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The Grasshopper, the Ant, and the Minimum Wage*

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Abstract

We study the macroeconomic impact of raising the minimum wage on the labor market and the distribution of wealth, with particular emphasis on the presence of labor market risk and how the minimum wage can potentially shape such risk. Building on the empirical observation of substantial wealth heterogeneity among minimum-wage workers, we set up a model with uninsurable idiosyncratic income risk, search frictions, and Nash bargaining. Our quantitative exercises suggest important non linearities on the impact of minimum-wage increases on the wealth distribution: while a moderate increase has little effect on job creation, slightly compresses the wage distribution, and exerts little impact on the distribution of wealth, a larger increase can have a non-negligible labor-market impact and generate more wealth inequality. This latter effect is mainly due to the enhancement of precautionary-savings motives among richer workers caused by the increased risk of job separation, which becomes stronger for wealthier individuals likely to earn the minimum wage in the future. We show that this precautionary-savings channel is quantitatively more important than the more traditional inequality channel, according to which wealth inequality rises because the fraction of the population earning a wage falls.

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The Grasshopper, singing
All summer long,
Now found winter stinging,
And ceased in his song.
Not a morsel or crumb in his cupboard–
So he shivered, and ceased in his song.
Miss Ant was his neighbor;
To her he went:
"O, you're rich from labor,
And I've not a cent.
Lend me food, and I vow I'll return it,
Though at present I have not a cent."
The Ant's not a lender,
I must confess.
Her heart's far from tender
To one in distress.
So she said: "Pray, how passed you the summer,
That in winter you come to distress?"
"I sang through the summer,"
Grasshopper said.
"But now I am glummer
Because I've no bread."
"So you sang!" sneered the Ant. "That relieves me.
Now it's winter–go dance for your bread!"

The Grasshopper and the Ant, Jean de La Fontaine

1 Introduction

The literature on the impact of the minimum wage is extensive. While the primary interest is on employment (Card and Krueger, 1994; Stigler, 1946), it has also tackled issues related to the distribution of wages (Autor et al., 2016), the returns to education (Bárány, 2016), firm profitability (Draca et al., 2011) or even the prices faced by consumers (Aaronson and French, 2007; Harasztosi and Lindner, 2019). Inequality is the main implicit concern behind all these questions, yet a lot needs to be understood about the distribution of wealth. A reason for the lack of empirical evidence on this matter may be due to scarcity of available data. Also, wealth being a stock variable—as opposed for instance to labor earnings—requires the time horizon of such studies to be longer and makes them subject to empirical challenges due to general equilibrium channels.[1]

At the same time, most models that study the impact of the minimum wage on employment in the literature do not consider the wealth of workers as a relevant variable that influences the impact of such policy.[2] In reality, minimum-wage workers are a heterogeneous group. Their differences may shape the impact of the minimum wage on their employability, not only because productivity may vary across workers, but also because labor supply income effects could be different too. Card and Krueger (2016) document in their book that 57.2% of workers affected by the 1990 minimum wage increase do not come from the first three family-income deciles and even 20.9% come from the last three deciles.[3] Moreover, Neumark et al. (2005) report non-parametric evidence that higher income families are more likely to experience a decrease in family income following a minimum-wage increase than poorer families.[4]

In this paper, we study the impact of the minimum wage on the distribution of wealth

[1]A recent paper by Hurst et al. (2022) argues that empirical studies of the impact of the minimum wage on employment only identify short-run effects. While we do not discuss the validity of empirical studies to identify long-run effects on employment in this paper, we do think that a long time horizon is required to understand the impact of the minimum wage on the distribution of wealth.

[2]Many of these models are search and matching models where savings decisions are not modelled and workers simply consume the income they receive each period. Examples can be found in the books by Manning (2003) and Cahuc et al. (2014).

[3]They also show that only 35.9% of minimum wage workers are the only wage earner in their family compared to 41.5% for the larger employed population, with 29.4% of minimum-wage workers being aged between 16 and 19 years old.

[4]A large literature suggests that employability of secondary earners in a family is related to the level of income of the primary earner. This dependence is important in many models of family labor supply such as Attanasio et al. (2008), Blundell et al. (2016), Bick and Fuchs-Schundeln (2018), Borella et al. (2022), Fukui et al. (2023), Guner et al. (2020), Mankart and Oikonomou (2016), Sánchez-Marcos and Bethencourt (2018), Santos Monteiro (2008), among others. It thus seems natural that the minimum wage has a heterogenous impact on employment, depending on the wealth of families.

and other labor market characteristics such as the level of employment and the dispersion of wages in the context of a theoretical model, taking into account that the minimum-wage worker group is subject to wealth heterogeneity. Because consumption and savings decisions are relevant to understand the distribution of wealth, we work in the context of the canonical Bewley model with uninsurable idiosyncratic income risk. We add to this framework ingredients from the macro labor literature (Mortensen and Pissarides, 1994) such as search and matching frictions on the labor market, Nash bargaining for wage determination, and idiosyncratic productivity shocks that lead to endogenous job separations, as Krusell et al. (2010) and Bils et al. (2011) did already. We introduce a minimum wage to the framework of Bils et al. (2011), which acts as a constraint on the Nash bargaining solution as in Flinn (2006).

The title of the paper is a reference to the famous fable that was originally written by Aesop in the ancient Greece and which was then rewritten in more modern times by de La Fontaine in France and Samaniego in Spain. In the fable, the grasshopper is a caricature of individuals who save too little, while the ant is a caricature of the other extreme, that is, individuals who save a lot. Our quantitative experiments suggest that moderate increases in the minimum wage do not impact the labor market and the distribution of wealth much, with a small compression of wages at the bottom of the distribution. However, larger increases in the minimum wage tend to destroy employment and raise wealth inequality despite the reduced dispersion in wages. A standard reason for the increased wealth inequality is due to the increase in unemployment—which implies that a larger share of workers do not earn a wage. However, we find that this traditional channel plays a minor role in our quantitative experiments (it even reduces inequality when this mechanism is alone), while two additional mechanisms contribute to the increase in wealth inequality in the economy and operate through a change in the saving behavior of workers. Hence, the minimum wage generates both *ants* and *grasshoppers* in our model.

First, the increase in the minimum wage produces *ants* because of the increased idiosyncratic risk that richer workers face. The reason is the following. The match surplus is smaller for richer workers because their marginal utility of consumption is smaller. This implies that an increase in the minimum wage first destroys the employment of richer agents for a given productivity level. Therefore, this increases the idiosyncratic risk richer workers face and induces them to save more for precautionary reasons.

Second, the increase in the minimum wage produces *grasshoppers* among workers who earn the minimum wage or workers who would likely earn it in a near future. The reason is the following. Absent a minimum wage, poorer workers are only able to negotiate a low wage because of the precautionary motive, which makes them more attached to the labor market (Pijoan-Mas, 2006; Marcet et al., 2007). Hence, they save more than other agents, not only because of precautionary reasons, but also because they need to improve their bargaining position to earn a higher wage. We call this latter effect the *dolling-up* effect.[5] Once a minimum wage is introduced, this relation between the earned wage and the asset position of agents disappears for minimum-wage workers, lowering their incentives to save more. As a consequence of these two channels, large increases in the minimum wage increase wealth inequality in spite of reducing wage inequality.

A recent paper by Aguiar et al. (2020) documents that poorer households do not tend to display higher consumption growth as precautionary models of consumption would predict, even though they are characterized by higher marginal propensities to consume. A possible explanation that they give to this apparent puzzle is preference heterogeneity: impatient poor agents are not characterized by higher consumption growth, but do display larger marginal propensities to consume. An interesting feature of our model is that minimum-wage workers fit into this category: because they are not subject to the *dolling up* effect, they behave as if they were more impatient according to their consumption Euler equation, while workers earning more than the minimum wage have extra incentives to save. Interestingly, this explanation is not due to the discount factor of poorer agents being lower for exogenous reasons. It is due to an endogenous mechanism that incites minimum-wage workers to behave differently in terms of consumption and saving decisions when compared to individuals who can negotiate their wage.

A paper related to our work is Gorry (2013). He studies the impact of the minimum wage on unemployment in a search and matching model with endogenous separations (as we do), with a particular emphasis on youth unemployment. In the model, young workers are willing to accept low wages because of the return on experience they obtain later in life. Gorry (2013) carefully calibrates the evolution of the job finding and separation rates and wage growth over the life cycle. An increase in the minimum wage primarily impacts young workers because of their inexperience, but it may also have detrimental effects in the long run because it prevents workers from accumulating experience. As a consequence, the increase in the minimum wage has non-linear effects: it has a small impact on aggregate unemployment if it only affects workers early in life, but it has larger effects when it starts hurting workers at more advanced ages.[6] Even though our analysis does not focus on life cycle, our paper complements the

[5]Table 1 documents the quantitative importance of this effect, which mainly affects very poor agents, reaching an annualized return of 8.7% for the bottom 1% in our benchmark economy.

[6]Larrain and Poblete (2007) study a similar idea in a model where workers with heterogeneous initial skills gain productivity through experience. They show that a minimum wage may force less skilled workers

results in Gorry (2013) by emphasizing the importance of the persistence of productivity shocks: we show that wages set slightly above the minimum are renegotiated downward when it is possible to hit the minimum in the future, thus attenuating the negative impact of the minimum wage on job creation, which typically occurs in a model with exogenous separations. Our mechanism is also consistent with the empirical evidence on the compression of wages that the minimum wage generates for the lower tail of the distribution. Lee (1999) and DiNardo et al. (1996) document that such phenomenon seemed to occur in the eighties when the federal minimum wage was lowered, especially in the case of women. Admittedly, the compression implied by our model is not as strong as the one documented in these papers, but more recent evidence by Autor et al. (2016) and Villacorta (2017) suggests that the impact of the minimum wage is substantially smaller than that reported by the earlier literature.[7] Moreover, our framework allows studying implications for the distribution of wealth because we explicitly model saving decisions, an aspect which has been neglected by most of the literature.[8]

Our paper is also related to the literature that analyzes the interaction between the minimum wage and hold-up problems (Acemoglu and Shimer, 1999; Bauducco and Janiak, 2018; Card et al., 2014). These papers suggest that, absent a minimum wage, when capital and labor are complements in the production function, firms choose to accumulate too little capital in order to negotiate lower wages (Cahuc et al., 2008). The presence of a binding minimum wage thus gives incentives for firms to accumulate capital closer to the socially optimal level in these models. The literature has thus studied how the hold-up problem shapes the relation between the minimum wage and the demand for capital. Here, we study its relation with the supply of capital by considering an extension of our model where the aggregate capital stock is endogenous. The minimum wage interacts with saving decisions and has an impact on the supply of capital: a binding minimum wage cancels the *dolling-up* effect, reducing the incentives to save for minimum-wage workers, but it also amplifies precautionary savings for richer workers. Because the precautionary saving channel is the strongest of the two, a significant raise in the minimum wage increases the aggregate stock

to remain longer in the sector not covered by the minimum wage. They also derive results in terms of wealth dispersion and conclude that this channel tends to increase wealth inequality.

[7] Using Brazilian data however, Engbom and Moser (2022) found significant compression of the wage distribution brought by the increase in the minimum wage in this country.

[8] The search and matching literature of the minimum wage typically relies on models with linear utility functions, which impedes the analysis of consumption and saving decisions and the impact of the minimum wage on the distribution of wealth. Nevertheless the recent paper by Berger et al. (2022)—which does not rely on a framework with search frictions—considers a concave utility function. Such assumption allows them to study welfare improvements of the minimum wage related to redistribution. Yet, they do not study implications for precautionary savings as we do here.

of capital. However, we show that allowing for the aggregate capital stock to adjust doesn't alter the impact of the minimum wage on unemployment much in our numerical exercises.

The rest of the paper is organized as follows. In Section 2, we study the impact of the minimum wage in the canonical Mortensen and Pissarides (1994) framework. Even though this benchmark does not include saving decisions and an endogenous distribution of wealth, the simplicity of its linear framework sheds light on the labor market impact of the minimum wage in the extended model of Section 3, which includes these elements. In particular, we document in the context of the linear framework that the minimum wage may compress the distribution of wages as wages above the minimum are renegotiated downwards. Moreover, we show that this mechanism attenuates the impact of the minimum wage on job creation. In Section 4, we present quantitative results about the impact of the minimum wage in the context of the extended model of Section 3, confirming the insights about wages and job creation from Section 2, with additional results in terms of the distributions of wealth and consumption: a large increase in the minimum wage increases wealth inequality because (i) it increases the saving effort of wealthier workers who suffer from an increase in labor market risk, and because (ii) the aforementioned *dolling up* effect is reduced in the case of poorer workers. Finally, Sections 5 and 6 respectively show the extension with endogenous capital stock and conclude.

