Persuading Multiple Audiences: An Information Design Approach to Banking Regulation *

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Abstract

A policy maker concerned with the potential default of a bank sequentially conducts an *asset quality review* and a *liquidity stress test* under the scrutiny of multiple types of market participants (audiences). Surprisingly, the optimal *comprehensive assessment* (asset quality review and stress test) is opaque when the bank has high-quality assets, and transparent when the bank has poor-quality assets. The optimal policy also imposes contingent recapitalizations. Without them, disclosure of information may backfire and the bank may fare worse than under laissez faire. To deal with sudden liquidity shocks the policy maker optimally designs a persuasion mechanism that resembles an *emergency lending* facility, which (a) provides funds to banks in exchange for assets, and (b) discloses information about the bank's liquidity. Interestingly, imposing capital requirements hurts the effectiveness of the emergency lending program. In fact, public and private sector interventions are substitutes, and combinations of the two are strictly suboptimal. There exists a non-monotone pecking order: the private sector funds banks with either high or poor-quality assets, while institutions with intermediate-quality assets participate in the government's emergency lending mechanism. My results shed light on the role information disclosure as a regulatory tool in environments with multiple audiences and multi-dimensional fundamentals.

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1 Introduction

How much information should policy makers disclose about banks' balance sheets during financial crises? What is the optimal degree of transparency needed to calm down the market? Should policy makers provide more information about banks with stronger fundamentals or about those with weaker fundamentals? What is the interplay between disclosure of information and the policy makers' role as *Lenders of Last Resort*?

Information disclosure has become a prominent tool in banking supervision since the global financial crisis. In February 2009, the Federal Reserve introduced the Supervisory Capital Assessment Program (SCAP) (commonly known as the Fed's *stress test*). The objective was to assess whether the capital buffers of the 19 largest bank holding companies were enough to sustain lending in the event of an unexpectedly severe recession, and to communicate these results to the public (Hirtle and Lehnert [2015]). Many scholars and policy makers believe that the disclosure of stress tests results was a critical inflection point in the financial crisis as it provided market participants with credible information about potential losses which helped restore market confidence (Bernanke [2013]).

Since their introduction, *stress tests* and *asset quality reviews* have been regularly conducted both in the Eurozone and in the US.[1] Despite the consensus that transparency may impose market discipline on the otherwise opaque banking system (Morgan [2002], Flannery et al. [2013]),[2] there exists fundamental disagreement concerning the amount of information that should be disclosed as well as the set of policies that should accompany such disclosures. While the *stress tests* conducted by the Fed, for example, have combined granular data with a pass/fail grade,[3] the European Central Bank decided in 2016 to not assign grades to banks in order to avoid stigmatization. Moreover, while both regulatory authorities complement their disclosures with capital requirements, American regulators have chosen to publicly announce their decisions while their European counterparts have opted for private recommendations.[4],[5]

A crucial difficulty associated with the design of such disclosures is the complexity of the interactions among the multiple types of market participants involved. When a policy maker discloses information about a bank, she speaks to multiple *audiences* who care about different aspects of the bank's private information. Think, for example, of potential investors interested in the LT profitability of the bank's assets; ST creditors concerned by the bank's liquidity position; speculators interested in the fate of the bank; counterparties exposed to a potential default; taxpayers concerned with the use of public funds if a bailout takes place; the

[1]See Morgan et al. [2014], Flannery et al. [2017] and Petrella and Resti [2013] for evidence on the effect of such disclosure policies. The first two papers show that the stress tests conducted in the US provided information not previously available to the rest of market participants. The last paper provides similar evidence for the tests conducted in the EU.

[2]See Babus and Farboodi [2018] for a theory where opacity endogenously emerges as part of banks' strategy to create information asymmetry with external investors.

[3]In 2018, the Fed introduced for the first time an intermediate third grade: *conditional non-objection*, assigned to Goldman Sachs and Morgan Stanley. Both bank holding companies had to cut by half the amount they intended to distribute among shareholders in order to avoid failing the test.

[4]The privacy policy does not apply to those companies publicly listed for which capital requirements count as inside information and must be disclosed.

[5]Goldstein and Sapra [2014] offer an excellent review of the costs associated with information disclosures.

bank itself, which strategically chooses its funding strategy in response to the information publicly disclosed, among others. As a result, an optimally designed disclosure policy must necessarily account for the strategic reactions it induces on these multiple audiences.

Despite the recent attention that stress tests have drawn from the theoretical literature, the natural question concerning the *optimal* degree of transparency of such exercises remains elusive. The reason behind this observation is the standard assumption, usually encountered in the literature, of a single audience for the policy maker's disclosure which, to a large extent, simplifies the problem. When this is the case, the optimal policy is *opaque* and consists of an action recommendation to the single audience.[6] In most cases this takes the form of a pass/fail test (i.e., a recommendation whether to keep pledging funds to the bank). With multiple audiences, however, disclosures intended for a particular audience are simultaneously *observed* by the rest of market participants, generating an endogenous reaction. As a result, it is no longer clear what the optimal degree of transparency of such disclosures should be. Put differently, a crucial ingredient to discuss about the optimal degree of transparency of regulatory disclosures is accounting for the strategic interaction between the multiple audiences concerned about the banks' (multi-dimensional) private information. This paper aims to shed light on this question and to inform the debate on the optimal design of such disclosures.

To tackle this issue, I consider the minimal model that preserves the richness of the problem. The model consists of a bank, a policy maker, and two audiences: long-term investors and short-term creditors (henceforth, LT investors and ST creditors). The bank has private information about two dimensions, namely, (i) the long-term profitability of its assets and (ii) its liquidity position. Throughout the paper I refer to these two variables as the bank's fundamentals. Uncertainty about the bank's fundamentals is gradually resolved. While the quality of the bank's assets is determined early, the amount of liquid funds is determined at a later stage after a shock materializes. The timing is meant to reflect the idea that the quality of the bank's assets depends on investment decisions made in the past, while the liquidity position of the bank is subject to shocks and may suddenly change. The policy maker's technology allows her to learn the realization of these variables and to disclose information to the multiple audiences.[7]

The first audience, LT investors, is primarily interested in learning about the profitability of the bank's assets (e.g., the amount of non-performing loans). The second audience, ST creditors, on the other hand, is concerned by the bank's liquidity and its ability to repay short-term debt. Nevertheless, LT investors also care about disclosures concerning the bank's liquidity, as such information affects ST creditors' beliefs about the bank's buffers and, hence, their decisions of whether to keep rolling-over the bank's debt. Given that ST creditors' claims are senior to those of LT investors, the latter may be wiped out if ST creditors stop pledging to the bank. Therefore, LT investors are *indirectly* affected by disclosures about the bank's liquidity. In turn, ST creditors *indirectly* care about the profitability of the bank's assets. Disclosures about this dimension determine how much funds LT investors are willing to pay for claims on the bank's assets, and hence the bank's ability to raise funds to cover liquidity shortages in the future. Thus, optimal disclosures have a fixed-

[6] This is a manifestation of the Revelation Principle (Myerson [1982], Myerson [1986]).

[7] As is standard in the *information design* literature, I assume that the policy maker has commitment power and chooses the information disclosure policy before observing the true realization of the bank's fundamentals.

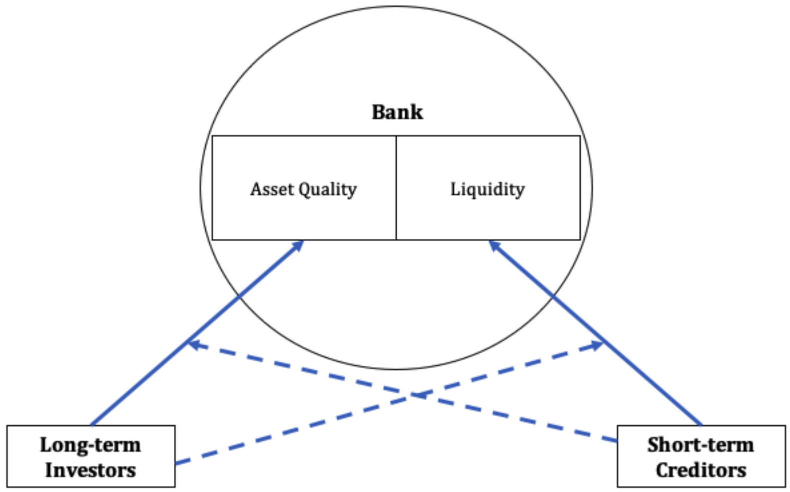

Figure 1: Persuading Multiple Audiences.

point structure in that disclosures about one of the dimensions (e.g., the profitability of the assets) account for the reaction of the audience who directly cares about it (LT investors), but *also* for the endogenous reaction of the audiences who *indirectly* care about that dimension (ST creditors).

The optimal *comprehensive assessment* is described as follows. The policy maker first examines the profitability of the bank's assets by conducting an *asset quality review*. When the profitability of the assets is above a threshold, the asset quality review assigns a *unique passing grade*. When the quality of the assets, instead, falls below such a threshold, the asset quality review assigns one of *multiple failing grades*. The optimal policy has a monotone-partitional structure in which adjacent quality levels are pooled together under the same grade. To improve the bank's chances of survival, and conditional on the bank having failed the asset quality review, the policy maker conducts a *liquidity stress test*. When the liquidity of the bank is sufficiently good, the bank is *passed*, which convinces ST creditors to keep rolling over the bank's debt. In the opposite case, the bank is given a *failing* grade, which prompts ST creditors to run.

The asymmetrical structure of the optimal comprehensive assessment (i.e., a unique passing grade and multiple failing grades) stems from the strategic interaction of the two audiences. When the profitability of the bank's assets is low there exists an endogenous *amplification effect* associated with increasing their *perceived* profitability. That is, increasing the perceived profitability of the assets induce LT investors to pay larger prices for claims on them. This increases the probability that the bank survives an eventual run of ST creditors as the bank can secure additional liquid funds. The increase in the survival probability then induces LT investors to offer an even higher price for claims on the assets, and so on. Thus, when the profitability of the assets is low, the interaction between both audiences generates an amplification mechanism which translate into the convexity of the probability of survival in the *perceived* profitability of the assets. The policy maker thus (strictly) prefers finer disclosure policies rather than coarser rules, similar to a risk-lover decision-maker who prefers lotteries over deterministic outcomes.

In contrast, when the profitability of the bank's assets is high, the bank may prevent default altogether by raising enough funds to persuade ST creditors that it has enough liquidity. Using a more transparent disclosure policy in this case does not help and, in fact, may hurt risk-sharing among banks with heterogeneous asset qualities. Thus, when the profitability of the bank's assets is sufficiently good, the optimal asset quality review assigns a unique and hence opaque passing grade.

Consistent with the qualitative properties of the optimal disclosure found in this paper, the recent empirical literature on stress tests has found evidence that banks with weaker fundamentals (i.e., riskier assets, more leverage, larger amounts of non-performing loans), are subject to more transparency than banks with stronger fundamentals (see e.g., Morgan et al. [2014], Flannery et al. [2017], and Ahnert et al. [2018]). The analysis proposed in this paper suggests that larger revisions in prices for weaker banks should not be interpreted as an anomaly but, instead, as a feature of optimal disclosures in environments with multiple audiences.

Crucially, I find that imposing contingent recapitalizations is key to the effectiveness of stress tests. Without recapitalizations, stress tests may backfire and prove even worse than the *laissez faire* policy. The intuition behind this result is that the bank's private information about the profitability of their assets induces a lemons problem when raising funds from LT inestors. Banks with high-quality assets have incentives to *separate* from those with weaker assets. They do this by retaining larger claims on their assets, and hence by exposing themselves to rollover risk. In the absence of government intervention, the threat of a run of ST creditors serves as a discipline device that curbs signalling incentives and induces banks to raise precautionary funds despite the lemons problems.[8]

Conducting stress tests reduces the subsequent probability of default by dissuading ST creditors from running for some liquidity levels. This, however, exacerbates the bank's incentives to signal, generating a countervailing effect that increases the likelihood of default. Recapitalizations are thus key to the effectiveness of stress tests as they substitute for the disciplining role of runs while keeping the benefits of regulatory disclosures. The policy maker achieves this by forbidding dividends if the bank fails to raise enough precautionary funds.

In the last part of the paper I study the interplay between information disclosure and the policy maker's role as LOLR. Specifically, I study the optimal design of emergency lending programs in the presence of asymmetric information, and the way disclosure of the bank's private information affects their design. To accommodate the practical concern that participation in such facilities reveals information about the bank's liquidity, I modify the baseline model and assume instead that the policy maker cannot conduct a stress test in period 2. In its place, the policy maker designs an emergency lending mechanism that offers to purchase some of the bank's assets and to publicly disclose information about the bank's liquidity position. That is, the policy maker *optimally* acts as the LOLR and, at the same time, try to persuade the market by disclosing relevant information regarding the bank's fundamentals.

[8] The disciplining role served by ST creditors has been described in the literature going back to Calomiris and Kahn [1991]. For recent developments, see Cheng and Milbradt [2012] and Eisenbach [2017].

The optimal emergency lending program asks the bank to (confidentially) report its private information and promises in return to assign a pass-fail grade. Contingent on assigning a passing grade, the policy maker purchases claims on the bank's assets. When the regulator announces the bank has passed the test, ST creditors find it optimal to keep rolling over the bank's debt. In turn, when the policy maker fails the bank, ST creditors stop rolling its debt.

To induce the different types to truthfully report their liquidity, the policy maker needs to compensate those types that are passed with lower probability, as otherwise no bank would truthfully report its private information. This compensation is done by offering higher prices (i.e., smaller discounts) for the bank's assets. The optimal emergency lending program offers most illiquid banks low passing probability but compensates them with higher prices for their assets, while more liquid banks are assigned a passing grade with higher probability but receive lower prices for their assets. In this manner, the policy maker improves the average liquidity position of banks receiving the passing grade, which persuades ST creditors to keep pledging funds to a *passing* bank.

I use the characterization of the optimal emergency lending program to show that private and public sector recapitalizations are *substitutes*. Interventions involving simultaneous pledging by both the private (LT investors) and the public sector (policy maker) are strictly suboptimal. In fact, imposing private sector recapitalizations undermines the effectiveness of the emergency lending program. A bank that retains a smaller fraction of its assets can be promised fewer funds by the policy maker. Given that the effectiveness of the program relies on compensating extremely vulnerable banks (which, again, are passed less often than more liquid banks) with higher prices for their assets, forcing the bank to sell a fraction of its assets to LT investor decreases the amount of funds that can be provided within the emergency lending program which deteriorates the policy maker elicitation capacity. Thus, having the bank raising funds from LT investors intensifies incentive compatibility issues. As a result, the optimal emergency lending mechanism minimizes any prior recapitalizations by the private sector.

The policy maker is thus confronted with the dilemma of choosing between private sector funding, which maximizes the price of the bank's securities by selling them before the liquidity shocks occur, and the government's emergency lending program, which asks the bank to report information about its liquidity buffers and then provide funds and reveals information to its ST creditors. I show that optimal comprehensive interventions display a non-monotone pecking order. Institutions with high-quality assets are passed by the asset quality review and are required to raise enough capital from LT investors. Banks with intermediate-quality assets are assigned one of multiple failing grades and are funded with the government's emergency lending program. Finally, institutions with extremely poor-quality assets are failed with one of multiple failing grades and are induced to seek private sector funding. The paper thus shows that the non-monotonicity in funding strategies need not be proof of sub-optimality. In fact, the non-monotone pecking order naturally arises when accounting for the strategic interaction of the multiple audiences involved.

The rest of the paper is organized as follows. Below, I wrap up the introduction with a brief review of the most pertinent literature. Section 2 presents the model. Section 3 describes the equilibria in the absence of government intervention. Section 4 studies the optimal comprehensive disclosure policy. Section 5 studies the case where the policy-maker designs an elicitation mechanism to learn the liquidity position of the bank. Proofs omitted in the text are in the Appendix or in the Supplementary Material.

Related literature. The paper is related to several strands of the literature. The first strand is the literature on *regulatory disclosures*. Close in spirit to this paper is the work by Faria-e Castro et al. [2016] who consider a model of information disclosure in an environment with runnable liabilities and asymmetric information. The paper finds a monotonic relationship between the government's fiscal capacity and the stress tests' degree of transparency. My paper complements theirs in several respects. First, I emphasize the interaction between multiple audiences, with heterogeneous objectives, who care about different aspects of the bank's multi-dimensional fundamentals. I show that this interaction gives rise to a novel amplification mechanism which translates into more transparent disclosures. Secondly, I rule out the possibility that the policy maker may use public funds to provide (imperfect) deposit insurance. ST credit usually takes the form of repo transactions, commercial paper, certificates of deposits, etc. Providing government insurance in such contexts is unlikely. Instead, I assume that the policy maker may act as a lender of last resort and purchase assets from the bank under a budget balance constraint that prevents the policy maker from losing money (in expectation). Finally, on a technical level, I consider fully flexible dynamic information structures (with a continuum of states).

Goldstein and Leitner [2018] consider the stress test design problem of a regulator who wishes to facilitate risk sharing among banks endowed with assets of heterogeneous qualities. My model complements theirs by analyzing an environment where the amount of additional funds needed by the bank is endogenously determined by the disclosure policy and the endogenous interaction between the multiple audiences. Orlov et al. [2017] consider the joint design of stress tests and capital requirements in a setting where multiple banks have correlated exposures to an exogenous shock. Inostroza and Pavan [2019] follow an adversarial approach and explore optimal disclosure policies with heterogeneously informed receivers. They show that optimal stress tests need not generate conformism in beliefs among market participants, but generate perfect coordination among their actions. Alvarez and Barlevy [2015] study the incentives of banks to disclose balance sheet (hard) information in a setting where the market is not able to observe the exposure to counterparty risks. In my model, banks cannot disclose hard information but may try to signal information through their funding strategy. Bouvard et al. [2015] study a model with rollover risk where a policy maker must choose between transparency and opacity but cannot ex-ante commit to a disclosure policy. In contrast, I assume the policy maker can fully commit to her disclosure policy and allow for fully-flexible information structures.

Optimal government interventions in markets plagued by adverse selection have been studied in Philippon and Skreta [2012], Tirole [2012], and Fuchs and Skrzypacz [2015]. These papers share the common feature that government interventions affect post-intervention outcomes and vice versa. The first two papers consider a static setting, and show that the policy maker optimally chooses to purchase low quality assets to *jump-start* a frozen market, permitting banks with better assets to receive funding from the private sector. The third paper considers a dynamic model in which low quality assets are sold first, which gradually improves the pool of legacy assets. I propose a model that shares the common feature of these papers. Namely, that the policy maker's emergency lending program generates endogenous participation constraints. In my model, however, the policy-maker may also engage in information design when trading with the bank.

The present paper also contributes to the extensive literature on security design with adverse selection, as in Myers and Majluf [1984], DeMarzo and Duffie [1999], and DeMarzo and Fishman [2007], among many others. Recent developments along these lines include Daley et al. [2016], who consider the effect of ratings on security issuance; Yang [2015], who studies security design when the buyer may acquire information about asset quality at a cost; Szydlowski [2018], who considers the problem of a firm that seeks financing and chooses both its information disclosure policy and the type of security it offers to investors; and Azarmsa and Cong [2018] who study the role of information in relationship finance. I adopt the framework of Nachman and Noe [1994], who consider the problem of a seller with private (but imperfect) information about the profitability of her assets, and who issues claims on them in exchange for funds that help her meet a former liability. I modify their setting by introducing an endogenous probability of default, which is determined in equilibrium. In contrast to their celebrated result, which shows existence of a unique equilibrium where all types of sellers pool over the same debt contract, I show that, in the current environment, there exist multiple equilibria.

Finally, this paper relates to the literature on *information design*. This literature can be traced back to Myerson [1986], who introduced the idea that, in a general class of dynamic games of incomplete information, the designer can restrict attention to private incentive-compatible action recommendations to agents. Recent developments include Kamenica and Gentzkow [2011], Kamenica and Gentzkow [2016], and Ely [2017]. These papers consider persuasion with a single receiver. Persuasion with multiple receivers is less studied. Calzolari and Pavan [2006a] consider an auction setting in which the sender is the initial owner of a good and where the different receivers are bidders in an upstream market who then resell in a downstream market. Related to this paper is Dworczak [2016], who offers an analysis of persuasion in environments with aftermarkets.Alonso and Camara [2016a] and Bardhi and Guo [2017] consider persuasion in a voting context, whereas Mathevet et al. [2016] and Taneva [2016] study persuasion in more general multi-receiver settings. Bergemann and Morris [2016a] and Bergemann and Morris [2016b] characterize the set of outcome distributions that can be sustained as Bayes-Nash equilibria under arbitrary information structures consistent with a given common prior. Alonso and Camara [2016b] study public persuasion in a setting with multiple receivers with heterogeneous priors. Kolotilin et al. [2017] consider a screening environment whereby the designer elicits the agents' private information prior to disclosing further information. Basak and Zhou [2017] and Doval and Ely [2017] study dynamic games in which the designer can control both the agents' information and the timing of their actions.

2 Model

Players and Actions. The economy consists of a bank, short-term (ST) creditors, long-term (LT) investors, and a policy maker. There are 3 periods, $T \equiv \{1,2,3\}$. The bank is risk-neutral and has two legacy assets: (i) a safe and liquid asset (e.g., treasuries, MBS, etc), and (ii) a risky and illiquid asset (e.g., a portfolio of loans).[9] Both assets mature in period 3. The risky asset delivers an observable stochastic cash flow, $y \in \mathbb{R}_+$, while the safe asset has a deterministic face value R. In period 1, to increase the amount of liquid funds available in period 2, the bank may sell claims on its assets to a competitive and risk-neutral set of LT investors. In period 2, the bank may suffer a temporary liquidity shock (described in detail below) that turns a fraction of the liquid asset illiquid, preventing the bank from selling a fraction of it.[10] Finally, on the liability side of the bank's balance sheet, a mass one of ST creditors, uniformly distributed on $[0,1]$, has a claim of \$1 due in period 2, which they may redeem at this period (*early*), or equal to R if they decide to *roll over* until $t=3$. Let $a_i \in \{0,1\}$ denote the action chosen by creditor i, where $a_i = 0$ represents the action of rolling over the bank's debt, and $a_i = 1$ the decision of withdrawing early (*running* on the bank). I denote by $A \in [0,1]$ the fraction of ST creditors withdrawing early.

