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Bernardo Guimaraes, Caio Machado y Ana Elisa Pereira.

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Dynamic Network Effects with Fear of Missing Out*

Bernardo Guimaraes[†] Caio Machado[‡] Ana Elisa Pereira[§]

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Abstract

We study the dynamics of network goods, such as social media, when agents experience fear of missing out (FOMO): the consumption of a popular product generates negative externalities for those who abstain. We incorporate FOMO into a model of network externalities with timing frictions. FOMO can give rise to product market traps, where agents get stuck consuming a good in equilibrium, but would prefer it not to exist. However, such traps may still be constrained efficient. When an economy is in a trap, large restrictions on social media raise welfare, but small restrictions might lower it. We show how experimental results can be used to assess efficiency through the lens of our model, and using estimates from the empirical literature, we find that TikTok and Instagram are in an inefficient equilibrium.

Keywords: FOMO, dynamic coordination, social networks, timing frictions.

JEL Classifications: C73, D62

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[†]Sao Paulo School of Economics - FGV, Brazil, bernardo.guimaraes@fgv.br.

[‡]Instituto de Economia, Pontificia Universidad Católica de Chile, caio.machado@uc.cl.

[§]School of Business and Economics, Universidad de los Andes, Chile, apereira@uandes.cl.

1. Introduction

Experts and policymakers are increasingly concerned about the impacts of social networks on well-being. They offer valuable opportunities for interaction but also bring unintended negative effects. Why might a product that is so widely consumed be harmful? There are two possible explanations: (i) some form of irrationality leads people to make choices that are not in their best interest, or (ii) consuming the product entails negative externalities.

One important externality is the fear of missing out (FOMO). When everyone is engaged in a social network, the cost of not participating becomes high. As a result, joining or staying in a network may be individually rational but not socially optimal. This can create product market traps: agents consume a good in equilibrium even though they would prefer the good not to exist.

However, scenarios in which a social network vanishes overnight are likely to be unrealistic. Consumer inertia, inattention, and other frictions tend to make transitions away from such equilibria – if possible at all – slow and path-dependent.

This paper studies the welfare implications of network goods in a dynamic model where missing out is costly and transition dynamics unfold gradually due to those frictions.

From a positive perspective, the negative externalities associated with missing out are equivalent to positive network externalities. The normative implications, however, are different. Fear of missing out can generate networks even when it would have been better for everyone if they had never formed – we call these product market traps. Once attention frictions are taken into account, though, not all such traps are inefficient. Small restrictions on product market traps may actually reduce welfare, whereas larger interventions tend to improve it. In addition, we derive a sufficient statistic to test whether product market traps are inefficient and apply it to the experimental results of Bursztyn et al. (2024) for TikTok and Instagram. In what follows, we describe our model and main findings in more detail.

Model. The model builds on the framework of Frankel and Pauzner (2000). Agents can be in or out of the network and are subject to an attention friction: they have opportunities to revise their actions given by a Poisson clock. Their instantaneous payoffs depend on a fundamental (intrinsic network quality) and on the actions of others. Joining a network can either increase or decrease the payoffs for users, but decreases the payoff for non-users, as the more agents are in a social network, the more the non-users miss out. We assume the existence of network effects, in the sense that an agent's valuation of the network increases with the number of users.

To get a unique equilibrium as in Frankel and Pauzner (2000), we assume the fundamental is subject to Brownian shocks and then focus on the limiting case where the volatility of those shocks goes to zero. We then characterize the unique equilibrium threshold. Agents choose to consume the network good whenever the network quality is above a fundamental that is decreasing in the size of the network user base.

Fear of missing out and positive network externalities have the same effect on the equilibrium threshold: both make individuals more willing to enter the network. As either effect strengthens, joining becomes optimal at lower fundamentals. Fear of missing out, in particular, can drive adoption even when the network is initially empty, and can lead to widespread use of a welfare-reducing network.

Product market traps. As a benchmark case, we start the welfare analysis by studying the problem of a social planner that maximizes the sum of agents' utility and is not subject to the same timing (attention) frictions as agents. This unconstrained planner can choose how many agents are in the network at each date.

A product market trap is defined as a situation where agents choose to be in the network when hit by the Poisson clock, but the unconstrained planner would like everyone to be out of it – which implies that everyone would be better off if the network vanished. If FOMO is strong enough, there is always a region where product market traps exist.

Constrained efficiency. We then study the problem of a constrained social planner that is subject to the same timing frictions as agents. The planner can choose whether agents hit by the Poisson shock consume the network good or not.

We show that depending on the type and magnitude of externalities faced by users and non-users (FOMO), the economy can feature either excessive entry (stay) or exit from the network. If users have preferences for exclusion – i.e., their utility of being in the network decreases as more agents use it – or if FOMO is large relative to positive consumption externalities, the economy is subject to inefficient entry: there is a region in which agents remain in a network, but constrained efficiency requires them to leave. If there are sufficiently large positive consumption externalities relative to FOMO, the result reverts.

Intuitively, in the presence of FOMO, transitioning out of a network is costly for the first ones to leave, as they miss out. Even if this transition is good in the long run, no one wants to be the first to start it – as when making decisions agents only care about what happens until their next chance to revise their decision – and hence agents get stuck consuming the good. This pushes the economy towards inefficient entry.

However, there is another force that can push in the opposite direction. When deciding whether to leave a large network, agents do not internalize the effect on users. If consumption externalities are positive, this pushes towards inefficient exit; if those consuming a good cause a negative externality on users (preferences for exclusion), this pushes towards inefficient entry. Hence, for the case of positive consumption externalities, the direction of the inefficiency depends on the strength of FOMO relative to that of externalities on users – and on the extent to which agents' horizon differs from the planner's horizon, which is tied to timing frictions.

Are product market traps (constrained) inefficient? Is a gradual transition away from a product market trap desirable? In order to understand this, we ask whether product market traps are always constrained inefficient and show that the answer is no. In some cases, agents would all be better off if the network ceased to exist, but it is inefficient to induce a movement in this direction whenever agents get the chance to choose. We establish conditions for this to happen.

Whenever there is a region of the state space where product market traps exist, there is also a region where they are constrained efficient. Perhaps surprisingly, we show that if positive consumption externalities are stronger than FOMO, then all product market traps are constrained efficient. Whenever a product market trap happens in that case, the constrained planner would not want to take the economy out of it. If there are preferences for exclusion or positive consumption externalities are weaker than FOMO, then whether product market traps are efficient depends on the state.

Intuitively, the concept of constrained efficiency considers the costs associated with a sluggish transition out of a network, which can be important given that agents experience FOMO during that period. By definition, the concept of product market traps ignores the FOMO experienced during the transition: a product market trap happens when agents say they would like a good not to exist (all of a sudden).

Relation to experimental evidence. Experimental research has measured the value attributed to social networks under the status quo and in counterfactual scenarios. We show how that evidence can be mapped into our model and what the existing empirical studies imply about product market traps and constrained efficiency.

The condition for constrained efficiency depends on the degree of timing frictions, which experimental papers typically do not observe or estimate. To circumvent this obstacle, we provide a necessary and sufficient condition for a market trap to be inefficient regardless of the degree of timing frictions. This condition implies that when the network has no value in the absence of users, constrained inefficiency arises only if agents are willing to pay more to eliminate the network entirely than they would require to give up their own access while others continue to participate.

We then feed our model with the experimental results of the deactivation study of Bursztyn et al. (2024) for TikTok and Instagram. Roughly, Bursztyn et al. (2024) estimate three statistics regarding TikTok and Instagram users in their experiment: (i) consumers' valuation of using the social media with the current number of users (individual consumer surplus); (ii) consumers' valuation of using social media conditional on other users participating in a large-scale deactivation program; (iii) consumers' valuation of not implementing a large-scale deactivation in which one must be outside of social media together with their peers (product market surplus). Among other things, they show that Instagram and TikTok are product market traps in their setting, as they estimate a negative product market surplus for both platforms.

We show how we can interpret their results through the lens of our model to recover key parameters from their data. This allows us to check whether our theoretical condition for a product market trap to be inefficient holds using the three statistics they compute.

For TikTok, we show that the condition for a product market trap to be inefficient for any degree of timing frictions holds, regardless of how large agents expect the deactivation study to be. For Instagram, it holds under mild assumptions about how many people would comply with a large-scale deactivation and the amount of timing frictions. Combining our model with their empirical results leads to the conclusion that Instagram's and TikTok's traps are constrained inefficient.

Time restrictions. We then ask whether partially restricting the time users of social media can spend on those platforms can increase welfare when the economy is stuck in a product market trap. This can capture, for instance, the ban of cell phones in schools, or large-scale restrictions implemented by national authorities.

If the economy is in a product market trap, then large restrictions improve welfare. However, if the authority lacks the instruments to enforce a sufficiently large time restriction, then imposing a (small) restriction may or may not be beneficial – depending on the state and parameters – even when the economy is in a product market trap.

Although all agents would prefer to live in a world without the network when in a product market trap, small restrictions cannot sufficiently approach that benchmark. If agents decide to continue using the network after a small restriction is imposed, they experience significant FOMO during the hours they are forced not to use social media, which can render small restrictions suboptimal.

We characterize a necessary and sufficient condition for *any* time restriction – no matter how small – to be welfare improving. We show how this condition can be verified using the experimental results of Bursztyn et al. (2024) alone and show that it holds for TikTok and holds for Instagram under the (mild) assumptions needed for Instagram traps to be constrained inefficient. Hence, our results support the idea that even incremental time restrictions would be welfare improving for the subjects of their study.

Related literature. Social exclusion activates brain regions also associated with physical pain (Eisenberger et al., 2003).[1] A body of psychological literature has documented the connection between fear of missing out and social media use, and

[1] For a review of the connections between physical and social pain in the brain and nervous system, see Eisenberger (2012).

examined its impact on well-being.[2] An empirical literature in economics has also evaluated the value people attribute to social media and its relation to mental health, addiction and self-control. Examples include Allcott et al. (2020), Mosquera et al. (2020), Allcott et al. (2022), Braghieri et al. (2022), Brynjolfsson et al. (2023), and Bursztyn et al. (2024). Aridor et al. (2024) present a survey of this fast-growing literature.

Among those papers, Bursztyn et al. (2024) is the closest to ours. They conduct field experiments to estimate the importance of FOMO for TikTok and Instagram. They emphasize that knowing the value individuals assign to a social network under the status quo is not sufficient for welfare analysis when social interaction effects are significant – a point also made, in a different context, by Bhattacharya et al. (2024).

Our paper is also related to the literature on negative consumption externalities. The core idea and policy implications of positional externalities are developed in Frank (1985, 2005, 2008). On the theoretical side, Pesendorfer (1995) develops a model of fashion cycles and shows that, under certain conditions, banning fashion can make consumers better off. Empirically, Bursztyn et al. (2018) show that platinum credit cards in Indonesia function as status goods and generate positional externalities. Luttmer (2005) and Bottan and Perez-Truglia (2022) provide evidence that individuals care about relative income and that having richer neighbors reduces their utility.

Our analysis also connects with the literature on social norms, in which individuals often comply with a norm because deviation entails a private cost such as reputational loss or social disapproval. Depending on the context, norms may sustain inefficient equilibria, perpetuating socially undesirable behavior, or may instead promote cooperation, trust, or other forms of prosocial behavior that help achieve efficient outcomes (see, e.g., Akerlof, 1980, Kandori, 1992, Bénabou and Tirole, 2006, Acemoglu and Jackson, 2017). FOMO can also be interpreted as a psychological cost of not complying with some social norm of activity or consumption. Higher adhesion to the norm increases the cost for non-compliers, creating a coordination motive.

Several papers have employed the framework in Frankel and Pauzner (2000) to

[2] Examples include Przybylski et al. (2013), Stead and Bibby (2017), and Wolniewicz et al. (2018). For a survey, see, e.g., Elhai et al. (2020).

study coordination dynamics in a variety of economic settings.[3] Closest to this paper, Guimaraes and Pereira (2016) analyze the welfare implications in a model with two competing networks. The distinctive feature of our paper is the negative externality on outsiders (fear of missing out). In our model, agents are ex-ante identical. Papers exploring heterogeneity in the framework of Frankel and Pauzner (2000) have shown that the basic insights about welfare from simple models also hold in environments with heterogeneous agents (Guimaraes and Pereira, 2017; Guimaraes and Jardanovski, 2022).

