INSTITUTO DE ECONOMÍA

TESIS de DOCTORADO

2014

Essays on the Political Economy of Fiscal Policy

José Carlos Tello.

www.economia.puc.cl

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
INSTITUTO DE ECONOMIA
DOCTORADO EN ECONOMIA

TESIS DE GRADO
DOCTORADO EN ECONOMIA

Tello Paredes, José Carlos

Agosto, 2014

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
INSTITUTO DE ECONOMIA
DOCTORADO EN ECONOMIA

Essays on the Political Economy of Fiscal Policy

José Carlos Tello Paredes

Comisión:

Presidente: Profesor Klaus Schmidt-Hebbel

Profesor Nicolás Figueroa

Profesor Francisco Gallego

Profesor Martin Besfamille

Santiago, Agosto de 2014

Essays on the Political Economy of Fiscal Policy

by

José Carlos Tello

August 2014

Abstract

This dissertation consists of two essays on the political economy of fiscal policy.

In the first essay, I analyze the persistent difference between legal and effective direct tax rate. This difference is explained by the level of tax enforcement. I consider that tax enforcement is a policy choice and study how society determines tax enforcement policy through a process of social preference aggregation. It find that tax enforcement causes an intertemporal conflict between workers and capitalists. In doing so, we seek a sharper answer to why direct tax enforcement varies across economies.

In the second essay, coauthored with Klaus Schmidt-Hebbel, we develop a dynamic general-equilibrium political-economy model for the optimal size and composition of public spending. An analytical solution is derived from majority voting for three government spending categories: public consumption goods and transfers, as well as productive government services. We establish conditions, in an environment of multi-dimensional voting, under which a non-monotonic, inverted U-shape relation between inequality and growth is obtained.

Thesis Supervisor: Klaus Schmidt-Hebbel
Professor of Economics

Contents

Acknowledgments

During the preparation of the thesis, a number of people deserve my gratitude and respect. First of all, my main advisor, professor Klaus Schmidt-Hebbel, for his guidance and inspiring coauthorship. Also I thank my advisors, professors Francisco Gallego, Nicolás Figueroa and Martin Besfamille, among other things for the opportunity to interact with both of them and encouraging me to finished this thesis.

Also at Pontifical Catholic University of Chile, I have had the opportunity to interact with other faculty members, Raimundo Soto, Rodrigo Cerda, Felipe Zurita, Francisco Rosende, Salvador Valdés, Rodrigo Fuentes, Aristides Torche and Ricardo Caballero, from whom I have learned a lot. I thank José Miguel Sánchez for his support at different stages of my graduate studies and for encouraging me to enroll in the PhD program and Vittorio Corbo for the opportunity to work with him as a teaching assistant.

I gratefully acknowledge the financial support of the Economics Institute, Catholic University of Chile, the Program MECESUP and the Department of Economics, Catholic University of Peru.

I also would like to thank all those helped me and encouraged me during the completion of my thesis. To Gabriel, Omar and Daniel my best friends in Santiago. To Yarela, Vesna, Carlos, Javiera, Marcela and all my classmates. I have enjoyed very much their company and remembered their advices. To many other students at PhD and MA program so many nice people. To Graciela for her patience and support in hard times, thanks for joining me in this process.

Mi tesis la dedico especialmente a Carlos, Teófila y Gladys, mi familia, que siempre están presentes en todos los lugares donde estoy. Todo su cariño, su esfuerzo y su optimismo fueron importantes en esta larga aventura.

Finally, I want to thank everyone who were close to me. All conversations and experiences helped me gain new insights beyond the economy.

Chapter 1

Incomplete Tax Enforcement in a Two-sector Economy

Incomplete Tax Enforcement in a Two-sector Economy*

Job Market Paper

José-Carlos Tello†

August 14, 2014

Abstract

This paper derives endogenous enforcement of income taxation as a result of political-economy decisions under repeated voting in general dynamic equilibria. The economy is characterized by two overlapping generations (workers and capitalists), two production sectors (a formal sector that uses capital and labor and pays accrued taxes fully, and an informal sector that only uses labor and pays taxes subject to a level of tax enforcement decided by majority voting), and a government (who collects taxes used to finance transfers to workers). Enforcement causes an intertemporal conflict between agents that differs in two Markov-perfect equilibria. In the myopic equilibrium, tax enforcement increases in capital because higher transfers compensate the decisive voter for higher tax payments. In the strategic equilibrium, tax enforcement declines with capital because the reduction of capital is used by the current decisive voter as a credible threat to the future voter. In steady state both Markov-perfect equilibria are determined by the negative impact of tax enforcement on capital. Steady-state enforcement is shown to be higher in the strategic equilibrium and in the social-planner equilibrium, compared to the myopic equilibrium. The model is extended to consider intra-generational inequality among workers that differ by productivity. For this case, the Markov-perfect equilibrium is unique and strategic, reflecting a coalition between capitalists and low-productivity workers. Higher inequality lowers capital and raises tax enforcement.

JEL Classification: D72, E62, H11, H31
Keywords: Tax Enforcement, Political Economy, Majority Voting, Overlapping Generations, Markov Equilibrium.

*I would like to thank Claudio Ferraz, Pablo Querubín as well as participants of Economic History and Cliometrics Lab (EH-Clio LAB) Annual Meeting - 2013 for comments.

†Comments: jctello@uc.cl

"The power to tax is taken for granted in most of mainstream public finance. Traditional research focuses on limits imposed by incentive constraints tied to asymmetric information, or sometimes political motives, rather than administrative capabilities of the state. Thus, public finance and taxation remains a relatively unexplored field." Besley, T. and T. Persson, Handbook of Public Economics, 2013, Vol 5, Chapter 2, p. 52

1 Introduction

As stated in the opening quotation by Besley and Persson (2013), countries show a marked difference in levels of direct tax revenue that is correlated with development status, but surprisingly uncorrelated with statutory (legal) tax rates. Richer countries tend to raise more tax revenue despite having statutory tax rates similar to their poorer counterparts (Besley and Persson, 2013).[1] This difference, as suggested by Gordon and Li (2009), reflects higher tax evasion in poorer countries due in large part to tax enforcement deficiencies. Generally, the literature on tax evasion shows an inverse relationship between state capacity and fiscal evasion. State capacity, however, is taken as exogenous.

The traditional focus of tax evasion, based on the seminal work of Allingham and Sandmo (1972)[2], finds an ambiguous relationship between the tax rate and evasion and an inverse relationship between tax enforcement – defined as the probability of detecting tax evasion weighted by the penalty for such evasion – and tax evasion, assuming an exogenous tax enforcement policy. Later extensions of the Allingham and Sandmo model show that the relationship between the tax rate and evasion is not ambiguous but is positive. This original inverse relationship between tax enforcement and evasion is upheld without variation.[3] The existence of an informal sector

[1]There are several ways of measuring representative tax rates, Besley and Persson (2013) uses top statutory income tax rate. Gordon and Lee (2005) discuss four representative personal income tax rates: the top statutory personal tax rate, estimated effective marginal tax rates, effective average tax rate on labor income, and a weighted average statutory individual income tax rate. Although none of these four measures is ideal.

[2]Where the decision to evade taxes is a choice made under uncertainty, taking into account the risk aversion and expected utility of the representative agent. These characteristics mean that the model can be thought of as a type of portfolio model.

[3]Sandmo (2012), Alm (2012), and Myles et al. (2010) review the extensive theoretical literature on tax evasion and they highlight i) the extension using occupational choice in the labor market as in Kesselman (1989), in

is a sufficient but not necessary condition for tax evasion. Rauch (1991), Dabla-Norris (2008) and de Paula and Scheinkman (2011) show existence of an informal sector by allowing agent management ability to be heterogeneous. Each agent can choose to start a firm in the formal sector, start a firm in the informal sector, or become a worker. The high costs of formalization (high taxes and/or capital costs) prevent low-skill firm owners from doing well in the formal sector and they therefore choose to operate informally.

Subsequent work on tax evasion endogenizes tax enforcement from a normative perspective. This literature, labeled under the category of optimal tax enforcement, shows that if the goal of the government is to maximize tax revenue to fund public spending, it is necessary to consider administrative costs as well as general distorting effects generated by the decision to evade (or pay) taxes, so that optimal policy depends on distorting factors such as: the regressiveness of the tax rates, the initial investment in detection technology, the existence of a labor market comprised by undocumented migrant workers, and the size of the informal economy, among others. Studies in this area include Border and Sobel (1987), Chander and Wilde (1998), Reinganum and Wilde (1985), Cuff et al. (2011) and Leal (2014).

In this paper we are interested in the problem of tax enforcement, defined as the inability of the state to raise taxes as a result of weak administrative execution rather than weak legislation. To our best understanding, only a small number of studies have analyzed tax enforcement from a political-economy perspective. Only Besfamille et al. (2013) have taken a political-economy approach to tax enforcement – the optimal level of indirect tax enforcement is derived in a static environment via majority rule. Their objective is to explain why indirect tax enforcement varies between markets. Firms compete à la Cournot and the government collects taxes in order to finance transfers. Individuals face the following trade-off: they want more transfers but this leads to higher prices for consumer goods. The paper explains the low level of enforcement in indirect taxes (e.g. sales taxes) whereby firms underreport sales and therefore pay fewer indirect taxes. Fiscal evasion is a firm decision, not a personal one, and firms do not vote. In equilibrium, tax enforcement positively affects prices and the decisive voter is the individual with

particular the consideration of general equilibrium effects in a static model, ii) the use of tax retention mechanisms at the origin, iii) the design of audit selection rules, iv) the incorporation of various aspects of behavioral economics and v) the inclusion of social norms. With regard to the empirical literature we should mention Cebula (1997), who shows that an increase in audits and evasion penalties can reduce evasion, and Andreoni et al. (1998) who review the empirical literature supporting the positive effect of tax rates on evasion.

median willingness to pay. Therefore the tax enforcement level in each market is determined by the parameter distribution of willingness to pay in that market. However, the search in the literature of positive economy on the direct tax enforcement was disappointing since we find no study on this matter. Even if the studies of Borck (2009), Roine, (2006), and Traxler (2009, 2012) deal with direct tax rate determination problem and relate it to the level of enforcement, unfortunately, here the level of tax enforcement is an exogenous policy. In these studies, higher tax enforcement can be attractive to the decisive voter given its positive impact on transfers. In particular, Roine (2006) shows that evasion can lead to a coalition between rich and poor in order to vote for a higher tax rate. The poor benefit from increased transfers and the rich are compensated with greater evasion.

Our approach is to explain deficiencies in direct tax enforcement in a dynamic environment as the result of a political conflict between agents rather than as the product of institutions inherited from the past. Moreover, we avoid the usual treatment of tax evasion where economic agents must choose a level of evasion, given enforcement capacity of the state, the technology of tax evasion, and the risk bearing in such technology. Instead, we consider that tax enforcement is a policy choice and study how society determines tax enforcement policy through a process of social preference aggregation. In doing so, we seek a sharper answer to why direct tax enforcement varies across economies. In this model tax enforcement policy is the result of an voting process and the amount of tax evasion is obtained directly as in a simple deterrence model.

For this purpose, we use a dynamic macroeconomic model with overlapping generations and two productive sectors, with tax enforcement problems in the traditional sector. We use a political economy model with Downsian voting to choose a tax enforcement policy in each period.[4] Since agents are forward-looking and rational, their vote in each election will be strategic. Each agent (representative of his cohort) has opposite political preferences depending on whether he is young (a worker) or old (a capitalist). The two sector production structure of the economy contributes to the polarity of agents' political preferences. We assume that the modern sector demands labor and capital, while the traditional sector only uses labor.[5] The coexistence of a

[4]It is difficult to justify a direct vote on tax enforcement policy, still, the policy convergence theorem allows us to use the median voter theorem result in a two-party competitive environment.

[5]Turnovsky and Basher (2009), Ihrig and Moe (2004), and Garca-Peñalosa and Turnosvky (2005) model a productive dual structure called formal and informal sector to characterize a developing economy. In our model,

modern and a traditional sector will imply that the factor income is affected in opposite ways by the same policy of tax enforcement. In the long run, tax enforcement is incomplete due to the political conflict between workers and capitalists, with workers preferring low values of enforcement while capitalists prefer the opposite.

We derive two forward-looking political equilibria. In the myopic equilibrium, the policy function is increasing with respect to capital; therefore tax enforcement increases in capital because higher transfers compensate the decisive voter for higher tax payments. In the strategic equilibrium, however, the policy function is decreasing with respect to capital, a result that reflects the believable threat of the current decisive voter regarding the choice of the future decisive voter. As the equilibrium is Markov-perfect, the only way to influence the myopic choice of the future voter is via a reduction in the level of capital stock, which is the result if the current decisive voter chooses a higher level of tax enforcement. Finally, in steady state, both Markov-perfect equilibria are determined by the negative impact that tax enforcement has on long run capital levels. This discouragement of capital accumulation is generated by the fact that workers have lower income and are the only agents who save. Steady-state enforcement is shown to be higher in the strategic equilibrium and in the social-planner equilibrium, compared to the myopic equilibrium. The model is extended to consider intra-generational inequality among workers that di?er by productivity. For this case, the Markov-perfect equilibrium is unique and strategic, reecting a coalition between capitalists and low-productivity workers. Higher inequality lowers capital and raises tax enforcement.

Another feature of the model is the analytical characterization of the political-economy equilibrium. To our knowledge, the work of Hassler et al. (2003, 2005, 2007) and Grossman and Helpman (1998) are the only dynamic political economy models that achieve an analytical solution as a Markov-perfect equilibrium. This is due to the complexity that results from policy endogenization in a dynamic environment. Our work, while highly stylized and simplified, contributes to the understanding of various aspects of tax enforcement policy dynamics because we offer a closed form analytical solution.

The paper continues as follows. In section 2 we describe the model, derive the equilibrium given a level of tax enforcement and show a closed solution of the capital accumulation equation

we took that duality but we label it as the modern sector and traditional sector because in our model agents pay income taxes, not firms.

and the respective indirect utility functions. In section 3, we introduce tax enforcement policy into the model. Section 4 presents a politico-economic equilibrium, wherein we characterize two Markov perfect equilibria, myopic and strategic. Then, in section 5, we extend the model by allowing the labor productivity level to differ for each agent. We conclude in section 6.

2 The Model

In this section we describe a simple model that yields a closed-form solution for economic equilibrium. The economy has one non-produced input (labor) and one produced good, which serves both as a capital input and as a consumption good. Each agent is endowed with one unit of labor in each period.

2.1 Firms

This economy has two sectors, a modern sector (M) and a traditional sector (T), and both sectors produce a homogeneous commodity with different technologies. The modern sector employs capital, K_M, and labor, L_M, and has constant returns to scale. The production function is Cobb-Douglas, $Y_M = A_M K_M^{\alpha} L_M^{1-\alpha}$, where $\alpha \in (0,1)$. On the other hand, the traditional sector employs only labor, L_T, and its production function is linear, $Y_T = A_T L_T$.[6] This treatment is similar to Turnovsky and Basher (2009), with the difference being that they adopt a production function with decreasing returns to scale in the traditional sector. Thus our traditional sector is labor intensive and its productivity can be considered a fixed factor.[7] Useful examples for the traditional sector include the market for agriculture, wherein land is the fixed factor, or the black-market, wherein the fixed factor is the technology available to produce consumption goods without accounting and financial records. The fact that the model allows only one final good facilitates the characterization of the economic equilibrium, and as we will later show, the political economy equilibrium as well. The assumption that the same good is produced

[6] The assumption of linear production function plays no role in the characterization of the equilibrium. In appendix A, we show that the economic equilibrium keeps the qualitative implications yet under the assumption of labor's diminishing marginal returns. However, the assumption of linear production simplifies the characterization of the equilibrium.

[7] Our model can be thought of as a simplified version of a two-sector two-good model with a degree of substitution between the goods. Kesselman (1989) developed a static intersectoral model for tax evasion in which the goods of each sector are imperfect substitutes in consumption.

both formally and informally is not unreasonable. Slemrod and Yitzhaki (2002) show that the simultaneous existence of formal and informal services (in the home or another small-scale sector) justify the one-good treatment.[8]

Given that the firms operate in a competitive environment, factor payments are equal to their respective marginal productivities

$$r = \alpha A_M K_M^{\alpha-1} L_M^{1-\alpha} \tag{1}$$

$$w_M = (1-\alpha) A_M K_M^{\alpha} L_M^{-\alpha} \tag{2}$$

$$w_T = A_T \tag{3}$$

Capital does not depreciate and the firms do not face cost of adjustment of capital stock. In this case, the investment decision of the modern firm is simple.

2.2 Government

The government collects taxes from capital and labor income, at rates τ^k and τ^l, respectively. We assume that taxes are implemented without evasion if capital and labor income come only from the modern sector; however, labor income from the traditional sector can only be partially observed by the government and the collected taxes are necessarily less than or equal to the legally established rates. So the effective tax rate on traditional labor income is $\tau^e \in [0, \tau^l]$. The difference between the de facto tax, τ^e, and the statutory tax, τ^l, depends on tax enforcement, $\phi \in [0, 1]$, which will be determined by a majority vote of rational self-interested individuals.

The traditional tax enforcement problem is one of asymmetric information but the issue here is more complicated.[9] Reported labor income may or may not be equal to real income -

[8] If the model had two sectors producing distinct goods, the existence of one sector is guaranteed, as the demand for each good is determined by the degree to which the goods are substitutes. Additionally, the aggregate demand would be a new determinant of the size of both the traditional and modern sectors. Nonetheless, in our model, labor market equilibrium determines participation in each sector even though measuring participation is not the formal objective of this paper.

[9] Mirrlees (1971) addresses the issue of asymmetric information between the government and taxpayers. In the society, there is income inequality due to the distribution of ability for individuals. The government wants to reduce this inequality and is interested in setting a tax rate that allows for a transfers scheme. However, skills are not observable (asymmetric information) and higher tax rates encourage individuals with greater skills to choose higher levels of leisure - thereby affecting economic efficiency.

but these reports can be reviewed by the collecting agency through costly audits. We assume that tax enforcement will positively affect the effective tax rate but in a decreasing way, namely the effective tax rate will be $\tau^e = \tau^l \phi^\lambda$, with $\lambda \in (0,1)$.[10] Moreover, increasing tax enforcement entails increased administrative costs because tax administration deals primarily with the matter of information gathering. Slemrod and Yitzhaki (2002) enumerate several reasons why information is asymmetric and costly. In particular, they note that income reports tend to be more honest in sectors whose transactions are documented or registered. In this model, modern sector transactions are recorded in the financial system, unlike the traditional sector whose transactions are in cash and therefore more difficult to trace. For this reason, administrative costs of increased enforcement in the modern sector would be low and constant, for convenience we consider these costs equal to zero. In the traditional sector, however, administrative costs will be positive and strictly increasing with respect to tax enforcement.[11] Therefore, we assume that the cost function is convex, and defined as $c = \bar{c}\phi^\rho$ with $\rho > 1$.

Additionally, we assume that government revenues are only used to finance transfers to young agents. This transfer mechanism is a particular generational policy where the old generation faces a tax and the young generation receives a transfer payment.[12] In the literature developed by Rangel (2003), this mechanism can be conceived as a forward intergenerational public good because government policy increases younger generation income (and therefore savings), which has a consequent positive impact on capital accumulation.[13] Given the parameter ϕ, if the after-tax interest rate is greater than the subjective discount rate, young-generation consumption (without considering transfers) is less than old-generation consumption and transfer payments can be seen as a consumption redistribution from rich to poor.

The government budget constraint is

$$G_t = \tau^k r_t K_t + \tau^l w_M L_M + (\tau^e - c) w_T L_T. \tag{4}$$

[10]This feature can be justified in a simple model of tax evasion which we will detail in the following section.

[11]In addition, this assumption will allow us to obtain interior solutions on one of the politico-economic equilibria.

[12]Kotlikoff (2002) shows how generational policy works in models of overlapping generations. In a general theoretical framework, he analyzes the impacts of transfers, taxes, and financing of public deficits on the actions taken by the different generations.

[13]Other examples of forward intergenerational goods are education and the environment, while a classic backward intergenerational good is the pension system. Boldrin and Montes (2005) show an example of the linkage between backward and forward intergenerational goods exchange. Several papers have studied investment in forward goods in an OLG model, for example, Kotlikoff and Rosenthal (1993) and Collard (2000).

2.3 Consumers

Agents live for two periods. In the first, they offer hours of effective labor, measured by θ[14], then make consumption and savings decisions based on their income and preferences. In the second period, agents consume all wealth (savings plus interest). In the first period, agents are labeled as workers because they earn labor income and in the second, agents are called capitalists because their income comes from savings in the previous period. Let N be the constant number of workers in each period.[15] Intertemporal preferences are captured by a separable logarithmic utility function.[16] Finally, assuming perfect intersectoral mobility of labor[17], the problem that faces each agent living at time t is the following

$$\max_{c_t, c_{t+1}, l_T, l_M} \ln c_t + \beta \ln c_{t+1} \quad s.t.$$

$$c_t + s_t = (1 - \tau^l) w_M l_M + (1 - \tau^e) w_T l_T + g_t \tag{5}$$

$$c_{t+1} = (1 + (1 - \tau^k) r_{t+1}) s_t \tag{6}$$

where c represents consumption, l_T is the effective number of labor hours in the traditional sector, l_M are the effective number of labor hours in the modern sector which satisfy $l_T + l_M = \theta$, $g_t = G_t/N$ is the transfer per worker, s_t is worker's savings and r_{t+1} is the rental rate.[18] The parameter $\beta \in (0, 1)$ is the subjective discount factor and τ^j, for $j = k, l, e$ denotes the tax rates according to income sources, as discussed in the previous subsection.

[14]In Section 5, we extend the model and allow intragenerational heterogeneity.

[15]We assume zero population growth.

[16]It is well-known that this function has a intertemporal elasticity of substitution equal to one. In this case, the savings decision is insensitive to changes in the interest rate, more precisely, the income effect is fully compensated by the substitution effect. This particular functional form will allow us to obtain a closed-form solution of the politico-economic equilibrium.

[17]Turnovsky and Basher (2009) says developing economies have rigid labor markets and its intersectoral mobility can be reduced. This rigidity may be due to deficiencies in institutions that do not promote coordination between employer and employee. We simplify the reality of economies and assume that mobility is perfect. Furthermore, the inclusion of an equation that captures the intersectoral dynamics can unnecessarily obscure our derivation and subsequent comparative analysis. Our assumption enables us to characterize a labor market whose equilibrium wage is determined by the low productivity of the traditional sector.

[18]Interest rate received by the agent is equal to the rental rate paid by firms under the assumption that the relative price of the capital is constant and equal to one, we assume that capital can be consumed.

2.4 Equilibrium with Exogenous Policy

Given that the workers chose their hours of effective labor for each sector, the respective after-tax wages must be equivalent in equilibrium.

$$(1-\tau^l)w_M = (1-\tau^e)w_T. \tag{7}$$

Given the first-order conditions of the agent's problem, consumption levels are

$$c_t = \frac{1}{1+\beta}\ \mathrm{I}_t, \tag{8}$$

$$c_{t+1} = (1+(1-\tau^k)r_{t+1})\frac{\beta}{1+\beta}\ \mathrm{I}_t, \tag{9}$$

where worker income, denoted as I_t, is the sum of labor income and government transfers

$$\mathrm{I}_t \equiv (1-\tau^e)w_T\theta + g_t. \tag{10}$$

Given equation (7), labor income is equal to the effective labor hours valued at market price, namely, the after-tax equilibrium wage. We know that before-tax wages in the traditional sector are equal to their respective marginal productivity, which, according to equation (3), is constant. Therefore the before-tax equilibrium wage in the modern sector will be equal to $w_M = \frac{(1-\tau^e)}{(1-\tau^l)}A_T$.

If we include the before-tax equilibrium wage in the equation for the marginal productivity of labor (2) we have labor demand of the modern sector, which is shown to depend linearly on the economy-wide capital stock.

$$L_M = \left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T(1-\tau^e)}\right]^{1/\alpha} K_M. \tag{11}$$

In this equation, an increase in aggregate capital stock increases the marginal productivity of labor, which in turn allows the modern sector to hire more labor. The intuition behind this dynamic response is straightforward. Since marginal productivity in the traditional sector is constant (3), any increase in the marginal productivity of labor in the modern sector requires a parallel increase in the amount of modern-sector labor, L_M, to ensure that the labor market clears.

On the other hand, an increase in the effective tax rate, τ^e, (the equivalent of more tax enforcement) reduces traditional labor income, leading agents to increase the hours of effective labor in the modern sector. The effects of changes in sectoral factor productivity on the traditional labor are well-known in the literature. An increase in modern (traditional) factor productivity, A_M (A_T), causes an increase in the respective labor income and therefore increases the modern (traditional) labor to the detriment of traditional (modern) labor. For a given level of aggregate labor demand (11), the equilibrium interest rate, r, will be constant. Due to a perfect intersectoral mobility of labor, the interest rate does not depend on the aggregate capital K_M. [19] We replace (11) in (1), which yields

$$r = \alpha A_M \left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T(1-\tau^e)} \right]^{\frac{(1-\alpha)}{\alpha}} . \tag{12}$$

The effective tax rate affects the interest rate positively via the supply of labor to the modern sector as discussed above. We also should mention that the adjustment in the labor market is responsible for the negative dependence of marginal return to capital with respect to A_T. An increase in A_T lowers the modern labor demand and the marginal return on capital will be lower because the marginal return depends positively on the other factor, labor.

Replacing the equilibrium wage (7) and modern-sector labor demand (11) in the equation for government revenue per worker (4), yields

$$g_t = \Psi(\tau^e, \tau^k, A_M, A_T)k_t + (\tau^e - c)A_T\theta \tag{13}$$

where $\Psi(\tau^e, \tau^k, A_M, A_T) \equiv \tau^k r_t + A_T \left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T(1-\tau^e)}\right]^{\frac{1}{\alpha}} \left\{\frac{\tau^l}{(1-\tau^l)}(1-\tau^e) - (\tau^e - c)\right\}$ and r_t defined by (12).

The level of government transfers per worker, g_t, has two components, one linearly linked to savings of workers born in previous period, k_t, and another related to taxes on labor income, which is discounted by administrative costs c. To improve the intuition of this observation, we consider two extreme cases. First, suppose that fiscal capacity is null, $\phi = 0$, in the traditional sector then the effective tax rate and administrative costs are also null ($\tau^e = 0$ and $c = 0$). In this case taxes are collected only from capital income in the modern sector. In the case that tax enforcement is complete, $\phi = 1$, if the administrative costs are lower than the labor tax rate, $\bar{c} < \tau^l = \tau^e$, tax collections will come from labor and capital income. Note that it is not possible to assert if the total tax collection is higher or lower in relation to the null tax enforcement.

