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Competition, Incentives, and the Distribution of Investments in Private School Markets

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COMPETITION, INCENTIVES, AND THE DISTRIBUTION OF INVESTMENTS IN PRIVATE SCHOOL MARKETS

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Competition, Incentives, and the Distribution of Investments in Private School Markets[*]

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Abstract

This paper develops a one-to-one matching model to analyze how different education funding regimes affect incentives and equilibrium allocations in competitive markets served by heterogeneous private providers. The main result is that alternative funding schemes change the relative incentives faced by schools with different productivities, dramatically altering equilibrium allocations and outcomes. The paper also explicitly characterizes equilibrium in markets served by for-profit and non-profit schools, an analysis that has not been made in previous literature. The basic version of the model is calibrated using data from Chile's education market and used to simulate the impact of alternative policy scenarios.

Keywords: Education funding, school competition, heterogeneous firms, for-profit and non-profit firms.

JEL codes: I21, I22, L33, D40

1   Introduction

This paper contributes to the literature on the distribution of education investments and outcomes (Becker and Tomes, 1979; Benabou, 2002; Cunha and Heckman, 2007) by studying the role of two-sided heterogeneity on the equilibrium allocations in competitive school markets. In particular, we analyze how equilibrium outcomes are affected by different policy regimes that change the distribution of education funding across

schools. The paper is also closely related to the theoretical and empirical discussion of the effect of school vouchers on school qualities and the allocations of students across different types of schools (Epple and Romano, 1998 and 2002; Neal, 2002 and 2008; McMillan, 2005; Ferreyra, 2007; Vial, 2008; Urquiola and Verhoogen, 2009; MacLeod and Urquiola, 2009).

While most of the literature has given considerable attention to heterogeneity across students (both in income and ability), relatively less emphasis has been put on the differences in productivity between schools, and to the impact that the distribution of school funding has on the investment and enrollment decisions of schools of different types.

The paper makes two main contributions. First, it develops a tractable model with heterogenous agents that can be used to analyze the effect of different funding regimes on equilibrium allocations and matches. Secondly, ublike the rest of the literature, it explicitly discusses the interactions between for-profit and non-profit private schools, and how these interactions are affected by the policy regime funding education.

The paper presents a one-to-one matching model between schools and students. Students differ in ability and income, while schools differ in productivity and, in the last part of the paper, on their objective function. In the model, education outcomes are a joint product of student ability and the school's endogenous quality, which depends on the school's productivity and its investment decision. More productive schools need to invest less to reach a given quality level. The paper studies how different funding regimes, ranging from a centralized scheme with homogenous education expenditures to fully private tuition regimes, affect the schools' investment and enrollment decisions, as well as determining the set of schools that operate in equilibrium and the equilibrium sizes of different sectors. The paper analyzes how changes in which the way schools are funded impact the intensive and extensive margins, by changing the competitive incentives both from within and from outside the market. The paper also analyzes the competition between private schools with different objective functions - profit and non-profit - and how their behavior is affected by the distribution of education expenditure and the policy regime. This is an important contribution, as the previous literature has never fully characterized equilibrium allocations in such a setup, and has focused on profit-maximizing schools. The model is calibrated to fit the observed outcomes in Chile's education market. Chile implemented competitive voucher school markets in 1980,and currently more than 50% of students are enrolled in private schools. The calibrated model is then used to simulate the effect of alternative policy scenarios on equilibrium outcomes and allocations.

In the context of the model, there are four margins which determine the overall productive efficiency of the human capital accumulation process, conditional on an aggregate level of education funding. This is, efficiency defined not in terms of maximizing overall welfare, but rather on terms of the level of average education outcomes for a given level of spending. The first one, extensively discussed in Becker and Tomes

(1979), Benabou (2002), and Cunha and Heckman (2007), deals with the distribution of parental investments across households, in particular when capital markets are not complete and poor families are credit constrained. The second margin refers to the set of operating schools, and whether, under the institutional setting, it coincides with the set of most productive education providers. For example, implementing a voucher system might allow the entry of productive private schools, by eliminating an artificial barrier given by the restricted access to public funding in "traditional" public school regimes. The third margin relates to the equilibrium matching function between students and schools. If student ability and school productivity are complements in the production of education outcomes, do more productive schools enroll more able students? How are matches related to income? How does this depend on the school's objective function? Finally, the fourth margin refers to school investments, and the associated school qualities. This has two related aspects. First, how much of what schools receive from parents is actually invested in providing higher education outcomes? Secondly, how are investments distributed across schools with different productivities? Are the largest investments made by the most productive schools?

The paper consciously leaves aside issues such as the choice of school scale (and, as a related issue, the composition of the school body and the interactions between students), or the role non-academic elements in the demand for education, such as religious preferences or transportation. While this are certainly important elements to fully understand the operation of school markets, we rely on the stylized structure that delivers powerful conclusions, while being simple enough to obtain explicit analytic solutions.

The main result of the paper is that the distribution of education expenditures across households determines the distribution of equilibrium investments made by schools with different productivity. Regimes that allow for heterogeneous tuition payments - relative, for example, to a flat voucher that provides uniform funding - typically increase efficiency by changing the incentives faced by productive profit-maximizing schools. Heterogeneity in tuition payments between parents increases leads to an equilibrium allocation in which a larger share of investment is made by more productive schools. This efficiency gain, however, is associated with larger inequality in outcomes.

More specifically, the paper's main findings can be summarized as follows. The first two findings deal with the equilibrium behavior of profit-maximizing schools, while the other two deal with school markets served by for-profit and non-profit schools.

A. In a market with flat vouchers in which schools are not allowed to charge extra tuition, there need not be assortative matching between more able students and more productive profit-maximizing schools, even if there are complementarieties in the production of education outcomes. Differences in education outcomes between schools only reflect differences in student ability. All schools are ex-post identical, their quality determined by the threat of entry of the marginal school. More productive schools make strictly smaller

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investments.

B. In a market in which tuition payments are fully private, the competitive (efficient) allocation will be characterized by assortative matching between more productive profit-maximizing schools and students with higher willingness to pay for education. However, there need not be sorting in ability and productivity. Differences in outcomes between schools reflect differences in student ability and income, as well as school productivity. Schools will differentiate, with more productive schools being of strictly higher quality. Qualities depend on threat of entry and competition within existing schools. The distribution of school investments is shifted towards more productive schools.

C. When non-profit schools that choose to maximize outcomes coexist with profit-maximizing schools in a market with flat voucher and no additional payments, the market will be segmented, as profit-maximizing schools will choose not to compete directly with the non-profit schools, leaving the ablest students in the non-profit sector. Positive assortative matching will occur between more able students and more productive non-profit schools. Differences in education outcomes between non-profit schools reflect differences in student ability and school productivity.

D. When tuition payments are heterogenous, profit-maximizing schools have an incentive to compete directly with the non-profit schools. The solution might be characterized by multiple equilibria.

The organization of the paper is as follows. Section 2 presents the basic results on schooling outcomes and equilibrium allocations in a competitive market with heterogenous profit-maximizing private schools and students, under three distinct regimes: uniform public funding, fully private funding, and a mixed regime in which parents can choose to privately complement public funds. Section 3 allows for privately-funded and publicly-funded schools to coexist and solves for the endogenous equilibrium sector sizes when public education funding is financed with taxes. Section 4 presents a calibration using data from Chile's education market and simulates various counterfactual education funding policies. Section 6 extends the analysis in Section 2 to account for non-profit schools. Section 7 concludes.

2 A basic framework on competition between private schools

This section provides a general description on the operation of competitive education markets served by heterogeneous private profit-maximizing schools with fixed capacity. In particular, we analyze how different education funding regimes, which allow for varying degrees of heterogeneity in education expenditure across households, change the incentives faced by schools, and how that affects their equilibrium decisions.

We begin by analyzing two polar cases. Section 2.2 describes competition and equilibrium allocations in a market in which education expenditures are perfectly homogeneous across all households. This is akin

to an economy with mandatory flat vouchers in which direct parental expenditures are forbidden. Section 2.3 analyzes the case in which parents freely determine their education expenditures, with no provision of centralized funding. Section 2.4 combines both polar cases by studying in which households receive a voucher that they can choose to complement with private tuition.

2.1 General Setup

2.1.1 Households

Households maximize the following utility function:

$$U_i = U(c_i, h_i)\tag{1}$$

where c_i is total household consumption and h_i is the education outcome of the household's only child, which can be interpreted as human capital. Education can only be obtained by attending a school, and students cannot be enrolled in more than one school.

As usual, $U_c, U_h > 0; U_{cc}, U_{hh} < 0; U_c(0, h), U_q(c, 0) \to \infty$.

Households differ on two aspects: their income level, m_i, and the ability of their only child, a_i. Income and ability are exogenously distributed across the population[1].

Notice that ability is exogenous at the time of the decision[2] and is summarized by a one-dimensional index that arguably captures cognitive and non-cognitive skills. While this might be seen as a simplification of the most recent research (which can be summarized in Cunha and Heckman, 2007), providing a stylized description of skill formation allows us to focus on the main contribution of the paper: namely, the role of heterogeneity on school providers, and the impact of different funding regimes in equilibrium outcomes.

Throughout the paper, the utility function is assumed to have the specific functional form:

$$U_i = \ln c_i + \beta \ln h_i\tag{2}$$

where β is a utility weight.

Education is a joint function of the quality of the school in which the student is enrolled, q_j, and the student's ability, a_i, where h_{ij} is the outcome if the student is enrolled in a school with quality q_j. As

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[1] In a related paper (Ferrada and Tapia, 2010) I extend the analysis to a dynamic overlapping generations model, and endogeneize income by making it a function of human capital and the adult´s labor decision.

[2] This does not imply that ability is given at birth. As the model is static, it is consistent with model in which ability was endogenously determined at an early age, and is already determined when children reach school age..

discussed in more detail below, quality is a choice variable for schools, and is perfectly observable for all parents.

$$h_{ij} = f(a_i, q_j) \tag{3}$$

, with $f_a, f_q > 0$; $f_{aq} > 0$; $f_{qq}, f_{aa} \leq 0$, and $f(0, q_j)$, $f(0, q_j) = 0$. Ability and school quality are complements in the production of education.

Throughout the paper, it is assumed that education outcomes are produced according to the specific functional form:

$$h_{ij} = a_i^\alpha q_j \tag{4}$$

Households live in a specific location, which defines the relevant market, and will choose among the schools operating in it. Implicitly, there are no transportation costs of attending different schools within a given geographic location, but those costs approach infinity for schools outside the location. Moreover, agents do not change residence if the quality of education is better elsewhere. N households (and thus, N students) live in a given location.

There are no capital markets, so parents cannot borrow or lend[3]. The absence of financial markets will imply that, in general, marginal rates of return for education investments will not be equated across households.

The household's budget constraint is:

$$m_i = c_i + p_{ij} h_{ij} \tag{5}$$

where c_i is the total value of consumption (the price of consumption is normalized to 1) and p_{ij} is the unit price of education outcomes faced by household i at school j.

Thus, the households's maximization problem can be written as:

$$\begin{aligned} \text{Max } U_i &= \ln c_i + \beta \ln h_i \\ s.t. \ m_i &= c_i + p_{ij} h_{ij} \end{aligned} \tag{6}$$

from where the demands for consumption and education can be written as:

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[3]This is also consistent with the notion that human capital is not contractible.

$$c_i^d = \frac{1}{1+\beta} m_i = (1-\kappa) m_i \tag{7}$$

$$h_{ij}^d = \frac{\beta}{1+\beta} \frac{m_i}{p_{ij}} = \kappa \frac{m_i}{p_{ij}} \tag{8}$$

The demand for education of household i, it it decides to attend school j, only depends on income, with total education expenditure $(p_{ij} h_{ij}^d)$ being a constant share κ of income.

2.1.2 Schools

All schools have the same production technology, and have the capacity to enroll only one student. Capacity cannot be expanded. Schools maximize profits and have no outside options.

Schools differ in their exogenous productivity, γ_j, which can be seen as a proxy for the skills of the owner/manager. Each owner/manager only runs one school[4].

The school´s endogenous quality, q_j, depends on the school's productivity, γ_j and the investment made the school, y_j, in the form of variable inputs (teachers, materials, books, etc.). The production technology for school quality can be written as

$$q_j = g(\gamma_j, y_j) \tag{9}$$

where $g_y, g_\gamma > 0, g_{yy}, g_{\gamma\gamma} \leq 0$, and $g_{\gamma y} \geq 0$. Also assume that $g(\gamma_j, 0) = 0$, so that a school that makes zero investment has zero quality (and thus, cannot produce human capital). The school's actual quality, perfectly observed by all students, then, is not directly the school's exogenous productivity, but the joint product of the school's productivity and the investment it decides to make.

For the moment, assume that there are no fixed costs of setting up a school. There are M potential schools, where $M > N$, so that some schools will not be matched with a student in equilibrium.

Throughout the paper, the school quality production function is assumed to be:

$$q_j = \gamma_j^\zeta y_j^\varphi \tag{10}$$

Notice that, for any non-zero level of investment, this implies that the production function of education outcomes is supermodular in ability and productivity.

$$h_{ij} = a_i \gamma_j^\zeta y_j^\varphi \tag{11}$$

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[4]It is very simple to extend the model to one in which managers can open more than one school.

The school's production function can be used to define a cost function, which determines the investment the school with productivity γ_j must make to get the outcome h_{ij} when matching with a student with ability a_i. The cost function is decreasing on student ability and school productivity.

$$y_j(a_i, h_{ij}) = \left(\frac{h_{ij}}{a_i^\alpha \gamma_j^\zeta} \right)^{1/\varphi} = C_j(h_{ij}) \tag{12}$$

The school´s maximization problem, then, is:

$$
\begin{aligned}
Max \ \pi \ &= \ p_{ij} h_{ij}^d - y_j = \kappa m_i - y_j \\
s.t \ y_j \ &= \ \left(\frac{h_{ij}}{a_i^\alpha \gamma_j^\zeta} \right)^{1/\varphi}
\end{aligned}
$$

Schools perfectly observe the characteristics of each household, as well as the productivity of all other schools. In the equilibrium allocations proposed below, schools will make simultaneous offers to the students they want to enroll.

The functional forms chosen in the paper will provide closed form solutions that will stress the main theoretical points that this paper tries to address. More general specifications can incorporate additional effects that are excluded here, such as interactions between students or private demands that depend on ability. However, the main qualitative results presented here are robust to alternative specifications.

2.2 Homogeneous education expenditure (mandatory vouchers)

The economy has three sources of exogenous heterogeneity: differences in productivity, on the side of schools, and differences in ability and income, on the side of students. The endogenous distribution of education outcomes will reflect those underlying sources of heterogeneity.

We begin the analysis by shutting down one of the sources of heterogeneity. In particular, this section analyzes an economy in which tuition payments are homogenous across the population, and where heterogeneity in the market only comes from the productivity of the schools and the ability of students. This can be interpreted as an economy where education is centrally funded through a flat voucher system. Each household receive an exogenous voucher v, which is used to pay tuition at the school in which the household enrolls the child. No additional transfers from households to schools are allowed, and households cannot spend the voucher on consumption.

In this setup, differences in willingness to pay across households (due to differences in income, student ability, or preferences) play no role in the allocation, as the transfer from the student to the school is

restricted. Households will try to enroll in the highest quality school that is willing to accept them (and will always prefer to enroll in a school than to stay unmatched). From the perspective of the school, the only relevant difference between households is the ability of the child. Also notice that, given the outside options for schools, the fact that no household would ever choose to leave a child out of school, and the absence of fixed costs, there will be always be N schools in the market.

Let $\mu : N \to M$ be a one-to-one matching function. This is, for each student $i, \mu(i)$ corresponds to his associated school, and $\mu(i) = \mu(k)$ is only true if $i = k$. As $M > N, \mu^{-1}(j)$ either corresponds to a student (for the N schools inside the market) or the empty set (for the $M - N$ schools that must be inside the market).

All schools will make simultaneous outcome offers to any student they choose. An offer can be defined as the education outcome, h_{ij}, promised by school j to student i. It is assumed school investments are perfectly observable and done at the same time as tuition payments. As mentioned earlier, school productivities and student abilities are perfectly observed and known. Define a given set of offers from all schools as an offer profile.

Proposition 1 *An equilibrium is an offer profile h from schools to students and a matching μ such that:*

There does not exist any student i, school j, and an offer h'_{ij} where:

(i) $h'_{ij} > h_{i\mu(i)}$

and

(ii) $\pi\left(a_i, \gamma_j, h'_{ij}\right) > \pi\left(a_{\mu^{-1}(j)}, \gamma_j, h_{\mu^{-1}(j), j}\right)$

For the cases in which $\mu^{-1}(j) = \varnothing, \pi\left(a_{\mu^{-1}(j)}, \gamma_j, q_{\mu^{-1}(j), j}\right) = 0.$

Notice that, for any offer h_{ij}, one can define an implicit price per unit of outcome, p_{ij}, as the ratio between the household´s total education expenditure, v, and h_{ij}.

$$p_{ij} = \frac{v}{h_{ij}} = \frac{v}{a_i^\alpha \gamma_j^\zeta y_j^\varphi}$$

Given school quality, student ability fully determines h_{ij}. As a consequence, results would be identical if one assumes that schools, instead of outcome offers, make quality offers, q_{ij}.

To solve for equilibrium, rank schools from high to low productivity, j from 1 to M, with school $j = 1$ having the largest productivity (γ_1). Define q_i^{ZP} as the maximum quality that can be reached by a school with productivity γ_i, when it fully invests the voucher and gets zero profits:

$$q_i^{ZP} = v^\varphi \gamma_i^\zeta \tag{13}$$

We assume, for simplicity, that schools that are indifferent between being inside and outside the market choose not to enter.

Proposition 2 *The stable market equilibrium can be characterized as follows:*

(a) Only the N more productive schools will operate

(b) All schools will have the same quality, $q^* = \left(y_j^*\right)^\varphi \gamma_j^\zeta = v^\varphi \gamma_{N+1}^\zeta = q_{N+1}^{ZP}$.

(c) All potential allocations between the N students and the N most productive schools are equilibria. There is no force that drives the economy towards assortative matching in productivity and ability. The only source of competition is the threat of entry of school γ_{n+1}.

(d) School investments are of the form $y_j^* = v \left(\dfrac{\gamma_{N+k}}{\gamma_j} \right)^{\frac{v}{\varphi}}$

(d) For any student i and matching school $\mu(i)$, the equilibrium outcome will be $h_{i\mu(i)}^* = a_i^\alpha v^\varphi \gamma_{N+1}^\zeta$, for all $i \in \{1..N\}, \mu(i) \in \{1..N\}$.

(e) In equilibrium, more productive schools and more able students are strictly better off.

Proof. See Appendix. ∎

Corollary 3 *More productive schools make strictly smaller investments.*

Proof. As in equilibrium, $y_j^* = v \left(\dfrac{\gamma_{N+k}}{\gamma_j} \right)^{\frac{v}{\varphi}}$, $\dfrac{\partial y}{\partial \gamma_j} = -\dfrac{v}{\varphi} \left(\dfrac{\gamma_{N+k}}{\gamma_j} \right)^{\frac{v}{\varphi}} \dfrac{1}{\gamma_j} < 0$ ∎

The key result is that schools are indifferent across all students, as more able students need to be exactly compensated for their ability differential with a higher $h_{i\mu(i)}^*$ offer. Schools gain nothing from a more productive match, as given threat of entry, productivity gains go completely to the student. Although ability and productivity are complements, schools receive the same payoff in any potential match.

Schools have no incentives to differentiate. Thus, all operating schools have the same homogenous quality, q^*, enough to prevent entry from the marginal school. All competitive pressures are driven by threat of entry. As attaining any given level of quality is less costly for more productive schools, schools with a higher γ need to invest strictly less to get q^*.