More broadly, the paper is related to the vast literature on network goods that followed the seminal work of Katz and Shapiro (1985).[4]

2. The Model

Time is continuous and indexed by $t \in [0, \infty)$. There is a unit mass of consumers that at each point can either consume a network good or not. One can think of this network as a social media network, such as TikTok or Instagram, although our results apply to other network goods as well.

The instantaneous payoff of an agent in the network is given by

$$u_{\text{in}}(\theta_t, n_t) = \theta_t + \gamma_{\text{in}} n_t, \tag{1}$$

where θ_t is a fundamental variable that encapsulates all aspects of network quality, n_t is the fraction of agents in the network, and γ_{in} is a constant that measures the importance of externalities for agents in the network.

The instantaneous payoff of an agent outside the network is

$$u_{\text{out}}(n_t) = \kappa - \underbrace{\gamma_{\text{out}} n_t}_{\text{FOMO}}, \tag{2}$$

where κ is a constant that captures the value assigned to time spent outside the network, and γ_{out} is a positive constant. The term $\gamma_{\text{out}} n_t$ measures the cost of missing

[3] Examples include business cycles (Guimaraes and Machado; 2018), technology adoption (Crouzet et al.; 2023) and currency trading (Plantin and Shin; 2018). See the survey in Guimaraes et al. (2020).

[4] A few examples are Caillaud and Jullien (2003), Argenziano (2008), Weyl (2010), Amir and Lazzati (2011) and Agur et al. (2022).

out (FOMO).

The instantaneous relative gain from being in the network is $\Delta u(\theta_t, n_t) = u_{\text{in}}(\theta_t, n_t) - u_{\text{out}}(n_t)$, implying that

$$\Delta u(\theta_t, n_t) = \theta_t - \kappa + (\gamma_{\text{in}} + \gamma_{\text{out}})n_t. \tag{3}$$

By being in the network, the agent gets externalities from others ($\gamma_{\text{in}} n_t$) and avoids missing out ($\gamma_{\text{out}} n_t$).

We do not restrict the sign of γ_{in}, allowing both for positive consumption externalities ($\gamma_{\text{in}} > 0$) and preferences for exclusion ($\gamma_{\text{in}} < 0$). However, we assume throughout the paper that there are positive network effects ($\gamma_{\text{in}} + \gamma_{\text{out}} > 0$), meaning that consumers' instantaneous valuation of the good (Δu) is increasing in the number of agents also consuming it.

Making rational decisions is costly, so people do not think whether they should join or quit a social network all the time. Sometimes, they stop to think about their choices. We capture that in the model by assuming that agents cannot choose their actions at every period. They receive opportunities to choose between being in or out of the network according to a Poisson clock (independent across agents) with arrival rate $\delta > 0$. Once they choose an action, they are locked in the chosen action until the Poisson shock hits again. We refer to $1/\delta$ as the degree of timing frictions, as it reflects the expected time agents are locked in a given action. We also refer to the action chosen by agents at date t by $a_t \in \{0, 1\}$, where $a_t = 1$ and $a_t = 0$ refer to the actions of choosing to be in or out of the network, respectively.

The fundamental θ_t is subject to stochastic shocks:

$$d\theta_t = \sigma dZ_t,$$

where dZ_t is a standard Brownian motion and $\sigma > 0$ is the volatility.

The discounted expected gain of choosing 1 instead of 0 at some date τ is given by

$$V_\tau = \int_\tau^\infty e^{-(\rho+\delta)(t-\tau)} \mathbb{E}_\tau[\Delta u(\theta_t, n_t)]\, dt,$$

where $\rho > 0$ is the discount rate of agents. An agent's decision affects her payoffs until she gets selected by the Poisson process again. Hence, agents effectively discount the future at rate $\rho + \delta$, since the probability of not being hit by the Poisson shock in an interval of length $(t - \tau)$ is $e^{-\delta(t-\tau)}$. They choose 1 if $V_\tau > 0$, 0 if $V_\tau < 0$ and are indifferent between the two actions if $V_\tau = 0$.

3. Decentralized Equilibrium

This model satisfies the conditions stated in Theorem 1 of Frankel and Pauzner (2000). Hence, it has a unique equilibrium characterized by a threshold $\theta^*(n_t)$. Agents with an opportunity to revise their actions choose 1 when $\theta_t > \theta^*(n_t)$ and 0 when $\theta_t < \theta^*(n_t)$. For every $n_\tau \in [0, 1]$, $\theta^*(n_\tau)$ must solve:

$$\int_\tau^\infty e^{-(\rho+\delta)(t-\tau)} \mathbb{E}[\Delta u(\theta_t, n_t) | \theta^*, \theta^*(n_\tau), n_\tau] \, dt = 0, \tag{4}$$

where the operator $\mathbb{E}\left[\cdot | \tilde{\theta}, \theta_\tau, n_\tau\right]$ denotes agents' expectation when the current state is (θ_τ, n_τ) and they expect others to play according to the threshold $\tilde{\theta}$. In other words, agents must be indifferent between both actions when choosing on the equilibrium threshold.

Throughout the paper, we will focus on the limiting case where $\sigma \to 0$, as in that case we can get a closed-form solution for the equilibrium threshold. Whenever we say that the equilibrium is characterized by a threshold, we mean that action 1 (0) is chosen when θ is above (below) that threshold.

Proposition 1. *As $\sigma \to 0$, the equilibrium threshold is given by*

$$\theta^*(n_t) = \kappa - \frac{\delta(\gamma_{out} + \gamma_{in})}{\rho + 2\delta} - \frac{\rho(\gamma_{out} + \gamma_{in})}{\rho + 2\delta} n_t. \tag{5}$$

The equilibrium is depicted in Figure 1. When the economy is to the right of the equilibrium threshold, agents join the network and n_t moves towards one. When it is to the left of the threshold, agents leave it and n_t moves towards zero. Whenever n_t is constant and equals zero or one, we say that the economy in steady state.

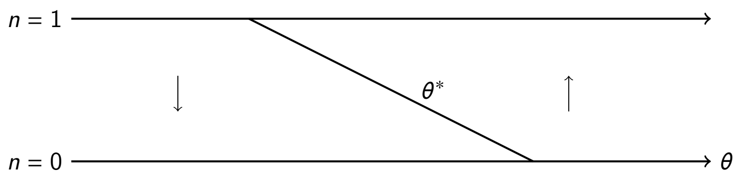

Figure 1: Equilibrium threshold θ^*.

Notes. Arrows indicate direction in which n_t moves in each region.

3.1. The Effect of FOMO

Proposition 1 shows that the effect of γ_{out} and γ_{in} on the equilibrium threshold are exactly the same. It does not matter whether agents are driven to the network by the desire to join friends or by the fear of missing out. As shown in Figure 2, an increase in γ_{out} or γ_{in} shifts the decentralized equilibrium threshold to the left (joining the network becomes more attractive for any n) and θ^* gets even smaller for larger n (since the fear of missing out and the positive network effects depend on n).

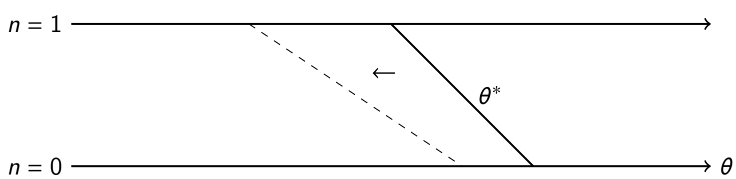

Figure 2: Effect of an increase in γ_{out} or γ_{in} on the equilibrium threshold.

Fear of missing out makes agents more prone to joining a network even if there is no one currently there, out of the expectation that the network might pick up in the future, before they have the chance to reoptimize, and they will miss out.

An arbitrarily large γ_{out} shifts the threshold arbitrarily far to the left, implying that even when the network is fundamentally uninteresting (i.e., θ is negative) and there are no complementarities (i.e., γ_{in} is zero or negative), fear of missing out alone can induce universal adoption.

4. The Planner's Problem

Now consider a social planner that maximizes the discounted sum of individual agents' payoffs (which depend on others' actions as well):

$$\mathbb{E}_\tau\left[\int_\tau^\infty e^{-\rho(t-\tau)} W(\theta_t, n_t)\, dt\right], \tag{6}$$

where

$$W(\theta, n) = n u_{\text{in}}(\theta, n) + (1-n) u_{\text{out}}(n). \tag{7}$$

We define the *constrained efficient* equilibrium as the solution to the planner's problem subject to the same timing frictions faced by agents, and the *unconstrained efficient* equilibrium as the solution in the absence of such frictions. The key distinction is that the unconstrained planner ignores transition costs. In what follows, we compare the decentralized equilibrium with the constrained and unconstrained efficient outcomes.

4.1. Unconstrained Efficiency and Product Market Traps

Due to FOMO, agents may choose to consume a network good in equilibrium, but they would all be better off if the product ceased to exist, which we refer to as a *product market trap*. Formally, we can define a product market trap in our setting as follows:

Definition. The economy is in a **product market trap** at date τ if, the unconstrained social planner would like to choose $n_t = 0$, but agents choose to be in the network ($a_t = 1$). Moreover, if the economy in a product market trap with $n_\tau = 1$, we say that the economy is in a **steady-state** product market trap.

The definition above implies that if the economy is in a product market trap in steady state, everyone would be better off if the network did not exist.

As a benchmark, it is useful to characterize under which circumstances such product market traps arise in our setting. To do so, we must compare the solution of the unconstrained planner to the decentralized equilibrium. Our equilibrium results focus on the case where $\sigma \to 0$, which for the purpose of computing the planner's solution is equivalent to $\sigma = 0$, since the planner's objective in (6) is continuous in σ for any path of n_t.

Replacing (1) and (2) in (7) we get

$$W(\theta, n) = \kappa + (\theta - \kappa - \gamma_{\text{out}})n + (\gamma_{\text{out}} + \gamma_{\text{in}})n^2. \tag{8}$$

Note that $W(\theta, n)$ is strictly convex in n, and therefore the unconstrained planner will either choose $n_t = 1$ or $n_t = 0$ at all dates. Hence, the planner chooses $n = 0$ forever whenever

$$\theta < \kappa - \gamma_{\text{in}} \equiv \theta^U, \tag{9}$$

and chooses $n = 1$ whenever $\theta > \theta^U$. In the absence of timing frictions, the planner does not take into account the fear of missing out, as γ_{out} does not affect θ^U. The planner is basically comparing a situation with all agents out of the network ($n_t = 0$, so the cost of missing out is zero) and a situation with all agents in ($n_1 = 1$, so nobody is missing out).

A product market trap is possible in some region of the state-space whenever $\theta^*(1) < \theta^U$, which after some algebra yields:

$$\gamma_{\text{out}} > \frac{\delta}{\rho + \delta}\gamma_{\text{in}}. \tag{10}$$

For instance, if that condition holds, then if $n_0 = 1$ and $\theta \in \left(\theta^*(1), \theta^U\right)$, the economy remains stuck in an equilibrium with $n = 1$ although everyone would be better off if all agents moved instantaneously out of the network. This is illustrated in Figure 3.

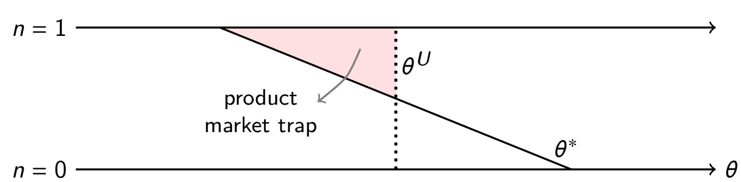

Figure 3: Product market traps.

Notes. The figure shows the equilibrium threshold θ^* and unconstrained planner's threshold θ^U when condition (10) holds.

Product market traps can only arise if there is FOMO, that is, if $\gamma_{\text{out}} > 0$ – if $\gamma_{\text{out}} = 0$, (10) can never be satisfied. When there are preferences for exclusion ($\gamma_{\text{in}} < 0$) and FOMO, that condition is automatically satisfied. With positive consumption externalities ($\gamma_{\text{in}} > 0$), the higher γ_{out}, the easier it is for the condition for product market traps to be satisfied. With enough FOMO, agents may individually find it optimal to join a network they would like not to exist because they fear facing the disutility of missing out.

