Monetary Policy in Small Open Economies and the International Zero Lower Bound

Marco Rojas*

Banco Central de Chile

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Abstract

How does the zero lower bound (ZLB) on the international interest rate affect monetary policy in small open economies (SOE)? When the Fed's rate was at the ZLB (2008-2015), data for several SOE show a significantly lower correlation between interest rates and inflation, which is at odds with the empirical regularity. This is explained in a model where the distribution of shocks that affect SOE changes when the international interest rate hits the ZLB. Two opposing channels affect the exchange rate. At the ZLB, the depreciating channel is amplified, while the appreciating channel is attenuated. Then, the SOE currency depreciates more than in a scenario without ZLB. This passes through to inflation, which affects SOE's ability to stabilize the economy as it cannot lower its interest rate as much. In an estimated model, this mechanism by itself can explain 26 percent of the lower correlation observed in the data.

Keywords: Small open economy, International spillovers, Monetary policy, Zero lower bound

JEL codes: F41, E52, E65

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1 Introduction

During the Great Recession and Covid Crisis, the Fed's rate hit the zero lower bound (ZLB), which had several and known implications for the U.S. economy (Eggertsson and Woodford, 2003; Gust et al., 2017). In addition, because this interest rate can be interpreted as the international interest rate, the ZLB may also affect other countries. Recent papers have studied how the ZLB in one economy can influence another economy in reaching it too (Caballero et al., 2020). Others have looked at how unconventional policies in the U.S. during this period affect emerging and small open economies (SOE) (Alpanda and Kabaca, 2020). However, little is known about how the ZLB restriction itself on the international interest rate may affect SOE.

This paper contributes to this question by studying monetary policy in SOE when the international interest rate is at the ZLB. The first part documents a novel fact in a key relationship for monetary policy. The correlation between domestic interest rates and inflation falls significantly for several SOE during the Fed's rate ZLB episode that took place amid, and in the aftermath, of the Great Recession. Second, the paper presents a simple model to rationalize this finding. At the international ZLB, there is additional depreciation in SOE that generates an increase in inflation at a moment when domestic interest rates are falling, which may break the usual positive correlation between these two variables. Next, this mechanism is included in a larger quantitative model that can explain part of the lower correlation observed in the data. Finally, this framework is used to understand the impact that the ZLB restriction on the international interest rate can have SOE.

Using sixteen small open economies with inflation-targeting central banks, I find that the correlation between core CPI inflation and policy rates is significantly lower during the international ZLB period when compared to normal times. On average, the correlation coefficient goes from 0.75 to 0.31. This finding is explained through the lens of a standard SOE model (Galí and Monacelli, 2005), where the large economy or Rest of the World (ROW), is affected by discount rate shocks. In this setup, I study the whole distribution of shocks that end up impacting the SOE, rather than an individual shock by its own (e.g., foreign demand). In particular, how this distribution may vary with the monetary policy characteristics in ROW. For instance, whether there is a zero bound restriction or not.

In the model, when ROW – interpreted as the U.S. – enters a recession, two channels affect a SOE. First, a large negative discount rate shock impacting U.S. households may cause a recession. This triggers a policy response that lowers the international interest rate. Ceteris paribus, this depreciates the U.S. dollar and appreciates the SOE currency. Second, U.S. output drops, which lowers the foreign demand that the SOE faces. Ceteris paribus, this depreciates the SOE currency. Therefore, total depreciation depends on which channel dominates. In this context, the model evaluates what is the differential effect on SOE from two scenarios. One where the international interest rate can move freely (No ZLB*) and one with a zero bound restriction (ZLB*). In the ZLB* scenario, the first effect is smaller and the second is larger compared to a No ZLB* scenario. This generates added depreciation of the SOE currency that may pass through overall inflation. At the same time, the SOE is trying to lower its interest rate due to the external crisis. However, because of this higher than otherwise inflation, the SOE cannot lower its rate as much or keep it low for long. This produces a weaker relationship between interest rate and inflation, and affects the ability the SOE has to combat the recession.

To better illustrate the channels and how they change in both scenarios, the model focuses on a simple case with complete markets and unitary intertemporal and intratemporal elasticities, which allow for clean analytical expressions. The first channel stays constant in either scenario, while the second channel becomes relatively more relevant in the ZLB* scenario. To understand this consider the following. Note that the risk-sharing condition in this context states that the value of marginal utilities in both economies must be equal to each other when measured in the same currency. And, that lowering the international interest rate affects the extent to which the second channel is absent or not. If there is no restriction, the shock is fully accommodated and there is no drop in the foreign demand faced by the small economy.

In the No ZLB* scenario, there is no change in U.S. output, therefore the discount rate shock in the U.S. lowers the value of marginal utility of U.S. households. Due to complete markets, the value of marginal utility of SOE households must be lowered too. This can be happen in two ways. Either by increasing contemporaneous consumption or by appreciating the currency of SOE. In equilibrium, both happen. The first is achieved by lowering the interest rate in SOE, and the second implies that imported inflation falls. Together they pin down a positive correlation between interest rates and CPI inflation.

In the ZLB* scenario, the international interest rate cannot fully accommodate the shock and U.S. output now drops which lowers the foreign demand faced by the SOE. The first channel is still present and in the same magnitude, so the only difference is larger depreciation, which makes CPI inflation to increase with respect to the previous scenario. Depending on the magnitude of the latter, this second channel may lower, cancel or outweigh the appreciation coming from the first channel. Because the interest rate falls by the same amount as in the No ZLB* scenario, the relationship with inflation is weaker, null or positive, respectively, in the ZLB* scenario. Thus, the simple model provides a rationale for why we observe a drop in the correlation between interest rate and inflation during the international ZLB.

Then, I embed this mechanism in a quantitative SOE model that builds on Justiniano and Preston (2010a,b), and adds local currency pricing for domestic firms when exporting (Gopinath et al., 2010) and forward guidance in the international interest rate, i^*. The purpose of this is to have a model that can match better the data. The model has incomplete markets, habit formation, and sticky wages and prices. In the SOE the law of one price does not necessarily hold for both imports and exports. There are retail firms that import at the competitive price, but have monopolistic power when setting their prices internally. Domestic producing firms set their prices in ROW currency when exporting. In addition to the discount rate shocks in ROW that explain the ZLB on the international interest rate, the model considers discount rate shocks in SOE too. Also, productivity, cost-push, monetary policy and risk-premium shocks are included. Finally, monetary policy in ROW contemplates forward guidance as it can characterize better what happened to the Fed's rate amid and in the aftermath of the Great Recession. For this, the model follows Del Negro et al. (2013) which proposes a Taylor rule that reacts to not only contemporaneous, but also past inflation.

To solve the model I focus on Australia as the SOE and U.S. as ROW. Most of the parameters come from Justiniano and Preston (2010a) and related literature, and some others are calibrated such that they match their average data counterparts (e.g. discount factor and average interest rate). To estimate the remaining parameters, I use the simulated method of moments. One key moment is the share of quarters i^* is at the ZLB, which relies heavily on the parametrization of the discount rate shocks to ROW households. Because of this non-linearity, the model cannot be solved using traditional perturbation methods, so instead it follows the approach in Guerrieri and Iacoviello (2015) that provides piecewise linear solutions.

To evaluate how the model performs, I quantify the ability the model has to explain the lower correlation between interest rate and core inflation observed in the data. I simulate the fully estimated model and compute the equivalent correlations to those of the data. The model can explain at least 26 percent of the drop in the correlation that happens when comparing periods where the international rate is not at the ZLB and periods where it is.

Finally, the quantitative model can be used to understand the implications on a SOE that a restricted international interest rate can have. I do this by studying impulse response functions from large discount rate shocks to U.S. households under two scenarios. The baseline scenario (ZLB*), and an alternative one where the international interest rate can be adjusted freely scenario (No ZLB*). For instance, under large shocks, it could become negative. When comparing these scenarios, the main result is verified: there is larger depreciation in SOE when the international interest rate is at the ZLB. This gets passed to imported and overall inflation, which together with a higher international interest rate, results in a higher domestic interest rate compared to a No ZLB* scenario. The ability to lower the interest rate further allows for output in the SOE to fall by less when facing the external recession.

This exercise illustrates how the monetary policy structure in the U.S. can affect small economies by producing abnormal exchange rate movements due to the mismatch between the structural shock, the policy response and the effects on activity.

This paper contributes to several strands of the literature. First, those that study the international spillover effects in SOE and emerging economies from monetary policy in the U.S. or other large economies (e.g., Eurozone). Some papers have examined the impacts of conventional monetary policy.[1] For example, they have used identified shocks to Fed's rate movements to study effects on the exchange rates. Others have estimated the effects of unconventional monetary policy.[2] For instance, they have implemented event study techniques to understand the effects on international bond yields from large-scale asset purchases done by the Fed. I contribute to this by studying the effect of a particular feature of Fed's monetary policy, which is that its main instrument cannot fall below zero, on monetary policy itself in SOE. My paper provides descriptive evidence on monetary policy in SOE during this period, and lends a theoretical rationale of why we observe a break in the relationship between two key variables that characterize monetary policy.

Second, the paper builds on the literature at the intersection of international economics and the ZLB on interest rates, either understood as a consequence of secular stagnation or as a transitory shock, which is produced, for example, because of a discount rate shock as in this paper.[3] When the ZLB is the result of long-term trends, Eggertsson et al. (2016)

[1] See, for example, Kalemli-Özcan (2019); Iacoviello and Navarro (2019); Albagli et al. (2019); Buch et al. (2019); Vicondoa (2019); Lakdawala et al. (2020); Miranda-Agrippino and Rey (2020).

[2] See, for example, Neely (2015); Curcuru et al. (2018); Gajewski et al. (2019).

[3] In addition, important contributions have been made in this intersection. For instance when ZLB

and Caballero et al. (2020) propose different models to study two symmetric economies, and what occurs when one enters secular stagnation. They predict that, under certain conditions, the ZLB in one country generates the other economy to reach it too. With a world economy structure like in this paper, Corsetti et al. (2019) challenge that prediction by studying a SOE affected by secular stagnation in ROW, and show that the SOE can isolate itself from it.

When the ZLB takes place as a transitory shock, Cook and Devereux (2013) study how the zero restriction generates odd exchange rate variations. This is in a model with symmetric economies and where the country of interest is the one initially affected by the ZLB. In this context, this paper fills in the gap in the literature by studying a SOE when the ZLB is foreign (as in Corsetti et al., 2019), but in the presence of a transitory shock (as in Cook and Devereux, 2013).

Finally, it relates to research about the impact of the ZLB on the economy, and the associated literature studying the effects of negative interest rates. Gust et al. (2017) study how the ZLB affected the U.S. during the Great Recession and restricted its ability to overcome the recession. They do this by using an alternative model where the Fed rate can be negative. Ulate (2021) and Lopez et al. (2020) examine the effects of negative interest rate and their impact to commercial banks. Sims and Wu (2021) study negative policy rates as a tool of unconventional monetary policy. I further this understanding by studying how the ZLB in one country spills over to other economies. For this, the paper compares the baseline scenario against an economy where the international interest rate can be negative.

The rest of the paper is organized as follows. Section 2 describes the data and presents descriptive evidence on what happens to monetary policy in several SOE during the international ZLB. Section 3 presents a simple model that delivers the main mechanism, which is then included in a quantitative model described in Section 4. Section 5 presents the parametrization of the model, together with the estimation of certain parameters and the solution method. Section 6 evaluates the performance of the model and carries out impulse response exercises. Finally, Section 7 concludes.

occurs within currency unions. See, for example, Gomes et al. (2015); Farhi and Werning (2016); Hettig and Müller (2018); Cook and Devereux (2019).

2 Inflation and interest rates in small open economies

This section studies whether the relationship between interest rates and inflation, in small open economies, changes during the period when the Fed's rate was at its zero lower bound during and in the aftermath of the Great Recession.

2.1 Data

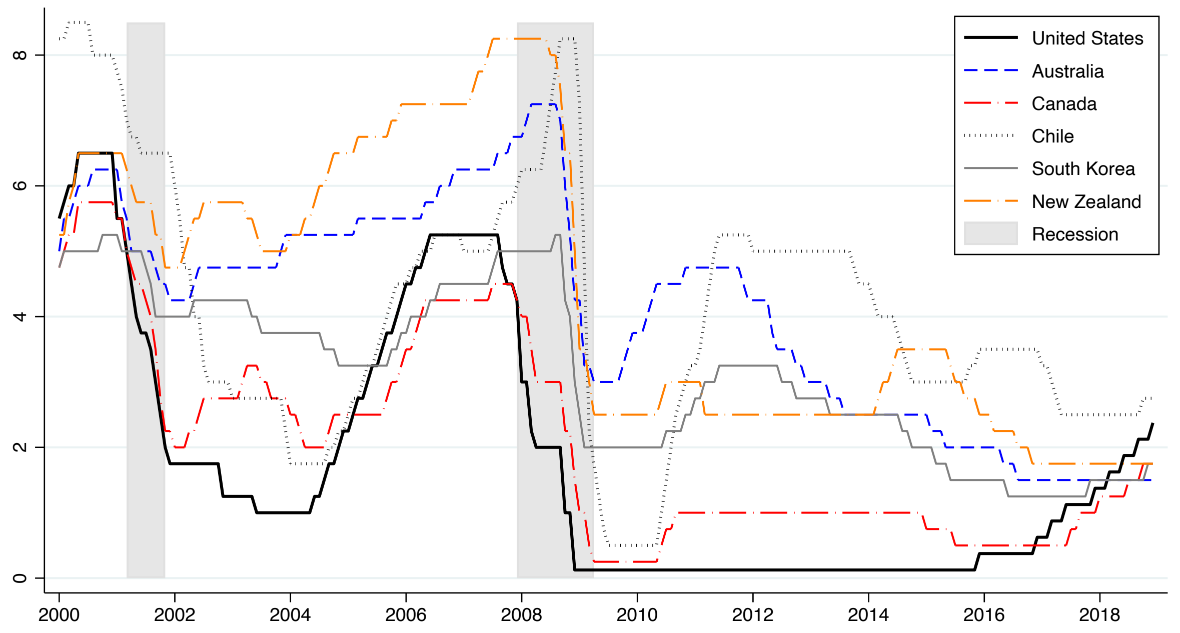
The main sources of data are the BIS statistics for country's policy rates, and OECD statistics for core CPI inflation indexes. The baseline uses CPI of all non-food non-energy items. I include all small open economies in the BIS dataset with at least 20 years of data between 1990 and 2019. SOE are defined as countries integrated with world markets, and whose policies do not affect world prices.[4] In particular, they take the international interest rate as given and cannot affect it. This leaves 16 countries at quarterly frequency, which are listed in Appendix Table A.3. All these countries have inflation-targeting central banks (Hammond, 2012).[5] More details are provided in Appendix A.

To provide context, Figure 1 shows the Fed's rate together with the policy rate of five SOE. The Fed's rate drops during the Dot-Com bubble and Great Recession (gray areas), which is accompanied by drops in the other policy rates too. This paper investigates the potential different mechanism taking place during – and after – the last recession when the Fed's rate hit – and stayed – at the ZLB, and how that may have affected SOE. If the Fed's rate is considered to be at the ZLB when it is below 0.25%, then the international ZLB period takes place between 2008Q4 and 2015Q4.

[4] See the definition in Deardoff's glossary of international economics.

[5] Switzerland is not included in that study, but it has an inflation-targeting central bank (See https://www.snb.ch/en/iabout/snb/id/snb_tasks).

Figure 1: Interest rate in U.S. and selected economies, 2000-2019



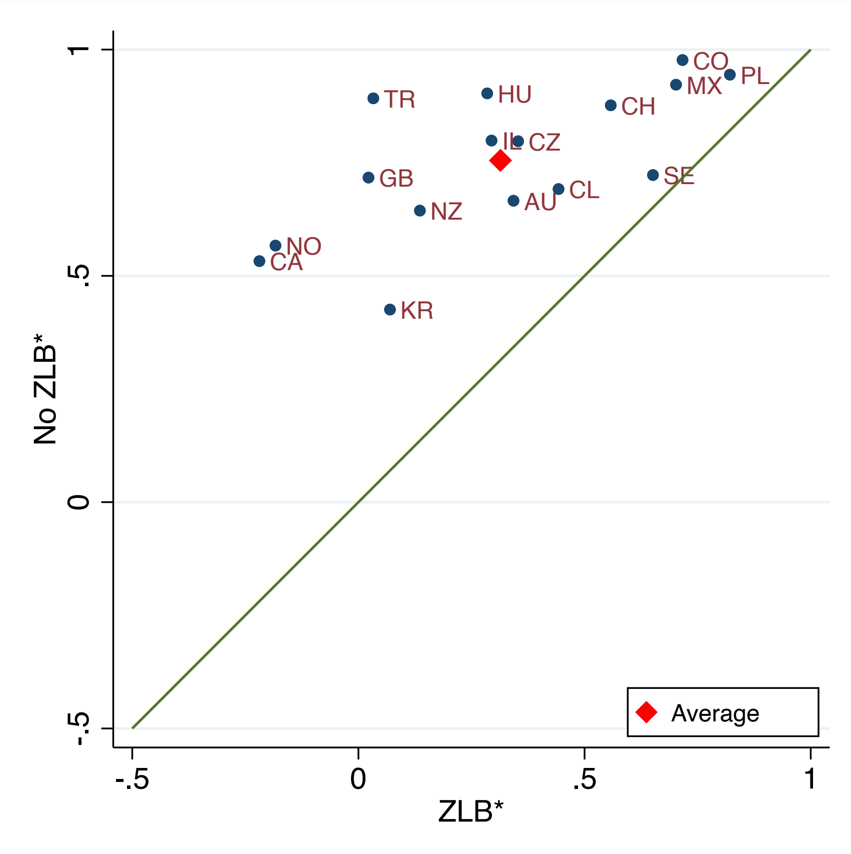
2.2 Inflation and interest rates during the international ZLB

One of the many empirical regularities in macroeconomies is the positive correlation between interest rates and different measures of inflation. When these are equilibrium rates in the financial markets, for instance in the Treasury bond market, this relationship is sometimes referred to as the Fisher relationship (Fisher, 1930). Despite many ways that monetary policy can be understood (e.g., using Taylor rules), studying their correlation is a simple model-free approach that can inform whether the relationship between key variables for monetary policy changes during the international ZLB.

