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Essays on Life-Cycle Savings and Consumption

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ESSAYS ON LIFE-CYCLE SAVINGS AND CONSUMPTION

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To my family

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Contents

List of Tables

List of Figures

Abstract

This dissertation is divided into four chapters. The first chapter includes a brief literature survey on life-cycle precautionary saving. It presents a stochastic life-cycle model of consumption and savings and summarizes the empirical evidence on the degree of precautionary saving.

The second chapter studies the savings behavior of households over the life-cycle by revisiting Gourinchas and Parker (2002). They conclude that "consumer behavior changes strikingly over the life cycle"[1] based on the age-profile of target level of liquid wealth. I find that target wealth behavior differs substantially from actual savings behavior in a finite-horizon consumption life-cycle model. While the target value of wealth is a good indicator of the overall direction to which the distribution of normalized cash-on-hand moves, it fails to describe the magnitude of household overall savings. Furthermore, the age-profile of the target value of liquid wealth depends crucially on the retirement age's consumption rules, retirement income risks, and on the systematic-age variation of the consumption functions. Each of these factors are implied by the model's assumptions instead of observed consumer behavior. Moreover, by definition, the target value of cash-on-hand is sensitive to small changes (within confidence intervals for such parameter values) in the model's parameters, such as the interest rate, when the marginal propensity to consume is less than one.

The third chapter examines the implications of the individual life-cycle behavior predicted by Gourinchas and Parker (2002) for average liquid wealth, average consumption and average marginal propensity in light of Carroll Carroll (2000). Here, I show that the appropriateness of a representative-agent depends on the age distribution of the population and on the assumed values of the preference and retirement rule parameters of the life-cycle model. Under the base-line parameter values, individuals optimally choose to accumulate a substantial amount of liquid wealth: they quickly adjust their consumption and saving decisions so as to avoid the regions where borrowing constraints are binding. In fact, households' probability to hold low amounts of liquid wealth is almost zero by age 45. As a result, all individuals have the same marginal propensity to consume in their mid-forties and up. Thus, the aggregate dynamics of a representative-consumer model possessing liquid wealth equal to the mean of the distribution would resemble the aggregate predictions of the life-cycle model for middle-aged individuals. However, I also find that the predicted distribution of cash-on-hand does not match the wealth holdings in microeconomic data despite the various combinations of parameter values that are considered. This discrepancy sheds light in the suitability of the life-cycle model to reproduce the observed saving behavior across U.S. households.

The fourth chapter evaluates the use of the endogenous grid-points solution method when estimating a stochastic life-cycle model. The main finding is that the numerical solution method to solve the consumer problem affects its structural estimation. The Monte Carlo results suggest that one must be cautious when adopting the endogeneous grid-points solution method when numerically minimizing the Simulated Method of Moments estimators' objective function. The mode of the SMM estimates for the coefficient of risk aversion is approximately zero when its true value is small.

[1] Gourinchas and Parker, 2002, p. 47.

Chapter 1

Literature Review on Precautionary Saving

1.1 Introduction

A vast amount of literature has investigated how individuals make optimal consumption and savings decisions over their lifetime. Modigliani and Brumberg (1954)'s life-cycle model has been the stem for most of the works on inter-temporal consumption and savings behavior. Most of the implications derived from this model are inter-related and are all ultimately derived from the proposition that agents keep the expected marginal utility of expenditure constant. Overall, the standard life-cycle model predicts that a rational forward-looking agent, who makes consumption and savings decisions taking into account her life-time resources, will save when young and deccumulate her assets when old. The main prediction of the standard life-cycle model, however, has been challenged by three empirical observations (mostly on U.S. data): excess sensibility of consumption to income (Carroll, Hall, and Zeldes, 1992; Shea, 1995), fall in consumption at retirement (Banks, Blundell, and Tanner, 1998; Bernheim, Skinner, and Weinberg, 2001), and slow deccumulation of assets by the elderly (Kotlikoff and Summers, 1981; Hurd, 1989). In an attempt to reconcile these empirical observations and the theoretical predictions, the life-cycle theory has been extended and generalized into different ways relaxing progressively the standard life-cycle assumptions as new economic methods have been developed over time. Similarly over the past two decades and incited by advances in numerical solution methods, most empirical works have emphasized in explaining and improving our understanding of the effect of uncertainty on household consumption and savings behavior. In particular, the qualitative and quantitative aspects of precautionary saving, i.e. the additional saving resulting from future uncertainty, have been explored in depth.

This literature survey provides a concise summary on precautionary saving. The remainder of this document is structured as follows. Section 2 presents a life-cycle model of consumption and savings under uncertainty, commonly used to study precautionary saving. Section 3 summarizes the empirical evidence on the degree of precautionary saving. Finally, Section 4 concludes.

1.2 Precautionary Saving

Precautionary saving is traditionally modeled as the outcome of a consumer's optimal inter-temporal allocation of resources under future uncertainty. Hall (1978) is one of the first to consider labor income uncertainty in Modigliani and Brumberg's life-cycle framework. Assuming quadratic preferences for tractability, he finds out that consumption follows a random walk. Because its implication for inter-temporal consumption and savings behavior are equivalent to those in the perfect foresight model, his model is known as the certainty-equivalent (CEQ) model. Nevertheless, the notion of precautionary saving first emerged in Leland (1968). Based on a two-period model, he theoretically concludes that risk aversion is not sufficient to guarantee positive precautionary savings. Subsequently, Kimball (1990) shows that any utility function with a positive third derivative results in extra savings compared to the CEQ case. He terms the additional savings behavior resulting from income uncertainty as "prudence." He further defines the degree of absolute prudence as $-\frac{u'''(c)}{u''(c)}$, where $u(c)$ is any utility function with a positive third derivative. Thus, the degree of absolute prudence measures the strength of the precautionary saving motive just as the degree of absolute risk aversion measures the intensity of risk aversion. Despite the lack of a closed-form solution implied by constant relative risk aversion (CRRA) preferences, Zeldes (1989) and Kimball (1990) (among others) argue that this type of preferences is the most realistic as it allows precautionary saving to depend on the level of individual wealth.

Based on the standard approach of time-invariant preferences,[1] consider the following discrete-time, life-cycle model of household consumption. Individuals live until age N (assumed to be exogenous and fixed), and their preferences are represented by the standard additively separable expected utility form:

$$E\left[\sum_{t=1}^{N} \beta^t \frac{C_t^{1-\rho}}{1-\rho} + \beta^{N+1} V_{N+1}(W_{N+1})\right] \tag{1.1}$$

where β is the time-discount factor, $\frac{1}{\rho}$ is the inter-temporal elasticity of substitution, C_t is the total consumption at age t, W_t is the total financial wealth at age t, and V_{N+1} is the value to the consumer of the remaining assets after death. At each age $t \in [1, N]$, the individual receives a stochastic income Y_t given by:

$$\begin{aligned} Y_t &= P_t U_t \\ P_t &= G_t P_{t-1} N_t \end{aligned} \tag{1.2}$$

where labor income is divided into a permanent component P_t and a transitory component U_t. The transitory shocks U_t are assumed to be independently and identically distributed; moreover, there is a non-negative probability of a zero-income event, i.e. $U_t = 0$ with probability $p \in [0, 1)$. U_t is otherwise log-normally distributed, $\ln U_t \sim N(0, \sigma_U^2)$. The permanent component of income P_t follows a random walk with drift G_t (predictable growth of income) and permanent shock N_t, which is independently and identically log-normally distributed, $\ln N_t \sim N(0, \sigma_N^2)$.[2] While N_t is meant to capture the effects of job changes, wage raises, and other persistent factors, U_t is meant to capture the effects of one-time bonuses, unemployment spells, and other transitory factors.

The consumer's goal at age τ is then to allocate resources between current consumption and

[1]Time-invariant preferences are assumed to avoid issues of time inconsistency analyzed by Strotz (1955), Phelps and Pollak (1968), Laibson (1997), and others. The model described in this section is essentially the same as the one in Zeldes (1989), Carroll (1997), and Gourinchas and Parker (2002). Furthermore, this study focuses on the standard life-cycle model and does not encompass the growing literature on behavioral models on saving and consumption decisions.

[2]It is worth noting that under this income process formulation, consumers will never choose to borrow against future labor income. As Carroll and Kimball (2001) show, precautionary saving motive can induce self-imposed liquidity constraints. In particular, they provide an example in which the behavior of a consumer facing a zero-income event is virtually indistinguishable from the behavior of a perfect foresight but liquidity-constrained consumer as the probability of zero-income event approaches zero.

savings for future consumption by solving:

$$V_\tau(X_\tau, P_\tau) = \max_{C_\tau,\ldots,C_N} E_\tau \left[\sum_{t=\tau}^{N} \beta^{t-\tau} \frac{C_t^{1-\rho}}{1-\rho} + \beta^{N+1-\tau} V_{N+1}(X_{N+1}, P_{N+1}) \right]$$

given the labor income process defined in (1.2) and subject to:

$$X_{t+1} = R(X_t - C_t) + Y_{t+1} \quad (1.3)$$

$$X_{N+1} \geq 0 \quad (1.4)$$

where R is the constant, after-tax, gross real interest rate of the only asset available in the economy, X_t is defined as the level of cash-on-hand (total liquid financial wealth) in period t, i.e. $X_t \equiv W_t + Y_t$, and the last inequality reflects the fact that the consumer cannot die in debt.

Note that the consumer's problem is homogeneous of degree $(1-\rho)$ in P_t. Thus, by denoting lowercase letters as normalized variables, e.g. $x_t \equiv \frac{X_t}{P_t}$, the Euler equation at age t is given by:

$$u'(c_t(x_t)) = \beta R E \left[u'(c_{t+1}(x_{t+1}) G_{t+1} N_{t+1}) \right] \quad (1.5)$$

where $c_t(x_t)$ is the optimal consumption function. Assuming $V_{N+1} = 0$, the optimal consumption function at the last period is $c_N(x_N) = x_N$. Hence, the set of optimal consumption rules at each t can be found by solving problem (1.5) recursively. Figure 1.1a displays the optimal consumption functions under the parameter values listed in Table 1.1. As can be seen, the consumption at N is linear and equal to the 45^o-line depicting the fact that it is optimal to consume all the remaining wealth in the last period of life. On the other hand, the consumption rules for $t < N$ are increasing, positive, and concave in normalized cash-on-hand.[3] Figure 1.1b shows the optimal consumption functions in the absence of income uncertainty. The consumption rules are increasing, positive, and linear in normalized cash-on-hand. The magnitude of precautionary saving is calculated as the difference between the consumption rules without and with income uncertainty, as depicted on Figure 1.1c. As can be observed, income uncertainty reduces the amount of optimal consumption, and this reduction is higher as the horizon recedes and for consumers with low levels of normalized cash-on-hand. In fact, precautionary saving is inversely related to cash-on-hand with CRRA preferences.

Furthermore, assuming an infinite-horizon ($N = \infty$) and plausible parameter values, Carroll, Hall, and Zeldes (1992), Carroll (1997), and Carroll (2019) show that consumers have a target level of normalized cash-on-hand in the presence of income uncertainty. He defines it as the level of normalized cash-on-hand at which cash-on-hand is expected to remain unchanged from age t to age $t+1$, that is:

$$\bar{x}_t = E_t \left[x_{t+1} | x_t = \bar{x}_t \right] \quad (1.6)$$

Carroll points out that households must be both impatient and prudent for the existence of this target in the presence of zero income risk. Impatience refers to consumers who would like to consumer more now than save for the future, and prudence in regards to the the propensity to prepare oneself in the presence of uncertainty.[4] As a result, households engage in "buffer-stock" savings behavior: they choose to accumulate some amount of wealth to cushion against income risks at each age t.

The implications of the "buffer-stock" saving model have been explored on observed wealth inequality, average marginal propensities to consume, business cycles, among other things. For instance,

[3] Carroll and Kimball (2001) show algebraically how income uncertainty creates non-linearities in the consumption function for CRRA preferences.

[4] Kimball (1990) defines prudence as the "the sensitivity of the optimal choice of a decision variable to risk." (p. 54)

Jappelli, Padula, and Pistaferri (2008) test the "buffer-stock" saving model on Italian working-age individuals and find no evidence for buffer-stock behavior even among young households. On the other hand, Carroll, Slacalek, and Tokuoka (2013) calibrate the "buffer-stock" model to match the wealth distribution in various European countries and find that the average marginal propensities to consume out of transitory shocks are between $0.1 - 0.4$. Similarly, Kaplan, Violante, and Weidner (2014) document that approximately 30% of US households are living "hand-to-mouth," i.e. with little or no liquid wealth, and as a result they consume all of their disposable income every period.

1.3 Empirical Evidence

1.3.1 Euler Equation Estimation

As surveyed by Browning and Lusardi (1996), the early empirical literature aiming to explore the strength of precautionary saving motives relied on estimating consumption Euler equations. Note that although an analytical closed-form solution for the life-cycle model in the previous section is not available, the solution must satisfy the Euler equation:

$$1 = R\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right] \tag{1.7}$$

Taking a first-order Taylor expansion and making some approximations (for a step by step derivation see Carroll (2001)), the Euler Equation becomes:

$$\begin{aligned} 0 &\approx (r-\delta) - \rho E_t\left[\Delta logC_{t+1}\right] \\ E_t\left[\Delta logC_{t+1}\right] &\approx \frac{1}{\rho}(r-\delta) \end{aligned} \tag{1.8}$$

where δ is the time preference rate from $\beta = \frac{1}{1+\delta}$, r is the interest rate from $R = (1+r)$, and $\Delta logC_{t+1} = logC_{t+1} - logC_t$. Finally, defining the expectation error as $\epsilon_{t+1} \equiv \Delta logC_{t+1} - E_t\left[\Delta logC_{t+1}\right]$,

$$\Delta logC_{t+1} \approx \frac{1}{\rho}(r-\delta) + \epsilon_{t+1}. \tag{1.9}$$

Hence, in order to estimate ρ, researchers often estimated regression equations of the form

$$\Delta logC_{t+1} = \alpha_0 + \alpha_1 E_t[r_{t+1}] + \epsilon_{t+1} \tag{1.10}$$

where α_1, the coefficient on the interest rate r, is interpreted as an estimate of the intertemporal elasticity of substitution, $\frac{1}{\rho}$. Browning and Lusardi (1996) report that only few studies using cross-sectional and time-series data have found significantly positive values of α_1. Moreover, as Carroll (2001) criticizes, the presence of precautionary saving invalidates most of these log-linearized estimations as higher-order terms on the approximation of the Euler equation (which are usually absorbed into the regression error term) are endogenous with respect to the first-order terms. In order to understand this criticism, consider the second-order approximation of the Euler equation (1.7):

$$\Delta logC_{t+1} \approx \frac{1}{\rho}(r-\delta) + \left(\frac{1+\rho}{2}\right) E_t[\Delta logC_{t+1}^2] + \epsilon_{t+1} \tag{1.11}$$

where $E_t[\Delta logC_{t+1}^2]$ is interpreted as a measure of volatility in consumption growth caused by the uncertainty of income. Carroll (2001) argues the theory implies that risks generate a positive correlation between interest rates and the variance of consumption growth through precautionary savings.

As consumers save more in order to reduce future consumption volatility, precautionary saving depresses interest rates in a steady state. Hence, empirical tests using the regression form (1.10) result in inconsistent estimations of α_1. Carroll further argues that the difficulty to find an observable and exogenous proxy for risk results in biased estimations, even when trying to estimate the regression form (1.11), invalidating most empirical tests using the usual log-linearized Euler equation approach.

1.3.2 Structural Estimation

As a response and fueled by advances in numerical solution methods, some researchers have opted to calibrate preference parameters on observable data and use these values to simulate the life-cycle consumption model under uncertainty.[5] The objective is to compare the simulated and observed consumption trajectories to evaluate the predictions of the life-cycle model. For instance, Zeldes (1989), Carroll, Hall, and Zeldes (1992), and Carroll, Hall, and Zeldes (1992) examine a finite-horizon consumption problem where households face labor income uncertainty without explicitly modeling retirement years. They find that both precautionary savings and marginal propensity to consume (MPC) are decreasing functions of the level of liquid wealth. While the MPC is similar to the one implied by the certainty-equivalent life-cycle hypothesis (CEQ-LCH) model for households with high wealth holdings relative to expected future income, the MPC is higher for households with low liquid wealth holdings relative to expected future income than for the rest of the population. Moreover, Carroll, Hall, and Zeldes (1992) and Carroll (1997) document that impatient households maintain a buffer-stock of assets as a response to income uncertainty over most of the working lifetime until approximately age 50, but impatience keeps buffer-stocks small. Similar results are found by Deaton (1991) who considers an infinite-horizon problem augmented by borrowing constraints. In particular, he finds that buffer-stock emerges in the presence of liquidity constraints as households save to guard against negative income shocks. Furthermore, Hubbard, Skinner, and Zeldes (1995) show that income uncertainty can create hump-shaped consumption age-profiles as individuals save early in life for precautionary reasons and dissave during retirement years.

More recent empirical studies follow the influential work of Gourinchas and Parker (2002) (henceforth GP). They take a step forward from the calibration technique by proposing a full dynamic structural modeling approach to estimate the importance of precautionary saving. The aim is to econometrically estimate the values of the preference parameters of a life-cycle model by matching simulated average age-profiles of consumption with those observed over the working ages of households in empirical data. The degree of the precautionary motive emerges as an estimate of the coefficient of relative risk aversion.

In order to understand GP's estimation methodology, recall the model in Section 2. The consumption for individual i at age t depends on the parameters of the problem ($\psi \in \Psi \subset R^s$), the realization of the permanent component of income (P_{it}) and the level of cash on hand (X_{it}). Thus, based on the model, the data-generating process for each age t can be assumed to be:

$$\ln C_{it} = \ln(C_t(X_{it}, P_{it}; \psi)) + \epsilon_{it}$$

where $\ln C_{it}$ is the observed log-consumption of individual i of age t and ϵ_{it} is an idiosyncratic shock. Due to the lack of a good quality panel data of consumption, assets, and income for individual households, GP propose to estimate the model based on the following condition for each age t:

$$E[\ln C_{it} - \ln C_t(\psi_0)] = 0$$

[5] Analytical solutions to the life-cycle problem in (1.1) does not generally exist (except for the exponential utility functions) when certainty equivalence does not hold; thus, the optimal consumption is solved employing numerical techniques.

where $\ln C_t(\psi)$ is the unconditional expectation of log-consumption at each age t and ψ_0 is the true parameter vector. Partitioning the parameter vector into first-stage ($\chi \in R^r$) and second-stage ($\theta \in \Theta \subset R^s$ where Θ is a compact set) parameters, the estimation procedure proceeds by first estimating χ using additional data and moments, and then estimating θ using the Simulated Method of Moments. Because $\ln C_t(\psi)$ does not have an analytic expression and depends on the parameters, it is simulated by solving the model numerically for L households and computing the mean of the simulated consumption profiles (for each age t). Thus, the SMM estimator solves:

$$\min_{\theta} g(\theta; \hat{\chi})' W g(\theta; \hat{\chi})$$

where W is a $T \times T$ weighting matrix and $g(\theta; \hat{\chi}) \in R^T$ is a vector with t^{th} element:

$$g_t(\theta; \hat{\chi}) = \ln \bar{C}_t - \ln \hat{C}_t(\theta; \hat{\chi})$$

where $\ln \bar{C}_t$ is the average consumption for age t observed in the empirical data and $\ln \hat{C}_t(\theta; \hat{\chi})$ is the simulated counterpart of $\ln C_t(\theta; \hat{\chi})$. Thus, the SMM estimator chooses θ that matches the means of the empirical and simulated distributions for each age t.[6]

Matching the simulated average age-profiles of consumption with those observed over the working ages of U.S. households in the Consumer Expenditure Survey (CEX), GP find that the average household has a coefficient of risk aversion of $0.1 - 0.6$ and a discount factor of $0.95 - 0.97$ depending on the various assumptions considered. Based on the values of the estimated parameters, GP conclude that the mean household exhibit "buffer-stock" behavior until around age 40. Following the simulation strategy used by GP, Cagetti (2003) structurally estimates the preference parameters of a life-cycle model of wealth accumulation by matching simulated median wealth profiles with those observed in the Panel Study of Income Dynamics and in the Survey of Consumer Finances. He finds larger values for the coefficient of risk aversion compared to the ones reported by GP, usually higher than 3.

It is worth noting that the quantitative results from these structural estimations depend on the model's assumptions and on choice of the moment conditions. Carroll and Kimball (2006) warn that the degree of income uncertainty faced by households affect the estimates of relative risk aversion in both Gourinchas and Parker (2002) and Cagetti (2003). Similarly, Michaelides and Ng (2000) point out the choice of moment conditions for the SMM estimation matters for the identification of the structural parameters when there are non-linearities and serial dependence on the data. Nevertheless, the methodology proposed by Gourinchas and Parker (2002) has been since applied by French (2005), Laibson, Repetto, and Tobacman (2007), De Nardi, French, and Jones (2010), French and Jones (2011), and Fella, Frache, and Koeniger (2016), among others.

1.3.3 Survey Evidence

Given the drawbacks mentioned above, some studies have focused their attention on survey evidence. This alternative approach consists on asking survey participants direct questions about their target level of precautionary wealth and hypothetical questions about their risk behavior in order to gather information about the intensity of their underlying preferences. For instance, Kimball, Sahm, and Shapiro (2008) construct a measurement of risk aversion based on survey responses about gambles over lifetime income in the 1992 and 1994 Health and Retirement Study. They ask individuals to choose between a job with a certain lifetime income and a risky job which can doble lifetime income or decrease it by a specific fraction with equal chances. Based on the responses, they construct a cardinal proxy of risk preference and report a relative risk aversion with a mean of 8.2 and a median of

[6] A full description of the estimation methodology can be found in Section 3 of Gourinchas and Parker (2002).

6.3. These estimates imply a much stronger precautionary saving motive than those found empirically when matching observed consumption or wealth data.

1.3.4 Conclusion

Although the qualitative aspects of the theory of precautionary savings behavior are now well-established, there is less agreement about the degree of precautionary saving motive in the empirical literature. Earlier studies use log-linearized Euler equations and consumption data to estimate the preference parameters, generally reporting a low risk aversion and thus a low precautionary saving motive. However, Carroll (2001) shows that the presence of precautionary saving invalidates most of these log-linearized estimations. On the other hand, Gourinchas and Parker (2002), Cagetti (2003), and other works following their proposed structural methodology find a degree of risk aversion that is less than the estimates obtained based using survey evidence. This unresolved discrepancy is important as the coefficient of risk aversion, and therefore the degree of precautionary saving motive, have implications on the optimal savings behavior of households and their reactions to policy changes.

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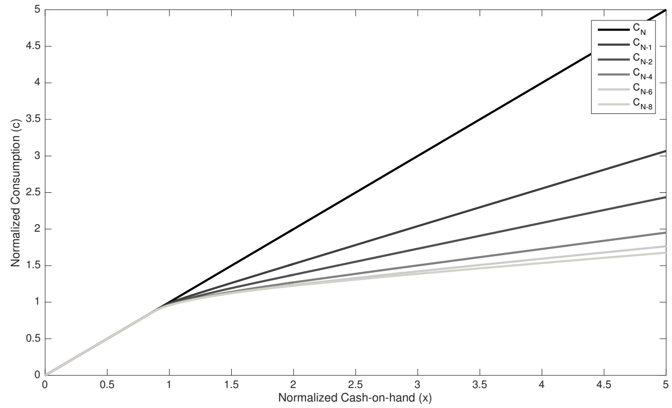
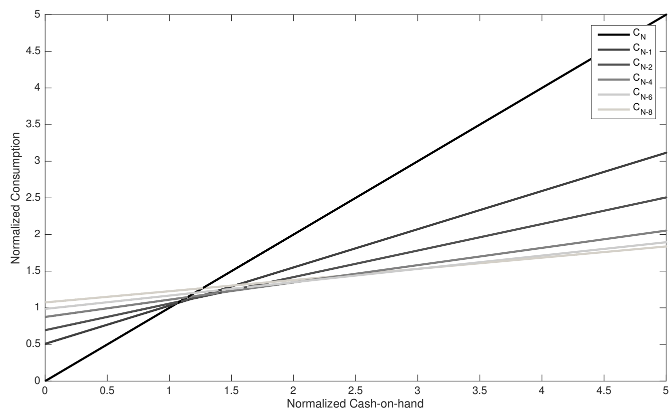
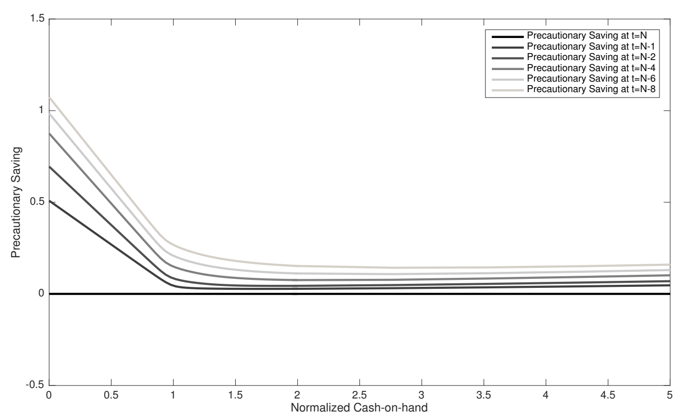


(a) Income uncertainty



(b) Perfect foresight



(c) Precautionary savings

Figure 1.1: Consumption Functions and Precautionary Savings (assuming $\rho = 0.514, \beta = 0.96, R = 1.0344$)

Parameter	Value	Source
ρ	0.514	Gourinchas and Parker (2002)
β	0.96	Gourinchas and Parker (2002)
R	1.03440	Moody's AAA municipal bonds, Jan 1980 - March 1993
σ_U^2	0.0440	Carroll and Samwick (1997), PSID 1981-1987
σ_N^2	0.0212	Carroll and Samwick (1997), PSID 1981-1987
p	0.00302	Carroll, Hall, and Zeldes (1992), PSID 1976-1985
G_t:		Gourinchas and Parker (2002), CEX 1980-1993
age	.32643678179	
age^2	$-.0148947085$	
age^3	.00036342384	
age^4	$-4.411685e^{-6}$	
age^5	$2.056916e^{-8}$	
$constant$	6.801368713	

Table 1.1: Parameter Values from Gourinchas and Parker (2002)

Chapter 2

Life-Cycle Savings Behavior

2.1 Introduction

The motivation of this chapter is to understand the savings behavior of households over the life-cycle. In order to do so, I revisit Gourinchas and Parker (2002) (henceforth GP), an influential article on asset accumulation and consumption over the life-cycle under income uncertainty. Based on their calculations on target wealth, GP conclude that "consumer behavior changes strikingly over the life-cycle. Young consumers behave as buffer-stock agents. Around age 40, the typical household starts accumulating liquid assets for retirement and its behavior mimics more closely that of a certainty equivalent consumer."[1] In this document, I show that this assertion is inaccurate. I do not identify an age at which consumer behavior changes strikingly over the life-cycle.

Precautionary saving is traditionally modeled as the outcome of a consumer's optimal inter-temporal allocation of resources under uncertainty of future income realizations. Hall (1978) is one of the first to consider labor income uncertainty in a life-cycle framework. Assuming quadratic preferences for tractability, he finds out that consumption follows a random walk. Because its implication for inter-temporal consumption and savings behavior are equivalent to those in the perfect foresight model, his model is known as the certainty-equivalent (CEQ) model. The notion of precautionary saving first emerged in Leland (1968). Based on a two-period model, he theoretically concludes that risk aversion is not sufficient to guarantee positive precautionary savings. Subsequently, Kimball (1990) shows that any utility function with a positive third derivative results in extra savings compared to the CEQ case. Despite the lack of a closed-form solution implied by constant relative risk aversion (CRRA) preferences, Zeldes (1989) and Kimball (1990), among others, argue that this type of preferences are the most realistic as it allows precautionary saving to depend on the level of individual wealth. Following Zeldes (1989), the literature has used numerical solution methods to calculate the optimal amount of precautionary saving in the presence future labor income.

Buffer-stock savings behavior was first documented by Carroll, Hall, and Zeldes (1992) and Carroll (1997). Using numerical solutions of an infinite-horizon model and plausible parameter values, Carroll finds that households have a target level of cash-on-hand to permanent income ratio in the presence of income uncertainty. He points out that households must be both impatient and prudent for the existence of this target. Impatience refers to consumers who would like to consumer more now than save for the future. Prudence is defined as the inclination to prepare oneself in the presence of uncertainty. As a result, households engage in "buffer-stock" behavior: they choose to accumulate some amount of wealth to cushion against income risks. In a recent work, Carroll (2019) further

[1] Gourinchas and Parker, 2002, p. 47.

provides sufficient theoretical conditions for the existence of the target level of cash-on-hand.

Following Carroll, GP calculate the target level of normalized cash-on-hand to study the savings behavior of U.S. households. Using their estimated parameter values, they observe that the target wealth is relatively constant until age 40 and increases substantially as households get older and closer to retirement age 65. From this one observation, GP conclude that there is a striking change of savings behavior over the life-cycle. However, it is unclear how the target wealth to permanent to income ratio describes the optimal savings behavior under a life-cycle model. The target itself can change "mildly" or "strikingly" with age as income growth and consumption rules change over the life-cycle.

The current document differentiates target wealth behavior from actual savings behavior by households based on GP's life-cycle model. The main finding is that while the target value of wealth is a good indicator of the overall direction to which the distribution of normalized cash-on-hand moves, it fails to describe the magnitude of household overall savings. Furthermore, the age-profile of the target value of liquid wealth depends crucially on the retirement age's consumption rules, retirement income risks, and on the systematic-age variation of the consumption functions. Each of these factors are implied by the model's assumptions instead of observed consumer behavior. Moreover, by definition, the target value of cash-on-hand is sensitive to small changes (within confidence intervals for such parameter values) in the model's parameters, such as the interest rate, when the marginal propensity to consume is less than one.

The remainder of this document is structured as follows. Section 2 describes the base-line life-cycle model estimated by GP. Section 3 characterizes household savings behavior and target wealth behavior. Section 4 concludes.

2.2 Life-Cycle Model in Gourinchas and Parker (2002)

2.2.1 The Model

GP consider the following discrete-time, life-cycle model of household consumption. Individuals live until age N and retire at age $T < N$. Both T and N are assumed to be exogenous and fixed. Preferences are represented by the standard additively separable expected utility form:

$$E\left[\sum_{t=1}^{N} \beta^t u(C_t, Z_t) + \beta^{N+1} V_{N+1}(W_{N+1})\right] \tag{2.1}$$

where β is the time-discount factor, C_t is the total consumption at age t, W_t is the total financial wealth at age t, Z_t is the vector of deterministic household characteristics at age t, and V_{N+1} is the value to the consumer of the remaining assets after death. Furthermore, the Bernoulli function is assumed to take the following form:

$$u(C, Z) = v(Z)\frac{C^{1-\rho}}{1-\rho}$$

where $\frac{1}{\rho}$ is the inter-temporal elasticity of substitution. At each age $t \in [1, T]$, the individual receives a stochastic income Y_t and maximizes (2.1) subject to:

$$\begin{aligned} W_{t+1} &= R(W_t + Y_t - C_t) \\ W_{N+1} &\geq 0 \end{aligned}$$

given an initial wealth level W_1 and where R is the constant, after-tax, gross real interest rate of the only asset available in the economy. Following Zeldes (1989) and Hall and Mishkin (1982), the labor income process is given by:

$$\begin{aligned} Y_t &= P_t U_t \\ P_t &= G_t P_{t-1} N_t \end{aligned} \tag{2.2}$$

where labor income is divided into a permanent component P_t and a transitory component U_t.[2] The transitory shocks U_t are assumed to be independently and identically distributed; moreover, there is a non-negative probability of a zero-income event, i.e. $U_t = 0$ with probability $p \in [0, 1)$. U_t is otherwise lognormally distributed, $\ln U_t \sim N(0, \sigma_U^2)$. The permanent component of income P_t follows a random walk with drift G_t (predictable growth of income) and permanent shock N_t, which is independently and identically lognormally distributed, $\ln N_t \sim N(0, \sigma_N^2)$. While N_t is meant to capture the effects of job changes, wage raises, and other persistent factors, U_t is meant to capture the effects of one-time bonuses, unemployment spells, and other transitory factors. Note that individuals will never choose to borrow against future labor income as long as the Inada condition $\lim_{c\to 0} = \infty$ is satisfied (the assumed income process has a zero lower bound which imposes a natural borrowing limit at 0).

GP make four additional assumptions. First, in order to reduce the number of state variables in the model, they assume that the age variations in $v(Z_t)$ are deterministic, common across households of the same age, and that they come from changes in family size. Second, stating that most of the retirement wealth in U.S. households is accumulated in illiquid assets (which are only available after retirement), GP assume that illiquid wealth accumulates exogenously, cannot be borrowed against, and that illiquid wealth in the first year of retirement is proportional to the last permanent component of income, i.e. $H_{T+1} \equiv hP_{T+1} = hP_T$. These assumptions eliminate both illiquid assets as a state variable and contributions to illiquid accounts as a control variable in the dynamic stochastic program. Third, invoking Bellman's optimality principle, the inter-temporal consumption problem is truncated at the age of retirement to avoid modelling the retirement period. Lastly, due to the truncation assumption, GP adopt the following retirement value function to condense the consumer's problem at retirement ages:

$$V_{T+1}(X_{T+1}, H_{T+1}, Z_{T+1}) = kv(Z_{T+1})(X_{T+1} + H_{T+1})^{1-\rho} \tag{2.3}$$

for some constant k and where X_t is defined as the level of cash-on-hand (total liquid financial wealth) in period t, i.e. $X_t \equiv W_t + Y_t$. According to GP, the functional form (2.3) is chosen to maintain the tractability of the problem and for being flexible enough to allow robustness checks.

In sum, the consumer's problem at age τ can be expressed as:

$$V_\tau(X_\tau, P_\tau, Z_\tau) = \max_{C_\tau,\ldots,C_T} E_\tau \left[\sum_{t=\tau}^{T} \beta^{t-\tau} v(Z_t) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau} kv(Z_{T+1})(X_{T+1} + hP_{T+1})^{1-\rho} \right]$$

given the labor income process defined in (2.2) and subject to:

$$X_{t+1} = R(X_t - C_t) + Y_{t+1} \tag{2.4}$$

$$X_{T+1} \geq 0 \tag{2.5}$$

where the last inequality reflects the borrowing constraint on liquid assets at retirement age, imposed by the assumption that illiquid wealth cannot be borrowed against.

[2]Labor income is defined as disposable income, net of Social Security taxes and contributions to illiquid accounts.

2.2.2 Numerical Solution

Since an analytical closed-form solution for the above problem is not available, GP solve the problem numerically by first normalizing all variables by the permanent component of income. They note that the particular functional form for the retirement function makes the household's consumption problem homogeneous of degree $(1-\rho)$ in P_t. Thus, by denoting lowercase letters as normalized variables, e.g. $x_t \equiv \frac{X_t}{P_t}$, the following Euler equation holds for ages $t < T$:

$$u'(c_t(x_t)) = \beta R E_t \left[\frac{v(Z_{t+1})}{v(Z_t)} u'(c_{t+1}(x_{t+1}) G_{t+1} N_{t+1}) \right] \tag{2.6}$$

where $c_t(x_t)$ is the optimal consumption function. In the last working period, the Euler equation is replaced by:

$$u'(c_T(x_T)) = \max \left\{ u'(x_T), \beta R \left[\frac{v(Z_{T+1})}{v(Z_T)} u'(c_{T+1}(x_{T+1})) \right] \right\} \tag{2.7}$$

since the illiquid wealth available in $T+1$ imposes a liquidity constraint on the cash-on-hand available at the age of retirement. Furthermore, under (2.3), the optimal retirement consumption rule is linear in total wealth. Hence, the normalized consumption rule at $T+1$ is expressed as:

$$\begin{aligned} \frac{C_{T+1}}{P_{T+1}} &= \gamma_1 \left(\frac{X_{T+1} + H_{T+1}}{P_{T+1}} \right) \\ c_{T+1} &= \gamma_1 (x_{T+1} + h) = \gamma_0 + \gamma_1 x_{T+1} \end{aligned} \tag{2.8}$$

where $\gamma_0 \equiv \gamma_1 h$ and γ_1 is the marginal propensity to consume out of liquid wealth. Thus, in order to find the set of optimal consumption rules for each age t, the problem can be solved recursively by first finding $c_T(x_T)$ in (2.7) by using (2.8). The optimal solutions to (2.7) and (2.6) then generate the consumption functions $c_{T-1}(x_{T-1}), ..., c_1(x_1)$.