However, moving all agents at once out of a network may not be a viable option. What happens if the planner is subject to timing frictions as the agents are? Are product market traps constrained inefficient?

4.2. Constrained Efficiency

A constrained planner subject to timing frictions chooses, at each date t, the proportion $\phi_t \in [0, 1]$ of agents that will pick action 1 (among those who receive a chance to switch actions).

We solve the planner's problem following the method proposed by Guimaraes et al. (2020). The planner's solution is also determined by a threshold. For agents that get the opportunity to revise their actions, the planner chooses 1 when $\theta_t > \theta^P(n_t)$ and 0 when $\theta_t < \theta^P(n_t)$. For every $n_\tau \in [0, 1]$, $\theta^P(n_\tau)$ must solve:

$$\int_\tau^\infty e^{-(\rho+\delta)(t-\tau)} \mathbb{E}\left[\frac{\partial W(\theta_t, n_t)}{\partial n} \middle| \theta^P, \theta^P(n_\tau), n_\tau \right] dt = 0, \tag{11}$$

The difference between (11) and (4) is that the planner's threshold considers $\partial W(\theta_t, n_t)/\partial n$ instead of $\Delta u(\theta_t, n_t)$. Hence, the solution to the constrained planner's problem is equivalent to the solution of the game where the relative gain of being in network 1 is

$$\frac{\partial W(\theta_t, n_t)}{\partial n_t} = \Delta u(\theta_t, n_t) + \underbrace{[\gamma_{\text{in}} n_t - \gamma_{\text{out}}(1 - n_t)]}_{\text{Externalities}} \tag{12}$$

An agent that joins a network generates two externalities: she raises the utility of the n_t agents in the network (second term of (12)) but reduces the utility of the $(1 - n_t)$ agents that are missing out (third term).

Proposition 2 characterizes the constrained efficient outcome.

Proposition 2. *As $\sigma \to 0$, the constrained efficient equilibrium is characterized by a* *threshold given by*

$$\theta^C(n_t) = \kappa + \gamma_{out} - \frac{2\delta(\gamma_{out} + \gamma_{in})}{\rho + 2\delta} - \frac{2\rho(\gamma_{out} + \gamma_{in})}{\rho + 2\delta} n_t. \tag{13}$$

The proof uses the fact that the solution of the constrained planner is equal to the equilibrium of a game where the relative payoff of being in the network is given by (12).

4.2.1. The Effect of FOMO

We had shown that γ_{out} and γ_{in} affect the equilibrium threshold in the same way. As can be seen in Proposition 2, this does not hold for the planner's solution. The parameters γ_{out} and γ_{in} have the same effect on the slope of the planner's threshold but shift the threshold in different ways.

The constrained efficient threshold is less inclined than the equilibrium threshold. Intuitively, the planner assigns a higher weight to the externalities. Agents' decisions consider the fundamental θ and the (positive and negative) externalities they receive from the others, while the planner takes into account everything agents value, but also internalizes the (positive and negative) externalities an agent generates on others. A similar point is made by Guimaraes and Pereira (2016) in a model with two competing networks and a coordination motive only.[5]

Inspecting the expressions for θ^* and θ^C in those propositions also shows that the effect of γ_{in} on the planner's and equilibrium thresholds are qualitatively similar (though the effect on the planner's threshold is larger). However, γ_{out} affects the planner's threshold in a different way.

Note that $\theta^C(1/2) = \kappa - \gamma_{\text{in}} = \theta^U$: For $n = 1/2$, the planner's threshold coincides with what it would be in the absence of timing frictions (equation 9). Hence it does not depend on γ_{out}.

Increases in γ_{out} rotate the (constrained) planner's threshold counter-clockwise around the point $\left(\theta^U, 1/2\right)$, as shown in Figure 4. When $n = 1$, reducing n implies some people will miss out, which is very costly at that point. But when $n = 0$, increasing n will have a negative effect on those who will be missing out, and they are many.

[5] Angeli (2024) extends this reasoning to a set where agents procrastinate.

Notably, $\theta^C(0)$ jumps to the right (while $\theta^*(0)$ jumps to the left) following an increase in γ_{out}. The planner becomes more reluctant to let people in a nascent network as FOMO increases, since it internalizes the negative externalities caused on the large mass $1-n$ currently out. However, once they are in, the planner is more reluctant to let them out, in the sense that lower fundamentals are required for emptying the network to be efficient.

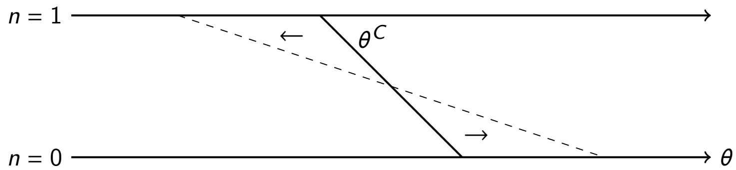

Figure 4: Effect of an increase in γ_{out} on the planner's threshold.

One implication is that a social planner would be particularly keen to prevent the rise of a network when γ_{out} is relevant. In practice, it may be difficult to assess the negative externalities of networks that do not yet exist, but whenever γ_{out} seems large, the planner would require strong fundamentals before allowing the network to grow.

4.2.2. Inefficient Entry and Exit

If agents consume the network good (i.e., choose $a_t = 1$) and the constrained planner would like them not to (i.e., to choose $a_t = 0$), we say there is inefficient entry. Similarly, we say there is inefficient exit when agents choose not to consume the network good when the constrained social planner would like them to do so.

Under which conditions can the economy feature inefficient entry or exit? To answer that question, it is useful to compute the difference between the constrained and the decentralized thresholds:

$$\tilde{\Delta}(n_0) \equiv \theta^C(n_0) - \theta^*(n_0) = \gamma_{\text{out}} - \frac{\delta(\gamma_{\text{out}} + \gamma_{\text{in}})}{\rho + 2\delta} - \frac{\rho(\gamma_{\text{out}} + \gamma_{\text{in}})}{\rho + 2\delta} n_0. \tag{14}$$

If $\tilde{\Delta}(1) > 0$, the constrained planner's threshold lies to the right of its decentralized counterpart, and therefore, there is a region of the state-space where agents stay in a network in equilibrium, but the planner would like them to move out of it (inefficient entry).[6] Likewise, if $\tilde{\Delta}(0) \leq 0$, there is a region of the state-space with inefficient exit. The next proposition shows when each case can happen.

Proposition 3. *Comparing the decentralized equilibrium and the constrained efficient solution leads to the following inefficiencies:*

1. *If $\gamma_{out} \geq \frac{\rho+\delta}{\delta}\gamma_{in}$, there is only a region with inefficient entry;*
2. *If $\gamma_{out} \leq \frac{\delta}{\rho+\delta}\gamma_{in}$, there is only a region with inefficient exit;*
3. *If $\frac{\delta}{\rho+\delta}\gamma_{in} < \gamma_{out} < \frac{\rho+\delta}{\delta}\gamma_{in}$, there is a region with inefficient entry and a region with inefficient exit.*

The condition for inefficient entry only always holds if agents have preferences for exclusion ($\gamma_{\text{in}} < 0$). Figure 5 shows a case with inefficient entry. In the shaded and dotted area, agents choose to enter or remain in the network and the economy converges to a situation where everyone is in the network ($n = 1$). The planner, however, would like agents to move out of the network at all dates, and hence the economy is stuck forever in a situation of inefficient entry. In the shaded but not dotted region, there is also inefficient entry, but eventually, as n_t moves up, agents' decisions coincide with the efficient allocation. Hence, the economy does not get trapped in an inefficient network in the long run, but it does build a large network that should not have been built in the first place.

To build intuition, consider $\gamma_{\text{in}} > 0$ and assume that condition for inefficient entry holds, so that FOMO is strong relative to positive consumption externalities. Assume the economy in point A in Figure 5, so that everyone is using the network ($n = 1$) but the constrained planner would like everyone to leave.

A transition out of the network requires the first agents to experience significant FOMO, but there are long-run gains. Therefore, if the planner's decision concerned only a single agent, FOMO would discourage moving that agent out. However, the planner considers transitioning all agents out gradually, and values the long run gains.

Agents, however, do not care about the long run, they only care about what happens until their next opportunity to change actions. They effectively discount the future at

[6] Since the planner's threshold is more inclined than the agents' one, if the planner's threshold is to the right of the decentralized one at $n = 1$, it lies to the right of it for any $n \in [0, 1]$.

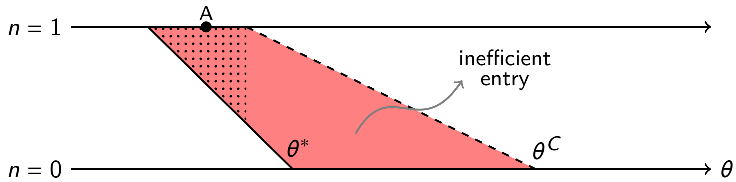

Figure 5: Equilibrium threshold θ^* and constrained efficient threshold θ^C when γ_{out} is large. The shaded area features inefficient entry and in the dotted area this inefficiency persists in the long run.

rate $\rho + \delta$. Hence they are unable to coordinate in a transition out of the network – no one wants to be first to leave the network – and they get trapped in it. The constrained planner avoids that outcome, as it discounts the future at rate ρ.[7]

Figure 6 shows what happens in a case where γ_{out} is sufficiently lower than γ_{in}, so that the two thresholds do not intersect.

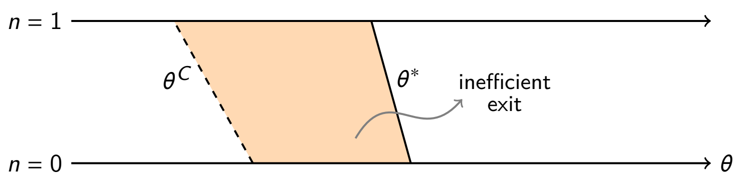

Figure 6: Equilibrium threshold θ^* and constrained efficient threshold θ^C with γ_{out} sufficiently lower than γ_{in}.

Note that a necessary condition for this condition to hold is $\gamma_{\text{in}} > 0$. In that case, the economy suffers purely from inefficient exit: the stream of positive externalities each agent that enters generates on the other members of the network more than compensates the extra negative externalities caused on outsiders due to FOMO. The planner internalizes that, and is thus more prone to inducing entry.

For completeness, Figure 7 illustrates an intermediate case of parameters, in which the planner's and agents' thresholds cross, creating regions of inefficient entry and exit. In both cases, as n moves, eventually the planner's decisions coincides with agents'.

Inspection of Proposition 3 and the condition in (10) shows that whenever for

[7] If $\delta \to 0$ (large timing frictions), agents' discounting approaches the planner's discounting. In that case, the condition for inefficient entry is not satisfied even for very large FOMO, and the gap between the planner's and agents' thresholds switches sign at $n = 1$ (still assuming $\gamma_{\text{in}} > 0$). In this case, the dynamic inefficiencies highlighted are small relative to the fact that agents do not internalize the effect of their absence on users.

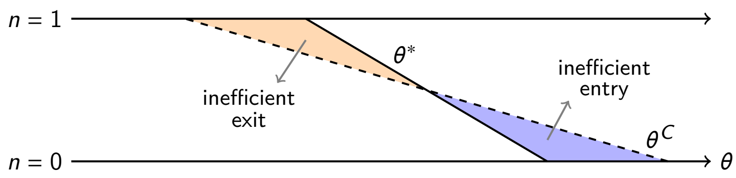

Figure 7: Equilibrium threshold θ^* and constrained efficient threshold θ^C with $\gamma_{\text{out}} = \gamma_{\text{in}}$.

product market traps are possible, there is a region with inefficient entry. However, product market traps might coexist with a region of inefficient exit. The next section helps to understand the difference between both notions of efficiency.

4.3. Are Product Market Traps Constrained Inefficient?

As the next proposition shows, product market traps are not always constrained inefficient:

Proposition 4. *In the model with $\sigma \to 0$, product traps might be constrained efficient. In particular,*

1. *If $\frac{\delta}{\rho+\delta}\gamma_{in} < \gamma_{out} \leq \gamma_{in}$, product market traps are always constrained efficient;*
2. *If $\gamma_{out} > \gamma_{in}$, there is a region in which product market traps are constrained efficient and a region of the state-space in which they are constrained inefficient.*

Hence, if there are positive consumption externalities ($\gamma_{\text{in}} > 0$) and those are stronger than FOMO ($\gamma_{\text{in}} \geq \gamma_{\text{out}}$), product market traps are certainly constrained efficient. If there are preferences for exclusion ($\gamma_{\text{in}} < 0$) or positive consumption externalities with large FOMO ($\gamma_{\text{out}} > \gamma_{\text{in}} > 0$), then whether or not product market traps are efficient depend on the state.