[19] The interest rate is constant but that does not mean that we are facing an endogenous growth model. This model lacks the appropriate externality to generate perpetual growth of both factors of production. The aggregate hours of effective labor do not increase over time unlike the capital stock. However, the aggregate capital, K_M, moves to reach its steady state level. See the subsection 3.5.

We deduce the workers after-tax income function by placing government transfers (13) in worker's income (10)

$$\mathrm{I}(\tau^e, \tau^k, A_M, A_T, k_t, \theta) = \Psi(\tau^e, \tau^k, A_M, A_T)k_t + (1-c)A_T\theta. \tag{14}$$

Since generations are homogeneous, government transfers associated with labor income are compensated in a one by one fashion by after-tax labor income. Therefore no worker receives transfers from another worker. As mentioned above, transfers are effectively intergenerational income redistribution from capitalists to workers. Note that the administrative costs of taxation, however, are borne by workers.

Additionally, net aggregate saving[20], $S_t = Ns_t - K_t$, is equal to net aggregate investment, $K_{t+1} - K_t$ if we assume, for simplicity, that population growth is zero. The intuition is simple - workers are the ones who save part of their income and since there is no heterogeneity between them savings must equal investment.[21] It follows that the dynamic equation of the capital per worker is

$$\frac{K_{t+1}}{N} = k_{t+1} = s_t = \left(\frac{\beta}{1+\beta} \times \mathrm{I}_t\right). \tag{15}$$

In a standard overlapping generations model the dynamic of capital is known to depend on the marginal product of labor. Here, the existence of modern and traditional sectors and the perfect intersectorial labor mobility change the typical capital dynamic. However, transfers, g, allow us to create a new mechanism for capital accumulation because they are financed by taxes on capital income of agents born at time $t-1$.[22] The distribution of labor per worker in the modern sector is simply equation (11) divided by N and since the total labor supply is constant, $N\theta$, traditional labor per worker is obtained as a residual.

Finally, economic equilibrium is defined as follows.

Definition 1. *Given the initial condition k_0 and the vector of policies $[\tau^e\ \tau^k\ \tau^l]'$, a Competitive Economic Equilibrium is defined as a sequence of allocations $\{c_{1,t}, c_{2,t+1}, k_{t+1}\}_{t=0}^{\infty}$ such that individual choices, which coincide with (8) and (9), and with I_t defined by (14), are consistent with the law of motion described in (15) and markets clear at all points in time.*

[20] The aggregate saving is the saving of worker minus the dissaving of the capitalist.

[21] Since the capital stock can only be operated by the modern sector in the future we will omit the subscript M.

[22] Remember that the transfers only are intended for workers. Hence $G_t = g_t N$.

An interesting and simple comparative static analysis of tax revenue allows us to simplify the exposition of the political economic equilibrium which is the main objective of the paper. An increase in the effective tax rate τ^e leads to an obvious increase in tax revenues from the traditional sector as long as costs of collection are less than the tax itself. However, tax revenues associated with the capital stock (in the modern sector) respond ambiguously. The term Ψ increases due to higher marginal productivity of capital analyzed above and, simultaneously, by the increase in modern sector labor supply. However, the same term Ψ decreases due to a reduction in the tax base caused by a lower after-tax equilibrium wage.

The effect of an increase in A_T, on tax revenues is also ambiguous due to changes between income sources. On one hand an increase in A_T causes an increase in the equilibrium wage and increased tax collection in the labor market. However, the opposite occurs in the modern sector as the demand of labor and the marginal productivity of capital are reduced (see equations (11) and (12), respectively). In contrast, an increase in A_M raises tax revenue as it increases both the marginal returns of capital and the demand for labor in the modern sector. Both effects are discussed in more detail above.

3 Tax Enforcement as Public Policy

In this section we study the choice of indirect tax enforcement as an endogenous policy instrument. In order to do so, we assume the existence of two parties that compete for office (Downsian Competition).[23] This majority choice should anticipate perfectly the current and future impact on economic equilibrium. Before proceeding we must define which of the two groups of agents is majority. If population growth is zero, the two groups have equal populations. However, a positive (negative) growth rate would indicate that workers (capitalists) are in the majority.

In this section we characterize the relationships of the main variables of the model with respect to ϕ and define sufficient conditions under which voter preferences of satisfy the single-

[23] Alternatively, we can assume that the numbers of candidates is endogenous, i.e., each citizen should decides his entry to electoral contest although his net benefits of winning an election are low (Citizen Candidate Model). In both cases, if all voters have single-peaked policy preferences then the winning political platform will be one that matches the policy preferred by a majority of voters. More precisely, this means that tax enforcement outcomes will be determined by the voter whose most preferred tax enforcement is the median in the distribution of most preferred levels of tax enforcement.

peaked property.

The relevant variable in the politico-economic equilibrium is the effective tax rate, which takes the functional form $\tau^e(\phi) = \tau^l \phi^\lambda$, which has a positive and decreasing derivative, $\lambda \in (0, 1)$. This function may correspond to a reduced form model of tax evasion in which agents choose the amount of evasion subject to a certain probability of being audited by the tax collection agency and subsequently fined. In this model, tax enforcement is represented by the probability of being audited and by the amount of the fine.[24] The proposed tax enforcement is static, as agents only live for two periods and second period income cannot be partially hidden. We assume that capital income can be observed without cost by the regulatory agency. In this context any learning on the part of the collection agency would be useless. Additionally, there is little empirical support for the idea that audit probability and fines improve collection. Only Cebula (1997) can support this idea, while Kleven et al. (2011) and Phillips (2014) show that including third-party reports as part of tax enforcement improves upon the seminal Allingham-Sandmo (1972) model which is the basis of my structural formulation of the effective tax rate.

As we mentioned earlier, the interest rate is constant and depends on tax enforcement but not on K.

Lemma 1. $r(\phi_t) = \alpha A_M \left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T(1-\tau^l\phi_t^\lambda)}\right]^{\frac{(1-\alpha)}{\alpha}}$ *and it is increasing in* ϕ.

Higher tax enforcement reduces the equilibrium wage which in turn displaces labor from the traditional sector to the modern sector. The increase of labor in the modern sector allows an increase in the marginal productivity of capital. This implies that $r(\phi)$ is increasing since the modern sector is the one that uses capital.

As we will show in the political equilibrium section, the interest rate will be the determining factor in the agent's vote and it is important to assume that the positive effect of tax enforcement on the interest rates is attenuated by increasingly large values.

Assumption 1. $r(\phi)$ *is concave, a sufficient condition is* $\tau^e_{\phi\phi} + \frac{(\tau^e_\phi)^2}{\alpha(1-\tau^e)} < 0$.

Proof. The details of this condition are shown in appendix C. □

Tax collection also depends on ϕ. Replacing the effective tax rate, $\tau^e(\phi) = \tau^l\phi^\lambda$, and interest rate, $r(\phi)$, in (13), yields

[24] In the appendix B shows a simple model that provides details of the above.

Lemma 2. *Tax collection is given by* $g(\phi_t, k_t) = \Psi(\phi_t)k_t + (\tau^e(\phi_t) - c(\phi_t))A_T\theta$*, where* $\Psi(\phi_t) \equiv \tau^k r_t(\phi_t) + A_T \left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T(1-\tau^e(\phi_t))}\right]^{\frac{1}{\alpha}} \left\{\frac{\tau^l}{(1-\tau^l)}(1-\tau^e(\phi_t)) - (\tau^e(\phi_t) - c(\phi_t))\right\}$ *and* $c(\phi) = c\phi^\rho$*.*

Labor tax collection increases with a higher tax enforcement even when we consider administrative costs. However, this increase tends to decline when tax enforcement is known (ex-ante) to be high. Given that the administrative cost function is convex and the effective tax rate is concave then the second term of equation $g(\phi, k)$ is concave. Namely, the increase in fines and/or probability of a successful audit becomes less auspicious.

Tax enforcement has an ambiguous effect on tax collection. Stronger tax enforcement produces two positive effects on Ψ, through an increased demand for labor in the modern sector and an increase in the marginal productivity of capital. However, these positive effects are counteracted by the negative effect of a lower equilibrium wage. To a lesser extent, administrative costs have a positive effect on Ψ because the tax collection to labor becomes more expensive than the tax collection on capital. We assume the following condition in order to define a political equilibrium without corner solutions.

Assumption 2. $\Psi(\phi)$ *is concave if* $\bar{c}\phi^\rho \left[1 + \rho(\alpha(\rho-1)(1-\frac{1}{\tau^e(\phi)}) - 1)(1-\frac{1}{\tau^e(\phi)})\right] < 1$.

Proof. The details of proof are shown in appendix D. □

In this way, tax revenue, $g(\phi, k)$, increases with tax enforcement until a certain point, then decreases at high levels of tax capacity, i.e. tax revenues form a Laffer Curve. This inverted U-shape is due to an increase in the effective tax rate while the statutory tax rates, τ^k and τ^l, remains constant.

Proposition 1. *Worker's income is composed of after-tax labor income plus transfers. Given that transfers are financed by tax revenue, the after-tax income of workers is* $I(\phi_t, k_t) = \Psi(\phi_t)k_t + (1 - c(\phi_t))A_T\theta$*.*

Worker's income benefits from an increase in capital stock through higher transfers. Recall that the current capital stock are simply the savings from the workers of previous generation, and should therefore be seen as an intergenerational redistribution. In addition, labor income is not directly affected by the tax rate τ^e because there is no heterogeneity between workers and therefore intragenerational redistribution is not possible. Proving that Ψ is concave is enough to prove that worker income is concave with respect to tax enforcement.

Then, by substituting the interest rate, $r(\phi)$, and after-tax worker's income, $\mathrm{I}(\phi, k)$, in equation (9)), the indirect utility function of the capitalist can be written as

$$V^C(\phi_t) = \ln\left(\frac{\beta}{1+\beta}(1 + (1-\tau^k)r(\phi_t))\mathrm{I}(\phi_{t-1}, k_{t-1})\right) \tag{16}$$

Note that capitalists are only interested in the current level of tax enforcement (ϕ_t). Even though the previous value of tax enforcement (ϕ_{t-1}) determines, via savings, his consumption, the capitalist takes as given this previous value because the social aggregation mechanism only permits him to take the future choice into account.[25]

Substituting after-tax income of workers, $\mathrm{I}(\phi, k)$, in equation (8) and (9) yields the indirect utility function of workers

$$V^W(\phi_t, \phi_{t+1}, k_t) = \ln\left(\frac{1}{1+\beta}\mathrm{I}(\phi_t, k_t)\right) + \beta \ln\left(\frac{\beta}{1+\beta}(1 + (1-\tau^k)r(\phi_{t+1}))\mathrm{I}(\phi_t, k_t)\right) \tag{17}$$

Workers, on the other hand, are interested in both the present and future value of tax enforcement. While current tax enforcement negative or positively affects labor income and savings, tax enforcement in period $t+1$ will positively affect his return on savings and thus his consumption in period $t+1$.

Finally, investment per worker (the capital stock available for the next period) also depends on tax enforcement.

Corollary 1. *Replacing worker's income $I(\phi, k)$ and $r(\phi)$ in (15) and the law of motion is $k_{t+1} \equiv H(\phi_t, k_t) = \frac{\beta}{1+\beta}\left[\Psi(\phi_t)k_t + (1 - c(\phi_t))A_T\theta\right]$.*

The original dynamics of the capital stock, in which the steady state is reached instantaneously, will be modified due to the fact that we add transfers to workers. As mentioned above, transfers have a component that depends linearly on the capital stock and such transfers are present in the worker income (note that workers are the only agents who save). Therefore, the dynamics of capital stock are determined by a difference equation (see Corollary 1).[26]

[25] In the next section, we define the Markov-perfect equilibrium. In such equilibrium, the previous elections are summarized in the current value of the state variable and, therefore, policy choices will only depend on the current value of the state variable.

[26] This model allows us to calculate analytically the transitional function of state variable and for these reason this function is not part of the solution of political equilibrium as if it is necessary in others models of dynamic voting.

Now we analyze the steady-state of capital given a tax enforcement $\bar{\phi}$ (suppose that tax enforcement take value of $\bar{\phi}$ by some mechanism of collective choice). Note that the capital stock is decisive in the political preferences of the next-period's worker. It is straightforward to see that the steady state is achieved if $\frac{\beta}{1+\beta}\Psi(\bar{\phi}) < 1$ and the steady-state capital stock is

$$k^{ss}(\bar{\phi}) = \frac{\frac{\beta}{1+\beta}(1 - c(\bar{\phi}))A_T\theta}{1 - \frac{\beta}{1+\beta}\Psi(\bar{\phi})}. \tag{18}$$

Given an arbitrary increase in tax enforcement, we ask, what is the effect on the steady-state level of the capital stock? The effect of tax enforcement is ambiguous. On the one hand, stronger tax enforcement negatively affects the labor income, the main determinant of saving, but on the other hand, increases the transfers via the factor Ψ. Note that as mentioned above, the factor Ψ is concave and a high value of ϕ can negatively affect transfers. Therefore, if $\bar{\phi} > \tilde{\phi}$ then the effect on the steady-state capital stock is negative. The threshold $\tilde{\phi}$ must therefore satisfy the following equation

$$(1 - c(\tilde{\phi}))\Psi_{\tilde{\phi}} - c_{\tilde{\phi}}\left(\frac{1+\beta}{\beta} - \Psi(\tilde{\phi})\right) = 0. \tag{19}$$

4 Politico-Economic Equilibrium

In this section, we define the equilibrium conditions for the dynamic social aggregation mechanism. Furthermore, we analytically characterize the equilibrium and we will analyze the effects of changes in some exogenous variables on the dynamic determination of tax enforcement. To do so, we use the Policy Convergence Theorem to aggregate preferences, and in this way, the result of tax enforcement will coincide with the median value in the distribution of most preferred tax-enforcement levels.

In a repeated voting setup, we restrict the analysis to Markov perfect equilibria where the state of the economy is summarized by the capital stock (minimum state variable), k_t. A policy function $\phi_t = F(k_t)$ is used to establish the optimal decisions of decisive voters given their strategic voting. Each agent lives for two periods and in the second period also has preferences for a certain tax-enforcement level. Therefore, agents are aware that their current vote may affect the next voting, i.e. voting is strategic.

This equilibrium is formulated recursively, with the basic requirement that the decisive voter chooses a ϕ that maximizes indirect utility.

In this model all agents have equal voting rights, and as such the identity of the median (decisive) voter depends on population growth. If population growth is positive (negative), the decisive voter is a worker (capitalist). In the first case, a strategically voting worker (strategic in the sense that the worker is aware that in the next period he will be a capitalist) is interested in ϕ'.[27] In the negative growth case, the decisive voter is a capitalist and therefore does not vote strategically in the sense that he is uninterested in subsequent period policy outcomes.

Given the positive population growth, the recursive political equilibrium is defined as follows[28]

Definition 2. *A Politico-Economic equilibrium is defined as a policy function* $F : \mathbb{R}_+ \to [0,1]$ *such that the following functional equation holds* $F(k) = \arg\max_\phi V^W(\phi, \phi', k)$ *subject to* $\phi' = F(k')$ *where* $k' = h(\phi, k)$ *is defined by Corollary 1 and* $V^W(\phi, \phi', k)$ *is the indirect utility of the current period worker.*

According to our definition of equilibrium, the only way to influence the decision of the next period is by changing the state variable k' which is determined by Corollary 1. The political equilibrium requires that $F(k)$ is a fixed point of the functional equation described in the Definition 2. Intuitively, if todays agents believe that the choice of ϕ' is defined according to $F(k')$, then the same function $F(k)$ will define the current choice.

We calculate of first-order condition of the recursive problem

$$\left\{ \frac{(1+\beta)}{I(\phi^W, k)} + \frac{\beta(1-\tau^k) r_\phi(F(k'))}{1+(1-\tau^k) r(F(k'))} F_k(k') \left(\frac{\beta}{1+\beta} \right) \right\} I_\phi(\phi^W, k) = 0 \tag{20}$$

We can see that there are two possible equilibria. In one, which we call the myopic equilibrium, the worker considers only the outcome of his vote on current period variable. This does not mean that the decisive voter ignores his influence in determining k', rather the worker decides it is optimal to set aside this capacity.

In the second type of equilibrium, which we call strategic, the workers vote so as to induce a favorable future vote. Workers prefer a high level enforcement in the next period, and they induce it by decreasing next-period capital stock. So workers effectively distort the policy ϕ.

[27]The prime symbol ($'$) denotes next periods value.

[28]$\mathbb{R}_+$ is subspace of $\mathbb{R}$ containing nonnegative real numbers.

To intuitively show that a decreasing policy function in k is an equilibrium, we outlined the following argument: If the decisive voter chooses tax enforcement in period t that results in a lower income and therefore in lower saving,[29] then tax enforcement in period $t+1$ will improve in accordance with a decreasing policy function and this result is accepted by all decisive voters because lower consumption in t will be offset by increased consumption in $t+1$. In contrast, the decisive voter receives no compensation if the policy function is increasing in k, therefore this policy function will not be appropriate under strategic voting.

4.1 Politico-Equilibrium I: Myopic

In this part, we consider the myopic equilibrium. This Markov perfect equilibrium is characterized in the following proposition.

Proposition 2. *The myopic equilibrium (ME), characterized by $F^{me}(k)$, is increasing in k if $I(\phi, k)$ is concave with respect to ϕ.*

Proof. If worker income is concave with respect to ϕ, the political decision must satisfy the following condition

$$\mathrm{I}_\phi(F^{me}(k), k)) \equiv \Psi_\phi(F^{me}(k_t))k_t - c_\phi(F^{me}(k_t))A_T\theta = 0,$$

given $k_0 \geqslant 0$, for all $t \geqslant 0$ and $k_{t+1} = H(F^{me}(k_t), k_t)$ is defined in Corollary 1. This condition shows that the worker should choose that policy for which the marginal benefit is equal to the marginal administrative cost.

Differentiating the condition $\mathrm{I}_\phi = 0$, and since $\mathrm{I}(\phi, k)$ is concave, we get

$$\frac{d\phi_t}{dk_t} = \frac{-\Psi_\phi}{I_{\phi\phi}} > 0 \tag{21}$$

since $\Psi_\phi = c_\phi A_T \theta \frac{1}{k_t} > 0$ by $\mathrm{I}_\phi = 0$. Then, we say that there is a function $F^{me} : \mathbb{R}^+ \to [0, 1]$ that it is increasing in k. $\square$

An exogenous increase in capital stock causes an increase in the marginal utility of policy ϕ and at the margin, workers can assume a higher marginal disutility or higher administrative costs for levels of enforcement. Therefore, the worker should take advantage by choosing a higher level of enforcement despite the higher costs of tax administration that he must face.

[29] This is because the relationship between income and saving is linear.

We now discuss the politico-economic equilibrium in steady state. The steady-state myopic equilibrium is characterized by two functions: the policy rule, $F^{me}(k)$, and the equation of steady state capital, $k^{ss}(\phi)$, see (18). We know that the policy function is increasing in k but we need to know the direction of movement of capital or the steady state value of capital with a fixed ϕ. For this reason, we analyze two cases.

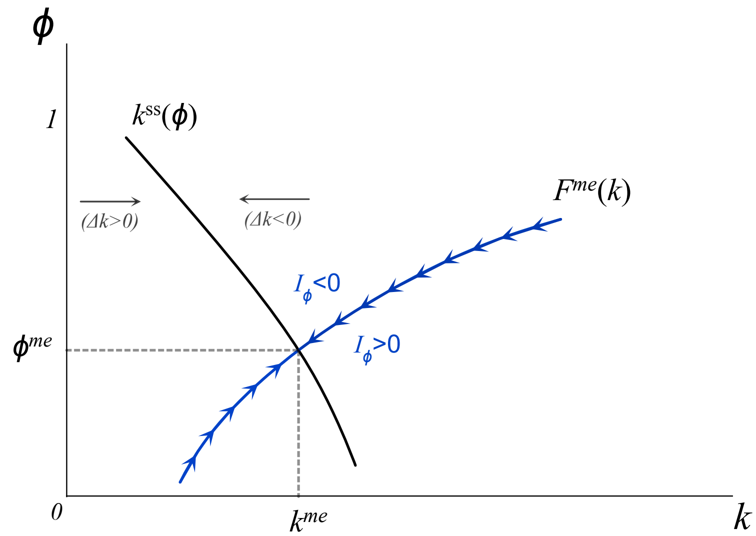

Figure 1: Myopic Equilibrium on Tax Enforcement

First, we assume that the effect of an increase in ϕ on the steady-state value of capital is negative.[30] In figure 1, if capital is to the left of the k^{ss} curve, given a fixed rate $\bar{\phi}$, capital must evolve towards the k^{ss} curve or towards its steady-state value $k^{ss}(\bar{\phi})$. We note that the choice of ϕ_t^{me} only passes through the locus defined by the policy function. Finally, the choice converges to tax enforcement ϕ^{me} where the capital reaches the steady-state level consistent with ϕ^{me}.

Now suppose that the effect of ϕ on steady state capital is ambiguous, i.e. $\tilde{\phi} > 0$. In this case, we face multiple equilibria, two of which are stable and one of which is unstable. The proof is straightforward and to achieve the goal we rely on Figure 2. Given that the policy function F^{me} is monotonic, there are only three points where the policy function and the k^{ss} curve intersect. Recall that the k^{ss} curve leads the evolution of the capital stock for a given ϕ so that intersection point 2 is unstable, meaning that a small perturbation of the equilibrium

[30]This assumption is equivalent to assuming a non-positive threshold. The threshold is defined by (19).

pushes ϕ and k towards points 1 or 3.

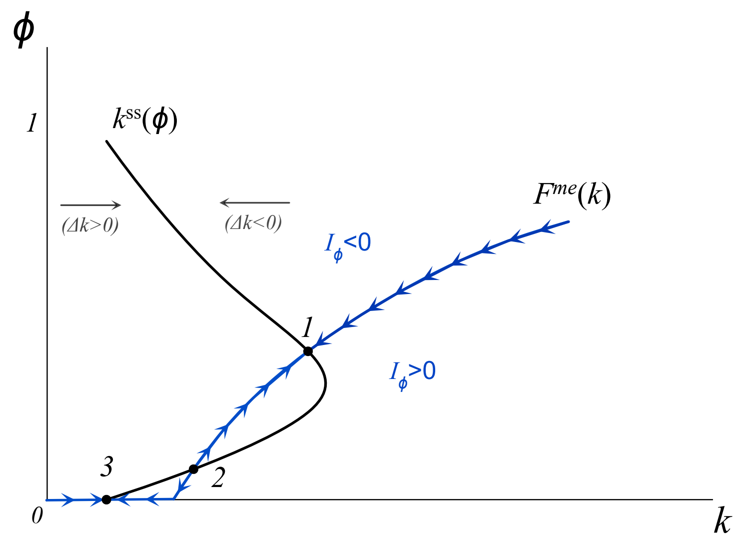

Figure 2: Myopic Equilibria on Tax Enforcement

It is also necessary to mention that the equilibrium level of enforcement may not take very high values due to the assumed inverted U-shape of transfers related to capital income taxes ($\Psi(\phi)k$). This feature is due to the policy chosen by the worker because this decision must provide a positive marginal benefit to compensate the marginal cost of administration.

4.2 Politico-Equilibrium II: Strategic

In this section we show a different equilibrium, whose main characteristic is that the policy function is not increasing in k. This feature is possible because the worker distorts their current period vote on tax enforcement in order to influence the next period vote.

Proposition 3. *There exist a continuum of differentiable policy functions as*

$$F^{se}(k;C) = \left(\frac{1}{\tau^l} \left(1 - \left[\frac{Ck^{-\frac{(1+\beta)}{\beta}} - 1}{(1-\tau^k)D} \right]^{\frac{-\alpha}{1-\alpha}} \right) \right)^{\frac{1}{\lambda}},$$

where C is a arbitrary constant of integration and $D \equiv \alpha A_M \left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T} \right]^{\frac{(1-\alpha)}{\alpha}}$.

Since this policy function is valid for $\phi \in [0,1]$ then state variable must satisfy the constraint $k \in \left[k_{min} = \left[\frac{1}{C} \left(1 + (1-\tau^l)^{\frac{-(1-\alpha)}{\alpha}} (1-\tau^k) D \right) \right]^{\frac{-\beta}{1+\beta}}, k_{max} = \left[\frac{1}{C} \left(1 + (1-\tau^k) D \right) \right]^{\frac{-\beta}{1+\beta}} \right]$.

Proof. See Appendix E. □

The intuition behind the fact that the policy function is not increasing in capital stock is the following: we imagine that the decisive voter wants to manipulate the subsequent election in order to obtain tax enforcement in next period higher than the case where he decides not to manipulate (myopic equilibrium). If so, the decisive voter does not choose the value of tax enforcement that maximizes his income. This decision allows for an increase (decrease) in capital stock in period $t+1$, assuming an initial position to the left (right) of the k^{ss} curve. If the proposed policy function is increasing in k (and assuming that we are on the left side of the k^{ss} curve) then tax enforcement in period $t+1$ is greater which is the aim of the present decisive voter. However, this outcome is not a strategic equilibrium. Since the voter deviates from the myopic choice, which maximizes income and savings, the decisive voter does not take full advantage of the proposed policy function (which is increasing in k). Therefore, a larger increase of the capital stock increases both present consumption, via increased income, and future consumption, via a higher tax enforcement and interest rate. Therefore, if the policy function is increasing in k the myopic equilibrium holds, but the strategic does not.

The constant of integration C is defined implicitly when the initial values of k and tax enforcement are replaced in the strategic equilibrium policy function F^{se}. Given k_0 and ϕ_0, we defined $\bar{C}$ in order to satisfy the following: $F^{se}(k_0; \bar{C}) = \phi_0$ or, by the implicit function theorem, $\bar{C} = \bar{C}(\phi_0, k_0)$.

However, the strategic equilibrium must induce a stable dynamic for capital stock. Substituting the strategic equilibrium policy function into the dynamic equation for capital, we find the following relationship

$$k_{t+1} = \tilde{H}(k_t) \equiv H(F^{se}(k_t; \bar{C}), k_t) \tag{22}$$

If a steady state exists, a necessary and sufficient condition for $k^{ss} < k_{max}$ is

$$\tilde{H}' = H_1 F_k^{se} + H_2 < 1 \quad \text{or} \quad F_k^{se} < \frac{1 - H_2}{H_1} \tag{23}$$

The partial derivatives H_1 and H_2 are obtained directly from $H(\phi, k)$ (see Corollary 1). We know that H_1 can be positive or negative. In fact, it will be positive (negative) if the initial value of capital is very low (high) or alternatively, this value is to the left (right) of the k^{ss} curve. Moreover, we know that $H_2 = \Psi \in (0, 1)$. Therefore, if $H_1 > 0$, then the dynamic of capital stock is stable and if $H_1 < 0$ then the stable steady state k^{ss} exists for all $\bar{C}$ that satisfy (23).