2 The impact of the minimum wage in a linear framework with endogenous job destruction

In this section, we show the job creation and destruction effects of an increase in the minimum wage and its wage compression effects in a standard search and matching model with endogenous separations *à la* Mortensen and Pissarides (1994). We call this version the *linear model*. The main upshot is that the negative impact of a minimum wage on the incentives to post new vacancies are attenuated by wage renegotiation: when agents anticipate the possibility of paying the minimum wage in the future, Nash bargaining results in a wage cut to wages above the minimum to dampen the harm on match surplus (in present value). This mechanism is reminiscent of the possible null impact of employment protection legislation on employment (Garibaldi and Violante, 2005; Lazear, 1990; Ljungqvist, 2002; Mortensen and Pissarides, 1999). This mechanism however does not prevent the nefarious consequences caused by job destruction: when job destruction increases because of the minimum wage, job creation is reduced since tenure on the job is shorter. A corollary result is that wage renegotiation also compresses the wage distribution since wages above the minimum are cut and the wage of minimum wage earners increases.

2.1 Model framework

There exists a continuum of workers of measure one and a continuum of firms, both sharing a common discount factor $\beta \equiv \frac{1}{1+r}$, where r denotes the interest rate. In any given period, workers can be employed or unemployed. Workers that are employed receive a wage w, that is subject to an exogenous minimum $\underline{w}$, and unemployed workers search for jobs, obtaining an unemployment benefit b. Firms have a technology that specializes in either production or vacancy, the former requiring the firm to be matched to a worker. Each firm in this economy is assumed to employ one worker, and can therefore be thought of as a job position that is either vacant or filled. Firms that undertake production pay their employee a wage w and those that are searching for a worker pay a vacancy flow cost c.

The job search process in this economy is characterized by a matching function that embodies labor search-frictions and determines the job finding and filling rates. In any period, an unemployed worker meets a firm with probability p and a vacant firm is matched to a searching worker with probability q. Let u denote the number of unemployed workers and v the number of job vacancies. The amount of matches in any period is assumed to be determined by the following matching function:

$$m\left(u,v\right) = \chi u^{\eta} v^{1-\eta}, \tag{1}$$

where χ is a scale parameter. As matches are assumed to occur at random, the probability that an unemployed worker finds a job is given by $p = \frac{m}{u}$ and, similarly, the probability that a vacancy is filled is $q = \frac{m}{v}$. Letting $\theta \equiv \frac{v}{u}$ denote the market tightness, the job filling rate q and finding rate p are respectively described by:

$$q\left(\theta\right) = \chi\theta^{-\eta}, \quad p\left(\theta\right) = \theta q\left(\theta\right). \tag{2}$$

A matched worker-firm pair produces a stochastic output of value x, corresponding to the firm's idiosyncratic productivity or match quality. All new jobs are assumed to be formed at a productivity level x_0. In any given period, a new productivity is drawn with probability ϕ from a distribution characterized by a cumulative distribution function $G(x)$ with a bounded support $[\underline{x}, \overline{x}]$. Note that under these assumptions, match quality x is a persistent but memoryless process. The wage paid by a firm in this context will be assumed to be a productivity contingent contract $w\left(x\right)$. This is equivalent to the assumption that wages are renegotiated whenever a new productivity shock hits the firm.

Match separation in this economy is assumed to be endogenous and occurs whenever either party finds it in their interest to break up the match, that is, whenever their outside option is more valuable to them than the job. Once a match ends, a new round of search is triggered for both the respective worker and firm. As shown by Mortensen and Pissarides (1994), separations in this context satisfy a reservation property: only matches of quality $x > R$ are continued, where R denotes the reservation productivity. Since a proportion ϕ of firms draw a new productivity each period, the inflow to unemployment is then given by $\phi G(R)(1-u)$, where $1-u$ is the amount of employed workers. Similarly, a proportion $p(\theta)$ of unemployed workers find a job every period, and therefore the outflow of unemployment is simply $p(\theta)\,u$. The steady state unemployment rate that equates these two flows is given by:

$$u = \frac{\phi G(R)}{p(\theta) + \phi G(R)}. \tag{3}$$

The value of employment is then defined by:

$$W(x) = w(x) + \beta\left[(1-\phi)\,W(x) + \phi\left(G(R)\,U + \int_R^{\bar{x}} W(z)\;dG(z)\right)\right], \tag{4}$$

where U denotes the value of unemployment and R the reservation productivity. The above expression accounts for the fact that a match receives a productivity shock with probability ϕ, in which case a new match quality is randomly drawn from the distribution characterized by G. Given the separation reservation property, whenever a match receives a productivity shock that generates a match quality below R, the worker enters unemployment. The value of a job to a productivity x match to a firm is analogously defined by:

$$J(x) = x - w(x) + \beta\left[(1-\phi)\,J(x) + \phi\left(G(R)\,V + \int_R^{\bar{x}} J(z)\;dG(z)\right)\right], \tag{5}$$

where x denotes the firm's output and V the value of vacancy. Let x_0 denote the productivity level of new jobs. Since unemployed workers meet a vacant firm with probability $p(\theta)$ each period, the value of unemployment is defined by:

$$U = b + \beta\Big[p(\theta)\,W(x_0) + (1-p(\theta))\,U\Big], \tag{6}$$

where b denotes the worker's unemployment insurance. Firms that search for workers incur a vacancy cost c each period and meet a worker with probability $q(\theta)$, and the value of vacancy is similarly defined by:

$$V = -c + \beta \left[q(\theta) J(x_0) + (1 - q(\theta)) V \right]. \tag{7}$$

Given that posting vacancies is costly, jobs must entitle rents in equilibrium, or otherwise there would be no incentive for firms to enter the market. In equilibrium, free-entry of firms occurs until it is no longer profitable to post a vacancy; that is,

$$V = 0 \tag{8}$$

holds in equilibrium. Since jobs generate a positive surplus and the worker-firm pair constitutes a bilateral monopoly, a surplus sharing rule must be specified.

As in Flinn (2006), we assume that wages are determined by constrained Nash-bargaining over the match's surplus. Let $H(x) \equiv W(x) - U$ and $J(x)$ respectively denote the worker's and firm's surplus obtained from a job. Note that since $V = 0$, the job's value and surplus to a firm are equivalent in equilibrium and these terms will be used interchangeably in what follows. The wage schedule is a productivity-contingent contract that solves the following problem:

$$\max_{w(x)} \; H(x)^{\gamma} J(x)^{1-\gamma} \quad \text{s.t. } w(x) \geq \underline{w}, \tag{9}$$

where $\gamma \in [0, 1]$ denotes the worker's bargaining power and $1 - \gamma$ that of the firm. Note that since the solution to the unconstrained problem is a wage schedule that is increasing in productivity, the minimum wage acts solely as a side constraint to the problem and is paid in low productivity matches. In particular, the minimum wage will be paid by the firm up to a productivity threshold x^*, at which the bargained wage coincides with the minimum wage.

2.2 Equilibrium and impact of the minimum wage

2.2.1 Wage compression

Recall that the wage schedule will be constrained by the minimum wage up to a productivity level x^*. When the wage is an interior solution to the bargaining problem (9), the worker and firm respectively obtain a constant fraction γ and $1 - \gamma$ of the match's surplus. That is, for $x > x^*$, the following condition is satisfied:

$$\frac{H(x)}{\gamma} = S(x) = \frac{J(x)}{1-\gamma}, \tag{10}$$

where $S(x) = H(x) + J(x)$ denotes the match's surplus. When the constraint in (9) is active (for $x < x^*$), the wage is set at the minimum wage level.

In Appendix A.1, we show that the following proposition characterizes negotiated wages:

Proposition 1. *In the linear model, when the minimum wage is binding for some of the jobs, the equilibrium wages $w(x)$ for productivity levels $x \in (x^*, \bar{x}]$ are given by the following expression:*

$$w(x) = \underbrace{\gamma x + (1-\gamma)\frac{r}{1+r}U}_{\substack{\text{endogenous separations} \\ \text{wage curve}}} - \underbrace{\frac{\phi}{r+\phi}\gamma \int_R^{x^*} (x^* - z)\, dG(z)}_{\text{minimum wage spillover}}, \tag{11}$$

Note that, by definition, bargained wages are an interior solution to (9) for all productivity $x > x^*$, and when the minimum wage is not binding, it holds that the separation thresholds R and x^* are equal, reducing the wage schedule to that of a standard endogenous separations model. In the general case, for matches of quality $x \in [R, x^*]$, the firm pays the minimum wage $\underline{w}$.

The expression in (11) can be understood easily by comparing it to the standard expression in Mortensen and Pissarides (1994): in an economy without a binding minimum wage, the wage expression would collapse to the first two terms in (11). In this case, the bargained wage is increasing in both productivity and in the worker's outside option, which is given by the value of unemployment. The last term in (11) is added when the minimum wage binds for some matches in the economy. Because firms obtain a lower share of the match surplus when the minimum wage binds, when the worker-firm pair negotiates, they internalize the expected surplus share loss that the firm would incur in the case of a future binding minimum wage. Appendix A.1 shows that the second bracketed term precisely accounts for this expected loss, which occurs with probability $\phi [G(x^*) - G(R)]$.[9]

The model therefore implies that a minimum wage has a negative spillover on the level of bargained wages, decreasing the overall wage schedule if market tightness is held constant.

[9]Notice that the persistence of productivity shocks influences the size of the spillover effect. If ϕ is close to zero, then productivity is highly persistent. In this case, the worker-firm pair anticipates that it is unlikely that the minimum wage will be paid in the future. They thus internalize such event in the negotiation to a lesser extent, implying a lower compression of the wage distribution.

This implies that the minimum wage also compresses the wage distribution: on the one hand, a binding minimum wage implies a larger lower bound on the wage schedule, and on the other, the spillover to bargained wages implies a decrease in the upper bound, generating lower wage dispersion overall.

2.2.2 Job creation, job destruction and the minimum wage

In the baseline framework of Mortensen and Pissarides (1994), a job creation and job destruction curve pin down the equilibrium market tightness and reservation value. Here, an additional condition is needed in order to determine the equilibrium triple (θ, R, x^*): the threshold at which bargained wages are an interior solution to the bargaining problem (9). In this section, we introduce these equilibrium curves in the presence of a binding minimum wage and derive comparative statics of the impact of an increase in the minimum wage on job creation and destruction.

The reservation productivity R is defined by a null job's value, i.e., $J(R) = 0$. We show in the Appendix A.2 that, in the presence of a binding minimum wage, the reservation productivity is determined by the following partial **job destruction curve**:

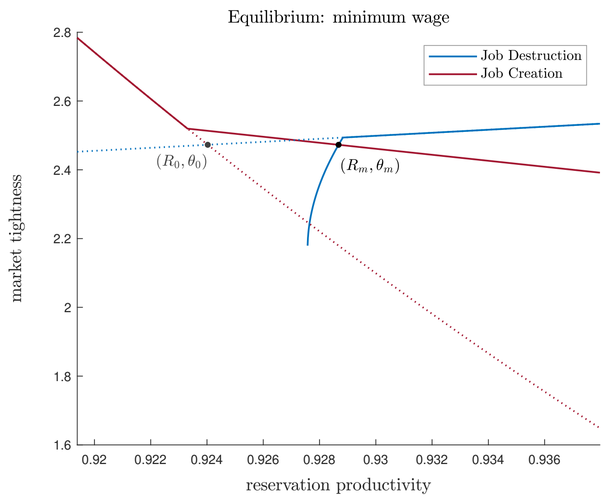
$$0 = R - \underline{w} + \frac{\phi}{r+\phi}\left(\int_R^{\bar{x}} (z-R)\ dG(z) - \int_{x^*}^{\bar{x}} \gamma\,(z-x^*)\ dG(z)\right). \tag{JD}$$

We use the term partial to highlight the fact that the above equation does not represent a direct relationship between the reservation productivity R and market tightness θ like it does in the baseline Mortensen and Pissarides (1994) model. Therefore, an additional condition linking the threshold x^* with market tightness will be required to represent the job destruction curve in the (R, θ) plane: the productivity threshold x^* is defined as the level at which wages are an interior solution to the bargaining problem (9).

