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Mechanisms and efficiency in permit markets

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# Mechanisms and efficiency in permit markets

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# Abstract

My thesis analyses mechanisms that we can find in the context of permit markets or water allocation. The first paper analyses the possibility to use the property rights as an instrument to reach efficiency when there is serial correlation and that the initial distribution is an absolute property rights. The third paper is an extension of Cramton-Gibbons-Klemperer (1987) to agents with concave valuations which is generally the case in the context of permits. We show that if agents are symmetric then equal share is one possible solution. In the case of the root function for two agents, we show that the maximum expected transfer is positive so an efficient mechanism exists. In the second paper, we analyze the effects of the interaction between private information and decision of capital in permit markets. Agents have to invest in a first period and in a second period, they submit a demand schedule for permits and a price is chosen to clear the market. We analyze the effects on the efficiency of the allocation and of the investment. We decompose the effects to analyze them and we also add the possibility to use subsidy to reach efficiency. Simulations are run as an illustration.

# Resumen

Mi tesis analiza mecanismos que podemos encontrar en el contexto de mercado de permisos o de asignación de agua. El primer paper analiza la posibilidad de usar los derechos de propiedad como instrumento para alcanzar la eficiencia cuando hay correlación serial y que la distribución inicial de los derechos es absoluta. El tercer paper es una extensión de Cramton-Gibbons-Klemperer (1987) con agentes con valuación cóncava, lo cual es generalmente el caso en contexto de permisos. Mostramos que si los agentes son simétricos entonces una solución posible es una distribución de porciones iguales. En el caso de la función raíz para dos agentes, mostramos que la transferencia esperada es positiva y por ende, que existe un mecanismo eficiente. En el segundo paper, analizamos los efectos de la interacción de la información privada y de la decisión de capital en los mercados de permisos. Agentes deben investir en el primer periodo y en el segundo periodo, presentan su demanda para los permisos y un precio está elegido para aclarar el mercado. Analizamos los efectos sobre la eficiencia de la asignación y de la inversión. Por eso, descomponemos los efectos y añadimos la posibilidad de usar un subsidio. Corremos simulaciones para ilustrar.

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Pasarán estos días como pasan
todos los días malos de la vida
Amainarán los vientos que te arrasan
Se estancará la sangre de tu herida

El alma errante volverá a su nido
Lo que ayer se perdió será encontrado
El sol será sin mancha concebido
y saldrá nuevamente en tu costado

Y dirás frente al mar: ¿Cómo he podido
anegado sin brújula y perdido
llegar a puerto con las velas rotas?

Y una voz te dirá: ¿Que no lo sabes?
El mismo viento que rompió tus naves
es el que hace volar a las gaviotas.

*El doliente*, Óscar Hahn

# Contents

# Chapter 1

# Efficient mechanism in a repeated game with serial correlation

We extend the problem *à* la Myerson Satterwhaite to a two-period game with initial absolute ownership. Private information affects future realization of the types through serial correlation. We show that if the correlation is defined by first-order stochastic dominance, then efficiency cannot be reached. We extend the game to the possibility to use the allocations as an instrument. The mechanism can rearrange the assignation in period two in exchange of a transfer. It is chosen to maximize the expected sum of payments. We show that, even though the new allocation changes the outside option and may change the critical types, the best the mechanism can do is to return to the situation of absolute property rights. Therefore, there is no mechanism that can be incentive compatible, ex-post efficient, budget balanced and that satisfies voluntary participation.

## 1.1 Introduction

Since the seminal paper of Myerson and Satterwhaite (1983, MS), it is well-known that private information hinders the ability of negotiating parties to achieve mutually beneficial exchange. When there is absolute ownership of the good and private information in a one-shot game, there is no mechanism that can be incentive compatible, individual rational, ex-post efficient and budget feasible. Efficiency cannot be reached without an external subsidy, such as asymmetric information can be understood as a transaction cost in Coase's tradition.

Of course, many real-world allocation problems are dynamic so we extend the problem to a two-period game with initial absolute ownership. In this context, private information may change over time or present situation may affect future realization of the types. For example, if a water right is given to the agent with the highest valuation in each time, we can expect that an agent with a high valuation today will have a high valuation tomorrow too. For this, we introduce serial correlation. Agent $i$'s type in period two depends on his realization in period one. We do not consider correlation between agents even though the probability that $i$ wins in period two depends on the type of agent $-i$ which itself depends on his type in period one. We develop the model without imposing a form to the correlation but to draw conclusions, we suppose first-order stochastic dominance. Positive correlation indicates that $\theta_{i2}$ increases with $\theta_{i1}$ and that the probability that $i$ wins in period two decreases with $\theta_{-i1}$.

Without correlation, the problem is simply the repetition of the game and we show that efficiency is not achievable. With correlation, period two is still a game *à* la MS. Critical types are zero or one and the sum of expected payments is negative. In period one, the payoff depends on the expected payoff in period two. The discounted part changes the slopes of the inside and outside options so the value of the critical types is not anymore obvious. However, we find that if correlation is defined by first-order stochastic dominance, then the critical types are corner solutions such as efficiency could not be attained.

For the moment, the unique instrument the mechanism uses is the transfers. They are determined in order to make agents tell the truth and participate in the game. Allocations are fixed and the mechanism indicates what proportion of the good each agent can use but the property does not change. Cramton, Gibbons and Klemperer (1987, CGK)show that with symmetric and independent private values, efficient dissolution can be realized if the initial endowment is close to equal sharing in a one-shot game. Our idea is to modify the mechanism letting the allocation in period two be a new control variable. Our initial intuition is that maybe an allocation close to equal sharing in period two would allow to achieve efficiency. In this case, the mechanism can rearrange the assignation in period two. Of course, agents pay or receive money for this. Therefore, the new assignation is chosen in order to maximize the expected sum of payments. The allocation affects both the inside and outside options. Outside option in period two depends directly on the new allocation. In period one it does not depend on it since agent goes out the game before period two. The allocation does not affect directly the payoff in period two but through the critical type. In effect, since outside option is modified by the new assignation, the critical type may change and depend on the allocation. We show that the optimal is to not change the initial situation of absolute property rights.
We illustrate our model with a uniform distribution $[0, 1]$ in period one and $[0, h(\theta_{i1})]$ in period two. Optimization leads to give again absolute ownership, so the best situation the mechanism can achieve is the one with a fixed allocation. It is not worth to use allocation as a control variable.

## 1.2 Related literature

Our analysis is related to the literature on partnership that had grown since Cramton, Gibbons and Klemperer (1987, CGK). They show that, at least with private values, the main obstacle to achieve efficiency is not really the presence of information asymmetries but rather the presence of asymmetries in endowments. With symmetric and independent private values, efficient dissolution can be realized if the initial endowment is close to equal sharing. Most of the papers studying dissolution after CGK are concerned with the presence of interdependent values across agents (Fieseler, Kittsteiner and Moldovanu 2003), with veto right (Compte and Jehiel 2009), buy-sell clause (Frutos and Kittsteiner 2008),buy-sell clause with investment (Li and Wolfstetter 2010) or even the structure of the partnership (Ornelas and Turner 2007). When only one of the two parties is informed and values are interdependent, the CGK's result that efficient trade is feasible whenever initial shares are equal falls. The subsidy required in a second best mechanism is minimal when the entire ownership is allocated initially to one of the parties. This happens because a mixed ownership does not alter any more the worst off type as it does in CGK environment (Jehiel and Pauzner 2006).
Figueroa and Skreta (2012) study a partnership in which valuations are private information but are drawn from different distributions. In this case they show that efficient dissolution is possible if critical types are equal, which in case of asymmetric distributions implies different property rights that can be really unequal, what is the opposite of the result with symmetric distributions. Segal and Whinston (2014, working paper)

study how the property rights affect the efficiency of bargaining and the final allocations in a static game. They establish a wide class of economic settings and property rights in which efficient bargaining is impossible. The second best when the first best cannot be reach is contrary to the intuition of CGK since they demonstrate that less extreme property rights may be worse than extreme cases.
Most of the papers study how to dissolve efficiently partnership. On the contrary, Kuribko and Lewis (2009) and Kuribko, Lewis, Liu and Song (2015) study how to design partnerships that survive until the project is complete. The latter is a more general version of the former and adds ex-ante investment. The economy consists in a supply chain that creates a capacity unit and agents that need this capacity. All agents and the manufacturer form a partnership such as if only one individual decides to not participate, the partnership is dissolved. In each period, the mechanism must distribute efficiently the unit of capacity, guarantee individual compatibility, voluntary participation and budget balanced. Control rights are defined at the beginning of each stage in such a way that if the partnership is dissolved, each agent is entitled with his control right to use capacity. Thus, control rights are adjusted to maintain each member's willing participation. The rights are allocated to minimize the breakup value of the partnership. In contrast, we use ownership to maximize the expected total payment in order to have feasibility. Control rights in our model are not established only in the case there is no exchange but in every period such as agents receive or pay money to part with or obtain right.

Mechanism design can be useful in public good problems. Grüner and Koriyama (2012) study the possibility to replace a voting mechanism by an efficient d'Aspremont-Gerard-Varet (AGV) mechanism in the provision of an indivisible public good. Neeman (1999) show that voluntary participation in a public good problem with asymmetric information depends a lot on the structure of property rights and finds a result similar to CGK.

Our paper is part of a growing literature on efficient dynamic mechanisms. Dynamic settings have been studied through extension of the Vickrey-Groves-Clarke mechanism (VCG) and the d'Aspremont-Gerard-Varet (AGV). Bergemann and Välimäki (2010) construct a dynamic pivot mechanism where agents have private information. They focus on social efficiency such as the planner has to allocate a good in each period in order to maximize the expected discounted sum of individual utilities. After every period, the expected transfer that an agent must pay coincides with the dynamic externality cost he imposes on the others. The type of $i$ in $(t+1)$ depends on his own type in $t$ and the allocation in $t$ but does not depend directly on the types of the other agents. They show that the dynamic pivot mechanism is ex-post incentive compatible, individually rational, ex-post efficient, budget feasible but not budget-balanced. Kakade, Lobel and Nazerzadeh (2013) present a mechanism based on the dynamic pivot mechanism of Bergemann and Välimäki in the context of online advertising and supply-chain contracting. They modify the periodic ex-post incentive compatibility by allowing agents to re-report the entire story of their types in each period. Athey and Segal (2013) focus on the budget-balancedness of an efficient mechanism. They extend both the VCG and the AGV mechanisms to dynamic settings. Using the logic of the AGV and under the assumption of independent types ( the distribution of each agent's private information is not directly affected by other agents' private information) they show how to balance the budget in a balanced team mechanism . The main idea is to give each agent in each period an incentive payment equal to the change in the expected present value of the other agents' utility caused by his current report.

Closer to our setting, Athey and Segal (2007), Athey and Miller (2007) , Skrzypacz and Toikka (2015) and Lamba (2013) study mechanism design in repeated trade. Athey and Segal (2007) introduce serial correlation between agent in a bilateral repeated game and analyse the conditions to make a mechanism efficient. They underline that today's reports have effect on future surplus such as the incentives for truth telling are undermined, which complicates a lot the mechanism. They focus their study on the importance of agents' patience and the game is infinitely repeated. Athey and Miller develop an infinitely repeated trade between agents with private information about their valuations that are iid. They analyse the feasibility of efficient trade with budget balance in different institutional environments as an external agent that can absorb deficits and surplus or a bank that allows to accumulate budget imbalances within a bounded range. The main problem in order to reach efficiency is the patience or impatience of the participants. They show that ex-ante budget balance (without any institutional restriction) can be attained if and only if the discount factor is at least one half. In our work, we are not interested in the degree of patience agents must have but the possibility to use property rights as an instrument. For this reason we restrict our setting to two periods and show that without correlation, the result of Athey and Miller for a two-periods game is that the discount factor must be at least one. Skrzypacz and Toikka use a mechanism *à* la MS in a repeated game, like a trade between the supplier and the buyer of a service which can be provided in several periods. Agents may have serial correlation or private information about the parameters of their type processes. They study how the feasibility of an efficient mechanism in repeated trade depend on the persistence of values, the private information about the evolution of uncertainty, the patience and the frequency of the trade. They focus on ex-ante budget balance.

Lamba also uses a repeated game between a buyer and a seller with changing private information. He studies the possible efficiency when budget balance is restricted to be ex-ante, interim or ex-post. Interim budget balance occurs when expected budget surplus is positive after each history. In a two-periods mechanism, he analyses the conditions the discount factor and the difference between agents' value must comply to fulfil efficiency. That is, if the buyer's valuation is $v$ and the seller's one is $c$, the condition is $v > c + M$ where $M$ solves the binding expected budget surplus constraint. He calculates the no-trade areas and finds like us that the discount factor must be at least one if $M$ is zero to have ex-ante budget balance.

Then, for $T$ periods, he develops a modified VCG called the collateral dynamic VCG. This is a reformulation of the VCG such as the transfers are normalized to guarantee the critical types the minimum possible expected utility in every period. Thus the mechanism extracts the maximum possible transfers, generating a collateral that, if it is big enough, can be used to ensure budget balance. Lamba gives the condition to make the mechanism efficient and interim budget balanced. In particular he studies the case of iid types, perfect persistence and a Markov evolution governed by a truncated normal.

Moreover, he studies the possibility to use "fluid" property rights but in a different context than ours. A buyer and a seller with iid types may exchange in an infinitely repeated game. In this particular case, Lamba develops a mechanism that is closed to CGK's work and permits to reach efficiency. In each period, the mechanism is realized in three steps. First, the seller hands over half the ownership of the good to the buyer in exchange of a fixed cost such as the outside option of each agent is half his valuation. Second, if the seller refuses to sell the share, trade breaks down forever. Third, a modified second price auction is run. Each agent announces his bid and the winner pays the other half of the loosing bid. He shows that it exists a fixed cost and a discounted factor less than one such as efficiency can be sustained. In contrast, we analyse how the mechanism changes if the allocation is a control variable in the second period and types are serially correlated.

## 1.3 General Model

The economy consists of two agents $i$ and $-i$ who share a divisible object of size 1. Agent $i$ has a share $r_1 = 1$ and agent $-i$ has $1 - r_1 = 0$ in period one that is fixed. In period two, agent $i$ has a share $r_2$ and agent $-i$ has $1 - r_2$ that can be flexible. A two-period mechanism establishes the portion of the good each one can use in order to reach efficiency. When the share is fixed, agents have the same property rights of the good in both periods but can pay or be paid to use more or less of the good. On the opposite, when the share in period two is flexible, agents are assigned new property rights and moreover, can pay or be paid to use more or less of the good. Of course the transfers in the second period take account of the new allocation since an agent will not agree to give part of his property rights for free. We define the linear function $q\theta_{it} - x_{it}$ where $\theta_{it} \in [0, \bar{\theta}_{it}]$,$\bar{\theta}_{it} \leq \infty$ is the type of agent $i$ in period $t$, $q$ is the probability to win and $x_{it}$ is the transfer for agent $i$ in period $t$. In period one, the total utility for agent $i$ is the linear function of payoff for period one plus the discounted payoff in period two whereas the utility in period two is only the linear function defined in period two. The discount factor $\delta \in (0, 1]$ is the same for both agent. We further assume that individuals seek to maximize their expected utilities.
Individuals have private information about their type. It is common knowledge that it is drawn independently from a distribution $F_{i1}(.)$ with density distribution $f_{i1}(.)$ in period 1. In period 2, serial correlation is introduced and represented by a family of cumulative distributions $\{F_{i2}(.|\theta_{i1})\}$. We do not need to impose a form for the serial correlation at the beginning. The correlation is not between agents but between periods for the same agent. We can understand the positive correlation as a learning-by-doing process where the individual's type increases with his past valuation. Negative correlation is less intuitive and could be interpreted as a feeling of satiety. If an agent had a high type in period one and used enough good, his type in the next period is lesser because he does not want more. We will focus on positive correlation since it is more intuitive. Without correlation, the mechanism only has to be repeated independently in each period (would have to be). We assume that the distribution has support $[0, 1]$ in the first period and $[0, \bar{\theta}_i]$in period 2 for agent $i$.

By the revelation principle, any outcome that is obtained from a bargaining process among the agents can be obtained as an equilibrium outcome of an incentive compatible direct revelation mechanism. Therefore, we focus on this mechanism without loss of generality. In our setting, the timing of the game is standard. In each period $t$, agents simultaneously report their type $\theta_t = \{\theta_{it}, \theta_{-it}\}$ and then receive an allocation, the use of the good and a money transfer. We use the term "allocation" for the property right $r_t$ agent $i$ has or receives, and "use of the good" $q_{it}$ for the proportion of the good he uses in a period.

***Definition.*** The mechanism $< q, r_2, x >$ is defined by three functions: the decision rule of use of the good $q : [0, \bar{\theta}_{it}] \times [0, \bar{\theta}_{-it}] \rightarrow [0, 1]$ , the allocation rule $r_2 : [0, \bar{\theta}_{i1}] \times [0, \bar{\theta}_{-i1}] \rightarrow [0, 1]$ and a money transfer $x : [0, \bar{\theta}_{it}] \times [0, \bar{\theta}_{-it}] \rightarrow \mathbb{R}$ for each $t = \{1, 2\}$. We define $X, Q$ as the expected value of $x, q$.

In period 2, the expected payoff is simply the difference between the expected utility obtained by the allocation and the expected transfer, such as:

$$V_{i2}(\theta_{i1}, \theta_{-i1}, \theta'_{i2}, r_2(\theta_{i1}, \theta_{-i1})) = \max_{\theta'_{i2}} Q_{i2}(\theta'_{i2}|\theta_{-i1})\theta_{i2} - X_{i2}(\theta'_{i2}|\theta_{-i1}) \tag{1.1}$$

The dependency is not a dependency as understood in probabilistic theory but just a link between variables that is significant and must not be forgotten. The expected payoff depends on $\theta_{-i1}$ because this type determines the type of $-i$ in period two. It depends on $r_2(\theta_{i1}, \theta_{-i1})$ since it is his new property rights assigned by the mechanism. The dependency with $\theta_{i1}$ is not so evident and comes from the critical type in period two that may depend on the type in period one. The probability to win depends on the other type and consequently, depends on the type of the other agent in period one. For coherency, the money transfer depends also on the other agent's first period type. For the moment, we note $r_2(\theta_{i1}, \theta_{-i1})$ because in the most general case, the allocation can depend on both realized types but it varies in function of the cases we study.

In period 1, the expected payoff is the sum of the expected payoff for period 1 and the discounted expected payoff for period 2. Since agent $i$ only knows his type $\theta_{i1}$ he must compute expectation on the possible values of $\theta_{-i1}$. Moreover, agent $i$ does not know his type in period 2 so we must take expectation on his own type which depends on his actual type. His probability to win in period 2 depends on $\theta_{-i2}$ which depends itself on $-i$'s type in period 1 that $i$ does not know.

$$V_{i1}(\theta_{i1}) = \max_{\theta'_{i1}} Q_{i1}(\theta'_{i1})\theta_{i1} - X_{i1}(\theta'_{i1}) + \delta\, \mathbb{E}_{\theta_{i2},\theta_{-i1}} V_{i2}(\theta_{i1}, \theta_{-i1}, \theta_{i2}, r_2(\theta_{i1}, \theta_{-i1})) \tag{1.2}$$

We are interested in a mechanism that is ex-post efficient, incentive compatible , budget balanced and that satisfies voluntary participation. We now define these characteristics.

***Definition.*** A mechanism $\langle q, x, r_2\rangle$ is ex post efficient if $q_i(\theta_i) = 1$ if and only if $\theta_i \geq \theta_{-i}$.

***Definition.*** A mechanism $\langle q, r_2, x\rangle$ is incentive compatible if for all $i$, $t = \{1, 2\}$ and $\theta_{it}, \hat{\theta}_{it} \in \left[0, \bar{\theta}_{it}\right]$

$$V_{it}(\theta_{it}) \geq V_{it}(\hat{\theta}_{it})$$

We are interested in mechanisms that ex ante do not request any outside subsidy. We do not restrict the mechanism to an exact budget balance since we can easily imagine that the money can be used for other activities.

***Definition*** A mechanism is ex ante budget balanced if

$$\sum_{t=1}^{2} \sum_{i} \mathbb{E}_\theta X_{it}(\theta_{it}) \geq 0$$

Voluntary participation requires that the expected payoff obtained in the mechanism is higher than the outside option. We define the outside option $V_{it}^0$ as the payoff agent i would receive if he stays with his allocation $r_1$. In period one the payoff is the proportion of the good he has by his type and his expected payoff in period two since he decided to go out the mechanism. Note that, since he does not participate in the game in period two, his allocation is his initial allocation, that is $r_2 = r_1$. In period two, the new allocation is already taken so his payoff depends on this allocation $r_2$.

***Definition.*** A mechanism $\langle q, x, r_2\rangle$ satisfies voluntary participation if

$$V_{i1}(\theta_{i1}) \geq r_1\theta_{i1} + \delta r_1 \mathbb{E}_{\theta_{i2}}\left[\theta_{i2}\right] := V_{i1}^0(\theta_{i1}) \tag{1.3}$$

and

$$V_{i2}(\theta_{i2}) \geq r_2\theta_{i2} := V_{i2}^0(\theta_{i2}) \tag{1.4}$$

Indeed, in order to induce voluntary participation of all agents, the mechanism must guarantee the participation of the most reluctant agent, that is to say the agent who wins the least in participating in the game.

***Definition.*** The critical type $\theta^*_{it}$ is the solution of the minimization of the difference between the expected payoff and the outside option: $\theta^*_{it} \in argmin \left[V_{it}(\theta_{it}) - V^0_{it}(\theta_{it})\right]$.

We will assume the outside option $V^0_{it}$ is concave, which together with the fact that $V_{it}(.)$ is convex makes the characterization of $\theta^*_{it}$ simple.
The solution of the critical type is interior if the derivative of the expected payoff in the first point is less than the derivative of the outside option and if the derivative of the expected payoff in the last point is more than the derivative of the outside option.

**Lemma 1** At stage $t$, the critical type is:

$$\theta^*_{it} = \begin{cases} x & \text{if } V^{'}_{it}|_{\theta_{it}=0} < V^{0'}_{it}|_{\theta_{it}=0} \text{ and } V^{'}_{it}|_{\theta_{it}=\bar{\theta}_{it}} > V^{0'}_{it}|_{\theta_{it}=\bar{\theta}_{it}} \\ 0 & \text{if } V^{'}_{it}|_{\theta_{it}=0} \geq V^{0'}_{it}|_{\theta_{it}=0} \\ \bar{\theta}_{it} & \text{if } V^{'}_{it}|_{\theta_{it}=\bar{\theta}_{it}} \leq V^{0'}_{it}|_{\theta_{it}=\bar{\theta}_{it}} \end{cases}$$

where $x$ is the solution to $V^{'}_{it}(x) = V^{0'}_{it}(x)$.

We now analyse both periods to better understand the problem. First, using voluntary participation and the definition of critical types, we see that if we evaluate the expected utility in the critical type, we can evaluate the outside option in that point, that is $V_{it}(\theta^*_{it}) = V^0_{it}(\theta^*_{it})$. Moreover, using standard techniques we can write the derivatives of the expected payoff and then use them with the fundamental theorem of calculus to have :

$$V^{'}_{i1}(\theta_{i1}) = Q_{i1}(\theta_{i1}) + \delta \frac{\partial}{\partial \theta_{i1}} \left[\mathbb{E}_{\theta_{i2}|\theta_{i1},\theta_{-i1}} V_{i2}(\theta_{i1}, \theta_{-i1}, \theta_{i2}, r_2(\theta_{i1}, \theta_{-i1}))\right] \tag{1.5}$$

$$V^{'}_{i2}(\theta_{i1}, \theta_{-i1}, \theta_{i2}, r_2(\theta_{i1}, \theta_{-i1})) = Q_{i2}(\theta_{i2}|\theta_{-i1}) \tag{1.6}$$

then

$$V_{i1}(\theta_{i1}) = V_{i1}(\theta^*_{i1}) + \int_{\theta^*_{i1}}^{\theta_{i1}} \left[Q_{i1}(s) + \delta \frac{\partial}{\partial s} \left[\mathbb{E}_{\theta_{i2}|s,\theta_{-i1}} V_{i2}(s, \theta_{-i1}, \theta_{i2}, r_2(s, \theta_{-i1}))\right]\right] ds \tag{1.7}$$

$$V_{i2}(\theta_{i1}, \theta_{-i1}, \theta_{i2}, r_2(\theta_{i1}, \theta_{-i1})) = V_{i2}(\theta^*_{i2}) + \int_{s=\theta^*_{i2}}^{\theta_{i2}} Q_{i2}(s|\theta_{-i1}) ds \tag{1.8}$$

In period two, the system is the same as in a one-period game. If $r_2 = 1$ *à* la MS, we easily see that $\theta^*_{i2} = 1$ since the probability to win cannot be more than unity and $\theta^*_{-i2} = 0$ since the probability cannot be less than zero. If the allocation in period two is an extreme situation, we know that the critical types are extreme points and that the sum of the expected payments is not positive. However, if the critical types are interior solution, it would be possible to have a positive sum of expected payments. For that, the allocation has to be different than all-or-nothing.

In period one, we have to consider the second part of (5). It is not anymore a MS game. In effect, in a one-period game, we would have that the critical type for the agent who possesses the good is one. In a two-period game, it could happen that the discounted part breaks the extreme-value situation because it changes the slope of curves. We need to study both slopes. For that, note that the critical type in period two may depend on the type in period one, so the expected payoff in period two may depend on $\theta_{i1}$ as signalled earlier. There are two possible effects of $\theta_{i1}$. One is on the distribution $f(\theta_{i2}|\theta_{i1})$, such as we take the derivative on the expectation with $\theta_{i1}$ fixed in $V_{i2}$. The other one is on the function $V_{i2}$ directly. We have

$$V_{i1}'(\theta_{i1}) = Q_{i1}(\theta_{i1}) + \delta\mathbb{E}_{\theta_{-i1}}\frac{\partial}{\partial x}\mathbb{E}_{\theta_{i2}|x}V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1})) + \delta\mathbb{E}_{\theta_{-i1},\theta_{i2}}\frac{\partial}{\partial\theta_{i1}}V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1})) \tag{1.9}$$

where we use $x$ to underline that we take the derivative inside the distribution function and that $\theta_{i1}$ inside the expected payoff is fixed.
It is important to develop the last term of (9) to study the possible values of the critical type. We first write $V_{i2}(.)$ using the fundamental theorem of calculus and then take the derivative.

$$\begin{aligned}
\mathbb{E}_{\theta_{-i1}}V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1})) &= \mathbb{E}_{\theta_{-i1}}\left[V_{i2}(\theta_{i2}^*(\theta_{i1})) + \int_{s=\theta_{i2}^*(\theta_{i1})}^{\theta_{i2}} Q_{i2}(s|\theta_{-i1})ds\right] \\
\mathbb{E}_{\theta_{-i1}}\frac{\partial V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1}))}{\partial\theta_{i1}} &= \mathbb{E}_{\theta_{-i1}}\left[\frac{\partial\theta_{i2}^*(\theta_{i1})}{\partial\theta_{i1}}\left[r_{i2} - Q_{i2}(\theta_{i2}^*(\theta_{i1})|\theta_{-i1})\right] + \int_{\theta_{i2}^*}^{\theta_{i2}} \cancelto{0}{\frac{\partial Q_{i2}(s)}{\partial\theta_{i1}}} ds^1\right]
\end{aligned} \tag{1.10}$$

where the last element of (10) is null because there is no cross-correlation.

**Lemma 2**: If the critical type in period two is an interior solution, or if the solution is a corner one and the interval of $\theta_{i2}$ is fixed, then (10) is null.

**Proof**: Suppose the expected value on $\theta_{-i1}$ of the derivative of $V_{i2}(.)$ with respect to $\theta_{i1}$ is defined as in (10). Then, there are three cases:

1) If the solution of the critical type in period two is interior, by definition the critical type is the solution of the equalization of $Q_{i2}(\theta_{i2})$ and $r_2$ so the derivative of the expected payment in period 2 with respect to $\theta_{i1}$ is null and this term disappear.

2) If the solution is a corner one and the interval of $\theta_{i2}$ does not depend on $\theta_{i1}$,that means it is fixed, we will have that the derivative of $\theta_{i2}^*$ with respect to $\theta_{i1}$ is null. In effect, if $\theta_{i2}^* = 0$, we have that the slope of the outside option is less than the slide of the inside option, that is $r_2 < Q_{i2}(\theta_{i2^*})$ but $\theta_{i2}^*$ does not depend on $\theta_{i1}$ so the term disappear. If $\theta_{i2}^* = \overline{\theta}_{i2}$, the slope of the inside option in $\theta_{i2}^*$ is less than $r_2$ but since the interval does not change with $\theta_{i1}$, the critical type does not change with $\theta_{i1}$ and the term is null too.