Figure 3 represents the policy function $F^{se}(k)$ given the constant $\bar{C}$ for a stable strategic equilibrium. A Markov perfect solution is possible because of its stationarity, i.e., the policy rule should also apply in the long run. Point A represents the strategic equilibrium in steady state.

Now we present a "one-stage deviation" argument to prove that a policy function decreasing in k is a credible strategy. In Figure 3, we assume that the current level of capital is k_0, then, if the worker deviates from its strategic choice, the best thing to do is to maximize income, i.e. choose ϕ_0' instead of ϕ_0. In the next period, the stock of capital available will be greater, $k_1' > k_1$, if the chosen tax enforcement in initial period was ϕ_0' given that income maximization implies savings maximization. Since the next-period decisive voter maintains a strategic choice, tax enforcement will be greater if the current decisive voter does not deviate from his choice, $\phi_1 > \phi_1'$. Finally, the policy function, $F^{se}(k)$, determines the optimal path of strategic choices where lower current consumption is offset by higher future consumption.[31]

4.3 Comparison of steady-state equilibria

Here we demonstrate that tax enforcement in the strategic equilibrium will always be greater than or equal to tax enforcement in the myopic equilibrium. We use the fact that the policy functions have opposite slopes.

Proposition 4. *Let* (k_A, ϕ_A) *be the strategic equilibrium and* (k_B, ϕ_B) *be the myopic equilibrium. Both equilibria are stable. Since* $F_k^{me} > 0$ *and* $F_k^{se} < 0$ *then* $k_A \leqslant k_B$ *and* $\phi_A > \phi_B$*.*

Proof. We use proof by contradiction. Suppose that $k_A > k_B$. Then, for each $k_B < k_0 < k_A$ there is a $\phi_0^{me} = F^{me}(k_0)$ and a $\phi_0^{se} = F^{se}(k_0)$ such that $\phi_0^{me} > \phi_0^{se}$ because we know $F_k^{me} > 0$ and $F_k^{se} < 0$ by equation (21) and Proposition 2, respectively. Remember that the choice of ϕ_0^{me} maximizes revenues and therefore savings. That is, we can say the following $H(\phi_0^{me}, k_0) = k_1' > k_1 = H(\phi_0^{se}, k_0)$. However, if $k_0 < k_A$ then the existence and stability of the strategic

[31] The argument can be reformulated if the initial capital exceeds the value of political equilibrium, i.e. $k_0 > k_A$, where point A represents the ordered pair (k_A, ϕ_A). See Figure 3.

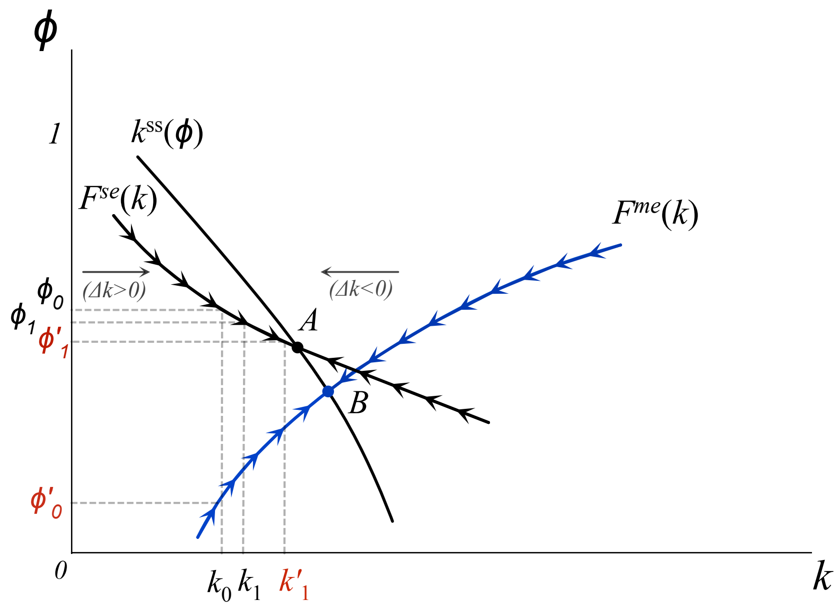

Figure 3: Myopic and Strategic Equilibrium on Tax Enforcement

equilibrium allows us to say that the savings in period 1 must be greater than the savings in period zero, namely $k_0 < k_1$, and it is straightforward to see that $k_B < k_0 < k_1'$. Finally, given $F_k^{me} > 0$ we have $\phi_B < \phi_0^{me} < \phi_1^{me}$. This outcome is a contradiction because ϕ_B is a stable myopic equilibrium and $\phi_A > \phi_B$. $\square$

As a benchmark result it is useful to analyze the optimal sequence $\{\phi^{pm}\}_{t=0}^{\infty}$ chosen by a benevolent social planner. A social planner maximizes a social utility function with a discount factor, β, equal to the subjective discount factor of agents. The maximization problem follows

$$\max_{\{\phi_t\}_{t=0}^{\infty}} U^S = \max_{\{\phi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\ln\left(\frac{1}{1+\beta}\mathrm{I}(\phi_t, k_t)\right) + \ln\left(\frac{\beta}{1+\beta}(1+(1-\tau^k)r(\phi_t))\mathrm{I}(\phi_{t-1}, k_{t-1})\right)\right)$$

given (ϕ_{-1}, k_{-1}).

Then, the first-order condition is

$$\overbrace{\frac{(1+\beta)\mathrm{I}_\phi(\phi_t^{sp}, k_t)}{\mathrm{I}(\phi_t^{sp}, k_t)}}^{\text{Today's workers}} + \overbrace{\epsilon(\phi_t^{sp})}^{\text{Today's capitalists}} + \overbrace{\frac{\beta(1+\beta)\mathrm{I}_k(\phi_{t+1}^{sp}, k_{t+1})}{\mathrm{I}(\phi_{t+1}^{sp}, k_{t+1})}}^{\text{Tomorrow's workers + capitalists}} H_\phi(\phi_t^{sp}, k_t) = 0,$$

for all $t = 0, 1, \ldots$, where $\epsilon(\phi) \equiv \frac{(1-\tau^k)r_\phi(\phi)}{1+(1-\tau^k)r(\phi)}$.

The first-order condition contains three decisions at the margin: (i) today's workers maximize their income, (ii) today's capitalists maximizes rental rate and (iii) tomorrow's workers and capitalists maximize their incomes. The benevolent social planner distorts individual optimal decisions in search of an optimal social solution. Because of the linear relationship between worker income and savings, this first-order condition can be written as

$$\mathrm{I}_{\phi}(\phi_t^{sp}, k_t)\left\{\frac{(1+\beta)}{\mathrm{I}(\phi_t^{sp}, k_t)} + \frac{\beta^2\Psi(\phi_{t+1}^{sp})}{\mathrm{I}(\phi_{t+1}^{sp}, k_{t+1})}\right\} + \epsilon(\phi_t^{sp}) = 0, \tag{24}$$

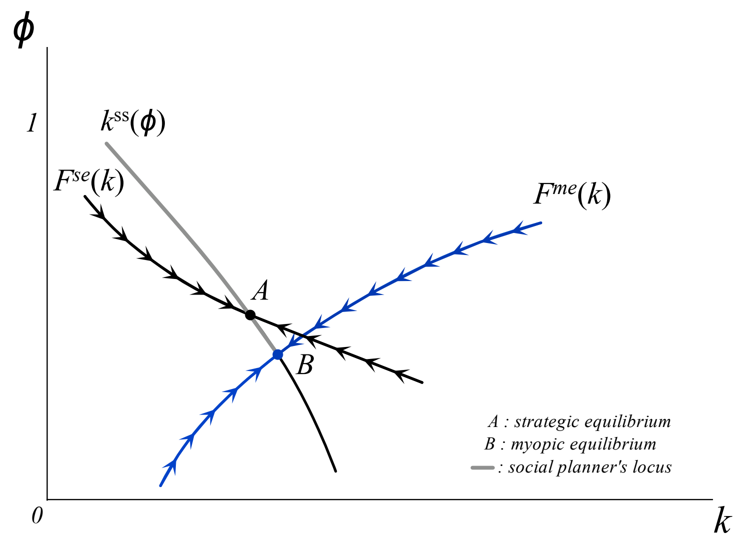

Figure 4: Equilibria on Tax Enforcement

It is straightforward to deduce that for all ϕ_{t+1}^{sp} and k_t, the derivative $\mathrm{I}_{\phi}(\phi_t^{sp}, k_t)$ will be strictly negative. Moreover, given $\mathrm{I}_{\phi} = 0$ and the assumption that worker income is concave, we can deduce that $\phi^{me} < \phi^{sp}$. The intuition is straightforward - it is necessary to reduce current worker income in order to fund transfers to future workers and increase current and future capitalist income. This conclusion is expected - governments require more resources to compensate all members of the current and future society.

However, we cannot rank the strategic and social planner equilibrium tax-enforcement levels. The strategic equilibrium depends on initial values k_0 and ϕ_0, which in turn defines the constant of integration $\bar{C}$. However, the equilibrium of the social planner does not depend on initial values if the difference equation (24) is stable. In Figure 4, we show the range of possible values of the social planner's equilibrium with the strategic and myopic equilibrium.

Tax enforcement is stronger in the strategic equilibrium than in the myopic equilibrium, and is therefore closer to the optimal level. Still, it would be counterproductive if the strategic enforcement takes on a larger than optimal value, which would even further reduce the long run capital level and formal sector production. Unfortunately, in our analysis it is not possible to analytically determine the conditions under which the equilibrium enforcement and capital stock levels are larger than what is socially optimal. Our analysis only allows us to point out that the myopic equilibrium is characterized by an over-accumulation of capital due to the labor income increase generated by the lower chosen level of tax enforcement.

4.4 Some Implications

Our analytical characterization of politico-economic equilibrium allows us to analyze some qualitative effects. This subsection examines implications for the effects of changes in capital tax rate, and modern technology.[32] As we have seen in the previous sections, in order to analyze the political equilibria in steady state we require only the policy functions of each equilibrium along with the k^{ss} curve, defined by (18).

The Effects of a Decrease of τ^k

The effects of a decrease in capital tax rate, τ^k, on the politico-economic equilibria are shown in Figure 5. A lower level of τ^k undoubtedly reduces the financing of transfers linked to capital stock, i.e. $\Psi_{\tau^k} > 0$. Because worker income is negatively affected, the impact on long-term saving or capital stock will be negative (see equation 14). Recall that workers are the only agents that can save. For this reason, the k^{ss} curve should shrink towards the origin of the capital-tax enforcement plane, the lower level of tax collection from capital cannot be undone by changing the enforcement choice due precisely because evasion does not exist in the modern sector (by assumption, even for capital income).

Now, consider the myopic equilibrium policy function F^{me}. This function is implicitly defined by marginal worker income of tax enforcement equal to zero, $\mathrm{I}_\phi(\phi^{me}, k) = 0$, i.e., when the worker decides to maximize income (see Proposition 2). It is straightforward to show that a decrease in τ^k reduces marginal worker income, I_ϕ, because marginal transfers shrink – given that the

[32]The effects of other variables on the functions that determine the balance are ambiguous and obscure the analysis, so we chose not to present the respective implications.

marginal transfers of enforcement policy depend linearly on the tax rate.[33] In this case, given the drop of τ^k, marginal income is negative, and the worker prefers a lower value of tax enforcement because marginal income is decreasing in tax enforcement. The worker demands a lower effective tax rate of labor income as compensation for lower transfers. More precisely, the policy function F^{me} shifts downward. Finally, in myopic equilibrium, a decrease of τ^k induces in steady state a reduction of tax enforcement as seen in point B'. However, the movement of capital stock is ambiguous because there are two opposing effects, a positive effect due to higher after-tax labor income (via lower tax enforcement), and a negative effect due to decreased transfer payments.

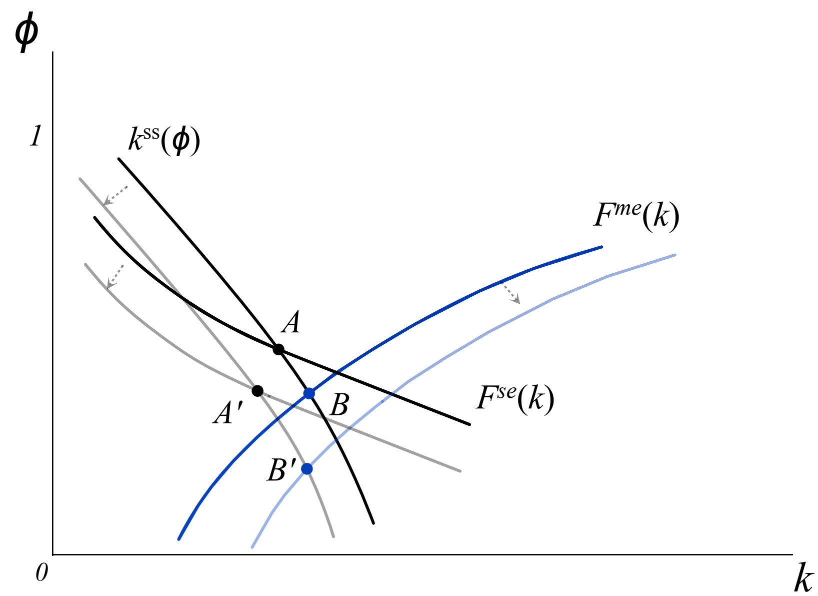

Figure 5: Effects of a Decrease of τ^k

However, in strategic equilibrium, the reduction of the capital tax rate allows an increase in the after-tax capital return because capitalists increase their consumption (see equation 17). In this case, the current voter (the worker) should not overly distort his choice since his future income will increase and he would like to reduce the future policy as compensation. More precisely, we can show that the partial derivative of the policy function, F^{se}, with respect to the tax rate is positive (see Appendix F). This result implies that the function F^{se} must shift

[33] Ψ_ϕ is calculated in Appendix D, it is straightforward to show that $\Psi_{\phi\tau^k} = r_\phi > 0$).

left. Figure 5 shows the movement of the policy function, the displacement of k^{ss} curve that we discussed previosuly, and new strategic equilibrium at Point A'. However, the results in steady state tax enforcement and capital stock are ambiguous. If the distortion in policy chosen by the worker is moderate (excessive), then savings will increase (decrease) and may (not) attenuate the negative effect on the stock of capital due to the decline in transfers.

As we have seen, capital movement is ambiguous in both political economy equilibria. To amplify the analysis we remember that in a general equilibrium model a reduction in taxes on capital causes an increase in long run capital stock and in accordance with the optimal taxation literature the long run tax rate on capital should be zero, given that such taxes distort present savings (and therefore future consumption) decisions.

This normative conclusion is due to the accumulative nature of capital, and the fact that any initial distortion in capital levels increases considerably over time making the taxation of labor income preferable.[34] Nonetheless, in our model, the tax rate on capital is not distortionary and the savings decision is therefore constant with regards to changes in the return rate (due to the assumption of logarithmic preferences we have eliminated the principal channel by which tax rates affect long run capital levels). In the myopic equilibrium it is possible that a reduction of the tax on capital has a positive effect on capital if the reduction in tax enforcement (on labor income) is sufficiently large in order to compensate for the reduction in transfers and therefore generate larger savings levels.

If we set aside the assumption of logarithmic preferences, the negative effect of a tax increase on capital is stronger and $k^s s$ curve should shift toward the origin. Unfortunately, the shift of the policy function can only be determined using numerical solutions. In the strategic equilibrium, as we have seen, it is also possible to obtain this positive capital response if the distortion in the worker-chosen policy is moderated by a decrease in tax rates on capital.

We have found no discussion in the fiscal evasion literature on the effect that a reduction of the capital tax rate would have on labor income tax enforcement. Nonetheless, in general equilibrium models of fiscal evasion, such as Kesselman (1989), and Fortin et al. (1997), the authors show that the relation between the tax rate and evasion may be positive contradictory

[34]Nonetheless, the work of Aiyagari (1994) and Conesa et al. (2009), use overlapping generation models with incomplete markets to show that the optimal tax on capital may be distinct from zero due to capital over-accumulation.

to the basic partial equilibrium result. In the case of Fortin et al. (1997), an increase in the payroll tax (or tax rate on profits) provokes less labor demand by formal firms and, as a result, less production.

Firms on the margin chose to informalize due to these increased labor costs. In Kesselman (1989), increased evasion produced by a tax increase is not only due to the shift in resources towards the evasive sector (as in Fortin et al (1997)), but is amplified by making the informal sector more productive through price effects the informally produced good costs less. Still, Kesselman uses a numerical evaluation to show that this effect of evasion is very small for a reasonable range of tax rates and suggests that other administrative or enforcement measures could have a greater effect on evasion.

In another branch of the literature, Cuff et al (2011) shows that the interaction between optimal tax rates and optimal tax enforcement is ambiguous and depends on the configuration of the labor market and the inclusion (exclusion) of undocumented or illegal immigrants. On the one hand, in a segmented equilibrium whereby domestic workers only work in the formal sector and undocumented workers only work in the informal sectors, a reduction in the tax rates entails a reduction in tax enforcement which in turn provokes an increase in both formal and informal salaries (which serves to maintain the segmented equilibrium). On the other hand, in any non-segmented equilibrium whereby domestic workers work in both sectors, the tax rates and enforcement policy should be substitutes as the equilibrium salary should be the same across sectors.

As we have seen, this ambiguity in the movement of tax enforcement is also obtained in our strategic equilibrium, although the mechanisms obviously differ. However, in the case of the myopic equilibrium, the impact on tax enforcement is negative, in other words, a decrease in the tax rate on capital also decreases the effective tax rate on labor which implies a higher level of evasion in the only sector in which it is possible: the traditional sector.[35]

The relevant part of our focus is that this result comes from an aggregation of social preferences which is sustainable in time. One noteworthy conclusion is that formal sector tax reduction policies may reduce fiscal evasion even as this desired effect may cause a reduction in formal

[35] Recall that in our model, evasion is only posible in the traditional sector. A relatively smaller modern sector, however, does not necessarily imply more evasion, but it does imply that a larger portion of the economy faces evasion, which is subject to the chosen enforcement policy. Stronger tax enforcement therefore reduces tax evasion relative to the traditional sector.

sector production due to decreased incentives to save or improve the long run capital stock. This ambiguous result is a product of a threat between present and future voters (strategic equilibrium). The coexistence of two income sources (or two productive sectors) is the assumption that allows the tax enforcement to be maintained over time as a consequence of political conflict. (Not necessarily for cost considerations, which would be a sufficient condition for incomplete enforcement in a static setting).

The Effects of an Increase of A_M

Steady state capital stock must increase since an increase in A_M leads to a higher marginal productivity of capital which in turn allows for enhanced transfers.[36] In Figure 6, k^{ss} curve must shift right.

Now, we consider the policy function in myopic equilibrium F^{me}. As mentioned in Proposition 2, in this policy function is defined implicitly the maximization of worker's income. If transfers related to capital stock, $\Psi(\phi)k$, are benefited by a technological leap in the modern sector, we can observe an increase in the level and slope of $\mathrm{I}(\phi, k)$. This increase in marginal income encourages the worker to prefer increased tax enforcement in order to take advantage of higher transfer payments. In Figure 6, F^{me} shifts upward and since transfers increase, steady-state capital stock must also increase and the k^{ss} curve should shift right. Tax enforcement increases unambiguously, but capital stock also increases (decreases) if the positive effect on steady state capital is large (modest).

Productivity improvements in the formal sector generate, via increased transfers, sufficient incentives for the decisive voter (worker) to prefer a higher level of enforcement even though labor income decreases. Still, it is possible that this improvement in tax enforcement is accompanied by a reduction in capital stock, which in turn implies a reduction in the size of the modern sector. As mentioned above, a reduction in long run capital stock has implications for the size of the modern sector and the economy in general but does not affect relative tax evasion in the traditional sector.

In the strategic equilibrium, an increase in A_M primarily benefits present and future capitalists and to a lesser extent, workers. Workers do not have sufficient incentives to change their policy preferences in any notable way due to the fact that expectations for future income have increased. We recall that the distortion is designed to provide an implicit contract be-

[36] In Appendix F we shows that $k^{ss}_{A_M} > 0$.

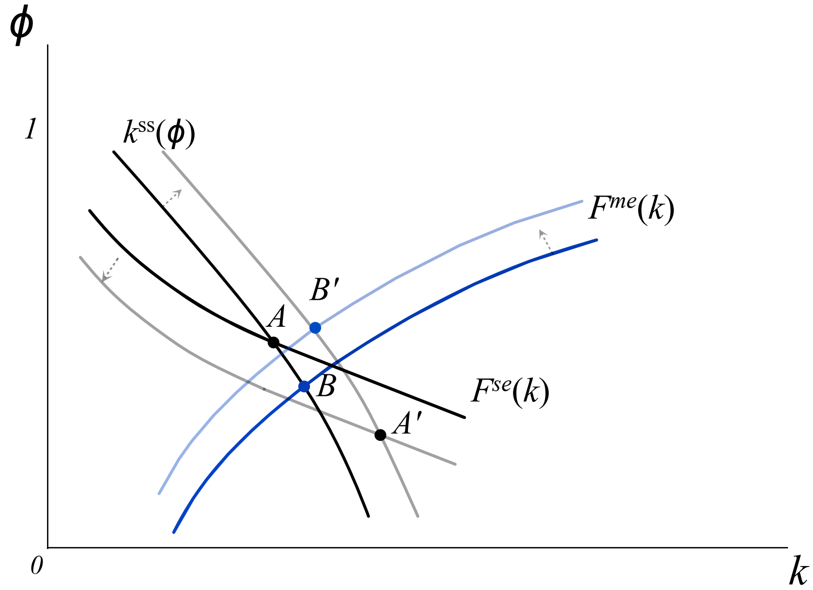

Figure 6: Effects of an Increase of A_M

tween generations. For this reason, workers choose a policy for tax enforcement that is higher than in the myopic case but lower than the case without an increase in A_M. That is, given an increased capital return, the policy function demands a lower ϕ. In plane (k, ϕ), the policy function F^{se} shifts down and as already mentioned, the steady-state level of capital improves due to an increase in modern technology, which in turn is due to increased transfers. Figure 6 shows that tax enforcement should fall and k should rise in the long term (see Point A'). This result may be counterintuitive but remember that an increase in productivity necessarily implies a greater productivity gap between sectors where capitalists benefit more than workers from this increase.Workers partially make up for this difference by chosing a lower level of tax enforcement (which leaves more post-tax income). Additionally, the modern sector increases in size due to the higher levels of long run capital. The endogenization of the policy generates results that compensate for (restrict) averse (favorable) circumstances for the decisive voter. In this case, productivity increases allow the worker to slightly distort his preferred level of tax enforcement.

5 Labor Productivity Inequality and Tax Enforcement

We now explore the effects of intragenerational inequality on the politico-economic equilibrium. Instead of the multiple equilibria presented in the previous case, here we have just one. Additionally, we find that the decisive voter is the worker with the lowest level of labor productivity in society and the policy function is decreasing in k, namely, there is a trade-off between the minimum productivity level and capital stock in steady state.

We assume that agents differ in effective labor hours or labor productivity. Each worker provides a quantity of labor, θ^i. These effective hours are distributed on the support $[\theta^{min}, \theta^{max}] \subset \mathbb{R}_+$. The distribution of effective hours is assumed to have mean θ.

Now worker income is redefined as

$$I^i(\phi_t, k_t) = \Psi(\phi_t)k_t + A_T\theta^i - c(\phi)A_T\theta + \tau^e(\phi)(\theta - \theta^i), \tag{25}$$

where the function $\Psi(\phi_t)$ is defined in Lemma 2. However, intragenerational heterogeneity is a new source of redistribution of worker income. This can be seen in the last term of worker income. A worker with low productivity (or productivity less than θ) will benefit from higher tax enforcement and, as suggested by Meltzer and Richard, (1981) more unequal income distributions induce larger redistribution policies. This worker also receives labor income and must bear the administrative costs of the tax collection.

In contrast, the capital stock per worker evolves according to the same function $H(\phi_t, k_t)$, as defined in Corollary 1. For this reason, the indirect utility functions of the capitalist and of the worker are modified only by replacing $I(\phi, k)$ by $I^i(\phi, k)$.

5.1 Strategic Equilibrium

Before proceeding to calculate the policy function we must first identify a decisive voter. As mentioned above, the only way to manipulate the vote in the next period is through the state variable. Therefore, the next step is to show that the median in the distribution of most-preferred tax-enforcement levels can be associated with a particular voter regardless of the value of the state variable.

Capitalists choose $\phi = 1$ regardless of the value of k and they have no incentive to manipulate future voting because they will not be part of the economy. Given that the rate of population growth is assumed to be positive, the capitalists group is smaller than the workers group. Therefore the decisive voter will be the worker that prefers the highest value of ϕ for all k.

Assumption 3. *The ranking of most-preferred tax-enforcement levels of workers is invariant with respect to the level of state variable.*

This monotonicity property implies that, in each election, the decisive worker is identified by the same level of productivity. Given this, the recursive political equilibrium is defined as follows

Definition 3. *A Politico-economic equilibrium is defined as a policy function $G : \mathbb{R}_+ \rightarrow [0,1]$ such that the following functional equation hold $G(k) = \arg\max_\phi V^W(\phi, \phi', k; \theta^{pv})$ subject to $\phi' = G(k')$ where $k' = H(\phi, k)$ and $V^W(\phi, \phi', k; \theta^{pv})$ is the indirect utility of the current decisive worker who is identified by his labor productivity θ^{pv}.*

The first-order condition for the recursive problem described in Definition 3 is

$$\left\{ \frac{(1+\beta)^2}{\beta I^{pv}(\phi,k)} + \frac{\beta}{(1+\beta)^2}\epsilon(\phi^{se-a\prime})G_k(k') \right\} H_\phi(\phi^{se-a}, k) - \frac{1+\beta}{I^{pv}(\phi,k)}\tau^e_\phi(\phi^{se-a\prime})A_T(\theta^{pv} - \theta) = 0 \tag{26}$$

where $\epsilon(\phi^{se-a\prime}) \equiv \frac{(1-\tau^k)r_\phi(\phi^{se-a\prime})}{1+(1-\tau^k)r(\phi^{se-a\prime})}$.

It is straightforward to see that the first-order condition collapses to the case where labor productivity is equal for all agents, $\theta^i = \theta$, for all i, see Proposition 1, Corollary 1 and equation (20).

Now we prove that the worker with the lowest level of labor productivity is the decisive voter.

Proposition 5. *If the rate of population growth is small enough such that the new generation of workers is greater than the previous generation by one worker there exist an equilibrium where $\theta^{pv} = \theta^{min}$ for all k, and the associated policy function $G(k)$ is decreasing.*

Proof. Since workers are the majority, the decisive voter (with preference for the median value of preferred tax enforcement) will be the worker who prefers the highest ϕ. Assumption (3) holds, this worker is identified only by their labor productivity regardless of value of the state variable, and the policy function, $G(k)$, calculated from condition (26) has a negative slope with respect to k and the policy choice of ϕ depends negatively on the level of labor productivity for all values of k (see Appendix E).

