are similar to the base model with independent valuations. When everybody applies ex-ante, the social Planner acquires more information and the opposite where all agents do not acquire ex-ante. This show that the conclusions reached before about the externalities do not depend on the different assumptions over the costs.

7.1.1. *When all agents apply to the special school, and $\eta \leq p$:*

In this case, we have that $pu_1 + (1-p)u_2 \geq 0$ for all agents. This implies that all agents a priori apply to the special school. Thus, increases in the information acquisition will imply that a fraction $(1-p)$ of the informed ones will not like the special school, causing less congestion to the system relative to the situation where nobody acquires.

About $\eta \leq p$, this assumption implies that, even if everyone acquires information, there will be congestion in the system (the probability of being accepted to the special school will always be less than one).

For this case, $\mathbb{P}$ represents the probability of being accepted in the special school. Because there is always exogenous congestion, we can define it as:

$$\mathbb{P} = \frac{\eta}{1-(1-p)Q}$$

Where η represents the supply of seats in the special school, and $1-(1-p)Q$ represents the proportion of the population that applies. This proportion is composed of everyone, except those who acquire information (Q) and not like the special school.

We review the three kinds of equilibrium relative to the agents' information: $Q = 0$, $Q = 1$, and mixed strategies—reviewing for each the necessary conditions not to deviate.

- $Q = 0$: to be in this case, we need that the utility of acquiring information to be less or equal to the one given for not doing it. This is equivalent to:

$$U_i\left(A_i \mid Q = 0\right) \leqslant U_i\left(NA_i \mid Q = 0\right)$$

Where A_i represents the acquisition of information by the agent and NA_i not doing it. In terms of the parameters of the model the equivalent equation is:

$$-c + \mathbb{P}pu_1 \leq \mathbb{P}(pu_1 + (1-p)u_2)$$

Replacing $\mathbb{P}$, $Q = 0$, and simplifying:

$$c^0 \geq \eta(1-p)\bar{u_2} \tag{7.1.1}$$

Where $\bar{u_2} = -u_2$. So, for a cost greater than this, there exists an equilibrium were nobody acquires information.

- $Q = 1$: to be in this case, the benefit of acquiring information must be greater than the one of not doing it. This is equivalent to:

$$U_i\left(A_i \mid Q = 1\right) \geqslant U_i\left(NA_i \mid Q = 1\right)$$

In terms of the parameters of the model:

$$-c + \mathbb{P}pu_1 \geq \mathbb{P}(pu_1 + (1-p)u_2)$$

Using the same form for $\mathbb{P}$, but replacing $Q = 1$ and simplifying, we obtain:

$$c^1 \leq (\frac{\eta}{p})(1-p)\bar{u_2} \tag{7.1.2}$$

For any cost smaller than the expression, there exists an equilibrium where all agents acquire information.

- Mixed Strategies: to be in an equilibrium of this kind, the agent must receive the same utility, independent of the action she finally chooses. This is expressed as:

$$U_i(A_i \mid Q) = U_i(NA_i \mid Q)$$

Which in terms of the parameters of the model is:

$$-c + \mathbb{P}pu_1 = \mathbb{P}(pu_1 + (1-p)u_2)$$

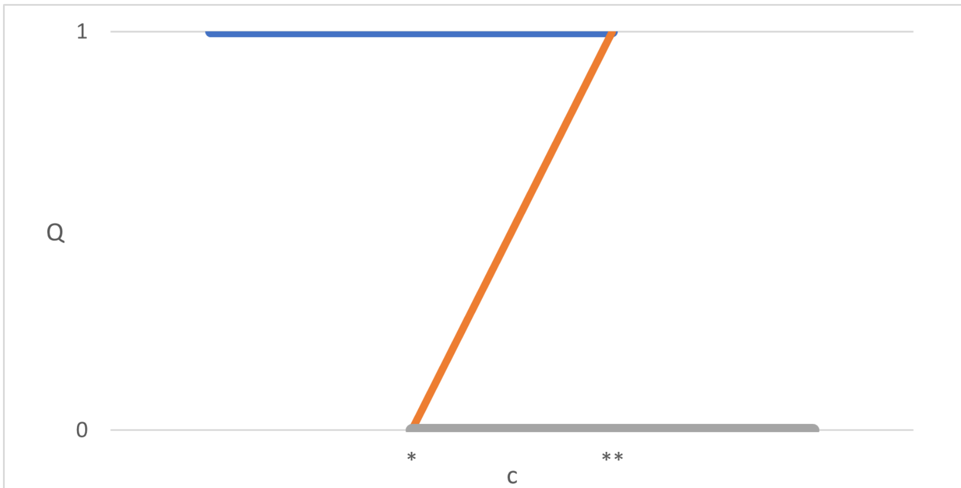
Now, simplifying and replacing $\mathbb{P} = \eta/(1 - Q(1-p))$, we obtain:

$$c^{ms} = \frac{n}{1-(1-p)Q}(1-p)\bar{u}_2 \tag{7.1.3}$$

Thus, to be in a mixed strategies equilibrium, we need that the cost of acquiring information be exactly equal to the expression. Expression that depends in a positive way of Q (i.e, greater the Q, the cost that allows us to be in a mixed strategy equilibrium must be higher too).