3) If the solution is a corner one and the interval is not fixed, the term will not disappear. In effect, if $\theta_{i2}^* = \overline{\theta}_{i2}$, it will change with $\theta_{i1}$ depending the sign of the correlation. Moreover, in this corner solution, $r_2 > Q_{i2}(\theta_{i2}^*)$ so the term in the bracket is positive. It is the only moment that the derivative of $V_{i2}$ with respect to $\theta_{i1}$ survives. ■

Once the critical types are determined, it is possible to determinate transfers. If the situation is *à* la MS, it is not necessary to calculate transfers since we know there is no efficient mechanism. In the other case, we calculate the payments in order to know if the mechanism can be budget feasible. The payments are defined by:

$$
\begin{aligned}
X_{i1}(\theta_{i1}) &= Q_{i1}(\theta_{i1}).\theta_{i1} - V_{i1}(\theta^*_{i1}) - \int_{s=\theta^*_{i1}}^{\theta_{i1}} V'_{i1}(s,r)ds + \delta \mathbb{E}_{\theta_{i2},\theta_{-i1}} V_{i2}(\theta_{i2},r) \\
X_{i2}(\theta_{i2}) &= Q_{i2}(\theta_{i2})\theta_{i2} - V_{i2}(\theta_{i2},r)
\end{aligned}
$$

And the expected payment in period one is :

$$
\begin{aligned}
&\mathbb{E}_{\theta_{i1}} X_{i1}(\theta_{i1}) + \mathbb{E}_{\theta_{-i1}} X_{-i1}(\theta_{-i1}) + \delta \mathbb{E}_{\theta_{i1},\theta_{-i1}} \left( \mathbb{E}_{\theta_{i2}} X_{i2}(\theta_{i2}) + \mathbb{E}_{\theta_{-i2}} X_{-i2}(\theta_{-i2}) \right) = \\
&\quad \mathbb{E}_{\theta_{i1}} \left( Q_{i1}(\theta_{i1}).\theta_{i1} - V_{i1}(\theta^*_{i1}) - \int_{s=\theta^*_{i1}}^{\theta_{i1}} V'_{i1}(s,r)ds + \delta\, \mathbb{E}_{\theta_{i2},\theta_{-i1}} V_{i2}(\theta_{i2},r)) \right) \\
&\quad + \delta\, \mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{i2}} \left( Q_{i2}(\theta_{i2})\theta_{i2} - V_{i2}(\theta_{i2},r) \right) \\
&\quad + \mathbb{E}_{\theta_{-i1}} \left( Q_{-i1}(\theta_{-i1}).\theta_{-i1} - V_{-i1}(\theta^*_{-i1}) - \int_{s=\theta^*_{-i1}}^{\theta_{-i1}} V'_{-i1}(s,r)ds + \delta\, \mathbb{E}_{\theta_{-i2},\theta_{i1}} V_{-i2}(\theta_{-i2},r) \right) \\
&\quad + \delta\, \mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{-i2}} \left( Q_{-i2}(\theta_{-i2})\theta_{-i2} - V_{-i2}(\theta_{-i2},r) \right)
\end{aligned}
$$

We can see that $\mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{i2}} V_{i2}$ disappears for both agents. In period one agent $i$ considers his future promised payoff but in the next period, he has to pay for it so in expected value, both cancel. We can simplify :

$$
\begin{aligned}
&= \mathbb{E}_{\theta_{i1},\theta_{-i1}} \left( Q_{i1}(\theta_{i1}).\theta_{i1} + Q_{-i1}(\theta_{-i1}).\theta_{-i1} \right) \\
&\quad + \ \delta\, \mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{i2}} \left[ Q_{i2}(\theta_{i2})\theta_{i2} + Q_{-i2}(\theta_{-i2})\theta_{-i2} \right] \\
&\quad + \ \mathbb{E}_{\theta_{i1},\theta_{-i}} \left[ -V_{i1}(\theta^*_{i1}) + -V_{-i1}(\theta^*_{-i1}) + \int_{s=\theta_{i1}}^{\theta^*_{i1}} V'_{i1}(s,r)ds - \mathbb{E}_{\theta_{-i1}} \int_{s=\theta^*_{-i1}}^{\theta_{-i1}} V'_{-i1}(s,r)ds \right]
\end{aligned}
\tag{1.11}
$$

The expected payment is a compound of the surplus of time one, of the full surplus of time two less the informational rents. There is not informational rent of period two because of the dynamics of the game.

## 1.4 Efficiency

First, we show that without correlation, the model is a repetition of MS result. Expected payment does not depend on the allocation in the second period so we do not have any useful instrument. Second, if we introduce correlation, we show that if the correlation is defined by first-order stochastic dominance, then critical types do not change and there is no efficient mechanism. Finally, if we make property right a control variable, we demonstrate that it is useless because the optimal is to choose extreme property right again.

### 1.4.1 Efficiency without serial correlation

Without correlation between periods, both games are games *à* la MS such as we already know that that $i's$ critical value is one and $-i's$ critical value is zero in period one. In period two, critical types depend on the assignation of the good. If the situation is not changed, the game is just repeated. We apply the result of Athey and Miller (2007) to a two-periods game to show that ex-ante budget feasibility cannot be reach unless the discount factor is one. Therefore, we want to know if feasibility can be reached using $r_2$ as an instrument between periods.

**Proposition**: In a two-periods game with two agents with absolute property rights and without serial correlation, the mechanism is incentive compatible, ex-post efficient, with voluntary participation and ex-ante budget feasible if and only if the discount factor is one.

**Proof:** See Appendix 1.

**Theorem**: In a two-periods game with two agents with absolute property rights in period one but a flexible property right in period two, it is helpless to use allocation in period two has an instrument if there is no serial correlation in the type. Thus, there is no mechanism that is incentive compatible, ex-post efficient, individual rational and budget feasible.

**Proof :** See Appendix 2.

### 1.4.2 Efficiency with serial correlation and fixed second-period allocations

We now introduce serial correlation. As we explained, correlation is simply defined as the dependency of the distribution of $\theta_{i2}$ to $\theta_{i1}$. First, we look for an efficient mechanism if $r_2$ is fixed such as the allocation is $r_1 = r_2 = 1$ in both periods. In this context of absolute property rights, the mechanism only uses payment as instrument so we need to find the critical types and then calculate expected payment to know if the mechanism can be budget-feasible. We know that the critical types in period two are extreme because the game is exactly a MS game. In period one, if the critical types are corner solutions, then the game is exactly *à* la MS and we already know that no efficient mechanism can be budget feasible. To determine the value of the critical types using Lemma (1), we study the derivatives of the expected payoff. For this we will use the particular form of stochastic dominance for the correlation.

**Assumption:** The distribution $F_{i2}(.|\theta_{i1})$ satisfies first-order stochastic dominance in $\theta_{i1}$, that is :

$$F_{i2}(x|\theta_{i1}) \leq F_{i2}(x|\theta'_{i1})\ \forall x \ \ if\ \theta_{i1} > \theta'_{i1} \quad \text{for positive correlation}$$

**Theorem :** In a two-period game with two agents with absolute property rights in both periods and serial positive correlation in the type, if correlation is defined by first-order stochastic dominance, then a mechanism cannot be incentive compatible, ex-post efficient, individual rational and budget feasible.

**Proof:** In the case of a fixed second-period allocation, $r_2$ does not appear in the list of dependent variables of the expected value of the payoff in period two because agents must take it as a data. We have:

$$V_{i1}^{'}(\theta_{i1}) = Q_{i1}(\theta_{i1}) + \delta \mathbb{E}_{\theta_{-i1}} \frac{\partial}{\partial x} \mathbb{E}_{\theta_{i2}|x} V_{i2}(\theta_{i1}, \theta_{-i1}, \theta_{i2}) + \delta \mathbb{E}_{\theta_{-i1}, \theta_{i2}} \frac{\partial}{\partial \theta_{i1}} V_{i2}(\theta_{i1}, \theta_{-i1}, \theta_{i2}) \tag{1.12}$$

$$V_{i1}^{0'}(\theta_{i1}) = r_1 + \delta\ r_1 \frac{\partial}{\partial x} \mathbb{E}_{\theta_{i2}|x}\left[\theta_{i2}\right] \tag{1.13}$$

In order to determine if the critical type is interior or not, we need to compare (12) and (13) in the maximum possible value of $\theta_{i1}$, that we assume is 1. Since $r_1 = 1$, we have that $Q_{i1}(1) = r_1$ so we compare the other elements. By Lemma (2), the last element of (12) is null if the interval of $\theta_{i2}$ is fixed. If it is not fixed, the critical type in period two depends on $\theta_{i1}$. However, using (10) and lemma (1) in $\theta_{i1} = 1$, the critical type in period two for agent i is the maximum possible value, which is by serial positive correlation higher than the maximum possible value of agent $-i$, so $Q_{i2}(\bar{\theta}_{i2}) = 1$ and (10) is null. We only have to compare the second elements of (12) and (13).

First, both are positive. In effect, by theorem we know that if $F(.|\theta_{i1})$ first-order stochastically dominates $F(.|\theta_{i1}^{'})$ then we have that for any nondecreasing function $H(\theta_{i2})$:

$$\int H(\theta_{i2}) f(\theta_{i2}|\theta_{i1}) d\theta_{i2} \geq \int H(\theta_{i2}) f(\theta_{i2}|\theta_{i1}^{'}) d\theta_{i2}$$

such as

$$\frac{\partial}{\partial \theta_{i1}} \left[ \int H(\theta_{i2}) f(\theta_{i2}|\theta_{i1}) d\theta_{i2} \right] \geq 0$$

In both cases, it is direct to see that $H(\theta_{i2})$ ($V_{i2}(.)$ and $\theta_{i2}$) is a nondecreasing function so both derivatives are positive.

Second, since $V_{i2}(\theta_{i1}=1,\theta_{-i1},\theta_{i2})$ and $\theta_{i2}$ do not depend on $\theta_{i1}$, we have to compare how they change with $\theta_{i2}$. By one hand, $\frac{\partial\theta_{i2}}{\partial\theta_{i2}}=1$, and by the other hand $\frac{\partial V_{i2}(1,\theta_{-i1},\theta_{i2})}{\partial\theta_{i2}}=\mathbb{E}_{\theta-i1}Q_{i2}(\theta_{i2}|\theta_{-i1})$ which is less than one so

$$\frac{\partial}{\partial\theta_{i1}}\mathbb{E}_{\theta_{i2}}V_{i2}(1,\theta_{-i1},\theta_{i2})\leq\frac{\partial}{\partial\theta_{i1}}\mathbb{E}_{\theta_{i2}}\left[\theta_{i2}\right]$$

Therefore, the derivative in $\theta_{i1}=1$ of the outside option is bigger than the derivative of the inside option. The critical type is not an interior solution but the same as in the Myerson-Sattherwaite case, $\theta^*_{i1}=1$. Moreover, it is direct to see that $\theta^*_{-i1}=0$ because his outside option is null. ■

### 1.4.3 Efficiency with serial correlation and flexible second-period allocations

Since the mechanism cannot be efficient if the unique tool is the payments, we modify the mechanism in order to make the allocation in period two an instrument. The redistribution of the good between both periods changes the outside option in the second period. The new allocation $r_2$ does not affect directly the expected payoff in period two but it affects the critical type because it changes the outside option, which changes the expected payoff. In period one, $r_2$ does not affect directly the outside option because the agent chooses to go out the game before the realization of the new allocation. However, it affects $V_1(.)$ through two effects. In one hand through $V_2(.)$ since $V_1(.)$ takes into count the future expected payoff. On the other hand, $r_2$ affects the critical type in period one since it affects the derivative of $V_2(.)$ which appears in $V_1^{'}(.)$. Notation here is very important and can be a little confusing. We will write $r_2$ as a dependent variable every time it is necessary to make clear where the allocation plays a role. The election of the allocation in the second period is made in order to maximize the expected payment (11). We rewrite it underlining the relations with $r_2$:

$$\begin{aligned}&\mathbb{E}_{\theta_{i1},\theta_{-i1}}\left(Q_{i1}(\theta_{i1}).\theta_{i1}+Q_{-i1}(\theta_{-i1}).\theta_{-i1}\right)\\&+\ \delta\ \mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{i2}}\left[Q_{i2}(\theta_{i2})\theta_{i2}+Q_{-i2}(\theta_{-i2})\theta_{-i2}\right]\\&+\ \mathbb{E}_{\theta_{i1},\theta_{-i}}\left[-V_{i1}(\theta^*_{i1}(r_2))-V_{-i1}(\theta^*_{-i1}(r_2))+\int_{s=\theta_{i1}}^{\theta^*_{i1}(r_2)}V_{i1}^{'}(s,r_2)ds-\int_{s=\theta^*_{-i1}(r_2)}^{\theta_{-i1}}V_{-i1}^{'}(s,r_2)ds\right]\end{aligned}\tag{1.14}$$

In the special case of $V_{i1}(\theta^*_{i1}(r_2))$, we do not need to write $V_{i1}(\theta^*_{i1}(r_2),r_2)$ because as it is usual in mechanism design, we use the expression of the outside option evaluated in the critical type and we already explained that the outside option does not depend directly on the allocation of period two. We see that only the last line depends on $r_2$, that is only the informational rent. Therefore, to maximize the expected payment, we need to choose the allocation that minimizes the informational rent.

**Theorem**: In a two-periods game with two agents with absolute property rights in period one but flexible allocation in period two and serial positive correlation in the types, if correlation is defined by first-order stochastic dominance, then a mechanism cannot be incentive compatible, ex-post efficient, individual rational and budget feasible.

**Proof**:

We are looking for the allocation $r_2$ that maximizes the last line of (14). We can separate the problem between agents. First we show that the problem can be simplified. If we analyse $i's$ informational rent, we have:

$$\mathbb{E}_{\theta_{i1},\theta_{-i}}\left[-V_{i1}(\theta^*_{i1}(r_2))+\int_{s=\theta_{i1}}^{\theta^*_{i1}(r_2)}V^{'}_{i1}(s,r_2)ds\right] \tag{1.15}$$

Taking the derivative with respect to $r_2$, we have to use Leibniz theorem:

$$\mathbb{E}_{\theta_{i1},\theta_{-i}}\left[-V^{'}_{i1}(\theta^*_{i1}(r_2))\frac{\partial\theta^*_{i1}}{\partial r_2}-\frac{\partial V_{i1}(\theta^*_{i1}(r_2))}{\partial r_2}+V^{'}_{i1}(\theta_{i1}(r_2),r_2)+\int_{s=\theta_{i1}}^{\theta^*_{i1}(r_2)}\frac{\partial V^{'}_{i1}(s,r_2)}{\partial r_2}ds\right] \tag{1.16}$$

The first term cancels with the third term and as we explained, the second term is null. Then, we only need to use the last term. Since it happens the same for the other agent, our problem is finally with the difference of the derivatives:

$$\mathbb{E}_{\theta_{i1},\theta_{-i}}\left[\int_{s=\theta_{i1}}^{\theta^*_{i1}(r_2)}\frac{\partial V^{'}_{i1}(s,r_2)}{\partial r_2}ds-\int_{s=\theta^{*(r_2)}_{-i1}}^{\theta_{-i1}}\frac{\partial V^{'}_{-i1}(s,r_2)}{\partial r_2}ds\right] \tag{1.17}$$

To study the derivatives with respect to $r_2$, we return to the expression of the derivative of the payoff in period one with respect to the type. Recall that:

$$V^{'}_{i1}(\theta_{i1})=Q_{i1}(\theta_{i1})+\underbrace{\delta\mathbb{E}_{\theta_{-i1}}\frac{\partial}{\partial x}\mathbb{E}_{\theta_{i2}|x}V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1}))}_{A}+\underbrace{\delta\mathbb{E}_{\theta_{-i1},\theta_{i2}}\frac{\partial}{\partial\theta_{i1}}V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1}))}_{B} \tag{1.18}$$

The first term does not depend on $r_2$ so we do not analyse it. We separate the study of $A$ and $B$. We first show that $A$ is independent of $r_2$.

- Study of A

For any distribution defined with positive correlation, we will have that $\frac{\partial}{\partial x}\mathbb{E}_{\theta_{i2}|x}V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1}))$ is independent of $r_2$.

In effect, since we are interesting in the derivative of the distribution $f(\theta_{i2}|\theta_{i1})$, we take the derivative with respect to the expected value of the payoff in period two with $\theta_{i1}$ fixed in $V_{i2}$. However, by (8), the payoff in period two depends on the allocation only through the critical type and the outside option. Then, taking the integral on $\theta_{i2}$, the terms that depend on the allocation do not depend on $x$ so the derivative with respect to $x$ is null. By(8), we have

$$\begin{aligned}V_{i2}(\theta_{i1},\theta_{-i1},\theta_{i2},r_2(\theta_{i1},\theta_{-i1}))&=r_2(\theta_{i1},\theta_{-i1})\theta^*_{i2}\left(r_2(\theta_{i1},\theta_{-i1}),\theta_{i1},\theta_{-i1}\right)+\int_{s=\theta^*_{i2}(r_2(\theta_{i1},\theta_{-i1}),\theta_{i1},\theta_{-i1})}^{\theta_{i2}}Q_{i2}(s|\theta_{-i1})ds\\&=r_2(\theta_{i1},\theta_{-i1}))\theta^*_{i2}\left(r_2(\theta_{i1},\theta_{-i1}),\theta_{i1},\theta_{-i1}\right)+M(\theta_{i2},\theta_{-i1})\\&\quad+N(\theta^*_{i2}\left(r_2(\theta_{i1},\theta_{-i1}),\theta_{i1},\theta_{-i1}\right),\theta_{-i1})\end{aligned}$$

where $M(.)$ and $N(.)$ are the integrated values. Taking expectation

$$\begin{aligned}
\mathbb{E}_{\theta_{i2}|x} V_{i2}(.) &= \int_{\theta_{i2}=0}^{\bar{\theta}_{i2}} r_2(\theta_{i1}, \theta_{-i1}))\theta_{i2}^* \left(r_2(\theta_{i1}, \theta_{-i1}), \theta_{i1}, \theta_{-i1}\right) f(\theta_{i2}|x) d\theta_{i2} \\
&+ \int_{\theta_{i2}=0}^{\bar{\theta}_{i2}} M(\theta_{i2}, \theta_{-i1}) f(\theta_{i2}|x) d\theta_{i2} \\
&+ \int_{\theta_{i2}=0}^{\bar{\theta}_{i2}} N(\theta_{i2}^* \left(r_2(\theta_{i1}, \theta_{-i1}), \theta_{i1}, \theta_{-i1}\right), \theta_{-i1}) f(\theta_{i2}|x) d\theta_{i2} \\
&= r_2(\theta_{i1}, \theta_{-i1})\theta_{i2}^* \left(r_2(\theta_{i1}, \theta_{-i1}), \theta_{i1}, \theta_{-i1}\right) \\
&+ N(\theta_{i2}^* \left(r_2(\theta_{i1}, \theta_{-i1}), \theta_{i1}, \theta_{-i1}\right), \theta_{-i1}) \\
&+ \int_{\theta_{i2}=0}^{\bar{\theta}_{i2}} M(\theta_{i2}, \theta_{-i1}) f(\theta_{i2}|x) d\theta_{i2}
\end{aligned}$$

Then, if we take the derivative with respect to $x$ ($\theta_{i1}$ in the function of distribution), only the last line survives and this does not depend on $r_2$. Therefore $A$ is independent of the allocation of the second period. The development for agent $-i$ is parallel.

- Study of B

Instead of studying $B$, we use the difference of this term for both agents and we call the derivative of this difference with respect to the allocation by the function $H(r_2)$.

Using (10) [2] for replacing the expression of $B$ and taking the derivative, we have

$$\begin{aligned}
H(r_2) &= \mathbb{E}_{\theta_{i1}, \theta_{-i1}} \int_{s=\theta_{i1}}^{\theta_{i1}^*(r_2)} \frac{\partial \theta_{i2}^*(s)}{\partial s} \left[r_2(s, \theta_{-i1}) - Q_{i2}(\theta_{i2}^*(s))\right] \, ds \\
&- \mathbb{E}_{\theta_{i1}, \theta_{-i1}} \int_{s=\theta_{-i1}^*(r_2)}^{\theta_{-i1}} \frac{\partial \theta_{-i2}^*(s)}{\partial s} \left[1 - r_2(\theta_{i1}, s) - Q_{-i2}(\theta_{-i2}^*(s)\right] ds
\end{aligned} \tag{1.19}$$

and we look for the allocation that maximizes (19).

By Lemma (2), we know that (19) is not null only if the critical type is the maximum possible value. In this particular case, we have that $V_{i2}^{'}(\bar{\theta}_{i2}(s)) \leq r_2(s, \theta_{-i1})$ and $V_{-i2}^{'}(\bar{\theta}_{-i2}(s)) \leq 1 - r_2(\theta_{i1}, s)$ , which is the same as $\bar{\theta}_{i2}(s) \leq V_{i2}^{'-1}(r_2(s, \theta_{-i1}))$ and $\bar{\theta}_{-i2}(s) \leq V_{-i2}^{'-1}(1 - r_2(\theta_{i1}, s))$. Noting $\hat{a}, \hat{b}$ the value of $\theta_{i1}, \theta_{-i1}$ such as $\bar{\theta}_{i2}(\hat{a}) = V_{i2}^{'-1}(r_2(\hat{a}, \theta_{-i1}))$ and $\bar{\theta}_{-i2}(\hat{b}) = V_{-i2}^{'-1}(1 - r_2(\theta_{i1}, \hat{b}))$, we can write:

$$\begin{aligned}
H(r_2) &= \mathbb{E}_{\theta_{i1}, \theta_{-i1}} \int_{s=\theta_{i1}}^{\min\{\theta_{i1}^*, \hat{a}\}} \frac{\partial \bar{\theta}_{i2}(s)}{\partial s} \left[r_2(s, \theta_{-i1}) - Q_{i2}(\bar{\theta}_{i2}(s))\right] \, ds \\
&- \mathbb{E}_{\theta_{i1}, \theta_{-i1}} \int_{s=\theta_{-i1}^*}^{\min\{\theta_{-i1}, \hat{b}\}} \frac{\partial \bar{\theta}_{-i2}(s)}{\partial s} \left[1 - r_2(\theta_{i1}, s) - Q_{-i2}(\bar{\theta}_{-i2}(s))\right] ds
\end{aligned} \tag{1.20}$$

When $\hat{a}$ or $\hat{b}$ are the minimum, the derivative with respect to $r_2$ is obtained by Leibniz. We show that the term caused by the dependence of $\hat{a}$ to $r_2$ is null. We develop it for agent $i$ but it is the same for agent $-i$. By Leibniz, the derivative with respect to $r_2$ of the first part of $H(r_2)$ is:

$$\frac{\partial}{\partial r_2} = \mathbb{E}_{\theta_{i1}, \theta_{-i1}} \left[ \int_{s=\theta_{i1}}^{\hat{a}} \frac{\partial \bar{\theta}_{i2}(s)}{\partial s} ds + \frac{\partial \bar{\theta}_{i2}(\hat{a})}{\partial s} \left[r_2(\hat{a}, \theta_{-i1}) - Q_{i2}(\bar{\theta}_{i2}(\hat{a}))\right] \frac{\partial \hat{a}(r_2)}{\partial r_2} \right]$$

[2] The expectation on $\theta_{i2}$ disappears because (10) does not depend on it.

but in $\hat{a}$, $\bar{\theta}_{i2}(\hat{a}) = V_{i2}^{\prime -1}(r_2(\hat{a}, \theta_{-i1}))$ so $Q_{i2}(\bar{\theta}_{i2}(\hat{a})) = V_{i2}' \left[V_{i2}^{\prime -1}(r_2(\hat{a}, \theta_{-i1}))\right] = r_2(\hat{a}, \theta_{-i1})$ and the last term is null. Thus, we do not need to take the derivative with respect to the variables of integration.

In order to take the derivative with respect to $r_2$, we use directional derivative:

$$\begin{aligned}
\frac{\partial}{\partial r_2} H(r_2) =& \frac{\partial}{\partial \epsilon}\bigg|_{\epsilon=0} H(r_2 + \epsilon\, g(\theta_{i1}, \theta_{-i1})) \\
=& \frac{\partial}{\partial \epsilon}\bigg|_{\epsilon=0} \mathbb{E}_{\theta_{i1},\theta_{-i1}} \int_{s=\theta_{i1}}^{\min\{\theta_{i1}^*, \hat{a}\}} \frac{\partial \bar{\theta}_{i2}(s)}{\partial s} \left[r_2(s, \theta_{-i1}) + \epsilon\, g(s, \theta_{-i1}) - Q_{i2}(\bar{\theta}_{i2}(s))\right] ds \\
&- \frac{\partial}{\partial \epsilon}\bigg|_{\epsilon=0} \mathbb{E}_{\theta_{i1},\theta_{-i1}} \int_{s=\theta_{-i1}^*}^{\min\{\theta_{-i1}, \hat{b}\}} \frac{\partial \bar{\theta}_{-i2}(s)}{\partial s} \left[1 - (r_2(\theta_{i1}, s) + \epsilon\, g(\theta_{i1}, s)) - Q_{-i2}(\bar{\theta}_{-i2}(s))\right] ds \\
=& \mathbb{E}_{\theta_{i1},\theta_{-i1}} \int_{s=\theta_{i1}}^{\min\{\theta_{i1}^*, \hat{a}\}} \frac{\partial \bar{\theta}_{i2}(s)}{\partial s} g(s, \theta_{-i1}) ds \\
&- \mathbb{E}_{\theta_{i1},\theta_{-i1}} \int_{s=\theta_{-i1}^*}^{\min\{\theta_{-i1}, \hat{b}\}} \frac{\partial \bar{\theta}_{-i2}(s)}{\partial s} \left[-g(\theta_{i1}, s)\right] ds
\end{aligned}$$

We see directly that the final equality is positive so $H(r_2)$ is increasing in $r_2$ : the maximization leads to choose $r_2 = 1$, which is the same as the initial distribution of property rights in the first period. The best we can choose is a fixed allocation. To finish the demonstration, we use the proof of the last theorem. ■

The model shows the impossibility of existence of a mechanism that would be incentive-compatible, ex-post efficient, individual rational and budget balanced if the correlation is defined by first-order stochastic dominance, no matter $r_2$ be flexible or not. We now illustrate our results with the uniform distribution.

## 1.5 Example : Uniform distribution

Types $\theta_{i1}$,$\theta_{i2}$ are drawn from a uniform distribution $[0, 1]$ in period one and $[0, h(\theta_{i1})]$ in period two with $\frac{\partial h(\theta_{i1})}{\partial \theta_{i1}} > 0$, that is positive correlation. We study fixed and flexible second-period allocations. To determinate if it exists an efficient mechanism, we must encounter the critical types.

### 1.5.1 fixed allocation $r_2$

Since in period 2 we do not have more periods, the system is the same as in a one-period game. With positive correlation, if $\theta_{i1} > \theta_{-i1}$, the maximum value $(h(\theta_{i1}))$ that $\theta_{i2}$ can reach is higher that the maximum of $\theta_{i2}$ such as $i$ is sure to win when his type is higher than $h(\theta_{-i1})$. In this case, the derivative of the expected payoff in period 2 is 1 and any type in this part is a critical type so we decide to use the first point, which is $h(\theta_{-i1})$. When $\theta_{i2} < h(\theta_{-i1})$, the critical type is the solution to $Q_{i2}(\theta_{i2}) = 1$ which is also $h(\theta_{-i1})$. On the other side, if $\theta_{i1} < \theta_{-i1}$ the probability to win is less than one but there are two cases. The maximum value of $Q_{i2}(\theta_{i2}|\theta_{-i1})$ is reached in $\theta_{i2} = h(\theta_{i1})$. If $Q_{i2}(h(\theta_{i1})|\theta_{-i1}) \leq r_i$, the critical type is the highest possible value whereas in the other case, is the solution to the equalization. Since $r_i = 1$, the second case does not occur and the critical type is $h(\theta_{i1})$. Thus we have:

$$\theta_{i2}^* = \begin{cases} h(\theta_{-i1}) & \text{if } \theta_{i1} \geq \theta_{-i1} \\ h(\theta_{i1}) & \text{if } \theta_{i1} < \theta_{-i1} \end{cases}$$

In period one, we compare (7) with (8). First, using (9), it is easy to see that the last element of (7) disappears. In effect, in $\theta_{i1} = 1$ positive correlation induces that $h(\theta_{i1}) > h(\theta_{-i1})$ and so $Q_{i2}(h(\theta_{i1})|\theta_{-i1}) = 1$.

Then, by stochastic dominance the derivative of the inside option is less than the derivative of the outside option so the critical type for $i$ in period one is one.
For agent $-i$, it is direct to see that since outside options in both periods are null, critical types are zero. In this special case, critical types are à la MS so we know that the mechanism would need an outside subsidy to be efficient.

We are interesting in positive correlation because it is more intuitive but it is interesting to underline that the results may change with negative correlation. In our case, if $Q_{i2}(\theta_{i2}^*|\theta_{-i1}) < 1$ and $\frac{\partial\theta_{i2}^*(\theta_{i1})}{\partial\theta_{i1}} < 0$, the last element of (7) is negative. On the other side, stochastic dominance with negative correlation implies that the second elements of (7) and (8) are both negative and that the first is bigger than the second.

We cannot conclude on the sign of the comparison between the derivatives of the outside and inside options. If the last element of (7) is negative enough, the result is the same as with positive correlation, that is to say the derivative of the outside option is bigger than the derivative of the expected payoff and the solution is a corner one. On the contrary, if it is small enough, the solution could be an interior one. We illustrate this result by an example.

**Example**

The types are drawn from a uniform function $[0,1]$ in the first period and $[0, h(\theta_{i1}) = a - b\theta_{i1}]$ in the second period with $a \geq b$. Agent $i$ owns the good.

The outside option is defined by :

$$V_{i1}^0 = \theta_{i1} + \delta \int_{\theta_{i2}=0}^{a-b\theta_{i1}} \theta_{i2}\frac{1}{a-b\theta_{i1}}d\theta_{i2}$$

such as

$$V_{i1}^{0'}|_{\theta_{i1}=1} = 1 - \delta\,\frac{b}{2}$$

In order to find the derivative of the expected payoff in period one, we need to develop the expressions. First,

$$V_{i2}(\theta_{i2}|\theta_{-i1}) = \theta_{i2}^* + \int_{s=\theta_{i2}^*}^{\theta_{i2}} Q_{i2}(s|\theta_{-i1})ds$$

$$\int_{s=\theta_{i2}^*}^{\theta_{i2}} Q_{i2}(s|\theta_{-i1})ds = \begin{cases} \displaystyle\int_{s=\theta_{i2}^*}^{\theta_{i2}} \frac{s}{a-b\theta_{i1}}ds & \text{if } \theta_{i2} < a - b\theta_{-i1} \\ \displaystyle\int_{s=\theta_{i2}^*}^{a-b\theta_{-i1}} \frac{s}{a-b\theta_{i1}}ds + \int_{s=a-b\theta_{-i1}}^{\theta_{i2}} 1ds & \text{ow} \end{cases}$$

then

$$V_{i2}(\theta_{i2}|\theta_{-i1}) = \begin{cases} \theta_{i2}^* + \dfrac{1}{a-b\theta_{-i1}}\left(\dfrac{1}{2}\theta_{i2}^2 - \dfrac{1}{2}\theta_{i2}^{*2}\right) & \text{if } \theta_{i2} < a - b\theta_{-i1} \\ \theta_{i2}^* + \theta_{i2} - \dfrac{1}{2}(a - b\theta_{-i1}) - \dfrac{1}{2}\dfrac{\theta_{i2}^{*2}}{a-b\theta_{-i1}} & \text{ow} \end{cases}$$

with

$$\theta_{i2}^* = \begin{cases} h(\theta_{-i1}) & \text{if } \theta_{i1} < \theta_{-i1} \\ h(\theta_{i1}) & \text{if } \theta_{i1} \geq \theta_{-i1} \end{cases}$$

$$\begin{aligned}\int_{\theta_{-i1}=0}^{1} V_{i2}(\theta_{i2}|\theta_{-i1})f(\theta_{-i1})d\theta_{-i1} &= \int_{\theta_{-i1}=0}^{\theta_{i1}} h(\theta_{i1}) + \frac{1}{2(a-b\theta_{-i1})}\theta_{i2}^2 - \frac{1}{2(a-b\theta_{-i1})}h(\theta_{i1})^2 d\theta_{-i1} \\ &+ \int_{\theta_{-i1}=\theta_{i1}}^{(a-\theta_{i2})/b} h(\theta_{-i1}) + \frac{1}{2(a-b\theta_{-i1})}\theta_{i2}^2 - \frac{1}{2(a-b\theta_{-i1})}h(\theta_{-i1})^2 d\theta_{-i1} \\ &+ \int_{\theta_{-i1}=(a-\theta_{i2})/b}^{1} h(\theta_{-i1}) + \theta_{i2} - \frac{a-b\theta_{-i1}}{2} - \frac{1}{2(a-b\theta_{-i1})}h(\theta_{-i1})^2 d\theta_{-i1}\end{aligned}$$

Calculating, we have :

$$\frac{\partial}{\partial\theta_{i1}}\int_{\theta_{i2}=0}^{a-b\theta_{i1}}\int_{\theta_{-i1}=0}^{1} V_{i2}(\theta_{i2}|\theta_{-i1})f(\theta_{-i1})d\theta_{-i1}f(\theta_{i2}|\theta_{i1})d\theta_{i2} = \frac{-19}{18}b + \frac{1}{18}a + \frac{2}{3}(b-a)ln(a-b) + \frac{2}{3}ln(a)$$

So we have :

$$V_{i1}^{0'}|_{\theta_{i1}=1} = 1 + \delta\ \frac{-b}{2}$$

$$V_{i1}^{'}|_{\theta_{i1}=1} = 1 + \delta\ \left(\frac{-19}{18}b + \frac{1}{18}a + \frac{2}{3}(b-a)ln(a-b) + \frac{2}{3}ln(a)\right)$$

If $\delta = 0.9$, $a = 2$, we have :

- if $b = 0.3$ : $V_{i1}^{0'}|_{\theta_{i1}=1} = 0.865$ and $V_{i1}^{'}|_{\theta_{i1}=1} = 0.9811$ so the critical type is interior
- if $b = 1.25$ : $V_{i1}^{0'}|_{\theta_{i1}=1} = 0.4375$ and $V_{i1}^{'}|_{\theta_{i1}=1} = 0.3547$ so the critical type is the corner solution 1

Therefore, with positive correlation, it is obvious that we have the same impossibility result than Myerson-Satterthwaite and with negative correlation, it could happen that the critical type is an interior solution. Positive correlation is more intuitive in our concern so we do not develop the model for negative correlation .