Since the policy function, $G(k)$, is decreasing in labor productivity, θ^{pv}, the worker with labor productivity equal to θ^{min} is the decisive voter. $\square$

Since workers are in the majority and all capitalists prefer total tax enforcement, the median value of ϕ preferred in each period corresponds to the highest preferred ϕ by workers. In Proposition 5, we show that the policy function is decreasing in labor productivity (or effective labor hours of the decisive worker) thus it is straightforward to deduce that a low-productivity worker will choose higher tax enforcement.

Corollary 2. *An increasing the growth rate of the population reduces tax enforcement chosen.*

If the rate of population growth is such that the current generation of workers is greater than the previous generation of workers by more than one, the decisive worker has lower preference for tax enforcement than the lowest-productivity worker.

Corollary 3. *The policy function $G(k)$ is decreasing in θ^{pv}.*

Proof. We are interested in the effect of an increase in the term θ^{pv} on $G(k)$. In Appendix E we demonstrate that $G_{\theta^i} < 0$, for all θ^i and, in particular, for θ^{pv}. $\square$

In the political spectrum, high and low productivity capitalists support the political platform of low productivity workers. Implicitly, all capitalists and relatively poor workers make up a political group. In future work, we plan to investigate whether inequality may be a result of some political mechanism in a model with tax enforcement and two-sector economy. Suppose the public policy positively affects θ^{min} but maintains the average value θ. In this case, the policy function indicates that lower tax enforcement should prevail because increased income for the decisive worker allows him to further distort current tax enforcement via strategic voting. Furthermore, capital stock only depends on mean value of productivity therefore the k^{ss} curve is the same. In this case, we have less tax enforcement but more capital stock. This result can be seen as an institutional trap. Improving the productivity of the poor helps increase the capital stock of the economy but worsens the effective collection of taxes on labor. Finally, intragenerational heterogeneity allows us to have unique Markov perfect equilibrium which may also be considered strategic equilibrium.

5.2 Comparison of myopic and strategic equilibria

Since the worker with the lowest labor productivity is the decisive voter, we can deduce that if the policy function satisfies the following restriction

$$\frac{(1+\beta)^4}{\beta^2 I^{pv}(\phi,k)\epsilon(\phi^{se-a\prime})} > -G_k(k') \tag{27}$$

then choice of tax enforcement will always be greater than the value of the myopic equilibrium without heterogeneity, i.e. $\phi^{se-a} > \phi^{me}$, for all t. Recall that in the model without heterogeneity the relationship between labor income and savings is linear, which implies that marginal revenue and marginal savings also have a linear relationship. More specifically $\mathrm{I}_\phi = \frac{1+\beta}{\beta} H_\phi$, given equation (26) and using restriction (27) we can deduce that $\mathrm{I}_\phi(\phi^{se-a}, k) < 0$ (remember that $\mathrm{I}_\phi(\phi^{me}, k) = 0$). Therefore, the ϕ^{se-a} will be greater than ϕ^{me} because of the marginal income is decreasing.

6 Conclusions

The literature of positive economy on tax enforcement levels is minimal. On one hand, the research of Borck (2009), Roine (2006) and Traxler (2009, 2012) evaluate the impact of tax enforcement level on the political economy equilibrium level of direct taxes, and even if enforcement is exogenous, these studies do show a relationship between enforcement and the tax rate. On the other hand, our understanding is that the only paper that treats tax enforcement as a policy choice (chosen by majority vote) is Besfamille et al. (2013) but their paper focuses on studying the indirect taxation. Therefore our study is the first political economy model to treat the enforcement level of direct taxation as endogenous.

The existence of a traditional sector, defined as a labor-intensive sector with less than 100 percent tax enforcement, can generate mixed results in terms of the impact of a change in enforcement policy on capital and labor income. This model shows that labor income may be negatively related to tax enforcement if the traditional sector is labor-intensive and if there is a perfect mobility of labor between two sectors. On the other hand, capital income (necessarily from the modern sector) grows in response to greater tax enforcement due to substitution between labor and capital. The mechanism is the following: an effective tax rate increase (necessarily in the traditional sector) causes labor to reallocate to the modern sector by raising the marginal productivity of capital. The model allows agents to choose their preferred level of ϕ and as expected, workers prefer a low level of tax enforcement while capitalists prefer the opposite. The resulting political conflict regarding the enforcement of taxation may yield an interesting trade-off between tax enforcement and capital accumulation.

We analytically characterize two Markov perfect equilibria: one myopic and another strategic. In the myopic equilibrium we derive a choice rule that is increasing in capital stock (if capital stock increases, decisive voters prefer more tax enforcement). The strategic equilibrium shows an implicit agreement among decisive voters, where the differentiable policy function is decreasing in capital stock in order to make the threat of the decisive voter credible. Therefore, for low values of capital stock, the path of tax enforcement defined by myopic equilibrium will be less than that which is defined by the strategic equilibrium. In the long term, we demonstrate that the chosen policy (level of tax enforcement) with strategic voting will be greater than the chosen policy (level of tax enforcement) in the myopic equilibrium. When we extend the model to include unequally distributed labor productivity, the Markov perfect equilibrium is unique and strategic and the decisive voter is the worker with the lowest labor productivity who demands a higher level of tax enforcement in the long term. Therefore, a political coalition forms between capitalists and the low-productivity worker. Improving the productivity of the least productive worker helps increase the capital stock of the economy but can worsen the effective collection of taxes on labor. We have identified some parameters affecting the dynamic of tax enforcement and we leave the task of confronting the model's predictions to data to future research. For example, a reduction in taxes on capital negatively affects tax enforcement in the myopic equilibrium but has ambiguous results in the strategic equilibrium. In the myopic equilibrium, this result is due to the fact that the income-maximizing worker faces negative marginal income with respect to tax enforcement and therefore chooses to reduce enforcement. In the strategic equilibrium, the worker can change his original policy choice and reduce the threat to future generations, given the higher levels of expected income in the next period. Nonetheless, the reduction in tax collection (for transfers) may result in lower capital accumulation and therefore less future income for workers, whom should be enticed to prefer a lower level of enforcement.

Tax evasion can be explained among other things by the level of enforcement applied to each source of tax revenue. Our main conclusion is that the diversity of levels of tax evasion may be explained by the conflict of interest between voters or social groups by the enforcement policy.

Moreover, incomplete tax enforcement can be sustained over time.

There are several natural extensions of the model. For example, related public policies such as backward-looking spending or social security fit within the general framework of the model but would require some minor changes. In this case, retirees would maintain their preference for complete tax enforcement independent of state variables because complete tax enforcement improve the profitability of savings as well as the overall pension system. Younger generations, on the other hand, benefit from lower tax enforcement in the current period (also independent of state variables) and prefer complete tax enforcement upon retirement. The independence of these preference with respect to state variables should probably motivate a replacement of the markov-perfect equilibrium concept with a stationary subgame perfect equilibrium. Another possible extension would be to study how the existence of relative prices in each sector might affect the public policy outcome. The simple model presented here may help us understand the numerical solution of a model with relative prices added. Finally, it would be interesting to consider changes in the way the model aggregates social preferences by either using probabilistic voting or by completely endoginizing fiscal policy (meaning tax enforcement as well as the tax rate are chosen via election).

7 References

Aiyagari, S., 1995. Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting, *Journal of Political Economy, 103(6), 1158-1175.*

Allingham, M. and A. Sandmo, 1972. Income Tax Evasion: A Theoretical Analysis, *Journal of Public Economics, 1, 323-338.*

Alm, J., 2012. Measuring, explaining, and controlling tax evasion: lessons from theory, experiments, and field studies, *International Tax and Public Finance, 19, 54-77.*

Andreoni, J., Erard, B. and J. Feinstein, 1998. Tax Compliance, *Journal of Economic Literature, 36, 818-860.*

Besley, T. and T. Persson, 2013. Taxation and Development, in: A.J. Auerbach, R. Chetty, M. Feldstein and E. Saez, eds., Handbook of Public Economics, Volume 5, Elsevier, 51-110.

Besfamille, M., De Donder, P. and J.-M. Lozachmeur, 2013. The Political Economy of the (Weak) Enforcement of Indirect Taxes. *Journal of Public Economic Theory, 15(6), 856-883.*

Boldrin, M. and A. Montes, 2005. The Intergenerational State Education and Pensions.*Review of Economic Studies, 72, 651-664.*

Border, K. and J. Sobel, 1987. Samourai Accountant: A Theory of Auditing and Plunder. *Review of Economics Studies, 54, 525-540.*

Borck, R. 2009. Voting on Redistribution with Tax Evasion. *Social Choice and Welfare, 32, 439-452.*

Cebula, R., 1997. An Empirical Analysis of the Impact of Government Tax and Auditing Policies on the Size of the Underground Economy: The Case of the United States, 1973-94, *American Journal of Economics and Sociology, 56 (2), 173-185.*

Chander, P, and L. Wilde, 1998. A General Characterization of Optimal Income Tax Enforcement. *Review of Economic Studies, 65, 165-183.*

Collard, D., 2000. Generational Transfers and The Generational Bargain. *Journal of International Development, 12, 453-462.*

Conesa, J., Kitao, S. and D. Krueger, 2009. Taxing Capital? Not a Bad Idea After All!, *American Economic Review, 99 (1), 25 - 48.*

Cuff, K., Marceau, N., Mongrain, S. and J. Roberts, 2011. Optimal Policies with an Informal Sector, *Journal of Public Economics, 95, 1280-1291.*

Dabla-Norris, E., M. Gradstein, and G. Inchauste, 2008. What Causes Firms to Hide Output? The Determinants of Informality. *Journal of Development Economics, 85, 1-27.*

de Paula, A. and J. Scheinkman, 2011. The Informal Sector: An Equilibrium Model and Some Empirical Evidence from Brazil. *The Review of Income and Wealth, 57, S8-S26.*

Fortin, B., N. Marceau, L. Savard, 1997. Taxation, Wage Controls and the Informal Sector. *Journal of Public Economics, 66, 293-312.*

García-Peñalosa and Turnosvky, 2005. Second-best Optimal Taxation of Capital and Labor in a Developing Economy. *Journal of Public Economics, 89, 1045-1074.*

Gordon, R. and Y. Lee, 2005. Tax Structure and Economic Growth, *Journal of Public Economics, 89, 10271043.*

Gordon, R. and W. Li, 2009. Tax Structures in Developing Countries: Many Puzzles and a Possible Explanation. *Journal of Public Economics, 93, 855 - 866.*

Grossman, G. and E. Helpman, 1998. Intergenerational Redistribution with Shrot-Lived Governments, *Economic Journal, 108, 1299-1329.*

Hassler, J., Rodríguez Mora, J., Storesletten, K. and F. Zilibotti, 2003. The Survival of the Welfare State, *American Economic Review, 93 (1), 87-112.*

Hassler, J., Krusell, P., Storesletten, K. and F. Zilibotti, 2005. The dynamics of government, *Journal of Monetary Economics, 52, 1331-1358.*

Hassler, J., Storesletten, K. and F. Zilibotti, 2007. Democratic public good provision, *Journal of Economic Theory, 133, 127-151.*

Ihrig, J. and K. Moe, 2004. Lurking in the Shadows: The Informal Sector and Government Policy. *Journal of Development Economics, 73, 541-557.*

Kesselman, J., 1989. Income Tax Evasion: An Intersectoral Analysis, *Journal of Public Economics, 38, 137-182.*

Kleven, H., Knudsen, M., Kreiner, C. Pedersen, S. and E. Saez, 2011. Unwilling or unable to cheat? Evidence from a tax audit experiment in Denmark. *Econometrica, 79, 651-692.*

Kotlikoff, L., 2002. Generational Policy, in: A. J. Auerbach and M. Feldstein, eds., Handbook of Public Economics, Volume 4, Elsevier, 1873-1932.

Kotlikoff, L. and R. Rosenthal, 1993. Some Inefficiency Implications of Generational Politics and Exchange. *Economics and Politics, 5, 27-42.*

Leal, J., 2014. Tax collection, the informal sector, and productivity, *Review of Economic*

Dynamics, 17, 262-286.

Meltzer, A. and S. Richard, 1981. A Rational Theory of the Size of Government. *Journal of Political Economy, 89, 914-027.*

Mirrlees, J. A., 1971. An Exploration in the Theory of Optimum Income Taxation. *The Review of Economic Studies, 38, 175-208.*

Myles, G., Hashimzade, N. and B. Tran-Nam, 2010. New Approaches to the Economics of Tax Evasion, *Exeter, United Kingdom: University of Exeter. Available at: http://people. exeter. ac. uk/gdmyles/papers/pdfs/Non-euEva. pdf*

Phillips, M., 2014. Deterrence vs Gamesmanship: Taxpayer response to targeted audits and endogenous penalties, *Journal of Economic Behavior and Organization, 100, 81-98.*

Rauch, J., 1991. Modelling the Informal Sector Formally. *Journal of Development Economics, 35 33-47.*

Rangel, A., 2003. Forward and Backward Intergenerational Goods: Why Is Social Security Good for the Environment? *The American Economic Review, 93, 813-834.*

Reinganum, J. and L. Wilde, 1985. Income Tax Compliance in a Principal-Agent Framework. *Journal of Public Economics, 26, 1-18.*

Roine, J., 2006. The Political Economy of not Paying Taxes. *Public Choice, 126, 107-134.*

Sandmo, A., 1981. Income Tax Evasion, Labour Supply and the Equity-Efficiency Trade-off. *Journal of Public Economics, 16, 265-288.*

Sandmo, A., 2012. An evasive topic: theorizing about the hidden economy, *International Tax and Public Finance, 19, 5-24.*

Slemrod, J., 2007. Cheating Ourselves: The Economics of Tax Evasion. *Journal of Economic Perspectives, 21, 25-48.*

Slemrod, J. and S. Yitzhaki, 2002. Tax Avoidance, Evasion, and Administration, in: A.J. Auerbach and M. Feldstein, eds., Handbook in Public Economics, Volume 3, Elsevier Science, 1423-1470.

Traxler, C., 2009. Voting over Taxes: the Case of Tax Evasion. *Public Choice, 140, 43-58.*

Traxler, C., 2012. Majority Voting and the Welfare Implications of Tax Avoidance. *Journal of Public Economics, 96, 1-9.*

Turnosvky, S. and A. Basher, 2009. Fiscal Policy and the Structure of Production in a Two-Sector Developing Economy. *Journal of Development Economics, 88, 205-216.*

Appendix

A: Economic Equilibrium with Diminishing Marginal Returns to Labor In the Traditional Sector

Let $Y_T = A_T L_T^{\beta}$, $\beta \in (0,1)$, be the production function of the traditional sector. In a competitive environment, the payment per hour of effective labor is equal to the respective marginal productivity

$$w_T = \beta A_T L_T^{\beta-1}$$

The agents face the following maximization problem

$$\max_{c_t, c_{t+1}, l_T, l_M} \ln c_t + \beta \ln c_{t+1} \quad s.t.$$

$$c_t + s_t = (1-\tau^l) w_M l_M + (1-\tau^e) A_T l_T^{\beta} + g_t$$

$$c_{t+1} = (1 + (1-\tau^k) r_{t+1}) s_t$$

Since the traditional production function does not have constant returns to scale, firm profits are distributed equally among workers. Therefore, each agent's revenue is comprised of salaries and benefit redistribution, i.e. $A_T l_T^{\beta}$.

Given that labor is perfectly mobile between sectors, both sectoral productivities must be equal

$$(1-\tau^l)(1-\alpha) A_M K_M^{\alpha} L_M^{-\alpha} = (1-\tau^e) \beta A_T L_T^{\beta-1}.$$

We know that $L_T + L_M = \theta N$, then

$$\frac{L_M^{\alpha}}{(\theta N - L_M)^{1-\beta}} = \frac{(1-\tau^l)(1-\alpha) A_M K_M^{\alpha}}{(1-\tau^e) \beta A_T}$$

We can check the respective labor demands the following derivatives.

$$L_M = L_M(\overset{+}{\tau^e}, \overset{-}{A_T}, \overset{-}{\beta}, \overset{-}{\alpha}, \overset{+}{A_M}, \overset{+}{K_M}),$$

where the symbols above variables indicate the sign of the respective partial derivatives. Demand for modern labor is an increasing function of K_M, as shown by the increase in marginal productivity of labor relative to marginal productivity in the traditional sector. However, this response is not linear unlike the case when traditional production is linear in labor, see equation (11).

Now we replace the function L_M in the marginal productivity of capital (1) and we get

$$r = \alpha A_M K_M^{\alpha-1}\left(L_M(\tau^e, A_T, \beta, \alpha, A_M, K_M)\right)^{1-\alpha} = r(\overset{+}{\tau^e}, \overset{-}{A_T}, \overset{-}{\beta}, \alpha, \overset{+}{A_M}, \overset{+}{K_M})$$

We can see that the partial derivatives in the interest rate coincide with those obtained in the equation (12).

B: Structural Form of Effective Tax Rate

Assuming that consumers are risk neutral, w is the gross taxpayer income, $e \in [0,1]$ is the fraction of income that is not reported to the tax authorities and τ is the tax rate. If tax evasion is not detected then the net taxpayer income is $[1-(1-e)\tau]w$. If tax evasion is detected then the taxpayer has to pay the tax τw and a fine, $\delta \in [0,1]$, which is a fraction of the amount of taxes evaded. In this case, net taxpayer income is $[1-(1+\delta e)\tau]w$. We assume that underreporting entails a cost of $c(e)$ per unit received. Assume that the function c is increasing and convex. Moreover, the tax authority audits each taxpayer with probability $p \in [0,1]$. Let V be the expected utility of taxpayer:

$$V = pU\left([1-(1+\delta e)\tau - c(e)]w\right) + (1-p)U\left([1-(1-e)\tau - c(e)]w\right)$$

If taxpayers are risk neutral, then U is linear and the first condition order is:

$$(1-p(1+\delta))\tau - c_e(e^*) = 0$$

If $c(e) = Be^\eta$ with $\eta > 1$ then $e^* = \left(\frac{1}{\eta B}(1-p(1+\delta))\tau\right)^{\frac{1}{\eta-1}}$. Replace e^* in V and get:

$$V^* = \left[1-\tau+\left(\frac{1}{\eta B}\right)^{\frac{1}{\eta-1}}\left[(1-p(1+\delta))\tau\right]^{\frac{\eta}{\eta-1}}\left(\frac{\eta-1}{\eta}\right)\right]w$$

Now we define the effective tax rate as $\tau^e(p) \equiv \tau - \left(\frac{1}{\eta B}\right)^{\frac{1}{\eta-1}}\left[(1-p(1+\delta))\tau\right]^{\frac{\eta}{\eta-1}}\left(\frac{\eta-1}{\eta}\right)$. The effective tax rate is strictly increasing and concave with respect to p.

$$\tau_p^e = \left(\frac{1}{\eta B}\right)^{\frac{1}{\eta-1}}\left[(1-p(1+\delta))\tau\right]^{\frac{1}{\eta-1}}(1+\delta)\tau > 0,\ \textit{and,}$$

$$\tau_{pp}^e = -\left(\frac{\eta-1}{\eta}\right)\left(\frac{1}{\eta B}\right)^{\frac{1}{\eta-1}}\left[(1-p(1+\delta))\tau\right]^{\frac{2-\eta}{\eta-1}}(1+\delta)^2\tau^2 < 0.$$

C: Sufficient Condition for Concavity of $\mathbf{r}(\phi_\mathbf{t})$

The second derivative of $r(\phi_t)$ is:

$$r_{\phi\phi} = r(\phi_t)\frac{(1-\alpha)}{\alpha}\frac{1}{1-\tau^e(\phi_t)}\left[\frac{1}{\alpha(1-\tau^e(\phi_t))}(\tau^e_\phi)^2 + \tau^e_{\phi\phi}\right]$$

If $\frac{1}{\alpha(1-\tau^e(\phi_t))}(\tau^e_\phi)^2 + \tau^e_{\phi\phi} < 0$, $r_{\phi\phi} < 0$.

D: Sufficient Condition for Ψ is Concave

The first derivative of $\Psi(\phi)$ is

$\Psi_\phi = \tau^k r_\phi + \frac{\tau^l}{1-\tau^l}A_T\left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T}\right]^{\frac{1}{\alpha}}\frac{(1-\alpha)}{\alpha}\frac{\tau^e_\phi}{(1-\tau^e)^{\frac{1}{\alpha}}} - A_T\left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T}\right]^{\frac{1}{\alpha}}\left\{\frac{(1-c_{\tau^e})\tau^e_\phi}{(1-\tau^e)^{\frac{1}{\alpha}}} + \frac{(\tau^e-c(\tau^e))\tau^e_\phi}{(1-\tau^e)^{\frac{1+\alpha}{\alpha}}\alpha}\right\}$

where $r_\phi = r(\phi_t)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{1-\tau^e}\right)\tau^e_\phi$.

The second derivative is

$$\Psi_{\phi\phi} = \tau^k r_{\phi\phi} + \frac{\tau^l}{1-\tau^l}A_T\left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T}\right]^{\frac{1}{\alpha}}\left\{\frac{\tau^e_{\phi\phi}}{(1-\tau^e)^{\frac{1}{\alpha}}} + \frac{(\tau^e_\phi)^2}{\alpha(1-\tau^e)^{\frac{(1+\alpha)}{\alpha}}}\right\} -$$

$$A_T\left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T}\right]^{\frac{1}{\alpha}}\left\{\left[\frac{(1-c_{\tau^e})}{(1-\tau^e)^{\frac{1}{\alpha}}} + \frac{(\tau^e-c(\tau^e))}{(1-\tau^e)^{\frac{1+\alpha}{\alpha}}\alpha}\right]\tau^e_{\phi\phi} - \frac{c_{\tau\tau}(\tau^e_\phi)^2}{(1-\tau^e)^{\frac{1}{\alpha}}} + \frac{2(1-c_{\tau^e})}{(1-\tau^e)^{\frac{1+\alpha}{\alpha}}}\frac{(\tau^e_\phi)^2}{\alpha} + \frac{(\tau^e-c(\tau^e))}{(1-\tau^e)^{\frac{1+2\alpha}{\alpha}}}\frac{(\tau^e_\phi)^2(1+\alpha)}{\alpha^2}\right\}$$

where $r_{\phi\phi} = \alpha A_M\left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T}\right]^{\frac{1-\alpha}{\alpha}}\left\{\frac{\tau^e_{\phi\phi}}{(1-\tau^e)^{\frac{1}{\alpha}}} + \frac{(\tau^e_\phi)^2}{\alpha(1-\tau^e)^{\frac{(1+\alpha)}{\alpha}}}\right\}$

The term $\left\{\tau^e_{\phi\phi} + \frac{(\tau^e_\phi)^2}{\alpha(1-\tau^e)}\right\}$ is negative by the assumption 1. For this reason, we need analyze the third term of $\Psi_{\phi\phi}$.

The third term can be written as

$$\left(\frac{(1-c_{\tau^e})}{(1-\tau^e)^{\frac{1}{\alpha}}} + \frac{(\tau^e-c(\tau^e))}{(1-\tau^e)^{\frac{1+\alpha}{\alpha}}\alpha}\right)\left(\tau^e_{\phi\phi} + \frac{(\tau^e_\phi)^2}{\alpha(1-\tau^e)}\right) + ...$$

$$\frac{(\tau^e_\phi)^2}{(1-\tau^e)^{\frac{1+2\alpha}{\alpha}}\alpha}\left[1 - c(\tau^e)[1+\rho(\alpha(\rho-1)(1-\frac{1}{\tau^e}) - 1)(1-\frac{1}{\tau^e})]\right]$$

We obtain

$\Psi_{\phi\phi} = A_T\left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T}\right]^{\frac{1}{\alpha}}\frac{1}{(1-\tau^e)^{\frac{1}{\alpha}}}\left\{(1-\tau^e)^{\frac{1}{\alpha}} - (1-c_{\tau^e}) + \frac{1}{\alpha(1-\tau^l)}\left(\tau^l(1-\tau^e)^{\frac{1}{\alpha}} - (\tau^e - c(\tau^e))\right)\right\} -$

$A_T\left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T}\right]^{\frac{1}{\alpha}}\frac{(\tau^e_\phi)^2}{(1-\tau^e)^{\frac{1+2\alpha}{\alpha}}\alpha}\left[1 - c(\tau^e)[1+\rho(\alpha(\rho-1)(1-\frac{1}{\tau^e}) - 1)(1-\frac{1}{\tau^e})]\right]$

Ψ is concave if $[1+\rho(\alpha(\rho-1)(1-\frac{1}{\tau^e}) - 1)(1-\frac{1}{\tau^e})] < c(\tau^e)^{-1}$.

E: Proof of Proposition 2

If $\mathrm{I}_\phi(\phi_t, k_t) = 0$ and the law of motion is $H(\phi, k)$, the first-order condition (20) can be expressed as

$$F_k(k_{t+1}) \equiv \frac{d\phi_{t+1}}{dk_{t+1}} = \frac{-(1+\beta)}{\beta k_{t+1}} \frac{(1+(1-\tau^k)r(\phi_{t+1}))}{(1-\tau^k)r_\phi}$$

If we changed variable $E \equiv 1 + (1-\tau^k)r(\phi_{t+1})$ then $\frac{dE}{d\phi_{t+1}} = (1-\tau^k)r_\phi$. Integrating both sides we obtain

$$\ln E = -\frac{1+\beta}{\beta} \ln k_{t+1} + F$$

where F is constant of integration. Given logaritmic equation and replacing E we obtain

$$(1-\tau^k)r(\phi_{t+1}) = \exp(F) k_{t+1}^{-\frac{1+\beta}{\beta}} - 1$$

Given $r(\phi)$ we obtain the policy function

$$\phi(k) = \left(\frac{1}{\tau^l} \left(1 - \left[\frac{Ck^{-\frac{(1+\beta)}{\beta}} - 1}{(1-\tau^k)D} \right]^{\frac{-\alpha}{1-\alpha}} \right) \right)^{\frac{1}{\lambda}},$$

where $D \equiv \alpha A_M \left[\frac{(1-\alpha)A_M(1-\tau^l)}{A_T} \right]^{\frac{(1-\alpha)}{\alpha}}$.

If $\phi_{t+1} = 0$ we define $k_{min} = \left[\frac{1}{C} \left(1 + (1-\tau^l)^{\frac{-(1-\alpha)}{\alpha}} (1-\tau^k)D \right) \right]^{\frac{-\beta}{1+\beta}}$ and if $\phi_{t+1} = 1$ then $k^{max} = \left[\frac{1}{C} \left(1 + (1-\tau^k)D \right) \right]^{\frac{-\beta}{1+\beta}}$.