Figure 2 plots the correlation between the average policy rate and year-ended core CPI inflation in a given quarter. This is done for two distinct periods. The correlation during periods when the international interest rate is not bound by zero is displayed in the y-axis. And the correlation during the international ZLB in the x-axis. For all countries, we observe a drop during the international ZLB as they all are above the 45° line. For some economies, this drop is small (see Sweden), but for most of them is a sizable drop (see Australia, Canada or Israel). The solid diamond shows that, on average, the correlation coefficient goes from 0.75 during normal times to 0.31during the international ZLB.

Table 1 accompanies the correlation coefficients with their standard deviation. This verifies that correlation coefficients during the international ZLB are statistically different to those during normal times (significance level of 5%), with the exception of Sweden. This finding is robust to using quarter-to-quarter inflation, using different measures for core CPI inflation and using headline instead of core CPI inflation. Appendix A performs these robustness checks.

Figure 2: Correlation of interest rate and year-ended core CPI inflation



Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the simple average correlation among all countries. The diagonal curve is a 45-degree line.

Discussion. The correlation coefficient can hardly tell something about causality between inflation and policy rates. A positive relationship can be viewed as a reaction of the policy rate to current inflation or expected inflation. Given the inflation targeting scheme, higher (lower) inflation requires a rise (drop) in the interest rate to keep inflation under control. And because it acts with a lag, we can still observe a positive relationship within a quarter. Alternatively, this positive correlation is also consistent with the Neo-Fisherian view that reverses this causality. Because agents in the economy care about real interest rates, the theory goes, a higher (lower) nominal interest rate will only have an effect through a higher (lower) inflation. Nominal interest rate equals inflation plus real interest rate, which in the long-run is unaffected by nominal variables.

Thus, the drop in this correlation can be viewed as one of these hypothesis becoming less strong during the international ZLB period. For instance, policy rates may also respond to output gap, to output growth, and – especially in open economies – to the exchange rate. If the international ZLB changes the distribution of shocks affecting SOE, such that the relative proportion of variables the policy rate responds to is different compared to normal times, then we can expect the relationship of interest rates to each of those variables to also change. Alternatively, nominal interest rates have an impact not only through demand in the short-run, but also through the supply side of the economy (Baqaee et al., 2021), which can end up affecting real variables in the long-run, and thus the one-to-one relationship between nominal variables. If the international ZLB exacerbates nominal rigidities, then we can expect the relationship between interest rates and other nominal variables to change too. The following section shows why small economies observe a weaker relationship between these two variables during the international ZLB period.

Table 1: Correlation of interest rate and year-ended core inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.67	0.34	IL - Israel	0.80	0.29
	(0.06)	(0.16)		(0.05)	(0.16)
CA - Canada	0.53	-0.22	KR - South Korea	0.43	0.07
	(0.07)	(0.21)		(0.11)	(0.19)
CH - Switzerland	0.88	0.56	MX - Mexico	0.92	0.70
	(0.04)	(0.13)		(0.04)	(0.11)
CL - Chile	0.69	0.44	NO - Norway	0.57	-0.18
	(0.08)	(0.14)		(0.07)	(0.21)
CO - Colombia	0.98	0.72	NZ - New Zealand	0.64	0.14
	(0.02)	(0.10)		(0.06)	(0.18)
CZ - Czechia	0.80	0.35	PL - Poland	0.94	0.82
	(0.06)	(0.15)		(0.03)	(0.08)
GB - Great Britain	0.72	0.02	SE - Sweden	0.72	0.65
	(0.06)	(0.19)		(0.06)	(0.11)
HU - Hungary	0.90	0.28	TR - Turkey	0.89	0.03
	(0.03)	(0.16)		(0.05)	(0.19)

Notes: This table reports sample correlations between core CPI inflation measures and interest rates for two periods at quarterly frequency. The ZLB* period is given by 2008Q4 to 2015Q4. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

3 Simple model

This section presents a simple model which is used to illustrate the main mechanism, and make sense of the data presented above. It is a simplified version of what is outlined in Section 4, and follows closely Galí and Monacelli (2005). However, instead of focusing on the effect of a given shock, the analysis looks at the entire distribution of shocks that, stemming from ROW, affect a SOE. And in particular, how that distribution may change with monetary policy features of ROW.

First, the world economy is presented with separate ROW and SOE blocks. Both contain households, firms and a central bank/government. Second, the log linearized equilibrium is derived. Finally, the international ZLB is analyzed by studying what happens after a one-time large negative discount rate shock hits ROW households.

3.1 World Economy

Time is indexed by t. The world economy is made of a large economy (or ROW) of size 1 and a SOE of size 0, indexed by R and S respectively. Given their relative sizes, ROW is in practice a closed economy. There is a unit mass of firms in each economy that can set their prices à la Calvo in their own currency, i.e., producer currency pricing. There are complete financial markets. International trade is frictionless and the law of one price (LOP) holds for individual goods. Because households have home bias, LOP fails to hold for consumption price indexes. ROW values are denoted with *.

ROW. There is a unit mass of households in ROW with the following utility function:

$$U^* = E_0 \sum_{t=0}^{\infty} \beta^t \exp(\nu_t^*) \left[\log C_t^* - \frac{N_t^{*1+\varphi}}{1+\varphi} \right]$$

where φ is the inverse of the Frisch elasticity and ν_t^* is a discount rate or preference shock that changes the relative weight given to marginal utility in period t with respect to $t+1$. This is the key shock that drives the mechanism and provokes a recession by making households extremely patient.[6] N_t^* is the labor supplied by the household. C_t^* is the consumption index that aggregates varieties produced by ROW firms: $C_t^* \equiv C_{R,t}^* \equiv \left(\int_0^1 C_{R,t}^*(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$, where $\epsilon > 1$ is the elasticity of substitution among differentiated goods, and $P_{R,t}^* = \left(\int_0^1 P_{R,t}^*(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ the price. Note that because of ROW's relative size, this is in practice a closed economy, which means $P_t^* = P_{R,t}^*$.

ROW households have access to a complete set of fully contingent claims. B_t^* is the nominal payoff in period $t+1$ of the portfolio of such claims held by ROW households at the end of period t. $\mathbb{Q}_{t,t+1}^*$ is the stochastic discount factor for a nominal payoffs in $t+1$

[6] This shock to characterize recessions that may lead interest rates to hit the zero lower bound have been used widely in the literature. See for example Gust et al. (2017); Christiano et al. (2015); Nakata (2016). Alternatively, a shock that drives agents away from risky assets into safe assets (à la Krishnamurthy and Vissing-Jorgensen (2012)) can also generate similar results in terms of bringing down output and prices simultaneously. In order to keep tractability and analytical expressions, this paper opts for a one-asset model in ROW. Further research is needed to understand what additional implications would this have on SOE.

from the perspective in t. Then, budget constraint is:

$$P_t^* C_t^* + E_t \mathbb{Q}_{t,t+1}^* B_{t+1}^* = W_t^* N_t^* + B_t^* + T_t^* + \Gamma_t^*.$$

where T_t^* are taxes/transfers and Γ_t^* are firms profits. Households maximize their utility subject to the budget constraints:

$$C_t^* N_t^{*\varphi} = \frac{W_t^*}{P_t^*} \qquad \text{and} \qquad \beta E_t \exp(\Delta \nu_{t+1}^*)(1+i_t^*) \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-1} \frac{P_t^*}{P_{t+1}^*} = 1, \tag{1}$$

where $1 + i_t^* = (E_t \mathbb{Q}_{t,t+1}^*)^{-1}$.

There is a continuum $[0, 1]$ of firms, where firm j produces with production function, $Y_t^*(j) = N_t^*(j)$. Firms enjoy monopolistic power, so there is a wage subsidy such that they charge marginal costs in steady state. If $\tau^* = \frac{1}{\epsilon}$, then $P_t^* = (1 - \tau^*)\frac{\epsilon}{\epsilon - 1} W_t^* = W_t^*$. This is financed with lump-sum tax to households T_t^*.

In each period, a share θ of firms cannot adjust their price, so for them $P_t^*(j) = P_{t-1}^*(j)$. The remaining $(1 - \theta)$ share set $\tilde{P}_t^*(j)$ to solve the following problem:

$$\max_{\tilde{P}_t^*(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \mathbb{Q}_{t,t+k}^* \left[\tilde{P}_t^*(j) Y_{t+k|t}^* - (1 - \tau^*) \Psi^* \left(Y_{t+k|t}^* \right) \right] \right\}. \tag{2}$$

where $\mathbb{Q}_{t,t+k}^* \equiv \beta^k (C_{t+k}^*/C_t^*)^{-1} (P_t^*/P_{t+k}^*)$, Ψ^* is the cost function, $Y_{t+k|t}^* = (\tilde{P}_t^*(j)/P_{t+k}^*)^{-\epsilon} Y_{t+k}^*$, and τ^* is the labor subsidy. Because of the relative size of ROW, $Y_t^* = C_t^*$.

Finally, the central bank at ROW has a stabilization objective of strict inflation target, $\bar{\Pi}_t^* = 1$ (Eggertsson and Woodford, 2003). The only tool available to attain such target is the nominal interest rate i_t^*. In this context, the simple model assesses two different scenarios:

$$\begin{aligned} &\text{(a) No ZLB}^*: i_t^* \in \mathbb{R} \\ &\text{(b) ZLB}^*: i_t^* \geq 0. \end{aligned} \tag{3}$$

SOE. In describing the SOE block, I omit the details that mirror those of the ROW, and just point out relevant differences.

There is a unit mass of households in SOE with the following utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right].$$

C_t is a consumption basket made of SOE and ROW goods: $C_t \equiv C_{S,t}^{1-\alpha} C_{R,t}^{\alpha}$, where $1-\alpha$ is the home bias.[7] In turn, $C_{S,t}$ and $C_{R,t}$ are indexes for the differentiated goods coming from SOE itself and ROW, respectively,

$$C_{S,t} \equiv \left(\int_0^1 C_{S,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \quad , \quad C_{R,t} \equiv \left(\int_0^1 C_{R,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

The prices corresponding to C_t, $C_{R,t}$, and $C_{S,t}$ are $P_t \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} P_{S,t}^{1-\alpha} P_{R,t}^{\alpha}$, $P_{S,t} \equiv \left(\int_0^1 P_{S,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$ and $P_{R,t} \equiv \left(\int_0^1 P_{R,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$, respectively. Budget constraints faced by SOE households are:

$$P_t C_t + E_t \mathbb{Q}_{t,t+1} B_{t+1} = W_t N_t + B_t + T_t + \Gamma_t.$$

SOE households maximize their utility subject to the budget constraints:

$$C_t N_t{}^{\varphi} = \frac{W_t}{P_t} \quad \text{and} \quad \beta E_t (1+i_t) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{P_t}{P_{t+1}} = 1,$$

where $1 + i_t = (E_t \mathbb{Q}_{t,t+1})^{-1}$.

There is a continuum $[0,1]$ of firms, where firm j produces with production function, $Y_t(j) = N_t(j)$. Firms enjoy monopolistic power, so there is a wage subsidy such that they charge optimal marginal costs in steady state: $\tau = \frac{1}{1-\alpha}$ (Corsetti and Pesenti, 2001). This is financed with a lump-sum tax to households T_t.

For a given differentiated product, the LOP holds, then

$$P_{S,t}(j) = \mathcal{E}_t P_{S,t}^*(j) \quad \text{and} \quad P_{R,t}(j) = \mathcal{E}_t P_{R,t}^*(j).$$

The nominal exchange rate is denoted by $\mathcal{E}_t$ and is defined as the price of one unit of ROW's currency in terms of SOE's currency (e.g. Chilean pesos per U.S. dollar). Then, an

[7]This aggregation is assuming that the elasticity of substitution between domestic and foreign goods is equal to one.

increase in $\mathcal{E}_t$ is a depreciation of SOE's currency. Given the preferences and the parity holding for individual goods prices, $P_{S,t} = \mathcal{E}_t P^*_{S,t}$ and $P_{R,t} = \mathcal{E}_t P^*_{R,t}$. However, due to home bias $P_t \neq \mathcal{E}_t P^*_t$.

Using households' preferences, the total demand for SOE goods is given by,

$$Y_t = \left(\frac{P_{S,t}}{P_t}\right)^{-1} \left[(1-\alpha)C_t + \alpha Q_t C^*_t\right]. \tag{4}$$

where $Q_t \equiv \dfrac{\mathcal{E}_t P^*_t}{P_t}$ is the real exchange rate (e.g. Chilean consumption baskets per U.S. basket). An increase (decrease) in Q_t is a real depreciation (appreciation) in the small economy.

In each period, a share θ of firms cannot adjust their price, so for them $P_{S,t}(j) = P_{S,t-1}(j)$. The remaining $(1-\theta)$ share set $\tilde{P}_{S,t}(j)$ to solve the following problem:

$$\max_{\tilde{P}_{S,t}(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{\mathbb{Q}_{t+k|t} \left[\tilde{P}_{S,t}(j) Y_{t+k|t} - (1-\tau)\Psi\left(Y_{t+k|t}\right)\right]\right\}, \tag{5}$$

where $Y_{t+k|t} = (\tilde{P}_{S,t}(j)/P_{S,t+k})^{-\epsilon} Y_{t+k}$.

Due to complete markets, the value of marginal utilities of households in SOE and ROW equal each other when priced in the same currency:

$$Q_t \cdot C_t^{-1} = \exp(\nu^*_t) C_t^{*-1}. \tag{6}$$

This expression is key to understand the mechanism described below.[8] Under constant prices (i.e., Q_t constant), when ROW households become patient (i.e., $\nu^*_t < 0$), even if ROW consumption does not fall, consumption in SOE increases. A drop in the value of marginal utility in ROW households requires an equally sized drop in SOE. Otherwise, given complete markets, gains from trade arise. Then, either a drop in marginal utility itself (through consumption), a drop in its price (through real appreciation) or both must occur. Due to home bias, this is effectively a rise in demand for SOE goods, which then rises domestic consumption.

Finally, the central bank at SOE maximizes households' utility subject to equilibrium conditions (4) (5) and (6). More details on its derivation are provided in Appendix B.

[8]It is however not necessary for the results, but it helps to obtain clean analytical solutions. The quantitative model of Section 4 dispenses with the complete markets assumption.

3.2 Log-linearized system

Next, the paper proceeds to present log-linear approximations to ROW and SOE blocks. Further equations, details and derivations appear in Appendix B. Lowercases denote percent deviations with respect to steady state ($x_t \equiv \log X_t - \log X$), with the exception of interest rates, i_t and i_t^*, that already correspond to percentages.

In ROW, the market clearing condition plus equations (1) and (2) are written as:

$$\begin{aligned} \pi_t^* &= \kappa^* y_t^* + \beta E_t \pi_{t+1}^* \\ c_t^* &= E_t c_{t+1}^* - \left(i_t^* - E_t \pi_{t+1}^* - r_t^{n*} \right), \\ y_t^* &= c_t^* \end{aligned} \tag{7}$$

where $r_t^{n*} = \rho - E_t \Delta \nu_{t+1}^*$, $\kappa^* \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\epsilon\varphi)}(1+\varphi)$, and $\rho \equiv 1/\beta - 1$. The next Lemma summarizes how the central bank in ROW implements its objective and determines equilibrium, depending on the size of the shock and the restriction (or lack of thereof) on its nominal interest rate.

Lemma 1 (ROW Implementation)**.** *Under a central bank in ROW with inflation target of zero percent, $\overline{\pi}^* = 0$, and equilibrium conditions in system (7):*

1. *If the nominal interest rate can take any value, $i_t^* \in \mathbb{R}$. Then, the target is implemented with $i_t^* = r_t^{n*}$, and output is stabilized, $y_t^* = 0$ for all t.*

2.(a) *If the nominal interest rate is bound by zero, $i_t^* \geq 0$, and $\rho \geq E_t \Delta \nu_{t+1}^*$. Then, the target is implemented with $i_t^* = r_t^{n*}$, and output is stabilized, $y_t^* = 0$ for all t.*

2.(b) *If the nominal interest rate is bound by zero, $i_t^* \geq 0$, and $\rho < E_t \Delta \nu_{t+1}^*$. Then, the target cannot be implemented, so $i_t^* = 0$, and $y_t^* < 0$.*

Next, in SOE, equations (4), (5) and (6) are written as:

$$\begin{aligned} \pi_{S,t} &= \kappa \left(c_t + \varphi y_t + \frac{\alpha}{1-\alpha} q_t \right) + \beta E_t \pi_{S,t+1}, \\ c_t &= y_t^* + q_t - \nu_t^*, \\ y_t &= (1-\alpha) c_t + \alpha y_t^* + \tilde{\alpha} q_t, \end{aligned} \tag{8}$$

where $\tilde{\alpha} \equiv \alpha(2-\alpha)/(1-\alpha)$ and $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\epsilon\varphi)}$.[9] The following Lemma summarizes

[9]SOE equilibrium conditions already consider $y_t^* = c_t^*$.

how the central bank at SOE implements its objective taking into consideration SOE's and ROW's equilibrium conditions.