In order to implement the proposed solution algorithm, a grid of 100 points between $[0, 40]$ is created for cash-on-hand, with 50 points between 0 and 2 following GP's discretization method. The finer grid for $x \in [0, 2]$ captures the curvature of the consumption rule at low values of cash-on-hand. Furthermore, to evaluate the expectation in (2.6), a two-dimensional Gauss-Hermite quadrature of order 12 is performed as GP.[3] Table 2.1 reports the assumed values for the variances of the income shocks (σ_U^2, σ_N^2), the probability of zero-income event (p), the initial distribution of liquid assets at age 26 (w_1), the gross real after tax interest rate (R), and the income and family-composition profiles (G_t, Z_t). Lastly, the age of retirement is set at 65, i.e., $T = 40$.

2.3 Results

2.3.1 Individual Consumption Behavior

Figure 2.1 reports the consumption rules of a typical consumer working from ages 26 to 65 ($t = 1, ..., T$), whose consumption at $T+1$ is characterized by (2.8). The preference parameters are set to $\beta = 0.960$, $\rho = 0.514$, $\gamma_0 = 0.001$, and $\gamma_1 = 0.071$. These are the base-line parameter values estimated from the structural estimation in Gourinchas and Parker (2002), which matches simulated average age-profiles of consumption with those observed over the working ages of U.S. households in the Consumer Expenditure Survey (CEX).[4]

[3] For more details, see Appendix A.

[4] See Chapter 4, Section 3, for a description of the Simulated Method of Moments Estimation.

As can be seen, normalized consumption is increasing, positive, and concave in normalized cash-on-hand. The consumption rules also change over the life-cycle, i.e. there is a systematic age-variation in consumption behavior. A typical young consumer has a low level of liquid wealth; as a result, her marginal propensity to consume is high as the consumption function is steep at low levels of cash-on-hand. Since labor income is expected to grow, young households prefer to borrow or save very little as more resources will be available in the future. Moreover, these parameter values imply a low level of illiquid wealth at $T+1$; thus, households must save for retirement as they age. As liquid wealth is accumulated to finance retirement years, consumption rules decrease as $t \to T$. Since most middle-aged households will have saved to a large extent for retirement purposes by then, their marginal propensity to consume will be low as the consumption function is relatively flat at high levels of cash-on-hand.

Alternatively, Figures 2.2a and 2.2b show the consumption functions for households facing two different retirement consumption rules $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$, respectively. Again, the optimal rules are concave in normalized cash-on-hand and vary systematically with age. With a higher level of illiquid wealth implied by γ_0, households do not accumulate as much liquid wealth as in Figure 2.1, and consumption functions decline less as the household ages. As can be seen, households accumulate less liquid wealth for retirement purposes as γ_0 increases: they can depend more on illiquid wealth to finance consumption when retired.

The model is further solved for $\rho = 1.5$ and $\rho = 3$, assuming $\gamma_0 = 0.01$, $\gamma_0 = 0.594$, and $\gamma_0 = 0.9$. Figure 2.3 reports the consumption policy functions for $\rho = 1.5$, and Figure 2.4 depicts the ones for $\rho = 3$. As can be seen, the consumption functions are concave in normalized cash-on-hand and vary systematically with age; however, as the coefficient of risk aversion increases, the consumption functions are lower for all ages since consumers save more for precautionary reasons.

2.3.2 Target Level of Normalized Cash-on-Hand

In order to answer how household savings behavior changes over the life-cycle, GP calculate the target level of cash-on-hand to permanent income ratio for each age. Following Carroll, Hall, and Zeldes (1992), Carroll (1997), and Carroll (2019), they define it as the level of normalized cash-on-hand at which cash-on-hand is expected to remain unchanged from age t to age $t+1$, that is:

$$\bar{x}_t = E_t\left[x_{t+1} | x_t = \bar{x}_t\right] \tag{2.9}$$

Since the target level of cash-on-hand is the fixed-point of the function $E_t\left[x_{t+1}|\, x_t\right]$, it is worth characterizing $E_t\left[x_{t+1}|\, x_t\right]$ first. Taking expectations on normalized cash-on-hand:

$$x_{t+1} = R(x_t - c_t)\frac{1}{G_{t+1}N_{t+1}} + U_{t+1} \tag{2.10}$$

$$E_t\left[x_{t+1}|\, x_t\right] = \frac{R}{G_{t+1}}(x_t - c_t)E_t\left[\frac{1}{N_{t+1}}\right] + E_t\left[U_{t+1}\right]. \tag{2.11}$$

Since the transitory income shock U_{t+1} is distributed lognormally with probability $(1-p)$ and is equal to 0 with probability p:

$$\ln U_t \sim N(0, \sigma_U^2) \quad \longrightarrow \quad E_t\left[U_{t+1}\right] = e^{0+\frac{1}{2}\sigma_U^2}(1-p) = e^{\frac{1}{2}\sigma_U^2}(1-p).$$

Similarly, for the inverse of the permanent income shock:

$$\ln\frac{1}{N_t} \sim N(0, \sigma_N^2) \quad \longrightarrow \quad E_t\left[\frac{1}{N_{t+1}}\right] = e^{0+\frac{1}{2}\sigma_N^2} = e^{\frac{1}{2}\sigma_N^2}$$

since if for random variable $\ln X \sim N(\mu, \sigma^2)$, then $\ln \frac{1}{X} \sim N(-\mu, \sigma^2)$. Thus, Equation (2.11) becomes

$$E_t\left[x_{t+1}|\, x_t\right] = \frac{R}{G_{t+1}}(x_t - c_t)E_t\left[\frac{1}{N_{t+1}}\right] + E_t\left[U_{t+1}\right] \tag{2.12}$$

$$E_t\left[x_{t+1}|\, x_t\right] = \frac{R}{G_{t+1}}(x_t - c_t)e^{\frac{1}{2}\sigma_N^2} + e^{\frac{1}{2}\sigma_U^2}(1-p). \tag{2.13}$$

It follows that

$$\frac{\partial E_t\left[x_{t+1}|x_t\right]}{\partial x_t} = \frac{R}{G_{t+1}}E_t\left[\frac{1}{N_{t+1}}\right]\left(1 - \frac{\partial c_t(x_t)}{\partial x_t}\right) \tag{2.14}$$

$$\frac{\partial E_t\left[x_{t+1}|x_t\right]}{\partial x_t} = \frac{R}{G_{t+1}}e^{\frac{1}{2}\sigma_N^2}\left(1 - \frac{\partial c_t(x_t)}{\partial x_t}\right). \tag{2.15}$$

Note that the slope of the function $E_t\left[x_{t+1}|\, x_t\right]$ at age t depends on the interest rate, the variance of the permanent income shocks, the income growth rate, and the marginal propensity to consume at age t, which in turn depends on preference parameters, retirement consumption rules, and variations in family size. Since the target level of cash-on-hand is the fixed-point of the function $E_t\left[x_{t+1}|\, x_t\right]$, it is also sensitive to the model's parameters and varies with age as income growth G_{t+1} and the marginal propensity to consume change over the life-cycle.

Figure 2.5a reports the function $E_t\left[x_{t+1}|\, x_t\right]$ for selected ages, assuming the base-line parameter values $\rho = 0.514$ and $\gamma_0 = 0.001$. The functions are constant up to a "kink-point" and then increasing in normalized cash-on-hand (these "kink-points" correspond to the same value of normalized cash-on-hand from the "kink-points" in the consumption functions). The points of intersection between the 45-degree line and each of the functions $E_t\left[x_{t+1}|\, x_t\right]$ correspond to the target level of normalized cash-on-hand, $\bar{x}_t$, for those ages.

To further characterize the target, it is useful to make a linear approximation of the consumption function such as:

$$c_t(x_t) = \begin{cases} x_t & for \quad x_t < x_t^* \\ (1-\psi_t)x_t^* + \psi_t x_t & for \quad x_t \geq x_t^* \end{cases} \tag{2.16}$$

where x_t^* corresponds to the value of normalized cash-on-hand at which households are no longer liquidity constrained, and ψ_t is the marginal propensity to consume at age t. Although individuals technically do not face liquidity constraints during their working ages, they do face a natural borrowing constraint. Their marginal propensity to consume is high for low levels of normalized cash-on-hand and low for high levels of normalize cash-on-hand. A linear approximation of the consumption function could thus give a simple but practical description of the target value of cash-on-hand.

From Equation (2.16), it follows that:

$$E_t\left[x_{t+1}|\, x_t\right] = \begin{cases} e^{\frac{1}{2}\sigma_U^2}(1-p) & for \quad x_t < x_t^* \\ \frac{R}{G_{t+1}}e^{\frac{1}{2}\sigma_N^2}(1-\psi_t)(x_t - x_t^*) + e^{\frac{1}{2}\sigma_U^2}(1-p) & for \quad x_t \geq x_t^*. \end{cases} \tag{2.17}$$

Calculating the fixed-point of the function above, the target level of cash-on-hand at age t is:

$$\bar{x}_t = \begin{cases} e^{\frac{1}{2}\sigma_U^2}(1-p) & for \quad x_t < x_t^* \\ \frac{e^{\frac{1}{2}\sigma_U^2}(1-p) - \frac{R}{G_{t+1}}e^{\frac{1}{2}\sigma_N^2}(1-\psi_t)x_t^*}{1 - \frac{R}{G_{t+1}}e^{\frac{1}{2}\sigma_N^2}(1-\psi_t)} & for \quad x_t \geq x_t^* \end{cases} \tag{2.18}$$

where $\frac{R}{G_{t+1}}e^{\frac{1}{2}\sigma_N^2}(1-\psi_t) < 1$ and $e^{\frac{1}{2}\sigma_U^2}(1-p) > \frac{R}{G_{t+1}}e^{\frac{1}{2}\sigma_N^2}(1-\psi_t)x_t^*$ for $\bar{x}_t$ to be positive. Observe that $\bar{x}_t$ is equal to the expected value of the transitory income shock when the fixed-point is located at low values of cash-on-hand. On the other hand, if $\bar{x}_t > x_t^*$, it will vary with age as income growth G_{t+1} and the marginal propensity to consume ψ_t change over the life-cycle.

Sensitivity of the Target level of Cash-on-hand to R and σ_N^2

Since the target level of cash-on-hand is defined as the fixed-point of the function (2.17), its stability will depend on the slope of $E_t\left[x_{t+1}|\, x_t\right]$ at $x_t = \bar{x}_t$. For low values of normalized cash-on-hand, the slope is zero and the function $E_t\left[x_{t+1}|\, x_t\right]$ is a constant. On the other hand, for high values of normalized cash-on-hand, the marginal propensity to consume is low and close to zero. Thus,

$$\begin{aligned} \frac{\partial E_t\left[x_{t+1}|x_t\right]}{\partial x_t} &= \frac{R}{G_{t+1}} e^{\frac{1}{2}\sigma_N^2}(1-\psi_t) \\ &\approx \frac{R}{G_{t+1}} e^{\frac{1}{2}\sigma_N^2} \end{aligned} \tag{2.19}$$

Note that this slope can be close to one at $x_t = \bar{x}_t$ for the parameters of the model. As a result, the target value of cash-on-hand can be sensitive to small variations to the model's parameters such as the interest rate and the variance of the permanent income shock.

Figure 2.5b displays the function $E_t\left[x_{t+1}|\, x_t\right]$ for selected ages when the interest rate is increased to $R = 1.038$, assuming $\rho = 0.514, \beta = 0.96, \gamma_0 = 0.001$ and $\gamma_1 = 0.071$. This small increase in R is within one standard deviation of the estimation of the interest rate reported in Gourinchas and Parker (2002). At age 35, the fixed-point is at 3.68, more than twice as high in value as the one when $R = 1.0344$.

On the other hand, Figure 2.6 shows that the the target value of cash-on-hand is not as sensitive to changes in σ_N^2 as it is to changes in R. Figure 2.6a reports the function $E_t\left[x_{t+1}|\, x_t\right]$ for selected ages when the variance of the permanent income shock is $\sigma_N^2 = 0.024$ instead of $\sigma_N^2 = 0.0212$. This change corresponds to an increase of one standard deviation of its estimated value reported in Gourinchas and Parker (2002). The target value of cash-on-hand at age 35 increases by 0.0034 compared to when $\sigma_N^2 = 0.0212$. Since the small change in σ_N^2 translates to an increase of only 0.0014 in the expected value of the permanent income shock, the target value of cash-on-hand remains almost unchanged. Alternatively, Figure 2.6b displays $E_t\left[x_{t+1}|\, x_t\right]$ when $\sigma_N^2 = 0.06$. An increase of fourteen standard deviation is necessary to increase the target value of normalized cash-on-hand at age 35 to 3.2667.

Sensitivity of the Target level of Cash-on-hand to γ_0

As discussed previously, the target level of cash-on-hand at age t depends on the marginal propensity to consume at age t, which in turn depends on the retirement rule parameters (among other parameters). Figure 2.7a displays the target level of cash-on-hand $\bar{x}_t$ over the life-cycle for $\gamma_0 = 0.001$. As can be seen, it varies with age as there is a systematic age-variation in the consumption rules. It is relatively constant until around age 40, and then it increases substantially. GP conclude from this graph that there is a striking change in savings behavior. Alternatively, Figures 2.7b and 2.7c show the age-profile of target wealth for $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$. As illiquid wealth increases implied by the value of γ_0, the target level of cash-on-hand is constant until around age 45 for $\gamma_0 = 0.594$ and until around age 50 for $\gamma_0 = 0.9$. As these figures reveal, the age-profile of target value of normalized cash-on-hand depends on the retirement consumption rule's parameters, i.e. it crucially depends on the retirement value function assumed by GP.

Sensitivity of the Target level of Cash-on-hand to Retirement Income

Since the age-profile of target value of normalized cash-on-hand depends on the retirement phase assumed by GP, it is worth examining the effect of retirement income and risks during retirement years. First, consider a life-cycle model with no retirement phase, but in which individuals die at age 87 ($N = 62$), receiving income shocks throughout their life-time with a constant income growth rate and no changes in family size. This alternative model corresponds to the finite-horizon version of the buffer-stock model in Carroll (2019). In this case, the consumption rules converge to a limiting consumption function as shown by Carroll and depicted in Figure 2.8b; thus, the age-profile of target value of cash-on-hand is relatively flat until a few years prior to N as seen in Figure 2.8a.

Alternatively, suppose individuals retire at age 65 ($T = 40$) but receive a proportion α of their labor income during retirement ages such that, for $t \geq T$:

$$x_{t+1} = \frac{R}{\Gamma N_{t+1}}(x_t - c_t) + \alpha U_{t+1} \tag{2.20}$$

where $0 < \alpha \leq 1$ is a retirement replacement rate. For $\alpha < 1$, there is a structural change in the life-cycle model as people need to save for retirement purposes when approaching age T. Consumption policy functions converge to different limiting functions before and after retirement age T. For instance, with a 70% replacement rate, the target value of cash-on-hand is constant until approximately age 50, increases gradually until age 65, and drops to a lower but constant value during retirement ages as shown in Figure 2.9a and consistent with the consumption rules in Figure 2.9b. Furthermore, note that when the replacement rate is 100% ($\alpha = 1$), this alternative specification is equivalent to the finite-horizon buffer-stock model mentioned previously. As the replacement rate decreases, the age-profile of the target value of cash-on-hand has a higher peak as shown in Figure 2.10. The peak is higher as households expect to receive less retirement income as the replacement rate decreases. Thus, consumption is lower over normalized cash-on-hand as people need to save for retirement purposes as they approach age T.

Next, consider the case in which individuals receive no income during retirement. This example is similar to GP's model under its base-line parameters with $\gamma_0 = 0.001$. With a constant income growth rate and no changes in family size, the target value of cash-on-hand is constant until age 35 and increases substantially until age 65, after which it falls to 0 as there is no income uncertainty during retirement ages as illustrated in Figure 2.11. Alternatively, by setting the income growth rate and family size to GP's values, the working ages of the age-profile of target cash-on-hand is similar to Figure 2.7a as shown in Figure 2.12.[5] Note that the assumed values for G_t create a "bump" in the age-profile of target value of cash-on-hand between ages 55 and 65 and heighten the slope at which the target value of cash-on-hand increases after age 40. Thus, the age-profile of target value of normalized cash-on-hand crucially depends on the retirement phase assumptions as well as the growth rate of labor income.

2.3.3 Savings Behavior

In order to characterize household savings behavior over the life-cycle, the model is first solved and simulated for the base-line parameter values. A sequence of $20,000$ income processes is generated over 40 years and households' initial financial wealth, w_1, is assumed to be lognormally distributed as in Gourinchas and Parker (2002).[6] The simulated distribution of normalized cash-on-hand at selected ages is displayed in Figure 2.13. It also reports the function $E_t[x_{t+1}|x_t]$.

As can seen, normalized cash-on-hand at age 26 is relatively tightly distributed near the "kink-point" of the function $E_t[x_{t+1}|x_t]$ with a mean value of 1.3179 and a median value of 1.1285. The bottom one forth of the distribution faces borrowing constraints and have high marginal propensities

[5]The age-profile of target level of cash-on-hand is not numerically exact since a death age of 87 implies a marginal propensity to consume $\gamma_1 = 0.0715$ instead of $\gamma_1 = 0.071$ at age 66.

[6]For more details, see Appendix A.

to consume (close to one) in contrast to the top three forths of the distribution. The target level of normalized cash-on-hand is 1.2470, which is greater than the 50^{th} percentile value of the distribution.

However, households quickly adjust their consumption and savings decisions so as to avoid the region where borrowing constraints are binding. At age 30, half of the distribution of normalized cash-on-hand is above the target value of 1.2634, with a mean value of 1.4061 and median value of 1.2755. By age 37, three forths of the cash-on-hand distribution is below the target level of cash-on-hand 1.8566, with a mean value of 1.5249 and a median value of 1.4085. Between ages 37 and 38, the target value of cash-on-hand increases by 0.6072, whereas the mean and median values increase by 0.0521 and 0.0411, respectively. By the late thirties, the target thus fails to describe the distribution of normalized cash-on-hand as its located at the right tail of the distribution. At age 40, most of the distribution of cash-on-hand is to the right of the "kink-point" of the function $E_t[x_{t+1}|x_t]$, but it is left to the target level of cash-on-hand 4.1525. In fact, the probability of normalized cash-on-hand being greater than the "kink-point" of the function $E_t[x_{t+1}|x_t]$ increases with age as seen in Figure 2.14. It rises from 87.6% to 98.5% between ages 26 and 40, reaching 100% by age 47. This result indicates that once a household reaches its forties, the probability that it will hold a low amount of liquid wealth is negligible; thus, it will continue to accumulate positive amounts of wealth as its marginal propensity to consume is low. During the forties and fifties, households continue accumulating cash-on-hand at a relatively constant rate, while the target level of cash-on-hand increases substantially. At age 45, the median value of the cash-on-hand distribution is 2.1510, which jumps to 7.5320 by age 60; meanwhile, the target level of cash-on-hand increases from 10.6717 to 105.3635 (not illustrated).

Figure 2.15 reports the age-profile of the target level of cash-on-hand with the mean, 25^{th} percentile, 50^{th} percentile, and 75^{th} percentile values of the distribution of normalized cash-on-hand for each age. As discussed above, most of the cash-on-hand distribution is close to its target value between ages 30 and 35 as seen in Figure 2.15a. As the target level of cash-on-hand increases substantially after age 37, households are expected to save and their cash to rise as $x_t < \bar{x}_t$. However, it is clear from these graphs that households do not exhibit a striking change in savings behavior as concluded by GP. The change in the distribution of normalized cash-on-hand is rather slow and gradual. Moreover, while the target drops drastically at around age 60, the distribution of normalized cash-on-hand keeps increasing. Furthermore, Figure 2.16a illustrates the mean, 25^{th} percentile, 50^{th} percentile, and 75^{th} percentile values of the distribution of normalized consumption for ages 26 to 65. Note that normalized consumption is tightly distributed between 0.8 and 1.01; thus, households' normalized consumption is relatively constant over the life-cycle. Alternatively, Figure 2.16b depicts the distribution of consumption (i.e. $C_t = c_t P_t$) between ages 26 and 65. The observed hump-shape of the consumption profile is mainly due to the fluctuations in the permanent component of income, i.e. the variations in the growth of rate of income, and partly due to the slight decrease in the normalized consumption profile after age 45.

Next, consider the special case in which the growth rate of income is constant over the life-cycle. As discussed previously, the age-profile of target level of cash-on-hand is sensitive to small changes in G_t, but the distribution of normalized cash-on-hand is similar to the case with variations in G_t as seen in Figure 2.17. In both scenarios, most households have low marginal propensity to consume by age 40; thus, their consumption and cash holdings are not as sensitive to changes in the growth rate of income as the target level of cash-on-hand.

Furthermore, consider the case in which households do not possess any initial financial wealth at age 26 as shown in Figure 2.18. Most of the young households have cash holdings that are lower than the target level of cash-on-hand. As target behavior indicates, households save and cash holding rises. As households reach age 30, most of the cash-on-hand distribution is close to its target value. However, as the target level increases substantially after age 38, households do not increase their savings at the same rate. In contrast, assume the initial financial wealth age 26 is equal to 2 for all households as depicted in Figure 2.19. In this alternative scenario, households have cash holdings higher than the target level of cash-on-hand. As target behavior suggests, households save less and decrease their cash holdings. Note that as soon as median household's normalize cash-on-hand reaches the target level at age 37, all households start saving as $\bar{x}_t$ increases substantially. Nonetheless, the change in the distribution of normalized cash-on-hand is again gradual. Thus, while the target level of cash-on-hand is a good indicator of the overall direction to which the distribution of normalized cash-on-hand moves, it fails to describe the magnitude of household savings.

Finally, Figure 2.20 reports the decomposition of total saving and wealth for each age into how much the mean household saves for precautionary motives and how much it saves for life-cycle or retirement motives. Following GP, total saving is defined as:

$$S_{i,t} = \frac{W_{i,t+1} - W_{i,t}}{R} = \frac{R-1}{R} W_{i,t} + Y_{i,t} - C_{i,t} \tag{2.21}$$

where $W_{i,t+1} = R(X_{i,t-1} - C_{i,t-1})$, and life-cycle saving is defined as:

$$S_{i,t}^{LC} = \frac{W_{i,t+1}^{LC} - W_{i,t}^{LC}}{R} \tag{2.22}$$

where W^{LC} is the financial wealth holdings from households that do not face any income risk: $N_t = E_t[N_t]$, $U_t = E_t[U_t]$. Lastly, precautionary savings is defined as:

$$S_{i,t}^{PS} = S_{i,t} - S_{i,t}^{LC}. \tag{2.23}$$

For the base-line parameter values, young households who do not face any income risk have negative life-cycle savings as they would like to borrow early in life. On the other hand, they hold a positive amount of savings in the presence of income risk; thus, precautionary saving is positive early in life. During their mid-forties, life-cycle saving exceeds precautionary saving which explains the negative value for buffer saving. Similarly, total wealth holdings by the mean household facing income uncertainty is higher than the wealth holdings (life-cycle wealth) by a mean household in the absence of income shocks displaying the presence of income risk.

2.4 Conclusion

This document explores the difference between target level of normalized cash-on-hand and actual savings behavior of households in a life-cycle model. Although the age-profile of target level of cash-on-hand pinpoints the direction which households' accumulate (de-accumulate) liquid wealth, it does not indicate its magnitude. Furthermore, the age-profile of the target value of liquid wealth depends crucially on the retirement age's consumption rules, retirement income risks, and on the systematic-age variation of the consumption functions. Each of these factors are implied by the model's assumptions instead of observed consumer behavior. Moreover, by definition, the target value of cash-on-hand is sensitive to small changes (within confidence intervals for such parameter values) in the model's parameters, such as the interest rate, when the marginal propensity to consume is less than one.

An interesting extension of the current work would be to consider the presence of health-care costs and survival risks during retirement years. Although labor income risks are not relevant for most of the retirees, the elderly face substantial medical expenditure risks. Despite the fact that a large fraction of health care costs risk is covered by Medicare, Medicaid and private health insurance, the elderly still face nursing home admission risks and its associated cost, as well as the risk of catastrophic medical expenses (Feenberg and Skinner, 1994; Palumbo, 1999; Hurd, 2002; French and Jones, 2004; De Nardi, French, and Jones, 2010; French and Jones, 2011). Assuming that consumers do not derive any utility from medical goods and services (they only "compensate" the affliction caused by bad health), the health-care cost uncertainty can induce a risk on the disposable income available to finance retirement consumption. As Carroll and Kimball (1996) show, income uncertainty also creates concavities in the consumption function which impinge on prior periods' consumption functions inducing precautionary savings. I expect that a richer representation of the retirement period would improve the understanding of households savings behavior over the life-cycle and contribute to the on-going discussion of the degree of precautionary savings motive in the empirical literature.

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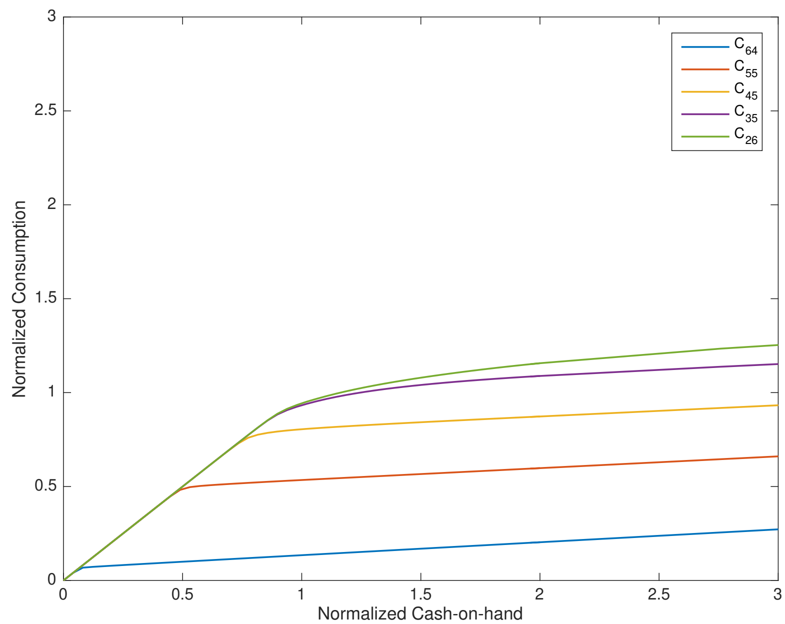
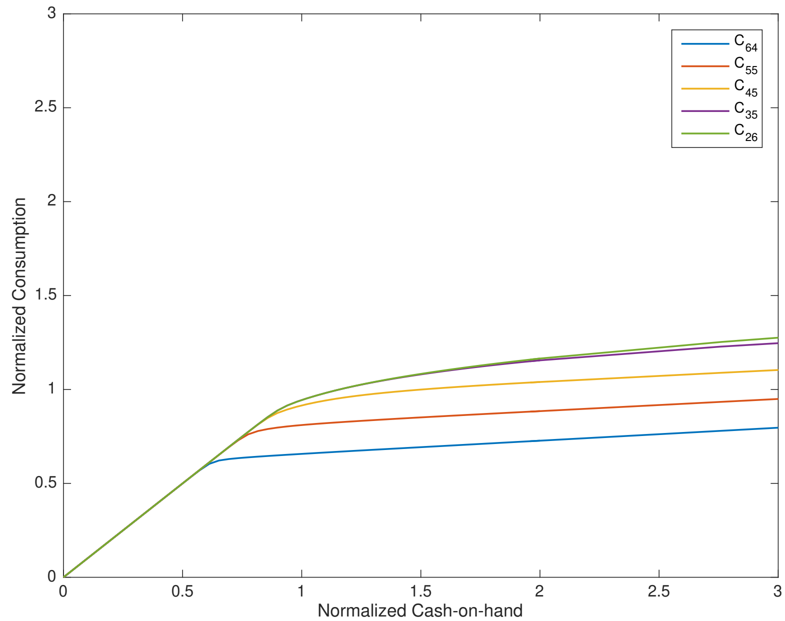
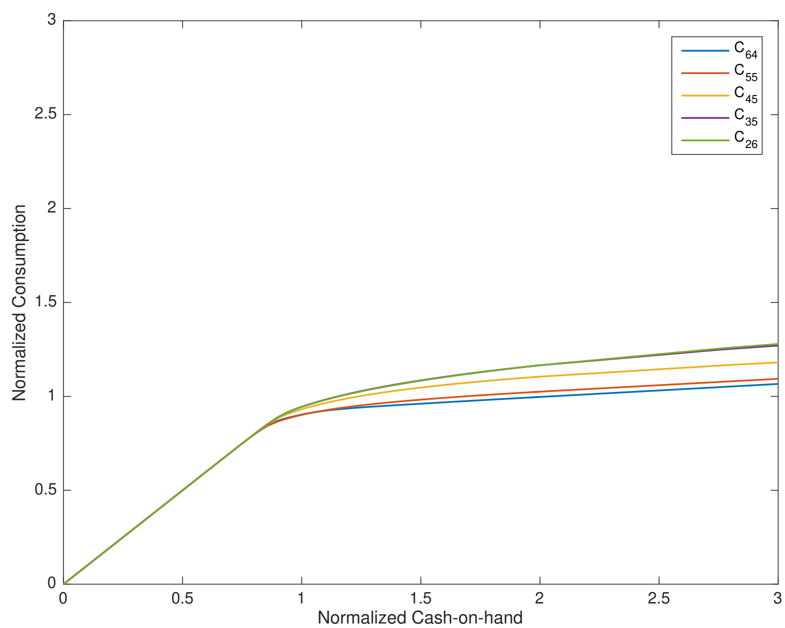


Figure 2.1: Consumption Functions for Base-line Parameters

$(\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$

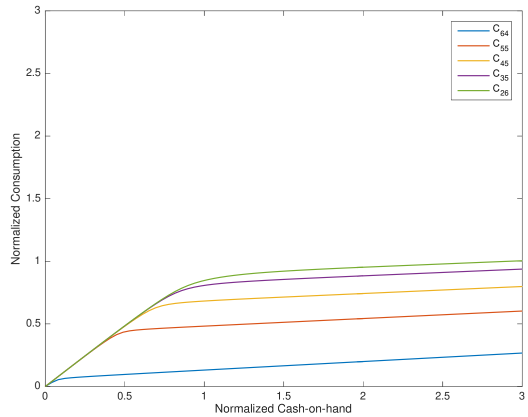


(a) $\gamma_0 = 0.594$

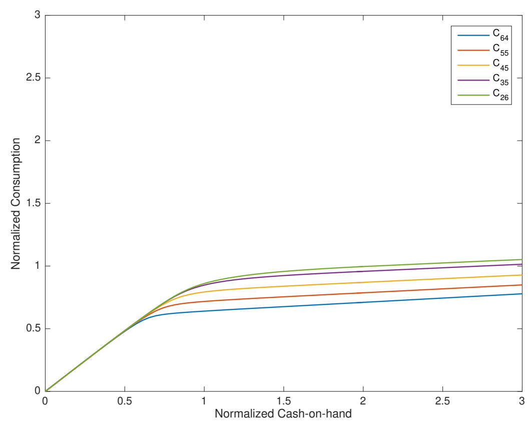


(b) $\gamma_0 = 0.9$

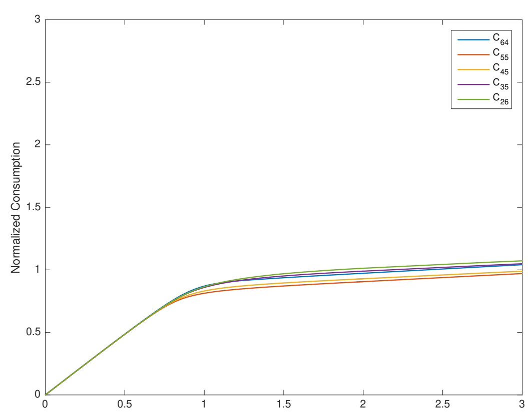
Figure 2.2: Consumption Functions for $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$ ($\rho = 0.514, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$)



(a) $\gamma_0 = 0.001$

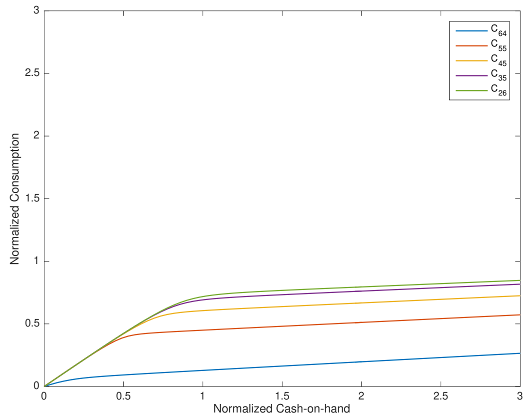


(b) $\gamma_0 = 0.594$

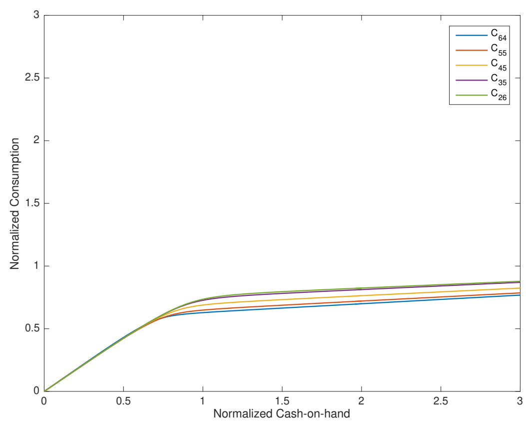


(c) $\gamma_0 = 0.9$

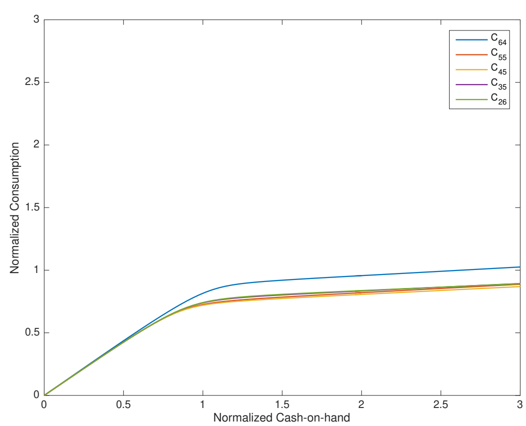
Figure 2.3: Consumption Functions for $\rho = 1.5$
$(\beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$