### 1.5.2 flexible allocation $r_2$

We allow the principal to use $r_{i2}$ as a control variable. Critical types are determined as usual. In period two, the derivative of the inside option is the probability to win. If $\theta_{i2}$ is greater than the maximum agent $-i$ can have, the probability to win is one. In other case, it is $\frac{\theta_{i2}}{h(\theta_{-i1})}$. There are three cases for the critical type. (GRAFICO??) If $\theta_{i1} > \theta_{-i1}$ it is the maximum possible agent $-i$ can have adjusted by the proportion of the good agent $i$ has: $r_{i2}h(\theta_{i1})$. If $\theta_{i1} < \theta_{-i1}$, the probability to win is less than one. If the probability to win agent $i$ evaluated in $h(\theta_{i1})$ is less than $r_{i2}$, the critical type of agent $i$ is the maximum value he can reach $h(\theta_{i1})$ whereas if it is higher, the critical type results from the equalization, and it is $r_{i2}h(\theta_{-i1})$. We have the same for agent $-i$ but we interchange agents. That is:

$$\theta_{i2}^* = \begin{cases} r_{i2}h(\theta_{-i1}) & \text{if } \theta_{i1} \geq \theta_{-i1} \\ h(\theta_{i1}) & \text{if } \theta_{i1} < \theta_{-i1} \text{ and } \dfrac{h(\theta_{i1})}{h(\theta_{-i1})} < r_{i2} \\ r_{i2}h(\theta_{-i1}) & \text{if } \theta_{i1} < \theta_{-i1} \text{ and } \dfrac{h(\theta_{i1})}{h(\theta_{-i1})} > r_{i2} \end{cases}$$

In period one, we compare the derivative of the inside and outside options. In element $B$, we have:

$$\frac{\partial V_{i2}(.)}{\partial \theta_{i1}} = \frac{\partial h(\theta_{i1})}{\partial \theta_{i1}} \left[ r_2(\theta_{i1}, \theta_{-i1}) - Q_{i2}(h(\theta_{i1})) \right] \text{ iif } \theta_{i1} < \theta_{-i1} \text{and } \frac{h(\theta_{i1})}{h(\theta_{-i1})} < r_2(\theta_{i1}, \theta_{-i1}) \tag{1.21}$$

$$\frac{\partial V_{-i2}(.)}{\partial \theta_{-i1}} = \frac{\partial h(\theta_{-i1})}{\partial \theta_{-i1}} \left[ (1 - r_2(\theta_{i1}, \theta_{-i1})) - Q_{-i2}(h(\theta_{-i1})) \right] \text{ iif } \theta_{-i1} < \theta_{i1} \text{and } \frac{h(\theta_{-i1})}{h(\theta_{i1})} < (1 - r_2(\theta_{i1}, \theta_{-i1})) \tag{1.22}$$

Since we want to study how it changes with $r_2$, we have to take the directional derivative as before. We choose the lineal form $h(\theta_{i1}) = b\theta_{i1}$. Any form of the type $h(\theta_{i1}) = b\theta_{i1}^n + a$ with $b > 0$,$a > 0$ and $n > 1$ gives the same result.

$$\begin{aligned} H(r) \quad = \quad & \int_{\theta_{-i1}=0}^{1} \int_{\theta_{i1}=0}^{1} \int_{\{s|s>\theta_{i1} \wedge r_{i2}(s,\theta_{-i1})\theta_{-i1}>s\}} b \left[ r_2(s, \theta_{-i1}) - \frac{s}{\theta_{-i1}} \right] ds \, d\theta_{i1} \, d\theta_{-i1} \\ & - \int_{\theta_{i1}=0}^{1} \int_{\theta_{-i1}=0}^{1} \int_{\{s|s<\theta_{-i1} \wedge (1-r_{i2}(\theta_{i1},s))\theta_{i1}>s\}} b \left[ (1 - r_2(\theta_{i1}, s)) - \frac{s}{\theta_{i1}} \right] ds \, d\theta_{-i1} \, d\theta_{i1} \end{aligned} \tag{1.23}$$

It is not possible to develop the calculus but it is easy to see that $H(r)$ is increasing in $r_{i2}$ such as the maximization leads to conclude that we have to put $r_{i2} = 1$ and $r_{-i2} = 0$ ! Therefore, the best we can do is the solution without having $r_2$ as a control variable and we already showed that we couldn't attain efficiency and budget balance.

## 1.6 Conclusion

We extended the absolute ownership problem to a two-period game with serial correlation. Without correlation, results are intuitive since the game is simply repeated twice. With correlation, the critical types can change in the first-period because of the relation between present and future. Yet, we show that the relation is not strong enough to break the impossibility result. Finally, we develop a game in which the property right is a control variable that can be manipulated by the regulator in order to minimize the informational rent. We show that the minimization results in the initial absolute ownership, concluding that it is useless to make the property rights a control variable. Our results are in the line of the works of Jehiel and Pauzner (2006)and Segal and Whinston (2014).

## 1.7 Appendix

### Proof 1

To demonstrate that the discount factor must be at least one, we follow the proof of Athey and Miller (2007). We define the ex-post gains from efficient trade $v^{get}(\theta)$ as the gains the group receives if he trades efficiently. The utility of agent $i$ in period $t$ is simply $u_i(\theta) = \theta_{it} q_{it} - x_{it}$ with $\theta$ his type, $q$ his probability to win and $x$ his payment. In period one, agents has their utility and a discounted promised utility for tomorrow $\delta w(\theta)$, whereas in period two, there is no future so agents receive only their utility. Patience defines the role of the promised utility. If agents are sufficiently patient, they are ready to pay more today because they know they will receive something tomorrow. Following the theorem 1 of Athey and Miller, we define the promised utilities. If we let $i$ be the "seller", that is $r_i = 1$, we have:

$$w_{-i}(\theta) = \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right]$$
$$w_i(\theta) = \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right] + \mathbb{E}\left[\theta_{i2}\right]$$

Then the payments are constructed as a VCG mechanism in period one whereas in period two, it is not possible to make agents pay more since there is no future:

$$x_{-i1} = \theta_{i1}q^*_{-i1}(\theta) + \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right]$$
$$x_{i1} = -\theta_{-i1}q^*_{-i1}(\theta) + \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right]$$
$$x_{-i2} = \theta_{i2}q^*_{-i2}(\theta)$$
$$x_{i2} = -\theta_{-i2}q^*_{-i2}(\theta)$$

where $q^*_{-it}(\theta) = 1$ if and only if $\theta_{-it} \geq \theta_{it}$.

Replacing in the utilities, we have:

$$u_{-i1} = (\theta_{-i1} - \theta_{i1})q^*_{-i1}(\theta) - \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right]$$
$$u_{i1} = \theta_{i1} + (\theta_{-i1} - \theta_{i1})q^*_{-i1}(\theta) - \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right]$$
$$u_{-i2} = (\theta_{-i1} - \theta_{i1})q^*_{-i2}(\theta)$$
$$u_{i2} = \theta_{i2} + (\theta_{-i1} - \theta_{i1})q^*_{-i2}(\theta)$$

Ex-post voluntary participation requires that the total utility of agent $-i$ be more than zero and that the total utility of agent $i$ be more than his outside option $\theta_{i1} + \delta\mathbb{E}\left[\theta_{i2}\right]$, that is

$$\begin{aligned} U_{-i}(\theta) &= u_{-i1} + \delta w_{-i} + \delta u_{-i2} \\ &= (\theta_{-i1} - \theta_{i1})q^*_{-i1}(\theta) + \delta(\theta_{-i1} - \theta_{i1})q^*_{-i2}(\theta) + \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right](\delta - 1) \geq 0 \\ U_i(\theta) &= \theta_{i1} + (\theta_{-i1} - \theta_{i1})q^*_{-i1}(\theta) + \delta\left(\theta_{i2} + (\theta_{-i1} - \theta_{i1})q^*_{-i2}(\theta)\right) + \delta\mathbb{E}\left[\theta_{i2}\right] + \frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right](\delta - 1) \geq \theta_{i1} + \delta\mathbb{E}\left[\theta_{i2}\right] \end{aligned}$$

Voluntary participation must exist in every possible type, in particular in the worst par $(\underline{\theta}_{-i}, \overline{\theta}_i)$. In this case, $-i$ does not receive the object and all the terms with $q^*_{-it}(\theta)$ are zero. Then, the condition is that $\frac{1}{2}\mathbb{E}\left[v^{get}(\theta)\right](\delta - 1) \geq 0$, which is satisfied only for $\delta$ more or equal to zero. ■

## Proof 2

The expected payoff in period two does not depend on the type in period one so the derivative is null. The derivative of the expected payoff in period one is only the probably to win. We have :

$$V_{i2}(\theta'_{i2}) = \max_{\theta'_{i2}} Q_{i2}(\theta'_{i2})\theta_{i2} - X_{i2}(\theta'_{i2})$$

$$V_{i1}(\theta'_{i1}) = \max_{\theta'_{i1}} Q_{i1}(\theta'_{i1})\theta_{i1} - X_{i1}(\theta'_{i1}) + \delta\mathbb{E}_{\theta_{i2}}V_{i2}(\theta_{i2}, r_2)$$

with

$$V'_{i1}(\theta_{i1}) = Q_{i1}(\theta_{i1})$$

$$V'_{i2}(\theta_{i2}) = Q_{i2}(\theta_{i1})$$

Critical types are defined as in the general model. In period two, the critical type can be an interior or a corner solution depending the value of $r_2$. Thus, the allocation in period two affects the value of the critical type by moving the outside option. In period one, since $r_{i1} = 1$ and $r_{-i1} = 0$, $\theta^*_{i1} = 1$ and $\theta^*_{-i1} = 0$. Thus, we can write the expected payoff in period one as:

$$V_{i1}(\theta_{i1}) = V_{i1}(1) - \int_{s=\theta_{i1}}^{\theta^*_{i1}=1} Q_{i1}(s)ds$$
$$V_{-i1}(\theta_{-i1}) = V_{-i1}(0) + \int_{s=0}^{\theta_{-i1}} Q_{-i1}(s)ds$$

with

$$\begin{aligned}
&V_{i1}(1) = V^0_{i1}(1) = r_i + \delta r_i \mathbb{E}_{\theta_{i2}}[\theta_{i2}] = 1 + \delta \mathbb{E}_{\theta_{i2}}[\theta_{i2}] \\
&V_{-i1}(0) = V^0_{-i1}(0) = 0
\end{aligned}$$

we have the payments:

$$\begin{aligned}
X_{i1}(\theta_{i1}) &= Q_{i1}(\theta_{i1})\theta_{i1} + \int_{s=\theta_{i1}}^{1} Q_{i1}(s)ds + \delta \mathbb{E}_{\theta_{i2}} V_{i2}(\theta_{i2}, r_2) - (1 + \delta \mathbb{E}_{\theta_{i2}}[\theta_{i2}]) \\
X_{-i1}(\theta_{-i1}) &= Q_{-i1}(\theta_{-i1})\theta_{-i1} - \int_{s=0}^{\theta_{-i1}} Q_{-i1}(s)ds + \delta \mathbb{E}_{\theta_{-i2}} V_{-i2}(\theta_{-i2}, r_2) \\
X_{i2}(\theta_{i2}) &= Q_{i2}(\theta_{i2})\theta_{i2} - V_{i2}(\theta_{i2}, r_2) \\
X_{-i2}(\theta_{-i2}) &= Q_{-i2}(\theta_{-i2})\theta_{-i2} - V_{-i2}(\theta_{-i2}, r_2)
\end{aligned}$$

We take the expectated sum of payments in period one:

$$\begin{aligned}
&\mathbb{E}_{\theta_{i1}} X_{i1}(\theta_{i1}) + \mathbb{E}_{\theta_{-i1}} X_{-i1}(\theta_{-i1}) + \delta \mathbb{E}_{\theta_{i1},\theta_{i2}} X_{i2}(\theta_{i2}) + \delta \mathbb{E}_{\theta_{-i1},\theta_{-i2}} X_{-i2}(\theta_{-i2}) = \\
&\mathbb{E}_{\theta_{i1}} \left[ Q_{i1}(\theta_{i1})\theta_{i1} + \int_{s=\theta_{i1}}^{1} Q_{i1}(s)ds - 1 - \delta \mathbb{E}_{\theta_{i2}}\left[\theta_{i2}\right] \right] \\
&+\mathbb{E}_{\theta_{-i1}} \left[ Q_{-i1}(\theta_{-i1})\theta_{-i1} - \int_{s=0} \theta_{-i1} Q_{-i1}(s)ds \right] \\
&+\delta \mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{i2}} \left[ Q_{i2}(\theta_{i2})\theta_{i2} + \delta \mathbb{E}_{\theta_{-i1},\theta_{i1},\theta_{-i2}} Q_{-i2}(\theta_{-i2})\theta_{-i2} \right] \\
&+\mathbb{E}_{\theta_{i1}} \delta \mathbb{E}_{\theta_{i2}} V_{i2}(\theta_{i2}, r_2) - \delta \mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{i2}} V_{i2}(\theta_{i2}, r_2) \\
&+\mathbb{E}_{\theta_{-i1}} \delta \mathbb{E}_{\theta_{-i2}} V_{-i2}(\theta_{-i2}, r_2) - \delta \mathbb{E}_{\theta_{i1},\theta_{-i1},\theta_{-i2}} V_{-i2}(\theta_{-i2}, r_2)
\end{aligned} \tag{1.24}$$

Only the two last lines may depend on $r_2$. However, since there is no correlation, the expectation on $\theta_{i1}$,$\theta_{-i1}$ and $\theta_{i2}$ is the same as the expectation on $\theta_{i2}$. Thus the two last lines disappear and we only have terms that do not depend on the control variable. It is not worth to use the allocation in period two if there is no correlation. In effect, without correlation, the new allocation does not depend on $\theta_{i1}$ and $\theta_{-i1}$ so the derivative of the expected payment in period one does not depend on the future allocation. ■

## 1.8 References


Athey, S. and Miller D., 2007. Efficiency in repeated trade with hidden valuations. Theoretical economics, 2:299-354.

Athey, S. and Segal, I., 2007. Designing efficient mechanisms for dynamic bilateral trading games. The american economic review, 97-2: 131-136

Athey, S. and Segal I., 2013. An efficient dynamic mechanism. Econometrica,81: 2463-2485.

Bergenamm, D. and Välimäki, J., 2010. The dynamic pivot mechanism. Econometrica, 78: 771-789.

Compte, O. and Jehiel, P., 2009.Veto constraint in mechanism design: inefficiency with correlated types. American economic journal: microeconomics,1(1):182-206

Cramton, P., Gibbons, R. and Klemperer, P., 1987. Dissolving a partnership efficiently. Econometrica, 55:615-632.

Fieseler, K., Kittsteiner, T. and Moldovanu, B.,2003. Partnerships, lemons and efficient trade. Journal of economic theory,113:223-234.

Figuera, N. and Skreta, V., 2012. Asymmetric partnerships. Economics Letters 115: 268-271.

Gruüner, H. and Koriyama, Y., 2011. Public goods, participation constraints and democracy: a possibility theorem. Games and economic behavior, 75: 152-167.

Jehiel, P. and Pauzner, A., 2006. Partnership dissolution with interdependent values. The RAND Journal of economics, 37(1):1-22.

Kakade, S., Lobel, I. and Nazerzadeh, H., 2013. Optimal dynamic mechanism and the virtual-pivot mechanism. Operation research,61(4): 837-854

Kuribko, N. and Lewis, T., 2009. Assigning property rights to insure efficient exchange and investment. Working Paper. Duke university

Kuribko, N. and Lewis, T., Liu, F. and Song, J., 2015. Long term partnership for achieving efficient capacity allocation. Working paper

Lamba, R., 2013. Repeated bargaining: a mechanism design approach. Working paper. Princeton University

Li, J. and Wolstetter, E., 2009. Partnership dissolution, complementarity and investment incentives. Oxford economic papers, 62: 529-552.

Myerson, R.B. and Satterthwaite, M.A., 1983. Efficient mechanisms for bilateral trading. Journal of economic theory, 29: 265-281.

Neeman, Z., 1999. Property rights and efficiency of voluntary bargaining under asymmetric information. Review of economic studies, 66: 679-691.

Ornelas, E. and Turner, J., 2007. Efficient dissolution of partnerships and the structure of control. Games and economic behavior, 60:187-199.

Segal, I. and Whinston, M., 2014. Property rights and the efficiency of bargaining. Working Paper.

Skrzypacz, A. and Toikka, J., 2015. Mechanisms for repeated trade. American Economic Journal: Microeconomics (forthcoming /a completar)

# Chapter 2

# Interaction of private information and capital in the allocation of emission permits

We use a two-period game in the context of permit allocation. In the first period agents must take investment decision and in the second period, they submit a permit demand schedule. This demand is defined following the supply schedule equilibrium literature. We analyse the effects of the interaction of the capital decision and the private information agents have about their type on the allocative efficiency and on the investment efficiency. We add the possibility to use capital and price subsidy and find that neither instrument allows us to reach both efficiency. To illustrate, the model is developed for the cases of substitutes and complements.

## 2.1 Introduction

How to emit less contamination is a vast issue that neither economists neither scientific have found the correct answer because of the complexities and the particularities of each country and culture. Different systems have emerged and we learn from these systems to improve them and establish better ones.

At the beginning of the history of contamination control command-and-control policies were usually used but experiences showed that it was not cost effective and had a large degree of inefficiency. (Tietenberg, 2002 and 2005) Under this strategy, the regulatory authority identifies pollution control technologies that are available and affordable for each pollutant and using this technology, determines the maximum allowable rate of discharges. Authority must review technologies and revise standards regularly. Innovation is not attractive for firms since there is a risk that the new technology they use will be consider as the standard in a future period. Another used program is the credit trading that is not a market but a command-and-control structure with more flexibility. When a firm discharges less than the authorized rate, he receives credits that he can sell to others who want to emit more pollutant. The authority must approve credit purchases so we cannot use the term market and transaction costs can be very high. Credit programs appeared in the emission reduction credit programs created under the Clean Air Act in 1976 [1].

[1] The EPA (environmental protection agency) established ambient standards in different regions, using the command-and-control tools. At the beginning, EPA had to prohibit the entrance of many new businesses in the regions where standards were not met. This policy was very unpopular as we can easily imagine so the authority created a form of trading. Present

In 1990, title IV was added to the Clean Air Act to reduce acid precipitation. The $SO_2$ (sulfur dioxide) allowance market is one of the most used and studied program. The objective was to reduce contamination to an annual aggregate cap in two stages. The final cap represents a 50 % reduction from 1980 emissions. The authority had to allocate 9 million allowances under a formula specified in the Clean Air Act. An allowance in this case was a ton of sulfur during a calendar year. Unused allowances could be carried forward into future years. They were full transferable, which means also that individuals may buy them to "retire" them from the market. The program is not based on an emission technology as in the credit one. This eliminates many of the costs and uncertainties present in the last policy. (Tietenberg 2002, Shabman et al 2002)
In an allowance market, authorities begin by selecting an ambient air or water quality objective, that is the total emission or effluent authorized during a period and divides it into a total number of allowances. An allowance can be seen as a ticket that authorize to discharge an exact amount of a pollutant during a specified period. Once the pollutant is emitted, the ticket is not any more valid. Participants can trade these allowances or permits, and that is why we usually call this policy as "cap-and-trade". Authority only chooses the cap (which is not easy of course) but then, firms have to choose how he controls its pollution: he can alter the production process, install a new equipment, reduce production or buy more allowances if he needs. The opportunity to trade makes allowances to be a valuable asset, therefore firms have incentives to diminish contamination. An allowance can be banked or/and borrowed, depending on the rules of the program, adding or not value to the permit. To work well, a competitive market must emerge, which is not automatic. Low costs of transaction is a well-known condition for example[2].
Permits market are usually observed in industrialized countries as the European Union-Emission trade scheme, EU-ETS or under the Clean Air Act in the US, but developing countries may also implement a strategy to protect their environment and diminish contamination. An example is the policy of abatement of total suspended particulate in Santiago, Chile. Implemented in 1992, the target was to abate contamination in Santiago, one of the most contaminated city of Latin America. The program was based on a proxy variable equal to the maximum emissions that a source could potentially emit in a given period of time[3]. Studies (Montero et al 2002, Coria 2007) show that abatement happened but the majority can be explained by the adoption of natural gas, which is explained by the lower cost of this fuel, not the environmental regulation.

Our paper focuses on two possible issues present in permits allocation through a mechanism in two stages. In the first stage, the society chooses to invest to abate his emissions. Agents do not know their type but distribution is public information. In the second stage, types are revealed such as agents may have private information and the regulator must assign the allowances. Therefore, two elements appear: the capital choice and the presence of private information. We do not allow trade between firms in order to

discharges where encouraged to voluntary diminish their emissions bellow the level and EPA certified these excess reduction as "credits". Then, new entrants needed to buy credits such as the total regional emissions were lower. For example, they needed to secure credits for 120 % of the emissions they would add. With time, credits became bankable and existing sources had been authorized to trade with other existing sources (which was prohibited initially).

[2]Moreover, there is a detail that authority must consider when it establishes a cap. If there is a shock, a cap can lead to politically unacceptable permit price increases. For example, in the program RECLAIM (regional clear air incentives market) in Los Angeles, an unexpected demand for power resulted in the use of older and more polluting plants. Since the supply of permits was fixed, the permit price increased a lot. A solution is to add a "safety valve", a predefined penalty on all emissions over the cap once prices exceeded a predefined threshold that would be lesser than the sanctions during normal situations. (Jacoby and Ellerman 2004)

[3]it would have been too costly to observe and monitor actual emissions

focus on the effects of the interaction between private information and capital on the investment decision and the social welfare.

Asymmetric information is usually present between the regulator and the firms. Firms have information about theirs costs that the social planner does not have. In an extension of a model of permit market, Montero (2004) compares the use of taxes with the use of permits in presence of imperfect information on costs and benefits. He shows that Weitzman's result[4] holds so a combination of instrument could be a better policy than a single-instrument policy. He also shows that permits are a better policy when the regulator cannot enforce perfectly. As Montero underlines, governments usually use simple regulatory designs so it is important to understand how the results change with imperfections.

The imperfections we analyse are the effects of the relation between types and capital on the investment in equilibrium, the allocation and the social welfare when agents are symmetric ex-ante. For this, after analysing the first best, we develop the case in which the agents do not have private information, that is they cannot hide their true type, but they can choose their capital individually, without taking care of the society as a whole. This allows us to underline the effect of the lack of coordination between agents. Then we develop the case in which agents have the possibility to reveal their type and so choose to lie through a demand function but the capital is chosen optimally for the group using the available information. Finally, we allow the agents to lie and choose themselves their capital, such as both elements are present. Agents choose capital or revelation different than the optimal because they realize they can manipulate the price and the allocation in their favour. In the lack of coordination case, we find that agents use their power market to manipulate the price and the capital in equilibrium is less than the optimum. When they can lie, the strategic behaviour case, we use the demand/supply theory to identify the optimal lie in equilibrium. Agents use their revelation to manipulate the allocation but as agents are symmetric ex-ante, the difference between the first best allocation and the allocation when they can lie is not very big as we will see. Finally, the strategic behaviour-lack of coordination case shows us how both elements interact, and we find that the capital is closer to the first best capital because agents have another tool to manipulate the allocation.

Since investment could not be observed, if firms can hide all or a part of their investment or technology, we analyse the cases where capital is not observed by others. In these cases, agents cannot use their power market since the social planner will use a fixed value of capital to determine the allocation. The lack of coordination case results in the first best value. When agents choose their capital, investment will be different.

We also analyse the effects of a capital subsidy and price subsidy. We show that the capital subsidy affects only the choice of capital so it is easy to determine the optimal subsidy whereas the price subsidy affects the lie and the capital in some direction that we analyse.

To be concrete, we present the results of each case for two different interaction between capital and private information, when they are substitutes and when they are complements. In particular we analyse the strategic behaviour-lack of coordination case to understand how the interaction changes the first-best results.

[4]Weitzman shows that the choice between both depends on the sign of the sum of the curvatures of the cost and benefit functions. If the marginal benefit function is steep and the marginal cost function is flat quantity regulation is preferred over price regulation, while price regulation is preferred over quantity regulation if the marginal benefit function is flat and the marginal cost function is steep

## 2.2 Related Literature

Our analysis is related to the literature of emission permits and imperfect markets. When markets work perfectly, the theorem of Coase applies and efficiency is reached. Montgomery (1972) proved theoretically that transferable permits would achieve a cost-effective allocation of control responsibility, independently of the initial allocation. However, Hahn and Stavins (2010) describe and review the literature about six elements that could impede the presence of the independence property between initial and final allocation in cap-and-trade systems: transaction costs, market power, uncertainty, conditional allowance allocations, non-cost minimizing behaviour by firms and differential regulatory treatment of firms. Transaction costs are a well-known impediment to the Coase's theorem. In the context of cap-and-trade systems, they are understood as search and information costs, bargaining and reaching a decision cost and the monitoring and enforcement cost. Results depend on the form of the marginal transaction cost. (see Stavins 1995, Montero 1998, Liski 2001, Cason and Gangadharan 2003) Market power precludes the independence property because of the faculty of agents with power to manipulate market price or the final product price.(Hahn 1984, Malueg and Yates 2009, Liski and Montero 2009, Misiolek and Elder 1989) Uncertainty about future permit price can lead to violation of the independence property because of the risk aversion of the firms and the transaction costs. Firms with small initial allocation may decide to over-invest in abatement technology in order to protect them against future high price whereas firms with large initial allocation may prefer to under-invest in order to hedge against future low price.(Badlursson and von dehr Fehr 2004, Betz and Gunnthorsdottir 2009 for laboratory support) The independence property is obviously affected only if uncertainty involves some limitation on transferability. The conditional allowance allocation changes the initial allocation and the vision about future of firms. For example, the output-based updating allocation is a production subsidy. Firms can anticipate to increase production because they will receive more permits in the next period. Emission-intensive products have lower marginal costs than under ordinary free allocations so they do not have the necessary incentive to substitute to less emitting products and so efficiency is reduced. Another issue is the non-cost-minimizing behaviour. Behavioural economics studied a lot this point: importance of the endowment effect which caused that individuals overvalue what they already have (Thaler 1980, Kahneman et al 1990, Murphy and Stranlund 2005), high degree of organizational complexity (Tschirhart 1984, Oates and Stassmann 1984), priority of the staff expenditures or managerial salary (Oates and Strassmann 1984), importance of elements not directly linked to profits as shame and moral (Parker 2006, Braithwaite 2002) or norms (May 2005) or for example the reluctance to engage in the first stages of a market. Ellerman (2000) calls this autarkic compliance and observes it in the early years of the $SO_2$ trading system. Some managers prefer to meet the emission target according to their initial allocation instead of buying and selling in the market. Finally, if firms receive different regulatory treatment, initial allocations can affect equilibrium allocations, performance, costs and therefore, efficiency.( Hahn and Noll 1983, Oates and Strassman 1984, Tschirhart 1984), Fullerton et al 1997)

They also give a detailed description of the empirical observation of the independence property. Their analysis is both descriptive and statistically depending on the availability of studies. Globally, the programs RECLAIM in California and the EU-ETC have statistically the proof of the independence property, despite some presence of uncertainty. However, some programs show some elements that impede a good support for the property, as the lead trading because of transaction costs, the CFC trading under Montreal protocol because of market power, the RGGI because of uncertainty and the Kyoto protocol article 17 because of transaction costs, market power but above all, because of non-cost-minimizing behaviour since countries do not act as firms. Therefore, the independence property is globally observed in the past or actual programs but the independence between initial and final allocations cannot be asserted with certainty.

In our model, allowances are distributed after auctioning off permits. There is an active debate about the choice of the initial allocation of permits. In particular the issue is to know if it is better to allocate auctioning or grandfathering the permits. Cramton and Kerr (2002) support the idea that auctioning permits is better than grandfathering them because it allows to reduce tax distortions and provides more incentives for innovation and more flexibility in distribution of costs. In effects, when auctioning, a permit gives revenue to the government that can use it to cut labor, payroll, capital or consumption taxes or to reduce the deficit.Cramton and Kerr underline that the argument that firms need to receive freely the permits to invest is quite empirically ambiguous. If permits are grandfathering, lobbies will fight and spend a lot of money to receive the biggest possible share of permits. Firms could decide to delay the investments to show a bigger marginal cost that would lead to more generous allowance allocation as compensation. Grandfathering is sometimes justified that if firms must pay a price for permits, the final product price will be increased. However, the price raises almost every time because even if industries do not pay for the allowances, the permits have a value that raises the final price. This can be illustrated by the case of one of the European largest carbon emitters, the German-based RWE. For instance, RWE received 30 percent of all German permits and was reprimanded by the German antitrust authority because of complaints about rising electricity prices (at 5 % per year). RWE's answer was : " while it may have received them for free from the government, they still had value in the market place". This opportunity cost is reflected in final price. (Goeree et al 2010). However, grandfathering had been seen as a good decision in the implementation of the abatement policy in Santiago Chile (Montero et al, 2002). In effect, they created incentive for participants to declare their emissions.