Lemma 2 (SOE Implementation)**.** *Under a central bank in SOE that maximizes (a second-order approximation to) households' utility subject to the system in* (8)*, the desired equilibrium is attained with the following optimal rule:*

$$\pi_{S,t} + \frac{1}{\epsilon}\Delta y_t = 0. \tag{9}$$

Proof. See Appendix B.

Note that given standard values for the elasticity of substitution across differentiated goods (e.g., $\epsilon = 8$), the optimal rule implies strong domestic price stabilization. It stays close to fully stable at the expense of movements in output

Finally, because the focus is on CPI inflation and interest rate, these can be written as:

$$\begin{aligned} \pi_t &= \pi_{S,t} + \frac{\alpha}{1-\alpha}(q_t - q_{t-1}), \\ i_t &= \rho + E_t\Delta c_{t+1} + E_t\pi_{t+1}. \end{aligned} \tag{10}$$

The first expression comes from the Euler equation and the second from the definition of P_t and Q_t.

Definition 1 (World equilibrium)**.** *The world economy is in equilibrium if, for a given series of discount rate shocks* $\{\nu_t^*\}_{t=0}^{\infty}$*, the following holds:*

(i) ROW variables $\{y_t^*, \pi_t^*\}$ *and* i_t^* *satisfy the system in* (7) *and one of the scenarios in Lemma 1. And,*

(ii) SOE variables $\{c_t, y_t, q_t, \pi_{S,t}, \pi_t\}$ *and* i_t *satisfy the system in* (8) *and* (10)*, and rule* (9)*.*

3.3 International ZLB

First, the model proceeds to derive the equilibria in ROW under each scenario. Then, it analyzes how each scenario affects the SOE differently. For that it begins by deriving supply and demand functions for SOE output.

To study the effect of the zero bound in the international interest rate, consider the following sequence of discount rate shocks $\{\nu_0^*, 0, 0, ...\}$. This series is known in ROW and SOE at the beginning of $t = 0$. In order to make the comparison among scenarios relevant, from Lemma 1 the shock must satisfy: $\rho > -\nu_0^*$. This ensures $r_0^{n*} < 0$. Given the monetary policies in both economies, it can be shown that this one-time shock in $t = 0$ only generates deviations from steady state in that period. Therefore, $E_0 x_{t+1} = E_0 x_{t+1}^* = 0$ and $E_0 i_{t+1} = E_0 i_{t+1}^* = \rho$ for all $t \geq 0$, where x_t is any variable in ROW or SOE.[10]

In scenario (a), i_0^* can take any value (No ZLB*), so the shock is fully absorbed such that no recession takes place, and the policy objective is attained. The equilibrium at ROW in this context is denoted with N and given by:

$$\pi_{N,0}^* = 0 \quad , \quad y_{N,0}^* = 0 \quad , \quad i_{N,0}^* = r_0^{n*} = \rho + \nu_0^*. \tag{11}$$

In scenario (b), i_0^* has a zero lower bound (ZLB*), so the response is halted for large enough shocks. The equilibrium at ROW in this context is denoted with Z and given by:

$$\pi_{Z,0}^* = \kappa^* r_0^{n*} \quad , \quad y_{Z,0}^* = r_0^{n*} \quad , \quad i_{Z,0}^* = 0. \tag{12}$$

To understand the differential effect on SOE, the following demand and supply curves for the output produced in SOE are derived from the system in (8):

$$-q_0 = (1-\alpha)\big(y_0^* - y_0 - (1-\alpha)\nu_0^*\big) \tag{DD}$$
$$-q_0 = (1-\alpha)\left(\varphi' y_0 + y_0^* - \nu_0^*\right) \tag{SS}$$

They are conveniently written to be drawn in the axis $(y_0, -q_0)$, so that schedules (DD) and (SS) have standard negative and positive slopes, respectively. Note that the domestic price of SOE output is negatively correlated to a real exchange rate appreciation. A rise in $p_{S,0}$ rises the overall price in the economy, p_0, which by the definition of the real exchange, lowers q_0. Gray curves in Figure 3 show both curves when $\nu_0^* = y_0^* = 0$.

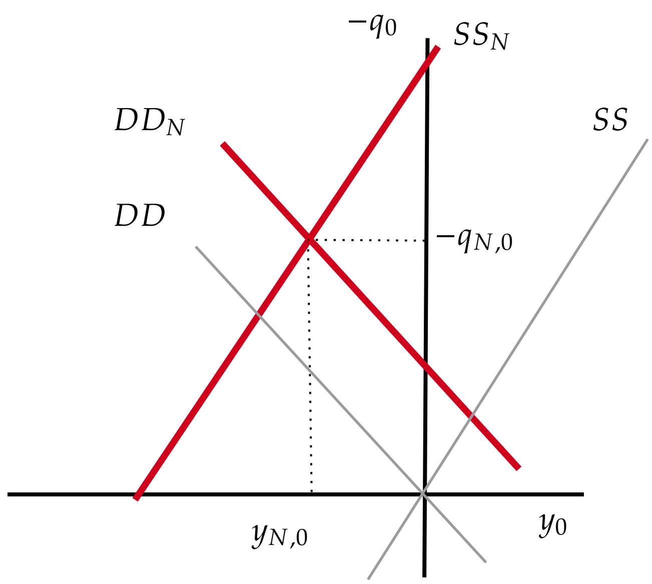
Note that in the No ZLB* scenario, only the structural shock affects the SOE. In the ZLB* scenario, it is also present, but there is an additional shock coming from a drop in y_0^*. Now, the model proceeds to assess how these two scenarios affect the SOE differently.

Figure 3 displays what happens in the No ZLB* scenario. Only the discount rate shock affects the SOE. From (6) the structural shock, under constant prices, generates upward pressure on domestic consumption. On one hand, this increases marginal costs by rising workers' opportunity cost, which pushes curve (SS) in. It shifts curve SS to SS_N. For a

[10]The dynamic system in the ROW block is not determinate, however, the proposed solution is indeed a *possible* equilibrium. In order to show that this also holds for a determinate model, Appendix B computes the same equilibria in a model with money-in-the utility and exogenous money supply.

given level of q_0, SOE's production falls. For a given level of y_0, SOE's competitiveness falls. On the other hand, it increases demand which pushes (DD) out. It shifts curve DD to DD_N. The overall drop in output occurs as the increase in marginal costs is larger than the increase in demand, which by (10) means a drop in interest rate. CPI inflation depends on output and real exchange rate, but is determined by the latter.[11] Thus, the appreciation means a drop in inflation. This pins down a positive relationship between interest rate and inflation.

Figure 3: Negative ROW discount rate shock under No ZLB* scenario



Intuitively, the structural shock generates a rise in the *relative demand* for SOE goods. This increases its relative price, so the real exchange rate appreciates. Because of this, imports into the SOE are cheaper which lowers overall inflation. In addition, the increase in consumption can only be achieved if nominal interest rates fall.[12] Given that this occurs despite the response of ROW's central bank, it is a *direct* channel that affects the SOE.

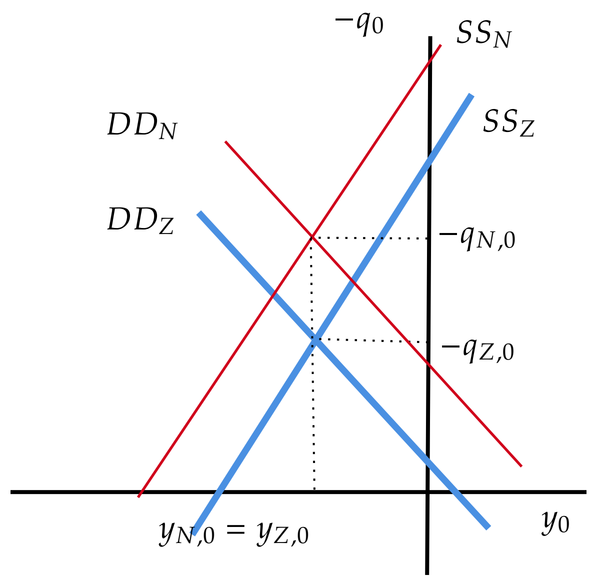
Figure 4 displays what happens in the ZLB* scenario. The effect of ν_0^* is still present, but now there is another channel given by the drop in y_0^*. From (4) it generates downward pressure on consumption. On one hand, this decreases marginal costs which pushes (SS) out compared to the previous scenario. It pushes curve SS_N to SS_Z. On the other hand,

[11] It can be shown that due to (9), variations in domestic inflation are very small, and given $\epsilon > 1$, the sign of CPI inflation is always determined by that of the real exchange rate.

[12] Alternatively, one could think in nominal terms. By the UIP, the drop in i_0^* goes partly to a drop in i_0 and partly to an expected depreciation, which requires current appreciation. So the drop in domestic interest rates is accompanied by a drop in inflation.

it decreases demand which pushes (DD) in compared to the previous scenario. It pushes curve DD_N to DD_Z. Given the simplified model, the drop in output in this scenario is the same as before, so is the drop in the domestic interest rate.[13] The difference comes from what happens to the real exchange rate which appreciates less than before, $q_Z > q_N$. This occurs because of the lower depreciation produced by $y_0^* < 0$, which turns into an increase in CPI inflation.

Figure 4: Negative ROW discount rate shock under ZLB* scenario



This new channel goes in the opposite direction, with respect to what happens from ν_0^*. Plus, there is no change in the interest rate. Then, in this scenario the positive relationship between inflation and interest rate in the SOE weakens. And it can potentially be zero or negative if, for example, y_0^* drops too severely. Figure B.1 and B.2 in the Appendix show the cases for zero and negative relationships, respectively.

Discussion. The comparison between both scenarios informs about how domestic variables behave depending on the restriction that the international interest rate may have or not. International recessions characterized by no restriction, either because it does not exist or because the recession is of smaller magnitude, delivers a positive relationship between interest rate and inflation. If those variables are positively correlated due to other domestic shocks, then no difference should arise in the data and the empirical regularity

[13] Specifically, a simplified model means complete markets, unitary elasticity of intertemporal substitution, unitary elasticity of substitution between domestic and foreign goods, and optimal mark-up in SOE.

holds. Conversely, an international recession characterized by the zero bound restriction delivers a weaker, null or negative relationship compared to the previous scenario. If the structural shock driving the recession is large enough, then a noticeable difference in the data may arise. This overall result is not dependent on the simplification of the model. Appendix B shows that the findings hold using a Taylor rule instead.

Section 2 presented descriptive evidence that can be explained by a mechanism presented here. Next, I include that mechanism into a quantitative model to quantify the ability it has in explaining the drop in the correlation.

4 Quantitative model

In this section, I incorporate the previous mechanism into a DSGE model that builds on Justiniano and Preston (2010a). The main departures from that model are the following. First, ROW's Taylor rule allows for forward guidance in the interest rate. This does not come as an insight from the simple model, but rather as a way to better fit the data as the original model does not contemplate the ZLB on i^*. Second, SOE firms invoice their exports in the currency of the ROW, which makes the LOP to not hold for goods produced domestically. In addition, the model no longer assumes a unitary elasticity of intertemporal substitution or unitary elasticity between domestic and foreign goods.

As in the simple model, the main source of the recession is due to a structural shock to the discount rate of ROW households (ν_t^* and ν_t). However, because the model attempts to match other moments in the data, it also includes risk-premium shocks ($\xi_{rp,t}$), productivity shocks ($\xi_{a,t}$), cost-push shocks to domestic and importing firms ($\xi_{cpS,t}$ and $\xi_{cpR,t}$) and monetary policy shocks ($\xi_{i,t}$). The model is presented from SOE's perspective, but differences with the ROW's counterpart are pointed out.

4.1 Households

There is a unit mass of households with the following utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \exp(\nu_t) \big[u(C_t, C_{t-1}) - v(N_t) \big]$$

where ν_t is the discount rate shock to SOE households. $u(C_t, C_{t-1}) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma}$ and $v(N_t) = \frac{N_t^{1+\varphi}}{1+\varphi}$, where $h \in (0,1)$ is an external habit coefficient. C_t is a consumption index,

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{S,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{R,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where η is the elasticity of substitution between SOE and ROW goods, and $(1-\alpha)$ is the home bias. C_t in Section 3 assumed $\eta = 1$. The corresponding price is $P_t = \left[(1-\alpha)P_{S,t}^{1-\eta} + \alpha P_{R,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$. Also, $C_{S,t}$ and $C_{R,t}$ are consumption indexes defined, together with corresponding prices, as in Section 3.

Households have access to bonds in SOE currency, D_t, and in ROW currency, B_t. Then, the budget constraint is

$$P_t C_t + D_t + \mathcal{E}_t B_t = (1 + i_{t-1})D_{t-1} + \mathcal{E}_t B_{t-1}(1 + i_{t-1}^*)\phi_t(A_t) + \Gamma_{S,t} + \Gamma_{R,t} + W_t N_t,$$

where the function $\phi_t(\cdot)$ is interpretable as a debt elastic interest rate premium given by:

$$\phi_t = \exp[-\chi A_t + \xi_{p,t}], \qquad \text{with } A_t \equiv \frac{\mathcal{E}_{t-1} B_{t-1}}{P_{t-1}},$$

where χ is the elasticity and $\xi_{rp,t}$ is a risk-premium shock. All households are assumed to start with the same initial wealth of 0, i.e. $A_{-1} = B_{-1} = 0$. The international interest rate is given by i_t^* and set by the central bank in ROW as it is explained below. $\Gamma_{S,t}$ and $\Gamma_{R,t}$ are profits from domestic and importing firms respectively.

The demand functions for each category are:

$$C_{S,t}(i) = \left(\frac{P_{S,t}(i)}{P_{S,t}}\right)^{-\epsilon} C_{S,t} \quad \text{and} \quad C_{R,t}(i) = \left(\frac{P_{R,t}(i)}{P_{R,t}}\right)^{-\epsilon} C_{R,t}$$

Optimal allocation of expenditure across domestic and foreign goods implies demand functions:

$$C_{S,t} = (1-\alpha)\left(\frac{P_{S,t}}{P_t}\right)^{-\eta} C_t \quad \text{and} \quad C_{R,t} = \alpha\left(\frac{P_{R,t}}{P_t}\right)^{-\eta} C_t$$

ROW households face a similar problem to the one outlined above, but with a few differences. As in Section 3, C_t^* aggregates differentiated goods instead of other composite goods. Because SOE is of negligible size, ROW own's debt is in zero net supply, so is SOE own's debt too. However, SOE can still access bonds denominates in ROW currency.

As in Justiniano and Preston (2010a), this model allows the intertemporal elasticity of substitution and habit formation coefficient to be different for ROW households, $\sigma^* \neq \sigma$ and $h^* \neq h$.

4.2 Optimal labor supply

Each domestic firm produces good j with technology $Y_t(j) = \exp(\xi_{a,t})N_t(j)$. The labor input used in production of j is an aggregation of different types of labor indexed by k:

$$N_t(j) = \left[\int_0^1 N_t(k)^{\frac{\epsilon_W - 1}{\epsilon_W}} dk\right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \tag{13}$$

where ϵ_W is the elasticity of substitution. Then, firm j's demand for each type of labor k is given by:

$$N_t^d(k) = N_t(j)\left(\frac{W_t(k)}{W_t}\right)^{-\epsilon_W},$$

where $W_t = \left[\int_0^1 W_t(k)^{1-\epsilon_W} dk\right]^{\frac{1}{1-\epsilon_W}}$. Households supply labor under monopolistic competition. A fraction $(1-\theta_W)$ of households set wages optimally, while a fraction θ_W adjusts according to the following rule:

$$W_t(k) = W_{t-1}(k)\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_W},$$

where $\gamma_W \in (0,1)$ is a degree of indexation to the previous period CPI inflation. Households solve the following problem when setting their wage $W_t(k)$:

$$\max_{\tilde{W}_t(k)} E_t \sum_{T \geq t} (\theta_W \beta)^{T-t} \left[\frac{\tilde{W}_t(k)}{P_T} N_{T|t}(k) \left(\frac{P_{T-1}}{P_{t-1}}\right)^{\gamma_W} \nu_T u_{1,T} - v_{1,T}(N_{T|t}(k))\right],$$

where $N_{T|t}(k) = N_T \left(\frac{\tilde{W}_t(k)}{W_T}\right)^{-\epsilon_W}$, $u_{1,T} \equiv \frac{\partial u}{\partial C_T}(C_T, C_{T-1})$, $v_T \equiv v(N_T)$ and $v_{1,T} \equiv \frac{\partial v}{\partial N_T}(N_T)$. ROW households solve a similar problem with parameters γ_W^* and θ_W^* instead.

4.3 Domestic producers

There is a continuum $[0,1]$ of monopolistically competitive domestic firms producing differentiated goods. They sell in the SOE where they set prices in their own currency ($P_{S,t}$), and sell in ROW where they set prices in the ROW currency ($P^*_{S,t}$). Namely, local or destination currency pricing for exports.