- Graphing the results as a function of the costs:

Figure 20



Where:

(*) Represents $c^0_{min} = \eta(1-p)\bar{u}_2$, or the minimum value were exists an equilibrium with no agent acquiring information.

(**) Represents $c^1_{max} = \frac{\eta}{p}(1-p)\bar{u}_2$, or the maximum value were exists an equilibrium with every agent acquiring information.

The orange line represents the combinations of (c, Q) compatible with equilibriums in mixed strategies. Figure 1 shows that a higher cost needs a higher proportion of people informed to be in a mixed strategy equilibrium. This could sound counterintuitive but comes as a consequence of the need to equal the utility of acquiring and not acquiring information.

Social Planner: as we saw before, she knows everything except each agent's valuation of the special school. Her utility comes from the sum of all information acquisition costs, the utility from the agents that acquire information, and expected utility from those who did not acquire. If we reorganize all those terms, use the fact that in this case always exist congestion (so $\mathbb{P} = \eta/(1-(1-p)Q)$), we can write:

$$U_{sp}(Q) = -cQ + Q\mathbb{P}[pu_1 - (pu_1 + (1-p)u_2)] + \mathbb{P}[pu_1 + (1-p)u_2] \tag{7.1.4}$$

To better understand the problem, we show and prove three Lemmas that will help us know the function's maximum and its relation with the cost.

Lemma A1: the maximum value of the social planner utility, $U_{sp}(Q^*)$, with $Q^* \in [0,1]$, is always in $Q^* = 0$ or $Q^* = 1$.

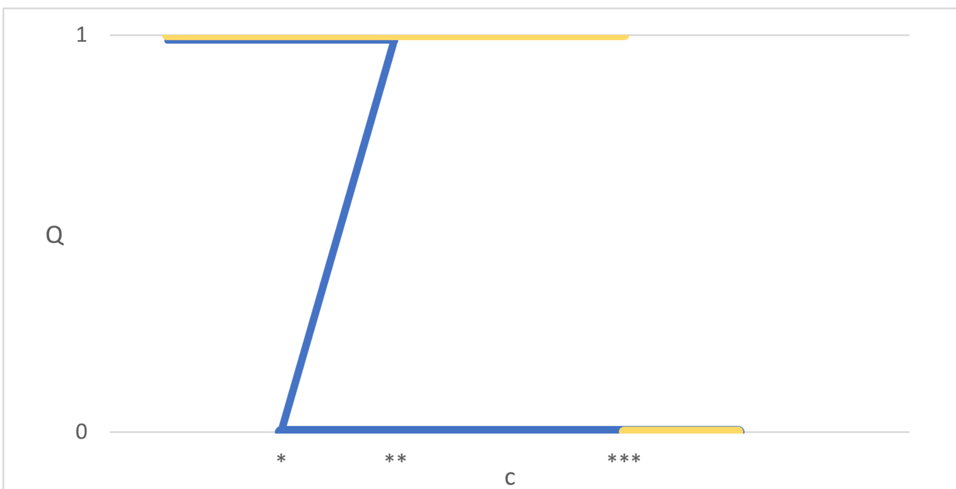
Lemma A2: There exist a level of cost $\tilde{c}$ such that if the actual cost is lower than that, everyone acquires information, and if it is higher than that, nobody acquires.

$$\exists\, \tilde{c} \text{ such that: } Q^* = 1 \text{ if } c \leq \tilde{c} \text{ and } Q^* = 0 \text{ if } c \geq \tilde{c}$$

Lemma A3: $\tilde{c} > c^1$, where c^1 represents the maximum level of cost where it can be an equilibrium that every agent acquires information.

The results of the Social Planner are in the yellow line of Figure 21. As we can note, the Social Planner internalizes the congestion, so she makes everyone acquire information until (***), which is the cost $\tilde{c}$ that we find in the Lemma A2, and prove in Lemma A3 that it was bigger than (**), or c^1. Which is the maximum cost compatible with a private equilibrium of everyone acquiring information.