(a) $\gamma_0 = 0.001$



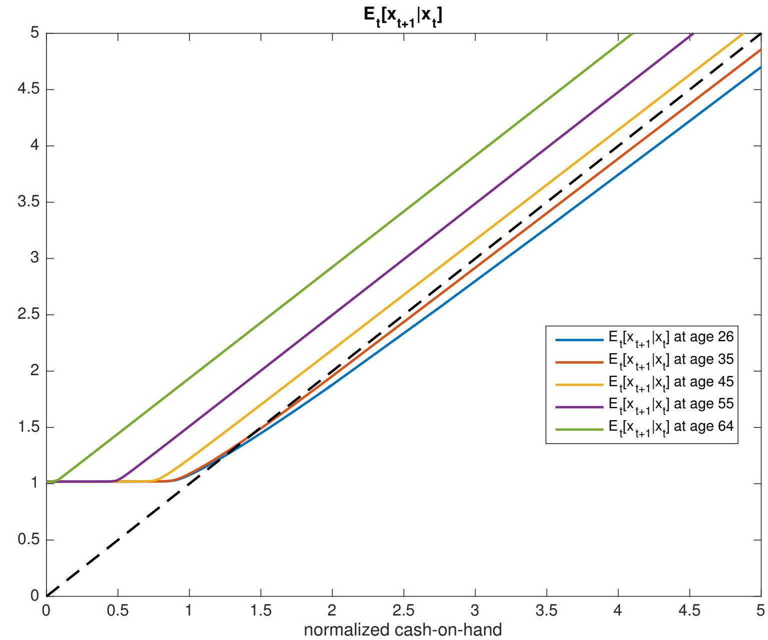
(b) $\gamma_0 = 0.594$



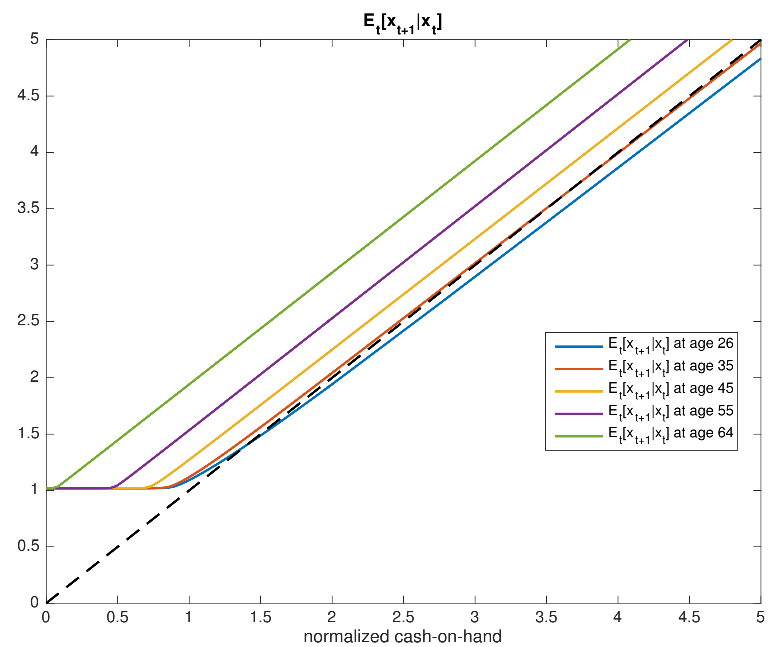
(c) $\gamma_0 = 0.9$

Figure 2.4: Consumption Functions for $\rho = 3$

$(\beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$

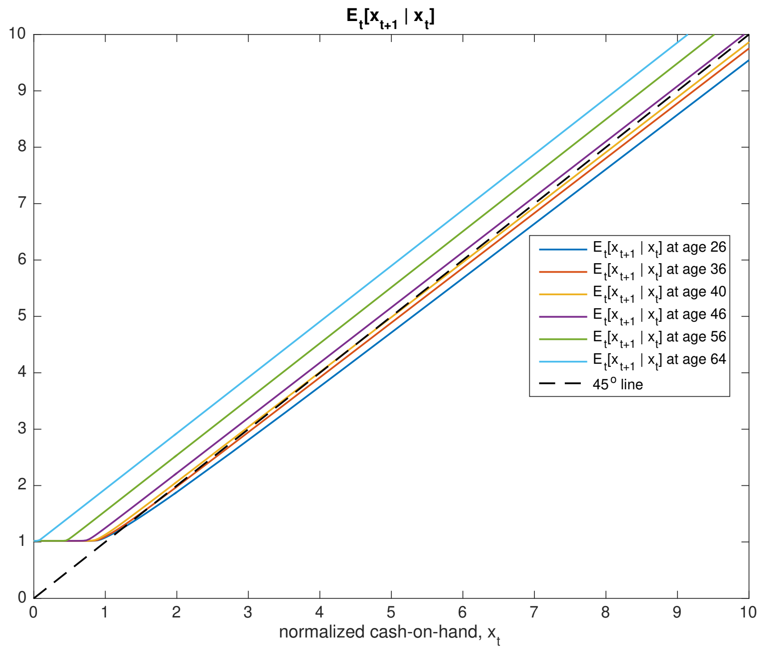


(a) $R = 1.0344$

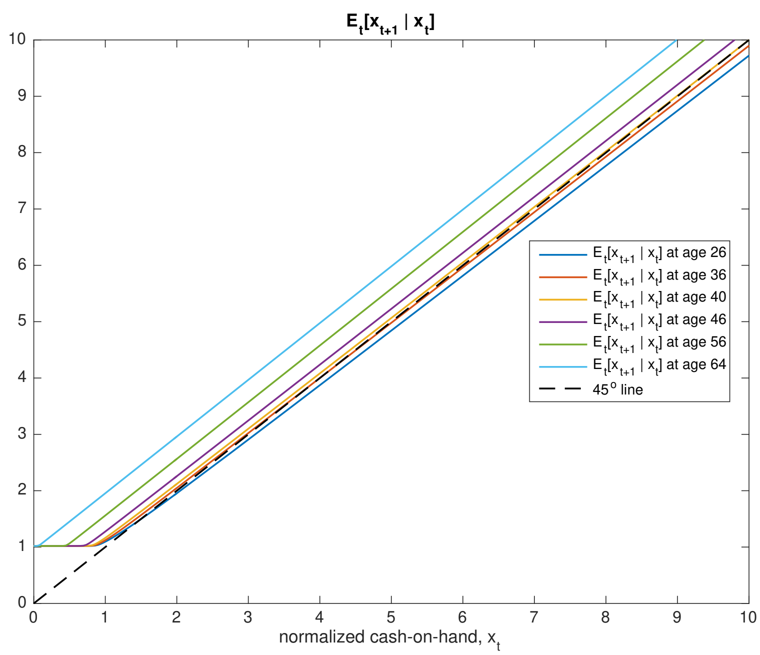


(b) $R = 1.038$

Figure 2.5: Function $E_t\left[x_{t+1}|\, x_t\right]$ for $R = 1.0344$ and $R = 1.038$ $(\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071)$

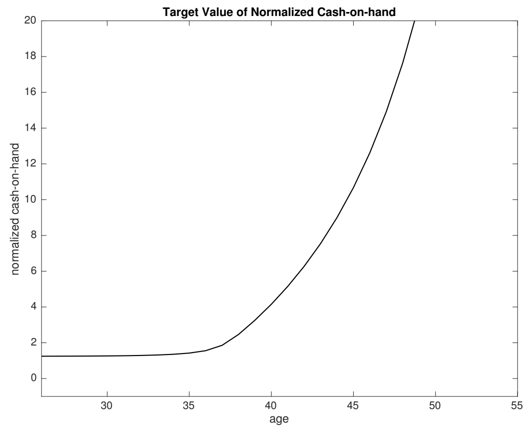


(a) $\sigma_N^2 = 0.024$

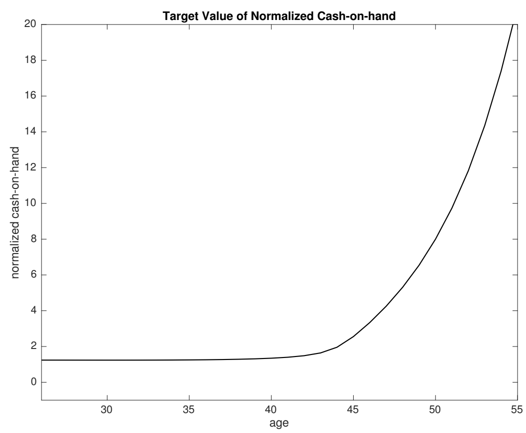


(b) $\sigma_N^2 = 0.06$

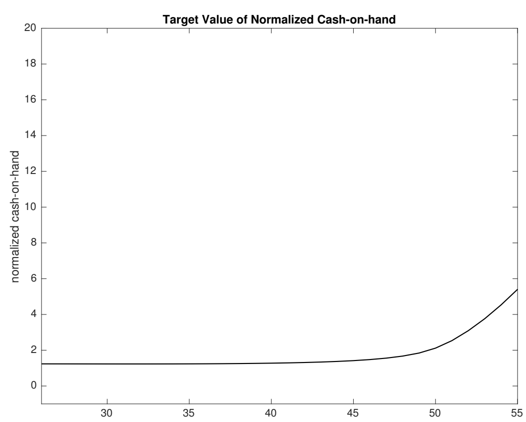
Figure 2.6: Function $E_t\left[x_{t+1}|\, x_t\right]$ for $\sigma_N^2 = 0.024$ and $\sigma_N^2 = 0.06$ $(\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071,\ R = 1.0344)$



(a) $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$

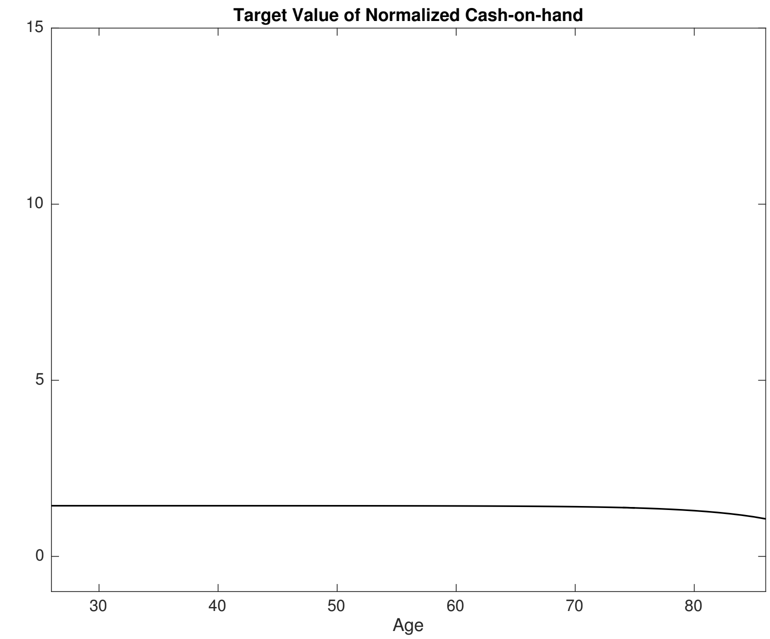


(b) $\rho = 0.514, \gamma_0 = 0.594, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$

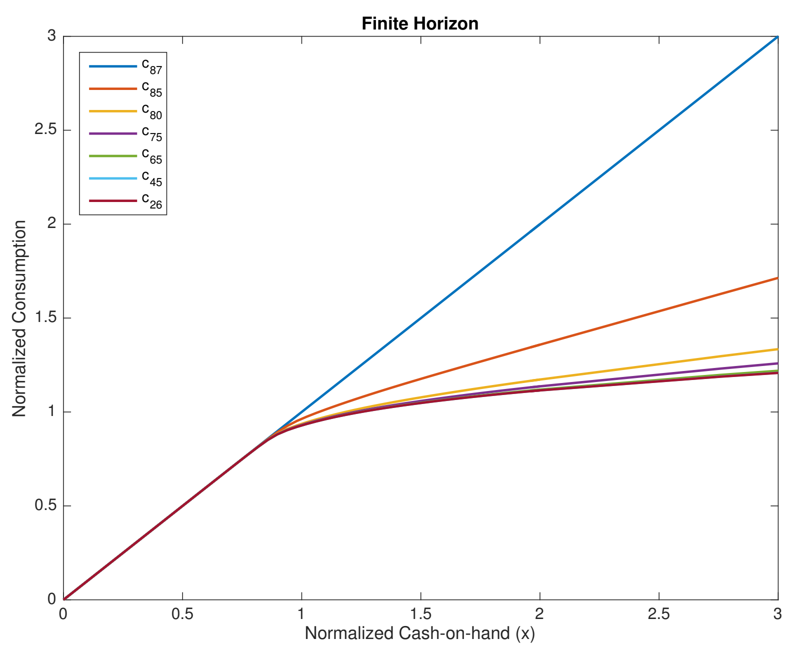


(c) $\rho = 0.514, \gamma_0 = 0.9, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$

Figure 2.7: Target Value of Normalized Cash-on-Hand



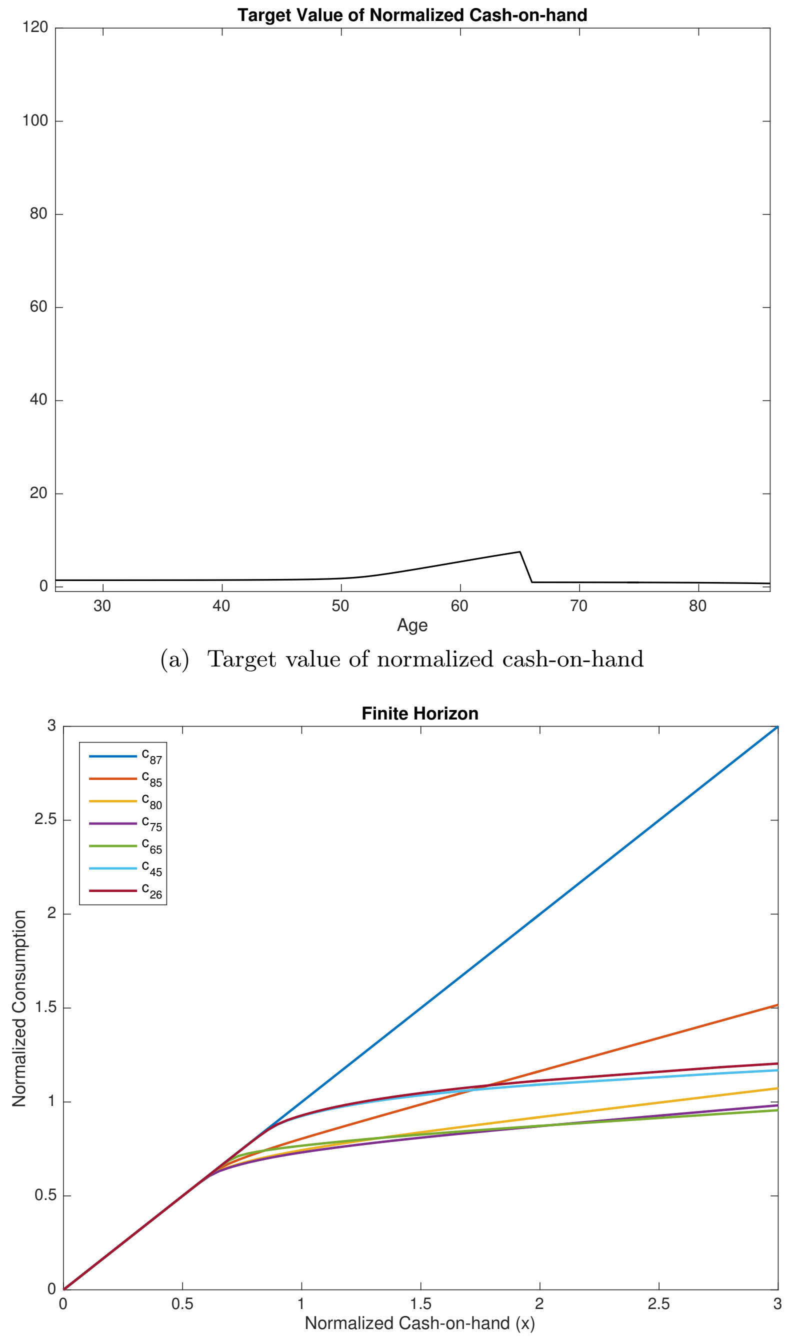
(a) Target value of normalized cash-on-hand



(b) Consumption policy functions

Figure 2.8: Target Cash-on-Hand and Consumption Functions assuming no Retirement Phase

For stochastic income assuming constant income growth rate and no changes in family size for parameter values $\rho = 0.514, \beta = 0.96, R = 1.0344$

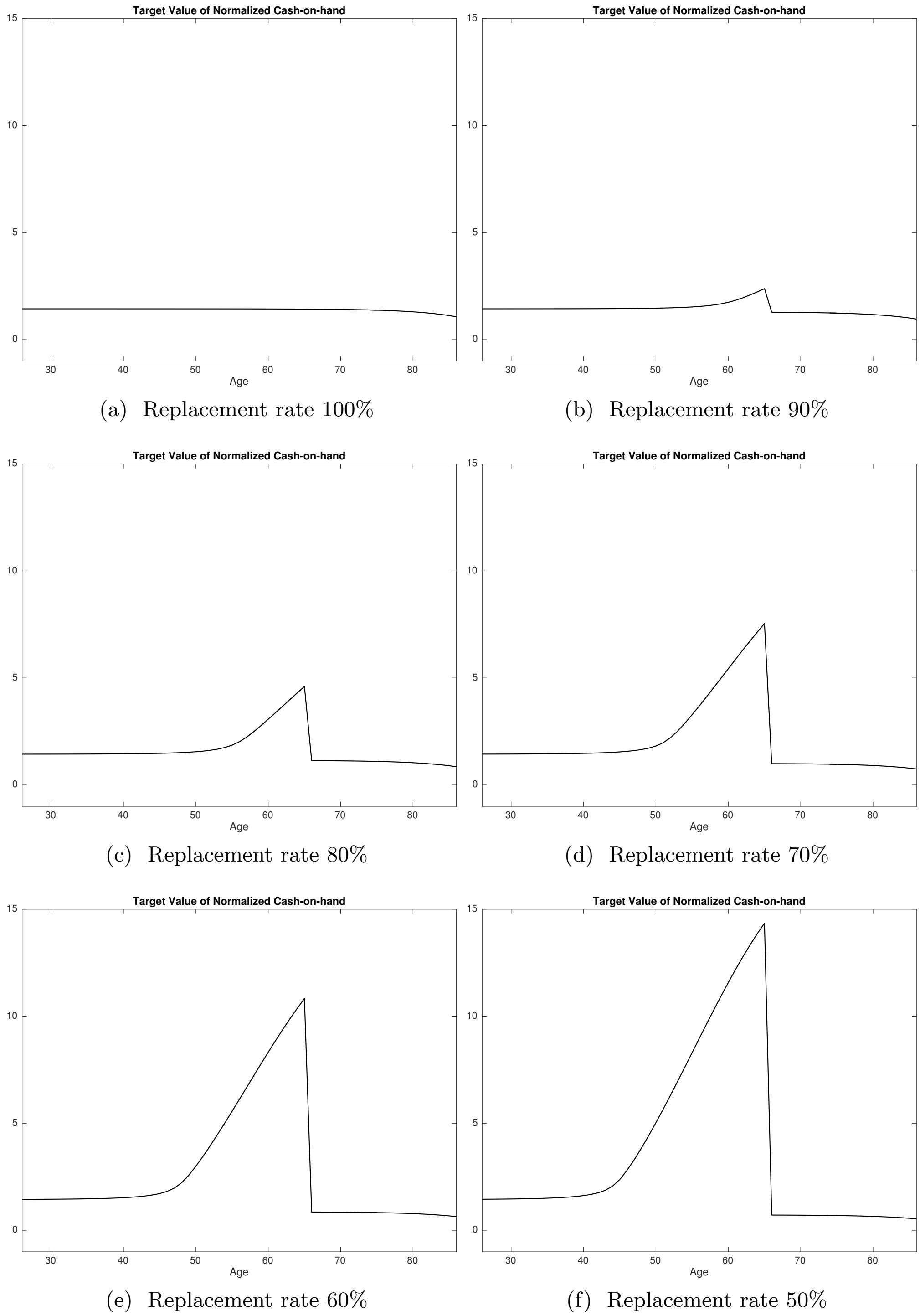


(a) Target value of normalized cash-on-hand

(b) Consumption policy functions

Figure 2.9: Target Cash-on-Hand and Consumption Functions assuming 70% Replacement Rate

For stochastic income during retirement, assuming constant income growth rate and no changes in family size during working ages, for parameter values $\rho = 0.514, \beta = 0.96, R = 1.0344$



(a) Replacement rate 100%

(b) Replacement rate 90%

(c) Replacement rate 80%

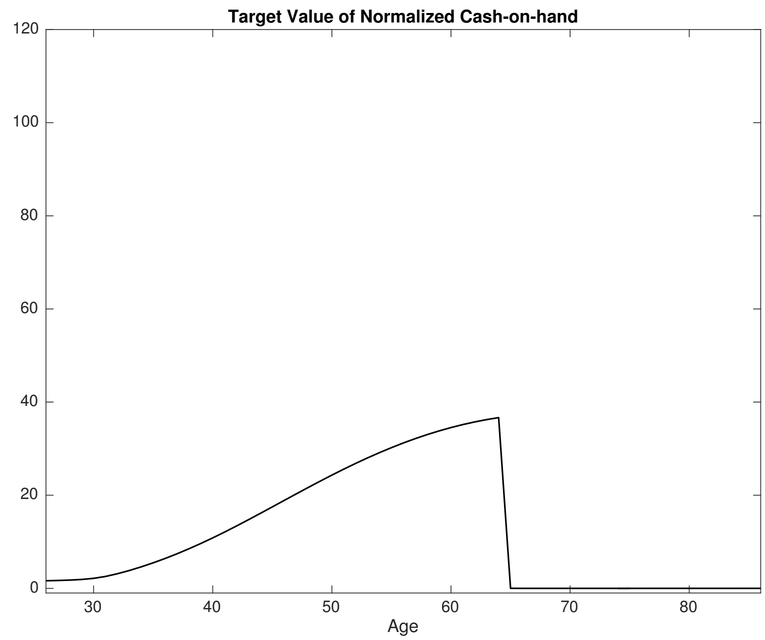
(d) Replacement rate 70%

(e) Replacement rate 60%

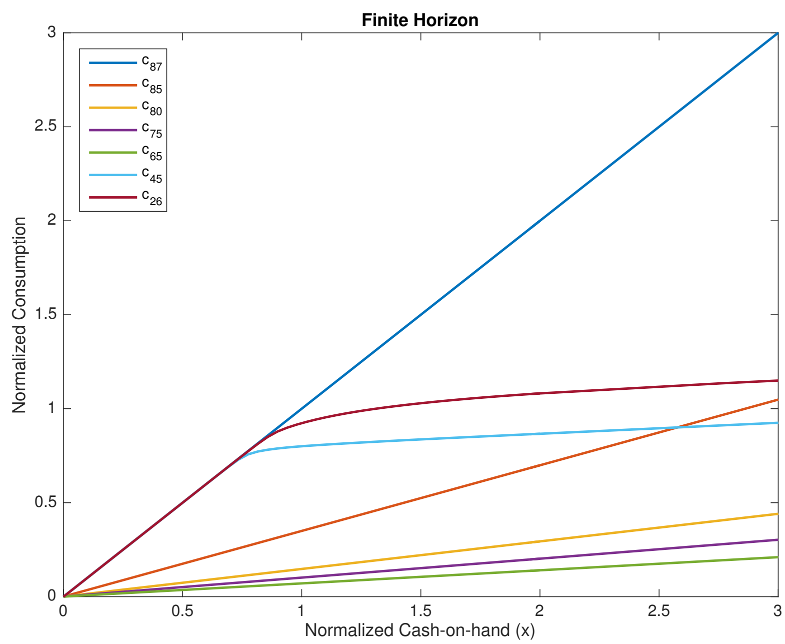
(f) Replacement rate 50%

Figure 2.10: Target Cash-on-Hand for Various Replacement Rates

For stochastic income during retirement, assuming constant income growth rate and no changes in family size during working ages, for parameter values $\rho = 0.514, \beta = 0.96, R = 1.0344$



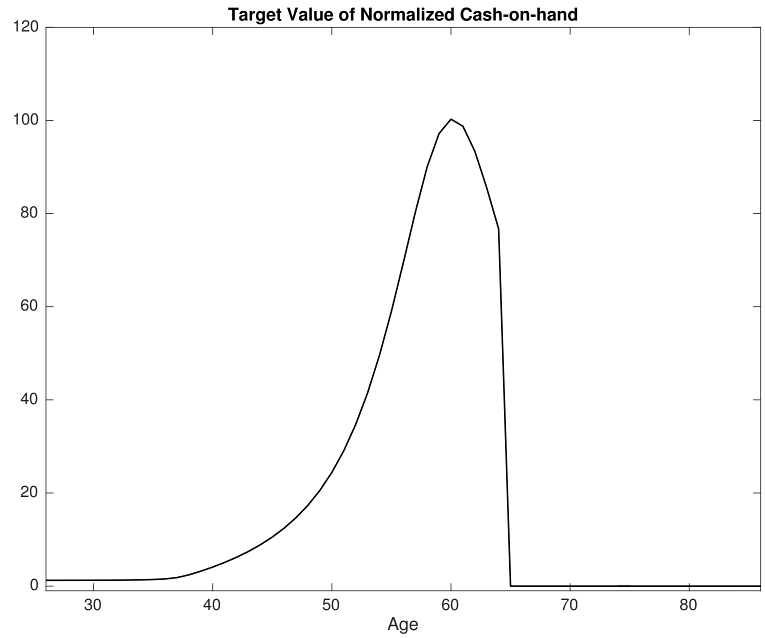
(a) Target value of normalized cash-on-hand



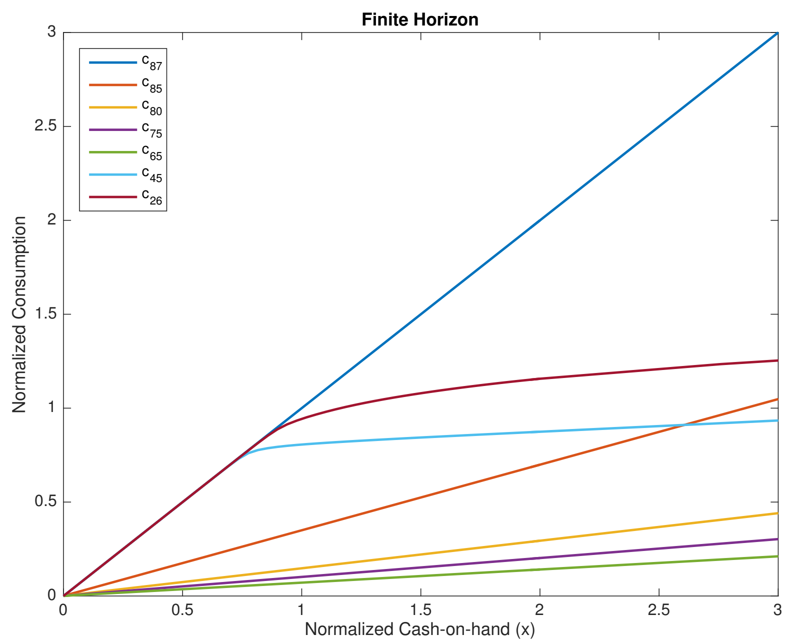
(b) Consumption policy functions

Figure 2.11: Target Cash-on-Hand and Consumption Functions with Perfect Foresight during Retirement

Assuming no retirement income, constant income growth rate and no changes in family size during working ages, for parameter values $\rho = 0.514, \beta = 0.96, R = 1.0344$



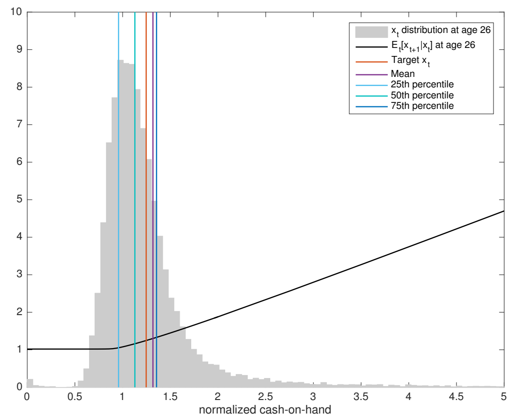
(a) Target value of normalized cash-on-hand



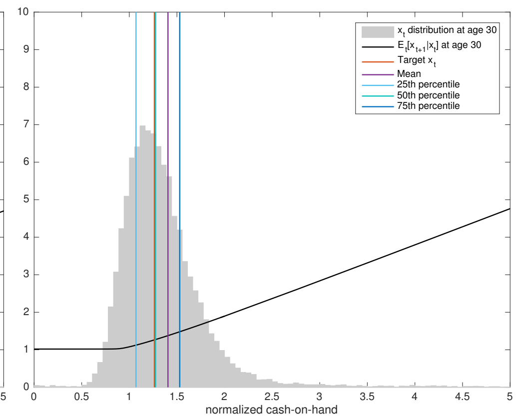
(b) Consumption policy functions

Figure 2.12: Target Cash-on-Hand and Consumption Functions without Retirement Income

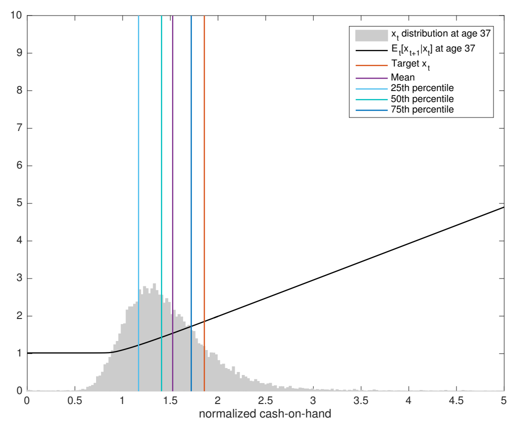
Assuming variations in income growth rate and changes in family size as in Gourinchas and Parker (2002) during working ages, for parameter values $\rho = 0.514, \beta = 0.96, R = 1.0344$



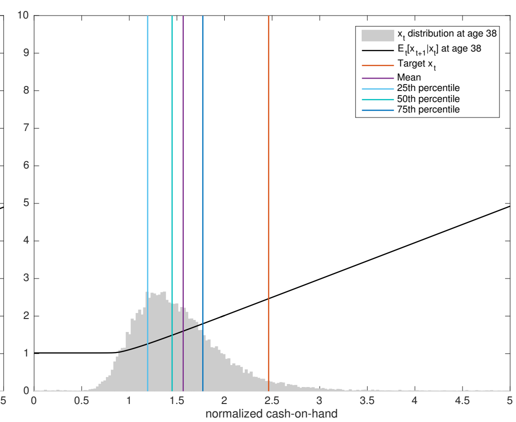
(a) Age 26



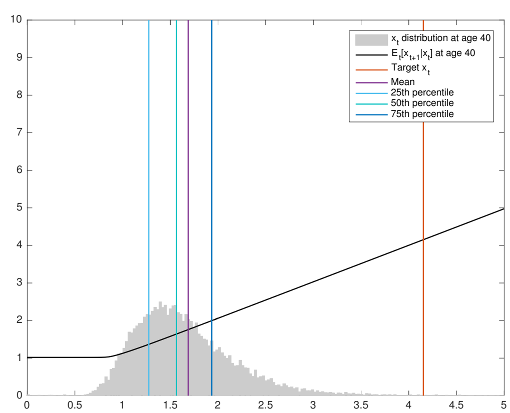
(b) Age 30



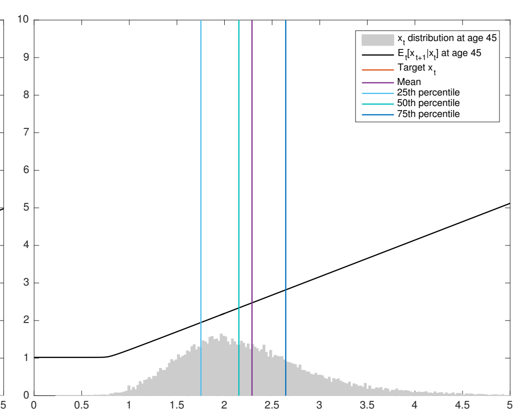
(c) Age 37



(d) Age 38



(e) Age 40



(f) Age 45

Figure 2.13: Function $E_t[x_{t+1}|x_t]$ and Distribution of Cash-on-Hand

Assuming variations in income growth rate and changes in family size as in Gourinchas and Parker (2002) for parameter values $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$

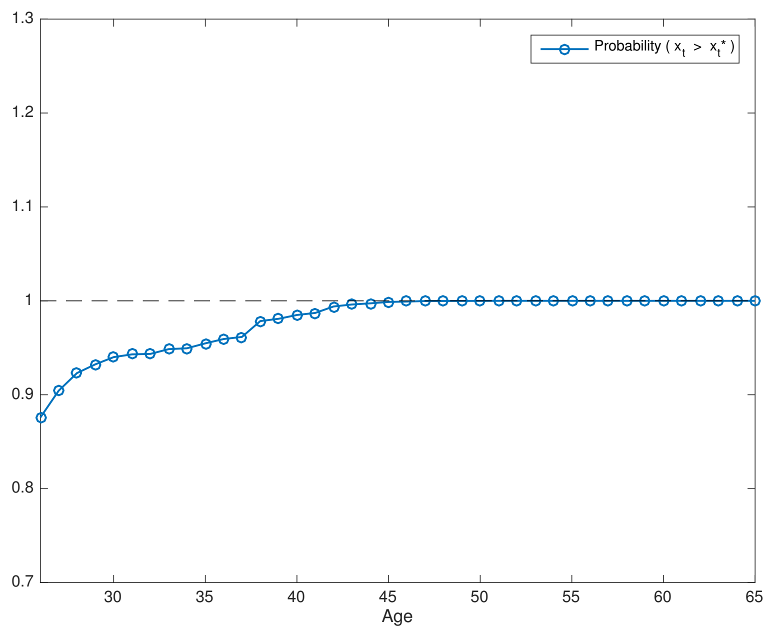
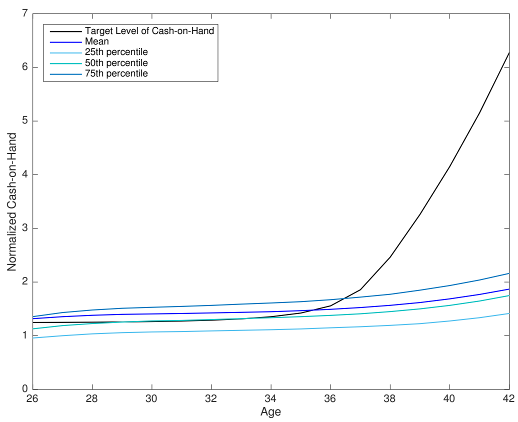
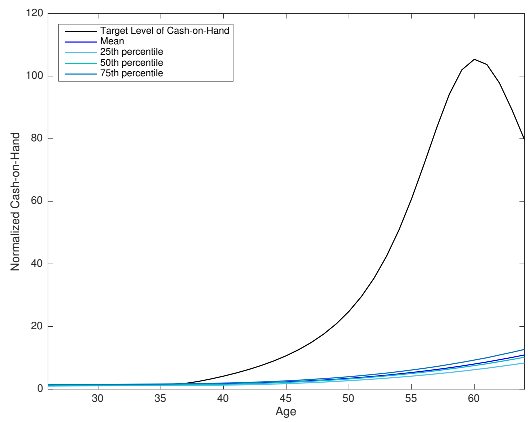


Figure 2.14: Probability of Holding High amounts of Cash-on-Hand

This probability is calculated as the probability of cash-holdings being greater than the "kink-point" of the consumption function at each age t. It considers $20,000$ simulated individuals who are followed throughout their life-cycle, based on the distribution of transitory and permanent income shocks. The parameters assumed are $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$.