As previously discussed, the threshold x^* is implicitly defined by $w(x^*) = \underline{w}$. This **interior negotiation solution** curve, henceforth INS, is defined by the wage equation (11) as:

$$\underline{w} = \gamma x^* + (1-\gamma)\frac{r}{1+r}U - \frac{\phi}{r+\phi}\gamma\int_R^{x^*} (x^*-z)\ dG(z), \tag{INS}$$

Figure 1: Equilibrium curves: minimum wage



with the value of unemployment U given by:

$$\frac{r}{1+r}U = \begin{cases} b + \dfrac{\gamma}{1-\gamma}c\theta & \text{if } x_0 \geq x^*. \\ b + \dfrac{\gamma}{1-\gamma}\dfrac{p(\theta)}{r+\phi}(x^* - R) & \text{if } x_0 < x^*. \end{cases}$$

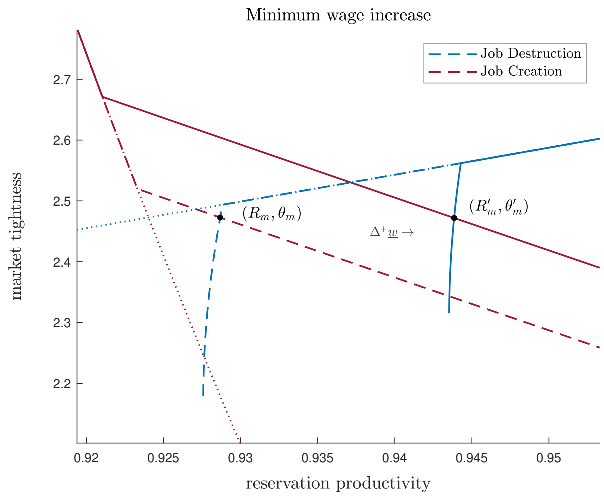
Analogously to the baseline model, Appendix A.2 shows that the free-entry condition $V = 0$, the value function of vacancy (7), and the value of a new job $J(x_0)$ imply that the **job creation** is described by:

$$\frac{c}{q(\theta)} = \frac{1}{r+\phi}\left[(x_0 - R) - \gamma(x_0 - x^*)\right]. \tag{JC}$$

In order to compare the equilibrium defined by the triplet (JD), (INS) and (JC) with the standard Mortensen and Pissarides (1994) framework that only includes a job creation and a job destruction curve, we choose to represent the following modified job destruction and creation curves: the (JD) curve and the (INS) condition jointly determine the modified job destruction curve, which slopes upward in the (R, θ) plane as in the standard model; and similarly the (JC) curve and the (INS) condition determine the analogue job creation curve.

The equilibrium curves are depicted in Figure 1 under a calibration similar to the one that follows Pissarides (2009). The dotted lines represent the equivalent curves in the baseline

Figure 2: Equilibrium curves: minimum wage increase



model without a minimum wage. Note that the job destruction curve no longer spans the whole productivity support as the minimum wage impedes wages from being bargained down in low productivity matches, which would be required to sustain a longer match duration. Since a larger market tightness implies higher bargained wages, the minimum wage is not binding in low productivity matches along the job destruction curve, merging this curve with its baseline counterpart. As can be seen in Figure 1, the job destruction curve becomes steeper when the minimum wage is binding, implying the termination of more marginal jobs. Turning to the job creation curve, note that the minimum wage spillover to bargained wages implies an increased profitability in new jobs, which dampens the fall in market tightness that would be implied by greater job destruction.

The baseline equilibrium without a minimum wage and the minimum-wage setting equilibrium are denoted respectively by (R_0, θ_0) and (R_m, θ_m) in Figure 1. As would be expected, match duration falls when the minimum wage is introduced, but market tightness reacts less due to the spillover on the bargained wage curve.

Figure 2 depicts an increase in the minimum wage, which further strengthens the effects mentioned previously. The increase in the minimum wage moves the job destruction curve to the right in Figure 2. The reservation productivity threshold goes up because some low productivity jobs are destroyed. This in turn reduces tenure on the job, reducing the incentives to post new vacancies. However, the increase in the minimum wage also moves the job creation curve upwards in Figure 2 because entry wages are renegotiated downward, attenuating the negative impact on vacancy posting.

It is actually easy to show that the only negative impact that the minimum wage has on job creation is due to the reduced tenure on the job brought by the minimum wage by calculating the total match surplus associated to new jobs:

$$\frac{r+\phi}{1+r}S(x_0) = x_0 - \frac{r}{1+r}U + \frac{\phi}{1+r}\int_R^{\bar{x}} S\left(z\right)\, dG\left(z\right). \tag{12}$$

The above equation emphasizes that, because wages are renegotiated upon entry if the minimum wage changes, any future profit loss due to higher future wages is neutralized by such renegotiation. This result is reminiscent of the literature on employment protection (Lazear, 1990), which showed that Nash bargaining may lower the negative impact of employment protection on job creation. As a result, the only downside of the minimum wage on job creation is due to the lower expected match duration, as the following comparative static shows:

$$\left(\eta\frac{r+\phi}{p\left(\theta\right)}+\gamma\right)c\frac{\partial\theta}{\partial\underline{w}} = -\frac{\phi}{r+\phi}S(R)g(R)\frac{\partial R}{\partial\underline{w}}. \tag{13}$$

In the Appendix A.3, we show that this comparative static can be obtained by combining the total surplus associated with new jobs in (12) together with the free entry condition.

Two additional observations are worth making about the comparative static (13). First, the impact on the labor market tightness depends on $g(R)$. This is relevant to understanding the quantitative results in Section 4 when the productivity state space will be discretized. Second, the impact is also increasing in $S(R)$: as the minimum wage goes up, the surplus of the marginal jobs at productivity R goes up too, implying a larger impact on job creation of an increase in the minimum wage at higher values of the minimum wage. This result will also be useful to understand the quantitative impact of large increases in the minimum wage in Section 4.

3 Introducing precautionary motives

3.1 Model extension

In this section, we extend the framework in Section 2 by introducing motives for saving. These are mainly due to consumption smoothing and the precautionary behavior of workers. To this end, we add a minimum wage to the model of Bils et al. (2011), which is a Bewley-Huggett-Aiyagari variant of the endogenous separations model of Mortensen and Pissarides (1994). Accordingly, we assume that the match's idiosyncratic productivity x follows an autoregressive process described by:

$$\log x' = \xi + \rho \log x + \varepsilon, \quad \varepsilon \sim N\left(0, \sigma_\varepsilon\right), \tag{14}$$

where next period variables are denoted with primes, ξ denotes the process's unconditional mean and ε an innovation to idiosyncratic match quality. Note that, unlike in the previous section, this process is by definition not memoryless. For simplicity, we assume that new jobs are created at mean productivity $x_0 = \bar{x}$. This is sufficient for the wage to be bargained in new matches in the exercises we perform. The matching technology remains unchanged and is described by Equation (1).

Workers can either be matched to a firm of productivity x or unemployed, and have preferences over consumption $c > 0$ and leisure $l \in \{0, 1\}$ described by:[10]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1 - \sigma} + B l_t \right], \tag{15}$$

where $\sigma > 0$ denotes the relative risk aversion coefficient and B the utility obtained from leisure. Employed workers are paid a wage w by the firm and those unemployed obtain an exogenous income flow b. Workers have access to a risk-free asset market, where borrowing is limited to a level $\underline{a} \leq 0$.

Aside from introducing risk-aversion and incomplete asset markets, the worker's problem is analogous to the previous section. In the exercises we perform, the unconstrained Nash-bargained wages are increasing over assets and productivity. The inclusion of a minimum wage constraint once again implies that match separations are determined by the firm and satisfy a reservation property. In particular, as the job's value is defined on the state pair (a, x), separations can be characterized by a threshold function $a^* (x)$ and occur in a match with productivity x whenever $a \geq a^* (x)$.

The employed worker's value function then solves the following problem:

$$\tilde{W}\left(w, a, x\right) = \max_{a'_e, c_e} \; u\left(c_e\right) + \beta \, \mathbb{E}_{x'|x} \left[U\left(a'_e\right) \mathbb{1}_{[a'_e > a^*(x')]} + W\left(a'_e, x'\right) \mathbb{1}_{[a'_e \leq a^*(x')]} \right], \tag{16}$$

[10] We only consider labor supply along the extensive margin, with employees obtaining no leisure utility.

subject to

$$c_e + a'_e = (1 + r)\, a + w,$$
$$a'_e \geq \underline{a}, \;\; c_e \geq 0,$$

where $\mathbb{1}_{[\cdot]}$ denotes the indicator function and $a^*(x)$ the asset separation-threshold in a match of productivity x. The value function W accounts for the fact that wages are bargained and is defined by:

$$W(a, x) \equiv \tilde{W}(\omega(a, x), a, x). \tag{17}$$

Let p denote probability of finding a job. Since new matches are formed at mean productivity $\overline{x}$, the unemployed worker's value function U solves:

$$U(a) = \max_{a'_u, c_u} \; u(c_u) + \beta \left[p(\theta) \cdot W(a'_u, \bar{x}) + (1 - p(\theta)) \cdot U(a'_u) \right], \tag{18}$$

subject to

$$c_u + a'_u = (1 + r)\, a + b,$$
$$a'_u \geq \underline{a}, \;\; c_u \geq 0.$$

As in the linear utility model, firms are risk-neutral and maximize the expected present value of profits, discounted at the market interest rate r. Given that separations are determined by the firm in the presence of a minimum wage, the job's value function is defined by:

$$\tilde{J}(w, a, x) = x - w + \frac{1}{1 + r} \mathbb{E}_{x'|x} \left[\max\langle J(a'_e, x'), V \rangle \right], \tag{19}$$

and J is defined analogously to the worker's problem:

$$J(a, x) \equiv \tilde{J}(\omega(a, x), a, x). \tag{20}$$

The job's value function (20) implicitly defines the separation-threshold function $a^*(x)$ by $J(a^*, x) = 0$. To determine the value of vacancy, recall that vacant firms are randomly matched to a searching worker in the unemployment pool. Since bargained wages depend on the worker's assets, the expected value of newly created jobs in turn depends on the unemployed worker distribution over assets $\Phi_u(a)$. The value of vacancy is then given by:

$$V = -\kappa + \beta \left[q(\theta) \int J(a'_u, \bar{x}) \, d\Phi_u(a'_u) + (1 - q(\theta)) V \right], \tag{21}$$

where q denotes the job-filling rate and κ the vacancy flow cost.

The evolution of measures of employed Φ_e and unemployed Φ_u workers is as follows. Let $\mathcal{A}$ and $\mathcal{X}$ respectively denote the set of all possible realizations of assets a and productivity x. For all $\mathcal{A}^0 \subset \mathcal{A}$ and $\mathcal{X}^0 \subset \mathcal{X}$, the measures Φ_e and Φ_u evolve as:

$$\begin{aligned} \Phi'_u\left(\mathcal{A}^0\right) = &(1 - p(\theta)) \int_{\mathcal{A}^0} \int_{\mathcal{A}} \mathbb{1}_{[a'=a'_u(a)]} \, d\Phi_u(a) \, da' \\ &+ \int_{\mathcal{A}^0} \int_{\mathcal{A},\mathcal{X}} \mathbb{1}_{[a'=a'_e(a,x), a'_e(a,x) > a^*(x')]} dF(x'|x) \, d\Phi_e(a,x) \, da' \end{aligned} \tag{22}$$

$$\begin{aligned} \Phi'_e\left(\mathcal{A}^0, \mathcal{X}^0\right) = &\int_{\mathcal{A}^0,\mathcal{X}^0} \int_{\mathcal{A},\mathcal{X}} \mathbb{1}_{[a'=a'_e, a'_e \le a^*(x')]} \, dF(x'|x) \, d\Phi_e(a,x) \, da' dx' \\ &+ p(\theta) \int_{\mathcal{A}^0} \int_{\mathcal{A}} \mathbb{1}_{[x'=\bar{x}, a'=a'_u]} \, d\Phi_u(a) \, da', \end{aligned} \tag{23}$$

where $F(x'|x)$ denotes the (exogenous) conditional cumulative density for productivity. The first and second terms of equation (22) reflect the movement within- and inflow to unemployment, respectively. The latter is determined by the productivity draw of an employed worker after asset holdings have been chosen: all those with $a' > a^*$ are fired. The two terms in equation (23) analogously describe movement within the employed-worker distribution and the inflow to employment.

3.2 Equilibrium

As before, the free entry condition $V = 0$ pins down the value of a vacancy and wages are assumed to be Nash-bargained every period, subject to the minimum-wage constraint. The wage schedule $\omega(a, x)$ solves the following problem:

$$\max_{w} \left[\tilde{W}(w, a, x) - U(a) \right]^{\gamma} \left[\tilde{J}(w, a, x) - V \right]^{1-\gamma}, \quad \text{subject to } w \ge \underline{w}, \tag{24}$$

where γ denotes the worker's bargaining power. As equilibrium wages in absence of a minimum wage are a non-decreasing function over states (a, x) under the chosen calibration (Bils et al., 2011), the minimum wage once again acts solely as a side constraint. That is, the minimum wage binds only in state pairs (a, x) where the unconstrained Nash-bargained wage would have been below $\underline{w}$. An interior solution to the above bargaining problem (24) satisfies:

$$\frac{\gamma}{W(a,x) - U(a)} u'(c_e(a,x)) = \frac{1-\gamma}{J(a,x) - V}. \tag{25}$$

These additional equilibrium conditions allow to define the stationary recursive competitive equilibrium, as follows:

Definition 1. *The stationary recursive competitive equilibrium consists of a set of value functions $\tilde{W}(w,a,x)$, $W(a,x)$, $U(a)$, $\tilde{J}(w,a,x)$, $J(a,x)$ and V; consumption policy functions $c_e(a,x)$ and $c_u(a)$; asset holdings policy functions $a'_e(a,x)$ and $a'_u(a)$; a separation policy function $a^*(x)$; a wage schedule $\omega(a,x)$; a law of motion for the distributions $(\Phi'_u, \Phi'_e) = T(\Phi_u, \Phi_e)$; and labor-market tightness θ such that:*

i) Given θ, ω and a^*, the value functions $\tilde{W}$, W, and U solve the consumer's problem described in (16)–(18) with associated policy functions a'_e, c_e, a'_u, and c_u.

ii) Given Φ_u, θ, ω, a'_e and a'_u, the value functions $\tilde{J}$, J, and V solve the firm's problem described in (20)–(21) with associated separation threshold $a^*(x)$.

iii) Given $\tilde{W}$, U, $\tilde{J}$, and V, ω solves the constrained Nash-bargaining problem (24).

iv) Given J and Φ_u, market tightness θ satisfies the free-entry condition (8), i.e. $V = 0$.

v) Given a'_e, a'_u, x^* and θ, Φ_u and Φ_e are stationary distributions described by T in (22)–(23).