Outcomes are solely driven by the productivity of the marginal school. As operating schools have no incentives to compete between themselves by providing offers that exceed the minimum offer required to prevent entry, the implicit equilibrium price faced by agent i, p_i^*, is simply the average production cost of the fringe school:

$$ p_i^* = \frac{v}{h_{ij}^*} = \frac{v}{a_i^\alpha v^\varphi \gamma_{N+1}^\zeta} = \frac{v^{1-\varphi}}{a_i^\alpha \gamma_{N+1}^\zeta} = AC_{i,N+1} \tag{14} $$

Households with higher ability face implicitly lower prices (they can get a better outcome out of the voucher). Differences in outcomes across students only reflect ability differences.

$$\frac{h_{i\mu(i)}}{h_{j\mu(j)}} = \frac{a_i^\alpha v^\varphi \gamma_{N+1}^\zeta}{a_{i+1}^\alpha v^\varphi \gamma_{N+1}^\zeta} = \left(\frac{a_i}{a_{i+1}}\right)^\alpha \tag{15}$$

Schools that have better outcomes need not to be intrinsically more productive: they do better only because they happen to enroll more able students. In this setup, outcome gaps between schools reveal no information about the schools' underlying characteristics, which can only be inferred by looking at school investments or profits. While there is a complementarity between more productive schools and more able students, schools receive no premium for providing higher quality. Thus, more productive schools have no incentives to invest more or to match with the more able students. This is different from the matching problem usually discussed in the literature (Becker, 1973) in at least three dimensions.

First, education outcomes are not a deterministic function of ability and productivity, but a result of the school's endogenous investment decision. Thus, outcomes are not only determined by the identity of the members of the match, but also by the decisions of the school, which are affected by competitive pressures.

The second difference lies in the nature of the output itself. Schools get no direct utility from the level of the outcome, which only provides utility to the student. Moreover, the outcome has no immediate market value (i.e., human capital is not contractible), so it cannot be converted into monetary units. Thus, students cannot directly transfer part of the outcome to the school. Ceteris paribus, matching more able kids with more productive schools increases average outcomes; however, schools would get no additional benefit from that allocation. As discussed in the next section, this implies that, even when direct transfers from students to schools are allowed, there is still need not be sorting in ability and productivity.

Complementarity in production will not lead to positive sorting if the production function implies that relative cost between schools is independent of the ability of the match, as is the case with the multiplicative production function presented here. Positive assortative matching in ability and productivity would require a stronger condition than supermodularity in this context. In particular, it would require that the relative cost of more productive schools is strictly decreasing in ability.

Finally, a flat voucher limits transfers between students and schools even more. All students can only transfer the voucher, even if privately they would choose to make larger (or smaller) payments.

What about efficiency? In terms of welfare, students with higher valuation on education outcomes are not allowed to reflect those preferences, so the market allocations cannot be Pareto efficient.

In terms of productive efficiency, all students, regardless of their ability, go to schools of the same quality. The lack of assortative matching, and the fact that schools do not fully invest the voucher (as they are

getting profits), implies that, given aggregate funding for education, the sum of educations outcomes is not maximized. Moreover, within the set of schools, investments are allocated inefficiently, as the bulk of them is made by low productivity schools. Free entry, however, is a force towards efficiency, guaranteeing that the only the most productive schools operate and providing the competitive pressure that determines equilibrium investments.

The only source of inequality in the economy comes from differences in ability. The distribution of educational attainment simply replicates the distribution of abilities across the population.

How would the argument change if schools were not limited to enroll only one student? In that setup, schools would have an incentive to attract additional students, and competition would not be solely be driven by threat of entry, but also from competition between operating schools. However, as long as schools cannot profitably expand capacity indefinitely, the qualitative argument presented here would still be valid.

2.3 Fully private funding

Now, we introduce income heterogeneity as a relevant element in the determination of market outcomes. Assume that education is now privately funded, with all household deciding how much to spend in education according to (6). There are no vouchers or any type of public funding. Given preferences, education expenditures will be a constant share of income for all agents.

As in the case with pure vouchers, only the N most productive schools will be able to operate, and the productivity of the fringe competitor will be relevant for setting equilibrium qualities. However, unlike the previous case, equilibrium qualities will not be solely determined by threat of entry. When tuition payments differ across students, more productive schools have an incentive to exploit their competitive advantage to match with consumers with higher willingness to pay. Equilibrium qualities will not only reflect productivity from the school outside the market (competitive pressure from outside), but also the productivities of schools within the market.

Once again, rank schools from high to low productivity, j from 1 to M, with school $j = 1$ having the largest productivity (γ_1). Do the same with students in terms of income, i from 1 to N, with student $i = 1$ having from the highest income households (m_1). a_i is the ability of the child coming from the household with income m_i. This implies, of course, that abilities are not ranked, unless there is perfect correlation between parental income and descendant ability.

As the demand for education has a unitary price elasticity, education expenditure for any given household does not depend on price. Thus, on the proposed equilibrium, schools take tuition payments for each household as given, and make simultaneous education outcome offers to those students they wish to enroll.

Once again, equilibrium is characterized by an offer profile h from schools to students and a matching μ, with the properties given in Definition 1.

Proposition 4 *The stable market equilibrium can be characterized as follows:*

(a) Only the N more productive schools will operate.

(b) There is strict assortative matching between more productive schools and higher income households, but not necessarily between productivity and ability. Competitive pressures come from outside the market (threat of entry) and between operating schools.

(c) School qualities can be written as

$$q_j^* = \kappa \left[(q_N^*)^{\frac{1}{\varphi}} + \sum_{i=1}^{N-j} (m_{N-i} - m_{N+1-i}) \gamma_{N+1-i}^{\frac{\zeta}{\varphi}} \right]^{\varphi}$$

, for $j = 1$ to $N - 1$, and $q_N^* = (\kappa m_N)^{\varphi} \gamma_{N+1}^{\zeta}$. More productive schools have strictly higher quality.

(d) For any student i and matching school $\mu(i) = i$, the equilibrium offer will be $h_{i\mu(i)}^* = a_i^{\alpha} q_{\mu(i)}^*$ for all $i \in \{1..N\}$.

(e) School investments can be written as $y_j^* = \left(\dfrac{q_j^*}{\gamma_{N+1-i}^{\zeta}} \right)^{\frac{1}{\varphi}}$. More productive schools might make larger investments.

(f) In equilibrium, more productive schools and more able students are strictly better off.

Proof. See Appendix ∎

The implicit equilibrium price described in (13), which only reflected ability differences, is not an equilibrium here. To see this, take any two students, i and k, and any given school j, $j < N + 1$. Without loss of generality, assume that $m_i > m_k$. At the equilibrium in Section 2.2, $p_i^{**} = \dfrac{\kappa m_i^{1-\varphi}}{a_i^{\alpha} \gamma_{N+1}^{\zeta}}$ and $p_k^{**} = \dfrac{\kappa m_k^{1-\varphi}}{a_k^{\alpha} \gamma_{N+1}^{\zeta}}$, with associated offers $h_i^{**} = (\kappa m_i)^{\varphi} a_i^{\alpha} \gamma_{N+1}^{\zeta}$ and $h_k^{**} = (\kappa m_k)^{\varphi} a_k^{\alpha} \gamma_{N+1}^{\zeta}$ From there, the profits of school j on each match would be:

$$
\begin{aligned}
\pi_{ij}^{**} &= \kappa m_i - \left(\frac{h_i^{**}}{a_i^{\alpha} \gamma_j^{\zeta}} \right)^{1/\varphi} = \kappa m_i \left(1 - \left(\frac{\gamma_{N+1}}{\gamma_i} \right)^{\frac{\upsilon}{\varphi}} \right) \\
\pi_{kj}^{**} &= \kappa m_k - \left(\frac{h_k^{**}}{a_k^{\alpha} \gamma_j^{\zeta}} \right)^{1/\varphi} = \kappa m_k \left(1 - \left(\frac{\gamma_{N+1}}{\gamma_j} \right)^{\frac{\upsilon}{\varphi}} \right)
\end{aligned}
\tag{16}
$$

As $m_i > m_k, \pi_{ij}^* > \pi_{kj}^*$. It is clear that this cannot be an equilibrium, as it implies that parents would

be indifferent across schools (they would all offer the same the same to a student with ability a_i and income m_i), but all schools would strictly prefer to match with students with higher income, as the mark-up per unit of tuition is constant. Competition between schools must drive implicit prices below the marginal cost of the fringe school, and, except for the lowest income student, equilibrium offers will be exceed the minimum level required to prevent entry. More productive schools have an advantage in producing better outcomes at a lower cost and end up serving the students with higher willingness to pay and having higher actual quality.

Thus, differences in quality between schools are more than proportional to the differences in income of the students they enroll. High-income students do not only do better because they can invest more, but because the equilibrium allocation implies that they are attached to high-productivity schools, where, controlling for ability, they face implicitly lower prices. This is a strong force towards inequality in outcomes, as income differences are magnified by the differences in the quality of the match.

For any school j, $j < N + 1$, q_j^* is the minimum quality that provides no incentives to its most direct competitor, $j + 1$, to make a better offer to attract student j. In equilibrium, school $j + 1$ makes exactly the same profits on its own equilibrium match, $j + 1$, than what it would get by providing q_j^* matching with student j. For school $j + 1$, the marginal cost of offering q_j^* instead of q_{j+1}^* equals the marginal revenue of enrolling student j instead of student $j + 1$. Thus, the marginal profit of deviating is zero. All other schools make strictly higher profits in their own equilibrium matches.

Once again, the complementarity of ability and productivity in the production of education outcomes does not drive the market to positive sorting on those dimensions. Productive schools match with students with high willingness to pay, who need not be those with high ability: the demand for education does not directly depend on ability, as substitution and income effects exactly cancel out at the household level.

This **does not** imply that the competitive solution is not a Pareto allocation. Given preferences, it is easy to show that the **competitive equilibrium in this case is indeed a Pareto allocation,** as complementariries are not directly valued by parents. For any given income level, a parent with a child with higher ability does not put a higher valuation on matching with a high productivity school than the parent of a low ability kid. Thus, allocations are **efficient in terms of maximizing welfare**, as they assign those parents with the highest valuation for education to the schools that can provide it a smaller cost.

Even if more able students were those who paid more (if, for example, ability and income had a perfect positive correlation), positive assortative matching would not emerge from the complementarieties in production, but as a by-product of the household's demand for education. Schools do not benefit from the outcome directly, but only through the valuation put on it by households. Transfers from households to schools can only come from the (exogenous) income with which they are endowed, not from the (endogenous) outcome they can generate.

What about productive efficiency (this is, efficiency in terms of the average human capital of the next generation) ? Given aggregate expenditure, the sum of outcomes is still not maximized, as in general there is no ability-productivity sorting and schools do not fully invest tuition. Nothing guarantees that more able students end up in schools with higher quality.

Overall, competitive incentives are stronger now, and go beyond mere threat of entry. Competition to attract richer students leads to differentiation, and implies that more productive schools have higher equilibrium qualities. While this not necessarily implies that more productive schools invest more[5], the distribution of investments is, relative to the voucher, always less biased towards low-productivity schools. A larger share of investments is made by more productive schools.

Inequality in final outcomes across students now comes from sources: differences in ability, differences in income, and differences in the productivity of the school in which student is enrolled. As more productive schools are matched in equilibrium with higher income students, richer students not only spend more, but can buy additional units of school quality at a smaller price. The same force that drives the economy towards efficiency increases inequality, exacerbating the impact of initial differences in income.

2.4   Vouchers with payments on top

We now combine the polar funding regimes described above. All parents still receive the voucher v, which can only be spent in education, but can freely complement it with additional out-of-pocket tuition expenditures. Notice that this is much closer in spirit to the original voucher proposal in Friedman (1953).

Once again, rank schools from high to low productivity, j from 1 to M, with school $j = 1$ having the largest productivity (γ_1). Do the same with students in terms of income, i from 1 to N, with student $i = 1$ having from the highest income households (m_1).

The household problem now becomes:

$$
\begin{aligned}
\underset{q^d,c}{Max U} &= U(c_i, h_{ij}^v + h_{ij}^d) & (17)\\
s.t.(i)\ m_i &= c_i + p_{ij} h_{ij}^d \\
(ii)\ h_{ij}^v &= \frac{v}{p_{ij}} \\
(iii) h_{ij}^d &\geq 0
\end{aligned}
$$

where h_{ij}^v is the outcome the student gets if he does not pay anything on top of the voucher and the second restriction reflects the fact that the household cannot have negative education expenditure (v.g., use

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[5] Roughly speaking, this depends on how productivity differences between schools comparew to income differences between households.

part of the voucher to finance consumption).

Parents will only be willing to make investments on top of the voucher if:

$$U_c(m_i, h^v_{ij}) < U_q(m_i, h^v_{ij})$$

This is, if the marginal utility of an additional unit of quality exceeds the marginal utility of consumption. In that case, we will have an interior solution, and $h^d > 0$.

If

$$U_c(m_i, h^v_{ij}) \geq U_q(m_i, h^v_{ij})$$

the third restriction binds, and $h^d = 0$.

Typically, and unless the voucher is too low, a segment of the population will decide not pay tuition beyond the voucher, while the rest will be willing to make additional investments.

It is straightforward to derive from (6) that tuition payments on top of the voucher can be written as

$$t_i = \left\{ \begin{array}{c} \kappa m_i - v \text{ if } m_i > \frac{v}{\kappa} \\ 0 \text{ if } m_i \leq \frac{v}{\kappa} \end{array} \right\} \tag{18}$$

The household's decision to invest privately only depends on income and the level of the voucher. Thus, we can define a threshold income level, $m^*(v) = \frac{v}{\kappa}$, above which households are willing to make positive education investments for a given voucher v. For each v, there is an associated $N^*(v)$, the number of households whose income exceeds $m^*(v)$, where $N \geq N^*(v) \geq 0$. Additional tuition payments are monotonic in m_i, being zero for $m_i < m^*$ and strictly increasing otherwise.

Households whose private demand for education exceeded the voucher pay up the difference, until their total tuition payment equals what they would had done privately. This is, for student i, $i < N^*$, $t_i + v = \kappa m_i$. Households whose private demand was below the level of the voucher make no additional payments.

Proposition 5 *When payments on top of the voucher are allowed, the equilibrium can be characterized as follows:*

(a) Only the N more productive schools will operate.

(b) The more productive $N^*(v)$ schools will receive private funding, while the remaining $N - N^*(v)$ will only receive the voucher.

(c) Schools that receive no additional payments have a standard quality, $q^* = \left(y^*_j\right)^\varphi \gamma^\zeta_j = v^\varphi \gamma^\zeta_{N+1} = q^{ZP}_{N+1}.$, with $j \in [N^*(v) + 1, N]$.

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(d) Students with income $m_i \leq m^*(v)$ pay nothing in addition to the voucher, and their outcomes are $h_{ij} = a_i^\alpha q^*$, *with* j, $i \in [N^*(v) + 1, N]$.

(e) For the $N^*(v)$ higher income households and higher productivity schools, there will be positive assortative matching in productivity and income. This is, $\mu(i) = i$, *for all* $i \leq N^*(v)$.

(f) School qualities in the N^* schools that receive additional payments can be written as $q_j^{**} = \left[(q^*)^{\frac{1}{\varphi}} + \sum (t_{N-1-j} - t_{N-j})\right]$ *for* $j \in [1, N^*(v) - 1]$.

Proof. This is simply an extension of what was discussed in the previous sections. (a) and (c) are direct from Proposition 2, while (b) is an application of Proposition 4. (d) comes from the demand for additional payments in 18.

(e) and (f) are again extensions of Proposition 4, with q_j^{**} still such that $\pi_{j+1}(q_j^{**}) = \pi_{j+1}(q_{j+1}^{**})$, which again implies that $\pi_j(q_j^{**}) > \pi_{j+1}(q_{j+1}^{**})$. As before, differences in tuition payment between student i and student $i + 1$ are associated to differences in school quality implicitly priced at the marginal cost of school $\mu(i + 1)$. ■

Take, for example, student $N^*(v)$, who comes from the lowest income household in the set of tuition paying households. Additional tuition payments for this student can be written as $t_{N^*} = \kappa m_{N^*} - v$. Thus, the quality in the school in which he enrolls, $N^*(v)$, will be $q_{N+1}^{**} = \left[(q^*)^{\frac{1}{\varphi}} + t_{N^*}\gamma_{N^*+1}^{\frac{\zeta}{\varphi}}\right]^\varphi$. School $N^*(v)+1$, the last school in the pure voucher sector, cannot make an offer that attracts the student and simultaneously augments his profits, as the investment needed to get q_{N+1}^{**} is $v\left(\frac{\gamma_{N+1}}{\gamma_{N^*+1}}\right)^{\frac{\zeta}{\varphi}} + t_{N^*}$. The school would be forced to invest the full extent of the additional tuition payments, getting zero marginal profits.

How is productive efficiency affected, relative to a case in which payments in addition to the voucher are restricted? The distribution of parental investments is shifted towards high–income households. This induces differentiation by high-productivity schools, who now exploit their productivity advantage, investing strictly more than what they did under the voucher. This effect on the intensive margin implies that, within the set of schools that receive parental payments, higher productivity schools also have higher actual quality. While the set of operating distribution of investments, and thus the extensive margin, is not changed, the model could be extended to allow for different opportunity costs across schools. If the opportunity cost is larger for high-productivity schools (because they have better outside options), then profits under a voucher regime might not be enough. In that context, allowing for additional payments, which increases their expected profits, might improve the productivity pool of operating schools.

3   Endogenous public funding

So far, we have discussed markets in which vouchers were mandatory and exogenous. This section relaxes both assumptions. First, it introduces explicit taxation to allow public-funding of schools. Secondly, and consistent with the operation of school market worldwide, it allows parents to choose whether they send their child to a school that receives public funding or to one that is funded privately.

First, we analyze an economy in which schools that are allowed to receive public funds are not eligible private tuition, and viceversa. This could be interpreted as a standard education system in which public funds can only be received by "public" schools.

Second, we relax the restrictions on eligibility, and allow all schools to receive public funds. This can be interpreted as the introduction of a voucher system. However, if a school chooses to receive a voucher, it can not receive private tuition.

The third scenario allows schools that receive a voucher to charge additional tuition, up to an exogenous limit.

In all cases, we solve for the equilibrium sizes and qualities of each education sector.

3.1   Public funding with restricted eligibility

The economy is the same as described in the previous section. As before, rank students in income, 1 to N, with m_1 being the largest income, with associated abilities a_{m_1} to a_{m_N} which are not ranked.

Assume there are two sets of potential schools. Set A schools, with productivities $\gamma_{a,1}$ to $\gamma_{a,M}$, receive funding directly from the government. The other set, B, with productivities $\gamma_{b,1}$ to $\gamma_{b,M}$, can only receive private funding. Schools that are eligible to receive public funding are not allowed to charge tuition. The underlying productivity distribution for both sets is the same. Technology is the same as before, and schools can still enroll only one student. All schools maximize profits.

Public education is financed through a flat income tax on households, τ. All households, regardless on whether they attend a school that receive public funding, pay the tax[6].

Households attending a publicly-financed school make no education expenditures, and can fully consume their after-tax income, $(1-\tau)m_i$. Agents who decide to stay in the paid sector, paying full tuition, spend κ of their after-tax income on education[7], and thus can only consume $(1-\tau)(1-\kappa)m_i$. The fact that staying in the paid sector implies a lower consumption means that, of course, quality in an operating paid school must

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[6] As income is assumed to be exogenous and households make no explicit labor decisions, potentially distortive effects of taxation are not considered here. In a related paper, Ferrada and Tapia (2010), we allow for distortive taxation in a similar setup.