F: Partial Derivatives of Policy Function F^{se}

Given the policy function $F^{se}(k)$ defined in Proposition 3, we calculate the following partial derivatives

$$F^{se}_{\tau^k} = \frac{1}{\lambda} \left(\frac{1}{\tau^l} \right)^{\frac{1}{\lambda}} \left(1 - \left[\frac{Ck^{-\left(\frac{1+\beta}{\beta}\right)} - 1}{(1-\tau^k)D} \right]^{\frac{-\alpha}{1-\alpha}} \right)^{\frac{1-\lambda}{\lambda}} \left[Ck^{-\left(\frac{1+\beta}{\beta}\right)} - 1 \right]^{\frac{-\alpha}{1-\alpha}} \frac{D^{\frac{\alpha}{1-\alpha}}}{(1-\tau^k)^{\frac{1}{1-\alpha}}} \frac{\alpha}{1-\alpha} > 0$$

$$F^{se}_{A_M} = -\frac{1}{\lambda} \left(\frac{1}{\tau^l} \right)^{\frac{1}{\lambda}} \left(1 - \left[\frac{Ck^{-\left(\frac{1+\beta}{\beta}\right)} - 1}{(1-\tau^k)D} \right]^{\frac{-\alpha}{1-\alpha}} \right)^{\frac{1-\lambda}{\lambda}} \left[Ck^{-\left(\frac{1+\beta}{\beta}\right)} - 1 \right]^{\frac{-\alpha}{1-\alpha}} \frac{(1-\tau^k)^{\frac{\alpha}{1-\alpha}}}{D^{\frac{1}{1-\alpha}}} \frac{\alpha}{1-\alpha} D_{A_M} < 0$$

where $D_{A_M} = \left(\frac{(1-\alpha)A_M(1-\tau^l)}{A_T} \right)^{\frac{1-\alpha}{\alpha}}$.

On the other hand, the accumulation equation is

$$k^{ss}(\phi^{ss}) = \frac{\frac{\beta}{1+\beta}(1 - c(\tau^e(\phi^{ss})))A_T\theta}{1 - \frac{\beta}{1+\beta}\Psi(\phi^{ss})}$$

We calculate partial derivative of $\Psi(\phi^{ss})$

$$\Psi_{A_M} = \tau^k r_{A_M} + A_T \left[\frac{(1-\alpha)(1-\tau^l)}{A_T(1-\tau^e(\phi^{ss}))} \right]^{\frac{1}{\alpha}} \left\{ \frac{\tau^l(1-\tau^e)}{1-\tau^l} - (\tau^e - c(\tau^e)) \right\} A_M^{\frac{(1-\alpha)}{\alpha}}$$

Given that $r_{A_M} = A_M^{\frac{1-\alpha}{\alpha}} \left[\frac{(1-\alpha)(1-\tau^l)}{A_T(1-\tau^e(\phi^{ss}))} \right]^{\frac{1-\alpha}{\alpha}} > 0$ and $\frac{\tau^l(1-\tau^e)}{1-\tau^l} - (\tau^e - c(\tau^e)) > 0$ then $\Psi_{A_M} > 0$. Therefore

$$k^{ss}_{A_M} = \frac{\frac{\beta}{1+\beta}(1 - c(\tau^e(\phi^{ss})))A_T\theta}{(1 - \frac{\beta}{1+\beta}\Psi(\phi^{ss}))^2} \frac{\beta}{1+\beta}\Psi_{A_M} > 0$$

Moreover, we calculate the following partial derivative

$$k^{ss}_{\tau^k} = \frac{\frac{\beta}{1+\beta}(1 - c(\tau^e(\phi^{ss})))A_T\theta}{(1 - \frac{\beta}{1+\beta}\Psi(\phi^{ss}))^2} \frac{\beta}{1+\beta} r(\phi^{ss}) > 0$$

G: Proof of Corollary 2

We write the first-order condition as

$$\frac{(1+\beta)I^{pv}_{\phi}(\phi,k)}{I^{pv}(\phi,k)} + \beta \frac{(1-\tau^k)r_\phi(\phi')}{1+(1-\tau^k)r(\phi')} \frac{d\phi'}{dk'} \frac{dk'}{d\phi} = 0$$

where $\frac{dk'}{d\phi} \equiv H_\phi(\phi, k)$ and $\frac{d\phi'}{dk'} \equiv G_k(k')$.

Then

$$\beta \frac{(1-\tau^k)r_\phi(\phi')}{1+(1-\tau^k)r(\phi')} d\phi' = -\frac{(1+\beta)I^{pv}_{\phi}(\phi,k)}{I^{pv}(\phi,k)} d\phi$$

We change variables. Let $E \equiv 1 + (1-\tau^k)r(\phi')$ and $M \equiv I^{pv}(\phi, k)$ then $\frac{dE}{d\phi'} = (1-\tau^k)r_\phi(\phi')$ and $\frac{dM}{d\phi} = I^{min}_\phi(\phi, k)$. Integrating both sides and we obtain

$$\beta \ln(1 + (1-\tau^k)r(\phi')) = -(1+\beta)\ln\left(\frac{(1+\beta)}{\beta}k' + (1-\tau^e(\phi))A_T(\theta^{pv} - \theta)\right) + \mathbb{C}$$

where $\mathbb{C}$ is the constant of integration. However, the latter equation is a functional equation, our goal is to find the function G that satisfies this condition. Furthermore, we know that the ϕ is also determined by the function G. We analize this condition in steady state, i.e., $k' = k = k^{ss}$. Therefore, $\phi^{ss} = G(k^{ss})$ then the functional equation is

$$\beta \ln(1 + (1-\tau^k)r(\phi^{ss})) = -(1+\beta)\ln\left(\frac{(1+\beta)}{\beta}k^{ss} + (1-\tau^e(\phi^{ss}))A_T(\theta^{pv} - \theta)\right) + \mathbb{C}$$

Differentiating around of ϕ^{ss} and k^{ss}, we obtain

$$\frac{d\phi^{ss}}{dk^{ss}} = \frac{-\left(\frac{(1+\beta)}{\beta}\right)^2}{I^{pv}\frac{(1-\tau^k)r_\phi}{1+(1-\tau^k)r(\phi^{ss})} - \frac{1+\beta}{\beta}A_T\tau^e_\phi(\theta^{pv}-\theta)} < 0$$

This slope is negative because $\theta^{pv} < \theta$.

Differentiating around of ϕ^{ss} and θ^{pv}

$$\frac{d\phi^{ss}}{d\theta^{pv}} = \frac{-\frac{(1+\beta)}{\beta}(1-\tau^e(\phi^{ss}))A_T I^{pv}}{I^{pv}\frac{(1-\tau^k)r_\phi}{1+(1-\tau^k)r(\phi^{ss})} - \frac{1+\beta}{\beta}A_T\tau^e_\phi(\theta^{pv}-\theta)} < 0$$

This slope is negative because $\theta^{pv} < \theta$.

Chapter 2

The Political Economy of Growth, Inequality, the Size and Composition of Government Spending

The Political Economy of Growth, Inequality, the Size and Composition of Government Spending*

Klaus Schmidt-Hebbel† and José-Carlos Tello‡

August 22, 2014

Abstract

This paper develops a dynamic general-equilibrium political-economy model for the optimal size and composition of public spending. An analytical solution is derived from majority voting for three government spending categories: public consumption goods and transfers (valued by households), as well as productive government services (complementing private capital in an endogenous-growth technology). Inequality is reflected by a discrete distribution of infinitely-lived agents that differ by their initial capital holdings. In contrast to the previous literature that derives monotonic (typically negative) relations between inequality and growth in one-dimensional voting environments, this paper establishes conditions, in an environment of multi-dimensional voting, under which a non-monotonic, inverted U-shape relation between inequality and growth is obtained. This more general result – that inequality and growth could be negatively or positively related – could be consistent with the ambiguous or inconclusive results documented in the empirical literature on the inequality-growth nexus. The paper also shows that the political-economy equilibrium obtained under multi-dimensional voting for the initial period is time-consistent.

JEL Classification: D72, E62, H11, H31
Keywords: inequality, endogenous growth, multidimensional voting, endogenous taxation.

*We thankfully acknowledge excellent comments received from Martin Besfamille, Nicolas Figueroa, and Francisco Gallego (members of Tello's Ph.D. dissertation committee) and from Sofia Bauducco, Waldo Mendoza, Facundo Piguillem, and participants at the SECHI 2011 meetings, LACEA-LAMES 2012 meetings, Peruvian Economic Association 2014 meetings, and a seminar at Catholic University of Chile.

†Department of Economics, Catholic University of Chile, email: kschmidt-hebbel@uc.cl

‡Catholic University of Peru, email: jctello@pucp.edu.pe

1 Introduction

The relation between inequality and growth has been widely analyzed in the analytical political-economy literature. A typical negative relation between inequality and growth has been established, based on the following mechanism. Higher inequality leads to a larger demand by the median voter for redistributive government spending, which is financed by higher distortionary taxation that reduces (physical or human) capital investment and growth. This negative relationship between inequality and growth is derived in Bertola (1993), Perotti (1993), Alesina and Rodrick (1994), Persson and Tabellini (1994), and Saint-Paul and Verdier (1997).[1] The theoretical framework of the aforementioned papers combines endogenous growth and the endogenous setting of a key policy instrument. However, the empirical literature based on cross-country data provides contradictory or inconclusive results on the sign of the inequality-growth link.[2]

Still within the median-voter framework, Saint-Paul and Verdier (1993) derive a positive relationship between inequality and growth, considering government spending on education as the only public spending category. Education spending has two effects: it is redistributive as the poor benefit from it to a larger extent than the rich and it raises growth because the production function is linear in human capital. This unambiguously positive effect of education on growth is the result of assuming non-distortionary taxation. However, the authors also suggest that it is possible to obtain an inverted U-shaped relationship between growth and inequality when education spending is financed by a sufficiently distortionary tax.

Other studies restrict the relationship between the median and the decisive voter by the inclusion of a wealth bias in the political system. Benabou (1996), Saint-Paul and Verdier (1996), and Lee and Roemer (1998) show that higher inequality may lead to less transfers and therefore higher growth if higher-income agents lobby against redistributive policy or limit access to voting by low-income agents. The latter mechanism can imply a U-shaped relation between inequality and redistribution.

Other studies focus on different mechanisms to majority voting that shape the inequality-growth nexus. Under asset-market imperfections a negative relationship between inequality and growth arises, as derived, for example, in Galor and Zeira (1993) and Benabou (1996). Here individuals endowed with low levels of inherited wealth cannot invest in human capital due to market imperfections. So the initial inequality of wealth continues generation after generation, restricting investment and therefore growth. Gupta (1990), Alesina and Perotti (1995), and Benhabib and Rustichini (1996) focus on the social instability that is

[1] See Ostry et al. (2014), Aghion et al. (1999), Benabou (1996), and Alesina and Perotti (1994) for a detailed review of the relevant theoretical and empirical literature.

[2] A negative empirical relation is documented in the cross-country studies conducted by Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1994) and Clarke (1992). Barro (2000) establishes a negative relation for low-income countries and a positive relation for high-income countries. A positive relation is reported in panel data studies by Perotti (1996), Benabou (2000), Forbes (2000), Milanovic (2000), Banerjee and Duflo (2003), and Benhabib and Spiegel (1998).

generated by income inequality as the cause of a higher demand for transfers or a lower demand for productive expenditure, which leads to lower growth.

The political-economy literature reviewed so far sets the relevant policy instrument in a one-dimensional policy. However, majority voting in a multi-dimensional setting opens the door to instability of the political equilibrium, due to the lack of a Condorcet winner policy, i.e., there is not a policy that beats any other feasible policy in a pairwise vote. The latter instability can be avoided by restricting the institutional environment for aggregating individual decisions, without restricting individual preferences. Shepsle (1979) and Shepsle and Weingast (1981) follow the latter strategy by developing a structure-induced political equilibrium.[3] In this framework the voting process is based on separate but simultaneous voting by each agent in n separate committees, one for each of n voting dimensions. Votes are aggregated for each dimension and the election outcome will be determined by the votes of the corresponding decisive voter. In each dimension, the median voter may vary or not and this depends on the preferences of the agents by government-spending categories. For example, Conde-Ruiz and Galasso (2005) characterize the political equilibrium of multi-dimensional public spending, applying the structure-induced equilibrium. There are two types of public expenditure in their overlapping-generations model: transfer payments from rich to poor and pension payments. In this model, the decisive voter for each expenditure category is different, because pensions are preferred by the older generation and transfers are preferred by the poor (both young and old).

Exogenous growth models do not allow derivation of an analytical solution of the class of dynamic political-economy models considered here because growth is not constant over time and therefore identification and decisions by future median voters are dependent on current policy decisions. Hence dynamic exogenous-growth models, like those by Krusell et al (1997), Krusell and Ríos-Rull (1999), and Azzimonti et al. (2006), are solved numerically. In contrast, the endogenous growth framework allows obtaining analytical solutions of political-economy equilibria. The latter setup allows identification of future median voters because the evolution of state variables preserves the ranking of voters over time.

In this paper we develop a dynamic general-equilibrium political-economy model for the optimal size and composition of public spending. Using an endogenous-growth technology, we derive an analytical solution based on majority voting for three government spending categories. We derive conditions under which a non-monotonic, inverted U-shaped relation between inequality and growth is obtained. This result extends the previous analytical literature in a median voter framework that was reviewed above, which derives a monotonic

[3]An alternative strategy for restricting the institutional environment without restricting individual preferences is adopted in agenda-setter models, where voter groups – the agenda setters – are allowed to submit policy proposals of their interest to general voting. For example, Persson et al. (2000) develop a static model with agenda setters for multi-dimensional voting over three categories of public spending: transfers, public goods, and payments to politicians. Other modeling strategies to avoid political equilibrium instability include probabilistic voting and using intermediate political preferences, as discussed by Persson and Tabellini (2000).

relationship under one-dimensional voting. Our more general result – that inequality and growth could be negatively or positively related – could be consistent with the ambiguous or inconclusive results documented in the empirical literature on the inequality-growth nexus.

Government spending falls on three expenditure categories: public consumption goods, transfers to households, and productive government services, which are financed by distortionary taxes. Inequality is reflected by a discrete distribution of infinitely-lived agents that differ by their initial capital holdings. Household preferences are homogeneous across households, who value positively private and public consumption goods, and benefit from lump-sum transfers from the government. Public consumption enters household utility in per capita terms as a rival and non-excludable consumption good.

Government services raise the productivity of capital and are subject to relative congestion, i.e., they are rival goods.[4] Productive government services are a complement of private capital in an endogenous-growth technology characterized by constant returns.[5]

The composition and aggregate level of spending is chosen endogenously by majority voting in a dynamic context. Here we follow Shepsle (1979) by attaining stability of the political equilibrium as a result of restricting the institutional environment for aggregating individual decisions, without restricting individual preferences. Voting cannot be restricted to aggregate spending or taxation because the median voter expresses a separate political demand for each component of public expenditure. In general equilibrium, the political demand for one government spending category affects the demand for the two other categories. Therefore the political demand for government transfers is lessened by the supply of other government spending categories that are valued by the median voter – a key feature for the main result of our model.

A key feature of this paper is the existence of an endogenous threshold level of inequality at which the median voter chooses a household transfer level of zero. At higher levels of inequality, the relation between inequality and growth is negative, as it is in much of the previous political-economy literature. At levels of inequality below the threshold level, negative transfers (i.e., taxes) would be chosen. This outcome is not sensible, so we restrict transfers to be zero at low levels of inequality. This implies that in the latter case voting is restricted to choosing optimal levels of two remaining spending categories: public consumption goods and productive government services.

This leads to the paper's main result. In the range where inequality is lower than the aforementioned threshold level and transfers are zero, the relationship between inequality

[4] Barro and Sala-i-Martin (1992) argue that most public goods are subject to congestion. Therefore pure public goods should be considered as useful but unrealistic benchmarks.

[5] Our model shares with Alesina and Rodrik (1994) and Persson and Tabellini (1994) (as well as other related work on political economy of government size and growth) two features: endogenous growth and public provision of a factor that enhances growth. However, we include two additional categories of government spending, focusing on a multi-dimensional political-economy equilibrium, in contrasts to the previous one-dimensional models.

and growth could be positive. While optimal provision of productive services is affected neither by inequality nor by transfers, optimal provision of public consumption goods is affected by inequality. When inequality exceeds the threshold level, lower income concentration leads to a smaller demand for transfers, lower taxation, and higher growth. However, when inequality falls below the threshold level, we obtain an ambiguous relation between income concentration and taxation, which is determined by the degree of substitution between private and public consumption goods in household utility. If private and public consumptions are complements (substitutes), a lower (higher) income concentration leads to a higher (lower) demand for public goods and hence higher (lower) overall taxation, which lowers (raises) growth.

We derive analytical policy functions, which in many cases imply obtaining closed-from solutions for the policy instruments. However, the latter results are obtained by voting at the initial period. Therefore the question is whether our political-economy equilibrium is time consistent. The solution is to implement successive votes; the time consistency is attained when beliefs about future policy rules match the current policy choice. We show that when focusing on Markov-perfect equilibria, the selected policy is stationary, i.e., time is not an argument of the policy function, which is only determined by key state variables. In our model, we derive the Generalized Euler Equation[6] (GEE) for a political equilibrium and conclude that the voting equilibrium attained at time zero satisfies the GEE and therefore is time consistent.

The outline of the paper is as follows. Section 2 describes the model in detail. Section 3 derives an analytical solution for the economic equilibrium, with an exogenous fiscal policy. In section 4 we characterize the full dynamic political-economy equilibrium with analytical solutions for the size and composition of government spending at time zero only, and then prove that this equilibrium is time consistent. Section 5 concludes.

2 The Model

The economy is populated by infinitely-lived households which consume a private and a public consumption good. While consumption preferences are homogeneous across households, they are heterogeneous in their initial capital endowment. Production technology reflects the use of capital, supplied by households, and productive services, supplied by the government. The government raises revenue by establishing a constant tax rate on capital and spends on three expenditure categories: public consumption, productive services, and lump-sum payments provided to each household. Next we present the model equations by agents.

[6]Klein et al (2008) derived a condition called Generalized Euler Equation. This condition captures, in equilibrium, the optimal deviation in the policy variable for the policy maker (in our model, the median voter maximizes utility). The latter deviation should match with the level described by the policy function (this function built on utility-maximizing agents considering the deviation of the policy variable).

2.1 Households

Infinitely-lived households differ by initial (physical and human) capital endowment $k_{i,0}$. Over time, households accumulate capital by investing. Intertemporal and intratemporal preferences are homogeneous. Intertemporal preferences of representative household i are reflected by a logarithmic utility function over $\mathbb{C}_{i,t}$, a CES aggregate over a private consumption good, $c_{i,t}$, and a public consumption good, $g_{1,t}$. Household income is the sum of net capital rents (the rental rate of capital, r_t, times outstanding capital, net of proportional capital income taxes at homogeneous tax rate τ_g) plus government transfer $g_{3,t}$.

Representative household i solves the following intertemporal optimization

$$\max_{\{c_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,t}(c_{i,t}, g_{1,t}) \tag{1}$$

subject to

$$c_{i,t} + k_{i,t+1} = (1-\tau_g)\, r_t k_{i,t} + g_{3,t}, \text{ given } k_{i,0}. \tag{2}$$

Aggregate intratemporal consumption is defined by CES preferences

$$\mathbb{C}_{i,t}(c_{i,t}, g_{1,t}) = \left(a c_{i,t}^{\rho} + (1-a) g_{1,t}^{\rho}\right)^{\frac{1}{\rho}}, \tag{3}$$

where $\frac{1}{1-\rho}$ is the intratemporal elasticity of substitution between consumption goods $(1 > \rho > -\infty)$, and a is the share parameter $(0 < a < 1)$.

The number of households, N, is exogenous and constant over time. Hence aggregate capital is $K_t = \sum_{i=1}^{N} k_{i,t}$, aggregate consumption is $C_t = \sum_{i=1}^{N} c_{i,t}$, and aggregate public spending is $G_t = \tau_g Y_t$.

Therefore the economy's aggregate resource constraint is given by

$$Y_t = C_t + K_{t+1} + G_t, \tag{4}$$

where Y is aggregate output.

2.2 Firms

Technology reflects the use of capital, supplied by households, and productive services, supplied free of charge by the government. Following an AK technology, production is linear in capital. Government productive services are a public good that is rival in the sense that each firms use of productive services is subject to congestion. Following Turnovsky (1996) the representative firm's per capita output is given by[7]

[7] Eicher and Turnosvsky (2000) present a general specification of production functions with congestion and the conditions that must be fulfilled by these functions to generate endogenous growth.

$$\frac{Y_t}{N} = y_t = A\left[\left(\frac{G_{2,t}}{k_t}\right)^{\sigma}\left(\frac{G_{2,t}}{K_t}\right)^{(1-\sigma)}\right]^{\gamma} k_t, \quad 0 \leqslant \sigma \leqslant 1; 0 < \gamma < 1, \tag{5}$$

where y_t is per capita output, $G_{2,t}$ is government productive services, k_t is per capita capital, σ is the congestion parameter associated to aggregate production services, γ is a parameter that reflects the positive but marginally declining productivity of capital with respect to productive services, and A is the linear capital productivity parameter.

Congestion in the use of government productive services is reflected at both firm level and the economy's aggregate level. If $0 \leqslant \sigma \leqslant 1$, productive services are rival because their use is associated to k, the use of capital at firm level, as well as to K, the economy's aggregate capital. Given $G_{2,t}$, a higher stock of firm-level capital and/or a higher stock of aggregate capital reduce the productivity of G_2.

Let's re-write equation (5) as:

$$y_t = A\left[h_t\left(\frac{K_t}{k_t}\right)^{\sigma}\right]^{\gamma} k_t, \tag{6}$$

where $h_t \equiv \frac{G_{2,t}}{K_t}$.

If $\sigma = 0$, congestion is proportional and capital productivity declines when G_2 grows at a lower rate than aggregate capital. If $\sigma = 1$, there is no congestion, production exhibits constant returns to scale with respect to k and G_2, and the latter is non-rival and non-excludible public good.

Firms interact in a perfectly competitive market for capital and take the aggregate capital and productive service as given, where the capital rental rate is determined by the marginal productivity of capital

$$r_t = Ah_t^{\gamma} - A\gamma h_t^{\gamma-1} h_t \sigma = Ah_t^{\gamma}(1 - \gamma\sigma). \tag{7}$$

We assume that the governments supply of services, $G_{2,t}$, grows proportionally to aggregate capital. Hence the ratio h_t is time-independent.

We note that the private marginal productivity of capital before taxes is larger than the social marginal productivity of capital. More precisely, a benevolent social planner is aware of the negative congestion externality and therefore chooses a socially optimal level of capital that is consistent with a lower (social) productivity of capital, equal to $Ah^{\gamma}(1 - \gamma)$. If the congestion increases (i.e., when σ declines), the distance of the private marginal productivity with respect to the social marginal productivity is higher due to over-investment of capital.

2.3 Government

The government raises tax revenue at zero costs and spends on three expenditure categories. At this stage we assume the tax rate on income, τ_g, to be time-invariant.

The government's budget constraint in per capita terms is the following:

$$\tau_g y_t = g_t = \sum_{j=1}^{3} g_{j,t}, \tag{8}$$

where $g_{j,t}$ $(j = 1, 2, 3)$ is any of the three expenditure types that were defined above.

In order to simplify derivation of the multi-dimension policy in section 4, we introduce fictitious separate tax rates τ_j $(j = 1, 2, 3)$ that correspond to each type of government expenditure, satisfying $g_{j,t} = \tau_j y_t, \forall j$, and $\sum_{j=1}^{3} \tau_j = \tau_g$.

Aggregate output of the economy is

$$Y_t = A \left(\frac{G_{2,t}}{K_t} \right)^{\gamma} K_t = A h^{\gamma} K_t. \tag{9}$$

As mentioned above, we assume that the government's expenditure on productive services, G_2, is proportional to aggregate capital. Considering the latter relation, the budget relation $G_{2,t} = \tau_2 Y_t$, and aggregate output equation (9), obtain

$$h \equiv \frac{G_2}{K} = \tau_2 A h^{\gamma}. \tag{10}$$

Therefore h is determined by $h = \tau_2^{\frac{1}{1-\gamma}} A^{\frac{1}{1-\gamma}}$.

Considering the government budget constraint, the fictitious tax rates, and the marginal product of capital, per capita output is derived as

$$y_t = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_t \tag{11}$$

Per capita government expenditures on each category are

$$g_{j,t} = \tau_j A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_t, \quad j = 1, 2, 3. \tag{12}$$

Finally, the rental rate of capital is the following

$$r_t = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} (1 - \gamma\sigma). \tag{13}$$

In sum, government spending on productive services raises marginal capital productivity but, as a result of the negative congestion externality due to the use of government services, private marginal capital productivity exceeds social productivity. Finally, a standard result from endogenous-growth theory, our economy's growth rate is time-invariant due to constant returns to capital, the economy's reproducible factor of production.

3 Economic Equilibrium

Each household solves her optimization problem, taking as given aggregate government spending, tax rates, and the rental rate of capital. Household i's Euler equation determines relative intertemporal private consumption as

$$\frac{c_{i,t+1}}{c_{i,t}} = \beta\left((1-\tau_g)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}(1-\gamma\sigma)\right) \equiv \Theta, \tag{14}$$

where Θ is the gross rate of growth of private consumption.

As a result of endogenous constant output growth, private consumption is also a time-invariant function of tax rates and parameters related to technology and preferences. Capital growth both at household i and the average level is also time invariant, and so is η_i, the ratio of capital held by household i and the average level of capital

$$\eta_i \equiv \frac{k_{i,t}}{k_t} = \frac{k_i}{k} \tag{15}$$

An important implication of the latter is that the relative ranking of households according to their capital wealth does not change over time and is unaffected by any exogenous or endogenous change in the determinants of capital productivity.[8]

After replacing Euler equation (14), expenditure levels $g_{j,t}$ as functions of their corresponding fictitious tax rates times output, and the rental rate of capital (13) in the household's budget constraint, we obtain household gross saving as

$$k_{i,t+1} = \Psi_i k_t - c_{i,t}, \tag{16}$$

donde $\Psi_i \equiv (1-\tau_g)A^{\frac{1}{\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}\left((1-\gamma\sigma)\eta_i + \frac{\tau_3}{1-\tau_g}\right)$.

Household gross saving in period t, $k_{i,t+1}$, is a positive function of period-t per capita capital endowment, k_t , and a negative function of period-t consumption, $c_{i,t}$. Hence, as is standard in an endogenous growth model, gross saving and the capital rental rate do not depend on future variables. Output, aggregate (and household) consumption, and aggregate (and household) capital growth at the common and time-invariant rate Θ. Hence the relation between consumption and capital is given by

$$c_{i,t} = (\Psi_i - \Theta)k_t \tag{17}$$

[8]In a neoclassical growth model with exogenous stationary growth but a dynamic time-variant growth path toward the steady-state equilibrium the household ranking by capital endowments is not affected by an exogenous change in tax rates. However, as shown by Krusell and Ríos-Rull (1999), when the tax rate is made endogenous by a political-economy decision in a neoclassical growth model, household rankings are altered in subsequent periods after the endogenous tax change takes place.

