Foster Care: A Dynamic Matching Approach *

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Abstract

This paper studies the two-sided, dynamic matching problem that occurs in the US foster care system. In this market, foster parents and foster children can form reversible foster matches, which may disrupt, continue in a reversible state, or transition into permanency via adoption. I first present an empirical analysis that yields four new stylized facts related to match transitions of children in foster care and their exit through adoption. Thereafter, I develop a two-sided dynamic matching model with five key features: (a) children are heterogeneous (with and without a disability), (b) children must be foster matched before being adopted, (c) children search for parents while foster matched to another parent, (d) parents receive a smaller per-period payoff when adopting than fostering (capturing the presence of a financial penalty on adoption), and (e) matches differ in their quality. I use the model to derive conditions for the stylized facts to arise in equilibrium and carry out predictions regarding match quality. An interesting insight is that the intrinsic disadvantage (being less preferred by foster parents) faced by children with a disability exacerbates due to the penalty. Moreover, I show that foster parents in high-quality matches (relative to foster parents in low-quality matches) might have fewer incentives to adopt.

JEL Classification. C78; D83

Keywords. Dynamic Matching · Reversible Matching · Sorting · Adoption · Foster Care

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1 Introduction

Each year more than a half-million children spend at least one day in the US foster care system, a federal program that costs taxpayers almost US$30 billion dollars annually. The foster care system provides out-of-home care for children removed from their homes due to abuse, maltreatment, neglect, or other reasons.[1] While in foster care, children are placed in foster family homes or institutional care and can be moved from one foster home to another or from a foster home to institutional care.[2] The stay in foster care is meant to be temporary until children can reunite with their birth families, but when reunification is not possible, children are relinquished for adoption.[3] In practice, each year, close to 18% of children in foster care are at risk of experience long-term care if they are not adopted. In fact, more than $20,000$ children leave foster care each year without an adoptive family, and out of those children, less than 3% will earn a college degree, and almost 20% will become homeless.[4]

Foster care can be viewed as a two-sided matching market, where foster parents have preferences over children, and social workers have preferences over foster parents (on behalf of children). In this market, foster parents and foster children can form reversible foster matches, which may disrupt, continue in a reversible state, or transition into permanency via adoption. Moreover, foster parents and foster children meet randomly over time, making matching decisions inherently dynamic. A crucial aspect of this environment is that foster matches are reversible, and so agents not only decide with whom to form a match but also which matches to disrupt. This introduces a new feature in the match decision process (absent in environments with only irreversible matches): agents must take into account that a partner might leave the foster match in the future.

For policymakers, the main concerns regarding the foster care system are match disruptions and permanency via adoption. First, research has shown that match disruptions have adverse effects on children, and it has become a priority for child welfare agencies to limit the match disruptions experienced by children.[5] And, since evidence suggests that adoption is a better

[1] A child can enter foster care for several reasons such as sexual or physical abuse, parents' drug or alcohol addictions, parents' incarceration, parents' inability to provide care, parents' death, inadequate housing, abandonment, child's behavioral problem, child's drug addiction, or child's alcohol addiction.

[2] Foster homes are private homes licensed to provide 24-hour care for children in a family-based environment. Institutional care are licensed facilities that provide 24-hour care for several children at once (groups from seven to twenty), and it includes group homes, shelter care, and other institutions.

[3] By federal law, if a child has been in foster care for at least 15 of the last 22 months, the process to terminate her parental rights must be started immediately. Further, a judge can decide to terminate parental rights at any moment in time if it is in the best interest of the child.

[4] Source: National Foster Youth Institute.

[5] Match disruptions experienced by children is part of the national outcome standards used by federal agencies to monitor the state's performance.

alternative than long-term foster care, policymakers had made significant efforts to increase the adoption rates of children through major federal policies.[6] However, my results suggest that limiting match disruptions might be counterproductive for the adoption goal: parents have incentives to foster a child indefinitely (without adopting) due to the presence of a financial penalty. First, the monthly payments received by parents (from the state child welfare agency) are lower as an adoptive parent than as a foster parent and often fall to zero. Second, parents are responsible for the medical and educational expenditures of adopted children. Thus, parents face the following trade-off when deciding to adopt: accept the adoption penalty in exchange for eliminating the likelihood that the child disrupts the match in the future. Hence, match disruptions play a crucial role in adoption by influencing the incentives of foster parents to adopt. The empirical literature supports this intuition; Argys and Duncan (2012) show that when the difference between the foster and adoption monthly payments decreases, the probability of adoption increases.

From a policy perspective, it is crucial to understand why certain children are more likely to have their matches disrupted and why certain children are less likely to be adopted. Besides, the presence of the adoption penalty might have a different effect on certain children, and it might influence the type of matches that transit to adoption (in terms of match quality). I distinguish children by whether they have a disability and study how this affects match disruption and adoption. I focus on disability for two reasons. First, most of the efforts made to increase adoption target children with a disability. Second, the adoption penalty might be higher for children with a disability as parents are responsible for potentially higher medical expenditures. Nevertheless, the model can be used to study the effect of other observable characteristics of the child, such as gender, race, and ethnicity.

This paper studies both, theoretically and empirically, the two-sided dynamic matching problem that occurs in the US foster care system. First, I present an empirical analysis that yields four new stylized facts related to match transitions of children in foster care and their exit through adoption. Second, I develop a two-sided dynamic matching model to disentangle the driving forces behind the stylized facts and derived other equilibrium properties. The main finding is that the presence of the financial penalty on adoption exacerbates the intrinsic disadvantage faced by children with a disability (being less preferred by parents), and it also creates incentives for high-quality matches to not transit from a reversible fostering to adoption.

Using a rich panel dataset, describing the universe of children relinquished for adoption in

[6] The Adoption and Safe Families Act of 1997 (ASFA), created the Adoption Incentive Program, which establishes performance bonuses to states that increase the adoption of children. The performance bonuses consisted of US$4,000 dollars per child plus an additional US$2,000 if the child has special needs (including disability). Later on, the Increasing Adoptions Act of 2008 increased the extra bonus to US$4,000 if the child has special needs.

the US foster care system over the period 2010 to 2016, I document the following stylized facts: (*1*) the presence of a disability decreases the probability that a child transits to permanency via adoption (becomes adoption matched), (*2*) the presence of a disability increases the probability that a foster placement is disrupted (foster match disruption), (*3*) the presence of a disability decreases the probability that a child transits from institutional care to a foster home (becomes foster matched), and (*4*) the presence of a disability increases the probability that a child transits from a foster home to institutional care (becomes unmatched).

To analyze how different forces interact in the agents' decisions of forming a foster match, disrupting a foster match, and transiting to permanency via adoption, I develop a dynamic matching model with search frictions (it takes time to find a match) and non-transferable utility (transfers are exogenously given). Children and parents can form two types of matches: foster (reversible) or adoption (irreversible). The setting assumes that (*a*) children are heterogeneous (with and without a disability), (*b*) agents must be foster matched before forming an adoption match, (*c*) parents receive a smaller per-period payoff when adoption matched than when foster matched, and (*d*) matches differ in their quality. Children and parents prefer matches of higher quality, and parents prefer children without a disability to children with a disability. The timing is as follows. Every period, when a child (unmatched or foster matched) and parent meet (unmatched only), agents draw a match quality. Before deciding whether to form a foster match, they observe only a noisy signal about this quality. A foster match forms if and only if both accept. If a new foster match forms, any old foster match dissolves. The uncertainty about the quality resolves once foster match forms, and it remains constant throughout the match. After observing the match quality, agents decide whether to destroy the foster match (and become unmatched), transit to an adoption match, or remain foster matched.

The model allows me to disentangle the driving forces behind the aforementioned stylized facts. More concretely, I establish sufficient conditions on primitives for these facts to emerge in equilibrium. One of the key features captured by the model is that a foster disruption can be the result of a *destruction* (child and parent become unmatched after the uncertainty resolves), or it can be the result of a *dissolution* (child forms a new foster match). Thus, foster match disruptions allow agents to avoid 'bad matches', and more importantly, it enables children to search for 'better matches' while in a foster environment. Concerning the child's observable characteristics, I find that foster disruptions involving children with a disability are mainly driven by destruction due to the uncertainty on the quality of the match, while foster disruptions affecting children without a disability are driven mostly by dissolution to improve the match quality. Moreover, I show that the increase in the probability of foster match disruption due to a disability (stylized fact 2) depends on two driving forces working in opposite directions. On the one hand, children with a disability are more likely (relative to children without a disability) to have a foster match destroyed, which itself makes them more likely to disrupt. On the other hand, I find that children with a disability are less likely (relative to children without a disability) to dissolve a foster match, which itself makes them less likely to disrupt. Hence, stylized fact 2 suggests that the former driving force prevails.

Another important insight of the model is that the decrease in the probability of becoming adoption matched due to a disability (stylized fact 1) arises for two reasons. First, I show that children with a disability are less likely to form a foster match because foster parents require higher signals to be willing to form a foster match with them. Second, parents foster matched to these children (relative to parents foster matched to children without a disability) have a greater incentive to remain in the reversible foster match and not transit to adoption. The reason is that the adoption penalty for children with a disability is higher, and the likelihood that they dissolve the match in the future is lower. Thus, the intrinsic disadvantage (being less preferred by foster parents) faced by children with a disability exacerbates in the presence of the adoption penalty.

Furthermore, the model allows me to obtain additional predictions that contribute to understanding the mechanics behind the match transitions and adoption of children. In particular, I analyze the impact of match quality on the probability of foster match disruption, the probability of becoming unmatched, and the probability of becoming adoption matched. Concerning foster match disruptions, I find that high-quality matches are less likely to disrupt. In this case, the driving forces of destruction and dissolution are aligned. Both the probability of destruction and the probability of dissolution are decreasing in the match quality. Surprisingly, I find that parents in high-quality matches might have fewer incentives to adopt. The result is driven by the fact that children in foster matches of high-quality have fewer incentives to dissolve the foster match in the future. Hence, the adoption penalty not only exacerbates the intrinsic disadvantage faced by children with a disability, but it also creates incentives for high-quality matches to not transit to adoption.

Related Literature. As a first attempt to analyze the foster care system, this paper contributes to the literature on dynamic matching and research on foster care. Most of the literature on dynamic matching with heterogeneous agents, analyze environments where matches do not reverse endogenously. Under this assumption, the literature has addressed issues regarding stability [Doval (2019)], matching algorithms and its implications on welfare [Üner (2010), Anderson et. al. (2015), Leshno (2017), Akbarpour et.al. (2019), Baccara et. al. (2019)], and positive assortative matching [Burdett and Coles (1997), Eeckhout (1999), Shimer and Smith (2000), Chade (2001), Chade (2006), Smith (2006)]. In these papers, agents face the tradeoff of whether form a match today or wait for a better partner. Now, if agents are allowed to form a match today and reverse it when a better partner arrives, an additional feature arises. In the presence of reversibility, agents must take into account that today's partner and the potential better partner of tomorrow might leave the match in the future. There is a small literature analyzing dynamic matching environments with reversibility of matches, but the focus is on stability and cooperative solution concepts [Damiano and Lam (2005), Kurino (2009), Kadam and Kotowski (2018), Liu (2018)]. This paper is more related to the literature on positive assortative matching analyzing two-sided markets with search frictions, heterogeneous agents, and irreversible matches. My project differs from the sorting research in two dimensions. First, I allow for irreversible and reversible matches. Second, instead of addressing positive sorting, I estimate stylized facts present on the market and establish sufficient conditions for these patterns to arise in equilibrium.

Regarding foster care, there is a vast literature analyzing the effect of children' characteristics on, placement disruption and adoption using a reduced-form approach [Wulczyn et.al. (2003), James (2004), Courtney and Wong (1996), Barth (1997), Snowden et. al. (2008)]. A few papers are studying the effect of the number of placements on adoption from a quantitative perspective. The results are ambiguous. Some studies found that the number of placements does not affect the probability of being adopted [Potter and Klein-Rothschild (2002), Park and Ryan (2009)]. On the contrary, other studies show that the number of placements negatively affect adoption [Smith (2003), Akin (2011)]. This paper contributes by proving a theoretical environment that analyzes how different forces interact in the agents' decisions of forming a foster-match, disrupting a foster-match, and transiting to permanency via adoption.

Organization of the Paper. The rest of the paper is organized as follows. Section 2 presents a background on the foster care system in the US. Section 3 estimates the stylized facts. Section 4 describes the theoretical environment. Section 5 introduces the recursive formulation and the equilibrium definition. Section 6 presents an equilibrium analysis. Section 7 presents the model predictions and the stylized facts in the theoretical model. Lastly, section 8 concludes. All proofs are in the appendix.

2 Background: Foster Care in the US

2.1 Overview

The foster care program is authorized by title IV-E of the Social Security Act and implemented under the Code of Federal Regulations. All states are eligible to participate in the program and receive federal funding. According to Rosinsky and Connelly (2016), the national spending on child welfare in 2014 was approximately \$29.1 billion dollars, out of which \$12.8 billion was federally funded, and the remaining was financed directly by states.[7] In practice, 47% of the national spending was destined to out-of-home placement expenditure, including monthly payments to foster parents and their training; and 17% was intended to finance adoption and guardianship programs, including monthly payments to adoptive parents and adoption fees.

Researchers and child welfare agencies have focused their attention on three significant issues: children's placements while in foster care, children' exit from foster care through adoption, and placement disruption.

Foster Homes and Institutional Care. Foster parents provide the highest source of out-of-home care.[8] For example, at the end of the federal fiscal year of 2014, the number of children in foster care was 415,129, out of which 79% were placed with foster parents, and 14% were placed in institutional care [U.S. DHHS (2014)]. Federal and state child welfare agencies have a strong preference for foster homes over institutional care. Furthermore, the Adoption Assistance and Child Welfare Act of 1980 (AACWA) requires children to be placed in the most family-like placement when possible. Research supports this preference. On the one hand, evidence shows that institutional care is between six to ten times more expensive than foster family homes [Barth (2002)]. Furthermore, research suggests that children placed with foster parents exhibit better short and long-run outcomes. In particular, children placed in institutional care have lower academic outcomes, lower levels of education, higher risk to engage in delinquent behavior, and a higher risk of criminal convictions when adults [Berrick et. al. (1993), Mech. et. al. (1994), Ryan et. al. (2008), Dregan and Gulliford (2012)]. In conclusion, institutional care is not only more expensive but also unable to support the healthy development of children.

Adoption and Long-term Care. At the end of the federal fiscal year of 2014, 18% of children in foster care had their parental rights terminated and waited to be adopted, out of which 41% were adopted [U.S. DHHS (2014)]. Research suggests that adoption is a better alternative to long-term care. First, maintaining a child in long-term care is more expensive than adoption [Barth (1993), Barth et. al. (2006), Hansen (2008)]. Second, adoption generates better outcomes for children. For example, Triseliotis (2002) and Hansen (2008) show that children who are adopted exhibit better social and educational outcomes. Since adoption from foster care is a major concern for policy markets, laws have been enacted to increase adoption. In particular,

[7] Federal fund sources include Title IV-E and Title IV-B of the Social Security Act, Medicaid, Social Services Block Grant, Temporary Assistance for Needy Families, and other federal grants and awards.

[8] Foster homes are divided in relative and non-relative. In a relative foster home, the foster parent is a relative or someone with a prior connection to the child who joins the program to care for a particular child. In a non-relative foster-home, the foster parent joins the program without prior connection to any child and later on is matched to a child to care for.

AACWA created the Adoption Assistance Program, which mandates states to make adoption assistance payments to parents who adopt children with special needs, including disability.[9]

Placement Disruption. Research shows that an increase in the number of placements can delay academic skills formation, increase problematic behavior among children, and increase the risk of delinquency among male children [Zima et. al. (2000), Newton et. al. (2000), Ryan and Testa (2005)]. At the end of the federal fiscal year of 2014, children exhibited in average 2.7 placements for a single foster care episode.[10] This is above the ideal number set by the Children's Bureau that defines adequate placement stability as limiting the number of placements for a child to no more than two for a single foster care episode. This paper focuses on the role that placement disruption plays on adoption, which has not been addressed in the literature.

2.2 Matching Process

Foster care is conducted and administered at the state level by Child Protective Services (CPS).[11] When an allegation concerning a child's well-being is received, CPS assigns a social worker to the case and initiates an investigation. If sufficient evidence supporting an accusation is found, the case is presented to a juvenile or family court, where a judge decides whether the child is removed from her birth-family home and placed in foster care.[12]

In most states, decisions concerning children' placements are made by social workers.[13] On behalf of a child, the social worker (a) searches and contacts foster parents, (b) arranges a meeting between the foster parent and child in order to obtain information of whether the foster parent is a good fit for the child, and (c) decides where to place the child. A placement in a foster home must be mutually agreed upon between the foster parent and social worker. The social worker can switch a child from one foster home to another or from a foster home to institutional care. Similarly, foster parents can request the child's removal from their home. Adoptions must be mutually agreed upon between the foster parent and social worker. Once the child is adopted, she exits foster care. It is essential to mention that an increasing number of states require parents to foster a child before adopting.[14]

[9] AACWA states that 'a child with special needs is a child who: can not be returned to her birth-family home, has a special condition such that the child can not be placed for adoption without providing assistance, and has not been able to be placed for adoption without assistance'.

[10] A child can enter foster care multiple times, each time a child enters foster care is a different foster care episode.

[11] Also known as Department of Children and Families, Department of Children and Family Services or Department of Human Services.

[12] If the social worker believes that the child is in serious or imminent danger; she is allowed to execute an emergency removal without the court's approval. Yet, the decision must be later on approved by the judge.

[13] In a small fraction of states, placement decisions are made by judges from juvenile or family courts.

[14] For example, some states mandate that the child must reside in the foster home for at least six months before

Foster parents must hold a license to provide care for children. The licensing process includes a home study and training requirements. The home study ensures that the foster parent's house is clean, in good condition, and free from danger. The initial training (15 to 30 hours of mandatory classes) addresses topics such as agency policies and procedures, roles and responsibilities of foster parents, and behavior management. Also, most states require ongoing post-training to maintain the license.

Foster parents receive financial transfers when a child is placed on their care, which differ on whether the parent is fostering or adopting. While in foster, the parent receives financial payments for the duration of the placement. If the child is adopted, the parent gets monthly financial payments until the child reaches at least 18 years old. Each state has its payment scheme, but as a rule-of-thumb, foster parents who provide care for a child with higher needs receive higher payments and adoption payments are lower than foster payments.[15]

3 Empirical Analysis

In this section, I motivate the key features of the two-sided dynamic matching model described in the next section with empirical analysis. Using data describing the universe of children in the US foster care system over the period 2010 to 2016, I document four new stylized facts about the matching process between foster children and foster parents.

3.1 Data and Descriptive Statistics

I use the 6-month Foster Care File from AFCARS,[16] which contains an unbalanced panel of all children in foster care (all fifty US states and the District of Columbia) between the federal fiscal years of 2010 and 2016.

The data track a child upon entry into the foster care system until the child exits the system, which could be due to reunification with birth parents or other relatives, adoption, emancipation, guardianship, transfer to another agency, runaway, or death. If a child exits foster care, both the exit manner and date of exit are indicated. The data additionally include a rich set of variables describing the child,[17] such as gender, race, ethnicity, disability, whether the child is

foster parents can adopt.

[15]For more detail on payment schemes, see DeVooght, K. and Blazey, D. (2013)

[16]Adoption and Foster Care Analysis and Reporting System (AFCARS) is a federally mandated data collection system. States are required to collect data on all children in foster care and all children adopted from foster care. This dataset was made available by the National Data Archive on Child Abuse and Neglect at Cornell University.

[17]Following Buckles (2013) and Brehm (2017), for all demographics I use the most recent record of each child since it updates all information.

federally funded by Title IV-E,[18], date of birth, date of most recent entry into foster care, and date of termination of parental rights (if applicable). To protect the confidentiality of the child, the date of birth is set to the 15th of the month and all dates are recoded to maintain consistent spans of time. The disability variable, which is the focus of my empirical analysis, indicates whether a child has been clinically diagnosed with a disability, clinically diagnosed without a disability, or not yet diagnosed.[19] I define **disability** as follows: a child has a disability if she has been clinically diagnosed with at least one disability, and a child has no disability otherwise.