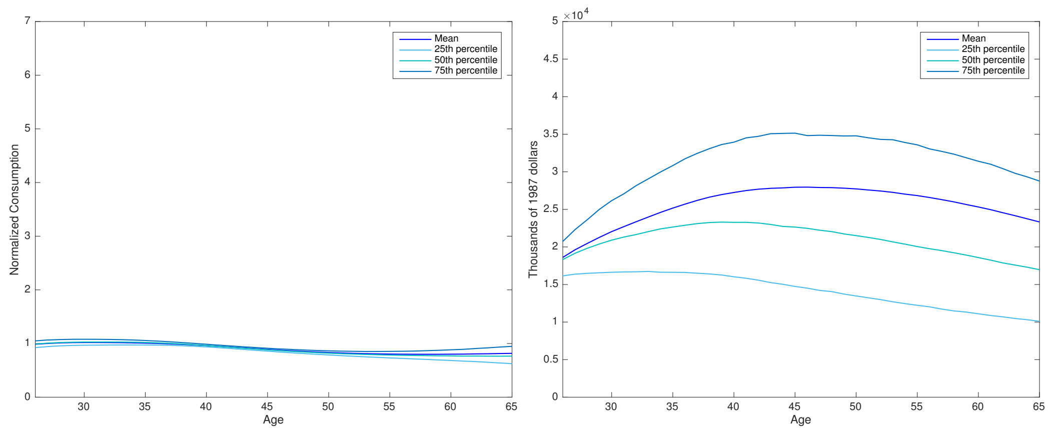
(a) Selected ages



(b) Working ages

Figure 2.15: Target Value and Distribution of Cash-on-Hand for Base-line Parameters

Target value of normalized cash-on-hand and selected percentiles of the distribution of cash-on-hand, assuming initial financial wealth is lognormally distributed with mean -2.794 and s.d. 1.784, variations in income growth rate and changes in family size as in Gourinchas and Parker (2002) for parameter values $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$

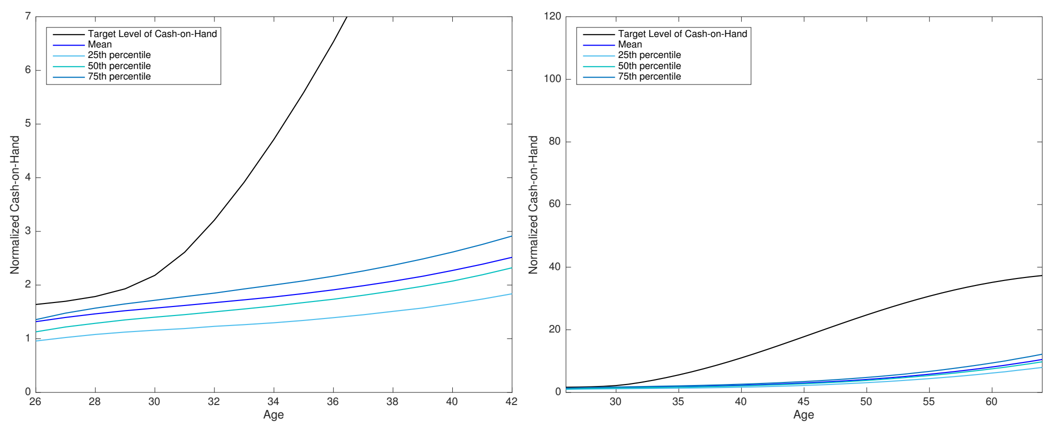


(a) Normalized Consumption c_t

(b) Consumption $C_t = c_t P_t$

Figure 2.16: Distribution of Consumption for Base-line Parameters

Selected percentiles of the distribution of normalized consumption and consumption in dollars, assuming initial financial wealth is lognormally distributed with mean -2.794 and s.d. 1.784, variations in income growth rate and changes in family size as in Gourinchas and Parker (2002) for parameter values $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$

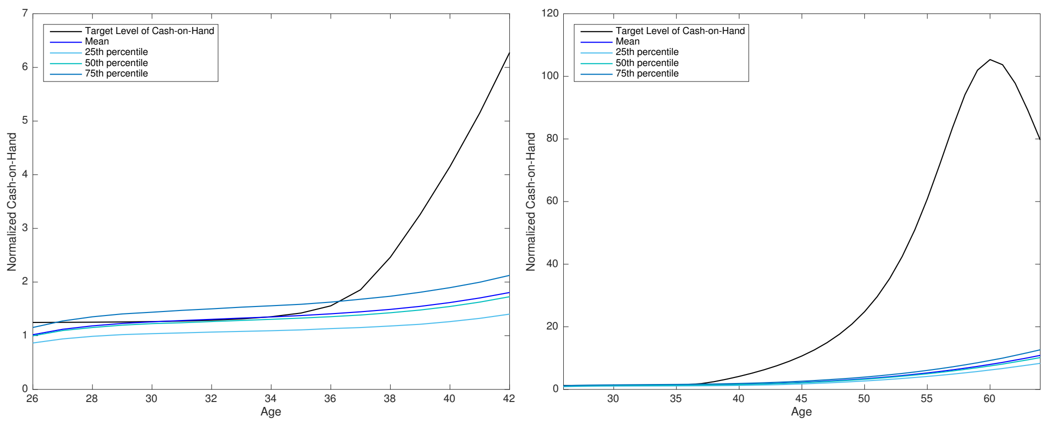


(a) Selected ages

(b) Working ages

Figure 2.17: Target Value and Distribution of Cash-on-Hand for constant G_t and Z_t

Target value of normalized cash-on-hand and selected percentiles of the distribution of cash-on-hand, assuming initial financial wealth is lognormally distributed with mean -2.794 and s.d. 1.784, constant income growth rate and no changes in family size for parameter values $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$

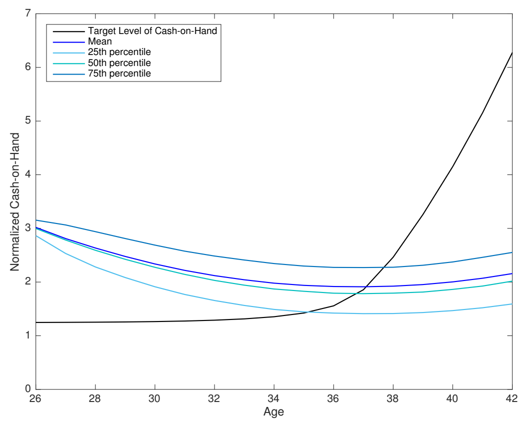


(a) Selected ages

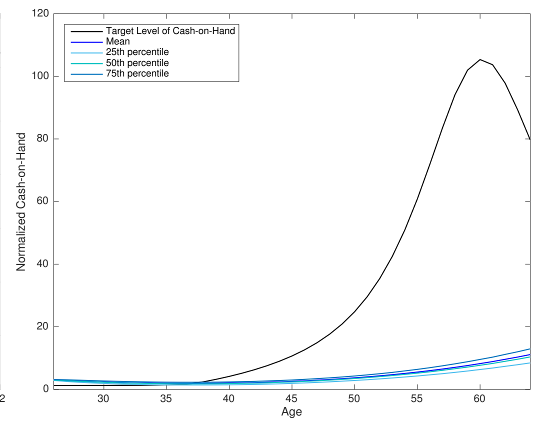
(b) Working ages

Figure 2.18: Target Value and Distribution of Cash-on-Hand for $w_1 = 0$

Target value of normalized cash-on-hand and selected percentiles of the distribution of cash-on-hand, assuming initial financial wealth is 0 for all households, variations in income growth rate and changes in family size as in Gourinchas and Parker (2002) for parameter values $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$



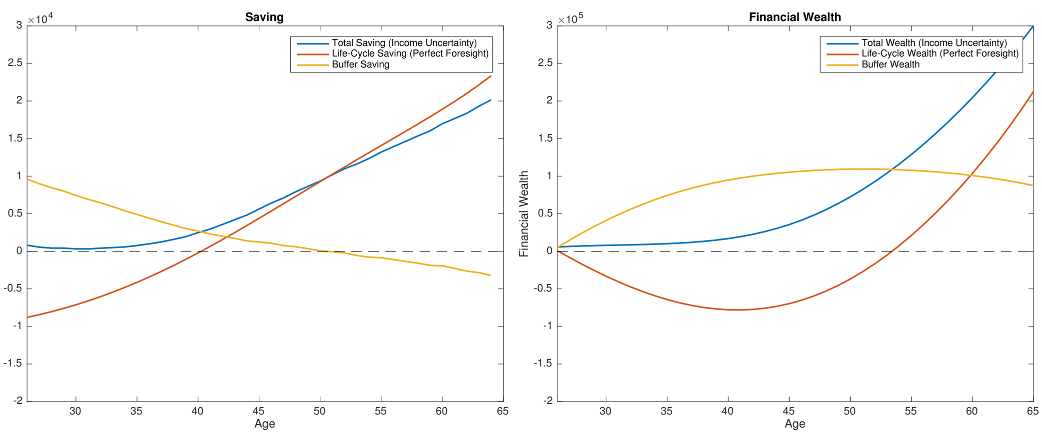
(a) Selected ages



(b) Working ages

Figure 2.19: Target Value and Distribution of Cash-on-Hand for $w_1 = 2$

Target value of normalized cash-on-hand and selected percentiles of the distribution of cash-on-hand, assuming initial financial wealth is 2 for all households, variations in income growth rate and changes in family size as in Gourinchas and Parker (2002) for parameter values $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$



(a) Savings (b) Financial wealth

Figure 2.20: Age-profiles of Mean Savings and Financial Wealth

($\rho = 0.514, \gamma_0 = 0.001,\ \beta = 0.96, \gamma_1 = 0.071,\ R = 1.0344$)

Parameter	Value	Source
R	1.03440	Moody's AAA municipal bonds, 01/1980 – 03/1993
σ_U^2	0.0440	Carroll and Samwick (1997), PSID 1981-1987
σ_N^2	0.0212	Carroll and Samwick (1997), PSID 1981-1987
p	0.00302	Carroll, Hall, and Zeldes (1992), PSID 1976-1985
$\bar{w}_1$	-2.7944810	Gourinchas and Parker (2002), CEX 1980-1993
σ_{w_1}	1.7838679	
G_t:		Gourinchas and Parker (2002), CEX 1980-1993
age	.32643678179	
age^2	$-.0148947085$	
age^3	.00036342384	
age^4	$-4.411685e^{-6}$	
age^5	$2.056916e^{-8}$	
$constant$	6.801368713	
Z_t:		Gourinchas and Parker (2002), CEX 1980-1993
age	0.13964975	
age^2	-0.0047742190	
age^3	$8.5155210e^{-5}$	
age^4	$-7.9110880e^{-7}$	
age^5	$2.9789550e^{-9}$	

Table 2.1: Parameter Values from Gourinchas and Parker (2002)

Chapter 3

Estimation of Consumption over the Life Cycle: Implications for a Representative Agent

3.1 Introduction

The aggregate implications of individual consumption and savings behavior have been addressed widely since the relationship between the two is essential for understanding the impacts of economic policies on interest rates, aggregate consumption and savings. A survey by Blundell and Stoker (2005) find that there is substantial evidence on individual heterogeneity which generates discrepancies between individual and aggregate variables. Thus, aggregation problems must be taken into account. In this chapter, I first explore the implications of the life-cycle behavior predicted by Gourinchas and Parker (2002) for average liquid wealth, average consumption, and average marginal propensity in light of Carroll (2000). Second, I assess the suitability of the life-cycle model by comparing the predicted distribution of liquid wealth to the one observed in microeconomic data across U.S. households. Lastly, I study the effect of the retirement rule assumptions by considering a stochastic retirement phase with replacement rates.

This chapter builds on the aggregation result of Carroll (2000). He claims that the "representative-consumer model should be abandoned in favor of a model that matches key microeconomics facts."[1] He argues that un-insurable risk prevents aggregation because risk causes concavities in the consumption policy function; in turn, this concavity implies that the distribution of wealth affects the average marginal propensity to consume and the level of aggregate consumption. Carroll further points out that the structure of wealth accumulation is important to understand the difference between the aggregate dynamics resulting from a stochastic microeconomic model and the one predicted by a representative-agent model. He states that a significant dispersion in aggregate wealth will predict an aggregate behavior that does not resemble the behavior of a representative agent whose wealth is equal to the mean of the distribution. As shown by Carroll, this discrepancy is a key factor in understanding the effects of economic policies, especially when the distribution of wealth across U.S. households, as well as in most modern economies, is extremely skewed and the mean marginal propensity to consume is high.

Carroll further suggests that the most appealing explanation of the skewness of the wealth distribution is one where there is a systematic age-variation in consumption behavior like the model

[1] Carroll, 2000, p. 110.

in Gourinchas and Parker (2002) (henceforth GP). Gourinchas (2000) explores the implications of individual life-cycle behavior on aggregate dynamics in a general equilibrium model following Krusell and Smith (1998). The calibration results in Gourinchas (2000) conclude that the distribution of capital holdings does not affect aggregate variables; however, this finding depends on the specifics of the setup and the computation of the approximate equilibrium.

I find three main results. First, the aggregation result of the life-cycle model depends on the age distribution of the population and on the assumed values of the preference and retirement rule parameters. For the base-line parameters, individuals optimally choose to accumulate a substantial amount of liquid wealth: they quickly adjust their consumption and saving decisions so as to avoid the regions where borrowing constraints are binding. In fact, households' probability to hold low amounts of liquid wealth is almost zero by age 45; as a result, all individuals have the same marginal propensity to consume in their mid-forties and up. Thus, the aggregate dynamics of a representative-consumer model possessing liquid wealth equal to the mean of the distribution would resemble the aggregate predictions of the life-cycle model for middle-aged individuals. Second, despite the various combinations of parameter values that are studied, I find that the predicted distribution of cash-on-hand does not match the one observed on microeconomic data. This discrepancy sheds light in the suitability of the life-cycle model to reproduce the observed savings behavior across U.S. households.[2] Lastly, these results suggest that the main weakness of GP's life-cycle model is its retirement rules assumptions as the model disregards consumption risks during retirement years.

The remainder of this document is structured as follows. Section 2 describes the life-cycle model estimated by GP. Section 3 shows how the model's aggregation prediction is sensitive to the assumed values of the preference and retirement rule parameters. Section 4 concludes.

3.2 Life-Cycle Model in Gourinchas and Parker (2002)

3.2.1 The Model

GP consider the following discrete-time, life-cycle model of household consumption. Individuals live until age N and retire at age $T < N$. Both T and N are assumed to be exogenous and fixed. Preferences are represented by the standard additively separable expected utility form:

$$E\left[\sum_{t=1}^{N} \beta^t u(C_t, Z_t) + \beta^{N+1} V_{N+1}(W_{N+1})\right] \tag{3.1}$$

where β is the time-discount factor, C_t is the total consumption at age t, W_t is the total financial wealth at age t, Z_t is the vector of deterministic household characteristics at age t, and V_{N+1} is the value to the consumer of the remaining assets after death. Furthermore, the Bernoulli function is assumed to take the following form:

$$u(C, Z) = v(Z)\frac{C^{1-\rho}}{1-\rho}$$

[2]This observation has also been pointed out by Deaton (1991), among others. Deaton states that most life-cycle models predict "the existence of substantial asset accumulation at least at some points of the life cycle"(p. 1222). He further reports that this prediction is challenged by multiple surveys that estimate that U.S. households own very few financial assets.

where $\frac{1}{\rho}$ is the inter-temporal elasticity of substitution. At each age $t \in [1, T]$, the individual receives a stochastic income Y_t and maximizes (3.1) subject to:

$$\begin{aligned} W_{t+1} &= R(W_t + Y_t - C_t) \\ W_{N+1} &\geq 0 \end{aligned}$$

given an initial wealth level W_1 and where R is the constant, after-tax, gross real interest rate of the only asset available in the economy. Following Zeldes (1989), the labor income process is given by:

$$\begin{aligned} Y_t &= P_t U_t \\ P_t &= G_t P_{t-1} N_t \end{aligned} \tag{3.2}$$

where labor income is divided into a permanent component P_t and a transitory component U_t.[3] The transitory shocks U_t are assumed to be independently and identically distributed; moreover, there is a non-negative probability of a zero-income event, i.e. $U_t = 0$ with probability $p \in [0, 1)$. U_t is otherwise log-normally distributed, $\ln U_t \sim N(0, \sigma_U^2)$. The permanent component of income P_t follows a random walk with drift G_t (predictable growth of income) and permanent shock N_t, which is independently and identically log-normally distributed, $\ln N_t \sim N(0, \sigma_N^2)$. While N_t is meant to capture the effects of job changes, wage raises, and other persistent factors, U_t is meant to capture the effects of one-time bonuses, unemployment spells, and other transitory factors. Note that individuals will never choose to borrow against future labor income as long as the Inada condition $\lim_{c \to 0} = \infty$ is satisfied (the assumed income process has a zero lower bound which imposes a natural borrowing limit at 0).

GP make four additional assumptions. First, in order to reduce the number of state variables in the model, they assume that the age variations in $v(Z_t)$ are deterministic, common across households of the same age, and that they come from changes in family size. Second, stating that most of the retirement wealth in U.S. households is accumulated in illiquid assets (which are only available after retirement), GP assume that illiquid wealth accumulates exogenously, cannot be borrowed against, and that illiquid wealth in the first year of retirement is proportional to the last permanent component of income, i.e. $H_{T+1} \equiv hP_{T+1} = hP_T$. These assumptions eliminate both illiquid assets as a state variable and contributions to illiquid accounts as a control variable in the dynamic stochastic program. Third, invoking Bellman's optimality principle, the inter-temporal consumption problem is truncated at the age of retirement to avoid modelling the retirement period. Lastly, due to the truncation assumption, GP adopt the following retirement value function to condense the consumer's problem at retirement ages:

$$V_{T+1}(X_{T+1}, H_{T+1}, Z_{T+1}) = kv(Z_{T+1})(X_{T+1} + H_{T+1})^{1-\rho} \tag{3.3}$$

for some constant k and where X_t is defined as the level of cash-on-hand (total liquid financial wealth) in period t, i.e. $X_t \equiv W_t + Y_t$. According to GP, the functional form (3.3) is chosen to maintain the tractability of the problem and for being flexible enough to allow robustness checks.

In sum, the consumer's problem at age τ can be expressed as:

$$V_\tau(X_\tau, P_\tau, Z_\tau) = \max_{C_\tau, \ldots, C_T} E_\tau \left[\sum_{t=\tau}^{T} \beta^{t-\tau} v(Z_t) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau} kv(Z_{T+1})(X_{T+1} + hP_{T+1})^{1-\rho} \right]$$

[3] Labor income is defined as disposable income, net of Social Security taxes and contributions to illiquid accounts.

given the labor income process defined in (3.2) and subject to:

$$X_{t+1} = R(X_t - C_t) + Y_{t+1} \tag{3.4}$$

$$X_{T+1} \geq 0 \tag{3.5}$$

where the last inequality reflects the borrowing constraint on liquid assets at retirement age, imposed by the assumption that illiquid wealth cannot be borrowed against.

3.2.2 Numerical Solution

Since an analytical closed-form solution for the above problem is not available, GP solve the problem numerically by first normalizing all variables by the permanent component of income. They note that the particular functional form for the retirement function makes the household's consumption problem homogeneous of degree $(1-\rho)$ in P_t. Thus, by denoting lowercase letters as normalized variables, e.g. $x_t \equiv \frac{X_t}{P_t}$, the following Euler equation holds for ages $t < T$:

$$u'(c_t(x_t)) = \beta R E_t \left[\frac{v(Z_{t+1})}{v(Z_t)} u'(c_{t+1}(x_{t+1}) G_{t+1} N_{t+1}) \right] \tag{3.6}$$

where $c_t(x_t)$ is the optimal consumption function. In the last working period, the Euler equation is replaced by:

$$u'(c_T(x_T)) = \max \left\{ u'(x_T), \beta R \left[\frac{v(Z_{T+1})}{v(Z_T)} u'(c_{T+1}(x_{T+1})) \right] \right\} \tag{3.7}$$

since the illiquid wealth available in $T+1$ imposes a liquidity constraint on the cash-on-hand available at the age of retirement. Furthermore, under (3.3), the optimal retirement consumption rule is linear in total wealth. Hence, the normalized consumption at $T+1$ is expressed as:

$$\begin{aligned} \frac{C_{T+1}}{P_{T+1}} &= \gamma_1 \left(\frac{X_{T+1} + H_{T+1}}{P_{T+1}} \right) \\ c_{T+1} &= \gamma_1 (x_{T+1} + h) \quad = \quad \gamma_0 + \gamma_1 x_{T+1} \end{aligned} \tag{3.8}$$

where $\gamma_0 \equiv \gamma_1 h$ and γ_1 is the marginal propensity to consume out of liquid wealth. Thus, in order to find the set of optimal consumption rules for each age t, the problem can be solved recursively by first finding $c_T(x_T)$ in (3.7) by using (3.8). The optimal solutions to (3.7) and (3.6) then generate the consumption functions $c_{T-1}(x_{T-1}), ..., c_1(x_1)$.

In order to implement the proposed solution algorithm, a grid of 100 points between $[0, 40]$ is created for cash-on-hand, with 50 points between 0 and 2 following GP's discretization method. The finer grid for $x \in [0, 2]$ captures the curvature of the consumption rule at low values of cash-on-hand. Furthermore, to evaluate the expectation in (3.6), a two dimensional Gauss-Hermite quadrature of order 12 is performed as GP.[4] Table 3.1 reports the assumed values for the variances of the income shocks (σ_U^2, σ_N^2), the probability of zero-income event (p), the initial distribution of liquid assets at age 26 (w_1), the gross real after tax interest rate (R), and the income and family-composition profiles (G_t, Z_t). Lastly, the age of retirement is set at 65, i.e., $T = 40$.

[4]For more details, see Appendix A.

3.3 Results

3.3.1 Individual Consumption Behavior

Figure 3.1 reports the individual consumption behavior of a typical consumer working from ages 26 to 65 ($t = 1, ..., T$), whose consumption at $T+1$ is characterized by (3.8). The preference parameters are set to $\beta = 0.960$, $\rho = 0.514$, $\gamma_0 = 0.001$, and $\gamma_1 = 0.071$. These are the base-line parameter values estimated from the structural estimation in Gourinchas and Parker (2002) which matches simulated average age-profiles of consumption with those observed over the working ages of U.S. households in the Consumer Expenditure Survey (CEX).[5]

As can be seen, normalized consumption is increasing, positive, and concave in normalized cash-on-hand. Moreover, consumer behavior changes over the life-cycle, i.e. there is a systematic age-variation in consumption behavior. A typical young consumer has a low level of liquid wealth; as a result, her marginal propensity to consume is high as the consumption function is steep at low levels of cash-on-hand. Since labor income is expected to grow, young individuals prefer to borrow or save very little as more resources will be available in the future; however, they optimally choose to accumulate wealth to guard against negative income shocks. They never consume all of their financial wealth even when x is low; however, they do consume most of their financial wealth at low values of normalized cash-on-hand. Moreover, these parameter values imply a low level of illiquid wealth at $T+1$; thus, individuals must save for retirement as they age. As liquid wealth is accumulated to finance retirement years, consumption rules decrease as $t \to T$. Since most middle-aged consumers will have saved to a large extent for retirement purposes by then, their marginal propensity to consume will be low as income uncertainty becomes negligible and the consumption function is relatively flat at high levels of cash-on-hand. In fact, households' probability to hold low amounts of liquid wealth decreases with age as seen in Figure 3.2. It is almost 0 by age 45. This result indicates that once a household reaches its mid-forties, the probability that it will hold low amounts of cash-on-hand in the future is negligible; thus, it will continue to accumulate positive amounts of wealth as its marginal propensity to consume is low.

Alternatively, Figures 3.3a and 3.3b show the consumption functions for households facing two different retirement consumption rules $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$, respectively. Again, the optimal rules are concave in normalized cash-on-hand and vary systematically with age. With a higher level of illiquid wealth implied by γ_0, households do not accumulate as much liquid wealth as in Figure 3.1, and consumption functions decline less as the household ages. As can be seen, households accumulate less liquid wealth for retirement purposes as γ_0 increases: they can depend more on illiquid wealth to finance consumption when retired.

The model is further solved for $\rho = 1.5$ and $\rho = 3$, assuming $\gamma_0 = 0.01$, $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$. Figure 3.4 reports the consumption policy functions and wealth distribution at retirement age for $\rho = 1.5$, and Figure 3.5 depicts the ones for $\rho = 3$. As can be seen, the consumption functions are concave in normalized cash-on-hand and vary systematically with age; however, as the coefficient of risk aversion increases, the consumption functions are lower for all ages since consumers save more for precautionary reasons.

3.3.2 Aggregate Behavior

Carroll (2000) argues that the concavity in the consumption functions imply that the dynamic of the distribution of wealth affects the average marginal propensity to consume and the level of aggregate consumption. A significant dispersion in aggregate wealth will predict an aggregate behavior that

[5]See Chapter 4, Section 3, for a description of the Simulated Method of Moments Estimation.

does not resemble the behavior of a representative agent whose wealth is equal to the mean of the distribution. To analyze the aggregate implications of GP's life-cycle model, the consumer problem is first solved and simulated for the base-line parameter values. A sequence of $20,000$ income processes is generated over 40 years. The economic aggregate to be considered and modeled in this section is average (arithmetic mean) cash-on-hand, average consumption, and average marginal propensity to consume over time.

Figure 3.6 reports the consumption function, the marginal propensity to consume, and the distribution of normalized cash-on-hand at various ages. At age 26, normalized cash-on-hand is distributed around the "kink-point" of the consumption function with a mean value of 1.3040 and a median value of 1.1288. The bottom one forth of the distribution faces borrowing constraints and have high marginal propensities to consume in contrast to the top three forths of the distribution. However, households seem to quickly adjust their consumption and savings decisions so as to avoid the region where borrowing constraints are binding. By age 35, most of the cash-on-hand distribution is in the flat region of the consumption function. At age 45, the distribution of normalized cash-on-hand is to the right of the "kink-point" of the consumption function. This implies that all households of a given age have the same marginal propensity to consume, which in this case is close to seven percent. As households age and accumulate liquid wealth, the distribution of cash-on-hand keeps gradually moving to the right. At age 55, the median value of the cash-on-hand distribution is 4.9834, which jumps to 10.9432 by age 65 (not illustrated). Thus, under the base-line parameter values, the aggregate behavior predicted by the life-cycle model would resemble the behavior of a representative-agent whose cash-on-hand is equal to the mean of the distribution starting at age 45.

However, note that the predicted distribution of cash-on-hand does not match the one observed across U.S. households during the years considered for the SMM estimation of the preference parameters in Gourinchas and Parker (2002). Figure 3.7a presents the distribution of liquid wealth at retirement age across U.S. households based on the Panel Study of Income Dynamics. This data is provided by GP, who construct measures of cash-on-hand in 1989 for households whose head has retired in 1991, 1992 or 1993 and is between 60 and 70 years old in 1992. An can be seen, the wealth distribution is highly skewed to the left. The median of the distribution is $\$40,447.41$ and the mean is $\$53,156.09$. Figure 3.7b shows the distribution of normalized cash-on-hand, which is also skewed to the left. Most of the distribution is below 3, with a median value of 0.95 and a mean value of 1.02. These values are well below the median of the cash-on-hand distribution at age 65 predicted by the life-cycle model with a $\gamma_0 = 0.01$. Recall that $\gamma_0 = 0.01$ imply a low level of illiquid wealth at retirement; thus, households in the life-cycle model must save enough wealth to finance their retirement years' consumption. Table 3.2 shows that the two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the predicted distribution for the base-line parameters and the empirical distribution are the same at a 5% significance level.

Figure 3.8 presents the simulated average age-profiles for consumption and cash-on-hand for the base-line parameters. The life-cycle profiles are constructed by calculating the arithmetic mean (averaging across $20,000$ households) for each age. The consumption profile increases until age 45 and then decreases, while cash-on-hand is increasing over the life-cycle.[6] As can be seen, the mean household accumulates a substantial amount of liquid wealth starting early in the life-cycle as discussed previously.

Alternatively, Figures 3.9 and 3.10 show the consumption function, the marginal propensity to consume, and the distribution of normalized cash-on-hand at various ages for $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$, respectively. As can be seen, the cash-on-hand distributions at age 26 in both cases are similar to the

[6]It is worth noting that the observed hump-shape of the consumption profile is heightened by variations in the growth of rate of income.

one in the base-line case. However, as γ_0 increases, households do not accumulate assets as quickly since they can rely on illiquid assets to finance consumption during retirement years. At age 45, there is still a small fraction of households in the area where borrowing constraints are binding. Only at age 65, the entire distribution of cash-on-hand is right of the "kink-point" of the consumption function when $\gamma_0 = 0.594$. On the other hand, there are still a few households at age 65 that face borrowing constraints when $\gamma_0 = 0.9$. Figure 3.11 confirms this result. With a higher level of illiquid wealth implied by γ_0, households do not accumulate as much liquid wealth as in Figure 3.8b. However, despite increasing the value of γ_0, the predicted distribution of cash-on-hand does not match the one observed in the PSID data. As can be seen, the median value of normalized cash-on-hand is around 4.9 and 2.4 for $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$, respectively. Table 3.2 shows that the two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the predicted distributions (for $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$, respectively) and the empirical distribution are the same at a 5% significance level. Similarly, the predicted distributions of normalized cash-on-hand when $\rho = 1.5$ and $\rho = 3$ (assuming $\gamma_0 = 0.001$ $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$) are also different from the observed distribution in the PSID data at a 5% significance level.

Furthermore, Figure 3.12 reports the average age-profile of the marginal propensity to consume for various value combinations of ρ and γ_0. It is calculated as the average across age of the slope of the normalized consumption function evaluated at the simulated normalized value of cash-on-hand at each age t. As can be seen, the average marginal propensities to consume converges to a relatively stable value around seven percent which is the marginal propensity to consume implied by the assumed retirement consumption rule. However, this convergence occurs faster as the level of illiquid wealth decreases and the coefficient of risk aversion rises. For instance, when $\rho = 0.514$, the average marginal propensity to consume reaches the stable value at age 45 when $\gamma_0 = 0.001$, at age 60 when $\gamma_0 = 0.594$, and at age 65 when $\gamma_0 = 0.9$. On the other hand, the average marginal propensity to consume converges to the stable value at age 35 when $\rho = 1.5$ and at age 30 when $\rho = 3$. This convergence further implies that all individuals have identical marginal propensities to consume at those mentioned ages. This result is further confirmed by Figure 3.13. It reports the standard deviation of the marginal propensity to consume at each age. For $\rho = 1.5$ and $\rho = 3$, the standard deviation converges rapidly to zero regardless of γ_0. The rapid convergence of the average marginal propensity to consume reflects that consumers accumulate enough liquid wealth to stay on the flat portion of the consumption function, and thus, avoiding the region where borrowing constraints are binding. Note, however, that the rate of convergence of the standard deviation depends on the value of γ_0 for individuals with $\rho = 0.514$. When expecting to receive a large amount of illiquid wealth at retirement (high value of γ_0), households do not accumulate as much liquid wealth. There is a significant fraction of households with low level of liquid wealth that boosts the value of average marginal propensity to consume at each age consistent with Figures 3.9 and 3.10.

3.3.3 Stochastic Retirement Years

The predicted consumption and savings behavior in the previous sections ignores the consumption risks associated with retirement years. Although labor income risks are not relevant for most of the retirees, the elderly face substantial medical expenditure risks (Feenberg and Skinner, 1994; Palumbo, 1999; Hurd, 2002; French and Jones, 2004; De Nardi, French, and Jones, 2010; French and Jones, 2011). Health-care cost uncertainty can induce risks on the disposable income available to finance retirement consumption, affecting individuals' consumption and saving decisions during their working ages. In this section, I explore the aggregate predictions of a modified version of the life-cycle model allowing for a simple stochastic retirement period.

Consider a life-cycle model in which individuals die at age 87 ($N = 62$), retire at age 65 ($T = 40$), but receive a proportion α of their income during retirement ages such that, for $t \geq T$:

$$x_{t+1} = \frac{R}{G_{t+1}N_{t+1}}(x_t - c_t) + \alpha U_{t+1} \tag{3.9}$$

where $0 < \alpha \leq 1$ is a retirement replacement rate.[7]

Figure 3.14 presents the average and the standard deviation of the marginal propensity to consume for ages 26 to 65 for various replacement rates. As can be seen, the average marginal propensities to consume converge to approximately 0.07 at age 65 for replacement rates $50\%, 60\%$, and 70%. This convergence implies that all individuals have the same identical marginal propensities to consume at age 65. However, as the replacement rate increases, this convergence is less obvious: the standard deviation of the marginal propensity to consume increase as well as the average marginal propensity to consume. When $\alpha = 1$, the average marginal propensity to consume is about 0.2 between ages 40 and 65, with a standard deviation of approximately 0.16. Furthermore, Figure 3.15 depicts the average age-profile of consumption and cash-on-hand. Until age 45, both the consumption and cash-on-hand age-profiles are almost identical regardless of the value of α. During the late-forties, households with higher replacement rates start to consume more and save less liquid wealth.

Figure 3.16 displays the consumption function, the marginal propensity to consume, and the distribution of normalized cash-on-hand at retirement age for various replacement rates. When $\alpha = 0.5$, the entire distribution of cash-on-hand lies on the flat region of the consumption, with an average propensity to consume of 0.075 and a standard deviation of 0.001 implying that all individuals have the same marginal propensity to consume at age 65. As the replacement rate increases, households accumulate less wealth during their working ages as they expect to receive a higher fraction of their income during retirement years. As a result, the predicted distribution of normalized cash-on-hand for $\alpha = 1$ is the one that most resembles the one observed in the PSID data, with a mean value of 1.620 and a median value of 1.521. However, for all these replacement rates, Table 3.3 shows that the two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the predicted distributions and the empirical distribution are the same at a 5% significance level. Furthermore, Table 3.4 shows that the average and the standard deviation of the marginal propensity to consume increase as the replacement rate increases. For instance, the average propensity to consume is 0.078 with a standard deviation of 0.006 for a replacement rate of 60%, and the average propensity to consume is 0.2126 with a standard deviation of 0.160 for a replacement rate of 100%. When $\alpha = 1$, there is a measurable fraction of households that face borrowing constraints that boosts both the value of the average marginal propensity to consume and its standard deviation.

3.4 Conclusion

The current chapter studies the implications of life-cycle behavior predicted by GP for aggregate dynamics. The non-linearity of the consumption functions and the systematic variation in age imply that the structure and dynamics of the wealth distribution are important in understanding aggregate behavior over the life-cycle. However, this study shows that the aggregation prediction of the life-cycle model in Gourinchas and Parker (2002) is sensitive to the assumed values of the preference parameters and the income profile during retirement years. For the base-line parameters, households optimally choose to accumulate a substantial amount of liquid wealth, moving the entire distribution of cash-on-hand to the region where the consumption function is flat. In fact, households' probability to hold low amounts of liquid wealth is almost zero by age 45; as a result, all individuals have the same marginal propensity to consume in their mid-forties and up. Thus, the

[7] When the replacement rate is 100% ($\alpha = 1$), this alternative model corresponds to the finite-horizon version of the buffer-stock model in Carroll (2019).

aggregate dynamics of a representative-consumer model possessing liquid wealth equal to the mean of the distribution would resemble the aggregate predictions of the life-cycle model for middle-aged individuals. The same conclusion extends for households with high coefficients of risk aversion. In contrast, when households expect to receive a large amount of illiquid wealth at retirement, there is a significant fraction of households in the region where borrowing constraints are binding as they do not accumulate as much liquid wealth. This, in turn, boosts the value of the aggregate marginal propensity to consume. Thus, the appropriateness of a representative-agent depends on the age distribution of the population and on the assumed values of the preference and retirement rule parameters. Furthermore, despite the various combinations of parameter values that are studied, the predicted distribution of cash-on-hand does not match the one observed across U.S. households. This discrepancy sheds light in the suitability of the life-cycle model to reproduce the observed savings behavior across U.S. households. Lastly, these results suggest that the main weakness of GP's life-cycle model is its retirement rules assumptions as the model disregards consumption risks during retirement years. A richer representation of the retirement period would improve the understanding of individuals' savings behavior over the life-cycle.