Each period a fraction θ_S of firms cannot adjust both prices optimally, and only adjust them according to the following rules:

$$P_{S,t}(j) = P_{S,t-1}(j)\left(\frac{P_{S,t-1}}{P_{S,t-2}}\right)^{\gamma_S} \quad \text{and} \quad P^*_{S,t}(j) = P^*_{S,t-1}(j)\left(\frac{P^*_{S,t-1}}{P^*_{S,t-2}}\right)^{\gamma^*_S},$$

where $\gamma_S \in (0,1)$ and $\gamma^*_S \in (0,1)$ are the degree of indexation to relevant inflation in the previous period. The other fraction $(1-\theta_S)$ of firms set prices optimally. They choose $\tilde{P}_{S,t}(j)$ for domestic sales and $\tilde{P}^*_{S,t}(j)$ for foreign sales to maximize the present discounted value of their nominal profits:

$$\max_{\tilde{P}_{S,t}(j),\tilde{P}^*_{S,t}(j)} E_t \sum_{T=t}^{\infty} \theta_S^{T-t}\mathbb{Q}_{t,T}\left[\tilde{P}_{S,t}(j)\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{\gamma_S} y_{S,T|t}(j) + \mathcal{E}_T\tilde{P}^*_{S,t}(j)\left(\frac{P^*_{S,T-1}}{P^*_{S,t-1}}\right)^{\gamma^*_S} y^*_{S,T|t}(j)\right.$$
$$\left. -W_T f^{-1}\left(\frac{y_{S,T|t}(j)+y^*_{S,T|t}(j)}{\xi_{a,t}}\right)\right],$$

where $y_{S,T|t}(j) = \left(\frac{\tilde{P}_{S,t}(j)}{P_{S,T}}\right)^{-\epsilon}\left(\frac{P_{S,T-1}}{P_{S,t-1}}\right)^{-\epsilon\gamma_S} C_{S,T}$ and $y^*_{S,T|t}(j) = \left(\frac{\tilde{P}^*_{S,t}(j)}{P^*_{S,T}}\right)^{-\epsilon}\left(\frac{P^*_{S,T-1}}{P^*_{S,t-1}}\right)^{-\epsilon\gamma^*_S} C^*_{S,T}$ are the demands faced in T when setting prices in t, and $\mathbb{Q}_{t,T} \equiv \beta^{T-t}\frac{\nu_T}{\nu_t}\left(\frac{C_T}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_T}\right)$ is the stochastic discount factor. In addition, these firms are subject to cost-push shocks, $\xi_{cpS,t}$.

Local currency pricing breaks the LOP of the S goods exported to ROW. It means that $P_{S,t}(j) \neq \mathcal{E}_t P^*_{S,t}(j)$. To keep track of this we define the deviations in the LOP for S goods, $\Psi_t \equiv \frac{\mathcal{E}_t P^*_{S,t}}{P_{S,t}}$, which is always equal to one in the simple model. It is important to note that terms of trade in this context are defined as $S_t \equiv \frac{P_{R,t}}{\mathcal{E}_t P^*_{S,t}}$.

ROW firms solve a similar problem with parameters γ^* and θ^*, but only selling in their own market from their perspective. And because it acts like as a closed economy, $P^*_{R,t} = P^*_t$. In addition, ROW demand for SOE good – though negligible from ROW's perspective – is given by

$$C^*_{S,t} = \left(\frac{P^*_{S,t}}{P^*_t}\right)^{-\lambda^*} C^*_t,$$

where λ^* is ROW's elasticity between ROW and SOE goods. These firms are also subject to cost-push shocks, $\xi^*_{cp,t}$.

4.4 Retail firms

Retail or importing firms are only present in the SOE. There is a continuum [0,1] of importing firms that buy ROW goods and sell them in SOE. The law of one price holds at the docks, however, in firms setting the price in terms of SOE currency they are monopolistically competitive. This pricing power leads to a violation of LOP, so $P_{R,t}(j) \neq P^*_{R,t}(j)\mathcal{E}_t$. To keep track of this we define the deviation in the LOP for R goods, $\Psi^*_t \equiv \dfrac{P_{R,t}}{\mathcal{E}_t P^*_t}$, which is always equal to one in the simple model.

A fraction $(1-\theta_R)$ of firms set prices optimally, while a fraction θ_R adjusts according to the following rule:

$$P_{R,t}(j) = P_{R,t-1}(j)\left(\frac{P_{R,t-1}}{P_{R,t-2}}\right)^{\gamma_R},$$

where $\gamma_R \in (0,1)$ is a degree of indexation to imported inflation in the previous period. The firm's price setting problem in t is to maximize their expected present discounted value of profits:

$$\max_{\tilde{P}_{R,t}(j)} E_t \sum_{T=t}^{\infty} \theta_R^{T-t} \mathbb{Q}_{t,T}\left[\tilde{P}_{R,t}(j)\left(\frac{P_{R,T-1}}{P_{R,t-1}}\right)^{\gamma_R} - \mathcal{E}_T P^*_{R,T}(j)\right] y_{R,T|t}(j),$$

where $y_{R,T|t}(j) = \left(\dfrac{\tilde{P}_{R,t}(j)}{P_{R,T}}\right)^{-\epsilon}\left(\dfrac{P_{R,T-1}}{P_{R,t-1}}\right)^{-\epsilon\gamma_R} C_{R,t}$. In addition, these firms are subject to cost-push shocks, $\xi_{cpR,t}$.

4.5 International risk sharing and prices

Optimality conditions of SOE households lead to the following equations to determine domestic and foreign bond allocations:

$$\exp(\nu_t)(C_t - hC_{t-1})^{-\sigma}\frac{1}{P_t} = \beta(1+i_t)E_t\left[\exp(\nu_{t+1})(C_{t+1} - hC_t)^{-\sigma}\frac{1}{P_{t+1}}\right],$$
$$\exp(\nu_t)(C_t - hC_{t-1})^{-\sigma}\frac{\mathcal{E}_t}{P_t} = \beta(1+i_t^*)E_t\left[\exp(\nu_{t+1})(C_{t+1} - hC_t)^{-\sigma}\phi_{t+1}\frac{\mathcal{E}_{t+1}}{P_{t+1}}\right].$$

Similarly for ROW households:

$$\exp(\nu_t^*)(C_t^* - h^*C_{t-1}^*)^{-\sigma^*}\frac{1}{P_t^*} = \beta(1+i_t^*)E_t\left[\exp(\nu_{t+1}^*)(C_{t+1}^* - h^*C_t^*)^{-\sigma^*}\frac{1}{P_{t+1}^*}\right].$$

Combining optimality conditions for ROW bonds, we arrive to the following incomplete risk-sharing condition,

$$\frac{\Phi_t}{\Phi_t^*}Q_t = \frac{E_t\Phi_{t+1}\mathcal{E}_{t+1}/P_{t+1}\Phi_{t+1}}{E_t\Phi_{t+1}^*/P_{t+1}^*},$$

where $\Phi_t \equiv \exp(\nu_t)(C_t - hC_{t-1})^{-\sigma}$. This expression reflects a similar idea to that in (6). Under constant prices, a discount rate shock to ROW households that lowers the relative value of current marginal utility (and that is uncorrelated to SOE households, i.e. $\nu_t = 0$) lowers the relative current marginal utility of SOE households too. This provokes a rise in the relative demand for SOE, which may require a contemporaneous increase in C_t. Note that it occurs even if consumption at the ROW does not change.

This expression can also be written as the uncovered interest parity condition (UIP):

$$(1+i_t) = (1+i_t^*)E_t\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\phi_t.$$

4.6 Monetary policy

ROW. The central bank follows a Taylor rule that has a zero lower bound and that can also capture forward guidance in its interest rate. For this, the model uses Del Negro et al. (2013) and defines $\tilde{i}^*_t$ as the shadow rate:

$$(1+\tilde{i}^*_t) = \left(1+\tilde{i}^*_{t-1}\right)^{\psi^*_i} \left(\left(\frac{P^*_t}{P^*_{t-4}}\right)^{\psi^*_\pi} \left(\frac{Y^*_t}{Y^*_{t-4}}\right)^{\psi^*_y}\right)^{1-\psi^*_i} \exp(\xi^*_{i,t}),$$
$$i^*_t = \max\left\{0, \tilde{i}^*_t\right\}, \tag{14}$$

where $\xi^*_{i,t}$ is a monetary policy shock. This rule captures forward guidance as a shock that happened in the past, say $t-3$, and that generated low prices and low output, is pushing interest rates down in t, even if the shock is no longer present.

SOE. The central bank in SOE sets the nominal interest rate i_t according to:

$$(1+i_t) = (1+i_{t-1})^{\psi_i} \left(\Pi_t^{\psi_\pi} Y_t^{\psi_y} \left(\frac{Y_t}{Y_{t-1}}\right)^{\psi_{\Delta y}} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\psi_{\Delta e}}\right)^{1-\psi_i} \exp(\xi_{i,t}),$$

where $\xi_{i,t}$ is a monetary policy shock. This is a rule that responds to CPI inflation, output, output growth and nominal depreciation. This is a good approximation to what monetary policy can be characterized in countries like Australia.

4.7 World equilibrium

Goods markets in SOE and in ROW clear:

$$Y_t = C_{S,t} + C^*_{S,t} \quad , \quad Y_{S,t} = C_{S,t} \quad , \quad Y_{R,t} = C_{R,t} \quad , \quad Y^*_t = C^*_t.$$

Asset market for bonds denominated in SOE currency clear:

$$D_t = 0.$$

Households in SOE are homogenous and they all start with the same wealth. And, given SOE's size, they do not trade with ROW.

Definition 2 (World equilibrium)**.** *An equilibrium is a set of*

(i) *Prices:* $\{P_{S,t}, P^*_{S,t}, P_{R,t}, P_t, \mathcal{E}_t, Q_t, W_t, P^*_t, W^*_t\}$

(ii) *Quantities:* $\{Y_t, Y_{S,t}(j), Y^*_{S,t}(j), Y_{R,t}(j), C_t, C_{S,t}, C_{R,t}, N_t, N^d_t, B_t, D_t, Y^*_t, Y^*_t(j), C^*_t, C^*_{S,t}, N^*_t, N^{d^*}_t, B^*_t\}$

(iii) *Price decisions:* $\{\tilde{P}_{S,t}, \tilde{P}^*_{S,t}, \tilde{P}_{R,t}, \tilde{W}_t, \tilde{P}^*_t, \tilde{W}^*_t, \}$

(iv) *Interest rates:* $\{i_t, i^*_t, \tilde{i}^*_t\}$

such that,

1. *Given prices,* $\{C_t, C_{S,t}, C_{R,t}, N_t, B_t, D_t\}$ *solve SOE households problem.*

2. *Given prices,* $\{C^*_t, N^*_t, B^*_t\}$ *solve ROW households problem.*

3. *Given prices and demand for labor,* $\{\tilde{W}_t\}$ *and* $\{\tilde{W}^*_t\}$ *solve SOE and ROW household wage-setting problems, respectively.*

4. *Given prices,* $\{Y_{S,t}, Y_{S,t}(j), Y^*_{S,t}, Y^*_{S,t}(j), Y_{R,t}, Y_{R,t}(j), Y_t, N^d_t\}$ *solve problem of domestic firms and of importing firms in SOE.*

5. *Given prices,* $\{Y^*_t, Y^*_t(j), N^{d^*}_t\}$ *solve problem of firms in ROW.*

6. *Given prices and demand for goods,* $\{\tilde{P}_{S,t}, \tilde{P}^*_{S,t}\}$ *and* $\{\tilde{P}_{R,t}\}$ *solve domestic and importing firms price-setting problems in SOE, respectively.*

7. *Given prices and demand for goods,* $\{\tilde{P}^*_t\}$ *solves firms price-setting problem in ROW.*

8. *Nominal interest rates in SOE* $\{i_t\}$ *and in ROW* $\{i^*_t, \tilde{i}^*_t\}$ *satisfy their corresponding Taylor rules.*

9. *Labor, goods and asset markets clear.*

5 Solution method and parametrization

The purpose of this section is to provide a solution method that takes into consideration the non-linearity in the model, and to provide a parametrization for the quantitative model.

Solution method. Given the occasionally binding constraint on i^*_t, the model is no longer linear. Thus traditional perturbation methods that rely on the model being always linear cannot be implemented. Therefore, I use the package (*OccBin*) and approach provided by Guerrieri and Iacoviello (2015) that uses a piecewise linear perturbation method that can accommodate non-linearities in the model. In particular, for each variable it delivers a perturbation based solution for when i_t^* binds and when it does not. This is also the solution method used for the estimation results described below.

Now that the model has been presented and that a solution method has been provided, the model proceeds to focus on one small open economy and the U.S. as the ROW for the parametrization. The SOE is Australia as it has had the same monetary policy framework since 1990, and was not affected as much by similar structural shock as the U.S. was.[14]

Parametrization. Following Justiniano and Preston (2010b), I assume all shocks x are AR(1) with persistence parameter ρ_x and standard deviation σ_x:

$$\log \xi_{x,t} = \rho_x \log \xi_{x,t-1} + \sigma_x u_t, \tag{15}$$

where $u_t \sim (0, 1)$ and i.i.d. The exceptions are each country's own cost-push shocks $(\xi_{cpS,t}, \xi^*_{cp,t})$ and monetary policy shocks $(\xi_{i,t}, \xi^*_{i,t})$, which are assumed to be i.i.d. ($\rho_x = 0$).

The parameters of the model are divided into two groups. The first group comes from the related literature and the second one is estimated using the simulated method of moments or calibrated using their data counterpart.

For the first group of parameters, and in order for the parametrization to not guide the results, I set several parameters in SOE and ROW equal to each other. The intertemporal elasticity of substitution, Frisch elasticity, and habit formation coefficient follow Gust et al. (2017). This means $\sigma = \sigma^* = 1$, $\varphi = \varphi^* = 2$ and $h = h^* = 0.70$. In their same vein, I set $\epsilon = \epsilon_W = \epsilon^* = \epsilon^*_W = 6$. The rest of common parameters come from the ROW block in Justiniano and Preston (2010a), which estimates a related model with the U.S. as ROW. This includes the parameters governing firms' cost-push and productivity shocks, the indexation parameters, and the probability of resetting prices for firms and wages for households. It is also assumed that domestic firms in SOE have the same indexation coefficient when selling at home or abroad. The details are found in Table 2.

The rest of the parameters for the small economy are taken from Justiniano and Preston (2010b), which estimates the parameters for Australia as a SOE. This is done as the SOE block of their model resembles most of the model presented above. It is worth noting that here we also follow the assumption that the elasticity of substitution between domestic and foreign goods is the same from both SOE's and ROW's perspective (i.e., $\eta = \lambda^*$).

[14] Canada is another potential candidate, but it is more likely to have been affected by similar shocks than the U.S. was, then not allowing a proper assessment of how the Great Recession in the U.S. affects a SOE via the zero restriction the international interest rate has.

Also, the parameters governing monetary policy and discount rate shocks are not equal to those in ROW, because these are estimated to match the U.S. data as is explained below.

The home bias parameter and discount factors are calibrated such that they correspond to their data average counterpart. I set $\alpha = 0.20$ as this is the average import to GDP ratio in Australia during this period. β and β^* are set such that annual steady state interest rates are 4% and 3%, respectively.

Finally, there are key parameters which are estimated to match relevant moments in the U.S. data. In particular, moments that are not be relevant in Justiniano and Preston (2010a) (e.g., share of quarters that i^* binds at zero) or that may be affected by the use of a different Taylor rule (e.g., correlation between i^* and y^*). Given the discussion of Section 3, the characteristics of the discount rate shock determine the extend of the ZLB in the international interest rate, which in the quantitative model is informed by parameters ρ^*_ν and σ^*_ν. And due to the inclusion of a different Taylor rule for ROW, its parameters are also estimated. These are ψ^*_i, ψ^*_π, ψ^*_y and σ^*_i.

Parameter estimation. To estimate the parameters, I use the simulated method of moments (SMM) as analytical expressions for the moments are not available given the non linearity of the international interest rate. Given the structure of the model, where ROW is in practice a closed economy, the set of parameters can be divided into the ones pertaining to ROW and to SOE separately, $\Theta = (\Theta^R, \Theta^S)$. This lowers the computational burden as only the ROW block is now solved for when estimating a subset of Θ^R.

In particular, I estimate $\bar{\Theta} \equiv (\sigma^*_\nu, \rho^*_\nu, \psi^*_i, \psi^*_\pi, \psi^*_y, \sigma^*_i) \subset \Theta^R$ by solving the following distance problem:

$$\widehat{\bar{\Theta}} = \arg\min_{\bar{\Theta}} \left[\mu(x_t) - \frac{1}{\mathcal{S}} \sum_{s=1}^{\mathcal{S}} \mu\left(x\left(\xi^s_t, \bar{\Theta} \right) \right) \right] \widehat{W}^{-1} \left[\mu(x_t) - \frac{1}{\mathcal{S}} \sum_{s=1}^{\mathcal{S}} \mu\left(x\left(\xi^s_t, \bar{\Theta} \right) \right) \right]'. \tag{16}$$

x_t is the observed data and $\mu(x_t)$ is a function that computes the moments that appear in column 'Data' of Table 3. ξ^s_t is a vector draw of random shocks for simulation s and $\mathcal{S}$ is the total number of simulations, which considers all shocks affecting ROW. The length of the shocks is the same as that of the data. $x\left(\xi^s_t, \bar{\Theta}\right)$ is the simulated data, which is the piecewise-linear solution obtained from the model under shocks ξ^s_t, parameters $\bar{\Theta}$ and the *OccBin* approach. $\widehat{W}$ is an estimate of the optimal weighting matrix (Ruge-Murcia, 2012). The data used in this estimation is described in Appendix D.

Discussion of estimation results. Panel B of Table 2 displays the results of the estimation procedure, and Table 3 shows the model-based moments obtained under the results.

The Taylor rule coefficients are in line with standard estimates for them, with the exception of the smoothing parameter (ψ_i^*) which is slightly higher. Justiniano and Preston (2010a) use data until 2007 and obtain $\widehat{\psi}_i^* = 0.85$. Given the mechanical persistence of i^* during the Great Recession, it is not surprising our estimate is higher. This is compounded by the forward guidance structure in (14).