Definition 1. *Given time-invariant tax rates τ_1, τ_2, τ_3 an initial capital distribution $k_{i,0}$ for all $i = 1, ..., N$; then an allocation $\{c_{i,t}, k_{i,t+1}\}_{t+0}^{\infty}$ represents a competitive equilibrium if and only if there is a price sequence $\{r_t\}_{t=0}^{\infty}$ such that agents solve (1) subject to budget constraint (2), the capital rental rate is given by (13), and the economy's resource constraint (4) is satisfied.*

The model's competitive economic equilibrium allows to write the agents' utility function in terms of the economy's growth rate Θ and initial consumption $\mathbb{C}_{i,0}$. Hence agent i's indirect utility function is given by

$$\mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0) \equiv \sum_{t=0}^{\infty} \beta^t \ln\left(\mathbb{C}_{i,0}\Theta^t\right) = \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,0} + \sum_{t=0}^{\infty} t\beta^t \ln \Theta, \tag{18}$$

where

$$\mathbb{C}_{i,0} = \left(a\left((\Psi_i - \Theta)k_0\right)^{\rho} + (1-a)(\tau_1 A^{\frac{1}{\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0)^{\rho}\right)^{\frac{1}{\rho}}. \tag{19}$$

4 Political-Economy Equilibrium

In this section we derive the political-economy equilibrium, which implies obtaining endogenous tax rates τ_j as a result of majority voting. Considering that voter heterogeneity is determined only by differences in initial capital endowments, we could consider applying the version of Median Voter Theorem (MVT) whose political preferences are characterized by the single-crossing property. By the single-crossing property we can sort voters by type and given that policy alternatives can be ordered from smallest to largest, the decisive voter is one whose number of voters who prefer a large alternative is equal to the number of voters who prefer a small alternative, i.e., the decisive voter is that endowed with a level of capital equal to the median value of the households' distribution of capital, k_{md}, that is $k_i = k_{md}$. In this case, there is a Condorcet winner policy that beats any other feasible policy in a pairwise vote and coincides with the preferred policy of the median voter.[9]

However, the standard MVT applies only when the policy variable is one-dimensional. When it is multi-dimensional, as it is our case, majority voting is cyclical, i.e., it is almost always feasible to derive an alternative policy preferred by a majority of voters.[10] [11]

To overcome the latter limitation of the standard MVT, Shepsle (1979) and Shepsle and Weingast (1981) propose the Structure-Induced Equilibrium (SIE). This political-economy equilibrium for multi-dimensional decisions results from an issue-by-issue voting game with

[9]See Myerson (1996) for more details of the two versions of MVT.

[10]Shepsle and Weingast (1982) discuss in detail the problem or curse of multi-dimensional majority voting that limits aplication of the MVT.

[11]Assuming that voters have intermediate preferences the MVT can be applied in multidimensional spaces.

commitment. Agents vote simultaneously but separately for each dimension of the policy under discussion. Votes are then socially aggregated over each issue by the decisive or median vote.

The definition of the SIE for our model is the following.

Definition 2. *Structure-Induced Equilibrium. An Structure-Induced Equilibrium is a policy vector* $\tau^* = (\tau_1^*, \tau_2^*, \tau_3^*)' \in [0,1]^3$ *such that*

$$\tau_1^* = \tau_1^{dc}(\tau_2^*, \tau_3^*),\ \ \tau_2^* = \tau_2^{dc}(\tau_1^*, \tau_3^*)\ \textit{and}\ \tau_3^* = \tau_3^{dc}(\tau_1^*, \tau_2^*),$$

where $\tau_j^{dc} = \arg\max_{\tau_j} \mathbb{V}(\tau_1, \tau_2, \tau_3; k_{dc,0}, k_0)$ *for j=1, 2, 3, and* $k_{dc,0}$ *is initial capital of the decisive voter. Considering that the space of alternatives is convex and the* τ_j^{dc} *functions are continuous, then, consistent with the Brouwer fixed-point theorem, an equilibrium is defined for* τ_j^**, for j=1, 2 and 3.*

This equilibrium implies applying a version of the MVT that requires satisfaction of the single-crossing property in each voting dimension. To this end we will define a property of political preferences that will enable us to directly apply the MVT. To this property we called marginal single-crossing utility.

Definition 3. *Property of Marginal Single-Crossing Utility. When the first derivatives of the indirect utility function* $\mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0)$ *satisfy*

$$\frac{\partial^2 \mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0)}{\partial k_{i,0} \partial \tau_j} > 0\ \textit{or} < 0,\ \textit{para todo}\ \tau_j \in [0,1]\ \textit{y}\ k_{i,0} > 0,$$

then this utility function satisfies the property of marginal single-crossing utility.

This monotonicity property implies that marginal utility changes monotonically in each policy dimension. The purpose of this property is to show that the marginal utility of voters shows monotonicity with respect to the variable that identifies the type of agent. Since agents are ordered completely and transitively in space $\mathbb{R}^+$ (nonnegative real numbers) according to their initial capital endowment, the existence of a Condorcet winner is possible if those agents who prefer a higher value of policy is not located between those who prefer a smaller value of policy. Following Shepsle (1979) and De Donder et al. (2012), we conclude that if indirect utility $\mathbb{V}$ satisfies the latter property, the decisive voters choice in each policy dimension is the median voters choice, i.e., where each $\tau_j^{dc} = \tau_j^{md}$.

Considering that our model is for a dynamic environment, first we assume that voting takes place in the first period under commitment and is therefore maintained over time. Then we will relax this assumption and show that the static policy result is dynamically consistent when assuming a Markov-perfect equilibrium.

4.1 Political-Economic Equilibrium under Voting in Period Zero

Here we assume voting to take place only once, in period $t = 0$, when society chooses by majority voting a policy τ^* that will be maintained over time. The policy choice results from maximizing indirect utility derived in equation (18).

We show in Appendix A that our indirect utility function satisfies the property of marginal single-crossing utility and we obtain the SIE

$$\tau_1^* = \tau_1^{md}(\tau_2^*, \tau_3^*),\ \tau_2^* = \tau_2^{md}(\tau_1^*, \tau_3^*)\ y\ \tau_3^* = \tau_3^{md}(\tau_1^*, \tau_2^*).$$

Therefore the social choice by majority voting of the policy variable coincides with the simultaneous choice yet separately (issue by issue), made by the decisive voter, which holds an initial level of capital that corresponds to the median level of the household distribution of capital endowments.

Lemma 1. *If the households indirect utility function defined by (18) satisfies the property of single-crossing marginal utility, then a political-economy equilibrium with voting in the initial period is defined by the policy vector $\tau^* = (\tau_1^*, \tau_2^*, \tau_3^*)' \in [0,1]^3$ where*

$$\tau_1^* = \frac{(1-\gamma)(1-\beta)}{(1+\phi)},$$

$$\tau_2^* = \gamma,$$

$$\tau_3^* = \frac{(1-\gamma)(1-\beta)(\phi - (\phi+\beta)(1-\gamma\sigma)\eta_{md})}{(1+\phi)(1-(1-\beta)(1-\gamma\sigma)\eta_{md})},$$

where $\phi = \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}}$ and $\eta_{md} = \frac{k_{md}}{k}$.

Proof. See Appendix A. □

Government provision of public consumption goods is financed by tax rate τ_1^*, which depends positively on the weight of public consumption goods in household preferences $(1-a)$ and the elasticity of substitution between private and public consumption goods $\frac{1}{1-\rho}$. When a is larger (smaller) than 0.5, i.e., when household weight attached to private goods is larger (smaller) than that attached to public goods, then a higher elasticity of substitution between both goods leads to a larger (smaller) demand, and therefore provision of public goods declines (increases). Hence if the degree of substitution between both goods rises, households prefer to reduce demand for the less-valued good. Note that, for now the choice of τ_1^* depends only on economic parameters and is therefore independent of the distribution of wealth and voting.

Government provision of productive services is financed by tax rate τ_2^*, which is exactly determined by parameter γ, the productivity of capital with respect to government services.

For the social planner, aggregate production if reflected by the function $Y = AG_2^{\gamma}K^{(1-\gamma)}$, where congestion in the use of government services is absent. Hence, from a social welfare perspective, parameter gamma represents the share of government productive services in a Cobb-Douglas production function.[12] It is well-known that in an economy with endogenous growth and where government services are a flow in a Cobb-Douglas production function, the tax rate that maximizes growth is identical to the rate that maximizes social welfare.[13] However, in contrasts to the literature that considers productive government flows services in an endogenous-growth model with a Cobb-Douglas technology, in our model τ_2^* does not maximize growth because of the presence of other government spending categories.

Corollary 1. *In a political-economy model of endogenous growth, Cobb-Douglas technology, CES preferences, and endogenous choice of different government spending categories by majority voting, the tax rate (that finances productive government services) that maximizes welfare of the decisive household is different from the tax rate that maximizes growth.*

Proof. This result is different from the result of the previous literature on endogenous growth and Cobb-Douglas technology (quoted in footnote 13), as reflected by the derivative of the growth with respect to τ_2

$$\frac{\partial\Theta}{\partial\tau_2} = \beta(1-\gamma\sigma)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}\left(\frac{(1-\tau_1-\tau_2-\tau_3)}{\tau_2}\frac{\gamma}{1-\gamma}-1\right). \tag{20}$$

Hence, for an economy with non-zero tax rates τ_1 and τ_3, the τ_2 tax rate that maximizes growth is given by

$$\widetilde{\tau}_2 = (1-\tau_1-\tau_3)\gamma$$

It is possible to obtain a value for tax rate τ_3 equal to zero for a given set of parameter values (in fact, we will consider this case below). However tax rate τ_1 cannot be zero because $0 < a, \gamma, \beta < 1$. Therefore the tax rate that maximizes growth, $\widetilde{\tau}_2$, is lower than the tax rate τ_2^*, selected in the political process. $\square$

[12]We have also considered an alternative transfer scheme to our lump-sum transfers. The alternative scheme allows for more distributive transfers, under which households with wealth levels that exceed the average wealth level get zero transfers and those with wealth below the average get transfers that are inversely proportional to their wealth. Hence individual household transfer g_3^i increases linearly with the reduction of relative wealth η_i. This feature adds an additional distortion to households' saving decisions. When assuming that the ratio η_i is distributed as a Fisk distribution with scale parameter equal to 1 and shape parameter equal to 1, transfers are defined as $(B(\eta_{md}))^{-1}(1-\frac{k^i}{k})$, where $B(\eta_{md}) \equiv \frac{1}{\eta_{md}} + ln(\frac{\eta_{md}}{\eta_{1+md}})$. We show that even though transfers affect the rate of growth, the voted τ_2 tax rate is equal to γ. For details, see Appendix E.

[13]For example, Barro (1990) and Barro and Sala-i-Martin (1992) derive endogenous-growth models with productive government flow services, for which they show that the level of government services supply that maximizes economic growth also maximizes social welfare. However, Misch et al. (2013) show that for a more general production technology exhibiting a factor substitution elasticity different from one, the latter equivalence is not satisfied anymore.

Therefore Corollary 1 proofs that the indirect utility-maximizing decisive voter, by voting for a $\widetilde{\tau}_2$ tax rate lower than γ, chooses a growth rate lower than the maximum she could attain because she also votes for tax financing of other types of government spending. In terms of her indirect utility, sacrificing growth by voting for a lower τ_2 is more than offset by higher consumption of private and public goods.

An increase in parameter γ leads to a change in the composition of government spending toward more government productive services, lowering spending on government consumption and transfers (see Lemma 1). This result is due to the fact that the marginal effect of government services on productivity, and hence on growth, rises with γ.

The third government spending category, transfers, with an income share equal to τ_3^*, is the only category that depends on η_{md}, the median-average wealth ratio. A reduction of this ratio reflects that the decisive voter is poorer relative to the average household. Let's define the measure of household inequality the term $1-\eta_{md}$. Not surprisingly, a higher level of inequality causes an unambiguous rise in the share of transfers in GDP, defined by τ_3^*, although it distorts saving decisions

$$\frac{\partial \tau_3^*}{\partial \eta_{md}} = \frac{-(1-\gamma)(1-\beta)\beta}{(1-(1-\beta)(1-\gamma\sigma)\eta_{md})^2} < 0. \tag{21}$$

This positive (negative) relation between the size of transfers and income concentration (η_{md}) is well established in the political economy literature. Meltzer and Richard (1981) in a static environment and Krusell and Ríos-Rull in a dynamic environment derive the latter relation. However, we assume that tax rates τ_j are restricted to the interval $[0,1]$ and all tax rates satisfy this condition other than transfers if and only if the inequality parameter η_{md} falls below a threshold level $\widetilde{\eta}$.

Corollary 2. *For a threshold level* $\widetilde{\eta} \equiv \frac{\phi}{(\phi+\beta)(1-\gamma\sigma)}$, *if* $\eta_{md} \in [0,\widetilde{\eta}]$ *then* $\tau_3^* \in [0,1]$.

Proof. Imposing a value of 0 for the numerator of τ_3^* (see Lemma 1) determines the threshold level $\widetilde{\eta}$. Hence if η_{md} tends to zero, then τ_3^* tends to $\frac{(1-\gamma)(\phi+\beta)}{1+\phi} < 1$. As transfers are monotonic, $\tau_3^* \in [0,1]$. □

Threshold level $\widetilde{\eta}$ (inequality measure $1-\widetilde{\eta}$) depends negatively (positively) on β. Hence if households get more impatient or β declines, the threshold level of inequality at which transfers are zero rises. More impatience leads to value growth less and therefore transfers attain a value of zero at a lower level of household inequality (Figure 1).

The degree of congestion in the use of public services (σ) and the share of productive government services in output (γ) have a positive effect on threshold level $\widetilde{\eta}$. Higher congestion (or σ decrease) raises the private marginal product of capital (at an intensity determined by the share of government services), hence increasing growth. Therefore the median voter will relinquish obtaining positive transfers at a higher level of inequality because she values growth more (Figure 1).

Finally the coefficient of public consumption in aggregate consumption, $1-a$, has a positive influence on the inequality threshold level. A higher weight of the public consumption good in private utility allows that the median voter demands both a higher transfer and a higher threshold of inequality, i.e., transfers cease to be positive from a higher level of $\widetilde{\eta}$. In both cases, the goal is to increase the private consumption good.

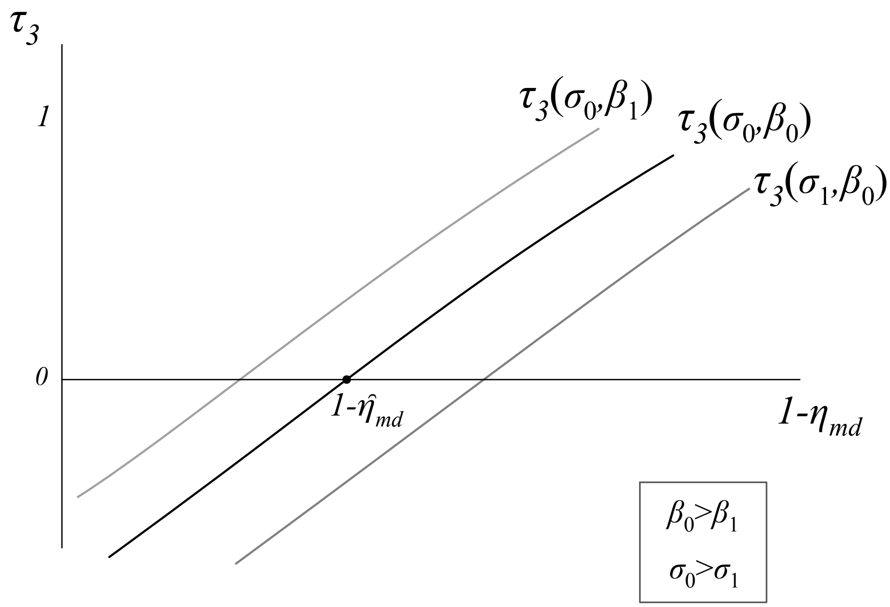

Figure 1: Equilibrium Relations between Inequality and Transfers for different Discount and Congestion Parameters

We show in Appendix A that the marginal utility of transfers is strictly decreasing in household wealth level k^i. We restrict τ_3 to be non-negative, because it does not make sense to pay a proportional tax on income to finance negative transfers, i.e., an additional tax. Therefore when income concentration is relatively low, i.e. when $\eta_{md} > \widetilde{\eta}$, transfers are set to zero and therefore $\tau_3^* = 0$.

Lemma 2. *Given indirect utility in (18), subject to the condition* $\eta_{md} > \widetilde{\eta}$ $(\tau_3 = 0)$*, then a political economy equilibrium attained under voting in the initial period is defined by vector* $\tau^{**} = (\tau_1^{**}, \tau_2^{**})' \in [0,1]^2$
where

$$\frac{a}{1-a}\left(\frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}}(1-\gamma\sigma)(1-\beta)\eta_{md}\right)^{\rho} = \frac{(1-\beta)(1-\gamma)}{\tau_1^{**}} - 1,$$

$$\tau_2^{**} = \gamma$$

Proof. See Appendix B. □

Although transfers are zero, the share of productive services in output is held at γ, i.e., it is determined by technology (5). Also if $\tau_1^{**} > 0$, then the voted policy does no maximize growth for the reasons discussed in the case of positive transfers. However, as opposed to the latter case, now it is not possible to derive a closed-form solution for τ_1^{**}. The only condition derived from the implicit function for τ_1^{**} in Lemma 2 is $\tau_1^{**} < (1-\beta)(1-\gamma)$.[14] This upper bound on τ_1^{**} is a sufficient condition for obtaining a positive level for after-tax household income, hence $\tau_1^{**} + \tau_2^{**} < 1$.

A value for η_{md} above the threshold level $\widetilde{\eta}$ implies zero transfers. Therefore we focus on the effect of η_{md} on τ_1^{**}. Differentiation of the implicit function for τ_1^{**} in Lemma 2 leads to an ambiguous sign of the partial derivative of τ_1^{**} with respect to η_{md}. We derive the condition under which the relation between the two latter variables can be signed.

Corollary 3. *The effect of income concentration coefficient η_{md} on the level of public consumption goods is determined by the intra-temporal elasticity of substitution between private and public consumption goods in utility.*

(I) For $\rho > 0$ (for an elasticity of substitution larger than 1), $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} < 0$.

(II) For $\rho < 0$ (for an elasticity of substitution smaller than 1), $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} > 0$.

Proof. See Appendix C. □

This corollary establishes two cases. We focus on the second case, when a lower concentration coefficient (i.e., a lower level of inequality) leads to a higher tax rate τ_1^{**} (to finance a larger level of public consumption goods) and hence to higher overall taxation, $\tau_g = \tau_1^{**} + \tau_3^{**}$. This case stands in contrast to the previous literature, which establishes a monotonic relation between income concentration, taxation, and the size of government, a relation that is upheld in case (I) of Corollary 3.

However, in case (II) of Corollary 3, we derive an ambiguous relation between income concentration and taxation. When two conditions are satisfied in our model (income concentration is relatively low, i.e., $\eta_{md} > \widetilde{\eta}$, therefore transfers are zero; and private and public consumptions are complements in utility, i.e., $\rho < 0$), then a reduction in income concentration leads to higher taxation for public goods and hence higher overall taxation.

The reason for our result is the following. In the traditional political-economy literature, less income concentration benefits the median voter who then votes for lower taxation and hence lowers transfers, under governments that spend only on transfers. In contrast, we consider three government-spending categories in our model. When income concentration is

[14] This inequality condition is derived from the fact that the left-hand side of the implicit equation for τ_1^{**} is always positive and therefore the right-hand side is also positive, implying this inequality.

relatively low ($\eta_{md} > \widetilde{\eta}$), transfers are set at zero and hence the median household votes only for two government-spending categories. Before she does so, consider first the economic equilibrium, before voting takes place. The median voter benefits from a larger capital endowment and hence higher income, raising her demand for private consumption goods, given the average level of capital. Now consider the political-economy equilibrium, when the median voter casts her vote for tax rates. Voting for tax rate τ_2^{**} is unaffected, according to Lemma 2. But voting for tax rate τ_1^{**} is determined by the degree of substitution between private and public consumption goods in household utility. When substitution is low ($\rho < 0$), both goods are complementary, and therefore a larger demand for private goods leads also to a larger demand of public consumption goods. Therefore the political-economy equilibrium implies a larger τ_1^{**} tax rate.

Now lets analyze the impact of income distribution on growth in our political-economy equilibrium. The endogenous growth rate is determined by

$$\Theta - 1 = \beta(1-\tau_g)(1-\gamma\sigma)A^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}} - 1, \tag{22}$$

where $(1-\tau_g) = (1-\gamma)\left(\beta + \frac{(1-\beta)(\beta+\phi)(1-\gamma\sigma)\eta_{md}}{1-(1-\beta)(1-\gamma\sigma)\eta_{md}}\right)$ if $\eta_{md} < \widetilde{\eta}$ and $(1-\tau_g) = (1-\gamma-\tau_1^{**}(\eta_{md}))$ if $\eta_{md} \geqslant \widetilde{\eta}$.

If income concentration is high (i.e., if $\eta_{md} < \widetilde{\eta}$), government spending falls on all three spending categories, including transfers. Then a reduction in inequality reduces the distributional conflict and the median voter choses a lower tax rate to finance transfers.

Consistent with the traditional literature (where government spending falls only on transfers), growth rises when inequality declines

$$\frac{\partial\Theta}{\partial\eta_{md}} = \frac{\beta(1-\gamma)(1-\beta)(\beta+\phi)(1-\gamma\sigma)^2A^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}}{(1-(1-\beta)(1-\gamma\sigma)\eta_{md})^2} > 0 \tag{23}$$

If income concentration is low (i.e., if $\eta_{md} \geqslant \widetilde{\eta}$), government spending falls on two spending categories, excluding transfers. Then a reduction in inequality has an ambiguous effect on the median voters choice of τ_1^{**}, and hence on growth, depending on the elasticity of substitution between consumption goods

$$\frac{\partial\Theta}{\partial\eta_{md}} = -(1-\gamma\sigma)A^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}\frac{\partial\tau_1^{**}}{\partial\eta_{md}} \tag{24}$$

where $\frac{\partial\tau_1^{**}}{\partial\eta_{md}} > (<)\ 0$, if $\rho < (>)\ 0$.

Figure 2 depicts the three economic and political-economy equilibrium relations between income concentration and growth derived in our model, consistent with equations (23) and (24). When income concentration is relatively high (i.e., if it exceeds threshold level $1-\widetilde{\eta}_{md}$), the median household votes for three tax rates. Then the relation between income concentration and growth is monotonically negative, because higher income concentration leads to higher taxation and hence to lower growth.

However, when income concentration is relatively low (i.e., if it falls below threshold level $1-\widetilde{\eta}_{md}$), the median household votes for two tax rates, excluding transfers. Then the relation between income concentration and growth is ambiguous, depending on consumption good substitution in household utility. If substitution is high ($\rho > 0$), the relation between income concentration and growth is monotonically positive. If substitution is low ($\rho < 0$), the relation is negative.

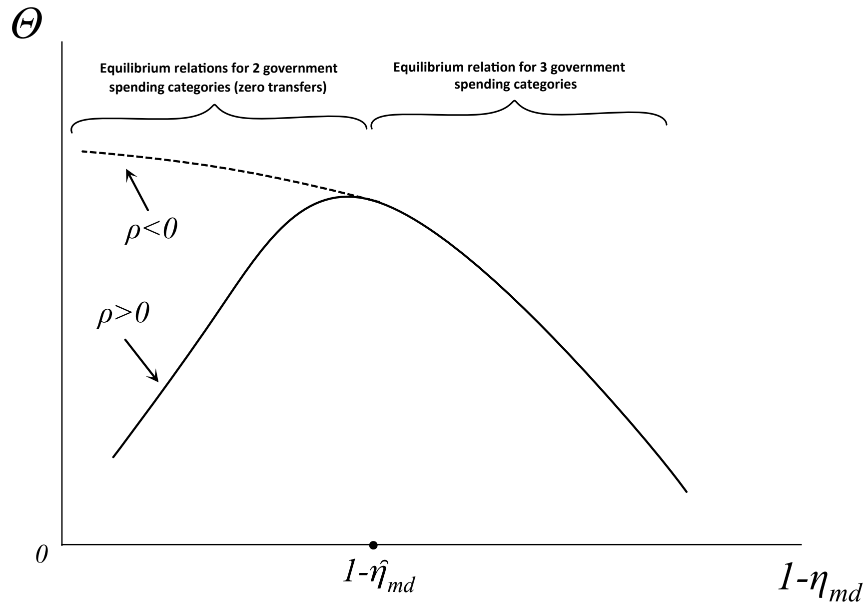

Figure 2: Inequality and Growth in Three Political-Economy Equilibria.

4.2 Political-Economic Equilibrium under Sequential Voting

Our previous equilibrium was derived under the assumption that voting takes place once in period zero, and therefore tax rates and government expenditure composition is held constant over time. Now we address the question if the latter policy is time-consistent, i.e., if society would repeat its period-0 choice if allowed to vote again at any period in the future.[15] One way to obtain time-consistent policies is allowing voting to take place in any

[15]Krusell et al. (1997) compare two equilibria, one with voting in period zero and other with sequential voting. A sequential voting can lead that the decisive voter faces more restrictions derived from imposing time consistency, lowering her welfare and hence implying a different vote to voting in period zero.

period t, under the condition that voting is shaped only by the relevant state variables in period t, without influence of past variables. This is a Markov-perfect equilibrium.[16]

In order to derive the latter equilibrium we re-write household i's indirect utility function in its recursive form. We assume that the policy vector voted in each period follows the rule $\tau = F(k_{dc}, k)$, where $\tau = (\tau_1 \ \tau_2 \ \tau_3)'$. This policy function is determined by two state variables: capital of the decisive voter k_{dc}, and society's average capital stock, k. Households expect that government policies regarding taxation and expenditure follow policy function F.

Considering the latter, households (including the decisive household) solve the following problem

$$\max_{\{k_{i,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,t}(c_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t), g_1(\tau_t, k_t)) \tag{25}$$

where $\mathbb{C}_i$ is defined by (3), $c_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t) = \psi_0(\tau)k + \psi_1(\tau)k_i - k_{i,t+1}$, and $g_{1,t}(\tau_t, k_t) = \psi_2(\tau)k_t$.[17]

Define function $\mathcal{H}^i(\tau, k_i, k)$ as the optimal saving response of household i. This function describes the response of households to a "one-shot deviation" on the part of the decisive voter in the following sense. If the current capital stock is k, current policy is τ, and the household i expects that future policy will be determined by the policy function $F(k_{dc}, k)$, then savings will be given by $\mathcal{H}^i(\tau, k_i, k)$.