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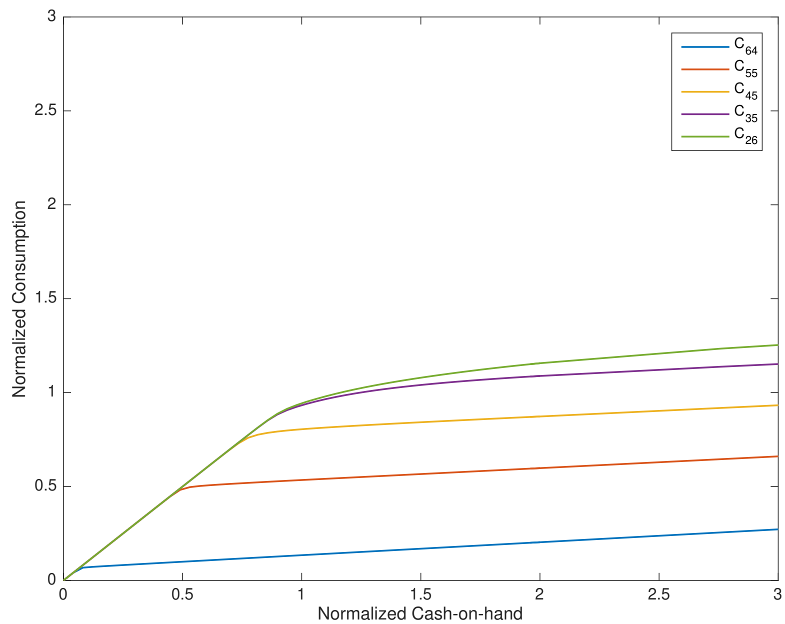


Figure 3.1: Consumption Function for Base-line Parameters
$(\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$

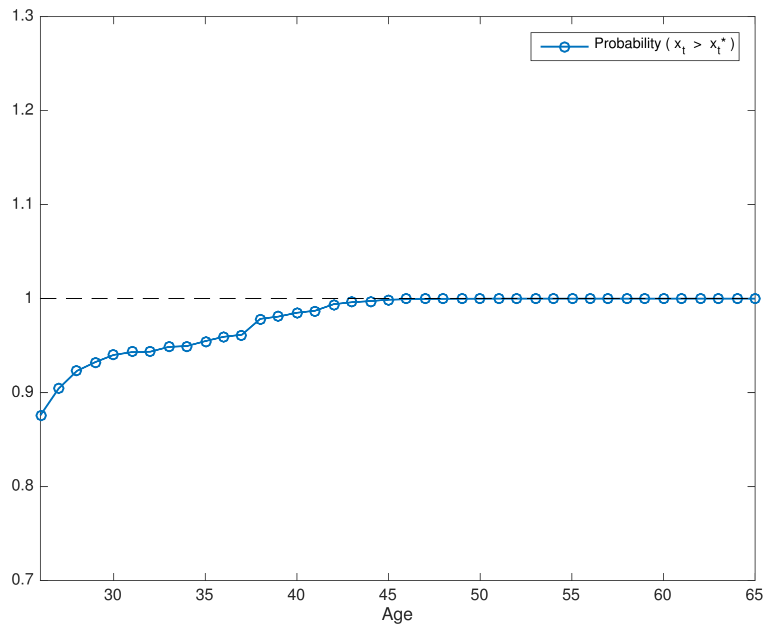
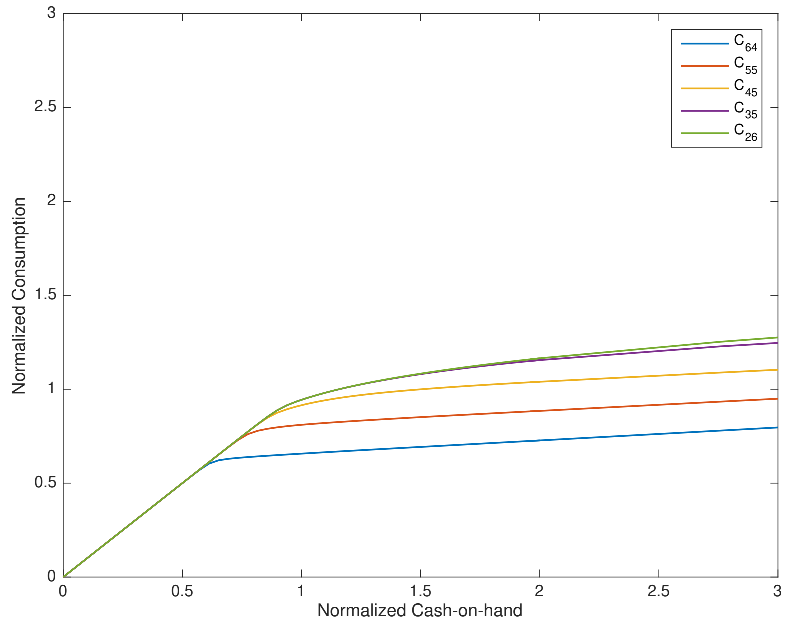
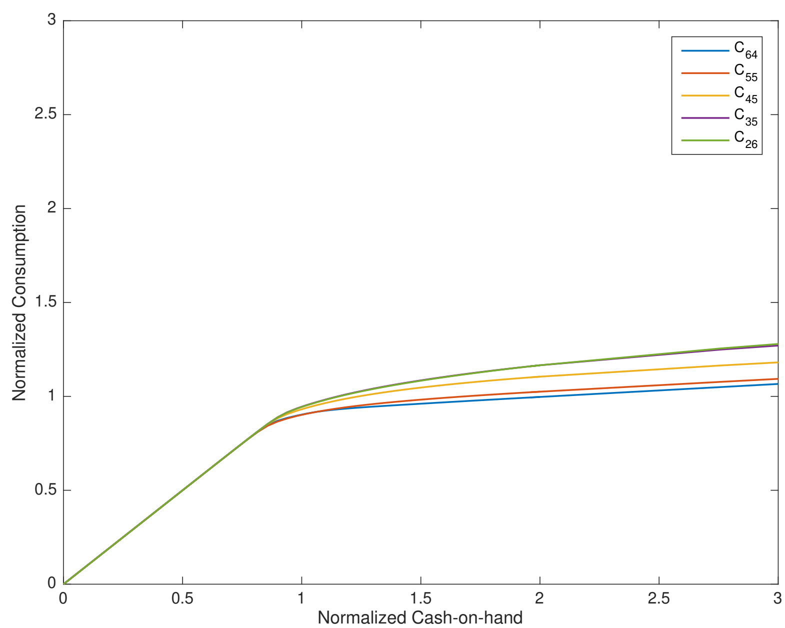


Figure 3.2: Probability of Holding High Amounts of Cash-on-Hand

This probability is calculated as the probability of normalized cash-holdings being greater than the "kink-point" of the consumption function at each age t. It considers $20,000$ simulated individuals who are followed throughout their life-cycle, based on the distribution of transitory and permanent income shocks. The parameters assumed are $\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$.

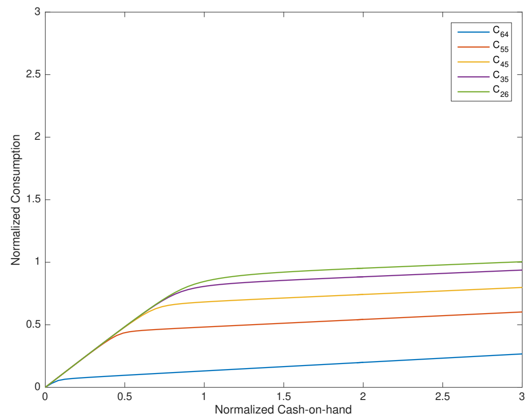


(a) $\gamma_0 = 0.594$

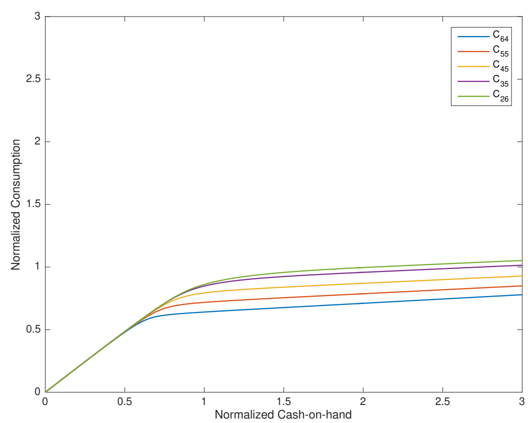


(b) $\gamma_0 = 0.9$

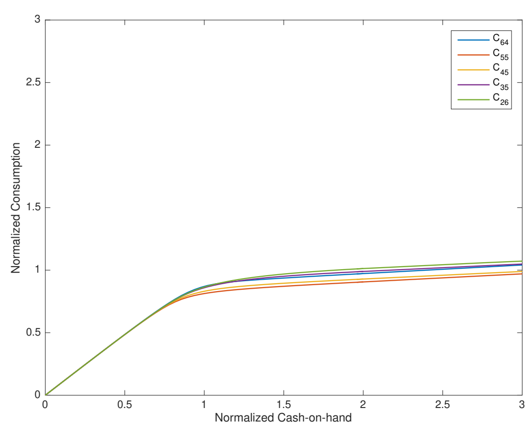
Figure 3.3: Consumption Functions for $\gamma_0 = 0.594$ and $\gamma_0 = 0.9$ ($\rho = 0.514, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$)



(a) $\gamma_0 = 0.001$

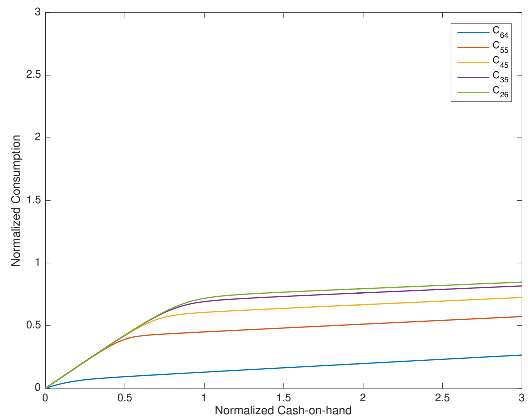


(b) $\gamma_0 = 0.594$

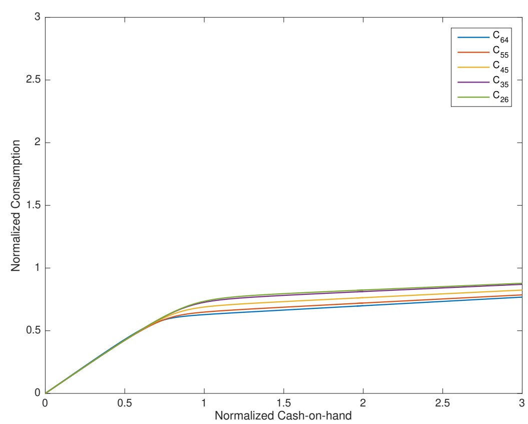


(c) $\gamma_0 = 0.9$

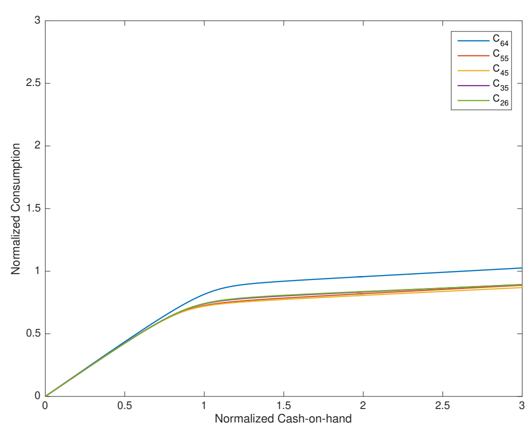
Figure 3.4: Consumption Functions for $\rho = 1.5$ ($\beta = 0.96, \gamma_1 = 0.071, R = 1.0344$)



(a) $\gamma_0 = 0.001$



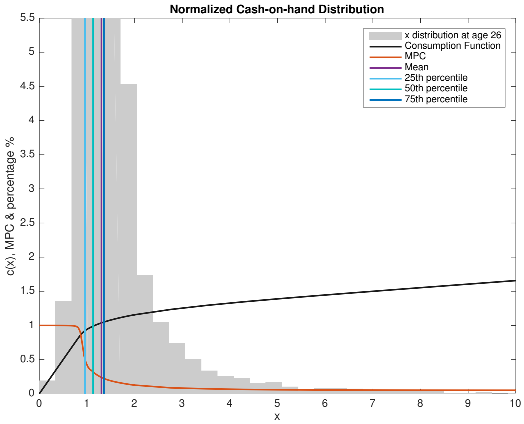
(b) $\gamma_0 = 0.594$



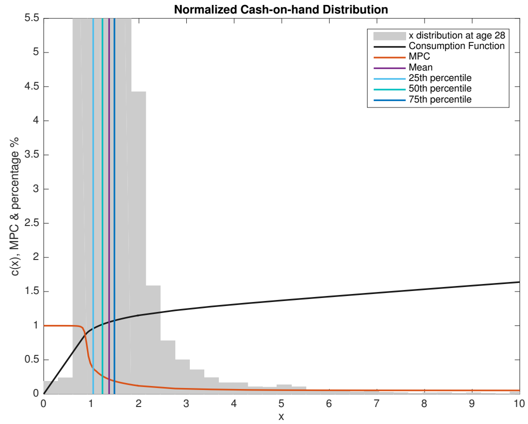
(c) $\gamma_0 = 0.9$

Figure 3.5: Consumption Functions for $\rho = 3$

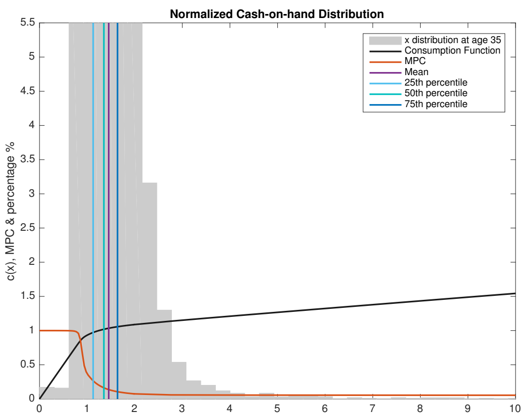
$(\beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$



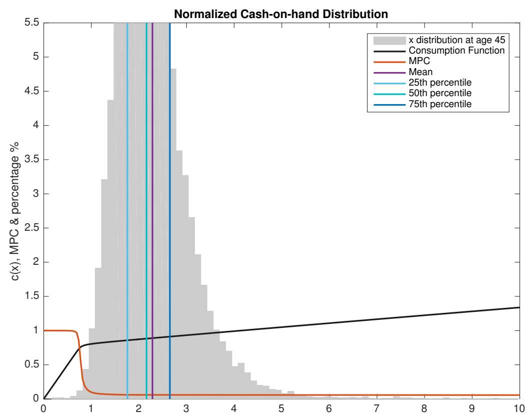
(a) Age 26



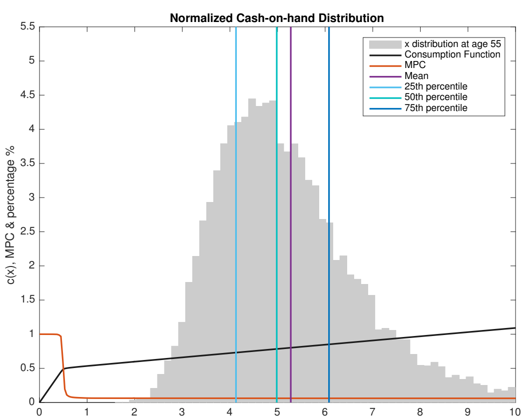
(b) Age 28



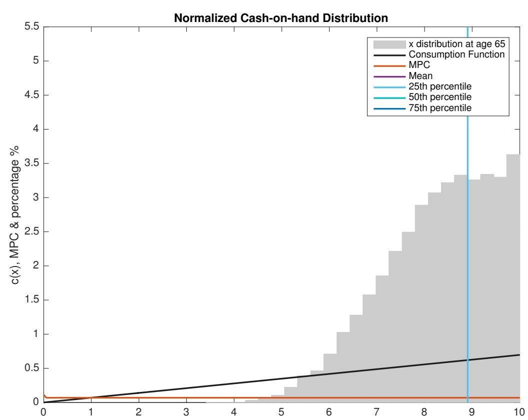
(c) Age 35



(d) Age 45

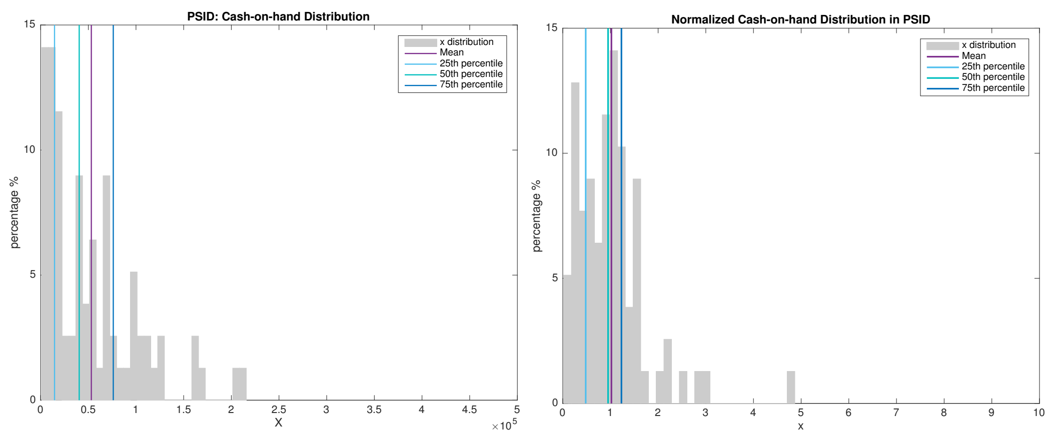


(e) Age 55



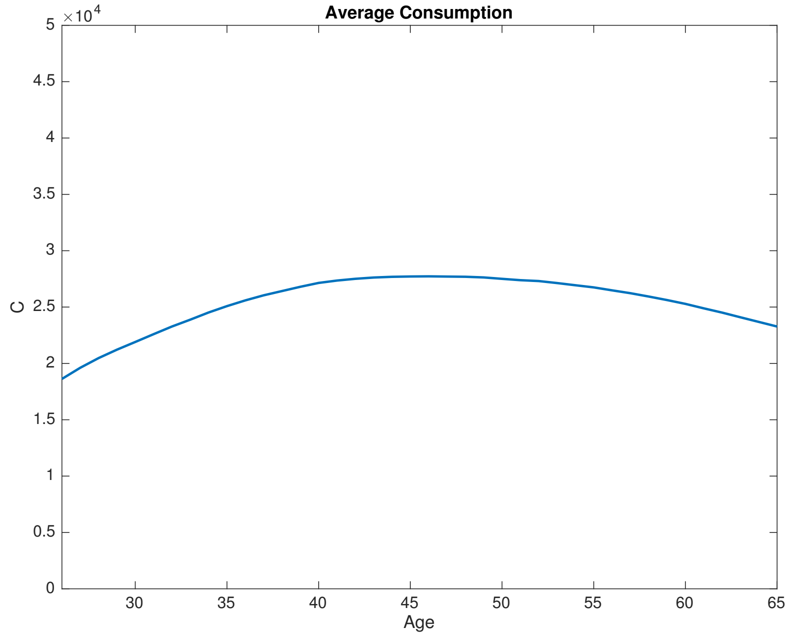
(f) Age 65

Figure 3.6: Consumption Function, MPC, and Distribution of Cash-on-Hand ($\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$)

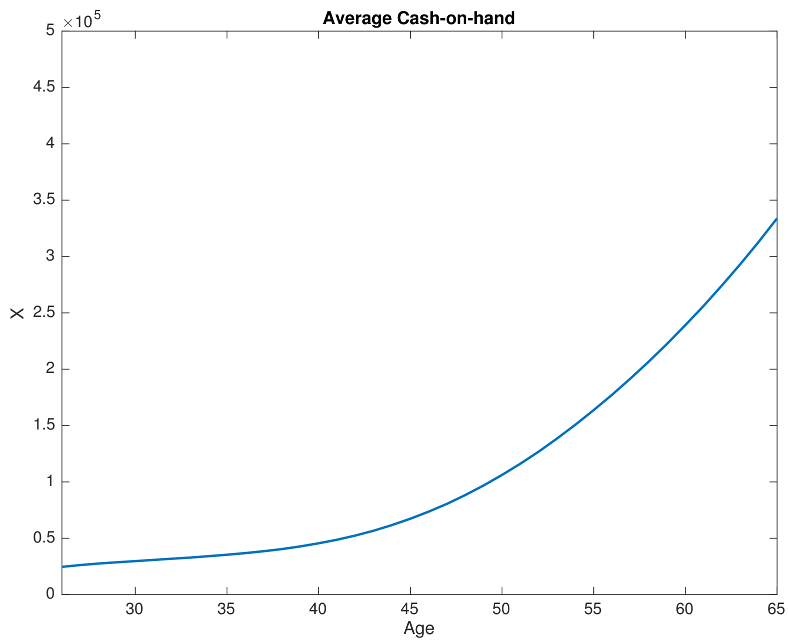


(a) Cash-on-hand (1987 US Dollars)   (b) Normalized Cash-on-hand

Figure 3.7: Distribution of Cash-on-Hand in PSID data



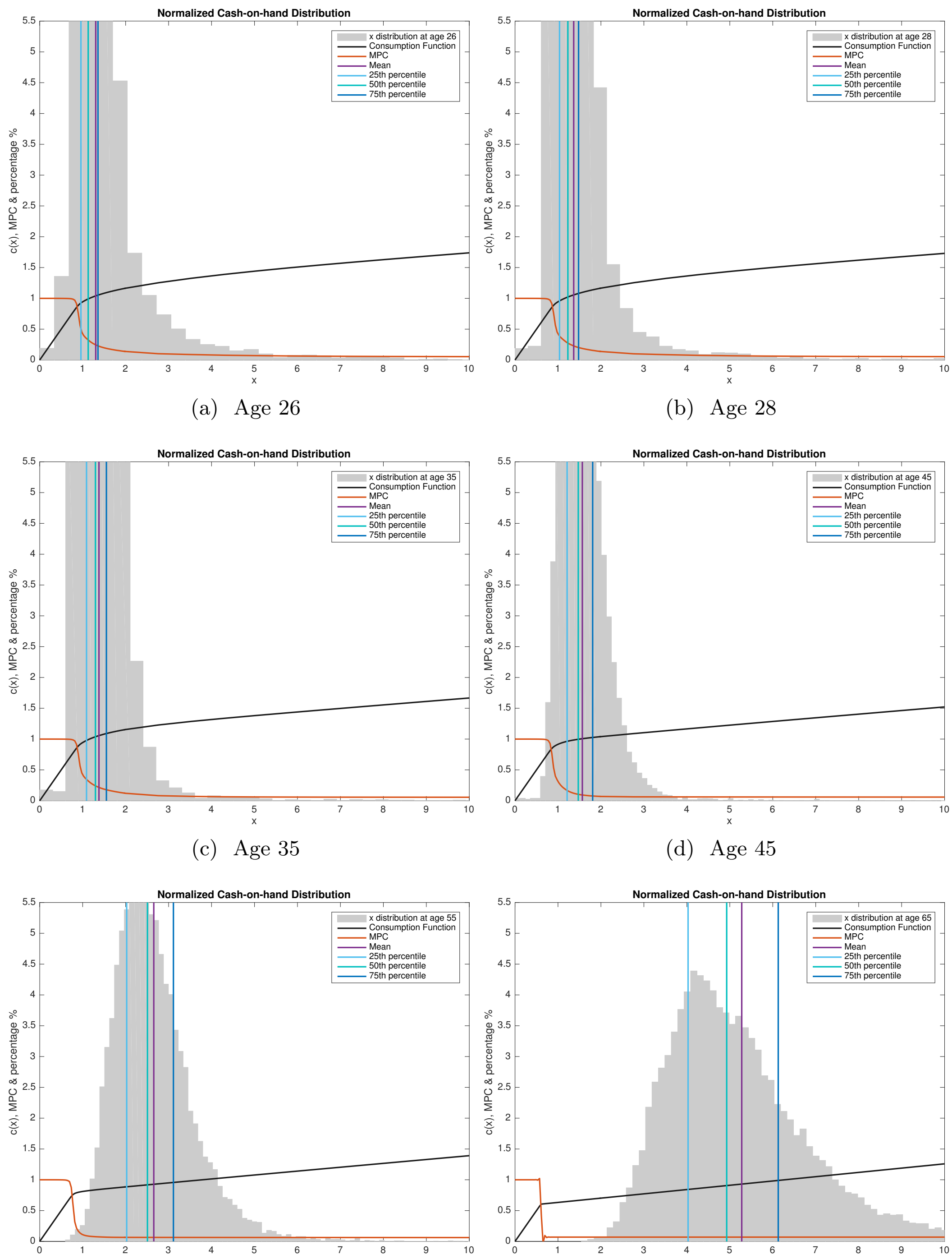
(a) Mean Consumption



(b) Mean Cash-on-hand

Figure 3.8: Age-Profiles of Mean C_t and Mean X_t

$(\rho = 0.514, \gamma_0 = 0.001, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$

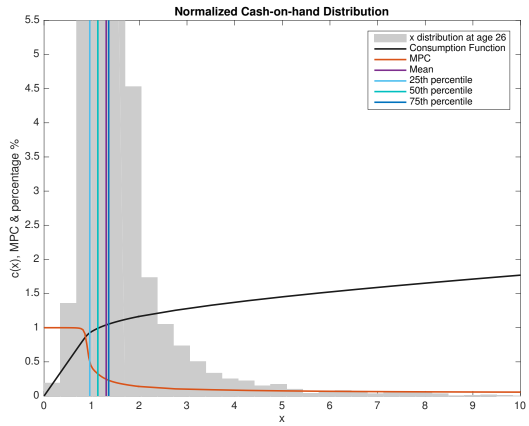


(a) Age 26 (b) Age 28

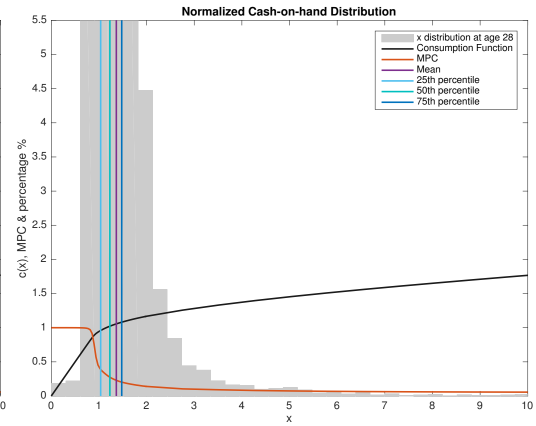
(c) Age 35 (d) Age 45

(e) Age 55 (f) Age 65

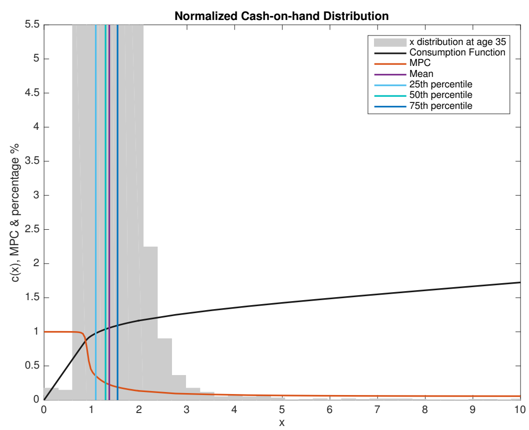
Figure 3.9: Consumption Function, MPC, and Distribution of Cash-on-Hand ($\rho = 0.514, \gamma_0 = 0.594, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$)



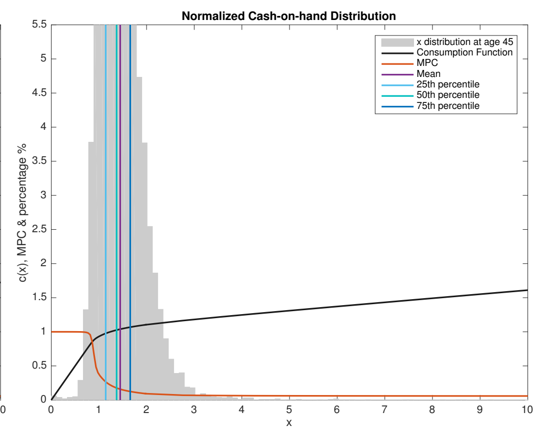
(a) Age 26



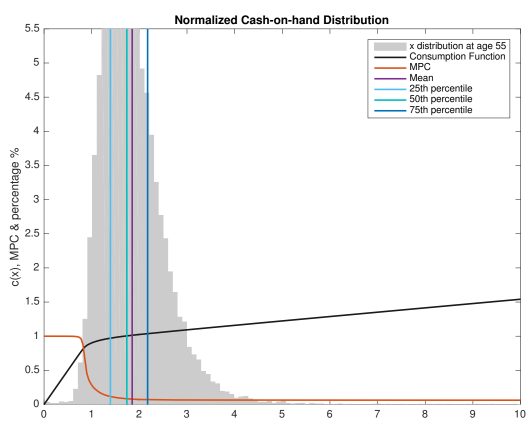
(b) Age 28



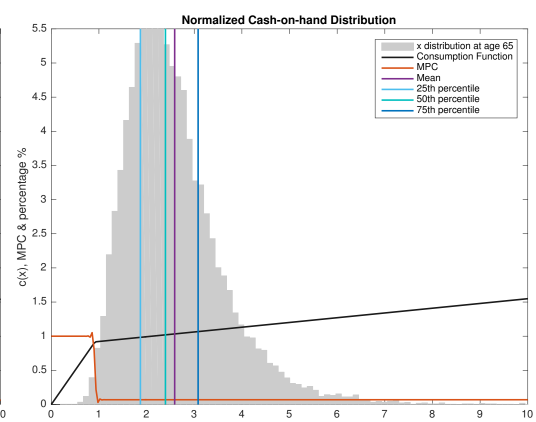
(c) Age 35



(d) Age 45

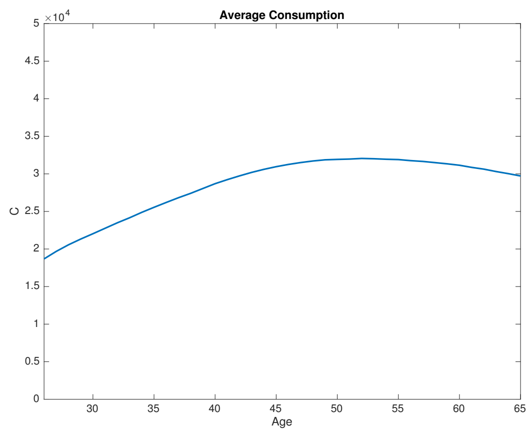


(e) Age 55

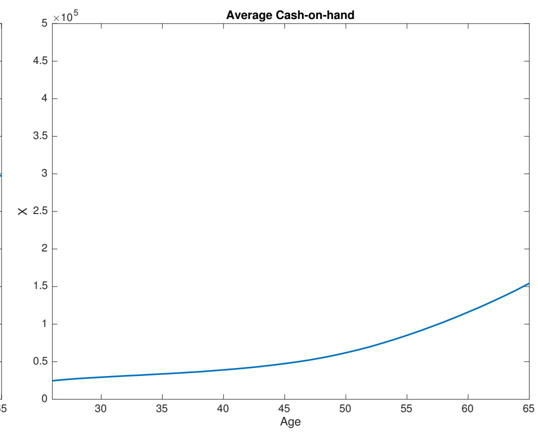


(f) Age 65

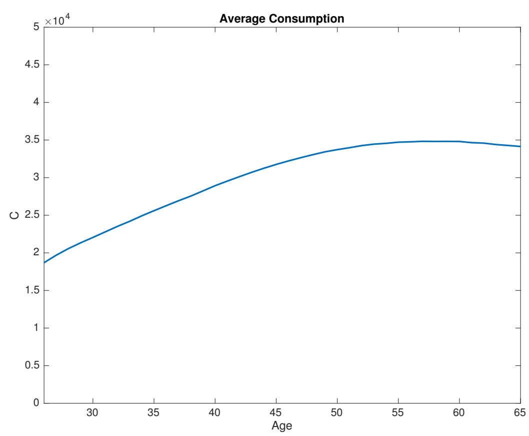
Figure 3.10: Consumption Function, MPC, and Distribution of Cash-on-Hand ($\rho = 0.514, \gamma_0 = 0.9, \beta = 0.96, \gamma_1 = 0.071, R = 1.0344$)



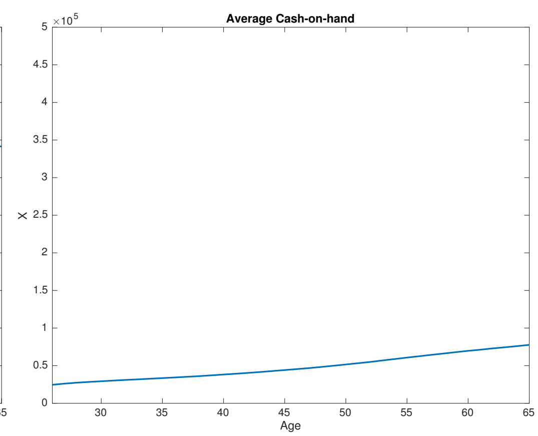
(a) Mean Consumption for $\rho = 0.514, \gamma_0 = 0.594$



(b) Mean Cash-on-hand for$\rho = 0.514, \gamma_0 = 0.594$



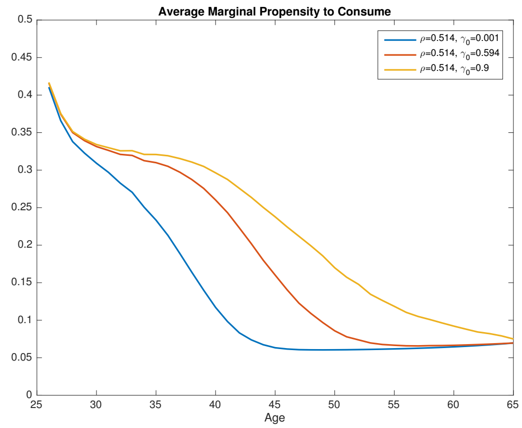
(c) Mean Consumption for $\rho = 0.514, \gamma_0 = 0.9$



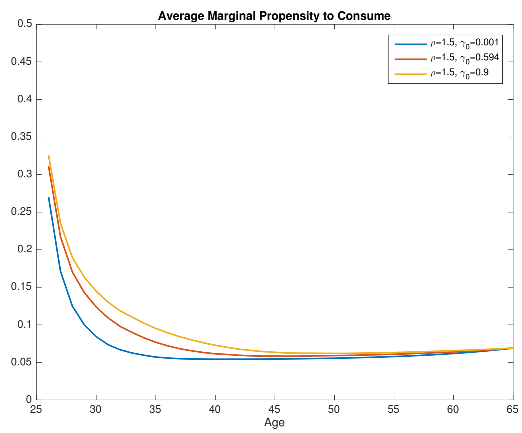
(d) Mean Cash-on-hand for $\rho = 0.514, \gamma_0 = 0.9$