The estimated persistence of the discount rate shocks of ROW pairs to those found in the literature. The standard deviation, though, is considerably lower (by around 5 times) when compared to the one obtained in Justiniano and Preston (2010a). This difference is expected as the mentioned paper does not take into consideration the existence of a lower bound and uses data up to 2007. Compared to studies that match moments in the U.S. and share of ZLB periods, our finding of $\sigma_\nu^* = 0.55$ is near to that of Nakata (2016) that finds a range between 0 and 0.40. Furthermore, our higher estimate is consistent with our estimated moment for the share of periods at the ZLB, which is of 16 percent compared to 6 in Nakata (2016).

By looking at Table 3, we can verify that the estimated parameters discussed above match the relevant moments reasonably well. One exception is the autocorrelation of inflation which is considerably higher in the model.[15] However, it is worth noting the close match to the mean share of quarters the international interest rate is at the ZLB. This moment has very high variance which affects its ability to be matched, thus in general we do not expect it to be as close.

[15] A potential remedy for this is to include γ^* into the parameters to be estimated too.

Table 2: Fixed and estimated parameter values

Panel A: Small open economy			
Coeff.	Description	Value	Source
β	Discount factor	0.99	4% interest rate
α	Openness	0.20	Average import/GDP
η	Elasticity of SOE demand	0.58	Justiniano and Preston (2010b)
ψ_i	Taylor rule, smoothing	0.84	" " " "
ψ_π	Taylor rule, inflation	1.83	" " " "
ψ_y	Taylor rule, output	0.09	" " " "
$\psi_{\Delta y}$	Taylor rule, output growth	0.74	" " " "
$\psi_{\Delta e}$	Taylor rule, nominal depreciation	0.14	" " " "
ρ_{rp}	Risk-premium, persistence	0.94	" " " "
σ_{rp}	Risk-premium, std. deviation	0.35	" " " "
ρ_v	Preferences, persistence	0.93	" " " "
σ_v	Preferences, std. deviation	0.16	" " " "
σ_{cpR}	Cost-push imports, std. deviation	1.58	" " " "
σ_i	Monetary policy, std. deviation	0.26	" " " "
θ_R	Calvo import prices	0.55	" " " "
γ_R	Index. import. prices	0.07	" " " "
χ	Elasticity of risk premium to debt	0.01	" " " "
ρ_a	Technology, persistence	0.93	Justiniano and Preston (2010a)
σ_a	Technology, std. deviation	0.47	" " " "
σ_{cpS}	Cost-push domestic, std. deviation	0.22	" " " "
γ_S	Index. dom. prices in SOE	0.58	" " " "
γ_S^*	Index. dom. prices in ROW	0.58	" " " "
γ_W	Index. wages	0.29	" " " "
θ_S	Calvo domestic prices	0.75	" " " "
θ_W	Calvo wages	0.75	" " " "
Panel B: Rest of the World			
Coeff.	Description	Value	Source
β^*	Discount factor	0.9925	3% interest rate
ψ_i^*	Taylor rule, smoothing	0.94	SMM
ψ_π^*	Taylor rule, inflation	1.38	SMM
ψ_y^*	Taylor rule, output	0.99	SMM
ρ_v^*	Preferences, persistence	0.88	SMM
σ_v^*	Preferences, std. deviation	0.55	SMM
σ_i^*	Monetary policy, std. deviation	0.00	SMM
ρ_a^*	Technology, persistence	0.93	Justiniano and Preston (2010a)
σ_a^*	Technology, std. deviation	0.47	" " " "
σ_{cp}^*	Cost-push, std. deviation	0.22	" " " "
θ^*	Calvo prices	0.75	" " " "
θ_W^*	Calvo wages	0.75	" " " "
γ^*	Index. prices	0.58	" " " "
γ_W^*	Index. wages	0.29	" " " "
λ^*	Elasticity ROW demand	0.58	" " " "

Table 3: Key moments: Data and Model

	Standard deviation				Autocorrelation				Corr w/ output			
	Data	Model	[2,	98]	Data	Model	[2,	98]	Data	Model	[2,	98]
Output (y^*)	1.22	1.56	1.05	2.29	0.89	0.87	0.80	0.92	–	–	–	–
Inflation (π^*)	0.23	0.27	0.22	0.33	0.12	0.62	0.45	0.74	0.45	0.52	0.25	0.73
Interest rate (i^*)	0.32	0.29	0.19	0.39	0.79	0.92	0.88	0.95	0.59	0.51	0.26	0.71
ZLB* (mean)	18.24	15.66	0.00	41.51	–	–	–	–	–	–	–	–

Notes: This table reports the moments used to solve (16), and the simulated moments under $\widehat{\Theta}^R$. It employs quarterly U.S. data between 1980-2019. The total number of simulations is $\mathcal{S} = 1,000$ where each one is of the same length as the data ($T = 160$) with a burning period of 100 quarters. Column headings $[2, 98]$ denote the confidence intervals. The last row is the share of periods that i^* in the data (or simulated model) is at the ZLB.

6 Results

This section quantifies the model's ability to explain the lower correlation between interest rate and inflation in Australia during the international ZLB. It also illustrates the implications of the quantitative model through impulse response functions after a large discount rate shock.

6.1 Correlations in the quantitative model

In order to know how the model performs in explaining what happens in SOE during the international ZLB, I simulate the model and compute the same correlations as in Table 1. To do so, I consider all shocks affecting both the ROW and SOE under the parametrization and structure given in Section 5. For each simulated economy I separate the periods between those when i^* is at the ZLB and those when i^* is not. Then, I calculate the contemporaneous correlation between interest rate and CPI inflation for the SOE, and with imported inflation too. Table 4 presents the comparison between the data and model.

Table 4: Correlation of interest rate and inflation: Data and Model

		Data	Model
$\rho(i, \pi)$	No ZLB*	0.6660	0.7288
	ZLB*	0.3429	0.6454
	% explained	25.82	

Notes: Column 1 of this table reports sample correlations between core year-ended CPI inflation and interest rates for Australia for two periods. The ZLB* period in the data is given by 2008Q4 to 2015Q4. Column 2 reports the correlation for the same elements in the quantitative. The ZLB* period in the model are all the quarters when (14) binds. The number of simulation is 1,000.

The model matches the drop in the correlation between ZLB* and No ZLB*. In particular, it explains around 26% of the drop observed in the data. Given the mechanism I exploit in this model and the extend of the international ZLB, it is reasonable to expect other shocks affecting Australia between 2008 and 2015 that may also play a role in explaining a lower correlation. For instance the flattening of the Phillips Curve in Australia (Ruberl et al., 2021). In addition, Appendix Table D.2 repeats this exercise for the correlation with quarterly inflation.

It is worth mentioning that the parametrization does not intent to match any moments for the SOE. Therefore, the share explained is likely a lower bound of how much this mechanism can explain. If parameters in the SOE were to be estimated such that they can match, on average, moments for Australia, then we would expect this percentage explained to be higher. For instance, the correlation between Australia's interest rate and CPI inflation.

The parameter estimation together with the model's ability to explain a significant part of the drop in the correlation point to the validity of this framework to understand what happens in a SOE when there is an external recession characterized by a binding international interest rate.

6.2 Impulse response functions

To understand the implications the international ZLB have on a SOE, I study impulse response functions stemming from discount rate shocks in two different scenarios. First, the baseline scenario where monetary policy follows (14). Second, an alternative scenario where i_t^* can potentially become negative. It means setting $i_t^* = \tilde{i}_t^*$ always in (14). This resembles scenario (a) in the simple model of Section 3.

To illustrate the differences that arise between both scenarios, this exercise considers an international interest rate that is bound at zero for 15 quarters. This is attained by a one-time disturbance to the discount rate shock of around 30%.[16]

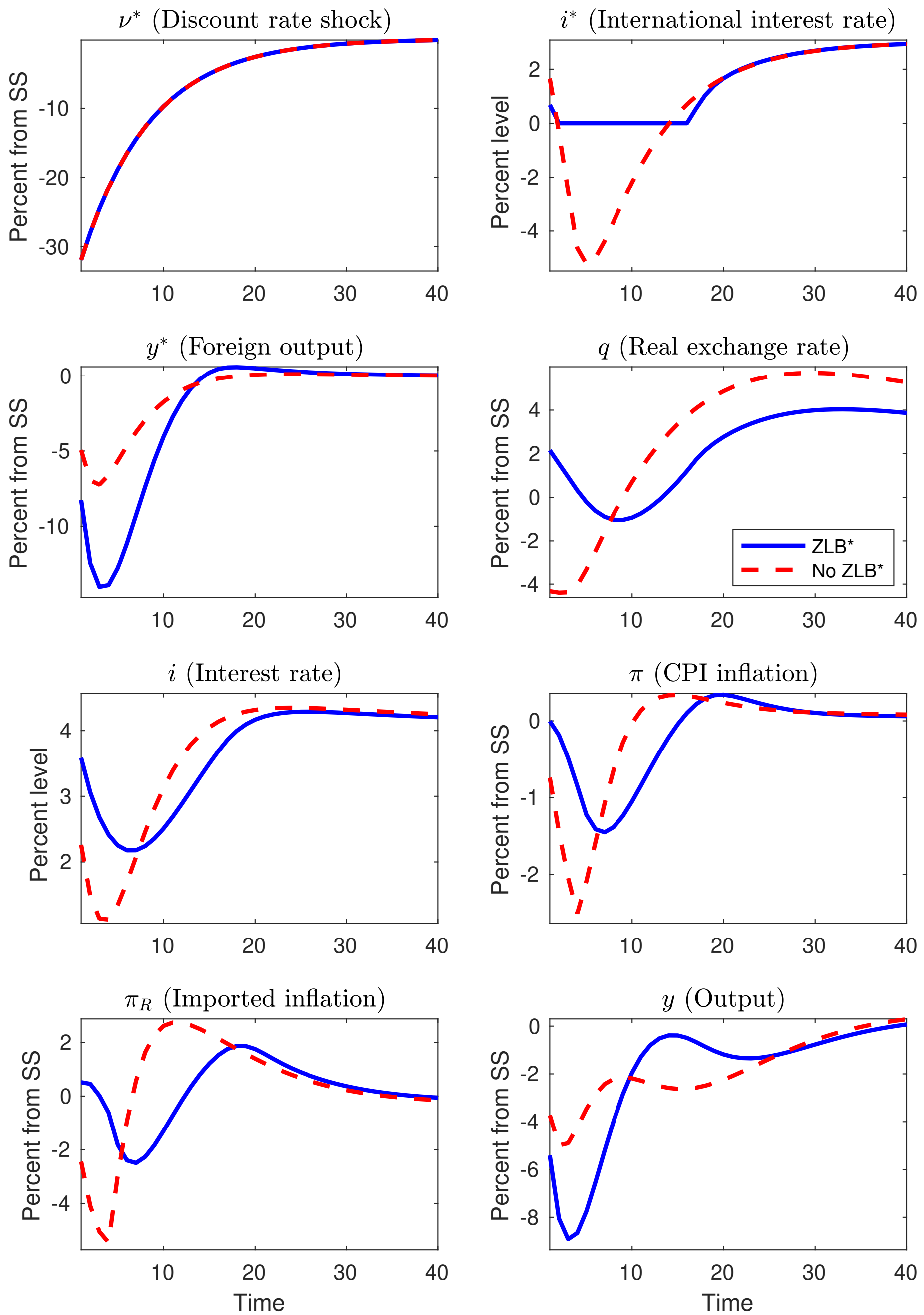
Figure 5 shows impulse responses after a large enough discount rate disturbance under both scenarios. Unsurprisingly, we can observe how the international interest rate goes into negative territory in the No ZLB* scenario. When this occurs, foreign output falls significantly less. With respect to the real exchange rate, we corroborate that the prediction made in Section 3 holds for the quantitative model as well. There is larger depreciation in the No ZLB* scenario with respect to the ZLB* scenario, as the solid line is above the dashed line.[17]

This exercise highlights how the zero bound restriction in the international interest rate affects the ability the SOE has in combating the external recession. It generates less appreciation by a contained drop in international interest rate and produces more depreciation by a larger drop in foreign output.

[16] There is nothing special about 15 quarters. Similar insights are obtained from shorter or longer international ZLB periods.

[17] In addition, there is *depreciation* under ZLB* and *appreciation* under No ZLB*, which is not an implication of the mechanism necessarily. The result from the simple model is that there is *more depreciation* under ZLB*, so their relative orders and not their levels.

Figure 5: Impulse responses to a foreign discount rate shock under ZLB* and No ZLB*



7 Conclusions

This paper studies what happens to small open economies in a context where the international interest rate is bound by zero. In particular, when this occurs as a result of a strong recession in the large economy. To understand what happens and guide the analysis, I study the only recent period when this has occurred. Namely, the period during and after the Great Recession where the Fed's rate, and therefore the international interest rate, was at the ZLB.

Using several SOE and different sources of data, I find that the usual positive relationship between interest rates and inflation weakens, breaks or flips during this period. This is explained by a model where the small economy is affected by two forces that have opposite effects on the exchange rate, which in turn can pass-through inflation. At the international ZLB, the relative size of these forces changes in such a way that breaks the usual positive relationship between inflation and interest rate.

This mechanism is embedded in a medium-size model for Australia and the US. Once the model is estimated and parametrized, it is used to quantitatively measure that 26 percent of the drop in the correlation is explained by this mechanism. Further research is needed to understand other aspects of SOE when the international interest rate is at the ZLB, such as their fiscal response, potential exchange rate interventions, or capital controls that may alleviate the added effect the international ZLB brings about.

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Online Appendix

A Additional results from Section 2

A.1 Data

For the baseline analysis, I use the following data sources:

Table A.1: Data (variables, description, source, period), several countries, 1990-2019

Variable	Description	Source	Period
BIS_CBPOL	Policy rate	BIS	1990M1:2019M12
CPGRLE01	CPI: All items non-food non-energy	OECD	1990Q1:2019Q4
CPALTT01	CPI: All items	OECD	1990Q1:2019Q4

Alternatively, I also use inflation data coming from each country's central bank:

Table A.2: Core CPI inflation measures, several countries, 1990-2019

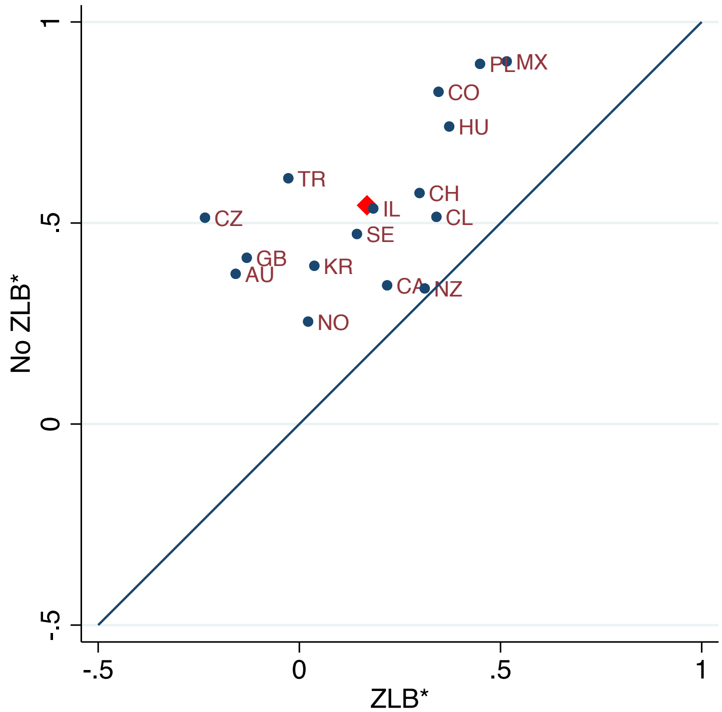
Variable/Table	Country	Source	Period
Table G1	AU - Australia	RBA	1990Q1:2019Q12
Table 18-10-0256-01	CA - Canada	Statistics Canada	1990Q1:2019Q4
TM15	CH - Switzerland	SNB	1990Q1:2019Q4
IPCSAE	CL - Chile	BCCh	1999Q1:2019Q4
Table 2.3.2	CO - Colombia	BanRep	1996Q1:2019Q4
IND9	CZ - Czechia	CNB	1996Q1:2019Q4
Table 7.4.2	KR - South Korea	BoK - ECOS	1990Q1:2019Q4
Table CP151	MX - Mexico	Banxico	1990Q1:2019Q4
Table HM1	NZ - New Zealand	RBNZ	1990Q1:2019Q4

Table A.3: Small open economies

Country	Time	No obs. (max)
AU - Australia	1990Q1:2019Q12	120
CA - Canada	1990Q1:2019Q4	120
CH - Switzerland	1990Q1:2019Q4	120
CL - Chile	1999Q1:2019Q4	84
CO - Colombia	1996Q1:2019Q4	96
CZ - Czechia	1996Q1:2019Q4	96
GB - Great Britain	1990Q1:2019Q4	120
HU - Hungary	1990Q1:2019Q4	120
IL - Israel	1990Q1:2019Q4	120
KR - South Korea	1990Q1:2019Q4	120
MX - Mexico	1990Q1:2019Q4	120
NO - Norway	1990Q1:2019Q4	120
NZ - New Zealand	1990Q1:2019Q4	120
PL - Poland	1996Q1:2019Q4	96
SE - Sweden	1990Q1:2019Q4	120
TR - Turkey	1995Q1:2019Q4	100

A.2 Additional figures and tables

Figure A.1: Correlation of interest rate and quarter-to-quarter core CPI inflation