The latter optimal response satisfies household i's first-order condition

$$\left(\frac{\mathbb{C}_{i,t+1}(\tau_{t+1}, k_{i,t+1}, k_{i,t+2}, k_{t+1})}{\mathbb{C}_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t)}\right)^{\rho} \left(\frac{c_{i,t+1}(\tau_{t+1}, k_{i,t+1}, k_{i,t+2}, k_{t+1})}{c_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t)}\right)^{1-\rho} = \beta\psi_1(\tau_{t+1}) \tag{26}$$

where $k_{i,t+1} = \mathcal{H}^i(\tau_t, k_{i,t}, k_t)$, $\tau_{t+1} = F(k_{i,t+1}, k_{t+1})$ and $k_{i,t+2} = \mathcal{H}^i(\tau_{t+1}, k_{i,t+1}, k_{t+1})$.

It is not possible to simplify (26) further because consumption growth rate is not necessarily constant over time because it depends on current and future tax rates. In addition, it is not yet possible to assure that aggregate capital growth is equal to consumption growth, which also implies that aggregate consumption growth $\mathbb{C}$ is equal to the growth rate of private consumption c. Hence it is not possible to simplify the latter first-order condition as we did in deriving equation (14).[18] As future tax rates determine current consumption,

[16] Time inconsistency arises in the absence of a mechanism or technology that allows commitment regarding the public policy choice. We do not consider sustainable equilibrium or reputation mechanisms in this model because we assume that governments implement the decisive voter's tax choice in any period and at zero cost. We also assume that current governments are not linked in any way to future governments, i.e., we do not consider government reelection.

[17] $\psi_0(\tau_t) \equiv A^{\frac{1}{1-\gamma}} \tau_{2,t}^{\frac{\gamma}{1-\gamma}} \tau_{3,t}$; $\psi_1(\tau_t) \equiv (1-\tau_{g,t})(1-\gamma\sigma)A^{\frac{1}{1-\gamma}} \tau_{2,t}^{\frac{\gamma}{1-\gamma}}$ y $\psi_2(\tau) \equiv \tau_{1,t} A^{\frac{1}{1-\gamma}} \tau_{2,t}^{\frac{\gamma}{1-\gamma}}$.

[18] For the special case of logarithmic aggregation function, the ratio of marginal utilities of private consumption c_{t+1}/c_t would not depend on public consumption good and the first-order condition would be simplified.

this opens the door to potential time inconsistency: the policy maker could change her tax policy after households have taken their current-period decisions. In our model, the policy maker is the median voter, who could face an incentive to revise his future tax choice in order to maximize his utility.

Considering optimal saving response $\mathcal{H}^i(\tau, k_i, k)$, the median voter chooses tax rates τ_j by solving the following problem expressed in its recursive form

$$v(k_{md}, k) = \max_{\tau_j} \ln \mathbb{C}_{md}(c_{md}, g_1) + \beta v(\mathcal{H}^{md}(k_{md}, \tau), \mathcal{H}(k, \tau)) \tag{27}$$

where $\mathcal{H}^{md}(k_{md}, \tau)$ and $\mathcal{H}(k, \tau)$ are the optimal saving responses of the median voter household and the average household, respectively.

We derive the first-order condition on for the political-economy problem of the median household in casting her decisive vote for tax rates, known as the Generalized Euler Equation (GEE), as defined in Klein et al. (2008).[19] The GEE reflects agreement between the median household's decision in period t with his own decision in period $t+1$ in setting tax rates $\tau_{j,t+1}$. Note that for this result to hold we assume (with Klein et al. 2008) that the choice of taxes in period t takes place before the economic choice of all households in period t. In other word, it is required to assume a weak sense of lack of commitment or that a intra-period commitment is needed because the government (i.e., the median voter) has to commit to her political choice after households take their saving decisions.

Definition 4. *Markov-perfect Political-Economy Equilibrium.*

Let $\tau = [\tau_1, \tau_2, \tau_3]' \in [0,1]^3$ the policy vector implemented in any period and $k_{md,0}$ as the median value of the initial distribution of capital. A Markov-perfect equilibrium is defined as a policy function $F : \mathbb{R}^2_+ \to [0,1]^3$ such that the following equations hold: (i) $F(k_{md}, k) = \arg\max_\tau v(k_{md}, k)$ where $\tau = F(k_{md}, k)$ and $k' = \mathcal{H}^i(\tau, k_i, k)$; and (ii) $\mathcal{H}^i(\tau, k_i, k)$ satisfies the condition (26).

Generally the literature has solved Markov-perfect equilibrium with numerical techniques, the absence of an analytical solution is due to the higher-order derivatives of decision rules. Both the policy functions and their derivatives have to satisfy the first-order conditions for both the current and future periods.[20]

In our case we conjecture the following solution for the policy function $F(k_{md}, k) = \tau^*$, that is, the policy of tax rates and expenditure composition that is constant over time, verifying if this policy satisfies the Markov-Perfect equilibrium definition. Our conjecture is based on analyzing first-order condition (26) for household i. If tax rates are constant over time, then individual consumption and gross saving, and the corresponding average levels, grow at a common rate determined by equation (14). Hence the latter variables do not depend on future capital, so that current saving is determined only by current, not by

[19]See Appendix D.

[20]Klein et al. (2008) propose a method of local approximations and compare its result with an alternative method based on Chebychev polynomials to evaluate first-order conditions for the steady-state equilibrium.

future variables. Note that this result also depends on having endogenous growth. If we had a technology of declining returns to capital, the dependence of current variables on future capital could not be avoided, and therefore the policy maker's (the median voter's) temptation to deviate from previous policy decisions would still be present.[21]

Proposition 1. *In an economy characterized by logarithmic inter-temporal preferences, CES intra-temporal preferences over private and public consumption goods, and constant marginal productivity of capital, the political-economy equilibrium with voting over the size and composition of government spending in period zero coincides with a Markov-perfect equilibrium of sequential voting.*

Proof. Full proof is provided in Appendix D. □

The idea of the proof is simple. Define $\tau*$ as the time-invariant policy to be chosen. In a model of endogenous growth, consumption, investment and public spending growth at a common constant rate $\Theta(\tau^*) - 1$ (see equation 14). Therefore optimal gross household saving is $k_{i,t+1} = \mathcal{H}^i(\tau, k_i) = \Theta(\tau^*)k_{i,t}$.

As function $\mathcal{H}^i(\tau^*, k_i)$ is twice differentiable, replace $\mathcal{H}^{'i}$ and $\mathcal{H}^{''i}$ in the GEE derived in Appendix D obtaining

$$\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} \Theta_{\tau_j^*} k - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau_j^{*'}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k \right\} = 0 \tag{28}$$

We derive the corresponding derivatives and obtain

$$(1-\beta)\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \frac{\Theta_{\tau_j^*}}{\Theta(\tau_j^*)} = 0 \tag{29}$$

Equation (29) coincides with the first-order condition of the political-economy equilibrium with voting in period zero. Considering that optimal government spending and its composition (i.e., tax rates) are independent of k_i and k, our conjecture about F is valid and, therefore, the equilibrium with voting in the initial period is time-consistent.

5 Conclusions

In this paper we have developed a dynamic general-equilibrium political-economy model for the optimal size and composition of public spending. An analytical solution has been derived from majority voting for the optimal level of three government-spending categories: public consumption goods and transfers (valued by households), as well as government productive services (complementing private capital in an endogenous-growth technology).

[21] Using numerical simulations, Krusell et al. (1997) illustrate how the choice of a Markov-Perfect equilibrium is constant in a model of endogenous growth.

Inequality is reflected by a discrete distribution of infinitely-lived agents that differ by their initial capital holdings. In contrast to the previous literature that derives monotonic (typically negative) relations between inequality and growth in one-dimensional voting environments, we have established conditions, in an environment of multi-dimensional voting, under which a non-monotonic, inverted U-shape relation between inequality and growth is obtained.

A key feature of this paper is the existence of an endogenous threshold level of inequality at which the median voter chooses a household transfer level of zero. At higher levels of inequality, the relation between inequality and growth is negative, as it is in much of the previous political-economy literature. At levels of inequality below the threshold level, negative transfers (i.e., taxes) would be chosen. This outcome is not sensible, so we restrict transfers to be zero at low levels of inequality. This implies that in the latter case voting is restricted to choosing optimal levels of two remaining spending categories: public consumption goods and productive government services.

This leads to the papers main result. In the range where inequality is lower than the aforementioned threshold level and transfers are zero, the relationship between inequality and growth could be positive. While optimal provision of productive services is affected neither by inequality nor by transfers, optimal provision of public consumption goods is affected by inequality. When inequality exceeds the threshold level, lower income concentration leads to a smaller demand for transfers, lower taxation, and higher growth. However, when inequality falls below the threshold level, we obtain an ambiguous relation between income concentration and taxation, which is determined by the degree of substitution between private and public consumption goods in household utility. If private and public consumptions are complements (substitutes), a lower (higher) income concentration leads to a higher (lower) demand for public goods and hence higher (lower) overall taxation, which lowers (raises) growth.

The latter main result extends the previous analytical literature that derives a monotonic relationship under one-dimensional voting. Our more general result that inequality and growth could be negatively or positively related could be consistent with the ambiguous or inconclusive results documented in the empirical literature on the inequality-growth nexus. We have derived analytical policy functions that characterize the political-economy equilibrium under multi-dimensional voting, which in many cases implied obtaining closed-from solution for the policy instruments. The latter results were obtained by voting at the initial period. Then we showed that when focusing on Markov-perfect equilibria, the policy voted at time zero is stationary, i.e., it is time consistent.

6 References

Alesina, A. and D. Rodrik, 1994. Distributive Politics and Economic Growth. *Quarterly Journal of Economics, 109, 465-490.*

Alesina, A. and R, Perotti, 1994. The Political Economy of Growth: A Critical Survey of the Recent Literature. *The World Bank Economic Review, 8, 351-371.*

Alesina, A. and R. Perotti, 1996. Income Distribution, Political Instability and Investment. *European Economic Review, 81, 1170-1189.*

Azzimonti, M., E. de Francisco, and P. Krusell, 2006. Median-voter Equilibria in The Neoclassical Growth Model under Aggregation. *Scandinavian Journal of Economics, 1008, 587-606.*

Banerjee, A. and E. Duflo, 2003. Inequality and Growth: What Can the Data Say? *Journal of Economic Growth, 8, 267-299.*

Barro, R., 1990. Government Spending in a Simple Model of Endogenous Growth. *Journal of Political Economy, 98 (5), S103-S125.*

Barro, R., 2000. Inequality and Growth in a Panel of Countries. *Journal of Economic Growth, 5, 5-32.*

Barro, R. and X. Sala-i-Martin, 1992. Public Finance in Models of Economic Growth. *Review of Economic Studies, 59, 645-661.*

Benabou, R., 2000. Unequal Socities: Income Distribution and the Social Contract. *American Economic Review, 90, 96-129.*

Benabou, R., 1996. Inequality and Growth, in: B. Bernanke and J. Rotemberg, eds., NBER Macroeconomics Annual 1996, Volume 11, MIT Press, 11-92.

Benhabib, J. and M. Spiegel, 1998. Cross-Country Growth Regressions. Working paper 97-20. C.V. Starr Center for Applied Economics, New York University.

Benhabib, J. and A. Rustichini, 1996. Social Conflict and Growth. *Journal of Economic Growth, 1,129146.*

Bertola, G., 1993. Factor Shares and Saving in Endogeneous Growth, *American Economic Review, 83, 5, 1184-1198.*

Clarke, G., 1992. More evidence on income distribution and growth. *Policy, Research and External Affairs - World Bank, Working Paper 1064.*

Conde-Ruiz, J. and V. Galasso, 2005. Positive arithmetic of the welfare state. *Journal of Public Economics, 89, 933-955.*

De Donder, P., Le Breton, M. and E. Pelusso, 2012. Majority Voting in Multidimensional Policy Spaces: Kramer and Shepsle versus Stackelberg. *Journal of Public Economic Theory, 14, 879-909.*

Eicher, T. and S. Turnovsky, 2000. Scale, Congestion and Growh. *Economica, 67, 325-346.*

Forbes, K., 2000. A Reassessment of the Relationship Between Inequality and Growth. *American Economic Review, 90, 869-887.*

Galor, O. and J. Zeira, 1993. Income Distribution and Macroeconomics. *Review of Economic Studies, 60, 35-52.*

Gupta, D. 1990. The Economics of Political Violence, Praeger, New York.

Krusell, P., Quadrini, V. y J.-V. R'ios-Rull, 1997. Politico-economic Equilibrium and Economic Growth. *Journal of Economic Dynamics and Control, 21, 243-272.*

Krusell, P. and J.-V. Ríos-Rull, 1999. On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model. *American Economic Review, 89 (5), 1156-1181.*

Klein, P., Krusell, P. and J.-V. Ríos-Rull, 2008. Time-Consistent Public Policy. *The Review of Economic Studies, 75, 789-808.*

Lee, W. and J. Roemer, 1998. Income Distribution, Redistributive Politics, and Economic Growth. *Journal of Economic Growth, 3, 217-240.*

Misch, F., Gemmell, N. and R. Kneller, 2013. Growth and Welfare Maximization in Models of Public Finance and Endogenous Growth. *Journal of Public Economic Theory, 1-29*

Milanovic, B., 2000. The median-voter hypothesis, income inequality, and income redistribution: an empirical test with the required data. *European Journal of Political Economy. 16, 367 - 410.*

Myerson, R., 1996. Fundamentals of Social Choice Theory, Center for Mathematical Studies in Economics and Management Science Northwestern University discussion paper number 1162.

Ostry, J., Berg, A. and C. Tsangarides, 2014. Redistribution, Inequality, and Growth. *IMF Staff Discussion Note, February.*

Perotti, R., 1996. Growth, Income Distribution, and Democracy: What the Data Say. *Journal of Economic Growth, 1: 149-187.*

Perotti, R., 1994. Income Distributions and Investment. *European Economic Review, 38, 827-835.*

Perotti, R., 1993. Political Equilibrium, Income Distribution and Growth. *Review of Economic Studies, 60, 755-776.*

Persson, T., Roland, G., Tabellini, G., 2000. Comparative politics and public finance. *Journal of Political Economy. 108, 1121 1161.*

Persson, T. and G. Tabellini, 2000. Political Economics: Explaining Economic Policy, Cambrigde, Mass.: MIT Press.

Persson, T. and G. Tabellini, 1994. Is Inequality Harmful for Growth? *American Economic Review, 84, 3, 600-621.*

Saint-Paul, G. and T. Verdier, 1997. Power, Distributive Conflicts and Multiple Growth Paths. *Journal of Economic Growth, 2, 155-168.*

Saint-Paul, G., and T. Verdier, 1996. Inequality, Redistribution and Growth: A Challenge to the Conventional Political Economy Approach. *European Economic Review 40, 719728.*

Saint-Paul, G and T. Verdier, 1993. Education, Democracy and Growth. *Journal of Development Economics, 42, 399-407.*

Shepsle, K., 1979. Institutional Arrangements and Equilibrium in Multidimensional Voting Models. *American Journal of Political Science, 23 (1), 27-59.*

Shepsle, K. and B. Weingast, 1981. Structure-Induced Equilibrium and Legislative Choice. *Public Choice, 37, 503-519.*

Shepsle, K. and B. Weingast, 1982. Institutionalizing Majority Rule: A Social Choice Theory with Policy Implications. *American Economic Review, 72, 367-371.*

Shepsle, K. and B. Weingast, 2012. Why so much stability? Majority Voting, Legislative Institutions, and Gordon Tullock. *Public Choice, 152, 83-95.*

Turnovsky, S., 1996. Optimal Tax, Debt, and Expenditure Policies in a Growing Economy. *Journal of Public Economics, 60, 21-44.*

Appendix A: Politico-Economic Equilibrium with Voting at Time Zero
We define the utility function of the decisive voter

$$\mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0) = \ln(\mathbb{C}_{i,0}) \sum_{t=0}^{\infty} \beta^t + \ln(\Theta) \sum_{t=0}^{\infty} t\beta^t = \mathbb{M}_0 \ln(\mathbb{C}_{i,0}) + \mathbb{M}_1 \ln(\Theta)$$

The first-order conditions are

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_1} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_1} = 0$$
$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_2} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_2} = 0$$
$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_3} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_3} = 0$$

Given $\mathbb{C}_{i,0} = \left(ac_t^{\rho} + (1-a)g_{1,t}^{\rho}\right)^{\frac{1}{\rho}}$, (10) and (11), we get

$$\frac{\partial c_{i,0}}{\partial \tau_1} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[(\beta - 1)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_1} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0$$

$$\frac{\partial c_{i,0}}{\partial \tau_2} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[\left(\frac{(1-\tau_1-\tau_2-\tau_3)}{\tau_2}\frac{\gamma}{1-\gamma} - 1\right)\left((1-\beta)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right) + \frac{\gamma}{1-\gamma}\frac{\tau_3}{\tau_2}\right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_2} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[\frac{\gamma}{1-\gamma}\frac{\tau_1}{\tau_2}\right]$$

$$\frac{\partial c_{i,0}}{\partial \tau_3} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[1 - (1-\beta)\frac{k_{i,0}}{k_0}\right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_3} = 0$$

$$\frac{\partial \ln \mathbb{C}_0}{\partial \tau_j} = \frac{1}{\mathbb{C}_0}\left(ac_{i,0}^{\rho-1}\frac{\partial c_{i,0}}{\partial \tau_j} + (1-a)g_0^{\rho-1}\frac{\partial g_0}{\partial \tau_j}\right),\ for\ j = 1, 2\ y\ 3.$$

Taking into account (10) we obtain

$$\frac{\partial \ln \Theta}{\partial \tau_2} = \frac{(1-\gamma\sigma)A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} \left(\frac{(1-\tau_1-\tau_2-\tau_3)}{\tau_2}\frac{\gamma}{1-\gamma} - 1\right)}{(1-\tau_1-\tau_2-\tau_3)(1-\gamma\sigma)A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

$$\frac{\partial \ln \Theta}{\partial \tau_j} = \frac{-(1-\gamma\sigma)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}}{(1-\tau_1-\tau_2-\tau_3)(1-\gamma\sigma)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}},\ para\ j=1\ y\ 3.$$

Replacing $\frac{\partial \ln \mathbb{C}_0}{\partial \tau_j}$ and $\frac{\partial \ln \Theta}{\partial \tau_j}$ in the first-order conditions

$$ac_{i,0}^{\rho-1}\left((\beta-1)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right)+(1-a)g_0^{\rho-1}-\frac{\mathbb{M}_1}{\mathbb{M}_0}\mathbb{C}_{i,0}^{\rho}\frac{1}{\Psi k_0}=0$$

$$ac_{i,0}^{\rho-1}\left[\mu\left((1-\beta)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right)+\frac{\tau_3}{\tau_2}\frac{\gamma}{1-\gamma}\right]+(1-a)g_0^{\rho-1}\frac{\tau_1}{\tau_2}\frac{\gamma}{1-\gamma}+\frac{\mathbb{M}_1}{\mathbb{M}_0}\mathbb{C}_{i,0}^{\rho}\frac{\mu}{\Psi k_0}=0$$

$$ac_{i,0}^{\rho-1}\left(1-(1-\beta)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right)-\frac{\mathbb{M}_1}{\mathbb{M}_0}\mathbb{C}_{i,0}^{\rho}\frac{1}{\Psi k_0}=0$$

where $\Psi=(1-\tau_1-\tau_2-\tau_3)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}$ and $\mu=\left(\frac{(1-\tau_1-\tau_2-\tau_3)}{\tau_2}\frac{\gamma}{1-\gamma}-1\right)$.

Marginal Single-Crossing Condition.

When multiplying both sides of later equations by k_0, it is straightforward to show that $\frac{\partial^2 \mathbb{V}(\tau_1,\tau_2,\tau_3;k_{i,0})k_0}{\partial k_{i,0}\partial \tau_j}>0\ for\ j=1,2$ and $\frac{\partial^2 \mathbb{V}(\tau_1,\tau_2,\tau_3;k_{i,0})k_0}{\partial k_{i,0}\partial \tau_3}<0$.

From the first-order conditions, the following results are obtained

$$\tau_1^*=\frac{(1-\gamma)}{(1+\phi)\frac{\mathbb{M}_0}{\mathbb{M}_1}(1+\frac{\mathbb{M}_1}{\mathbb{M}_0})}$$

$$\tau_2^*=\gamma$$

$$\tau_3^*=\frac{(1-\gamma)\left((1-\beta)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\left(1-(1+\phi)\frac{\mathbb{M}_0}{\mathbb{M}_1}(1+\frac{\mathbb{M}_1}{\mathbb{M}_0})\right)+\phi\right)}{(1+\phi)\frac{\mathbb{M}_0}{\mathbb{M}_1}(1+\frac{\mathbb{M}_1}{\mathbb{M}_0})\left(1-(1-\beta)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right)}$$

where $\phi=\left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}}$.

Convergence of ratio $\mathbb{M}_0\ /\ \mathbb{M}_1$.

Given that $0<\beta<1$ we know that $\mathbb{M}_0\equiv\sum_{t=0}^{\infty}\beta^t=\frac{1}{1-\beta}$ and $\mathbb{M}_1\equiv\sum_{t=0}^{\infty}t\beta^t=\frac{\beta}{(1-\beta)^2}$.

$$\begin{aligned}\mathbb{M}_1 &= 0+\beta+2\beta^2+3\beta^3+...\\ &= \beta(1+2\beta+3\beta^2+4\beta^3+...)\\ &= \beta(1+\beta+\beta^2+\beta^3+...\\ &\qquad\quad \beta+\beta^2+\beta^3+...\\ &\qquad\qquad +\beta^2+\beta^3+... \quad)\end{aligned}$$

Let $S=(1+\beta+\beta^2+\beta^3+...)$ then

$$\begin{aligned}\mathbb{M}_1 &= \beta(1+\beta+\beta^2+\beta^3+...)S\\ &= \beta\frac{1}{1-\beta}\frac{1}{1-\beta}.\end{aligned}$$

Therefore, if we replace the ratio $\frac{\mathbb{M}_1}{\mathbb{M}_0}=\frac{\beta}{1-\beta}$ in the political equilibrium, we obtain

$$\tau_1^*=\frac{(1-\gamma)(1-\beta)}{(1+\phi)}$$

$$\tau_2^*=\gamma$$

$$\tau_3^*=\frac{(1-\gamma)(1-\beta)(\phi-(\phi+\beta)(1-\gamma\sigma)\eta_{md})}{(1+\phi)(1-(1-\beta)(1-\gamma\sigma)\eta_{md})}$$

Appendix B: Politico-Economic Equilibrium with Voting at Time Zero and with $\tau_3 = 0$

We define the new utility function of the decisive voter

$$\mathbb{V}(\tau_1,\tau_2;k_{i,0},k_0)\equiv\sum_{t=0}^{\infty}\beta^t\ln\left(\mathbb{C}_{i,0}\Theta^t\right)=\sum_{t=0}^{\infty}\beta^t\ln\mathbb{C}_{i,0}+\sum_{t=0}^{\infty}t\beta^t\ln\Theta$$

where $\mathbb{C}_{i,0} = \left(a\widetilde{c}_{i,0}^{\rho}+(1-a)(\tau_1 A^{\frac{1}{\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}k_0)^{\rho}\right)^{\frac{1}{\rho}}$; $\widetilde{c}_{i,0} = (1-\tau_1-\tau_2)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}(1-\gamma\sigma)k_{i,0}-\widetilde{\Theta}k_{i,0}$; and $\widetilde{\Theta}=\beta(1-\tau_1\tau_2)(1-\gamma\sigma)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}$.

The first-order conditions are

$$\mathbb{M}_0\frac{\partial\ln\mathbb{C}_0}{\partial\tau_1}+\mathbb{M}_1\frac{\partial\ln\widetilde{\Theta}}{\partial\tau_1}=0$$

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_2} + \mathbb{M}_1 \frac{\partial \ln \widetilde{\Theta}}{\partial \tau_2} = 0$$

After the corresponding substitutions, we get

$$ac_{i,0}^{\rho-1}\left((\beta-1)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right) + (1-a)g_0^{\rho-1} - \frac{\beta}{1-\beta}\mathbb{C}_{i,0}^{\rho}\frac{1}{\widetilde{\Psi}k_0} = 0$$

$$ac_{i,0}^{\rho-1}\left[\widetilde{\mu}\left((1-\beta)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right) + \frac{\tau_3}{\tau_2}\frac{\gamma}{1-\gamma}\right] + (1-a)g_0^{\rho-1}\frac{\tau_1}{\tau_2}\frac{\gamma}{1-\gamma} + \frac{\beta}{1-\beta}\mathbb{C}_{i,0}^{\rho}\frac{\widetilde{\mu}}{\widetilde{\Psi}k_0} = 0$$

where $\widetilde{\Psi} = (1-\tau_1-\tau_2)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}$ y $\widetilde{\mu} = \left(\frac{(1-\tau_1-\tau_2)}{\tau_2}\frac{\gamma}{1-\gamma} - 1\right)$.

With a little algebra, we get

$$\frac{a}{1-a}\left(\frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}}(1-\gamma\sigma)(1-\beta)\eta_{md}\right)^{\rho} = \frac{(1-\beta)(1-\gamma)}{\tau_1^{**}} - 1$$

$$\tau_2^{**} = \gamma$$

Appendix C: Differentiation of the equation that characterizes the relationship between τ_1^{**} and η_{md}

Given equation $\frac{a}{1-a}\left(\frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}}(1-\gamma\sigma)(1-\beta)\eta_{md}\right)^{\rho} = \frac{(1-\beta)(1-\gamma)}{\tau_1^{**}} - 1$, the left-hand side is always positive because parameters a, β, γ, σ y τ_1 belong to $[0,1]$. Given the latter, the value of τ_1^{**} must satisfy the following condition

$$\tau_1^{**} < (1-\beta)(1-\gamma).$$

Now we differentiate the equation that characterizes τ_1^{**} and obtain

$$\frac{\partial \tau_1^{**}}{\partial \eta_{md}} = \frac{\frac{a}{1-a}\rho\left(\frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}}\right)^{\rho}\left((1-\gamma\sigma)(1-\beta)\right)^{\rho}(\eta_{md})^{\rho-1}}{-\frac{(1-\beta)(1-\gamma)}{(\tau_1^{**})^2} + \frac{a}{1-a}\frac{\rho(1-\gamma)}{(\tau_2^{**})^2}\left(\frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}}\right)^{\rho-1}\left((1-\gamma\sigma)(1-\beta)\right)^{\rho}(\eta_{md})^{\rho}}$$

The denominator of $\frac{\partial \tau_1^{**}}{\partial \eta_{md}}$ can be re-written taking the equation that characterizes τ_1^{**} and we get

$$\frac{\partial \tau_1^{**}}{\partial \eta_{md}} = -\frac{\frac{a}{1-a}\rho\left(\frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}}\right)^{\rho}\left((1-\gamma\sigma)(1-\beta)\right)^{\rho}(\eta_{md})^{\rho-1}(\tau_1^{**})^2(1-\tau_1^{**}-\gamma)}{(1-\gamma)\left[(1-\beta)(1-\gamma)(1-\rho) + (\rho-(1-\beta))\tau_1^{**}\right]}$$

The sign of the numerator depends on the value of ρ. Now let's analyze the sign of the denominator

$$(1-\beta)(1-\gamma)(1-\rho)+(\rho-(1-\beta))\tau_1^{**}$$

Case I: $(1-\beta)<\rho<1$, the first term of the denominator is positive and the term $\rho-(1-\beta)$ is also positive. Therefore the denominator is positive and $\frac{\partial \tau_1^{**}}{\partial \eta_{md}}<0$.