Figure 3.11: Age-Profiles of Mean C_t and Mean X_t

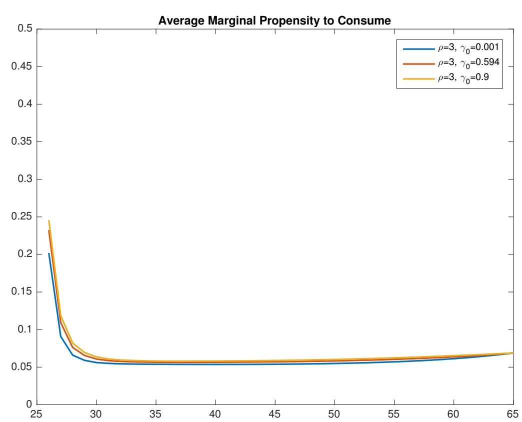
$(\beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$



(a) $\rho = 0.514$



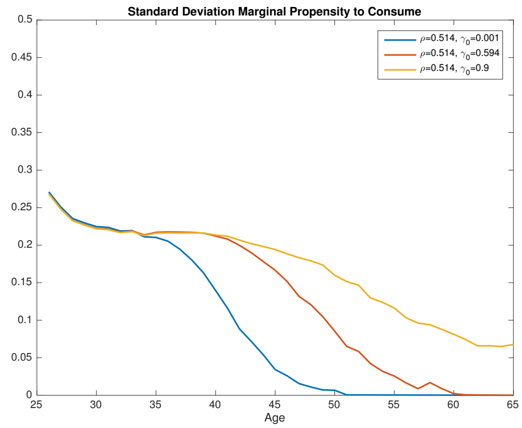
(b) $\rho = 1.5$



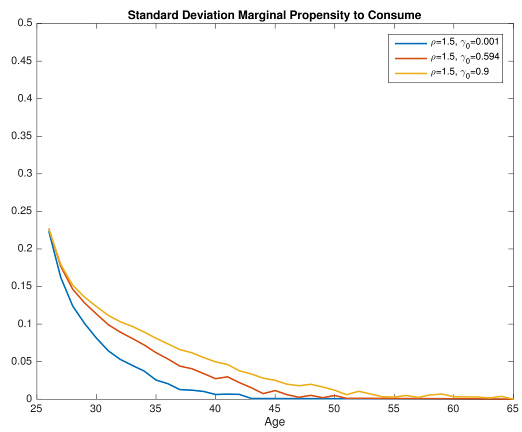
(c) $\rho = 3$

Figure 3.12: Average Marginal Propensity to Consume

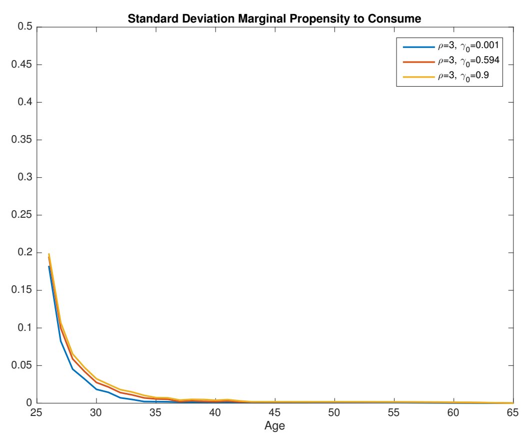
$(\beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$



(a) $\rho = 0.514$



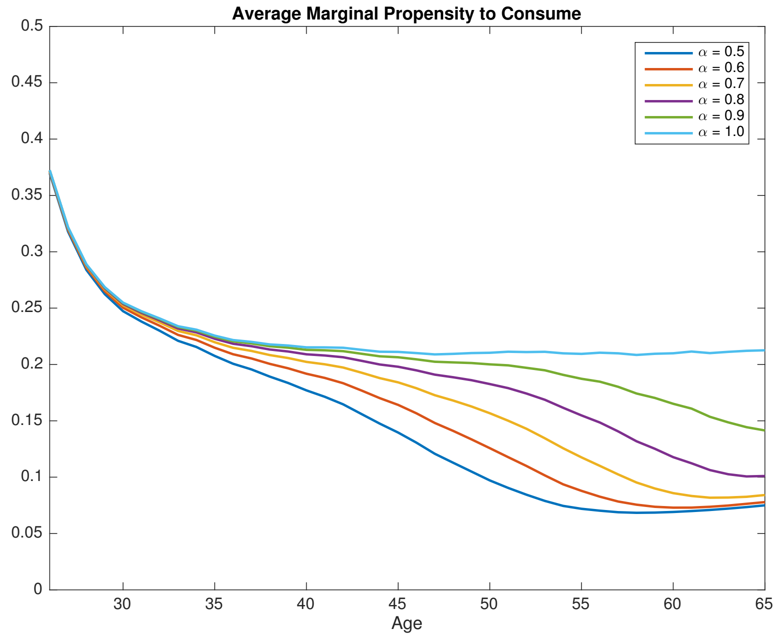
(b) $\rho = 1.5$



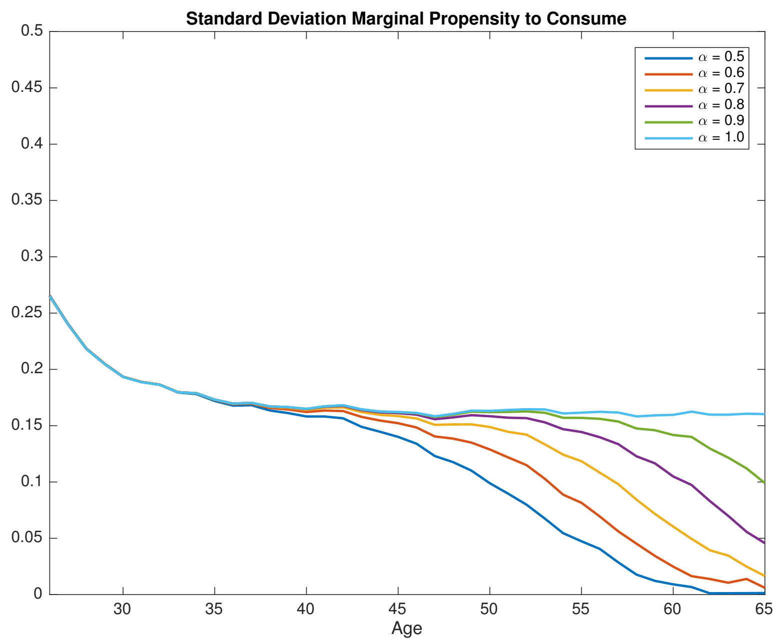
(c) $\rho = 3$

Figure 3.13: Standard Deviation of Marginal Propensity to Consume

$(\beta = 0.96, \gamma_1 = 0.071, R = 1.0344)$



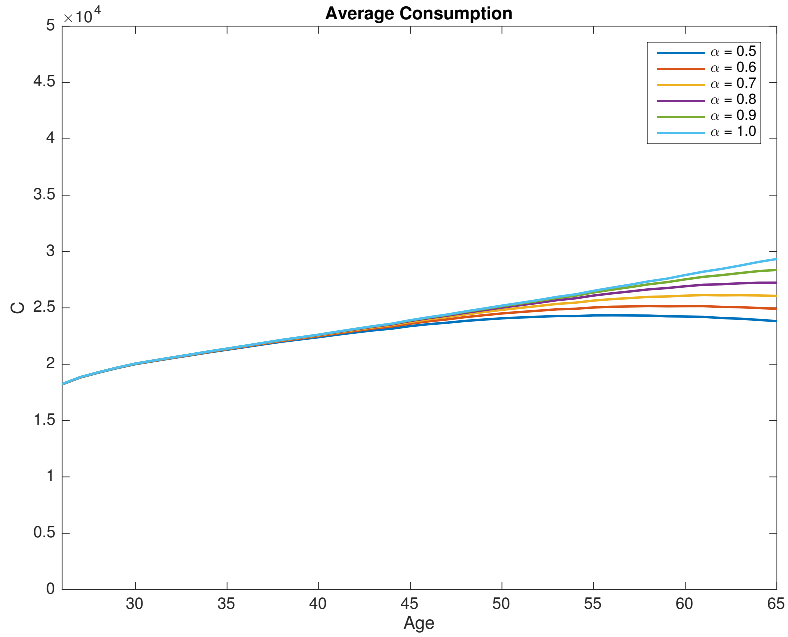
(a) Average MPC



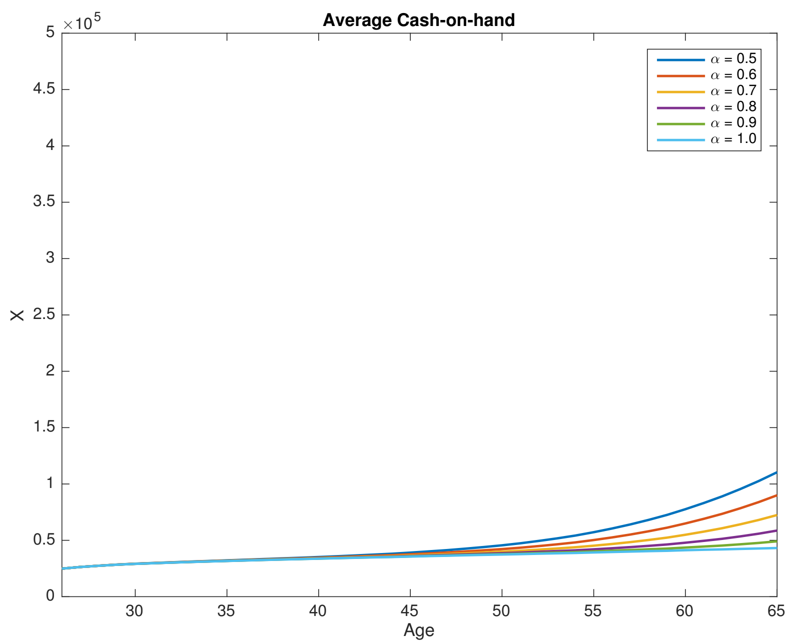
(b) Standard Deviation MPC

Figure 3.14: Average and Standard Deviation MPC with Replacement Rates

Assuming death age 87($N = 62$), retirement age 65($T = 40$), $\rho = 0.514, \beta = 0.96$



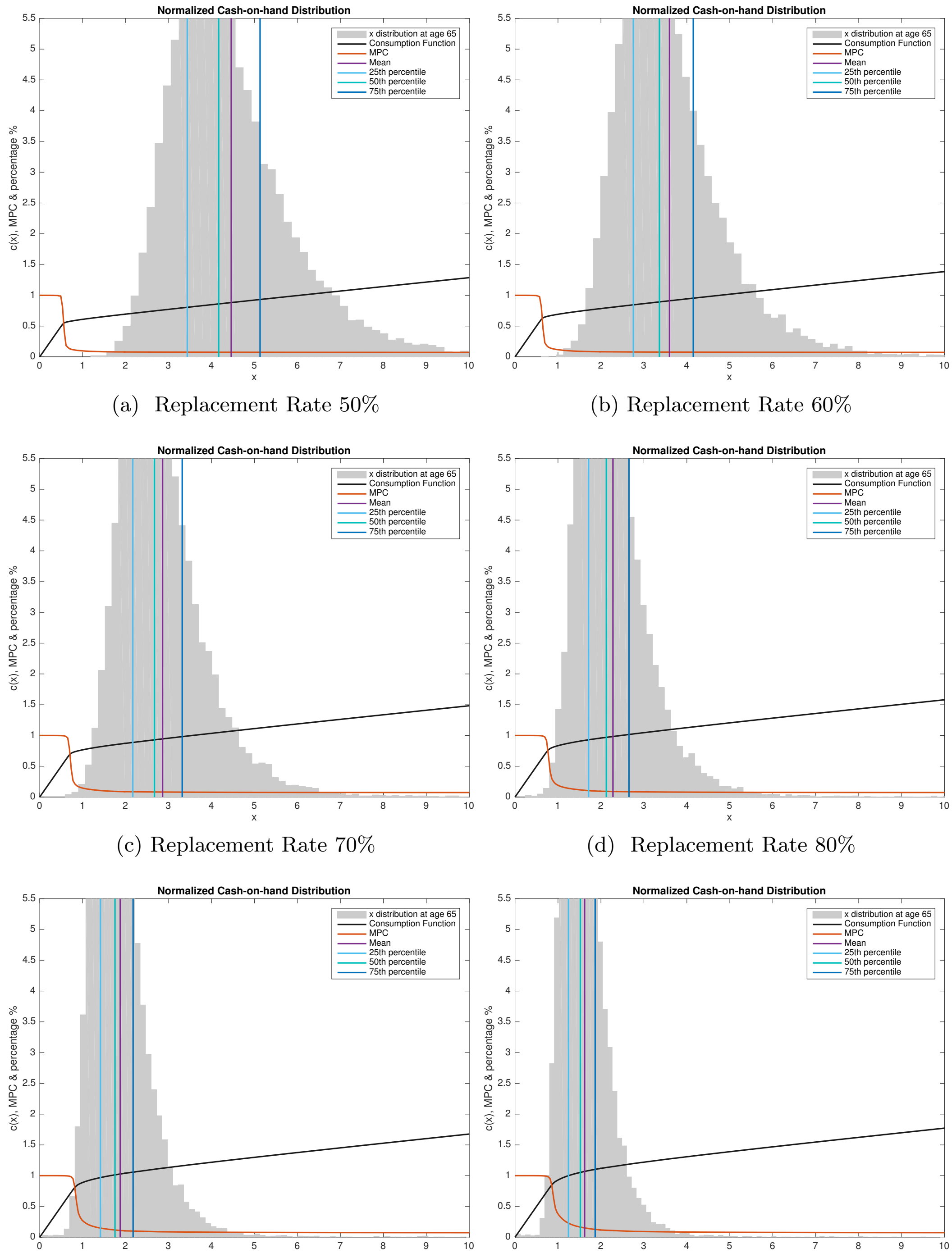
(a) Mean Consumption



(b) Mean Cash-on-hand

Figure 3.15: Age-Profiles of Mean C_t and Mean X_t with Replacement Rates

Assuming death age 87($N = 62$), retirement age 65($T = 40$), $\rho = 0.514, \beta = 0.96$



(a) Replacement Rate 50%

(b) Replacement Rate 60%

(c) Replacement Rate 70%

(d) Replacement Rate 80%

(e) Replacement Rate 90%

(f) Replacement Rate 100%

Figure 3.16: Consumption Function, MPC, and Distribution of Cash-on-Hand at age 65 ($\rho = 0.514, \beta = 0.96, R = 1.0344$)

Parameter	Value	Source
R	1.03440	Moody's AAA municipal bonds, Jan 1980 - March 1993
σ_U^2	0.0440	Carroll and Samwick (1997), PSID 1981-1987
σ_N^2	0.0212	Carroll and Samwick (1997), PSID 1981-1987
p	0.00302	Carroll, Hall, and Zeldes (1992), PSID 1976-1985
$\bar{w}_1$	-2.7944810	Gourinchas and Parker (2002), CEX 1980-1993
σ_{w_1}	1.7838679	
G_t:		Gourinchas and Parker (2002), CEX 1980-1993
age	.32643678179	
age^2	$-.0148947085$	
age^3	.00036342384	
age^4	$-4.411685e^{-6}$	
age^5	$2.056916e^{-8}$	
$constant$	6.801368713	
Z_t:		Gourinchas and Parker (2002), CEX 1980-1993
age	0.13964975	
age^2	-0.0047742190	
age^3	$8.5155210e^{-5}$	
age^4	$-7.9110880e^{-7}$	
age^5	$2.9789550e^{-9}$	

Table 3.1: Parameter Values from Gourinchas and Parker (2002)

Parameters	25^{th}	50^{th}	75^{th}	KS Statistic	P-value
$\rho = 0.514, \gamma_0 = 0.001$	8.905	10.943	13.674	0.999	0.0000
$\rho = 0.514, \gamma_0 = 0.594$	4.028	4.928	6.128	0.948	0.0000
$\rho = 0.514, \gamma_0 = 0.9$	1.870	2.396	3.082	0.747	0.0000
$\rho = 1.5, \gamma_0 = 0.001$	10.804	13.620	17.580	0.999	0.0000
$\rho = 1.5, \gamma_0 = 0.594$	5.996	7.649	9.981	0.9856	0.0000
$\rho = 1.5, \gamma_0 = 0.9$	3.690	4.831	6.450	0.9285	0.0000
$\rho = 3, \gamma_0 = 0.001$	12.003	15.396	20.242	0.9999	0.0000
$\rho = 3, \gamma_0 = 0.594$	7.607	9.932	13.3130	0.987	0.0000
$\rho = 3, \gamma_0 = 0.9$	5.512	7.390	10.123	0.978	0.0000
PSID Data	0.481	0.950	1.230	–	–

Table 3.2: Distribution of Cash-on-Hand at Retirement Age and Kolmogorov-Smirnov Test

Selected percentiles for the predicted distribution of normalized cash-on-hand assuming $\beta = 0.96, \gamma_1 = 0.071, R = 1.0344$. The two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the empirical and simulated distributions are the same distribution at a 5% significance level.

α	25^{th}	50^{th}	75^{th}	KS Statistic	P-value
50%	3.435	4.167	5.130	0.935	0.0000
60%	2.754	3.361	4.150	0.894	0.0000
70%	2.168	2.671	3.317	0.837	0.0000
80%	1.712	2.126	2.652	0.709	0.0000
90%	1.414	1.752	2.174	0.622	0.0000
100%	1.245	1.521	1.866	0.526	0.0000
PSID Data	0.4811	0.9503	1.2303	–	–

Table 3.3: Distribution of Cash-on-Hand at Retirement Age and Kolmogorov-Smirnov Test

Selected percentiles for the predicted distribution of normalized cash-on-hand for various replacement rates, assuming $\rho = 0.514, \beta = 0.96, R = 1.0344$. The two-sample Kolmogorov-Smirnov test rejects the null hypothesis that the empirical and simulated distributions are the same distribution at a 5% significance level.

Replacement Rate	Mean MPC	Standard Deviation of MPC
$\alpha = 50\%$	0.075	0.0014
$\alpha = 60\%$	0.078	0.0060
$\alpha = 70\%$	0.084	0.0165
$\alpha = 80\%$	0.101	0.0456
$\alpha = 90\%$	0.142	0.0990
$\alpha = 100\%$	0.213	0.1602

Table 3.4: Average and Standard Deviation of MPC at Retirement Age

Chapter 4

Implementing the Simulated Method of Moments: A Cautionary Note

4.1 Introduction

Estimating structural models often involve a non-linear optimization with respect to unknown parameters. For instance, in order to implement a structural estimation of a dynamic stochastic life-cycle model, the procedure involves two layers of optimization. The inner layer solves the model numerically using an optimization algorithm; it typically involves solving the model's Euler equations through backward induction and calculating numerical integration over random variables. Once the model is solved and simulated, the outer layer minimizes the objective function of the estimator as a function of the parameters. As Low and Meghir (2017) discuss, the main obstacle with structural estimation is the absent of an analytical relationship between the model's control variables and the unknown parameters; thus, in order to evaluate the marginal effect of the parameters on the model's dependent variables, the entire model needs to be re-solved.

The current document finds that the numerical solution method to the stochastic life-cycle problem can affect its structural estimation. Most recent empirical studies on life-cycle models use the Simulated Method of Moments (SMM) first developed by McFadden (1989) and Pakes and Pollard (1989). Gourinchas and Parker (2002) (henceforth GP) are the first to estimate the structural parameters of a dynamic stochastic model of life-cycle consumption with exogenous and stochastic labor income processes using SMM.[1] Due to the non-linearity of the consumption problem, the traditional root-finding solution to the Euler equations is computationally burdensome; thus, in order to reduce computation time, recent empirical works have performed structural estimations using the endogeneous grid-points solution method proposed in Carroll (2006).[2] The Monte Carlo results from this study suggest that although this alternative solution method is effective when solving the life-cycle model, one must be cautious when adopting it when numerically minimizing the SMM estimators' objective function. I find that the mode of the SMM estimates for the coefficient of risk aversion is approximately zero when its true value is small. The "zero trap" refers to this estimation result, i.e. the estimated parameter is zero even though the true parameter value is positive but close to zero. This result is related to other potential problems with SMM. Carroll and Kimball (2006) warn that the degree of income uncertainty faced by households affect the estimates of relative risk aversion in Gourinchas and Parker (2002). Similarly, Michaelides and Ng (2000) and Low and Meghir (2017) point out that the choice of moment conditions for the SMM estimation matters for the identification of the structural parameters when there are non-linearities and serial dependence on the data.

[1]Following GP, SMM on life-cycle models has also been performed by Cagetti (2003), French (2005), Laibson, Repetto, and Tobacman (2007), De Nardi, French, and Jones (2010), Low, Meghir, and Pistaferri (2010), French and Jones (2011), Bucciol (2012), and Fella, Frache, and Koeniger (2016), among many others.

[2]The endogenous grid-points solution method has been applied in structural estimations by Abe, Inakura, and Yamada (2007), Jørgensen (2013), Jørgensen (2014), Jørgensen (2017), Bucciol (2012), Rendon and Quella-Isla (2015), Yamana (2016), Yao, Fagereng, and Natvik (2015), Fella, Frache, and Koeniger (2016), Liu (2017), and Crawford and O'Dea (2020), among others.

The remainder of this document is structured as follows. Section 2 presents the life-cycle model in Gourinchas and Parker (2002). Section 3 describes the Simulated Method of Moments Estimation. Section 4 reports the Monte Carlo experiment results and characterizes the "zero trap". Section 5 concludes the analysis.

4.2 Life-Cycle Model

Gourinchas and Parker (2002) consider the following discrete-time, life-cycle model of household consumption. Individuals live until age N and retire at age $T < N$. In this standard model, both T and N are assumed to be exogenous and fixed. Preferences are represented by the standard additively separable expected utility form:

$$E\left[\sum_{t=1}^{N}\beta^t u(C_t, Z_t) + \beta^{N+1}V_{N+1}(W_{N+1})\right] \tag{4.1}$$

where β is the time-discount factor, C_t is total consumption at age t, W_t is total financial wealth, Z_t is a vector of deterministic household characteristics, and V_{N+1} is the value to the consumer of the remaining assets after age N, allowing for any bequest motive. Furthermore, the Bernoulli function is assumed to take the following form:

$$u(C, Z) = v(Z)\frac{C^{1-\rho}}{1-\rho} \tag{4.2}$$

where $\rho > 0$ and $\frac{1}{\rho}$ is the inter-temporal elasticity of substitution. At each age $t \in [1, T]$, the individual receives a stochastic income Y_t and maximizes (4.1) subject to:

$$W_{t+1} = R(W_t + Y_t - C_t) \tag{4.3}$$
$$W_{N+1} \geq 0 \tag{4.4}$$

given an initial wealth level W_1 and where R is the constant, after-tax, gross real interest rate of the only asset available in the economy. Following Zeldes (1989), the labor income process is given by:

$$Y_t = P_t U_t \tag{4.5}$$
$$P_t = G_t P_{t-1} N_t \tag{4.6}$$

where labor income Y_t is divided into a permanent component P_t and a transitory component U_t.[3] The transitory shocks U_t are assumed to be independently and identically distributed; moreover, there is a non-negative probability of a zero-income event, i.e. $U_t = 0$ with probability $p \in [0, 1)$. U_t is otherwise log-normally distributed, $\ln U_t \sim N(0, \sigma_U^2)$. The permanent component of income P_t follows a random walk with drift G_t and permanent shock N_t, which is also independently and identically log-normally distributed, $\ln N_t \sim N(0, \sigma_N^2)$.[4]

GP make four additional assumptions in order to estimate this model with household data. First, in order to reduce the number of state variables in the model, they assume that the age variations in $v(Z_t)$ are deterministic, common across households of the same age, and that they come from changes in family size. Second, stating that most of the retirement wealth in U.S. households is accumulated in illiquid assets (which are only available after retirement), GP assume that illiquid wealth accumulates exogenously, cannot be borrowed against, and that illiquid wealth in the first year of retirement is proportional to the last permanent component of income, i.e. $H_{T+1} \equiv hP_{T+1} = hP_T$. These assumptions eliminate both illiquid assets as a state variable and contributions to illiquid accounts as a control variable in the dynamic stochastic program. Third, invoking Bellman's optimality principle, the inter-temporal consumption problem is truncated at the age of retirement; thus, there is no need to define the sources of risks during retirement ages and the functional form

[3] Labor income is defined as disposable income, net of Social Security taxes and contributions to illiquid accounts.

[4] It is worth noting that under this income process formulation, consumers will never choose to borrow against future labor income. As Carroll and Kimball (2006) show, precautionary saving motive can induce self-imposed liquidity constraints. In particular, they provide an example in which the behavior of a consumer facing a zero-income event is virtually indistinguishable from the behavior of a perfect foresight but liquidity-constrained consumer as the probability of zero-income event approaches zero.

to capture any bequest motive. Lastly, due to the truncation assumption, GP adopt the following retirement value function V_{T+1} to condense the consumer's problem at retirement ages:

$$V_{T+1}(X_{T+1}, H_{T+1}, Z_{T+1}) = kv(Z_{T+1})(X_{T+1} + H_{T+1})^{1-\rho} \tag{4.7}$$

for some constant k and where X_t is cash-on-hand in period t, defined as total liquid financial wealth: $X_t = W_t + Y_t$. The functional form (4.7) is chosen by GP to maintain the tractability of the problem and for being flexible enough to allow robustness checks. In sum, the consumer's problem at age τ can be expressed as:

$$V_\tau(X_\tau, P_\tau, Z_\tau) = \max_{C_\tau,\ldots,C_T} E_\tau \left[\sum_{t=\tau}^{T} \beta^{t-\tau} v(Z_t) \frac{C_t^{1-\rho}}{1-\rho} + \beta^{T+1-\tau} kv(Z_{T+1})(X_{T+1} + hP_{T+1})^{1-\rho} \right] \tag{4.8}$$

given the labor income process defined in (4.5) and subject to:

$$X_{t+1} = R(X_t - C_t) + Y_{t+1} \tag{4.9}$$

$$X_{T+1} \geq 0. \tag{4.10}$$

where the last inequality reflects the borrowing constraint on liquid assets at retirement age, imposed by the assumption that illiquid wealth cannot be borrowed against.

Since an analytical closed-form solution for the above problem is not available, it can be solved numerically by first normalizing all variables by the permanent component of income. GP note that the particular functional form for the retirement function makes the household's consumption problem homogeneous of degree $(1-\rho)$ in P_t. Thus, by denoting lowercase letters as normalized variables, e.g. $x_t \equiv \frac{X_t}{P_t}$, the following Euler equation holds for ages $t < T$:

$$u'(c_t(x_t)) = \beta R E_t \left[\frac{v(Z_{t+1})}{v(Z_t)} u'(c_{t+1}(x_{t+1}) G_{t+1} N_{t+1}) \right] \tag{4.11}$$

where $c_t(x_t)$ is the optimal consumption function. The Euler equation in the last working period is

$$u'(c_T(x_T)) = \max \left\{ u'(x_T), \beta R \left[\frac{v(Z_{T+1})}{v(Z_T)} u'(c_{T+1}(x_{T+1})) \right] \right\} \tag{4.12}$$

since the assumption that illiquid wealth cannot be borrowed against imposes a liquidity constraint on the total financial wealth available at the age of retirement. Furthermore, under (4.7), the optimal consumption at retirement is linear in total wealth. Hence, the normalized consumption in $T+1$ is expressed as:

$$\begin{aligned} \frac{C_{T+1}}{P_{T+1}} &= \gamma_1 \left(\frac{X_{T+1} + H_{T+1}}{P_{T+1}} \right) \\ c_{T+1} &= \gamma_1 (x_{T+1} + h) \\ &= \gamma_0 + \gamma_1 x_{T+1} \end{aligned} \tag{4.13}$$

where $\gamma_0 \equiv \gamma_1 h$ and γ_1 is the marginal propensity to consume out of liquid wealth. Thus, in order to find the set of optimal consumption rules for each age t, the problem can be solved recursively by first finding $c_T(x_T)$ in (4.12) by using (4.13). The optimal solutions to (4.12) and (4.11) then generate the consumption functions $c_{T-1}(x_{T-1}), ..., c_1(x_1)$.[5]

4.3 Simulated Method of Moments Estimation (SMM)

Based on the life-cycle model in Gourinchas and Parker (2002), the consumption for individual i at age t depends on the parameters of the problem ($\psi \in \Psi \subset R^s$), the realization of the permanent component of income (P_{it}) and the level of cash on hand (X_{it}). Thus, based on the model, the data-generating process for each age t can be assumed to be:

$$\ln C_{it} = \ln(C_t(X_{it}, P_{it}; \psi)) + \epsilon_{it}$$

[5] For more details, see Appendix A.

where $\ln C_{it}$ is the observed log-consumption of individual i of age t and ϵ_{it} is an idiosyncratic shock. Due to the lack of a good quality panel data of consumption, assets, and income for individual households, GP propose to estimate the model based on the following condition for each age t:

$$E[\ln C_{it} - \ln C_t(\psi_0)] = 0$$

where $\ln C_t(\psi)$ is the unconditional expectation of log-consumption at each age t and ψ_0 is the true parameter vector. Due to the difficulty of estimating all the parameters in one step, GP partition the parameter vector into first-stage ($\chi \in R^r$) and second-stage ($\theta \in \Theta \subset R^s$ where Θ is a compact set) parameters; the estimation procedure proceeds by first estimating χ using additional data and moments, and then estimating θ using the Simulated Method of Moments.

The first-stage parameters χ consist of the variances of the permanent and transitory shocks (σ_U, σ_N), the probability of unemployment (p), the gross real after tax interest rate (R), the initial distribution of liquid assets at age 26, and the family-composition and income profiles. Although each of these parameters are estimated separately using different data, GP interpret them as GMM estimators. Thus, the parameters χ are estimated according to the moment condition $E[\mu(\chi)]$, where $\mu \in R^r$. The first-stage sample moments can then be defined as $m(\chi) = \frac{1}{J}\sum_{j=1}^{J} \mu_j(\chi)$, where J is the number of observations for the first-stage.

Because $\ln C_t(\psi)$ does not have an analytic expression and depends on the parameters, it is simulated by solving the model numerically for L households and computing the mean of the simulated consumption profiles (for each age t). Thus, the SMM estimator solves:

$$\min_{\theta} g(\theta; \hat{\chi})' W g(\theta; \hat{\chi}) \tag{4.14}$$

where W is a $T \times T$ weighting matrix and $g(\theta; \hat{\chi}) \in R^T$ is a vector with t^{th} element:

$$g_t(\theta; \hat{\chi}) = \ln \bar{C}_t - \ln \hat{C}_t(\theta; \hat{\chi}) \tag{4.15}$$

where $\ln \bar{C}_t$ is the average consumption for age t observed in the empirical data and $\ln \hat{C}_t(\theta; \hat{\chi})$ is the simulated counterpart of $\ln C_t(\theta; \hat{\chi})$. The SMM estimator then chooses θ that matches the means of the empirical and simulated distributions for each age t.[6]

4.4 Monte Carlo Experiment

To evaluate the SMM estimator, the life-cycle consumption and saving model described above is used as the structural model to simulate data. The dimension of the second-stage parameters θ is two: $\theta = \{\beta, \rho\}$. The retirement consumption rule's parameters are fixed exogeneously: $\gamma_0 = 0.594$, $\gamma_1 = 0.077$. The Monte Carlo experiment consists of 500 simulations ($m = 1, ..., 500$). For the m^{th} Monte Carlo:

1. Set the true parameters θ_0 equal to base-line parameter values estimated from the SMM structural estimation in Gourinchas and Parker (2002): $\beta_0 = 0.96$ and $\rho_0 = 0.514$.
2. Solve the consumption problem numerically using Equations (4.11) and (4.12), based on the parameter values listed in Table 4.1.
3. Generate sequence of income processes for $1,000$ households; for each household l, income shocks $U_{l,t}$ and $N_{l,t}$ are generated for $t = 1, ..., T$.
4. Calculate the consumption profile for each household l facing these income shocks by using the optimal consumption functions from Step 2.
5. Calculate the average of the logarithm of the simulated consumption profiles for each age t across all 1000 simulated households.
6. Generate a sequence of income processes for $20,000$ households over T years.
7. Begin at initial guess parameter values: $\theta^0 = \{0.959, 0.513\}$.

[6]See Appendix B for MATLAB codes.

8. For a given θ^i, generate the log consumption profiles for all 20,000 households and calculate its mean.
9. Calculate moment condition (4.15) and minimize objective function (4.14).
10. Update m and go to Step 3.

In order to solve the consumption model in Section 4.2, the continuous state variable x must be discretized. The standard solution method calculates their values at a finite grid of possible values of normalized cash-on-hand: $\{x^j\}_{j=1}^J \subset [0, x^{\max}]$. With known c_{t+1}, the standard approach solves a numerical root-finding routine to find, for each value of cash-on-hand on the grid x^j, the associated consumption c^j that satisfies the Euler equation (4.11). The points $\{x^j, c^j\}$ are then used to generate a interpolated approximation to c_t. Given the interpolated function c_t, the solution for the previous periods is then found by backward recursion. GP suggest a grid of 100 points between $[0, 40]$ for normalized cash-on-hand, with 50 points between 0 and 2.

However, as the standard solution method has proven to be computationally burdensome, the Monte Carlo experiments are done following the endogeneous grid-points solution method described by Carroll (2006). He proposes an alternative approach that does not require numerical root-finding and saves a substantial amount of computational time. Instead of using a grid of values for cash-on-hand, this alternative approach uses an exogenous grid of values for end-of-period assets (i.e. $a_t = x_t - c_t$): $\{a^j\}_{j=1}^J \subset [0, a^{\max}]$. Noting that $x_{t+1} = \frac{R}{G_{t+1}N_{t+1}} a_t + U_{t+1}$, the Euler equation (4.11) can be expressed as

$$u'(c_t(x_t)) = \beta R E_t \left[\frac{v(Z_{t+1})}{v(Z_t)} u' \left(c_{t+1} \left(\frac{R}{G_{t+1}N_{t+1}} a_t + U_{t+1} \right) G_{t+1} N_{t+1} \right) \right]. \tag{4.16}$$

With c_{t+1} in hand, the alternative approach calculates, for each value of end-of-period asset on the grid a^j, the associated consumption c^j based on equation (4.16). Further note that the dynamic budget constraint implies that $x^j = a^j + c^j$. Thus, the grid-points for the state variable x are endogenously generated from the exogenous grid of end-of-period assets a. The points $\{x^j, c^j\}$ are then used as before to construct an approximation to the consumption function c_t. Given the interpolated function c_t, the solution for the previous periods is found by backward recursion.[7] Furthermore, to evaluate the expectation in (4.16), a two dimensional Gauss-Hermite quadrature of order 12 is performed as GP.

In order to minimize the SMM objective function (4.14), the solver `fminsearch` in MATLAB is used. It is a non-linear programming solver that uses the Nelder-Mead simplex search method proposed by Lagarias, Reeds, M. H. Wright, and P. E. Wright (1998). Based on the life-cycle model in section 2, the parameter space is defined as $\Theta = \{(\beta, \rho) \in \mathbb{R}_+^2 : 0 < \beta < 1, \rho > 0\}$. The algorithm is stopped when both the change in the value of the objective function and the norm of the parameter vector during a step are less than 1.0×10^{-6}. The calculations are performed on an 3.2 and 3.6 GHz Intel Core $i7$ processor.