Therefore, in some cases, agents may think they would all be better off if the network ceased to exist, but it may still be inefficient to induce a movement towards emptying the network. Intuitively, the constrained planner takes into account the timing frictions and that if it induces a movement outside of the network, many agents will experience FOMO during the transition. Hence, agents may say they would prefer to live in a world where the network did not exist; but yet, they would dislike a policy that induces everyone to move out of the network when they get a chance to reoptimize.

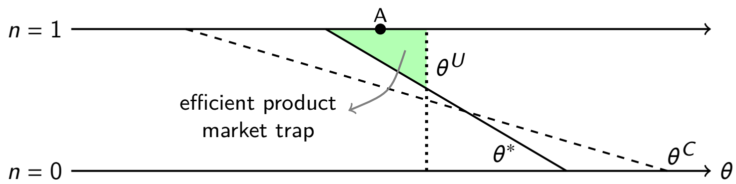

Figure 8: Efficient product market traps for $\gamma_{\text{out}} \in \left(\frac{\delta}{\rho+\delta}\gamma_{\text{in}}, \gamma_{\text{in}}\right)$.

Figure 8 illustrates a case where $\gamma_{\text{in}} > 0$ and FOMO is a large enough to create product market traps but is still below γ_{in}. Consider point A in that figure, so that everyone is in the network. If possible, the (unconstrained) planner would move everyone immediately to $n = 0$, and all agents would be better off. However, not only the economy is stuck at a point with lower welfare, but because of the timing frictions, a movement towards $n = 0$ would actually be worse than status quo. In fact, when the economy hits $\theta^*(1)$, agents will leave the network and that will be inefficient owing to agents not internalizing the externalities on others during the transition period.

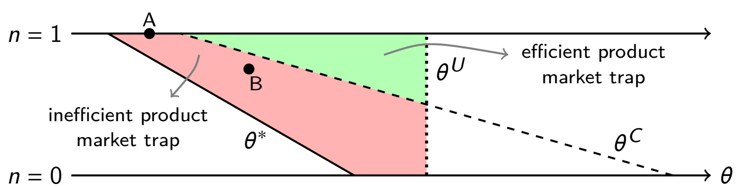

Figure 9: Efficient and inefficient product market traps for $\gamma_{\text{out}} > \frac{\rho+\delta}{\delta}\gamma_{\text{in}}$, which implies that $\theta^*(0) < \theta^U$.

If FOMO is sufficiently large, $\gamma_{\text{out}} > \gamma_{\text{in}}$, product markets traps may be inefficient, and such a case is illustrated in Figure 9. In the left-most shaded region, the economy is stuck in a product market that is inefficient, as both the constrained and unconstrained planners would prefer agents to move out of the network. In point A, the inefficiency is persistent. In point B, eventually the network grows enough, so that it becomes constrained inefficient to move agents out of the network once the network becomes large enough.

For completeness, Figure 10 shows a case where both inefficient product market traps and inefficient exit are possible. In that case, inefficient product market traps can only exist if the network is not so large, as for large n positive consumption externalities

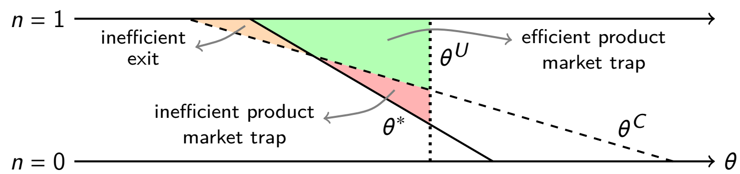

Figure 10: Coexistence of efficient product market traps with inefficient exit.

Notes. It is assumed that $\gamma_{\text{in}} < \gamma_{\text{out}} < \frac{\rho+\delta}{\delta}\gamma_{\text{in}}$, which implies that $\theta^*(0) > \theta^U$ and $\theta^*(1) > \theta^C(1)$.

are large enough for the economy to feature inefficient exit instead.

4.4. How Would Heterogeneity Affect Results?

We have shown that fear of missing out itself can give rise to product market traps. Heterogeneity is not needed to take the economy to an inefficient situation. Even in a world of ex-ante identical agents and no sizable shocks, we may build an inefficient network, one we would prefer not to exist.

However, preferences are likely heterogeneous in reality. Guimaraes and Pereira (2017) and Guimaraes and Jardanovski (2022) have studied how heterogeneity affects the decentralized equilibrium and the planner's solution in the framework of Frankel and Pauzner (2000). Different types will be associated with different equilibrium thresholds, but strategic complementarities will push thresholds close to each other. In the limit $\sigma \to 0$, thresholds of different types might coincide for intermediate values of n_t. Fear of missing out would affect agents' thresholds as shown in Figures 2 and 4.

Since mapping theoretical results with heterogeneous preferences to existing empirical evidence is not straightforward, we do not pursue this extension here and instead focus on the case of homogeneous agents.

5. Relation to Experimental Evidence

Experimental research can assess the value individuals place on participating in a social network under the status quo. This is typically done by eliciting how much money a person would require to forgo access to the network for a given period. Several studies have done so.[8] This information is necessary but insufficient for welfare analysis, since it does not reveal whether individuals value the network for its intrinsic utility (θ), the positive externalities of participation (γ_{in}), or the negative effects of exclusion (γ_{out}).

We now clarify what additional information is needed for welfare analysis and show how to use experimental data to assess efficiency. In essence, researchers must measure how individuals value the network under counterfactual scenarios in which the network size changes – not just under the status quo. This is more difficult than simply estimating willingness to pay for continued access, but some studies have taken this path. After presenting the theoretical results, we will use the empirical results from Bursztyn et al. (2024) to provide a quantitative evaluation of efficiency.

For a given value of the fundamental θ_0, define

$$\Delta(x_{\text{in}}, x_{\text{out}}) = u_{\text{in}}(\theta_0, x_{\text{in}}) - u_{\text{out}}(x_{\text{out}}) \tag{15}$$

That is, $\Delta(x_{\text{in}}, x_{\text{out}})$ represents the difference in utility flow from being in the network when x_{in} agents are participating versus being outside the network when x_{out} agents are participating, evaluated at the current level of fundamentals (θ_0). Experiments that elicit the compensation required for an agent to leave a mature network effectively estimate $\Delta(1,1)$ (assuming $n = 1$ in a mature network).

Proposition 5. *Consider an economy with $\sigma \to 0$ and suppose it is in steady state with $n = 1$ and $\theta = \theta_0$. Then, for any $\xi \in [0, 1)$:*

1. *The economy is in a product market trap if, and only if,*

$$\Delta(1, \xi) - \xi\Delta(1, 1) < 0. \tag{16}$$

2. *This product market trap is inefficient for any degree of timing frictions ($1/\delta$) if, and only if,*

$$\theta_0 - \kappa + \gamma_{out} + 2\gamma_{in} \leq 0. \tag{17}$$

[8] Examples include Allcott et al. (2020), Mosquera et al. (2020), Allcott et al. (2022), and Brynjolfsson et al. (2023).

3. In terms of experimental data, condition (17) holds if, and only if,

$$\Sigma \equiv \Delta(1,\xi) - \xi\Delta(1,1) + [\Delta(1,1) - \Delta(\xi,\xi)] \leq 0. \tag{18}$$

The first statement shows a condition for a product market trap. For $\xi = 0$, it boils down to $\Delta(1,0) < 0$, which means $u_{\text{in}}(\theta_0, 1) < u_{\text{out}}(0)$. Intuitively, we are in a product market trap if agents would prefer to shut down the network. Experiments, however, might not be able to assess a counterfactual scenario with an empty network, but might attempt to measure the value of the network in a scenario with less people ($\xi \in (0,1)$).

If $\Delta(1,\xi) < 0$, agents would surely prefer to live in a world without the network, since whenever agents choose to be in the network, $\Delta(1,1) \geq 0$, and hence (16) holds in that case. However, a product market trap is still possible with $\Delta(1,\xi) > 0$ for some $\xi > 0$. For instance, consider $\xi = 1/2$. If $\Delta(1,1/2) > 0$, agents prefer to maintain the status quo where they are in the network and everyone else is there instead of leaving the network with half of their peers. However, it still possible that if more people left in the counterfactual scenario, they would prefer it to the status quo, as they would face less FOMO in that counterfactual.

Now, suppose we know an economy is in a product market trap. Is the equilibrium constrained efficient? Theoretically, the answer depends on the degree of timing frictions ($1/\delta$), which will typically not be assessed in an experiment. The second statement of the proposition shows a necessary and sufficient condition for a product market trap to be inefficient for any value of δ.

The third statement presents the condition for an inefficient product market trap in terms of values that can be elicited through experiments. The difference between this condition in (18) and the condition for a product market trap in (16) is the term in square brackets, and is equivalent to $(1-\xi)(\gamma_{\text{in}} + \gamma_{\text{out}})$. A product market trap is more likely to be efficient when γ_{in} and γ_{out} are large, as both affect the condition in the same way. The constrained planner cares about the positive externalities that would be forgone (which scales with γ_{in}) and the negative externalities that would be incurred during the transition toward $n = 0$ (which scales with γ_{out}), and hence is not willing to induce a gradual movement out of the network if those externalities combined are large.

To get intuition, it is convenient to analyze the case $\xi = 0$ and $\Delta(0, 0) = 0$ (the network is irrelevant if nobody is there). The condition for inefficiency (for any value of δ) in the third statement becomes $-\Delta(1, 0) > \Delta(1, 1)$. Hence, constrained inefficiency requires that the amount agents are willing to pay to shut down the network is larger than what they would require to give up the network conditional on everyone else staying in.

Note that ρ does not enter the expressions in Proposition 5. Our results are sufficient to determine whether a social network is a product market trap and whether the situation is constrained inefficient for any time discount rate.

5.1. Application to TikTok and Instagram

We now use Proposition 5 to interpret the experimental results from Bursztyn et al. (2024) through the lens of our model. Their deactivation study conducted with college students is meant to estimate different dimensions of agents' payoffs.

Outcomes of experiments feature significant heterogeneity in responses, which cannot be captured by our model because all agents are ex-ante identical. So, as a first approximation, we will simply map the average responses in their experiments to the payoffs in the model, abstracting from heterogeneity in preferences.

We assume that before the experiment is conducted, all the network relevant for each participant is on TikTok and Instagram ($n = 1$), and that the only users college students care about are those in their own college. If students cared about people in their network beyond those in their college, this would translate into higher values of ξ in the counterfactual scenarios where some deactivate, and as will be shown, this would reinforce our findings. Finally, since we consider $\sigma \to 0$ in the model, we assume the fundamental is nearly constant at some value θ_0.[9]

In what follows, assume time is measured in years and define $C \equiv \left(1 - e^{-\rho/12}\right)/\rho$, which represents the utility gain of increasing flow utility in one unit for a month. The utility functions are assumed to represent willingness to pay, and hence are denominated in dollars.

[9] We are also assuming that our linear payoffs are a good approximation for agents' payoffs, as it would not be possible to capture curvature in agents' valuations from the experiment in Bursztyn et al. (2024).

The experiment of Bursztyn et al. (2024) consists of three steps.

1. *Valuation Keeping Network (VKN)*: the authors aim to estimate how much agents value the network by eliciting the monetary payment they would require to deactivate their account for a month, assuming they are the only ones in their network to do so. That is, it is a standard measure of *individual* consumer surplus, and is equal to $C\Delta(1, 1)$ in our model.

2. *Valuation Removing Network (VRN)*: they elicit the payment agents require to deactivate their social network for a month taking as given the implementation of a large-scale deactivation program. We denote by ξ the fraction of students that remain in the network in case of a large-scale deactivation. In the context of our model, this equals $C\Delta(\xi, \xi)$.

3. *Product Market Surplus (PMS)*: they estimate how much participants would require to have a large-scale deactivation program implemented for all participating students (including themselves) for a month, which is equal to $C\Delta(1, \xi)$ in our model.