Laboratory studies analyse firms' behaviour faced with different initial allocation systems. Benz and Ehrhart (2007) study the reliability of prices generated by different allocation policies for $CO_2$ allowances under the EU-emissions trading system (EU-ETS). Goeree et al (2010) compare grandfathering versus auctioning in the initial allocation of allowances, with the existence of a secondary spot market. They consider high and low emitters (natural gas versus coal for example). The experiment's result in the case of grandfathering shows that the permit price in the spot market is much higher than competitive predictions because of market power exercised by the high emitters who receive many allowances because of their history. On the opposite, permit price tends to competitive levels when permits are auctioning. This initial allocation is more favourable to low emitters and revenues for auction can be used by the government. Wrake et al (2010) proposes a laboratory experiment to test the hypothesis that the choice between an auction and free allocation should not influence firms' production choice nor consumer prices. They find that the concept of opportunity cost is not intuitively easy to consider but that agents learn with time.
In practice, auctions are more and more used. Auction is present in the EU-ETS but in a hybrid system and countries have to choice to implement it or not. In the Phase I (2005-7), auction share was limited up to 5 percent and only 4 countries (Denmark, Hungary, Ireland and Lithuania) decided to auction off parts of their permits. In phase II (2008-12), the limit was 10 percent and only six countries (Germany, UK, Netherlands, Lithuania, Hungary, Austria and Denmark) decided to auction. In phase III (2013-20), auction is expected to take a larger place since it is estimated that up to half of the allowances may be auctioned. In 2013, over 40 percent of the allowances were auctioned. (European Commission) In the US, the Regional Greenhouse Gas Initiative (RGGI) is a good example of the use of auctions since more than 90 per cent of allowances are auctioned. As in the EU-ETS, it employs uniform price sealed-bid auctions. The

auction revenues are used for strategic energy initiatives. In Australia, the Australian proposal for a Carbon Pollution Reduction scheme proposes to auction a significant share of total permits.(Betz et al 2010) Our goal is not to compare different systems of initial allocations but to analyse the effects of the interaction between private information and investment in the initial allocation in a system of auctions. Since auctions seem to take more importance, it is justified to choose this system.

Firms usually have private information that the regulator and other firms do not have. The authority would like to have access to this information to establish the efficient level of pollution. Mechanism can help to make firms to reveal the truth. Kwerel (1977) proposes a truth-telling mechanism in which the regulator distributes a fix quantity of permits and then pays a subsidy of s per permit to any firm holding allowances in excess of its emission. Since the permit market is perfectly competitive, firms equate marginal abatement cost to the market price and keep a number of licenses just to cover their emissions. Montero (2008) underlines that Kwerel's mechanism is efficient as long as permits are auctioned off and the auction is competitive but that there are a lot of other (inefficient) equilibria that are more profitable for firms. Under free allocation, the firms' best response is to overreport their demand curves to ensure the maximum possible number of licenses and subsidy level. Montero proposes another mechanism. Supply of permits is endogenous to the problem. Firms declare a nonincreasing demand schedule and the regulator clears the auction. To make the firms tell the truth, a fraction of the auction revenues gets back to the firm. The mechanism follows the VCG's logic: each firm pays for its residual damage to all other agents. In this mechanism, a permit market is unnecessary since the first best is reached.
Ollikka (2014) compares the policy of establishing a tax (price) with the decision of a constant quantity regulation following the work of Weitzman (1974) adding private information about firms' uncertain abatement costs which are correlated between firms. In this model, the firms and regulator know the distribution functions and firms receive a noisy signal about their own abatement costs in a previous period. In the first period an auction is ruled such as firms simultaneously submit a demand schedule and then the regulator sets the clearing price. Permits are allocated and firms pay. The next period is called the regulation stage. Firms learn their cost parameter during this period. If the policy is the quantity regulation, the supply of permits is fixed (a cap) at the level of initial allocation and firms can trade with other agents. If the policy is the price regulation, firms can purchase more permits from the regulator or sell permits back at the announced uniform price established in the first stage. Ollikka shows that if quantity regulation is used, the mechanism is incentive compatible if the positive correlation between emissions reduction benefits and costs is not too high. On the other hand, if the constant price regulation is used, the mechanism is incentive compatible if the negative correlation between emissions reductions benefits and costs is relatively high.

The propensity of a policy to incentive innovation is an important questions in abatement instruments. Since the objective of the policy is to reduce globally the emissions, regulators have to induce as much as possible, the use of cleaner technologies. In the literature, we find three kinds of analyses: theoretical, empirical and experimental. Empirically, few studies manage to use available data to analyse the effects of different alternative policies. Among them, Jaffe and Stavins (1995) develop an empirical model to compare the effects of different instruments on the diffusion of a new technology. They compare taxes, subsidies and technology standards in the case of insulation of new home construction. They find that subsidies let a greater adoption of new technology than taxes and they observe that technology standards are useless. In the $NO_x$ US market, Fowlie (2010) finds that deregulated firms[5] are less likely to choose more capital

[5] "Regulated" firms are faced with a "rate of return" regulation, that is the regulators set rates to allow utility to recover

intensive compliance options because of the absence of regulation.

Since data are not easy to find, economists use laboratory experiments to analyse the behaviour of agents in decision choice of abatement. Gangadharan et al (2012) analyse whether emissions market encourage optimal investment in laboratory when information about investing is made public. They find that firms tend to over-invest in abatement equipment. Camocho-Cuena et al (2012) analyse the impact of different initial allocation systems (grandfathering versus auctioning) and different auction schemes on the efficiency of technology adoption. Firms have different initial technologies but the new technology is the same for all firms. They find that investment patterns are close to the theoretical first-best allocation and that any initial allocation mechanism outperforms the others. This result goes against the preference of auctions on grandfathering (Cramton and Kerr 2002). Moreover, they find that individual investment decision is mainly determined by the initial technology, with some under-investment by inefficient firms (the "dirtiest") and some over-investment by less inefficient firms (the "cleanest"). Suter et al (2013) utilise laboratory experiments to analyse the hypothesis that limited trading in water quality trading schemes can be explained by over-investment when participants are not numerous and investments are done with large capital. They find over-investment in abatement technology across different treatment scenarios, especially when participants face limited abatement opportunities and are risk averse. Van Koten (2015) proposes an experiment to test the behaviour of agents if they receive freely the quantity of permits they need (over-allocated) or zero (under-allocated). The game is repeated 10 rounds. The results of the first six rounds are similar to the literature, with inefficient decisions that are biased by allocation. However, from the sixth round, the deviation from the optimal abatement is close to zero for both groups, showing an important learning effect. Therefore, agents learn from the repetition and initial allocation does not matter.

Theoretically the decision of investment has been studied too. We usually consider that market-based approaches provide the most effective long-term incentives for innovation. In effect, any abatement generates revenues or reduces costs, as the selling of a permit, obtaining a subsidy or avoiding a tax. On the opposite, once a firm satisfied a technology standard, it does not have incentive to use a cleaner technology. Even worse, it has uncertainty about future standards. Montero (2002) compares the incentives to invest created by the four main used instruments, emission standards, performance standards, tradable permits and auctioned permits in the case of an oligopoly market and output markets. He shows that tradable and auctioned permits provide the same incentives and are similar to those under emissions standards but greater than those under performance standards in case of perfect competition. However, if the market is characterised by an oligopoly, standards can offer greater incentives than permits. In an oligopoly appears a strategic effect, which is the influence of the investment decision of one firm on the other firm's choice of output. With standards, strategic effects are always positive because investment is always a cost-reducing innovation that allows to increase output and profits. On the opposite, under permits, strategic effects from the output market are negative since the investment "spills over" through the permit market reducing the rival's costs and therefore, increasing his output. Other elements can impede an efficient investment. Uncertainty can be one obstacle: under tradable permits, investment can be reduced because permit prices are typically random and the investment can be irreversible (Xepapadeas 1999, Chao and Wilson 1993). It can aslo be due to the entry and exit of polluting firms (Baldursson Karatzas and von de Fehr 1999). Moreover, grandfathering permits may provide less incentive to invest than a constant emissions tax because the marginal abatement costs decrease as firms invest, reducing the permit price and so the benefits of investment (Milliman and Prince 1989, Jung et al 1996). Zhao (2003) analyses the incentives to invest when abatement costs are uncertain. They compare the effects of this uncertainty on permits and charges and find that investment

prudently incurred operating costs whereas "unregulated" firms must recover their investment in the market.

incentive decreases in cost uncertainties but more under emissions charges than under permits.

Requate and Unold (2003) compare the adoption of technology in different abatement policies. In contrast to Milliman and Prince (1989) and Jung et al (1996), they do not assume that all firms adopt the new technology but determine the number of firms adopting it as an endogenous variable, in case regulator does or does not anticipate the adoption of the new technology. Any mechanism can be elected as the best one.

Insley (2003) analyses the optimal choice of investment by a firm facing pollution regulation and applies her model to the $SO_2$ policy of the 1990 Clean Air Act Amendment in the US, the Acid Rain Program. She considers investment decision as a real option and the price of allowances follows geometric Brownian motion. Firms can choose to invest in installing a scrubber and can decide to halt the construction. She estimates the critical allowance prices for a representative electric plant and studies how these prices may vary with changes in volatility and flexibility in scrubber installation and operation. Firms' expectation of these variables is very significant to understand investment decision. Fischer, Parry and Pizer (2003) compare the welfare effects of different mechanisms when technological innovation is endogenous. The game is in three stages. In the first stage, one of the firms is the innovator and decides how much to invest in R&D to develop a better abatement technology. In the second stage, the other firms decide whether to adopt this technology and pay a royalty fee or to use a costless imitation of the technology that is not fully equivalent. In the third stage, the firms choose emission abatement in order to minimize costs given an emission tax or a permit price. Two effects are important: the abatement cost effect and the emission payment effect. The former is the cost savings caused by the effect of the new technology on the marginal abatement cost. The latter appears when permits are auctioned off and the permit price is lowered because of the reduced marginal abatement cost. The results, theoretically and by simulation, reject the possibility to order the policy either on the criterion of welfare gains or the amount of induced innovation.

## 2.3 General model

We study a model with n firms and two stages. The firms' production causes pollution that must be authorized through permits. Agents have private information about their type $\theta_i \in [0,1]$ which comes from a function of distribution $F(.)$ with density function $f(.)$. We also introduce investment of capital $K_i$ in the model. This capital is decided and paid in period one and is used in period two in order to improve the function of production. Investment allows a better use of permits, that is the firm can produce more with the same quantity of rights or he can buy less permits to produce the same quantity than before investment.

We denote by $w_i$ the quantity of emission permits an agent $i$ owns. With this, firm $i$ is authorized to produce emissions $e(\theta_i, q_i) = w_i$ where $q_i$ is the production. We suppose perfect enforcement such as we use $w_i$ for the emission in firms' production. We make the assumption that the emissions are decreasing in $\theta$ and are increasing and convex in the production, that is $e_\theta < 0, e_q > 0, e_{qq} > 0$. $\theta$ is private information and captures special characteristics of the firm. Convexity can be justified by the use of older/ more polluting technologies.

From emissions $e(\theta_i, q_i)$ we can analyse the value of pollution permits. Considering $e(\theta_i, q_i) = w_i$ which defines $q_i(\theta_i, w_i)$, we find that $q_w = \dfrac{1}{e_q} > 0$ and $q_{ww} = \dfrac{-e_{qq} q_w^2}{e_q} < 0$ such as the production increases with allowances and is concave. Each added unit of permits gives less production.

Firms can invest in capital $K_i$ to improve their technologies and decrease their emission. Therefore, capital and private information interact in the production.

Firm $i$ has a payoff $\pi_i = p.q(w_i, \theta_i, K_i) - \lambda w_i$ where $p$ is the price of the good in the final market, $\lambda$ is the price of the permits which is determined by the emission market and is the Lagrangian parameter. Since we are not looking the final product, $p$ is a parameter and we fix it to $p = 1$ to simplify. We could develop the model using the benefit but it is not possible to solve. To be able to analyse and solve the model, we need to use a form for $q(w_i, \theta_i, K_i)$ that must be increasing and concave in $w_i$ as we explained and that must be linear in $w_i$ in order to solve. We use $q(w_i, \theta_i, K_i) = f(\theta_i, K_i)w_i - \gamma\frac{w_i^2}{2}$, where $f(.)$ specifies a form of relation between private information and investment. We consider the special cases of substitutes[6] $f(\theta_i, K_i) = \theta_i + K_i$ and complements $f(\theta_i, K_i) = \theta_i K_i$. Private information and investment are substitutes when machines are not the unique cause of contamination. For example, a firm must add a chemical in the process that raises the emissions. The products he uses are private information and before investing in better engines, he could change the inputs. Another On the other hand, private information and capital are complements when the type or private information of the firm is linked with the capital. For example, his type is the organization or timing of the production thus it affects directly the capital.

Inefficiency can come from the possibility to lie and the decision of investment. Since we want to study the interaction, we separate the effects and analyse four cases. The benchmark is the first-best in which the real types are known so the allocation of allowances is efficient and the group chooses the investment of each one efficiently. To analyse the effect of capital choice, we let the regulator known real types but each agent chooses separately his investment, without considering the society. Then, to study the effect of private information, agents can lie about their types whereas investment is chosen by the group. Finally, the interaction between private information and capital is analysed letting each agent choose his investment and be able to lie about his type.

### 2.3.1 First-best

As usual, the problem is solved backward. In period two, the welfare is maximized and the allocation of permits determined, given $K_i, K_j$. In period one, investment is chosen. The utility is the expected discounted period-two utility less the cost of capital. To simplify and without loss of generality, the discount factor is one so we omit it.

In period two, the social planner solves :

$$\begin{aligned} \max_{w_1,\dots,w_n} \sum_i f(\theta_i, K_i)w_i - \gamma\frac{w_i^2}{2} \\ s.t. \sum_i w_i = 1 \\ w_i \geq 0 \ \forall i \end{aligned} \tag{2.1}$$

In period one, the problem we are interested in is:

$$\max_{\{K_1,\dots,K_n\}} \mathbb{E}\left[\sum_i f(\theta_i, K_i)w_i - \gamma\frac{w_i^2}{2}\right] - \sum_i c(K_i) \tag{2.2}$$

where $c(.)$ is the cost function of the investment.

[6] Analyse would be the same using the more general formula $f(\theta_i, K_i) = \alpha\theta_i + \beta K_i + \epsilon$.

In period two, the resolution for an interior solution leads to:

$$w_i(\theta, K) = \frac{1}{n} + \frac{(n-1)f(\theta_i, K_i) - \sum_{j \neq i} f(\theta_j, K_j)}{n\gamma}$$
$$p = \lambda(\theta, K) = \frac{\sum_i f(\theta_i, K_i) - \gamma}{n}$$

where $\lambda$ is the Lagrangian parameter and the shadow price.
Proof: see Appendix 1 for the KKT problem

The allocation of emissions is one nth and a term of correction which depends on the types, the capital and the parameter of concavity. As it is intuitive, the allocation of emission permits for agent $i$ is increasing in his type and capital and is decreasing in the others' type and capital. It is decreasing in the factor of concavity because as the function of utility is more concave, it is less helpful to give more allowances. The price of the permit is increasing in the type and investment of all agents. As type and investment extend, firm $i$ wants to produce more but he needs more permits so price increases.

**Theorem 1:** In a game of $n$ agents defined by the problems (1) and (2), the first-best marginal cost in equilibrium is given by :

$$c'(K_{i,fb}) = \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w(\theta, K)\right] \tag{2.3}$$

**Proof:** see Appendix 2
In this case, all agents have a capital such as the marginal cost is equal to the expected value of the allocation adjusted by the change in capital of the function $f(\theta_i, K_i)$.

As it is well-known, a Vickrey-Clarke-Grooves mechanism (VCG) would allow the regulator to obtain the first-best. The VCG payment for agent i is the difference between the social welfare without i when i is absent from the decision allocation and the social welfare without i when i is present in the decision allocation, that is:

$$t_i^{vcg} = SW_{-i}(0, \theta_{-i}) - SW_{-i}(\theta_i, \theta_{-i}) \tag{2.4}$$

where

$$SW_{-i}(0, \theta_{-i}) = \sum_{j \neq i} \left\{ f(\theta_j, K_j) w_j(-i) - \gamma \frac{w_i^2(-i)}{2} \right\} \tag{2.5}$$

$$SW_{-i}(\theta_i, \theta_{-i}) = \sum_{j \neq i} \left\{ f(\theta_j, K_j) w_j(.) - \gamma \frac{w_i^2(.)}{2} \right\} \tag{2.6}$$

where $w_j(.)$ is the allocation for agent j given that i is present and $w_j(-i)$ is his allocation when i is absent in the decision.
If the regulator imposes this payment, each agent would consider the social utility and therefore, will act as the regulator and efficiency will be reached. To prove it, we define the utility of agent i and then verify that

he chooses the capital of first-best:

$$
\begin{aligned}
U_i(\theta_i) &= f(\theta_i, K_i)w_i - \gamma\frac{w_i^2}{2} - t_i^{vcg} \\
&= f_i(\theta_i, K_i) - \gamma\frac{w_i^2}{2} - \sum_{j\neq i}\left\{f(\theta_j, K_j)w_j(-i) - \gamma\frac{w_i^2(-i)}{2}\right\} + \sum_{j\neq i}\left\{f(\theta_j, K_j)w_j(.) - \gamma\frac{w_i^2(.)}{2}\right\} \\
&= \underbrace{\sum_i f(\theta_i, K_i)w_i - \gamma\frac{w_i^2}{2}}_{SW} - \underbrace{\sum_{j\neq i}\left\{f(\theta_j, K_j)w_j(-i) - \gamma\frac{w_i^2(-i)}{2}\right\}}_{\text{constant that does not depend on agent i}}
\end{aligned}
$$

The utility of an agent is the social welfare of the society less a constant that does not depend on his decision such as if he maximizes his utility, he maximizes the social utility.
When agent decides his investment:

$$\max_{K_i} \mathbb{E}\left[U_i(\theta_i)\right] - c(K_i)$$

Replacing the problem is:

$$\max_{K_i} \mathbb{E}\left[\sum_i f(\theta_i, K_i)w_i - \gamma\frac{w_i^2}{2} - \sum_{j\neq i}\left\{f(\theta_j, K_j)w_j(-i) - \gamma\frac{w_i^2(-i)}{2}\right\}\right] - c(K_i)$$

The FOC is:

$$c'(K_i) = \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}w_i(\theta, K) + f(\theta_i, K_i)\frac{\partial w_i}{\partial K_i} - \gamma w_i\frac{\partial w_i}{\partial K_i} + \sum_{j\neq i}\left(f(\theta_j, K_j)\frac{\partial w_j}{\partial K_i} - \gamma w_j\frac{\partial w_j}{\partial K_i}\right)\right]$$

but $f(\theta_i, K_i) - \gamma w_i = f(\theta_j, K_j) - \gamma w_j \;\; \forall j \neq i$, we can reduce the expression to:

$$
\begin{aligned}
c'(K_i) &= \mathbb{E}\left[\sum_j \left(f(\theta_i, K_i) - \gamma w_i\right)\frac{\partial w_j}{\partial K_i} + \frac{\partial f(\theta_i, K_i)}{\partial K_i}w_i(\theta, K)\right] \\
&= \mathbb{E}\left[\left(f(\theta_i, K_i) - \gamma w_i\right)\left[\sum_j \frac{\partial w_j}{\partial K_i}\right] + \frac{\partial f(\theta_i, K_i)}{\partial K_i}w_i(\theta, K)\right]
\end{aligned}
$$

since the total available allowances is 1, we have $\frac{\partial w_i}{\partial K_i} = -\sum_{j\neq i}\frac{\partial w_j}{\partial K_i}$ and finally

$$c'(K_i) = \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}w_i(\theta, K)\right]$$

which is the first-best solution.

The problem with VCG mechanism is that usually, they are not budget feasible so they are not attractive for a policy-maker. Environment is in general not seen as the most important issue in the society so taking money from other areas could not be accepted.
We focus on second-best solutions and analyse the effect on the investment and the social welfare.

### 2.3.2 Lack of coordination

When agents can choose their own investment as it is usually the case, they will not consider the welfare of the group. To focus on this effect, we suppose that the regulator does know their type so the distribution of allowances in period two is determined as in the first best but with the capital found in this case. In period one, agent i solves:

$$\max_{K_i} \mathbb{E}\left[f(\theta_i, K_i)w_i(\theta, K) - \gamma\frac{w_i^2(\theta, K)}{2} - \lambda(\theta, K)w_i(\theta, K)\right] - c(K_i)$$

where $\lambda(.)$ and $w_i(.)$ come from the first-best.

**Theorem 2:** In our game, if agents cannot lie but they do choose their investment individually, then the capital in equilibrium comes from:

$$c'(K_{lc}) = \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}w(\theta, K_{lc}) - \frac{\partial \lambda}{\partial K}w(\theta, K_{lc})\right] = \frac{n-1}{n}\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}w(\theta, K_{lc})\right] \tag{2.7}$$

**Proof:** see Appendix 3

The marginal cost is not anymore the expected value of the allocation but a fraction of it. Since agents choose their own investment, they realize they have market power to change the price so a price effect appears. They prefer to invest less.

### 2.3.3 Strategic behaviour

When agents can lie, we want to analyse it. We use the theoretical tool of demand schedule equilibrium, following the literature of supply function equilibrium that begun from Klemperer et al (1989) and in particular the work of Vives (2011). Agents make a demand $a_i(\theta_i)$ that is a function that depends on the real types and the capital. For analytical reason, we consider that agents lie symmetrically in equilibrium. In period two, the problem is solved with the available information, that is with $a_i$ instead of $\theta_i$. Allowances and shadow price are thus:

$$w_i(a(\theta, K), K) = \frac{1}{n} + \frac{(n-1)f(a_i(\theta_i, K), K_i) - \sum_{j\neq i} f(a_j(\theta_j, K), K_j)}{n\gamma}$$
$$p = \lambda(a(\theta, K), K) = \frac{\sum_i f(a_i(\theta_i, K), K_i) - \gamma}{n}$$

In period one, the individual chooses how to lie maximizing his function of utility. He knows that the social planner will use the information he reveals so he takes advantage of it:

$$\max_{a_i} \mathbb{E}\left[f(\theta_i, K_i)w_i(a_i(\theta_i, K), K) - \gamma\frac{w_i^2(a_i(\theta_i, K)}{2} - \lambda(a(\theta, K), K)w_i(a(\theta, K), K)\right]$$

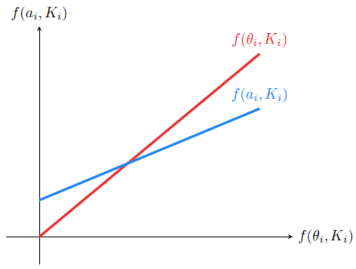
Note that $w_i(.)$ and $\lambda(.)$ are function of $a_i$. We develop the demand function equilibrium in the appendix. To be able to solve and following Vives (2011), we use a linear guess in a symmetric equilibrium.

**Theorem 3:** In our game, if agents can reveal a different type than their real one, then the demand function in equilibrium is the solution to:

$$f(a_i(\theta_i, K), K_i) = \frac{n}{n+1}f(\theta_i, K_i) - \frac{1}{n(n-1)}\gamma + \frac{1}{n^2(n+1)}\mathbb{E}\left[f(\theta_i, K_i)\right] + \frac{1}{n^2}\sum_{j\neq i}\mathbb{E}[f(\theta_j, K_j)] \tag{2.8}$$

**Proof:** see Appendix 4

The demand function can be understood as a change in the slope and the initial point:




When agents decide to lie, two changes appear. The first one is the jump in the initial point caused by the constant. This constant is composed by the three last terms. The slope changes to $\frac{n}{n+1}$ which is less than one. This means that the regulator will see agents that are less different than they are.

**Lemma:** In our game, if agents can reveal a different type than their real one, then there is underinvestment in equilibrium.

**Proof:** In the demand function, the constant is the same for all agents so it disappears in the allocation decision. From the slope, we known that agents will appear more homogeneous than they are in reality to the regulator. Therefore, the regulator will choose to assign permits that are more homogeneous. Since agents receive permits that are more similar, they have less incentive to invest.

Without specifying a form for the interaction between private information and capital, we cannot analyse exactly how the demand changes with the other variables. We note that as $n$ tends to infinity, the demand function tends to the real type as agents lost their market power.

Once we know how the agents are going to lie, we can solve for the capital using this information. The problem is same as earlier but instead of $\theta_i$, it is $a_i(.)$.

**Theorem 4:** In our game, when agents can lie but the capital is decided optimally for the society using the revealed information, then the investment is the solution to:

$$c'(K_i) = \mathbb{E}\left[\sum_j \frac{\partial f(a_j(\theta, K), K_j)}{\partial K_i} w_j(a(\theta, K), K)\right] \tag{2.9}$$

and if we replace $w_j(a(\theta, K), K)$ and the derivative of $f(a_j(\theta, K), K_j)$ by their expressions we have:

$$c'(K_i) = \frac{1}{n}\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}\right] + \frac{1}{n\gamma}\left(\frac{n}{n+1}\right)^2 \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}\left((n-1)f(\theta_i, K_i) - \sum_{j\neq i} f(\theta_j, K_j)\right)\right] \tag{2.10}$$

**Proof:** see Appendix 5

The comparison with the first-best is not direct and will depend on the form of the interaction between type and capital.

### 2.3.4 Strategic behaviour and lack of coordination

Since we want to analyse the interaction between private information and the capital decision, we now let agents to do what they want. Since agents can lie and choose their investment, they know they will demand optimally in the second period. The problem of optimization of agent i is:

$$\max_{K_i} \mathbb{E}_{\theta_i} \max_{a_i} \mathbb{E}_{\theta_{j\neq i}}\left[f(\theta_i, K_i) w_i(a_i(\theta_i, K), K) - \gamma\frac{w_i^2(a_i(\theta_i, K))}{2} - \lambda(a_i(\theta_i, K)) w_i(a_i(\theta_i, K))\right] - c(K_i)$$

**Theorem 5:** In our game, if agents can reveal a demand function and choose themselves their capital, then the investment in equilibrium is the solution to:

$$c'(K_{sblc}) = \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(a(\theta, K), K)\right] - \frac{1}{n^3}\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}\right] \tag{2.11}$$

and if we replace $w_i(a(\theta, K), K)$ by his expression :

$$c'(K_i) = \frac{n^2-1}{n^3}\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}\right] + \frac{1}{\gamma(n+1)}\mathbb{E}\left[(n-1)\frac{\partial f(\theta_i, K_i)}{\partial K_i} f(\theta_i, K_i) - \sum_{j \neq i} \frac{\partial f(\theta_i, K_i)}{\partial K_i} f(\theta_j, K_j)\right] \tag{2.12}$$

**Proof:** see Appendix 6

### 2.3.5 Investment is not observable

We now analyse the interaction if the social planner cannot observe the capital. Obviously, he cannot observe it when agents choose it but if capital is chosen optimally by the group, we consider he sees it since he decides the level himself.
Therefore, the first best and the case of strategic behaviour are the same as in the general model. We are interested in the cases of lack of coordination and of strategic behaviour-lack of coordination.

#### Lack of coordination

Agents cannot lie about their type since the social planner observes it. They can choose the level of investment but they know that the social planner will not observe it but will use a prediction $K^*$. Thus agents lost the market power compared to the general model. They maximize

$$\max_{K_i} \mathbb{E}\left[f(\theta_i, K_i) w_i(\theta, K^*) - \gamma \frac{w_i^2(\theta, K^*)}{2} - \lambda(\theta, K^*) w_i(\theta, K^*) - c(K_i)\right] \tag{2.13}$$

**Theorem 6:** In our game, if investment is not observable and agents cannot lie but can choose their own capital, then the investment in equilibrium is the same than in the first-best. Therefore, the social welfare is the first-best situation.

**Proof:** We see that $w_i(.)$ does not depend on the investment agent i chooses so the FOC is reduced to:

$$\mathbb{E}\left[\left.\frac{\partial f(\theta_i, K_i)}{\partial K_i}\right|_{K_i = K^*} w_i(\theta, K^*)\right] = c'(K^*) \tag{2.14}$$

Since agents are symmetric and they lost their market power, the marginal cost is the same as in the first best. Moreover, since agents cannot lie, the social welfare in the same as in the first-best. ■

#### Strategic behaviour and lack of coordination

Agent i must decide how he is going to lie in the second time of the game and he must choose his capital. Since investment is not observable, $i$ plans to deviate from the social planner equilibrium prediction $K^*$

thinking that other agents play with $K^*$.
His problem of maximization is:

$$\max_{a_i} \mathbb{E}\left[f(\theta_i, K_i)w_i(a_i, a_{-i}(\theta, K^*), K^*) - \gamma\frac{w_i^2(ai, a_{-i}(\theta, K^*), K^*)}{2} - \lambda(a_i, a_{-i}(\theta, K^*), K^*))w_i(a_i, a_{-i}(\theta, K^*), K^*))\right]$$

The maximization leads to the demand function where $i$ considers that the other agents do not deviate.

**Theorem 7:** In our game, if investment is not observable when agents can lie and choose their own capital, then the demand function in equilibrium is the solution to:

$$f(a_i, K^*) = \frac{n}{n+1}f(\theta_i, K_i) - \frac{1}{n(n-1)}\gamma + \frac{1}{(n+1)}\mathbb{E}[f(\theta, K^*)] \tag{2.15}$$

**Proof:** see Appendix 7

Once we have the demand function, agent $i$ chooses his level of capital:

$$\max_{K_i} \mathbb{E}\Big[f(\theta_i, K_i)w_i(a_i(\theta, K_i, K^*), a_{-i}(\theta, K^*), K^*) - \gamma\frac{w_i^2(a_i(\theta, K_i, K^*), a_{-i}(\theta, K^*), K^*)}{2}$$
$$- \lambda(a_i(\theta, K_i, K^*), a_{-i}(\theta, K^*), K^*)w_i(a_i(\theta, K_i, K^*), a_{-i}(\theta, K^*), K^*) - c(K_i)\Big]$$

**Theorem 8:** In our game, if investment is not observable, and agents can lie and choose their own capital, then the capital in equilibrium is given by:

$$c'(K^*) = \mathbb{E}\left[\frac{1}{n}\frac{\partial f(\theta_i, K_i)}{\partial K_i} + \frac{1}{(n+1)\gamma}\frac{\partial f(\theta_i, K_i)}{\partial K_i}\left((n-1)f(\theta_i, K_i) - \sum_{j\neq i} f(\theta_j, K_j)\right)\right]\Bigg|_{K=K^*}$$

**Proof:** The capital $K_i$ affects only $f(\theta_i, K_i)$ and the demand function. However as we used in the general model, since $a_i$ is chosen optimally, by the envelop theorem, terms disappear. Therefore, the FOC simply is:

$$\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}w_i(a_i(\theta, K_i, K^*), a_{-i}(\theta, K^*), K^*)\right] = c'(K_i)$$

Replacing the expression of the allocation:

$$\begin{aligned} w_i(a_i(\theta, K_i, K^*), a_{-i}(\theta, K^*), K^*) =& \frac{1}{n} + \frac{1}{n\gamma}\Bigg[(n-1)\left(f(\theta_i, K_i)\frac{n}{n+1} - \frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}(f(\theta, K^*))\right) \\ & - \sum_{j\neq i}\left(\frac{n}{n+1}f(\theta_j, K_j) - \frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}(f(\theta, K^*))\right)\Bigg] \\ =& \frac{1}{n} + \frac{1}{(n+1)\gamma}\left((n-1)f(\theta_i, K_i) - \sum_{j\neq i} f(\theta_j, K_j)\right) \end{aligned}$$

we find the marginal cost. ■

## 2.4 Subsidies

We can analyse what happens in our model if the regulator decides to put a subsidy to the price or to the capital in order to get closer to the first-best. We develop the model for both type of subsidies. For the price subsidy, it is quite difficult to analyse the theoretical results and the simulation will help us to understand what happens. In effect, the subsidy affects the demand function but it is not clear in which direction. With a capital subsidy, we will see that the subsidy does not affect the lie so the model is quite similar to the general model.