Figure 21



7.1.2. When all agents apply to the special school, and $\eta \geq p$:

As in the first case, $pu_1 + (1-p)u_2 \geq 0$ for all the agents. So, increases in the information acquisition will imply less congestion to the system. The difference with the last case is that $\eta \geq p$, to understand the effects of this, we can go to the situation where all agents acquire information. We will see that the fraction of people liking the special school will be p, so there will be no congestion. Because of this, congestion is endogenous to the model, so the probability of entering the special school is:

$$\mathbb{P} = \min\left(\frac{\eta}{1 - Q(1-p)}, 1\right)$$

In every equilibrium, we will check under what conditions of the parameters there is congestion or not.

We revise the three kinds of equilibriums relative to the information acquired by the agents: $Q = 0$, $Q = 1$, and mixed strategies.

- $Q = 0$: to be in this case, we need that the utility of acquiring information be less or equal to the one given by not acquiring. This is:

$$U_i(A_i \mid Q = 0) \leqslant U_i(NA_i \mid Q = 0)$$

In terms of the parameters:

$$-c + \mathbb{P}pu_1 \leq \mathbb{P}(pu_1 + (1-p)u_2)$$

Now, checking both cases of $\mathbb{P}$:

i) Congestion, or $\mathbb{P} = \eta/(1-(1-p)Q)$:

$$-c + \frac{n}{1-(1-p)Q}pu_1 \leq \frac{n}{1-(1-p)Q}(pu_1 + (1-p)u_2)$$

Replacing that $Q = 0$ and solving for the cost, we obtain:

$$c^0 \geq \eta(1-p)\bar{u_2} \tag{7.1.5}$$

For a cost greater than $\eta(1-p)\bar{u_2}$, exists a equilibrium in which no agent acquire information.

ii) No congestion, or $\mathbb{P} = 1$: we cannot be in this case and have $Q = 0$ at the same time. When nobody acquires information, all the agents will apply to the special school, those generating congestion.

- $Q = 1$: to be in this case, we need that the benefit of acquiring information must be greater than the one of not doing it. This is equivalent to:

$$U_i(A_i \mid Q = 1) \geqslant U_i(NA_i \mid Q = 1)$$

In terms of the parameters of the model:

$$-c + \mathbb{P}pu_1 \geq \mathbb{P}(pu_1 + (1-p)u_2)$$

Now checking both cases of $\mathbb{P}$:

i) Congestion or $\mathbb{P} = \eta/(1-(1-p)Q)$: because $\eta \geq p$, it is not possible to be in this case with $Q = 1$ at the same time. If everyone acquires information, only a fraction p of the agents will apply. At the same time, $p \leq \eta$, so there will be an excess of seats in the special school.

ii) No congestion, or $\mathbb{P} = 1$: solving the problem for the cost, we obtain:

$$-c + pu_1 \geq pu_1 + (1-p)u_2$$

$$c^1 \leq (1-p)\bar{u_2} \tag{7.1.6}$$

There is no congestion with $Q = 1$, thus we can say is like solving the problem for each agent individually, or to put it in another way, the externality disappears.

- Mixed Strategies: to be in an equilibrium of this kind, the agent must receive the same utility independent of the strategy. This could be expressed as:

$$U_i(A_i \mid Q) = U_i(NA_i \mid Q)$$

In terms of the parameters:

$$-c + \mathbb{P}pu_1 = \mathbb{P}(pu_1 + (1-p)u_2)$$

Now checking both cases of $\mathbb{P}$:

i) Congestion or $\mathbb{P} = \eta/(1-(1-p)Q)$:

$$-c = \frac{\eta}{1-(1-p)Q}(1-p)u_2$$

Solving for the cost:

$$c^{ms} = \frac{\eta(1-p)\bar{u}_2}{1-(1-p)Q} \tag{7.1.7}$$

All this is subject to be in the case where $\mathbb{P} \leq 1$. Now we check for which levels of Q this is the case:

$$\frac{\eta}{1-(1-p)Q} \leq 1$$

$$Q \leq \frac{1-\eta}{1-p}$$

We can define:

$$Q^{max} = \frac{1-\eta}{1-p}$$

Which is always less than one, because $\eta > p$. It is also useful to note that if we replace Q^{max} in the equation for the cost, we obtain:

$$c = \frac{\eta(1-p)\bar{u_2}}{1-(1-p)Q^{max}}$$

$$c^{max} = (1-p)\bar{u_2}$$

Which is the maximum cost compatible with the equilibrium in $Q = 1$.

ii) No congestion, or $\mathbb{P} = 1$: to be in this case, we need that $Q \geq Q^{max}$. Now, solving the problem for the cost:

$$-c + pu_1 = pu_1 + (1-p)u_2$$

We get:

$$c = (1-p)\bar{u_2} \tag{7.1.8}$$

These being a non-generic case.