In short,

$$\frac{VKN}{C} = \Delta(1, 1), \quad \frac{VRN}{C} = \Delta(\xi, \xi), \quad \frac{PMS}{C} = \Delta(1, \xi). \tag{19}$$

Moreover, using those relations we can rewrite the condition for an inefficient trap regardless of timing frictions in (18) as

$$C\Sigma = PMS + (1 - \xi)VKN - VRN \leq 0.$$

We still need to discuss what values of ξ are consistent with their experiment. Let $\hat{\xi} \equiv 1 - \xi$ denote the fraction that deactivates in a scenario where the large-scale deactivation is implemented. We cannot directly infer $\hat{\xi}$ from their experiment. However, we can set lower and upper bounds for it that are consistent with the instructions received by participants. In Step 2, they inform participants that the large-scale deactivation will take place only if they can recruit at least two thirds of students in their college. Moreover, they also tell participants that 90% of students are expected

	Bursztyn et al. (2024)			Implied parameters		
	VKN	VRN	PMS	γ_{out}	γ_{in}	$\theta_0 - \kappa$
TikTok	55.18	39.22	-23.91	1426.59	-1138.71	375.66
Instagram	47.02	37.06	-6.34	962.48	-782.83	385.76

Table 1: Mapping of results in Bursztyn et al. (2024).

Notes. Results from Bursztyn et al. (2024) represent average responses. It is assumed that $\rho = 0.05$, $\xi = 1/3$ in the right panel. The implied parameters are computed using (19) and (20).

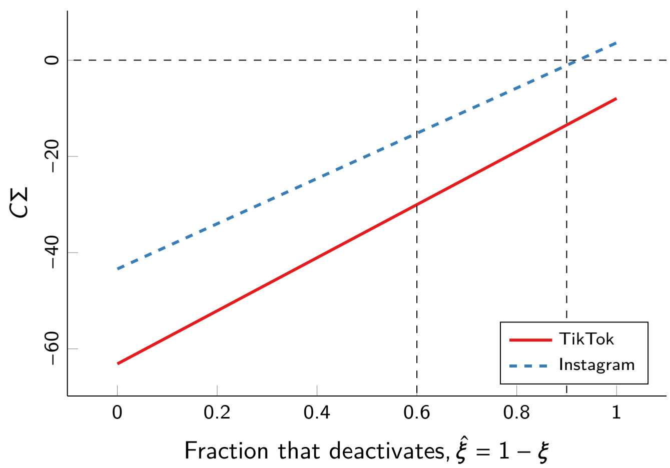

Figure 11: Sufficient condition for product market trap to be inefficient.

Notes. If $\Sigma \leq 0$ (or equivalently, $C\Sigma \leq 0$), the product market trap is constrained inefficient. The vertical lines represent the bounds 0.6 and 0.9 discussed in the text.

to comply with a possible deactivation. Hence, it makes sense to assume that students answer their survey with a value of $\hat{\xi}$ between $2/3 \times 0.9 = 0.6$ and 0.9 in mind.

The left panel of Table 1 shows the mean values of VKN, VRN and PMS estimated by the authors for TikTok and Instagram. Those are then used in Figure 11 to plot $C\Sigma$ under diferent values for the number of students (expected) to deactivate in case a large-scale deactivation program takes place, $\hat{\xi} = 1 - \xi$. The bounds 0.6 and 0.9 for $\hat{\xi}$ are represented as vertical lines.

Note that, for TikTok, regardless of $\hat{\xi}$, the condition $C\Sigma \leq 0$ holds, and hence TikTok's traps are constrained inefficient. For Instagram, it can be efficient or inefficient, depending on $\hat{\xi}$ and δ. However, within the relevant bounds for $\hat{\xi}$, Instagram traps are also constrained inefficient, regardless of timing frictions.[10]

[10]We numerically verified that even if we made $\hat{\xi} = 1$, with $\rho = 0.05$, we would need to assume that $1/\delta$ is larger than 70 to make the Instagram trap constrained efficient, which is unreasonable

As a last remark, we can also recover the parameters implied by the experimental data. Using (15) and the expressions for the utilities, we can write the following system of equations:

$$\begin{cases} \Delta(1,1) = \theta_0 - \kappa + \gamma_{\text{in}} + \gamma_{\text{out}}, \\ \Delta(\xi,\xi) = \theta_0 - \kappa + (\gamma_{\text{in}} + \gamma_{\text{out}})\xi, \\ \Delta(1,\xi) = \theta_0 - \kappa + \gamma_{\text{in}} + \gamma_{\text{out}}\xi, \end{cases}$$

which yields

$$\gamma_{\text{out}} = \frac{\Delta(1,1) - \Delta(1,\xi)}{1-\xi}\ , \ \gamma_{\text{in}} = \frac{\Delta(1,\xi) - \Delta(\xi,\xi)}{1-\xi}\ , \ \theta_0 - \kappa = \frac{\Delta(\xi,\xi) - \xi\Delta(1,1)}{1-\xi}. \quad (20)$$

The right panel of Table 1 shows the implied parameters γ_{out}, γ_{in} and $\theta_0 - \kappa$ assuming $\rho = 0.05$, $\xi = 1/3$ and using the relations in (19). As emphasized by Bursztyn et al. (2024), their results imply that agents have preference for exclusion ($\gamma_{\text{in}} < 0$). However, due to FOMO, there are positive network effects ($\gamma_{\text{out}} + \gamma_{\text{in}} > 0$).

6. Time Restrictions on Social Media Usage

An instrument often available to authorities is to restrict the time agents can spend on social media. Some national governments have considered limiting the amount of time teenagers spend on social media, with limits that can get to as little as forty daily minutes (depending on users' age) being imposed in China. Several countries and states restrict the usage of cell phones within schools, thereby reducing the amount of time students can spend on social networks. We now modify the baseline model to study those interventions.

The key difference is that we now assume that when an agent i is hit by the Poisson shock, she can choose the share of her (spare) hours $\ell_i \in [0, \overline{\ell}]$ she spends on social media, the network good. The parameter $\overline{\ell} \in [0, 1]$ is the maximum fraction of her time an agent is allowed to spend on social media. We interpret a policy that restricts social media usage as setting $\overline{\ell} < 1$.

(remember $1/\delta$ represents the expected number of years until an agent is selected by the Poisson process to reoptimize).

Once an agent chooses a ℓ_i, she is locked in that decision until the Poisson shock hits again. For each interval $[t, t+dt]$ an agent is locked in with a given ℓ_i: she spends $\ell_i dt$ random units of time using the network and $(1-\ell_i)dt$ random units of time not using it. At each date t, an agent is using the network with probability ℓ_i. To ease the exposition, we assume the tie-breaking convention that agents choose $\ell_i = 0$ whenever indifferent among any ℓ_i.

Denote by ℓ_{it} the last choice of agent i before date t. Then, the total mass of agents using the network at date t is $L_t = \int_0^1 \ell_{it}\, di$. If agent i is using the social network, we assume she gets a flow payoff of

$$\hat{u}_{\text{in}}\left(\hat{\theta}_t, L_t\right) = \hat{\theta}_t + \hat{\gamma}_{\text{in}} L_t,$$

and if the agent is not using it, she gets

$$\hat{u}_{\text{out}}(L_t) = \hat{\kappa} - \hat{\gamma}_{\text{out}} L_t.$$

As before, $\hat{\gamma}_{\text{out}} > 0$ and $\hat{\gamma}_{\text{in}} \in \mathbb{R}$ represent FOMO and consumption externalities, respectively, $\hat{\theta}_t$ is a fundamental variable that encapsulates all aspects of network quality and $\hat{\kappa}$ represents the direct utility of time spent outside the network. Moreover, $\hat{\theta}_t$ follows a Brownian motion with volatility $\hat{\sigma}$ and, as before, to get a unique equilibrium, and we focus hereafter on the limiting case where volatility vanishes, that is, $\hat{\sigma} \to 0$.

Agents maximize their discounted expected payoff. Since their choice of ℓ_i at a given date only affects their payoffs until they are selected again by the Poisson process, an agent selected at a time τ chooses ℓ_i to maximize

$$Q_\tau(\ell_i) = \int_\tau^\infty e^{-(\rho+\delta)(t-\tau)} \mathbb{E}_\tau\left[\hat{U}\left(\hat{\theta}_t, L_t, \ell_i\right)\right] dt,$$

where $\hat{U}\left(\hat{\theta}, L, \ell_i\right) = \ell_i \hat{u}_{\text{in}}\left(\hat{\theta}_t, L_t\right) + (1-\ell_i)\hat{u}_{\text{out}}(L_t)$. Note that $Q_\tau(\ell_i)$ is linear in ℓ_i, and hence, under the tie-breaking convention that agents choose $\ell_i = 0$ when indifferent, agents will either choose $\ell_i = 0$ or $\ell_i = \overline{\ell}$.[11]

Therefore, we can set our game as a binary action game as before, where action 0 is interpreted as choosing $\ell = 0$ and action 1 is interpreted as choosing $\ell_i = \overline{\ell}$. Using n_t

[11]The baseline model is thus equivalent to this extension with $\overline{\ell} = 1$.

to denote the fraction of agents locked in action 1 (as before), we have that $L_t = \overline{\ell} n_t$. Hence, the flow payoff of being locked in action 1 is

$$\begin{aligned} u_{\text{in}}\Big(\hat{\theta}_t, n_t\Big) &= \overline{\ell}\hat{u}_{\text{in}}\Big(\hat{\theta}_t, \overline{\ell} n_t\Big) + (1-\overline{\ell})\hat{u}_{\text{out}}\Big(\overline{\ell} n_t\Big) \\ &= \overline{\ell}\hat{\theta}_t + \Big(1-\overline{\ell}\Big)\hat{\kappa} + \Big[(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\overline{\ell}^2 - \overline{\ell}\hat{\gamma}_{\text{out}}\Big] n. \end{aligned}$$

The flow payoff of being locked in action 0 is

$$u_{\text{out}}(n_t) = \hat{u}_{\text{out}}\Big(\overline{\ell} n_t\Big) = \hat{\kappa} - \hat{\gamma}_{\text{out}}\overline{\ell} n_t.$$

Hence, by setting

$$\gamma_{\text{in}} = (\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\overline{\ell}^2 - \overline{\ell}\hat{\gamma}_{\text{out}}, \quad \gamma_{\text{out}} = \hat{\gamma}_{\text{out}}\overline{\ell}, \quad \theta_t = \overline{\ell}\hat{\theta}_t + \Big(1-\overline{\ell}\Big)\hat{\kappa}, \quad \kappa = \hat{\kappa}, \tag{21}$$

we make the model of Section 2 equivalent to the model with time restrictions. Note that if $\overline{\ell} = 1$ (no time restrictions), all the "hat" variables are equal to their counterparts without hats. Also, note that $\gamma_{\text{out}} + \gamma_{\text{in}} = \overline{\ell}^2(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})$, and hence the assumption of network externalities holds if, and only if, $\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}} > 0$, except for the trivial case where $\overline{\ell} = 0$. We maintain the assumption that $\gamma_{\text{out}} + \gamma_{\text{in}} > 0$ in this extension.

It is convenient to define the equilibrium threshold in terms of $\hat{\theta}_t$, which we denote by $\hat{\theta}^*(n)$. It satisfies $\theta^*(n) = \overline{\ell}\hat{\theta}^*(n) + \Big(1-\overline{\ell}\Big)\hat{\kappa}$, which after using (5) and (21), yields

$$\hat{\theta}^*(n) = \hat{\kappa} - \frac{\delta(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\overline{\ell}}{\rho + 2\delta} - \frac{\rho(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\overline{\ell}}{\rho + 2\delta} n. \tag{22}$$

That is, in equilibrium, agents choose action 1 ($\ell = \overline{\ell}$) if $\hat{\theta}_t > \hat{\theta}^*(n_t)$ and action 0 ($\ell = 0$) otherwise. It is also useful to apply the analogous procedure for the constrained planner's threshold $\theta^C(n)$, yielding the following threshold for the constrained planner in terms of $\hat{\theta}_t$:[12]

$$\hat{\theta}^C(n) = \hat{\kappa} + \hat{\gamma}_{\text{out}} - \frac{2\delta(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\overline{\ell}}{\rho + 2\delta} - \frac{2\rho(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\overline{\ell}}{\rho + 2\delta} n. \tag{23}$$

[12]The interpretation of this threshold is that, for a given fraction of time that users can spend in the network, $\overline{\ell}$, the social optimum would be that agents join the network for $\hat{\theta} > \hat{\theta}^C$ and stay out for $\hat{\theta} < \hat{\theta}^C$.