In addition to the equilibrium Definition 1, it is worth presenting illustrative Euler equation associated to the employed and unemployed workers' problem, as these provide insights on the agents' optimal saving decisions. This in turn sheds light on how the minimum wage affects the distribution of wealth in the economy, given that wealth accumulation is the consequence of past saving decisions. These conditions are respectively given by[11]

$$u'(c_e\,(a,x)) = \beta R \mathbb{E}_{x'|x}\left[u'(c_e\,(a'_e,x'))\left(1+\frac{\partial_a\omega(a'_e,x')}{R}\right)\mathbb{1}_{[a'_e\leq a^*(x')]} + u'\,(c_u\,(a'_e,x'))\,\mathbb{1}_{[a'_e>a^*(x')]}\right], \quad (26)$$

$$u'(c_u(a)) = \beta R\left[pu'(c_e(a'_u,\bar{x}))\left(1+\frac{\partial_a\omega(a'_u,\bar{x})}{R}\right)+(1-p)u'(c_u(a'_u))\right]. \quad (27)$$

These Euler equations characterize the saving decisions of unconstrained agents.[12] They include a non-standard term in the form of the derivatives of the negotiated wage with respect to the asset position of agents, denoted by $\partial_a\omega(a',x')$. The presence of these terms is due to the fact that negotiated wages depend on the worker's wealth, resulting in a upward schedule conditional on productivity in our specification.

At low wealth, the marginal utility of consumption is large, especially when agents are close to the borrowing limit. These agents are thus willing to accept lower wages because they are more attached to the labor market as compared to wealthier agents. Because agents anticipate that they are able to negotiate higher wages when they save more, their saving decision is distorted upward, explaining the presence of the derivatives $\partial_a\omega$ in the Euler equations (26) and (27). We call this extra incentive to save the *dolling-up* effect. In principle, the *dolling-up* effect would tend to decrease the dispersion of wealth since it's stronger for poorer agents—those close to the borrowing limit. However, when agents anticipate that it's likely they will earn the minimum wage in the future, the *dolling-up* effect becomes weaker since it's probable their future wage will not be negotiated. An increase in the minimum wage may thus contribute to an increase in wealth inequality if it lowers the incentives to save for a larger group of employees.

While the minimum wage may lower the incentives to save for agents close to the borrowing limit (because it weakens the *dolling-up* effect), it may also increase the incentives to save for wealthier agents for precautionary reasons. This can be seen by remembering that wealthier agents are characterized by a lower match surplus (Krusell et al., 2010; Bils et al., 2011). Thus, the inefficient separations generated by an increase in the minimum wage first hit wealthier workers, who suddenly face a larger labor market risk than poorer

[11]Note that in the worker's problem (16), the indicator function in the continuation value would generally imply the presence of non-differentiable kinks in the value function W, rendering it hard to apply first-order conditions. However, following Clausen and Strub (2020), we conjecture that a lower support function can be constructed in order to ensure differentiability at optimal choices, whenever the wage-schedule is smooth. Additionally, we verify in our numerical exercises that the stochastic process for productivity x effectively convexifies the value functions.

[12]In the case of constrained agents, the decision for assets in the next period is such that the borrowing constraint is binding in the problems (16) and (18).

agents with the same idiosyncratic productivity. In the Euler equations (26) and (27), this increased labor market risk operates through the indicator functions $\mathbb{1}_{[.]}$, which imply a larger probability of future unemployment for wealthier workers (holding the idiosyncratic productivity constant). This additional hike in precautionary savings by wealthy workers—induced by the separation risk associated to an increase in the minimum wage—also contributes to increasing the steady state wealth inequality.

4 Quantitative analysis

In this section, we study the impact of an increase in the minimum wage from a quantitative perspective. We document consequences in terms of unemployment, wage inequality, and the distribution of wealth. We calibrate the model introduced in Section 3 to the United States by closely following Bils et al. (2011) and considering the share of workers earning the minimum wage in this country, which is 1.6% in the calibrated economy. We then consider two types of increases in the minimum wage that we label "small" and "large". Given that Hornstein et al. (2011) showed that the dispersion in wages generated by search and matching models is small, the large increase in the minimum wage is considered large mainly because of the portion of the workforce it covers.[13] The small increase pushes the minimum up by 2% and has a limited impact on employment and the distribution of wealth. Such change in the minimum wage takes the share of minimum wage workers to 5.9% of employment. The large change increases the level of the minimum wage by 10%, covering 9.7% of employees, with a noticeable impact on aggregate employment and inequality.

4.1 Numerical resolution

4.1.1 Calibration

We calibrate the baseline model following the benchmark specification of Bils et al. (2011). In this specification, the authors set the consumer's relative risk aversion coefficient σ equal

[13]In Table A1 in the appendix, we report the ratio of the mean wage to the minimum. It is 1.3 in the benchmark economy. This ratio is higher than the 1.05 of the baseline search model in Hornstein et al. (2011), but it is still low compared to the 1.7-1.9 target that Hornstein et al. (2011) recommends. Hornstein et al. (2011) suggest that one could increase this gap to be more in line with the data by extending the model to consider human capital accumulation on the job or even a larger risk aversion coefficient. Moreover, our model does not include any form of *ex-ante* heterogeneity that could generate non-residual wage dispersion. Hence, the increase in the fraction of the population covered by the minimum wage in the model is a better indicator of the stringency of the impact of the minimum wage than the actual increase in the minimum wage.

to one, the discount factor to $\beta = 0.99477$ in order to set monthly time periods, and the monthly interest rate r to target a yearly rate of 6%. Following Shimer (2005), they set b equal to 0.4 and obtain a leisure value of $B = 0.15$. We maintain these parameters. We also follow them in normalizing market tightness to one, and target their unemployment and separation rates of 6% and 2%, respectively. These rates imply a steady-state job finding rate p of 31.33%, and in turn, a matching function scale factor of $\chi = 0.3133$. Both the matching elasticity η and bargaining power γ are set to 0.5 for simplicity.

The persistence of log productivity is set to $\rho = 0.97$, and the innovation's standard deviation to $\sigma_\varepsilon = 0.12$ and the minimum-wage to $\underline{w} = 0.873$ in order to target the separation rate of 2% and share of minimum-wage earners of 1.6%, which is in line with data from the BLS. We pin down the value of ξ by normalizing mean productivity to one ($\mathbb{E}[x] = 1$). The productivity's process is approximated as a 15-state Markov chain using the method of Tauchen (1986). Also, following Bils et al. (2011), we consider a credit limit of $\underline{a} = -6$ (which they set to roughly six months' worth of labor income). The described parameters and grid choices imply a vacancy flow cost of $\kappa = 0.523$ in our baseline steady-state.

4.1.2 Solution method

To solve the steady-state equilibrium, one must obtain fixed-points for the separation policy function $a^*(x)$ and the wage schedule $\omega(a, x)$. The worker's problem is solved by extending the endogenous grid method (EGM) of Carroll (2006) to the current framework. This requires computing the wage schedule's derivative in the Euler equations (26) and (27). In order to do this, we approximate the wages schedule using shape-preserving cubic splines, and compute the corresponding piece-wise polynomial's derivative.[14] The broad solution algorithm is as follows:[15]

Algorithm 1. ***Steady-state solution***

i) Guess the market tightness θ.

ii) Guess a separation-threshold function $a^*(x)$.

[14]For stability, we take a smooth approximation of the wage schedule and its derivative with respect to wealth when interpolating. Furthermore, due to the potential scale difference between the job's value implied by equations (20) and (24) for a given wage-schedule guess, the wage schedule updates in step vi) require considerable dampening. The separation-threshold and wage schedule updates can be used to refine the guesses in the appropriate steps to speed up the algorithm.

[15]We verify that the solution to the agent's problem using this method agrees with that of an off-the-shelf grid-choice VFI solution, which, albeit vis-a-vis less precise and more intensive in computing time, requires no approximation for the wage-schedule derivative.

iii) *Guess a wage schedule* $\omega(a, x)$.

iv) *Compute the wage schedule's derivative* $\partial\omega_a(a, x)$ *and solve the agent's problem using the EGM on the agent's Euler Equations* (26) *and* (27), *accounting for the credit constraint* $a' \geq \underline{a}$.

v) *Compute the job's value function by iterating on* (19).

vi) *Compute the wage schedule implied by the Nash-bargaining FOC* (25) *and job's value function* (19). *Impose the minimum wage constraint where appropriate.*

vii) *Update the wage schedule and return to step iv) until convergence.*

viii) *Compute and update the separation threshold policy* $a^*(x)$ *implied by* $J(a^*, x) = 0$. *Return to step iii) until convergence.*

ix) *Compute the stationary distribution by iterating on* (22) *and* (23).

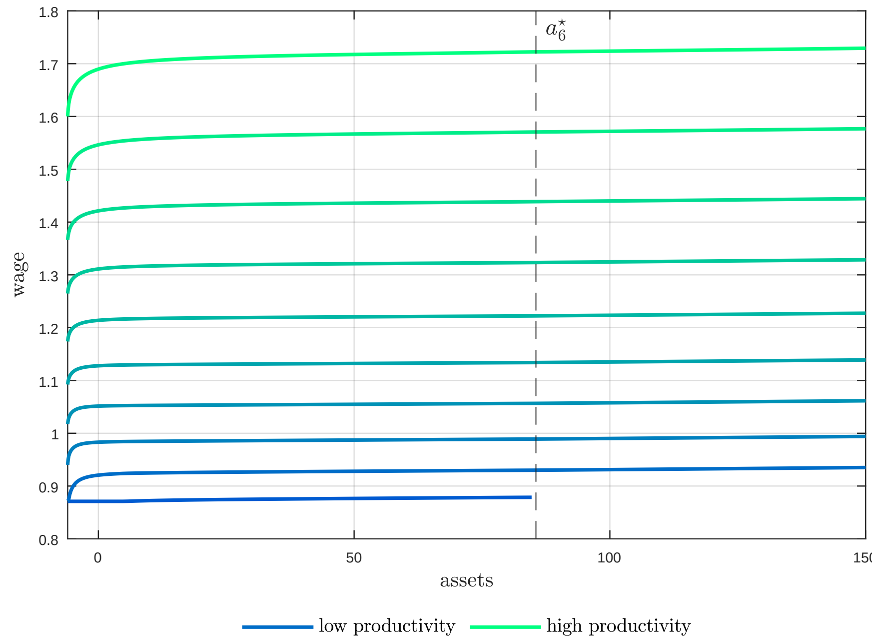
x) *Compute the value of vacancy using* (21) *and update market tightness* θ. *Return to step ii) until convergence.*

It is worth recalling that the presence of a binding minimum wage can potentially render the worker's problem non-convex. The intuition for this is the following: since the job's value is decreasing in the employee's wealth, for a given productivity, a binding minimum wage may generate inefficient separations that lead to termination of a match only for wealthy workers—distorting their savings incentives around that threshold. We document this possibility in Section 4.3.1 below. In general, the solution method proposed by Iskhakov et al. (2017) to deal with non-convexities in the value function for a discrete-continuous choice model could be applied here to deal with these issues. However, it turns out that given the parameterization we rely on, the dispersion of the distribution for productivity is sufficient to convexify the worker's problem so that there exists no multiplicity of local-optima satisfying the Euler equations, in line with the kink-smoothing effect of a taste shock in Iskhakov et al. (2017).

4.2 Policy function and equilibrium wages

We now present the policy functions and equilibrium wage relations approximated using Algorithm 1. Figure 3 displays the relation between wages and the two state variables in the program of an employed worker, namely assets and the idiosyncratic productivity shock. The former is plotted on the x-axis, while different values for the latter correspond to different lines in the graphs: the line at the bottom gives the relation between wages and assets for

Figure 3: Equilibrium wage schedule: baseline



Notes: Figure shows the equilibrium wage schedule $\omega(a, x)$ in the baseline model. The dashed line denotes the separation asset-threshold $a^\star(x)$ for employed workers of productivity x_6.

the lowest productivity realization, while the line at the top gives the same relation for the highest productivity realization. The vertical dashed line in the graph corresponds to the level of assets beyond which an employed worker of productivity x_6 (the lowest productivity realization that achieves employment) is laid off: this is the reason why the bottom line doesn't extend beyond the vertical dashed line.