[7] The maximization problem for a household attending a privately-funded school is the same as before.

be strictly larger than the one received in the tax-financed sector. In particular, any agent i will strictly prefer to attend a paid school if:

$$(1 - \kappa)^{1/\kappa} q_{B,i} > q_{A,i} \tag{19}$$

where $q_{B,i}$ identifies the quality of the school he would attend in the paid-sector and $q_{A,i}$ is the quality in the tax-financed sector.

This gives a competitive advantage to schools that receive public funding, who can provide strictly less quality than their privately-funded competitors. As the set of schools eligible to receive public funds is exogenously determined, this competitive advantage is an exogenous imposition of the policy regime.

Is this a model of public schools? No, as it is still assumed that schools that receive public funding maximize profits. This might approach the behavior of public schools that face little incentives or competitive pressures, but this need not be necessarily the case. Moreover, the process under which schools become eligible to receive public funding is not described. Indeed, the more productive "public" schools, if they are indeed profit-maximizers, would be better off if they were in the set of fully-paid private schools, as they could make larger profits there. In that sense, this has be to be seen in the first step towards developing a model that explicitly derives the maximization problem for (exogenously determined?) public schools, as well as incorporating non-profit private institutions. Despite this shortcoming, the setup still captures the notion of two distribution of schools that, in a sense, operate in separate markets, and which face different competitive pressures

Total public funding for education is equal to total taxation revenue:

$$F(\tau) = \tau \sum_{i=1}^{N} m_i \tag{20}$$

How much is received by each school (and student) that gets public funding, however, depends on the size of the sector. The larger the number of school-students matches that receive public funding, the smaller the individual endowment:

$$v_s = \frac{F(\tau)}{s}$$

where s is the number of students that decide to attend the tax-financed sector. Thus, individual funding becomes an endogenous variable, determined by the relative size of both sectors.

From our previous discussion, we know that, if the market is served by both types of schools, higher income households will choose to attend the fully-paid sector, and that positive assortative matching in

income and productivity holds therein.

Define as j^* as the index associated to the lowest income student that enrolls in the private sector in equilibrium. Thus, j^* is also the number of full tuition schools that operate in equilibrium, with $N - j^*$ being the equilibrium number of schools that receive public funds.

Thus, equilibrium public funding per student (school) can be written as:

$$v_{(N-j^*)} = \frac{F(\tau)}{N - j^*} \tag{21}$$

As in Section 2.1, equilibrium quality of the schools in the tax-financed sector will be given by threat of entry of the marginal school in Set A:

$$q^*_{A,i} = v^{\varphi}_{(N-j^*)} \gamma^{\zeta}_{A,N+1-j^*}, \ i > j^* \tag{22}$$

where j^* is the income index defined above and $\gamma^a_{(N-j^*)+1}$ is the productivity of the marginal school in the sector (there are $N - j^*$ operating schools in Set A).

Using Proposition 4, equilibrium quality of school i can be written as:

$$q^*_{B,j^*} \ = \ \left[((1-\tau)\,\kappa m_{j^*})\gamma^{\frac{\zeta}{\varphi}}_{B,j^*+1} \right]^{\varphi} \tag{23}$$

$$q^*_{B,i} \ = \ \left[(q^*_{B,j^*})^{\frac{1}{\varphi}} + (1-\tau) \sum_{k=j^*-1}^{k=i} (m_k - m_{k+1})\gamma^{\frac{\zeta}{\varphi}}_{k+1} \right]^{\varphi} , i < j^* \tag{24}$$

where $(1-\tau)\,\kappa m_{j^*}$ is the tuition paid by student j^* and $\gamma^b_{j^*+1}$ is the productivity of the marginal school in Set B. As before, equilibrium qualities are such that schools are exactly indifferent between enrolling their equilibrium match and the student immediately above in the income distribution. Differences in equilibrium quality between two consecutive schools reflect the difference in income between the students they enroll, priced at the marginal cost of the school with lower productivity.

To solve for the equilibrium j^*, use (21) and (22) to define $q_{A,j}$ and $qB_{B,j}$ as the equilibrium qualities of the schools that student j would attend if he was the marginal student in both sectors. If student j was the highest income student to attend the public sector, the equilibrium quality of the school in which he enrolls would be:

$$q_{A,j} = v^{\varphi}_{(N+1-j)} \gamma^{\zeta}_{A,N+1-j} \tag{25}$$

simply reflecting the fact that the size of the "public" sector would then be $N + 1 - j$.

If student j is the lowest income household in the fully-paid sector, the quality of the school in which he enrolls would be:

$$q_{B,j} = ((1-\tau)\,\kappa m_j)^\varphi \gamma_{B,j+1}^\zeta \tag{26}$$

just enough to make the marginal private school (with productivity $\gamma_{B,j+1}$) get zero profits.

Notice that $q_{A,j}$ is strictly increasing in j for two reasons. First, a higher j is associated to a smaller number of students in the public sector, so individual funding per school is higher. Second, the quality of the marginal public school increases as the number of operating public schools falls. By a similar argument, $q_{B,,j}$ is strictly decreasing in j, as tuition payments and the productivity of the marginal private school fall. Thus, if an interior solution exists (this is, one in which both sectors operate in equilibrium), the equilibrium will be unique.

Thus, for any student j that in equilibrium chooses to attend the private sector, it must be true that:

$$q_{A,j} \le (1-\kappa)^{1/\kappa} q_{B,j} \tag{27}$$

with the reverse being true for any student that enrolls in a public school.

Thus, in any interior solution, j^* must simultaneously satisfy

$$v_{(N+1-j^*)}^\varphi \gamma_{A,N+1-j^*}^\zeta \le (1-\kappa)^{1/\kappa}\left[((1-\tau)\,\kappa m_{j^*})^\varphi \gamma_{B,j^*+1}^\zeta\right] \tag{28}$$

and

$$v_{(N-j^*)}^\varphi \gamma_{A,N-j^*}^\zeta \ge (1-\kappa)^{1/\kappa}\left[((1-\tau)\,\kappa m_{j^*+1})^\varphi \gamma_{B,j^*+2}^\zeta\right] \tag{29}$$

with one of the inequalities being strict. For j^* to be the last student to enroll in the private sector, he must, at least weakly, obtain more utility-adjusted quality in the private sector, with the reverse being true for student j^*+1, the highest income student to attend the "public" sector. Notice that, by the properties of $q_{A,j}$ and $q_{B,j}$, all students $i < j^*$ are also better off by attending the full-tuition sector, while students $i > j^*+1$ will strictly prefer to enroll in the public sector.

This, all households with income above m_{j^*} are matched with fully-paid schools with productivity above γ_{B,j^*+1}. As mentioned earlier, in general $\gamma_{A,N+1-j^*} \ne \gamma_{B,j^*+1}$, so the productivity of the marginal school will not be the same in both sectors. This implies that the set of operating schools will typically not be formed by the N most productive schools in sets A and B. For example, if, consistent with the data, j^* is relatively small, so that the number of private schools is smaller, $\gamma_{A,N+1-j^*} < \gamma_{B,j^*+1}$ Private schools

that are left out of the market have higher productivity than the least productive operating "public" school. Restrictions on the allocation of public funds leave potentially productive schools of the market. Notice that the argument does not rely on "public" schools operating as for-profit (or rent-seekers), and that it would still hold if they fully invest their revenue.

If "public" schools do observe rent-seeking behavior, increases in the tax rate and public financing have a perverse effect on quality, as they increases entry barriers and the productivity gap between both sectors.

3.2   Implementing a voucher system

Assume now that all schools are eligible to receive public funds in the form of a voucher. However, if a school receives a voucher, it cannot charge additional tuition from parents. This is, any school can choose whether it wants to operate as a voucher school or as school financed by private tuition. This eliminates the artificial distinction between entrepreneurs of types A and B introduced in the previous section. It is clear that, just like in the equilibrium without taxation, only the set of N more productive schools will operate, as barriers to entry have been removed in both sectors. Unlike the previous case, whether a school operates in sector A (voucher) or B (private) , is not determined exogenously, but is an equilibrium result.

As in the previous case, if both a voucher and a fully-paid sector operate in equilibrium, higher income households will be the ones that enroll to the fully-paid sector, with positive assortative matching in income and productivity within the sector.

As shown in Section 2.1, equilibrium quality of the schools in the voucher sector, q_A , is simply given by threat of entry:

$$q_{A,i}^* = v_{N-k^*}^\varphi \gamma_{N+1}^\zeta, \; i > k^* \tag{30}$$

where k^* is the income index of the lowest income student that attends the fully-paid sector. This is, k^* is the size of the private tuition sector, while the voucher sector has size $N - k^*$.

Competition within the fully-paid sector is just an extension of what was discussed in Section 2.2. Thus, the quality of school i in the fully paid sector must be:

$$q_{B,k^*}^* = \left[(q_A^*)^{\frac{1}{\varphi}} + ((1-\tau)\,\kappa m_{k^*} - v)\gamma_{k^*+1}^{\frac{\zeta}{\varphi}} \right]^\varphi \tag{31}$$

$$q_{B,i}^* = \left[q_{B,k^*}^* + (1-\tau)\,\kappa \sum_{l=k^*-1}^{i} (m_l - m_{l+1})\gamma_{l+1}^{\frac{\zeta}{\varphi}} \right]^\varphi \tag{32}$$

$$i < k^*$$

with k^* as defined above. This is, equilibrium qualities make school i indifferent between their own equilibrium match, student i, and the student immediately above in the income distribution, $i-1$.

Finally, using the fact that the voucher sector has $N-k^*$ students, the endogenous voucher can be written as:

$$v_{N-k^*} = \frac{F(\tau)}{(N-k^*)} \tag{33}$$

By a similar argument as the one used in the previous case, if a non-corner solution exists, the equilibrium size of the private tuition sector, j^*, satisfies:

$$v_{N+1-k^*}^{\varphi} \gamma_{N+1}^{\zeta} \leq (1-\kappa)^{1/\kappa} \left[v_{N+1-k^*} \gamma_{N+1}^{\frac{\zeta}{\varphi}} + ((1-\tau)\kappa m_{k^*} - v)\gamma_{k^*+1}^{\frac{\zeta}{\varphi}} \right]^{\varphi} \tag{34}$$

and

$$v_{N-k^*}^{\varphi} \gamma_{N+1}^{\zeta} \geq (1-\kappa)^{1/\kappa} \left[v_{N-k^*} \gamma_{N+1}^{\frac{\zeta}{\varphi}} + ((1-\tau)\kappa m_{k^*} - v)\gamma_{k^*+2}^{\frac{\zeta}{\varphi}} \right]^{\varphi} \tag{35a}$$

with one of the inequalities being strict. As before, the left-hand side term of both equations is school quality in the voucher sector, which is strictly increasing in j, as the same aggregate funding is distributed among $N-j$ students. The right-hand side, utility-adjusted school quality in the private sector, is strictly decreasing in j, as both income and school productivity are decreasing in j. Student k^*, and all students $i < k^*$, are better off by attending the full-tuition sector, while agent k^*+1 prefers to attend a voucher school, as do all students $i > k^*+1$.

The most productive school in the voucher sector, γ_{j^*+1}, is the marginal school for the fully-paid sector, as it gets the same profits in the voucher sector than what it would get if it enrolled the first student paying tuition. Endogenous equilibrium quality must jump discretely between the voucher sector and the worst school in the fully-paid sector, as the first student that pays must be compensated with a significant increase in quality for the consumption loss of paying for education directly. Profits, on the other hand, will typically move smoothly from sector to sector, as the worst school in the private sector faces intense competition from the best voucher school.

Unlike the previous case, voucher and fully-paid schools come from the same set. This guarantees that the most productive outside the market, γ_{N+1}, is strictly less productive than all operating schools. Thus, the productivity of the marginal school in the voucher sector does not deteriorate as the sector expands. This implies that, for a given tax rate, typically $k^* < j^*$,as shown numerically in Section 3.4: there will be less students paying private tuition in this case. When restrictions on access to public funds are lifted, the

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voucher sector expands, as it is now formed by schools by higher productivity, which attract more students by providing larger equilibrium quality. Quality in the voucher sector is strictly larger when all schools become eligible to receive, as competitive pressures become fiercer and only the best schools can survive. How would the analysis change if "public" schools are not rent-seekers, but rather supply all the quality they can given their resources? The adoption of the voucher and the potential increase in competitive pressures would not change their behavior. However, it would be still be true that low-productivity schools would be forced our of the market by any private school with a larger productivity that was outside the market under the previous policy.

The discussion presented in this section highlights the impact of school vouchers. Voucher foster competition, and can increase overall quality relative to a case in which access to public funding is limited. However, that does not guarantee that quality gains will be significant, or that the overall level of education will be high, as equilibrium qualities are scaled by the productivity of the marginal school. If threat of entry is limited, or competitors within the market have low productivity, competitive pressures on productive profit-maximizers will be weak, and they will have no incentives to provide a high quality. For instance, the effect on quality will be limited if the quality of public schools is low and the supply of productive private producers is relatively scarce. In that sense, this provides a rationalization for having strong public schools. Society might like to have public schooling for several reasons besides pure skills formation, such as the transmission of values, social cohesion, etc. The argument here is that a strong public sector fosters competition, providing stronger incentives for private schools not only in the voucher sector, but also for those that operate with fully private tuition.

3.3   Vouchers with payments on top

Keeping the same setup as in the previous example, assume now that payments on top of the voucher are allowed, up to a maximum (exogenous) level, $\bar{t}$. This is, any school that charges the parents additional tuition beyond $\bar{t}$ cannot receive a voucher[8].

Once again, only the set of N more productive schools will operate, as there are no barriers to entry. Now, there are potentially 3 sectors: voucher-only schools, schools that receive a voucher and charge additional tuition, and schools that only receive private tuition. It is clear that, by extension of the previous arguments, participation in each sector will be solely determined by income, in the household case, and by productivity, on the school side.

As shown in Section 2.1, equilibrium quality of the schools in the pure voucher sector, sector A, is simply

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[8]If there are no limits on the tuition that can be charged with a voucher, the analysis is trivial. All students will receive the voucher and pay additiional tuition (if any) according to their demand for education.

given by threat of entry :

$$q^*_{A,j} = v^\varphi_{N-l^*}\gamma^\zeta_{N+1} = q^*_A, \; j \geq N - p^* \tag{36}$$

where p^* is the income index for the lowest income student that decides to top the voucher and l^* is the income index for the lowest income student that does not receive a voucher. This is, l^* is the size of the private tuition sector, while the complete voucher sector has size $N - l^*$, with an associated equilibrium voucher v_{N-l^*}. $N - p^*$ schools only receive a voucher, while $p^* - l^*$ receive both a voucher and parental payments. We assume that parameters are such that $N > p^* > k^*$, so all sectors operate in equilibrium.

Competition in the sectors that receive parental payments follows the discussion in Sections 2.2 and 2.3.

Label the sector that receives a voucher and additional tuition payments below $\bar{t}$ as sector B. Any given school i operating in sector B provides an equilibrium quality:

$$q^*_{B,i} = \left[(q^*_A)^{\frac{1}{\varphi}} + \sum_{j=1}^{j=N-p^*-i} (t_{N-p^*-j} - t_{N+1-p^*-j})\gamma^{\frac{\zeta}{\varphi}}_{N+1-p*-j}\right]^\varphi \tag{37}$$

$$\text{for all } N - l^* < i < N - p^*$$

with p^* and k^* as defined above, and t_{N-p^*-j} is the payment made in addition to the voucher, as derived in Section 2.3:

$$t_{N-p^*-j} = \left\{\begin{array}{ll} (1-\tau)\,\kappa m_{N-p^*-j} - v_{N-l^*} & \text{if } 0 < (1-\tau)\,\kappa m_{N-p^*-j} - v_{N-l^*} \leq \bar{t} \\ \bar{t} & \text{if } (1-\tau)\,\kappa m_{N-p^*-j} - v_{N-l^*} > \bar{t} \end{array}\right\}$$

Given a voucher, only parents whose private willingness to pay exceeds the voucher will be willing to make payments out of their pocket. Among that group, no parent that chooses to invest less than $\bar{t}$ will move to the fully-paid sector (sector C), as he would be worse off for certain: he would sacrifice consumption (as he loses the voucher and pays full out-of-pocket tuition) and end up with worse school quality (as his private tuition payments are smaller than the total expenditure he would get if he remained in sector B). The choice between sectors B and C, then, is only an issue for those households that are bound by the constraint on topping, $\bar{t}$. Even then, moving to sector C is not optimal for an agent that is marginally bound - this is, one who would like to invest privately an amount that is not significantly higher than $\bar{t}$, as the budget set is discontinuous at that point. The consumption loss associated to foregoing the voucher (and paying tuition directly) is not compensated, in terms of utility, if the increase in quality is only marginal, which will be the case if the household will only spend privately an amount that is not significantly higher that the total

expenditure he would make if he kept the voucher. Thus, there will exist an income range of agents that will optimally choose to remain constrained at $\bar{t}$, instead of abandoning the voucher sector.

In the sector that only receives payments from the parents, equilibrium qualities must be:

$$q^*_{C,i} = \left[\begin{array}{c} \left(q^*_{B,l^*+1}\right)^{\frac{1}{\varphi}} + \left((1-\tau)\,\kappa m_{l^*} - v - \bar{t}\right)\gamma^{\frac{\varsigma}{\varphi}}_{l^*+1} \\ +(1-\tau)\kappa \sum\limits_{j=l^*-1}^{i} (m_j - m_{j+1})\gamma^{\frac{\varsigma}{\varphi}}_{j+1} \end{array} \right]^{\varphi} , i \leq l^* \tag{38}$$

Quality is constant in Sector A, where all schools receive a voucher and no parental payments. As parents begin to pay, quality becomes an increasing function of school productivity and parental income, until tuition payments are constrained by $\bar{t}$, and quality becomes constant in the upper end of Sector B. Quality jumps discretely for the first school in the fully-private sector, and then continues to grow monotonically with school productivity and parental income.

Finally, using the fact that the complete voucher sector has $N - l^*$ students, the endogenous voucher can again be written as:

$$v_{N-l^*} = \frac{F(\tau)}{(N - l^*)} \tag{39}$$

To solve for the equilibrium, recall that, given a voucher, the choice of whether agents enrolled in the voucher sector pay additional tuition or not (this is, if they locate in Sector A or B) will only depend on income, as seen in Section 2.4. The solution for the equilibrium voucher is simply an extension of the previous cases. If a non-corner solution exists, the equilibrium size of the private tuition sector, l^*, satisfies:

$$q_{B,l^*} \leq \frac{(1-\tau)\,m(1-\kappa)}{m - \bar{t}} \left[q^c_{C,l^*}\right]^{\beta} \tag{40}$$

and

$$\left[q^B_{B,l^*+1}\right]^{\beta} \geq \frac{(1-\tau)\,m(1-\kappa)}{m - \bar{t}} \left[q^c_{C,l^*+1}\right]^{\beta} \tag{41a}$$

where the inequalities reflect the fact that the last agent in the private sector, l^*, would always be constrained by the tuition cap if he were to enroll in the voucher sector. Just as before, the left-hand side of both inequalities is monotonically increasing in income, while the right-hand side is monotonically decreasing. If an interior solution exists and voucher and non-voucher sectors exist, l^* and the equilibrium voucher is unique.. Given the income distribution, that solution in turn determines p^*.

How does the introduction of parental payments on top of the voucher affect the results? For a given tax

rate, the size of the voucher sector will increase ($l^* < k^*$) with $\bar{t} > 0$. The policy also enhances competition within the voucher sector, and increases investment in more productive voucher schools, enhancing efficiency (recall how, with a flat voucher with no topping, school investments are strictly decreasing in productivity). A flat voucher system that does not allow for additional payments fosters competition, as it allows entry from productive schools and provides competitive pressures in the form of threat of entry. In the context of this model, it also provides full equality in the quality of profit-maximizing attended by students enrolled in the voucher sector. However, it does not provide incentives for strong competition between operating schools, and allocates investments inefficiently, as the larger share is made by the least productive schools.