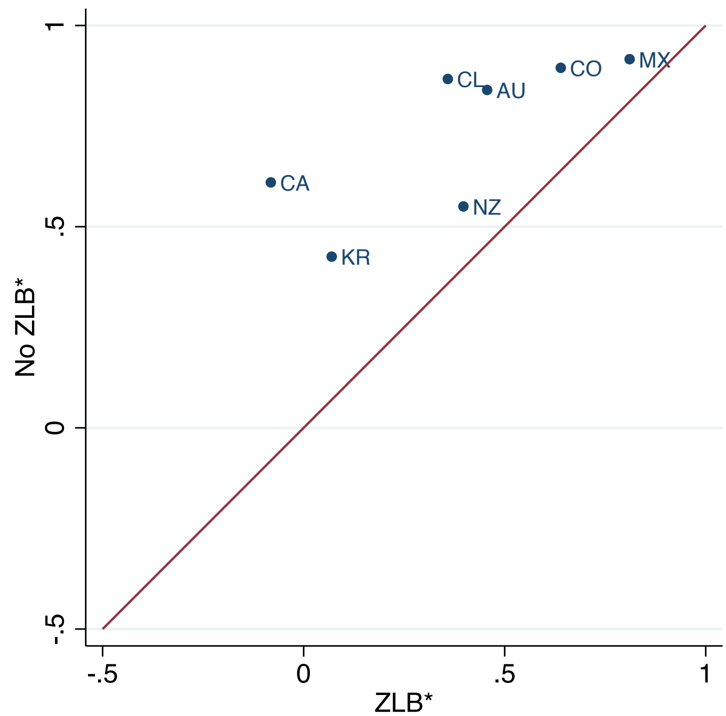
Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Table A.4: Correlation of interest rate and quarter-to-quarter core CPI inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.37	-0.16	IL - Israel	0.54	0.18
	(0.08)	(0.21)		(0.08)	(0.17)
CA - Canada	0.34	0.22	KR - South Korea	0.39	0.04
	(0.09)	(0.17)		(0.11)	(0.19)
CH - Switzerland	0.57	0.30	MX - Mexico	0.90	0.52
	(0.07)	(0.16)		(0.04)	(0.13)
CL - Chile	0.52	0.34	NO - Norway	0.25	0.02
	(0.10)	(0.16)		(0.09)	(0.19)
CO - Colombia	0.83	0.35	NZ - New Zealand	0.34	0.31
	(0.05)	(0.16)		(0.09)	(0.16)
CZ - Czechia	0.51	-0.23	PL - Poland	0.90	0.45
	(0.09)	(0.21)		(0.04)	(0.14)
GB - Great Britain	0.41	-0.13	SE - Sweden	0.47	0.14
	(0.08)	(0.20)		(0.08)	(0.18)
HU - Hungary	0.74	0.37	TR - Turkey	0.61	-0.03
	(0.05)	(0.15)		(0.10)	(0.20)

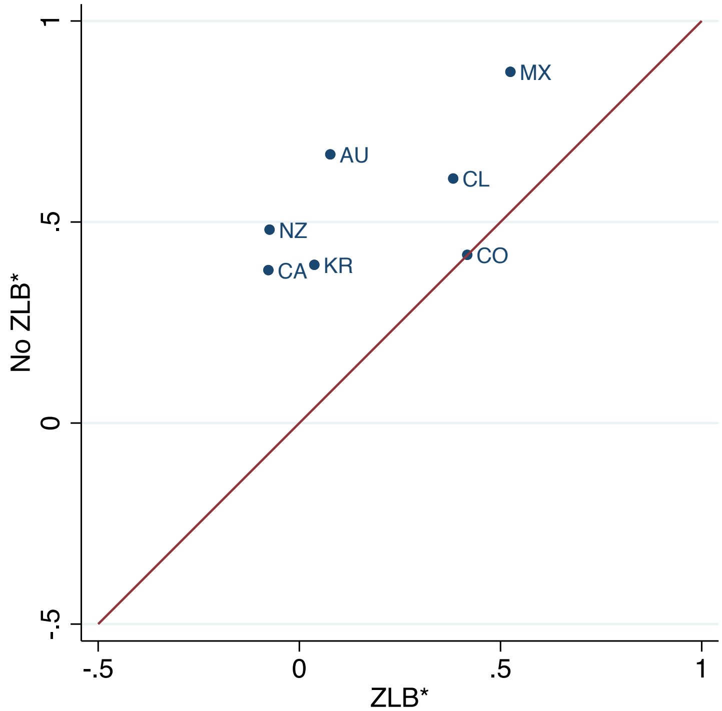
Notes: This figure reports correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1-2019Q4. ZLB*: 2008Q4-2015Q4. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Figure A.2: Correlation of interest rate and year-ended core CPI inflation, Central bank data



Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Figure A.3: Correlation of interest rate and quarter-to-quarter core CPI inflation, Central bank data



Notes: This figure plots correlations between core CPI inflation and interest rates for two periods at quarterly frequency between 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Table A.5: Correlation of interest rate and core CPI inflation, Central bank data

Panel A: Year-ended inflation

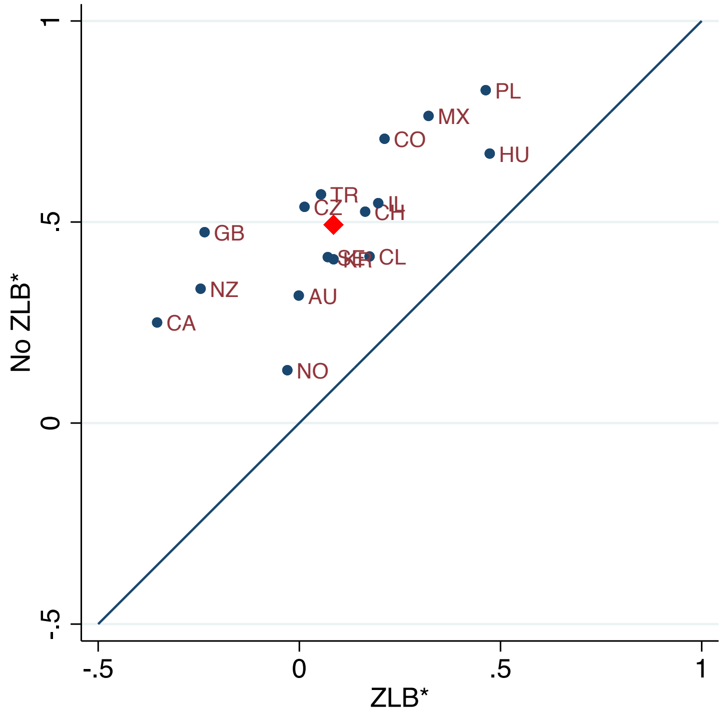
	No ZLB*	ZLB*
AU - Australia	0.84	0.46
	(0.04)	(0.14)
CA - Canada	0.61	-0.08
	(0.07)	(0.20)
CL - Chile	0.87	0.36
	(0.05)	(0.15)
CO - Colombia	0.89	0.64
	(0.04)	(0.12)
KR - South Korea	0.43	0.07
	(0.11)	(0.19)
MX - Mexico	0.92	0.81
	(0.04)	(0.08)
NZ - New Zealand	0.55	0.40
	(0.08)	(0.15)

Panel B: Quarter-to-quarter inflation

	No ZLB*	ZLB*
AU - Australia	0.67	0.08
	(0.06)	(0.18)
CA - Canada	0.38	-0.08
	(0.08)	(0.20)
CL - Chile	0.61	0.38
	(0.08)	(0.15)
CO - Colombia	0.42	0.42
	(0.11)	(0.15)
KR - South Korea	0.39	0.04
	(0.11)	(0.19)
MX - Mexico	0.87	0.52
	(0.05)	(0.13)
NZ - New Zealand	0.48	-0.07
	(0.11)	(0.20)

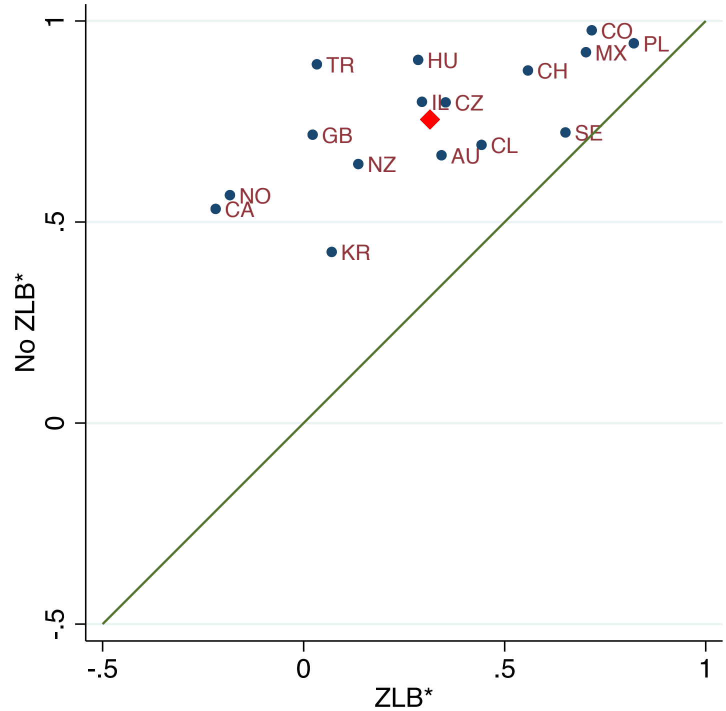
Notes: This table reports correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Figure A.4: Correlation of interest rate and year-ended headline CPI inflation



Notes: This figure plots correlations between headline CPI inflation and interest rates for two periodsat quarterly frequency during 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Figure A.5: Correlation of interest rate and quarter-to-quarter headline CPI inflation



Notes: This figure plots correlations between headline CPI inflation and interest rates for two periodsat quarterly frequency during 1990Q1–2019Q4. ZLB*: 2008Q4–2015Q4. The solid diamond marks the average correlation among all countries.

Table A.6: Correlation of interest rate and headline CPI inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.65	0.62	IL - Israel	0.82	0.64
	(0.06)	(0.12)		(0.05)	(0.12)
CA - Canada	0.53	0.48	KR - South Korea	0.58	0.59
	(0.07)	(0.14)		(0.09)	(0.12)
CH - Switzerland	0.85	0.74	MX - Mexico	0.92	0.77
	(0.04)	(0.10)		(0.04)	(0.09)
CL - Chile	0.70	0.65	NO - Norway	0.32	0.26
	(0.07)	(0.11)		(0.09)	(0.17)
CO - Colombia	0.97	0.89	NZ - New Zealand	0.63	0.10
	(0.02)	(0.06)		(0.06)	(0.18)
CZ - Czech Republic	0.87	0.49	PL - Poland	0.94	0.91
	(0.04)	(0.14)		(0.03)	(0.06)
GB - Great Britain	0.77	0.29	SE - Sweden	0.62	0.72
	(0.05)	(0.16)		(0.07)	(0.10)
HU - Hungary	0.90	0.77	TR - Turkey	0.87	0.30
	(0.03)	(0.09)		(0.06)	(0.16)

Notes: The tables report correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1-2019Q4. ZLB*: 2008Q4-2015Q4. Core inflation comes from country's central bank or statistical agency. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Table A.7: Correlation of interest rate and headline CPI inflation

	No ZLB*	ZLB*		No ZLB*	ZLB*
AU - Australia	0.32	-0.00	IL - Israel	0.55	0.20
	(0.09)	(0.19)		(0.08)	(0.17)
CA - Canada	0.25	-0.35	KR - South Korea	0.41	0.09
	(0.09)	(0.22)		(0.11)	(0.18)
CH - Switzerland	0.53	0.16	MX - Mexico	0.76	0.32
	(0.07)	(0.18)		(0.07)	(0.16)
CL - Chile	0.41	0.17	NO - Norway	0.13	-0.03
	(0.10)	(0.17)		(0.10)	(0.20)
CO - Colombia	0.71	0.21	NZ - New Zealand	0.33	-0.25
	(0.07)	(0.17)		(0.09)	(0.21)
CZ - Czech Republic	0.54	0.01	PL - Poland	0.83	0.46
	(0.08)	(0.19)		(0.05)	(0.14)
GB - Great Britain	0.47	-0.24	SE - Sweden	0.41	0.07
	(0.08)	(0.21)		(0.08)	(0.19)
HU - Hungary	0.67	0.47	TR - Turkey	0.57	0.05
	(0.06)	(0.14)		(0.10)	(0.19)

Notes: The tables report correlations between core CPI inflation and interest rates for two periods at quarterly frequency during 1990Q1-2019Q4. ZLB*: 2008Q4-2015Q4. Core inflation comes from country's central bank or statistical agency. The standard error is given by $\sqrt{(1-r^2)/(n-2)}$, where r is the correlation coefficient and n the sample size.

Table A.8: Policy rate movements, selected countries

	No ZLB*		ZLB*	
	No.	%	No.	%
AU - Australia				
Drop	35	13.6	13	16.6
No change	202	78.6	58	74.4
Hike	20	7.8	7	9.0
CA - Canada				
Drop	21	21.7	6	10.5
No change	56	57.7	48	84.2
Hike	20	20.6	3	5.3
CL - Chile				
Drop	34	20.6	16	18.8
No change	102	61.8	56	65.9
Hike	29	17.6	13	15.29
KR - South Korea				
Drop	15	10.3	10	11.8
No change	117	80.7	70	82.4
Hike	13	9.0	5	5.9
NZ - New Zealand				
Drop	16	15.1	9	15.8
No change	68	64.2	42	73.7
Hike	22	20.1	6	10.5

Notes: This table reports the frequency, and corresponding shares, of policy rate movements during monetary policy meetings. ZLB*: 2008Q4–2015Q4.

B Additional results from Section 3

B.1 Model under Taylor rule

Here I present a modified version of the two-period SOE model in Section 3. Instead of optimal monetary policy in SOE, here I assume that SOE central bank follows a Taylor rule:

$$i_t = \rho + \psi_\pi \pi_t + \psi_y y_t,$$

Then, the equations that define the equilibrium are given by:

$$\begin{aligned}
y_0 &= (1-\alpha)c_0 + \alpha y_0^* + \tilde{\alpha} q_0, \\
c_0 &= y_0^* + q_0 - \nu_0^*, \\
\pi_{S,0} &= \kappa\left(c_0 + \varphi y_0 + \frac{\alpha}{1-\alpha} q_0\right), \\
\pi_0 &= \pi_{S,0} + \frac{\alpha}{1-\alpha} q_0, \\
i_0 &= \rho + \psi_y y_0 + \psi_\pi \pi_0, \\
c_0 &= -(i_0 - E_0 \pi_1 - \rho).
\end{aligned}$$

The corresponding demand and supply functions are now:

$$q_0 = (1-\alpha)(y_0 - y_0^* + (1-\alpha)\nu_0^*) \tag{DD'}$$

$$q_0 = -\frac{1-\alpha}{A}\left[By_0 + (1+\kappa\psi_\pi)(y_0^* - \nu_0^*)\right] \tag{SS'}$$

where $A \equiv (\kappa + \alpha)\psi_\pi + 1 > 1$ and $B \equiv \psi_y + \kappa\varphi\psi_\pi \geq 0$. Curve (DD') is the same as curve (DD), but now the supply curve (SS') depends on how the Taylor rule responds to inflation and output. Note that if we let $\psi_\pi = 0$ and $\psi_y = \varphi'$, we are back at the baseline scenario.

These curves can be used to find the equilibrium as function of foreign shocks, y_0^* and ν_0^*:

$$\begin{aligned} y_0 &= \frac{\alpha}{A+B}(A-\psi_\pi)\nu_0^* + \frac{\alpha\psi_\pi}{A+B}y_0^* \\ q_0 &= (1-\alpha)\left[\frac{\kappa\psi_\pi + 1 + B(1-\alpha)}{A+B}\nu_0^* - \frac{\kappa\psi_\pi + 1 + B}{A+B}y_0^*\right]. \end{aligned} \tag{B.1}$$

Compared to the baseline, now the response on output from external shocks is ambiguous and depends on the parameters. And, when going from a No ZLB* to a ZLB* scenario, output falls.