The graph shows that wages are increasing in both productivity and assets (when workers do not earn the minimum wage). The former relation already characterizes the standard Mortensen and Pissarides (1994) framework: because productivity increases the match surplus, it also allows workers to negotiate wages further away from the threat point. As in Krusell et al. (2010) and Bils et al. (2011), wages are increasing in assets because of a wealth effect: for lower values of assets, the marginal utility of consumption is higher, increasing the attachment to the labor market and resulting in lower negotiated wages.

It is worth noting the larger concavity of the wage relations for lower values of assets. It is due to the fact that workers holding such levels of wealth are close to their borrowing limit. This strengthens their precautionary behavior, making their consumption function more concave to attempt to move away from the borrowing limit, while their consumption function becomes closer to a linear function for larger values of wealth, approaching the slope of the perfect foresight consumption function (Carroll, 2001). The concavity of the consumption policy functions in turn influences the concavity of the wage relations because of the wealth effect that enters wage negotiation. As Figure A1 suggests—which displays the consumption policy functions—the concavity of the wage relations coincide with the concavity of the consumption functions.

Table 1: Quantitative importance of the *dolling-up* effect: marginal return for several wealth categories of employed workers

Wealth	Annualized return
Bottom 1%	8.72%
Bottom 5%	3.18%
Bottom 10%	2.01%
Bottom 10-20%	0.48%
Bottom 20-30%	0.31%

Note: the table shows the annualized average dolling-up rate (defined as the average derivative of wage schedule with respect to assets, $\partial_a \omega(a, x)$) within each wealth group for the calibrated economy. These figures can be compared to the annualized risk-free interest rate of $r = 6\%$ in the economy.

As previously discussed, the positive slope of the wage functions in Figure 3 provides workers with additional motives to accumulate assets as a means to renegotiate higher wages in the future. The Euler equations (26) and (27) in Section 3 illustrate these incentives more formally. As evident in Figure 3, this *dolling-up* effect is strongest for low values of wealth, where the concavity of the wage functions is more pronounced. Table 1 shows the quantitative importance of this effect in the calibrated economy. Specifically, it gives the average value of the derivatives $\partial_a \omega(a, x)$ in equation (26) for different wealth categories of employees,[16] presented as annualized rates to facilitate its comparison to the annualized risk-free interest rate of $r = 6\%$ in the economy. One can notice in the table that the *dolling-up* effect is strongest for the bottom 1%—for which the respective return is 8.7%, above the value of the interest rate of 6%—and quickly dies out as one considers wealthier groups, remaining large through the bottom 5%.

[16] Notice that these derivatives are zero for unemployed workers given that they do not earn a wage. It is also zero for minimum wage workers.

4.3 The impact of an increase in the minimum wage

4.3.1 Labor market

Table 2 shows the impact on labor-market variables of a small and large increase in the minimum wage. The first column displays equilibrium aggregates for the calibrated model. As argued previously, increasing the minimum wage prevents workers from bargaining down their wages to extend match duration and causes the destruction of low-productivity jobs. These become unprofitable for firms in the face of the now higher wage expense. In the model, the extent to which the minimum wage induces job separations crucially depends on its impact on the expected value of a job and the distribution of employed workers around the separation thresholds—the mass of workers subject to increased job-loss risk.

Table 2: Labor market impact of a minimum wage increase

	Minimum wage		
	Baseline	+2%	+10%
Unemployment rate	5.9%	5.9%	9.8%
Separation rate	1.96%	1.97%	3.37%
Share earning minimum wage	1.6%	5.9%	9.7%
Job finding rate	31.3%	31.3%	31.0%
Mean wage	1.13	1.13	1.15
Wage dispersion	0.20	0.20	0.19

Note: the table shows equilibrium labor-market aggregates for the baseline model, the case of a 2% minimum wage increase, and a 10% minimum wage increase.

As has been argued in the empirical literature, employment effects of the minimum wage differ whether one considers small or large increases (Clemens and Strain, 2021; Jardim et al., 2022). Column 2 of this table shows that the model generates little aggregate labor-market movement in response to a moderate minimum-wage increase. Separations only marginally increase in this case, and there is virtually no effect on unemployment despite the substantial increase in the share of minimum-wage earners. The reason behind this is twofold. As can be noted by the separation threshold in Figure 3, the discretization of productivity in our baseline numerical exercise yields a solution where most productivity levels are either profitable or unprofitable for the firm regardless of the worker's asset holdings—implying that only a small fraction of workers are potentially affected by small minimum-wage increases. Furthermore, as shown in Figure A2, the minimum wage generates a negative spillover on bargained wages for non-minimum-wage earners, which acts as a countervailing effect for low-productivity jobs. This result echos the dampening effect of wage bargaining on job creation of the linear utility model of Section 2.

Column 3 of Table 2 displays the impact of a large increase in the minimum wage in the model, namely, a 10% increase. As would be expected, unemployment effects are significant in this case. The large increase substantially reduces the profitability of low productivity firms—destroying an entire productivity level worth of jobs—and results in a sizeable increase in job separations. Note that, by construction, the discretization of productivity in the model limits the (already large) increase in minimum-wage earners, as the bargained wages in the next productivity level are not close to the minimum. [htp!]

Figure 4 shows how the equilibrium relations between wages and the state variables are affected by the small (top graph) and large (bottom graph) increases in the minimum wage. The vertical solid line in both graphs corresponds to the separation threshold that characterizes the benchmark economy (the vertical dashed line in Figure 3). Both graphs also include the wage relations of the benchmark economy as a comparison (the solid lines), while the dashed and dotted lines correspond to the wage relations in the case of a 2% and 10% increase in the minimum wage, respectively. This figure shows the aforementioned wage compression at the bottom of the distribution. It also shows that wages right above the minimum are also renegotiated downwards (to a lesser extent in the case of the small increase) and that those further up the distribution do not appear to be significantly affected.[17] Figure A2 in the appendix displays the wage distributions for each productivity level, confirming these insights.

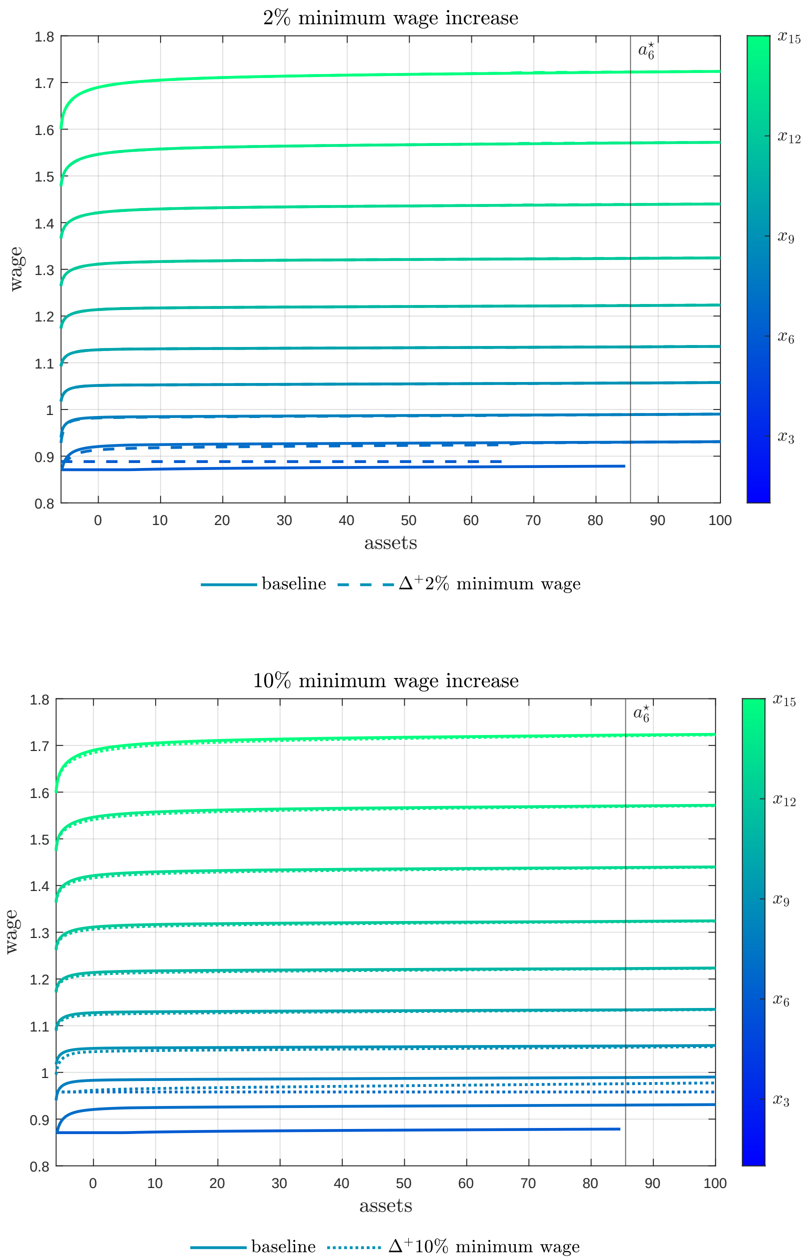
[htp!]

A (relevant) missing piece of information in Figure 4 is the distribution of workers on the assets-productivity plane. To understand how the increase in the minimum wage may shape labor market risk differently across the distribution of wealth, we present the share of workers earning the minimum across deciles of the wealth distribution in Table 3.[18] The table shows that, while the minimum wage is binding for the first two deciles of the wealth distribution in the benchmark economy, the higher minimum wage levels considered in the last two columns cover a significantly larger portion of the population in the other deciles. The information

[17]Notice that, in the linear model, wage compression at the top is stronger because this model was relying on a Poisson process for the process of idiosyncratic productivity. When we approximate a persistent autoregressive process for the economies in Figure 4, wage compression is lower at the top because at high productivity it's unlikely to hit the minimum wage in a near future. Figure A2 does show in more detail that there exists some compression at the top, but at a lower magnitude.

[18]The figures displayed in Table 3 are not directly comparable with the numbers from Card and Krueger (2016) noted in the Introduction, because these regard family-income deciles while we consider wealth deciles here.

Figure 4: Equilibrium wage schedule, minimum-wage increases



Notes: Figure shows the equilibrium wage schedules over assets (x-axis) and productivity levels for the baseline (solid lines), a 2% minimum-wage increase (top panel,dashed lines), and a 10% minimum-wage increase (bottom panel,dotted lines). Blue lines give the wage schedule for lower productivities, while green lines consider higher productivities.

Table 3: Share of workers earning the minimum wage by wealth decile

	Minimum wage		
	Baseline	+2%	+10%
Decile 1	8.3%	8.3%	12.8%
Decile 2	6.0%	6.8%	10.7%
Decile 3	0.0%	6.2%	10.0%
Decile 4	0.0%	6.0%	9.6%
Decile 5	0.0%	5.7%	9.4%
Decile 6	0.0%	5.6%	9.2%
Decile 7	0.0%	5.5%	9.0%
Decile 8	0.0%	5.4%	8.9%
Decile 9	0.0%	5.3%	8.7%
Decile 10	0.0%	0.9%	8.4%

Note: the table shows share of agents in each decile earning the minimum wage for the baseline model, the case of a 2% minimum wage increase, and a 10% minimum wage increase.

contained in the last two columns is consistent with labor-market risk being larger for richer workers, especially in the case of the large minimum wage increase. This impact on the risk of job separation for richer individuals in turn enhances their precautionary saving behavior, with consequences for the dispersion in wealth, as we now illustrate below.

4.3.2 Wealth inequality

Table 4 shows results for the steady-state distributions of wealth and consumption given the increases in the minimum wage in our two exercises. The main observation is that, while the 2% increase in the minimum wage does not affect the dispersion of assets significantly, the larger 10% increase in the minimum wage increases the dispersion of wealth by a great deal. Such changes also translates to the dispersion in consumption, to a lower extent though, given permanent income theory.