4   Calibration of the model with profit-maximzing schools: The case of Chile

This section provides empirical content to the theoretical model presented in Sections 2 and 3. In order to so, I calibrate the model to match the features of Chile's education market, using information on enrollment, income, tuition payments, and policy variables. Chile is a natural candidate for this exercise, as it has a nationwide, universal voucher program since the early 1980s. More than 50% of students are currently enrolled in private schools, of which more than 60% are for-profit institutions.

The calibrated model is then used to simulate alternative policy scenarios, such as changes in the voucher or modifications in the rule that allows to top up. This is, we analyze how different funding schemes affect the behavior of heterogeneous households and schools, and how this is reflected in allocations and outcomes. Section 3.1 presents a brief overview of the Chilean experience with vouchers. Sections 3.2 to 3.4 present the calibration exercise.

4.1   Educational vouchers in Chile: A historical overview

The desire to introduce competitive elements in the provision of school education seemed a natural step in the context of the widespread, pro-market program of economic reforms conducted in Chile since the late 1970s. Up to that point, the Ministry of Education directly controlled the funding, hiring, and investment decisions, and the general management of 90% of the country's schools. The relatively small private sector was almost fully funded by the students' parents, and thus in practice serviced only the population with highest income.

Between 1980 and 1981, the government decided to introduce a nation-wide reform on the school system. The property and management of public schools was transferred to the local government (municipalities),

while teachers were no longer considered public employees, with the rigidities associated to that condition, but part of the private sector. This new status increased the degrees of freedom in school management, allowing public schools to adjust the number of teachers to changes in demand and market conditions. Funding of the schools still came from the central government, but now in the form of a voucher per student enrolled. Private schools, for profit or non-profit, were eligible to receive the voucher. However, they were not allowed to make additional charges and had to finance their fixed entry costs by themselves.

However, the initial impulse on the reform soon came to a halt. As soon as 1983, Chile was immersed on one of its more severe economic crisis in recorded history. In that context, public spending in education fell was more than 25% as a share of GDP. The real value of the voucher fell strongly in a context of relatively high inflation. At the same time, the central government began to cover the deficit of municipalized schools, authorization for the entry of new private schools was conditioned to the presence of "excess demand" conditions, and local governments were instructed to avoid the firing of teachers (majors were directly appointed by the central government). The value of the voucher remained low until the mid-1990s, arguably preventing the entry of new private schools (the number and enrollment of private schools remained roughly constant from 1987 to 1997).

Competition suffered a new blow in 1991, as new legislation severely reduced teachers' mobility in the public sector, with wages now being determined outside the market. Some flexibility in wages and hiring/firing was reintroduced in 1995, but to this day labor regulation between public and private schools remains vastly different.

The last significant reforms have favored competition. In 1993, schools were allowed to charge tuition on top of the voucher. Although the voucher decreases proportionally with the tuition, this reform permitted schools to increase their revenue per student. This, together with the increase in the value of the voucher itself, is probably behind the robust growth of private schools in recent years.

4.2   Calibration strategy

As mentioned before, in 1994 voucher schools were allowed to charge additional tuition, up to an upper limit above which they are no longer eligible to receive a voucher. As discussed in Tapia (2010), this policy change should be associated with significant steady-state effects on the behavior of schools and students.

The calibration strategy exploits this change in policy to identify the model´s parameters, using data from various sources to characterize two steady-states, before and after the reform. In particular, we use information on schools, households, and the policy setup for 1992, a decade after a voucher system was adopted in Chile and 2 years before parents were allowed to top up the voucher, and 2006, more than a

decade after the new policy was implemented.

4.2.1 Observable parameters

For simplicity, the calibrated model assumes that all schools are private profit-maximizers. This is, we do not account explicitly for the distinct behavior of public schools or non-profit private schools. While this is certainly an important omission, working only with profit-maximizing schools facilitates the analysis still highlights the effect of competitive pressures under different setups. Moreover, and as discussed below, even under this restrictive assumption the model is able to replicate relevant feature of Chile's education system. Despite this, a more complete calibration, including different types of schools, is part of this research agenda.

From the data, we can directly obtain the income distribution and the education policy parameters (voucher size and maximum topping), as well as an estimate for the preference parameter β.

Ideally, all data could be collected from the SIMCE school and students database, which compiles individual test scores for the nationwide SIMCE test as well as detailed individual information on family background, tuition expenditures, and school characteristics. The information contained on SIMCE is a census of all students and schools that take the mandatory test on a given year.

However, although the SIMCE test was implemented in the late 1980s, student level data is only available since the late 1990s. Moreover, while parental income information is available at the school level for 1992, it is only provided in terms of broad income categories. Thus, to obtain a more precise representation of the distribution of income in Chile, income for both years is obtained from CASEN, a biannual nationwide household survey that includes detailed information on income and expenditure. From there, I collect information on total monetary household income (including government direct transfers) and the school in which their children where enrolled for 39,699 households in 1992 and 52,327 households in 2006. To get a more precise representation of students and schools that can be seen as existing within the same geographical "market", I only take data from the Metropolitan Region, the most densely populated urban area in the country. As the theoretical model implies that all schools enroll only one student, I calculate the average income across students all enrolled in a particular school, and thus take each school average as the representative student[9]. This leaves us with 1,525 "household-schools" in 1992 and 1,868 in 2006. Within this sample, in 1992 79.8% of the household-student pairs are in the voucher sector, while the remaining 20.2% are schools with fully-private tuition[10]. For 2006, the shares are 89.98% and 10.02%, respectively.

Information on the level of the voucher received by parents for both years is obtained from the Ministry of Education´s website. Relative to the mean income in each sample, the voucher almost doubles between

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[9] Calculating the average income for each survey at the household level data instead of a the school level, does not make a significant difference.

[10] As way of comparison, the same calculation done at the household level yields shares of 79.6% and 20.4%.

1992 and 2006. The same source is used to calculate $\bar{t}$, the maximum topping in 2006. In the model, parents are allowed to top up until $\bar{t}$, losing the voucher if they go beyond. In the Chilean school system, the procedure is slightly more complex, as the voucher is reduced proportionally as parents begin to pay tuition. For instance, if a parent pays $\bar{t}$ he receives a voucher that is 35% lower than the voucher he would receive if he paid nothing. To account for this, I set $\bar{t}$ not at its actual level, but at a smaller level that takes into account the effect of the reduction of the voucher on the overall funding received by the school. The model does not take into account any general equilibrium or funding considerations, so the level of the individual voucher is kept constant regardless of total enrollment in the sector.

The utility weight, β, is calibrated using the share of income paid in tuition by parents surveyed in the 2005 SIMCE database. Notice that this parameter can only be recovered from the behavior of parents that pay full tuition in non-voucher private schools. In the data, the income share associated to tuition in non-voucher schools, $\kappa = \frac{\beta}{1+\beta}$, equals 0.09, from where $\beta = 0.1$.

Conditional on a given distribution of school productivities, the model predicts that, even in the absence of topping, the share of students enrolled in the voucher sector should grow between 1992 and 2006, as the voucher has grown significantly in absolute real terms and, more importantly, as a share of each year´s mean income. Although the size of the voucher is modest relative to mean income - and indeed, quite below the tuition share in a private setup- the fact that economy's median income is roughly half of the mean implies that voucher represents a sizeable share of income for a large part of the population. Moreover, in 2006, the combination of the voucher plus topping up exceeds what the agent with mean income would invest privately.

Table 1: Observed parameters		
Observed model parameters		Source
Mean income per household, 1992 (Nominal $)	348,385	School averages in CASEN, 1992. Metropolitan Region.
Mean income per household, 2006 (Nominal $)	835,970	School averages in CASEN, 2005. Metropolitan Region.
Voucher relative to mean income, 1992	0.017	Calculated using information from the Ministry of Education and Casen, 1992
Voucher relative to mean income, 2006	0.029	Calculated using information from the Ministry of Education and Casen, 2006
Maximum private tuition allowed in voucher schools, 2006 (relative to mean income)	0.063	Calculated using information from the Ministry of Education and Casen, 2006
Share of income invested in education	0.087	Calculated using data from SIMCE 2005 for students enrolled in fully private schools
β	0.1	Calculated from share of income invested in education

4.2.2 Recovering the unobservable parameters: Using the sectors relative sizes

The model still has several unobserved parameters. Namely, the coefficients in the education production technology (φ, the coefficient associated to school investments; ζ, the coefficient associated to school productivity, and α, the coefficient associated to ability), plus the underlying distributions of student ability and school productivity. As discussed above, however, the equilibrium matches between students and schools are independent of student ability, and will only depend on parental income and on the equilibrium quality provided by schools. While equilibrium school quality is still unobservable, the theoretical model provides precise predictions on how, given equilibrium qualities and the model´s observable parameters, parents will choose to allocate between sectors.

Thus, we can use the observable information on income and equilibrium matches- specifically, the relative size of the voucher and non-voucher sectors- to recover the equilibrium qualities, which in turn depend on the relevant production technology parameters (ω and φ) and the underlying distribution of school productivities.

The calibration exercise finds the parameters of the production function and the distribution of school

productivities that best match the observed allocations across sectors for both 1992 and 2005. To do so, I minimize the sum of the square errors between the shares predicted by the model and the actual shares observed in the data for both years.

For each potential productivity distribution, the model is jointly simulated for both years for all possible combinations of ζ and φ between 0.1 and 1, in 0.1 intervals. Thus, for each steady-state, the model is simulated 100 times for every potential productivity distribution. The distribution of school productivities is assumed to be Pareto. The set of the potential schools is twice the size of the households set (this is, it is assumed that only 50% of potential schools can actually operate in equilibrium).

For any given random variable x that follows a Pareto distribution, the cumulated distribution function can be written as:

$$F(x) = 1 - (\frac{x_m}{x})^{\psi} \text{ for } x \geq x_m$$

$$0, \text{ for } x < x_m$$

, where x_m is the distribution´s lower bound and $\psi > 2$ is inversely related to variance (if $\psi = 2$, the variance is infinite). For all distributions, mean productivity across all potential schools is normalized to one[11]. Distributions vary in their dispersion as measured by ψ, ranging from homogenous schools with constant productivity to extremely skewed and wide distributions when ψ approaches 2.

Tables 2 and 3 show the simulated shares of the voucher sector for each year for various parameters values (in both tables, ζ, the coefficient associated to school productivity, is 0.9). As mentioned earlier, holding everything else constant, an increase in φ reduces the size of the voucher sector, as differences in income (which in equilibrium are associated to differences in school investment) are now associated to larger differences in outcome. Thus, agents who would privately would invest than the voucher (including the topping, in 2006) receive a larger premium in terms of school quality. A similar thing occurs with the dispersion of school productivities. As schools become more heterogeneous, the difference in quality between attending voucher and privately-paid schools does not only involve the difference in investment, but also the gap between the school's ex ante productivities.

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[11]It can be shown that the choice of mean for the productivity distribution does not make a difference in equilibrium shares, as it only scales quality in both sectors.

Table 2: Simulated and actual sizes of the voucher sector, 1992				
Investment productivity parameter (ω)	Homogeneous schools	Pareto with $\psi = 5$	Pareto with $\psi = 3$	Actual share
0.3	0.97	0.85	0.79	0.79
0.4	0.90	0.78	0.73	0.79
0.5	0.82	0.72	0.67	0.79
0.6	0.74	0.64	0.60	0.79
0.7	0.66	0.57	0.54	0.79

Table 3: Simulated and actual sizes of the voucher sector, 2006				
Investment productivity parameter (ω)	Homogeneous schools	Pareto with $\psi = 5$	Pareto with $\psi = 3$	Actual share
0.3	0.99	0.96	0.91	0.90
0.4	0.99	0.94	0.90	0.90
0.5	0.98	0.93	0.89	0.90
0.6	0.96	0.91	0.88	0.90
0.7	0.95	0.90	0.87	0.90

When schools are assumed to be homogeneous and the concavity in the return of investments is high, differences in equilibrium qualities are too small to justify the consumption loss of choosing private education for almost all agents. In 2006, the model with homogeneous schools always overpredicts the share of students in the voucher sector, even when allowing for high returns on investments. In general, as the production function becomes less concave, and thus investments become more productive, the simulated response of the increase in the voucher and the addition of topping up becomes too large when compared to the data.

The best simultaneous fit is attained when assuming a relatively concave return on investment ($\varphi = 0.3$) and large coefficient on school productivity ($\zeta = 0.9$), combined with a large dispersion in school productivities ($\psi = 3$). With those parameters, the model nicely matches the actual size of the voucher in both 1992 and 2006: the model almost perfectly nails the share in 1992 (the predicted share is 79.5%, 0.03% below the actual number) and slightly misses the 2006 share (91% vs 90%).

Table 4 : Calibrated parameters	
φ	0.3
ζ	0.9
ψ	3

How does the model do when predicting results outside those for which it has been specifically calibrated for? I simulate the out-of-pocket education expenditure for all parents in the economy for 2006, and compare it with the actual figure that can be calculated using information for all respondents in the 2005 SIMCE database. This number does not only include the tuition payments made by parents whose children attend fully-private schools, but also topping-up tuition payments made in the voucher sector. In the 2006 sample, the average parent spends 4.3% of his income in out-of-pocket tuition payments - a figure which combines households that range from those spending nothing out of their pocket (those that only invest the voucher) to those paying full tuition of 9% of income on average. The model does a good job in predicting this value, slightly underestimating it at 3.9% (Table 5). Thus, although the model is calibrated to replicate the overall shares of each sector, it also seems to be able to replicate with reasonable precision the distribution of out-of-pocket expenditures within the voucher sector.

Table 5: Actual and simulated outcomes in the calibrated model		
	Calibrated model	Actual data
Size of the voucher sector, 1992	79.5%	79.8%
Size of the voucher sector, 2005	91.2%	89.8%
Out-of-pocket expenditure as a share of income, 2005	3.9%	4.3%

4.2.3 Test scores

The previous sections implicitly calibrated the model to generate equilibrium school qualities that, given income, policy parameters, and preferences, were able to replicate the allocations across sectors observed in the data. It seems natural to try to extend the calibration to replicate the only observable measure of actual education outcomes available in the data: the results of the standardized SIMCE test.

There is one big problem with this. Methodologically, the test is not comparable between 1992 and 2005, and only ordinal inferences can be made by comparing results across time. Cardinal comparisons between SIMCE tests across time are only possible since the late 1990s, when the test was re-designed. In fact, one cannot use the 1992 and 2005 tests to say whether the gap between different types of schools has narrowed or widened, or even to quantify whether any given school is better or worse in absolute terms in 2006 relative

to how it was in 1992.

In any case, and with that significant caveat in mind, I try to calibrate the model in 2006 to replicate the observed normalized average test scores in the voucher and non-voucher sector, and then see whether the predicted normalized differences for 1992 are consistent with the data.

As test scores are not explicitly incorporated in the model, I assume that they are a concave function of education outcomes,

$$test = (h_{ij})^{\varrho} = \left(a_i^{\alpha} q_j^*\right)^{\varrho}$$

where again q_j are the equilibrium school qualities and a is student ability, which I assume is normally distributed across the population. ϱ is calibrated to replicate the actual normalized test score difference between non-voucher and voucher schools in 2005, which equals 1.71. The ϱ that replicates this difference equals 0.09. Then, the calibrated model is used to the simulate the normalized difference for 1992. As seen in the table below, the model does a poor job, predicting that the normalized gap in 1992 should have been much smaller than it actually was. However, and for the reasons mentioned above, there is no reason to assume that there exists a unique ϱ that can simultaneously generate the outcomes for what are in practice two different tests.

Table 6: Test scores		
	Calibrated mode	Actual data
Normalized test score gap, fully-paid and private schools, 2005	1.71	1.71
Normalized test score gap, fully-paid and private schools, 1992	1.48	1.68

4.3 Simulations and Policy Exercises Using the Calibrated Model

This section uses the calibrated model to perform several counterfactual policy exercises for the 2005-2006 data. Results are presented in Tables 7A to 8B. The first column of Table 7A presents the results for the baseline parameters used in the calibration exercise, in terms of the outcomes per student and the expenditure shares in each sector. The first policy exercise, presented in the second column of Table 7A, eliminates the 1994 policy reform, so that payments on top of the voucher are no longer allowed. The level of the voucher is not affected. As a result, the size of the voucher sector falls from 91% to 84%, as relatively rich parents are now better off funding education privately than being limited with the voucher.

Overall education investments fall significantly, as all parents that decide to stay in the voucher and were previously topping the voucher can no longer do so. This reduction in investment is not compensated by the larger investments made by the parents who shift to the private sector (Figure 1).Average expenditure as a share of income (including vouchers) falls from 8.5% to 6.9%. Parental out-of-pocket expenditures decrease from 3.8% to 1.5%. Overall outcomes fall, a result of the reduction in aggregate investment and the loss of efficiency in the voucher sector. Outcomes also fall in each sector. In the voucher sector, this is caused by the reduction on parental expenditure and the reduction in the investments made by the more productive schools (Figure 2). The reduction of outcomes in the private sector is due to the composition change brought by the shift of relatively low income households into that sector, which bring the sector´s average down. Quality decreases for all schools in the voucher sector that were receiving payments and increases for the schools that now become fully-paid (Figure 3). The first column of Table 8A presents the effect of the policy on different income quartiles. Households in the first income quartile are virtually unaffected, as they still receive the same voucher and made almost no additional payments to begin with. Educations outcomes are most affected in the next two quartiles, as households remain in the voucher sector but are no longer allowed to make the tuition payments they were choosing before.
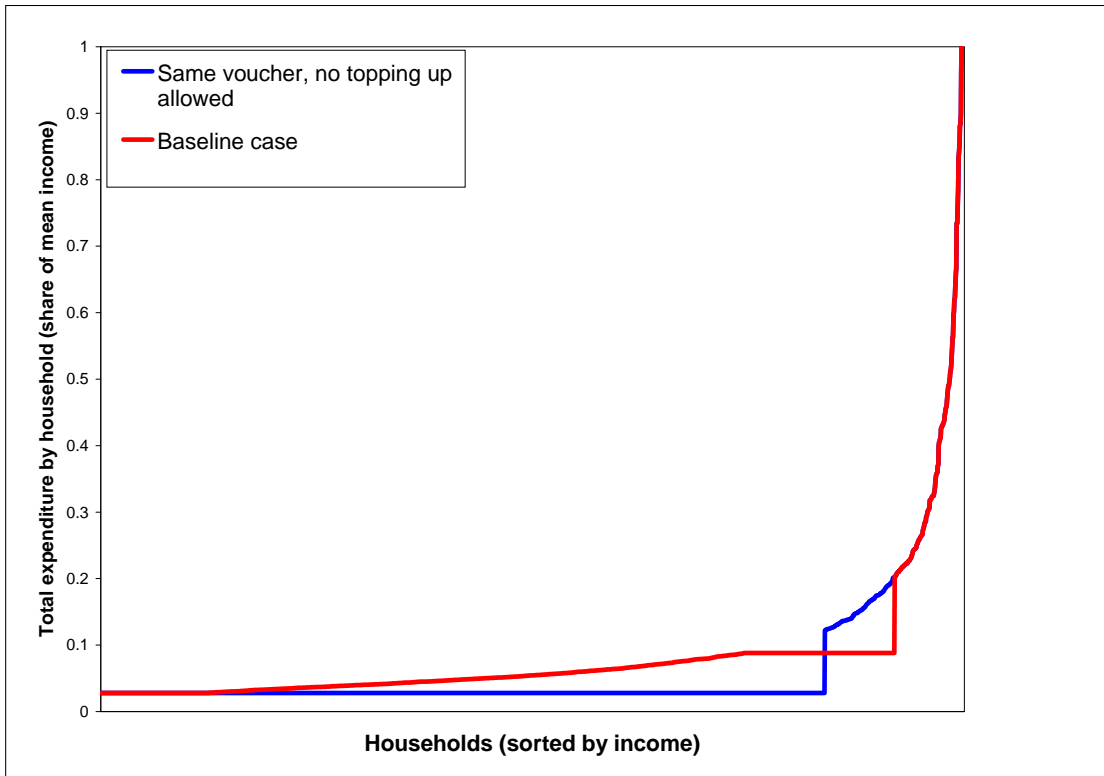