Fundamentals. The fundamentals of the bank's balance sheet are captured by the vector (ω, y). The variable y represents the risky asset's cash flows, drawn from the absolutely continuous cdf F^y with support $\mathbb{R}_+$. The variable ω represents the bank's liquidity. More specifically, $\omega \in \Omega \equiv [0,1]$ represents the fraction of the safe asset that the bank can sell during period 2 to repay its obligations. A value of $\omega < 1$ can be interpreted as an unexpected liquidity shock which reduces the amount liquid funds available at $t=2$ to ω (e.g., haircuts imposed in the repo market). I assume that the fraction of the safe asset that is not liquidated in period 2 becomes available at $t=3$ and can be used to repay ST creditors who decide to roll over. Thus, ω represents a *temporary* liquidity shock. This assumption is made for simplicity.[11]

Default. If the fraction of ST creditors withdrawing early is larger than the bank's available cash (i.e., if $\omega < A$), bankruptcy is triggered. In that case, the bank's risky asset is liquidated. The risky asset's liquidation value is $l < 1$.[12]

Precautionary Fund-Raising. To reduce the probability of default, the bank may raise funds at $t=2$ by selling claims on its risky asset to LT investors. If the bank raises P units of funds, the amount of cash available to repay early withdrawals is given by $\omega + P$.

Exogenous Information. There is *gradual resolution of uncertainty.* At $t=1$, the bank's long-term cashflows, y, are drawn from F^y. The cashflow realization cannot be observed by any market participant.

[9] In the case of loans, the illiquidity assumption captures the idea that the bank has a technology to monitor the borrowers which cannot be easily transferred to external investors.

[10] Alternatively, there exists an additional stochastic obligation that needs to be paid in period 2.

[11] A model where a fraction $1-\omega$ of the safe asset is destroyed during the interim period (i.e., a permanent liquidity shock) can be captured in the present specification by assuming that the range of F^y is given by $[1-\omega, \infty)$.

[12] The rich environment of this paper allows to treat the value of the asset upon bankruptcy and the amount the bank can get by selling its asset differently. The former is captured by l and represents the amount that can be raised once the bank has filed for bankruptcy. The latter is captured with the fund-raising game described in detail below.

The bank then learns a private signal θ about y, and forms beliefs about the realization of y according to the conditional cdf F^y_θ (resp., pdf f^y_θ), where θ belongs to the set $\Theta = \{\theta_L, \theta_H\}$, with $\theta_H > \theta_L$. I refer to θ as the bank's type. The policy maker, LT investors, and ST creditors share a common prior $\mu_\theta \in \Delta\Theta$ about the bank's type. I assume that the conditional pdf f^y_θ satisfies log-supermodularity in (y, θ) (or, equivalently, that cashflows are ordered according to MLRP). The liquidity shock ω is drawn from $F^\omega \in \Delta\Omega$ at the beginning of period 2 and is only observed by the bank. These assumptions reflect the idea that the profitability of the bank's asset depends on investment decisions made in the past, while the bank's liquidity is subject to unexpected shocks and may suddenly change. All market participants share the prior belief F^ω about the bank's liquidity shock. I denote by λ the mass point at $\omega = 1$.[13]

Payoffs. For simplicity, I assume no discounting. If the bank raises P units of money during the second period, draws a liquidity shock ω, and a fraction A of ST creditors withdraws early, it survives as long as the available funds are greater than its obligations, i.e., $\omega + P \geq A$. In such a case, the bank uses the remaining cash to buy a bond and obtain a payoff of $R(P+\omega-A)$ at $t = 3$. Thus, the bank's payoff when it raises P units of cash in period 2, cash flows are $\tilde{y}$ during period 3, the liquidity shock is ω, and faces a fraction A of early withdrawals, is given by

$$\begin{aligned} U(P, \tilde{y}, \omega, A) &= R\left((P+\omega-A) + ((1-\omega)-(1-A)) + \frac{\tilde{y}}{R}\right) \times 1\{P+\omega \geq A\} \\ &= (PR+\tilde{y}) \times 1\{P+\omega \geq A\}. \end{aligned} \tag{1}$$

ST creditors' payoffs are as follows. The utility from withdrawing early is normalized to 0. The utility from withdrawing late is denoted by $u_i(\tilde{\omega}, A)$, where $\tilde{\omega}$ represents the total amount of available cash held by the bank at $t = 2$. The utility $u_i(\tilde{\omega}, A)$ is measurable with respect to the bank's fate

$$u(\tilde{\omega}, A) = g(\tilde{\omega}, A; l, R)\, 1\{\tilde{\omega} \geq A\} + b(\tilde{\omega}, A; l, R)\, 1\{\tilde{\omega} < A\},$$

where $g(\tilde{\omega}, A; l, R) > 0 > b(\tilde{\omega}, A; l, R)$ for all $\tilde{\omega}, A, l, R$, with g and b are non-decreasing in $\tilde{\omega}, l, R$. I suppress the dependence of $u_i(\cdot)$ on l, R for notational convenience.[14]

Finally, I assume large social costs associated with the default of the *systemically important* bank. The policy maker obtains a positive payoff $W_0(A)$ when default is successfully avoided, and a payoff of 0 when that is not the case, with $W_0(\cdot)$ non-increasing.[15]

$$U^P(\tilde{\omega}, A) = W_0(A) \times 1\{\tilde{\omega} \geq A\}.$$

[13] The assumption that y and ω are independent does not mean that the bank's liquidity and asset profitability are uncorrelated. In fact, the price P that the bank is able to obtain by selling claims on its asset correlates with its underlying quality. Thus, banks with a better assets, in the absence of information frictions, are able to secure more liquid funds at short notice.

[14] A natural example is $u_i(\tilde{\omega}, A) \equiv \frac{\tilde{\omega}+l}{A} \times 1\{\tilde{\omega} < A\} + R \times 1\{\tilde{\omega} \geq A\} - 1$. That is, if the amount of withdrawals exceeds the bank's liquidity, $\tilde{\omega}$, the bank defaults and ST creditors obtain a proportional fraction of their claim $\frac{\tilde{\omega}+l}{A}$. Otherwise, if the bank survives, they receive R.

[15] In this model, all banks are solvent but may potentially become illiquid. A richer model that accommodates for insolvent banks can be encompassed by allowing negative realizations of y. The predictions in the present specification easily extend to the richer environment as long as the policy maker obtains a negative utility from having an insolvent bank surviving.

Fund-raising Stage. After observing its asset quality type θ in period 1, the bank proposes to LT investors a security $s[\theta]$, which corresponds to a claim on future cashflows realizations of the risky asset. Formally, $s[\theta]$ belongs to $S \equiv \{s : \mathbb{R}_+ \to \mathbb{R}_+ \text{ s.t: (LL),(M),(MR)}\}$ where (LL) $0 \leq s(y) \leq y,\ \forall y \geq 0$; (M) s non-decreasing; and (MR) $y - s$ non-decreasing. [16] The security s may represent an equity stake, a debt contract, or any (arbitrary) hybrid of two.

The market observes the security $s[\theta]$ and prices it according to the available public information. I assume that the claims promised to LT investors are subordinated to those of ST creditors, and hence are repaid *only if* the bank avoids default. Thus, security s should not be mis-interpreted as *securitizing* the bank's asset. I discuss the possibility of securitizations later on. LT investors purchasing security s, at price P, when the fraction of early withdrawals is A, and fundamentals are given by (y, ω), thus obtain[17]

$$v(P, A, y, \omega) = \frac{s(y)}{R} \times 1\{\omega + P \geq A\} - P. \tag{2}$$

Intervention Policies. The policy maker concerned with the possibility that the bank defaults may choose to intervene. The policy maker has a technology that allows her to disclose information to all market participants. The assumption of *gradual resolution of uncertainty* implies that the designer may disclose information about the cashflows at $t = 1$, after y has been determined, but can disclose information about the liquidity shock ω only at $t = 2$, after ω has been drawn. I denote by Γ^y the disclosure policy about the profitability of the bank's assets, y, and refer to it as the *asset quality review,* and by Γ^ω the liquidity examination conducted in period 2 about the bank's liquidity, which I dub the *stress test*. In addition to the information revealed by the asset quality review, the policy maker may impose a recapitalization requirement according to the rule $\mathscr{R}$ which specifies the amount that must be raised as a function of the information disclosed by Γ^y. I assume that the technology needed to conduct the asset quality review Γ^y is time-demanding and cannot be postponed until the liquidity shock takes place, since then the policy maker might not be able to disclose information on time, before ST creditors choose whether to withdraw early. Moreover, I assume that any information learned while conducting the asset quality review Γ^y becomes public. That is, the policy maker cannot choose to learn information about y and not share it with market participants.[18] Finally, I assume that the policy maker cannot commit at $t = 1$ to the stress test Γ^ω she will conduct in period 2.[19] Given that the bank is systemically important, promises made in period 1 to conduct a specific stress test Γ^ω during period 2 are not credible. The

[16] The first constraint represents *limited liability* and states that a security $s \in S$ is in fact a sharing rule. The *monotonicity* condition, requires that the security is non-decreasing in the asset's cashflows, since otherwise the bank would have the option of requesting (risk free) credit to a third party to boost the cashflow realization and thus decrease the amount owed to LT investors. Finally, the last constraint imposes that the share of cashflows kept by the bank is non-decreasing for otherwise the bank would have incentives to *burn* part of the cashflows to improve her payoff.

[17] Equations (1) and (2) assume that the proceeds obtained by selling security s are used first to repay ST creditors withdrawing early. Any un-used amount is distributed among the bank's *former* shareholders.

[18] Any information produced by the regulator leaks and, therefore, if the policy-maker wants the rest of market participants not to learn some information she does not produce it in the first place. A similar assumption is made by Faria-e Castro et al. [2016].

[19] In other words, the policy-maker has *intra-temporal* commitment power but not *inter-temporal* commitment power. That is, the policy-maker *can* commit to disclose information according to asset quality review Γ^y (resp. Γ^ω) in period 1 (resp. 2), but not to the liquidity stress test she will conduct in period 2, at $t = 1$.

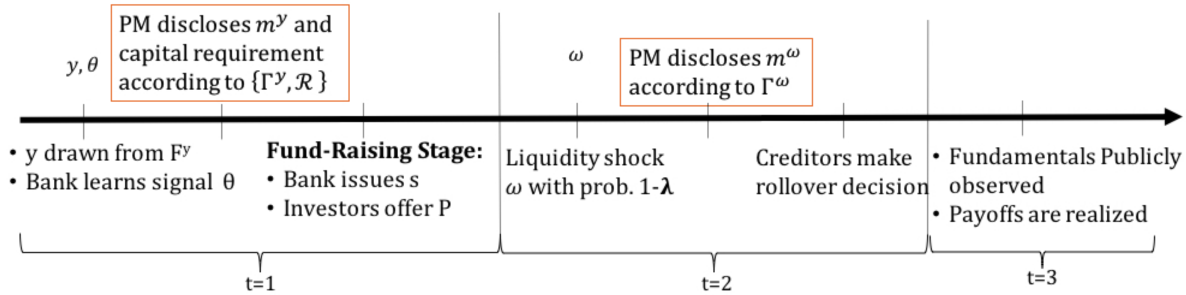

Figure 2: Timing.

policy maker chooses in period 2 the stress test that maximizes her payoff. [20]

Timing. The sequence of events is as follows:

Period 0. The policy maker designs the asset quality review and recapitalization policy, $\{\Gamma^y, \mathscr{R}\}$, and publicly announces it.

Period 1. (a) y is drawn from F^y. (b) No agent observes y but the bank observes a private signal θ about y. (c) The policy maker discloses information m^y and recapitalizations according to $\{\Gamma^y, \mathscr{R}\}$. (d) The bank sells security $s \in S$ to LT investors at price $P \geq 0$. **Period 2.** (a) The policy maker designs the (sequentially optimal) stress test Γ^ω and publicly announce it; (b) ω is drawn from F^ω; (c) The policy maker conducts stress test Γ^ω and discloses information m^ω; (d) ST creditors observe P and m^ω, and decide whether withdraw early or late; (e) The bank liquidates a fraction of her bond and its fate is determined according to whether $\omega + P$ is greater than the fraction of early withdrawals, A.

Period 3. Conditional on the bank's survival, (a) ST creditors that rolled over are paid back; (b)y is materialized and $s(y)$ is paid to LT investors, and bank's former shareholders obtain $y - s(y)$.

3 Laissez Faire

3.1 Fund-raising Stage

In this section, I study the case where the policy maker does not intervene. I show that, when the liquidity shock is severe the asset market may freeze leaving the bank vulnerable to a run of ST creditors. Moreover, under stringent conditions, *market freezing* may become the unique equilibrium outcome of the fund-raising stage.

The bank after observing its asset quality type θ enters the *fund-raising stage* by approaching LT investors. The bank offers to them claims on its asset to raise precautionary funds. I follow an adversarial approach, and assume that, when multiple action profiles are consistent with equilibrium play, ST creditors coordinate on the

[20] This assumption is also consistent with a narrative where the policy maker designing the asset quality review is different from the one designing the stress test. Such a narrative resonates well with the European experience where the policy maker conducting the asset quality reviews (European Central Bank) differs from the one in charge of running the stress tests (European Banking Authority, in cooperation with the European Single Resolution Board).

most aggressive outcome (from the perspective of the bank) consistent with the rationality of both audiences. This conservative approach is consistent with the standard practice in stress test design of contemplating adverse scenarios to assess the resilience of the bank.

The adversarial approach implies that ST creditors stop rolling over the bank's debt whenever withdrawing early is a best response to everyone else withdrawing early. That is, each ST creditor withdraws early when

$$\mathbb{E}\left(u\left(\omega+P,1\right)\right)=\int_{0}^{1}u\left(\omega+P,1\right)F^{\omega}\left(d\omega\right)\leq 0. \tag{3}$$

I assume that the distribution of the liquidity shock is severe so that, if the bank does not raise additional funds, ST creditors withdraw early (i.e., inequality (3) holds for $P=0$).

Assumption 1. $\mathbb{E}\left(u\left(\omega,1\right)\right)\leq 0$.

Define $K\geq 0$ as the minimum amount of capital that needs to be raised to persuade ST creditors to keep rolling over the bank's debt under adversarial coordination. That is,

$$K\equiv\inf\left\{P\geq 0:\mathbb{E}\left(u\left(\omega+P,1\right)\right)>0\right\}.$$

Thus, the bank may convince ST creditors that it is liquid by raising K units of precautionary funds during the fund-raising stage. Define then $\bar{A}\left(P\right)$ as the most aggressive fraction of early withdrawals, for a given recapitalization level P. From the definition of K, we have that $\bar{A}(P)=1\{P\leq K\}$. Next, I assume that, if θ were observable, then only type θ_H would be able to raise K.

Assumption 2. $\frac{1}{R}\mathbb{E}_L(y)<K\leq\frac{1}{R}\mathbb{E}_H\left(y\right)<1$.

By the end of period 2, ST creditors perfectly observe the bank's recapitalization and decide whether to rollover the bank's debt. If the bank raises at least K units of capital, then no ST creditor withdraws early, allowing the bank to survive. On the other hand, if the amount raised is smaller than K, then all ST creditors withdraw early, in which case the survival of the bank depends on the amount of capital raised and on the realization of the liquidity shock ω. The maximal price that LT investors are thus willing to pay for any security s is

$$P(s;\mu)\equiv\sup\left\{p\geq 0:\frac{\mathbb{E}_{\mu}\left(s\right)}{R}\mathbb{P}\left\{\omega+p\geq A\left(p\right)\right\}\geq p\right\}, \tag{4}$$

where $\mathbb{E}_{\mu}\left(s\right)$ is the expected value of security s when the market holds beliefs $\mu\in\Delta\Theta$ about the bank's type.[21] The next assumption will be used for certain results, for it favors tractability.

[21] Note that the definition of $P(s,\mu)$ implies that, in case the equation

$$\frac{\mathbb{E}_{\mu}(s)}{R}\mathbb{P}\left\{\omega+p\geq A\left(p\right)\right\}=p, \tag{5}$$

admits multiple solutions, the selected one is the one associated with the largest price. This selection has a game-theoretic foundation similar in spirit to the one encountered in *Bertrand* competition models. Namely, if the market reached a price $\tilde{P}<P(s;\mu)$ satisfying 5, a buyer might deviate and offer a larger price $\hat{P}$ for which the LHS of equation 5 is strictly greater than the RHS, and obtain a positive gain in the process. Such a deviation would be willingly accepted by the bank. As a result, $\tilde{P}$ would be inconsistent with equilibrium play. $P(s;\mu)$ is thus the unique price consistent with competitive markets and immune to such deviations.

Assumption 3. *The prior distribution of the liquidity shock ω, F^{ω}, is concave.*

Assumption 3 reflects the idea that the liquidity problem is severe. Intuitively, when F^{ω} is concave, low liquidity levels are more likely to occur. When this is the case, LT investors refuse to fund any project with NPV below K. To see this, note that, for any $p \in (0,K)$,

$$\frac{\mathbb{E}_{\mu}(s)}{R}\mathbb{P}\{\omega + p \geq A(p)\} < 1 - F^{\omega}(1-p) < p$$

where the first inequality obtains from assumption 2 which implies that, for any $\mu \in [0,1]$, $\mathbb{E}_{\mu}(s)/R < 1$. The second inequality is a consequence of assumption 3. Thus, the expected payoff that LT investors obtain from purchasing security s is no greater than the price, and therefore they refuse to purchase it.

This result is a consequence of the interaction between the two audiences. In fact, when assumption 3 holds (and under an adversarial approach), LT investors believe that it is plausible that ST creditors run on the bank. The interaction of both audiences then generates a negative feedback cycle as LT investors *price in* the bank's probability of default. This depresses the price the latter are willing to pay for s, which makes a run of ST creditors more likely. This feeds back as the probability of default increases which translates into an even lower price, and so on. Hence, when assumption 3 holds, the bank survives only if the price collected is at least K.

3.2 Solution Concept

I assume that renegotiation between ST creditors and the bank is not feasible. Given the speed of events and the dispersion of ST creditors, renegotiation is, in most cases, unviable.[22] The government's most preferred outcome consists of all bank types raising enough precatuionary funds to avoid bankruptcy. As is usually the case with signaling games, the fund-raising game may be plagued with multiple equilibria. To focus on equilibria which take into account the propensity of banks to deviate, I restrict attention to PBE satisfying the D1 criterion, and I refer to them hereafter as equilibria.

Let $V(P,s,\theta)$ be the utility of a bank of type θ, selling a security s and receiving funds in the amount of P. Without government intervention, the bank's payoff can be written as

$$\begin{aligned} V(P,s,\theta) &\equiv \mathbb{E}\left((PR+y-s)\,1_{\{\omega+P\geq A(P)\}}|\theta\right) \\ &= (PR+\mathbb{E}_{\theta}(y-s))\,\mathbb{P}\{\omega \geq A(P)-P\}. \end{aligned} \tag{6}$$

I say that $\left\{\{s^{\star}_{\theta}\}_{\theta\in\Theta}, \mu^{\star}, P^{\star}, A^{\star}\right\}$ is an equilibrium of the fund-raising game if:

$$\begin{aligned} &\text{[Sequential Rationality]:} & s^{\star}_{\theta} \in\; & \arg\max_{s} V\left(P^{\star}(s),\theta,s\right) \\ &\text{[Competitive Investors]:} & P^{\star}(s) = \sup\; & \left\{p \geq 0 : \frac{\mathbb{E}_{\mu^{\star}(s)}(s)}{R}\mathbb{P}\{\omega + p \geq A^{\star}(p)\} \geq p\right\} \\ &\text{[Adversarial Coordination]:} & A^{\star}(P) =\; & 1\{P<K\},\ \forall P \geq 0 \\ &\text{[Belief Consistency]:} & \mu^{\star}(s)\; & \text{computed according to Bayes rule on-path} \end{aligned}$$

[22] Landier and Ueda [2009] make a similar assumption.

Additionally, I impose that off-path beliefs associated with securities not observed in equilibrium, assign all probability weight to the asset quality type with the greatest propensity to deviate to them (see the Appendix).

3.3 Equilibrium Characterization.

Next, we characterize the set of equilibria that arise in the fund-raising game under a laissez faire policy. The main result of this section shows that, when the bank's liquidity problem is severe, the asset market freezes. I show that, under assumption 3, the only type of equilibria in the fund-raising game are pooling equilibria, where both bank types issue the same debt contract. If the expected profitability of the asset of the type L-bank is low, then there exists an equilibrium where the asset market freezes and no security is issued by any type. Furthermore, when in addition the average quality of the bank's asset is low, then market freezing becomes the unique equilibrium of the fund-raising game. As a result, all bank types default with probability 1 (note that, under assumption 3, $\lambda = 0$). Finally, I show that when the expected profitability of a type L-bank is sufficiently good, the unique equilibrium of the game has both bank types placing a debt contract which collects enough funds to dissuade ST creditors from running.

Proposition 1. *The following properties are true:*

1. *All pooling equilibria are in debt contracts (i.e., $s_{pool} = \min\{y, d\}$, $d \geq 0$). Moreover, $P\left(s_{pool}\right) \leq K$.*
2. *If assumption (2) holds, then in any separating equilibrium, the security issued by type H satisfies that $P\left(s_H^{sep}\right) \leq \mathbb{E}_L(y) < KR$.*
3. *Suppose that assumption (3) holds. Then,*
   (a) *(Market freeze) If, in addition, assumption (2) also holds, then there is an equilibrium where $s_\theta = \mathbf{0}$ for all $\theta \in \Theta$. Moreover, if $\frac{\mathbb{E}(y)}{R} < K$, this is the unique equilibrium.*
   (b) *(Risk-Sharing) If $\frac{\mathbb{E}(y)}{R} \geq K$, then there exists an equilibrium where $s_\theta = \min\left\{y, D^{pool}\right\}$ with D^{pool} defined as the unique solution to $\frac{1}{R}\mathbb{E}\left(\min\left\{y, d^{pool}\right\}\right) = K$. Moreover, if $\frac{\mathbb{E}_L(y)}{R} \geq K$, this is the unique equilibrium.*

Proposition1 extends the results in Nachman and Noe [1994] to the current environment where the probability of default is endogenously determined by the interaction between the two audiences, ST creditors and LT investors.[23] The proposition shows that the celebrated uniqueness result of the former paper may break in the current environment. Nonetheless, some properties remain true. Part (1) of the proposition shows that pooling equilibria necessarily occur over debt debt contracts. Part (2) shows that, in any separating equilibria, type-H raises an amount below K and hence remains vulnerable to rollover risk despite the fact that the value of its asset is above the cutoff K.

[23]The model in Nachman and Noe [1994] assumes that the seller of the asset (i.e., the bank in our environment) survives w.p. 1 if the latter raises an exogenous amount K, and defaults, also with certainty, if the bank does not raise this amount.