For each period (semester in the data) that a child remains in foster care, the data provide specific information about the last placement of the child during that period, including the start date of the placement. These placements are classified as: pre-adoptive home, non-relative foster home, relative foster home, group home, institution, supervised independent living, and runaway. Using these variables, I define a child as being **foster matched** in a given period if the child is placed in a pre-adoptive home, a non-relative foster home, or a relative foster home.[20] I define a child as being **unmatched** in a given period if the child is placed in a group home or institution.

To maintain a consistent estimation sample, I restrict the sample to children younger than age 16 whose parental rights have been terminated. The latter restriction is to ensure that children are eligible for adoption, and the former excludes older children who often exit through legal emancipation. I also restrict the sample to children who are either foster matched or unmatched. This leaves a full sample of $451,967$ children (sample A). Besides, I create two subsamples. The first subsample (sample B) keeps only those child-period observations such that the child is foster matched at the beginning of the period and still in foster care at the end of the period. The second subsample (sample C) keeps only those child-period observations such that the child is unmatched at the beginning of the period and still in foster care at the end of the period. Table 1 presents summary statistics for the full sample and the two subsamples. Appendix table A1 presents these summary statistics conditioned on, the variable of interest, child's disability.

In table 1 (sample A), children are, on average, almost 7 years old and have had their parental rights terminated for 17 months. Out of all children, 41 percent have been diagnosed with a disability. In a given period, 93 percent of children are foster matched, with the average duration of that match being 16 months. I say a child **becomes adoption matched** if she exits the system through adoption. On average, 28 percent of children become adoption matched in each pe-

[18] Title IV-E is a federal program through which states receive reimbursement of payments made on behalf of eligible children.

[19] A child can be diagnosed with more than one disability; unfortunately data do not allow to quantify either the number of disabilities nor the severity.

[20] It is important to mention that foster parents are not identifiable; when a child is placed in a foster home only family structure, foster parents' race and foster parents' year of birth are reported.

Table 1: Descriptive Statistics, All Samples

	Sample A		Sample B		Sample C	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Share becomes adoption matched	0.28	0.45	-	-	-	-
Share foster matched	0.93	0.25	1.00	0.00	0.00	0.00
Share becomes foster matched	-	-	-	-	0.24	0.42
Share becomes unmatched	-	-	0.02	0.14	-	-
Share disrupts foster match	-	-	0.19	0.39	-	-
Age in years	6.80	4.43	6.81	4.36	12.17	2.80
Share Disability	0.41	0.49	0.43	0.50	0.68	0.47
Share Male	0.53	0.50	0.52	0.50	0.63	0.48
Share White	0.43	0.50	0.42	0.49	0.44	0.50
Share Black	0.24	0.43	0.26	0.44	0.27	0.44
Share Hispanic	0.22	0.41	0.22	0.42	0.20	0.40
Share Receiving Title IV-E	0.48	0.50	0.51	0.50	0.47	0.50
Months in foster care	34.87	24.38	34.86	24.86	53.10	36.72
Months since termination of parental rights	17.09	22.62	16.23	22.15	41.99	36.86
Child-period observations ending in adoption	12.46	11.85	-	-	-	-
Months in current placement	16.06	15.72	17.31	16.32	10.85	13.99
Child-period observations foster matched	16.44	15.78	-	-	-	-
Number of child-period observations	1,165,818		659,253		65,970	
Number of children	451,967		312,028		24,783	

Notes: Data are from Adoption and Foster Care Analysis and Reporting System (AFCARS). Means and standard deviations are calculated for child-period observations. Sample A is the full sample containing all children younger than age 16 whose parental rights have been terminated and who are either foster matched or unmatched. Sample B and Sample C are subsamples of A. Sample B (sample C) keeps only those child-period observations such that the child is foster matched (unmatched) at the beginning of the period and still in foster care at the end of the period.

riod. I say a child **becomes unmatched** if conditional on being foster matched at the beginning of a period she is unmatched at the end of the same period. Conditional on starting the period foster matched (sample B), the probability that a child becomes unmatched is 2 percent. Now, I say a child **becomes foster matched** if conditional on being unmatched at the beginning of a period she is foster matched at the end of the same period. Conditional on starting the period unmatched (sample C), the probability that a child becomes foster matched is 24 percent. It is important to highlight that the probabilities at which children become foster matched or unmatched are affected by the rates at which foster matches are disrupted. I say a child's **foster match disrupts** if conditional on being foster matched at the beginning of a period the child is no longer foster matched to the same parent at the end of the period.[21] Table 1 (sample B) shows that foster matches disrupt with probability 19 percent. In practice, a disruption can arise for different reasons such as the social worker decides to move the child to institutional care, the parent requests the removal of the child, or the social worker finds a more suitable foster parent for the child and decides to move the child. Unfortunately, the dataset does not contain this information.

To evaluate these transitions conditional on the child's demographics, I specify an econometric framework in the next subsection that will provide a set of stylized facts that will motivate the key features of the two-sided dynamic matching model develop in section 4.

3.2 Empirical Specifications and Stylized Facts

I estimate the effect of disability on four outcomes: (a) the probability that a child becomes adoption matched, (b) the probability that a foster match disrupts, (c) the probability that a child becomes foster matched, and (d) the probability that a child becomes unmatched. For each outcome, I estimate the following linear probability model:

$$y_{ijt} = \alpha + \gamma\, disability_i + \beta X_i + \theta Z_{it} + \xi_j + \lambda_t + \epsilon_{ijt} \tag{1}$$

where y_{ijt} is an indicator for the outcome of child i in state j at period t. $disability_i$ is an indicator equal to one if child i has been clinically diagnosed with at least one disability and zero otherwise. X_i is a vector of time-invariant characteristics of child i such as gender, race, ethnicity, and whether the child is federally funded through Title IV-E. Z_{it} is a vector of time-varying characteristics of child i including age in months, number of months in foster care, and number of months since parental rights have been terminated. I include a vector of state fixed-effects, ξ_j, to control for unobserved state characteristics and a vector of period fixed-effects, λ_t, to control for time-trends.

[21] Even though, foster parents are not identifiable, a variable recording the number of placements allows me to identify whether the child is being fostered by the same parent.

Table 2: Stylized Facts from Foster Care: Effect of Disability

	Becomes Adoption matched (1)	**Foster matched (2)**	**Disrupts Foster match (3)**	**Becomes Foster matched (4)**	**Becomes Unmatched (5)**
Disability (γ)	-0.059***	-0.043***	0.023***	-0.045***	0.011***
	(0.005)	(0.002)	(0.002)	(0.006)	(0.001)
Mean of dependent variable	0.279	0.934	0.185	0.236	0.021
Number of child-period observations	1,165,818	1,165,818	659,253	65,970	659,253

Notes: Data are from Adoption and Foster Care Analysis and Reporting System (AFCARS). All specifications control for child's demographics, states indicators and period indicators. The first and second columns consider sample A, third and fifth columns use sample B, and the fourth column uses sample C. Standard errors are cluster at the state-period level and shown in parentheses. ***$P < 0.01$; **$P < 0.05$; *$P < 0.10$.

Stylized Fact 1: Disability Decreases the Probability of Becoming Adoption Matched. From the data, the rates of adoption match formation of children with and without a disability are 0.22 and 0.32, respectively (see Table A1). To evaluate whether this effect is significant conditional on other demographics, I use sample A to estimate equation (1) where the dependent variable y_{ijt} is equal to one if child i in state j is adopted in period t and zero if she either remains in foster care or exits through any other manner. Table 2 column 1 shows that children with a disability are less likely to become adoption matched than children without a disability. Specifically, I find that disability decreases the probability of becoming adoption matched by nearly 6 percent.

As many states require parents to foster a child before an adoption can take place, the fact that children with a disability are less likely to become adoption matched might be driven by the fact that these children are less likely to be foster matched in the first place. To analyze this, I estimate a version of equation (1) where the dependent variable y_{ijt} is redefined to take the value of one if child i in state j is foster matched in period t and zero otherwise. As in adoption, the coefficient on disability is negative (Table 2 column 2). While this is suggestive, the theoretical model will allow me to identify separately, the effect of being foster matched and the effect of transiting from a foster match to an adoption match, on the total probability of being adopted. Specifically, the model will show that children with a disability are less likely to become adoption matched not only because they are less likely to be foster matched, but they are also less likely to transit from a foster match to an adoption match.

Stylized Fact 2: Disability Increases the Probability of Foster Match Disruption. From the data, foster matches constituted by children with and without a disability disrupt at rates 0.19 and 0.18, respectively (see Table A1). Using sample B, I estimate equation (1) where the dependent variable y_{ijt} is equal to one if child i in state j has her foster match disrupted in period t and zero otherwise. Here, the vector Z_{it} includes the number of months that the child has been in her current foster match and what type of foster match it is (i.e., whether a pre-adoptive home, non-relative foster home or relative foster home).

Table 2 column 3 shows that children with a disability are more likely to have their foster match disrupted than children without a disability. Specifically, I find that disability increases the probability of disrupting a foster match by more than 2 percent. Even though, the dataset does not allow to identify the reason of the disruption, the theoretical model will separately identify two types of disruptions: the child transits from foster matched to unmatched (from foster home to institutional care), and the child transits from a foster match to another foster match (from foster home to foster home). In the former case, I say the foster match is **destroyed**, and in the latter case, I say the foster match is **dissolved**. The theoretical model will show that these two forces behind disruptions work on opposition directions since children with a disability are more likely to destroy but less likely to dissolve.

Stylized Fact 3: Disability Decreases the Probability of Becoming Foster Matched. From the data, the rates of foster match formation (conditional on starting the period unmatched) of children with and without a disability are 0.22 and 0.28, respectively (see Table A1). To study the effect of disability on the probability of becoming foster matched, I use sample C to estimate equation (1) where a dependent variable y_{ijt} equal one indicates that child i in state j becomes foster matched in period t and zero otherwise. In this specification, the vector Z_{it} now additionally includes the number of months that the child has been in her current unmatched state and where she is currently living (i.e., whether a group home or institution).

Table 2 column 4 shows that disability decreases the probability of becoming foster matched by 5 percent. That is, children with a disability are less likely to become foster matched than children without a disability. The theoretical model will show that this probability is driven by the fact that disability decreases the probability that a child finds a parent willing to foster her, and if they do, disability increases the probability that the foster match is later on destroyed.

Stylized Fact 4: Disability Increases the Probability of Becoming Unmatched. From the data, the rates of unmatched formation (conditional on starting the period foster matched) of children with and without a disability are 0.03 and 0.01, respectively (see Table A1). Here I use sample B to estimate equation (1) where a dependent variable y_{ijt} equal one indicates child i in state j becomes unmatched in period t and zero otherwise. As in the previous estimation, Z_{it} includes the number of months that the child has been in her current foster match and the type of foster match.

As we can see from Table 2 column 5, disability increases the probability of becoming unmatched. In the theoretical model, the probability of becoming unmatched will depend on the rate at which foster matches disrupt and the probability that a child finds a parent willing to foster her. Thus, behind this stylized, there are driving forces working on opposite directions, as in the case of disruptions.

Other Demographics. Table A2 exhibits the complete results of all regressions. The effect of one more year of age is the same (in terms of sign) to the effect of a disability. Similarly, being a male has a similar pattern to disability, except that it decreases the probability of disruption. Now, an interesting result is that the probability of becoming adoption matched is decreasing in the length of time that a child remains in foster care since her parental rights have been terminated. This is very similar to the documented evidence relating to unemployment spells and job finding rates. On the one hand, the child's behavior might become 'more difficult' the longer she stays in foster care, searching for an adoptive family. On the other hand, parents might interpret a long wait as a signal that those children might be 'difficult'. As future research, it would be interesting to build a model incorporating these features and analyze these two effects.

4 Model

In this section, I develop a two-sided dynamic matching model to analyze how different forces interact in the agents' decisions of forming a foster match, disrupting a foster match, and forming an adoption match. Later on, I will use the model to show that some of the stylized facts estimated in the previous section are driven by forces working in opposite directions. Also, I will also obtain predictions regarding match quality.

Time t is discrete with an infinite-horizon. One side of the market is populated by **children** who differ in an observable attribute $x \in X = \{x_1, x_2\}$ where x_1 denotes a child with a disability, x_2 indicates a child without a disability, and $x_1 < x_2$.[22] Each period, a strictly positive mass of children ρ enters the market and each child draws an attribute from a full support probability

[22] Even though social workers decide on behalf of children, I assume that children make their own decisions. The assumption behind this is that children and social workers have the same preferences. The drawback is that the model abstracts from any private incentives that social workers might have, yet given that social workers are trained to ensure children' well-being, it seems like a reasonable assumption to begin with.

distribution $l : X \to [0, 1]$. The other side of the market is constituted by homogeneous **parents**. Every period, parents make entry/exit decisions and the mass of parents out of the market is strictly positive. I refer to a child as 'she' and a parent as 'he'.

Children and parents who are in the market can be unmatched or matched. Let $u^p \geq 0$ denote the endogenous distribution of unmatched parents in the market, and $u^c : X \to \mathbb{R}_+$ denote the endogenous distribution of unmatched children in the market. Matches are formed between children and parents, one-to-one, and heterogeneous in their **match quality** denoted as $q \in Q = \{q_1, q_2, ..., q_N\}$ where $q_1 < q_2 < ... < q_N$.[23] Further, I define two types of matches: **foster matches** (reversible) and **adoption matches** (irreversible). Agents who form a foster match (hereafter *f-match*) remain in the market, while agents who form an adoption match (hereafter *a-match*) leave the market. Hence, I define an endogenous distribution over f-matches denoted as $m : X \times Q \to \mathbb{R}_+$ where $m(x, q)$ is the time invariant mass of f-matches of quality q involving a child x. Thus, the aggregate state of the economy is summarized by the triple $\phi = (u^p, u^c, m)$.

All agents are risk-neutral and discount future at rate $\beta \in (0, 1)$. Payoffs for unmatched children are normalized to zero. For children who are f-matched or a-matched, payoffs are given by the real-valued function $b^c(x, q, z)$ where $z \in \{0, 1\}$, $z = 0$ indicates an f-match, and $z = 1$ indicates an a-match. I denote $b^c(x, q, 0)$ and $b^c(x, q, 1)$ as $b^c(x, q)$ and $b^c_a(x, q)$, respectively. I assume that children' payoff function satisfies the following properties:

Assumption 1 (Children' payoffs)**.** *(a) $b^c_a(x, q) > b^c(x, q) \geq 0$ for all (x, q); (b) $b^c(x, q)$ and $b^c_a(x, q)$ are decreasing in x for all q; (c) $b^c(x, q)$ and $b^c_a(x, q)$ are increasing in q for all x; (d) $b^c(x, q') > b^c_a(x, q)$ whenever $q' > q$ for all x; (e) $b^c(x, q, z)$ is supermodular in (x, z) for all q; (f) $b^c(x, q, z)$ is submodular in (x, q) for all z; and (g) $b^c(x_1, q') - b^c_a(x_1, q) > b^c(x_2, q') - b^c_a(x_2, q)$ whenever $q' > q$.*

Assumption 1(a) captures that children are better-off with a foster parent than in institutional care, and better-off when adopted than fostered. 1(b) reflects that children with a disability benefit more from the family environment and emotional stability provided by foster and adoption. The intuition that children are better-off in high-quality matches is addressed in 1(c). Assumption 1(d) states that children prefer to be f-matched when the quality is high than a-matched when the quality is low. 1(e) imposes that the gain of being adopted is greater for children without a disability, and 1(f) captures that the gain of being in high-quality matches is greater for children with a disability. Lastly, assumption 1(g) implies that the gain of being in an f-match of high-quality versus being in an a-match of low-quality is geater for children with a disability.

[23] Match quality captures other factors affecting the match independent of the child's attribute, such as the emotional bond between the child and parent, and the relationship between the parent and the child's birth family.

Payoffs for parents out of the market are normalized to zero. Parents incur on a per-period cost $k > 0$ to hold a license and stay in the market. Parents who are f-matched or a-matched receive payoffs according to the real-valued functions $\varphi(x, q, z)$ and $\tau(x, z)$ representing the preferences of parents over children and the financial transfers received (in utility units) from the child welfare agency, respectively. Additionally, I define a net payoff function $b^p(x, q, z)$ as follows:

$$b^p(x,q,z) = \begin{cases} \varphi(x,q,z) + \tau(x,z) - k & \text{if } z = 0 \\ \varphi(x,q,z) + \tau(x,z) & \text{if } z = 1 \end{cases}$$

where $b^p(x, q, 0)$ and $b^p(x, q, 1)$ are denoted as $b^p(x, q)$ and $b^p_a(x, q)$, respectively. The analogous follows for functions φ and τ. Moreover, I assume that the parents' net payoff function satisfies the following properties:

Assumption 2 (Parents' payoffs)**.** *(a) $b^p(x, q) > b^p_a(x, q)$ for all (x, q); (b) $b^p(x, q)$ and $b^p_a(x, q)$ are increasing in x for all q; (c) $b^p(x, q)$ and $b^p_a(x, q)$ are increasing in q for all x; (d) $b^p(x, q, z)$ is log-supermodular in (x, z) for all q; and (e) $b^p(x, q, z)$ is log-submodular in (q, z) for all x.*

Assumption 2(a) reflects the presence of the adoption penalty. 2(b) captures the intuition that parents prefer children without a disability to children with a disability. 2(c) reflects that parents in high-quality matches benefit more from fostering or adopting than parents in low-quality matches. Now, the term $1 - \frac{b^p_a(x,q)}{b^p(x,q)}$ represents the adoption penalty. Assumption 2(d) captures that the adoption penalty is higher for children with a disability. Lastly, assumption 2(e) imposes that the adoption penalty is increasing in the match quality. Intuitively, you can think that parents in f-matches of low-quality need to make higher efforts to meet the necessary standards impose by the child welfare agency, and once they adopt those standards vanish.

Figure 1 exhibits the timeline within a period. Each period is divided into four stages: destruction and a-matching, entry and exit, search and f-matching, and payoff realization.

Figure 1: Timeline

I start by describing the **search and f-matching stage**. Children and unmatched parents meet over time according to a meeting technology that can be described in terms of the parents-to-children ratio (i.e. market 'tightness'). Children search while unmatched or f-matched but not while a-match. Parents search only while unmatched. Specifically, a child meets an unmatched parent according to the meeting technology $\pi^c(\theta)$, where the market tightness θ is given by:

$$\theta = \frac{u^p}{\sum_x u^c(x) + \sum_q m(x,q)}. \tag{2}$$

I assume that $\pi^c : \mathbb{R}_+ \to [0, \overline{\pi}^c]$ is a strictly increasing and strictly concave function such that $\pi^c(0) = 0$ and $1 \geq \overline{\pi}^c$. Similarly, an unmatched parent meets a child with probability $\pi^p(\theta)$. I assume that $\pi^p : \mathbb{R}_+ \to [0,1]$ is a strictly decreasing and convex function such that $\pi^p(\theta) = \frac{\pi^c(\theta)}{\theta}$ and $\pi^p(0) = 1$. Now, a parent can meet a child who is unmatched or f-matched when the quality is q. Refer to child x in an f-match of quality q as **child (x,q)**, and refer to child x who is unmatched as **child (x,$\mathbf{q_0}$)**. To make reference to a child's match status, I define an auxiliary set $\bar{Q} = Q \cup \{q_0\}$. Similarly, **parent (x,q)** is a parent f-matched to child x when the quality is q. Thus, an unmatched parent meets a **child (x,$\bar{\mathbf{q}}$)** according to the probability distribution $\hat{m}(x,\bar{q})$ where:

$$\hat{m}(x,\bar{q}) = \begin{cases} \frac{u^c(x)}{\sum_x u^c(x) + \sum_q m(x,q)} & \text{if } \bar{q} = q_0 \\ \frac{m(x,q)}{\sum_x u^c(x) + \sum_q m(x,q)} & \text{if } \bar{q} = q \end{cases} \tag{3}$$

Therefore, an unmatched parent meets an unmatched child x with total probability $\pi^p(\theta) \cdot \hat{m}(x,q_0)$. Similarly, an unmatched parent meets a child x f-matched when the quality is q with total probability $\pi^p(\theta) \cdot \hat{m}(x,q)$. When a child and parent meet, a match quality q is drawn from the full support probability distribution $h : Q \to [0,1]$. A match quality is constant through the duration of the f-match, and learned through experience. Before forming an f-match, agents observe a noisy signal $s \in S = \{s_1, s_2, ..., s_N\}$ generating a full support conditional probability distribution $g(q|s)$ such that $G(q|s') \leq G(q|s)$ whenever $s' > s$. After observing the signal, agents announce simultaneously 'foster' or 'reject'. An f-match is formed if and only if both agents announce foster. If a new f-match is formed, any old f-match dissolves.