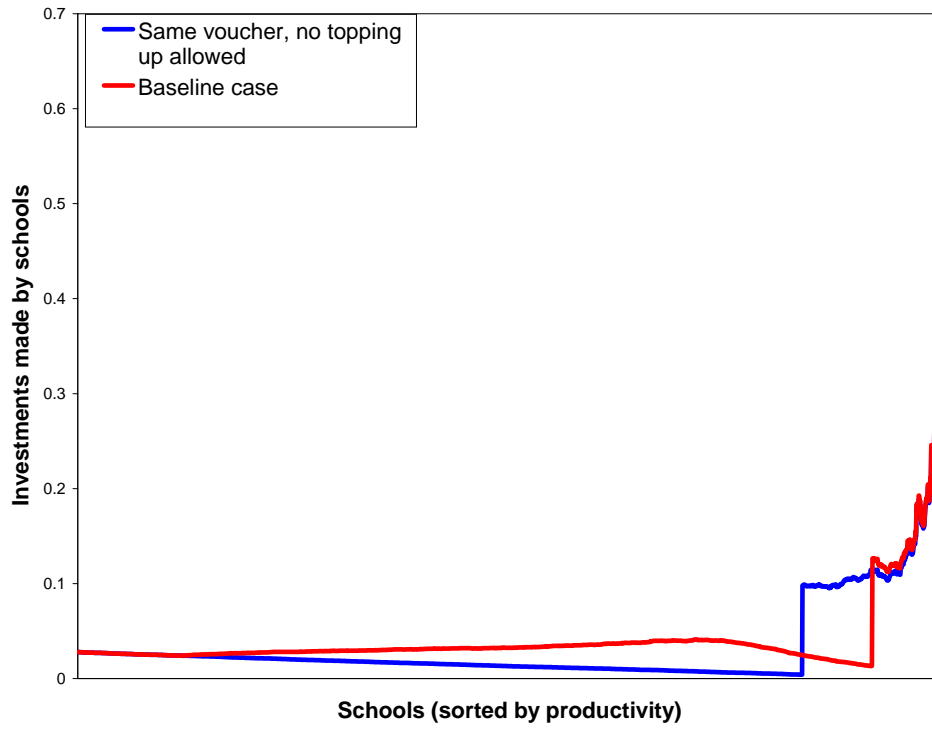Figure 1: Education expenditures by household
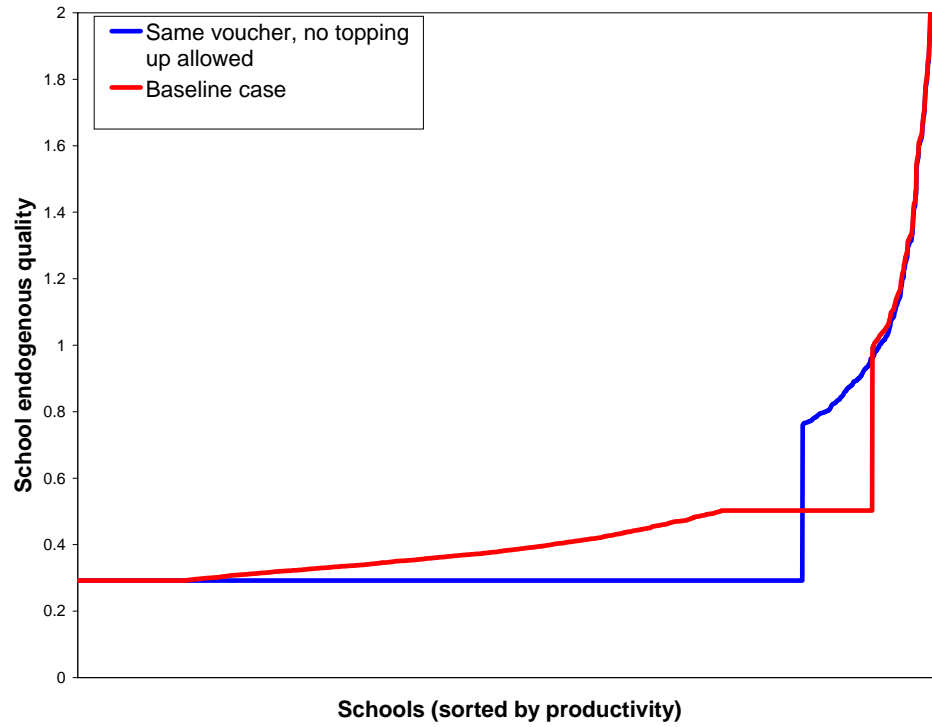
Figure 2: Investments by schools

Figure 3: Equilibrium school qualities

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The second exercise relaxes the restrictions on payments on top by doubling the current cap on parental tuition payments in the voucher sector. As before, the level of the voucher stays the same. Results are shown in the third column of Tables 7A and 8A. Relative to the baseline case, the size of the voucher sector grows from 91% to 95%, as some agents are now better off reducing their out of pocket expenditures and moving to the voucher sector (Figure 4). Average investments as a share of income goes from 8.5% to 8.8%. Larger out of pocket expenditures within the voucher sector, by parents who were bound by the previous cap, are somewhat offset by the reduction in investment from those parents who abandon the fully-paid sector (Figure 4). Regarding school investments (Figure 5), the upper end of the schools that initially were in the voucher sector now invest more, while investments fall significantly for the schools that move from the private to the voucher sector. As a result, the same pattern is observed in school qualities (Figure 6). Overall outcomes increase as a result of the increase in overall investments, although the result is dampened by the fact that investments are shifted towards schools with relatively less productivity. While the increase in outcomes in the voucher sector is driven by the increase in overall investment within the sector and the entry of more productive schools, better average outcomes in the fully-private sector are once again explained by the change in composition. Regarding income quartiles, the first three quartiles, who were not bounded by the tuition cap in the voucher sector, remain unaffected by the new policy. Inequality increases as higher income students improve relative to the rest.
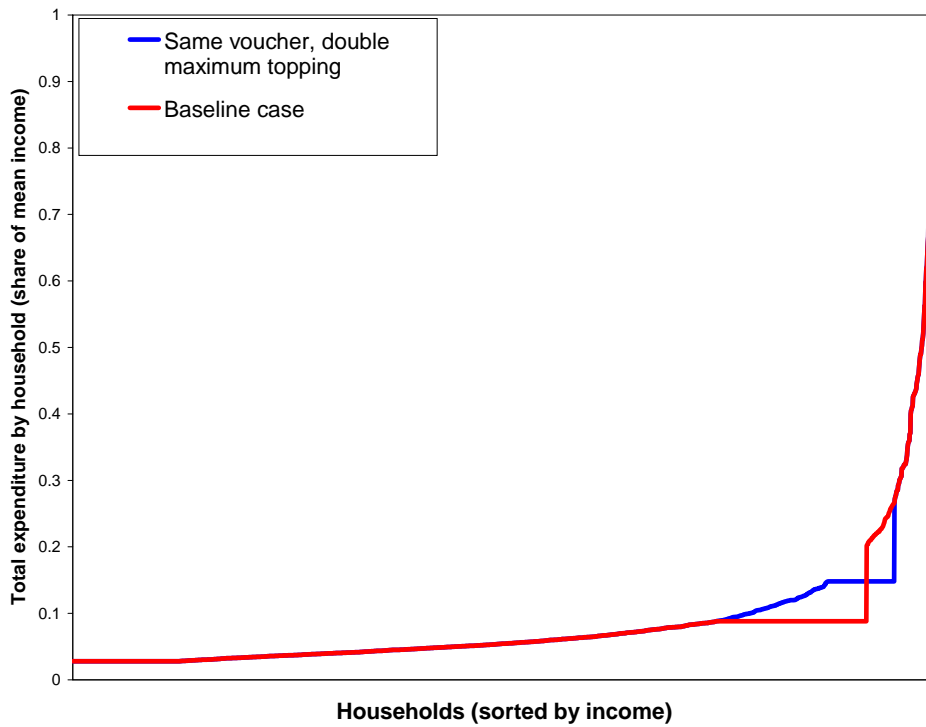

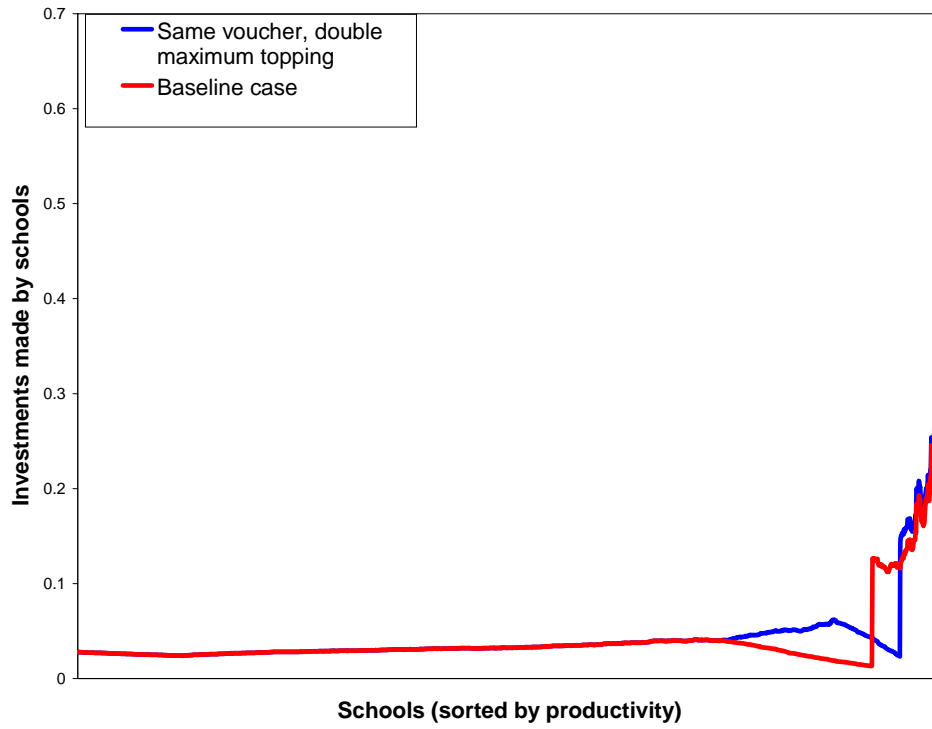Figure 4: Education expenditures by household
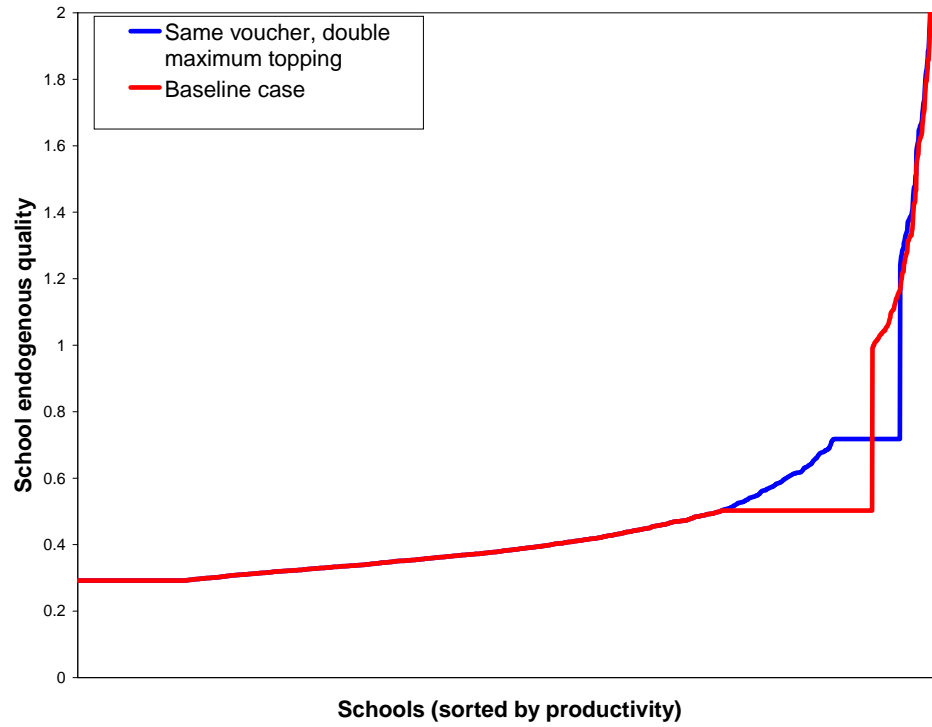
38

Figure 5: Investments by school

Figure 6: Equlibrium school qualities

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The third exercise doubles the level of the voucher, while keeping the absolute level of the cap on topping constant. Results are presented in the first column of Tables 7B and 8B, as well as one Figures 7 to 9. Relative to the baseline case, the size of the voucher sector once again increases, although slightly less than when the policy targeted the cap on tuition. The effects on aggregate expenditure, however, are much larger, as this policy has impact over a much larger base (Figure 7). Relative to the baseline case, average expenditure as a share of income jumps from 8.5% to 9.4%. Out-of-pocket expenditures, though, fall significantly, as the larger voucher partially crowds out parental expenditure. Despite the significant differences in expenditure between both cases, average outcomes are only marginally above those attained with a policy that relaxes the tuition cap. This is basically due the reduction in efficiency associated to the shift of investment towards least productive schools, as seen in Figures 8 and 9. As expected, this policy also has very different distributive implications than the policy that relaxes the tuition cap. While changes in the tuition cap did not affect the outcomes of the first quartile, this group is significantly benefited by an increase in the voucher. Outcomes in the second quartile are almost unaffected (a combination of crowding out of parental expenditure and efficiency loss), while outcomes in the third quartile actually fall. The distribution of outcomes becomes more egalitarian.
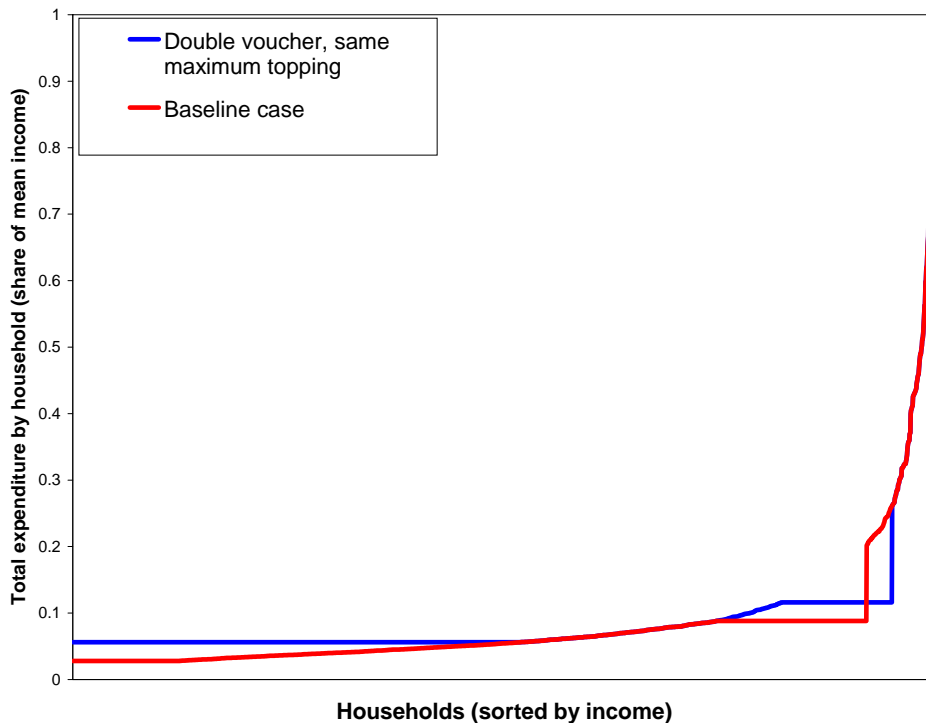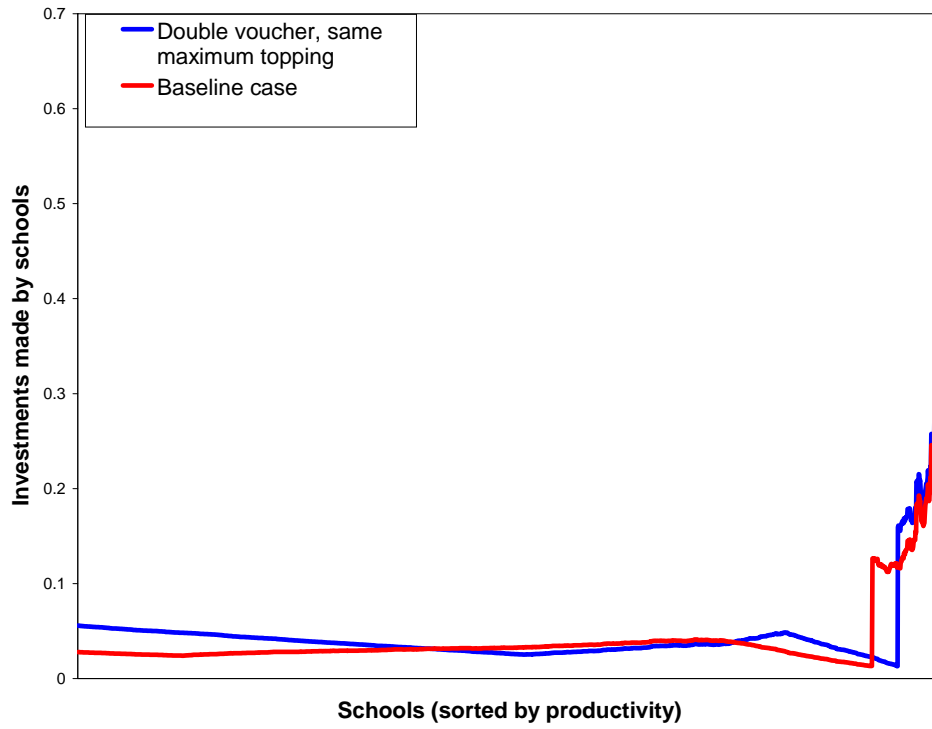

Figure 7: Expenditures by household

Figure 8: Investments by School
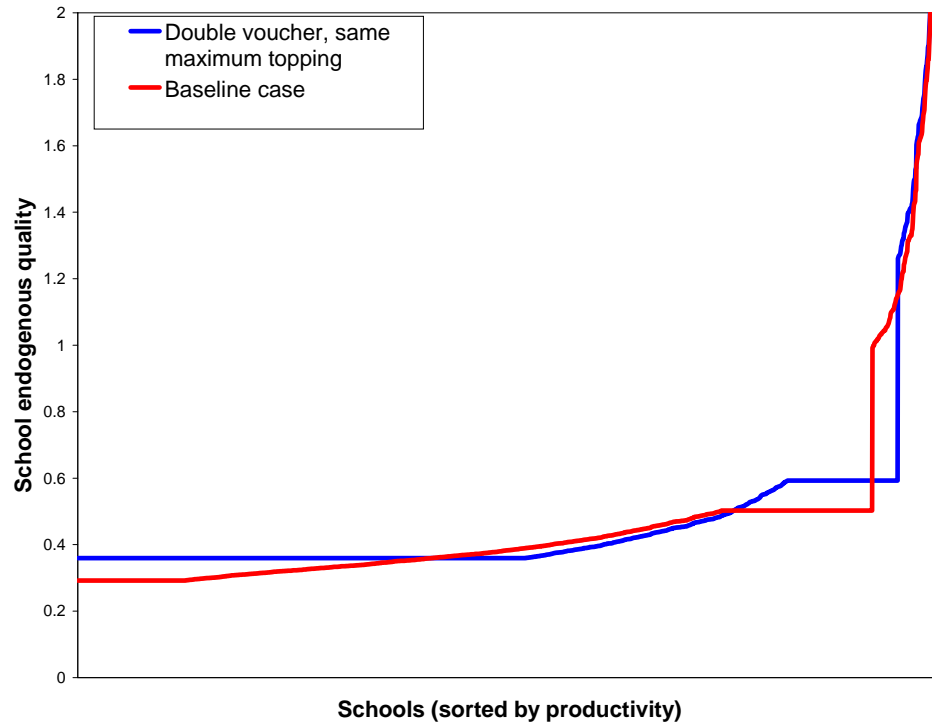
Figure 9: Equilibrium school qualities

The last two columns of Tables 7B and 8B present two polar policy exercises that illustrate how the distribution of educational expenditures across the population affects the distribution of outcomes. The first case eliminates the voucher, so that each parent must finance education privately[12] The second case forces all parents to receive a voucher, and forbids any kind of additional payment. The voucher is set to the average expenditure in the no voucher case, so that the economy's aggregate expenditure is the same in both scenarios. This simulated voucher is much larger than the actual voucher used in the baseline or the previous counterfactual scenarios.