Finally, part (3) shows that, when the bank expects a severe liquidity shock, the existence of a type with sufficiently poor assets may freeze the asset market, preventing both types of bank from raising precautionary funds. LT investors' ability to foresee the possibility of a run and to price assets accordingly, together with the incentives of type H-banks to separate from type L, induce a *fire sale* effect so severe that no bank is able to raise funds. Futhermore, if the aggregate expected quality of banks' assets is lower than the threshold required to dissuade ST creditors from running, K, market freezing becomes the unique equlibrium of the fund-raising stage.

4 Comprehensive Assessment

The policy maker concerned with the potential freeze of the asset market intervenes by conducting a *comprehensive assessment*. In period 1, the policy maker performs an *asset quality review,* $\Gamma^y = \{M^y, \pi^y\}$, characterized by an arbitrary set of scores M^y and a disclosure policy $\pi^y : \mathbb{R}_+ \to \Delta M^y$. Conditional on the score $m^y \in M^y$, the policy maker specifies a *recapitalization* rule $\mathscr{R}(\cdot|m^y) : \mathbb{R}_+ \to [0,1]$ where, for any $P \in \mathbb{R}_+$, $\mathscr{R}(P|m^y)$ represents the maximal fraction of the bank's payoff the latter is allowed to distribute as dividends, as a function of the the level of capital raised during the fund-raising stage, P.[24],[25] The recapitalization rule emphasized below is consistent with the *capital conservation buffers* recommended by Basel III which restrict the amount of dividends as a function of the bank's capital.

Consider the family of recapitalization rules satisfying

$$\mathscr{R}(P) = \begin{cases} 1 & P > C \\ \alpha & P \leq C, \end{cases}$$

with $\alpha, C \in [0,1]$. Any such a rule can be interpreted as a *minimal recapitalization* C, which if not met limits the dividends to a fraction α of the bank's profit. The optimal recapitalization policy can be described as a minimal recapitalization policy. The decision of allowing shareholders to distribute only a fraction of the bank's profit serves the purpose of enforcing the policy maker's recommendation.[26] [27]

In period 2, the policy maker conducts a stress test $\Gamma^\omega[P] = \{M^\omega, \pi^\omega[P]\}$ which discloses information about the bank's liquidity shock according to the rule $\pi^\omega[P] : \Omega \to \Delta M^\omega$. Hereafter, I refer to the combination of an asset quality review, a recapitalization rule, and a stress test, $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$, as a *comprehensive*

[24]The assumption that $\mathscr{R}(P|m^y)$ takes only deterministic values is without loss of optimality. In fact, the solution presented below *implements* the optimal policy in an environment where the policy maker also has the authority to also choose the bank's recapitalization strategy.

[25]That $\mathscr{R}$ does not depend directly on y is wlog. Under this assumption, the induced beliefs about the quality of the asset depend only on m^y, and not on $\mathscr{R}$. A similar assumption is made by Orlov et al. [2017].

[26]Recapitalizations can be interpreted in different ways in this *one-shot* framework. The favored interpretation is that it represents a limit on the amount of dividends if the bank fails to raise the required level of capital. It could also represent the decision of selling the firm to another institution and $1-\alpha$, in that case, represents the discount applied to the value of the bank.

[27]Although the designer has the authority to impose recapitalizations, she cannot repudiate the contracts the bank agrees upon with the investors. That is, investors preserve their claims on the future cashflows even if the bank does not comply with the recapitalization requirement.

assessment. An implicit assumption is therefore that the government cannot inject any type of funds to insulate ST creditors from the liquidity shock. She may only disclose information and impose recapitalizations. I relax this assumption in the next section.

4.1 Period 1

In period 1, the profitability level y is determined and the policy maker discloses score m^y according to π^y and set recapitalization requirements $\mathscr{R}[m^y]$. The bank then approaches LT investors and offers a security s. The latter, after observing the security issued by the bank, form beliefs $\mu \in \Delta\Theta$ about its type. I denote by $P_\mu(s;m^y)$ the competitive price offered to the bank. LT investors, who expect the designer to disclose information about ω according to $\Gamma^\omega[P] = \{M^\omega, \pi^\omega[P]\}$, price the asset according to

$$P_\mu(s;m^y) \equiv \sup\left\{P : \frac{\mathbb{E}_\mu(s;m^y)}{R}\int_{\Omega\times M^\omega} 1\{\omega+P \geq A(P,m^\omega)\}\,\pi^\omega(dm^\omega|\omega;P)\,F^\omega(d\omega) \geq P\right\}, \tag{7}$$

where $\bar{A}(P,m^\omega)$ represents the most aggressive fraction of early withdrawals when the bank raises P units of additional capital and the designer discloses information m^ω about ω.

4.2 Period 2

After the liquidity shock ω materializes and the capital raised by the bank P has been observed by all market participants, the designer conducts the stress test $\Gamma^\omega[P]$. Assume that score $m^\omega \in M^\omega$ is publicly disclosed and let $F^{\omega|m}(\cdot|m^\omega)$ be the posterior conditional cdf characterizing the beliefs about ω (computed according to Bayes rule). Denote by $\mathbb{E}(u(\omega+P,1)|m^\omega)$ the expected posterior utility of a ST creditor who observes the public score m^ω and believes the rest of ST creditors will run on the bank. That is,

$$\mathbb{E}(u(\omega+P,1)|m^\omega) \equiv \int_\Omega u(\omega+P,1)\,F^{\omega|m}(d\omega|m^\omega).$$

I refer to it as the *posterior expected adversarial utility*. The most aggressive fraction of early withdrawals faced by the bank is then given by

$$\bar{A}(P,m^\omega;\Gamma^\omega) = 1\{\mathbb{E}(u(\omega+P,1)|m^\omega) \leq 0\}.$$

That is, under adversarial coordination, ST creditors run on the bank unless it becomes dominant not to do so. As a result, the payoff that a type θ-bank obtains when (a) information m^y is disclosed at $t=1$, (b) the recapitalization requirement is $\mathscr{R}[m^y]$, and (c) it issues a security s at price P, is

$$\begin{aligned} V(s,P,\theta;m^y,\mathscr{R}) &= \mathscr{R}(P|m^y)\,(PR+\mathbb{E}_\theta(y-s|m^y)) \times \\ &\quad \times \left(\int_{\Omega\times M^\omega} 1\left\{\omega+P \geq \bar{A}(P,m^\omega)\right\}\pi^\omega(dm^\omega|\omega;P)\,F^\omega(d\omega)\right). \end{aligned}$$

4.3 Stress Test

To characterize the optimal comprehensive assessment $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$ we proceed by backward induction. The approach below borrows from Gentzkow and Kamenica [2016] who characterize arbitrary disclosures by the induced distribution of posterior expectations.[28]

Under an adversarial approach, finding the optimal stress test is equivalent to finding the distribution of posterior expected adversarial utilities that maximizes the mass assigned to the event $\{\omega : \mathbb{E}(u(\omega+P,1)|m^\omega) > 0\}$. Intuitively, under adversarial coordination, and when ST creditors have homogenous beliefs, the policy maker's task reduces to convincing ST creditors that rolling over is a dominant strategy. That is, that their expected payoff if they rollover the bank's debt is positive, even if the rest of ST creditors choose to run on the bank.[29]

The solution to the designer's problem is given by the monotone-binary policy $\Gamma^\omega_\star = (\{0,1\}, \pi^\omega_\star)$ that satisfies

$$\pi^\omega_\star(0|\omega) = 1\{\omega \geq \bar{\omega}(P)\},$$

where $\bar{\omega}(P)$ represents the liquidity cutoff defined as

$$\bar{\omega}(P) \equiv \inf\{\tilde{\omega} \geq 0 : \mathbb{E}(u(\omega+P,1)|\omega \geq \tilde{\omega}) > 0\}. \tag{8}$$

The optimal stress test can thus be interpreted as a pass-fail announcement, where given the level of recapitalization P, the policy maker assigns a *passing* grade ($m^\omega = 0$) when the bank's liquidity is above the cutoff $\bar{\omega}(P)$. Proposition 2 below summarizes these findings.

Proposition 2. *Fix the amount of capital $P \geq 0$. Then, the optimal stress test $\Gamma^\omega_\star[P]$ consists of a monotone pass-fail test with cutoff $\bar{\omega}(P)$, such that $\Gamma^\omega_\star(P) = (\{0,1\}, \pi^\omega_\star[P])$, with $\pi^\omega_\star(0|\omega;P) = 1_{\{\omega \geq \bar{\omega}(P)\}}$. The cutoff $\bar{\omega}(\cdot)$ is non-increasing with P.*

Under the optimal policy, when the policy maker announces that the bank passed the stress test ($m^\omega = 0$), all ST creditors rollover the bank's debt, and survival occurs with certainty. When, instead, the bank fails the stress test $\Gamma^\omega[P]$ ($m^\omega = 1$), all ST creditors withdraw early and the bank defaults. In fact, note that $\bar{\omega}(P) < 1 - P$ and therefore failing the stress test, which implies that $\omega < \bar{\omega}(P)$, induces bank failure with certainty.

4.4 Asset Quality Review

We now characterize the optimal policy $\{\Gamma^y, \mathscr{R}\}$ conducted in period 1, taking into account the (sequentially rational) stress test $\Gamma^\omega[P]$ that follows in period 2. The optimal policy $\{\Gamma^y, \mathscr{R}\}$ combine a disclosure about the profitability of the bank's asset y, along with a recommendation to the bank to raise a minimal amount of precautionary funds.

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[28]The approach is described in detail in the Online Supplement.

[29]Inostroza and Pavan [2019] show, in an environment with heterogeneous beliefs, that the optimal disclosure policy perfectly coordinates ST creditors actions. The current model can be modified to accommodate heterogeneous beliefs. The current specifications captures the perfect coordination property while simplifying the intrincancies of characterizing the optimal policy in the richer environment.

The policy maker asks the bank to raise an amount equivalent to the minimum between the capital cutoff which prevents posterior runs, K, and the expected price of the entire asset $\bar{P}\left(\mathbb{E}\left(y|m^y\right)\right)$, where

$$\bar{P}(z) \equiv \sup\left\{P \geq 0 : \frac{z}{R}\mathbb{P}\left\{\omega \geq \bar{\omega}\left(P\right)\right\} \geq P\right\} \tag{9}$$

represents the (maximal) fair price consistent with selling a security with expected value of $z \geq 0$, accounting for the endogenous probability of default. Given the policy maker's commitment to limit the dividends when the bank does not meet the capital cutoff, the game played between the bank and LT investors becomes similar to the one in Nachman and Noe [1994] and, therefore, under the unique equilibrium of the fund-raising stage, both asset quality types pool and offer a debt security $s_\star^{\text{pool}}$ satisfying $\mathbb{E}\left(s_\star^{\text{pool}}|m^y\right) = R\min\left\{K, \bar{P}\left(\mathbb{E}\left(y|m^y\right)\right)\right\}$.

On-path, recapitalization requirements are always obeyed. Whenever possible, the optimal policy asks the bank to raise K to persuade ST creditors to keep rolling over the bank's debt. When this is not possible, i.e., when the value of the assets falls below K, the regulator asks the bank to raise as much precautionary funds as possible.

Proposition 3. *For any score m^y disclosed by the asset quality review $\Gamma^y = \{M^y, \pi^y\}$, consider the recapitalization rule:*

$$\mathscr{R}\left(P|m^y\right) = \begin{cases} 1, & P > \min\left\{K, \bar{P}\left(\mathbb{E}\left(y|m^y\right)\right)\right\} \\ \alpha, & \textit{otherwise.} \end{cases} \tag{10}$$

(a) If $\alpha \leq \alpha^*\left(\mathbb{E}\left(y|m^y\right)\right)$, then $s_L^* = s_H^* = \min\left(y, \tilde{D}\right)$, with $\tilde{D}$ such that $\mathbb{E}\left(\min\left(y, \tilde{D}\right)\right) = K$, is an equilibrium of the fund-raising stage.

(b) If $\alpha = 0$, then $s_L^* = s_H^* = \min\left(y, \tilde{D}\right)$ is the unique equilibrium of the fund-raising stage.

Recapitalizations are instrumental to implement the optimal policy. An asset quality review that reveals that the expected cashflows are greater than K (i.e., $\frac{1}{R}\mathbb{E}(y|m^y) \geq K$), but does not impose recapitalizations, need not guarantee that all bank types raise enough funds to prevent default. In fact, in the absence of recapitalizations, default may occur with positive probability, across all equilibria, even if under the laissez faire policy the bank would have survived with certainty.

The reason behind this finding is that the bank's residual private information induces a *lemons problem* during the fund-raising game. In fact, type θ_H has an incentive to separate from type θ_L (by issuing a different security) to avoid *underpricing*. If the probability of default is low enough, type θ_H may prefer to expose itself to rollover risk, by raising less funds than K, to signal its type. The fact that type θ_H is more optimistic about its asset implies that it is relatively more willing to risk default to retain a larger fraction of it.

The threat of a run of ST creditors imposes *discipline* on the bank during the fund-raising stage and mitigate the separation incentives. However, when market participants expect the policy maker to conduct a stress test in the future, their assessment about ST creditors' response becomes more optimistic. This, in turn, makes it easier for type θ_H to separate from type θ_L, since rollover risk is now mitigated, generating a countervailing effect that increases the probability of default and erases the benefits of having the technology to disclose information in the future. Imposing precautionary recapitalizations is thus key to the effectiveness of stress tests. The recapitalizations substitute for the disciplining role of ST creditors, while keeping the benefits of information disclosures.

The next proposition shows that a policy maker endowed with a technology to conduct stress tests but who does not impose recapitalizations may fare worse than a policy maker that does not intervene at all.

Condition 1. The distribution of liquidity shocks F^{ω} and ST creditors' payoff functions g and b satisfy

(a) F^{ω} is convex over $[0,1]$.

(b) f^{ω} is continuous at $\omega=0$, with $f^{\omega}(0)=0$.

(c) $\lim_{P\to K^-} 1-F^{\omega}(1-P)<\bar{\phi}\equiv\frac{\mathbb{E}_H(y-s_D)-\mathbb{E}_L(y-s_D)}{\mathbb{E}_H(y)-\mathbb{E}_L(y)}$, where $s_D\equiv\min\{y,D\}$ with $\mathbb{E}(s_D)=KR$.

Proposition 4. *Assume Condition 1 holds, then without recapitalizations, under the sequentially optimal stress test* Γ^{ω}*, default occurs with positive probability across all equilibria of the fund-raising stage. In contrast, under the laissez-faire policy, the probability of default reduces to* 0*.*

Assumptions (a) and (b) in Condition 1 jointly imply that, under the sequentially optimal stress test Γ^{ω}, both bank types find it optimal to deviate from the pooling equilibrium where both raise K (recall that, as proved in Proposition 1, banks raise at most K when pooling, and type θ_H always raises strictly less than K in any separating equilibrium). Intuitively, assumptions (a) and (b) guarantee that, by issuing a debt contract that raises slightly less than the amount K, the probability of default barely increases above 0. Such a deviation is always interpreted by the market as coming from type H. Given that type H has a better asset than type L, the former is relatively more willing to risk defaulting to signal its quality. However, if the probability of default increases significantly when the bank does not raise the amount K, the benefits from signaling may be outweighed by the likelihood of default. Assumptions (a) and (b) guarantee that the probability of default behaves smoothly around the threshold K and therefore the former effect dominates. Thus, small deviations are interpreted as coming from type θ_H and are therefore priced accordingly. Both types then have incentives to deviate from the situation where both raise K.

Assumption (c), on the other hand, implies that, in the *absence* of any disclosure, the probability of default substantially increases if the bank does not meet the cutoff K. This effect imposes discipline on the banks as it urges them to raise enough precautionary funds to dissuade ST creditors from running. Under assumption (c), both bank types pool over the debt contract $s_D=\min\{y,D\}$, with $\mathbb{E}(s_D)=KR$ and, as a result, they avoid default with certainty.[30]

Surprisingly, having the technology to conduct stress tests may be detrimental if these disclosures are not accompanied by additional policies to complement them. To prevent separation among asset quality types the policy maker has to punish banks that, despite being able to raise enough precautionary funds to prevent future runs (i.e., amount K), choose not to do so.[31] By imposing recapitalizations, the policy maker retains the benefits of having a technology to conduct stress tests and avoids the costs of dissipating pooling incentives.

[30]Note that, e.g., when ST creditors' payoff functions g and b are invariant in ω, assumption (c) is equivalent to requiring that the ratio $\frac{g}{g+|b|}<\bar{\phi}$.

[31]If the policy maker were able to commit to the liquidity stress test Γ^{ω} when designing the asset quality review Γ^{y}, she might threaten the bank to conduct an adversarial stress test if the latter were to raise less funds than desired. These threats, however, would require the policy maker to minimize the (off-path) probability of survival if the bank failed to raise enough capital. The approach

4.5 Optimal Comprehensive Assessment

The analysis so far characterizes the optimal recapitalization policy $\mathscr{R}[m^y]$ for any score m^y disclosed under Γ^y. We now proceed to the characterization of the optimal asset quality review Γ^y_* taking into account the optimal policies $\{\mathscr{R}, \Gamma^\omega\}$.

Any score m^y disclosed with positive probability induces a posterior expectation of y, $\mathbb{E}(y|m^y)$. Let G^{Γ^y} be the distribution of posterior expectations induced by an arbitrary asset quality review Γ^y. The set of possible distributions of posterior expectations that can be induced with a disclosure policy coincides with the set of distributions which are a mean-preserving contraction of the prior F^y (Blackwell [1953], Gentzkow and Kamenica [2016]).

Fix a score m^y. Proposition 3 implies that the policy maker chooses a minimal recapitalization rule so that the cutoff defining whether the bank survives or not is given by $\bar{\omega}(\bar{P}(\mathbb{E}(y|m^y)))$. Recall that the function $\bar{\omega}(P)$ identifies the critical value of the liquidity shock below which the bank defaults when the capital raised at $t = 1$ is equal to P. This value is equal to 0 for any $P \geq K$ since at these prices the probability of default is 0.

4.5.1 Convexity and the Probability of Survival

Define $\phi(\tau)$ as the probability that the bank survives conditional on selling a security with expected value of τ. That is,

$$\phi(\tau) \equiv \mathbb{P}\{\omega \geq \bar{\omega}(\bar{P}(\tau))\} = 1 - F^\omega(\bar{\omega}(\bar{P}(\tau))).$$

Lemma 1 below shows that, under some conditions, ϕ is convex over the critical region $(0, KR)$.

Assumption 4. *ST creditors' payoff functions $g(\omega, A; l, R)$ and $b(\omega, A; l, R)$ are invariant in ω.*

That the payoff of withdrawing late and seeing the bank survive, $g(\omega, A; l, R)$, does not depend on ω is a natural assumption as ST creditors' payoff in the long-run should not be associated to ST liquidity shocks. That the payoff of withdrawing late and seeing the bank default, $b(\omega, A; l, R)$, does not depend on ω is more restrictive but may be justified on the grounds of large bankruptcy costs. This assumption is implicitly assumed in the banking literature that follows Rochet and Vives [2004] where ST creditors' decision of withdrawing early or late only depends on their assessment of the probability of default. Assumption (4) generalizes the assumption on those earlier models by allowing the payoffs g and b to depend on the size of early withdrawals A, the liquidation value l, and the returns on ST creditors' contracts R.

Lemma 1. *Under assumptions 3 and 4, ϕ is convex in $(0, KR)$.*

The convexity of ϕ is a consequence of the interaction of the two audiences: ST creditors and LT investors. When the expected value of the security sold to LT investors, τ, increases by $\varepsilon > 0$, the probability that the bank survives increases, as the bank becomes resilient to more stringent liquidity shocks and hence more

followed in this paper (i.e., lack of inter-temporal commitment) implies that the optimal policy will not be sustained with non-credible threats.

resilient to potential runs of ST creditors. The increase in the probability of survival feeds back and increases the expected value of the bank's security to LT investors and hence the price they are willing to offer. The larger price paid by LT investors further increases the probability of survival, and so on. As a result, in the absence of other forces, this amplification mechanism implies that the probability that the bank survives is convex in the expected quality of the bank's security.

However, there exists another force at play. Increasing the amount collected by the bank decreases the critical threshold above which the bank passes the stress test in period 2. Decreasing the critical threshold then increases the probability of survival, but this effect need not occur in a convex manner. Lemma 1 identifies primitive sufficient conditions under which the combination of both forces induces a convex probability of survival.

4.5.2 The Optimal Asset Quality Review

The designer's objective function can be written as

$$\mathbb{E}\left(W_0\left(1_{\{\omega<\bar{\omega}(\bar{P}(\mathbb{E}(y|m^y)))\}}\right)1_{\{\omega\geq\bar{\omega}(\bar{P}(\mathbb{E}(y|m^y)))\}}\right),$$

or equivalently as

$$W_0\left(0\right)\times\phi\left(\mathbb{E}\left(y|m^y\right)\right),$$

where we have used the fact that stress test Γ^ω coordinates all ST creditors to rollover whenever assigning the passing grade. Thus, the policy maker's problem reduces to

$$\begin{aligned}\max_{G^{\Gamma_y}} \quad & \int_0^\infty \phi\left(\tau\right)G^{\Gamma^y}\left(d\tau\right)\\ \text{s.t:} \quad & F^y \succeq_{\text{MPS}} G^{\Gamma^y}.\end{aligned}$$

As the next theorem shows, the optimal asset quality review Γ_*^y consists of a monotone partition signal, where different values of y are pooled (if at all) with adjacent realizations (i.e., within the same interval). I show that when the LT profitability of the asset is good enough, above a cutoff y^+, the asset quality review Γ_*^y assigns a unique (and hence *opaque*) passing grade. When the LT profitability is low, i.e., falls below the cutoff y^+, Γ_*^y is more *transparent* and assigns one of multiple failing grades. Moreover, I show that whenever the probability that the bank survives, $\phi\left(\tau\right)$, is convex in $(0,KR)$, then Γ_*^y fully discloses the realization of y for any realization below a cutoff y^+, and pool all realizations above y^+ under a single message, m_+^y. The posterior expectation induced by message m_+^y satisfies $\frac{1}{R}\mathbb{E}\left(y|y\geq y^+\right)\geq K$. Thus, y^+ corresponds to the lowest cutoff that allows the bank to raise enough capital to persuade ST creditors to rollover the bank's debt, under the prior beliefs characterized by F^ω.