During the **payoff realization stage**, agents in a newly formed f-match perfectly observe the match quality q. Once a match quality is complete information, payoffs received during the remaining duration of the f-match are known.

Next, at the beginning of the **destruction and a-matching stage**, a child is adopted (by a relative) with exogenous probability $\delta_x \in (0,1)$ where $\delta_{x_2} \geq \delta_{x_1}$.[24] If a child is adopted by a relative, the f-match destroys. If an f-match remains, then the child and parent announce simultaneously 'adoption', 'destroy', or 'remain'. An f-match is destroyed if and only if at least one agent announces destroy, and an a-match takes place if and only if both agents announce adoption. If an f-match destroys, the parent remains unmatched that period and the child searches that

[24] In some cases, relatives reach out when they learn about the situation and request to adopt the child. Child welfare agencies have strong preferences for relatives.

period. Agents who form an a-match receive adoption payoffs to perpetuity, and those children adopted by a relative receive $b^c_a(x, q_N)$ to perpetuity. I assume the match quality q remains the same when transitioning from f-matched to a-matched.

Lastly, at the beginning of the **entry and exit stage**, a mass of new children enters the market and parents make entry/exit decisions. Parents and children who enter the market remain unmatched that period. I assume that only unmatched parents can decide to exit the market. Furthermore, agents who formed an a-match during the previous stage leave the market.

I restrict attention to stationary pure symmetric Markov strategies. Strategies depend on the aggregate state of the economy $\phi = (u^p, u^c, m)$, but to simplify notation, I suppress it. For each parent, a strategy consists of the tuple (in, out, f^p, d^p, a^p) where $in \in \{0, 1\}$ and $out = 1 - in$ are the entry and exit strategies such that $in = 1$ when the parent enters the market (analogous for out), $f^p : X \times S \to \{0, 1\}$ is the decision to form an f-match such that $f^p(x, s) = 1$ if and only if the parent announces foster after meeting child x and observing signal s, $d^p : X \times Q \to \{0, 1\}$ is the decision to destroy the f-match such that $d^p(x, q) = 1$ when parent (x, q) announces destroy, and $a^p : X \times Q \to \{0, 1\}$ is the decision to form an a-match such that $a^p(x, q) = 1$ when parent (x, q) announces adoption. For each child x, a strategy consists of the triple (f^x, d^x, a^x) where $f^x : \bar{Q} \times S \to \{0, 1\}$ is the decision to form a new f-match such that $f^x(\bar{q}, s) = 1$ when child $(x, \bar{q})$ announces foster after observing signal s, $d^x : Q \to \{0, 1\}$ is the decision to destroy the f-match such that $d^x(q) = 1$ when child (x, q) announces destroy, and $a^x : Q \to \{0, 1\}$ is the decision to form an a-match such that $a^x(q) = 1$ when child (x, q) announces adoption. As an abuse of notation, I denote the foster formation, destruction, and adoption strategies as $f^c(x, \bar{q}, s)$, $d^c(x, q)$ and $a^c(x, q)$. Moreover, let $d(x, q) = d^c(x, q) + (1 - d^c(x, q))\, d^p(x, q)$ and $a(x, q) = a^c(x, q)\, a^p(x, q)$.

I express the f-match formation decisions on terms of whom a particular agent is willing to form an f-match with. I call these sets **foster sets**. For each child $(x, \bar{q})$, let $F^c(x, \bar{q})$ denote the set of signals such that she is willing to form an f-match. Similarly, for parents let $F^p(x)$ denote the set of signals such that he is willing to form an f-match with child x. Formally, foster sets for children and parents are defined as follows, $F^c(x, \bar{q}) = \{s \in S | f^c(x, \bar{q}, s) = 1\}$ and $F^p(x) = \{s \in S | f^p(x, s) = 1\}$. Moreover, the f-matching correspondence is defined as:

Definition 4.1. *A* ***foster-matching correspondence*** *is a map $\mathcal{M} : X \times \bar{Q} \mapsto S$ such that $s \in \mathcal{M}(x, \bar{q})$ if and only if $s \in F^c(x, \bar{q})$ and $s \in F^p(x)$.*

5 Recursive Formulation and Equilibrium

In this section, I present the value functions for children and parents, define the aggregate state of the economy, and present the equilibrium definition used in this environment.

5.1 Value Functions

5.1.1 Value Functions for Children

Let $\mathcal{C}(x,\bar{q})$ denote the value function for child $(x,\bar{q})$ at the end of a period, where $\mathcal{C}(x,q_0)$ represents the value for child x who is unmatched and $\mathcal{C}(x,q)$ represents the value for child x who is f-matched when the quality is q. To simplify the presentation of the value functions, I define $\hat{\mathcal{C}}(x,\bar{q})$ as the value for child $(x,\bar{q})$ at the beginning of the search and f-matching stage and specified by equation (4). At the beginning of the search and f-matching stage, child $(x,\bar{q})$ meets an unmatched parent with endogenous probability $\pi^c(\theta)$. If no meeting takes place, the status-quo is preserved and she receives the continuation value $\mathcal{C}(x,\bar{q})$. If a meeting takes place, a noisy signal s is observed, where $f(s)$ is the probability distribution over signals derived from $h(q)$ and $g(q|s)$. Suppose agents meet and observe a common signal s such that at least one agent announces reject, then the status-quo is preserved. Now, suppose agents meet and observe signal s such that both agents announce foster, then the child receives the conditional expected value $\mathbb{E}_s[\mathcal{C}(x,q)] = \sum_q \mathcal{C}(x,q)\ g(q|s)$.

$$\hat{\mathcal{C}}(x,\bar{q}) = \Big(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,\bar{q})} f(s)\Big)\, \mathcal{C}(x,\bar{q}) + \pi^c(\theta) \sum_{\mathcal{M}(x,\bar{q})} \mathbb{E}_s[\mathcal{C}(x,q)]f(s) \tag{4}$$

Child $(x,\bar{q})$ announces foster after observing signal s if and only if the conditional expected value of forming a new f-match is greater than the continuation value of the status-quo i.e. $s \in F^c(x,\bar{q})$ if and only if $\mathbb{E}_s[\mathcal{C}(x,q)] \geq \mathcal{C}(x,\bar{q})$.[25]

For a child x who is unmatched at the end of a period, the value function is equal to $\mathcal{C}(x,q_0) = 0 + \beta\ \delta_x \frac{b_a^c(x,q_N)}{1-\beta} + \beta\ (1-\delta_x)\ \hat{\mathcal{C}}(x,q_0)$, or equivalently:

$$\mathcal{C}(x,q_0) = \frac{\beta\ \delta_x \frac{b_a^c(x,q_N)}{1-\beta} + \beta\ (1-\delta_x)\ \beta\ \pi^c(\theta) \sum_{\mathcal{M}(x,q_0)}\ \mathbb{E}_s[\mathcal{C}(x,q)]f(s)}{1 - \beta\ (1-\delta_x)\ \Big(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q_0)} f(s)\Big)} \tag{5}$$

Next, consider a child x f-matched when the quality is q at the end of a period. Child (x,q)'s value function is specified by equation (6). In the current period, she receives the f-match payoff $b^c(x,q)$. At the beginning of the next period, she is adopted by a relative with probability δ_x. If the f-match remains, child and parent decide between transit to an a-match, destroy the f-match, or remain f-matched. In each case, child (x,q)'s possible continuation values are: she receives $\frac{b_a^c(x,q)}{1-\beta}$ when transiting to adoption, she receives the search value $\hat{\mathcal{C}}(x,q_0)$ when the f-match destroys, and she receives the search value $\hat{\mathcal{C}}(x,q)$ when the f-match remains.

[25] I assume all agents make decisions supposing their decision is payoff relevant

$$\mathcal{C}(x,q) = b^c(x,q) + \beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta} + \beta(1-\delta_x)\Big[d^p(x,q)\,\hat{\mathcal{C}}(x,q_0) + a^p(x,q)\max\left\{\frac{b^c_a(x,q)}{1-\beta}\,,\hat{\mathcal{C}}(x,q_0)\,,\hat{\mathcal{C}}(x,q)\right\} + \Big(1-d^p(x,q)-a^p(x,q)\Big)\max\left\{\hat{\mathcal{C}}(x,q_0)\,,\hat{\mathcal{C}}(x,q)\right\}\Big] \tag{6}$$

Child (x,q) chooses adoption if and only if the value of being adopted is greater than the value of continue searching while unmatched and the value of continue searching while f-matched when the quality is q. Thus, a child faces the following trade-off: receive a higher adoption payoff but forgo the opportunity of finding a 'better' match. Similarly, child (x,q) chooses destroy if and only if the value of searching while unmatched is greater than the value of being adopted and the value of continue searching while f-matched. Intuitively, when a child decides to destroy the f-match, she is destroying a 'bad' match.

5.1.2 Value Functions for Parents

Let $\mathcal{P}^u$ denote the value function for an unmatched parent at the end of a period, and let $\mathcal{P}(x,q)$ denote the value function for parent (x,q) at the end of a period. First, consider a parent who is unmatched, then his value function is presented in equation (7). In the current period, the unmatched parent incurs in the per-period cost k of holding a license. Next period, he decides between stay or exit the market. When the parent exits his payoff is zero, and when the parent stays he meets a child with probability $\pi^p(\theta)$. If no meeting takes place, the parent remains unmatched. When a meeting takes place, a child is drawn from the endogenous probability distribution $\hat{m}(x,\bar{q})$ defined in equation (3). After a meeting with child $(x,\bar{q})$ has taken place, agents observe a noisy signal s. Suppose agents observe s such that at least one agent announces reject, then the parent remains unmatched. On the contrary, suppose agents observe s such that both agents announce foster, then the parent receives the conditional expected value $\mathbb{E}_s[\mathcal{P}(x,q)]$.

$$\mathcal{P}^u = \max\left\{0, \frac{-k+\beta\ \ \pi^p(\theta)\sum_{\mathcal{M}(x,\bar{q})}\sum_{x,\bar{q}}\mathbb{E}_s[\mathcal{P}(x,q)]\hat{m}(x,\bar{q})f(s)}{1-\beta\Big(1-\pi^p(\theta)\sum_{\mathcal{M}(x,\bar{q})}\sum_{x,\bar{q}}\hat{m}(x,\bar{q})f(s)\Big)}\right\} \tag{7}$$

An unmatched parent forms an f-match with child $(x,\bar{q})$ after observing signal s if and only if the conditional expected value of forming the f-match is greater than the unmatched value i.e. $s \in F^p(x)$ if and only if $\mathbb{E}_s\big[\mathcal{P}(x,q)\big] \geq \mathcal{P}^u$. Further, a parent enters the market if and only if the expected benefit of being in the market is greater than the cost of the license, i.e. $in = 1$ if and only if $\mathcal{P}^u \geq 0$.

At the end of a period, a parent f-matched to child x when the quality is q has a value function given by equation (8). In the current period, he receives the f-match payoff $b^p(x,q)$. At the beginning of next period, he becomes unmatched with exogenous probability δ_x. If the f-match remains, child and parent decide between transit to adoption, destroy the f-matched or remain f-matched. For each possible outcome, the continuation values for the parent are the following. When transiting to adoption, he receives $\frac{b^p_a(x,q)}{1-\beta}$. When the f-match destroys, he receives the unmatched value $\mathcal{P}^u$. Lastly, when the f-match remains, his continuation value depends on the outcome of the search and f-matching stage: with probability $\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)$ he becomes unmatched due to the child forming a new f-match, and with probability $\big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\big)$ the f-match remains.

$$\begin{aligned}\mathcal{P}(x,q) &= b^p(x,q) + \beta\,\delta_x\,\mathcal{P}^u + \beta(1-\delta_x)\Bigg[d^c(x,q)\,\mathcal{P}^u \\ &+ a^c(x,q)\max\left\{\frac{b^p_a(x,q)}{1-\beta}\,,\mathcal{P}^u\,,\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\Big)\,\mathcal{P}(x,q) + \pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\,\mathcal{P}^u\right\} \\ &+ \Big(1-d^c(x,q)-a^c(x,q)\Big)\max\left\{\mathcal{P}^u\,,\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\Big)\,\mathcal{P}(x,q) + \pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\,\mathcal{P}^u\right\}\Bigg]\end{aligned} \tag{8}$$

A parent f-matched to child x when the quality is q announces adoption if and only if the value of adopting is greater than the unmatched value and the value of continue f-matched. When deciding to adopt, a parent faces the following trade-off: he eliminates the likelihood that the f-match is destroyed or dissolved, but forgoes part of the per-period payoff. Similarly, a parent f-matched to child x when the quality is q announces destroy if and only if the value of begin unmatched is greater than the value of adopting and the value of continue f-matched. Intuitively, when a parent decides to destroy the f-match, he is destroying a 'bad' match.

5.2 Aggregate State of the Economy

The distribution of unmatched parents in the market depends on the entry and exit strategies of parents. Thus, the stationary mass of unmatched parents u^p satisfies the following inequality:

$$\pi^p\left(\frac{u^p}{\sum_x u^c(x) + \sum_q m(x,q)}\right) \leq \frac{k}{\beta\,\sum_{\mathcal{M}(x,\bar{q})}\sum \mathbb{E}_s[\mathcal{P}(x,q)]\hat{m}(x,\bar{q})f(s)} \tag{9}$$

with equality if u^p is strictly positive. For distributions $u^c(x)$ and $m(x,q)$ to be time invariant, the mass destruction and mass creation must exactly balance (see equations in appendix A).

5.3 Definition of Equilibrium

Definition 5.1. *A **foster care equilibrium** consists of tuple* $(\mathcal{M}, d^c, d^p, a^c, a^p, in, \mathcal{C}, \mathcal{P}^u, \mathcal{P}, \phi)$ *such that the following properties are satisfied:*

(1) ***Value Functions.*** (*a*) *Given* $(\mathcal{M}, d^c, d^p, a^c, a^p, \phi)$*, the value function* $\mathcal{C}(x, q_0)$ *is specified by* (5) *and the value function* $\mathcal{C}(x, q)$ *is specified by* (6). (*b*) *Given* $(\mathcal{M}, d^c, d^p, a^c, a^p, in, \phi)$*, value function* $\mathcal{P}^u$ *is specified by* (7) *and value function* $\mathcal{P}(x, q)$ *is specified by* (8).

(2) ***Strategies.*** (*a*) *Given* $(\mathcal{M}, d^p, a^p, \mathcal{C}, \phi)$*,* $a^c(x, q)$ *is one if and only if* $\frac{b^c_a(x,q)}{1-\beta} > \max\{\hat{\mathcal{C}}(x, q_0), \hat{\mathcal{C}}(x, q)\}$ *and* $d^c(x, q)$ *is one if and only if* $\hat{\mathcal{C}}(x, q_0) > \max\{\frac{b^c_a(x,q)}{1-\beta}, \hat{\mathcal{C}}(x, q)\}$. (*b*) *Given* $(\mathcal{M}, d^c, a^c, \mathcal{P}^u, \mathcal{P}, \phi)$*,* in *is one if and only if* $\mathcal{P}^u \geq 0$*,* $a^p(x, q)$ *is one if and only if* $\frac{b^p_a(x,q)}{1-\beta} > \max\{\mathcal{P}^u, \mathcal{P}(x, q)\}$*, and* $d^p(x, q)$ *is one if and only if* $\mathcal{P}^u > \max\{\frac{b^p_a(x,q)}{1-\beta}, \mathcal{P}(x, q)\}$. (*c*) *Given* $(d^c, d^p, a^c, a^p, \mathcal{C}, \mathcal{P}^u, \mathcal{P}, \phi)$*,* $s \in \mathcal{M}(x, \bar{q})$ *if and only if* $\mathbb{E}_s[\mathcal{C}(x, q)] \geq \mathcal{C}(x, \bar{q})$ *and* $\mathbb{E}_s[\mathcal{P}(x, q)] \geq \mathcal{P}^u$.

(3) ***Aggregate state of the economy.*** (*3a*) *Given* $(\mathcal{M}, d^c, d^p, a^c, a^p, in, \mathcal{P}^u, \mathcal{P}, u^c, m)$*,* u^p *satisfies equation* (9). (*3b*) *Given* $(\mathcal{M}, d^c, d^p, a^c, a^p)$*, for each* x*,* $u^c(x)$ *and* $\{m(x, q_i)\}_{i=1}^N$ *solve the system of equations given by* (*A.1*) *and* (*A.2*).

6 Equilibrium Analysis

In this section, I derive equilibrium properties that I will use later on to ensure that the stylized facts estimated in section 3 arise in equilibrium and carry out model predictions. The analysis focuses on foster care equilibria with a positive mass of parents in the market i.e $u^p > 0$ which implies that $\mathcal{P}^u = 0$ [from equations (7) and (9)]. Moreover, I assume that for each child, there is at least one signal such that parents receive a positive expected foster payoff. Formally:

Assumption 3. *For each* x*, there exists* $\hat{s}$ *such that* $\mathbb{E}_{\hat{s}}[b^p(x, q)] \geq 0$.

I start by describing the destruction strategies of children and parents. Lemma 6.1(i) states that, in any foster care equilibrium, child x does not destroy an f-match of quality q if $b^c(x, q)$ is non-negative. In terms of the model, if the payoff of being f-matched is equal or higher than the unmatched payoff (normalized to zero), then a child prefers to continue searching while f-matched than continue searching unmatched. This result follows from the assumption that the meeting probability is the same for unmatched and f-matched children.

Lemma 6.1 (Destruction Strategies of Children and Parents)**.** *Assume parents' payoffs satisfy assumption 2(a). Then, in any foster care equilibrium:*

(*i*) *for each* (x, q)*,* $d^c(x, q) = 0$ *if* $b^c(x, q) \geq 0$.

(*ii*) *for each* (x, q), $d^p(x, q) = 1$ *if and only if* $b^p(x, q) < 0$.

Proof. See appendix B.1. □

Lemma 6.1(ii) shows that a parent destroys an f-match of quality q with child x if and only if $b^p(x, q)$ is negative. The fact that f-matches with negative net payoffs destroy is intuitive. In words, if providing care for a child is not as rewarding as being out of the market (receive the normalize payoff zero), then a parent prefers to request the removal of the child and search for another child. Now, since parents can not search while f-matched, an interesting question is why they never destroy f-matches with non-negative net payoff. Intuitively, parents might have incentives to destroy these f-matches in exchange for an opportunity of finding a 'better' child. A key assumption driving the result is the unbounded mass of parents outside the market who can freely enter. In equilibrium, this feature of the model equalizes the value of searching for a child in the market with the value of being out of the market. If the child welfare agencies were to impose strong restrictions on entry (as politicians have proposed it), the result could break down due to a potential small mass of parents in the market. That is, parents might have incentives to destroy f-matches even if the net f-match payoff is non-negative.

Proposition 6.1 (Destruction: Effect of Disability and Match Quality)**.** *Assume children' payoffs satisfy assumption 1(a), and parents' payoffs satisfy assumptions 2(a) to (c). Then, in any foster care equilibrium:*

(*i*) *for all* $j \in \{c, p\}$ *and* q, *if* $d^j(x_2, q) = 1$ *then* $d^j(x_1, q) = 1$. *Hence,* $d(x_1, q) \geq d(x_2, q)$ *for all* q.

(*ii*) *for all* $j \in \{c, p\}$ *and* x, *if* $d^j(x, q) = 1$ *then* $d^j(x, q') = 1$ *whenever* $q' < q$. *Hence,* $d(x, q') \geq d(x, q)$ *whenever* $q' < q$ *for all* x.

Proof. See appendix B.2. □

Proposition 6.1 exhibits how the destruction of f-matches varies with disability and match quality. First, I show that f-matches involving children with a disability destroy more than f-matches involving children without a disability. Formally, if the f-match (x_2, q) is destroyed then the f-match (x_1, q) is also destroyed. Recall that, an f-match can be destroyed by either the child or the parent, $d(x, q) = d^c(x, q) + (1 - d^c(x, q))\, d^p(x, q)$. By assumption 1(a) and lemma 6.1(i), it follows that children never destroy an f-match. Thus, in equilibrium, the destruction is driven by parents, which is consistent with the anecdotal evidence suggesting that when a child moves from foster home to institutional care is generally due to the request of the foster parent. Now, by assumption 2(b) and lemma 6.1(ii), it follows that if $d^p(x_2, q) = 1$ then $d^p(x_1, q) = 1$ for all q. In words, if the utility gain received by parents when providing care for a child without a disability respect to a child with a disability $[\varphi(x_2, q) - \varphi(x_1, q) \geq 0]$ is equal or higher than the transfer gain of providing care for a child with a disability over a child without a disability $[\tau(x_1) - \tau(x_2) \geq 0]$, then parents f-matched to children with a disability destroy at least as much as parents f-matched to children without a disability. Loosely speaking, if the transfer is not enough to compensate for the preferences of parents, f-matches involving children with a disability will destroy more than f-matches involving children without a disability.