### 2.4.1 Subsidy to the capital

The problem does not change a lot. The regulator helps agents such as they do not pay the totality of the cost of the capital but a fraction $(1-s)$. Since the subsidy acts on the capital cost, it is easy to understand that it only affects the marginal cost and therefore, the final decision of investment.

**Theorem 9**: In our game, if the social planner decides to put a subsidy to the capital, then the marginal cost in equilibrium are given by:

$$c'(K_z^s) = \frac{1}{1-s} c'(K_z) \tag{2.16}$$

where $z = LC, SB, SBLC, SBLC$ k unknown

**Proof:** The new problem in each case is simply the problem without subsidy but with $(1-s)c(K_i)$ instead of $c(K_i)$.

**Theorem 10:** In our game, if the social planner decides to put a subsidy to the capital then capital efficiency can be reached but not the allocative efficiency.

**Proof:** Using Theorem 9 and the first-best capital, we easily show that it exists a subsidy such as agents choose the optimal capital so capital efficiency is reached. However, agents lie so the allocation is still inefficient.

### 2.4.2 Subsidy to the price

If the regulator decides to put a subsidy on the price, it would affect the demand function so the analysis is more complicated.

#### lack of coordination

**Theorem 11:** If there is a subsidy to the price and agents can choose their own investment but cannot lie, then the capital in equilibrium is given by:

$$c'(K_i) = \frac{n-(1-s)}{n} \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(\theta, K)\right] \tag{2.17}$$

**Proof:** The problem agent i solves is:

$$\max_{K_i} \mathbb{E}\left[f(\theta_i, K_i) w_i(\theta, K) - \gamma \frac{w_i(\theta, K)^2}{2} - (1-s) p(\theta, K) w_i(\theta, K)\right] - c(K_i) \tag{2.18}$$

Taking the derivative, we easily find the marginal cost in equilibrium. ■

**Strategic behaviour**

In this case we need to determinate the demand function first. Agent i solves:

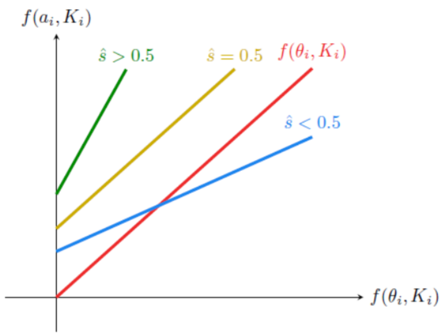
$$\max_{a_i} \mathbb{E}\left[ f(\theta_i, K_i) w_i(a_i(\theta, K), K) - \gamma \frac{w_i(a_i(\theta, K), K)^2}{2} - (1-s) p(a_i(\theta, K), K) w_i(a_i(\theta, K), K) \right] \tag{2.19}$$

**Theorem 12:** When there is a price subsidy and agents can lie but the capital is decided for the society, then the demand function is the solution to:

$$f(a_i(\theta, K), K) = \frac{n}{n-2s+1} f(\theta_i, K_i) - \frac{s(n-2)+1}{(1-s)(n-1)n}\gamma + \frac{s(n-2)+1}{(1-s)n(n-s)} \sum_{j \neq i} \mathbb{E}\left[f(\theta_j, K_j)\right]$$
$$+ \frac{(sn-2s+1)^2}{(1-s)n(n-s)(n-2s+1)} \mathbb{E}\left[f(\theta_i, K_i)\right]$$

**Proof:** see Appendix 4 but using the new problem.

We easily see that, as before, as $n$ tends to infinity, the lie tends to true type. As before, we analyse the demand function:




As before, the constant changes the initial value. Now, the slope depends on the value of the subsidy. If the subsidy is less than 0.5 the demand function as the same form as without subsidy. If the subsidy is more than 0.5 then the slope is more than one so the regulator will see agents that are more heterogeneous than they are in reality. If the subsidy is 0.5 then the slope is one.

We would like to know how the subsidy affects the demand function but since the capital depends on the subsidy, we would need to replace before taking the derivative. We will analyse when we analyse the cases of substitutes and complements.

Once we know the demand function, the regulator can determinate the allocation and the capital as in the first-best but using the available information:

$$\max_{K_{i=1\dots n}} \mathbb{E}\left[ \sum_i f(a_i(\theta, K), K) w_i(a_i(\theta, K), K) - \gamma \frac{w_i^2(a_i(\theta, K), K)}{2} \right] - \sum_i c(K_i)$$

Developing the FOC and cancelling terms because of the envelop theorem, we finally the marginal cost.

**Theorem 12:** When there is a subsidy price and agents can lie but the capital is chosen for the society, then the investment in equilibrium is:

$$c'(K_i) = \mathbb{E}\left[ \sum_j \frac{\partial f(a_j(\theta, K), K_j)}{\partial K_i} w_j(a(\theta, K), K) \right]$$

Developing the terms, we can show that the marginal cost is:

$$c'(K_i) = \frac{1}{n(1-s)} \mathbb{E}\left[ \frac{\partial f(\theta_i, K_i)}{\partial K_i} \right]$$
$$+ \frac{1}{n\gamma} \left( \frac{n}{n-2s+1} \right)^2 \mathbb{E}\left[ \frac{\partial f(\theta_i, K_i)}{\partial K_i} \left\{ (n-1) f(\theta_i, K_i) - \sum_{j \neq i} f(\theta_j, K_j) \right\} \right]$$

**Proof:** see Appendix 8

**Strategic behaviour and lack of coordination**

The problem of agent $i$ is :

$$\max_{K_i} \mathbb{E}_{\theta_i} \max_{a_i} \mathbb{E}_{\theta_{j\neq i}} \left[ f(\theta_i, K_i) w_i(a_i(\theta, K), K) - \gamma \frac{w_i^2(a_i, K)}{2} - (1-s) p(a_i, K) w_i(a_i, K) \right] - c(K_i) \qquad (2.20)$$

**Theorem 14:** When there is a price subsidy, and agents can lie and choose their own investment, then the marginal cost in equilibrium is:

$$\begin{aligned} c'(K) = & \frac{n^2 - 2ns + 2s - 1}{n^2(n-s)} \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}\right] \\ & + \frac{(n+1)(1-s)}{\gamma(n-2s+1)^2} \mathbb{E}\left[(n-1) f(\theta_i, K_i) \frac{\partial f(\theta_i, K_i)}{\partial K_i} - \sum_{j\neq i} f(\theta_j, K_j) \frac{\partial f(\theta_i, K_i)}{\partial K_i}\right] \\ & + \frac{s(n-1)^2}{\gamma(n-2s+1)^2} \left( \mathbb{E}\left[f(\theta_i, K_i) \frac{\partial f(\theta_i, K_i)}{\partial K_i}\right] - \mathbb{E}\left[f(\theta_i, K_i) \mathbb{E}\left[\frac{\partial f(\theta_i, K_i}{\partial K_i}\right]\right]\right) \end{aligned}$$

**Theorem 15:** In our game, when the regulator can use a price subsidy, then allocative efficiency can be reached with $\hat{s} = 0.5$ whereas investment efficiency is reached with another subsidy, such as both efficiency cannot be reached at the same time.

**Proof:** Using the allocation expression, we see that $\hat{s} = 0.5$ leads to efficient. Using theorems 13 and 14, we easily show that it exists a subsidy such as agents choose the optimal investment but it is not 0.5.

## 2.5 Substitutes and complements

In order to understand what happens, we illustrate our analysis with the two cases exposed at the beginning: type and investment are substitutes[7] $f(\theta_i, K_i) = \theta_i + K_i$ or complements $f(\theta_i, K_i) = \theta_i K_i$. For now on, we use the cost function $c(K_i) = \frac{1}{2} K_i^2$ such as we can determine the investment and the demand function in equilibrium in each case. For the simulation, we have to impose a distribution function and the value of the parameter $\gamma$. We use the uniform distribution function between $[1, 2]$ because for the complement case, it does not make sense to use a type that can be equal to zero. For $\gamma$, it is important to note that the parameter must be such as $f(\theta_i, K_i) w_i(\theta, K) - \gamma \frac{w_i(\theta, K)^2}{2}$ is positive in all the cases studied. For this reason, we choose $\gamma = 0.6$.

### 2.5.1 Substitutes

The type does not affect directly the choice of the capital, since capital and type are substitutes. This happens because each choice acts as a separate problem. The capital of the first best is simply one nth whereas it is smaller when agents choose individually the investment because of the price effect underlined in

[7]The results do not change fundamentally if we use a more general form as $\alpha\theta_i + \beta K_i + \eta$

| Without subsidy | Investment | Demand function |
|---|---|---|
| FB | $K_{FB} = \frac{1}{n}$ | $a_i(.) = \theta_i$ |
| LC | $K_{LC} = \frac{1}{n} - \frac{1}{n^2}$ | $a_i(.) = \theta_i$ |
| SB | $K_{SB} = \frac{1}{n}$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}[\theta]$ |
| SBLC | $K_{SBLC} = \frac{1}{n} - \frac{1}{n^3}$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}[\theta]$ |
| SBLC k unknown | $K_{SBLC}$ k unknown $= \frac{1}{n}$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}[\theta]$ |

the general case. When agents can lie, the demand function in equilibrium does not depend on the capital. However, before equilibrium, the demand function is :

$$a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{n-1}{n^2}K_i + \frac{1}{n^2}\sum_{j\neq i} K_j - \frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}[\theta] \tag{2.21}$$

The first part shows that $a_i$ is increasing in the real type. It is decreasing in his own investment and increasing in the investment of the other. In effect, the increase of $K_i$ has two effects. In one hand, the marginal productivity of each unit of permit is higher so he could take advantage of it and high his demand. On the other hand, agent $i$ would receive more rights if $K_i$ is higher but the price $\lambda$ increases too. In this case, he could prefer lying downwards to not increase the price he would pay. Thus, it seems the price effect dominates, agent $i$ chooses to low his revelation to compensate the rise of the price. When $K_j$ increases, $i$ knows he would receive less rights because the others are more productive and that the price would rise also. In this case, the effect that dominates is the first one such as $i$ decides to lie upwards. For the same reason, the function $a_i$ is increasing in the sum of the expected value of the real types of the others. Finally, it is decreasing in the factor $\gamma$ which is a parameter of concavity. By one side, the allocation of rights and the shadow price are decreasing in the parameter so we could imagine that the individual wants to compensate it. However, if the function of utility is more concave, the agent values less the additional unit of right so finally, he prefers to lie downwards to reduce even more the price . Since agents are symmetric, the capital is the same for all in equilibrium and effects cancel. The investment in this case is the same as in the first-best but since agents lie, the social welfare will be different. Therefore we see that agents use the instrument of demand function to manipulate the price and the allocation but since types and capital are substitutes and that agents do not choose the capital, the capital is the same than in the first best. Finally, the capital when agents can lie and decide themselves their investment is less than in the first-best but more than in the lack of coordination case. This means that when agents can use both instrument, they take advantage of both: they lie to manipulate the price and the allocation and they choose a lesser capital to reduce the price. When the capital is not observed, since the social planner will use a given capital, it is useless to use it as an instrument. Therefore the capital is the same as in the first best and the social welfare will be the same as in the strategic behaviour case since agents lie.

***Corollary:*** If types and investment are substitutes and if agents can reveal a false type and can choose their own capital, then the investment in equilibrium is less than the first-best investment: $K_{SBLC} = \frac{1}{n} - \frac{1}{n^3}$.

***Proof:***

To understand precisely why the capital is not the first-best capital in the SBLC case, we can write the marginal cost as:

$$c'(K_{sblc}) = \mathbb{E}[w_i(a_i(\theta_i, K))] - \mathbb{E}\left[w_i(a_i(\theta_i, K)) \underbrace{\left[\frac{\partial \lambda}{\partial K_i} + \sum_{j \neq i} \frac{\partial \lambda}{\partial a_j} \frac{\partial a_j}{\partial K_i}\right]}_{A}\right]$$

$$+ \mathbb{E}\left[(\theta_i - a_i) \underbrace{\left[\frac{\partial w_i(a_i(\theta_i, K))}{\partial K_i} + \sum_{j \neq i} \frac{\partial w_i(a_i(\theta_i, K))}{\partial a_j} \frac{\partial a_j}{\partial K_i}\right]}_{B}\right]$$

We analyse each part:

- $\mathbb{E}[w_i(a_i(\theta_i, K))]$

  When agents lie, the allocation depends on this lie. However, we show that in expectation, the lie does not matter:

$$w_i(a_i(\theta, K), K) = \frac{\gamma + (n-1)a_i(.) - \sum_{j \neq i} a_j(.)}{n\gamma}$$

  but we have:

$$\begin{aligned}
(n-1)a_i(.) - \sum_{j \neq i} a_j(.) &= (n-1)\frac{n}{n+1}\theta_i + (n-1)\left[-\frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}[\theta]\right] \\
&\quad - \sum_{j \neq i} \frac{n}{n+1}\theta_j - (n-1)\left[-\frac{1}{n(n-1)}\gamma + \frac{1}{n+1}\mathbb{E}[\theta]\right] \\
&= \frac{n}{n+1}\left[(n-1)\theta_i - \sum_{j \neq i} \theta_j\right]
\end{aligned}$$

  therefore

$$\mathbb{E}[w_i(a_i(\theta_i, K))] = \frac{1}{n} = \mathbb{E}[w_{i,FB}(\theta_i, K)] \tag{2.22}$$

- $\mathbb{E}\left[w_i(a_i(\theta_i, K)) \underbrace{\left[\frac{\partial \lambda}{\partial K_i} + \sum_{j \neq i} \frac{\partial \lambda}{\partial a_j} \frac{\partial a_j}{\partial K_i}\right]}_{A}\right]$

  Part $A$ shows the effect of a marginal change in $K_i$ on the price. The first element is the direct effect and the second element is the indirect effect on the price through the change in the function of demand of the other agents. Each derivative is positive: the price increases with the capital and the revealed type of each agent, and as we analysed, the function $a_j$ increases with $K_i$. Thus, the total effect of $A$

is positive and with the minus sign, the agent tends to ask for less capital in order to reduce the price. We have:

$$\frac{\partial \lambda}{\partial K_i} = \frac{1}{n}$$
$$\frac{\partial \lambda}{\partial a_j} = \frac{1}{n}$$
$$\frac{\partial a_j}{\partial K_i} = \frac{1}{n^2}$$

using this with the expression of the allocation we have

$$\mathbb{E}\left[ w_i(a_i(\theta_i, K)) \left[ \frac{\partial \lambda}{\partial K_i} + \sum_{j \neq i} \frac{\partial \lambda}{\partial a_j} \frac{\partial a_j}{\partial K_i} \right] \right] = \frac{n^2 + n - 1}{n^4} \tag{2.23}$$

which is positive but we have a negative sign in the expression.

- $\mathbb{E}\left[ (\theta_i - a_i) \underbrace{\left[ \frac{\partial w_i(a_i(\theta_i, K))}{\partial K_i} + \sum_{j \neq i} \frac{\partial w_i(a_i(\theta_i, K))}{\partial a_j} \frac{\partial a_j}{\partial K_i} \right]}_{B} \right]$

Finally, part $B$ shows the effect of a marginal change in $K_i$ on the allocation for agent $i$. As for A, there is the direct effect on $w_i(.)$ which is positive and the indirect effect through the change in the revealed type by the others. As before, the derivative of $a_j$ to $K_i$ is positive, but the allocation of agent $i$ diminishes with the revelation of the others such as the total effect could be positive or negative. Developing, we have:

$$\frac{\partial w_i(a(.))}{\partial K_i} = \frac{n-1}{n\gamma}$$
$$\frac{\partial w_i(a(.))}{\partial a_j} = -\frac{1}{n\gamma}$$
$$\frac{\partial a_j(.)}{\partial K_i} = \frac{1}{n^2}$$

We also have:

$$\theta_i - a_i(.) = \frac{1}{n+1}\theta + \frac{1}{n(n-1)}\gamma - \frac{1}{n+1}\mathbb{E}[\theta] + \frac{n-1}{n^2}K_i - \frac{1}{n^2}\sum_{j \neq i} K_j$$

Replacing and noting that in equilibrium $K_i = K_j \ \forall \ i \neq j$, we finally have:

$$\mathbb{E}\left[ (\theta_i - a_i) \left[ \frac{\partial w_i(a_i(\theta_i, K))}{\partial K_i} + \sum_{j \neq i} \frac{\partial w_i(a_i(\theta_i, K))}{\partial a_j} \frac{\partial a_j}{\partial K_i} \right] \right] = \frac{n^2 - 1}{n^4} \tag{2.24}$$

which is positive, so the marginal change in capital tends to increase the allocation for this agent.

Therefore the capital in SBLC case is :

$$\begin{aligned} K_{SBLC} &= \frac{1}{n} - \underbrace{\frac{n^2 + n - 1}{n^4}}_{\text{price effect}} + \underbrace{\frac{n^2 - 1}{n^4}}_{\text{allocation effect}} \\ &= \frac{1}{n} - \frac{1}{n^3} \\ &= K_{FB} - \frac{1}{n^3} \end{aligned}$$

The price effect is bigger than the allocation effect such as finally, the chosen capital is less than the first-best capital.

***Corollary:*** If types and capital are substitutes and agents can lie and choose their own investment, then the difference between the SBLC allocation and the FB allocation is less than an upper bound:

$$|w_i(a_i(.), K) - w_{FB}(\theta, K)| < \frac{n-1}{n(n+1)\gamma}$$

***Proof***: We can express the allocation in function of the first-best allocation. Using $w_i(a_i(.))$ it is easy to see that

$$w_i(a_i(.), K) = w_{FB}(\theta, K) - \frac{1}{n(n+1)\gamma}\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right]$$

Then we have:

$$\begin{aligned}|w_i(a_i(.), K) - w_{FB}(\theta, K)| &= \frac{1}{n(n+1)\gamma}\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right]\\ &\leq \frac{n-1}{n(n+1)\gamma} sop(\theta)\\ &\leq \frac{n-1}{n(n+1)\gamma}\end{aligned}$$

The difference between the allocations of FB and of SBLC is very small but it exists. We notice that as $n$ tends to infinity, both capital and allocation of SBLC tends to the FB results.
The size of the difference can be explained by the symmetry. Since we analyse when agents are symmetric, their lie are very similar and finally only the parameter on the true type is not removed. Other terms just cancel.

Then, when the social planner decide to give a subsidy to the capital, the objective is to make the investment closer to the first best value.

***Corollary:*** When types and capital are substitutes, if the social planner decides to put a capital subsidy then the capital decision is defined by :

$$K_z^s = \frac{1}{1-s}K_z \tag{2.25}$$

where $z = LC, SB, SBLC, SBLC$ k unknown

***Proof:*** Just replace in the general model.

***Corollary:*** If types and capital are substitutes, and agents can lie and choose their own investment, then the first best capital can be reached using the optimal capital subsidy $s^{opt} = \frac{1}{n}$. However, the social welfare is not the first-best social welfare.

**Proof:** Since we have $K_{SBLC}^s = \frac{1}{1-s}K_{SBLC}$ we want $s$ such as :

$$\frac{1}{1-s}\left(\frac{1}{n} - \frac{1}{n^3}\right) = \frac{1}{n} \tag{2.26}$$

so $s^{opt} = \frac{1}{n}$. As the number of agents increase, the subsidy needed to reach the first best capital decreases. The allocation is not changed with the capital subsidy, so even if we have the FB capital, agents still lie and the social welfare is not the FB social welfare.

Now we see what happens if the social planner decides to put a subsidy $\hat{s}$ to the price. Using the general model, results are:

| Price subsidy $\hat{s}$ | Investment | Demand function |
|---|---|---|
| LC | $K_{LC}^{\hat{s}} = \frac{n-(1-\hat{s})}{n^2}$ | $a_i(.) = \theta_i$ |
| SB | $K_{SB}^{\hat{s}} = \frac{1}{n(1-s)}$ | $a_i(\theta_i, K_i) = \frac{n}{n-2\hat{s}+1}\theta_i - \frac{\hat{s}n-2\hat{s}+1}{(1-\hat{s})(n-1)n}\gamma + \frac{\hat{s}}{1-\hat{s}}K + \frac{n\hat{s}-2\hat{s}+1}{(1-\hat{s})(n-2\hat{s}+1)}\mathbb{E}[\theta]$ |
| SBLC | $K_{SBLC}^{\hat{s}} = \frac{n^2+4n\hat{s}+2\hat{s}-1}{n^2(n-\hat{s})}$ | $a_i(\theta_i, K_i) = \frac{n}{n-2\hat{s}+1}\theta_i - \frac{\hat{s}n-2\hat{s}+1}{(1-\hat{s})(n-1)n}\gamma + \frac{\hat{s}}{1-\hat{s}}K + \frac{n\hat{s}-2\hat{s}+1}{(1-\hat{s})(n-2\hat{s}+1)}\mathbb{E}[\theta]$ |
| SBLC k unknown | $K_{SBLC\text{ k unknown}}^{\hat{s}} = \frac{1}{n}$ | $a_i(\theta_i, K_i) = \frac{n}{n-2\hat{s}+1}\theta_i - \frac{\hat{s}n-2\hat{s}+1}{(1-\hat{s})(n-1)n}\gamma + \frac{\hat{s}}{1-\hat{s}}K + \frac{n\hat{s}-2\hat{s}+1}{(1-\hat{s})(n-2\hat{s}+1)}\mathbb{E}[\theta]$ |

When the social planner decides to use a subsidy to the price, the effects are quite different because this subsidy does affect the demand function. The total effect on the social welfare is less obvious.
Following our analysis of the SBLC, we notice that with a subsidy to the price, the capital now affects the lie. A bigger investment implies an increase of the lie, showing that the agent wants to take advantage of a bigger marginal productivity to try to have more right. Moreover, the effect is increasing in the subsidy. We can show that each factor is increasing in the subsidy:

$$\frac{\partial}{\partial \hat{s}}\left[\frac{n}{n-2\hat{s}+1}\right] = \frac{2n}{(-2\hat{s}+n+1)^2} > 0$$
$$\frac{\partial}{\partial \hat{s}}\left[\frac{\hat{s}n-2\hat{s}+1}{(1-\hat{s})(n-1)n}\right] = \frac{1}{n(1-\hat{s})^2} > 0$$
$$\frac{\partial}{\partial \hat{s}}\left[\frac{\hat{s}(n-2)+1}{(1-\hat{s})(n-2\hat{s}+1)}\right] = \frac{n^2-2n\hat{s}^2+(1-2\hat{s})^2}{(1-\hat{s})^2(n-2\hat{s}+1)^2} > 0$$
$$\frac{\partial}{\partial \hat{s}}\left[\frac{\hat{s}}{1-\hat{s}}K_{SBLC}^{\hat{s}}(\hat{s})\right] = \frac{n^3+n^2(8-5\hat{s})\hat{s}+n(-6\hat{s}^2+4\hat{s}-1)-\hat{s}^2}{n^2(1-\hat{s})^2(n-\hat{s})^2} > 0$$

The global effect depends on the value of $\gamma$ since it is the only parameter with a minus sign.

***Corollary:*** If $\gamma = 0.6$ and $\theta$ is issued from a uniform distribution in $[1, 2]$, then the subsidy to the price makes the agent lie upward.

***Proof:*** Replacing the value of the parameter and using $\theta_i = 1$ as the worst possible value, the derivative is minimized in $n = 2$ and $s = 0$ and is positive, so the derivative is always positive. ■

***Corollary:*** When types and capital are substitutes, agents can lie and choose their capital, and that there is a price subsidy, then the capital can be written as a function of the first-best:

$$K^{\hat{s}}_{SBLC} = \frac{1}{n} + \frac{5n\hat{s} + 2\hat{s} - 1}{n^2(n - \hat{s})} \tag{2.27}$$

such as the optimal subsidy is $\hat{s}^{opt} = \dfrac{1}{5n+2}$. We note that as $n$ increases, the subsidy needed reduces.

***Proof:*** As before we can write the expression of the capital as:

$$c'(K^{\hat{s}}_{sblc}) = \mathbb{E}[w_i(a_i(\theta_i, K))] - \mathbb{E}\left[(1-\hat{s})w_i(a_i(\theta_i, K)) \underbrace{\left[\frac{\partial \lambda}{\partial K_i} + \sum_{j \neq i} \frac{\partial \lambda}{\partial a_j}\frac{\partial a_j}{\partial K_i}\right]}_{A}\right]$$
$$+ \mathbb{E}\left[(\theta_i - a_i + \hat{s}\left[a_i(.) - \gamma w_i(a_i(\theta_i, K))\right] \underbrace{\left[\frac{\partial w_i(a_i(\theta_i, K))}{\partial K_i} + \sum_{j \neq i} \frac{\partial w_i(a_i(\theta_i, K))}{\partial a_j}\frac{\partial a_j}{\partial K_i}\right]}_{B}\right]$$

The subsidy does not affect the first part but it does affect the others. On the price effect, since agents pay only a part of the total price, the effect is proportional to this part. On the allocation effect, we have, as without subsidy, $(\theta_i - a_i(.))$ on one side but it appears $\hat{s}\,(a_i(.) - \gamma w_i(a_i(.), K))$ on the other side, which is the derivative of the individual utility function multiplied by the subsidy. This second element appears because, when using the Envelop theorem, it leaves a remaining term. This term is positive by construction so $B$ is more important. It is not possible to conclude without developing the calculus but the intuition is that the price effect (which is negative) is less important and the allocation effect (which is positive) is bigger so the marginal cost gets closer to the FB marginal cost. Replacing, we have the final expression.

***Corollary:*** When types and capital are substitutes, the price subsidy can reduce the difference between the allocation of SBLC and the FB allocation.

***Proof:*** We can write the new allocation as a function of the FB:

$$w_i^{\hat{s}}(a_i(.)) = w_{i,FB}(\theta, K) - \frac{(1-2\hat{s})}{n(n-2\hat{s}+1)\gamma}\left[(n-1)\theta_i - \sum_{j \neq i}\theta_j\right] \tag{2.28}$$

and we can check that $\dfrac{\partial}{\partial \hat{s}}\left[\dfrac{1-2\hat{s}}{n-2\hat{s}+1}\right] = -\dfrac{2n}{(n-2\hat{s}+1)^2} < 0$ so the subsidy help us to reach the FB allocation.

***Corollary:*** When types and capital are substitutes and agents can lie and choose their investment, there is no price subsidy that allows to reach the first best social welfare.

***Proof:*** To reach the FB social welfare, we need to have the FB capital and the FB allocation. However, the optimal subsidy to reach the FB allocation is always 0.5 whereas it is $\dfrac{1}{5n+2}$ for the capital so the subsidy cannot be optimal for both objectives. ■

### 2.5.2 Complements

| Without subsidy | Investment | Demand function |
|---|---|---|
| FB | $K_{FB} = \frac{\gamma}{n\gamma - (n-1)V(\theta)}\mathbb{E}[\theta]$ | $a_i(.) = \theta_i$ |
| LC | $K_{LC} = \frac{(n-1)\gamma}{n^2\gamma - (n-1)^2V(\theta)}\mathbb{E}[\theta]$ | $a_i(.) = \theta_i$ |
| SB | $K_{SB} = \frac{\gamma(n+1)^2}{n\left[\gamma(n+1)^2 - (n-1)nV(\theta)\right]}\mathbb{E}[\theta]$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma\frac{1}{K_i} + \frac{1}{n+1}\mathbb{E}[\theta]$ |
| SBLC | $K_{SBLC} = \frac{(n^2-1)(n+1)\gamma}{n^3\left[\gamma(n+1) - (n-1)V(\theta)\right]}\mathbb{E}[\theta]$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma\frac{1}{K_i} + \frac{1}{n+1}\mathbb{E}[\theta]$ |
| SBLC k unknown | $K_{\text{k unknown}} = \frac{(n+1)\gamma}{n\left[\gamma(n+1) - (n-1)V(\theta)\right]}\mathbb{E}[\theta]$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma\frac{1}{K_i} + \frac{1}{n+1}\mathbb{E}[\theta]$ |

When types and capital are complement, the distribution function does affect the investment. The equilibrium investment is increasing in the expected value and the variance of the type.
In equilibrium, the lie does depend on the capital. As before, we can see how the demand function moves before equilibrium is :

$$a_i(\theta_i, K) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\frac{\gamma}{K_i} + \frac{1}{n^2}\mathbb{E}[\theta]\frac{\sum_{j\neq i} K_j}{K_i} + \frac{1}{n^2(n+1)}\mathbb{E}[\theta] \tag{2.29}$$

The first part shows that $a_i$ is increasing in the real type. The second element shows that it is increasing in his own investment but the third element shows the opposite. This underlines both effects of an increase in $K_i$. In one hand, the marginal productivity of each unit of rights is higher so he could take advantage of it and high his lie. On the other hand, agent $i$ would receive more permits if $K_i$ is higher but the price $\lambda$ increases too. In this case, he could prefer lying downwards to not increase the price he would pay. We cannot conclude which effect dominates.
When $K_j$ increases, $i$ knows he would receive less permits because the others are more productive and that the price would rise also. In this case, the effect that dominates is the first one such as $i$ decides to lie upwards. For the same reason, the function $a_i$ is increasing in the expected value of the real type. Finally, it is decreasing in the factor $\gamma$ which is a parameter of concavity. By one side, the allocation and the shadow price are decreasing in the parameter so we could imagine that the individual wants to compensate it. However, if the function of utility is more concave, the agent values less the additional unit of right so finally, he prefers to lie downwards to reduce even more the price .
In equilibrium, the complementary between investment and demand function appears through the parameter with $\gamma$ such as the demand function is increasing in the investment.

***Corollary:*** When types and capital are complements, agents can lie and choose their own capital, then the equilibrium investment is

$$K_{SBLC} = \frac{(n^2-1)(n+1)\gamma}{n^3\left[\gamma(n+1) - (n-1)V(\theta)\right]}\mathbb{E}[\theta]$$

.