Theorem 1. *The optimal asset quality review Γ_*^y consists of a monotone partitional signal: There exists a monotone partition $\mathscr{P}=\left\{m_i^y\equiv[y_i,y_{i+1})\right\}_{i\in I}$ of $\mathbb{R}_+$ such that $\Gamma_*^y=\left\{\left\{m_i^y\right\}_{i\in I},\pi^y\right\}$, with $\mathbb{E}(y|m_i^y)<\mathbb{E}(y|m_j^y)$ for all $i<j$, and $\mathbb{E}\left(y|m_{I+1}^y\right)\geq K$. Furthermore, if $\phi\left(\cdot\right)$ is convex over $(0,KR)$, then $\Gamma_*^y=\left\{\left\{[0,y^+]\cup m_{pass}^y\right\},\pi^y\right\}$,*

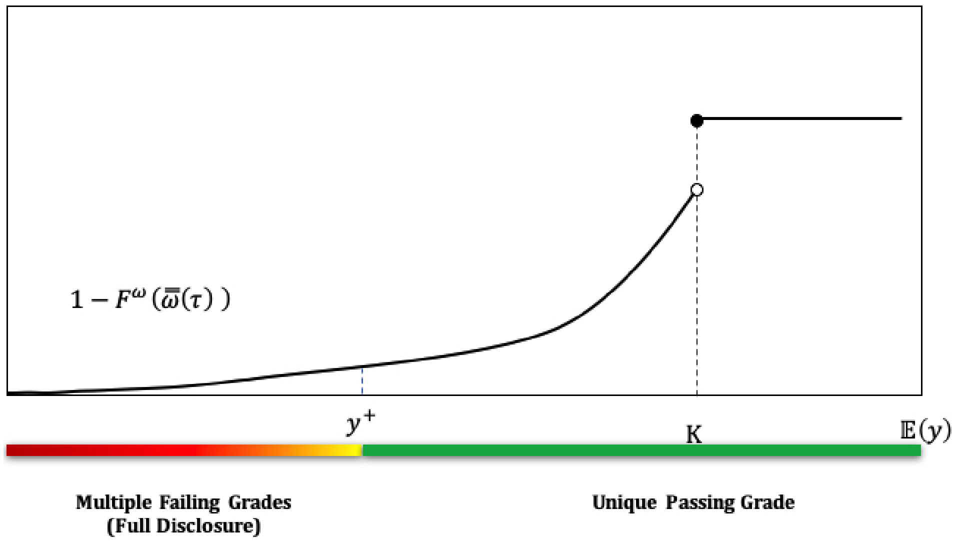

Figure 3: Optimal Asset Quality Review under Assumption 3.

with $\pi^y(\tilde{y}|y) = 1\{\tilde{y} = y\}$ *and* $\pi^y\left(m^y_{pass}|y\right) = 1\{y \geq y^+\}$ *for all* $\tilde{y} \in [0, y^+]$*, and all* $y \geq 0$*, where* y^+ *is defined by:*

$$y^+ \equiv \inf\left\{y \geq 0 : \int_y^{\max\{KR,\mathbb{E}(y)\}} (F^y(y) - F^y(\tau))\, d\tau + \int_{\max\{KR,\mathbb{E}(y)\}}^{\infty} (1 - F^y(\tau))\, d\tau \geq 0\right\}. \tag{11}$$

Congruent with the qualitative properties of the disclosure policy found in this paper, the empirical literature has consistently found evidence that banks with weaker fundamentals (i.e., riskier assets, more leverage, and larger amounts of non-performing loans), are subject to more transparency than banks with stronger fundamentals when conducting stress tests (see Morgan et al. [2014], Flannery et al. [2017], and Ahnert et al. [2018]). Thus, larger revisions in prices of weaker banks, after disclosure of their private information, should not be interpreted as an anomaly but, instead, as a feature of optimal disclosures with multiple audiences.

Theorem 1, along with the former results, imply that under assumptions 3 and 4, the optimal comprehensive policy $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$ has a simple structure. The policy Ψ assigns a single passing grade to all banks that meet a minimum standard in terms of profitability of their assets. This grade should be thought of as passing the policy maker's test on the asset quality dimension. Any bank failing to meet this minimal standard receives a grade that fully reveals the quality of its assets. The policy Ψ also specifies a recapitalization rule that asks the bank to either raise enough funds to prevent a ST creditors' run, or to sell the whole asset to LT investors when its quality is low. Finally, the optimal policy entails a follow-up stress test on the bank's liquidity position which takes the form of a monotone pass-fail test that fails all banks with a liquidity position below an optimal cut-off, and passes the other.

Corollary 1. *The optimal comprehensive policy* $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$ *can be sequentially implemented by:*

(1) Conducting an asset quality review which (i) assigns a passing grade (m^y_{pass}*) to all banks with assets generating cash-flow above* y^+*, and assigns a failing grade* m^y_i *to any assets delivering cash-flows*

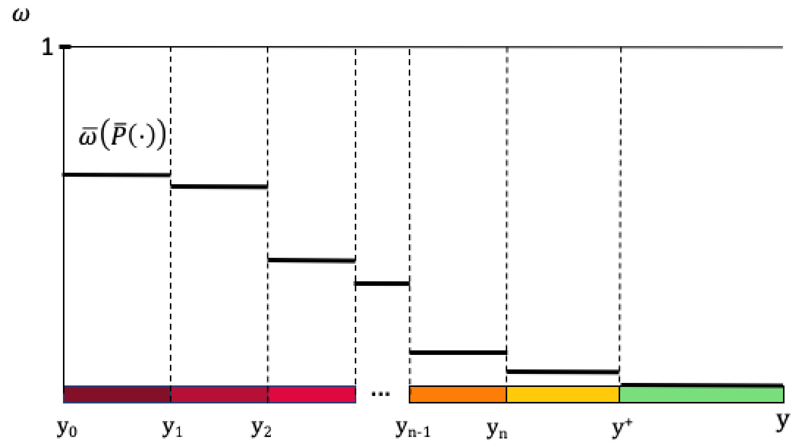

Figure 4: Optimal Comprehensive Assessment $\Psi = \{\Gamma^y, \mathscr{R}, \Gamma^\omega\}$.

$y \in (y_i, y_{i+1}]$, *and (ii) imposing recapitalizations which dictates that the bank raises K when receiving the passing grade, and to sell the asset when falling below cut-off* y^+.

(2) Conducting a liquidity stress test that informs ST creditors of wether the liquidity shock is above the cut-off $\bar{\omega}(\bar{P}(\mathbb{E}(y|m^y)))$.

The optimal asset quality review Γ^y pools all cash-flows realizations above y^+ so that the induced posterior mean, $\mathbb{E}(y|\{y > y^+\})$, is greater than K and, hence, all ST creditors are dissuaded from running. Using a more transparent disclosure policy for high values of y does not generate any benefits and, in fact, may hinder risk-sharing among banks with heterogeneous asset qualities. Thus, when the LT profitability of the assets of a bank is above y^+, the optimal asset quality review assigns an *opaque* (and unique) pass grade.

When the profitability level y falls below y^+, the optimal policy specifies multiple failing grades. The intuition for this result is that, as explained in subsection 4.5.1, there exists an endogenous amplification effect associated with increasing the perceived profitability of the bank's asset which originates in the interaction between the multiple audiences. Indeed, increasing the perceived profitability of the bank's asset induces LT investors to pledge a larger amount of funds. This induces a smaller probability of failure, which feeds back and induce LT investors to pledge an even larger amount. As a result, the interaction of both audiences generates a virtuous cycle which *convexifies* of the probability of survival $\phi(\cdot)$ in the perceived profitability of the asset. Given that $\phi(\cdot)$ is also determined by shape of the prior F^ω, and ST creditors' payoff functions g and b, the resulting shape of ϕ need not satisfy global convexity. In the regions of (strict) local convexity the policy maker prefers to separate different profitability levels y under different signals similar to a risk-lover agent who prefers to separate different states rather than pooling them together under a single realization. As a result, and perhaps surprisingly, the optimal asset quality review is *more transparent* when banks have poor quality assets.

As proved in Lemma 1, when $f^\omega(\cdot)$ does not increase too fast, and the payoff functions g and b are constant, the amplification effect dominates and the induced probability of survival is globally convex. Whenever this is the case, the policy maker prefers a fully revealing policy that perfectly discloses the profitability of the bank's asset y, for any $y < y^+$. As a result, the optimal asset quality review becomes fully transparent for banks with low quality assets and fully opaque for banks with high quality assets.

Example 1 in the Online Supplement shows that the optimality of multiple failing grades is a consequence of the interaction between multiple audiences. When LT investors are protected against the potential default of the bank, which completely eliminates the amplification effect, and the prior distribution of the liquidity shock is uniform on $[0,1]$, the optimal asset quality review corresponds to a monotone pass-fail test.

5 Information and the Lender of Last Resort

5.1 Emergency Lending Mechanism

This section studies the interplay between *information disclosure* and the policy maker's role as *lender of last resort* (LOLR). To accommodate the practical concern that participation in emergency lending programs reveals information about the bank's liquidity position, I consider interventions wherein the bank has private information regarding its ST obligations. The policy maker does not possess a technology to learn about the bank's liquidity position in a timely manner. In practice, conducting stress tests may takes several months while liquidity shocks that compromise the bank's buffers may suddenly transpire. I thus assume that the policy maker needs to rely on information directly reported by the bank when such liquidity shocks take place.

Additionally, I relax the assumption that the policy maker cannot use public funds to aid the bank. Nonetheless, in the spirit of the paper, I retain the feature that public funds are limited. Specifically, I assume that the policy maker may purchase securities from the bank using taxpayers' money, but under a *budget balance* constraint that requires the price to not exceed the fair price of the securities, taking into consideration the probability of default. This assumption is consistent with the *Bagehot principle* which states that the policy maker should act a LOLR and provide funds to institutions under distress, in exchange of good assets and at a discount. Interventions in the form of direct transfers and government guarantees are thus ruled out by design.[32]

The timing of the game remains similar to the one in Section 4, with two modifications. First, I dispense with the assumption that the bank has *residual private information* with respect to the quality of its asset (i.e., that the bank observes signal $\theta \in \Theta$). This assumption simplifies the exposition and provides some traction. I show at the end that the optimal mechanism is robust to the bank observing residual private information with respect to its asset and therefore can be implemented in such richer environments. Secondly, as anticipated above, I assume that instead of conducting a stress test Γ^{ω} at $t = 2$, the policy maker runs an emergency

[32]Rigorously, the budget balance constraint implies that the policy maker's intervention must not incur a deficit neither *on* nor *off* the equilibrium path. Interventions in the form of guarantees require that the policy maker sets aside a large budget to insure the corresponding creditors (which takes place off the equilibrium path). Allowing for guarantees then opens the door to uncertainty whether the policy maker has, in fact, the resources to fulfill her promise if the occasion requires it.

lending mechanism Υ^{ω}, which asks the bank to report the magnitude of its liquidity shortage (equivalently, its private information ω). Conditional on the report, the mechanism (i) purchases claims on the bank's asset, and (ii) discloses information to all market participants according to the rule π^{ω} (explained in detail below).

An important modeling choice is whether banks may approach LT investors in period 2 after interacting with the policy maker.[33] Introducing the possibility of approaching the asset market after dealing with the policy maker implies that the emergency lending mechanism has participation constraints that are endogenous to the mechanism itself. In fact, a bank that chooses to not deal within the emergency lending program faces a market whose beliefs depend on Υ^{ω} itself. In order to simplify the exposition I focus, in the main text, on the case where the asset market remains closed during period 2 and hence avoid the intricacies of dealing with endogenous participation constraints. I show in the Appendix that the solution described in the main text also holds in the richer environment with an asset market at $t = 2$.[34]

A (comprehensive) intervention $\Psi = \{\Gamma^{y}, \mathscr{R}, \Upsilon^{\omega}\}$ thus consists of an asset quality review Γ^{y}, a recapitalization policy $\mathscr{R}$, and an emergency lending mechanism Υ^{ω}. Hereafter, I refer to Ψ as a *persuasion mechanism* as it combines a *bayesian persuasion* component with the usual tools from the mechanism design literature.[35]

5.2 The Policy Maker's Problem

Suppose that the bank has successfully raised P units of capital by selling security $s^{\star}$ to LT investors in period 1. Recall that, if the recapitalization P is such that ST creditor's (ex-ante) expected payoff under the *Laissez Faire* regime

$$\bar{U}_{\mathrm{LF}}(P) \equiv \int_0^1 u(\omega + P, 1) F^{\omega}(d\omega) \leq 0, \tag{12}$$

then, under adversarial coordination, all ST creditors withdraw early in the absence of the policy maker's intervention.

The policy maker runs an emergency lending mechanism $\Upsilon^{\omega} = \{\{M^{\omega}, \pi^{\omega}\}, t, s\}$, where $\pi^{\omega} : \Omega \to \Delta M^{\omega}$ corresponds to a disclosure rule which asks the bank to report its liquidity position ω. Conditional on its report $\tilde{\omega}$, the policy maker offers to purchase a claim on the bank's asset $s[\tilde{\omega}] \in \mathscr{S}$, with $s(y|\tilde{\omega}) \in [0, y - s^{\star}(y)]$ for all y, at a price $t(\tilde{\omega}) \geq 0$. In addition, Υ^{ω} discloses a score m^{ω} to all market participants according to the disclosure policy $\pi^{\omega}[\tilde{\omega}] \in \Delta M^{\omega}$.

I assume that the mechanism Υ^{ω} must be (interim) (i) *incentive compatible* and (ii) *individually rational*. That is, (i) the bank must be at least as well-off by disclosing its private information than by reporting any other value of ω, and (ii) the bank must be at least as well-off by participating in the mechanism than by opting out of it.

[33] Note that, in the framework of Section 4, introducing a second round to deal with LT investors in period 2 is inconsequential as (a) banks that pass the stress test do not need to raise funds, and (b) banks that fail the stress test inevitably face a run of ST creditors and therefore no additional funds can be raised.

[34] Mechanism with endogenous participation constraints have been studied before by Philippon and Skreta [2012] and Tirole [2012]. The main difference with respect to those earlier papers is that the current paper assumes that the policy maker may control/design the information that is passed on to the asset market.

[35] The persuasion mechanism term was first used by Kolotilin et al. [2017] in a single-receiver environment with no transfers.

5.2.1 Severe Market Conditions and Necessity of Emergency Lending

I restrict attention to the case where the expected quality of the bank's asset is sufficiently low so that the policy maker cannot set recapitalizations to dissuade ST creditors from running on the bank. That is,

$$\frac{1}{R}\mathbb{E}\left(y|m^{y}\right) \quad < \quad K. \tag{13}$$

This assumption means that $\bar{U}_{\mathrm{LF}}(P) < 0$ for any $P \leq \frac{1}{R}\mathbb{E}\left(y|m^{y}\right)$. That is, the bank is unable to persuade ST creditors to keep rolling over even if it sells its whole asset. This is the interesting case as otherwise the policy maker may avoid bank failure (with certainty) by imposing recapitalizations in period 1. When inequality (13) holds, the policy maker's ability to prevent bank failure thus depends on her capacity to elicit information within the emergency lending mechanism.

Next, I impose an upper bound on the amount of funds the policy maker can dedicate to purchase assets within Υ^{ω}. In particular, I assume that there exists an upper bound $B < 1$ on the amount funds the policy maker has at her disposal. This means that

$$t\left(\omega\right) \in \mathscr{T}\left(B\right) \equiv \left\{t : \Omega \to [0,B]\right\}. \tag{14}$$

This constraint rules out situations in which the policy maker alone can solve the bank's liquidity problem (recall that $\omega \in \Omega = [0,1]$). This assumption is made on empirical grounds. Policy makers rarely have the resources to solve banks' liquidity problems by acting alone. Their interventions usually require some degree of coordination with external creditors and investors.

Finally, I impose the additional constraint that F^{ω} is continuous on $[0,1]$. This assumption implies that there are no mass points.

5.2.2 Constraints of the Emergency Lending Mechanism

An argument similar to the one establishing the *Revelation Principle* implies that it is without loss of optimality to restrict attention to policies where the scores announced to ST creditors take the form of action recommendations.[36] Thus, we can restrict the analysis to disclosure policies $\pi\left[\omega\right] \in \Delta\left\{0,1\right\}$, where message $m^{\omega} = 0$ is interpreted as the recommendation to rollover the bank's debt (equivalently, *passing* the policy maker's test) and $m^{\omega} = 1$ as the recommendation to stop pledging funds to the bank (*failing* the test). We distinguish between the price paid by the policy maker when she passes the bank, $t_0\left(\tilde{\omega}\right)$, and the price offered when failing it, $t_1\left(\tilde{\omega}\right)$. ST creditors then *obey* the policy maker's announcement as long as

$$\mathbb{E}\left(u\left(P+t_0(\omega),\omega,1\right)|m^{\omega}=0\right) = \frac{\int_{\Omega} u\left(\omega+P+t_0(\omega),1\right)\pi^{\omega}(0|\omega)dF^{\omega}(\omega)}{\int_{\Omega}\pi^{\omega}(0|\omega)dF^{\omega}(\omega)} > 0, \tag{15}$$

and

$$\mathbb{E}\left(u\left(P+t_1(\omega),\omega,1\right)|m^{\omega}=1\right) = \frac{\int_{\Omega} u\left(\omega+P+t_1(\omega),1\right)\pi^{\omega}(1|\omega)dF^{\omega}(\omega)}{\int_{\Omega}\pi^{\omega}(1|\omega)dF^{\omega}(\omega)} \leq 0. \tag{16}$$

[36]This is a consequence of assuming (a) adversarial coordination and (b) homogeneity in ST creditors' beliefs. Together these assumption imply that, for any arbitrary score disclosed by the policy maker $\tilde{m}$, the size of withdrawals $A\left(\tilde{m}\right) \in \{0,1\}$.

Hereafter, I refer to conditions (15) and (16) as *obedience constraints.* Let $U_{P,s^\star}(\tilde{\omega};\omega)$ be the utility of a bank with private information ω, which raises P units of capital in period 1 by selling security $s^\star$, and reports $\tilde{\omega}$. That is,

$$\begin{aligned} U_{P,s^\star}(\tilde{\omega};\omega) \quad \equiv \quad & \sum_{m^\omega\in\{0,1\}} \pi^\omega(m^\omega|\tilde{\omega})\, 1\{\omega+P+t_{m^\omega}(\tilde{\omega})\geq A(P,m^\omega)\}\times \\ & \times\left((P+t_{m^\omega}(\tilde{\omega}))R+\mathbb{E}_\theta(y-s^\star-s[\tilde{\omega}])\right) \end{aligned}$$

where $A(P,m^\omega)$ corresponds to the most aggressive fraction of withdrawals consistent with observing the bank raising P and the policy maker disclosing message $m^\omega\in\{0,1\}$. That the mechanism satisfies incentive compatibility then translates to

$$\omega \in \arg\max_{\tilde{\omega}\in\Omega} U_{P,s^\star}(\tilde{\omega};\omega), \forall\omega. \tag{17}$$

Finally, I impose that the price paid by the policy maker not exceed the fair price of the security purchased from the bank, accounting for all the information publicly disclosed at this point (i.e., m^y and m^ω), and the endogenous probability of default. This means that

$$t_{m^\omega}(\omega) \quad\leq\quad \frac{\mathbb{E}(s(y;\omega)|m^y)}{R} 1\{\omega+P+t_{m^\omega}(\omega)\geq A(P,m^\omega)\}, \forall\omega,\forall m^\omega. \tag{18}$$

Constraint (18) is novel and deserves some explanation. This constraint is consistent with requirements usually imposed by policy makers to protect taxpayers.[37] Instead, just as the *Bagehot principle* dictates, the policy maker provides funds against sound assets, and at a discount to dissuade those borrowers that are not genuinely in need. Second, the constraint prevents the policy maker from consistently running losses and therefore makes the emergency lending program sustainable. Finally, this constraint incorporates the idea that disclosing information about the bank's fundamentals affects the amount of funds the policy maker may pay for the bank's securities. Hereafter, I refer to (18) as the *fair price constraint.*

The policy maker's problem can then be summarized as finding the disclosure policy $\{M^\omega,\pi^\omega\}$ and transfers $\{t_{m^\omega}\}_{m^\omega\in\{0,1\}}$ that maximize the ex-ante probability of bank survival under obedience, incentive compatibility, and fair price constraints.

$$\begin{aligned} \max_{\{\{\{0,1\},\pi^\omega\},t_0,t_1\in\mathscr{T}(B)\}} \quad & \int_0^{1-P} \pi^\omega(0|\omega)\,dF^\omega(\omega) \\ \text{s.t:} \quad & (15),(16),(17),(18). \end{aligned} \tag{19}$$

5.2.3 Towards a Characterization of the Optimal Policy

Denote banks with $\omega<1-P$ as *vulnerable* banks, and those with $\omega\geq 1-P$ as *safe.* As shown in section 4, the policy maker's (unconstrained) optimal disclosure policy consists in failing all banks with a liquidity position below the cutoff $\bar{\omega}(P)$ so that banks with liquidity positions above $\bar{\omega}(P)$ may survive. This logic,

[37] Section 13(3) of the Federal Reserve act states, with respect to the design of emergency lending programs, that "*Such policies and procedures shall be designed to ensure that the security for emergency loans is sufficient to protect taxpayers from losses*."

however, does not work in the current environment where information about ω has to be self-reported by the bank. In fact, no *vulnerable* bank would ever report its true type if this leads the policy maker to recommend ST creditors to run with probability 1.

In order to solve this conflict, the policy maker may offer *less liquid* banks to purchase their assets at better prices (i.e., lower discounts) in exchange for a lower *passing* probability. This implies that *more liquid* (but still vulnerable) banks receive a lower price for their assets but a higher probability of passing the policy maker's test. The fact that the latter are also the ones that require less funds to withstand liquidity shocks induces a virtuous cycle as it enhances the perceived liquidity of all banks receiving a passing grade. These banks (more liquid but still vulnerable) would default if a run takes place in the absence of a deal with the policy maker, which alleviates incentive constraints.

Lemma 2. *It is without loss of optimality to restrict attention to mechanisms that satisfy:*

(i) $t_1(\omega) = 0$ *for all* ω.

(ii) $s[\omega] = y - s^\star$ *for all* $\omega \leq 1 - P$.

(iii) $s[\omega] = \mathbf{0}$ *for all* $\omega > 1 - P$.