Proposition 6.1(ii) states that if the f-match (x, q) is destroyed, then all f-matches (x, q') such that $q' < q$ are also destroyed. In words, if a parent f-matched to child x when the quality is q is not willing to continue providing care, then a parent f-matched to child x when the quality is lower than q is also not willing to continue providing care. This follows from assumption 2(c) and lemma 6.1(ii).

Lemma 6.2 (F-match Formation Strategies of Unmatched Children)**.** *For each (x, s), in any foster care equilibrium, $s \in F^c(x, q_0)$ if $\mathbb{E}_s[b^c(x, q)] \geq 0$.*

Proof. See appendix B.3. □

Lemma 6.2 shows that an unmatched child x announces foster, after observing signal s, if $\mathbb{E}_s[b^c(x, q)]$ is non-negative. That is, if the conditional expected payoff of being f-matched is equal or higher than the unmatched payoff, children prefer to become f-matched than remain unmatched. The result follows from the reversibility of the f-matches. Intuitively, since children can always destroy, they are willing to experiment and form an f-match if they believe it is at least as good as staying in institutional care. Now, note that children might be willing to announce foster even if the conditional expected f-match payoff is negative. This is due to the continuation value: even if the conditional expected f-match payoff $\mathbb{E}_s[b^c(x, q)]$ is negative, the conditional expected value of being f-matched $\mathbb{E}_s[\mathcal{C}(x, q)]$ might still be non-negative. Yet, given assumption 1(a), this situation will not arise in this environment.

Assumption 4. *For each (s, x), if $\mathbb{E}_s[b^p(x, q)] < 0$ then the following condition on primitives holds:*

$$\sum_q b^p(x, q)\; g(q|s) + \beta\;(1 - \delta_x) \sum_q \max\left\{\frac{b^p_a(x, q)}{1 - \beta}\,, 0\,, \frac{b^p(x, q)}{1 - \beta\;(1 - \delta_x)}\right\} g(q|s) < 0$$

To establish the stylized facts later on, I ensure that parents' strategies satisfy the following: (1) if a parent is willing to form an f-match with child x_1 after observing signal s, then he is also willing to form an f-match with child x_2 after observing signal s; and (2) if a parent is willing to adopt child x_1 when the quality is q, then he is also willing to adopt child x_2 when the quality is q. Since (1) might contradict (2), I impose assumption 4 which allows me to characterize parents' f-match formation strategies using the per-period payoffs. This assumption ensures that, if the conditional expected payoff received by a parent f-matched to child (x, q) is negative then the conditional expected value of being f-matched to child (x, q) is also negative.

Lemma 6.3 (F-match Formation Strategies of Parents)**.** *Assume parents' payoffs satisfy assumption 4. For each (x, s), in any foster care equilibrium, $s \in F^p(x)$ if and only if $\mathbb{E}_s[b^p(x, q)] \geq 0$.*

Proof. See appendix B.4. □

Lemma 6.3 shows that an unmatched parent announces foster, after observing signal s, if and only if the conditional expected payoff of being f-matched is non-negative. An unmatched parent announces foster if the conditional expected payoff of being f-matched is non-negative due to the reversibility of f-matches. Specifically, given that parents can destroy an f-match freely, if $\mathbb{E}_s[b^p(x, q)]$ is non-negative it follows that $\mathbb{E}_s[\mathcal{P}(x, q)]$ is also non-negative.

Proposition 6.2 (F-match Formation Involving Unmatched Children: Effect of Disability and Match Quality)**.** *Assume children' payoffs satisfy assumption 1(a), parents' payoffs satisfy assumptions 2(b)(c), 3 and 4. Then, in any foster care equilibrium:*

(*i*) *$F^c(x_1, q_0) = F^c(x_2, q_0) = S$ and if $s \in F^p(x_1)$ then $s \in F^p(x_2)$. Moreover, $\mathcal{M}(x, q_0)$ is non-empty for all x, and $\mathcal{M}(x_1, q_0) \subseteq \mathcal{M}(x_2, q_0)$.*

(*ii*) *for all x, if $s \in F^p(x)$ then $s' \in F^p(x)$ whenever $s' > s$. Hence, if $s \in \mathcal{M}(x, q_0)$ then $s' \in \mathcal{M}(x, q_0)$ whenever $s' > s$ for all x.*

Proof. See appendix B.5. □

Proposition 6.2 exhibits how the formation of f-matches involving unmatched children varies with disability and match quality. Recall that f-matches must be mutually agreed upon, that is, $s \in \mathcal{M}(x, q_0)$ if and only if $s \in F^p(x)$ and $s \in F^c(x, q_0)$. By assumption 1(a) and lemma 6.2, it follows that children always announce foster after observing signal s. Intuitively, as the law requires, children are placed in foster family homes whenever possible. Thus, the formation of an f-match depends on the parent's decision. First I show that conditional on observing signal s, if a parent is willing to foster a child with a disability, then he must also be willing to foster a child without a disability [if $s \in F^p(x_1)$ then $s \in F^p(x_2)$]. This follows from assumption 2(b) and lemma 6.3. In words, if the utility gain received by parents when providing care for a child without a disability respect to a child with a disability $[\varphi(x_2, q) - \varphi(x_1, q) \geq 0]$ is not compensated by the transfer gain of providing care for a child with a disability over a child without a disability $[\tau(x_1) - \tau(x_2) \geq 0]$, then in equilibrium children with a disability are less likely to find a parent willing to foster them. Second, I show that if a parent announces foster after meeting child x and observing signal s, then he also announces foster after observing signal s' such that $s' > s$ [if $s \in F^p(x)$ then $s' \in F^p(x)$ whenever $s' > s$]. The result follows from assumption 2(c) and lemma 6.3. Since $G(q|s)$ first-order stochastically dominates $G(q|s')$, it follows that the conditional expected value received by a parent when fostering a child is increasing in the signal.

Lemma 6.4 (Adoption Strategies of Parents)**.** *Assume parents' payoffs satisfy assumptions 2(a)(b) and 2(d)(e). Then, the adoption strategies of parents exhibit the following properties.*

(i) for each (x, q), if $b_a^p(x, q) > 0$ and $\frac{b_a^p(x,q)}{b^p(x,q)} > \frac{1-\beta}{1-\beta(1-\delta_x)}$ then $a^p(x, q) = 1$.

(ii) for all q, if $\sum_{\mathcal{M}(x_2,q)} f(s) \geq \sum_{\mathcal{M}(x_1,q)} f(s)$ then the best-response of parents satisfies the following: if $a^p(x_1, q) = 1$ then $a^p(x_2, q) = 1$.

(iii) for all x, if $\sum_{\mathcal{M}(x,q')} f(s) \geq \sum_{\mathcal{M}(x_1,q)} f(s)$ and $b_a^p(x, q') > 0$ whenever $q' < q$ then the best-response of parents satisfies the following: if $a^p(x, q) = 1$ then $a^p(x, q') = 1$.

Proof. See appendix B.6. □

Lemma 6.4 presents some properties of the adoption strategies of parents. In item (i), for each (x, q), if the a-match payoff is positive and the exogenous probability that the child leaves the f-match is sufficiently high $\delta_x > \frac{b^p(x,q)-b_a^p(x,q)}{b_a^p(x,q)} \cdot \frac{1-\beta}{\beta}$, then a parent always announces adoption. The intuition is as follows. When deciding whether to adopt, the parent faces the following trade-off: forgo part of the per-period payoff in exchange to eliminate the likelihood that the child is removed from his care. Thus, if the probability of being adopted by a relative is sufficiently high, then the parent is willing to transit to adoption and receive a smaller payoff. If policymakers were to increase the barriers for relatives to adopt (as it has been discussed), the result suggests that it might be counterproductive on the incentives of foster parents to adopt.

The adoption strategies of parents strongly depend on the probability that children leave the f-match. Fix some match quality q. Item (ii) states that, if f-matches (x_1, q) dissolve more than f-matches (x_2, q), then parents have greater incentives to adopt children without a disability than children with a disability. The result follows from assumption 2(d). Parents are more willing to adopt children without a disability because they forgo a smaller share of payoff in exchange to eliminate a bigger dissolution probability. Note that assumption 2(d) might still hold even if the penalty on the financial transfer is smaller for children with a disability $\frac{\tau_a(x_1)}{\tau(x_1)} > \frac{\tau_a(x_2)}{\tau(x_2)}$. Lastly, item (iii) shows that parents might have more incentives to choose adoption when the match quality is low than when it is high. The driving forces of this result are the dissolution rates of high versus low-quality matches and the assumption that the adoption penalty is increasing in the match quality [assumption 2(e)]. In words, if children in high-quality matches have fewer incentives to dissolve, then the best response of parents is to adopt them less.

Proposition 6.3 (Dissolution: Effect of Disability and Match Quality)**.** *Suppose* $Q = \{q_1, q_2\}$ *and* $S = \{s_1, s_2\}$*. Assume children' payoffs satisfy assumptions 1(a)(c)(d), and parents' payoffs satisfy assumptions 2(a)(b)(c), 3 and 4. Then, in any foster care equilibrium:*

(i) $\sum_{\mathcal{M}(x_2,q)} f(s) \geq \sum_{\mathcal{M}(x_1,q)} f(s)$ *for all* q*.*

(ii) $\sum_{\mathcal{M}(x,q_1)} f(s) \geq \sum_{\mathcal{M}(x,q_2)} f(s) = 0$ *for all* x*.*

Proof. See appendix B.7. □

Proposition 6.3 exhibits how dissolution outcomes vary with disability and match quality. Item (i) states that children without a disability are more likely to dissolve an f-match than children with a disability. The result is driven by the parents' decision: children without a disability are more demanded by foster parents [Proposition 6.2(i)]. Item (ii) shows that low-quality matches dissolve more than high-quality matches. The result is driven by the children' decision. By assumption 1(d), children value more quality than the adoption status, thus they have no incentives to dissolve high-quality matches.

Assumption 5. *Suppose* $Q = \{q_1, q_2\}$ *and* $S = \{s_1, s_2\}$*, children' payoffs satisfy the following:*
(a) $\frac{\delta_{x_1}}{\delta_{x_2}} > \frac{b^c_a(x_2,q_2) - b^c_a(x_2,q_1)}{b^c_a(x_1,q_2) - b^c_a(x_1,q_1)}$;
(b) $\frac{\{b^c(x_1,q_2) - b^c_a(x_1,q_1)\}(1-\beta(1-\delta_{x_1})) - \{b^c_a(x_1,q_2) - b^c(x_1,q_2)\}\beta(1-\delta_{x_1})}{1-\beta(1-\delta_{x_1})} > b^c(x_2, q_2) - b^c_a(x_2, q_1)$;
(c) $\frac{\{b^c(x_1,q_2) - b^c_a(x_1,q_1)\}\beta\delta_{x_1} - \{b^c_a(x_1,q_2) - b^c(x_1,q_2)\}(1-\beta)}{g(q_2|s_1)} > b^c(x_2, q_2)(1-\beta) + b^c_a(x_2, q_2)\beta - b^c_a(x_2, q_1)$.

Due to proposition 6.3(i), children with a disability are more willing to announce adoption after observing a low-quality match because their search opportunities are lower. However, the intuition suggests that social workers might be pickier when searching for an adoptive parent for a child with a disability since these children benefit more from higher quality matches. Thus, to ensure that this intuition arises in equilibrium, I impose stronger conditions specify in assumptions 5(a) to (c). These conditions will help to ensure that if children with a disability are willing to give up the opportunity of continue searching for a high-quality match, then children without a disability will also be willing to give up this opportunity.

Proposition 6.4 (Adoption: Effect of Disability and Match Quality)**.** *Suppose* $Q = \{q_1, q_2\}$ *and* $S = \{s_1, s_2\}$*. Assume children' payoffs satisfy assumptions 1(a) to (g) and 5(a) to (c), parents' payoffs satisfy assumptions 2(a) to (e), 3 and 4. Then, in any foster care equilibrium:*

(*i*) $a(x_2, q) \geq a(x_1, q)$ *for all* q.

(*ii*) *for each* x, *if* $b^p_a(x, q_1) > 0$ *and* $\frac{b^p_a(x,q_2)}{b^p(x,q_2)} \leq \frac{1-\beta}{1-\beta(1-\delta_x)}$ *then* $a(x, q_1) \geq a(x, q_2) = 0$.

Proof. See appendix B.8. □

Proposition 6.4 exhibits how adoption outcomes vary with disability and match quality. Item (i) states that children with a disability transit to adoption less than children without a disability. Both parents' and children's decisions drive the result. Item (ii) shows that if the probability that the child leaves the f-match is sufficiently low, then high-quality matches do not transit to adoption due to the parents' decision. Thus, high-quality matches transit to adoption less than low-quality matches.

7 Stylized Facts and Model Predictions

In this section, I identify the driving forces behind the empirical results estimated in section 3 and establish sufficient conditions on primitives for these facts to emerge in equilibrium. Also, I analyze the impact of match quality on the probability of becoming adoption matched, the probability of foster match disruption, and the probability of becoming unmatched.

7.1 Probability of Becoming Adoption Matched

The probability that child x becomes adoption matched depends on whether she is unmatched or f-matched when the quality is quality q. First, consider child x who is unmatched at the beginning of a period. Let $A(x, q_0)$ denote the **probability that unmatched child x becomes a-matched** next period, defined as:

$$A(x, q_0) = \delta_x + \big(1 - \delta_x\big)\, \pi^c(\theta) \sum_{\mathcal{M}(x,q_0)} f(s) \sum_{q'} g(q'|s)\, a(x, q')$$

The probability that child (x, q_0) is adopted endogenously depends on two events. First, the child forms an f-match during the search and f-matching stage. Second, both agents announce adoption after observing some quality q.

Consider child x who is, at the beginning of a period (after the exogenous adoption is realized), f-matched when the quality is q. Let $A(x, q)$ denote the **probability that child x f-matched**

when the quality is q becomes a-matched next period, defined as:

$$A(x,q) = a(x,q) + d(x,q) \underbrace{\left[\delta_x + \left(1-\delta_x\right)\pi^c(\theta) \sum_{\mathcal{M}(x,q_0)} f(s) \sum_{q'} g(q'|s)\, a(x,q')\right]}_{A(x,q_0)} + \left(1 - a(x,q) - d(x,q)\right) \left[\delta_x + \left(1-\delta_x\right)\pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s) \sum_{q'} g(q'|s)\, a(x,q')\right]$$

The probability that child (x,q) is adopted endogenously can be decomposed in three events. First, the f-match (x,q) transits to adoption. Second, the f-match (x,q) destroys, and the unmatched child transits to an a-match with another parent. Third, the f-match (x,q) remains, but the child dissolves the f-match and transits to an a-match with another parent.

The effect of disability on the probability of becoming adoption matched estimated in section 3 (coefficient γ) is an average across match status and match qualities. Thus, using the unmatched distribution of children and the endogenous distribution over f-matches, the **probability that child x becomes a-matched** is:

$$\gamma^1(x) = A(x,q_0) \cdot \frac{u^c(x)}{u^c(x) + \sum_q m(x,q)} + \sum_q A(x,q) \cdot \frac{m(x,q)}{u^c(x) + \sum_q m(x,q)}$$

Stylized fact 1 holds if and only if $\gamma^1(x_2) \geq \gamma^1(x_1)$. In words, children with disability, relative to children without disability, are less likely to become a-matched. Corollary 7.1 presents the sufficient conditions for stylized fact 1 to arise in equilibrium. The result makes use of propositions 6.1(i), 6.2(i), 6.3(i), and 6.4(i).

Corollary 7.1 (Stylized Fact 1: Disability Decreases the Probability of Becoming Adoption Matched)**.** *Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Assume children' payoffs satisfy assumptions 1(a) to (g) and 5(a) to (c), parents' payoffs satisfy assumptions 2(a) to (e), 3 and 4. Then, in any foster care equilibrium, $\gamma^1(x_2) \geq \gamma^1(x_1)$ if $\frac{\delta_{x_2} - \delta_{x_1}}{1-\delta_{x_1}} > \overline{\pi}$.*

Proof. See Appendix C.1. □

I show that $A(x_2,q_0) \geq A(x_1,q_0)$ and $A(x_2,q) \geq A(x_1,q)$ for all q. The first inequality follows from propositions 6.2(i) and 6.4(i). In words, unmatched children with a disability are less likely to form an f-matched, and if they do, they are less likely to transit to adoption. For the second inequality, the propositions mentioned above are not sufficient. Consider the case where children with a disability have their f-match destroyed and children without a disability remain f-matched. Fix some match quality q, then $A(x_1,q) = A(x_1,q_0)$ and

$A(x_2, q) = \delta_{x_2} + \big(1 - \delta_{x_2}\big)\, \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_{q'} g(q'|s)\, a(x_2, q')$. To establish that the second equation is greater than the first, I impose an upper bound on the meeting probability.

Corollary 7.2 (Probability of Becoming Adopted is Decreasing in the Match Quality)**.** *Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Assume children' payoffs satisfy assumptions 1(a) to (g) and 5(a) to (c), parents' payoffs satisfy assumptions 2(a) to (e), 3 and 4. For each child x, in any foster care equilibrium, $A(x, q_1) \geq A(x, q_2)$ if $b^p_a(x, q_1) > 0$ and $\frac{b^p_a(x,q_2)}{b^p(x,q_2)} \leq \frac{1-\beta}{1-\beta(1-\delta_x)}$.*

Proof. See Appendix C.2. □

To have a better understanding of the mechanics behind the adoption of children, corollary 7.2 presents the impact of match quality on the probability of becoming adoption matched. The result make use of propositions 6.1(ii), 6.3(ii), and 6.4(ii). In the presence of the adoption penalty, when the exhibited conditions are satisfied, high-quality matches are less likely to transit to an a-match than low-quality matches. Intuitively, if the disruption of high-quality matches is low enough, then parents have no incentives to choose adoption. Note that this result strongly depends on the presence of the adoption penalty, that is, in the absence of the adoption penalty it breaks down.

7.2 Probability of Foster Match Disruption

Consider child x who is, at the beginning of a period, f-matched when the quality is q. Let $D(x, q)$ denote the **probability that f-match** (x, q) **disrupts** within a period, defined as follows:

$$D(x,q) = (1-\delta_x)(1-a(x,q))\Big[\underbrace{d(x,q)}_{destruction} + \underbrace{\big(1-d(x,q)\big)\ \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)}_{dissolution}\Big]$$

The probability that an f-match (x, q) disrupts is discomposed in two events. First, if the f-match (x, q) destroys during the destruction and a-matching stage. Second, if the f-match (x, q) remains but, during the search and f-matching stage, child x forms a new f-match with some parent after observing signal s. Intuitively, an f-match disrupts because it is 'bad' (f-match destroys) or due to the search of the child to find a 'better' parent (f-match dissolves).

Using the endogenous distribution over f-matches, the **probability that an f-match involving child x disrupts** is:

$$\gamma^2(x) = \sum_q D(x,q) \cdot \frac{m(x,q)}{\sum_q m(x,q)}$$

Stylized fact 2 holds if and only if $\gamma^2(x_1) \geq \gamma^2(x_2)$. In words, children with a disability, relative to children without a disability, are more likely to have an f-match disrupted. This depends on two forces working on opposite directions, and the empirical result sheds light on which of the driving forces prevails in equilibrium. On the one hand, proposition 6.1(i) shows that children with a disability are more likely to have an f-matched destroyed, which by itself makes them more likely to disrupt. On the contrary, proposition 6.3(i) shows that children with a disability are less likely to dissolve an f-match, which by itself makes them less likely to disrupt. In other words, foster disruptions involving children with a disability are mainly driven by destruction due to the uncertainty on the quality of the match, while foster disruptions affecting children without a disability are driven mostly by dissolution to improve the match quality.