- Graphing the results as a function of the costs: Where:

(*) Represents $c^0_{min} = \eta(1-p)\bar{u}_2$, or the minimum value were exists an equilibrium with no agent acquiring information.

(**) Represents the cost $c^{max} = (1-p)\bar{u}_2$, which is the level of cost consistent with $Q = Q^{max}$,

Figure 22

and is also the maximum level of cost compatible with a equilibrium were every agent acquires information.

As in the previous one, the orange line represents combinations of (c, Q) that are compatible with equilibriums in mixed strategies.

Social Planner: her utility is similar to the one of the previous case. The main difference is that here congestion is endogenous, so now the probability $\mathbb{P} = min(1, \eta/(1-(1-p)Q))$. It is:

$$U_{sp}(Q) = -cQ + Q\mathbb{P}[pu_1 - (pu_1 + (1-p)u_2)] + \mathbb{P}[pu_1 + (1-p)u_2] \tag{7.1.9}$$

Now we will show and proof the three Lemmas with and without congestion.

- Case with Congestion: $\mathbb{P} = \eta/(1-(1-p)Q)$ and $Q \leq Q^{max}$

  Lemma A4: the maximum value of the social planner utility, $U_{sp}(Q^*)$, with $Q^* \in [0, Q^{max}]$ is always in $Q^* = 0$ or $Q^* = Q^{max}$. All this subject to $Q \leq Q^{max} = (1-\eta)/(1-p)$.

  Lemma A5: There exist a level of cost $\tilde{c}$ such that if the actual cost is lower than that, a fraction of the agents, at least equal to $Q = Q^{max}$ acquires information, and if it is higher than that, nobody acquires.

  $$\exists\, \tilde{c} \text{ such that: } Q^* = (1-\eta)/(1-p) \text{ if } c \leq \tilde{c} \text{ and } Q^* = 0 \text{ if } c \geq \tilde{c}$$

  Lemma A6: $\tilde{c} > c^1$, where c^1 represents the maximum level of cost where it can be an equilibrium that every agent acquires information.

- Case without congestion: $\mathbb{P} = 1$ and $Q > (1-\eta)(1-p)$

  The utility of the social planner with $\mathbb{P} = 1$ is:

  $$U_{sp}(Q) = -cQ + Q[pu_1 - (1-p)u_2] + [pu_1 + (1-p)u_2]$$

Which is lineal with respect of Q, so it has only one maximum. To check this, we can check if its second derivative is equal to zero. The first one is:

$$\frac{\partial U_{sp}(Q)}{\partial Q} = -c + [pu_1 - (1-p)u_2]$$

It is easy to note that it's second derivative with respect to Q is equal to zero. Thus, the problem is convex in this case (not strictly convex as before). The social planner will acquire information for everyone if the cost is smaller than:

$$c^{sp}_{max} = pu_1 + (1-p)\bar{u}_2$$

Now, to see if the Social Planner acquires more information than the agents by their own, we will compare if $c^{sp}_{max} \geq c^{max}$, were c^{max} was the maximum level of cost consistent with the a equilibrium were everyone acquires information.

$$pu_1 + (1-p)\bar{u}_2 \geq (1-p)\bar{u}_2$$

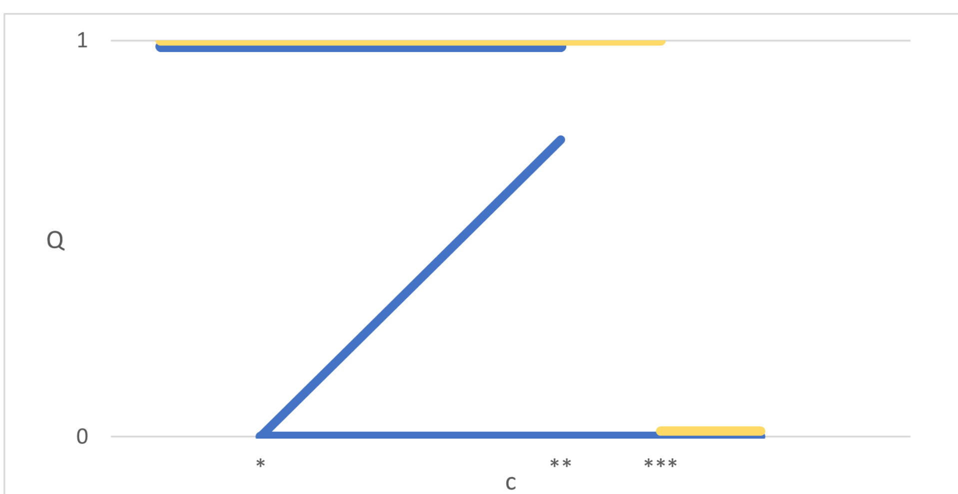
$$pu_1 \geq 0$$

Thus, the Social Planner always acquires more information than the agents on their own.