Three mechanisms explain the increase in the dispersion of assets brought by the large increase in the minimum wage. First, as was noted in Table 2, the large increase in the minimum wage also brings a large increase in the rate of unemployment: given that unemployed workers do not earn a wage, wealth dispersion mechanically increases because of the rise in unemployment. This is an old argument against the minimum wage: Stigler (1946) rested upon it to claim that the minimum wage generates more poverty. However, this first—standard—channel is not enough to explain the increase in asset dispersion, as one notices from the table that assets increase on average too. Second, as previously discussed

Table 4: Distributional effects of a minimum wage increase

Assets	Minimum wage Baseline	+2%	+10%
Mean			
Unemployed	11.4	11.5	15.1
Employed	15.6	15.6	19.3
All	15.4	15.4	18.9
Standard deviation			
Unemployed	12.7	12.9	15.0
Employed	13.5	13.5	15.7
All	13.5	13.5	15.7

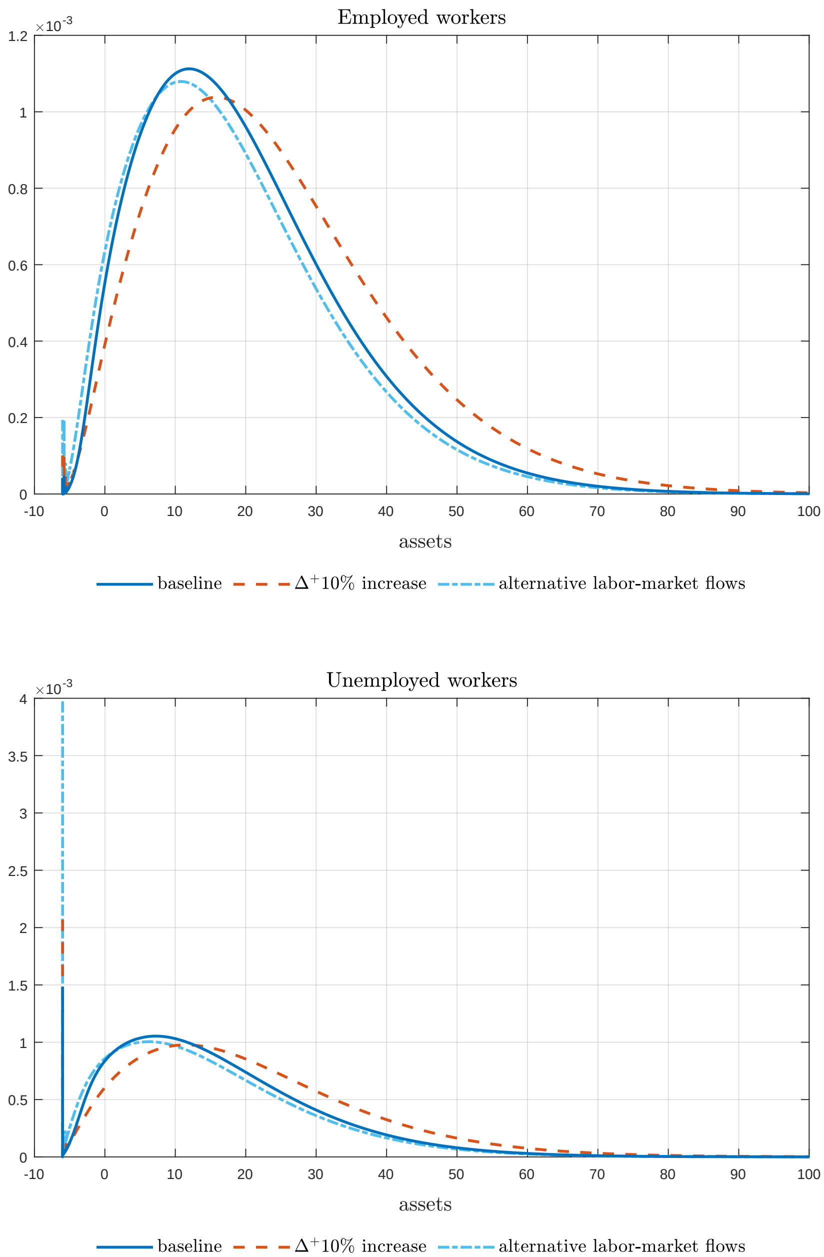
Consumption	Minimum wage Baseline	+2%	+10%
Mean			
Unemployed	1.07	1.07	1.09
Employed	1.17	1.17	1.18
All	1.16	1.16	1.17
Standard deviation			
Unemployed	0.15	0.15	0.16
Employed	0.12	0.12	0.13
All	0.13	0.13	0.14

Note: the table shows statistics characterizing the distributions of assets and consumption for the baseline model, a 2% minimum wage increase, and a 10% minimum wage increase. The last column summarizes the decomposition exercise (alternative labor-market flows) of Figure 5.

and noted in Figure 4, wealthier workers (for a given level of productivity) are the first to be affected by an increase in the minimum wage in terms of job separation. This implies that labor-market risk especially increases for these workers, enhancing their incentives to save for precautionary reasons, i.e., this second channel increases the incentives to save for wealthier individuals. Third, the share of minimum-wage workers increases. These workers are less subject to the *dolling-up* effect. This last channel reduces the incentives to save at the bottom of the distribution.

To illustrate the importance of the latter two channels as drivers of the increased wealth inequality, as compared to the former traditional one, Figure 5 presents a decomposition exercise that tries to identify changes in the distribution of assets brought by the former channel. Distributions of assets for the employed are displayed at the top of the figure and for the unemployed at the bottom. Both graphs consider three situations. The solid line gives the distribution of assets in the case of the calibrated economy; the dashed line gives the distribution in the case of a 10% higher minimum wage; and the dashed-dotted line con-

Figure 5: Asset distribution comparison



Notes: Figure shows the asset distribution of employed (top panel) and unemployed workers (bottom panel). The solid and dashed lines respectively display the baseline and 10% minimum-wage increase equilibrium outcomes. The dashed-dotted line displays the invariant distribution generated by the labor-market transition probabilities associated with a 10% minimum wage increase while preserving the baseline consumption/savings policy functions (note that the worker's behavior is not optimal in this exercise).

siders the stationary distribution of assets in an economy where the labor-market transition probabilities are the ones of the 10%-larger minimum-wage economy, while preserving the consumption/saving policy functions of the calibrated economy. We can thus understand the quantitative importance of the traditional channel by comparing the solid line distribution with the dashed-dotted line distribution, as it only considers the changes in employment brought by the minimum wage. As one can notice on the figure, the traditional channel shifts the distribution to the left, leaving a larger mass of agents on the borrowing limit, but marginally reducing the dispersion of wealth: the standard deviation of assets is 13.3 for this distribution as compared to the 13.5 in the baseline economy given in Table 4. The channels based on precautionary savings and the *dolling-up* effect thus contribute more to the increase in wealth inequality when increasing the minimum wage by 10%.

4.3.3 Welfare

Finally, Figure 6 shows the change in welfare for the small and large increases in the minimum wage, depending on the asset position of agents and their idiosyncratic productivity. The top graph considers the change by 2% in the minimum wage, while the bottom graph considers the 10% increase. The welfare calculations are expressed in terms of the consumption equivalent variation and compares workers facing the same state variables in two steady states (the baseline economy and the steady state of an economy where the minimum wage is 2% or 10% higher).

The graphs show that welfare effects are generally small, especially in the case of the 2% increase. One can notice that the small increase in the minimum wage favors workers subject to the lowest productivity realization—up to the new separation threshold (depicted by the dashed vertical line), who enjoy a direct increase in the wage they earn. This welfare increase is smaller for wealthier workers because the marginal utility of consumption decreases with wealth. A very small fraction of workers on the next productivity level also benefits from the increase in the minimum wage. These are the workers with the lowest amounts of assets, for which the minimum wage suddenly binds:[19] on top of experiencing a wage increase—that makes the borrowing limit less stringent—the minimum wage alleviates the extra savings incentives brought by the *dolling-up* effect.

The rest of the workers, however, suffer welfare losses. Two groups of workers are particularly affected. First, the wage renegotiation that occurs for wages just above the minimum is detrimental for workers close to the borrowing limit. While such renegotiation has no impact on lifetime utility in the context of the linear model of Section 2 (when job separation

[19] An inspection of Figure 4 reveals that the minimum wage suddenly binds for these workers.

remains unchanged), the borrowing limit in the model of Section 3 impedes workers from smoothing consumption over time. Thus, the wage renegotiation has a non-neutral impact in this context because it brings the level of wealth closer to the borrowing limit. This channel affects more workers with a lower productivity realization and a lower level of wealth, for whom the marginal utility of income is larger, as can be seen on the top graph of Figure 6.

Second, wealthier workers who are hit by the drop in the separation threshold are impacted to a greater extent.[20] These workers suddenly face increased labor-market risk because the probability that they will be laid off in a near future goes up. This channel affects workers with a lower productivity realization more because their marginal utility of income is larger. Note that unemployed workers are less impacted than low-productivity workers because separation is not their primary concern: they care more about finding a job first, and their expected productivity is larger upon hiring than the one of low-productivity workers.

The interpretation of the welfare results in the case of the large increase in the minimum wage (bottom graph in Figure 6) is straightforward. In this case, the increase in the job separation probability is felt by all workers, primarily those with a larger marginal utility of income, that is, those with less assets and a lower productivity. Logically, the welfare losses in the case of the 10% increase in the minimum wage are also substantially larger than in the former case.

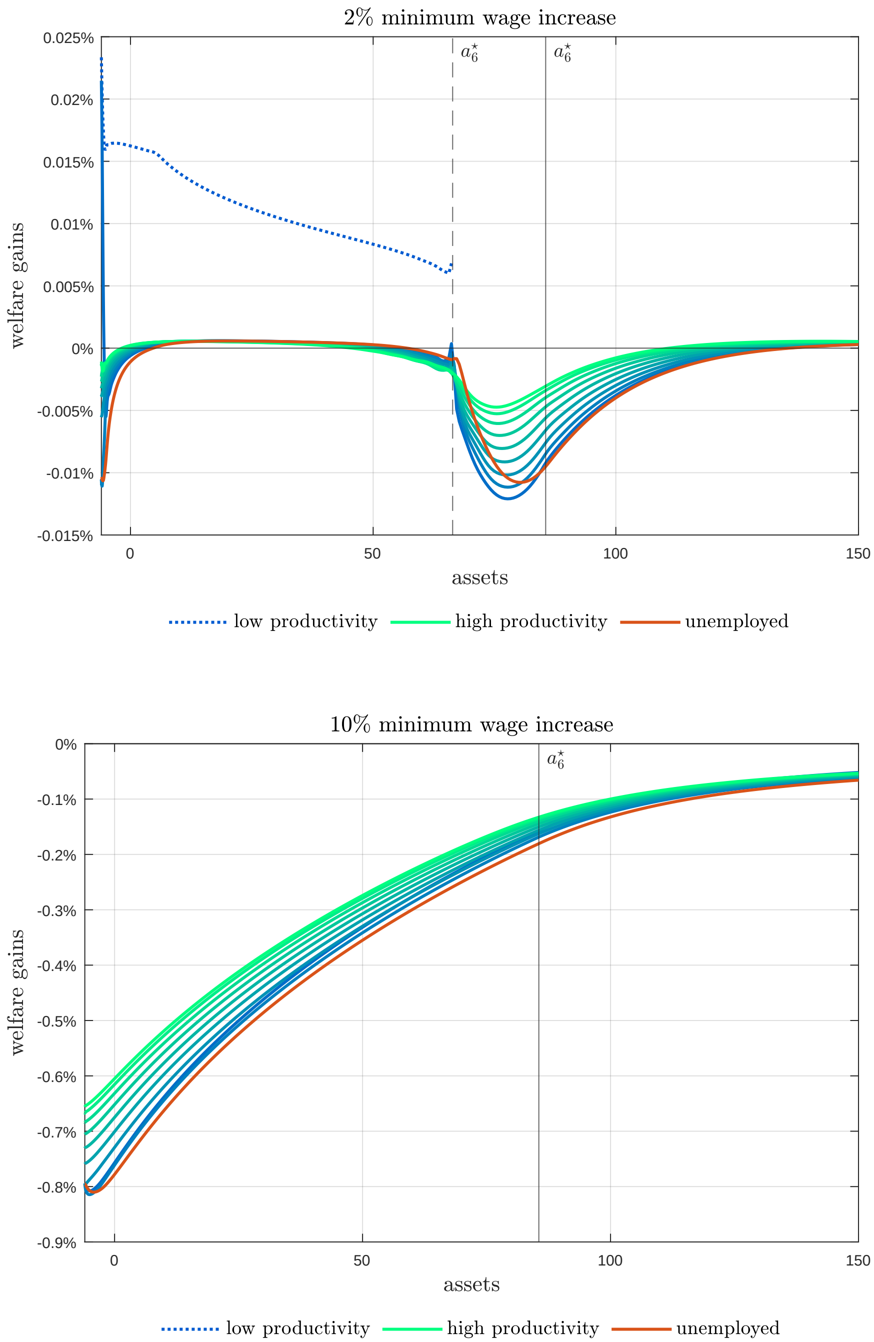
5 Introducing Capital

In the previous sections, we showed that a large increase in the minimum wage can produce an increase in the dispersion of wealth, mainly because precautionary savings by agents with larger wealth is enhanced. This increased saving effort also contributes towards an increase in aggregate savings. It is thus natural to ask if the increased aggregate savings can limit the negative impact the minimum wage has on employment if the aggregate capital stock goes up: when capital and labor are complements in the production function, an increase in the stock of capital increases the marginal product of labor and produces incentives to post vacancies.[21] For this reason, we now consider an extension of the model in Section 3 to include the presence of capital.