Property (i) states that, within the emergency lending program, disclosures and transfers are *complements.* In fact, no bank receiving a failing grade should receive funding from the emergency lending program The intuition behind this claim is that purchasing securities from a bank that has received a failing grade makes both the obedience constraint (16) and the incentive compatibility constraint (17) harder to satisfy and does not increase the set of banks that survive. Thus, the policy maker is better off by not purchasing assets from such bank types.[38]

Claim (ii) states that vulnerable banks sell all their remaining claims on their asset to the policy maker. The proof follows from noting that vulnerable banks fail if a run takes place and they are not helped by the policy maker. Thus, the latter who control not only the transfers but also ST creditors' behavior (through the disclosure policy π^ω), has all the bargaining power and is therefore better off by offering to purchase $s[\omega] = y - s^\star$ to all $\omega < 1 - P$. This allows her to offer higher prices for the securities under the fair price constraint.[39]

Finally, claim (iii) states that safe banks (i.e., with $\omega \geq 1 - P$) never participate in the emergency lending program. To see this, note that a bank with $\omega > 1 - P$ survives without government aid, even if a run takes place, and therefore it does not need to participate in the program. On the other had, offering to purchase assets from this bank type is costly for the policy maker as the bank's outside option, for any security s, is given by $\mathbb{E}(s|m^y)$ (with certainty). Paying this amount (in expectation) would induce other vulnerable banks

[38] In fact, consider any feasible mechanism $\Upsilon^\omega = \{\{\{0,1\}, \pi^\omega\}, t_0, t_1\}$ with $t_1(\tilde{\omega}) > 0$ for some $\tilde{\omega}$ which survives even when failed by the policy maker (i.e., $\tilde{\omega} + P + t_1(\tilde{\omega}) > 1 = A(P, m^\omega = 1)$). Such a mechanism can be improved upon by a similar mechanism $\hat{\Upsilon}^\omega = \{\{\{0,1\}, \hat{\pi}^\omega\}, \hat{t}_0, \hat{t}_1\}$ slightly modified so that such a bank passes the test instead. That is, $\hat{\pi}^\omega[\omega] = \pi^\omega[\omega]$, $\hat{t}_0(\omega) = t_0(\omega)$, and $\hat{t}_1(\omega) = 0$ for all $\omega \neq \tilde{\omega}$ and $\hat{\pi}^\omega(0|\tilde{\omega}) = 1$, $\hat{t}_0(\tilde{\omega}) = t_1(\omega)$. The new mechanism satisfies all the feasibility constraints, as $\tilde{\omega} + P + t_1(\tilde{\omega}) > 1$ implies that $\tilde{\omega} + P + \hat{t}_0(\omega) > 1$.

[39] This property need not be satisfied for type-H banks. To see this, note that it might be in the interest of the policy-maker to offer type-H banks to retain a fraction of their asset. This might be useful to alleviate incentive constraints.

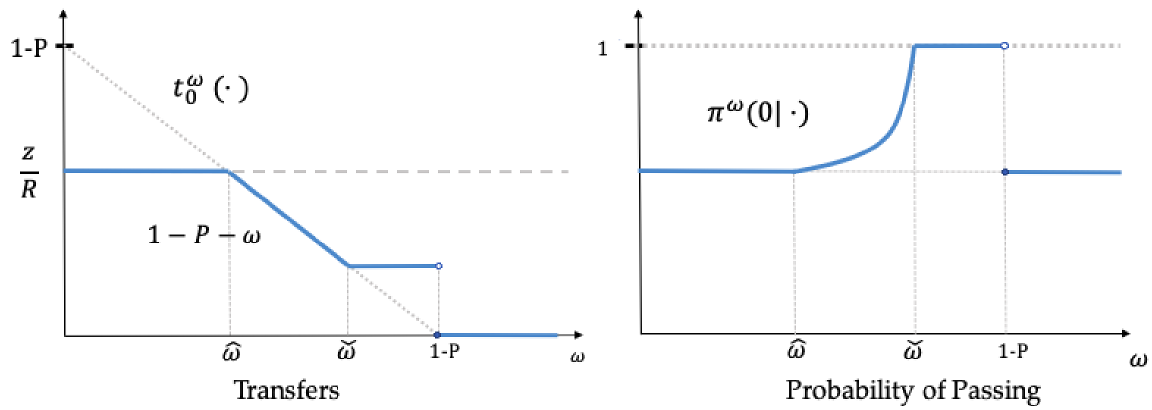

Figure 5: Optimal Emergency Lending mechanism under observable of asset quality.

to mimic, compromising incentive compatibility. As a result, the policy maker chooses not to trade with such banks (i.e., $s[\omega] = \mathbf{0}$, for all $\omega \geq 1-P$).

5.3 Optimal Emergency Lending Mechanism for fixed P

Equipped with the characterization in Lemma 2, we characterize the optimal Emergency Lending mechanism.

Proposition 5. *Suppose that the bank raises $P < K$ in period 1. Then, the optimal emergency lending mechanism, $\Upsilon^\omega[P]$, is characterized by:*

$$t_0^\omega(\omega;P) = \begin{cases} \frac{z}{R} & \omega < \hat{\omega} \\ 1-P-\omega & \omega \in [\hat{\omega}, \check{\omega}] \\ 1-P-\check{\omega} & \omega \in (\check{\omega}, 1-P) \\ 0 & \omega \geq 1-P \end{cases}, \quad \pi(0|\omega;P) = \begin{cases} \frac{(1-\check{\omega})R}{PR+z} & \omega < \hat{\omega} \\ \frac{1-\check{\omega}}{1-\omega} & \omega \in [\hat{\omega}, \check{\omega}] \\ 1 & \omega \in (\check{\omega}, 1-P) \\ \frac{(1-\check{\omega})R}{PR+z} & \omega \geq 1-P \end{cases} \tag{20}$$

where $z \equiv \mathbb{E}(y - s^\star(y)|m^y)$, $\hat{\omega} \equiv 1-P-\frac{z}{R}$, and $\check{\omega}$ is implicitly defined by

$$g \times \left(\int_{\hat{\omega}}^{\check{\omega}} \frac{dF^\omega(\omega)}{(1-\omega)R} + \frac{F^\omega(1-P) - F^\omega(\check{\omega})}{(1-\check{\omega})R} \right) = \int_0^{\hat{\omega}} \frac{|b|\, dF^\omega(\omega)}{PR+z} - g\frac{(1-F^\omega(1-P))}{PR+z}.$$

The optimal Emergency Lending program, characterized in Proposition 5, is illustrated in Figure 5. To persuade ST creditors to follow the recommendation to rollover the bank's debt, the policy-maker modifies the likelihood of the bank's survival. The upper bound on the price that can be pledged by the policy maker implies that banks with a liquidity position smaller than $\hat{\omega} = 1-P-\mathbb{E}(y-s^\star(y)|m^y)/R$ default when all ST creditors withdraw early. The policy maker then minimizes the passing probability assigned to these liquidity types and compensates them by paying them the maximal feasible price.

All banks with liquidity positions above $\hat{\omega}$ receive enough funds to prevent default under an adversarial withdrawal. Incentive compatibility among vulnerable banks induces a negative relationship between the passing probability and the price paid by the policy maker. Banks with a liquidity shock $\omega \in [\hat{\omega}, \check{\omega})$ receive the smallest price that allows them to survive a run. The level $\check{\omega}$ is chosen so that obedience constraints are satisfied. Intuitively, the smaller the value of $\check{\omega}$, the more liquidity-types receive the maximal passing probability and, hence, the larger the aggregate survival probability. The optimal mechanism chooses the minimal value of $\check{\omega}$ consistent with the obedience constraints.

5.4 Private and Public Sector Interventions are Substitutes

I use the construction of the optimal emergency lending mechanism Υ^{ω} to show that imposing capital requirements in period 1 is detrimental for its effectiveness. Asking the bank to increase its recapitalization in period 1 from P to $P' > P$ induces an ex-antes probability of survival strictly smaller. As a result, a policy maker implementing an emergency lending program strictly prefers to minimize the recapitalizations imposed in period 1.

The intuition behind this result follows from two observations. First, asking the bank to raise funds from LT investors in period 1 depletes the fraction of the assets available to sell within the emergency lending mechanism. The fair-price constraint then implies that the amount the policy maker can pledge to the bank within the program is smaller. This is detrimental for the the policy maker's elicitation capacity. I dub this effect the transfer channel. Second, raising a larger amount of funds in period 1 implies that the measure of safe banks increases. This implies that incentive compatibility constraints become more demanding and, as a result, the policy maker is forced to pass vulnerable banks with smaller probability. I call this effect the incentive channel. Interestingly, both channels reinforce each other and together outweigh the effect of having a larger measure of safe banks (recall that a safe bank always survives).

In contrast, when the value of the remaining claims on the bank's asset is sufficiently small, the policy maker may not be able to successfully elicit information about the bank's liquidity position. As discussed in the former section, the key trade-off that allows the regulator to induce the bank to self report its liquidity position involves a negative relation between the amount of funds offered to the bank, and the probability of assigning a passing grade. When the maximal amount than can be pledged by the policy maker is too low to discourage most vulnerable banks from mimicking more liquid types, the policy maker does not induce truthful reporting and, therefore, cannot engage in persuasion.

Let $\underline{z}$ be the minimal expected value of the bank's remaining claims necessary for information elicitation. That is,

$$\underline{z} \equiv \inf_{z} \left\{ z \geq 0 : \frac{b}{z} F^{\omega} \left(1 - \frac{z}{R} \right) + g \int_{1-\frac{z}{R}}^{1} \frac{dF^{\omega}(\omega)}{(1-\omega) R} \geq 0 \right\}. \tag{21}$$

Proposition 6. *Fix $P \geq 0$ and suppose that $\mathbb{E}(y - s^{\star}(y) | m^{y}) \geq \underline{z}$. Then, increasing the amount raised from*

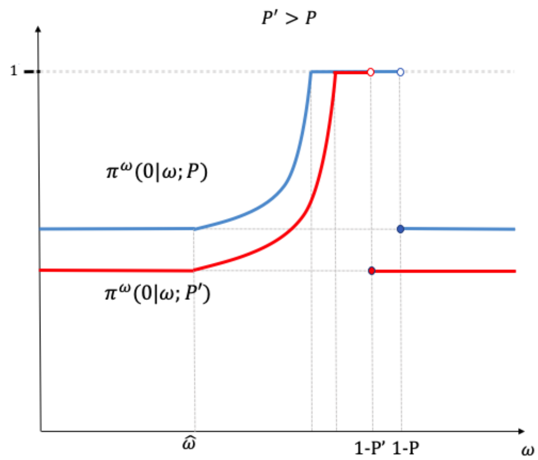

Figure 6: Passing probabilities $\pi(0|\omega;P)$ and $\pi(0|\omega;P')$ with $P < P'$.

$P > 0$ to $P' > P$ decreases the ex-ante probability of survival. That is,

$$\int_0^{1-P} \pi^{\omega}(0|\omega;P)dF^{\omega}(\omega) + 1 - F^{\omega}(1-P) > \int_0^{1-P'} \pi^{\omega}(0|\omega;P')dF^{\omega}(\omega) + 1 - F^{\omega}\left(1-P'\right).$$

Figure 6 depicts this last result. The content of proposition 6 is that private and public sector funding are, as a matter of fact, substitutes. The policy maker is strictly better off by choosing at most one of these options than any combination of the two. When the value of the bank's asset $\mathbb{E}(y|m^y)$ is smaller than $\underline{z}$ the emergency lending program in period 2 is not be able to elicit information from the bank. Consequently, in this case the policy maker imposes recapitalizations during period 1. Instead, for any value of $\mathbb{E}(y|m^y)$ in $[\underline{z}, KR)$ (recall that the analysis has assumed inequality 13), the policy maker optimally chooses to avoid recapitalizations during the fund-raising stage in period 1 and runs the emergency lending mechanism $\Upsilon[P=0]$.

5.5 Period 1: Asset Quality Review

In period 1 the policy maker jointly designs the asset quality review Γ^y and recapitalization requirements that precede the emergency lending mechanism Υ^{ω}. As I show below, the policy-maker faces an important trade-off when designing the recapitalizations rule to impose in the first period. On the one hand, as shown above, smaller recapitalizations allow the bank to retain a greater fraction of the asset on its balance sheet. This increases the price that the policy maker may offer to the bank thus enhancing the effectiveness of the liquidity provision program $\Upsilon^{\omega,\theta}$. On the other hand, more stringent recapitalizations permit the bank to raise capital before the liquidity shock materializes. This helps decrease the premium the bank has to pay to compensate for rollover risk.

The next theorem completes the analysis by characterizing the structure of the optimal persuasion mechanism as a function of the quality of the bank's asset.

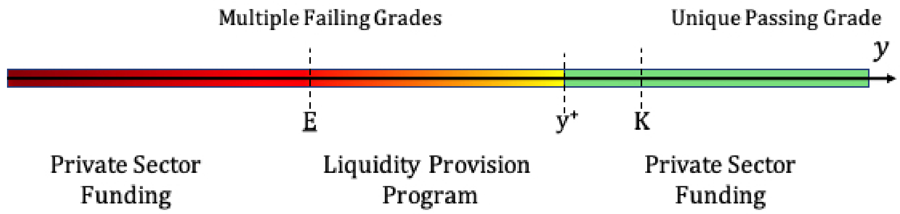

Figure 7: Structure optimal comprehensive intervention.

Theorem 2. *The optimal persuasion mechanism $\hat{\Psi} = (\Gamma^y, \mathscr{R}, \Upsilon^\omega)$ is characterized by a monotone partition $\mathscr{P} = \{(y_i, y_{i+1}]\}_{i\in I}$ of $\mathbb{R}_+$ such that the optimal asset quality review $\Gamma^y = \left\{\left\{m_i^y\right\}_{i\in I}, \pi^y\right\}$ satisfies $\mathbb{E}(y|m_i^y) < \mathbb{E}(y|m_j^y)$ for all $i < j$. Moreover, the highest interval always include y^+. Furthermore,*

(1) If $y \geq y^+$, the policy-maker passes the bank and sets recapitalizations according to the policy $R_\alpha(P) = 1\{P < K\}$, $\alpha > 0$.

(2) If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \in (\underline{z}, K)$, the bank participates in the emergency lending program characterized by $t_0^\star\left(\omega; \mathbb{E}(y|m_i^y)\right), \pi_0^\star\left(\omega; \mathbb{E}(y|m_i^y)\right)$, where $t_0^\star$ and $\pi_0^\star$ are as defined in (20).

(3) If $y \in (y_i, y_{i+1}]$ with $\mathbb{E}(y|m_i^y) \leq \underline{z}$, the policy maker imposes recapitalizations according to $R_{\tilde{\alpha}}(P) = 1\{P < \bar{P}(z)\}$, $\tilde{\alpha} > 0$, and no emergency lending program is used.

Theorem 2 shows that the optimal comprehensive policy features a non-monotone pecking order. Institutions with high-quality assets are given a passing grade by the asset quality review Γ^y, and are required to raise enough capital from the private sector to persuade ST creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades and are funded with the government's optimal emergency lending program. Finally, institutions with extremely poor-quality assets, are failed with multiple failing grades and are induced to seek funding from the private sector by imposing recapitalizations.

Theorem 2 informs the policy debate by showing that non-monotone relations between asset profitability and the source of funding that institutions receive, need not be a proof of sub-optimality. In fact, they are expected to arise in environments where emergency lending mechanisms are optimally designed. In contrast, as highlighted before, simultaneous pledging by both the public and the private sector is, indeed, evidence of sub-optimality.

Furthermore, the analysis shows that recapitalization rules need not be part of an optimal policy. In opposition to the results found in section 4 that advocate for the use of recapitalization policies, theorem2 offers a message of caution. If the policy maker does not possess the technology to react in a quickly manner to liquidity events to implement a liquidity stress test to alleviate pessimistic assessment of ST creditors, then recapitalization policies are costly and undesirable. Such rules deplete the amount of assets that the bank may use as collateral to obtain emergency lending negatively affecting her capacity to elicit information about the bank's liquidity needs, and therefore her ability to persuade ST creditors to keep pledging to the bank.

5.6 Discussion

At the beginning of this Section we made the assumption that the bank did not possess any residual information with respect to the profitability of its asset (i.e., that the bank did not observe signal θ). This assumption was made to make the arguments as transparent as possible. Nonetheless, the optimal persuasion mechanism described above is, in fact, robust to the bank's private information. In fact, the emergency lending mechanism Υ^{ω} characterized in theorem 2 (as part of the persuasion mechanism $\hat{\Psi} = (\Gamma^{y}, \mathscr{R}, \Upsilon^{\omega})$) remains incentive compatible, even if the bank observes a noisy signal about the profitability of its asset, θ. The reason behind this fact is that, when $\hat{\Psi}$ is optimally designed, all banks tparticipating in the emergency lending mechanism Υ^{ω} are vulnerable and require the policy maker's aid to survive an eventual run of ST creditors. This is not the case, e.g., when the bank raises $P > 0$ units of funds during period 1. In fact, for any amount $P > 0$ raised during period 1, safe banks (i.e., those with $\omega > 1 - P$) with a bad signal about their assets, θ_L, have incentives to pretend to be vulnerable in order to exchange their assets for transfers within the emergency lending program. As a result, the policy maker is forced to limit the amount of funds that can be provided within Υ^{ω}, deteriorating its effectiveness. The optimal persuasion mechanism is robust to this critique by making sure that, conditional on participating in the emergency lending mechanism, the bank is vulnerable and requires government aid.

6 Conclusions

This paper studies interventions aimed at stabilizing financial institutions subject to rollover risk. I consider a rich environment which emphasizes the interaction among multiple audiences who care about different aspects of the bank's multi-dimensional fundamentals. I show that complementing disclosure policies with minimal recapitalizations is essential to maximizing the probability of the bank's survival. Perhaps surprisingly, the optimal review is *opaque* when the institution has high-quality assets and assigns a unique pass grade. In contrast, the optimal review is more *transparent* with banks with low-quality assets, in which case multiple failing grades are assigned to the bank as a function of the precise quality of the assets, which also triggers a follow-up stress test on the bank's liquidity position.

When the policy-maker lacks the ability to examine the bank's liquidity position and, hence, needs to elicit information from the bank, the initial asset quality review is followed by an emergency lending program, whereby the government offers to buy assets from the bank, in exchange of cash and a public disclosure of the bank's liquidity. I show that, in this case, imposing recapitalizations undermines the effectiveness of the government's liquidity program. I also show that simultaneous pledging by the government and the private sector is suboptimal. I find that optimal comprehensive policies feature a non-monotone pecking order: Institutions with high-quality assets are given a pass grade by the asset quality review that assess the LT profitability of the bank's assets and are required to raise enough capital from the private sector to persuade ST creditors to rollover its debt. Banks with intermediate-quality assets, in turn, are assigned one of multiple failing grades, and are funded with the government's emergency lending program. Finally, institutions with extremely poor-quality assets are failed with multiple failing grades and are induced to seek private sector financing.

The above results are worth extending in several directions. The analysis presented assumes the policy maker knows the distribution of future liquidity shocks when she designs the optimal comprehensive policy. Such knowledge may come from previous experience with banks of similar fundamentals. While this is a natural starting point, there are many environments in which it is more appropriate to assume that the designer lacks information about the joint distribution of the underlying fundamentals. In future work, it would be interesting to investigate the optimal disclosure policy in such situations. One idea is to apply a robust approach to the policy maker's problem, whereby the designer expects nature to select the information structure that minimizes her payoff. The characterization of the optimal policy in this environment is highly relevant both from a theoretical standpoint and for the associated policy implications.

The analysis also assumes that uncertainty regarding the bank's liquidity is resolved after the bank approaches LT investors. However, ST creditors' runs are intrinsically dynamic phenomena. If the fundamentals are partially persistent over time, the optimal policy must also specify the timing of disclosures. In future work, it would be interesting to extend the analysis in this direction.

Appendix A: Laissez Faire

D1 Refinement. Define first the set of *best responses* to an arbitrary security s, $BR(s)$, as the set of prices which are consistent with rationality of the investors under some belief about the asset quality type of the bank:[40]

$$BR(s) \equiv \left\{ P : \frac{\mathbb{E}_H(s)}{R} \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P \right\}.$$

Define then,

$$\mathscr{D}(\theta|s) \equiv \left\{P \in BR(s) : V(P, s, \theta) > V\left(P^\star\left(s^\star_\theta\right), s^\star_\theta, \theta\right)\right\}$$

$$\mathscr{D}^0(\theta|s) \equiv \left\{P \in BR(s) : V(P, s, \theta) = V\left(P^\star\left(s^\star_\theta\right), s^\star_\theta, \theta\right)\right\}.$$

The profile $\left\{\left\{s^\star_\theta\right\}_{\theta\in\Theta}, \mu^\star, P^\star, A^\star\right\}$ satisfies the D1 criterion if for any security $s \in S$ with $s \neq s_*(\theta)$ all $\theta \in \Theta$, $\mu_*(s)$ is such that $\forall \theta, \theta' \left(\mathscr{D}(\theta|s) \cup \mathscr{D}^0(\theta|s)\right) \subset \mathscr{D}(\theta'|s)\right) \Rightarrow \mu_*(\theta|s) = 0$.

Definition 1. We say a function $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies *single crossing from above* (SCFA), if the following holds true: if there exists some $y \in Y$ such that $g(y) < 0$, then $\forall \tilde{y} > y$, $g(\tilde{y}) \leq 0$. Similarly, we say that $h : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies *single crossing from below* (SCFB), if the following holds true: if there exists some $y \in Y$ such that $h(y) > 0$, then $\forall \tilde{y} > y$, $h(\tilde{y}) \geq 0$.

Lemma 3. *Suppose that $g : Y \subseteq \mathbb{R} \to \mathbb{R}$ satisfies SCFA and that $f(y,t)$ is log-supermodular for all $(y,t) \in Y \times T \subseteq \mathbb{R}^2$. Define $\phi(t) \equiv \int_Y g(y) f(y,t) dy$ and let $y_0 \equiv \inf\{y \in Y : g(y) < 0\}$.Then, $\forall \tilde{t} > t \in T$:*

$$\phi(\tilde{t}) = 0 \Rightarrow \phi(t) > 0.$$

Proof. That $f(y,t)$ is log-SM implies that $\frac{f(\cdot,t)}{f(\cdot,\tilde{t})}$ is non-increasing. Then,

$$\begin{aligned} \phi(t) &= \int_Y 1\{y \leq y_0\} g(y) \frac{f(y,t)}{f(y,\tilde{t})} f(y,\tilde{t}) dy + \int_Y 1\{y > y_0\} g(y) \frac{f(y,t)}{f(y,\tilde{t})} f(y,\tilde{t}) dy \\ &\geq \left(\frac{f(y_0,t)}{f(y_0,\tilde{t})}\right) \phi(\tilde{t}) \end{aligned}$$

which implies the result. □

Proof of Proposition 1.

The proof offered below applies regardless of whether the designer has disclosed information about the fundamentals (y, ω).