Because there is no congestion, there are no externalities to take into account by the social Planner. So the solution is the same as agents acting on their own.

The results of the Social Planner are presented in the yellow line of Figure 23. As in the previous case, the Social Planner internalizes the congestion, so it makes everyone acquire information until (***), which is the level of cost $\tilde{c}$ that we find in Lemma A5, and prove in Lemma A6 that was bigger than (**), or c^1, which is the maximum cost compatible with a private equilibrium with $Q = 1$.

Figure 23



7.1.3. When agents do not apply to the special school, and $\eta \leq p$:

Now we have that $pu_1 + (1-p)u_2 < 0$ for all the agents. Agents a priori do not apply, so increases in the information acquisition will imply more applications coming into the special school. Also, $\eta \leq p$. To understand the effects of this, we can go to a situation where all agents acquire information; there, p or the fraction of people who like the special school will be greater than its capacity. Because of that, congestion is now endogenous.

Here only the ones who acquire information and like the school will apply, so now $\mathbb{P}$ is:

$$\mathbb{P} = \min(\frac{\eta}{Qp}, 1)$$

In every one of the three possible equilibriums, we test each version of $\mathbb{P}$.

- $Q = 0$: to be in this case, we need that the utility of acquiring information be less or equal to not being informed. This is:

  $$U_i\left(A_i \mid Q=0\right) \leqslant U_i\left(NA_i \mid Q=0\right)$$

  In terms of the parameters of the model:

  $$-c + \mathbb{P}pu_1 \leq 0$$

  Now checking both cases of $\mathbb{P}$:

  i) Congestion, or $\mathbb{P} = \eta/Qp$: it is not possible to be in this case and have congestion at the same time. This can be seen by replacing $Q = 0$ in the formula for $\mathbb{P}$.

  ii) No congestion, or $\mathbb{P} = 1$:

  $$-c + pu_1 \leq 0$$

  Solving for the cost:

  $$c^0 \geq pu_1 \tag{7.1.10}$$

  For costs higher than pu_1 people will not acquire information.

- $Q = 1$: to be in this case, we need that the benefit of acquiring information must be greater than the one of not acquiring.

  $$U_i\left(A_i \mid Q=1\right) \geqslant U_i\left(NA_i \mid Q=1\right)$$

  In terms of the parameters of the model:

  $$-c + \mathbb{P}pu_1 \geq 0$$

  Now checking both cases of $\mathbb{P}$:

  i) Congestion, or $\mathbb{P} = \eta/Qp$: replacing also $Q = 1$ in the previous equation we can obtain the expression:

  $$c^1 \leq \eta u_1 \tag{7.1.11}$$

  For every cost smaller than that, there exists an equilibrium in which all agents acquire information.

ii) No congestion, or $\mathbb{P} = 1$: this case is not possible with $Q = 1$ at the same time. If all acquire information, the fraction of agents who like the special school will be bigger than η, generating congestion.

- Mixed Strategies: to be in an equilibrium of this kind, the agent must receive the same utility, independent of the strategy. This could be expressed as:

$$U_i(A_i \mid Q) = U_i(NA_i \mid Q)$$

In terms of the parameters of the model:

$$-c + \mathbb{P}pu_1 = 0$$

Now checking both cases of $\mathbb{P}$:

i) Congestion or $\mathbb{P} = \eta/Qp$:

$$-c + \frac{\eta}{Qp}pu_1$$

Solving for the cost:

$$c^{ms} = \frac{\eta u_1}{Q} \tag{7.1.12}$$

All this is subject to be in the case where $\mathbb{P} \leq 1$. Now we check for which levels of Q this is the case:

$$\frac{\eta}{Qp} \leq 1$$

$$(\eta/p) \leq Q$$

We can define:

$$Q^{min} = (\eta/p)$$

For any Q less than Q^{min} there will be no congestion. It is useful to note that if we replace Q^{min} in the equation for the cost, we obtain:

$$c = pu_1$$

Which is the minimum cost compatible with the equilibrium $Q = 0$.