[20]The drop in the separation threshold is illustrated by the vertical solid line being replaced by the vertical dashed line.

[21]In the model by Bauducco and Janiak (2018), an increase in the minimum wage improves the holdup problem on capital (Acemoglu and Shimer, 1999) and limits the negative impact of the minimum wage on the incentives to post vacancies because of the complementarity between capital and labor in the production function.

Figure 6: Welfare effects, minimum-wage increase



Notes: Figure shows the steady-state welfare effects across asset holdings of a 2% minimum wage increase (top panel) and a 10% minimum wage increase (bottom panel) with respect to the baseline economy. Blue lines denote the welfare effects for lower productivity levels (the lowest productivity level employed in the baseline economy is dotted), green lines those for higher productivity levels, and the orange line denotes those for unemployed workers.

We consider a final good sector, where a representative firm produces a final good by combining capital K and intermediary goods in quantity X. These intermediary goods are produced by firms that use labor. The production function of the final good firm is the following:

$$Y = ZK^{\alpha}X^{1-\alpha},$$

where $\alpha \in (0,1)$ is a parameter, Y is the quantity of the final good produced, and Z the total factor productivity. Both markets for capital and intermediary goods are competitive. The price of intermediary goods is denoted by p and $(r+\delta)$ is the marginal cost of capital the representative firm faces, where δ is the capital depreciation rate, and the interest rate r—that was exogenous in Section 3—is now endogenous and determined by capital-market clearing. The first-order conditions for the representative firm are

$$Z(1-\alpha)\left(\frac{K}{X}\right)^{\alpha} = p,$$

and

$$Z\alpha\left(\frac{X}{K}\right)^{1-\alpha} = r+\delta.$$

The firms producing the intermediary goods are those firms we studied in the model of Section 3, with the subtlety that now they sell their goods at the price p. Their value is now determined by the following modification to equation (20):

$$\tilde{J}\left(w,a,x\right) = px - w + \frac{1}{1+r}\,\mathbb{E}_{x'|x}\left[\max\langle J\left(a'_e, x'\right), V\rangle\right].$$

Thus, the specification here nests the model of Section 3 as a particular case of this more general framework, which corresponds to the case with $p=1$. The market-clearing conditions for the intermediary-goods and capital markets in the steady state are given by:

$$X = \int_{\mathcal{A}}\int_{\mathcal{X}} x d\Phi_e\left(a,x\right),$$

and

$$K = \int_{\mathcal{A}}\int_{\mathcal{X}} a d\Phi_e\left(a,x\right) + \int_{\mathcal{A}} a d\Phi_u\left(a\right).$$

The steady-state equilibrium definition and computation follow analogously from Section 3, which can be readily extended to account for the two new market-clearing conditions and optimality conditions of the representative firm previously described. Following Krusell et al.

(2010), we set $\alpha = 0.3$ to target a capital share of 0.3 and set the annualized depreciation-rate to $\delta = 12.5\%$ to target an investment-to-output ratio of 0.2.[22] We set the discount factor at $\beta = 0.995$ to target the annualized interest-rate of 6% of Section 3, maintain the baseline minimum-wage level at $\underline{w} = 0.873$, and set $Z = 0.5195$ and $\kappa = 0.562$ to normalize $\theta = 1$ and $p = 1$ in the baseline economy.

Table 5 shows how the labor-market impact of the minimum wage varies when one allows the aggregate capital stock to adjust. The columns denoted "Fixed capital" consider the model of Section 3, while the columns denoted by "Flexible capital" consider the model with the endogenous capital stock extension.[23]

Table 5: Labor market impact of a minimum-wage increase with endogenous capital stock

		Minimum wage			
		Fixed capital		**Flexible capital**	
	Baseline	+2%	+10%	+2%	+10%
Unemployment rate	5.95%	6.17%	9.79%	6.17%	9.72%
Separation rate	1.98%	2.06%	3.37%	2.06%	3.37%
Share earning minimum wage	1.43%	5.54%	9.67%	5.53%	9.67%
Job finding rate	31.3%	31.3%	31.0%	31.3%	31.3%
Mean wage	1.13	1.14	1.15	1.14	1.16
Wage dispersion	0.20	0.20	0.19	0.20	0.19

Note: the table shows equilibrium labor-market aggregates for the baseline model, the case of a 2% minimum wage increase, and a 10% minimum wage increase. Minimum wage impacts are shown for both fixed and variable capital-stock models.

The main message from Table 5 is that the results of Section 3 are not affected much when one allows the capital stock to adjust. The main reason behind this is that the equilibrium prices r and p do not respond much to minimum-wage increases in this extended model.

[22]It is common to consider a ten percent annual capital depreciation rate when calibrating macro models. The 12.5% may thus seem a bit high. However, Gomme and Rupert (2007) documented that depreciation rates of equipment and software have been rising.

[23]In order to target the same annualized interest-rate of 6%, the parameters used in this model extension differ slightly from those of Section 3. Because this required increasing the discount factor and vacancy flow-cost, the impact on unemployment is a bit larger in Table 5 than the one reported in the analogous Table 2 of Section 3.

6 Conclusion

We have studied the impact of the minimum wage on the distribution of wealth and the labor market. Our framework is the Bewley-Huggett-Aiyagari model extended with search and matching frictions on the labor market. The model predicts an impact of the minimum wage on the labor market in line with part of the empirical literature on the topic: modest increases have little impact on employment, with a small compression of wages at the bottom of the distribution, while a larger increase significantly affects unemployment. We find that large increases of the minimum wage also amplify wealth inequality. According to a traditional economic mechanism, more inequality appears in this case because fewer workers earn wages due to the increase in unemployment. However, our simulations suggest that this channel is quantitatively less important than the combination of two other mechanisms. First, the minimum wage increases the precautionary savings of wealthy workers because it puts their jobs more at risk than the jobs of poorer workers. Second, absent a minimum wage, poor workers have incentives to improve their financial position to renegotiate higher wages in the model: the minimum wage thus decreases the incentives to save for poorer workers, given that some of them are ensured to earn the minimum wage. Given that the incentives to save are increased by the minimum wage at the top of the distribution and decreased at the bottom, wealth inequality is exacerbated by the minimum wage. As in the fable by de La Fontaine, the minimum wage thus sets the scene with both ants and grasshoppers in the economy.

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Appendix

A Linear utility model

This appendix derives the main results for the linear utility model when the minimum wage is binding. When the minimum wage does not bind, the model reduces to that of Mortensen and Pissarides (1994). In this case, Nash-bargaining implies that both the worker and firm obtain a fraction equal to their bargaining power of the match's total surplus, $S(x) = W(x) - U + J(x) - V$. As this surplus is increasing productivity, separations are privately efficient and satisfy a reservation property—namely that all matches with productivity below some level R (defined by $S(R) = 0$) are destroyed. Furthermore, the resulting wage schedule is also increasing in productivity.

A.1 Equilibrium wages

Let $H(x) \equiv W(x) - U$ and $J(x)$ respectively denote surplus of a match to the worker and firm. Note that the latter is equivalent the job's value function when free-entry holds, i.e., when $V = 0$. Wages solve the following constrained Nash-bargaining problem:

$$\max_{w(x)} \; H(x)^{\gamma} J(x)^{1-\gamma} \quad \text{s.t. } w(x) \geq \underline{w}. \tag{A1}$$

Given that wages are increasing in the unconstrained problem, the minimum wage acts solely as a side constraint to this problem. When it binds, a sightly modified reservation property holds. To see this, consider a match with productivity $x = R$ and null surplus in an economy without a minimum wage. The imposition of a minimum wage $\underline{w} > w(R)$ increases the respective worker's surplus but decreases that of the firm to $J(R) < V = 0$, resulting in the destruction of this match. That is, job destruction will now be unilaterally determined by firms and the reservation productivity will be given by $J(R) = 0$.

Rearranging the value functions (4) and (5) results in the following expressions for the firm's and worker's surpluses:

$$J(x) = \frac{1+r}{r+\phi}\left[x - w(x) + \frac{\phi}{1+r}\int_R^{\bar{x}} J(z)\, dG(z)\right] \text{ and} \tag{A2}$$

$$H(x) = \frac{1+r}{r+\phi}\left[w(x) - \frac{r}{1+r}U + \frac{\phi}{1+r}\int_R^{\bar{x}} H(z)\, dG(z)\right]. \tag{A3}$$

Consequently, total match surplus can be written as:

$$S(x) = \frac{1+r}{r+\phi}\left[x - \frac{r}{1+r}U + \frac{\phi}{1+r}\int_R^{\bar{x}} S(z)\ dG(z)\right]. \tag{A4}$$

Let x^* denote the productivity level up to which the minimum wage is binding. The first order condition of the Nash-bargaining problem (A1) for productivity $(x^*, \bar{x}]$ implies that surplus is shared according to:

$$\frac{H(x)}{\gamma} = S(x) = \frac{J(x)}{1-\gamma}. \tag{A5}$$

This result and the surplus equations imply that the bargained wage for all $x \in [x^*, \bar{x}]$ can then be written as:

$$w(x) = \gamma x + (1-\gamma)\frac{r}{1+r}U + \frac{\gamma\phi}{1+r}\int_R^{x^*} J(z)\ dG(z) - \frac{(1-\gamma)\phi}{1+r}\int_R^{x^*} H(z)\ dG(z). \tag{A6}$$

To evaluate the above integral, note first that for productivity $x \in [R, x^*]$, the worker receives the minimum wage $\underline{w}$ and therefore his surplus from employment is constant over that range. Furthermore, as the minimum wage and the bargained wage coincide at $x = x^*$, the surplus sharing rule (A5) implies that the worker's surplus $H(x)$ for all $x \in [R, x^*]$ satisfies:

$$H(x^*) = \frac{\gamma}{1-\gamma}J(x^*). \tag{A7}$$

Secondly, the firm pays the minimum wage $\underline{w}$ in low productivity matches, which is constant over x. Therefore, the job's surplus (A2) implies that $J'(x) = \frac{1+r}{r+\phi}$ for $x \in [R, x^*]$, and since job destruction is given by $J(R) = 0$, it follows that the job's value over this productivity range can be written as:

$$\underline{J}(x) = \frac{1+r}{r+\phi}(x - R), \tag{A8}$$

where $\underline{J}(x)$ denotes the job's value function when the minimum wage $\underline{w}$ is paid. Using these results, one can rewrite the bargained wages (A6) as:

$$w(x) = \gamma x + (1-\gamma)\frac{r}{1+r}U - \frac{\phi}{r+\phi}\gamma\int_R^{x^*}(x^* - z)\ dG(z). \tag{A9}$$

The above expression reflects the fact that firms internalize the lost surplus share in low productivity matches generated by a binding minimum wage. To see this, note that since $J(R) = 0$, match surplus is given by $S(R) = H(R)$ at the reservation threshold. Furthermore, given that the worker's surplus is constant and equal to $H(x^*)$ for $x \in [R, x^*]$, it follows from (A5) that match surplus at the reservation productivity R satisfies:

$$S(R) = \frac{\gamma}{1-\gamma}\frac{r+\delta}{r+\phi}(x^* - R), \tag{A10}$$

which is positive when the minimum wage binds. Noting from equation (A4) that $S'(x) = \frac{1+r}{r+\phi}$, it follows from the equation (A10) that the match's surplus can be written as:

$$S(x) = \frac{1}{1-\gamma}\frac{1+r}{r+\phi}\Big((x-R) - \gamma(x - x^*)\Big). \tag{A11}$$

Therefore, the surplus share a firm loses due to the imposition of a minimum wage with respect to the share it could capture if wages where freely bargained is given by:

$$\underline{J}(x) - (1-\gamma)S(x) = -\frac{(1+r)\gamma}{r+\phi}(x^* - x), \tag{A12}$$

which—in expected present value—is precisely the spillover term in the wage equation (A9).