In contrast to what happens to output, real exchange rate reacts in the same way as it did in the baseline when we compare a No ZLB* to a ZLB* scenario. From (B.1), we can observe that a depreciation takes place for the whole parameter space. In fact,

$$q_{Z,0} - q_{N,0} = -\frac{\kappa\psi_\pi + 1 + B}{A+B}\underbrace{(y_{Z,0}^* - y_{N,0}^*)}_{<0} > 0.$$

In addition, we can compute domestic inflation, CPI inflation and interest rate:

$$\begin{aligned} \pi_{S,0} &= \kappa\left[\frac{\alpha\psi_\pi(1+\varphi)}{A+B}y_0^* + \frac{\alpha(\varphi A - B) - \alpha\psi_\pi(1+\varphi)}{A+B}\nu_0^*\right] \\ \pi_0 &= -\alpha\frac{\psi_y + 1}{A+B}y_0^* + \alpha\frac{\kappa(\varphi A - B) + \psi_y + 1 - \alpha B}{A+B}\nu_0^* \\ i_0 &= \rho + \frac{\alpha}{A+B}\left[(\psi_\pi + B)\nu_0^* - \psi_\pi y_0^*\right] \end{aligned}$$

As predicted in the baseline model, CPI inflation increases from a No ZLB* to a ZLB* scenario. Now, interest rate increases even if the drop in output is stronger in the No ZLB* scenario. However, because CPI inflation is rising, while output is falling, the increase in interest rate is substantially smaller when faced with negative foreign output shocks, $y_0^* < 0$. We can compare:

$$\frac{\partial i_0}{\partial y_0^*} = -\alpha\frac{\psi_\pi}{A+B} \quad > \quad \frac{\partial \pi_0}{\partial y_0^*} = -\alpha\frac{\psi_\pi + 1}{A+B}$$

B.2 Derivation of optimal target rule in SOE

The second-order approximation to the utility function of SOE households in the two-period model corresponds to:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[c_t - \frac{y_t}{\mu} - \frac{1}{2}\frac{1+\varphi}{\mu} y_t^2 - \frac{1}{2}\frac{\epsilon}{\kappa\mu}\pi_{S,t}^2 \right] + t.i.p. + \mathcal{O}(\|\zeta\|^3).$$

The problem that the monetary authority at SOE solves is:

$$\max_{c_t, y_t, q_t, \pi_{S,t}} U_0 \quad \text{subject to system in (8)}$$

The FOC are:

$$\begin{aligned} 1 - \kappa\lambda_{1,t} + \lambda_{2,t} - (1-\alpha)\lambda_{3,t} &= 0 \\ \lambda_{1,t}\kappa\frac{\alpha}{1-\alpha} + \lambda_{2,t} + \tilde{\alpha}\lambda_{3,t} &= 0 \\ -\frac{1}{\mu} - \frac{1+\varphi}{\mu} y_t - \kappa\varphi\lambda_{1,t} + \lambda_{3,t} &= 0 \\ -\frac{\epsilon}{\kappa\mu}\pi_{S,t} + \lambda_{1,t} - \lambda_{1,t-1} &= 0 \end{aligned}$$

Solving this system and imposing $\mu = \frac{1}{1-\alpha}$, we arrive to (9) in the main text.

B.3 Model with money-in-the-utility

Consider the following utility function,

$$U(C_t^*, M_t^*/P_t^*, N_t^*) = \log C_t^* + \log\left(\frac{M_t^*}{P_t^*}\right) - \frac{N_t^{*1+\varphi}}{1+\varphi}$$

where M_t^* are money holdings. The budget constraint in this context is:

$$P_t^* C_t^* + \mathbb{Q}_t^* B_t^* + M_t^* = W_t^* N_t^* + B_{t-1}^* + M_{t-1}^* - T_t^* + \Gamma_t^*.$$

From the first-order conditions we can derive the following demand function:

$$\frac{M_t^*}{P_t^*} = \frac{Y_t^*}{1 - \mathbb{Q}_t^*}.$$

Doing a first-order log approximation we can arrive to:

$$l_t^* \equiv m_t^* - p_t^* = y_t^* - \frac{1}{\rho} i_t^*.$$

Then, by combining the Euler equation and Phillips curve in (8), together with the demand for money and noting that, $l^*_{t-1} = l^*_t + \pi^*_t - \Delta m^*_t$, we can derive the following system:

$$\underbrace{\begin{pmatrix} 1+1/\rho & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}}_{A_0} \begin{pmatrix} y^*_t \\ \pi^*_t \\ l^*_{t-1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1/\rho & 1/\rho & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{A_1} E_t \begin{pmatrix} y^*_{t+1} \\ \pi^*_{t+1} \\ l^*_t \end{pmatrix} + \underbrace{\begin{pmatrix} 1/\rho & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}}_{B} \begin{pmatrix} \rho - E_t \Delta v^*_{t+1} \\ \Delta m^*_t \end{pmatrix}$$

It can be shown that, for any relevant parametrization, $A_0^{-1}A_1$ has two eigenvalues inside the unit circle and one outside. This means that there is a unique and stationary solution in the system.

Under initial conditions, $l^*_{-1} = m_{-1} = 0$, the ZLB* equilibrium in (11) is achieved by setting: $m^*_{N,0} = -(\rho + v^*_0)/\rho$. The No ZLB* equilibrium in (12) is achieved by setting: $m^*_{Z,0} = (\kappa + 1)(\rho + v^*_0)$.

B.4 Derivation of (DD) and (SS)

To solve for all the endogenous variables $(c_0, q_0, y_0, \pi_{S,0})$ we proceed as follows. First, we replace the optimal rule into the NKPC:

$$\begin{aligned} -\frac{1}{\epsilon} y_0 &= \kappa \left(c_0 + \varphi y_0 + \frac{\alpha}{1-\alpha} q_0 \right) \\ 0 &= \left(\kappa \frac{\alpha}{1-\alpha} \right) q_0 + \left(\kappa\varphi + \frac{1}{\epsilon} \right) y_0 + \kappa c_0 \end{aligned}$$

Now we replace the risk-sharing condition into the expression above and solve for q_0:

$$\begin{aligned} 0 &= \left(\kappa \frac{\alpha}{1-\alpha} \right) q_0 + \left(\kappa\varphi + \frac{1}{\epsilon} \right) y_0 + \kappa \left(y^*_0 + q_0 - v^*_0 \right) \\ 0 &= \frac{1}{1-\alpha} q_0 + \varphi' y_0 + (y^*_0 - v^*_0) \\ q_0 &= -(1-\alpha) \left(\varphi' y_0 + y^*_0 - v^*_0 \right) \end{aligned} \tag{SS}$$

where $\varphi' \equiv \varphi + \frac{1}{\epsilon\kappa}$. Now, the risk sharing condition into the demand function:

$$\begin{aligned} y_0 &= (1-\alpha)(y^*_0 + q_0 - v^*_0) + \alpha y^*_0 + \tilde{\alpha} q_0 \\ [(1-\alpha) + \tilde{\alpha}]\, q_0 &= y_0 - y^*_0 + (1-\alpha) v^*_0 \\ q_0 &= (1-\alpha)\big(y_0 - y^*_0 + (1-\alpha) v^*_0 \big) \end{aligned} \tag{DD}$$

This leads to,

$$y_0 = \frac{\alpha}{1+\varphi'} v^*_0 \quad \text{and} \quad q_0 = (1-\alpha)\frac{1+\varphi'(1-\alpha)}{1+\varphi'} v^*_0 - (1-\alpha) y^*_0 \quad \text{and} \quad \pi_{S,0} = -\frac{1}{\epsilon}\frac{\alpha}{1+\varphi'} v^*_0 \tag{B.2}$$

B.5 Additional figures

Figure B.1: Negative ROW discount rate shock under ZLB* scenario and $y_0^* << 0$

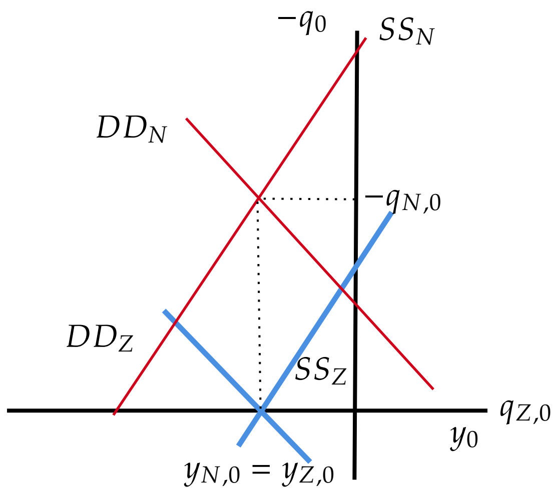
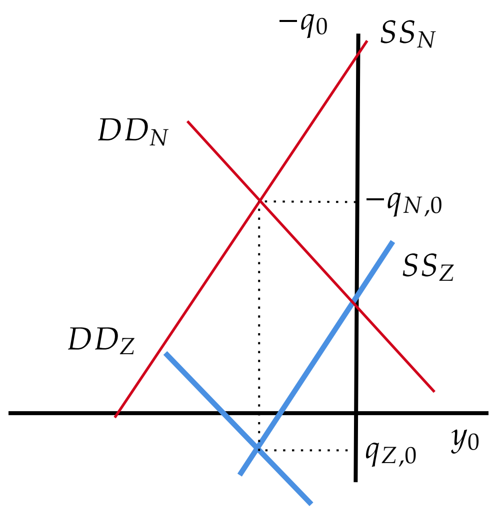


Figure B.2: Negative ROW discount rate shock under ZLB* scenario and $y_0^* <<< 0$



C Additional results from Section 4

C.1 Derivation of first-order approximations

C.1.1 Domestic firms selling in SOE

Taking the first-order condition with respect to $\tilde{P}_{S,t}$ leads to:

$$E_t \sum_{T=t}^{\infty} \theta_S^{T-t} \mathbb{Q}_{t,T} y_{S,T|t}(i) \left[\left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S} - \frac{\epsilon}{\epsilon-1} \frac{W_T}{\xi_{a,T}} \frac{1}{\tilde{P}_{S,t}} \right] = 0$$

$$\tilde{P}_{S,t} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \frac{1}{P_T} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S} y_{S,T|t}(i) = \frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \overline{MC}_T y_{S,T|t}(i)$$

$$\frac{\tilde{P}_{S,t}}{P_{S,t}} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \frac{P_{S,t}}{P_T} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S(1-\epsilon)} {P_{S,T}}^{\epsilon} C_{S,T} = \frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \overline{MC}_T \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{-\gamma_S\epsilon} {P_{S,T}}^{\epsilon} C_{S,T}$$

$$\frac{\tilde{P}_{S,t}}{P_{S,t}} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \frac{\nu_T}{C_T^{\sigma}} Q_T \overline{P}_{S,T} \left(\frac{P_{S,T}}{P_{S,t}} \right)^{\epsilon-1} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S(1-\epsilon)} C_{S,T} = \frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \frac{\nu_T}{C_T^{\sigma}} \overline{MC}_T \left(\frac{P_{S,T}}{P_{S,t}} \right)^{\epsilon} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{-\gamma_S\epsilon} C_{S,T}$$

where $\overline{MC}_T = \frac{1}{\xi_{a,T}} \frac{W_T}{P_T}$ and $\overline{P}_{S,T} = \frac{P_{S,T}}{P_T}$.

Doing a first-order approximation:

$$\frac{1}{1-\beta\theta_S} (\tilde{p}^*_{S,t} - p^*_{S,t}) = E_t \sum_{T \geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}_{S,T} + (p_{S,T} - p_{S,t}) - \gamma_S (p_{S,T-1} - p_{S,t-1}) \right)$$
$$= E_t \sum_{T \geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}_{S,T} \right) + \frac{\beta\theta_S}{1-\beta\theta_S} E_t \sum_{T \geq t} (\beta\theta_S)^{T-t} (\pi_{S,T+1} - \gamma_S \pi_{S,T})$$

The price index for exported goods in ROW:

$${P_{S,t}}^{1-\epsilon} = \left[(1-\theta_S) {\tilde{P}_{S,t}}^{1-\epsilon} + \theta_S \left(P^*_{S,t-1} \left(\frac{P_{S,t-1}}{P_{S,t-2}} \right)^{\gamma_S} \right)^{1-\epsilon} \right]$$

$$\frac{\tilde{P}_{S,t}}{P^*_{S,t}} = \left[\frac{1 - \theta_S {\Pi_{S,t}}^{\epsilon-1} \Pi_{S,t-1}^{\gamma_S(1-\epsilon)}}{1-\theta_S} \right]^{\frac{1}{1-\epsilon}}$$

Doing a first-order approximation:

$$\tilde{p}_{S,t} - p_{S,t} = \frac{\theta_S}{1-\theta_S} (\pi_{S,t} - \gamma_S \pi_{S,t-1})$$

Putting both first-order approximations together:

$$\frac{\theta_S}{(1-\beta\theta_S)(1-\theta_S)} (\pi_{S,t} - \gamma_S \pi_{S,t-1}) = \overline{mc}_t - \overline{p}_{S,t} - q_t + \frac{\beta\theta_S}{1-\beta\theta_S} E_t (\pi_{S,t+1} - \gamma_S \pi_{S,t}) + E_t \sum_{T \geq t+1} (\beta\theta_S)^{T-t} \big(\overline{mc}_T$$
$$- \overline{p}_{S,T} - q_T \big) + \frac{\beta\theta_S}{1-\beta\theta_S} E_t \sum_{T \geq t+1} (\beta\theta_S)^{T-t} (\pi_{S,T+1} - \gamma_S \pi_{S,T})$$

Doing it in $t+1$, multiplied by $\beta\theta$ and then replaced back into the original equation leads to:

$$\pi_{S,t} - \gamma_S \pi_{S,t-1} = \kappa_S(\overline{mc}_t - \overline{p}_{S,t}) + \beta E_t(\pi_{S,t+1} - \gamma_S \pi_{S,t})$$

And, by looking at the price index,

$$\begin{aligned} P_t^{1-\eta} &= (1-\alpha)P_{S,t}^{1-\alpha} + \alpha P_{R,t}^{1-\alpha} \\ 1 &= (1-\alpha)\left(\frac{P_{S,t}}{P_t}\right)^{1-\eta} + \alpha\left(\frac{P_{R,t}}{P_t}\right)^{1-\eta} \\ \Rightarrow \bar{p}_{S,t} = p_{S,t} - p_t &= -\frac{\alpha}{1-\alpha}(q_t + \psi_t^*) \end{aligned}$$

C.1.2 Domestic firms selling in ROW

Taking the first-order condition with respect to $\tilde{P}^*_{S,t}$ leads to:

$$E_t \sum_{T=t}^{\infty} \theta_S^{T-t} \mathbb{Q}_{t,T} y^*_{S,T|t}(i) \left[\mathcal{E}_t \left(\frac{P^*_{S,T-1}}{P^*_{S,t-1}} \right)^{\gamma_S^*} - \frac{\epsilon}{\epsilon-1} \frac{W_T}{\xi_{a,T}} \frac{1}{\tilde{P}^*_{S,t}} \right] = 0$$

$$\tilde{P}^*_{S,t} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \frac{\mathcal{E}_T}{P_T} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S^*} y^*_{S,T|t}(i) = \frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \overline{MC}_T y^*_{S,T|t}(i)$$

$$\frac{\tilde{P}^*_{S,t}}{P^*_{S,t}} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} Q_T \frac{P^*_{S,t}}{P^*_T} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S^*(1-\epsilon)} {P^*_{S,T}}^{\epsilon} C^*_{S,T} = \frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \nu_T C_T^{-\sigma} \overline{MC}_T \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{-\gamma_S^*\epsilon} {P^*_{S,T}}^{\epsilon} C^*_{S,T}$$

$$\frac{\tilde{P}^*_{S,t}}{P^*_{S,t}} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \frac{\nu_T}{C_T^{\sigma}} Q_T \overline{P}^*_{S,T} \left(\frac{P^*_{S,T}}{P^*_{S,t}} \right)^{\epsilon-1} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{\gamma_S^*(1-\epsilon)} C^*_{S,T} = \frac{\epsilon}{\epsilon-1} E_t \sum_{T=t}^{\infty} (\beta\theta_S)^{T-t} \frac{\nu_T}{C_T^{\sigma}} \overline{MC}_T \left(\frac{P^*_{S,T}}{P^*_{S,t}} \right)^{\epsilon} \left(\frac{P_{S,T-1}}{P_{S,t-1}} \right)^{-\gamma_S^*\epsilon} C^*_{S,T}$$

where $\overline{MC}_T = \frac{1}{\xi_{a,T}} \frac{W_T}{P_T}$ and $\overline{P}^*_{S,T} = \frac{P^*_{S,T}}{P^*_T}$.