6.1. Optimal Time Restrictions

Consider a situation where initially, agents face no constraint in the time they spend on social media, that is $\overline{\ell} = 1$. We then consider the problem of an authority that can impose a usage restriction, limiting the maximum fraction of their spare time agents can spend on social media, that is, the authority chooses a new value for $\overline{\ell}$. The authority, however, is unable to fully prohibit agents from using the network, and cannot choose $\overline{\ell}$ below a cutoff $\underline{r} \in (0, 1)$. For instance, a school may be unable to restrict students' access to social media at home, and hence choosing $\overline{\ell} = 0$ is not feasible, which implies $\underline{r} > 0$.

We can then write the authority's problem as follows. The authority chooses $\overline{\ell} \in [\underline{r}, 1]$ to maximize the discounted sum of agents' utility (6), subject to (21) and to agents playing according to the decentralized threshold $\hat{\theta}^*(n_t)$ given by (22). Note that now the parameters γ_{out}, γ_{in} and the variable θ_t are not exogenous, but depend on the chosen policy.

Before determining the solution to that problem, it is useful to discuss the trade-offs facing the authority. Lowering $\overline{\ell}$ have direct and indirect effects. The direct effect is the impact of time restrictions on agents' utility taking as given the path of network quality and size, $\hat{\theta}_t$ and n_t. For non-users, lowering $\overline{\ell}$ always increases their utility as long as $n > 0$, since for $n > 0$

$$\frac{du_{\text{out}}}{d\overline{\ell}} = -\hat{\gamma}_{\text{out}} n < 0.$$

This is because, when usage is restricted, non-users automatically experience less FOMO. For users, the direct effect is more nuanced, since

$$\frac{du_{\text{in}}}{d\overline{\ell}} = \hat{u}_{\text{in}} - \hat{u}_{\text{out}} + \overline{\ell}\frac{d\hat{u}_{\text{in}}}{d\overline{\ell}} + \left(1 - \overline{\ell}\right)\frac{d\hat{u}_{\text{out}}}{d\overline{\ell}} = \hat{\theta} - \hat{\kappa} + \left[\left(2\overline{\ell} - 1\right)\hat{\gamma}_{\text{out}} + 2\overline{\ell}\hat{\gamma}_{\text{in}}\right]n. \quad (24)$$

If a single agent were to spend more time on social media, she would get the utility differential $\hat{u}_{\text{in}} - \hat{u}_{\text{out}}$. However, if all users get to spend more time on social media, that changes the consumption externalities users face during the fraction of time $\overline{\ell}$ they are on social media, captured by $\overline{\ell}\frac{d\hat{u}_{\text{in}}}{d\overline{\ell}}$ above. Also, they face less FOMO in the fraction $1 - \overline{\ell}$ of the time they are not on social media, captured by the term $\left(1 - \overline{\ell}\right)\frac{d\hat{u}_{\text{out}}}{d\overline{\ell}}$.

Whether the direct effect for users is positive or negative depends on the state of the economy and parameters.

The indirect effect refers to the fact that by changing $\overline{\ell}$, the equilibrium threshold $\hat{\theta}^*(n)$ changes, possibly changing the path of n_t. Differentiating (22) with respect to $\overline{\ell}$ we get

$$\frac{d\hat{\theta}^*}{d\overline{\ell}} = -\frac{(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})(\rho n + \delta)}{\rho + 2\delta} < 0.$$

Therefore, imposing more strict time limits (lowering $\overline{\ell}$) increases the equilibrium threshold, possibly leading to lower values of n_t.

We now present and discuss the solution to the authority's problem, denoting by ℓ^* the value of $\overline{\ell}$ that maximizes the authority's objective function. We start with the case where the economy is in a constrained inefficient product market trap before the policy:

Proposition 6. *Suppose that at date zero there are no time restrictions ($\overline{\ell} = 1$) and the economy is in a steady-state product market trap that is constrained inefficient, that is, $n_0 = 1$ and $\hat{\theta}_0 \in \left(\hat{\theta}^*(1), \hat{\theta}^C(1)\right)$. Define*

$$\tilde{r} \equiv \frac{(2\delta + \rho)(\hat{\kappa} - \hat{\theta}_0)}{(\hat{\gamma}_{out} + \hat{\gamma}_{in})(\delta + \rho)} < 1 \tag{25}$$

and

$$r^\dagger \equiv \frac{\hat{\kappa} - \hat{\gamma}_{in} - \hat{\theta}_0}{\hat{\gamma}_{out} + \hat{\gamma}_{in}}. \tag{26}$$

The optimal time restriction policy $\ell^* \in [\underline{r}, 1]$ is as follows:

1. *For $\hat{\theta}_0 \leq \hat{\kappa} - \hat{\gamma}_{out} - 2\hat{\gamma}_{in}$, the planner always imposes the most stringent time restriction, $\ell^* = \underline{r}$.*

2. *For $\hat{\theta}_0 > \hat{\kappa} - \hat{\gamma}_{out} - 2\hat{\gamma}_{in}$, the planner only imposes time restrictions if the policy space $[\underline{r}, 1]$ is sufficiently large,*

$$\ell^* = \begin{cases} \underline{r} & \text{if } \underline{r} < \max\left\{\tilde{r}, r^\dagger\right\}, \\ 1 & \text{if } \underline{r} > \max\left\{\tilde{r}, r^\dagger\right\}, \end{cases}$$

with $\max\left\{\tilde{r}, r^\dagger\right\} \in (0, 1)$.

Moreover, if $\underline{r} < \tilde{r}$, the optimal policy causes a switch in the path of n_t, with $n_t = e^{-\delta t}$, and if $\underline{r} > \tilde{r}$, $n_t = 1$ for all $t \geq 0$.

One reason a time restriction might not be optimal – even when the economy is stuck in an inefficient product market trap – is that mild restrictions keep all agents as users of the network and hence make agents experience FOMO during the hours they are unable to access it. Therefore, if the authority lacks sufficient policy space to induce a shift away from network usage, it may find it optimal to just let agents continue to use the network freely, as to prevent agents' feelings of missing out when they reach their time limit.

However, in some cases, the authority may still prefer to impose time limits even if they do not induce exit from network. Consider case 2 of Proposition 6 with $\tilde{r} < r^{\dagger}$, as illustrated in Figure 12. If the planner sets any $\bar{\ell} > \tilde{r}$, the path of n_t remains unchanged, and the only effect of a restriction is the direct utility impact on users given by (24), since $n_0 = 1$. Note that $du_{\text{in}}/d\bar{\ell}$ increases with $\bar{\ell}$. If the planner's policy space is very limited ($\underline{r}$ is large), that derivative is positive for any feasible $\bar{\ell}$. In that case, imposing time restrictions (reducing $\bar{\ell}$) harms welfare: agents would still spend too much time in the network, so the induced cost of FOMO of (modest) restrictions is too large. Now, as one can verify, for smaller values of $\bar{\ell}$, $du_{\text{in}}/d\bar{\ell}$ becomes negative at $n = 1$.[13] If the planner has enough margin to impose restrictions ($\underline{r} < r^{\dagger}$), it can limit network usage enough so that the negative effect of FOMO is not so large, and welfare increases even without a shift in the path of n. For intermediate values $\underline{r} \in \left(\tilde{r}, r^{\dagger}\right)$, although the planner is unable to induce exit, the optimal policy is to impose the most string restriction $\underline{r}$. That is also the optimal policy when $\underline{r} \leq \tilde{r}$, but in that case, the policy does cause a shift in the path of n, leading to even larger welfare gains. Agents face FOMO during the transition out of the network, but that effect vanishes in the long run.

If parameters are such that we are in case 1 of Proposition 6 – which is only possible when there is a sufficiently strong preference for exclusion, $\gamma_{\text{in}} < -\frac{\delta}{\rho+3\delta}\gamma_{\text{out}}$ – any reduction in network usage, no matter how small, is always socially optimal.[14]

[13] Evaluating $du_{\text{in}}/d\bar{\ell}$ at $n = 1$ and $\bar{\ell} = 0$, we get that it equals $\hat{\theta} - \hat{\kappa} - \hat{\gamma}_{\text{out}}$. Under the assumption that the economy in a steady-state product market trap when $\bar{\ell} = 1$, $\hat{\theta} - \hat{\kappa} - \hat{\gamma}_{\text{out}} < -(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}}) < 0$, where the first inequality uses that $\hat{\theta} < \theta^U = \hat{\kappa} - \hat{\gamma}_{\text{in}}$.

[14] Proposition 6 concerns product market traps, so in the absence of interventions, $\hat{\theta}_0 > \hat{\theta}^*(1)$.

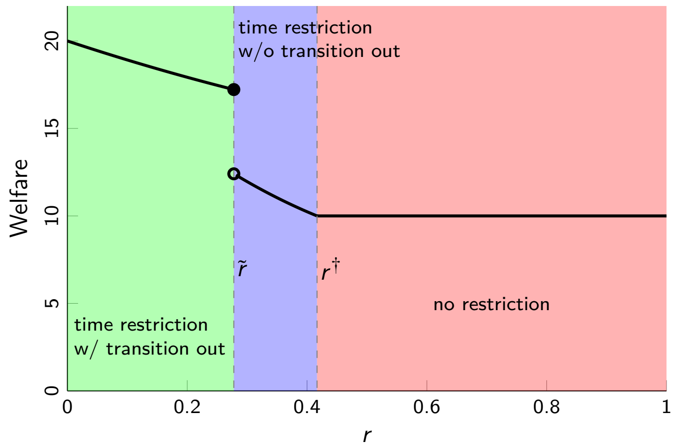

Figure 12: Welfare under optimal policy in the case of $\hat{\theta}_0 > \hat{\kappa} - \hat{\gamma}_{\text{out}} - 2\hat{\gamma}_{\text{in}}$. A numerical example with $\tilde{r} < r^\dagger$.

Notes. The following parameters were used: $\hat{\theta}_0 = 0.8$, $\hat{\kappa} = 1.0$, $\hat{\gamma}_{\text{in}} = -0.3$, $\hat{\gamma}_{\text{out}} = 1.5$, $\delta = 0.1$, $\rho = 0.05$, $n_0 = 1.0$.

This is because the marginal benefit of reducing negative consumption externalities on users always more than compensate the FOMO suffered by agents in their time off the network. Consequently, the planner is happy to impose the most stringent time restriction possible, regardless of whether that causes a shift in the path of n or not.

For completeness, Proposition B.1 in Appendix B presents the optimal time restriction in an economy that, before any intervention, is in an efficient product market trap in a mature network, that is, $\hat{\theta}_0 \in \left(\hat{\theta}^C(1), \hat{\kappa} - \hat{\gamma}_{\text{in}}\right)$ and $n_0 = 1$. In that case, the result is qualitatively the same as the one in case 2 of Proposition 6.

6.2. Relation to Experimental Evidence

Consider an experiment conducted in a setting with no time restrictions for users and all the assumptions employed in Section 5. The following proposition shows how experimental data can be used to assess the optimality of time restrictions.

Proposition 7. *Consider an economy that is in a product market without any intervention ($\bar{\ell} = 1$). Then,* (18) *is a necessary and sufficient condition for any time*

Hence, case 1 in the proposition is only possible if $\hat{\theta}^*(1)\Big|_{\bar{\ell}=1} < \hat{\kappa} - \hat{\gamma}_0 - 2\hat{\gamma}_1$, which simplifies to $\gamma_{\text{in}} < -\frac{\delta}{\rho+3\delta}\gamma_{\text{out}}$.

restriction to be welfare improving.

In other words, we can check whether any time restriction is desirable by checking the same condition we used to detect inefficient product market traps regardless of timing frictions. Therefore, if a product market is inefficient for any degree of timing frictions, any time restriction – however small – is desirable.

Our model then supports the idea that time restrictions for TikTok and Instagram would increase the welfare of the subjects considered in Bursztyn et al. (2024).

7. Final Remarks

Several countries have implemented nationwide bans or strict restrictions on the use of mobile phones in schools, including Brazil, China, Finland, France, Italy, Malaysia, the Netherlands, and New Zealand. In addition, many states and provinces around the world have adopted similar policies. Other countries have introduced guidelines or measures aimed at discouraging phone use in educational settings. The regulation of mobile phone use in schools has become a prominent policy issue across the globe.