Figure 10: Expenditures by household

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[12] Remember that there are no fixed operation costs, so there is no minimum revenue threshold for a school to be viable.

Figure 11: Investments by school

Figure 12: Equilibrium school qualities

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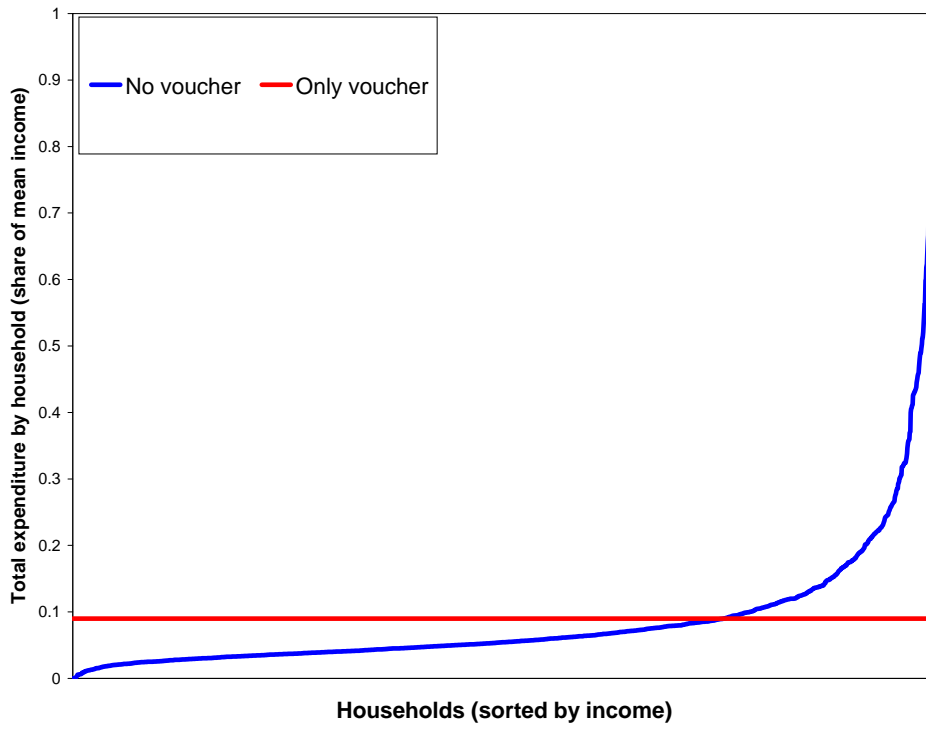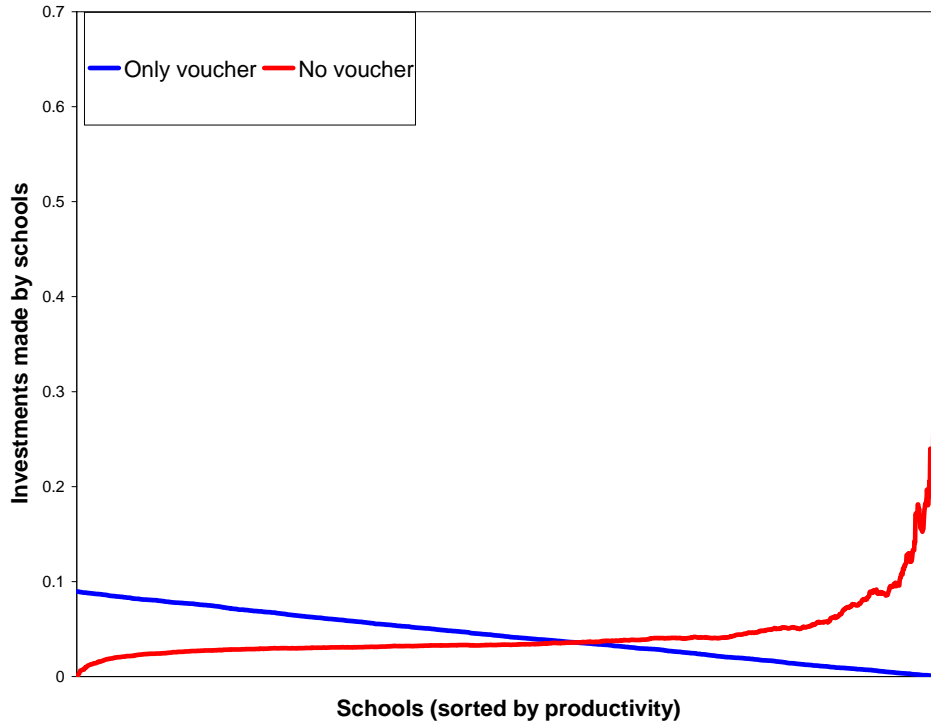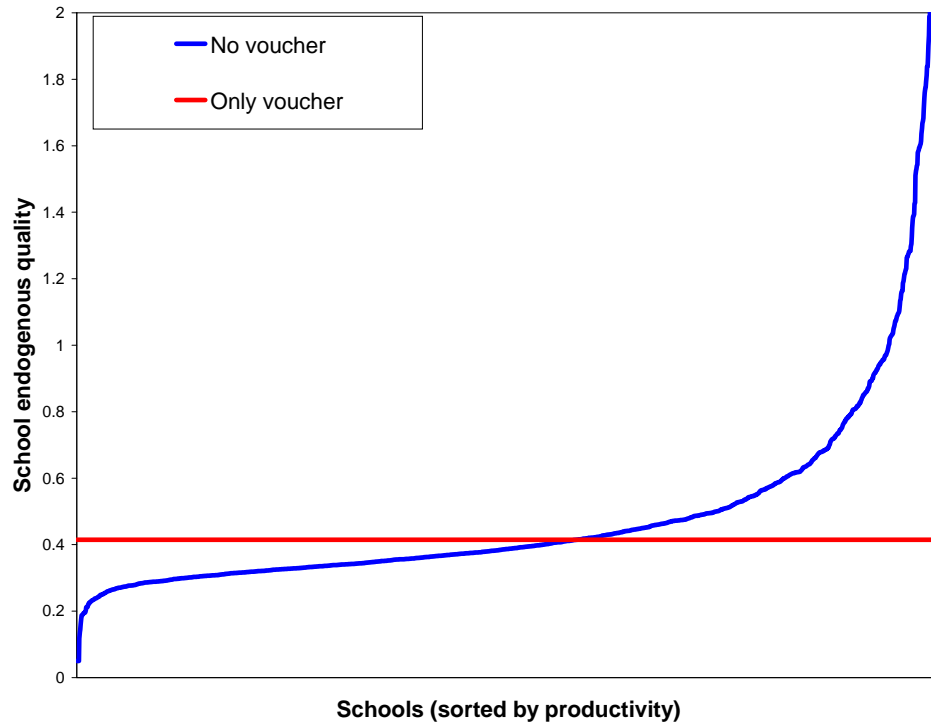Notice that, give concavity on the production function and the fact that ability is uniformly distributed across the population, in a model with homogeneous schools education would tend to be higher with an equalization of expenditure across parents, rather than with the extreme differentiation brought by eliminating the voucher and making each parent invest privately. However, the introduction of school heterogeneity reverses the result, as a consequence of different incentives faced by schools in both cases and the way in which investments are allocated across schools of different productivity (see Figures 11 and 12). While, controlling for overall expenditure, aggregate outcomes on the no-voucher case are similar to the ones observed in the baseline case, simulated outcomes are on average much worse when a uniform (and relatively high) tuition is imposed across the population. Unsuprisingly, a policy that eliminates the voucher exacerbates inequality, significantly worsening outcomes in the first income quartile.

Table 7A: Simulations of the baseline model			
	Baseline case	No topping up	Double topping up
Size of the voucher sector	0.91	0.83	0.95
Out-of-pocket expenditure as a share of income	0.038	0.014	0.041
Average outcome, private sector	1.31	1.01	1.64
Average outcome, voucher sector	0.33	0.25	0.36
Average outcome, overall	0.411	0.372	0.421
Average expenditure as a share of mean income, voucher sector	0.091	0.091	0.091
Average expenditure as a share of mean income, private sector	0.082	0.048	0.087
Average expenditure as a share of mean income, overall	0.085	0.070	0.088

Table 7B: Simulations of the baseline model			
	Double voucher	No voucher	Only voucher
Size of the voucher sector	0.94	0	1
Out-of-pocket expenditure as a share of income	0.016	0.09	0
Average outcome, private sector	1.61	—	—-
Average outcome, voucher sector	0.36	—	—
Average outcome, overall	0.422	0.428	0.358
Average expenditure as a share of mean income, voucher sector	0.090	—	—-
Average expenditure as a share of mean income, private sector	0.096	—	—
Average expenditure as a share of mean income, overall	0.094	0.090	0.09

Table 8a: Simulated outcomes			
	Baseline case	No topping up	Double topping up
Average outcome, first quartile	0.257	0.250	0.257
Average outcome, second quartile	0.304	0.253	0.304
Average outcome, third quartile	0.374	0.251	0.375
Average outcome, fourth quartile	0.73	0.754	0.770
Average expenditure as a share of mean income, first quartile	0.104	0.096	0.105
Average expenditure as a share of mean income, second quartile	0.091	0.056	0.091
Average expenditure as a share of mean income, third quartile	0.091	0.037	0.091
Average expenditure as a share of mean income, fourth quartile	0.079	0.079	0.084

| Table 8b: Simulated outcomes	Double voucher	No voucher	Only voucher
Average outcome, first quartile	0.308	0.249	0.355
Average outcome, second quartile	0.311	0.308	0.359
Average outcome, third quartile	0.355	0.377	0.356
Average outcome, fourth quartile	0.732	0.800	0.356
Average expenditure as a share of mean income, first quartile	0.193	0.091	0.310
Average expenditure as a share of mean income, second quartile	0.112	0.091	0.181
Average expenditure as a share of mean income, third quartile	0.091	0.091	0.118
Average expenditure as a share of mean income, fourth quartile	0.080	0.091	0.0367
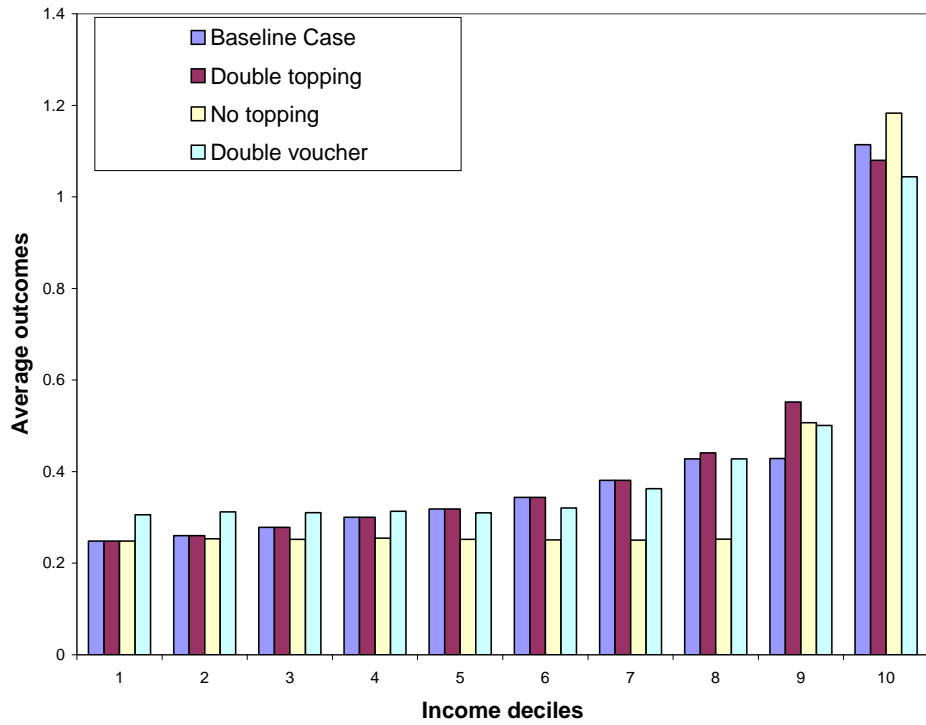
Figure 13: Outcomes by Decile

5 Extending the model: Non-profit schools

The model discussed so far, and most of the literature, assumes that the objective function of private education providers is to maximize profits. However, a significant number of private providers in education markets operate as non-profit institutions. This section extends the analysis presented in the previous part, and solves for the equilibrium allocations in markets that are served by private schools with different objective functions. As discussed below, the distribution of education expenditures will be crucial for the distribution of students across school of different types. Our strategy follows the one in Section 2. First, we characterize the equilibrium allocations in a market in which tuition payments are completely homogenous. In the setup of the model, that regime leads to complete separation between for-profit and non-profit schools, with the latter serving the more able students. Then, we introduce heterogeneity in tuition payments, and show how now profit-maximizing might compete directly against non-profit schools.

What is the objective function of a non-profit school? In the context of this paper, we define a non-profit school as one that always exhaust its budget, reinvesting all of its revenue. Moreover, we assume that non-profit schools maximize education outcomes. Thus, for a given level of tuition, they will strictly prefer to enroll student with higher ability.

However, outcome maximization need not be the objective for all non-profit schools. In a voucher setup, they might maximize their utility by providing the best outcomes to disadvantaged, low ability kids. When receiving private funding, they might put a cap on tuition, as their objective function could be to try to provide the best education for low-income households. Whatever the objective function of non-profit schools actually is, it will not only determine their own behavior, but also the response of profit-maximizing and households and, as a result, the features of the equilibrium allocation .

The maximization problem for nonprofit schools can be written as:

$$Max \ h_{ij}$$
$$\text{s.t. to } h_{ij} = a_i^\alpha y_j^\varphi \gamma_j^\zeta$$
$$y_j \leq p_{ij} h_{ij}^d$$

As before, non-profit schools differ in their exogenous productivity, γ. Technology and the rest of the setup are the same as in the profit-maximizing case discussed in Section 2. Schools are only financed through tuition, either directly from parents or through a voucher. The outside option of non-profit schools is normalized to zero. There are N students.

5.1 Homogeneous education expenditure (mandatory vouchers)

Again, we begin by analyzing equilibrium allocations in a market with homogeneous tuition expenditures. Thus, from the perspective of the school, students only differ in their ability.

5.1.1 A market where non-profit schools exist

Initially, we consider an economy in where only non-profit schools exist.

Proposition 6 *In a market where schooling is only financed with flat vouchers, and where only nonprofit schools operate, all schools will invest the full extent of the voucher, thus reaching their highest feasible quality. This is, for any operating school with exogenous productivity γ_j, its equilibrium quality will be $q_j^* = v^\varphi \gamma_j^\zeta$.*

Proof. This result comes directly from the schools objective function. Non-profit schools do not get direct utility from the funds they receive, but only indirectly through the effect of those funds in the maximization of education outcomes. As education outcomes are monotonic on school investments, non-profit schools will fully invest any tuition payments they receive. ∎

Given that schools always invest the voucher completely, relative qualities only depend on relative exogenous productivities. Outcomes, then, only depend on the exogenous characteristics of the agents in the match. The matching problem becomes a simple application of Becker (1973), where ability and productivity are complements, and the utility of both schools and students is increasing in the output of their match.

Proposition 7 *The stable market equilibrium can be characterized as follows:*

(a) Only the N more productive schools will operate.

(b) There will be positive assortative matching between productive non-profit schools and able students. The matching function can be written as $\mu(i) = i$.

(c) For any student i and matching school $\mu(i) = i$, the equilibrium outcome will be $h^*_{i\mu(i)} = v^\varphi a_i^\alpha \gamma_i^\zeta$, for all $i \in \{1..N\}$.

Proof. (a) is a straightforward extension of the previous proposition.

For (b), take any school with productivity γ_j, and any pair of students with ability a_i and a_k, such that $a_i > a_k$.

As any school invests the full extent of the voucher, it must be true that outcomes are higher in the match with the more able student, $h_{ji} = q_j^* a_i^\alpha > h_{jk} = q_j^* a_k^\alpha \Rightarrow w\left(a_i, \gamma_j, q_{ij}^{ZP}\right) > w\left(a_k, \gamma_j, q_{ik}^{ZP}\right)$. All schools strictly prefer to match with more able students.

Now, take any student with ability a_i and any pairs of schools with productivity γ_j and γ_k, $\gamma_j > \gamma_k$. As $q_j^* > q_k^*$, the student would strictly prefer to attend the school with the highest productivity.

Assume that the set of equilibrium allocations does not exhibit positive sorting. Take two potential equilibrium matches, $\mu(i) = j$ and $\mu(j) = i$, with $i < j$. The output of the proposed matches are $h_{ij} = q_j^* a_i$ and $h_{ji} = q_i^* a_j$, respectively. This is not an equilibrium allocation, as student i and school j could match and get $h_{ii} = q_i^* a_i^\alpha$. School i would be better off as $h_{ii} = q_i^* a_i^\alpha > h_{ji} = q_i^* a_j$, given that $a_i > a_j$. Student i would be better off, as $h_{ii} = q_i^* a_i^\alpha > h_{ij} = q_j^* a_i^\alpha$, given that $\gamma_i > \gamma_j$. Thus, both agents have incentives to deviate, and the initial matches break, so they cannot be part of an equilibrium allocation. Any allocation without positive assortative matching in ability and productivity is not a stable equilibrium. ∎

In this setup, nonprofit schools with better performance have the best outcomes not only because they enroll the more able students, but also because they are intrinsically more productive. Differences in outcomes between students with ability a_i and $a_{i+1}, a_i > a_{i+1}$, are more than proportional to their ability differences, reflecting also the productivity gap between the schools in which they enroll.

$$\frac{h_{i\mu(i)}}{h_{j\mu(j)}} = \frac{v^\varphi a_i^\alpha \gamma_i^\zeta}{v^\varphi a_j^\alpha \gamma_j^\zeta} = \left(\frac{a_i}{aj}\right)^\alpha \left(\frac{\gamma_i}{\gamma j}\right)^\zeta, \text{ where } a_i > a_j \Rightarrow \gamma_i > \gamma_j \tag{43}$$

5.1.2 A market with non-profit and profit-maximizing schools

We now incorporate profit-maximizing schools into the market, and allow both types of schools to exist simultaneously and compete for the same funding.