A.2 Equilibrium curves

To obtain the job destruction curve, recall that matches separate when $J(R) = 0$ when the minimum wage is binding. Note that the job's value function (A8) implies that $\underline{J}'(x) = \frac{1+r}{r+\phi}$ over productivity levels $[R, x^*]$, where the minimum wage is paid. Since the job's value is given by $J(x) = (1-\gamma)S(x)$ for all productivity $x \in [x^*, \bar{x}]$ by virtue of the surplus sharing rule (A5), the job's surplus equation (A2) can be restated as:

$$\frac{r+\phi}{1+r}J(x) = x - w(x) + \frac{\phi}{r+\phi}\left(\int_R^{\bar{x}}(z-R)\,dG(z) - \gamma\int_{x^*}^{\bar{x}}(z-x^*)\,dG(z)\right). \tag{A13}$$

Given that the minimum wage $\underline{w}$ is paid at the reservation productivity R when the constraint in (A1) binds, the **job destruction equation** can be written as:

$$0 = R - \underline{w} + \frac{\phi}{r+\phi}\left(\int_R^{\bar{x}}(z-R)\,dG(z) - \gamma\int_{x^*}^{\bar{x}}(z-x^*)\,dG(z)\right). \tag{JD}$$

A new equilibrium condition is required in order to pin down the value of x^*. This condition is given by the threshold x^* at which **bargained wages** coincide with the minimum wage. Using the wage equation (A9), this condition—the interior negotiated solution condition (hereon INS)—is given by:

$$\underline{w} = \gamma x^* + (1-\gamma)\frac{r}{1+r}U - \gamma\frac{\phi}{r+\phi}\int_R^{x^*}(x^* - z)\, dG(z)\,. \tag{INS}$$

Note that two potential cases arise when the minimum wage is binding: if starting productivity is high enough to satisfy $x_0 \geq x^*$, wages are bargained and the surplus is shared according to (A5) in a new job; if a new job is formed at a low productivity $x_0 < x^*$, the worker is paid the minimum wage and his surplus is given by (A7). To pin the value of unemployment when wages are bargained in new matches, note that the free-entry condition $V = 0$ and vacancy value (7) imply that $\frac{c}{q(\theta)} = \beta J(x_0)$. It then follows from the Nash-bargaining surplus sharing rule (A5) that the value of unemployment (6) for all $x \in (x^*, \bar{x}]$ can be written as:

$$\frac{r}{1+r}U = b + \frac{\gamma}{1-\gamma}c\theta, \tag{A14}$$

which follows from the fact that $p(\theta) = \theta q(\theta)$. If the initial productivity is low enough for the minimum wage to be paid in new matches, it follows from Equation A7 implies that the new workers' surplus is equal to a proportion $\frac{\gamma}{1-\gamma}$ of $\underline{J}(x^*)$ (as defined in Equation A8). Therefore, the value of unemployment can be readily characterized as:

$$\frac{r}{1+r}U = \begin{cases} b + \dfrac{\gamma}{1-\gamma}c\theta & \text{if } x_0 \geq x^*. \\ b + \dfrac{\gamma}{1-\gamma}\dfrac{p(\theta)}{r+\phi}(x^* - R) & \text{if } x_0 < x^*. \end{cases} \tag{A15}$$

Analogously, since the firm pays the minimum wage over $x \in [R, x^*]$—valuing the job at $\underline{J}(x)$ defined by (A8)—and shares the surplus according to (A5) otherwise, the **job creation condition** here is given by:

$$\frac{c}{q(\theta)} = \begin{cases} \dfrac{1}{r+\phi}\Big((x_0 - R) - \gamma(x_0 - x^*)\Big) & \text{if } x_0 \geq x^*. \\ \dfrac{1}{r+\phi}(x_0 - R) & \text{if } x_0 < x^*. \end{cases} \tag{JC}$$

A.3 Job creation impact of the minimum wage

Assuming that x_0 is high enough for the wage to be bargained in new matches, the value of unemployment (A15) implies that the match surplus (A4) in new matches is determined by:

$$\frac{r+\phi}{1+r}S(x_0) = x_0 - b - \frac{\gamma}{1-\gamma}c\theta + \frac{\phi}{1+r}\int_R^{\bar{x}} S(z)\ dG(z)\,. \tag{A16}$$

Given the assumption for initial productivity x_0, the surplus of is shared according to Equation (A5). It follows from the free-entry condition (7) that said surplus can be written as:

$$c = \frac{q(\theta)}{1+r}(1-\gamma)S(x_0). \tag{A17}$$

Combining these equations results in:

$$\frac{c}{q(\theta)}\frac{r+\phi}{1-\gamma} = x_0 - b - \frac{\gamma}{1-\gamma}c\theta + \frac{\phi}{1+r}\int_R^{\bar{x}} S(z)\ dG(z)\,. \tag{A18}$$

Differentiation with respect to the minimum wage $\underline{w}$ then yields:

$$\left(\eta\frac{r+\phi}{p(\theta)}+\gamma\right)c\frac{\partial\theta}{\partial\underline{w}} = -\frac{\phi}{r+\phi}S(R)g(R)\frac{\partial R}{\partial\underline{w}}, \tag{A19}$$

As Equation A10 shows, $S(R)$ is positive when the minimum wage policy binds. Consequently, the above equation implies a fall in market tightness in response to an increase in the minimum wage, given that the reservation threshold increases ($\frac{\partial R}{\partial\underline{w}} > 0$) due to the induced marginal job-destruction effect.

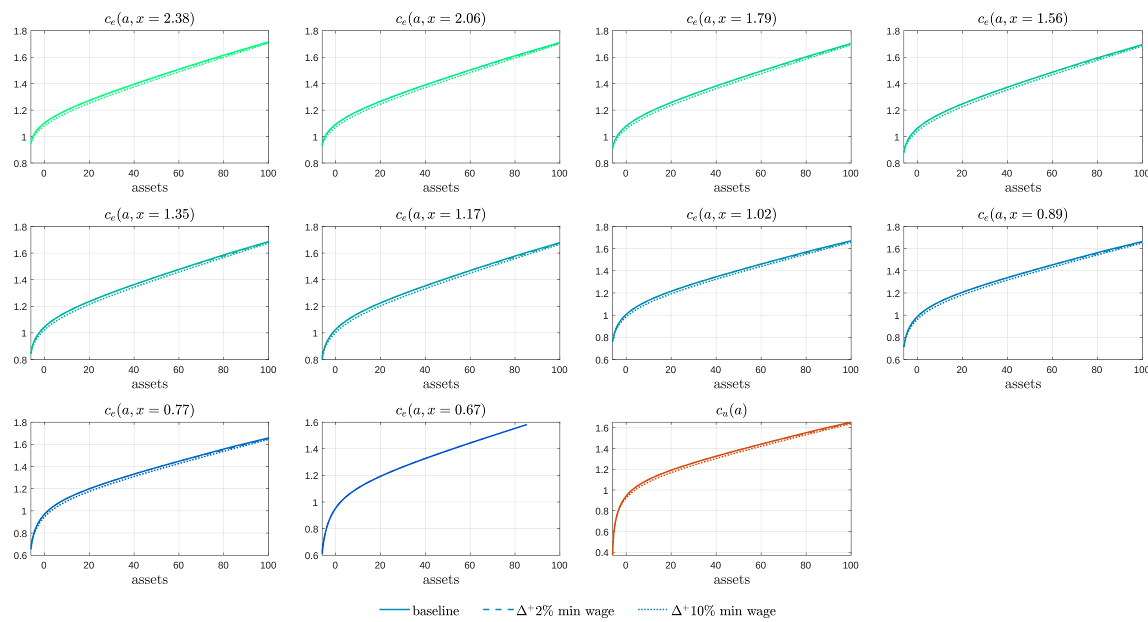
Appendix Tables & Figures

Table A1: Wage dispersion

	Minimum wage		
	Baseline	+2%	+10%
Mean-min ratio	1.30	1.28	1.20
50-min ratio	1.21	1.19	1.17
50-10 ratio	1.14	1.15	1.17
90-50 ratio	1.36	1.36	1.27

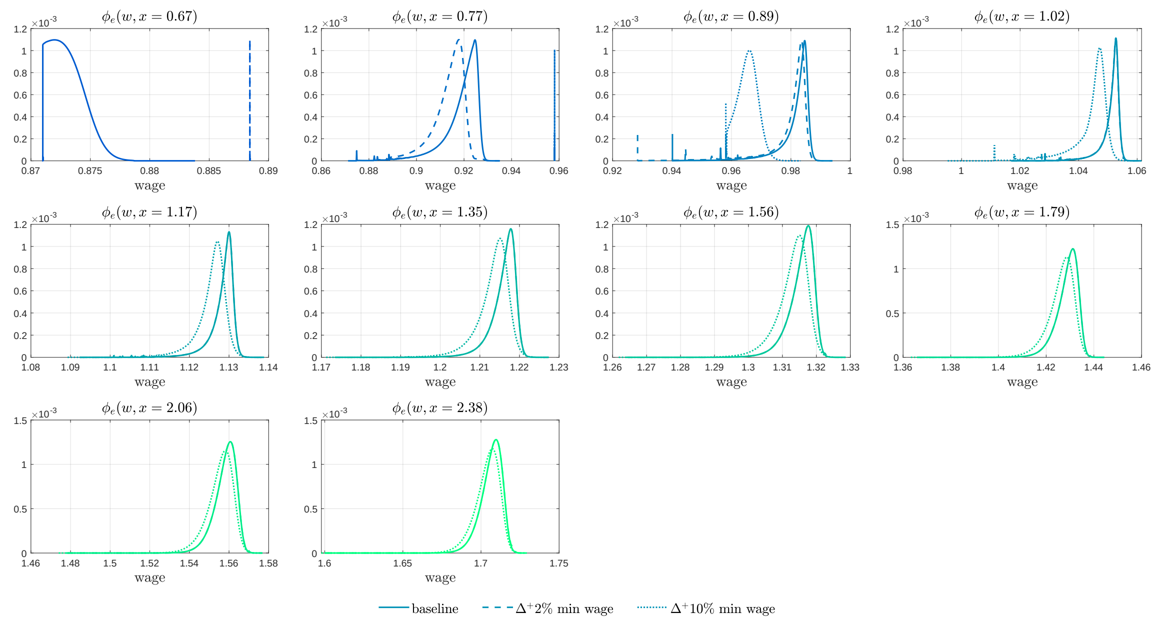
Note: Table shows wage ratios the baseline model, the case of a 2% minimum-wage increase, and a 10% minimum-wage increase.

Figure A1: Consumption policy function



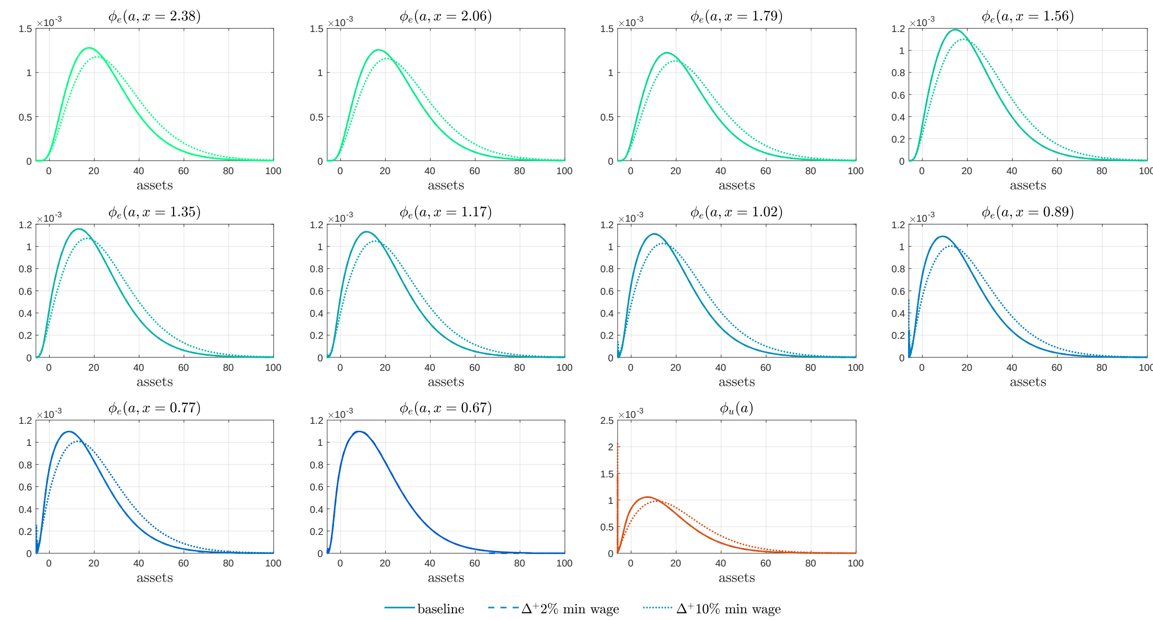
Notes: Figure shows the consumption policy over productivity (ordered from most to least productive on a green to blue color-map) and employment states (unemployed workers in orange) for the baseline model (solid line), a 2% minimum-wage increase (dashed line), and a 10% minimum-wage increase (dotted line).

Figure A2: Wage distribution



Notes: Figure shows the distribution of wages over assets (x-axis) and across productivity levels (ordered from least to most productive on a blue to green color-map) for the baseline model (solid line), a 2% minimum-wage increase (dashed line), and a 10% minimum-wage increase (dotted line).

Figure A3: Asset distribution



Notes: Figure shows the distribution of assets conditional on productivity (ordered from most to least productive on a green to blue color-map) and employment state (unemployed workers in orange) for the baseline model (solid line), a 2% minimum-wage increase (dashed line), and a 10% minimum-wage increase (dotted line).