4.4.1 Results

Table 4.2 reports the results of the Monte Carlo experiment for the SMM estimator using moment conditions (4.15). The SMM estimator has a tendency to underestimate ρ. Its mean across 500 experiments is 0.4606 while its median is 0.4545 with a standard deviation of 0.4250 and a mean squared error of 0.1831. On the the hand, the SMM estimate of β is more precise with a mean of 0.9592 and a median of 0.9609, with standard deviation of 0.0077 and a mean squared error of 0.0001 across the 500 experiments.

Figure 4.1 sums up the properties of the SMM estimators for the life-cycle consumption model. Figure 4.1a displays the density plot of ρ. As can be seen, the mode of the SMM estimates for ρ is near 0, far from the true value of 0.514. This explains the smaller mean and median biases reported in Table 4.2. In contrast, the mode of the SMM estimates for β is closer to 0.9667 as displayed in Figure 4.1b. In fact, the mode values of ρ and β are related as can be seen in Figure 4.2: when $\hat{\rho} \approx 0$, then $\hat{\beta} = 0.9667 = 1/R$. Moreover, when β and ρ are estimated together, the estimates are inversely related.

Estimating the preference parameters separately improves the efficiency of the SMM estimates significantly. As displayed in Figure 4.3, the estimated values are centered around the true value of the parameters for both β an ρ, respectively. Across 500 experiments, the estimates for β have a mean of 0.9599 and a median of

[7] See Appendix B for MATLAB codes.

0.9600 with a standard deviation of 0.0001 when fixing $\rho = 0.514$. On the other hand, the estimates for ρ have a mean value of 0.5131 and a median value of 0.5123 with a standard deviation of 0.0477 when fixing β to its true parameter value of 0.96. These results are aligned with the Monte Carlo results in Jørgensen (2013); he evaluates the endogenous grid-points method when estimating an infinite horizon version of the consumption model through (partial) Maximum Likelihood. Jorgensen finds that the Monte Carlo standard deviation of the $\hat{\rho}$ estimate is 0.006 when $\beta = 0.95$.

Next, the Monte Carlo experiments are repeated assuming that the true parameter values are $\theta_0 = (1.5, 0.96)$ and $\theta_0 = (4, 0.96)$. When the preference parameters are estimated jointly, the SMM estimates of ρ have a mean of 1.4540 and 3.9772 respectively, as reported in Table 4.2. Similarly, the estimated values of β have a mean and a median approximately equal to its true value of 0.96 for both sets of Monte Carlo experiments. As presented in Figures 4.4 and 4.5, the SMM estimates for ρ and β are centered around the true value of the parameters for both β an ρ. Moreover, Figure 4.6 shows that the inverse relationship between the preference parameter is preserved regardless of the true value of ρ_0.

4.4.2 The Zero Trap

Recall when the true value of the coefficient of risk aversion is assumed to be 0.514, the Monte Carlo experiments find that the mode of the SMM estimates is ($\hat{\beta} = 0.9667, \hat{\rho} = 0.0000$), far from the true value of $\rho_0 = 0.514$. The "zero trap" refers to this estimation anomaly when ρ_0 is small, and it emerges from the adoption of the endogenous grid-points solution method when solving the life-cycle consumption problem.[8]

In order to implement SMM, the life-cycle consumption model must be first solved numerically. For instance, in the last working period, the individual solves the following optimality condition:

$$c_T(x_T)^{-\rho} = \max\left\{x_T^{-\rho}, \beta R\left[\frac{v(Z_{T+1})}{v(Z_T)} c_{T+1}(x_{T+1})^{-\rho}\right]\right\}$$

This is a root-finding problem as $x_{T+1} = R(x_T - c_T)$. Thus,

$$c_T = \min\left\{x_T, \frac{1}{(R\beta)^{\frac{1}{\rho}}\left(\frac{v(Z_{T+1})}{v(Z_T)}\right)^{\frac{1}{\rho}} + \gamma_1 R}(\gamma_0 + \gamma_1 R x_T)\right\} \tag{4.17}$$

since $c_{T+1} = \gamma_0 + \gamma_1 x_{T+1}$. Alternatively, the endogenous grid-points solution method solves

$$\begin{aligned} c_T(x_T)^{-\rho} &= \beta R\left[\frac{v(Z_{T+1})}{v(Z_T)} c_{T+1}(Ra_T)^{-\rho}\right] \\ c_T(x_T) &= \frac{1}{(R\beta)^{\frac{1}{\rho}}\left(\frac{v(Z_{T+1})}{v(Z_T)}\right)^{\frac{1}{\rho}}}(\gamma_0 + \gamma_1 R a_T). \end{aligned} \tag{4.18}$$

Note that as ρ approaches 0, the consumption function (4.18) goes to positive infinity since its denominator approaches 0 when $R\beta < 1$ (the limit of the consumption function is finite when $R\beta = 1$ and is equal to zero when $R\beta > 1$).[9] Similarly, for any $t < T$,

$$c_t(x_t) = \frac{1}{(R\beta)^{\frac{1}{\rho}}\left(\frac{v(Z_{t+1})}{v(Z_t)}\right)^{\frac{1}{\rho}}}\left(E_t\left[c_{T+1}\left(\frac{R}{G_{t+1}N_{t+1}}a_t + U_{t+1}\right)^{-\rho}(G_{t+1}N_{t+1})^{-\rho}\right]^{-\frac{1}{\rho}}\right). \tag{4.19}$$

As previously stated, the Monte Carlo experiments use the endogenous grid-points solution to solve the optimality conditions. Therefore, since the SMM estimation matches finite positive consumption means and the

[8]As the Monte Carlo results show previously, this problem only arises when jointly estimating β and ρ.

[9]When implementing the endogenous grid-points solution method, liquidity constraints are handled by adding 0 as the lowest point in the grid over a_t since $a_t = 0$ implies $c_t = x_t$; thus by adding the interpolation point $(x_t, c_t) = (0, 0)$ ensures the correct implementation of the liquidity constraints.

parameter space is $\Theta = \{(\beta, \rho) \in \mathbb{R}^2_+ : 0 < \beta < 1, \rho > 0\}$, the optimization algorithm can result in estimates $(\hat{\beta} = \frac{1}{R}, \hat{\rho} \approx 0)$ when solving the consumption problem through backward induction. Since $\hat{\rho} \approx 0$ implies $u(c) = c$, the parameter space must be compactified. One way to avoid the "zero trap" when ρ is small is restricting the parameter space, such that $\Theta_\varepsilon = \{(\beta, \rho) \in \mathbb{R}^2_+ : 0 < \beta < 1, \rho > \varepsilon\}$ for any fixed $\varepsilon > 0$. This alternative compactification of the parameter space would force the SMM parameters to stay out of the "zero trap" but is not a satisfying solution. Since the life-cycle model's utility function restricts ρ to be positive, choosing any positive value for ε is rather arbitrary.

Nonetheless, when the SMM estimation is performed using the traditional root-finding solution method, the estimations do not fall into the "zero trap" since the limit of the consumption functions as ρ approaches 0 is finite as seen in equation (4.17). Although the first term in the denominator approaches 0 as $\rho \to 0$, the second term is constant; thus, as long as $\frac{\gamma_0}{\gamma_1 R} > 0$, the solution to the root-finding problem is $c_T(x_T) = x_T$.

For instance, Table 4.3 displays the SMM estimation results from two separate Monte Carlo exercises. The true parameter values are assumed to be $\rho_0 = 0.514$ and $\beta_0 = 0.96$, and the life-cycle model is solved using both the endogeneous grid-points and root-finding solution methods. As can be seen, the SMM estimates using the endogenous grid-points solution are $\hat{\theta} = (0.0000, 0.9667)$ for both Monte Carlo exercises. The minimum of the SMM objective function is reached at those parameter values as seen in Figure 4.7. However, the SMM estimations using the root-finding solution method do not fall into the "zero trap," although the $\hat{\rho}$ estimates are far from the true parameter values. Lastly, Figure 4.8 shows the SMM objective function when the true parameter values are assumed to be $\rho_0 = 4$ and $\beta_0 = 0.96$ using the endogeneous grid-points solution method. As can be seen, the "zero trap" is not a problem when the true value of ρ is positive and far from zero.

4.5 Conclusion

The Monte Carlo results from this study suggests that one must be cautious when adopting the endogenous grid-points solution method when numerically calculating SMM estimates in a life-cycle model. Even though, this solution approach is computationally more efficient than the root-finding method to solve the consumption model, the optimization algorithm of the SMM estimations can fall into the "zero trap" when the true parameter values are near zero. It would be interesting to check whether this estimation anomaly extends to other simulation estimators and to find a general, non ad-hoc solution to compactify the parameter space.

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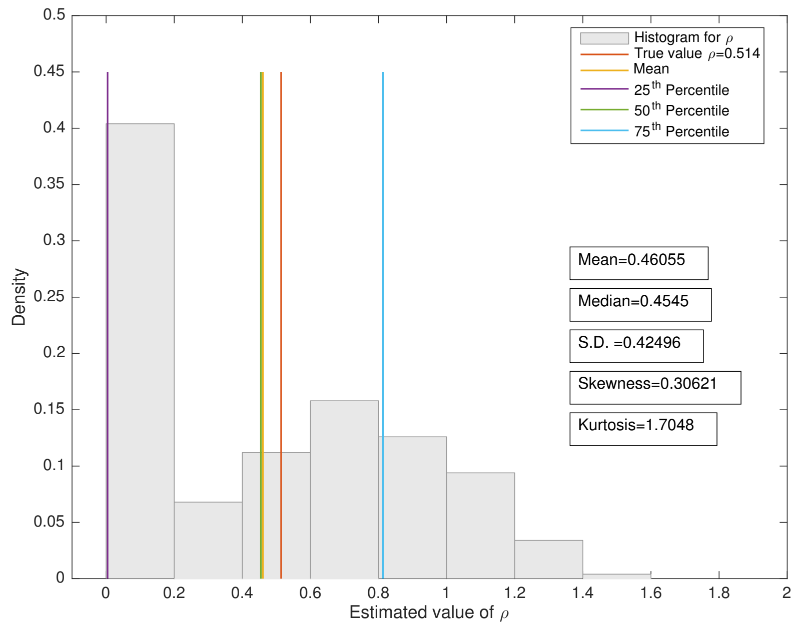
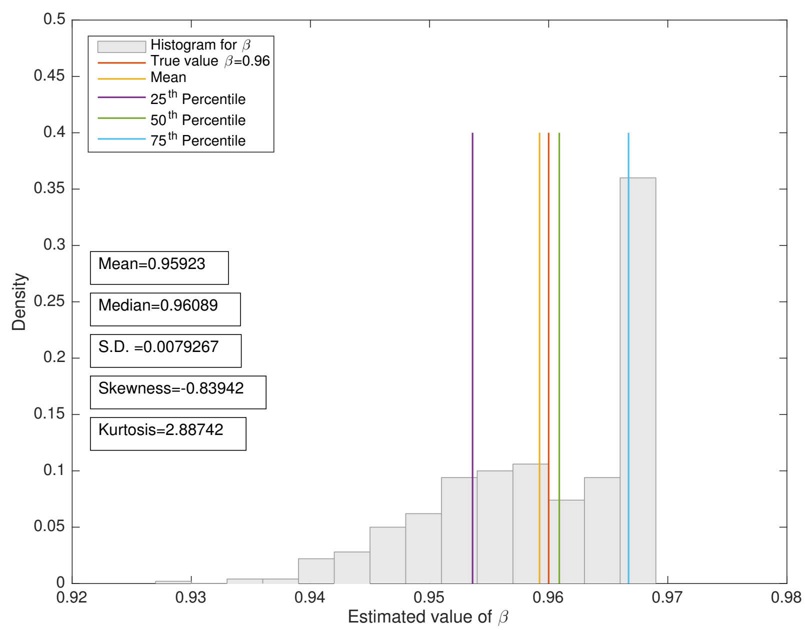


(a) Density for $\hat{\rho}$ from 500 Monte Carlo Experiments



(b) Density for $\hat{\beta}$ from 500 Monte Carlo Experiments

Figure 4.1: SMM Estimates Density

SMM estimates from 500 Monte Carlo experiments for true parameters $\theta_0 = \{0.96, 0.514\}$

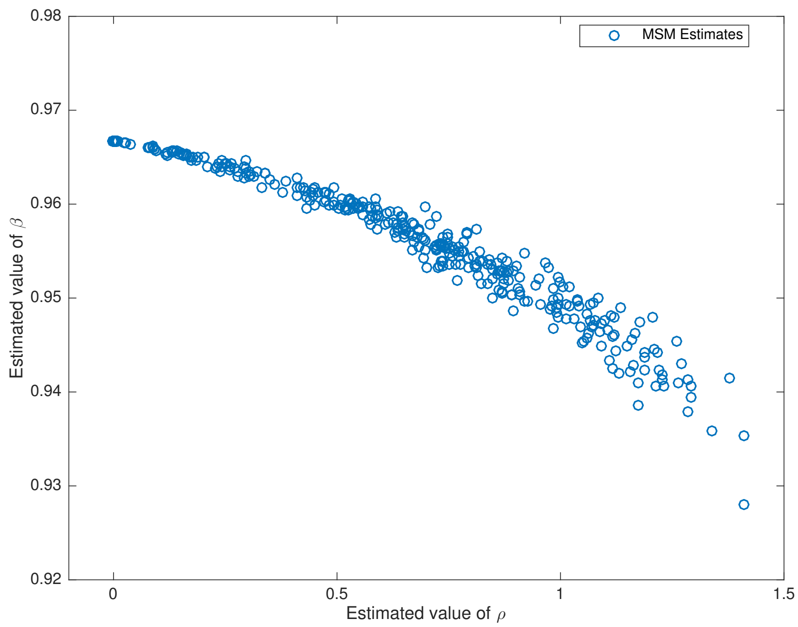
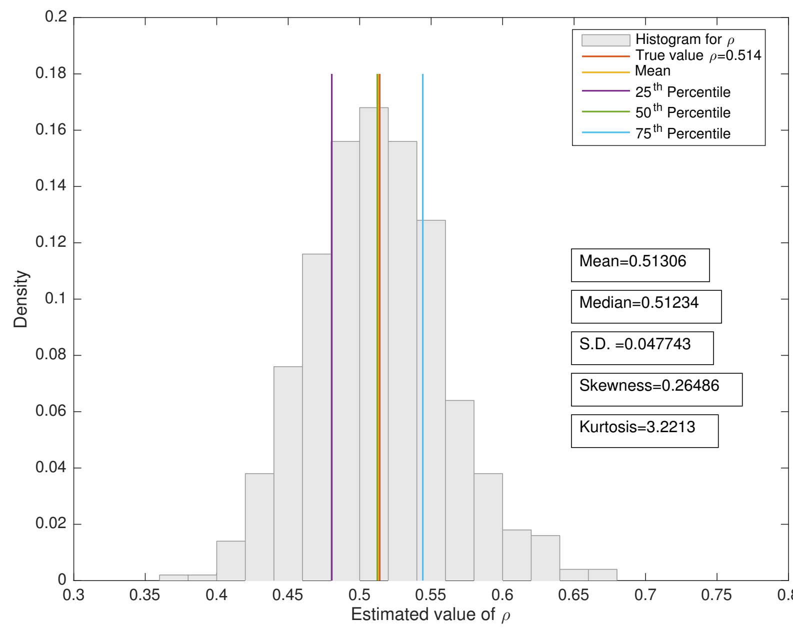
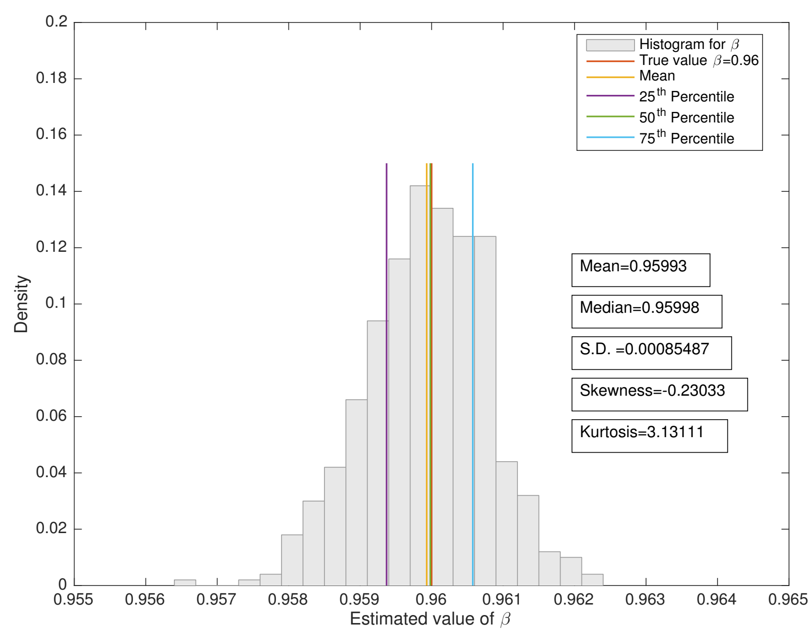


Figure 4.2: Relationship between SMM Estimates

Relationship between $\hat{\beta}$ and $\hat{\rho}$ from 500 Monte Carlo experiments for true parameters $\theta_0 = \{0.96, 0.514\}$

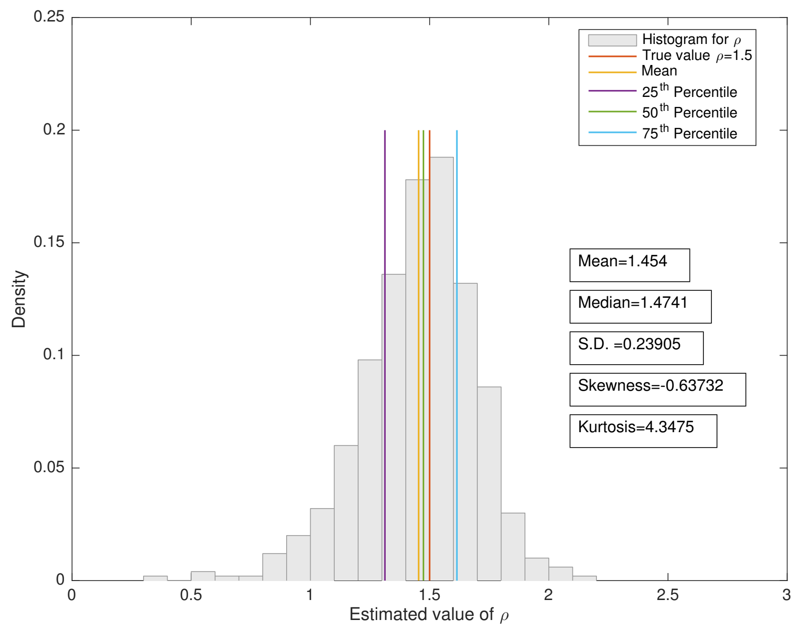


(a) Density for $\hat{\rho}$ fixing $\beta = \beta_0$

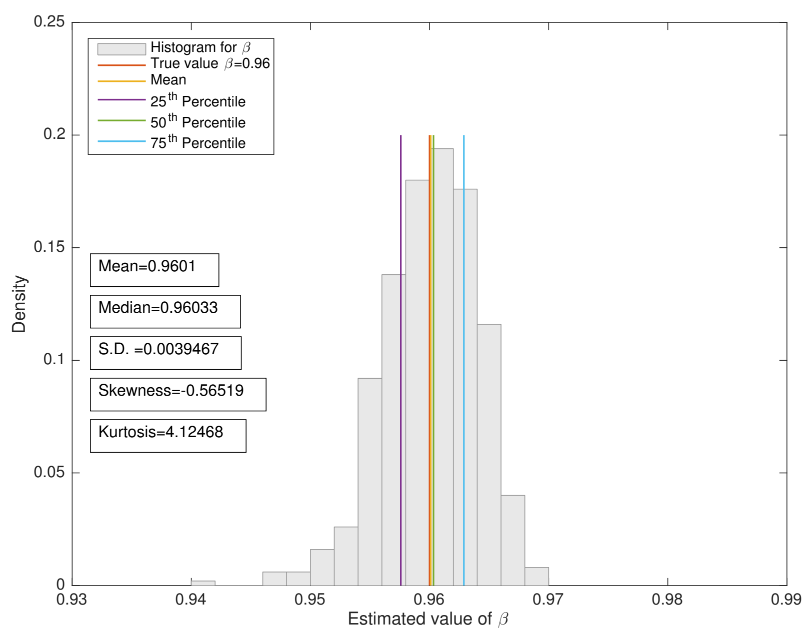


(b) Density for $\hat{\beta}$ fixing $\rho = \rho_0$

Figure 4.3: SMM Estimates Density for $\theta_0 = \{0.96, 0.514\}$ (SMM estimates from 500 Monte Carlo experiments)



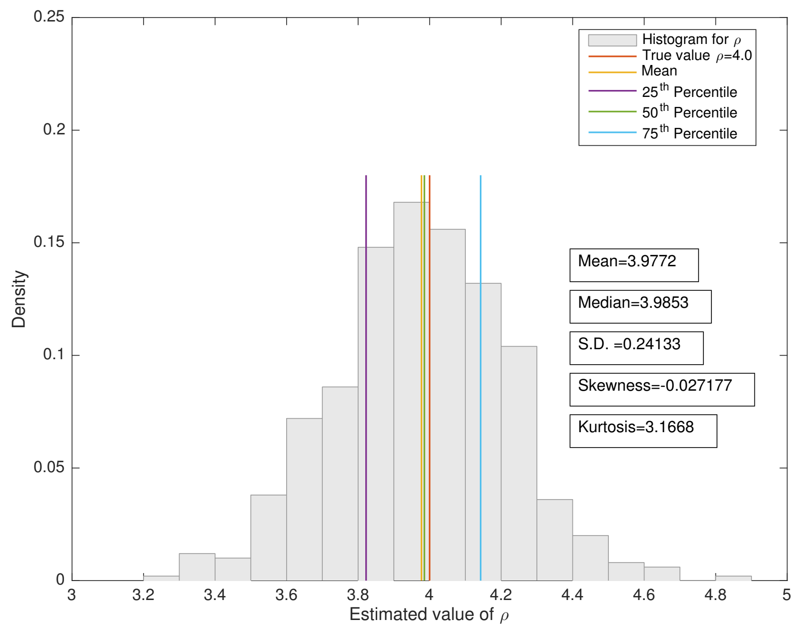
(a) Density for estimates $\hat{\rho}$



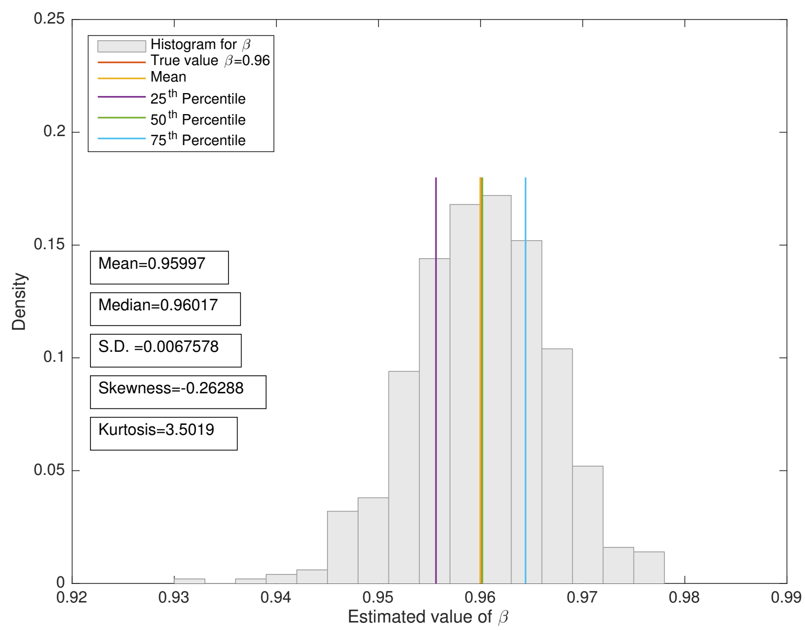
(b) Density for estimates $\hat{\beta}$

Figure 4.4: SMM Estimates for $\theta_0 = \{0.96, 1.5\}$

(SMM Estimates from 500 Monte Carlo experiment)



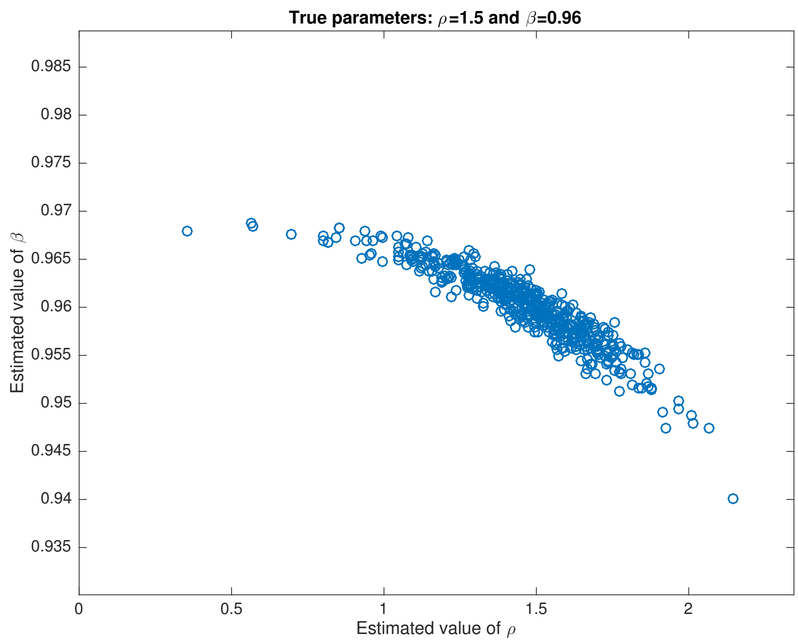
(a) Density for estimates $\hat{\rho}$



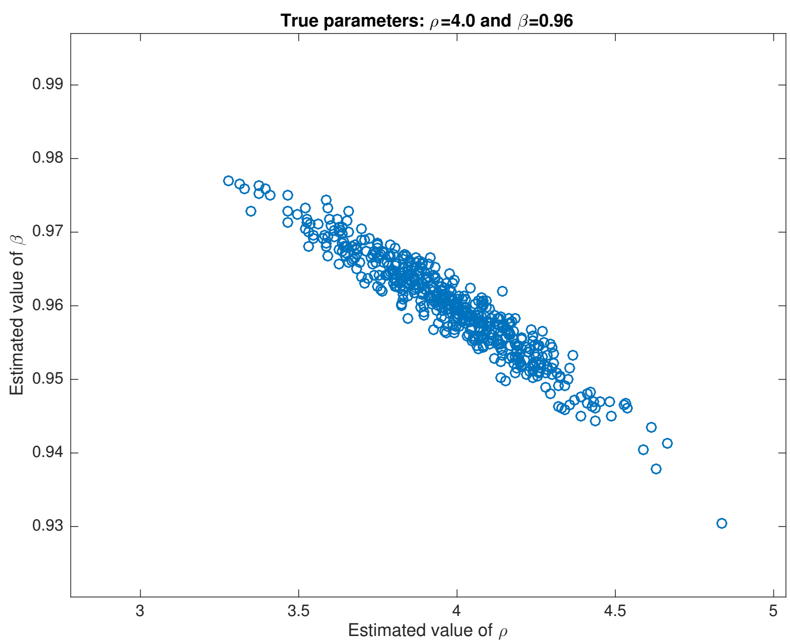
(b) Density for estimates $\hat{\beta}$

Figure 4.5: SMM Estimates for $\theta_0 = \{0.96, 4\}$

(SMM Estimates from 500 Monte Carlo experiment)



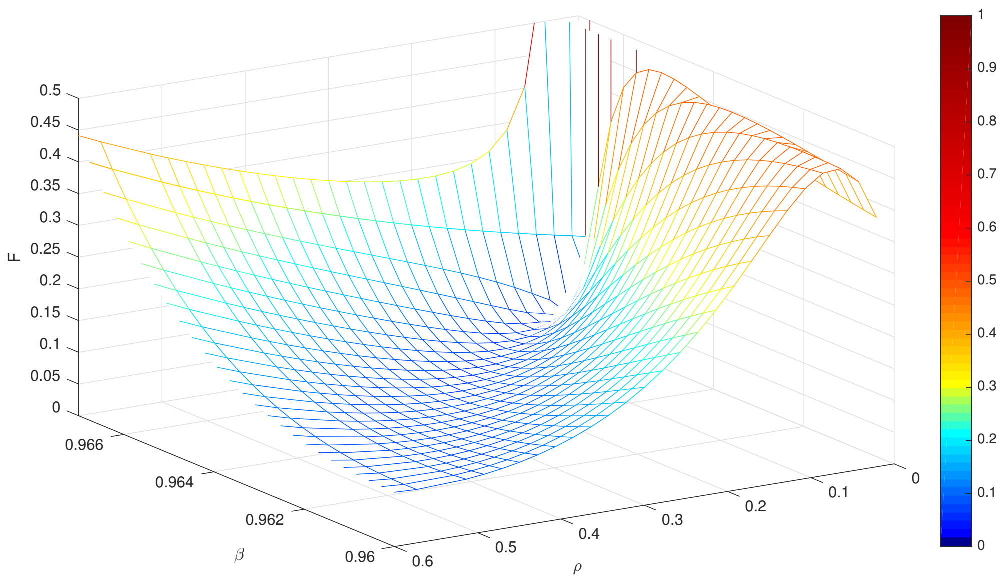
(a) Relationship between $\hat{\beta}$ and $\hat{\rho}$ for true parameters $\theta_0 = \{0.96, 1.5\}$



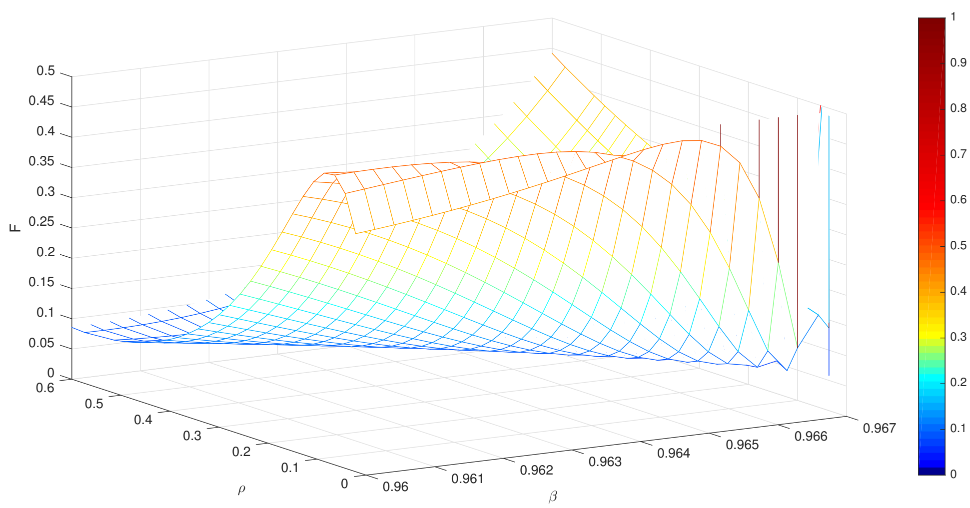
(b) Relationship between $\hat{\beta}$ and $\hat{\rho}$ for true parameters $\theta_0 = \{0.96, 4.0\}$

Figure 4.6: Relationship between SMM Estimates

(SMM Estimates from 500 Monte Carlo experiment)



(a) View 1



(b) View 2

Figure 4.7: SMM Objective Function for $\theta_0 = \{0.96, 0.514\}$

(SMM objective function as a function of β and ρ for Monte Carlo exercise #1)

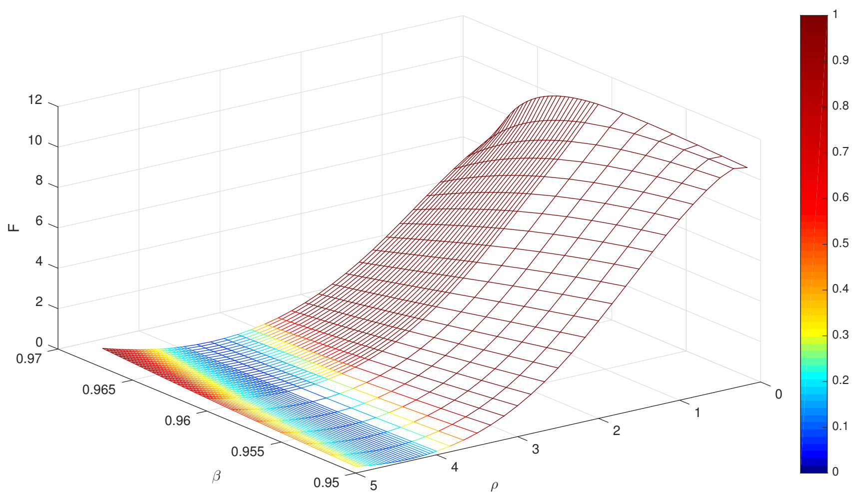


Figure 4.8: SMM Objective Function for $\theta_0 = \{0.96, 4\}$

(SMM objective function as a function of β and ρ)

Parameter	Value	Source
R	1.03440	Moody's AAA municipal bonds, Jan 1980 - March 1993
σ_U^2	0.0440	Carroll and Samwick (1997), PSID 1981 – 1987
σ_N^2	0.0212	Carroll and Samwick (1997), PSID 1981 – 1987
p	0.00302	Carroll, Hall, and Zeldes (1992), PSID 1976 – 1985
$\bar{w}_1$	-2.7944810	Gourinchas and Parker (2002), CEX 1980 – 1993
σ_{w_1}	1.7838679	
G_t:		Gourinchas and Parker (2002), CEX 1980 – 1993
age	.32643678179	
age^2	$-.0148947085$	
age^3	.00036342384	
age^4	$-4.411685e^{-6}$	
age^5	$2.056916e^{-8}$	
$constant$	6.801368713	
Z_t:		Gourinchas and Parker (2002), CEX 1980 – 1993
age	0.13964975	
age^2	-0.0047742190	
age^3	$8.5155210e^{-5}$	
age^4	$-7.9110880e^{-7}$	
age^5	$2.9789550e^{-9}$	

Table 4.1: First-stage Parameter Values from Gourinchas and Parker (2002)

$\theta_0 = (0.96, 0.514)$	Mean	Median	S.D.	MSE
Joint Estimation: $\hat{\theta} = (\hat{\beta}, \hat{\rho})$				
$\hat{\beta}$	0.9592	0.9609	0.0079	0.0001
$\hat{\rho}$	0.4606	0.4545	0.4250	0.1831
Individual Estimation: fixing $\rho = 0.514$				
$\hat{\beta}$	0.9599	0.9600	0.0001	0.0000
Individual Estimation: fixing $\beta = 0.96$				
$\hat{\rho}$	0.5131	0.5123	0.0477	0.0023
$\theta_0 = (0.96, 1.5)$	Mean	Median	S.D.	MSE
Joint Estimation: $\hat{\theta} = (\hat{\beta}, \hat{\rho})$				
$\hat{\beta}$	0.9601	0.9603	0.0039	0.0000
$\hat{\rho}$	1.4540	1.4741	0.2390	0.0591
Individual Estimation: fixing $\rho = 1.5$				
$\hat{\beta}$	0.9598	0.9599	0.0013	0.0000
Individual Estimation: fixing $\beta = 0.96$				
$\hat{\rho}$	1.4899	1.4891	0.0749	0.0057
$\theta_0 = (0.96, 4)$	Mean	Median	S.D.	MSE
Joint Estimation: $\hat{\theta} = (\hat{\beta}, \hat{\rho})$				
$\hat{\beta}$	0.9600	0.9602	0.0068	0.0000
$\hat{\rho}$	3.9772	3.9853	0.2413	0.0586
Individual Estimation: fixing $\rho = 4$				
$\hat{\beta}$	0.9598	0.9598	0.0023	0.0000
Individual Estimation: fixing $\beta = 0.96$				
$\hat{\rho}$	3.9905	3.9874	0.0822	0.0068

Table 4.2: Results from 500 Monte Carlo Experiments

$\theta_0 = (0.96, 0.514)$	Endogeneous grid-points	Root-Finding
Monte Carlo Exercise #1:		
$\hat{\beta}$	0.9667	0.9664
$\hat{\rho}$	4.547×10^{-7}	0.0496
Monte Carlo Exercise #2:		
$\hat{\beta}$	0.9667	0.9661
$\hat{\rho}$	2.220×10^{-5}	0.0531

Table 4.3: SMM estimations from two Monte Carlo experiments when $\theta_0 = \{0.96, 0.514\}$

Appendices

Appendix A

Numerical Solution to the Consumer Problem

This appendix outlines the MATLAB codes used in this dissertation in order to solve and simulate the life-cycle problem in Gourinchas and Parker (2002). The codes are divided into three main files: `SettingUp.m`, `BackwardInduction.m`, and `Simulation.m`. These m-files should be ran in that specific order.