Corollary 7.3 presents sufficient conditions for stylized fact 2 to arise in equilibrium. The result makes use of propositions 6.1(i), 6.3, and 6.4(i).

Corollary 7.3 (Stylized Fact 2: Disability Increases the Probability of Foster Match Disruption)**.** *Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Assume children' payoffs satisfy assumptions 1(a) to (g) and 5(a) to (c), parents' payoffs satisfy assumptions 2(a) to (e), 3 and 4. Then, in any foster care equilibrium, $\gamma^2(x_1) \geq \gamma^2(x_2)$ if $\frac{\delta_{x_2}-\delta_{x_1}}{1-\delta_{x_1}} \geq f(s_1)$.*

Proof. See Appendix C.3. □

For each match quality q, I show that $D(x_1, q) \geq D(x_2, q)$. Fix some quality q, suppose that f-matches (x_1, q) destroy and f-matches (x_2, q) do not destroy, then $d(x_1, q) \geq \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s)$. In words, due to the search friction, the destruction rate of children with a disability is higher than the dissolution rate of children without a disability. The interesting case is when both f-matches are not destroyed, then $\sum_{\mathcal{M}(x_2,q)} f(s) \geq \sum_{\mathcal{M}(x_1,q)} f(s)$ [by proposition 6.3(i)]. Hence, to ensure that $D(x_1, q) \geq D(x_2, q)$, I assume $\frac{\delta_{x_2}-\delta_{x_1}}{1-\delta_{x_1}} \geq f(s_1)$. This condition imposes a maximum bound on the rate at which f-matches (x_2, q) dissolve respect to the exogenous adoption rate. Loosely speaking, children without a disability are less likely to have an f-match disrupted because their dissolution rate is bounded above by the rate at which they exit the market exogenously (adjusted by the exit rate of children with a disability).

Corollary 7.4 (Probability of Foster Match Disruption is Decreasing in the Match Quality)**.** *Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Assume children' payoffs satisfy assumptions 1(a)(c)(d), parents' payoffs satisfy assumptions 2(a)(c), 3 and 4. For each child x, in any foster care equilibrium, if $a(x, q_1) = 0$ and $a(x, q_2) = 0$ then $D(x, q_1) \geq D(x, q_2)$.*

Proof. See Appendix C.4. □

Corollary 7.4 exhibits the impact of the match quality in the probability of f-match disruption. The result follows from propositions 6.1(ii) and 6.3(ii). Restricting attention to the case where f-matches do not exit through adoption, I show that f-matches of high-quality are less likely to disrupt than f-matches of low-quality because the driving forces of destruction and dissolution are aligned. Specifically, as long as agents' payoffs are increasing in the match quality (along with other conditions), the probability of destruction and the probability of dissolution are decreasing in the match quality. I focus on the case where f-matches do not exit through adoption since it is crucial to understand what happens to children if they remain in the market.

7.3 Probability of Becoming Foster Matched

Consider child x who is unmatched at the beginning of a period. Let $\gamma^3(x)$ denote the **probability that child x becomes f-matched** next period, defined as follows:

$$\gamma^3(x) = \pi^c(\theta) \sum_{\mathcal{M}(x,q_0)} f(s) \sum_q g(q|s) \big(1 - d(x,q)\big)$$

Stylized fact 3 holds if and only if $\gamma^3(x_2) \geq \gamma^3(x_1)$. Corollary 7.5 describes the sufficient conditions for stylized fact 3 to arise in equilibrium. The result follows from propositions 6.1(i) and 6.2(i). First children with a disability are less likely to form an f-match. Second, if they form an f-match, children with a disability are more likely to have it destroyed.

Corollary 7.5 (Stylized Fact 3: Disability Decreases the Probability of Becoming Foster Matched)**.** *Assume children' payoffs satisfy assumption 1(a), parents' payoffs satisfy assumptions 2(a)(b), 3 and 4. Then, in any foster care equilibrium, $\gamma^3(x_2) \geq \gamma^3(x_1)$.*

Proof. See Appendix C.5. □

7.4 Probability of Becoming Unmatched

Consider child x who is, at the beginning of a period, f-matched when the quality is q. Let $U(x,q)$ denote the **probability that child x f-matched when the quality is q becomes unmatched** next period, defined as follows:

$$U(x,q) = (1-\delta_x)\,\big(1 - a(x,q)\big)\Bigg\{ \underbrace{d(x,q)}_{destruction} \underbrace{\Big[1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q_0)} f(s) \sum_{q'} g(q'|s) \big(1 - d(x,q')\big)\Big]}_{1-\gamma^3(x)}$$
$$+ \big(1 - d(x,q)\big) \underbrace{\pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)}_{dissolution} \sum_{q'} g(q'|s)\, d(x,q') \Bigg\}$$

A child x f-matched when the quality is q becomes unmatched in two cases. First, if the f-match (x, q) is destroyed and she remains unmatched after the search and f-matching stage. Second, if the f-match (x, q) dissolves and the new f-match is later on destroyed.

Using the endogenous distribution over f-matches, the **probability that child x becomes unmatched** is:

$$\gamma^4(x) = \sum_q U(x, q) \cdot \frac{m(x, q)}{\sum_q m(x, q)}$$

Stylized fact 4 holds if and only if $\gamma^4(x_1) \geq \gamma^4(x_2)$. In words, children with a disability, relative to children without a disability, are more likely to become unmatched. There are potentially two driving forces working on opposite directions in this case. On the one hand, by proposition 6.1(i) and corollary 7.5, children with a disability are more likely to destroy an f-match and more likely to remain unmatched, which makes them more likely to become unmatched. On the other hand, by propositions 6.3(i) and 6.1(i), children with a disability are less likely to dissolve an f-match but are more likely to destroy the new f-match later on, thus is not clear who is more likely to become unmatched.

Corollary 7.6 presents sufficient conditions for stylized fact 4 to arise in equilibrium. The result makes use of propositions 6.1(i), 6.2(i), 6.3, and 6.4(i).

Corollary 7.6 (Stylized Fact 4: Disability Increases the Probability of Becoming Unmatched)**.** *Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Assume children' payoffs satisfy assumptions 1(a) to (g) and 5(a) to (c), parents' payoffs satisfy assumptions 2(a) to (e), 3 and 4. Then, in any foster care equilibrium, $\gamma^4(x_1) \geq \gamma^4(x_2)$ if $\frac{\delta_{x_2} - \delta_{x_1}}{1 - \delta_{x_1}} \geq f(s_1)$ and $\frac{1 - \delta_{x_1}}{2 - \delta_{x_1} - \delta_{x_2}} > \overline{\pi}$.*

Proof. See Appendix C.6. □

For each match quality q, I show that $U(x_1, q) \geq U(x_2, q)$. As for stylized fact 2, when both f-matches are not destroyed, it follows that $\pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \geq \pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s)$ [by proposition 6.3(i)]. Hence, to ensure that $U(x_1, q) \geq U(x_2, q)$ holds, I also impose condition $\frac{\delta_{x_2} - \delta_{x_1}}{1 - \delta_{x_1}} \geq f(s_1)$. In addition, in this stylized fact a new interesting situation arises when children with a disability have their f-match destroyed and children without a disability remain f-matched. Thus, fixing a match quality q, $U(x_2, q) = (1 - \delta_{x_2}) \left(\pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_{q'} g(q'|s)\, d(x_2, q')\right)$ and $U(x_1, q) = (1 - \delta_{x_1}) \left(1 - \gamma^3(x_1)\right)$. To establish that the second equation is bigger than the first I impose an upper bound on the meeting probability. This creates a lower bound on the probability that a child with a disability remains unmatched, and an upper bound on the probability that a child without a disability dissolves an f-match (taking the risk that it is destroyed later on).

Corollary 7.7 (Model Prediction: Probability of Becoming Unmatched is Decreasing in the Match Quality). *Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Assume children' payoffs satisfy assumptions 1(a)(c)(d), parents' payoffs satisfy assumptions 2(a)(c), 3 and 4. For each child x, in any foster care equilibrium, $U(x, q_1) \geq U(x, q_2)$ if $a(x, q_1) = 0$ and $a(x, q_2) = 0$.*

Proof. See Appendix C.7. □

Corollary 7.7 exhibits the impact of match quality on the probability of becoming unmatched. The result follows from propositions 6.1(ii) and 6.3(ii). I show that children in high-quality matches are less likely to become unmatched than children in low-quality matches because both the probability of destruction and the probability of dissolution are decreasing in the quality.

8 Concluding Remarks

This paper provides an extensive analysis of the match transitions and exit through adoption of children relinquished for adoption in the US foster care system. For policymakers, the primary concerns regarding foster care are to decrease foster match disruptions and to increase permanency via adoption. Yet, I find that foster match disrupts play a crucial role in adoption by influencing the incentives of foster parents to adopt. Due to the presence of the financial penalty on adoption, parents face the following trade-off when deciding to adopt: accept the penalty in exchange for eliminating the likelihood that the child disrupts the foster match in the future. Moreover, I show that foster disruptions allow agents to avoid 'bad matches', and more importantly, enables children to search for 'better matches' while in a foster environment.

Concerning the child's observable characteristics, I show that foster disruptions involving children with a disability are mainly driven by the uncertainty on the quality of the match, while foster disruptions involving children without a disability are driven to improve the match quality. Also, I find that high-quality matches are less likely to be disrupted. For adoption, I show that the adoption penalty not only exacerbates the intrinsic disadvantage faced by children with a disability, but it also creates incentives for high-quality matches to not transit to adoption.

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A Appendix: Omitted Equations

For each x, $u^c(x)$ satisfies the following equality:

$$\underbrace{u^c(x)\left\{\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} f(s)\sum_q\Big[\delta_x+(1-\delta_x)(1-d(x,q))g(q|s)\Big]+\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} f(s)\Big)\delta_x\right\}}_{\textit{mass destruction}}=$$
$$\underbrace{\sum_q m(x,q)(1-\delta_x)\left\{\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\sum_{q'} g(q'|s)\;d(x,q')+\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\Big)d(x,q)\right\}+\rho\, l(x)}_{\textit{mass creation}} \tag{A.1}$$

For each (x,q), $m(x,q)$ satisfies the following equality:

$$\underbrace{m(x,q)\left\{\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)+\Big(1-\pi^c(\theta_t)\sum_{\mathcal{M}(x,q)} f(s)\Big)\Big[\delta_x+(1-\delta_x)\;d(x,q)\;a(x,q)\Big]\right\}}_{\textit{mass destruction}}=$$
$$\underbrace{\begin{aligned}&u^c(x)\;\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} f(s)g(q|s)\;(1-\delta_x)\Big(1-d(x,q)\Big)\;\Big(1-a(x,q)\Big)\\&+\sum_{q'} m(x,q')\;\pi^c(\theta)\sum_{\mathcal{M}(x,q')} f(s)g(q|s)(1-\delta_x)\Big(1-d(x,q)\Big)\;\Big(1-a(x,q)\Big)\end{aligned}}_{\textit{mass creation}} \tag{A.2}$$

B Appendix: Proofs of Equilibrium Analysis

B.1 Proof of Lemma 6.1

(i) Fix (x,q) and assume that $b^c(x,q)$ is non-negative. By contradiction, suppose $d^c(x,q)=1$ then, by the equilibrium definition, it follows that $\hat{\mathcal{C}}(x,q_0)>\hat{\mathcal{C}}(x,q)$. Equivalently, the following inequality must hold:

$$\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} f(s)\Big)\,\mathcal{C}(x,q_0)+\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)}\mathbb{E}_s[\mathcal{C}(x,q)]f(s)>$$
$$\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\Big)\,\mathcal{C}(x,q)+\pi^c(\theta)\sum_{\mathcal{M}(x,q)}\mathbb{E}_s[\mathcal{C}(x,q)]f(s) \tag{B.1}$$

By assumption $\hat{\mathcal{C}}(x,q_0) > \hat{\mathcal{C}}(x,q)$, the value function for child x f-matched when the quality is q is equal to:

$$\mathcal{C}(x,q) = b^c(x,q) + \beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta} + \\ \beta(1-\delta_x)\Big[\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} f(s)\Big)\,\mathcal{C}(x,q_0) + \pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} \mathbb{E}_s[\mathcal{C}(x,q)]f(s)\Big]$$

Since $b^c(x,q)$ is non-negative, it follows that:

$$\begin{aligned}\mathcal{C}(x,q) &= b^c(x,q) + \beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta} \\ &\quad + \beta(1-\delta_x)\Big[\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} f(s)\Big)\,\mathcal{C}(x,q_0) + \pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} \mathbb{E}_s[\mathcal{C}(x,q)]f(s)\Big] \\ &\geq \beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta} + \beta(1-\delta_x)\Big[\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} f(s)\Big)\,\mathcal{C}(x,q_0) + \pi^c(\theta)\sum_{\mathcal{M}(x,q_0)} \mathbb{E}_s[\mathcal{C}(x,q)]f(s)\Big] \\ &= \mathcal{C}(x,q_0)\end{aligned}$$

Before continuing, let me do the following remark. In equilibrium, $s \in \mathcal{M}(x,\bar{q})$ if and only if $\mathbb{E}_s[\mathcal{C}(x,q)] \geq \mathcal{C}(x,\bar{q})$ and $\mathbb{E}_s[\mathcal{P}(x,q)] \geq \mathcal{P}^u$. Thus, if $\mathcal{C}(x,q) \geq \mathcal{C}(x,q_0)$ then $\mathcal{M}(x,q) \subseteq \mathcal{M}(x,q_0)$. Now, I show that $\mathcal{C}(x,q) \geq \mathcal{C}(x,q_0)$ contradicts $\hat{\mathcal{C}}(x,q_0) > \hat{\mathcal{C}}(x,q)$. For this, I analyze two cases: $\mathcal{C}(x,q) = \mathcal{C}(x,q_0)$ and $\mathcal{C}(x,q) > \mathcal{C}(x,q_0)$.

► Case 1: If $\mathcal{C}(x,q) = \mathcal{C}(x,q_0)$ then $\mathcal{M}(x,q) = \mathcal{M}(x,q_0)$. Thus $\hat{\mathcal{C}}(x,q) = \hat{\mathcal{C}}(x,q_0)$ which implies that $d^c(x,q) = 0$. A contradiction.

► Case 2: If $\mathcal{C}(x,q) > \mathcal{C}(x,q_0)$ then $\mathcal{M}(x,q) \subset \mathcal{M}(x,q_0)$. Here I define the set $\hat{\mathcal{M}}(x,q) =$

$\{s \in S | s \in \mathcal{M}(x, q_0) \setminus \mathcal{M}(x, q)\}$. Thus, the following holds:

$$
\begin{aligned}
\hat{\mathcal{C}}(x, q) =& \Big(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)\Big)\, \mathcal{C}(x, q) + \pi^c(\theta) \sum_{\mathcal{M}(x,q)} \mathbb{E}_s[\mathcal{C}(x, q)] f(s) \\
=& \Big(1 - \pi^c(\theta)\Big)\, \mathcal{C}(x, q) + \pi^c(\theta) \sum_{s \notin \mathcal{M}(x,q)} f(s)\, \mathcal{C}(x, q) + \pi^c(\theta) \sum_{\mathcal{M}(x,q)} \mathbb{E}_s[\mathcal{C}(x, q)] f(s) \\
=& \Big(1 - \pi^c(\theta)\Big)\, \mathcal{C}(x, q) + \pi^c(\theta) \sum_{s \notin \mathcal{M}(x,q_0)} f(s)\, \mathcal{C}(x, q) + \pi^c(\theta) \sum_{s \in \hat{\mathcal{M}}(x,q)} f(s)\, \mathcal{C}(x, q) \\
& + \pi^c(\theta) \sum_{\mathcal{M}(x,q)} \mathbb{E}_s[\mathcal{C}(x, q)] f(s) \\
>& \Big(1 - \pi^c(\theta)\Big)\, \mathcal{C}(x, q_0) + \pi^c(\theta) \sum_{s \notin \mathcal{M}(x,q_0)} f(s)\, \mathcal{C}(x, q_0) + \pi^c(\theta) \sum_{s \in \hat{\mathcal{M}}(x,q)} f(s)\, \mathcal{C}(x, q) \\
& + \pi^c(\theta) \sum_{\mathcal{M}(x,q)} \mathbb{E}_s[\mathcal{C}(x, q)] f(s)
\end{aligned}
$$

By definition, if $s \in \hat{\mathcal{M}}(x, q)$ then $\mathcal{C}(x, q) > \mathbb{E}_s[\mathcal{C}(x, q)] > \mathcal{C}(x, q_0)$. Thus, the following holds:

$$
\begin{aligned}
\hat{\mathcal{C}}(x, q) >& \Big(1 - \pi^c(\theta)\Big)\, \mathcal{C}(x, q_0) + \pi^c(\theta) \sum_{s \notin \mathcal{M}(x,q_0)} f(s)\, \mathcal{C}(x, q_0) + \pi^c(\theta) \sum_{s \in \hat{\mathcal{M}}(x,q)} f(s)\, \mathcal{C}(x, q) \\
& + \pi^c(\theta) \sum_{\mathcal{M}(x,q)} \mathbb{E}_s[\mathcal{C}(x, q)] f(s) \\
>& \Big(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q_0)} f(s)\Big)\, \mathcal{C}(x, q_0) + \pi^c(\theta) \sum_{\mathcal{M}(x,q_0)} \mathbb{E}_s[\mathcal{C}(x, q)] f(s) = \hat{\mathcal{C}}(x, q_0)
\end{aligned}
$$

which contradicts equation B.1. Hence, if $b^c(x, q) \geq 0$ then $d^c(x, q) = 0$.

(ii) ($\Rightarrow$) Fix (x, q). Assume $d^p(x, q) = 1$ then the following inequality must hold:

$$
0 > \max \left\{ \Big(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)\Big) \cdot \frac{b^p(x, q)}{1 - \beta(1 - \delta_x)\big(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)\big)}, \frac{b^p_a(x, q)}{1 - \beta} \right\}
$$

By contradiction, suppose $b^p(x, q)$ is non-negative. Since $1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s) \geq 0$ and $1 - \beta(1 - \delta_x)\big(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)\big) \geq 0$, there is a contradiction. Hence, $d^p(x, q) = 1$ only if $b^p(x, q)$ is negative.

($\Leftarrow$) Fix (x, q) and assume that $b^p(x, q)$ is negative. By contradiction, suppose $d^p(x, q) = 0$. There are two possible cases: $a^p(x, q) = 1$ or $a^p(x, q) = 0$.

► Case 1: If $a^p(x, q) = 1$ then $\frac{b^p_a(x,q)}{1-\beta} > 0$ must hold. Since $b^p(x, q)$ is negative then, by assumption 2(a), $b^p_a(x,q)$ is also negative. Hence, there is a contradiction.

▶ <u>Case 2:</u> If $a^p(x,q)=0$ and $d^p(x,q)=0$, then the following inequality must hold:

$$\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\Big)\cdot\frac{b^p(x,q)}{1-\beta(1-\delta_x)\big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\big)}\geq \max\left\{0\,,\,\frac{b^p_a(x,q)}{1-\beta}\right\}$$

Since $b^p(x,q)$ is negative then, by assumption 2(a), $b^p_a(x,q)$ is also negative. Thus, it must be that $d^p(x,q)=1$.

B.2 Proof of Proposition 6.1

By assumption 1(a) and lemma 6.1(i), it follows that $d^c(x,q)=0$ for all (x,q). This implies that the total probability of destruction of an f-match (x,q) is $d(x,q)=d^p(x,q)$. In words, the destruction of f-matches depends on the destruction strategies of parents.

(i) Fix some quality q. Suppose a parent f-matched to child x_2 when the quality is q chooses $d^p(x_2,q)=1$. Then, by lemma 6.1(ii), $b^p(x_2,q)$ is negative. By assumption 2(b), $b^p(x_1,q)$ is also negative. Thus, by lemma 6.1(ii), $d^p(x_1,q)=1$. Hence, $d(x_1,q)\geq d(x_2,q)$.

(ii) Fix some child x. Suppose a parent f-matched to child x when the quality is q chooses $d^p(x,q)=1$. Then, by lemma 6.1(ii), $b^p(x,q)$ is negative. Now, consider some q' such that $q'<q$. By assumption 2(c), $b^p(x,q')$ is also negative. Thus, by lemma 6.1(ii), $d^p(x,q')=1$. Hence, $d(x,q')\geq d(x,q)$ whenever $q'<q$.