***Proof:*** As for substitutes, we can write the marginal cost as:

$$c'(K_{i,sblc}) = \mathbb{E}[\theta_i w_i(a(\theta,K),K)] + \underbrace{\mathbb{E}\left[K(\theta_i - a_i(\theta,K))\left(\frac{\partial w_i(a(\theta,K),K)}{\partial K_i} + \sum_{j\neq i}\frac{\partial w_i(a(\theta,K),K)}{\partial a_j(\theta,K)}\frac{\partial a_j(\theta,K)}{\partial K_i}\right)\right]}_{A}$$
$$- \underbrace{\mathbb{E}\left[w_i(a(\theta,K),K)\left(\frac{\partial\lambda}{\partial K_i} + \sum_{j\neq i}\frac{\partial\lambda}{\partial a_j(\theta,K)}\frac{\partial a_j(\theta,K)}{\partial K_i}\right)\right]}_{B}$$

We analyse by part:

- $\mathbb{E}[\theta_i w_i(a(\theta,K),K)]$

  Using the expression of the demand function, we can write the allocation in function of the real type. As for substitutes, since agents lie symmetrically, the allocation is not very different of the FB allocation:

$$w_i(a_i(\theta,K),K) = \frac{1}{n} + \frac{1}{n\gamma}\frac{n}{n+1}K\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right] \tag{2.30}$$

  such as we have

$$\mathbb{E}[\theta_i w_i(a(\theta,K),K)] = \frac{1}{n}\mathbb{E}[\theta] + \frac{n-1}{n\gamma}\frac{n}{n+1}KV(\theta) \tag{2.31}$$

  The difference with the first best appears on the second term, with the factor $n/(n-1)$ which is not present in the FB.

- $\mathbb{E}\left[K(\theta_i - a_i(\theta,K))\left(\frac{\partial w_i(a(\theta,K),K)}{\partial K_i} + \sum_{j\neq i}\frac{\partial w_i(a(\theta,K),K)}{\partial a_j(\theta,K)}\frac{\partial a_j(\theta,K)}{\partial K_i}\right)\right] = A$

  This is the expected value of the marginal effect of a change in the capital by the difference between the real type and the demand function. The direct effect of an increase in capital of $i$ is an increase of his allocation of emission rights. However, there is an indirect effect. If the capital of $i$ increases, agents $j$ would increase their function of demand to counteract, and the allocation of $i$ diminishes if the revealed type of the others increases. The total effect can be positive or negative.
  Noting that:

$$\frac{\partial w_i(a(.))}{\partial K_i} = a_i(\theta,K)\frac{(n-1)}{n\gamma}$$
$$\frac{\partial w_i(a(.))}{\partial a_j} = -\frac{K_j}{n\gamma}$$
$$\frac{\partial a_j(.)}{\partial K_i} = \frac{\mathbb{E}[\theta]}{n^2K_j}$$

  Replacing we finally have:

$$A = \frac{n-1}{(n+1)^2\gamma}KV(\theta) - \frac{\gamma}{n^3(n-1)K} + \mathbb{E}[\theta]\left(\frac{n^3+n^2-n-1}{n^4(n+1)}\right)$$

- $\mathbb{E}\left[w_i(a(\theta,K),K)\left(\frac{\partial\lambda}{\partial K_i} + \sum_{j\neq i}\frac{\partial\lambda}{\partial a_j(\theta,K)}\frac{\partial a_j(\theta,K)}{\partial K_i}\right)\right] = B$

B is the price effect. The direct effect is the increase of the shadow price if the capital of $i$ rises. The indirect effect is the change of the price through the changes in the demand function of the others caused by the capital. As we explained for A, agents $j$ ask more $a_j(.)$ if the $i$'s capital rises, and the price increases also. Therefore, the total effect is positive which with the minus sign, indicates that the total price effect diminishes $c'(K_i)$.
Noting that:

$$\frac{\partial \lambda}{\partial K_i} = \frac{a_i(.)}{n}$$
$$\frac{\partial \lambda}{\partial a_j} = \frac{K_j}{n}$$
$$\frac{\partial a_j}{\partial K_i} = \frac{\mathbb{E}[\theta]}{n^2 K_j}$$

Replacing, we finally have:

$$B = \frac{(n-1)}{\gamma(n+1)^2} KV(\theta) - \frac{\gamma}{n^3(n-1)K} + \frac{n^2+n-1}{n^4}\mathbb{E}[\theta] \tag{2.32}$$

- Adding parts A and B we find:

$$A - B = -\frac{1}{n^3}\mathbb{E}[\theta] \tag{2.33}$$

such as the marginal cost is:

$$c'(K_{SBLC}) = \left(\frac{1}{n} - \frac{\mathbf{1}}{\mathbf{n^3}}\right)\mathbb{E}[\theta] + \frac{n-1}{n\gamma}\frac{\mathbf{n}}{\mathbf{n+1}}KV(\theta) \tag{2.34}$$

where the terms in bold font indicate the difference with the FB. Since we are using $c(K) = \frac{1}{2}K^2$ we then need to extract the capital:

$$K_{SBLC} = \frac{(n^2-1)(n+1)\gamma}{n^3\left[\gamma(n+1) - (n-1)V(\theta)\right]}\mathbb{E}[\theta] \tag{2.35}$$

***Corollary:*** When types and capital are complements and agents can lie and choose their own capital, then the difference between the allocation and the first-best allocation is less than the upperbound:

$$|w_i(a(\theta, K), K) - w_{i,FB}(\theta)| < \frac{n-1}{(n+1)\gamma}K$$

***Proof:*** We can rewrite the allocation :

$$w_i(a(\theta, K), K) = w_{i,FB}(\theta) - \frac{1}{(n+1)\gamma}K\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right]$$

so we have:

$$\begin{aligned}
|w_i(a(\theta, K), K) - w_{i,FB}(\theta)| &= \frac{1}{(n+1)\gamma}K\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right] \\
&\leq \frac{1}{(n+1)\gamma}K(n-1)sop(\theta) \\
&\leq \frac{n-1}{(n+1)\gamma}K
\end{aligned}$$

As for substitutes, the difference would not be very big. However, since the capital appears because of the complementary between capital and type, the effect of the inefficiency is multiplied. As we will see in the simulations, the differences in percentage are bigger in this case than in the substitutes case.

As before, we can analyse the effect of putting a capital subsidy. Since the capital appears in both side of the marginal cost equilibrium equation, the investment result is more complicated than earlier :

| Capital subsidy s | Investment | Demand function |
|---|---|---|
| LC | $K^s_{LC} = \frac{(n-1)\gamma\mathbb{E}[\theta]}{n^2(1-s)\gamma - (n-1)^2 V(\theta)}$ | $a_i(.) = \theta_i$ |
| SB | $K^s_{SB} = \frac{(n+1)^2\gamma\mathbb{E}[\theta]}{n\left[(n+1)^2(1-s)\gamma - n(n-1)V(\theta)\right]}$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma\frac{1}{K_i} + \frac{1}{n+1}\mathbb{E}[\theta]$ |
| SBLC | $K^s_{SBLC} = \frac{(n^2-1)(n+1)\gamma\mathbb{E}[\theta]}{n^3\left[(n+1)(1-s)\gamma - (n-1)V(\theta)\right]}$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma\frac{1}{K_i} + \frac{1}{n+1}\mathbb{E}[\theta]$ |
| SBLC k unknown | $K^s_{\text{k unknown}} = \frac{(n+1)\gamma\mathbb{E}[\theta]}{n\left[(n+1)(1-s)\gamma - (n-1)V(\theta)\right]}$ | $a_i(\theta_i, K_i) = \frac{n}{n+1}\theta_i - \frac{1}{n(n-1)}\gamma\frac{1}{K_i} + \frac{1}{n+1}\mathbb{E}[\theta]$ |

The lie is not affected directly by the subsidy but through the capital. The lie is the same as in the case without subsidy. For SBLC we can calculate the optimal subsidy easily comparing both capital.

***Corollary:*** When types and capital are complements and agents can lie and choose their own investment, a capital subsidy can help us to reach the first-best capital. If types are described by the uniform distribution $[1, 2]$ and $\gamma = 0.6$ then this subsidy is decreasing in the number of participants.

***Proof:*** The optimal subsidy comes from:

$$K^s_{SBLC} = \frac{(n^2-1)(n+1)\gamma\mathbb{E}[\theta]}{n^3\left[(n+1)(1-s)\gamma - (n-1)V(\theta)\right]} = K_{FB} = \frac{\gamma\mathbb{E}[\theta]}{n\gamma - (n-1)V(\theta)}$$

such as:

$$s^{opt} = \frac{\gamma(n^2+1) + V(\theta)\left[n^3 - 2n^2 + 1\right]}{\gamma n^3(n+1)}$$

and we can check that :

$$\frac{\partial s^{opt}}{\partial n} = -\frac{2\gamma n(n+1)^2 + V(\theta)[n^4 - 4n^3 - 2n^2 + 4n + 3]}{\gamma n^4(n+1)^2}$$

where $n^4 - 4n^3 - 2n^2 + 4n + 3$ is positive for $n \geq 5$ and in our case with $\gamma = 0.6$ and $V(\theta) = 1/12$ the derivative is always negative, so the subsidy goes to zero as n increases, as expected.

It is possible to reach the first-best capital with the subsidy but since agents lie, the allocation will not be the first best one so the social welfare will be different.

Now, we analyse the results if the social planner decides to put a subsidy to the price.

| Price subsidy | Investment | Demand function |
|---|---|---|
| LC | $K^{\hat{s}}_{LC} = \dfrac{(n-(1-\hat{s}))(n-1)\gamma\mathbb{E}[\theta]}{n^3\gamma-(n-(1-\hat{s}))(n-1)^2V(\theta)}$ | $a_i(.) = \theta_i$ |
| SB | $K^{\hat{s}}_{SB} = \dfrac{(n-2\hat{s}+1)\gamma\mathbb{E}[\theta]}{n(1-s)\left[\gamma(n-2s+1)-n(n-1)V(\theta)\right]}$ | $a_i(.) = \dfrac{n}{n-2s+1}\theta_i - \dfrac{sn-2s+1}{(1-s)(n-1)n}\dfrac{\gamma}{K} + \dfrac{sn-2s+1}{(1-s)(n-2s+1)}\mathbb{E}[\theta]$ |
| SBLC | $K^{\hat{s}}_{SBLC} = \dfrac{(n-2\hat{s}+1)^2\gamma\mathbb{E}[\theta]}{n(1-\hat{s})\left[(n-2\hat{s}+1)^2\gamma-n(n-1)V(\theta)\right]}$ | $a_i(.) = \dfrac{n}{n-2s+1}\theta_i - \dfrac{sn-2s+1}{(1-s)(n-1)n}\dfrac{\gamma}{K} + \dfrac{sn-2s+1}{(1-s)(n-2s+1)}\mathbb{E}[\theta]$ |
| SBLC k unknown | $K^{\hat{s}}_{\text{k unknown}} = \dfrac{(n-2\hat{s}+1)\gamma\mathbb{E}[\theta]}{n\left[(n-2\hat{s}+1)\gamma-(n-1)V(\theta)\right]}$ | $a_i(.) = \dfrac{n}{n-2s+1}\theta_i - \dfrac{sn-2s+1}{(1-s)(n-1)n}\dfrac{\gamma}{K} + \dfrac{sn-2s+1}{(1-s)(n-2s+1)}\mathbb{E}[\theta]$ |

A subsidy to the price affects the lie and through it, the equilibrium capital. Contrary to the capital, there is no new element, the subsidy affects every parameter. Since the factors are the same as with substitutes, we already showed the sign of the derivative. The subsidy intensifies each parameter. For the case of SBLC, we can see that the price subsidy affects the capital differently than the capital subsidy.

***Corollary:*** When types and capital are complements and agents can lie and choose their own investment, it exists a price subsidy $\hat{s}$ such as $K^{\hat{s}}_{SBLC} = K_{FB}$.

***Proof***: We could calculate the optimal subsidy but the solution is a huge expression so we prefer to show that without subsidy, $K_{SBLC}$ is less than $K_{FB}$ whereas with a subsidy of 0,9 the opposite happens so it exists a subsidy $\hat{s}$ such as they are equal.

- if $\hat{s} = 0$:
$$\frac{(n+1)2}{n\left[(n+1)^2\gamma-n(n-1)V(\theta)\right]} < \frac{1}{n\gamma-(n-1)V(\theta)}$$
since we see that $n(n+1)^2\gamma-(n+1)^2(n-1)V(\theta) < n(n+1)^2\gamma-n^2(n-1)V(\theta)$

- if $\hat{s} = 0.9$:
$$\frac{10(n-0.8)^2}{n\left[(n-0.8)^2\gamma-n(n-1)V(\theta)\right]} > \frac{1}{n\gamma-(n-1)V(\theta)}$$
since we see that: $10\left[n\gamma(n-0.8)^2-(n-1)(n-0.8)^2V(\theta)\right] > n\gamma(n-0.8)^2-(n-1)n^2V(\theta)$.

On another side, we can see what happens for the allocation.

***Corollary:*** When types and capital are substitutes, the price subsidy can reduce the difference between the allocation of SBLC and the FB allocation.

***Proof:*** Contrary to substitutes, the capital appears in the allocation so the difference is more complicated

$$w_i(a_i(.)) = \frac{1}{n} + \frac{1}{n\gamma}K^{\hat{s}}_{SBLC}\frac{n}{n-2s+1}\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right]$$

$$w_{FB} = \frac{1}{n} + \frac{1}{n\gamma}K_{FB}\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right]$$

Replacing we have:

$$w_i(a_i(.)) = w_{FB} - \frac{1}{n\gamma}\left[(n-1)\theta_i - \sum_{j\neq i}\theta_j\right]\left[K_{FB} - \frac{n}{n-2s+1}K^{\hat{s}}_{SBLC}\right] \tag{2.36}$$

If $K^{\hat{s}}_{SBLC} = K_{FB}$, we have the same than for substitutes. The derivative $\frac{\partial}{\partial\hat{s}}\left[\frac{1-2\hat{s}}{n-2\hat{s}+1}\right] = -\frac{2n}{(n-2\hat{s}+1)^2} < 0$ so the subsidy help us to reach the FB allocation. Since we show that the subsidy helps us to reach the first best capital, the subsidy helps us to reduce the difference of allocation.

***Corollary:*** When types and capital are complements and agents can lie and choose their investment, there is no price subsidy that allows to reach the first best social welfare.

***Proof:*** To reach the FB social welfare, we need to have the FB capital and the FB allocation. However, the optimal subsidy to reach the FB allocation cannot be the same than for the capital since we need to have at the same time $K^{\hat{s}}_{SBLC} = K_{FB}$ and $\frac{n}{n-2s+1}K^{\hat{s}}_{SBLC} = K_{FB}$.

### 2.5.3 Simulation results

To simulate results, we first calculate the investment decision in equilibrium, the demand functions and then the social welfare :

$$SW = \mathbb{E}\left[\sum_i\left\{f(\theta_i,K)w_i(\theta,K) - \gamma\frac{w_i(\theta,K)^2}{2}\right\} - \sum_i\frac{1}{2}K^2\right] \tag{2.37}$$

for the FB and the LC cases with the correspondent capital, and:

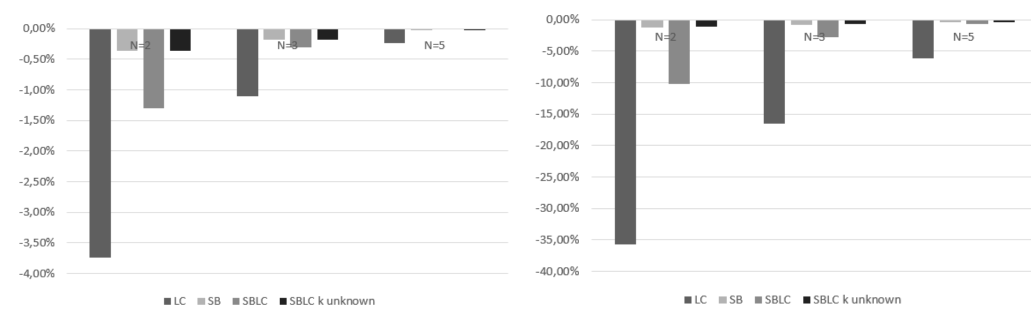
$$SW = \mathbb{E}\left[\sum_i\left\{f(\theta_i,K)w_i(a(\theta,K),K) - \gamma\frac{w_i(a(\theta,K),K)^2}{2}\right\} - \sum_i\frac{1}{2}K^2\right] \tag{2.38}$$

for the SB and SBLC cases where the allocation depends on the demand function, not the real type.

We are interested in the percent differences with respect to the first-best. Tables are available in Appendix (9).

Figure 1 shows the results for the case without subsidy. As expected, differences decrease with the total number of participants and from 5 agents, differences are almost null. Differences are small and we note that they are bigger in case of complements than substitutes but the logic is the same. In case of lack of coordination, since agents cannot lie, the unique instrument they have to manipulate the result in their favour is the capital. They choose a smaller capital to reduce the price and this results in inefficiency. If they cannot choose their own capital but can lie (strategic behaviour), the inefficiency is reduced a lot. They manipulate the allocation through their revelation but since agents are symmetric the difference in allocation and then in social welfare, is relatively small. However, when they can use both instrument, the inefficiency is bigger than in strategic behaviour and smaller than in lack of coordination. They do not need to choose a capital as small as in lack of coordination since they can use the other instrument.

When investment is not observable, we already explained that with substitutes, the social welfare is the same than the strategic behaviour case. With complements, the lie is the same but the capital is a little different such as the social welfare is barely smaller.




(a) Substitutes

(b) Complements

Figure 2.1: Percent differences with the FB when there is no subsidy

Now, Figures 2 and 3 show how the social welfares vary when we add the subsidy to the capital. For one side, the subsidy allows to get closer to the FB-investment but for other side, the capital cost increases. For the substitutes, in case of lack of coordination, it is easy to calculate that the optimal subsidy is 1/n so the difference with the first best decreases and then increases. In case of strategic behaviour, the difference increases such as it is useless to use a subsidy to the capital. In effect the difference with the first best without subsidy is very small so adding a subsidy is costly. In the case of strategic behaviour-lack of coordination, the subsidy is useful for low value of subsidy but then, the increase in utility generated by more capital does not compensate the increase in the capital cost such as the subsidy is too high. The logic for the complements is the same. It is interesting to note that with a subsidy of 0.5, the cost of capital is so high that the social welfare is negative in the cases of strategic behaviour, strategic behaviour and lack of coordination. The figures allow us to have an idea of the optimal subsidies.
Figures 4 and 5 present the effects of a price subsidy. The logic is the same as before but with the associated optimal subsidy. The subsidy needed in the case that agent can lie is smaller than in the case they cannot, which is intuitive since without subsidy, the social welfare is initially closer to the first-best

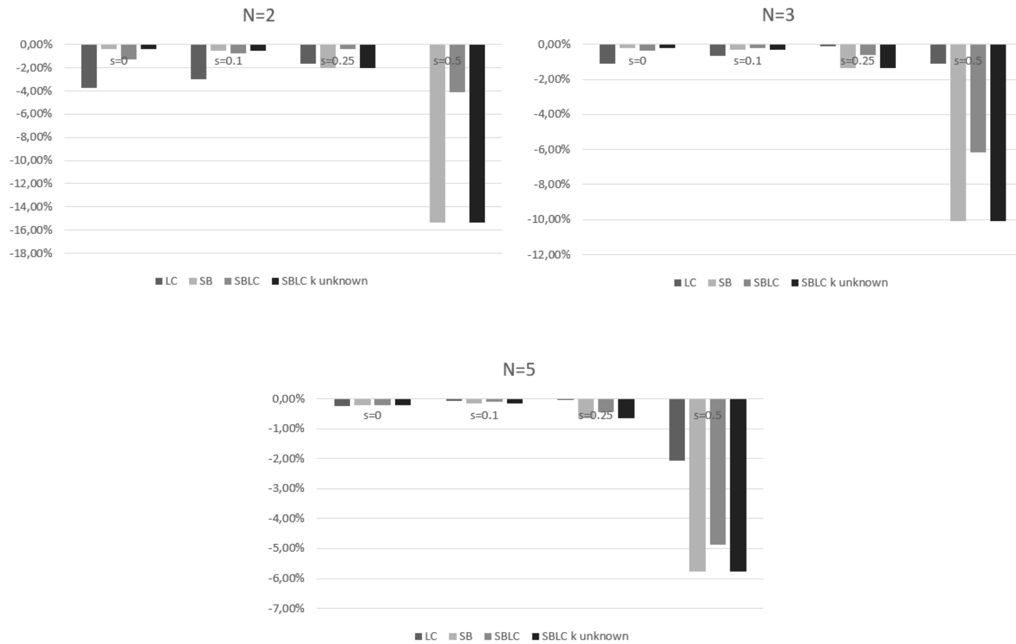



Figure 2.2: Effects of the capital subsidy with substitutes

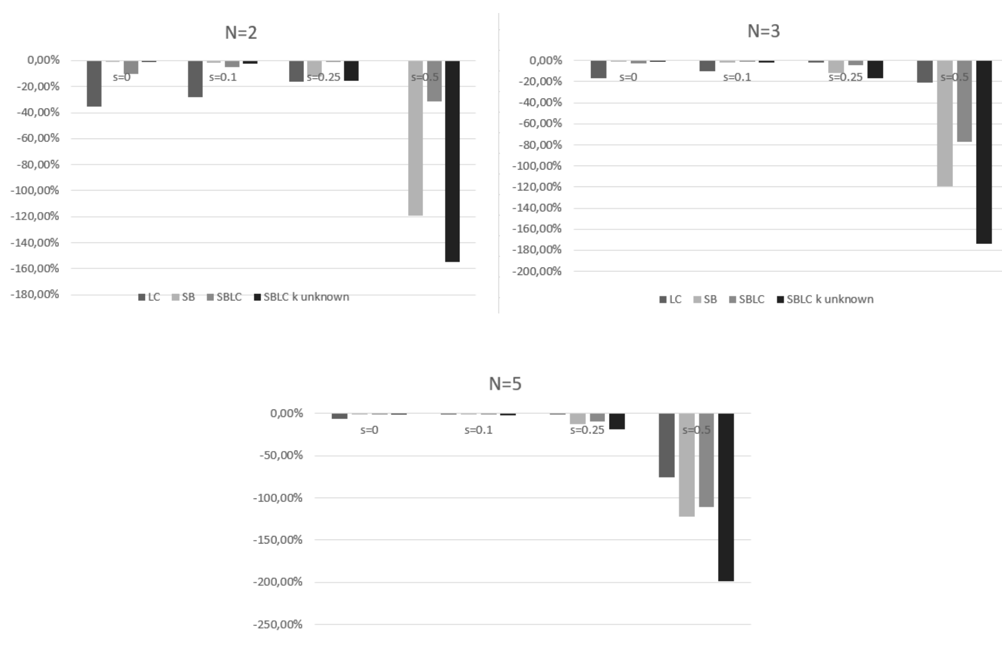



Figure 2.3: Effects of the capital subsidy with complements

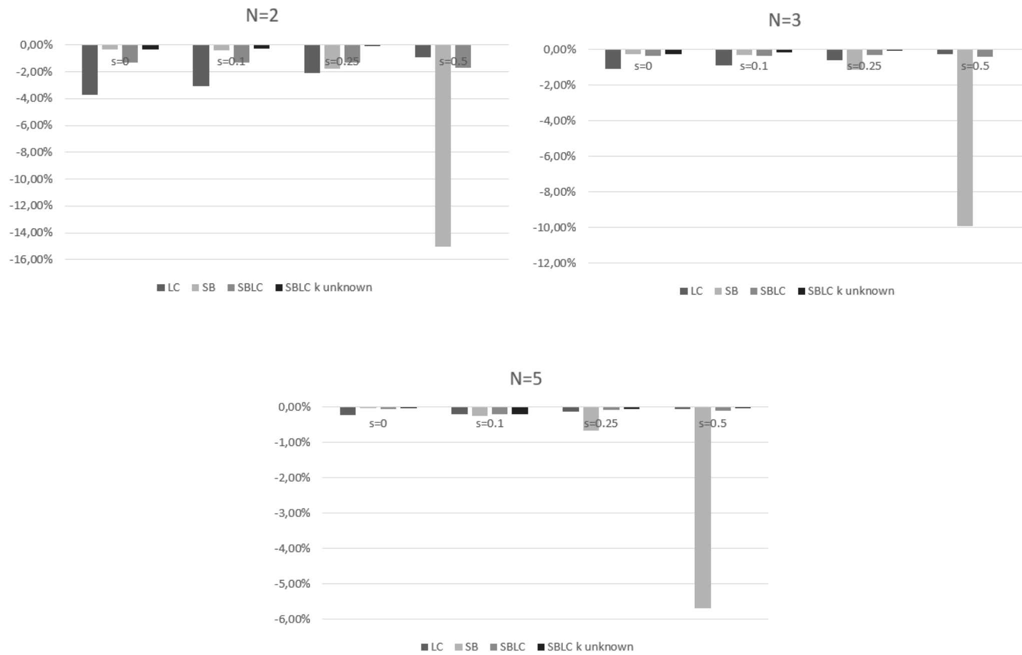



Figure 2.4: Effects of the price subsidy with substitutes

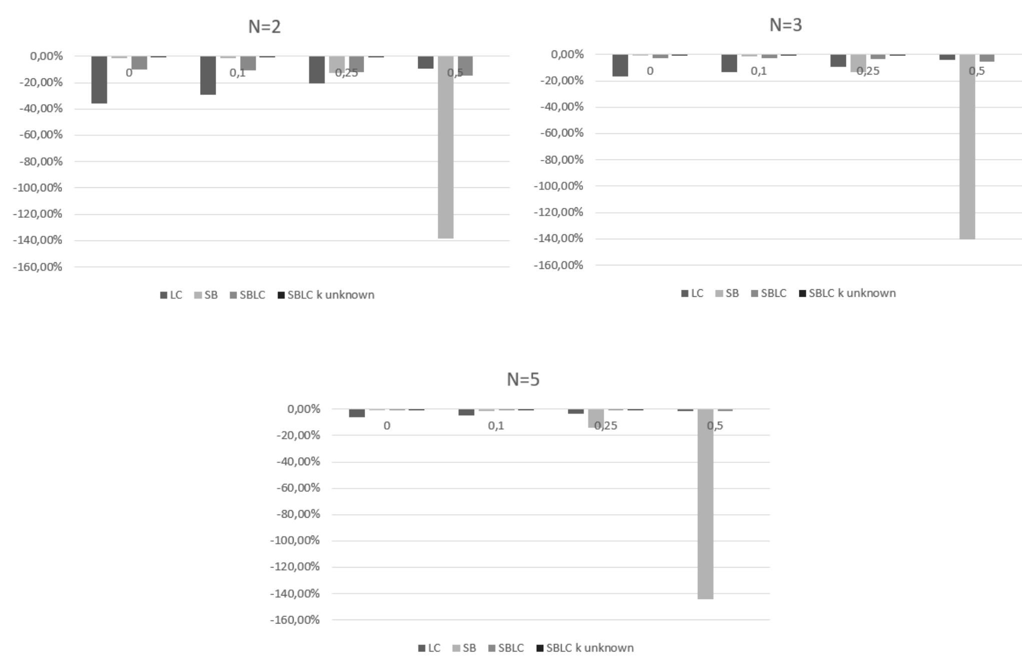



Figure 2.5: Effects of the price subsidy with complements

## 2.6 Appendix

### Appendix 1

In period two, the problem is:

$$\max_{w_1,\dots,w_n} \sum_i \left[ f(\theta_i, K_i)w_i - \gamma \frac{w_i^2}{2} \right]$$
$$s.t. \sum_i w_i = 1$$
$$w_i \geq 0 \; \forall \; i$$

Using the KKT theory, we rewrite the problem as:

$$\min_{w_i} - \sum_i f(\theta_i, K_i)w_i + \sum_i \gamma \frac{w_i^2}{2}$$
$$s.t. \sum_i w_i = 1 \;\; (\lambda)$$
$$w_i \geq 0 \; \forall \; i \; (\mu_i)$$

The Lagrangian is :

$$\mathscr{L} = - \sum_i f(\theta_i, K_i)w_i + \sum_i \gamma \frac{w_i^2}{2} + \lambda \left( \sum_i w_i - 1 \right) + \sum_i \mu_i(-w_i)$$
$$s.t. \sum_i w_i = 1$$
$$w_i \geq 0 \; \forall \; i$$
$$\mu_i(-w_i) = 0 \; \forall \; i$$
$$\mu_i \geq 0 \; \forall \; i$$

The FOC are:

$$\frac{\partial \mathscr{L}}{\partial w_i} = -f(\theta_i, K_i) + \gamma w_i + \lambda - \mu_i = 0 \rightarrow w_i = \frac{f(\theta_i, K_i) - \lambda + \mu_i}{\gamma}$$

$$\sum_i w_i = \frac{\sum_i f(\theta_i, K_i) - \lambda n + \sum_i \mu_i}{\gamma} = 1 \rightarrow \lambda = \frac{\sum_i f(\theta_i, K_i) + \sum_i \mu_i - \gamma}{n}$$

Using both results, we have

$$w_i = \frac{1}{\gamma n} \left[ (n-1) f(\theta_i, K_i) - \sum_{j \neq i} f(\theta_j, K_j) + (n-1)\mu_i - \sum_{j \neq i} \mu_j + \gamma \right]$$

$$w_i \geq 0 \rightarrow (n-1) \left[ f(\theta_i, K_i) + \mu_i \right] \geq \sum_{j \neq i} f(\theta_j, K_j) + \sum_{j \neq i} \mu_j$$

Then we analyse the cases:

- $\mu_i = 0 \; \forall \; i$
  if $\mu_i = 0 \rightarrow w_i$ is free $\rightarrow w_i = \dfrac{1}{\gamma n} \left[ (n-1) f(\theta_i, K_i) - \sum_{j \neq i} f(\theta_j, K_j) + \gamma \right]$ but we need to check the no negativity constraint : $(n-1) f(\theta_i, K_i) + \gamma \geq \sum_{j \neq i} f(\theta_j, K_j)$

- $\mu_i > 0 \; \forall \; i$
  if $\mu_i > 0 \rightarrow w_i = 0 \rightarrow \sum_i w_i = 0 \rightarrow$ Not possible

- $\mu_{\bar{i}} > 0 \; \forall \; \bar{i} \in Z$
  We suppose that there are z agents with $\mu_i > 0$ and n-z with $\mu_i = 0$:

  - $\bar{i} \in Z$: $\mu_{\bar{i}} > 0 \rightarrow w_{\bar{i}} = 0$
  - $j \in R = N/Z$: $\mu_j = 0 \rightarrow w_j$ is free

  We have $\sum_i w_i = \sum_{j\in R} = \dfrac{1}{\gamma}\left[\sum_{j\in R} f(\theta_j, K_j) - \lambda(n-Z) + 0\right] = 1$ such as $\lambda = \dfrac{\sum_{j\in R} f(\theta_j, K_j) - \gamma}{n-Z}$.
  Replacing we also have $w_j = \dfrac{1}{\gamma(n-Z)}\left[(n-Z-1)f(\theta_j, K_j) - \sum_{k\neq jk\in R} f(\theta_k, K_k) + \gamma\right]$.
  Finally, the condition of no negativity is: $w_j \geq 0 \rightarrow (n-Z-1)f(\theta_j, K_j) + \gamma \geq \sum_{k\neq jk\in Z} f(\theta_k, K_k)$.