Proposition 8 *Regardless of their objective functions, the market will be served by the N most productive schools.*

Proof. As before, rank the productivity of potential entrepreneurs from γ_1 to γ_M. From the set N more productive schools, take any school j with productivity γ_j. For any voucher level v, γ_j can attain a quality level q_J^* that is above the maximum feasible level $q_{N+1}^{\max} = v\gamma_{N+1}$ of the most productive school in the set $\{\gamma_{N+1}, ..., \gamma_M\}$. Then, any student would strictly prefer to attend school j, and school j gets a non-negative payoff. As this is true for all $\gamma_j \in \{\gamma_1, ..., \gamma_N\}$, and schools can only enroll one student, the set of operating schools is $\{\gamma_{N+1}, ..., \gamma_M\}$. ∎

The discussion above has shown that, regardless of the degree of competitive pressures, operating non-profit schools will choose to invest the total amount of the voucher, operating at the limit of their technological possibilities, and will always prefer to match with more able students. Thus, their behavior will not change when they are in the same market as profit-maximizing schools.

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Proposition 1, on the other hand, states that the behavior of profit-maximizing schools is solely driven by competitive pressures. First, they place no value on the ability of the student they match with. Second, they invest just enough to reach the minimum quality that leaves the marginal school outside the market. But operating non-profit schools behave exactly like the fringe competitor, in the sense that they operate at zero profits, and are at least as productive as it.

Proposition 9 *Profit-maximizing schools will choose not to compete directly with the non-profit schools, leaving the ablest students in the non-profit sector.*

Proposition 10 *The market equilibrium can be characterized as:*

(a) Only the N more productive schools will operate, with N_p profit-maximizing schools and $N_{np} = N - N_p$ non-profit schools. Label the non-profit schools, sorted by productivity, as i', with productivities γ'_1 to $\gamma'_{N_{np}}$.

(b) There will be positive assortative matching between the N_{np} non-profit schools and the N_{np} most able students. The matching function can be written as $\mu(i) = i'$, with $i \in \{1, ..N_{np}\}$ The remaining N_p students will be enrolled in the profit-maximizing sector. Within the profit-maximizing sector, any allocation of schools and students is an equilibrium.

(c) For any student i enrolled in the non-profit sector and matching school $\mu(i) = i'$, the equilibrium outcome will be $h^*_{i\mu(i)} = v^\varphi a_i^\alpha \gamma_i^\zeta$, for all $i \in \{1, .., N_{np}\}$. For any student j enrolled in the profit-maximizing sector and matching school $\mu(j)$, the equilibrium outcome will be $h^*_{j\mu(j)} = v^\varphi a_i^\alpha \gamma_{N+1}^\zeta$, for all

$j \in \{N_{np} + 1, .., N\}$.

(d) More productive schools and more able students get strictly higher payoffs.

Proof. (a) comes directly from the previous proof.

Assume (b) and (c) hold. For any operating profit-maximizing school with productivity γ_j, its profits when matched with student i are $\pi\left(a_i, \gamma_j, q^*_{N+1}\right) = v\left(1 - \left(\frac{\gamma_{N+1}}{\gamma_j}\right)^{\frac{\zeta}{\varphi}}\right)$ for any $i \in \{N_{np} + 1, .., N\}$. Notice that, as discussed in Section 2, profits do not depend on the ability of the student in the match.

If the profit-maximizing school wanted to get a student that is enrolled in a non-profit school with productivity γ'_i, it would have to at least have the same quality as that school, $q^*_{i'} = v\gamma'_i$. Profits would then be $\pi\left(a_i, \gamma_j, q^*_{N+1}\right) = v\left(1 - \left(\frac{\gamma'_i}{\gamma_j}\right)^{\frac{\zeta}{\varphi}}\right)$. As, for all operating non-profit schools, $\gamma'_i > \gamma_{N+1}$, $\pi\left(a_i, \gamma_j, q^*_{i'}\right) < \pi\left(a_i, \gamma_j, q^*_{N+1}\right)$, for all $i' \in \{1, .., N_{np}\}$. Profit-maximizing schools have no incentives to deviate from the proposed equilibrium and enroll the more able students.

For the non-profit sector, the existence of assortative matching in ability and productivity between was already shown.

Take the best student enrolled in the profit-maximizing sector, with ability $a_{N_{np}+1}$. In the proposed equilibrium, $a_{N_{np}+1} < a_{N_{np}}$, where $a_{N_{np}}$ is the ability of the least able student in the non-profit sector. But then all non-profit schools strictly prefer to match with the student with ability $a_{N_{np}}$, and thus strictly prefer to match with any student in the non-profit sector than with any student in the profit-maximizing sector.

Students in the profit-maximizing sector would prefer to attend the non-profit sector, where schools have strictly larger quality. However, non-profit schools would never enroll them.

Finally, students in the non-profit sector would never deviate to the profit-maximizing sector, as school qualities there are strictly lower.

As all matches are stable, the equilibrium is as described in (b) and (c) ∎

Non-profit schools are tough competitors, in the sense that they always exhaust their budget to attain their maximum feasible quality. Profit-maximizing have no incentives to compete against them, as that would imply incurring in a higher cost with no increase in revenue. This leads to a market that is strictly separated, with non-profit schools serving the ablest students and profit-maximizing schools choosing to serve low-ability students.

Differences in outcomes in the non-profit sector reflect differences in both ability and school productivity, while in the profit sector they only reflect differences in ability. Non-profit schools are fully differentiated, while all profit-maximizing schools choose to have the same (strictly lower) quality.

What about productive efficiency? It is easy to see that the most efficient feasible allocation - in terms of maximizing aggregate education outcomes given the distribution of aggregate investment[13] - would be to have assortative matching, with all schools investing the full extent of the voucher. It is clear that non-profit schools will, indeed, lead to a feasible efficient allocation in terms of education outcomes, under the preferences assumed for them here. That is clearly not true for profit-maximizing schools. The differences in productivities between profit-maximizing schools implies that better schools will never invest the full extent of the voucher, and thus will always attain a smaller quality than their potential.

However, the outcomes in an economy that makes profit-maximizing schools eligible for a voucher are at least as efficient as those of an economy in which only non-profit schools are eligible. If a profit-maximizing school is able to operate successfully, its quality must be strictly larger than the feasible quality of the best non-profit school that is outside the market. Thus, banning profit-maximizing schools from receiving vouchers would only increase "efficiency" in the sense that all education funding would be invested in augmenting school quality. However, overall productive efficiency would be hurt, as average school qualities would decrease as a result of the deterioration in average productivity.

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[13]Notice that, as investments in more able students are always more productive an investment plan with flat vouchers can never be fully efficient.

5.2 Fully private funding

When schools are allowed to charge tuition to parents with different willingness to pay, the problem becomes more complex. When all parents paid exactly the same tuition, it was optimal for profit-maximizing to yield to non-profit schools, as directly competing against them implied incurring in a higher cost that brought no additional revenue. That need no longer be true when parents are willing to pay different levels of tuition, as larger costs from matching with particular students might be compensated by an even larger increase in revenue. Thus, relative to the previous case, profit-maximizing schools will have an incentive to compete directly with non-profit schools, and the strict segmentation discussed earlier need not hold in equilibrium.

In general, equilibrium can not be explicitly characterized, as it will depend on the joint distribution of ability and income, on the parents side, and the distribution of productivities between profit-maximizing and non-profit schools.

To get a more precise characterization of the features of the market equilibrium, we impose the strong assumption that income and ability are perfectly correlated. If this were not the case, the problem becomes less tractable, though the main features of the equilibrium allocation and the proposed solution method would still hold. For simplicity, we also assume that, when two schools of different type make the same offer, the student chooses to enroll in a profit-maximizing school.

Proposition 11 *Regardless of their objective functions, the market will be served by the N most productive schools. Only the N more productive schools will operate, with N_p profit-maximizing schools and $N_{np} = N - N_p$ non-profit schools.*

Regardless of the distribution of tuition payments, as long as there are no barriers to entry[14], the market will be served by the most efficient schools, even if they have different objective functions. The introduction of heterogeneity in tuition payments, however, breaks the strict separation between for-profit and non-profit schools that was an equilibrium feature in the previous case.

Proposition 12 *Profit-maximizing schools might choose to compete directly against non-profit schools.*

Proof. We illustrate this through an example. Take a market with 3 schools. γ_1 to γ_3. γ_1 and γ_3 are for-profit, γ_2 is no profit. There are two students, with income (ability) $m_1(a_1)$ and $m_2(a_2)$. As established in Proposition 9, under a pure voucher regime, the for-profit school with productivity γ_1 would choose not to compete with the non-profit school γ_2 for the student with income m_1, as it can get higher profits by matching with the next student and competing directly against γ_3. This no longer need be true if parents

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[14] In a wide sense, ranging from limits on the eligibility of funding to fixed costs.

pay different tuition. If school 1 decides to enroll the first student, it has provide an offer that is at least as good as the best offer that school 2 will be willing to be made. As school 2 is non-profit, that offer always exhausts the school´s budget: : $h_{i\mu(i)}^* = (\kappa m_1)^\varphi a_1^\alpha \gamma_2^\zeta$. In that case, the profits for school 1 will be $\pi = \kappa m_1 \left(1 - \left(\frac{\gamma_2}{\gamma_1} \right)^{\frac{v}{\varphi}} \right)$. Alternatively, it can match with student 2, providing an offer that is at least as good as the best offer of school 3, the marginal school outside the market: $h_{i\mu(i)}^* = (\kappa m_2)^\varphi a_2^\alpha \gamma_3^\zeta$. If so, the profits for school 1 are $\pi = \kappa m_2 \left(1 - \left(\frac{\gamma_3}{\gamma_1} \right)^{\frac{v}{\varphi}} \right)$. Revenue is larger in the first case, but costs are smaller in the second. Which of the matches maximizes profits depends on the productivity and income parameters. Thus, unlike the pure voucher case, competing directly against a non-profit school can be optimal. ∎

This extends the argument presented in Section 2 regarding the behavior of profit-maximizing schools under different distributions for tuition payments. As education expenditures become heterogeneous, profit-maximizing schools have an incentive to differentiate in order to attract students with a higher willingness to pay. In this case, differences in tuition might provide the incentive for profit-maximizing schools to compete directly against their non-profit counterparts. The behavior of non-profits schools, on the other hand, is not affected by competitive incentives, as they will always reinvest the full extent of their revenue. As discussed below, this guarantees that schools will be strictly sorted within each type , but does not guarantee that there will strict sorting across the complete set of schools.

Proposition 13 *In any equilibrium, it will be true that:*

(a) Within the set of non-profit schools, there will be positive sorting in school productivity and student income: more productive schools will enroll students with a higher willingness to pay. This is, take any 2 non-profit schools with productivities γ_j and γ_k, $\gamma_j > \gamma_k$, with associated matches with incomes m_i and m_l, respectively. In any equilibrium allocation, it must be true that $m_i > m_l$.

(b) As a consequence, within the set of non-profit schools, more productive schools will have strictly larger equilibrium qualities. Non-profit schools completely exhaust their budget.

(c) Within the set of profit-maximizing schools, there will be positive sorting in school productivity and student income: more productive schools will enroll students with a higher willingness to pay. This is, take any 2 profit-maximizing schools with productivities γ_j and γ_k, $\gamma_j > \gamma_k$, with associated matches with incomes m_i and m_l, respectively. In any equilibrium allocation, it must be true that $m_i > m_l$.

(d) As a consequence, within the set of profit-maximizing schools, more productive schools will have strictly larger equilibrium qualities.

(e) A profit-maximizing school will never have an equilibrium match that has a higher income that the equilibrium match of a non-profit school with higher productivity. However, a non-profit school might have

an equilibrium match that has a higher income that the equilibrium match of a profit-maximizing school with higher productivity

Proof. (a) and (b): Directly from the objective function of non-profit schools and Proposition 7.

(c): Directly from Proposition 4.

(d): For first part of the statement: Take any 2 schools, $\gamma_j > \gamma_k$, where γ_j is non-profit and γ_k for-profit. Assume that the statement is not true, so the equilibrium matches for schools γ_j are student with income m_l and m_i, respectively, with $m_i > m_l$. That clearly can not be an equilibrium, as the non-profit school is better off matching with m_i, and can provide an offer that is strictly larger than the best offer that γ_k can make without getting negative profits: $(\kappa m_i)^\varphi a_i^\alpha \gamma_j^\zeta > (\kappa m_i)^\varphi a_i^\alpha \gamma_k^\zeta$. A for-profit school can never successfully compete with a more productive non-profit school.

For the second part, see the proof for Proposition 12. ∎

The last part of the statement is the most interesting, as it lies at the core of the difference in behavior between both types of schools. A profit-maximizing school can never make a better offer than the one made by a non-profit school with higher productivity. As non-profit schools are always fully investing revenue, making a better offer would imply negative profits for the profit-maximizing school.

Within their feasible set of matches, profit-maximizing schools make a joint choice on the match and investment that maximizes their profits. That choice might imply yielding to a non-profit school with lower productivity, as was clearly seen in the extreme case in which tuition payments were pre-determined and uniform. Thus, in equilibrium, a non-profit school might have an equilibrium match that has a higher income that the equilibrium match of a profit-maximizing school with higher productivity, if that is the optimal choice of the for-profit school.

Proposition 14 *In any equilibrium, payoffs will be such that:*

(a) If a student with income m_i and ability a_i is enrolled in a non-profit school with productivity γ_j, the outcome of the match will be $h_{i\mu(i)}^* = (\kappa m_i)^\varphi a_i^\alpha \gamma_j^\zeta$.

(b) If a student with income m_i and ability a_i is enrolled in a profit-maximizing school with productivity γ_k, the outcome of the match will depend on the productivity and type of the school that is matched with student m_{i+1}, the one immediately below in the income distribution. The equilibrium outcome must be such that the school enrolling m_{i+1} has no incentive to provide a better offer to student m_i.

(b.1) If the school enrolling student m_{i+1} is a non-profit school with productivity γ_s, then $h_{i\mu(i)}^* = (\kappa m_i)^\varphi a_i^\alpha \gamma_j^\zeta$.

(b.2) If the school enrolling student m_{i+1} is a profit-maximizing school with productivity γ_s, then $h_{i\mu(i)}^* =$

$$a_i^\alpha \left[(\kappa m_i - \kappa m_{i+1}) \gamma_s^\zeta + \left(\frac{h_{i+1\mu(i+1)}^*}{a_{i+1}^\alpha} \right)^{\frac{1}{\varphi}} \right]^\varphi$$

(c) Higher income students and more productive schools are strictly better off.

Proof. (a): Directly from the objective function of a non-profit school

(b.1) Any offer below $h_{i\mu(i)}^* = \kappa m_i a_i \gamma_s$ would allow the non-profit school with productivity γ_s to make an offer to student m_i that makes both better off. Any offer above $h_{i\mu(i)}^* = \kappa m_i a_i \gamma_s$ only increases the cost for school γ_k, without any additional revenue.

(b.2) Any offer below $h_{i\mu(i)}^* = a_i^\alpha \left[(\kappa m_i - \kappa m_{i+1}) \gamma_s^\zeta + \left(\frac{h_{i+1\mu(i+1)}^*}{a_{i+1}^\alpha} \right)^{\frac{1}{\varphi}} \right]^\varphi$ would allow the non-profit school with productivity γ_s to make an offer to student m_i that would be accepted by the student and increase profits for γ_s. Any offer above $h_{i\mu(i)}^* = a_i^\alpha \left[(\kappa m_i - \kappa m_{i+1}) \gamma_s^\zeta + \left(\frac{h_{i+1\mu(i+1)}^*}{a_{i+1}^\alpha} \right)^{\frac{1}{\varphi}} \right]^\varphi$ only increases the cost for school γ_k, without any additional revenue. ∎

Once again, the behavior of non-profit schools is not driven by competitive pressures. In any equilibrium match, they provide their maximum feasible quality. In any equilibrium match, profit-maximizing schools will provide just the quality to prevent being outbid by their most direct competitor, as they have no incentives to provide moire.

A market in which tuition payments are heterogenous provides additional incentives for profit-maximizing schools, and can lead to them competing directly with non-profit schools. This is simply an application of the discussion in Section 2. While the behavior of non-profit schools is not affected by the distribution of tuition payments, profit-maximizing schools will be sensitive to the heterogeneity in tuition payments. Regimes that allow for greater heterogeneity, such as authorizing schools to charge tuition on top of the voucher, foster competition between schools of different types.

5.2.1 Solving for equilibrium: Numerical example

This subsection solves numerically for the equilibrium allocations in a market with for-profit and non-profit schools, using the results presented in Section 5.2. The equilibrium is solved recursively, starting with the behavior of the least productive profit-maximizing school, and then moving up the productivity distribution. For each potential allocation of students across schools, we use Proposition 14 to calculate the equilibrium qualities and outcomes that would hold if such an allocation was indeed the equilibrium one. For simplicity, we assume that $\alpha = \zeta = \varphi = 1$.

There are 7 potential schools, with productivities ranked from γ_1 to γ_7, γ_1 being the largest. There are 6 households, with income m_1 to m_6, m_1 being the largest. Ability is perfectly correlated with income, so non-profit schools strictly prefer to match with higher income households.

Assume that school types (profit or non-profit) alternate along the productivity distribution, with the best school being for-profit. This is, the most productive school, γ_1, is for-profit, while the one immediately below it, γ_2, is non-profit, the one after that for-profit, and so on. As there are 7 schools and only 6 students, any equilibrium will leave the least productive school, γ_7, out of the market. In any equilibrium, there will be 3 profit-maximizing schools (γ_1, γ_3, γ_5) and 3 non-profit schools (γ_2, γ_4, γ_6).

As stated before, the objective function of non-profit schools implies that will fully reinvest the tuition paid by the student they enroll. Profit-maximizing schools, on the other hand, will decide an optimal level of investment conditional on the behavior of their competitors.

Take the least productive profit-maximizing school in the market, γ_5. It is clear that, in any equilibrium allocation, this school can never get a better match (i.e., enroll a higher income student) than any other school that has a larger productivity, as that school can always profitably make a better offer. However, γ_5 can choose whether it competes against it most direct competitor (γ_6, the least productive non-profit school) or against the marginal school, γ_7. This is, the profit-maximizing school can decide to enroll the student with income m_5 - and invest enough to make a better offer than the one made by non-profit school γ_6− or to enroll the student with the lowest income, m_6, investing enough to leave γ_7 out of the market. As equilibrium qualities only depend on the productivities and income of schools and students that rank below any given equilibrium match, we do not need to account for all other schools and students to get an get explicit expressions for these two scenarios.

In allocation A, school γ_5 enrolls student m_5, by offering at least the same quality that would be offered by the non-profit school γ_6 (for simplicity, assume that the students chooses to enroll in the profit-maximizing school when both schools make the same offer):

$$q_{5,A} = \kappa m_5 \gamma_6$$

Under that allocation, the profits for school γ_5 are:

$$\pi_{5,A} = \kappa m_5 - \kappa \frac{m_5 \gamma_6}{\gamma_5}$$

In allocation B, the profit-maximizing school matches with student m_6, and faces direct competition from γ_7, the marginal school. Thus, the equilibrium quality would be the one that keeps the marginal school outside the market

$$q_{5,B} = \kappa m_6 \gamma_7$$

with associated profit:

$$\pi_{5,B} = \kappa m_6 - \kappa \frac{m_6 \gamma_7}{\gamma_5}$$

Comparing profits in both cases, γ_5 would prefer to enroll the higher income student and directly compete against the non-profit school if and only if:

$$\pi_{5,A} - \pi_{6,A} > 0 \rightarrow m_5 \left(1 - \frac{\gamma_6}{\gamma_5}\right) - m_6 \left(1 - \frac{\gamma_6}{\gamma_5}\right) > 0$$

This is, the profit-maximizing school analyzes whether the revenue gain of enrolling a higher income student compensates the higher cost of directly competing again a school with relatively larger productivity.