[40] First-order stochastic dominance (which is implied by MLRP) means that

$$\left\{P > 0 : \frac{\mathbb{E}_H(s)}{R} \times \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P\right\} = \bigcup_{\mu\in\Delta\Theta} \left\{P > 0 : \frac{\mathbb{E}(s;\mu)}{R} \times \mathbb{P}\left\{\omega + P \geq A^\star(P)\right\} \geq P\right\}$$

Assume that the survival probability can be written as $\mathbb{P}\left\{\omega \geq \omega^{\sharp}(\tau)\right\}$, where $\omega^{\sharp}(\cdot)$ represents a non-increasing function, continuously differentiable for all $\tau < K$, and with $\omega^{\sharp}(\tau) = 0$, for all $\tau \geq K$. In the context of Section 3, $\omega^{\sharp}(P) = A^{\star}(P) - P$, while in the context of section 4, $\omega^{\sharp} = \bar{\omega}$. Define $\Pi(\tau)$ as the set of prices which induce a non-negative profit to investors when a security of expected value τ is purchased. That is

$$\Pi(\tau) \equiv \left\{P \geq 0 : \frac{\tau}{R}\mathbb{P}\left\{\omega \geq \omega^{\sharp}(P)\right\} \geq P\right\}.$$

Part 1. Suppose that there exists an equilibrium of the fund-raising game, $\{\{\sigma_\theta\}_\theta, \mu, P, A\}$, and any non-trivial security $\hat{s} \in S$ with $\sigma_\theta(\hat{s}) > 0$, for all $\theta \in \Theta$. Suppose by contradiction that $\hat{s}$ is not a debt contract. Define the debt security $s_D \equiv \min\{y, D\}$ where D is such that $\mathbb{E}_H(s_D - \hat{s}) = 0$. Note that $s_D - \hat{s}$ satisfies *single crossing from above* (SCFA) and hence lemma 3 implies that $\mathbb{E}_L(s_D - \hat{s}) > 0 = \mathbb{E}_H(s_D - \hat{s})$. Thus,

$$\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D) > \mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s}). \tag{22}$$

Next, let $P^{\sharp}(\tau) \equiv \sup \Pi(\tau)$ and define $\Delta V_\theta(P)$ as the difference in payoffs for bank θ obtained by switching to security s_D, and sell it at price P, instead of issuing security $\hat{s}$ and receiving the market price $P(\hat{s}) \equiv P^{\sharp}\left(\mathbb{E}_{\hat{\mu}}(\hat{s})\right)$, with $\hat{\mu} = \frac{\sigma_H(\hat{s})}{\sigma_L(\hat{s}) + \sigma_H(\hat{s})} \in (0,1)$. That is,

$$\begin{aligned}
\Delta V_\theta\left(\tilde{P}\right) &= V\left(\tilde{P}, s_D, \theta\right) - V\left(P(\hat{s}), \hat{s}, \theta\right) \\
&= \left(\tilde{P}R + \mathbb{E}_\theta(y - s_D)\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(\tilde{P}\right)\right\} - \left(P(\hat{s})R + \mathbb{E}_\theta(y - \hat{s})\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(P(\hat{s})\right)\right\},
\end{aligned}$$

Inequality (22) together with the fact that $y - s_D$ and $y - \hat{s}$ are monotone then imply that

$$\begin{aligned}
\Delta V_H\left(\tilde{P}\right) - \Delta V_L\left(\tilde{P}\right) &= \left(\mathbb{E}_H(y - s_D) - \mathbb{E}_L(y - s_D)\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(\tilde{P}\right)\right\} \\
&\quad - \left(\mathbb{E}_H(y - \hat{s}) - \mathbb{E}_L(y - \hat{s})\right)\mathbb{P}\left\{\omega \geq \omega^{\sharp}\left(P(\hat{s})\right)\right\} \\
&> 0, \ \forall \tilde{P} \geq P(\hat{s}).
\end{aligned} \tag{23}$$

Next, the fact that F^{ω} is non-decreasing and right-continuous implies that $\Pi(\tau)$ is compact and strictly increasing for any $\tau \geq 0$.[41] Thus,

$$P(\hat{s}) = \max\ \Pi\left(\mathbb{E}_{\hat{\mu}}(\hat{s})\right) < \max\ \Pi\left(\mathbb{E}_H(\hat{s})\right) = \max\ BR(s_D),$$

where the first equality follows from the compacity of Π and the definition of $P(\hat{s})$. The inequality arises from the strict monotonicity of Π and the MLRP order of θ. The second equality is by definition of $BR(\cdot)$ and the construction of s_D.

Finally, note that $\mathbb{E}_L(y - \hat{s}) > \mathbb{E}_L(y - s_D)$ implies that $\Delta V_L(P(\hat{s})) < 0$. By construction, we also have that $\Delta V_H(P(\hat{s})) = 0$, which together with the fact that $P(\hat{s}) < P^{\sharp}(\mathbb{E}_H(s_D))$ and the result in (23) imply that

$$\mathscr{D}(\theta_L|s_D) \cup \mathscr{D}^0(\theta_L|s_D) \subset \mathscr{D}(\theta_H|s_D).$$

[41] We say that a correspondence $\varphi : \mathbb{R}_+ \to 2^{\mathbb{R}_+}$ is strictly increasing if, for any τ, $\tau' \in \mathbb{R}_+$, with $\tau < \tau'$, $\varphi(\tau) \subsetneq \varphi(\tau')$.

As a consequence, market beliefs consistent with D1 must necessarily assign $\mu(\theta_H|s_D) = 1$. This implies that the market prices s_D at $P^\sharp(\mathbb{E}_H(s_D)) > P(\hat{s})$ and therefore both types have incentives to deviate and issue s_D instead, which contradicts the assumption that $\{\{\sigma_\theta\}_\theta, \mu, P, A\}$ is an equilibrium.

Next, we prove that the price of any debt security placed by both types in equilibrium, cannot be larger than K. Suppose that there exists an equilibrium $\{\{\sigma_\theta\}_\theta, \mu, P, A\}$ with $\sigma_\theta(s_d \equiv \min\{y, d\}) > 0$ for all $\theta \in \Theta$. Let $\mu_d \equiv \frac{\sigma_H(s_d)}{\sigma_L(s_d)+\sigma_H(s_d)}$ and define $P(s_d) \equiv P^\sharp\left(\mathbb{E}_{\mu_d}(s_d)\right)$. Assume by contradiction that $P(s_d) > K$. Consider the alternative debt contract $s_\varepsilon = \min\{y, d-\varepsilon\}$ with $\varepsilon > 0$ small so that (a) $\mathbb{E}_H(s_\varepsilon) > \mathbb{E}_{\mu_d}(s_d)$, and (b) $\frac{1}{R}\mathbb{E}_{\mu_d}(s_D - s_\varepsilon) < P(s_D) - K$. We show that type θ_H can profitably deviate to s_ε. Observe that $s_d - s_\varepsilon$ is an increasing function. FOSD then implies that $\mathbb{E}_H(s_d - s_\varepsilon) > \mathbb{E}_L(s_d - s_\varepsilon)$, or equivalently,

$$\mathbb{E}_H(y - s_\varepsilon) - \mathbb{E}_L(y - s_\varepsilon) > \mathbb{E}_H(y - s_d) - \mathbb{E}_L(y - s_d). \tag{24}$$

Similar to the analysis above, let $\Delta V_\theta\left(\tilde{P}\right) \equiv V\left(\tilde{P}, s_\varepsilon, \theta\right) - V\left(P(s_d), s_d, \theta\right)$. Inequality 24 implies that:

$$\begin{aligned}
\Delta V_H\left(\tilde{P}\right) - \Delta V_L\left(\tilde{P}\right) &= \left(\mathbb{E}_H(y - s_\varepsilon) - \mathbb{E}_L(y - s_\varepsilon)\right) \times \mathbb{P}\left\{\omega \geq \omega^\sharp\left(\tilde{P}\right)\right\} \\
&\quad - \left(\mathbb{E}_H(y - s_d) - \mathbb{E}_L(y - s_d)\right) \times \underbrace{\mathbb{P}\left\{\omega \geq \omega^\sharp\left(P(s_d)\right)\right\}}_{=1} \\
&> 0, \ \forall \tilde{P} \geq K. \qquad (25)
\end{aligned}$$

Next, that $\mathbb{E}_H(s_\varepsilon) > \mathbb{E}_{\mu_d}(s_d)$ implies

$$\Pi\left(\mathbb{E}_{\mu_d}(s_d)\right) \subsetneq \Pi\left(\mathbb{E}_H(s_\varepsilon)\right) = BR(s_\varepsilon),$$

and hence $P(s_d)$ is contained in $BR(s_\varepsilon)$. Moreover, given that s_ε is smaller than s_d, we must have that $\Delta V_\theta(P(s_d)) > 0$ for both $\theta \in \Theta$. Finally, let

$$\tilde{P}_\varepsilon \equiv P(s_D) - \frac{\mathbb{E}_{\mu_d}(s_d - s_\varepsilon)}{R}.$$

Condition (b) above implies that $\tilde{P}_\varepsilon \in [K, P(s_d))$. Moreover, by construction, $\Delta V_H\left(\tilde{P}_\varepsilon\right) > 0 > V_L\left(\tilde{P}_\varepsilon\right)$. Thus, $\mathscr{D}(\theta_L|s_\varepsilon) \cup \mathscr{D}^0(\theta_L|s_\varepsilon) \subset \mathscr{D}(\theta_H|s_\varepsilon)$, and consequently market beliefs consistent with D1 must assign $\mu(\theta_H|s_\varepsilon) = 1$, which together with condition (a) implies that both types can profitably deviate to s_ε. This is a contradiction and therefore $P(s_d) \leq K$.

Part 2. In any equilibrium in which there exists a security s_H issued only by type θ_H (i.e., $\sigma_H(s_H) > 0 = \sigma_L(s_H)$), we must have that $P(s_H) \leq \mathbb{E}_L(y) < KR$. To see this, assume by contradiction that $P(s_H) > \frac{1}{R}\mathbb{E}_L(y)$. Denote by s_L any security issued with positive probability by type L. Observe that the separating nature of the equilibrium requires that:

$$P(s_L) = \max \Pi\left(\mathbb{E}_L(s_L)\right) < \frac{\mathbb{E}_L(s_L)}{R},$$

as $\mathbb{E}_L(s_L) < KR$, from assumption 2. Hence,

$$P(s_H)R > \mathbb{E}_L(y) > P(s_L)R + \mathbb{E}_L(y - s_L) \tag{26}$$

As a result, type θ_L has incentives to mimic type θ_H. To see this, note that

$$\begin{aligned} V(P(s_H), s_H, \theta_L) - V(P(s_L), s_L, \theta_L) &= (P(s_H)R + \mathbb{E}_L(y - s_H)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_H))\right\} \\ &\quad - (P(s_L)R + \mathbb{E}_L(y - s_L)) \times \mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_L))\right\} \\ &> (P(s_H)R + \mathbb{E}_L(y - s_H) - (P(s_L)R + \mathbb{E}_L(y - s_L))) \times \\ &\quad \times \mathbb{P}\left\{\omega \geq \omega^\sharp(P(s_L))\right\} \\ &> 0, \end{aligned}$$

where the first inequality arises from the fact that $P(s_H) > P(s_L)$ and the monotonicity of $\omega^\sharp$. The second inequality, in turn, is a consequence of equation (26). This is a contradiction and hence $P(s_H) \leq \frac{1}{R}\mathbb{E}_L(y|m^y) < K$. $\square$

Part 3. To see part (a), we prove that there exists an equilibrium where $s_\theta = \mathbf{0}$ for all $\theta \in \Theta$. Consider the deviation to any security $\hat{s}$ satisfying $\frac{\mathbb{E}_\mu(\hat{s})}{R} \geq K$ for some $\mu \in (0,1]$, which is the only relevant case since, by assumption 2, $\frac{1}{R}\mathbb{E}_L(y) < K$ which implies that the market would never fund a security only issued by θ_L. Observe that $BR(\hat{s}) = \left[K, \frac{1}{R}\mathbb{E}_H(\hat{s})\right]$, since any $P < K$ induces default with certainty when assumption 3 holds, and any $P \geq K$ dissuades all ST creditors from running, and hence prevents default w.p. 1. Next, note that, for any price $P \in BR(\hat{s})$, type θ_L can profitably deviate and place security $\hat{s}$. In fact,

$$V(P, \hat{s}, \theta_L) = (PR + \mathbb{E}_L(y - s)) \times \underbrace{\mathbb{P}\left\{\omega \geq \omega^\sharp(P)\right\}}_{=1} > 0.$$

Thus, $\mathscr{D}(\theta_L; \hat{s}) = BR(\hat{s})$, implying that market beliefs that assign $\mu(\theta_L, s) = 1$ for any such $s \in S$ are consistent with D1. This amounts to say that any feasible deviation is always attributed to type L, and therefore no type gets funded. If $\frac{1}{R}\mathbb{E}(y) < K$ instead, then part 1 and 2 above, together with the observation that, when assumption (3) holds, $\Pi(\mathbb{E}(y)) = \{0\}$ imply that $s_\theta = \mathbf{0}$ for all $\theta \in \Theta$ is the unique equilibrium.

Finally, to see part (b), assume that $\frac{1}{R}\mathbb{E}(y) \geq K$. The result follows directly from Theorem 4 in Nachman and Noe [1994].

Appendix B: Comprehensive Assessment

Proof of Proposition 4.

First, we prove that under the laissez-faire policy there exists an equilibrium of the fund-raising stage where both bank types pool over the debt contact $s_D \equiv \min\{y, D\}$, with D chosen so that $\frac{\mathbb{E}(s_D)}{R} = K$. At this equilibrium, the market keep its prior belief about θ, μ_0, when observing security s_D and thus offers a payoff equal to K for s_D.

To see that this is, in fact, an equilibrium fix an arbitrary security $\tilde{s}(\cdot)$ and define $\Delta V_\theta (P|\tilde{s})$ as the differential payoff obtained by type θ by switching from security s_D to $\tilde{s}$ and receiving a price P for the latter. That is,

$$\Delta V_\theta (P|\tilde{s}) \equiv (PR + \mathbb{E}_\theta (y - \tilde{s}))\, \phi (P) - (KR + \mathbb{E}_\theta (y - s_D)),$$

where $\phi(P) = 1 - F^{\omega}(1-P)$. We show that beliefs that assign probability 1 to the type being $\theta = L$ are consistent with D1. Clearly, under such beliefs no bank type deviates. The next claim reduces the set of deviations that need to be considered.

Claim 1. Fix an arbitrary security $s \in \mathscr{S}$, let $s_d \equiv \min\{y, d\}$ be such that $\mathbb{E}_H (s - s_d) = 0$. Then, $\Delta V_L (P|s_d) < \Delta V_L (P|s)$.

Proof. For any $s_0 \in \mathscr{S}$,

$$\Delta V_H (P|s_0) - \Delta V_L (P|s_0) = (\mathbb{E}_H (y - s_0) - \mathbb{E}_L (y - s_0))\, \phi (P) - (\mathbb{E}_H (y - s_D) - \mathbb{E}_L (y - s_D)). \tag{27}$$

By virtue of Lemma 3 and the definition of s_d, we have that $\mathbb{E}_L (s - s_d) < 0$. By evaluating (27) at $s_0 = s_d$, we conclude that $\Delta V_L (P|s_d) < \Delta V_L (P|s)$. □

Claim 1 implies that the only deviations that need to be considered are those to debt contracts. Indeed, for any security $s \in \mathscr{S}$, the *equivalent debt* security s_d reduces the set of prices that would induce type L to deviate while keeping the set of prices for type H unchanged since, by definition, $\Delta V_H (P|s_d) = \Delta V_H (P|s)$. Under the D1 criterion, off-path beliefs at any security s, must assign all weight to the bank type with the largest set of prices consistent with a profitable deviation.[42] Recall that we are trying to show that, for any off-path belief, type θ_L has the greatest incentives to deviate. Claim 1 then shows that it is enough to restrict attention to debt contracts as these are the securities that minimize the set of price consistent with a profitable deviation for type θ_L.

Consider first deviations to debt contracts $\tilde{s} = \min\left\{y, \tilde{d}\right\}$ with $\mathbb{E}_H (\tilde{s} - s_D) > 0$. In this case, for any $P \geq K$,

$$\Delta V_\theta (P|\tilde{s}) = (P - K) R - \mathbb{E}_\theta (\tilde{s} - s_D),\ \theta \in \{L, H\}.$$

The fact that $\tilde{s}$ is a debt contract implies that $\tilde{s} - s_D$ is weakly increasing and therefore FOSD (implied by MLRP) means that $\mathbb{E}_H (\tilde{s} - s_D) > \mathbb{E}_L (\tilde{s} - s_D)$. As a result, there exists a price $\hat{P} > K$ for which

$$\Delta V_L \left(\hat{P}|\tilde{s}\right) > 0 > \Delta V_H \left(\hat{P}|\tilde{s}\right).$$

This implies that beliefs satisfying $\mu (\tilde{s}) = 1\{\theta = \theta_L\}$ are consistent with D1.

Now consider the case where $\tilde{s}$ is a debt contract with $\mathbb{E}_H (\tilde{s} - s_D) \leq 0$. For any $P \geq K$, we have that

$$\Delta V_\theta (P|\tilde{s}) = (P - K) R + \mathbb{E}_\theta (s_D - \tilde{s}),\ \theta \in \{\theta_L, \theta_H\}.$$

[42] To be precise, the set of relevant prices are those in $BR(s) = \left\{P \geq 0 : \frac{\mathbb{E}_H(s)}{R} \phi (P) \geq P\right\}$ (see the equilibrium definition in the Appendix). This set remains unchanged when considering the equivalent debt security s_d, by construction.

That $\tilde{s}$ is a debt contract implies that $s_D - \tilde{s}$ is positive and weakly increasing. Thus, $\Delta V_\theta (K|\tilde{s}) \geq 0$ for all θ. Next, for any $P < K$,

$$\begin{aligned}
\Delta V_\theta (P|\tilde{s}) &= (\mathbb{E}_H (y-\tilde{s}) - \mathbb{E}_L (y-\tilde{s}))\, \phi(P) - (\mathbb{E}_H (y-s_D) - \mathbb{E}_L (y-s_D)) \\
&< (\mathbb{E}_H (y-\tilde{s}) - \mathbb{E}_L (y-\tilde{s}))\, \bar{\phi} - (\mathbb{E}_H (y-s_D) - \mathbb{E}_L (y-s_D)) \\
&= \left(\frac{\mathbb{E}_H (y-\tilde{s}) - \mathbb{E}_L (y-\tilde{s})}{\mathbb{E}_H (y) - \mathbb{E}_L (y)} - 1 \right) (\mathbb{E}_H (y-s_D) - \mathbb{E}_L (y-s_D)) \\
&< 0,
\end{aligned}$$

where the first inequality follows from assumption (c) in Condition 1. The second equality is by definition of $\bar{\phi}$. The last inequality follows from noting that $\mathbb{E}_H (\tilde{s}) - \mathbb{E}_L (\tilde{s}) > 0$ since $\tilde{s}$ is monotone and signals are ordered according to MLRP. As a result, $\mathscr{D}(\theta_L|\tilde{s}) \supseteq \mathscr{D}(\theta_H|\tilde{s})$ and, therefore, beliefs satisfying $\mu(\tilde{s}) = 1\{\theta = \theta_L\}$ are consistent with D1. This completes the proof that s_D is an equilibrium of the fund-raising stage.

Next, we prove that, under the sequentially optimal stress test Γ^ω, having both bank types pooling over the security s_D cannot be an equilibrium outcome. To show this, we prove that there exists a profitable deviation. In fact, consider the security $s_\varepsilon = \min\{y, D-\varepsilon\}$ with $\varepsilon > 0$ small. Similarly to the analysis above, define $\Delta V_\theta^{\Gamma^\omega} (P|\tilde{s})$ as the differential payoff obtained by type θ by switching from security s_D to s_ε and receiving a price P, when the policy maker runs the sequentially optimal stress test Γ^ω. That is,

$$\Delta V_\theta^{\Gamma^\omega} (P|s_\varepsilon) \equiv (PR + \mathbb{E}_\theta (y-\tilde{s}))\, \hat{\phi}(P) - (KR + \mathbb{E}_\theta (y-s_D)),$$

where $\hat{\phi}(P) = \mathbb{P}\{\omega \geq \bar{\omega}(P)\} = 1 - F^\omega(\bar{\omega}(P))$. For any $P \geq K$, we have that

$$\Delta V_\theta^{\Gamma^\omega} (P|s_\varepsilon) = (P-K)R + \mathbb{E}_\theta (s_D - s_\varepsilon),\ \theta \in \{\theta_L, \theta_H\}.$$

Thus, $\Delta V_\theta^{\Gamma^\omega} (P|s_\varepsilon) > 0$ for any $P \geq K$, and any θ. Next, note that

$$\Delta V_H^{\Gamma^\omega} (K|s_\varepsilon) - \Delta V_L^{\Gamma^\omega} (K|s_\varepsilon) = \mathbb{E}_H (s_D - s_\varepsilon) - \mathbb{E}_L (s_D - s_\varepsilon) > 0,$$

as $s_D - s_\varepsilon$ is non-decreasing. We prove that, under the assumptions in Condition 1, there exists a price $P_\varepsilon < K$ satisfying that $\Delta V_H^{\Gamma^\omega} (P_\varepsilon|s_\varepsilon) > 0 > \Delta V_L^{\Gamma^\omega} (P_\varepsilon|s_\varepsilon)$. To see this, let $\tilde{P}_\varepsilon < K$ be defined as the unique solution to $\Delta V_H^{\Gamma^\omega} (P|s_\varepsilon) = 0$. Note that the definition of $\bar{\omega}(\cdot)$ implies that $\lim_{P \to K^-} \hat{\phi}(P) = 1$ and, therefore, $\lim_{\varepsilon \to 0^+} \tilde{P}_\varepsilon = K$. Next, we rewrite $\Delta V_\theta^{\Gamma^\omega} (\tilde{P}_\varepsilon|s_\varepsilon)$ using the first-order Taylor expansion as

$$\Delta V_\theta^{\Gamma^\omega} (\tilde{P}_\varepsilon|s_\varepsilon) = \Delta V_\theta^{\Gamma^\omega} (K|s_\varepsilon) + \partial_P^- \Delta V_\theta^{\Gamma^\omega} (K|s_\varepsilon) (\tilde{P}_\varepsilon - K) + o(\tilde{P}_\varepsilon - K),$$

where $\partial_P^- \Delta V_\theta^{\Gamma^\omega} (K|s_\varepsilon) \equiv \lim_{P \to K^-} \lim_{\xi \to 0} \frac{\Delta V_\theta^{\Gamma^\omega} (P|s_\varepsilon) - \Delta V_\theta^{\Gamma^\omega} (P-\xi|s_\varepsilon)}{\xi}$ represents the *left* derivative of $\Delta V_\theta^{\Gamma^\omega} (P|s_\varepsilon)$ at K. Thus, we can express

$$\Delta V_L^{\Gamma^\omega} (\tilde{P}_\varepsilon|s_\varepsilon) = \Delta V_L^{\Gamma^\omega} (K|s_\varepsilon) - \partial_P^- \Delta V_L^{\Gamma^\omega} (K|s_\varepsilon) \underbrace{\frac{\Delta V_H^{\Gamma^\omega} (K|s_\varepsilon) + o(\tilde{P}_\varepsilon - K)}{\partial_P^- \Delta V_H^{\Gamma^\omega} (K|s_\varepsilon)}}_{= K - \tilde{P}_\varepsilon} + o(\tilde{P}_\varepsilon - K).$$

Next, assumption (b) in Condition 1, together with the fact $\lim_{P\to K^-} \bar{\omega}(P)=0$, imply that

$$\lim_{P\to K^-} \hat{\phi}'(P) = \lim_{P\to K^-} f^{\omega}(\bar{\omega}(P))\,\bar{\omega}'(P) = 0,$$

which in turn implies that

$$\frac{\partial_P^- \Delta V_L^{\Gamma^\omega}(K|s_\varepsilon)}{\partial_P^- \Delta V_H^{\Gamma^\omega}(K|s_\varepsilon)} = \lim_{P\to K^-} \frac{R\phi(P) + (KR + \mathbb{E}_L(y - s_\varepsilon))\,\hat{\phi}'(P)}{R\phi(P) + (KR + \mathbb{E}_H(y - s_\varepsilon))\,\hat{\phi}'(P)} = 1 > \frac{\Delta V_L^{\Gamma^\omega}(K|s_\varepsilon)}{\Delta V_H^{\Gamma^\omega}(K|s_\varepsilon)}.$$

Thus, by choosing $\varepsilon = \tilde{\varepsilon}$ sufficiently close to 0 we obtain that $\Delta V_L^{\Gamma^\omega}\left(\tilde{P}_{\tilde{\varepsilon}}|s_{\tilde{\varepsilon}}\right) < 0 = \Delta V_H^{\Gamma^\omega}\left(\tilde{P}_{\tilde{\varepsilon}}|s_{\tilde{\varepsilon}}\right)$.