ii) No congestion, or $\mathbb{P} = 1$: to be in this case, we need that $Q \leq Q^{min}$. Now, solving the problem for the cost:

$$-c + 1pu_1 = 0$$

$$c = pu_1 \tag{7.1.13}$$

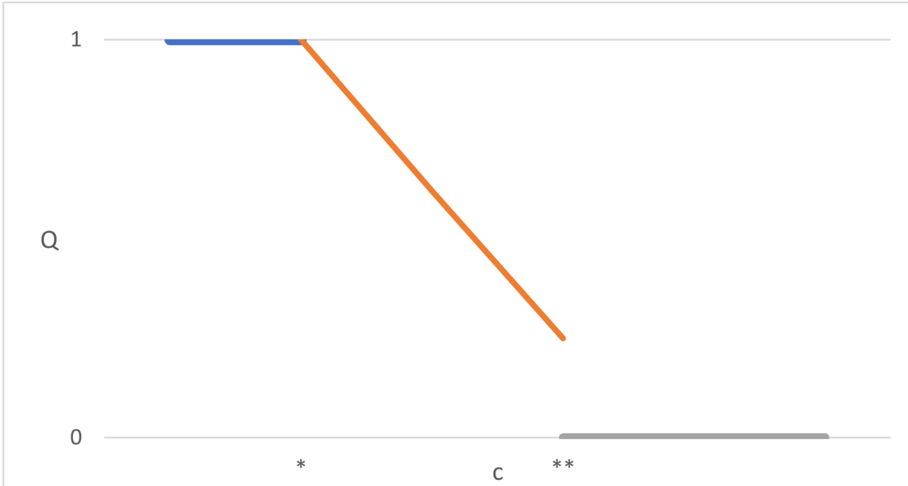
These being a non-generic case.

- Graphing the results as a function of the costs:

Where:

(*) Represents $c^1_{max} = \eta u_1$, or the maximum value were exists an equilibrium with all agents acquiring information.

Figure 24



(**) Represents $c^0_{min} = pu_1$, which is the minimum level of cost consistent with an equilibrium with no agent acquiring information.

Social Planner : her utility comes from the sum of all the information acquisition costs, the utility from the agents that acquire information, and the expected utility from those who did not, which is equal to zero. In term of the model parameters, this is:

$$U_{sp}(Q) = -cQ + \mathbb{P}Qpu_1$$

Were $\mathbb{P} = min(1, \eta/Qp)$. Now, we are going to analyze the case with and without congestion for every possible equilibrium. We will not use any lemma as on the previous cases because the utility function here is always linear in Q, having a clear maximum.

- Congestion, $\mathbb{P} = \eta/Qp$ and $Q > Q^{min}$.

  The utility of the social Planner is:

  $$U_{sp}(Q) = -cQ + \eta u_1$$

  Which is a linear function of Q. Now its easy to see that $Q^* = Q^{min} = \eta/p$. Taking as given the congestion, this is the best point for the Social Planner.

- No congestion, $\mathbb{P} = 1$ and $Q < Q^{min}$.

  The utility of the social Planner is:

  $$U_{sp}(Q) = -cQ + Qpu_1$$

  Which is also linear on Q. If we derive with respect to Q:

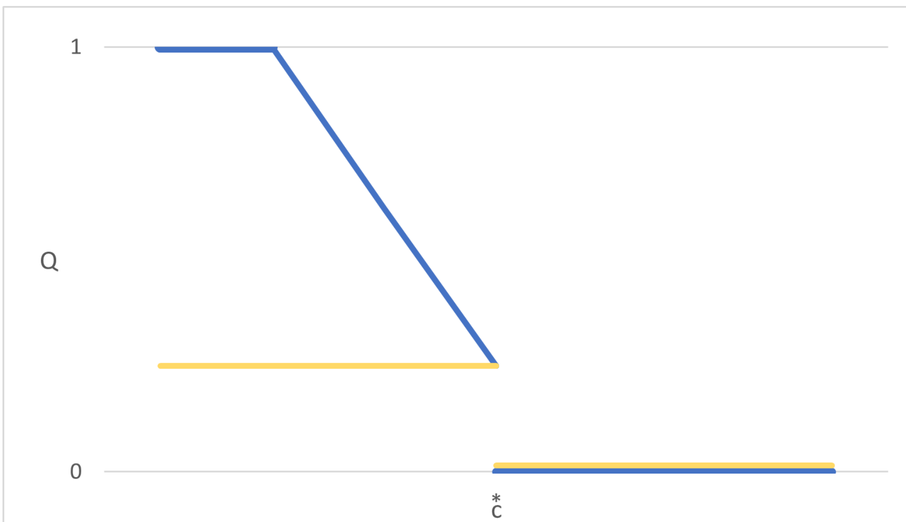
  $$\frac{\partial U_{sp}(Q)}{\partial Q} = -c + pu_1 = 0$$

  If we equal to zero, we obtain: $c = pu_1$. This is equal to the individual problem, so every time the cost is greater than the expected benefit of acquiring information, the social Planner will acquire information.

Joining the results of both situations, starting from $Q = 0$ and $c < pu_1$, we can see that it will be optimal for the social Planner to acquire information for everyone. This will lead us to the case with congestion so that the Planner will choose the fraction $Q = Q^{min} = \eta/p$.