The optimal choice made by the least productive profit-maximizing school determines the set of potential equilibrium choices available to the next profit-maximizing school, γ_5. If the optimal decision for school γ_5 is to compete directly against the non-profit school γ_6, γ_3 has to choose if it competes with the next non-profit school (γ_4), enrolling the student with income m_3, or if moves downwards the income distribution to student m_4, competing directly with γ_5. It is easy to show that, while feasible, allocations in which γ_3 enrolls m_5 or m_6 will never be an equilibrium: as stated earlier, strict sorting in income and productivity still exists between the set of profit-maximizing schools.

If the optimal choice for school γ_5 is not to compete with non-profit school γ_6 and to enroll student m_6, school γ_3 has 3, instead of 2, potential equilibrium choices: enroll student m_3 (again competing against the non-profit γ_4); enroll student m_4 (now competing directly with non-profit γ_6) or enroll student m_5 (with γ_5 as its direct competitor).

Finally, the optimal choices of schools γ_5 and γ_5 determine the set of potential equilibrium enrollments available to the best profit-maximizing school, γ_1. The complete set of potential equilibrium allocation decisions, and the recursive nature of the problem, is illustrated in Figure 15. If all schools strictly prefer one of their potential equilibrium choices, the market will have a unique equilibrium allocation. If at least one school is indifferent between two or more potential choices, the market will have multiple equilibria.

In this simple case with only 6 schools, the number of potential equilibrium allocations is 21. In fact, the number of potential equilibrium allocations grows exponentially with the number of schools. For example, keeping the same setup in which school types alternate, doubling the number of schools to 12 increases the set of potential equilibrium allocations to 117. Moreover, the number of potential allocations does not only depend on the number of for-profit and non-profit schools, but also on the specific distribution of school types along the productivity ranking.

We solve the model numerically for 3 cases, following the recursive solution strategy discussed above.

In the first case, differences in productivity across schools and income across households are large. In the second case, while income differences are still large, the distribution of productivities is more homogeneous. The third case is one in which differences in both productivity and income are small.

In the first case, differences between consecutive for-profit/non-profit schools are large, so that, for any profit-maximizing school, the revenue gain from matching with a high income household offsets the higher cost of competing against a non-profit school. Under the assumed parameters, the unique competitive equilibrium is one in which there is strict positive sorting in school productivity and student income.

In the second case, each profit-maximizing school chooses to yield to the non-profit school immediately below, as now the cost of competing directly against them is significantly higher and is not fully offset by the increase in revenue of having a better match. The matching function associated to the unique competitive solution no longer exhibits strict sorting on productivity and income.

In the third case, differences in income between households are small, so that the revenue gain of enrolling a richer parent is modest. In this case, even the most productive profit-maximizing prefers to avoid competition with the worst non-profit school, as the revenue gain is too small to justify the cost. Just as in the case in which tuition payments were perfectly homogeneous, there is complete segmentation between non-profit and profit-maximizing schools, with non-profit schools providing strictly higher qualities and enrolling high income students while profit-maximizing schools choose to provide lower qualities and serve low income households.

Case 1: Positive sorting on productivity and income			
Productivity		Income	
γ_1 (for-profit)	1	m_1	1
γ_2 (non-profit)	0.8	m_2	0.85
γ_3 (for-profit)	0.7	m_3	0.7
γ_4 (non-profit)	0.5	m_4	0.55
γ_5 (for-profit)	0.4	m_5	0.4
γ_6 (non-profit)	0.2	m_6	0.25
γ_7 (for-profit)	0.1		
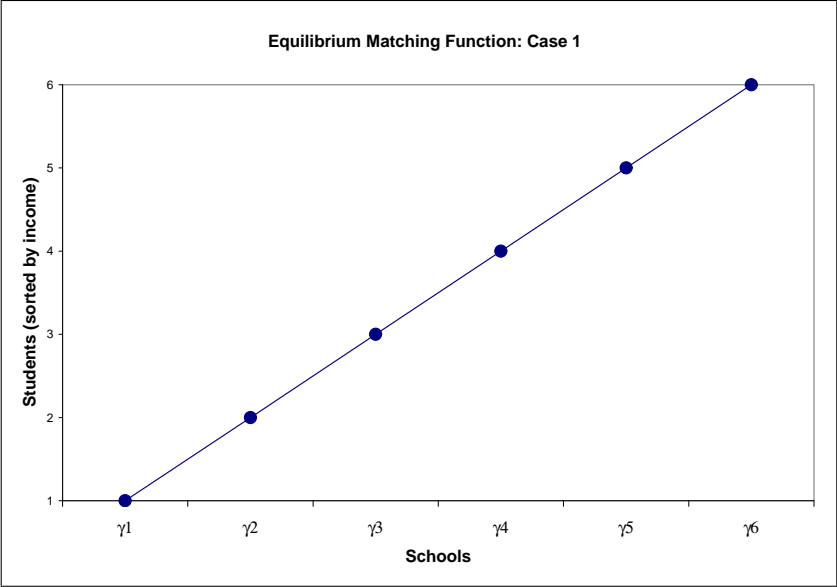
Figure 15: Matching Function, Case 1

Case 2: Partial segmentation			
Productivity		Income	
γ_1 (for-profit)	1	m_1	1
γ_2 (non-profit)	0.9	m_2	0.85
γ_3 (for-profit)	0.8	m_3	0.7
γ_4 (non-profit)	0.7	m_4	0.55
γ_5 (for-profit)	0.6	m_5	0.4
γ_6 (non-profit)	0.5	m_6	0.25
γ_7 (for-profit)	0.4		
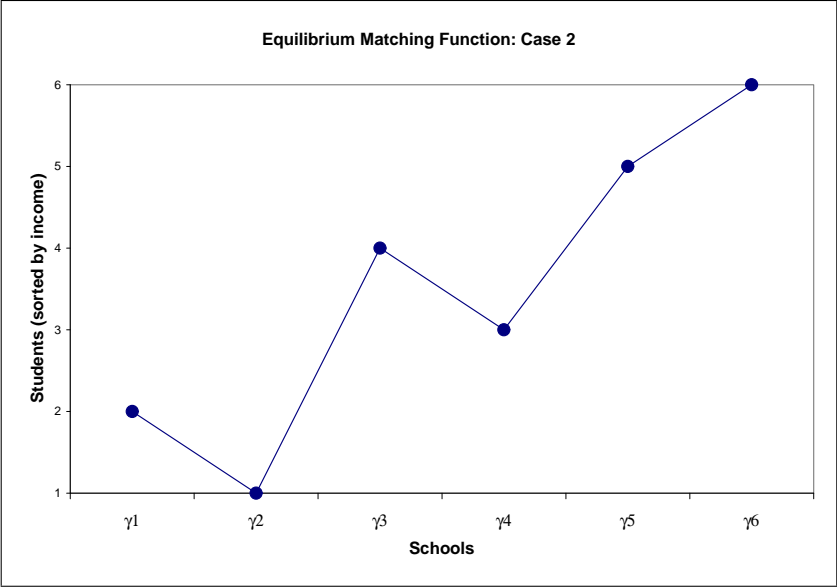
Figure 16: Matching function, Case 2

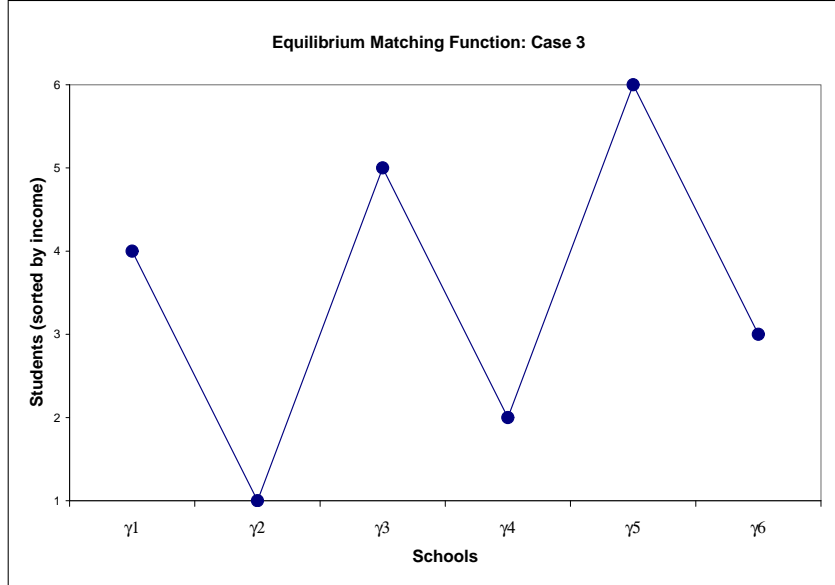| Case 3: Complete segmentation			
Productivity		Income	
γ_1 (for-profit)	1	m_1	1
γ_2 (non-profit)	0.95	m_2	0.97
γ_3 (for-profit)	0.9	m_3	0.95
γ_4 (non-profit)	0.85	m_4	0.93
γ_5 (for-profit)	0.8	m_5	0.91
γ_6 (non-profit)	0.75	m_6	0.89
γ_7 (for-profit)	0.7		

Figure 17: Matching Function, Case 3

6 Conclusions and Future Research

This paper has developed a model to analyze the behavior of an education market with heterogeneous households and schools under different funding schemes and competitive structures. The first part focused on how equilibrium matches and outcomes are affected by the way in which schools are funded, as well as the role of restrictions to entry. When schools differ in productivity, changes in the funding structure directly affects the type of competitive pressures and incentives they face, with important consequences on the overall allocation of student and schooling investments and, hence, on equilibrium school qualities and outcomes. The second section of the paper focuses on the competition between for-profit and non-profit schools, highlighting their different response to changes in competitive incentives and the policy regime.

There are several extensions that can be done on this basic framework, which are part of my future research agenda. The model so far deals basically with steady states, and has not taken into account market dynamics. A natural extension would be to analyze the behavior of schools in time, analyzing school entry and exit, and the effect of changes in the policy regime over such dynamics. A second set of extensions deals with providing a more general analysis of the static problem, allowing schools to choose their scale endogenously and incorporating non-academic aspects of schooling such as religion, amenities, and location.

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Appendix

Proof. Proposition 2.

For (a): Suppose that is not true. Take any school outside the set of N most productive schools, with productivity γ_{N+k}. Assume that the school is inside the market making positive profits, matched with any given student i. Take any school in the set of N most productive schools, with productivity γ_{N-j}. Assume that the school is outside the market. By definition, $\gamma_{N-j} > \gamma_{N+k}$.

The maximum quality that can be reached by school $N + k$, by fully investing the voucher, is $q_{N+k}^{ZP} = v^{\varphi} \gamma_{N+k}^{\zeta}$. School γ_{N-j} can obtain q_{N+k}^{ZP} by investing just $v \left(\frac{\gamma_{N+k}}{\gamma_{N-j}} \right)^{\frac{v}{\varphi}}$, which is strictly smaller than v. As γ_{N+k} is making positive profits, it must be investing less than the voucher, and offering a quality q_{N+k} that is strictly smaller than q_{N+k}^{ZP}. But then school γ_{N-j} can invest $v \left(\frac{\gamma_{N+k}}{\gamma_{N-j}} \right)^{\frac{v}{\varphi}}$, and offer quality q_{N+k}^{ZP}. As $q_{N+k}^{ZP} > q_{N+k}$, student i would be better off moving to school γ_{N-j}. School γ_{N-j} would be getting positive profits, and school γ_{N+k} cannot provide a better offer and get non-negative profits. Therefore, the proposed allocation cannot be an equilibrium. Only the N more productive can get strictly positive profits in equilibrium.

For (b) and (c). Take any arbitrary matching allocation between the N most productive schools and the N students, with $q^* = y_j \gamma_j = v \gamma_{N+1}$. We will show that no one has incentives to deviate.

(i) Students: If all schools provide q^*, students are exactly indifferent across all matches. Thus, in any arbitrary allocation, student i has no incentives to move to another school.

(ii) Operating schools: The profits for any given school γ_j are $\pi \left(a_i, \gamma_j, h_{i\mu(i)}^* \right) = v - v \left(\frac{\gamma_{N+1}}{\gamma_j} \right)^{\frac{v}{\varphi}}$ for $j \in \{1, .., N\}$. This is, profits are not a function of the characteristics of the match (a_i), but only of the school's productivity relative to the productivity of the fringe competitor. Schools have the same profit in any potential match.

In any equilibrium match, the revenue of school γ_j is v, and its cost $v \left(\frac{\gamma_{N+1}}{\gamma_j} \right)^{\frac{v}{\varphi}}$. What if school γ_j deviates to $q^{**} > q^*$? All students would then strictly prefer school j to all competitors. But as the school is restricted to enroll only one student, its revenue would not change, while its cost would be strictly larger. Thus, deviations above q^* are not profitable, as revenue remains unchanged while costs increase.

What if school γ_j deviates to $q^{**} < q^*$? The fringe competitor, school γ_{N+1}, could then provide $q^{**} + \varepsilon$, and leave γ_j out of the market while still making a positive profit. Thus, deviations below q^* are not profitable, as they trigger entry by the marginal school.

(iii) Schools outside the market: q^* is the maximum quality that can be supplied by the best school outside the market by investing the full voucher v. But then school γ_{N+1} would make zero profits, and thus

has no incentives to enter the market. Any other school γ_j, $\gamma_j < \gamma_{N+1}$, would get strictly negative profits at q^*.

For (d): Direct implication of (b).

For (e): From (d), $h^*_{i\mu(i)} = a_i^\alpha v^\varphi \gamma_{N+1}^\zeta$, which is increasing in a_i. From (2), $\pi\left(a_i, \gamma_j, h^*_{i\mu(i)}\right) = v\left(1 - \left(\frac{\gamma_{N+1}}{\gamma_j}\right)^{\frac{v}{\varphi}}\right)$, which is increasing in γ_j. ∎

Proof. Proposition 4.

(a) Straightforward.

(b) to (e): We can show that under the proposed qualities no school has a profitable deviation.

Take any school with productivity $\gamma_j > \gamma_{N+1}$, that in the proposed equilibrium is matched with a student with income m_j. Equilibrium profits can be written as

$$\pi_j^* = \kappa m_j - y_j^* = \kappa m_j - \kappa \left[\frac{m_N \gamma_{N+1}^{\frac{\varsigma}{\varphi}} + \sum_{i=1}^{N-j} (m_{N-i} - m_{N+1-i}) \gamma_{N+1-i}^{\frac{\varsigma}{\varphi}}}{\gamma_j^{\frac{\varsigma}{\varphi}}}\right]$$

Can the school find a profitable deviation?

Suppose the school tries to enroll the student immediately above in the income distribution, which has income m_{j-1}. The school must provide a quality that is at least as good as q_{j-1}^*, the equilibrium quality of the school in which student $m - 1$ is enrolled. Profits would then be:

$$\pi_j(m_{j-1}) = \kappa m_{j-1} - y_j^{**} = \kappa m_{j-1} - \kappa \left[\frac{m_N \gamma_{N+1}^{\frac{\varsigma}{\varphi}} + \sum_{i=1}^{N-j+1} (m_{N-i} - m_{N+1-i}) \gamma_{N+1-i}^{\frac{\varsigma}{\varphi}}}{\gamma_j^{\frac{\varsigma}{\varphi}}}\right]$$

Comparing profits, we can see that the school would get exactly the same profits in both cases, and thus has no incentives to deviate:

$$
\begin{aligned}
\pi_j^* - \pi_j(m_{j-1}) &= \\
&(\kappa m_j - \kappa m_{j-1}) + \left(\frac{q_{j-1}^*}{\gamma_j^\varsigma}\right)^{\frac{1}{\varphi}} . - \left(\frac{q_j^*}{\gamma_j^\varsigma}\right)^{\frac{1}{\varphi}} \\
&= (\kappa m_j - \kappa m_{j-1}) + \kappa \left[\frac{(m_{j-1} - m_j) \gamma_j^{\frac{\varsigma}{\varphi}}}{\gamma_j^{\frac{\varsigma}{\varphi}}}\right] \\
&= (\kappa m_j - \kappa m_{j-1}) + \kappa \left[(m_{j-1} - m_j)\right] = 0
\end{aligned}
$$

At the equilibrium qualities, the marginal revenue that the school would gain from enrolling a richer student is exactly offset by the additional cost of providing higher quality. In fact, if the school tried to go even further upwards the income distribution, and enroll the student with income m_{j-2}, its profits would actually decrease:

$$
\begin{aligned}
\pi_j^* - \pi_j(m_{j-2}) &= (\kappa m_j - \kappa m_{j-2}) + \left(\frac{q_{j-2}^*}{\gamma_j^\zeta}\right)^{\frac{1}{\varphi}} . - \left(\frac{q_j^*}{\gamma_j^\zeta}\right)^{\frac{1}{\varphi}} \\
&= (\kappa m_j - \kappa m_{j-2}) + \kappa\left[\frac{(m_{j-1} - m_j)\gamma_j^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right] + \kappa\left[\frac{(m_{j-2} - m_{j-1})\gamma_{j-1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right] \\
&= (\kappa m_j - \kappa m_{j-1}) + \kappa\left[\frac{(m_{j-1} - m_j)\gamma_j^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right] \\
&\quad + (\kappa m_{j-1} - \kappa m_{j-2}) + \kappa\left[\frac{(m_{j-2} - m_{j-1})\gamma_{j-1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right] \\
&= (\kappa m_j - \kappa m_{j-1}) + \kappa(m_{j-1} - m_j) + (\kappa m_{j-1} - \kappa m_{j-2})\left(1 - \frac{\gamma_{j-1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right) \\
&= (\kappa m_{j-1} - \kappa m_{j-2})\left(1 - \frac{\gamma_{j-1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right) > 0 \text{ as } \kappa m_{j-1} < \kappa m_{j-2} \text{ and } \frac{\gamma_{j-1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}} > 1
\end{aligned}
$$

Profits would be smaller as the quality increase between students $j-2$ and $j-1$ is implicitly priced at the average cost of school $j-1$, which is strictly smaller than the marginal cost of j. As the argument can be easily extended for any student $j-s, s > 1$, there are no profitable deviations in enrolling higher income students. What about enrolling a student with lower income? If school j tries to get student $j+1$, it has to offer at least q_{j+1}^* :

$$
\begin{aligned}
\pi_j^* - \pi_j(m_{j+1}) &= \kappa(m_j - m_{j+1}) + \left(\frac{q_{j+1}^*}{\gamma_j^\zeta}\right)^{\frac{1}{\varphi}} . - \left(\frac{q_j^*}{\gamma_j^\zeta}\right)^{\frac{1}{\varphi}} \\
&= (m_j - m_{j+1}) - \left[\frac{(m_j - m_{j+1})\gamma_{j+1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right] \\
&= (m_j - m_{j+1})\left(1 - \frac{\gamma_{j+1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}}\right) > 0 \text{ as } m_j > m_{j-1} \text{ and } \frac{\gamma_{j+1}^{\frac{\zeta}{\varphi}}}{\gamma_j^{\frac{\zeta}{\varphi}}} < 1.
\end{aligned}
$$

As the reduction in revenue is larger than cost savings, no school has incentive to move downwards the

income distribution.

Thus, at the proposed equilibrium qualities, it is true that, for any $j < N + 1$:

$$\pi_j^* = \pi_j(q_{j-1}^*)$$

$$\pi_j^* > \pi_j(q_{j-s}^*), \ s > 1$$

$$\pi_j^* > \pi_j(q_{j+s}^*), \ s \geq 1$$

There are no profitable deviations outside of the equilibrium matches.

The qualities will be sustained in equilibrium by competitive pressures and the incentives the school faces. Given any equilibrium match, an offer below $h_{i\mu(i)}^* = a_i^\alpha q_i^*$ would allow the school immediately below to make a better offer to student i and make profit. Therefore, equilibrium quality cannot be below q_i^*, as the school would lose the student. Any offer above $h_{i\mu(i)}^* = a_i^\alpha q_i^*$ only increases the cost for school γ_i, without any additional revenue. Therefore, the equilibrium quality will not be above q_i^*, as the school has no incentive to incur in that cost. ∎