B.3 Proof of Lemma 6.2

Fix x. In any foster care equilibrium, $s\in F^c(x,q_0)$ if and only if $\mathbb{E}_s[\mathcal{C}(x,q)]\geq\mathcal{C}(x,q_0)$. Now, I show that if $\mathbb{E}_s[b^c(x,q)]\geq 0$ then $\mathbb{E}_s[\mathcal{C}(x,q)]\geq\mathcal{C}(x,q_0)$, for all $s\in S$. Fix s and consider the conditional expected value $\mathbb{E}_s[\mathcal{C}(x,q)]$ given by:

$$\mathbb{E}_s[\mathcal{C}(x,q)]=\sum_q b^c(x,q)g(q|s)+\beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta}+\beta(1-\delta_x)\sum_q\Bigg[d^p(x,q)\hat{\mathcal{C}}(x,q_0)+$$
$$a^p(x,q)\max\left\{\frac{b^p_a(x,q)}{1-\beta}\,,\hat{\mathcal{C}}(x,q_0)\,,\hat{\mathcal{C}}(x,q)\right\}+\Big(1-d^p(x,q)-a^p(x,q)\Big)\max\left\{\hat{\mathcal{C}}(x,q_0)\,,\hat{\mathcal{C}}(x,q)\right\}\Bigg]g(q|s)$$

Since the destruction of f-matches is unilateral, the conditional expected value $\mathbb{E}_s[\mathcal{C}(x,q)]$ is bounded bellow by $\sum_y b^c(x,q)g(q|s)+\beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta}+\beta(1-\delta_x)\,\hat{\mathcal{C}}(x,q_0)$. Given that $\sum_q b^c(x,q)g(q|s)$

is non-negative, the following inequality holds:

$$\mathbb{E}_s[\mathcal{C}(x,q)] \geq \sum_y b^c(x,q)g(q|s) + \beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta} + \beta(1-\delta_x)\,\hat{\mathcal{C}}(x,q_0)$$
$$\geq \beta\,\delta_x\,\frac{b^c_a(x,q_N)}{1-\beta} + \beta(1-\delta_x)\,\hat{\mathcal{C}}(x,q_0) = \mathcal{C}(x,q_0)$$

Hence, if $\mathbb{E}_s[b^c(x,q)] \geq 0$ then $\mathbb{E}_s[\mathcal{C}(x,q)] \geq \mathcal{C}(x,q_0)$.

B.4 Proof of Lemma 6.3

($\Rightarrow$) Fix x. In any foster care equilibrium, $s \in F^p(x)$ if and only if $\mathbb{E}_s[\mathcal{P}(x,q)] \geq 0$. I show that if $\mathbb{E}_s[b^p(x,q)] \geq 0$ then $\mathbb{E}_s[\mathcal{P}(x,q)] \geq 0$. Fix s and consider the conditional expected value $\mathbb{E}_s[\mathcal{P}(x,q)]$ given by:

$$\mathbb{E}_s[\mathcal{P}(x,q)] = \sum_q b^p(x,q)g(q|s)$$
$$+\,\beta(1-\delta_x)\sum_q \Bigg[a^c(x,q)\max\left\{\frac{b^p_a(x,q)}{1-\beta}\,,0\,,\frac{\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)}f(s)\Big)\,b^p(x,q)}{1-\beta\,(1-\delta_x)\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)}f(s)\Big)}\right\}$$
$$+\Big(1-d^p(x,q)-a^p(x,q)\Big)\max\left\{0\,,\frac{\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)}f(s)\Big)b^p(x,q)}{1-\beta\,(1-\delta_x)\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)}f(s)\Big)}\right\}\Bigg]g(q|s)$$

Since f-match destruction is unilateral, the conditional expected value $\mathbb{E}_s[\mathcal{P}(x,q)]$ is bounded bellow by $\sum_q b^p(x,q)g(y|s)$. Hence, if $\mathbb{E}_s[b^p(x,q)] \geq 0$ then $\mathbb{E}_s[\mathcal{P}(x,q)] \geq 0$.

($\Leftarrow$) Fix (x,s). I show that, if $\mathbb{E}_s[b^p(x,q)]$ is negative then $\mathbb{E}_s[\mathcal{P}(x,q)]$ is also negative. First, note that $\sum_q \mathcal{P}(x,q)g(q|s)$ is bounded above by the following expression:

$$\sum_q \overline{\mathcal{P}(x,q)}g(q|s) \;=\; \sum_q b^p(x,q)g(q|s) \;+\; \beta(1\;-\;\delta_x)\sum_q\left[\max\left\{\frac{b^p_a(x,q)}{1-\beta}\,,0\,,\frac{b^p(x,q)}{1-\beta\,(1-\delta_x)}\right\}\right]g(q|s)$$

Since $\sum_q b^p(x,q)g(q|s)$ is negative, by assumption 4, $\sum_q \overline{\mathcal{P}(x,q)}g(q|s)$ is also negative.

B.5 Proof of Proposition 6.2

Fix child x. By definition, f-matches must be mutually agreed upon $s \in \mathcal{M}(x,q_0)$ if and only if $s \in F^c(x,q_0)$ and $s \in F^p(x)$. First, I analyze f-match formation strategies of children. By assumption 1(a), it follows that $\mathbb{E}_s[b^c(x,q)] \geq 0$ for all $s \in S$. Hence, by lemma 6.2, $F^c(x,q_0) = S$.

(i) Fix signal s, I show that if $s \in F^p(x_1)$ then $s \in F^p(x_2)$. Suppose $s \in F^p(x_1)$ then, by lemma 6.3, it follows that $\mathbb{E}_s[b^p(x_1, q)]$ must be non-negative. Since $b^p(x_2, q) \geq b^p(x_1, q)$ for all $q \in Q$ [assumption 2(b)], then $\mathbb{E}_s[b^p(x_2, q)]$ is also non-negative. Thus, by lemma 6.3, $s \in F^p(x_2)$. By assumption 3, it follows that $F^p(x_1)$ and $F^p(x_2)$ are non-empty. Hence, $\mathcal{M}(x, q_0)$ is non-empty for all x, and $\mathcal{M}(x_1, q_0) \subseteq \mathcal{M}(x_2, q_0)$.

(ii) Fix child x. Consider signals s and s' such that $s' > s$. I show that, if $s \in F^p(x)$ then $s' \in F^p(x)$. Suppose $s \in F^p(x)$ then, by lemma 6.3, it follows that $\mathbb{E}_s[b^p(x, q)]$ is non-negative. Given that $G(q|s') \leq G(q|s)$ and $b^p(x, q)$ is increasing in q [assumption 2(c)], it follows that $\mathbb{E}_{s'}[b^p(x, q)]$ is also non-negative. Hence, by lemma 6.3, $s' \in F^p(x)$. Hence, if $s \in \mathcal{M}(x, q_0)$ then $s' \in \mathcal{M}(x, q_0)$.

B.6 Proof of Lemma 6.4

Assume 2(a). A parent f-matched to child x when the quality is q announces adoption if and only if the following inequalities hold:

$$\frac{b_a^p(x, q)}{1-\beta} > 0 \tag{B.2}$$

$$\frac{b_a^p(x, q)}{1-\beta} > \left(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)\right) \cdot \frac{b^p(x, q)}{1 - \beta(1-\delta_x)\left(1 - \pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)\right)} \tag{B.3}$$

(i) Fix (x, q). Assume $b_a^p(x, q)$ is positive then $a^p(x, q) = 1$ if and only if inequality B.3 holds. The right-hand side of this inequality is decreasing in $\pi^c(\theta) \sum_{\mathcal{M}(x,q)} f(s)$. Thus, for $a^p(x, q)$ to take value one independent of the endogenous objects $\pi^c(\theta)$ and $\mathcal{M}(x, q)$, the following inequality must hold:

$$\frac{b_a^p(x, q)}{b(x, q)} > \frac{1-\beta}{1-\beta(1-\delta_x)}$$

Or, equivalently $\delta_x > \frac{b^p(x,q) - b_a^p(x,q)}{b_a^p(x,q)} \frac{1-\beta}{\beta}$.

(ii) Consider a parent f-matched to child x_1 when the quality is q. Assume $a^p(x_1, q) = 1$, then inequalities B.2 and B.3 hold for $x = x_1$. By assumption 2(b), it follows that $b_a^p(x_2, q) > 0$. Hence, $a^p(x_1, q) = 1$ implies $a^p(x_2, q) = 1$ if the following inequalities holds:

$$\frac{b_a^p(x_2,q)}{b^p(x_2,q)} > \frac{b_a^p(x_1,q)}{b^p(x_1,q)} \quad \text{and} \quad \frac{(1-\beta)\left(1-\pi^c(\theta)\sum_{\mathcal{M}(x_1,q)} f(s)\right)}{1-\beta(1-\delta_{x_1})\left(1-\pi^c(\theta)\sum_{\mathcal{M}(x_1,q)} f(s)\right)} \geq \frac{(1-\beta)\left(1-\pi^c(\theta)\sum_{\mathcal{M}(x_2,q)} f(s)\right)}{1-\beta(1-\delta_{x_2})\left(1-\pi^c(\theta)\sum_{\mathcal{M}(x_2,q)} f(s)\right)}$$

By assumption 2(d), the first inequality holds. Since $\delta_{x_2} \geq \delta_{x_1}$ and $\sum_{\mathcal{M}(x_2,q)} f(s) \geq \sum_{\mathcal{M}(x_1,q)} f(s)$, then the second inequality holds.

(iii) Consider a parent f-matched to child x when the quality is q. Assume $a^p(x,q) = 1$, then inequalities B.2 and B.3 hold. Also, consider a parent f-matched to child x when the quality is q' such that $q' < q$. Since $b^p(x,q') \geq 0$, then $a^p(x,q) = 1$ implies $a^p(x,q') = 1$ if the following inequalities holds:

$$\frac{b_a^p(x,q')}{b^p(x,q')} > \frac{b_a^p(x,q)}{b^p(x,q)} \qquad \text{and} \qquad \frac{(1-\beta)\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\Big)}{1-\beta(1-\delta_x)\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\Big)} \geq \frac{(1-\beta)\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q')} f(s)\Big)}{1-\beta(1-\delta_x)\Big(1-\pi^c(\theta)\sum_{\mathcal{M}(x,q')} f(s)\Big)}$$

By assumption 2(e), the first inequality always holds. The second inequality holds since $\sum_{\mathcal{M}(x,q')} f(s) \geq \sum_{\mathcal{M}(x,q)} f(s)$ by assumption.

B.7 Proof of Proposition 6.3

First I show that, as a best-response, children with and without a disability choose the same dissolution strategy, and both are more willing to dissolve an f-match of low-quality q_1 than a high-quality match q_2. Formally:

Lemma B.1 (Dissolution Strategies of Children)**.** *Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Assume children' payoffs satisfy assumptions 1(a)(c)(d). If $d(x,q_1) \geq d(x,q_2)$ for all x, then the dissolution strategies of children are $F^c(x_1,q_1) = F^c(x_2,q_1) = \{s_1,s_2\}$ and $F^c(x_1,q_2) = F^c(x_2,q_2) = \{\emptyset\}$.*

Proof. Assume $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. For each x, $F^c(x,q_2) \cap F^c(x,q_1) = \{s_1,s_2\}$ or $F^c(x,q_2) \cap F^c(x,q_1) = \{\emptyset\}$. The reason is the following. For each signal s, $s \in F^c(x,q_2)$ if and only if $\mathbb{E}_s[\mathcal{C}(x,q)] = \mathcal{C}(x,q_1)g(q_1|s) + \mathcal{C}(x,q_2)g(q_2|s) \geq \mathcal{C}(x,q_2)$. Then, it must be that $\mathcal{C}(x,q_1) \geq \mathcal{C}(x,q_2)$ independent of the distributions. Now, if $\mathcal{C}(x,q_1) > \mathcal{C}(x,q_2)$ then $s \notin F^c(x,q_1)$, and if $\mathcal{C}(x,q_1) = \mathcal{C}(x,q_2)$ then $s \in F^c(x,q_1)$. Hence, there are three possible cases: (1) $F^c(x,q_2) = F^c(x,q_1) = \{s_1,s_2\}$, (2) $F^c(x,q_2) = \{s_1,s_2\}$ and $F^c(x,q_1) = \{\emptyset\}$, and (3) $F^c(x,q_2) = \{\emptyset\}$ and $F^c(x,q_1) = \{s_1,s_2\}$.

Fix x. I show that $\mathcal{C}(x,q_2) > \mathcal{C}(x,q_1)$ holds, thus only the third case is feasible. Since $d(x,q_1) \geq d(x,q_2)$, the following cases might arise:

► Case a: Suppose $a(x,q_1) = 1$ then $\mathcal{C}(x,q_1) = b^c(x,q_1) + \beta\,\delta_x \frac{b_a^c(x,q_2)}{1-\beta} + \beta(1-\delta_x)\frac{b_a^c(x,q_1)}{1-\beta}$

(a1) if $a(x,q_2) = 1$ then $\mathcal{C}(x,q_2) - \mathcal{C}(x,q_1) = b^c(x,q_2) - b^c(x,q_1) + \beta(1-\delta_x)\Big[\frac{b_a^c(x,q_2)}{1-\beta} - \frac{b_a^c(x,q_1)}{1-\beta}\Big]$

(a2) if $a(x,q_2) = 0$ and $d(x,q_2) = 0$ then $\mathcal{C}(x,q_2) - \mathcal{C}(x,q_1) = b^c(x,q_2) - b^c(x,q_1) + \beta(1-\delta_x)\Big[\hat{\mathcal{C}}(x,q_2) - \frac{b_a^c(x,q_1)}{1-\beta}\Big]$

► Case b: Suppose $d(x,q_1) = 1$ then $\mathcal{C}(x,q_1) = b^c(x,q_1) + \beta\,\delta_x \frac{b_a^c(x,q_2)}{1-\beta} + \beta(1-\delta_x)\hat{\mathcal{C}}(x,q_0)$

(b1) if $d(x,q_2) = 1$ then $\mathcal{C}(x,q_2) - \mathcal{C}(x,q_1) = b^c(x,q_2) - b^c(x,q_1)$

(b2) if $a(x,q_2)=1$ then $\mathcal{C}(x,q_2)-\mathcal{C}(x,q_1)=b^c(x,q_2)-b^c(x,q_1)+\beta(1-\delta_x)\left[\frac{b^c_a(x,q_2)}{1-\beta}-\hat{\mathcal{C}}(x,q_0)\right]$

(b3) if $a(x,q_2)=0$ and $d(x,q_2)=0$ then $\mathcal{C}(x,q_2)-\mathcal{C}(x,q_1)=b^c(x,q_2)-b^c(x,q_1)+\beta(1-\delta_x)\left[\hat{\mathcal{C}}(x,q_2)-\hat{\mathcal{C}}(x,q_0)\right]$

► <u>Case c:</u> Suppose $a(x,q_1)=0$ and $d(x,q_1)=0$ then $\mathcal{C}(x,q_1)=b^c(x,q_1)+\beta\,\delta_x\frac{b^c_a(x,q_2)}{1-\beta}+\beta(1-\delta_x)\hat{\mathcal{C}}(x,q_1)$

(c1) if $a(x,q_2)=1$ then $\mathcal{C}(x,q_2)-\mathcal{C}(x,q_1)=b^c(x,q_2)-b^c(x,q_1)+\beta(1-\delta_x)\left[\frac{b^c_a(x,q_2)}{1-\beta}-\hat{\mathcal{C}}(x,q_1)\right]$

(c2) if $a(x,q_2)=0$ and $d(x,q_2)=0$ then $\mathcal{C}(x,q_2)-\mathcal{C}(x,q_1)=b^c(x,q_2)-b^c(x,q_1)+\beta(1-\delta_x)\left[\hat{\mathcal{C}}(x,q_2)-\hat{\mathcal{C}}(x,q_1)\right]$

Assume 1(a)(c), then $\mathcal{C}(x,q_2)-\mathcal{C}(x,q_1)>0$ in cases (a1), and (b1). For case (b3), if $d(x,q_2)=0$ then it must be that $\hat{\mathcal{C}}(x,q_2)\geq\hat{\mathcal{C}}(x,q_0)$. Thus, by assumption 1(a)(c) it follows that $\mathcal{C}(x,q_2)-\mathcal{C}(x,q_1)>0$ in case (b3). By assumption 1(d) it follows that $\frac{b^c_a(x,q_2)}{1-\beta}\geq\hat{\mathcal{C}}(x,\bar{q})$ for all q. Hence, by assumptions 1(a)(c)(d) it follows that $\mathcal{C}(x,q_2)-\mathcal{C}(x,q_1)>0$ for all the other cases.

Therefore, $F^c(x_1,q_2)=F^c(x_2,q_2)=\{\emptyset\}$ and $F^c(x_1,q_1)=F^c(x_2,q_1)=\{s_1,s_2\}$. □

Now, I show Proposition 6.3. Suppose $Q=\{q_1,q_2\}$ and $S=\{s_1,s_2\}$. First, by assumptions 1(a) and 2(a)(c), it follows that proposition 6.1(ii) holds. Thus, for children the result in lemma B.1 holds, that is, $F^c(x,q_2)=\{\emptyset\}$ and $F^c(x,q_1)=\{s_1,s_2\}$ for all x. For parents, by assumptions 2(b), 3 and 4, it follows that proposition 6.2(i) holds, that is, $F^p(x)$ is non-empty for all x, and $F^p(x_1)\subseteq F^p(x_2)$. Moreover, by adding assumption 2(c), proposition 6.2(ii) holds. That is, if $s_1\in F^p(x)$ then $s_2\in F^p(x)$ for all x.

Given that $s\in\mathcal{M}(x,q)$ if and only if $s\in F^c(x,q)$ and $s\in F^p(x)$, it follows that (a) $\mathcal{M}(x,q_1)$ is non-empty for all x, (b) $\mathcal{M}(x,q_2)=\{\emptyset\}$ for all x, (c) $\mathcal{M}(x_1,q_1)\subseteq\mathcal{M}(x_2,q_1)$, and (d) $s_1\in\mathcal{M}(x,q_1)$ implies $s_2\in\mathcal{M}(x,q_1)$ for all x.

Since $\mathcal{M}(x,q_2)=\{\emptyset\}$ for all x, then $\sum_{\mathcal{M}(x,q_2)}f(s)=0$ for all x. Hence, $\sum_{\mathcal{M}(x,q_1)}f(s)\geq\sum_{\mathcal{M}(x,q_2)}f(s)$. Now, since $\mathcal{M}(x_1,q_1)\subseteq\mathcal{M}(x_2,q_1)$ then $\sum_{\mathcal{M}(x_2,q)}f(s)\geq\sum_{\mathcal{M}(x_1,q)}f(s)$ for all q.

B.8 Proof of Proposition 6.4

First I show the following lemma:

Lemma B.2 (Adoption Strategies of Children). *Suppose $Q=\{q_1,q_2\}$ and $S=\{s_1,s_2\}$. Assume children' payoffs satisfy assumptions 1(a) to (g), and 5(a) to (c). Moreover, suppose the following (a) $d(x,q_2)=0$ for all x, (b) $\mathcal{M}(x,q_1)$ is non-empty for all x, (c) $\mathcal{M}(x,q_2)$ is empty for all x, (d) $\mathcal{M}(x_1,q_1)\subseteq\mathcal{M}(x_2,q_1)$, (e) $s_1\in\mathcal{M}(x,q_1)$ implies $s_2\in\mathcal{M}(x,q_1)$ for all x, and (f) $a^p(x_2,q)\geq a^p(x_1,q)$*

for all q. Then, the adoption strategies of children are $a^c(x_2, q) \geq a^c(x_1, q)$ for all q, and $1 = a^c(x, q_2) \geq a^c(x, q_1)$ for all x.

Proof. Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Consider child x f-matched when the quality is q. Since $d^c(x, q) = 0$ [by assumption 1(a) and lemma 6.1], then she announce adoption if and only if:

$$\frac{b_a^c(x,q)}{1-\beta} > \frac{\big(b^c(x,q) + \beta\delta_x \frac{b_a^c(x,q_2)}{1-\beta}\big)\big(1 - \pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\big) + \pi^c(\theta)\sum_{\mathcal{M}(x,q)} \mathbb{E}_s[\mathcal{C}(x,q)]f(s)}{1 - \beta(1-\delta_x)\big(1 - \pi^c(\theta)\sum_{\mathcal{M}(x,q)} f(s)\big)} \quad \text{(B.4)}$$

Since $\mathcal{M}(x, q_2) = \{\emptyset\}$, inequality B.4 is equal to:

$$\frac{b_a^c(x,q_2)}{1-\beta} > \frac{b^c(x,q_2) + \beta\delta_x \frac{b_a^c(x,q_2)}{1-\beta}}{1-\beta\,(1-\delta_x)}$$

By assumption 1(a), this inequality holds. Hence, $a^c(x, q_2) = 1$ for all x.