The results of the Social Planner are presented in the yellow line of Figure 25. As we can note, the externality goes in the other direction; here is a negative one. So the Social Planner will make a fraction (η/p), which is always less than one, acquire information. The point until the Social Planner acquires for this fraction (*) is the same that the agents chose by themselves: $c^0_{min} = pu_1$, which is the minimum level of cost consistent with an equilibrium with no agent acquiring information. Over that point, it is too costly.

Figure 25



7.1.4. When agents do not apply to the special school, and $\eta > p$:

As in the last case, $pu_1 + (1-p)u_2 < 0$ for all the agents, the difference is that now $\eta > p$. To understand the effects of this in the result, we can go to a situation where all agents acquire information. We will see that the fraction p of people that like the special school will always be smaller than η, or the number of seats available at the special school. Since there is no congestion, $\mathbb{P} = 1$, and we will see no externalities.

Now, checking the three kinds of equilibriums:

- $Q = 0$: to be in this case, we need that the utility of acquiring information be less or equal to the one of not being informed. This is:

$$U_i\left(A_i \mid Q=0\right) \leqslant U_i\left(NA_i \mid Q=0\right)$$

In terms of the parameters:

$$-c + \mathbb{P}pu_1 \leq 0$$

With $\mathbb{P} = 1$ we have that:

$$c^0 \geq pu_1 \tag{7.1.14}$$

For costs higher than pu_1 people will not acquire information, just as in the previous case.

- $Q = 1$: to be in this case, we need that the benefit of acquiring information be greater than that of not doing it.

$$U_i\left(A_i \mid Q=1\right) \geqslant U_i\left(NA_i \mid Q=1\right)$$

In terms of the parameters:

$$-c + \mathbb{P}pu_1 \geq 0$$

Now replacing $\mathbb{P} = 1$:

$$c^1 \leq pu_1 \tag{7.1.15}$$

- Mixed Strategies: to be in an equilibrium of this kind, independent of what the agent chooses, she must receive the same utility. This could be expressed as:

$$U_i(A_i \mid Q) = U_i(NA_i \mid Q)$$

In terms of the parameters of the model:

$$-c + \mathbb{P}pu_1 = 0$$

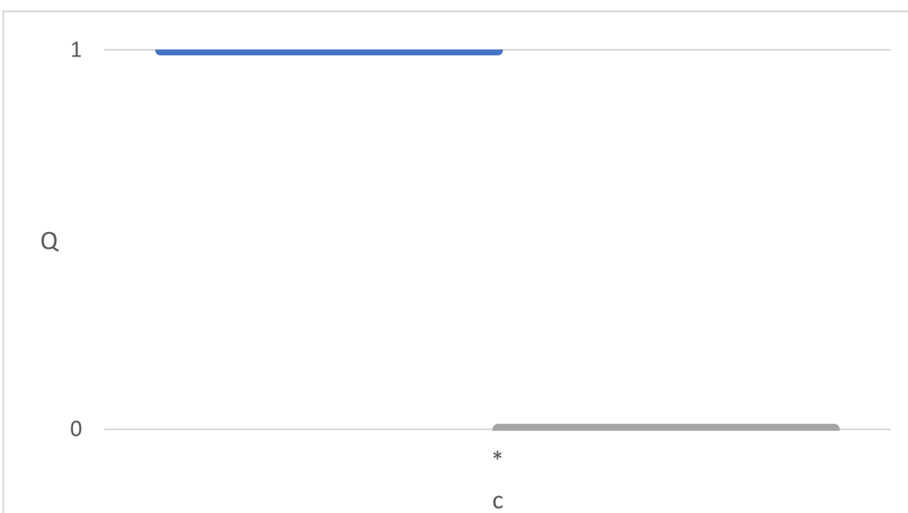
Now replacing $\mathbb{P} = 1$:

$$c^{ms} = pu_1 \tag{7.1.16}$$

Which is a non-generic case.

- Graphing the results as a function of the costs:

Figure 26



Social Planner : her utility now is the same as in the previous case.

$$U_{sp}(Q) = -cQ + \mathbb{P}Qpu_1$$

The main difference is that there is never congestion in this case, so the probability of being accepted in the special school is always equal to one.

As in the previous case with no congestion, a cost greater than $\tilde{c} = pu_1$ implies nobody acquires, and less than that, that every agent acquires.

The results of the Social Planner are presented in the yellow line on Figure 27. As we said before, in this case, there is no congestion. Since there are no other founts for externalities in this model, the private solution is the same that the one proposed by the Social Planner.