At the core of the debate is the use of social media. While behavioral concerns – such as addiction and attention capture – are central, the issue goes beyond individual psychology. Social media platforms are shaped by powerful network effects. Their value increases as more people use them, but at the same time, they generate negative externalities for non-users.

This paper derives positive and normative implications of network effects in a setting where networks form and evolve gradually. The distinguishing feature of the model is the presence of negative externalities on non-users – the cost of missing out.

We show that negative externalities on non-users alone can give rise to welfare-reducing networks with universal adoption. Once such a network is established, small restrictions may be harmful, even though large interventions would raise welfare. The model provides a framework for quantitatively assessing the desirability of restriction policies and platform bans.

Although the paper is motivated by the debate on social media, the model applies more broadly to dynamic settings with strategic complementarities and negative externalities on non-users. For instance, teenagers may coordinate around social norms that encourage risky or aggressive behavior, where failure to comply leads to exclusion from the group. Once most individuals conform, the cost of deviating might be too high. We show that in this environment, a sudden and complete change in social norms would be desirable but small interventions may backfire.

A. Proofs

A.1. Proof of Proposition 1

Our model satisfies the conditions in Theorem 1 of Frankel and Pauzner (2000), and thus has a unique equilibrium characterized by a threhshold $\theta^*(n_0)$. With $\sigma \to 0$, Theorem 2 in Frankel and Pauzner (2000) can be applied to our setting, and the equilibrium threshold is thus given by

$$\int_0^{n_0} \left(\frac{\nu}{n_0}\right)^{\frac{\rho}{\delta}} \Delta u(\theta^*, \nu) d\nu + \int_{n_0}^1 \left(\frac{1-\nu}{1-n_0}\right)^{\frac{\rho}{\delta}} \Delta u(\theta^*, \nu) d\nu = 0, \tag{A.1}$$

where $\Delta u(\cdot)$ is given by (3). Solving for θ^* yields the claim. $\square$

A.2. Proof of Proposition 2

From Guimaraes et al. (2020), the solution of the (constrained) social planner's problem is the equilibrium of our game with individual payoffs replaced by (12). We can then use (12) to replace $\Delta u(\cdot)$ by $\frac{\partial W(\theta^C, \nu)}{\partial n}$ in (A.1) and solve it for θ^C to get the stated result. $\square$

A.3. Proof of Proposition 3

First statement. Using (14), note that if $\tilde{\Delta}(1) \geq 0$, then $\tilde{\Delta}(n) > 0$ for all $n \in [0, 1)$, since $\tilde{\Delta}(n)$ is strictly decreasing in n. Therefore, whenever $\tilde{\Delta}(1) \geq 0$, $\theta^C(n) \geq \theta^*(n)$ for all $n \in [0, 1]$, with strict inequality for $n < 1$, and hence there is only a region of inefficient entry. Rearranging $\tilde{\Delta}(1) \geq 0$, we get $\gamma_{\text{out}} \geq \frac{\rho+\delta}{\delta}\gamma_{\text{in}}$.

Second statement. Using (14), note that if $\tilde{\Delta}(0) \leq 0$, then $\tilde{\Delta}(n) < 0$ for all $n \in (0, 1]$, since $\tilde{\Delta}(n)$ is strictly decreasing in n. Therefore, whenever $\tilde{\Delta}(0) \leq 0$, $\theta^C(n) \leq \theta^*(n)$ for all $n \in [0, 1]$, with strict inequality for $n > 0$, and hence there is only a region of inefficient exit. Rearranging $\tilde{\Delta}(0) \leq 0$, we get $\gamma_{\text{out}} \leq \frac{\delta}{\rho+\delta}\gamma_{\text{in}}$.

Third statement. From the proof the previous statements, whenever $\frac{\delta}{\rho+\delta}\gamma_{\text{in}} < \gamma_{\text{out}} < \frac{\rho+\delta}{\delta}\gamma_{\text{in}}$, $\tilde{\Delta}(1) < 0$ and $\tilde{\Delta}(0) > 0$. This implies that $\theta^*(n)$ and $\theta^C(n)$ intersect for some $n = \hat{n} \in (0, 1)$. Hence, $\theta^C(n) < \theta^*(n)$ for $n > \hat{n}$ and we have inefficient exit in that case. For $n < \hat{n}$, $\theta^C(n) > \theta^*(n)$ and we have inefficient entry. $\square$

A.4. Proof of Proposition 4

First statement. Note that $\theta^C(1/2) = \theta^U$, and hence $\theta^C(n) < \theta^U$ for $n > 1/2$ and $\theta^C(n) > \theta^U$ for $n < 1/2$. Therefore, if there is a product market trap with $n = \tilde{n} \leq 1/2$, we must have $\theta^*(\tilde{n}) < \theta^U \leq \theta^C(\tilde{n})$, and hence this product market trap is constrained inefficient. Thus, a necessary condition for product market traps to be always constrained efficient is that $\theta^*(1/2) \geq \theta^U$, so that product market traps with $n \leq 1/2$ never happen. Using (5) and (9) and simplifying, the condition $\theta^*(1/2) \geq \theta^U$ can be rewritten as $\gamma_{\text{in}} \geq \gamma_{\text{out}}$.

We now check under which conditions any product market traps with $n > 1/2$ satisfies $\theta^C(n) \leq \theta^*(n)$, so that every product market trap with $n > 1/2$ is constrained efficient. We then need to check whether $\tilde{\Delta}(n) \leq 0$ (see equation (14)), for all $n > 1/2$. Since $\tilde{\Delta}(n)$ is strictly decreasing in n, it then suffices to check under which conditions $\tilde{\Delta}(1/2) \leq 0$, which after rearranging yields, again, $\gamma_{\text{in}} \geq \gamma_{\text{out}}$. Therefore, if $\gamma_{\text{in}} \geq \gamma_{\text{out}}$, there is no region in which inefficient product market traps arise. The condition $\gamma_{\text{out}} > \frac{\delta}{\rho+\delta}\gamma_{\text{in}}$ is equivalent to $\theta^*(1) < \theta^U$, which is the necessary and sufficient condition for a region with product market traps to exist.

Second statement. Using the proof of the first statement, note that $\gamma_{\text{out}} > \gamma_{\text{in}}$ implies $\theta^*(1/2) < \theta^U$. Hence, there is a $\tilde{\theta} \in \left(\theta^*(1/2), \theta^U\right)$ such that with $n = 1/2$ and $\theta = \tilde{\theta}$, the economy is in a product market trap. Since $\theta^C(1/2) = \theta^U$, $\tilde{\theta} < \theta^C(1/2) = \theta^U$, and hence such trap is inefficient (and by continuity, the same holds if n is in some neighborhood of $n = 1/2$ and θ is in some neighborhood of $\tilde{\theta}$, so that the set of states in which inefficient product market traps happen contains infinite elements).

To see why constrained efficient product market can happen with $\gamma_{\text{out}} > \gamma_{\text{in}}$, note that $\theta^C(1) < \theta^U$ and that, with $\gamma_{\text{out}} > \gamma_{\text{in}}$, $\theta^*(1) < \theta^U$. Hence, a constrained efficient product market trap happen for $\theta < \theta^U$ but sufficiently close θ^U and $n = 1$ or sufficiently close to one. $\square$

A.5. Proof of Proposition 5

First statement. We start replacing the expressions for agents' utilities, (1) and (2), in equation (15):

$$\Delta(x_{\text{in}}, x_{\text{out}}) = \theta_0 - \kappa + \gamma_{\text{in}} x_{\text{in}} + \gamma_{\text{out}} x_{\text{out}}. \tag{A.2}$$

Hence, condition (16) can be written as

$$\theta_0 - \kappa + \gamma_{\text{in}} + \gamma_{\text{out}}\xi - \xi(\theta_0 - \kappa + \gamma_{\text{in}} + \gamma_{\text{out}}) < 0,$$

which simplifies to $\theta_0 < \kappa - \gamma_{\text{in}} = \theta^U$, and thus the unconstrained would prefer agents not to join the network if that condition holds. Since the proposition assumes the economy is in a steady state with $n = 1$, it must be that $\theta_0 \geq \theta^*(1)$ and agents choose to remain in the network in the decentralized equilibrium, implying that the economy is in a product market trap when (16) holds.

Second statement. For a product market trap with $n = 1$ to be inefficient we need $\theta_0 < \theta^C(1)$, which, using (13) and simplifying becomes

$$\tilde{\Sigma} \equiv 2\delta(\theta_0 - \kappa) + (\gamma_{\text{out}} + \theta_0 - \kappa)\rho + 2\gamma_{\text{in}}(\delta + \rho) < 0. \tag{A.3}$$

Note that, $\frac{\partial \tilde{\Sigma}}{\partial \delta} = 2(\theta_0 - \kappa + \gamma_{\text{in}}) < 0$ whenever the economy is in a product market trap, since in those cases $\theta_0 < \kappa - \gamma_{\text{in}}$. Hence, for (A.3) to hold for any $\delta > 0$ in a product market trap, it suffices to check that $\tilde{\Sigma} \leq 0$ for $\delta = 0$. Replacing $\delta = 0$ in $\tilde{\Sigma} \leq 0$ yields (17).

Third statement. Replacing (A.2) in (18) and simplifying, we obtain equation (17). $\square$

A.6. Proof of Proposition 6

Throughout this proof, we use the notation $\hat{\theta}^*(1;\ell') \equiv \hat{\theta}^*(1)\big|_{\bar{\ell}=\ell'}$ and $\hat{\theta}^C(1;\ell') \equiv \hat{\theta}^C(1)\big|_{\bar{\ell}=\ell'}$.

To determine the optimal time restriction policy, the planner must solve the following problem:

$$\max_{\overline{\ell}\in[\underline{r},1]}\int_0^\infty e^{-\rho t}W(\theta_t,n_t)dt$$

subject to

$$n_t=\begin{cases}n_0e^{-\delta t} & \text{if } \hat{\theta}_t\le\hat{\theta}^*\Big(n_t;\overline{\ell}\Big),\\ 1-(1-n_0)e^{-\delta t} & \text{if } \hat{\theta}_t>\hat{\theta}^*\Big(n_t;\overline{\ell}\Big),\end{cases}\tag{A.4}$$

and to (21), with $W(\cdot)$ and $\hat{\theta}^*(\cdot)$ given by (8) and (22), respectively. Using (21), we can rewrite the objective function as

$$\int_0^\infty e^{-\rho t}\Big[n_t^2(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})\overline{\ell}^2-n_t\Big(\hat{\kappa}+\hat{\gamma}_{\text{out}}-\hat{\theta}_t\Big)\overline{\ell}+\hat{\kappa}\Big]dt.\tag{A.5}$$

Suppose that, in the absence of any restrictions (with $\overline{\ell}=1$), the economy is in an inefficient product market trap with $n_0=1$ and hence $\hat{\theta}_0\in\Big(\hat{\theta}^*(1;1),\hat{\theta}^C(1;1)\Big)$. From (22), note that $\hat{\theta}^*(\cdot)$ is decreasing in $\overline{\ell}$, and hence $\hat{\theta}_0>\hat{\theta}^*\Big(1;\overline{\ell}\Big)$ if and only if

$$\overline{\ell}>\frac{(\rho+2\delta)\Big(\hat{\kappa}-\hat{\theta}_0\Big)}{(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})(\delta+\rho)}=\tilde{r}.\tag{A.6}$$

Suppose the planner sets some $\overline{\ell}\in[\underline{r},\tilde{r}]$. In this case, (A.4) implies $n_t=n_0e^{-\delta t}$, and the objective function in (A.5) evaluated at $n_0=1$ can be rewritten as

$$\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0;\overline{\ell}\Big)\equiv\frac{\overline{\ell}^2(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})}{2\delta+\rho}+\frac{\overline{\ell}\Big(\hat{\theta}_0-\hat{\gamma}_{\text{out}}-\hat{\kappa}\Big)}{\rho+\delta}+\frac{\hat{\kappa}}{\rho}.\tag{A.7}$$

One can verify that

$$\frac{d\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0;\overline{\ell}\Big)}{d\overline{\ell}}=\frac{2\overline{\ell}(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})}{2\delta+\rho}+\frac{\hat{\theta}_0-\hat{\kappa}-\hat{\gamma}_{\text{out}}}{\rho+\delta},\tag{A.8}$$

which is strictly increasing in $\hat{\theta}_0$ and $\overline{\ell}$. Hence, for any $\hat{\theta}_0\in\Big(\hat{\theta}^*(1;1),\hat{\theta}^C(1;1)\Big)$ and $\overline{\ell}\in[0,1]$,

$$\frac{d\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0;\overline{\ell}\Big)}{d\overline{\ell}}<\frac{2(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})}{2\delta+\rho}+\frac{\hat{\theta}^C(1;1)-\hat{\kappa}-\hat{\gamma}_{\text{out}}}{\rho+\delta}=0,$$

where the equality above follows from (23). Therefore, if the optimal $\overline{\ell}$ lies in the interval $[\underline{r},\tilde{r}]$, then $\ell^*=\underline{r}$. Also note that if $\hat{\kappa}<\hat{\theta}_0$, then $\tilde{r}<0$ and there is no $\overline{\ell}>0$

that can trigger a switch in the path of n.