## Appendix 2

In period one, the problem is:

$$\max_{K_1,\dots,K_n} \mathbb{E}\left[\sum_i f(\theta_i, K_i)\right] - \sum_i c(K_i)$$

FOC:

$$\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(\theta, K) + \sum_j \left(f(\theta_j, K_j) - \gamma w_j(\theta, K)\right) \frac{\partial w_j(\theta, K)}{\partial K_i}\right] = c'(K_i)$$

However, using Appendix 1, we have that $f(\theta_i, K_i) - \gamma w_i(\theta, K) = f(\theta_j, K_j) - \gamma w_j(\theta, K)$ for all $i, j$, and since $w_1 + w_2 + ... + w_n = 1$, we know that $\dfrac{\partial w_i}{\partial K_i} = -\sum_{j\neq i} \dfrac{\partial w_j}{\partial K_i}$, so we can simplify:

$$\sum_j \left(f(\theta_j, K_j) - \gamma w_j(\theta, K)\right) \frac{\partial w_j(\theta, K)}{\partial K_i} = 0$$

and finally:

$$c'(K_1) = \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(\theta, K)\right] \tag{2.39}$$

## Appendix 3

In period one, agent is maximizes his utility, that is his functions of utility less the cost of water and the cost of investment:

$$\max_{K_i} \mathbb{E}\left[f(\theta_i, K_i) w_i(\theta, K) - \gamma \frac{w_i^2(\theta, K)}{2} - \lambda(\theta, K) w_i(\theta, K) - c(K_i)\right]$$

FOC:

$$\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(\theta, K) - \frac{\partial \lambda(\theta, K)}{\partial K_i} w_i(\theta, K) + \left[f(\theta_i, K_i) - \gamma w_i(\theta, K) - \lambda(\theta, K)\right] \frac{\partial w_i(\theta, K)}{\partial K_i}\right] = c'(K_i)$$

but, as before, $\lambda = f(\theta_i, K_i) - \gamma w_i$, so

$$c'(K_i) = \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(\theta, K) - \frac{\partial \lambda(\theta, K)}{\partial K_i} w_i(\theta, K)\right]$$

Notice that $\dfrac{\partial \lambda(\theta, K)}{\partial K_i} = \dfrac{1}{n}\dfrac{\partial f(\theta_i, K_i)}{\partial K_i}$, so:

$$c'(K_i) = \frac{n-1}{n}\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(\theta, K)\right]$$

## Appendix 4

Agent $i$ knows he can lie in period two, so he first determines how he will lie. For this, he maximizes his utility :

$$\max_{a_i} \mathbb{E}_{\theta_{j\neq i}}\left[f(\theta_i, K_i)w_i(a, K) - \gamma\frac{w_i^2(a, K)}{2} - \lambda(a, K)w_i(a, K)\right]$$

FOC:

$$\mathbb{E}\left[f(\theta_i, K_i)\frac{\partial w_i}{\partial a_i} - \gamma w_i(a, K)\frac{\partial w_i(a, K)}{\partial a_i} - \lambda(a, K)\frac{\partial w_i(a, K)}{\partial a_i} - w_i(a, K)\frac{\partial \lambda(a, K)}{\partial a_i}\right] = 0$$

but $\lambda = f(a_i, K_i) - \gamma w_i(a, K)$, so

$$\mathbb{E}\left[(f(\theta_i, K_i) - f(a_i, K_i))\frac{\partial w_i(a, K)}{\partial a_i} - w_i(a, K)\frac{\partial \lambda(a, K)}{\partial a_i}\right] = 0$$

Noting that $\dfrac{\partial w_i(a, K)}{\partial a_i} = \dfrac{n-1}{n\gamma}\dfrac{\partial f(a_i, K_i)}{\partial a_i}$ and $\dfrac{\partial \lambda(a, K)}{\partial a_i} = \dfrac{1}{n}\dfrac{\partial f(a_i, K_i)}{\partial a_i}$, and using the expression of $w_i(a, K)$ we have that agent i solves:

$$f(a_i(\theta, K), K_i) = \frac{n}{n+1}f(\theta_i, K_i) - \frac{1}{(n-1)(n+1)}\gamma + \frac{1}{(n-1)(n+1)}\mathbb{E}\left[\sum_{j\neq i} f(a_j(\theta, K), K_j)\right]$$

To solve, we use the technique of "guess and verify". Agents are symmetric so we are looking for a symmetric equilibrium. The guess we use is:

$$f(a_j, K_j) = cf(\theta_j, K_j) + e\gamma + d\sum_{l\neq j}\mathbb{E}\left[f(\theta_l, K_l)\right] + g\mathbb{E}\left[f(\theta_j, K_j)\right]$$

Replacing in $f(a_i(\theta, K), K_i)$ and solving for the parameters, we finally have:

$$f(a_i(\theta, K) = \frac{n}{n+1}f(\theta_i, K_i) - \frac{1}{n(n-1)}\gamma + \frac{1}{n^2(n+1)}\mathbb{E}\left[f(\theta_i, K_i)\right] + \frac{1}{n^2}\sum_{j\neq i}\mathbb{E}\left[f(\theta_j, K_j)\right]$$

## Appendix 5

Once we know the function $a_i(\theta, K)$, we can determine the investment. The logic is the same as for the first-best but we use $a_i$ instead of the real type. The main difference is that the lie depends on the capital. Now, the problem is:

$$\max_{K_1,\ldots,K_n} \mathbb{E}\left[\sum_i f(a_i(\theta, K), K_i)w_i(a_i(\theta, K), K_i) - \gamma\frac{w_i^2(a_i(\theta, K), K_i)}{2} - \sum_i c(K_i)\right]$$

FOC of $K_i$:

$$\begin{aligned}\mathbb{E}\Bigg[&\frac{\partial f(a_i(\theta, K), K_i}{\partial K_i}w_i(a(\theta, K), K) + \sum_{j\neq i}\frac{\partial f(a_j(\theta, K), K_j)}{\partial K_i}w_j(a(\theta, K), K)\\ &+ \left[f(a_i(\theta, K), K_i) - \gamma w_i(a(\theta, K), K)\right]\sum_j \frac{\partial w_j(\theta, K)}{\partial K_i}\\ &+ \left[f(a_i(\theta, K), K_i) - \gamma w_i(a(\theta, K), K)\right]\sum_l\sum_j \frac{\partial w_j(\theta, K)}{\partial a_l}\frac{\partial a_l(\theta, K)}{\partial K_i} = c'(K_i)\end{aligned}$$

We used that $f(a_i, K_i) - \gamma w_i(a(\theta, K), K) = f(a_j, K_j) - \gamma w_j(a(\theta, K), K)) = \lambda$ for all $i, j$. Moreover, since $\sum_i w_i(.) = 1$, we have that $\dfrac{\partial w_i(a(\theta, K), K)}{\partial K_i} = -\sum_{j\neq i} \dfrac{\partial w_j(a(\theta, K), K)}{\partial K_i}$ and $\dfrac{\partial w_i(a(\theta, K), K)}{\partial a_i} = -\sum_{j\neq i} \dfrac{\partial w_j(a(\theta, K), K)}{\partial a_i}$ for all $i$ such as, for all $j$:

$$\sum_j \frac{\partial w_j(\theta, K)}{\partial K_i} = 0$$

$$\sum_l \sum_j \frac{\partial w_j(\theta, K)}{\partial a_l} \frac{\partial a_l(\theta, K)}{\partial K_i} = 0 \tag{2.40}$$

which reduces the FOC to:

$$\mathbb{E}\left[\sum_j \frac{\partial f(a_j(\theta, K), K_j)}{\partial K_i} w_j(a(\theta, K), K)\right] = c'(K_i) \tag{2.41}$$

Note that:

$$\frac{\partial f(a_i(\theta_i, K), K_i)}{\partial K_i} = \frac{n}{n+1} \frac{\partial f(\theta_i, K_i)}{\partial K_i} + \frac{1}{n^2(n+1)} \frac{\partial \mathbb{E}(f(\theta_i, K_i))}{\partial K_i}$$

$$\frac{\partial f(a_j(\theta_j, K), K_j)}{\partial K_i} = \frac{1}{n^2} \frac{\partial \mathbb{E}(f(\theta_i, K_i))}{\partial K_i} \tag{2.42}$$

such as we can replace and we have:

$$\begin{aligned} c'(K_i) = \quad & \mathbb{E}\left[\frac{n}{n+1} \frac{\partial f(\theta_i, K_i)}{\partial K_i} w_i(a(\theta, K), K) + \frac{1}{n^2(n+1)} \frac{\partial \mathbb{E}(f(\theta_i, K_i))}{\partial K_i} w_i(a(\theta, K), K)\right. \\ & \qquad \left. + \sum_{j\neq i} \frac{1}{n^2} \frac{\partial \mathbb{E}(f(\theta_i, K_i))}{\partial K_i} w_j(a(\theta, K), K))\right] \end{aligned}$$

If we develop the allocation we have:

$$\begin{aligned} w_i(a(\theta, K), K) &= \frac{1}{n} + \frac{1}{n\gamma}\left((n-1) f(a(\theta_i, K), K_i) - \sum_{j\neq i} f(a(\theta_j, K), K_j)\right) \\ &= \frac{1}{n\gamma}\left[\gamma + \frac{n(n-1)}{n+1} f(\theta_i, K_i) - \frac{1}{n}\gamma + \frac{n-1}{n^2(n+1)} \mathbb{E}[f(\theta_i, K_i)] + \frac{n-1}{n^2} \sum_{j\neq i} \mathbb{E}[f(\theta_j, K_j)]\right. \\ &\quad \left. - \sum_{j\neq i} \frac{n}{n+1} f(\theta_j, K_j) + \frac{1}{n}\gamma - \sum_{j\neq i} \frac{1}{n^2(n+1)} \mathbb{E}[f(\theta_j, K_j)] - \sum_{j\neq i} \frac{1}{n^2} \sum_{l\neq j} \mathbb{E}[f(\theta_l, K_l)]\right] \\ &= \frac{1}{n\gamma}\left[\gamma + \frac{n}{n+1}\left((n-1) f(\theta_i, K_i) - \sum_{j\neq i} f(\theta_j, K_j)\right)\right. \\ &\quad + \frac{1}{n^2(n+1)}\left((n-1)\mathbb{E}[f(\theta_i, K_i)] - \sum_{j\neq i} \mathbb{E}[f(\theta_j, K_j)]\right) \\ &\quad \left. + \frac{1}{n^2} \sum_{j\neq i}\left((n-1)\mathbb{E}[f(\theta_j, K_j)] - \sum_{l\neq j} \mathbb{E}[f(\theta_l, K_l)]\right)\right] \end{aligned}$$

In equilibrium, we will have $K_i = K_j \; \forall i \neq j$, so $\mathbb{E}[f(\theta_i, K)] = \mathbb{E}[f(\theta_j, K)]$, and the two last lines disappear:

$$w_i(a(\theta_i, K) K_i) = \frac{1}{n\gamma}\left[\gamma + \frac{n}{n+1}\left((n-1) f(\theta_i, K) - \sum_{j\neq i} f(\theta_j, K)\right)\right] \tag{2.43}$$

Using it in the marginal cost we have:

$$\begin{aligned} c'(K_i) =& \frac{1}{n}\mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}\right] \\ &+ \frac{1}{n\gamma}\left(\frac{n}{n+1}\right)^2 \mathbb{E}\left[\frac{\partial f(\theta_i, K_i)}{\partial K_i}\left((n-1)f(\theta_i, K_i) - \sum_{j\neq i} f(\theta_j, K_j)\right)\right] \\ &+ \frac{1}{n\gamma}\frac{1}{n(n+1)^2}\mathbb{E}\left[\frac{\partial \mathbb{E}[f(\theta_i, K_i)]}{\partial K_i}\left((n-1)f(\theta_i, K_i) - \sum_{j\neq i} f(\theta_j, K_j)\right)\right] \\ &+ \frac{1}{n\gamma}\frac{1}{n(n+1)}\mathbb{E}\left[\frac{\partial \mathbb{E}[f(\theta_i, K_i)]}{\partial K_i}\sum_{j\neq i}\left((n-1)f(\theta_j, K_j) - \sum_{l\neq j} f(\theta_l, K_l)\right)\right] \end{aligned}$$

The two last lines disappear. In effect, $\frac{\partial \mathbb{E}[f(\theta_i, K_i)]}{\partial K_i}$ is a constant that can be put outside the parentheses. Since $K_i = K_j \ \forall i \neq j$ in equilibrium, $\mathbb{E}[f(\theta_i, K_i)] = \mathbb{E}[f(\theta_j, K_j)]$ and the parentheses of the two last lines are null.

## Appendix 6

Agent i chooses his investment and knows that in period two, he will lie "optimally". Thus, his problem cam be written as:

$$\max_{K_i} \mathbb{E}_{\theta_i} \max_{a_i} \mathbb{E}_{\theta_{j\neq i}} \left[f(\theta_i, K_i)w_i(a(\theta, K), K) - \gamma\frac{w_i^2(a(\theta, K), K)}{2} - \lambda(a(\theta, K), K)w_i(a(\theta, K), K)\right] - c'(K_i)$$

FOC:

$$\begin{aligned} \mathbb{E}\Bigg[&\frac{\partial f(\theta_i, K_i)}{\partial K_i}w_i(a(\theta, K), K) + [f(\theta_i, K_i) - \gamma w_i(a(\theta, K), K)\lambda(a(\theta, K), K)]\frac{\partial w_i(a(\theta, K), K)}{\partial K_i} \\ &+ [f(\theta_i, K_i) - \gamma w_i(a(\theta, K), K) - \lambda(a(\theta, K), K)]\sum_j \frac{\partial w_j(a(\theta, K), K)}{\partial a_j(\theta, K)}\frac{a_j(\theta, K)}{\partial K_i} \\ &-w_i(a(\theta, K), K)\left[\frac{\partial \lambda(a(\theta, K), K)}{\partial K_i} + \sum_j \frac{\partial \lambda(a(\theta, K), K)}{\partial a_j}\frac{\partial a_j(\theta, K)}{\partial K_i}\right]\Bigg] = c'(K_i) \end{aligned}$$

We know that $\lambda(a(\theta, K), K) = f(a_i, K_i) - \gamma w_i(a(\theta, K), K)$, so we can replace and eliminate the terms with $\gamma w_i(a(\theta, K), K)$.

Then, we know that $a_i$ comes from the maximization, so by the Envelop Theorem, we have that

$$\begin{aligned} &f(\theta_i, K_i)\frac{\partial w_i(a(\theta, K), K)}{\partial a_i} - \gamma w_i(a(\theta, K), K)\frac{\partial w_i(a(\theta, K), K)}{\partial a_i} \\ &- w_i(a(\theta, K), K)\frac{\partial \lambda(a(\theta, K), K)}{\partial a_i} - \lambda\frac{\partial w_i(a(\theta, K), K)}{\partial a_i} = 0 \end{aligned}$$

Therefore, the marginal cost is:

$$\begin{aligned} c'(K_i) =& \mathbb{E}\Bigg[\frac{\partial f(\theta_i, K_i)}{\partial K_i}w_i(a(\theta, K), K) + [f(\theta_i, K_i) - f(a_i(\theta, K), K_i)]\left(\frac{\partial w_i(a(\theta, K), K)}{\partial K_i} + \sum_{j\neq i}\frac{\partial w_i(a(\theta, K), K)}{\partial a_j}\frac{\partial}{\partial K_i}\right) \\ &-w_i(a(\theta, K), K)\left(\frac{\partial \lambda(a(\theta, K), K)}{\partial K_i} + \sum_{j\neq i}\frac{\partial \lambda(a(\theta, K), K)}{\partial a_j}\frac{\partial a_j(\theta, K)}{\partial K_i}\right)\Bigg] \end{aligned}$$

Replacing the derivative, note that:

$$\frac{\partial w_i(a(\theta,K),K)}{\partial K_i} = \frac{n-1}{n\gamma}\frac{\partial f(a_i,K_i)}{\partial K_i} \tag{2.44}$$

$$\frac{\partial \lambda(a(\theta,K),K)}{\partial K_i} = \frac{1}{n}\frac{\partial f(a_i,K_i)}{\partial K_i} \tag{2.45}$$

We have:

$$\begin{aligned} c'(K_i) =& \mathbb{E}\left[\frac{\partial f(\theta_i,K_i)}{\partial K_i} w_i(a(\theta,K),K)\right. \\ &+ [[f(\theta_i,K_i) - f(a_i(\theta,K),K_i)]\left(\frac{n-1}{n\gamma}\frac{\partial f(a(\theta,K),K)}{\partial K_i} + \sum_{j\neq i}\frac{-1}{n\gamma}\frac{\partial f(a(\theta,K),K)}{\partial a_j}\frac{\partial a_j(\theta,K)}{\partial K_i}\right) \\ &\left. - w_i(a(\theta,K),K)\left(\frac{1}{n}\frac{\partial f(a_i(\theta,K),K_i)}{\partial K_i} + \sum_{j\neq i}\frac{1}{n}\frac{\partial f(a_j(\theta,K),K_j)}{\partial a_j}\frac{\partial a_j(\theta,K)}{\partial K_i}\right)\right] \end{aligned}$$

To replace the derivative of $a$, note that $\frac{\partial f(a_j(\theta,K),K_j}{\partial a_j}\frac{a_j(\theta,K)}{\partial K_i} = \frac{1}{n^2}\frac{\mathbb{E}f(\theta_i,K_i)}{\partial K_i}$. Therefore, developing and replacing we finally have:

$$\begin{aligned} c'(K_i) =& \mathbb{E}\left[\frac{\partial f(\theta_i,K_i)}{\partial K_i} w_i(a(\theta,K),K)\right] - \frac{1}{n^3}\mathbb{E}\left[\frac{\partial f(\theta_i,K_i)}{\partial K_i}\right] \\ &+ \frac{n-1}{\gamma(n+1)^2}\mathbb{E}\left[\frac{\partial f(\theta_i,K_i)}{\partial K_i} f(\theta_i,K_i) - \frac{\partial f(\theta_i,K_i)}{\partial K_i}\mathbb{E}(f(\theta_i,K_i))\right] \\ &- \frac{1}{\gamma(n+1)^2}\mathbb{E}\left[(n-1)\frac{\partial f(\theta_i,K_i)}{\partial K_i} f(\theta_i,K_i) - \sum_{j\neq i}\frac{\partial f(\theta_i,K_i)}{\partial K_i} f(\theta_j,K_j)\right] \end{aligned}$$

It is easy to see that the two last lines are zero. Finally, using the expression of $w_i(a(\theta,K),K)$, we have:

$$\begin{aligned} c'(K_i) =& \left(\frac{1}{n} - \frac{1}{n^3}\right)\mathbb{E}\left[\frac{\partial f(\theta_i,K_i)}{\partial K_i}\right] \\ &+ \frac{1}{\gamma(n+1)}\mathbb{E}\left[(n-1)\frac{\partial f(\theta_i,K_i)}{\partial K_i} f(\theta_i,K_i) - \sum_{j\neq i}\frac{\partial f(\theta_i,K_i)}{\partial K_i} f(\theta_j,K_j)\right] \end{aligned}$$

## Appendix 7

The problem of maximization of agent $i$ is:

$$\max_{a_i}\mathbb{E}\left[f(\theta_i,K_i)w_i(a_i,a_{-i}(\theta,K^*),K^*) - \gamma\frac{w_i^2(ai,a_{-i}(\theta,K^*),K^*)}{2} - \lambda(a_i,a_{-i}(\theta,K^*),K^*))w_i(a_i,a_{-i}(\theta,K^*),K^*))\right] \tag{2.46}$$

The FOC is:

$$\begin{aligned} &\mathbb{E}\left[f(\theta_i,K_i)\frac{\partial w_i(a_i,a_{-i}(\theta,K^*),K^*))}{\partial a_i} - \gamma w_i(a_i,a_{-i}(\theta,K^*),K^*))\frac{\partial w_i(a_i,a_{-i}(\theta,K^*),K^*))}{\partial a_i}\right. \\ &\left. - \lambda(a_i,a_{-i}(\theta,K^*),K^*))\frac{\partial w_i(a_i,a_{-i}(\theta,K^*),K^*))}{\partial a_i} - w_i(a_i,a_{-i}(\theta,K^*),K^*))\frac{\partial \lambda(a_i,a_{-i}(\theta,K^*),K^*))}{\partial a_i}\right] = 0 \end{aligned}$$

Since $\lambda(a_i,a_{-i}(\theta,K^*),K^*)) = f(a_i,K_i) - \gamma w_i(a_i,a_{-i}(\theta,K^*),K^*))$, we have:

$$\mathbb{E}\left[(f(\theta_i,K_i) - f(a_i,K_i))\frac{\partial w_i(a_i,a_{-i}(\theta,K^*),K^*)}{\partial a_i} - w_i(a_i,a_{-i}(\theta,K^*),K^*))\frac{\partial \lambda(a_i,a_{-i}(\theta,K^*),K^*))}{\partial a_i}\right] = 0$$

Replacing:

$$\mathbb{E}_{-i}\left[(f(\theta_i,K_i)-f(a_i,K^*))\frac{n-1}{n\gamma}\frac{\partial f(a_i,K^*)}{\partial a_i}-\frac{1}{n}\frac{\partial f(a_i,K^*)}{\partial a_i}\left(\frac{1}{n}+\frac{(n-1)f(a_i,K^*)-\sum_{j\neq i}f(a_j,K^*)}{n\gamma}\right)\right]=0$$

Reorganizing, we have the expression of $f(a_i,K^*)$:

$$f(a_i,K^*)=\frac{n}{n+1}f(\theta_i,K_i)-\frac{1}{(n-1)(n+1)}\gamma+\frac{1}{(n-1)(n+1)}\sum_{j\neq i}\mathbb{E}\left[f(a_j,K^*)\right]$$

Agents $j$ act as if all the agents had $K^*$. We use what we did in the general model:

$$f(a_j,K^*)=\frac{n}{n+1}f(\theta_j,K^*)-\frac{1}{n(n-1)}\gamma+\frac{1}{n^2(n+1)}\mathbb{E}[f(\theta_j,K^*)]+\frac{1}{n^2}\sum_{l\neq j}\mathbb{E}[f(\theta_l,K^*)]$$

Since agents are symmetric,

$$f(a_j,K^*)=\frac{n}{n+1}f(\theta_j,K^*)-\frac{1}{n(n-1)}\gamma+\frac{1}{(n+1)}\mathbb{E}[f(\theta,K^*)] \tag{2.47}$$

Replacing in $f(a_i,K^*)$, we finally have:

$$f(a_i,K^*)=\frac{n}{n+1}f(\theta_i,K_i)-\frac{1}{n(n-1)}\gamma+\frac{1}{(n+1)}\mathbb{E}[f(\theta,K^*)] \tag{2.48}$$

## Appendix 8

The development of the calculus is the same as for the general case but using the new demand function.

Note that:

$$\frac{\partial f(a_i(\theta,K),K_i)}{\partial K_i}=\frac{n}{n-2s+1}\frac{\partial f(\theta_i,K_i)}{\partial K_i}+\frac{(sn-2s+1)^2}{(s-1)n(s-n)(n-2s+1)}\frac{\partial \mathbb{E}[f(\theta_i,K_i)]}{\partial K_i}$$

$$\frac{\partial f(a_j(\theta,K),K_j)}{\partial K_i}=\frac{sn-2s+1}{(s-1)n(s-n)}\frac{\partial \mathbb{E}[f(\theta_i,K_i)]}{\partial K_i}$$

and the allocation

$$w_i(a(\theta,K),K)=\frac{1}{n\gamma}\left[\gamma+\frac{n}{n-2s+1}\left\{(n-1)f(\theta_i,K_i)-\sum_{j\neq i}f(\theta_j,K_j)\right\}\right]$$

Replacing and using as in Appendix 5 that in equilibrium $K_i=K_j$ so $\mathbb{E}[f(\theta_i,K_i)]=\mathbb{E}[f(\theta_j,K_j)]$, we find the final expression. ■

## Appendix 9

We present here the tables of data that we use to make the figures.

Table 2.1: Differences with respect to the first best for the substitutes without subsidy

| Cases | N=2 | N=3 | N=5 |
|---|---|---|---|
| LC | -3,74% | -1,10% | -0,23% |
| SB | -0,36% | -0,18% | -0,02% |
| SBLC | -1,30% | -0,30% | -0,01% |
| SBLC k unknown | -0,36% | -0,18% | -0,02% |

Table 2.2: Differences with respect to the first best for the complements without subsidy

| Cases | N=2 | N=3 | N=5 |
|---|---|---|---|
| LC | 35,72% | 16,50% | 6,06% |
| SB | 1,20% | 0,84% | 0,43% |
| SBLC | 10,18% | 2,79% | 0,67% |
| SBLC k unknown | 1,09% | 0,72% | 0,34% |

Table 2.3: Differences with respect to the first best for substitutes with capital subsidy $s$

| N | Case | s=0 | s=0.1 | s=0.25 | s=0.5 |
|---|---|---|---|---|---|
| N=2 | LC | -3,75% | -2,96% | -1,66% | 0,00% |
| | SB | -0,36% | -0,55% | -2,03% | -15,38% |
| | SBLC | -1,30% | -0,78% | -0,37% | -4,12% |
| | SBLC k unknown | -0,36% | -0,55% | -2,03% | -15,38% |
| N=3 | LC | -1,10% | -0,67% | -0,12% | -1,10% |
| | SB | -0,21% | -0,33% | -1,37% | -10,07% |
| | SBLC | -0,33% | -0,21% | -0,61% | -6,17% |
| | SBLC k unknown | -0,21% | -0,33% | -1,37% | -10,07% |
| N=5 | LC | -0,24% | -0,07% | -0,02% | -2,06% |
| | SB | -0,21% | -0,15% | -0,63% | -5,76% |
| | SBLC | -0,22% | -0,10% | -0,45% | -4,87% |
| | SBLC k unknown | -0,21% | -0,15% | -0,63% | -5,76% |

Table 2.4: Differences with respect to the first best for complements with capital subsidy $s$

| N | Case | s=0 | s=0.1 | s=0.25 | s=0.5 |
|---|---|---|---|---|---|
| N=2 | LC | -35,60% | -28,35% | -16,24% | 0,00% |
| | SB | -1,17% | -1,82% | -12,14% | -119,09% |
| | SBLC | -10,06% | -5,30% | -1,07% | -31,13% |
| | SBLC k unknown | -1,07% | -2,37% | -15,65% | -155,06% |
| N=3 | LC | -16,38% | -9,92% | -1,81% | -20,60% |
| | SB | -0,82% | -1,45% | -11,74% | -119,11% |
| | SBLC | -2,72% | -0,77% | -4,33% | -77,02% |
| | SBLC k unknown | -0,70% | -2,16% | -16,64% | -174,28% |
| N=5 | LC | -5,99% | -1,72% | -0,98% | -75,71% |
| | SB | -0,41% | -1,26% | -12,35% | -122,42% |
| | SBLC | -0,65% | -0,77% | -9,98% | -110,71% |
| | SBLC k unknown | -0,33% | -2,09% | -18,55% | -199,03% |

Table 2.5: Differences with respect to the first best for substitutes with price subsidy $\hat{s}$

| N | Case | $\hat{s}$=0 | $\hat{s}$=0.1 | $\hat{s}$=0.25 | $\hat{s}$=0.5 |
|---|---|---|---|---|---|
| N=2 | LC | -3,75% | -3,04% | -2,11% | -0,94% |
| | SB | -0,35% | -0,43% | -1,77% | -15,04% |
| | SBLC | -1,29% | -1,29% | -1,33% | -1,67% |
| | SBLC k unknown | -0,35% | -0,25% | -0,11% | 0,00% |
| N=3 | LC | -1,10% | -0,89% | -0,62% | -0,27% |
| | SB | -0,23% | -0,29% | -1,13% | -9,89% |
| | SBLC | -0,35% | -0,33% | -0,28% | -0,40% |
| | SBLC k unknown | -0,23% | -0,17% | -0,03% | 0,00% |
| N=5 | LC | -0,23% | -0,19% | -0,14% | -0,06% |
| | SB | -0,03% | -0,25% | -0,65% | -5,69% |
| | SBLC | -0,04% | -0,20% | -0,08% | -0,11% |
| | SBLC k unknown | -0,03% | -0,19% | -0,04% | -0,01% |

Table 2.6: Differences with respect to the first best for complements with price subsidy $\hat{s}$

| N | Case | $\hat{s}$=0 | $\hat{s}$=0.1 | $\hat{s}$=0.25 | $\hat{s}$=0.5 |
|---|---|---|---|---|---|
| N=2 | LC | -35,62% | -29,11% | -20,43% | -9,26% |
| | SB | -1,18% | -1,58% | -12,69% | -138,62% |
| | SBLC | -10,08% | -10,73% | -11,85% | -14,80% |
| | SBLC k unknown | -1,08% | -0,80% | -0,37% | 0,00% |
| N=3 | LC | -16,53% | -13,40% | -9,25% | -4,17% |
| | SB | -0,85% | -1,28% | -13,22% | -140,45% |
| | SBLC | -2,81% | -3,01% | -3,48% | -5,14% |
| | SBLC k unknown | -0,73% | -0,49% | -0,20% | 0,00% |
| N=5 | LC | -6,12% | -4,93% | -3,35% | -1,36% |
| | SB | -0,44% | -1,20% | -13,80% | -144,25% |
| | SBLC | -0,69% | -0,72% | -0,85% | -1,36% |
| | SBLC k unknown | -0,35% | -0,23% | -0,08% | 0,00% |

## 2.7 References


- Baldursson, F.M. and N.H.M. von der Fehr, 2004. Price Volatility and Risk Exposure in Market-Based Environmental Policy Instruments. Journal of Environmental Economics and Management 48: 682-704.

- Baldursson, M.F., Karatzas, I., von der Fehr, N.-H.M., 1999. Prices vs. quantities: the irrelevance of irreversibility. Mimeo, University of Olso.

- Benz, E. and Ehrhart, K.-M.,2007. Which allocation rule generates true price signals for the CO2 allowance market? Proceedings of the 2007 ECEEE Summer Study – Saving energy –Just Do It!; 4–9 June 2007, Colle sur Loup, France, pp. 125–134.