A.1 The Consumer Problem

Since an analytical closed-form solution for the consumer problem is not available, it must be solved numerically. Recall the Euler equation for ages $t < T$:

$$u'(c_t(x_t)) = \beta R E_t \left[\frac{v(Z_{t+1})}{v(Z_t)} u'(c_{t+1}(x_{t+1}) G_{t+1} N_{t+1}) \right] \tag{A.1}$$

where $c_t(x_t)$ is the optimal consumption function. At age T, the Euler equation is replaced by:

$$u'(c_T(x_T)) = \max \left\{ u'(x_T), \beta R \left[\frac{v(Z_{T+1})}{v(Z_T)} u'(c_{T+1}(x_{T+1})) \right] \right\} \tag{A.2}$$

where $c_{T+1} = \gamma_0 + \gamma_1 x_{T+1}$. In order to find the set of optimal consumption rules for each age t, the problem is solved recursively by first finding $c_T(x_T)$ in (A.2). The optimal solutions to (A.2) and (A.1) then generate the consumption functions $c_{T-1}(x_{T-1}), ..., c_1(x_1)$.

A.2 Setting-up Parameters and other Inputs

The `settingup.m` file starts by declaring the global variables needed to solve the consumer problem (explained in the next section). Next, the code sets up the model's parameters and generates a grid for normalized cash-on-hand. Then, the income growth rates are constructed as well as the ratio of the shift in marginal utility caused by changes in family size. Finally, the code performs a two-dimensional Gauss-Hermite quadrature of order 12 for both transitory and permanent income shocks. Please read the comments below when considering the `settingup.m` file:

1. Following GP's discretization method, a grid of 100 points between $[0, 40]$ is created for normalized cash-on-hand, with 50 points between 0 and 2. The finer grid for $x \in [0, 2]$ captures the curvature of the consumption rule at low values of cash-on-hand. The code uses the `linspace` function from MATLAB.
2. GP construct smooth age-profiles for the logarithm of income and for the shift in marginal utility caused by family size using fifth polynomials in age.

(a) The age-profile of the logarithm of income is constructed as:

$$\ln Y_t = b_1(age) + b_2(age)^2 + b_3(age)^3 + b_4(age)^4 + b_5(age)^5 + b_6$$

where the vector of coefficients $ypoly = [b_1, b_2, b_3, b_4, b_5, b_6]$ is estimated and provided by GP. The growth rate of income is calculated as

$$G_{t+1} = \frac{Y_{t+1}}{Y_t}$$

(b) In Appendix C of Gourinchas and Parker (2002), GP define the shift in marginal utility caused by family size as:

$$v(Z_t)^{1/p} = k \exp(Z_t)$$

Its smooth profile is constructed as:

$$v(Z_t)^{1/p} = a_1(age) + a_2(age)^2 + a_3(age)^3 + a_4(age)^4 + a_5(age)^5$$

where the vector of coefficients $fampoly = [a_1, a_2, a_3, a_4, a_5]$ is estimated and provided by GP. The ratio of shift in marginal utilities caused by changes in family size is then caculated as:

$$\left(\frac{v(Z_t)}{v(Z_{t-1})}\right)^{\frac{1}{\rho}} = \frac{\exp(Z_t)}{\exp(Z_{t-1})}$$

3. Since the labor income shocks N and U are both log-normally distributed, a two dimensional Gauss-Hermite quadrature of order 12 is performed following GP. The function `gauher.m` constructs the nodes and weights that will be used to approximate the expectation in (A.1) when solving the consumer problem in the next section.

A.3 Solving the Model

The `BackwardInduction.m` file solves the consumer model through backward induction. The solution algorithm involves two main steps:

1. The policy rule $c_T(x_T)$ is found by solving Equation (A.2) where $c_{T+1} = \gamma_0 + \gamma_1 x_{T+1}$. In order to find $c_T(x_T)$, the `fsolve` root-finding routine in MATLAB is used. It requires Equation (A.2) to be expressed as:

$$EET = u^{'-1}\left(\max\left\{u^{'}(x_T), \beta R\left[\frac{v(Z_{T+1})}{v(Z_T)}u^{'}(c_{T+1}(x_{T+1}))\right]\right\}\right) - c_T(x_T)$$

which is recorded in the function `EET`. This is the function to be solved by `fsolve`, given an initial guess (e.g. $\frac{x_T}{2}$).

2. Once $c_T(x_T)$ is found, Equation (A.1) can be solved sequentially to find $c_{T-1}(x_{T-1})$, $c_{T-2}(x_{T-2})$, ..., $c_1(x_1)$. First, in order to approximate the expectation in the Euler equation, Equation (A.1) needs to be slightly modified to use the Gauss-Hermite quadrature nodes and weights. It can be rewritten as:

$$u'(c_t(x_t)) = \beta R \frac{v(Z_{t+1})}{v(Z_t)}(pE_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1})|U_{t+1} = 0] + (1-p)E_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1})|U_{t+1} > 0])$$

where

$$E_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1}|U_{t+1} = 0] = \int u'\left(c_{t+1}\left((x_t - c_t)\frac{R}{G_{t+1}N}\right)G_{t+1}N\right)dF(N)$$

$$E_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1}|U_{t+1} > 0] = \int u'\left(c_{t+1}\left((x_t - c_t)\frac{R}{G_{t+1}N} + U\right)G_{t+1}N\right)dF(U)dF(N)$$

Since U and N are log-normally distributed:

$$E_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1}|U_{t+1}=0] = \int u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}N}\right)G_{t+1}N\right)dF(N)$$
$$= \int u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}N}\right)G_{t+1}N\right)f(N)dN$$
$$= \int u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}e^{\sigma_N\bar{n}}}\right)G_{t+1}e^{\sigma_N\bar{n}}\right)\frac{e^{-\frac{1}{2}\bar{n}^2}}{\sqrt{2\pi}}d\bar{n}$$
$$= \int \frac{1}{\sqrt{\pi}}u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}}e^{-\sqrt{2}\sigma_N n}\right)G_{t+1}e^{\sqrt{2}\sigma_N n}\right)e^{-n^2}dn \tag{A.3}$$

where the last equation uses the change of variables $n=\frac{\bar{n}}{\sqrt{2}}$, and

$$E_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1}|U_{t+1}=0] = \int u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}N}+U\right)G_{t+1}N\right)dF(N)$$
$$= \int u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}N}+U\right)G_{t+1}N\right)f(N)f(U)dNdU$$
$$= \int u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}e^{\sigma_n\bar{n}}}+e^{\sigma_U\bar{u}}\right)G_{t+1}e^{\sigma_n\bar{n}}\right)\frac{e^{-\frac{1}{2}\bar{n}^2}}{\sqrt{2\pi}}\frac{e^{-\frac{1}{2}\bar{u}^2}}{\sqrt{2\pi}}d\bar{n}d\bar{u}$$
$$= \int \frac{1}{\pi}u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}}e^{-\sqrt{2}\sigma_N n}+e^{\sqrt{2}\sigma_U u}\right)G_{t+1}e^{\sqrt{2}\sigma_N n}\right)e^{-n^2}e^{-u^2}dndu \tag{A.4}$$

where the last equation uses the change of variables $n=\frac{\bar{n}}{\sqrt{2}}$ and $u=\frac{\bar{u}}{\sqrt{2}}$.

Equations (A.3) and (A.4) can then be approximated as a weighted-sum using the Gauss-Hermite quadrature nodes (n_j, u_i) and weights (ω_j, ω_i):

$$E_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1}|U_{t+1}=0]$$
$$\approx \sum_{j=1}^{12}\frac{1}{\sqrt{\pi}}u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}}e^{-\sqrt{2}\sigma_N n_j}\right)G_{t+1}e^{\sqrt{2}\sigma_N n_j}\right)\omega_j \tag{A.5}$$

$$E_t[u'(c_{t+1}(x_{t+1})G_{t+1}N_{t+1}|U_{t+1}=0]$$
$$\approx \sum_{j=1}^{12}\sum_{i=1}^{12}\frac{1}{\pi}u'\left(c_{t+1}\left((x_t-c_t)\frac{R}{G_{t+1}}e^{-\sqrt{2}\sigma_N n_j}+e^{\sqrt{2}\sigma_U u_i}\right)G_{t+1}e^{\sqrt{2}\sigma_N n_j}\right)\omega_j\omega_i \tag{A.6}$$

Equations (A.5) and (A.6) are recorded in the function `EEworkingages`. This is the function to be solved by `fsolve`, given an initial guess (e.g. $\frac{x_t}{2}$).

A.4 Simulation

The `Simulation.m` file simulates the life-cycle model after the consumer problem is solved. The code first generates 20,000 initial financial wealth values from a lognormal distribution. Then, a sequence of 20,000 income processes is generated over 40 years. The initial permanent component of income is assumed to be 18690.96 following GP. Given the initial parameters, the simulation starts from $t = 1$ to $t = T$ using the budget constraint and the interpolation function `Cdash.m`. Lastly, non-normalized variables are constructed by multiplying the respective normalized variables by P_t.

A.5 MATLAB Codes

A.5.1 SettingUp.m File

```
% 1. Declare global variables needed to solve life-cycle model:
    global T R rho beta gamma0 gamma1 sN sU probUnEmp gridx ...
           ZList GList GListT ZListT nodes weights Cons Cashonhand t

% 2. Set up model parameters:
    T          = 40;             % retirement age
    R          = 1.0344;         % interest rate
    beta       = 0.960;          % time discount factor
    rho        = 0.514;          % coefficient of risk aversion
    gamma0     = 0.001;          % retirement rule: illiquid wealth
    gamma1     = 0.071;          % retirement rule: MPC out of wealth
    sN         = sqrt(0.0212);   % stand. dev. of permanent income shock
    sU         = sqrt(0.0440);   % stand. dev. of transitory income shock
    probUnEmp = 0.00302;         % probability of zero income

% 3. Create grid for normalized cash-on-hand:
    xmin = 0.000001;             % minimum value for cash-on-hand
    xmax = 40;                   % maximum value for cash-on-hand
    xint = 2;                    % Interim value for cash-on-hand
    xn   = 100;                  % total number of points in grid
    xn1  = xn/2;
    xn2  = xn-xn1+1;
    ngridx1 = linspace(xmin,xint,xn1)';
    ngridx2 = linspace(xint,xmax,xn2)';
    gridx   = [ngridx1; ngridx2(2:end)];

% 4. Set up income growth rate and changes in family size:

  % Age vector to use with polynomials:
    age    = 26:1:26+T;
    agep   = [age' (age.^2)' (age.^3)' (age.^4)' (age.^5)'];
    agep1  = [agep ones(size(agep,1),1)];

  % Polynomials for income and family size:
    ypoly   = [.32643678179      -.0148947085      .00036342384...
               -4.411685e-06    2.056916e-08      6.801368713];
    fampoly = [0.13964975        -0.0047742190     8.5155210e-005...
               -7.9110880e-007   2.9789550e-009];

  % Income growth rate:
    lnY=agep1*ypoly';                      % log(income)
    Y= exp(lnY);                           % income
    for i=2:length(Y)
      YY(i,:)=Y(i,:)/Y(i-1,:);
    end
    GList(1:T)=YY(2:length(Y),:);          % G_{t+1}
    GListT=GList(T);

  % Variations in family size:
    Z= agep*fampoly';                      % marginal utility (family size)
    for i=2:size(Z,1)
      ZZ(i,:)=exp(Z(i,:))/exp(Z(i-1,:));
    end
```

```
    ZList(1:T)=ZZ(2:length(Z),:);          %[v(Z_t)/v(Z_{t-1})]^{1/p}
    ZListT=ZList(T);

% 5. Set up income shocks using Gauss Hermite Quadrature of order 12:
    [nodes,weights] = gauher(12);
```

A.5.2 `BackwardInduction.m` File

```
% Backward Induction
Cons=zeros(length(gridx),T);
Cashonhand=zeros(length(gridx),T);

Cashonhand(1:end,T) = gridx;
Cons(1:end,T)= fsolve(@EET,gridx/2,optimset('Display', 'off'));
for t=T-1:-1:1
  disp(['period', num2str(t)]);
  Cons(1:end,t)= fsolve(@EEworkingages,gridx/2,optimset('Display','off'));
  Cashonhand(1:end,t) = gridx;
end
```

A.5.3 Simulation.m File

```
% Simulate Model
   MM=T;                                   % Length of simulated series
   nsimppl=20000;                          % Number of simulated households

% 1. Initial financial wealth (w26) is lognormally distribution:
   sb=1.7838670;                    % standard deviation
   mu_b=-2.7944810;                 % mean
   bb0 = randn(1,nsimppl);          % generate w26 from Normal distribution
   bb0 = exp(bb0*sb)*exp(mu_b);     % transform to lognormal

% 2. Permanet Income Shock:
   Nt   = randn(MM,nsimppl);        % generate N from Normal distribution
   Nt   = exp(Nt*sN);               % transform to lognormal

% 3. TransitoryIncome Shock:
   Ut   = randn(MM,nsimppl);        % generate U from Normal distribution
   Ut   = exp(Ut*sU);               % transform to lognormal
   v    = rand(MM,nsimppl);         % take into account Ut=0 with prob. p
   Ut   = (v>probUnEmp).*Ut;

% 4. Simulation:
   cc=zeros(MM,nsimppl);  xx=zeros(MM,nsimppl);
   aa=zeros(MM,nsimppl);  bb=zeros(MM,nsimppl);
   yy=zeros(MM,nsimppl);  pp=zeros(MM,nsimppl);

   for nn=1:nsimppl
     pp(1,nn)   = 18690.96*Nt(1,nn);
     bb(1,nn)   = bb0(1,nn);
     yy(1,nn)   = Ut(1,nn);
     xx(1,nn)   = bb(1,nn) + yy(1,nn);
     cc(1,nn)   = max(realmin,Cdash(xx(1,nn),Cons(:,1),Cashonhand(:,1)));
     aa(1,nn)   = xx(1,nn)-cc(1,nn);
   end
   for t=2:MM
     for nn=1:nsimppl
     pp(t,nn)   = pp(t-1,nn)*GList(t-1)*Nt(t,nn);
     bb(t,nn)   = (R/(GList(t-1)*Nt(t,nn)))*aa(t-1,nn);
     yy(t,nn)   = Ut(t,nn);
     xx(t,nn)   = bb(t,nn) + yy(t,nn);
     cc(t,nn)   = max(realmin,Cdash(xx(t,nn),Cons(:,t),Cashonhand(:,t)));
     aa(t,nn)   = xx(t,nn)-cc(t,nn);
     end
   end

% 5. Non-normalized Consumption, Cash-on-hand, and Income:
   C=pp.*cc;
   X=pp.*xx;
   Y=pp.*yy;
```

A.5.4 Functions for SettingUp.m, BackwardInduction.m, Simulation.m

gauher.m

```
function [x,w] = gauher(n)
% Input : n- order of quadrature
% Output: x- nodes of quadrature (n x 1); w- weights (n x 1)

 x=zeros(n,1);
 w=zeros(n,1);
 maxit=10;
 pima=(pi)^(-1/4);
 m=(n+1)/2;

 i=1;
 while i<=m
      if i==1;
           z= sqrt(2*n+1)-1.85575*((2*n+1)^(-0.16667));
      elseif i==2;
           z= z-1.14*((n^0.426))/z;
      elseif i==3;
           z= 1.86*z-0.86*x(1);
      elseif i==4;
           z= 1.91*z-0.91*x(2);
      else
           z=2.0*z-x(i-2);
      end

      its=1;
      while its<= maxit;
           p1=pima;
           p2=0.0;
           j=1;
           while j<=n;
            p3=p2;
            p2=p1;
            p1=z*sqrt(2.0/j)*p2-sqrt((j-1)/j)*p3;
            j=j+1;
           end
      pp=sqrt(2*n)*p2;
      z1=z;
      z=z1-p1/pp;
      if abs(z-z1) <= eps;
           break;
      end;
      its=its+1;
      end

      if (its>maxit);
      disp('too many iterations in gauher');
      end;

      x(i)=z;
      x(n+1-i)=-z;
      w(i)=2.0/(pp*pp);
      w(n+1-i)=w(i);
      i=i+1;
```

end
end

EET.m

```
% Root-finding for Euler Equation at age T
function f = EET(c)
global  gridx rho beta gamma0 gamma1 R ZListT
f=invlambda(max(lambda(gridx,rho), beta.*R.*ZListT.^rho...
    *lambda((gamma0 + gamma1.*(R.*(gridx-c))),rho)),rho)-c;
end
```

EEworkingages.m

```
% Root-finding for Euler Equation at age t<T
function f = EEworkingages(c)
global ZList beta R rho x t GList gridx Cons Cashonhand sN sU...
       probUnEmp nodes weights
% when U>0
    EpU=0;
    for j=1:length(nodes)
    for i=1:length(nodes)
        xdashU = (gridx-c).*R./GList(t).*exp(-sqrt(2)*sN*nodes(j))...
                 + exp(sqrt(2)*sU*nodes(i));
        cdashU = max(realmin,Cdash(xdashU,Cons(:,t+1),Cashonhand(:,t+1)));
        EpU    = EpU + (1/pi) * lambda(cdashU*GList(t)*exp(sqrt(2)...
                 *sN*nodes(j)),rho) * weights(j)*weights(i);
    end
    end
% when U=0:
    Ep0=0;
    for j=1:length(nodes)
        xdash0 = (gridx-c).* R./GList(t).* exp(-sqrt(2)*sN*nodes(j));
        cdash0 = max(realmin,Cdash(xdash0,Cons(:,t+1),Cashonhand(:,t+1)));
        Ep0    = Ep0 + (1/pi^(0.5))*lambda(cdash0*GList(t)*exp(sqrt(2)...
                 *sN*nodes(j)),rho) * weights(j);
    end
Ep = beta.*R.*ZList(t).^rho*(probUnEmp*Ep0 + (1-probUnEmp)*EpU );
f=invlambda(Ep,rho)-c;
%f=invlambda(max(lambda(gridx,rho),Ep),rho)-c;
end
```

lambda.m

```
% Marginal Utility
function lamb = lambda(c,rho)
if (abs(rho-1.0)<sqrt(eps));
    lamb=c.^(-1);
else
    lamb=c.^(-rho);
end
end
```

invlambda.m

```
% Inverse Marginal Utility
function invlamb = invlambda(cc,rho)
if (abs(rho-1.0)<sqrt(eps));
    invlamb=cc.^(-1);
else
    invlamb=cc.^(-(1/rho));
end
end
```

Cdash.m

```
% Generates next-period consumption by interpolating c_{t+1}(x_{t+1})
function c=Cdash(xdash,C,X)
xtp1=X(:,end);      % Cash-on-hand vector in period t+1
ctp1=C(:,end);      % Consumption  vector in period t+1
c = zeros(size(xdash));

% extrapolate above when xdash is greater than maximum value of xtp1:
iAbove = xdash>=xtp1(end);
slopeAbove = (ctp1(end)-ctp1(end-1))/(xtp1(end)-xtp1(end-1));
c(iAbove)  = ctp1(end) + (xdash(iAbove)-xtp1(end))*slopeAbove;

% extrapolate below when xdash is lower than minimum value of xtp1:
iBelow = xdash<=xtp1(1);
slopeBelow =  (ctp1(2)-ctp1(1))/(xtp1(2)-xtp1(1));
c(iBelow) = ctp1(1) + (xdash(iBelow)-xtp1(1))*slopeBelow;

% interpolate when xdash is in between min and max value of xtp1:
iInterp = ~(iAbove | iBelow);
c(iInterp) = interp1(xtp1,ctp1,xdash(iInterp));
end
```

Appendix B

Estimation of the Life-Cycle Problem

This appendix outlines the MATLAB codes to estimate the life-cycle model through the Simulated Method of Moments. The main file is called `SmmEstimation.m`. For the Monte Carlo experiments in Chapter 4, these codes are iterated 500 times using simulated data for $1,000$ households and the endogeneous grid-points solution method to solve the consumer problem. The MATLAB code for this alternative solution method is included in the last section of this appendix as `endogenousgridsolution.m`.

B.1 Simulated Method of Moments

Recall that the SMM estimator solves the objective function:

$$\min_{\theta} g(\theta;\hat{\chi})' W g(\theta;\hat{\chi}) \tag{B.1}$$

where W is a $T \times T$ weighting matrix and $g(\theta;\hat{\chi}) \in R^T$ is a vector with t^{th} element:

$$g_t(\theta;\hat{\chi}) = \ln \bar{C}_t - \ln \hat{C}_t(\theta;\hat{\chi}) \tag{B.2}$$

where $\ln \bar{C}_t$ is the average consumption for age t observed in the empirical data and $\ln \hat{C}_t(\theta;\hat{\chi})$ is the simulated counterpart of $\ln C_t(\theta;\hat{\chi})$. Thus, the SMM estimator chooses θ that matches the means of the empirical and simulated distributions for each age t.

B.2 SMM estimation

The `SMMestimation.m` file begins by setting up the model's parameters needed to perform the SMM estimation. It calls the `setupdata.m` file to load the consumption data and to generate the matrix Ω (needed to construct the weighting matrix $W = \Omega^{-1}$). The SMM estimation is performed by calling the `smm` function. This function minimizes the objective function (B.1) and calculates the variance of the estimated parameters. The output "resultSMM" gives the values of estimated parameters, their standard errors, the value of the objective function at the estimated parameters, and the over-identifying restrictions.

B.2.1 The `smm` Function

Please read the comments below for an outline of the `smm` function:

1. The function starts off by declaring the global variables needed to estimate the consumption problem, setting up the parameter vector to be used as an initial guess for the SMM optimization problem, generating income growth rates and shifts in marginal utility caused by changes in family size, and performing a two-dimensional Gauss-Hermite quadrature of order 12 for both transitory and permanent income shocks.
2. The SMM estimation is performed by using `fminsearch`, a nonlinear programming solver from MATLAB. The objective function to be minimized is recorded in the function `ObjectFunction`.
3. `ObjectFunction` solves and simulates the consumer problem by calling `BackwardInduction.m` and `Simulation.m` (from Appendix A). The file then calculates the geometric mean of the simulated consumption profiles and constructs the SMM objective function (B.1). Alternatively, the model can be solved using the endogeneous

grid-points solution method in the `endogenousgridsolution.m` file. In order to do so, "gridx" must be replaced by a grid of the end-of-period assets (i.e. $a_t = x_t - c_t$) : $\{a^j\}_{j=1}^J \subset [0, a^{\max}]$.

4. Once the SMM parameters are estimated, the `smm` function calculates the standard error of the estimated parameters and the over-identifying restrictions based on Appendix B of Gourinchas and Parker (2002). The function `dscore` calculates the derivatives of the moment conditions with respect to the parameters. It uses the `moments` function whose output is the moment conditions in (B.2).

B.3 MATLAB Codes

B.3.1 SMMestimation.m File

```
clear;
clc;

% Parameters:
T = 40;
R      = 1.0344;
beta   = 0.96;
rho    = 0.514;
gamma0 = 0.001;
gamma1 = 0.077;
probUnEmp = 0.00302;

% Grid for cash-on-hand
xmin = 0.000001; xmax = 40;    xint = 2;
xn   = 100;
xn1  = xn/2;
xn2  = xn-xn1+1;
ngridx1 = linspace(xmin,xint,xn1)';
ngridx2 = linspace(xint,xmax,xn2)';
gridx  = [ngridx1; ngridx2(2:end)];

% Vector of parameters used as initial guess
mu= [rho beta gamma0 gamma1 R];

% Fixes parameters (R is always fixed)
fixedgamma=0;
if fixedgamma==1
        fixed={[gamma0] [gamma1] [R]}; %#ok<NBRAK>
else
        fixed={[] [] R};
end

% Loads consumption data and generates Omega and itm
setup_data;
data_logC=mean_logC;


% SMM Estimation
resultSMM = smm(mu,fixed,probUnEmp,gridx,T,data_logC,Omega,itm);
```

B.3.2 Setupdata.m File

```
% 1. Loads households data and mean log(consumption)
data = xlsread('GPCEXdata');
mean_logC=log(xlsread('GPCEXdata_mean'));

consumption=data(:,1);
age=data(:,2);
NumObs=length(consumption);

% 2. Calculates Omega
v = zeros(NumObs,1);
for i=1:NumObs
    v(i)=(log(consumption(i))-mean_logC(age(i)-25))^2;
end

vAge=zeros(T,NumObs+1);
for i=1:NumObs
for agei=1:T
    if data(i,2)-25==agei
        vAge(agei,1:1+i)=[vAge(agei,1:i) v(i)];
    else
    end
end
end

Sum_vAge=zeros(T,1);
nAge=zeros(T,1);
for agei=1:T
    Sum_vAge(agei,1)=sum(vAge(agei,:));
    nAge(agei,1)=sum(data(:,2)==agei+25);
end
itm = sum(nAge)/size(nAge,1);    % average number of obs per age in data

Omegadiag=Sum_vAge./nAge;
Omega=diag(Omegadiag);
```

B.3.3 smm.m Function

```
function result=smm(mu, fixed ,probUE, xgrid ,T, data_logC ,Omega, itm)

global sN sU ZList GList nodes weights Cons Cashonhand...
       probUnEmp ZListT gridx x

% Sets Initial Parameters: non-negativity restruction
theta = [sqrt(mu(1)) sqrt(-log(mu(2))) sqrt(mu(3))...
           sqrt(-log(mu(4))) sqrt(mu(5))];

% Fixing parameters
gg0 = [fixed{1}]; gg1 = [fixed{2}]; RR= [fixed{3}];
if isempty(gg0) == 0.0;
    theta = theta([1 2 4 5]);
    if isempty(gg1) == 0.0;     theta = theta([1 2 4]);
        if isempty(RR) == 0.0; theta = theta([1 2]);  end
    elseif isempty(gg1) == 1;
        if isempty(RR) == 0.0; theta = theta([1 2 3]);  end
    end
elseif isempty(gg0) == 1.0
    if isempty(gg1) == 0.0;      theta = theta([1 2 3 5]);
        if isempty(RR) == 0.0; theta = theta([1 2 3]);  end
    elseif isempty(gg1) == 1.0;
        if isempty(RR) == 0.0; theta = theta([1 2 3 4]);  end
    end
end

%Income growth and family size:
age=26:1:26+T;
agep=[age' (age.^2)' (age.^3)' (age.^4)' (age.^5)'];
agep1=[agep ones(size(agep,1),1)];

fampoly = [0.13964975  -0.0047742190   8.5155210e-005...
           -7.9110880e-007   2.9789550e-009];
Ypoly    = [.32643678179   -.0148947085     .00036342384...
           -4.411685e-06     2.056916e-08      6.801368713];

%Family size
Z= agep*fampoly';
for i=2:size(Z,1)
    ZZ(i,:)=exp(Z(i,:))/exp(Z(i-1,:)); %#ok<AGROW>
end
ZList(1:40)=ZZ(2:length(Z),:);
ZListT=ZList(T);

%Income Growth
Y= exp(agep1*Ypoly');
for i=2:length(Y)
YY(i,:)=Y(i,:)/Y(i-1,:); %#ok<AGROW>
end
GList(1:40)=YY(2:length(Y),:);

% Income shocks
probUnEmp=probUE;
sN   = sqrt(0.0212);    sU = sqrt(0.0440);
[nodes,weights] = gauher(12);
```

```
% Grid for cash-on-hand
gridx=xgrid;

% ESTIMATION
W=Omega^(-1);
options=optimset('Display','iter','MaxFunEvals',5000,'MaxIter',5000,...
                 'tolx',1.0000e-6,'tolfun',1.0000e-6);

[theta, fval, ~,~] = fminsearch(@ObjectFunction,theta,options,fixed,...
                                 data_logC,W,T);

if size(theta,2)==2
disp([(theta(1))^2  exp(-(theta(2)^2)) ]);
elseif size(theta,2)==4
disp([(theta(1))^2  exp(-(theta(2)^2)) (theta(3))^2  exp(-(theta(4)^2))]);
else
end

%Asymptotic Variance-Covariance Matrix
F = ObjectFunction(theta,fixed,mean_logC,W,T); %#ok<NASGU>

nsimppl=20000;
tau = itm/nsimppl;

D = dscores(@MPscoresV,theta,fixed,data_logC,W,T);
g = moments(theta,theta,fixed,data_logC,W,T);

W=Omega^(-1);

%Params Variance
invDWD     = inv(D'*W*D);
VV_robust = invDWD*(1+tau)/itm;  %#ok<MINV>

%Overidentification Test:
Ov = itm/(1+tauuuu )*g'*W*g;

%Parameters
paramsR = [(theta(1))^2, exp(-(theta(2)^2))];
if numel(theta)>2; paramsR = [paramsR (theta(3))^2];
    if numel(theta)>3; paramsR = [paramsR exp(-(theta(4)^2))];
        if numel(theta)>4; paramsR = [paramsR (theta(5))^2];
        end
    end
end

result.paramsR   = paramsR;
result.fixed     = fixed;
result.moment    = fval;
result.seRobust  = sqrt(diag(VV_robust))';
result.OvR       = Ov;

end


function F=ObjectFunction(theta,fixed,data_logC,W,T)

global  ZList  GList  t  Cons Cashonhand R  sN sU nodes...
        weights rho beta gamma0 gamma1 pUE ZListT x gridx probUnEmp
```

```
rho    = (theta(1))^2;
beta   = exp(-(theta(2))^2);
gamma0 = [fixed{1}];
gamma1 = [fixed{2}];
R      = [fixed{3}];

if numel(theta) == 5;
gamma0 = (theta(3))^2;
gamma1 = exp(-(theta(4))^2);
R      = (theta(5))^2;
elseif numel(theta) == 4;
if ~isempty([fixed{1}]); gamma1=exp(-(theta(3))^2); R=(theta(4))^2;end
if ~isempty([fixed{2}]); gamma0=(theta(3))^2; R=(theta(4))^2;end
if ~isempty([fixed{3}]); gamma0=(theta(3))^2; gamma1=exp(-(theta(4))^2);end
elseif numel(theta) == 3;
if  isempty([fixed{1}]); gamma0=(theta(3))^2;end
if  isempty([fixed{2}]); gamma1=exp(-(theta(3))^2);end
if  isempty([fixed{3}]); R=(theta(3))^2;end
end


% Solves Model:
x=gridx;
BackwardInduction;

% Simulates Model;
Simulation;

sim_logC=zeros(T,1);
log_cons=log(C);
for i=1:T
    sim_logC(i)=sum(log_cons(i,:))/nsimppl;
end

% SMM Objective Function:
g=data_logC-sim_logC;
F=g'*W*g;

end


function g=moments(theta,fixed,data_logC,T)

global  ZList  GList  t  Cons Cashonhand R  sN sU  nodes...
        weights rho beta gamma0 gamma1 pUE ZListT x gridx probUnEmp

rho    = (theta(1))^2;
beta   = exp(-(theta(2))^2);
gamma0 = [fixed{1}];
gamma1 = [fixed{2}];
R      = [fixed{3}];

if numel(theta) == 5;
gamma0 = (theta(3))^2;
gamma1 = exp(-(theta(4))^2);
R      = (theta(5))^2;
elseif numel(theta) == 4;
```

```
if ~isempty([fixed{1}]); gamma1=exp(-(theta(3))^2); R=(theta(4))^2;end
if ~isempty([fixed{2}]); gamma0=(theta(3))^2; R=(theta(4))^2;end
if ~isempty([fixed{3}]); gamma0=(theta(3))^2; gamma1=exp(-(theta(4))^2);end
elseif numel(theta) == 3;
if  isempty([fixed{1}]); gamma0=(theta(3))^2;end
if  isempty([fixed{2}]); gamma1=exp(-(theta(3))^2);end
if  isempty([fixed{3}]); R=(theta(3))^2;end
end

% Solves Model:
x=gridx;
BackwardInduction;

% Simulates Model;
Simulation;

sim_logC=zeros(T,1);
log_cons=log(C);
for i=1:T
    sim_logC(i)=sum(log_cons(i,:))/nsimppl;
end

% MSM moment function:
g=data_logC-sim_logC;

end


function dscore = dscores(f,x,varargin)
  h = max(abs(x),1).*(eps^(1/3));
  xh1 = x + h;
  xh0 = x - h;
  h = xh1 - xh0;
  for j=1:length(x)
    xx = x;
    xx(j) = xh1(j); f1=feval(f,xx,varargin{:});
    xx(j) = xh0(j); f0=feval(f,xx,varargin{:});
    dscore(:,j) = (f1-f0)/h(j); %#ok<AGROW>
  end
end


function g = MPscoresV(x,varargin)
    g = moments(x,varargin{:});
end
```

B.3.4 endogenousgridsolution.m File

```
% Endogenous Gridpoints Solution Method:
Cons              = zeros(length(grida(:,T))+1,T);
Cashonhand        = zeros(length(grida(:,T))+1,T);
Cons(2:end,T)     = invlambda(beta*R*ZList(T).^rho.*lambda((gamma0 ...
                                    + gamma1*(R.*(grida))),rho),rho);
Cashonhand(2:end,T) = grida(:,T) + Cons(2:end,T);

for t=T-1:-1:1
disp(['period', num2str(t)]);
EpU=0;
for j=1:length(shocks)
for i=1:length(shocks)
xdashU = grida.*R./GList(t).*exp(-sqrt(2)*sN*shocks(j)) ...
                                            + exp(sqrt(2)*sU*shocks(i));
cdashU = max(realmin,Cdash(xdashU,Cons(2:end,t+1),Cashonhand(2:end,t+1)));
EpU    = EpU + (1/pi) *lambda(cdashU*GList(t)*exp(sqrt(2)...
                               *sN*shocks(j)),rho)*weights(j)*weights(i);
end
end

Ep0=0;
for j=1:length(shocks)
xdash0 = grida.*R./GList(t).*exp(-sqrt(2)*sN*shocks(j));
cdash0 = max(realmin,Cdash(xdash0,Cons(2:end,t+1),Cashonhand(2:end,t+1)));
p0     = Ep0 + (1/pi^(0.5))*lambda(cdash0*GList(t)*exp(sqrt(2)...
                               *sN*shocks(j)),rho)*weights(j);
end

Ep = beta.*R.*ZList(t).^rho*( pUE*Ep0 + (1-pUE)*EpU );
Cons(2:end,t)       = invlambda(Ep,rho);
Cashonhand(2:end,t)= grida+Cons(2:end,t);
end
```