Finally, consider $\varepsilon = \tilde{\tilde{\varepsilon}}$ sufficiently small so that $\frac{\mathbb{E}_H\left(s_{\tilde{\tilde{\varepsilon}}}\right)}{R} > K$. Note that assumption (a) in Condition 1 then implies that $BR\left(s_{\tilde{\tilde{\varepsilon}}}\right) = \left[0, \frac{\mathbb{E}_H\left(s_{\tilde{\tilde{\varepsilon}}}\right)}{R}\right]$. By picking $\varepsilon = \min\left\{\tilde{\varepsilon}, \tilde{\tilde{\varepsilon}}\right\}$ we then have that $\mathscr{D}(\theta_H|s_\varepsilon) \supsetneq \mathscr{D}(\theta_L|\varepsilon)$. As a consequence, beliefs consistent with D1 necessarily assign $\mu(s_\varepsilon) = 1\{\theta = \theta_H\}$ and therefore such a deviation receives a price $P = \frac{\mathbb{E}_H\left(s_{\tilde{\tilde{\varepsilon}}}\right)}{R} > K$ which leads both types to choose s_ε over s_D. This proves that s_D cannot be an equilibrium. The rest of the proof follows from results (1) and (2) in Proposition 1 which show that (i) any pooling contract always delivers a price weakly smaller than K, and that (ii) in any separating equilibrium, type H always raises less than K.[43] □

Proof of Lemma *1*.

The proof is standard and hence relegated to the Online Supplement. □

Proof of Theorem 1.

Define Z^{Γ^y} as the auxiliary function that allows to take mean-preserving contractions of F^y. That is, for any mean-preserving contraction G^{Γ^y}, Z^{Γ^y} is defined so that $G^{\Gamma^y} = F^y + Z^{\Gamma^y}$. Any such $Z^{\Gamma^y} : \mathbb{R}_+ \to \mathbb{R}$ must respect the following conditions: $F^y + Z^{\Gamma^y}$ *(i) belongs to* $[0,1]$*, (ii) is non-decreasing, (iii) is right-continuous, and satisfies*

$$(iv)\ \int_0^{\tilde{y}} Z^{\Gamma^y}(y)dy \le 0\ \ (\forall \tilde{y} \ge 0),\ \int_0^{\infty} Z^{\Gamma^y}(y)dy = 0,\ Z^{\Gamma^y}(\infty) = 0.$$

I define the set of all such auxiliary functions as

$$\mathscr{Z}(F^y) \equiv \{Z : \mathbb{R}_+ \to \mathbb{R} : (i) - (iv)\}.$$

We can thus rewrite the policy maker's period 1 problem in terms of Z^{Γ^y} as

$$\max_{Z^{\Gamma^y}} \quad \int_0^{\infty} \phi(\tau) Z^{\Gamma^y}(d\tau)$$
$$\text{s.t:} \quad Z^{\Gamma^y} \in \mathscr{Z}(F^y).$$

We first prove that ϕ satisfies the following properties: ϕ is (a) continuous, (b) non-decreasing, and (c) satisfies $\phi(0) = 0$, and $\phi(y) = 1$ for all $y \ge KR$. That ϕ is continuous comes from the fact that (i) $\bar{\omega}(\cdot)$ is continuously

[43] Note that the proof of Proposition 1 is general and works not only for the laissez faire policy but also under the sequentially rational stress test Γ^ω.

differentiable, (ii) $F^{\omega}(\cdot)$ admits a density and has at most one mass point at $\omega = 1$, and (iii) $\bar{P}$ is continuous. To see this last point, we apply the *maximum theorem* to the definition of $\bar{P}$:

$$\begin{aligned} \bar{P}(\tau) &= \max P \\ \text{s.t:} \quad & P \in \Gamma(\tau) \equiv \left\{ P \geq 0 : \frac{\tau}{R} \mathbb{P}\{\omega \geq \bar{\omega}(P)\} \geq P \right\} \end{aligned}$$

where $\Gamma(\cdot)$ is a compact valued and continuous correspondence. To see (b), we note that $\bar{P}$ is non-decreasing and that $\bar{\omega}$ is non-increasing which implies the result. Finally, (c) is by definition of functions $\bar{P}$ and $\bar{\omega}$. Conditions (a)-(c) guarantee that ϕ satisfies the *regularity* assumption in Dworczak and Martini [2019]. That the optimal disclosure policy consists of monotone partitions thus follows proposition 2 in their paper. Next, to prove that the highest partition includes KR, we observe that using integration by parts, we can rewrite the policy maker's objective function as:

$$\int_0^{\infty} \phi(y) Z(dy) = -\left(\left(1 - \lim_{y \to KR^-} \phi(y) \right) Z(KR) + \int_0^{\infty} \phi'(y) Z(y) dy \right).$$

As a result, the designer's problem is equivalent to:

$$\begin{aligned} \min_{Z} \quad & \left(1 - \lim_{y \to KR^-} \phi(y) \right) Z(KR) + \int_0^{\infty} \phi'(y) Z(y)\, dy \\ \text{s.t:} \quad & Z \in \mathscr{Z}(F^y). \end{aligned} \tag{28}$$

Conditions (b) and (c) then imply that it is optimal to choose $Z(y) = 1 - F^y(y)$ for all $y \geq KR$. This implies that KR will be included in the highest partition cell.

Next, we show that when ϕ is convex, the optimal policy takes a simple form: there exists a threshold $y^+ \geq 0$ and Γ_*^y fully disclose y for any $y \in [0, y^+)$ and pools under the same score all $y \geq y^+$.

Lemma 4. *ϕ convex implies that the optimal choice of Z is given by:*

$$Z_*(y) = \begin{cases} 0 & y < y^+ \\ F^y(y^+) - F^y(y) & y \in [y^+, KR) \\ 1 - F^y(y) & y \geq KR, \end{cases} \tag{29}$$

where

$$y^+ = \inf\left\{ y \geq 0 : \int_y^{KR} (F^y(y) - F^y(\tau))\, d\tau + \int_{KR}^{\infty} (1 - F^y(\tau))\, d\tau \geq 0 \right\}.$$

Proof. That ϕ is convex implies that ϕ is differentiable for *almost all* $y \geq 0$, with ϕ' non-decreasing over $[0, KR)$. Moreover, the definition of K implies that $\phi' = 0$ for all $y \geq KR$.

Next, take any function $X \in \mathscr{Z}(F^y)$. We prove that either (a) $X = Z$ for (almost) all $y \geq 0$, or else (b) X is dominated. Assume that $X \neq Z$. The constraint that $F^y(y) + X(y) \leq 1$ for all $y \geq 0$, together with the requirement that $\int_0^{\infty} X(y) dy = 0$, impose a lower bound on $\int_0^{KR} X(y) dy$. In fact, we must have that $\int_{KR}^{\infty} X(y)dy \leq \int_{KR}^{\infty}(1-F^y(y))\,dy$, and hence $\int_0^{KR} X(y)dy \geq -\int_{KR}^{\infty}(1-F^y(y))\,dy$. We prove that the last inequality must be binding for any non-dominated policy X.

Step 1. *If $\int_0^{KR} X(y)dy > -\int_{KR}^{\infty}(1-F^y(y))\,dy$, then X is necessarily dominated.*

If $\int_0^{KR} X(y)dy > -\int_{KR}^{\infty}(1-F^y(y))\,dy$ then there exist alternative feasible policies that allocate more (negative) mass to the interval $[0,KR)$, which improves the objective function since ϕ' is positive (recall the policy maker's optimization problem in (28)). One of such improvements is

$$X^{\delta}(y) \equiv \begin{cases} (1-\delta)X(y)+\delta(-F^y(y)) & y<KR \\ X(y) & KR \leq y < KR+\xi(\delta), \\ \varepsilon(\delta)X(y)+(1-\varepsilon(\delta))(1-F^y(y)) & y \geq KR+\xi(\delta) \end{cases}$$

where, for each $\delta>0$, $\varepsilon(\delta), \xi(\delta)>0$ are chosen so that[44]

$$\int_0^{\infty} X^{\delta}(y)\,dy = 0.$$

Note that $X^{\delta} \in \mathscr{X}(F^y)$ for δ small. In fact, $F^y + X^{\delta} = (1-\delta)(X+F^y)$ *(i) belongs to* $[0,1-\delta] \subset [0,1]$*;* (ii) is right-continuous (inherited from X); (iii) non-decreasing, which follows from $X \in \mathscr{X}(F^y)$ and the fact that

$$\lim_{y\to KR^-}(1-\delta)(X(y)+F^y(y)) < (1-\delta)(X(KR)+F^y(KR));$$

and (iv) $X^{\delta}(y)$ satisfies

$$\int_0^{\tilde{y}} X^{\delta}(y)\,dy \leq \int_0^{\tilde{y}} X(y)\,dy \leq 0, \text{ for all } y,$$

and

$$X^{\delta}(\infty) = 0 = \int_0^{\infty} X^{\delta}(y)\,dy,$$

by construction.

Clearly,

$$\int_0^{\infty} \phi'(y)X^{\delta}(y)\,dy < \int_0^{\infty} \phi'(y)X(y)\,dy,$$

which, together with the fact that $X^{\delta}(KR) = X(KR)$, imply that X^{δ} dominates X. Which proves Step 1.

Assume then that $\int_0^{KR} X(y)dy = -\int_{KR}^{\infty}(1-F^y(y))\,dy$. For any $Z \in \mathscr{X}(F^y)$, let

$$y_0(Z) \equiv \sup\{y \in [0,KR) : Z(y) \geq 0\}.$$

For convenience, I omit hereafter the dependence of y_0 on Z. Next, fix any $\varepsilon, \Delta > 0$ small, and construct the alternative policy

[44] $\varepsilon(\delta)$ and $\xi(\delta)$ are well defined for $\delta>0$ small as $\int_0^{KR} X(y)dy > -\int_{KR}^{\infty}(1-F^y(y))\,dy$, and therefore $\left|\int_0^{KR} X(y)dy\right| < \int_{KR}^{\infty}(1-F^y(y))\,dy$, which implies that $\int_{KR}^{\infty} X(y)dy < \int_{KR}^{\infty}(1-F^y(y))\,dy$.

$$X^{\varepsilon,\Delta}(y) \equiv \begin{cases} 0 & y < y_0+\Delta+\varepsilon \\ F^y(y_0+\Delta+\varepsilon) - F^y(y) & y \in [y_0+\Delta+\varepsilon, \hat{y}(\varepsilon)) \\ F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon)) & y \in \left[\hat{y}(\varepsilon), \hat{\hat{y}}(\varepsilon)\right) \\ X(y) & y \geq \hat{\hat{y}}(\varepsilon), \end{cases}$$

where $\hat{y}(\varepsilon), \hat{\hat{y}}(\varepsilon)$ are implicitly defined by

$$F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon)) = X\left(\hat{\hat{y}}(\varepsilon)\right) \tag{30}$$

$$\int_0^\infty X^{\varepsilon,\Delta}(y)\,dy = 0. \tag{31}$$

Assume that ε and Δ are small enough so that $\hat{\hat{y}}(\varepsilon) < KR$.[45]

Step 2. *For ε,Δ small, $X^{\varepsilon,\Delta}$ dominates X.*

To see this, consider the policy maker's payoff under policy $X^{\varepsilon,\Delta}$,

$$\bar{U}^{\varepsilon,\Delta} \equiv \left(1 - \lim_{y\to KR^-}\phi(y)\right) X^{\varepsilon,\Delta}(KR) + \int_0^\infty \phi'(y) X^{\varepsilon,\Delta}(y)\,dy.$$

Next, differentiating (31) with respect to ε we obtain

$$\begin{aligned} 0 &= \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\hat{y}'(\varepsilon) + f^y(y_0+\Delta+\varepsilon)\left(\hat{y}(\varepsilon) - (y_0+\Delta+\varepsilon)\right) + \\ &\quad + \left(f^y(y_0+\Delta+\varepsilon) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\right)\left(\hat{\hat{y}}(\varepsilon) - \hat{y}(\varepsilon)\right) + \\ &\quad + \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\left(\hat{\hat{y}}'(\varepsilon) - \hat{y}'(\varepsilon)\right) - X\left(\hat{\hat{y}}(\varepsilon)\right)\hat{\hat{y}}'(\varepsilon) \\ &= f^y(y_0+\Delta+\varepsilon)\left(\hat{\hat{y}}(\varepsilon) - (y_0+\Delta+\varepsilon)\right) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\left(\hat{\hat{y}}(\varepsilon) - \hat{y}(\varepsilon)\right), \end{aligned} \tag{32}$$

where the second equality obtains from (30). Therefore,

$$\begin{aligned} \frac{\partial}{\partial\varepsilon}\bar{U}^{\varepsilon,\Delta} &= \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\phi'(\hat{y}(\varepsilon))\hat{y}'(\varepsilon) + f^y(y_0+\Delta+\varepsilon)\int_{y_0+\Delta+\varepsilon}^{\hat{y}(\varepsilon)}\phi'(y)\,dy + \\ &\quad + \left(f^y(y_0+\Delta+\varepsilon) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\right)\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(\hat{y}(\varepsilon))\right) + \\ &\quad + \left(F^y(y_0+\Delta+\varepsilon) - F^y(\hat{y}(\varepsilon))\right)\left(\phi'\left(\hat{\hat{y}}(\varepsilon)\right)\hat{\hat{y}}'(\varepsilon) - \phi'(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\right) + \\ &\quad - X\left(\hat{\hat{y}}(\varepsilon)\right)\phi'\left(\hat{\hat{y}}(\varepsilon)\right)\hat{\hat{y}}'(\varepsilon) \\ &= f^y(y_0+\Delta+\varepsilon)\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(y_0+\Delta+\varepsilon)\right) - f^y(\hat{y}(\varepsilon))\hat{y}'(\varepsilon)\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(\hat{y}(\varepsilon))\right) \\ &= f^y(y_0+\Delta+\varepsilon)\left\{\frac{\left(\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(y_0+\Delta+\varepsilon)\right)}{\left(\hat{\hat{y}}(\varepsilon) - (y_0+\Delta+\varepsilon)\right)} - \frac{\phi\left(\hat{\hat{y}}(\varepsilon)\right) - \phi(\hat{y}(\varepsilon))}{\left(\hat{\hat{y}}(\varepsilon) - \hat{y}(\varepsilon)\right)}\right\} \times \\ &\quad \times \left(\hat{\hat{y}}(\varepsilon) - (y_0+\Delta+\varepsilon)\right) \\ &< 0. \end{aligned}$$

[45] Obviously, $\hat{y}(\varepsilon)$ and $\hat{\hat{y}}(\varepsilon)$ are also functions of Δ. I omit this dependence as Δ is kept fixed throughout the proof. To see that both functions are well defined, note that the definition of y^+, together with Step1 in the proof, imply that $y_0 < y^+$. The requirement that $X + F^y$ is increasing then implies that there must exist a point within (y_0, KR) where $X^{\varepsilon,\Delta}$ crosses from below X. As a result, $\hat{y}(\varepsilon)$ and $\hat{\hat{y}}(\varepsilon)$ are well defined provided that ε and Δ are small enough.

The first equality comes from the definition of $\bar{U}^{\varepsilon,\Delta}$ and $X^{\varepsilon,\Delta}$. The second equality obtains from using (30). The third equality arises from plugging in the identity found in (32). Finally, the inequality is a consequence of the convexity of ϕ. As a result, any policy X different from Z_* is dominated. This proves the lemma.

That $Z_*(y) = 0$ for all $y \leq y^+$ implies that $G(y) = F^y(y)$ for any such y, or equivalently, that G coincides with the full-disclosure policy for all $y \leq y^+$. On the other hand, that $G(y) = F^y(y^+) - F^y(y)$ for all $y \in (y^+, KR)$, and $G(y) = 1$ for all $y \geq \max KR$, means that the optimal policy pools all the realizations of y above y^+ under a single message, so that the induced posterior mean is at least KR. □

Appendix C: Elicitation Mechanisms

Proof of Proposition 5.

Fix a message m^y disclosed with positive probability under Γ^y and a price $P \geq 0$. Let $z \equiv \mathbb{E}(y - s^\star(y)|m^y)$. The policy maker's problem can then be written as:

$$
\begin{aligned}
\max_{\{V, t_0(\cdot), \pi(0|\cdot)\}} \quad & \int_0^{1-P} \pi(0|\omega)\, dF^\omega(\omega) \\
\text{s.t:} \quad (i) \quad & \int_0^{1-P} \left(b1_{\{P+t_0^\omega(\omega)+\omega<1\}} + g1_{\{P+t_0^\omega(\omega)+\omega\geq 1\}} \right) \pi(0|\omega)\, dF^\omega(\omega) + \\
& +\pi^s g\left(1 - F^\omega(1-P)\right)) \geq 0 \\
(ii) \quad & V \times \left(\int_0^{1-P} |b|\, \pi(0|\omega)\, dF^\omega(d\omega) - g\pi^s \left(1 - F^\omega(1-P)\right) \right) \leq |\bar{U}_{\text{LF}}(P)| \\
(iii) \quad & \pi(0|\omega)\left(P + t_0(\omega)\right) R = V, \ \forall \omega \leq 1-P \\
(iv) \quad & \pi(0|\omega) = \pi^s \leq \frac{V}{PR+z}, \ \ \forall \omega \geq 1-P \\
(v) \quad & t_0(\omega) \leq \frac{z}{R}, \ \ \forall \theta \in \Theta
\end{aligned} \tag{33}
$$

where the first two constraints are the obedience constraints associated with messages 0 and 1, respectively, and the last three correspond to incentive compatibility constraints. Constraint (iii) imposes that the payoff of any bank reporting a liquidity position below $1-P$ must be the same, (iv) guarantees that vulnerable banks do not have incentives to mimic safe banks, and (v) simultaneously guarantees that (a) safe banks do not want to mimic vulnerable banks, and (b) that the fair-price constraint is satisfied. Note that (v) does not include a liquidity discount. This is a consequence of the fact that the policy maker only make transfers conditional on obtaining a passing grade.

Let $\hat{\omega} \equiv 1 - P - \frac{z}{R}$. Define next the auxiliary variable ρ as follows:

$$
\rho \equiv \frac{\left|\frac{b}{g}\right| F^\omega(\hat{\omega})}{PR+z} - \frac{1 - F^\omega(1-P)}{PR+z_\theta}. \tag{34}
$$

I characterize the optimal screening mechanism as a function of the value of ρ. Assume first that

$$
\rho \in \left(\frac{F^\omega(1-P) - F^\omega(\hat{\omega})}{PR+z}, \int_{\hat{\omega}}^{1-P} \frac{dF^\omega(\omega)}{(1-\omega)R} \right).
$$

Note that inequality (iv) must bind since doing so relaxes (i) and (ii), does not affect neither (iii) nor (v), and therefore allows to improve the policy maker's objective function. Next, constraint (iii) implies that we can write the policy maker's problem as a function only of V and t_0. Thus, the set of relevant constraints is given by:

$$\begin{aligned}
(i') \quad & \int_0^{1-P} \frac{b1_{\{P+t_0(\omega)+\omega<1\}} + g1_{\{P+t_0(\omega)+\omega\geq 1\}}}{(P+t_0(\omega))R} dF^{\omega}(\omega) + g\frac{(1-F^{\omega}(1-P))}{PR+z} \geq 0 \\
(ii') \quad & V\int_0^{1-P} \frac{|b|\, dF^{\omega}(\omega)}{(P+t_0(\omega))R} \leq |\bar{U}_{\text{LF}}(P)| + Vg\frac{(1-F^{\omega}(1-P))}{PR+z} \\
(iv') \quad & \pi^s = \frac{V}{PR+z} \\
(v) \quad & t_0(\omega) \leq \frac{z}{R} \\
(vi) \quad & \frac{V}{(P+t_0(\omega))R} \leq 1, \ \ \forall \omega \leq 1-P
\end{aligned}$$

where the new constraint (vi) is added so that probabilities are well defined.

Next, I characterize the optimal mechanism $\Upsilon^{\omega}[P] = \{t_m(\cdot;P), \pi^{\omega}(m|\cdot;P)\}_{m\in\{0,1\}}$. I assume existence, which I then verify by constructing the optimal mechanism.