- Betz R., Seifert S., Cramton P. and Kerr S., 2010. Auctioning greenhouse gas emissions permits in Australia. The Australian Journal of agricultural and resource economics 54, 219-238

- Betz, R. and A. Gunnthorsdottir., 2009. Modeling Emissions Markets Experimentally: The Impact of Price Uncertainty. Unpublished Manuscript

- Braithwaite, J., 2002. Restorative Justice and Responsive Regulation. Oxford, United Kingdom: Oxford University Press

- Cason T.N. and L. Gangadharan, 2003. Transactions Costs in Tradable Permit Markets: An Experimental Study of Pollution Market Designs. Journal of Regulatory Economics 23:145-165

- Camacho-Cuena E., Requate T. and Waichman I., 2012. Investment incentives under emission trading: an experimental study. Environmental resource economics 53: 229-249

- Chao, H.-P., Wilson, R., 1993. Option value of emission allowances. Journal of Regulatory Economics 5, 233–24

- Coria, J., 2007. Environmental policy, fuel prices and the switching to natural gas in Santiago Chile. LACEEP Working Paper Series.

- Cramton P. and Kerr S., 2002. Tradeable carbon permit auctions: how and why to auction not grandfather. Enercy policy 30, 333-345

- Ellerman, A. Denny, From Autarkic to Market-based Compliance: Learning from our Mistakes, in Kosobud, Richard F. (ed.), Emissions Trading: Environmental Policy's New Approach, John Wiley & Sons, Inc., 2000, pp 190-215.

- Fischer C., Parry I. and Pizer W., 2003. Instrument choice for environmental protection when technological innovation is endogenous. Journal of environmental economics and management 45: 523-545

- Fowlie M., 2010. Emissions trading, electricity restructuring and investment in pollution abatement. American Economic Review 100: 837-869

- Gangadharan L., Farrell A. and Croso R., 2012. Investment decisions and emissions reductions: results from experiments in emissions trading. List JA, Price M (eds) Handbook on experimental economics and the environment. Edward Elgar Publishing Ltd., Cheltenham

- Goeree J., Palmer K., Holt C., Shobe W. and Burtraw D., 2010. An experimental study of auctions versus grandfathering to assign pollution permits. Journal of the European economic association 8 (2-3): 514-525
- Hahn, R.W., 1984. Market Power and Transferable Property Rights. Quarterly Journal of Economics 99:753-765
- Hahn, R.W., and R.G. Noll, 1983. Barriers to Implementing Tradable Air Pollution Permits: Problems of Regulatory Interactions. Yale Journal on Regulation 1:63-91
- Hahn R. and Stavins R., 2010. The effect of allowance allocations on cap-and-trade system performance.
- Harrison D. and Radov D., 2002. Evaluation of alternative initial allocation mechanisms in a European union greenhouse gas emissions allowance trading scheme. Prepared for DG Environment, European Commission
- Insley M., 2003. On the option to invest in pollution control under a regime of tradable emissions allowances. Canadian journal of economics 36, 4: 860-883
- Jaffe A. and Stavins R., 1995. Dynamic incentives of environmental regulations: the effects of alternative policy instruments on technology diffusion. Journal of environmental economics and management 29: 43-63
- Jung, Ch., Krutilla, K., Boyd, R., 1996. Incentives for advanced pollution abatement technology at the industry level: An evaluation of policy alternatives. Journal of Environmental Economics and Management 30, 95–111
- Kahneman, D., J. L. Knetsch, and R.H. Thaler, 1990. Experimental Tests of the Endowment Effect and the Coase Theorem. Journal of Political Economy 98:1325-1348.
- Klemperer, P. and Mere, M., 1989. Supply function equilibria in oligopoly under uncertainty. Econometrica 57,6: 1243-1277
- Kwerel, E. 1977. To Tell the Truth: Imperfect Information and Optimal Pollution Control. Review of Economic Studies, 44(3): 595–601
- Liski, M., 2001. Thin Versus Thick CO2 Market. Journal of Environmental Economics and Management 41:295-311.
- Liski, M. and J.P. Montero, 2009. Market Power in an Exhaustible Resource Market: The Case of Storable Pollution Permits. Discussion Paper No. 276, Helsinki Center of Economic Research
- Malueg, D.A. and A.J. Yates, 2009. Bilateral Oligopoly, Private Information, and Pollution Permit Markets. Environmental & Resource Economics 43:553-572
- May P., 2005. Compliance Motivations: Perspectives of Farmers, Homebuilders, and Marine Facilities. Law and Policy 27:317-347
- Milliman, S.R., Prince, R., 1989. Firms incentives to promote technological change in pollution control. Journal of Environmental Economics and Management 17: 247–265

- Misiolek, W.S. and Elder, H.W., 1989. Exclusionary Manipulation of Markets for Pollution Rights. Journal of Environmental Economics and Management 16:156-166.
- Montero J.P., 1998. Marketable Pollution Permits with Uncertainty and Transaction Costs. Resource and Energy Economics 20:27-50
- Montero JP., 2002. Permits, standards and technology innovation. Journal of environmental economics and management 44: 23-44
- Montero, J.P., Sanchez J.M. and Katz R., 2002. A Market-based environmental policy experiment in Chile. Journal of law and economics 45: 267- 287
- Montero J.P., 2004. Markets for environmental protection: design and performance. Estudios de Economia 31: 79-99
- Montero, J.P., 2008. A simple auction mechanism for the optimal allocation of the commons. American economic review 98: 496-518
- Montgomery D.W., 1972. Markets in Licenses and Efficient Pollution Control Programs. Journal of Economic Theory 5, 395.
- Murphy, J.J. and Stranlund, J.K.,2005. A Laboratory Investigation of Compliance Behavior under Tradable Emissions Rights: Implications for Targeted Enforcement. Working Paper 2005-1, University of Massachusetts Amherst, Department of Resource Economics. Forthcoming in the Journal of Environmental Economics and Management.
- Oates, W.E. and Strassmann, D.L., 1984. Effluent Fees and Market Structure. Journal of Public Economics 24:29-46.
- Ollikka, 2014 Prices vs quantities when information is incomplete and asymmetric. PhD thesis
- Parker, C., 2006. The Compliance Trap: The Moral Message in Responsive Regulatory Enforcement. Law and Society Review 40:591-622.
- Requate T. and Unold W., 2003. Environmental policy incentives to adopt advanced abatement technology: will the true ranking please stand up? European economic review 47: 125-146
- Shabman L., Stephenson K. and Shobe W., 2002. Trading programs for environmental management: reflection on the air and water experience. Environmental practice 4: 153-162
- Stavins, R.N., 1995. Transaction Costs and Tradeable Permits. Journal of Environmental Economics and Management 29:133-148.
- Suter J.F., Spraggon J.M. and Poe G.L., 2013. Thin and lumpy: an experimental investigation of water quality trading. Water resources and economics 1: 36-60
- Thaler, R., 1980. Toward a Positive Theory of Consumer Choice. Journal of Economic Behavior and Organization 1:39-60
- Tietenberg T., 2003. The tradable permits approach t protecting the commons: lessons for climate change. Oxford review of economic policy, 19 (3)

- Tietenberg T., 2005. Tradable Permits in Principle and Practice. The Penn State Environmental Law Review 14: 251-281
- Tschirhart, J.T. 1984. Transferable Discharge Permits and Profit-Maximizing Behavior. In: Crocker, T.D. (Ed.), Economic Perspectives on Acid Deposition Control. Stoneham, Massachusetts: Butterworth Publishers.
- van Koten S., 2015. Do emission trading schemes facilitate efficient investments? an experimental study. Working Papers - Robert Schuman centre for advanced studies and Loyola de Palacio programme on energy policy
- Vives, X., 2011. Strategic Supply Function Competition with Private Information. Econometrica 79, 6: 1919-1966.
- Weitzman, M.L., 1974. Prices vs. Quantities. Review of Economic Studies 41(4): 477–91.
- World Bank, 2010. Guidance notes on tools for pollution management, Market-based instruments/ economic incentives
- Wrake M., Myers E., Burtraw D., Mandell S. and Holt C., 2010. Opportunity cost for free allocations of emissions permits: an experimental analysis. Environmental resource economics 46: 331-336
- Xepapadeas, A., 1999. Environmental policy and firm behavior: abatement investment and location decisions under uncertainty and irreversibility. Working Paper T0243, NBER
- Zhao J., 2003. Irreversible abatement investment under cost uncertainties: tradable emission permits and emissions charges. Journal of public economics 87: 2765-2789

# Chapter 3

# Dissolving a partnership with concave valuation

Cramton, Gibbons and Klemperer (1987) show that if a group of symmetric agents has ex-ante the same proportion of an asset, then it is possible to give complete control to the partner with the highest ex-post valuation. However, what happens if agents have decreasing marginal utility and so, efficiency leads to share the good between them? For the general case, we show that if agents are symmetric, then ex-ante equal sharing is optimum. In the case of the root function, we show that the maximum expected transfer is positive. Therefore, there is a mechanism that is incentive compatible, ex-post efficient, budget balanced and that satisfies voluntary participation.

## 3.1 Introduction

Since the seminal papers of Myerson and Satterwhaite (1983, MS), it is well-known that private information hinders the ability of negotiating parties to achieve mutually beneficial exchange. When there is absolute ownership of the good and private information in a one-shot game, there is no mechanism that can be incentive compatible, individual rational, ex-post efficient and budget feasible. Efficiency cannot be reached without an external subsidy and therefore asymmetric information can be understood as a transaction cost in Coase's tradition. Cramton, Gibbons and Klemperer (1987, CGK) show in a symmetric environment that if a group of agents has ex-ante the same proportion of an asset, then it exists a mechanism that is incentive compatible, ex-post efficient, budget balanced and that satisfies voluntary participation . They conclude that, at least with private values, the main obstacle to achieve efficiency is not really the presence of information asymmetries but rather the presence of asymmetries in endowments. For asymmetric environment, Figueroa and Skreta (2012) show that efficient dissolution is possible if critical types are equal, which in case of asymmetric distributions implies different property rights that can be really unequal, what is the opposite of the result with symmetric distributions.

As an extension of CGK setup, we analyse how the result changes if agents have decreasing marginal utility for the good. For example in Chile, Article 186 of the Water Code establishes that the individuals who share a same canal or source of water can set up association or community of water in order to distribute the water, to construct or improve the construction of catchment and the necessary aqueducts. If the State constructs a dam, the association is the "owner" and is responsible to allocate the water. If we define the water available as "the" right, it is more probable that agents want a share of the right but not the entire right. It would be more costly and useless for them to obtain all the right if they need only a part. In order to model this idea, we make agents' utility to depend not lineally on the probability to win the good but rather use a function that is concave. In effect, the relationship between water withdrawals and benefits is concave for industries and farmers (Rosegrant et.al., 2000), and can be decreasing after a threshold. We assume a concave and increasing function of benefit, that is we suppose that agents would never receive a quantity of water that is too much and damage their production. In this situation, the optimal redistribution of the good does not give anymore the good to the agent with the highest valuation but share between participants. In the case of the root-function, we show that the maximum expected transfer is obtained in the equal initial share as in CGK and it is positive. Therefore, there is a mechanism that is incentive compatible, ex-post efficient, budget balanced and that satisfies voluntary participation.

## 3.2 Related Literature

Our analysis is related to the literature on partnership that had grown since CGK. Papers studying dissolution generally modify the environment of CGK. Fieseler, Kittsteiner and Moldovanu (2003) are concerned with the presence of interdependent values across agents. They show that if valuations are positively interdependent, efficient trade is possible if and only if it exists a price p such that all types of buyer and seller agree to trade at this price whereas if valuations are negatively interdependent, efficient trade can take place even in cases where gains from trade are not common. When only one of the two parties is informed and values are interdependent, the CGK's result that efficient trade is feasible whenever initial shares are equal falls. The subsidy required in a second best mechanism is minimal when the entire ownership is allocated initially to one of the parties. This happens because a mixed ownership does not alter any more the worst off type as it does in CGK environment (Jehiel and Pauzner 2006). The introduction of veto right is also a significant issue in the literature. Compte and Jehiel (2009) study a situation where agents privately know the realization of their outside option and can use a veto right to receive at least their outside option which is correlated with others' outside option. In this case, they show that inefficiencies are inevitable whatever the exact form of correlation. If a buy-sell clause is introduced, the determination of the proposer is crucial for efficiency (Frutos and Kittsteiner 2008) and change if investment is introduced (Li and Wolfstetter 2010). It is also possible to give a special form to the structure of the partnership. If there is only one active partner and the others are silent, ownership may be significant. A two-agents partnership is not dissoluble but if there are more than two partners, an extreme ownership will be dissoluble if the owner is a silent partner. If the owner is active, the situation is à la MS (Ornelas and Turner 2007).

Segal and Whinston (2014, working paper) study how the property rights affect the efficiency of bargaining and the final allocations in a static game. They establish a wide class of economic settings and property rights in which efficient bargaining is impossible. The second best when the first best cannot be reach is contrary to the intuition of CGK since they demonstrate that less extreme property rights may be worse than extreme cases.

Loertscher and Wesser (2015) extend the result of Figueroa and Skreta for interdependent agents and also analyze an objective function that is not the traditional ex-post efficiency but a convex combination of revenue and social surplus. This is useful if the mechanism has to generate positive revenue, for example to cover legal expenses. The partnership is constituted by an arbitrary number of agents with nonidentical type distributions and interdependent values. They study the optimal dissolution for any given initial ownership structure and then suppose that the designer can choose the initial ownership and solve for the optimal ownership structure also. They show that symmetric ownership is always optimal when all agents draw their types from the same distribution but typically not otherwise. When distributions are different, the optimal ownership structure depends on the distributions, the weight on revenue in the objective function and the importance of interdependency. In contrast to the private values case where, as observed by Figueroa and Skreta (2012), all agents have strictly positive shares, the asymmetry under interdependent values may result in an extreme ownership structure where some agents get zero shares.

Most of the papers study how to dissolve efficiently partnership. On the contrary, Kuribko and Lewis (2009) and Kuribko, Lewis, Liu and Song (2015) study how to design partnerships that survive until the project is complete. The latter is a more general version of the former and adds ex-ante investment. The economy consists in a supply chain that creates a capacity unit and agents that need this capacity. All agents and the manufacturer form a partnership such as if only one individual decides to not participate, the partnership is dissolved. In each period, the mechanism must distribute efficiently the unit of capacity, guarantee individual compatibility, voluntary participation and budget balanced. Control rights are defined at the beginning of each stage in such a way that if the partnership is dissolved, each agent is entitled with his control right to use capacity. Thus, control rights are adjusted to maintain each member's willing participation. The rights are allocated to minimize the breakup value of the partnership.

Mechanism design can be useful in public good problems. Grüner and Koriyama (2012) study the possibility to replace a voting mechanism by an efficient d'Aspremont-Gerard-Varet (AGV) mechanism in the provision of an indivisible public good. Neeman (1999) show that voluntary participation in a public good problem with asymmetric information depends a lot on the structure of property rights. In the context of a society composed by n inhabitants and a firm that has to pollute to produce, he considers that the firm has a property right $\gamma \in (0, 1)$ and the residents have $1 - \gamma$. Thus the firm can produce up to his right and residents have the right to insist that production does not exceed $\gamma$. In this context, Neeman defines "efficient" property rights structures and find a result similar to CGK since no agent must have property right that is too strong. When efficient property rights do not exist for an economy, he shows that if some agents are coercing into participating in the mechanism, the degree of coercion that is needed is small and converges to zero as the economy gets large.

Chou, Liang and Wu (2012) extend the model of CGK to the case that agents desire only a fraction of the partnership and efficiency warrants multiple owners. The setup is very different than ours since the multiple owners situation is not the result of the optimal allocation of the good but the consequence that agents only desire a portion of the good. The good is separated in K identical fractions and the agents with the K highest valuations receive one unit. The payoff of an agent is zero if her share is lower than a common given level and it is her valuation, her type if the share is greater or equal to that level. They show that the possibility of achieving efficiency and the total surplus are increasing in the total number of agents which initial share is less that the threshold, that is the potential buyers, and in the largest number of partners who can be awarded with a valuable share if the partnership is dissolved efficiently, that is the number of blocks distributed. Both are indicating more concentration of the partnership so the result they find goes in the opposite direction than CGK.

## 3.3 General Model

The economy consists of n agents, $n = 1...n$ who share a divisible object of size one in a one-period game. Initially, agent $i$ has a portion $r_i$ of the good. We define the linear function $\theta_i g(q_i) - x_i$ where $\theta_i \in [0,1]$ is the type of agent $i$, $q_i$ is the probability to obtain the good and $x_i$ is the transfer function. The function $g(.)$ is concave such as agents have decreasing returns to scale.
Information is known privately but it is common knowledge that it is drawn independently from a distribution $F(.)$ with density function $f(.)$. We assume that individuals seek to maximize their expected utilities.

By the revelation principle, any outcome that is obtained from a bargaining process among the agents can be obtained as an equilibrium outcome of an incentive compatible direct revelation mechanism. Therefore, we focus on the direct mechanism without loss of generality. The timing of the game is standard. First agents simultaneously report their type $\theta = \{\theta_1, ..., \theta_n\}$ and then they receive an allocation and a money transfer.

***Definition*** **:** The mechanism $< q, x >$ is defined by two functions: the decision rule of allocation $q : [0,1]^n \to [0,1]$ and the money transfer $x : [0,1]^n \to \mathbb{R}$.

***Definition*** **:** The expected value of the monetary transfer is $X_i = \mathbb{E}_{\theta_{-i}}[x_i(\theta)]$ and the expected value of the allocation is defined by $\mathbb{E}_{\theta_{-i}}[g(q_i(\theta)]$ where $\theta_{-i} = \{\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n\}$.

The expected payoff is the difference between the expected utility obtained by the allocation and the expected transfer:

$$V_i(\theta_i) = \max_{\theta_i^{'}} \theta_i \mathbb{E}_{\theta_{-i}}\left[g(q_i(\theta_i^{'}, \theta_{-i}))\right] - X_i(\theta_i^{'}) \tag{3.1}$$

We are interested in a mechanism that is ex-post efficient, incentive compatible, budget balanced and that satisfies voluntary participation. We now define these characteristics.

***Definition*****:** A mechanism $< q, x >$ is ex post efficient if $q$ is the solution to the maximization of the total expected payoff.

***Definition*****:** A mechanism $< q, x >$ is incentive compatible if for all $i$ and $\theta_i, \hat{\theta}_i \in [0,1]$,

$$V_i(\theta_i) \geq V_i(\hat{\theta}_i) \tag{3.2}$$

We are interested in mechanisms that ex ante do not request any outside subsidy. We do not restrict our mechanism to exact budget balance since we an easily imagine that money can be used for other activities.

***Definition*****:** A mechanism is ex ante budget balanced if

$$\sum_i \mathbb{E}_{\theta_i} X_i(\theta_i) \geq 0$$

Voluntary participation requires that the expected payoff obtained in the mechanism is higher than the outside option given by $\theta_i g(r_i)$.

***Definition*****:** A mechanism satisfies voluntary participation if

$$V_i(\theta_i) \geq \theta_i g(r_i) \tag{3.3}$$

In order to induce voluntary participation of all agents, the mechanism must guarantee the participation of the most reluctant agent, that is the agent who wins the least in participating in the game.

***Definition*:** The critical type $\theta_i^*$ is the solution to the minimization of the difference between the expected payoff and the outside option : $\theta_i^* \in argmin[V_i(\theta_i) - \theta_i g(r_i)]$.

We assume that the outside option is concave, which together with the fact that $V_i(\theta_i)$ is convex makes the characterization of $\theta_i^*$ simple. The solution of the critical type is interior if the derivative of the expected payoff in zero is less than the derivative of the outside option and if the derivative of the expected payoff in one is bigger than the derivative of the outside option.

**Lemma** The critical type is given by:

$$\theta_i^* = \begin{cases} x & \text{if } V_i^{'}|_{\theta_i=0} < g(r_i) \text{ and } V_i^{'}|_{\theta_i=1} > g(r_i) \\ 0 & \text{if } V_i^{'}|_{\theta_{it}=0} \geq g(r_i) \\ 1 & \text{if } V_i^{'}|_{\theta_i=1} \leq g(r_i) \end{cases}$$

where $x$ is the solution to $V_i^{'}(x) = g(r_i)$.

By the revenue-equivalence theorem (Myerson, 1981), all incentive compatible mechanisms that implement the same allocation rules generate the same expected payoff up to a constant. Since the VCG mechanism is efficient and interim compatible, we can use it without loss of generality. We know that any payment of the form $x_i(\theta_i, \theta_{-i}) = -\sum_{j\neq i} \theta_j g\left(q_j(\theta)\right) + h_i(\theta_{-i})$ is incentive compatible in dominant strategy and that the constant $h$ is defined to induce the voluntary participation of the critical types. Since we must have $\theta_i^* g\left(q_i(\theta_i^*, \theta_{-i})\right) - x_i(\theta_i^*, \theta_{-i}) = \theta_i^* g(r_i)$, the constant is $h_i(\theta_{-i}) = \theta_i^* g\left(q_i(\theta_i^*, \theta_{-i})\right) + \sum_{j\neq i} \theta_j g\left(q_j(\theta_i^*, \theta_{-i})\right) - \theta_i^* g(r_i)$. Then, the generalized payment is:

$$x_i(\theta) = -\sum_{j\neq i} \theta_j g\left(q_j(\theta)\right) + \theta_i^* g\left(q_i(\theta_i^*, \theta_{-i})\right) + \sum_{j\neq i} \theta_j g\left(q_j(\theta_j^*, \theta_{-i})\right) - \theta_i^* g(r_i) \tag{3.4}$$

Since we are interested in an ex ante budget balanced mechanism, we need the total expected payment:

$$\begin{aligned} \sum_i \mathbb{E}_{\theta_i} X_i(\theta_i) &= \sum_i \mathbb{E}_\theta \left[ -\sum_{j\neq i} \theta_j g\left(q_j(\theta)\right) + \theta_i^* g\left(q_i(\theta_i^*, \theta_{-i})\right) + \sum_{j\neq i} \theta_j g\left(q_j(\theta_j^*, \theta_{-i})\right) - \theta_i^* g(r_i) \right] \\ &= \sum_i \mathbb{E}_\theta \left[ -\sum_{j\neq i} \theta_j g\left(q_j(\theta)\right) + \sum_{j\neq i} \theta_j g\left(q_j(\theta_j^*, \theta_{-i})\right) \right] \end{aligned} \tag{3.5}$$

because by Lemma, $\mathbb{E}_{\theta_{-i}}\left[\theta_i^* g\left(q_i(\theta_i^*, \theta_{-i})\right)\right] = \theta_i^* g(r_i)$.

***Proposition 1:*** In a n-agent game, if types are defined by a function of distribution $F_i$ and utility is characterized by a concave function $g_i(.)$, then the optimal allocation is such that $\theta_i^*(r_i)\dfrac{\partial g_i(r_i)}{\partial r_i} = \theta_j^*(r_j)\dfrac{\partial g_j(r_j)}{\partial r_j}$ for all $i \neq j$.

*Proof:* We want to maximize the expected total payment, which is equivalent to minimize the expected total payoff of the agents. The problem is

$$\begin{gathered} min\ \mathbb{E}_\theta\left[\textstyle\sum_i V_i(\theta_i)\right] \\ \text{s.t. } \textstyle\sum_i q_i = 1 \end{gathered}$$

Using standard techniques,we have:

$$V_i(\theta_i) = \theta_i^*(r_i)g(r_i) + \int_{x=\theta_i^*(r_i)}^{1} V_i^{'}(x)dx \tag{3.6}$$

Using the Lagrangian, we have the FOC:

$$\begin{aligned}\lambda &= \frac{\partial\theta_i^*}{\partial r_i}g(r_i) + \frac{\partial g(r_i)}{\partial r_i}\theta_i^*(r_i) - V_i^{'}(\theta_i^*)\frac{\theta_i^*}{\partial r} \\ &= \theta_i^*(r_i)\frac{\partial g(r_i)}{\partial r_i} + \frac{\partial\theta_i^*}{\partial r}\left[g(r_i) - V_i^{'}(\theta_i^*)\right]\end{aligned}$$

The second term of the right-hand side is always null: if $\theta_i^*$ is a solution corner that it does not depend on $r_i$ and $\frac{\partial\theta_i^*}{\partial r} = 0$; if $\theta_i^*$ is an interior solution, then by Lemma, it is the solution to $V_i^{'}(\theta_i^*) = g(r_i)$.

Then, the FOC leads to the condition:

$$\theta_i^*(r_i)\frac{\partial g(r_i)}{\partial r_i} = \theta_j^*(r_j)\frac{\partial g(r_j)}{\partial r_j} \text{ for all } i \neq j$$

■

***Proposition 2:*** If agents are symmetric and have the same function of utility, that is $F_i = F$ and $g_i = g$, then $r_i = r = \frac{1}{n}$ is one of the solution to the problem of maximization, but it could exist others. In this case, the critical types are all equal.

*Proof:* We have : $\theta_i^*(r_i)\frac{\partial g(r_i)}{\partial r_i} = \theta_j^*(r_j)\frac{\partial g(r_j)}{\partial r_j}$ If $r_i = r_j$ then the derivative of $g(.)$ are equal on both side and we have $\theta_i^* = \theta_j^*$. ■

With asymmetry in the distribution function, we cannot directly conclude on the critical types. In Figueroa and Skreta (2012), the optimum is reached in $\theta_i^* = \theta_j^*$ which happens if $g(.)$ is linear. However, the concavity adds a constraint.

Since we know that $r = \frac{1}{n}$ is one of the solution, we focus now on this result and analyse the sign of the total expected payment. The total expected payment is $\mathbb{E}_\theta\left[-\sum_{j\neq i}\theta_j g\left(q_j(\theta)\right) + \sum_{j\neq i}\theta_j g\left(q_j(\theta_j^*, \theta_{-i})\right)\right]$ which can rewritten as:

$$\sum_i \mathbb{E}_{\theta_i} X_i(\theta_i) = \sum_{i=1}^{n}\sum_{j\neq i}\left(\mathbb{E}_{\theta_{-j}}\theta_i\left[g_i(q_i(\theta_j^*, \theta_{-j})) - \mathbb{E}_{\theta_j} g_i(q_i(\theta))\right]\right) \tag{3.7}$$

## 3.4 Case of the root-function

We study the case that $g(x) = \sqrt{x}$. Using the uniform distribution for the type, we show that the mechanism does not require an exterior subsidy to be ex-post efficient, incentive compatible and to satisfy voluntary participation.

First, efficiency is the solution to the problem of optimization:

$$\max_{qi, q_{-i}} \theta_i\sqrt{q_i} + \theta_{-i}\sqrt{q_{-i}}$$

$$\text{subject to } q_{-i} = 1 - q_i$$

so the solution is $q_i(\theta_i, \theta_{-i}) = \dfrac{\theta_i^2}{\theta_i^2 + \theta_{-i}^2}$.

To determine the critical type, we use the derivative of the expected values:

$$\begin{aligned}
V_i^{'}(\theta_i, \theta_{-i}) &= \theta_i \left[ ln\left(1 + \sqrt{1 + \theta_i^2}\right) - ln\left(\theta_i\right) \right] \\
V_i^{0'}(\theta_i, \theta_{-i}) &= \sqrt{r} \\
V_{-i}^{0'}(\theta_i, \theta_{-i}) &= \sqrt{1 - r}
\end{aligned}$$

We know by the general model that $r = 0.5$ is a possible solution. We now show that the expected total generalized payment is positive.

***Proposition*:** In a two-player game, if types are defined by a uniform function and utility is defined by the root function, then the maximum ex-ante budget revenue is reached in an equal share allocation $r = 0.5$. Moreover, the expected value in this point is positive, so there is a mechanism that is efficient, incentive compatible, ex ante budget balanced and that satisfies voluntary participation.

*Proof.* First, using (5) the expected value of the total generalized payment is:

$$\begin{aligned}
T(\theta_i, \theta_{-i}) &= \mathbb{E}_{\theta_i}\mathbb{E}_{\theta_{-i}} \left[ \frac{\theta_{-i}^2}{\sqrt{\theta_i^{*2} + \theta_{-i}^2}} + \frac{\sqrt{\theta_i^2}}{\sqrt{\theta_i^2 + \theta_{-i}^{*2}}} - \sqrt{\theta_i^2 + \theta_{-i}^2} \right] \\
&= \sqrt{\theta^{*2} + 1} - \theta^{*2} ln\left(\sqrt{\theta^{*2} + 1} + 1\right) + \theta^{*2} ln(\theta^*) - \frac{\sqrt{2}}{3} - \frac{1}{3} ln(1 + \sqrt{2}) \qquad (3.8)
\end{aligned}$$

Using $r = 0.5$, we obtain the critical type $\theta^* \simeq 0.4741$ and find that $T(\theta_i, \theta_{-i})$ is positive.

□

## 3.5 References


Chou E., Lang M-Y. and Wu C-T., 2012. Partnership dissolution when efficiency warrants multiple owners. Working Paper

Compte, O. and Jehiel, P., 2009.Veto constraint in mechanism design: inefficiency with correlated types. American economic journal: microeconomics,1(1):182-206

Cramton, P., Gibbons, R. and Klemperer, P., 1987. Dissolving a partnership efficiently. Econometrica, 55:615-632.

Fieseler, K., Kittsteiner, T. and Moldovanu, B.,2003. Partnerships, lemons and efficient trade. Journal of economic theory,113:223-234.

Figuera, N. and Skreta, V., 2012. Asymmetric partnerships. Economics Letters 115: 268-271.

Gruüner, H. and Koriyama, Y., 2011. Public goods, participation constraints and democracy: a possibility theorem. Games and economic behavior, 75: 152-167.

Jehiel, P. and Pauzner, A., 2006. Partnership dissolution with interdependent values. The RAND Journal of economics, 37(1):1-22.

Kuribko, N. and Lewis, T., 2009. Assigning property rights to insure efficient exchange and investment. Working Paper. Duke university

Kuribko, N. and Lewis, T., Liu, F. and Song, J., 2015. Long term partnership for achieving efficient capacity allocation. Working paper

Li, J. and Wolstetter, E., 2009. Partnership dissolution, complementarity and investment incentives. Oxford economic papers, 62: 529-552.

Loertscher S. and Wasser C., 2015. Optimal structure and dissolution of partnerships. Working Paper

Myerson, R.B. and Satterthwaite, M.A., 1983. Efficient mechanisms for bilateral trading. Journal of economic theory, 29: 265-281.

Neeman, Z., 1999. Property rights and efficiency of voluntary bargaining under asymmetric information. Review of economic studies, 66: 679-691.

Ornelas, E. and Turner, J., 2007. Efficient dissolution of partnerships and the structure of control. Games and economic behavior, 60:187-199.

Rosegrant M.W., Ringler C., McKinney D.C., Cai X., Keller A. and Donoso G., 2000. Integrated economic-hydrologic water modeling at the basin scale: the Maipo river basin. Agricultural Economics, 24:33-46.