Case II: $0<\rho<\ (1-\beta)$, the first term of the denominator is positive and the term $\rho-(1-\beta)$ is negative. By contradiction, if $\tau_1^{**}>\frac{(\rho-1)}{(\rho-(1-\beta))}(1-\beta)(1-\gamma)$ then the denominator is negative. However, given that $\frac{(\rho-1)}{(\rho-(1-\beta))}>1$, this contradicts the restriction $\tau_1^{**}<(1-\beta)(1-\gamma)$. Therefore the denominator can not be negative and $\frac{\partial \tau_1^{**}}{\partial \eta_{md}}<0$.

Case III: $-\infty<\rho<0$, the first term of the denominator is positive and the term $\rho-(1-\beta)$ is negative. If $\tau_1^{**}<\frac{(\rho-1)}{(\rho-(1-\beta))}(1-\beta)(1-\gamma)$, the denominator and numerator are negative and therefore the $\frac{\partial \tau_1^{**}}{\partial \eta_{md}}>0$.

Appendix D: Politico-Economic Equilibrium with Sequential Voting.

The median voter faces the following problem in recursive version

$$v(k_{md},k)=\max_{\tau}\ln \mathbb{C}_{md}\left(c_{md},g_1\right)+\beta v(\mathcal{H}^{md}(k_{md},\tau),\mathcal{H}(k,\tau))$$

where $\mathcal{H}^{med}(k_{md},\tau)$ and $\mathcal{H}(k,\tau)$ are optimal decisions of gross saving of the average and median agent, respectively; c_{md} and g_1 are the consumption function of the median agent and the supply function of the public good, respectively. The latter functions depend on the political variable and the state variables

$$c_{md}=\psi_0(\tau)k+\psi_1(\tau)k_{md}-\mathcal{H}^{med}(k_{md},\tau)$$

and

$$g_1=\psi_2(\tau)k,$$

where $\psi_0(\tau)\equiv(1-\tau)A^{\frac{1}{\alpha}}\tau_2^{\frac{1-\alpha}{\alpha}}(1-\alpha+\frac{\tau_3}{1-\tau})$, $\psi_1(\tau)\equiv(1-\tau)\alpha A^{\frac{1}{\alpha}}\tau_2^{\frac{1-\alpha}{\alpha}}$, and $\psi_2(\tau)\equiv\tau_1 A^{\frac{1}{\alpha}}\tau_2^{\frac{1-\alpha}{\alpha}}$.

To obtain the generalized Euler equation (GEE) of the median voter, we derive the first-order conditions

$$\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j}+\beta v_k'\mathcal{H}_{\tau_j}+\beta v_{k_{md}}'\mathcal{H}_{\tau_j}^{md}=0 \qquad j=1,2,3.$$

Now we derive the Bellman equation with respect to k and k_{md}

$$v_k=\frac{\partial \ln \mathbb{C}_{md}}{\partial k}+\beta v_k'\mathcal{H}_k \qquad j=1,2,3.$$

$$v_{k_{md}} = \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} + \beta v'_{k_{md}} \mathcal{H}^{md}_{k_{md}} \qquad j = 1, 2, 3.$$

From the first-order conditions we obtain

$$\beta v'_k = \left(-\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} - \beta v'_{k_{md}} \mathcal{H}^{md}_{\tau_j} \right) \frac{1}{\mathcal{H}_{\tau_j}} \qquad j = 1, 2, 3.$$

Then v_k is

$$v_k = \frac{\partial \ln \mathbb{C}_{md}}{\partial k} - \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} - \beta v'_{k_{md}} \mathcal{H}^{md}_{\tau_j} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} \qquad j = 1, 2, 3.$$

From v_k and $v_{k_{md}}$ we get

$$\beta v'_{k_{md}} = \left(v_{k_{md}} - \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} \right) \frac{1}{\mathcal{H}^{md}_{k_{md}}} \qquad j = 1, 2, 3.$$

Now v_k can be expressed as

$$v_k = \frac{\partial \ln \mathbb{C}_{md}}{\partial k} - \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} - \left(v_{k_{md}} - \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} \right) \frac{\mathcal{H}^{md}_{\tau_j}}{\mathcal{H}^{md}_{k_{md}}} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} \qquad j = 1, 2, 3.$$

Shifting the latter equation one period forward

$$v'_k = \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau'_j} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} - \left(v'_{k_{md}} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \right) \frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \qquad j = 1, 2, 3.$$

Replacing the latter equation in the first-order conditions, we obtain

$$\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau'_j} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \right\} \mathcal{H}_{\tau_j} + ...$$

$$\beta v'_{k_{md}} \left(\mathcal{H}^{md}_{\tau_j} - \frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \mathcal{H}_{\tau_j} \right) = 0 \qquad j = 1, 2, 3.$$

From the latter equation we obtain an expression for $\beta v'_{k_{md}}$, and we replace in $v_{k_{md}}$ as follows

$$v_{k_{md}} = \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} - \left[\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau'_j} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \right\} \mathcal{H}_{\tau_j} \right] \times$$

$$\left\{\mathcal{H}^{md}_{\tau_j} - \frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}}\frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}}\mathcal{H}_{\tau_j}\right\}^{-1}\mathcal{H}^{md}_{k_{md}}. \quad j = 1, 2, 3.$$

Shifting the latter equation one period forward and replacing in the first-order conditions we get the GEE of the median voter

$$\Omega_j + \beta\left[\frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} - \Omega'_j \times \left\{\mathcal{H}^{md}_{\tau_j} - \frac{\mathcal{H}^{md}_{\tau''_j}}{\mathcal{H}^{md}_{k''_{md}}}\frac{\mathcal{H}_{k''}}{\mathcal{H}_{\tau''_j}}\mathcal{H}_{\tau_j}\right\}^{-1}\mathcal{H}^{md}_{k'_{md}}\right]\left(\mathcal{H}^{md}_{\tau_j} - \frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}}\frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}}\mathcal{H}_{\tau_j}\right) = 0 \quad j = 1, 2, 3.$$

where

$$\Omega_j \equiv \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} + \beta\left\{\frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau'_j}\frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}}\frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}}\frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}}\right\}\mathcal{H}_{\tau_j}$$

Case: Generalized Euler Equation with $\mathbf{F}(\mathbf{k_i}, \mathbf{k}) = \tau^*$

Now we derive a particular outcome of the GEE. Given that tax rate τ^* is constant, all variables grow at the same rate and we obtain

$$k' = \Theta(\tau^*)k,$$
$$c' = \psi_0(\tau^*)k + \psi_1(\tau^*)k_i - \Theta(\tau^*)k, and \;\text{ y}$$
$$\mathbb{C}'_i/\mathbb{C}_i = c'_i/c_i = \Theta(\tau^*).$$

The derivatives of functions $\mathcal{H}^i$ and $\mathcal{H}$ are

$$\mathcal{H}_{\tau^*_j} = \Theta_{\tau^*_j}k$$
$$\mathcal{H}^i_{\tau^*_j} = \Theta_{\tau^*_j}k_i$$
$$\mathcal{H}_{\tau^{*\prime}_j} = \Theta_{\tau^{*\prime}_j}k'$$
$$\mathcal{H}^i_{\tau^{*\prime}_j} = \Theta_{\tau^{*\prime}_j}k'_i$$
$$\mathcal{H}_{k'} = \mathcal{H}^i_{k'_i} = \Theta(\tau^{*\prime})$$

If we substitute the latter derivatives into the GEE, the GEE is reduced to the expression Ω_j since $\mathcal{H}^{md}_{\tau_j} - \frac{\mathcal{H}^{md}_{\tau'_j}}{\mathcal{H}^{md}_{k'_{md}}}\frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}}\mathcal{H}_{\tau_j} = 0$. This simplification is possible because the conjecture that the tax rate is constant allows us to deduce that individual and aggregate saving grow at the same rate and, therefore, it is redundant to consider k_i and k in the value function. The ratio k_i/k is constant for all t and, therefore, k is a linear transformation of k_i.

The GEE is

$$\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} \Theta_{\tau_j^*} k - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau_j^{*\prime}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k \right\} = 0$$

We derive the corresponding derivatives and obtain

$$\begin{aligned}
\frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} \Theta_{\tau_j^*} k &= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} k' (\mathbb{C}'_{md})^{-\rho} \left\{ a(c'_{md})^{\rho-1} \psi_0(\tau') + (1-a)(g'_1)^{\rho-1} \sigma(\tau') \right\} \\
&= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} (\mathbb{C}'_{md})^{-\rho} \left\{ a(c'_{md})^{\rho-1} \psi_0(\tau') k' + (1-a)(g'_1)^{\rho-1} \sigma(\tau') k' \right\}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k &= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} k'_{md} (\mathbb{C}'_{md})^{-\rho} \left\{ a(c'_{md})^{\rho-1} \psi_1(\tau') \right\} \\
&= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} (\mathbb{C}'_{md})^{-\rho} \left\{ a(c'_{md})^{\rho-1} \psi_1(\tau') k'_{md} \right\}
\end{aligned}$$

Therefore

$$\frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} \Theta_{\tau_j^*} k + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k = \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} (\mathbb{C}'_{md})^{-\rho} (\mathbb{C}'_{md})^{\rho} = \frac{\Theta_{\tau_j^*}}{\Theta(\tau)}$$

Moreover

$$\begin{aligned}
\frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau_j^{*\prime}} &= (\mathbb{C}'_{md})^{-\rho} \left\{ a(c'_{md})^{\rho-1} \frac{\partial \psi_0}{\partial \tau'} k' + (1-a)(g'_1)^{\rho-1} \frac{\partial \sigma}{\partial \tau'} k' \right\} \\
&= (\mathbb{C}_{md})^{-\rho} (\Theta(\tau))^{\rho} \Theta(\tau) (\Theta(\tau))^{\rho-1} \left\{ a(c_{md})^{\rho-1} \frac{\partial \psi_0}{\partial \tau'} k + (1-a)(g_1)^{\rho-1} \frac{\partial \sigma}{\partial \tau'} k \right\} \\
&= (\mathbb{C}_{md})^{-\rho} \left\{ a(c_{md})^{\rho-1} \frac{\partial \psi_0}{\partial \tau} k + (1-a)(g_1)^{\rho-1} \frac{\partial \sigma}{\partial \tau} k \right\} = \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*}
\end{aligned}$$

Finally, the GEE of the median voter is

$$(1-\beta) \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \frac{\Theta_{\tau_j^*}}{\Theta(\tau_j^*)} = 0.$$

Appendix E: Politico-Economic Equilibrium with Voting at Time Zero and Conditional Transfers

Agent i solves the following intertemporal problem

$$\max_{\{c_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,t}(c_{i,t}, g_{1,t})$$

subject to

$$c_{i,t} + k_{i,t+1} = (1-\tau_g)\, r_t k_{i,t} + g^i_{3,t}, \text{ dado } k_{i,0},$$

Now transfers g^i_3 are financed by a linear tax rate τ_3 (all agents pay without exception) and those resources are distributed only to the poor. The poor are those endowed with a capital stock below mean, $k_i \leqslant k$. Furthermore, the distribution of transfers to the poor will be inversely proportional to k_i/k. Transfers must satisfy the following condition

$$g^i_{3,t} = g\left(1 - \frac{k_{i,t}}{k_t}\right), \text{where } \ g > 0$$

Tax revenue should be equal to transfers delivered, i.e., $\tau_3 y_t = \int_0^k g(1-s)f(s)ds$. Assuming that ratio $\eta = \frac{k_i}{k}$ follows a Fisk or log-logistic distribution with scale parameter $\alpha = \eta_{md}$ and shape parameter $\beta = 1$. In this case, the probability distribution and cumulative distribution functions are $f(s) = \frac{1}{(\eta_{md}+s)^2}$ and $F(s) = \frac{s}{\eta_{md}+s}$, respectively. Transfers are equal to

$$\begin{aligned}\int_0^k g(1-s)\frac{1}{(\eta_{md}+s)^2}ds &= g\left\{\int_0^k \frac{1}{(\eta_{md}+s)^2} - \int_0^k \frac{s}{(\eta_{md}+s)^2}\right\} ds \\ &= g\left\{\frac{1}{\eta_{md}} + log\left(\frac{\eta_{md}}{1+\eta_{md}}\right)\right\} = gB(\eta_{md})\end{aligned}$$

Given that transfers must be equal to tax revenue then $g = \tau_3 y_t (B(\eta_{md}))^{-1}$. Finally, transfers are determined by

$$g^i_{3,t} = \tau_3 y_t (B(\eta_{md}))^{-1}\left(1 - \frac{k_{i,t}}{k_t}\right)$$

As shown in subsection 2.2, marginal productivity of capital is $A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}(1-\gamma\sigma)$ and production per capita is $y_t = A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}k$.

Each agent solves his optimization problem taking as given equilibrium levels of public spending, tax rates, and factor prices. The Euler equation for each agent i is

$$\Theta \equiv \frac{c_{i,t+1}}{c_{i,t}} = \beta\left((1-\tau_g)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}(1-\gamma\sigma) - \tau_3 A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}(B(\eta_{md}))^{-1}\right).$$

Since we are dealing with a model of endogenous growth, Θ reflects that growth will be affected by transfers unlike the model where transfers are *lump sum* (See Appendix A).

Replacing the Euler equation, government expenditure and marginal productivity of capital in the agent's budget constraint, we obtain

$$c_{i,t} = (\Psi - \Theta)k_t$$

where $\Psi \equiv (1-\tau_g)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}\left((1-\gamma\sigma)\eta_i + \frac{\tau_3}{(1-\tau_g)}(B(\eta_{md}))^{-1}(1-\eta_i)\right)$.

The competitive economic equilibrium allows us to write the utility function of the agent depending on Θ and $\mathbb{C}_0$. Therefore, the problem faced by the decisive agent is the following

$$\max_{\tau_1,\tau_2,\tau_3} \mathbb{V}(\tau_1,\tau_2,\tau_3;k_{i,0},k_0) \equiv \sum_{t=0}^{\infty}\beta^t \ln\left(\mathbb{C}_{i,0}\Theta^t\right) = \sum_{t=0}^{\infty}\beta^t \ln \mathbb{C}_{i,0} + \sum_{t=0}^{\infty} t\beta^t \ln\Theta$$

where $\mathbb{C}_{i,0} = \left(a\left((\Psi-\Theta)k_0\right)^\rho + (1-a)(\tau_1 A^{\frac{1}{\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}k_0)^\rho\right)^{\frac{1}{\rho}}$.

Let $\mathbb{M}_0 = \sum_{t=0}^{\infty}\beta^t$ and $\mathbb{M}_1 \sum_{t=0}^{\infty} t\beta^t$ then the first-order conditions are

$$\mathbb{M}_0\frac{\partial \ln \mathbb{C}_0}{\partial \tau_1} + \mathbb{M}_1\frac{\partial \ln\Theta}{\partial\tau_1} = 0$$

$$\mathbb{M}_0\frac{\partial \ln \mathbb{C}_0}{\partial \tau_2} + \mathbb{M}_1\frac{\partial \ln\Theta}{\partial\tau_2} = 0$$

$$\mathbb{M}_0\frac{\partial \ln \mathbb{C}_0}{\partial \tau_3} + \mathbb{M}_1\frac{\partial \ln\Theta}{\partial\tau_3} = 0$$

Given $\mathbb{C}_{i,0}$, Θ and Ψ, we obtain

$$\frac{\partial c_{i,0}}{\partial\tau_1} = A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}k_0\left[(\beta-1)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right]$$

$$\frac{\partial g_{1,0}}{\partial\tau_1} = A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}k_0$$

$$\frac{\partial c_{i,0}}{\partial\tau_2} = A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}k_0\left[\left(\frac{(1-\tau_1-\tau_2-\tau_3)}{\tau_2}\frac{\gamma}{1-\gamma}-1\right)\left((1-\beta)(1-\gamma\sigma)\frac{k_{i,0}}{k_0}\right)+\frac{\gamma}{1-\gamma}\frac{\tau_3}{\tau_2}\right]$$

$$\frac{\partial g_{1,0}}{\partial\tau_2} = A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}k_0\left[\frac{\gamma}{1-\gamma}\frac{\tau_1}{\tau_2}\right]$$

$$\frac{\partial c_{i,0}}{\partial \tau_3} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[(B(\eta_{md}))^{-1} (1-(1-\beta)) \frac{k_{i,0}}{k_0} + (\beta - 1)(1-\gamma\sigma) \frac{k_i}{k} \right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_3} = 0$$

$$\frac{\ln \mathbb{C}_0}{\partial \tau_j} = \frac{1}{\mathbb{C}_0} \left(a c_{i,0}^{\rho-1} \frac{\partial c_{i,0}}{\partial \tau_j} + (1-a) g_0^{\rho-1} \frac{\partial g_0}{\partial \tau_j} \right), \ para\ j = 1, 2\ y\ 3.$$

$$\frac{\partial \ln \Theta}{\partial \tau_1} = \frac{-(1-\gamma\sigma) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}{\left((1-\tau_1-\tau_2-\tau_3)(1-\gamma\sigma) - \tau_3 (B(\eta_{md}))^{-1}\right) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

$$\frac{\partial \ln \Theta}{\partial \tau_2} = \frac{\left[\left(\frac{(1-\tau_1-\tau_2-\tau_3)}{\tau_2} \frac{\gamma}{1-\gamma} - 1 \right) (1-\gamma\sigma) - \frac{\gamma}{1-\gamma} \frac{\tau_3}{\tau_2} (B(\eta_{md}))^{-1} \right] A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}{\left((1-\tau_1-\tau_2-\tau_3)(1-\gamma\sigma) - \tau_3 (B(\eta_{md}))^{-1}\right) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

$$\frac{\partial \ln \Theta}{\partial \tau_3} = \frac{-\left\{ (1-\gamma\sigma) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} + (B(\eta_{md}))^{-1} \right\}}{\left((1-\tau_1-\tau_2-\tau_3)(1-\gamma\sigma) - \tau_3 (B(\eta_{md}))^{-1}\right) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

Substituting the latter equations in the first-order conditions, we obtain

$$a c_{i,0}^{\rho-1} \left((\beta-1)(1-\gamma\sigma) \frac{k_{i,0}}{k_0} \right) + (1-a) g_0^{\rho-1} - \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^{\rho} \frac{(1-\gamma\sigma)}{\left((1-\tau_g)(1-\gamma\sigma) - \tau_3 (B(\eta_{md}))^{-1}\right) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0} = 0$$

$$a c_{i,0}^{\rho-1} \left[\mu \left((1-\beta)(1-\gamma\sigma) \frac{k_{i,0}}{k_0} \right) + \frac{\tau_3}{\tau_2} \frac{\gamma}{1-\gamma} (B(\eta_{md}))^{-1} \left(1 - (1-\beta) \frac{k_i}{k} \right) \right] +$$

$$(1-a) g_0^{\rho-1} \frac{\tau_1}{\tau_2} \frac{\gamma}{1-\gamma} + \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^{\rho} \frac{(1-\gamma\sigma)\mu - \frac{\tau_3}{\tau_2} (B(\eta_{md}))^{-1}}{\left((1-\tau_g)(1-\gamma\sigma) - \tau_3 (B(\eta_{md}))^{-1}\right) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0} = 0$$

$$a c_{i,0}^{\rho-1} \left(((B(\eta_{md}))^{-1} (1-(1-\beta) \frac{k_{i,0}}{k_0}) + (\beta-1)(1-\gamma\sigma) \frac{k_{i,0}}{k_0} \right) -$$

$$\frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^{\rho} \frac{(1-\gamma\sigma) + (B(\eta_{md}))^{-1}}{\left((1-\tau_g)(1-\gamma\sigma) - \tau_3 (B(\eta_{md}))^{-1}\right) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0} = 0$$

where $\mu = \left(\frac{(1-\tau_1-\tau_2-\tau_3)}{\tau_2} \frac{\gamma}{1-\gamma} - 1 \right)$.

After some algebra and considering that $\frac{\mathbb{M}_1}{\mathbb{M}_0} = \frac{\beta}{1+\beta}$ (see appendix A), we obtain the following

$$\tau_1^* = \frac{(1-\gamma)(1-\beta)}{\beta(1+\phi)}\left(1+(1+\beta)\frac{\zeta+\phi\nu^{\frac{\rho}{1-\rho}}}{((1-\gamma\sigma)+(B(\eta_{md}))^{-1})\beta(1+\phi)}\right)^{-1}$$

$$\tau_2^* = \gamma$$

$$\tau_3^* = \frac{(1-\gamma)\phi(1+\beta)\nu^{\frac{1}{1-\rho}}\left(1+(1+\beta)\frac{\zeta+\phi\nu^{\frac{\rho}{1-\rho}}}{((1-\gamma\sigma)+(B(\eta_{md}))^{-1})\beta(1+\phi)}\right)^{-1}}{\beta(1+\phi)}+$$

$$(1-\gamma)(1-\beta)(1-\gamma\sigma)\frac{k_i}{k}\left(\frac{\left(1+(1+\beta)\frac{\zeta+\phi\nu^{\frac{\rho}{1-\rho}}}{((1-\gamma\sigma)+(B(\eta_{md}))^{-1})\beta(1+\phi)}\right)^{-1}}{\beta(1+\phi)}-1\right)$$

where $\phi \equiv \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}}$, $\chi \equiv (B(\eta_{md}))^{-1}(1-(1-\beta)\frac{k_i}{k}) - (1-\beta)(1-\gamma\sigma)\frac{k_i}{k}$,
$\nu \equiv (1-\beta)(1-\gamma\sigma)\frac{k_i}{k} + \chi\frac{(1-\gamma\sigma)}{(1-\gamma\sigma)+(B(\eta_{md}))^{-1}}$, y $\zeta \equiv \frac{(1-\beta)(1-\gamma\sigma)\frac{k_i}{k}+\chi(1-\gamma\sigma)}{(1-\beta)(1-\gamma\sigma)\frac{k_i}{k}+\chi\frac{(1-\gamma\sigma)}{(1-\gamma\sigma)+(B(\eta_{md}))^{-1}}}$.

For every unit of tax revenue, the amount allocated to productive services is equal to γ.

Appendix F: Politico-Economic Equilibrium with Voting at Time Zero under Constant Returns of Capital

In this context, we face an endogenous growth model a la Romer (1986). The production function for a representative firm is

$$y_t = Ak^{\alpha}\bar{k}^{1-\alpha},$$

where $\bar{k}$ is the aggregate stock of capital and Romer asumes that it is proportional to aggregate stock of knowledge. There are only two types of public expenditures: $g_1 = \tau_1 y_t$ is public good consumption and $g_3 = \tau_3 y_t$ are lump-sum transfers distributed to all individuals.

Agent i solves the following intertemporal problem

$$\max_{\{c_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,t}(c_{i,t}, g_{1,t})$$

subject to

$$c_{i,t} + k_{i,t+1} = (1-\tau_g)(w_t + r_t k_{i,t}) + g_{3,t}, \text{ given } k_{i,0},$$

where the agent's income is taxed at a linear tax $t_g = \tau_1 + \tau_3$.

Since firms operate in competitive factor market, factor prices are determined by

$$w_t = (1-\alpha)A\bar{k}$$
$$r_t = \alpha A$$

The Euler equation for each agent is

$$\Theta = \beta(1-\tau_1-\tau_3)\alpha A$$

Replacing the Euler equation in the budget constraint, we get

$$c_{i,t} = (1-\tau_1-\tau_3)A\bar{k}\left((1-\alpha)+\alpha\frac{k_i}{k}+\frac{\tau_3}{1-\tau_1-\tau_3}\right)-\Theta k_i$$

The utility function of the decisive voter is

$$\mathbb{V}(\tau_1,\tau_3;k_{i,0},k_0) \equiv \sum_{t=0}^{\infty}\beta^t \ln\left(\mathbb{C}_{i,0}\Theta^t\right) = \sum_{t=0}^{\infty}\beta^t \ln \mathbb{C}_{i,0} + \sum_{t=0}^{\infty} t\beta^t \ln\Theta$$

where $\mathbb{C}_{i,0} = \left(ac_{i,0}^{\rho}+(1-a)(\tau_1 A\bar{k}_0)^{\rho}\right)^{\frac{1}{\rho}}$; $c_{i,0} = (1-\tau_1-\tau_3)A((1-\alpha)\bar{k}_0+\alpha k_{i,0}+\frac{\tau_3}{1-\tau_1-\tau_3}\bar{k}_0)-\Theta k_{i,0}$; and $\Theta = \beta(1-\tau_1-\tau_3)\alpha A$.

The first-order conditions are

$$ac_{i,0}^{\rho-1}\left(-(1-\alpha)-\alpha(1-\beta)\frac{k_i}{k}\right)+(1-a)g_{1,0}^{\rho-1}-\frac{\mathbb{M}_1}{\mathbb{M}_0}\mathbb{C}_{i,0}^{\rho}\frac{\alpha}{((1-\tau_1-\tau_3)\alpha A+1)\bar{k}_0}=0$$

$$ac_{i,0}^{\rho-1}\alpha\left(1-(1-\beta)\frac{k_i}{k}\right)-\frac{\mathbb{M}_1}{\mathbb{M}_0}\mathbb{C}_{i,0}^{\rho}\frac{\alpha}{((1-\tau_1-\tau_3)\alpha A+1)\bar{k}_0}=0$$

Given that $\frac{\mathbb{M}_1}{\mathbb{M}_0} = \frac{\beta}{1+\beta}$ (see appendix A) and after the corresponding substitutions, we obtain

$$\tau_1^* = \frac{1-\beta}{1+\phi},$$

$$\tau_3^* = 1-\frac{\beta}{\alpha\epsilon_i}-\tau_1^*,$$

where $\epsilon_i \equiv 1-(1-\beta)\frac{k_{i,0}}{k_0}$.