Doing a first-order approximation:

$$\begin{aligned} \frac{1}{1-\beta\theta_S}(\tilde{p}^*_{S,t} - p^*_{S,t}) &= E_t \sum_{T\geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}^*_{S,T} - q_T + (p^*_{S,T} - p^*_{S,t}) - \gamma_S^*(p^*_{S,T-1} - p^*_{S,t-1}) \right) \\ &= E_t \sum_{T\geq t} (\beta\theta_S)^{T-t} \left(\overline{mc}_T - \overline{p}^*_{S,T} - q_T \right) + \frac{\beta\theta_S}{1-\beta\theta_S} E_t \sum_{T\geq t} (\beta\theta_S)^{T-t} (\pi^*_{S,T+1} - \gamma_S^* \pi^*_{S,T}) \end{aligned}$$

The price index for exported goods in ROW:

$$\begin{aligned} {P^*_{S,t}}^{1-\epsilon} &= \left[(1-\theta_S){\tilde{P}^*_{S,t}}^{1-\epsilon} + \theta_S \left(P^*_{S,t-1} \left(\frac{P^*_{S,t-1}}{P^*_{S,t-2}} \right)^{\gamma_S^*} \right)^{1-\epsilon} \right] \\ \frac{\tilde{P}^*_{S,t}}{P^*_{S,t}} &= \left[\frac{1 - \theta_S {\Pi^*_{S,t}}^{\epsilon-1} \Pi_{S,t-1}^{\gamma_S^*(1-\epsilon)}}{1-\theta_S} \right]^{\frac{1}{1-\epsilon}} \end{aligned}$$

Doing a first-order approximation:

$$\tilde{p}^*_{S,t} - p^*_{S,t} = \frac{\theta_S}{1-\theta_S}(\pi^*_{S,t} - \gamma^*_S \pi^*_{S,t-1})$$

Putting both first-order approximations together:

$$\frac{\theta_S}{(1-\beta\theta_S)(1-\theta_S)}(\pi^*_{S,t} - \gamma^*_S\pi^*_{S,t-1}) = \overline{mc}_t - \overline{p}^*_{S,t} - q_t + \frac{\beta\theta_S}{1-\beta\theta_S}E_t(\pi^*_{S,t+1} - \gamma^*_S\pi^*_{S,t}) + E_t \sum_{T\geq t+1}(\beta\theta_S)^{T-t}\big(\overline{mc}_T - \overline{p}^*_{S,T} - q_T\big) + \frac{\beta\theta_S}{1-\beta\theta_S}E_t\sum_{T\geq t+1}(\beta\theta_S)^{T-t}(\pi^*_{S,T+1} - \gamma^*_S\pi^*_{S,T})$$

Doing it in $t+1$, multiplied by $\beta\theta$ and then replaced back into the original equation leads to:

$$\pi^*_{S,t} - \gamma^*_S\pi^*_{S,t-1} = \kappa_S(\overline{mc}_t - \overline{p}^*_{S,t} - q_t) + \beta E_t(\pi^*_{S,t+1} - \gamma^*_S\pi^*_{S,t})$$

where

$$\begin{aligned}\overline{p}^*_{S,t} + q_t &= \psi_t + (p_{S,t} - p_t) \\ &= \psi_t - \frac{\alpha}{1-\alpha}(q_t + \psi^*_t).\end{aligned}$$

C.1.3 Importing firms

Taking the first-order condition with respect to $\tilde{P}_{R,t}$ leads to:

$$E_t\sum_{T=t}^{\infty}\theta_R^{T-t}\mathbb{Q}_{t,T}y_{R,T|t}(i)\left[\left(\frac{P_{R,T-1}}{P_{R,t-1}}\right)^{\gamma_R} - \frac{\epsilon}{\epsilon-1}\mathcal{E}_t\frac{P^*_{R,t}(i)}{\tilde{P}_{R,t}}\right] = 0$$

$$\tilde{P}_{R,t}E_t\sum_{T=t}^{\infty}(\beta\theta_R)^{T-t}\nu_T C_T^{-\sigma}\frac{1}{P_T}\left(\frac{P_{R,T-1}}{P_{R,t-1}}\right)^{\gamma_R}y_{R,T|t}(i) = \frac{\epsilon}{\epsilon-1}E_t\sum_{T=t}^{\infty}(\beta\theta_R)^{T-t}\nu_T C_T^{-\sigma}\mathcal{E}_t\frac{P^*_t}{P_t}y_{R,T|t}(i)$$

$$\frac{\tilde{P}_{R,t}}{P_{R,t}}E_t\sum_{T=t}^{\infty}(\beta\theta_R)^{T-t}\nu_T C_T^{-\sigma}\frac{P_{R,t}}{P_{R,T}}\overline{P}_{R,T}\left(\frac{P_{R,T-1}}{P_{R,t-1}}\right)^{\gamma_R}y_{R,T|t}(i) = \frac{\epsilon}{\epsilon-1}E_t\sum_{T=t}^{\infty}(\beta\theta_R)^{T-t}\nu_T C_T^{-\sigma}\frac{\overline{P}_{R,T}}{\Psi^*_t}y_{R,T|t}(i)$$

where $\overline{P}^*_{S,T} = \frac{P^*_{S,T}}{P^*_T}$. A first-order approximation:

$$\begin{aligned}\frac{1}{1-\beta\theta_R}(\tilde{p}_{R,t} - p_{R,t}) &= E_t\sum_{T\geq t}(\beta\theta_R)^{T-t}\left[(p_{R,T} - p_{R,t}) - \gamma_R(p_{R,T-1} - p_{R,t-1}) - \psi^*_t\right] \\ &= E_t\sum_{T\geq t}(\beta\theta_R)^{T-t}(-\psi^*_t) + \frac{\beta\theta_R}{1-\beta\theta_R}E_t\sum_{T\geq t}(\beta\theta_R)^{T-t}(\pi_{R,T+1} - \gamma_R\pi_{R,T})\end{aligned}$$

Following similar steps to the ones above lead to the NKPC for importing goods.

C.1.4 Optimal labor supply

Taking the first-order condition with respect to $\tilde{W}_t$ leads to:

$$
\begin{aligned}
E_t \sum_{T=t} (\theta_W \beta)^{T-t} N_{T|t}(k) \left[\frac{\tilde{W}_t}{P_T} \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\gamma_W} \nu_t u_{1,T} - \frac{\epsilon_W}{\epsilon_W - 1} v_{1,T}(N_{T|t}(k)) \right] &= 0 \\
E_t \sum_{T=t} (\theta_W \beta)^{T-t} N_{T|t}(k) \left[\frac{\tilde{W}_t}{W_t} \frac{W_t}{P_T} \Pi_{T-1,t-1}^{\gamma_W} \nu_t u_{1,T} - \frac{\epsilon_W}{\epsilon_W - 1} v_{1,T}(N_{T|t}(k)) \right] &= 0
\end{aligned}
$$

Doing a first-order approximation:

$$
\frac{1}{1-\theta_W \beta}(\tilde{w}_t - w_t) + E_t \sum_{T=t}^{\infty} (\theta_W \beta)^{T-t} \left[\zeta_T + \gamma_W \pi_{T-1,t-1} + \nu_t + \hat{u}_{1,T} - \hat{v}_{1,T|t} - \sum_{k=1}^{T-t} \pi_{W,t+k} \right]
$$

where

$$
\begin{aligned}
v_{1,T}(N_{T|t}(k)) &= N_{T|t}(k)^{\varphi} = \left[\left(\frac{\tilde{W}_t}{W_t} \cdot \frac{W_t}{W_T} \right)^{-\theta_W} N_T \right]^{\varphi} \\
\Rightarrow \hat{v}_{1,T|t} &= -\theta_W \varphi \left[(\tilde{w}_t - w_t) + (w_t - w_T) \right] + \varphi n_T \\
&= -\theta_W \varphi \left[(\tilde{w}_t - w_t) - \sum_{k=1}^{T-t} \pi_{W,t+k} \right] + \varphi (y_T - \xi_{a,T})
\end{aligned}
$$

Back into the first order approximation:

$$
\frac{1+\theta_W \varphi}{1-\theta_W \beta}(\tilde{w}_t - w_t) + E_t \sum_{T=t}^{\infty} (\theta_W \beta)^{T-t} \left[\zeta_T + \gamma_W \pi_{T-1,t-1} + \nu_t + \hat{u}_{1,T} - \varphi n_T - -(1+\theta_W \varphi) \sum_{k=1}^{T-t} \pi_{W,t+k} \right] = 0
$$

$$
\begin{aligned}
\frac{\theta_W}{(1-\theta_W \beta)(1-\theta_W)}(\pi_{W,t} - \gamma_W \pi_{t-1}) + \frac{1}{1+\theta_W \varphi} E_t \sum_{T=t}^{\infty} (\theta_W \beta)^{T-t} (\zeta_T + \nu_t + \hat{u}_{1,T} - \varphi n_T -) \\
- E_t \sum_{T=t}^{\infty} (\theta_W \beta)^{T-t} \sum_{k=1}^{T-t} \pi_{W,t+k} + \gamma_W E_t \sum_{T=t}^{\infty} (\theta_W \beta)^{T-t} \pi_{T-1,t-1} = 0
\end{aligned}
$$

Then, in $t+1$:

$$
\begin{aligned}
\frac{\theta_W}{(1-\theta_W \beta)(1-\theta_W)}(\pi_{W,t+1} - \gamma_W \pi_t) + \frac{1}{1+\theta_W \varphi} E_{t+1} \sum_{T=t+1}^{\infty} (\theta_W \beta)^{T-t-1} (\zeta_T + \nu_t + \hat{u}_{1,T} - \varphi n_T -) \\
- E_{t+1} \sum_{T=t+1}^{\infty} (\theta_W \beta)^{T-t-1} \sum_{k=1}^{T-t-1} \pi_{W,t+1+k} + \gamma_W E_{t+1} \sum_{T=t+1}^{\infty} (\theta_W \beta)^{T-t-1} \pi_{T-1,t} = 0 \\
\frac{\theta_W^2 \beta}{(1-\theta_W \beta)(1-\theta_W)}(E_t \pi_{W,t+1} - \gamma_W \pi_t) + \frac{1}{1+\theta_W \varphi} E_t \sum_{T=t+1}^{\infty} (\theta_W \beta)^{T-t-1} (\zeta_T + \nu_t + \hat{u}_{1,T} - \varphi n_T) \\
- E_t \sum_{T=t}^{\infty} (\theta_W \beta)^{T-t} \sum_{k=1}^{T-t} \pi_{W,t+k} + \frac{\theta_W \beta}{1-\theta_W \beta} E_t \pi_{W,t+1} + \gamma_W E_t \sum_{T=t}^{\infty} (\theta_W \beta)^{T-t} \pi_{T-1,t-1} - \frac{\gamma_W \theta_W \beta}{1-\theta_W \beta} = 0
\end{aligned}
$$

Substracting the last terms of expressions in t and $t+1$:

$$\frac{\theta_W}{(1-\theta_W\beta)(1-\theta_W)}\left[(\pi_{W,t}-\gamma_W\pi_{t-1})-\theta_W\beta(E_t\pi_{W,t+1}-\gamma_W\pi_t)\right]-\frac{\theta_W\beta}{1-\theta_W\beta}E_t\pi_{W,t+1}$$
$$+\frac{1}{1+\theta_W\varphi}(\zeta_t+\nu_t+\hat{u}_{1,t}-\varphi n_t-)+\frac{\gamma_W\theta_W\beta}{1-\theta_W\beta}\pi_t = 0$$
$$\Rightarrow \pi_{W,t}-\gamma_w\pi_{t-1}=\beta(E_t\pi_{W,t+1}-\gamma_W\pi_t)+\kappa_W\left[\varphi y_t-\varphi\xi_{a,t}+\nu_t+\frac{\sigma}{1-h}(c_t-hc_{t-1})-\zeta_t\right]$$

where $\kappa_W=\frac{(1-\theta_W\beta)(1-\theta_W)}{\theta_W(1+\epsilon_W\varphi)}$

C.2 Log-linearized equilibrium

The ROW is characterized by the following set of equations:

1. Euler equation

$$c_t^*-h^*c_{t-1}^*=E_t(c_{t+1}^*-h^*c_t^*)-\sigma^{*-1}(1-h^*)(i_t^*-E_t\pi_{t+1}^*+E_t\Delta\nu_{t+1}^*)$$

2. Firms price setting

$$\pi_t^*-\gamma^*\pi_{t-1}^*=\beta^*E_t(\pi_{t+1}^*-\gamma^*\pi_t^*)+\kappa^*(\zeta_t^*-\xi_{a,t}^*)+\xi_{cp,t}^*$$

where $\kappa^*=\frac{(1-\theta^*)(1-\beta^*\theta^*)}{\theta^*}$

3. Household price setting

$$\pi_{W,t}^*-\gamma_W^*\pi_{t-1}^*=\beta^*E_t(\pi_{W,t+1}^*-\gamma_W^*\pi_t^*)+\kappa_W^*\left[\varphi^*y_t^*-\varphi^*\xi_{a,t}^*+\frac{\sigma^*}{1-h^*}(c_t^*-h^*c_{t-1}^*)-\zeta_t^*\right]$$

where $\kappa_W^*=\frac{(1-\theta^*)(1-\beta^*\theta^*)}{\theta^*}\frac{1}{1+\epsilon_W^*\varphi^*}$

4. Real wages law of motion

$$\zeta_t^*=\zeta_{t-1}^*+\pi_{W,t}^*-\pi_t^*$$

5. Monetary policy

$$\tilde{i}_t^*=\psi_i^*i_{t-1}^*+(1-\psi_i^*)\left[\psi_\pi^*(\pi_t^*+\pi_{t-1}^*+\pi_{t-2}^*+\pi_{t-3}^*)+\psi_y^*(y_t^*-y_{t-4}^*)\right]+\xi_{i,t}^*$$
$$i_t^*=\max\{0,\tilde{i}_t^*\}$$

The SOE is characterized by the following set of equations:

1. Euler equation:

$$c_t-hc_{t-1}=E_t(c_{t+1}-hc_t)-\sigma^{-1}(1-h)(i_t-E_t\pi_{t+1}-E_t\Delta\nu_{t+1})$$

2. Market clearing:

$$y_t = (1-\alpha)c_t + \alpha y_t^* + \eta\alpha(q_t + \psi_t^*) - \lambda\alpha(\psi_t^* - s_t)$$

3. Households' wage setting

$$\pi_{W,t} - \gamma_W \pi_{t-1} = \beta E_t(\pi_{W,t+1} - \gamma_W \pi_t) + \kappa_W \left[\varphi y_t - \varphi \xi_{a,t} + \frac{\sigma}{1-h}(c_t - hc_{t-1}) - \zeta_t\right]$$

where $\kappa_W = \frac{(1-\theta_W)(1-\beta\theta_W)}{\theta_W}\frac{1}{1+\epsilon_W\varphi}$

4. Domestic firms price setting at SOE

$$\pi_{S,t} - \gamma_S \pi_{S,t-1} = \beta E_t(\pi_{S,t+1} - \gamma_S \pi_{S,t}) + \kappa_S \left(\zeta_t - \xi_{a,t} + \frac{\alpha}{1-\alpha}(q_t + \psi_t^*)\right) + \xi_{cpS,t}$$

where $\kappa_S = \frac{(1-\theta_S)(1-\beta\theta_S)}{\theta_S}$

5. Domestic firms price setting at ROW

$$\pi_{S,t}^* - \gamma_S^* \pi_{S,t-1}^* = \beta E_t(\pi_{S,t+1}^* - \gamma_S^* \pi_{S,t}^*) + \kappa_S \left(\zeta_t - \xi_{a,t} + \frac{\alpha}{1-\alpha}(q_t + \psi_t^*) - \psi_t\right) + \xi_{cpS,t}$$

6. Retail firms price setting

$$\pi_{R,t} - \gamma_R \pi_{R,t-1} = \beta E_t(\pi_{R,t+1} - \gamma_R \pi_{R,t}) + \kappa_R(-\psi_t^*) + \xi_{cpR,t}$$

where $\kappa_R = \frac{(1-\theta_R)(1-\beta\theta_R)}{\theta_R}$

7. Terms of trade

$$s_t = p_{R,t} - e_t - p_{S,t}^*$$

8. Domestic inflation

$$\pi_{S,t} = p_{S,t} - p_{S,t-1}$$

9. Imported inflation

$$\pi_{R,t} = p_{R,t} - p_{R,t-1}$$

10. CPI Inflation

$$\pi_t = \pi_{S,t} + \alpha(s_t - s_{t-1}) + \alpha(\psi_t - \psi_{t-1})$$

11. Risk sharing

$$\frac{\sigma}{1-h}\left(E_t\Delta c_{t+1} - h\Delta c_t\right) = \frac{\sigma^*}{1-h^*}\left(E_t\Delta y_{t+1}^* - h\Delta y_t^*\right) + E_t\left[\Delta q_{t+1} - \Delta \nu_{t+1}^* + \Delta \nu_{t+1} - \chi a_t - \xi_{rp,t}\right]$$

12. Real exchange rate

$$q_t = e_t + p_t^* - p_t$$

13. LOP gap for exports

$$\psi_t = e_t + p_{S,t}^* - p_{S,t}$$

14. LOP gap for imports

$$\psi_t^* = p_{R,t} - e_t - p_t^*$$

15. Real wages law of motion

$$\zeta_t = \zeta_{t-1} + \pi_{W,t} - \pi_t$$

16. Budget constraint

$$c_t + a_t = \beta^{-1} a_{t-1} - \frac{\alpha}{1-\alpha}(q_t + \psi_t^*) + \psi_t + y_t$$

17. Taylor rule

$$i_t = \psi_i i_{t-1}(1-\psi_i)(\psi_\pi \pi_t + \psi_y y_t + \psi_{\Delta y}\Delta y_t + \psi_{\Delta e}\Delta e_t) + \xi_{i,t}$$

D Additional results from Section 5 and Section 6

D.1 Data

Here I provide details on the macroeconomic data for Australia and the US, including sources and period.

Table D.1: Data (variables, description, source, period), U.S. and Australia, 1990-2019

Variable	Description	Source	Period
Australia			
AUSGDPRQDSMEI	Constant Price Gross Domestic Product	FRED	1990Q1:2019Q4
NAEXKP02AUQ189S	Constant Price Final Private Consumption	FRED	1990Q1:2019Q4
POPTOTAUA647NWDB	Population	FRED	1990:2019
GCPIXVIQP	Core CPI quarterly inflation (Table G.1)	RBA	1990Q1:2019Q4
GCPIXVIYP	Core CPI yearly inflation (Table G.1)	RBA	1990Q1:2019Q4
A2298279F	Imported consumption goods price index (Tables 4, 5 and 6)	ABS	1990Q1:2019Q4
BIS_CBPOL	Policy rate	BIS	1990M1:2019M12
DEXUSAL	Nominal exchange rate US$/AU$	FRED	1990Q1:2019Q4
United States			
A939RX0Q048SBEA	Real gross domestic product per capita	FRED	1990Q1:2019Q4
FEDFUNDS	Effective Federal Funds Rate	FRED	1990Q1:2019Q4
CPILFESL	Core CPI	FRED	1990Q1:2019Q4

D.2 Additional tables and figures

Table D.2: Correlation of interest rate and inflation: Data and Model

		Data	Model
$\rho(i,\pi)$	No ZLB*	0.3735	0.5473
	ZLB*	-0.1583	0.4647
	% explained	15.52	

Notes: Column 1 of this table reports sample correlations between core quarterly CPI inflation and interest rates for Australia for two periods. The ZLB* period in the data is given by 2008Q4 to 2015Q4. Column 2 reports the correlation for the same elements in the quantitative. The ZLB* period in the model are all the quarters when (14) binds. The number of simulation is 1,000.

Figure D.1: Impulse responses to a foreign discount rate shock under ZLB* and No ZLB*