Assume $\mathcal{M}(x, q_1)$ is non-empty for all x, $\mathcal{M}(x_1, q_1) \subseteq \mathcal{M}(x_2, q_1)$, $s_1 \in \mathcal{M}(x, q_1)$ implies $s_2 \in \mathcal{M}(x, q_1)$ for all x. Thus, there are three possible dissolution outcomes (1) $\mathcal{M}(x_1, q_1) = \{s_1, s_2\}$ and $\mathcal{M}(x_2, q_1) = \{s_1, s_2\}$, and (2) $\mathcal{M}(x_1, q_1) = \{s_2\}$ and $\mathcal{M}(x_2, q_1) = \{s_2\}$, and (3) $\mathcal{M}(x_1, q_1) = \{s_2\}$ and $\mathcal{M}(x_2, q_1) = \{s_1, s_2\}$.

▶ Case 1: Suppose $\mathcal{M}(x_1, q_1) = \{s_1, s_2\}$ and $\mathcal{M}(x_2, q_1) = \{s_1, s_2\}$. Fixing (x, q_1), inequality B.4 is equal to:

$$b_a^c(x,q_1)\Big\{\big(1-\pi^c(\theta)\big)\big(1-\beta\big) + \beta\delta_x\big(1-\pi^c(\theta)\big) + \pi^c(\theta)\big(g(q_2|s_1)f(s_1) + g(q_2|s_2)f(s_2)\big)\Big\} >$$
$$\Big(b^c(x,q_1) + \beta\delta_x\frac{b_a^c(x,q_2)}{1-\beta}\Big)\big(1-\pi^c(\theta)\big)\big(1-\beta\big) + \mathcal{C}(x,q_2)\,\pi^c(\theta)\big(g(q_2|s_1)f(s_1) + g(q_2|s_2)f(s_2)\big)\big(1-\beta\big)$$
$$\text{(B.5)}$$

Since $d(x, q_2) = 0$ and given the strategies of parents, the value function $\mathcal{C}(x, q_2)$ can take two values $\mathcal{C}(x, q_2) = b^c(x, q_2) + \beta\frac{b_a^c(x,q_2)}{1-\beta}$ or $\mathcal{C}(x, q_2) = \frac{b^c(x,q_2) + \beta\delta_x \frac{b_a^c(x,q_2)}{1-\beta}}{1-\beta(1-\delta_x)}$. Now, since $a^p(x_2, q) \geq a^p(x_1, q)$ for all q, I analyze the following sub-cases:

- Case 1a: Suppose $a^p(x_1, q_2) = 1$ and $a^p(x_2, q_2) = 1$. Child x announces adoption if and only if the following inequality holds:

$$\Big\{b_a^c(x,q_1) - b^c(x,q_1)\Big\}\big(1-\pi^c(\theta)\big)\big(1-\beta\big) > \Big\{b_a^c(x,q_2) - b_a^c(x,q_1)\Big\}\beta\delta_x\big(1-\pi^c(\theta)\big) +$$
$$\Big\{b^c(x,q_2)(1-\beta) + b_a^c(x,q_2)\beta - b_a^c(x,q_1)\Big\}\big(g(q_2|s_1)f(s_1) + g(q_2|s_2)f(s_2)\big)\pi^c(\theta) \quad \text{(B.6)}$$

where:

$$b_a^c(x,q_1) - b^c(x,q_1) > 0 \text{ by assumption 1(a)}$$
$$b_a^c(x,q_2) - b_a^c(x,q_1) > 0 \text{ by assumption 1(c)}$$
$$b^c(x,q_2)(1-\beta) + b_a^c(x,q_2)\beta - b_a^c(x,q_1) > 0 \text{ by assumptions 1(c)(d)}$$

Now, I show that if equation B.6 holds for child x_1 then it also holds for child x_2. By assumption 1(e), the following inequality holds:

$$\Big\{b_a^c(x_2,q_1)-b^c(x_2,q_1)\Big\}\big(1-\pi^c(\theta)\big)\big(1-\beta\big) \geq \Big\{b_a^c(x_1,q_1)-b^c(x_1,q_1)\Big\}\big(1-\pi^c(\theta)\big)\big(1-\beta\big) \quad \text{(B.7)}$$

By assumptions 1(f) and 5(a), the following inequality holds:

$$\Big\{b_a^c(x_1,q_2) - b_a^c(x_1,q_1)\Big\}\beta\delta_{x_1}\big(1-\pi^c(\theta)\big) \geq \Big\{b_a^c(x_2,q_2) - b_a^c(x_2,q_1)\Big\}\beta\delta_{x_2}\big(1-\pi^c(\theta)\big) \quad \text{(B.8)}$$

By assumptions 1(f)(g), the following inequality holds:

$$\Big\{b^c(x_1,q_2)(1-\beta) + b_a^c(x_1,q_2)\beta - b_a^c(x_1,q_1)\Big\}\big(g(q_2|s_1)f(s_1) + g(q_2|s_2)f(s_2)\big)\pi^c(\theta) \geq \\ \Big\{b^c(x_2,q_2)(1-\beta) + b_a^c(x_2,q_2)\beta - b_a^c(x_2,q_1)\Big\}\big(g(q_2|s_1)f(s_1) + g(q_2|s_2)f(s_2)\big)\pi^c(\theta) \quad \text{(B.9)}$$

Hence, if $a^c(x_1,q_1) = 1$ then $a^c(x_2,q_1) = 1$.

- Case 1b: Suppose $a^p(x_1,q_2) = 0$ and $a^p(x_2,q_2) = 0$. Child x announces adoption if and only if the following inequality holds:

$$\Big\{b_a^c(x,q_1) - b^c(x,q_1)\Big\}\big(1-\pi^c(\theta)\big)\big(1-\beta\big) > \Big\{b_a^c(x,q_2) - b_a^c(x,q_1)\Big\}\beta\delta_x\big(1-\pi^c(\theta)\big) + \\ \Big\{\frac{b^c(x,q_2)(1-\beta)}{1-\beta(1-\delta_x)} + \frac{b_a^c(x,q_2)\beta\ \delta x}{1-\beta(1-\delta_x)} - b_a^c(x,q_1)\Big\}\big(g(q_2|s_1)f(s_1) + g(q_2|s_2)f(s_2)\big)\pi^c(\theta) \quad \text{(B.10)}$$

Now, I show that if equation B.10 holds for child x_1 then it also holds for child x_2. Since equations B.7 and B.8 hold, then I check whether the following inequality is satisfied:

$$\Big[b^c(x_1,q_2)(1-\beta)+b_a^c(x_1,q_2)\beta-b_a^c(x_1,q_1)-\{b^c(x_1,q_2)-b^c(x_1,q_1)\}\beta(1-\delta_{x_1})\Big]\big(1-\beta+\beta\delta_{x_2}\big) \geq \\ \Big[b^c(x_2,q_2)(1-\beta)+b_a^c(x_2,q_2)\beta-b_a^c(x_2,q_1)-\{b^c(x_2,q_2)-b^c(x_2,q_1)\}\beta(1-\delta_{x_2})\Big]\big(1-\beta+\beta\delta_{x_1}\big) \quad \text{(B.11)}$$

After some algebra, this inequality holds given assumptions 1(f)(g) and 5(a). Hence, if $a^c(x_1,q_1) = 1$ then $a^c(x_2,q_1) = 1$.

- Case 1c: Suppose $a^p(x_1, q_2) = 0$ and $a^p(x_2, q_2) = 1$. I show that if equation B.10 holds for child x_1 then equation B.6 holds for child x_2. Since equations B.7 and B.8 hold, I check whether the following inequality is satisfied:

$$b^c(x_1, q_2)(1-\beta) + b_a^c(x_1, q_2)\beta - b_a^c(x_1, q_1) - \beta(1-\delta_{x_1})\Big[b_a^c(x_1, q_2) - b_a^c(x_1, q_1)\Big] \geq$$
$$b^c(x_2, q_2)(1-\beta)+b_a^c(x_2, q_2)\beta-b_a^c(x_2, q_1)-\beta(1-\delta_{x_1})\Big[b^c(x_2, q_2)(1-\beta)+b_a^c(x_2, q_2)\beta-b_a^c(x_2, q_1)\Big] \tag{B.12}$$

After some algebra, this inequality holds given assumptions 1(b)(f)(g) and 5(b). Hence, if $a^c(x_1, q_1) = 1$ then $a^c(x_2, q_1) = 1$.

▶ Case 2: Suppose $\mathcal{M}(x_1, q_1) = \{s_2\}$ and $\mathcal{M}(x_2, q_1) = \{s_2\}$. Fixing (x, q_1), inequality B.4 is equal to:

$$b_a^c(x, q_1)\Big\{\big(1-\pi^c(\theta)f(s_2)\big)(1-\beta) + \beta\delta_x\big(1-\pi^c(\theta)f(s_2)\big) + \pi^c(\theta)g(q_2|s_2)f(s_2)\Big\}$$
$$> \left(b^c(x, q_1) + \beta\delta_x\frac{b_a^c(x, q_2)}{1-\beta}\right)\big(1-\pi^c(\theta)f(s_2)\big)(1-\beta) + \mathcal{C}(x, q_2)\,\pi^c(\theta)g(q_2|s_2)f(s_2)(1-\beta) \tag{B.13}$$

As in the previous case, I analyze the following sub-cases:

- Case 2a: Suppose $a^p(x_1, q_2) = 1$ and $a^p(x_2, q_2) = 1$. Child x announces adoption if and only if the following inequality holds:

$$\Big\{b_a^c(x, q_1) - b^c(x, q_1)\Big\}\big(1-\pi^c(\theta)f(s_2)\big)(1-\beta) > \Big\{b_a^c(x, q_2) - b_a^c(x, q_1)\Big\}\beta\delta_x\big(1-\pi^c(\theta)f(s_2)\big) +$$
$$\Big\{b^c(x, q_2)(1-\beta) + b_a^c(x, q_2)\beta - b_a^c(x, q_1)\Big\}g(q_2|s_2)f(s_2)\pi^c(\theta) \tag{B.14}$$

By equations B.7, B.8 and B.9, it follows that if equation B.14 holds for child x_1 then it also holds for child x_2. Hence, if $a^c(x_1, q_1) = 1$ then $a^c(x_2, q_1) = 1$.

- Case 2b: Suppose $a^p(x_1, q_2) = 0$ and $a^p(x_2, q_2) = 0$. Child x announces adoption if and only if the following inequality holds:

$$\Big\{b_a^c(x, q_1) - b^c(x, q_1)\Big\}\big(1-\pi^c(\theta)f(s_2)\big)(1-\beta) > \Big\{b_a^c(x, q_2) - b_a^c(x, q_1)\Big\}\beta\delta_x\big(1-\pi^c(\theta)f(s_2)\big) +$$
$$\Big\{\frac{b^c(x, q_2)(1-\beta)}{1-\beta(1-\delta_x)} + \frac{b_a^c(x, q_2)\beta\ \delta x}{1-\beta(1-\delta_x)} - b_a^c(x, q_1)\Big\}g(q_2|s_2)f(s_2)\big)\pi^c(\theta) \tag{B.15}$$

By equations B.7, B.8 and B.11, it follows that if equation B.15 holds for child x_1 then it also holds for child x_2. Hence, if $a^c(x_1, q_1) = 1$ then $a^c(x_2, q_1) = 1$.

- Case 2c: Suppose $a^p(x_1, q_2) = 0$ and $a^p(x_2, q_2) = 1$. By equations B.7, B.8 and B.12, it follows that if equation B.15 holds for child x_1 then equation B.14 holds for child x_2. Hence, if $a^c(x_1, q_1) = 1$ then $a^c(x_2, q_1) = 1$.

▶ Case 3: Suppose $\mathcal{M}(x_1, q_1) = \{s_2\}$ and $\mathcal{M}(x_2, q_1) = \{s_1, s_2\}$.

- Case 3a: Suppose $a^p(x_1, q_2) = 1$ and $a^p(x_2, q_2) = 1$. I show that if equation B.14 holds for child x_1 then equation B.6 holds for child x_2. After some algebra, since equations B.7, B.8 and B.9 hold, it suffices to check whether the following inequality holds:

$$\left\{b_a^c(x_1, q_2) - b_a^c(x_1, q_1)\right\} \beta\delta_{x_1} \geq \left\{b_a^c(x_1, q_2) - b^c(x_1, q_1)\right\}(1-\beta)+ \\ \left\{b^c(x_2, q_2)(1-\beta) + b_a^c(x_2, q_2)\beta - b_a^c(x_2, q_1)\right\}g(q_2|s_1) \quad \text{(B.16)}$$

  This inequality is satisfied by assumption 5(c). Hence, if $a^c(x_1, q_1) = 1$ then $a^c(x_2, q_1) = 1$.

- Case 3b: Suppose $a^p(x_1, q_2) = 0$ and $a^p(x_2, q_2) = 0$. I show that if equation B.15 holds for child x_1 then equation B.10 holds for child x_2. After some algebra, since equations B.7, B.8 and B.9 hold, it suffices to check whether the following inequality holds:

$$\left\{b_a^c(x_1, q_2) - b_a^c(x_1, q_1)\right\} \beta\delta_{x_1} \geq \left\{b_a^c(x_1, q_2) - b^c(x_1, q_1)\right\}(1-\beta)+ \\ \left\{\frac{b^c(x_2, q_2)(1-\beta)}{1-\beta(1-\delta_{x_2})} + \frac{b_a^c(x_2, q_2)\beta\delta_{x_2}}{1-\beta(1-\delta_{x_2})} - b_a^c(x_2, q_1)\right\}g(q_2|s_1) \quad \text{(B.17)}$$

  This inequality is satisfied by assumption 5(c). Hence, if $a^c(x_1, q_1) = 1$ then $a^c(x_2, q_1) = 1$.

- Case 3c: Suppose $a^p(x_1, q_2) = 0$ and $a^p(x_2, q_2) = 1$. Since equations B.7, B.8, B.12 and B.16 hold, it follows that if equation B.15 holds for child x_1 then equation B.6 holds for child x_2. Hence, if $a^c(x_1, q_1) = 1$ then $a^c(x_2, q_1) = 1$.

□

Now, I show Proposition 6.4. Suppose $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. I start by showing that statements (a) to (f) hold. By proposition, 1(a), 2(a)(c) and 3, it follows that $d(x, q_2) = 0$ for all q. Further, then results in proposition 6.3 hold by assumptions 1(a)(c)(d), 2(a)(b)(c), 3 and 4.

Now, fixing q, I show that if $a^p(x_1, q) = 1$ then $a^p(x_2, q) = 1$. Assume that $a^p(x_1, q) = 1$, then it must be that $b^p(x_1, q) > 0$. By assumption 2(b), it follows that $b^p(x_2, q) > 0$. Suppose $\frac{b_a^p(x_1,q)}{b^p(x_1,q)} > \frac{1-\beta}{1-\beta(1-\delta_{x_1})}$, then $a(x_1, q) = 1$ [lemma 6.4(i)]. Since $\delta_{x_2} \geq \delta_{x_1}$ and assumption 2(d) holds, then $\frac{b_a^p(x_2,q)}{b^p(x_2,q)} \geq \frac{b_a^p(x_1,q)}{b^p(x_1,q)} > \frac{1-\beta}{1-\beta(1-\delta_{x_1})} \geq \frac{1-\beta}{1-\beta(1-\delta_{x_2})}$. Thus, by lemma 6.4(i), $a^p(x_2, q) = 1$. Now, since proposition 6.3(i) holds, the result in lemma lemma 6.4(ii) follows. Hence, $a^p(x_2,q) \geq a^p(x_1,q)$ for all q.

Given the above, results in lemma B.2 hold. That is, $a^c(x_2,q) \geq a^c(x_1,q)$ for all q, and $1 = a^c(x,q_2) \geq a^c(x,q_1)$ for all x.

(i) Fix q, by definition, $a(x,q) = 1$ if and only if $a^c(x,q) = 1$ and $a^p(x,q) = 1$. Since $a^c(x_2,q) \geq a^c(x_1,q)$ and $a^p(x_2,q) \geq a^p(x_1,q)$, then $a(x_2,q) \geq a(x_1,q)$.

(ii) Fix x. Suppose $b^p_a(x,q_1) > 0$, then $b^p_a(x,q_2) > 0$ [by assumption 2(c)]. Since $\frac{b^p_a(x,q_2)}{b^p(x,q_2)} \leq \frac{1-\beta}{1-\beta(1-\delta_x)}$ and $\mathcal{M}(x,q_2)$ is empty, then $a^p(x,q_2) = 0$. Thus, $a^p(x,q_1) \geq a^p(x,q_2) = 0$. Since $a^c(x,q_2) \geq a^c(x,q_1)$, it follows that $a(x,q_1) \geq a(x,q_2) = 0$.

C Appendix: Proofs of Stylized Facts and Model Predictions

C.1 Proof of Corollary 7.1

Assume $Q = \{q_1,q_2\}$ and $S = \{s_1,s_2\}$. Given the definition of $\gamma^1(x)$ it suffices to show that $A(x_2,q_0) - A(x_1,q_0) \geq 0$ and $A(x_2,q) - A(x_1,q) \geq 0$ for all q.

Consider unmatched children, the result follows from propositions 6.2(i) and 6.4(i). That is, $\mathcal{M}(x_1,q_0) \subseteq \mathcal{M}(x_2,q_0)$ and $a(x_2,q) \geq a(x_1,q)$ for all q. Consider the set $\hat{\mathcal{M}}(x_2,q_0)$ defined as $\hat{\mathcal{M}}(x_2,q_0) = \{s \in S | s \in \mathcal{M}(x_2,q_0) \setminus \mathcal{M}(x_1,q_0)\}$, then the following inequality holds:

$$\begin{aligned}
A(x_2,q_0) &= \delta_{x_2} + (1-\delta_{x_2})\, \pi^c(\theta) \sum_{\mathcal{M}(x_2,q_0)} f(s) \sum_q g(q|s)\;\; a(x_2,q) \\
&\geq \delta_{x_2} + (1-\delta_{x_2})\, \pi^c(\theta) \sum_{\mathcal{M}(x_2,q_0)} f(s) \sum_q g(q|s)\;\; a(x_1,q) \\
&\geq \delta_{x_2} + (1-\delta_{x_2})\, \pi^c(\theta) \Big[\sum_{\hat{\mathcal{M}}(x_2,q_0)} f(s) \sum_q g(q|s)\; a(x_1,q)) + \sum_{\mathcal{M}(x_1,q_0)} f(s) \sum_q g(q|s)\; a(x_1,q) \Big] \\
&\geq \delta_{x_2} + (1-\delta_{x_2})\, \pi^c(\theta) \sum_{\mathcal{M}(x_1,q_0)} f(s) \sum_q g(q|s)\; a(x_1,q) \\
&\geq \delta_{x_1} + (1-\delta_{x_1})\, \pi^c(\theta) \sum_{\mathcal{M}(x_1,q_0)} f(s) \sum_q g(q|s)\; a(x_1,q) = A(x_1,q_0)
\end{aligned}$$

Now, consider f-matched children and fix a match quality q. By propositions 6.1(i) and 6.4(i), there are three cases to analyze:

► Case 1: Suppose $a(x_1,q) = a(x_2,q) = 1$ then $A(x_2,q) - A(x_1,q) = 0$

► Case 2: Suppose $d(x_1,q) = 1$.

- Case 2a: Suppose $a(x_2, q) = 1$ then $A(x_2, q) - A(x_1, q) = 1 - A(x_1, q_0) \geq 0$.

- Case 2b: Suppose $d(x_2, q) = 1$ then $A(x_2, q) - A(x_1, q) = A(x_2, q_0) - A(x_1, q_0) \geq 0$

- Case 2c: Suppose $a(x_2, q) = d(x_2, q) = 0$ then:

$$A(x_2, q) - A(x_1, q) = \delta_{x_2} + (1 - \delta_{x_2})\, \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_q g(q|s)\;\; a(x_2, q) - A(x_1, q_0)$$

  It suffices to show that the following inequality holds $\delta_{x_2} > \delta_{x_1} + (1 - \delta_{x_1})\, \pi^c(\theta)$. Since $\frac{\delta_{x_2} - \delta_{x_1}}{(1-\delta_{x_1})} > \overline{\pi}$ it is satisfied.