Figure 27

7.2. *Proofs:*

Proof lemma A1 A function that has its maximum values in its extremes is convex. For a derivable function, if its second derivative with respect to one argument is greater than zero, this function is convex. We proceed to prove that the Social Planner's utility function meets these criteria for every $Q \in [0, 1]$.

First derivative with respect to Q:

$$\frac{\partial U_{sp}(Q)}{\partial Q} = -c + \left[\frac{\eta}{1 - Q(1-p)} + \frac{Q\eta(1-p)}{(1 - Q(1-p))^2}\right](1-p)\bar{u}_2 + \frac{\eta(1-p)}{(1-Q(1-p))^2}\left[pu_1 - (1-p)\bar{u}_2\right]$$

The second derivative with respect to Q is:

$$\begin{aligned}\frac{\partial^2 U_{sp}(Q)}{\partial Q \partial Q} = \frac{\eta(1-p)^2\bar{u}_2}{(1-Q(1-p))^2} + \left[\frac{\eta(1-p)}{(1-Q(1-p))^2} + \frac{Q\eta(1-p)^2 2}{(1-Q(1-p))^3}\right](1-p)\bar{u}_2 \\ + \frac{\eta(1-p)^2 \cdot 2}{(1-Q(1-p))^3} \cdot \left[pu_1 - (1-p)\bar{u}_2\right]\end{aligned} \tag{7.2.1}$$

Because the restrictions we impose on the parameters ($u_1 \geq 0, \bar{u}_2 \leq 0, \eta \in (0,1), p \in (0,1), pu_1 - (1-p)\bar{u}_2 \geq 0$ and $\eta \leq p$, we can see that: $1 - Q(1-p) \geq 0 \; \forall \; Q$, then this second derivative is greater than zero for all $Q \in [0, 1]$.

Therefore, we can conclude that the problem, in this case, is a convex one; thus, Lemma A1 is proved.

■

Proof lemma A2: taking as given the results of Lemma A1, the solution is always $Q = 0$ or $Q = 1$. With this, we can now go on and find the point where the utilities obtained by the social planner choosing each option are equivalent:

$$U_{sp}(0) = U_{sp}(1)$$

Which in terms of the parameters is:

$$0 + 0 + \frac{\eta}{1}(pu_1 - (1-p)\bar{u}_2) = -c + \frac{\eta}{p}(1-p)\bar{u}_2 + \frac{\eta}{p}(pu_1 - (1-p)\bar{u}_2)$$

Now, calling the cost $\tilde{c}$ and solving:

$$\tilde{c} = \frac{n}{p}(1-p)p[u_1 + \bar{u}_2] \tag{7.2.2}$$

Expression that is always greater than zero. With which the Lemma A2 is proven. ■

Proof lemma A3: From the previous calculations, we know that:

$$c_1^{max} = \frac{\eta}{p}(1-p)\bar{u}_2$$

Now, we compare it with $\tilde{c}$:

$$\frac{n}{p}(1-p)p[u_1 + \bar{u}_2] > \frac{\eta}{p}(1-p)\bar{u}_2$$

Rearranging terms, it can be written as:

$$pu_1 - (1-p)\bar{u}_2 > 0 \tag{7.2.3}$$

A condition that is always true because agents ex-ante, agents always apply to the special school. Then, Lemma A3 is proved. ■

Proof lemma A4 We use the same logic as in the previous case with exogenous congestion. Also, the derivatives are the same. The only difference is that we can assure the strict convexity of the problem for every $Q \in [0, Q^{max}]$.

Proof lemma A5 taking as given the results of the Lemma A4, we have that the solution is always $Q = 0$ or $Q = Q^{max}$. With this, we can now go on and find the point under which the utilities of every option are equivalent for the social planner. This can be written down like;

$$U_{sp}(0) = U_{sp}((1-\eta)/(1-p))$$

Which in terms of the parameters of the model is:

$$0 + 0 + \frac{\eta}{1}(pu_1 - (1-p)\bar{u}_2) = -c\frac{1-\eta}{1-p} - \frac{1-\eta}{1-p}(1-p)\bar{u}_2 + (pu_1 + (1-p)u_2)$$

Calling the cost $\tilde{c}$ and solving:

$$\tilde{c} = p(1-p)(u_1 + \bar{u}_2) \tag{7.2.4}$$

Expression that is always greater than zero. ■

Proof lemma A6: Here we check if $\tilde{c}$ is greater than c^1:

$$p(1-p)(u_1+\bar{u}_2) > (1-p)\bar{u}_2$$

Rearranging terms, it can be written as:

$$pu_1 - (1-p)\bar{u}_2 > 0 \tag{7.2.5}$$

Condition that is always true in this case. Then, the Lemma A6 is proved. ■