Claim 0: Constraint (vi) must *essentially* bind for at least some $\omega \in \Omega$. Rigorously,

$$\sup_{\omega\in\Omega} \frac{V}{(P+t_0(\omega))R} = 1. \tag{35}$$

I show that, if equation (35) is not satisfied under Υ^{ω}, then there exists another feasible mechanism $\tilde{\Upsilon}^{\omega}$ that dominates Υ^{ω}. To prove this, suppose by contradiction that, under Υ^{ω},

$$\delta \equiv 1 - \sup_{\omega\in\Omega} \frac{V}{(P+t_0(\omega))R} > 0.$$

Define then the mechanism $\tilde{\Upsilon}^{\omega}[P] = \left\{\tilde{t}_m(\cdot;P), \tilde{\pi}^{\omega,\theta}(m|\cdot;P)\right\}_{m\in\{0,1\}}$ such that (a) $\tilde{t}_m(\cdot;P) \equiv t_m(\cdot;P)$ for all $m \in \{0,1\}$, (b) $\tilde{\pi}^{\omega}(0|\cdot;P) \equiv \frac{\pi^{\omega}(0|\cdot;P)}{1-\delta}$, and (c) $\tilde{\pi}^s = \frac{\pi^s}{1-\delta}$. The new mechanism $\tilde{\Upsilon}^{\omega,P}[P]$ satisfies (i)-(v), and does strictly better than $\Upsilon^{\omega}[P]$, which proves the claim.

Claim 1: $t_0(\omega;P) = \frac{z}{R}$ for all $\omega < \hat{\omega}$.

To see this, let $\Upsilon^{\omega}[P] = \{t_m^{\omega}(\cdot;P), \pi^{\omega}(m|\cdot;P)\}_{m\in\{0,1\}}$. Suppose that the claim is not true. I show that then it is possible to find another mechanism which strictly improves upon Υ^{ω}.

Consider the alternative mechanism $\Upsilon^{\varepsilon}[P] = \{t_m^{\varepsilon}(\cdot;P), \pi^{\omega}(m|\cdot;P)\}_{m\in\{0,1\}}$ which offers the alternative price t_0^{ε} that modifies the value of t_0^{ω} for values of $\omega \leq \hat{\omega}$ in the following way:

$$t_0^{\varepsilon}(\omega;P) \equiv \begin{cases} \varepsilon z + (1-\varepsilon)t_0(\omega;P) & \omega \leq \hat{\omega} \\ t_0(\omega;P) & \omega > \hat{\omega}. \end{cases}$$

Let V^{ε} be the value of V which preserves the value of the LHS in (ii'). That is,

$$V^{\varepsilon}\int_0^{1-P} \frac{|b|\, dF^{\omega}(\omega)}{\left(P+t_0^{\varepsilon}(\omega)\right)R} = V\int_0^{1-P} \frac{|b|\, dF^{\omega}(\omega)}{(P+t_0(\omega))R}.$$

This perturbation relaxes (i') since $b<0$ and $t_0^{\varepsilon}(\omega;\theta) \geq t_0^{\omega}(\omega;\theta)$ for all $\omega \leq \hat{\omega} \leq 1-P-t_0^{\omega}(\omega)$.[46] The perturbation also increases the value of V, which then relaxes (ii') since the RHS increases while the LHS remains constant (by construction). Constraint (v) is never affected by this perturbation, while (vi) is strictly relaxed as $t_0^{\varepsilon}(\omega) \geq t_0^{\omega}(\omega)$ for all ω. As a result, constraint (vi) does not longer bind and, hence, by the result in claim 0, the designer can do strictly better with another mechanism.

Claim 2: $\exists \check{\omega} \in [\hat{\omega}, 1-P]$ so that $t_0^{\omega}(\omega;P) = \max\{1-\omega-P, 1-\check{\omega}-P\}$ for all $\omega \in [\hat{\omega}, 1-P]$.

Consider an arbitrary pricing policy $\tilde{t}_0(\cdot;P)$. Construct the alternative policy $t_0(\omega;P) \equiv \max\{1-\omega-P, 1-\tilde{\omega}-P\}$ for all $\omega \in [\hat{\omega}, 1-P]$, where $\tilde{\omega}$ is chosen so that

$$\int_{\hat{\omega}}^{1-P} \frac{dF^{\omega}(d\omega)}{P+\tilde{t}_0^{\omega}(\omega)} = \int_{\hat{\omega}}^{1-P} \frac{dF^{\omega}(\omega)}{\max\{1-\omega, 1-\tilde{\omega}\}}.$$

I claim that the policy induced by t_0 dominates the one induced by $\tilde{t}_0$. To see this this, note that constraints (i'), (ii'), (v) remain unchanged under the alternative policy, but constraint (vi) relaxes. In fact,

$$\sup_{\omega\in[\hat{\omega},1-P]} \left\{ \frac{V}{\left(P+t_0^{\omega}(\omega)\right)R} \right\} \leq \sup_{\omega\in[\hat{\omega},1-P]} \left\{ \frac{V}{\left(P+\tilde{t}_0^{\omega}(\omega)\right)R} \right\} \leq 1.$$

The first inequality is strict if $F^{\omega}\left(\left\{\omega \in [\hat{\omega}, 1-P] : \tilde{t}_0^{\omega}(\omega) \neq t_0^{\omega}(\omega)\right\}\right) > 0.\square$

Claim 3: Constraint (i') must bind.

This constraint corresponds to obedience constraint (15), and requires that ST creditors have an incentive to follow the recommendation to keep rolling over the bank's debt. By contradiction, assume that this is not the case. Then,

$$\int_0^{\hat{\omega}} \frac{b dF^{\omega}(d\omega)}{(P+t_0(\omega))R} + \int_{\hat{\omega}}^{1-P} \frac{g dF^{\omega}(\omega)}{(P+t_0(\omega))R} + g\frac{(1-F^{\omega}(1-P))}{(PR+z)} > 0. \tag{36}$$

This implies that either (ii') or (vi) must be binding, or otherwise the policy maker would strictly increase the passing probability $\pi^{\omega}(0|\omega) = \frac{V}{(P+t_0(\omega))R}$.

Suppose first that (ii') is the binding constraint. Consider the following deviation from Υ^{ω}. Modify t_0^{ω} between $[\hat{\omega}, 1-P]$ as follows. $\tilde{t}_0^{\varepsilon} = \max\{1-\omega-P, 1-\check{\omega}^{\varepsilon}-P\}$, where $\check{\omega}^{\varepsilon} < \check{\omega}$ satisfies that

$$\int_{\hat{\omega}}^{1-P} \frac{dF^{\omega}(\omega)}{P+\tilde{t}_0^{\varepsilon}(\omega)} = \int_{\hat{\omega}}^{1-P} \frac{dF^{\omega}(\omega)}{P+t_0(\omega)} - \varepsilon,$$

for some $\varepsilon > 0$ small enough so that inequality 36 is respected[47]. Next, let $\tilde{V}(\varepsilon)$ be the maximal value that V may take under the new policy so that (ii') remains unchanged. That is,

$$\tilde{V}(\varepsilon)\left(\int_0^{\hat{\omega}} \frac{|b| dF^{\omega}(\omega)}{(P+t_0(\omega))R} + \int_{\hat{\omega}}^{1-P} \frac{|b| dF^{\omega}(\omega)}{\left(P+\tilde{t}_0^{\varepsilon}(\omega)\right)R} - g\frac{1-F^{\omega}(1-P)}{PR+z}\right) = C, \tag{37}$$

[46] Note that, for any $\omega \in \Omega$, $1-P-t_0^{\omega,\theta}(\omega;\theta) \geq 1-P-\frac{z_\theta}{R} = \hat{\omega}_\theta$

[47] The existence of such ε comes from (34) and (36). Together, they imply

$$\int_{\hat{\omega}}^{1-P} \frac{dF^{\omega}(\omega)}{P+t_0(\omega)} > \rho > \frac{F^{\omega}(1-P)-F^{\omega}(\hat{\omega})}{P+z}.$$

where $C > 0$ is a constant. Next, differentiating (37) against ε and then taking the limit from the right as ε goes to 0, we get:

$$\lim_{\varepsilon \downarrow 0} \tilde{V}'(\varepsilon) = \frac{\tilde{V}(0)\,|b|}{\left(|b| \int_0^{1-P} \frac{dF^{\omega}(\omega)}{(P+t_0(\omega))R} - g\frac{1-F^{\omega}(1-P)}{PR+z}\right)}.$$

This allows us to compute the effect of such a perturbation on the policy maker's payoff

$$W(\varepsilon) \equiv \tilde{V}(\varepsilon) \int_0^{1-P} \frac{dF^{\omega}(d\omega)}{\left(P + t_0^{\varepsilon}(\omega)\right) R} + 1 - F^{\omega}(1-P)$$

for small values of ε. In fact,

$$\begin{aligned}
\lim_{\varepsilon \downarrow 0} \frac{dW}{d\varepsilon} &\propto \left(\lim_{\varepsilon \downarrow 0} \tilde{V}'(\varepsilon)\right) \cdot \frac{W(0)}{\tilde{V}(0)} - \tilde{V}(0) \\
&= \frac{V^2}{C}(g-b)\frac{1-F^{\omega}(1-P)}{PR+z} \\
&> 0.
\end{aligned}$$

which contradicts the optimality of Υ^{ω}.

Next, assume that (vi) is the binding constraint (which determines the value of V). Consider the alternative policy

$$\widehat{t}_0^{\varepsilon}(\omega; P) \equiv \begin{cases} z/R & \omega \leq \hat{\omega} \\ \max\left\{1-\omega-P, 1-\breve{\omega}_{\theta}^{\varepsilon}-P\right\} & \omega \in (\hat{\omega}, 1-P], \\ 0 & \omega > 1-P \end{cases}$$

with $\breve{\omega}^{\varepsilon} = \breve{\omega} - \varepsilon$ and ε small enough so that (ii') is still satisfied. Let $\check{V}^{\varepsilon}$ be the maximal value that V may take under the new policy so that (vi) is still satisfied. That is,

$$\frac{\check{V}^{\varepsilon}}{1-\breve{\omega}^{\varepsilon}} = \frac{V}{1-\breve{\omega}}.$$

This implies that $\check{V}^{\varepsilon} > V$ and hence $\widehat{\pi}^{\varepsilon}(0|\omega) \equiv \frac{\check{V}^{\varepsilon}}{\left(P+\widehat{t}_0^{\varepsilon}(\omega)\right)R} > \pi(0|\omega)$ for all $\omega \leq \breve{\omega}$ and $\widehat{\pi}^{\varepsilon}(0|\omega) = \pi(0|\omega)$ for all $\omega > \breve{\omega}$, and hence the policy-maker's payoff must increase. This is a contradiction and hence (i') must be satisfied with equality. □

The last claim then implies that

$$\int_{\hat{\omega}}^{1-P} \frac{dF^{\omega}(\omega)}{(P+t_0(\omega))R} = \rho \in \left(\frac{F^{\omega}(1-P) - F^{\omega}(\hat{\omega})}{PR + \frac{z}{R}}, \int_{\hat{\omega}}^{1-P} \frac{F^{\omega}(d\omega)}{(1-\omega)R}\right).$$

Therefore, we choose $t_0(\omega; P)$ in $[\hat{\omega}, 1-P]$ among all the policies satisfying (i') so that V is largest. Let $\breve{\omega}$ be implicitly defined by

$$\int_{\hat{\omega}}^{\breve{\omega}} \frac{dF^{\omega}(\omega)}{(1-\omega)R} + \frac{F^{\omega}(1-P) - F^{\omega}(\breve{\omega})}{(1-\breve{\omega})R} = \int_0^{\breve{\omega}} \frac{|b|\,dF^{\omega}(\omega)}{g\left(P+\frac{z}{R}\right)R} - \frac{1-F^{\omega}(1-P)}{PR+z}.$$

That is, $\breve{\omega}$ is the cutoff defining the price t_0 which maximizes $\min_{\omega \leq 1-P} (P + t_0(\omega))R$ while still respecting (i').

Claim 4: $V=(1-\check{\omega})R$.

To see this, note that constraint (ii') is satisfied with strict inequality. In fact, that $\bar{U}_{\text{LF}}(P)<0$ for all $P<\frac{\mathbb{E}(y|m^y)}{R}$ implies that

$$\begin{aligned}
& \int_0^{1-P} b\left(1-\pi^{\omega}(0|\omega)\right)dF^{\omega}(\omega)+g\left(1-\pi^{s}\right)\left(1-F^{\omega}(1-P)\right) \\
= & \int_0^{1-\frac{z}{R}}\left(1-\frac{V}{z}\right)bdF(\omega)+\int_{1-\frac{z}{R}}^{\check{\omega}}\left(1-\frac{V}{(1-\omega)R}\right)bdF^{\omega}(\omega) \\
& +\int_{\check{\omega}}^{1-P}\left(1-\frac{V}{(1-\check{\omega})R}\right)bdF^{\omega}(\omega)+g\left(1-\frac{V}{z}\right)\left(1-F^{\omega}(1-P)\right) \\
< & \left(1-\frac{V}{z}\right)\left(\int_0^{1-\frac{z}{R}}bdF(\omega)+g\left(1-F^{\omega}\left(1-\frac{z}{R}\right)\right)\right) \\
= & \left(1-\frac{V}{z}\right)\bar{U}_{\text{LF}}\left(\frac{z}{R}\right) \\
\leq & \ 0.
\end{aligned}$$

As a consequence, the constraint defining the value of V is (iii). Therefore, at the optimum,

$$V=\inf\left\{\left(P+t_0^{\omega}(\omega;P)\right)R:\omega\leq 1-P\right\}=(1-\check{\omega})R.$$

The optimal policy is thus given by:

$$t_0(\omega;P)=\begin{cases}\frac{z}{R} & \omega<\hat{\omega}\\ 1-P-\omega & \omega\in[\hat{\omega},\check{\omega}]\\ 1-P-\check{\omega} & \omega\in(\check{\omega},1-P)\\ 0 & \omega\geq 1-P\end{cases},\quad \pi(0|\omega;P)=\begin{cases}\frac{(1-\check{\omega})R}{PR+z} & \omega<\hat{\omega}\\ \frac{1-\check{\omega}}{1-\omega} & \omega\in[\hat{\omega},\check{\omega}]\\ 1 & \omega\in(\check{\omega},1-P)\\ \frac{(1-\check{\omega})R}{PR+z} & \omega\geq 1-P\end{cases}$$

Finally, assume that

$$\rho\geq\int_{\hat{\omega}}^{1-P}\frac{dF^{\omega}(\omega)}{(1-\omega)R}. \tag{38}$$

Then, the designer in unable to successfully dissuade ST creditors from running on the bank with positive probability. In other words, $\pi(0|\cdot)=\mathbf{0}$. To see this, rewrite the inequality 38 as:

$$\int_0^{\hat{\omega}}\frac{|b|dF^{\omega}(d\omega)}{(PR+z)}-g\frac{1-F^{\omega}(1-P)}{PR+z}\geq g\int_{\hat{\omega}}^{1-P}\frac{dF^{\omega}(\omega)}{(1-\omega)R},$$

or equivalently,

$$\mathbb{E}\left(u(\omega,P,1)|0\right)=\int_0^{\hat{\omega}}\frac{bdF^{\omega}(\omega)}{(PR+z)}+\int_{\hat{\omega}}^{1-P}\frac{gdF^{\omega}(\omega)}{(1-\omega)R}+\frac{g\left(1-F^{\omega}(1-P)\right)}{PR+z}\leq 0.$$

That is, ST creditors obtain a negative payoff if they pledge to the bank (and the rest does not), even if the designer were to offer enough funds so that every bank with $\omega>\hat{\omega}$ survives the liquidity shortage caused by all ST creditors refraining from rolling over the bank's debt. As a result, under the most adversarial equilibrium all ST creditors run on the bank. The policy maker thus cannot engage in disclosing informative messages about the bank's liquidity position, and may only try to increase the likelihood of the bank's survival by purchasing claims on its asset. The optimal strategy for the policy maker consists of purchasing the totality of the remaining claims on the asset at the largest price allowed by *fair price* constraint. Thus, the government purchases $y-s^{\star}$ at price $t^{\emptyset}$ defined by:

$$t^{\emptyset} \equiv \sup\{\tau \leq B : z\mathbb{P}\{\omega+P+\tau \geq 1\} \geq \tau\}.\square$$

Proof of Proposition 6

Suppose that the bank sells security $s^{\star}$ to LT investors and raises $P \geq 0$. Clearly, $PR+\mathbb{E}(y-s^{\star}(y)|m^{y}) \leq \mathbb{E}(y|m^{y})$. Let $z \equiv \mathbb{E}(y-s^{\star}(y)|m^{y})$ and suppose that $PR=\mathbb{E}(y|m^{y})-z$. This last assumption overstate the true value of P and therefore increases the policy maker's payoff. Consider the function

$$\begin{aligned}
\varphi^{+}(P,\breve{\omega}) &\equiv \int_{0}^{1-P}\left(b1_{\{P+t_{0}^{\omega}(\omega)+\omega<1\}}+g1_{\{P+t_{0}^{\omega}(\omega)+\omega\geq 1\}}\right)\pi^{\omega}(0|\omega)\,dF^{\omega}(\omega)+\\
&\quad +\pi^{s}g\left(1-F^{\omega}(1-P)\right)\\
&= \frac{b}{PR+z}F\left(1-P-\frac{z}{R}\right)+\\
&\quad +g\left(\int_{1-P-\frac{z}{R}}^{\breve{\omega}}\frac{dF^{\omega}(\omega)}{(1-\omega)R}+\frac{F^{\omega}(1-P)-F^{\omega}(\breve{\omega})}{(1-\breve{\omega})R}+\frac{(1-F^{\omega}(1-P))}{PR+z}\right).
\end{aligned}$$

φ^{+} corresponds to the expected payoff of a ST creditor, at the optimal mechanism, after $m^{\omega,\theta}=0$ (i.e., pass) is disclosed. Function φ^{+} can be shown to decrease with P (or equivalently, increases with z) if we keep the rest of variables (other than z) constant, since $(1-\breve{\omega})R<PR+z$ (the case in which $(1-\breve{\omega})R=\mathbb{E}(y|m^{y})$ corresponds to the situation in which the policy maker can avoid default altogether without disclosing any information and thus is not considered here). This implies that constraint (i) in (33) is relaxed when we decrease the value of P, or equivalently, when we increase the value z. Decreasing P (increasing z) also relaxes constraints (ii) and (v), and does not affect neither (iii), nor (iv). To see the first point, consider the following function:

$$\begin{aligned}
\varphi^{-}(P,\breve{\omega},\bar{V}) &\equiv \int_{0}^{1-P}b\left(1-\pi^{\omega}(0|\omega)\right)dF^{\omega}(\omega)+\\
&\quad +g\left(1-F^{\omega}(1-P)\right)(1-\pi^{s})\\
&= \int_{0}^{1-P-\frac{z}{R}}\left(1-\frac{\bar{V}}{\mathbb{E}(y|m^{y})}\right)bdF(\omega)+\int_{1-P-\frac{z}{R}}^{\breve{\omega}}\left(1-\frac{V}{(1-\omega)R}\right)bdF^{\omega}(\omega)\\
&\quad +\int_{\breve{\omega}}^{1-P}\left(1-\frac{V}{(1-\breve{\omega})R}\right)bdF^{\omega}(\omega)+g\left(1-F^{\omega}(1-P)\right)\left(1-\frac{\bar{V}}{\mathbb{E}(y|m^{y})}\right).
\end{aligned}$$

φ^{-} corresponds to the expected payoff of ST creditors, at the optimal elicitation mechanism, after $m^{\omega,\theta}=1$ (fail) is disclosed. Function φ^{-} increases with P if we keep the rest of variables (other than z) constant. As a result, reducing P relaxes constraint (ii) in (33). Finally to see that (iii) is not affected by reductions of P, observe that for every reduction of P in the amount of Δ, the maximal price that may be pledged by the policy maker (determined by constraint (v) in (33)) increases by Δ. Thus, the policy maker can replicate the effect of P by increasing the price paid by the securities, t_0^{ω}, in the same amount.

Next, define $\check{\omega}(P)$ as the optimal cutoff associated with any price $P \in \left[0, \frac{\mathbb{E}(y|m^y)}{R}\right]$, as in (**??**). That is, $\check{\omega}(P)$ is chosen so that $\varphi^+(P, \check{\omega}(P)) = 0$. Consider the case where $P = 0$. The optimal elicitation mechanism is then given by:

$$(t_0^{\omega}(\omega; P=0), \pi^{\omega}(0|\omega; P=0)) = \begin{cases} \frac{\mathbb{E}(y|m^y)}{R},\ \frac{(1-\check{\omega}(0))R}{\mathbb{E}(y|m^y)} & \omega < 1 - \frac{\mathbb{E}(y|m^y)}{R} \\ 1-\omega,\ \frac{1-\check{\omega}(0)}{(1-\omega)} & \omega \in \left[1 - \frac{\mathbb{E}(y|m^y)}{R}, \check{\omega}(0)\right] \\ 1-\check{\omega}(0),\ 1 & \omega \in (\check{\omega}(0), 1]. \end{cases}$$

Choose any alternative policy in which the bank raises a price $\tilde{P} \in (0, 1-\check{\omega}(0))$ from LT investors. That φ^+ decreases with P implies that $\check{\omega}(\tilde{P}) > \tilde{\omega}(0)$, since $\check{\omega}(\tilde{P})$ satisfies $\varphi^+(\tilde{P}, \check{\omega}(\tilde{P})) = 0$. This means that $\pi^{\omega}(0|\omega; 0) > \pi^{\omega}(0|\omega; \tilde{P})$ for all $\omega \le \check{\omega}(\tilde{P})$, and $\pi^{\omega}(0|\omega; 0) = 1$ for all $\omega > \check{\omega}(\tilde{P})$. As a result, the policy-maker's payoff is strictly greater at $P = 0$. Finally, consider the case where $\tilde{P} \le 1 - \check{\omega}(0)$. We note that:

$$\begin{aligned} \frac{\int_0^{1-\frac{\mathbb{E}(y|m^y)}{R}} b\,dF^{\omega}(\omega)}{\mathbb{E}(y|m^y)} + g\left(\int_{1-\frac{\mathbb{E}(y|m^y)}{R}}^{1-P} \frac{dF^{\omega}(\omega)}{(1-\omega)R} d\omega + \frac{(1-F^{\omega}(1-P))}{\mathbb{E}(y|m^y)}\right) &< \varphi^+(0, \check{\omega}(0)) \\ &= 0, \end{aligned}$$

which means that the policy-maker is unable to convince ST creditors to keep pledging to the bank, regardless of her chosen elicitation mechanism. Thus, the best emergency lending program sets $P = 0$, which confirms that the optimal intervention will never involve the government and the private sector at the same time.□

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