▶ Case 3: Suppose $a(x_1, q) = 0$ and $d(x_1, q) = 0$.

- Case 3a: Suppose $a(x_2, q) = 1$ then:

$$A(x_2, q) - A(x_1, q) = 1 - \underbrace{\Big(\delta_{x_1} = (1 - \delta_{x_1})\, \pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s) \sum_q g(q|s)\;\; a(x_1, q)\Big)}_{\leq 1} \geq 0$$

- Case 3b: Suppose $a(x_2, q) = d(x_2, q) = 0$ then:

$$A(x_2, q) - A(x_1, q) = \delta_{x_2} + (1 - \delta_{x_2})\, \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_q g(q|s)\;\; a(x_2, q) - \\ \delta_{x_1} - (1 - \delta_{x_1})\, \pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s) \sum_q g(q|s)\;\; a(x_1, q)$$

  By proposition 6.3(i) and 6.4(i), the following inequality holds:

$$\sum_{\mathcal{M}(x_2,q)} f(s) \sum_q g(q|s)\;\; a(x_2, q) \geq \sum_{\mathcal{M}(x_1,q)} f(s) \sum_q g(q|s)\;\; a(x_1, q) \tag{C.1}$$

  Hence $A(x_2, q) - A(x_1, q) \geq 0$.

C.2 Proof of Corollary 7.2

Assume $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Fix child x, by proposition 6.3(ii) and 6.4(ii) follows that $A(x, q_2) = \delta_x$. For match quality q_1, by proposition 6.1(ii), there are three cases to analyze:

▶ Case 1: Suppose $a(x, q_1) = 1$ then $A(x, q_1) = 1$. Thus, $A(x, q_1) \geq A(x, q_2)$.

▶ Case 2: Suppose $d(x, q_1) = 1$ then $A(x, q_1) = \delta_x + (1 - \delta_x)\, A(x, q_0)$. Thus, $A(x, q_1) \geq A(x, q_2)$.

▶ Case 3: Suppose $a(x, q_1) = d(x, q_1) = 0$ then:

$$A(x, q_1) = \delta_x + \Big(1 - \delta_x\Big)\pi^c(\theta) \sum_{\mathcal{M}(x,q_1)} f(s) \sum_{q'} g(q'|s)a(x, q')$$

Thus, $A(x, q_1) \geq A(x, q_2)$.

C.3 Proof of Corollary 7.3

Assume $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Given the definition of $\gamma^2(x)$ it suffices to show that $D(x_1, q) - D(x_2, q) \geq 0$ for all q.

Fix a match quality q. By propositions 6.1(i) and 6.4(i), there are three cases to analyze:

▶ Case 1: Suppose $a(x_1, q) = a(x_2, q) = 1$ then $D(x_1, q) - D(x_2, q) = 0$

▶ Case 2: Suppose $d(x_1, q) = 1$ then:

$$D(x_1, q) - D(x_2, q) = (1-\delta_{x_1}) - (1-\delta_{x_2})(1-a(x_2, q))\Big[d(x_2, q) + \big(1-d(x_2, q)\big)\ \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s)\Big]$$

Since $\delta_{x_2} \geq \delta_{x_1}$ and $1 \geq (1 - a(x_2, q))\Big[d(x_2, q) + \big(1 - d(x_2, q)\big)\ \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s)\Big] \geq 0$ it follows that $D(x_1, q) - D(x_2, q) \geq 0$.

▶ Case 3: Suppose $d(x_1, q) = a(x_1, q) = 0$ then:

$$\begin{aligned} D(x_1, q) - D(x_2, q) = (1 - \delta_{x_1})\pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s) \\ - (1 - \delta_{x_2})(1 - a(x_2, q))\Big[d(x_2, q) + \big(1 - d(x_2, q)\big)\ \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s)\Big] \end{aligned}$$

- Case 3a: Suppose $a(x_2, q) = 1$ then $D(x_1, q) - D(x_2, q) = (1 - \delta_{x_1})\pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s) \geq 0$.

- Case 3b: Suppose $a(x_2, q) = d(x_2, q) = 0$ then:

$$D(x_1, q) - D(x_2, q) = (1 - \delta_{x_1})\pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s) - (1 - \delta_{x_2})\pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s)$$

  For match quality q_2, the proof of proposition 6.3 shows that $\mathcal{M}(x, q_2) = \{\emptyset\}$ for all x. Hence, $D(x_1, q_2) - D(x_2, q_2) = 0$.

  For match quality q_1, since $1 \geq \sum_{\mathcal{M}(x_2,q_1)} f(s)$, it suffices to check that the following inequality holds:

$$(1-\delta_{x_1})\pi^c(\theta)\sum_{\mathcal{M}(x_1,q_1)} f(s) - (1-\delta_{x_2})\pi^c(\theta) \geq 0$$

The proof of proposition 6.3 shows that $\mathcal{M}(x_1, q_1)$ is non-empty, and $\mathcal{M}(x_1, q_1) = \{s_1, s_2\}$ or $\mathcal{M}(x_1, q_1) = \{s_2\}$. In the first case, $D(x_1, q_1) - D(x_2, q_1) = (1-\delta_{x_1}) - (1-\delta_{x_2}) \geq 0$. In the second case, $D(x_1, q_1) - D(x_2, q_1) = (1-\delta_{x_1})\pi^c(\theta)f(s_2) - (1-\delta_{x_2})\pi^c(\theta)$ which is positive if and only if $f(s_2) \geq \frac{(1-\delta_{x_2})}{(1-\delta_{x_1})}$.

C.4 Proof of Corollary 7.4

Assume $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Fix child x, and suppose that $a(x, q_1) = a(x, q_2) = 0$. By propositions 6.1(ii) and 6.3(ii) it follows that for all x, $d(x, q_1) \geq d(x, q_2)$ and $\sum_{\mathcal{M}(x,q_1)} f(s) \geq \sum_{\mathcal{M}(x,q_2)} f(s) = 0$ respectively. Hence, the following inequality holds:

$$\begin{aligned}
D(x, q_1) &= (1-\delta_x)\Big[d(x, q_1) + \big(1 - d(x, q_1)\big)\,\pi^c(\theta)\sum_{\mathcal{M}(x,q_1)} f(s)\Big] \\
&\geq (1-\delta_x)\Big[d(x, q_2) + \big(1 - d(x, q_2)\big)\,\pi^c(\theta)\sum_{\mathcal{M}(x,q_1)} f(s)\Big] \\
&\geq (1-\delta_x)\Big[d(x, q_2) + \big(1 - d(x, q_2)\big)\,\pi^c(\theta)\sum_{\mathcal{M}(x,q_2)} f(s)\Big] = D(x, q_2)
\end{aligned}$$

C.5 Proof of Corollary 7.5

The result follows from propositions 6.1(i) and 6.2(i). That is, $d(x_1, q) \geq d(x_2, q)$ for all q, and $\mathcal{M}(x_1, q_0) \subseteq \mathcal{M}(x_2, q_0)$. Let $\hat{\mathcal{M}}(x_2, q_0) = \{s \in S | s \in \mathcal{M}(x_2, q_0) \setminus \mathcal{M}(x_1, q_0)\}$, then the following inequality holds:

$$\begin{aligned}
\gamma^3(x_2) &= \pi^c(\theta)\sum_{\mathcal{M}(x_2,q_0)} f(s)\sum_q g(q|s)\,\big(1 - d(x_2, q)\big) \\
&\geq \pi^c(\theta)\sum_{\mathcal{M}(x_2,q_0)} f(s)\sum_q g(q|s)\,\big(1 - d(x_1, q)\big) \\
&\geq \pi^c(\theta)\Big[\sum_{\hat{\mathcal{M}}(x_2,q_0)} f(s)\sum_q g(q|s)\,\big(1 - d(x_1, q)\big) + \sum_{\mathcal{M}(x_1,q_0)} f(s)\sum_q g(q|s)\,\big(1 - d(x_1, q)\big)\Big] \\
&\geq \pi^c(\theta)\sum_{\mathcal{M}(x_1,q_0)} f(s)\sum_q g(q|s)\,\big(1 - d(x_1, q)\big) = \gamma^3(x_1)
\end{aligned}$$

Hence, $\gamma^3(x_2) \geq \gamma^3(x_1)$.

C.6 Proof of Corollary 7.6

Assume $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Given the definition of $\gamma^4(x)$ it suffices to show that $U(x_1, q) - U(x_2, q) \geq 0$ for all q.

Fix a match quality q. By propositions 6.1(i) and 6.4(i), there are three cases to analyze:

▶ Case 1: Suppose $a(x_1, q) = a(x_2, q) = 1$ then $U(x_1, q) - U(x_2, q) = 0$

▶ Case 2: Suppose $d(x_1, q) = 1$ then:

$$U(x_1, q) - U(x_2, q) = (1 - \delta_{x_1})\Big(1 - \gamma^3(x_1)\Big)$$
$$-(1-\delta_{x_2})\ \big(1-a(x_2, q)\big)\Big\{d(x_2, q)\Big(1-\gamma^3(x_2)\Big)+\big(1-d(x_2, q)\big)\ \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_{q'} g(q'|s)\ d(x_2, q')\Big\}$$

- Case 2a: Suppose $a(x_2, q) = 1$ then $U(x_1, q) - U(x_2, q) = (1 - \delta_{x_1})(1 - \gamma^3(x_1)) \geq 0$.

- Case 2b: Suppose $d(x_2, q) = 1$ then $U(x_1, q) - U(x_2, q) = (1 - \delta_{x_1})(1 - \gamma^3(x_1)) - (1 - \delta_{x_2})(1 - \gamma^3(x_2))$. By corollary 7.5 it follows that $U(x_1, q) - U(x_2, q) \geq 0$.

- Case 2c: Suppose $a(x_2, q) = d(x_2, q) = 0$ then:

$$U(x_1, q) - U(x_2, q) = (1 - \delta_{x_1})(1 - \gamma^3(x_1)) - (1 - \delta_{x_2})\pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_{q'} g(q'|s)\ d(x_2, q')$$

  For match quality q_2, the proof of proposition 6.3 shows that $\mathcal{M}(x, q_2) = \{\emptyset\}$ for all x. Hence, $U(x_1, q_2) - U(x_2, q_2) = (1 - \delta_{x_1})(1 - \gamma^3(x_1)) \geq 0$.
  For match quality q_1, since $(1-\pi^c(\theta)) \geq (1-\gamma^3(x_1))$ and $1 \geq \sum_{\mathcal{M}(x_2,q_1)} f(s) \sum_{q'} g(q'|s)\ d(x_2, q')$, it suffices to check that the following inequality holds:

$$(1 - \delta_{x_1})(1 - \pi^c(\theta)) - (1 - \delta_{x_2})\pi^c(\theta) \geq 0$$

  which holds if and only if $\frac{1-\delta_{x_1}}{2-\delta_{x_1}-\delta_{x_2}} \geq \overline{\pi}$.

▶ Case 3: Suppose $a(x_1, q) = 0$ and $d(x_1, q) = 0$ then:

$$U(x_1, q) - U(x_2, q) = (1 - \delta_{x_1})\pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s) \sum_{q'} g(q'|s)\ d(x_1, q')$$
$$-(1-\delta_{x_2})\ \big(1-a(x_2, q)\big)\Big\{d(x_2, q)\Big(1-\gamma^3(x_2)\Big)+\big(1-d(x_2, q)\big)\ \pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_{q'} g(q'|s)\ d(x_2, q')\Big\}$$

- Case 3a: Suppose $a(x_2, q) = 1$ then:

$$U(x_1, q) - U(x_2, q) = (1 - \delta_{x_1})\pi^c(\theta) \sum_{\mathcal{M}(x_1,q)} f(s) \sum_{q'} g(q'|s)\, d(x_1, q') \geq 0$$

- Case 3b: Suppose $a(x_2, q) = d(x_2, q) = 0$ then:

$$\begin{aligned} U(x_1, q) - U(x_2, q) = (1 - \delta_{x_1})\pi^c(\theta) &\sum_{\mathcal{M}(x_1,q)} f(s) \sum_{q'} g(q'|s)\, d(x_1, q') \\ &- (1 - \delta_{x_2})\pi^c(\theta) \sum_{\mathcal{M}(x_2,q)} f(s) \sum_{q'} g(q'|s)\, d(x_2, q') \end{aligned}$$

For match quality q_2, proposition 6.3 states that $\mathcal{M}(x, q_2) = \{\emptyset\}$ for all x. Hence, $U(x_1, q) - U(x_2, q) = 0$.

For match quality q_1, since $1 \geq \sum_{\mathcal{M}(x_2,q_1)} f(s)$, it suffices to check that the following inequality holds:

$$(1 - \delta_{x_1})\pi^c(\theta) \sum_{\mathcal{M}(x_1,q_1)} f(s) - (1 - \delta_{x_2})\pi^c(\theta) \geq 0$$

The proof of proposition 6.3 shows that $\mathcal{M}(x_1, q_1)$ is non-empty, and $\mathcal{M}(x_1, q_1) = \{s_1, s_2\}$ or $\mathcal{M}(x_1, q_1) = \{s_2\}$. In the first case, $D(x_1, q_1) - D(x_2, q_1) = (1 - \delta_{x_1}) - (1 - \delta_{x_2}) \geq 0$. In the second case, $D(x_1, q_1) - D(x_2, q_1) = (1 - \delta_{x_1})\pi^c(\theta)f(s_2) - (1 - \delta_{x_2})\pi^c(\theta)$ which is positive if and only if $f(s_2) \geq \frac{(1-\delta_{x_2})}{(1-\delta_{x_1})}$.

C.7 Proof of Corollary 7.7

Assume $Q = \{q_1, q_2\}$ and $S = \{s_1, s_2\}$. Fix child x, and suppose that $a(x, q_1) = 0$, $a(x, q_2) = 0$. By propositions 6.1(ii) and 6.3(ii) it follows that $d(x, q_1) \geq d(x, q_2)$ and $\sum_{\mathcal{M}(x,q_1)} f(s) \geq \sum_{\mathcal{M}(x,q_2)} f(s) = 0$ respectively. Hence, the following inequality holds:

$$\begin{aligned} U(x, q_1) &= (1 - \delta_x)\Big\{d(x, q_1)\,\big(1 - \gamma^3(x)\big) + \big(1 - d(x, q_1)\big)\,\pi^c(\theta) \sum_{\mathcal{M}(x,q_1)} f(s) \sum_{q'} g(q'|s)\, d(x, q')\Big\} \\ &\geq (1 - \delta_x)\Big\{d(x, q_1)\,\big(1 - \gamma^3(x)\big) + \big(1 - d(x, q_1)\big)\,\underbrace{\pi^c(\theta) \sum_{\mathcal{M}(x,q_2)} f(s) \sum_{q'} g(q'|s)\, d(x, q')}_{=0}\Big\} \\ &\geq (1 - \delta_x)\, d(x, q_2)\,\big(1 - \gamma^3(x)\big) = U(x, q_2) \end{aligned}$$

Table A1: Descriptive Statistics by Disability, All Samples

	Sample A		**Sample B**		**Sample C**	
Disability	Yes	No	Yes	No	Yes	No
Share becomes adoption matched	0.22	0.32	-	-	-	-
	(0.41)	(0.47)				
Share foster matched	0.89	0.96	1.00	1.00	0.00	0.00
	(0.31)	(0.19)	(0.00)	(0.00)	(0.00)	(0.00)
Share becomes foster matched	-	-	-	-	0.22	0.28
					(0.41)	(0.45)
Share becomes unmatched	-	-	0.03	0.01	-	-
			(0.18)	(0.11)		
Share disrupts foster match	-	-	0.19	0.18	-	-
			(0.40)	(0.38)		
Age in years	7.96	6.01	7.79	6.07	12.34	11.81
	(4.45)	(4.23)	(4.35)	(4.22)	(2.56)	(3.22)
Share Male	0.57	0.50	0.60	0.50	0.65	0.58
	(0.50)	(0.50)	(0.50)	(0.50)	(0.48)	(0.49)
Share White	0.43	0.44	0.41	0.42	0.45	0.42
	(0.49)	(0.50)	(0.49)	(0.50)	(0.50)	(0.49)
Share Black	0.25	0.23	0.27	0.24	0.26	0.29
	(0.43)	(0.42)	(0.44)	(0.43)	(0.44)	(0.45)
Share Hispanic	0.22	0.22	0.22	0.23	0.19	0.21
	(0.41)	(0.41)	(0.41)	(0.42)	(0.39)	(0.41)
Share Receiving Title IV-E	0.49	0.47	0.53	0.50	0.45	0.50
	(0.50)	(0.50)	(0.50)	(0.50)	(0.50)	(0.50)
Months in foster care	41.20	30.53	40.91	30.23	55.94	46.99
	(28.66)	(19.80)	(28.62)	(20.37)	(37.34)	(34.54)
Months since termination of parental rights	21.71	13.91	20.09	13.29	44.14	37.34
	(26.53)	(18.83)	(25.27)	(18.92)	(37.02)	(36.09)
Child-period observations ending in adoption	14.62	11.44	-	-	-	-
	(14.30)	(10.35)				
Months in the current placement	16.67	15.64	18.18	16.64	11.03	10.47
	(17.32)	(14.51)	(18.16)	(14.72)	(14.67)	(12.38)
Child-period observations foster matched	17.38	15.85	-	-	-	-
	(17.50)	(14.54)				
Number of child-period observations	474,824	690,994	285,519	373,734	45,073	20,897

Notes: Data are from Adoption and Foster Care Analysis and Reporting System (AFCARS). Means and standard deviations are calculated for child-period observations. Sample A is the full sample containing all children younger than age 16 whose parental rights have been terminated and who are either foster matched or unmatched. Sample B and Sample C are subsamples of A. Sample B (sample C) keeps only those child-period observations such that the child is foster matched (unmatched) at the beginning of the period and still in foster care at the end of the period.

Table A2: Regression Output

	Becomes Adoption matched (1)	**Foster matched (2)**	**Disrupts Foster match (3)**	**Becomes Foster matched (4)**	**Becomes Unmatched (5)**
Age in years	-0.002***	-0.001***	0.001***	-0.002***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Disability	-0.059***	-0.043***	0.023***	-0.045***	0.011***
	(0.005)	(0.002)	(0.002)	(0.006)	(0.001)
Male	-0.011***	-0.017***	-0.003***	-0.030***	0.003***
	(0.001)	(0.001)	(0.001)	(0.004)	(0.000)
White	0.022***	-0.004***	-0.006**	-0.002	0.001
	(0.002)	(0.001)	(0.002)	(0.007)	(0.001)
Black	-0.025***	0.001	0.005	-0.009	0.000
	(0.003)	(0.002)	(0.003)	(0.007)	(0.001)
Hispanic	0.007***	0.003***	-0.004	-0.007	-0.001
	(0.003)	(0.001)	(0.002)	(0.009)	(0.001)
Receiving Title IV-E	-0.079***	-0.009***	-0.002	0.013***	-0.000
	(0.005)	(0.003)	(0.002)	(0.004)	(0.001)
Months in foster care	0.002***	0.000***	-0.000***	-0.000***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Months since termination of parental rights	-0.001***	-0.002***	0.000***	-0.001***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Months in the current placement	-	-	-0.002***	-0.001***	-0.001***
			(0.000)	(0.000)	(0.000)
Pre-adoptive home	-	-	-0.154***	-	-0.017***
			(0.008)		(0.008)
Relative home	-	-	-0.070***	-	-0.011***
			(0.004)		(0.001)
Institution	-	-	-	-0.020***	-
				(0.005)	
Mean of dependent variable	0.279	0.934	0.185	0.236	0.021
Number of child-period observations	1,165,818	1,165,818	659,253	65,970	659,253
R-square	0.073	0.152	0.113	0.053	0.046

Notes: Data are from Adoption and Foster Care Analysis and Reporting System (AFCARS). All specifications control for child's demographics, states indicators and period indicators. The first and second columns consider sample A, third and fifth columns use sample B, and the fourth column uses sample C. Standard errors are cluster at the state-period level and shown in parentheses. ***$P < 0.01$; **$P < 0.05$; *$P < 0.10$.