Suppose now that the planner sets $\overline{\ell}$ above $\tilde{r}$, so $n_t = 1$ for all t. Using (21) and the restriction in (A.4), the objective function (A.5), considering that $n_0 = 1$, can be written as

$$\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big) \equiv \frac{(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\overline{\ell}^2 + \Big(\hat{\theta}_0 - \hat{\kappa} - \hat{\gamma}_{\text{out}}\Big)\overline{\ell} + \hat{\kappa}}{\rho}, \tag{A.9}$$

and we have that

$$\frac{d\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)}{d\overline{\ell}} = \frac{1}{\rho}\Big[\hat{\theta}_0 - \hat{\kappa} - \hat{\gamma}_{\text{out}} + 2\overline{\ell}(\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}})\Big]. \tag{A.10}$$

Since (A.9) is strictly convex in $\overline{\ell}$, $\text{argmax}_{\ell \in [\ell', 1]} \mathcal{W}_{n\uparrow} \in \{\ell', 1\}$, for any ℓ'. Now note that the expression in (A.10) is strictly increasing in $\overline{\ell}$. Hence, for $\hat{\theta}_0 \leq \hat{\kappa} - \hat{\gamma}_{\text{out}} - 2\hat{\gamma}_{\text{in}}$, we have that

$$\frac{d\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)}{d\overline{\ell}} < \left.\frac{d\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)}{d\overline{\ell}}\right|_{\overline{\ell}=1} = \frac{1}{\rho}\Big(\hat{\theta}_0 - \hat{\kappa} + \hat{\gamma}_{\text{out}} + 2\hat{\gamma}_{\text{in}}\Big) \leq 0$$

for all $\overline{\ell} < 1$, and thus (A.9), as well as (A.7), is strictly decreasing in $\overline{\ell}$. This concludes the proof that for $\hat{\theta}_0 \leq \hat{\kappa} - \hat{\gamma}_{\text{out}} - 2\hat{\gamma}_{\text{in}}$, $\ell^* = \underline{r}$ (case 1).

Assume hereafter that $\hat{\theta}_0 > \hat{\kappa} - \hat{\gamma}_{\text{out}} - 2\hat{\gamma}_{\text{in}}$. Note from (A.7) and (A.9) that both $\mathcal{W}_{n\downarrow}$ and $\mathcal{W}_{n\uparrow}$ are quadratic functions of $\overline{\ell}$, and recall that $\mathcal{W}_{n\downarrow}$ is strictly decreasing in $\overline{\ell}$ for $\overline{\ell} \in [0, 1]$ and $\hat{\theta}_0 \in \Big(\hat{\theta}^*(1; 1), \hat{\theta}^C(1; 1)\Big)$. Also, note that, for $\hat{\theta}_0 \in \Big(\hat{\theta}^*(1; 1), \hat{\theta}^C(1; 1)\Big)$,

$$\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; 0\Big) = \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 0\Big) = \frac{\hat{\kappa}}{\rho}$$

and

$$\left.\frac{d\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)}{d\overline{\ell}}\right|_{\overline{\ell}=0} < \left.\frac{d\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)}{d\overline{\ell}}\right|_{\overline{\ell}=0} < 0,$$

where the first inequality holds because $\left.\frac{d\mathcal{W}_{n\downarrow}(\hat{\theta}_0; \overline{\ell})}{d\overline{\ell}}\right|_{\overline{\ell}=0} - \left.\frac{d\mathcal{W}_{n\uparrow}(\hat{\theta}_0; \overline{\ell})}{d\overline{\ell}}\right|_{\overline{\ell}=0}$ is strictly decreasing in $\hat{\theta}_0$ and positive even when evaluated at $\hat{\theta}_0 = \hat{\theta}^C(1; 1)$, and the second inequality holds because $\left.\frac{d\mathcal{W}_{n\downarrow}(\hat{\theta}_0; \overline{\ell})}{d\overline{\ell}}\right|_{\overline{\ell}=0}$ is increasing in $\hat{\theta}_0$, and is negative even at $\hat{\theta}_0 = \hat{\theta}^C(1; 1)$.

Moreover, one can verify that

$$\Delta_1\Big(\hat{\theta}_0\Big) \equiv \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big) - \mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; 1\Big) = \frac{\delta}{\rho+\delta}\left[\frac{\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}}}{\rho + 2\delta} + \frac{\hat{\theta}_0 + \hat{\gamma}_{\text{in}} - \hat{\kappa}}{\rho}\right],$$

which is increasing in $\hat{\theta}_0$, and that $\Delta_1\Big(\hat{\theta}^C(1;1)\Big) = 0$. Therefore, given $\hat{\theta}_0 \in \Big(\hat{\theta}^*(1;1), \hat{\theta}^C(1;1)\Big)$, $\mathcal{W}_{n\uparrow}\Big(\hat{\theta}; \overline{\ell}\Big)$ does not cross $\mathcal{W}_{n\downarrow}\Big(\hat{\theta}; \overline{\ell}\Big)$ as $\overline{\ell}$ goes from zero to one and hence $\mathcal{W}_{n\uparrow}\Big(\hat{\theta}; \overline{\ell}\Big) < \mathcal{W}_{n\downarrow}\Big(\hat{\theta}; \overline{\ell}\Big)$ for all $\overline{\ell} \in (0, 1)$. This implies that whenever $\underline{r} \leq \tilde{r}$, $\ell^* = \underline{r}$.

Suppose now that $\underline{r} > \tilde{r}$. Then $\ell^* \in \{\underline{r}, 1\}$, given the convexity of $\mathcal{W}_{n\uparrow}$ in $\overline{\ell}$. The planner then chooses $\ell^* = 1$ if $\underline{r} > r^\dagger$ and $\ell^* = \underline{r}$ if $\underline{r} \in \Big(\tilde{r}, r^\dagger\Big)$, where $r^\dagger$ solves $\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big) = \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; r^\dagger\Big)$ in the interval $(0, 1)$ and is thus given by

$$r^\dagger \equiv \frac{\hat{\kappa} - \hat{\theta}_0 - \hat{\gamma}_{\text{in}}}{\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}}}. \tag{A.11}$$

If $\underline{r} = r^\dagger$, the planner is indifferent between $\overline{\ell} = 1$ and $\overline{\ell} = \underline{r}$. Note that $\tilde{r}$ in (A.6) is decreasing in $\hat{\theta}_0$ and $\tilde{r} = 1$ for $\hat{\theta}_0 = \hat{\theta}^*(1;1)$, so $\tilde{r} < 1$ for any $\hat{\theta}_0 \in \Big(\hat{\theta}^*(1;1), \hat{\theta}^C(1;1)\Big)$. Also, $r^\dagger \in (0, 1)$ for $\hat{\theta}_0 \in \Big(\hat{\kappa} - \hat{\gamma}_{\text{out}} - 2\hat{\gamma}_{\text{in}}, \hat{\theta}^C(1;1)\Big)$. Therefore, $\max\Big\{\tilde{r}, r^\dagger\Big\} \in (0, 1)$ for any $\hat{\theta}_0 > \hat{\kappa} - \hat{\gamma}_{\text{out}} - 2\hat{\gamma}_{\text{in}}$. This concludes the proof of the second statement in Proposition 6.

Finally, note that there is a switch in the path of n_t if, and only if, $\overline{\ell} \leq \tilde{r}$ (see (A.6)), so for $\underline{r} > \tilde{r}$, a switch is not achievable. Moreover, from the two first statements of the proposition, whenever $\underline{r} < \tilde{r}$, $\ell^* = \underline{r}$ and a switch in the path of n_t happens. □

A.7. Proof of Proposition 7

From Proposition 5, remember that (18) can be rewritten as (17), which after using (21) with $\overline{\ell} = 1$ yields

$$\hat{\theta}_0 \leq \hat{\kappa} - \hat{\gamma}_{\text{out}} - 2\hat{\gamma}_{\text{in}}. \tag{A.12}$$

Consider an economy that is in a product market trap with $\overline{\ell} = 1$. We first prove that if any time restriction increases welfare, then condition (A.12) holds. Suppose any time restriction is welfare improving. Then, from the results in Appendix B, we cannot be in a constrained efficient product market trap. This follows from Proposition

B.1 and the fact that if $\hat{\theta}_0 = \hat{\theta}^C(1)$ (the only case of efficient product market traps not directly considered in that proposition), a marginal decrease in $\overline{\ell}$ at $\overline{\ell} = 1$ reduces welfare (see equation (B.6)). Hence, we are in a constrained inefficient product market trap. But then, condition (A.12) must hold, by Proposition 6.

Suppose now condition (A.12) holds. Then, we are in an inefficient product market trap by Proposition 5 and the fact with $\overline{\ell} = 1$ the hatted variables equal their not-hatted counterparts (see equation (21)). Hence, the optimal policy is given by case 1 of Proposition 6, which implies that any time restriction increases welfare. □

B. Optimal Time Restrictions in Efficient Traps

We now present the optimal time restriction policy when the economy is initially in an efficient product market trap. Before, we define the following variables:

$$r_E \equiv \max\Big\{r^\dagger, \min\{\tilde{r}, \check{r}\}\Big\},$$

where $\tilde{r}$ is given by (25), $r^\dagger$ by (26) and $\check{r}$ is given by

$$\check{r} \equiv \frac{\tilde{r}}{2} + \frac{\rho + 2\delta}{2(\rho+\delta)}\left[\frac{\hat{\gamma}_{\text{out}}}{\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}}} - \sqrt{\left(r^\dagger - \frac{\rho+2\delta}{\rho}\right)\left(r^\dagger - \frac{\rho}{\rho+2\delta}\right)}\right]. \tag{B.1}$$

Proposition B.1. *Suppose that at date zero there are no time restrictions ($\overline{\ell} = 1$) and the economy is in a steady-state product market trap that is constrained efficient with $n_0 = 1$ and $\hat{\theta}_0 \in \left(\max\left\{\hat{\theta}^*(1), \hat{\theta}^C(1)\right\}, \hat{\theta}^U\right)$, where $\hat{\theta}^U \equiv \hat{\kappa} - \hat{\gamma}_{in}$ is the unconstrained efficient threshold. In this case, it is optimal to impose a time restriction if, and only if, the policy space $[\underline{r}, 1]$ is sufficiently large. The optimal policy is:*

$$\overline{\ell}^* = \begin{cases} 1 & \text{if } \underline{r} > r_E, \\ \underline{r} & \text{if } \underline{r} < r_E, \end{cases} \tag{B.2}$$

with $r_E \in (0, 1)$.

Proof. Suppose that, in the absence of any restrictions ($\overline{\ell} = 1$), the economy is in an efficient product market trap, with $n_0 = 1$ and $\hat{\theta}_0 \in \left(\max\left\{\hat{\theta}^*(1), \hat{\theta}^C(1)\right\}, \hat{\theta}^U\right)$. Hence, throughout this proof, consider $n_0 = 1$ and that $\hat{\theta}_0$ is in that range. The proof follows closely the proof of Proposition 6. The planner solves the same problem, considering the different initial conditions. The problem can be written as

$$\max_{\overline{\ell}\in[\underline{r},1]} \mathcal{W}\Big(\hat{\theta}_0;\overline{\ell}\Big) = \begin{cases} \mathcal{W}_{n\downarrow}(\hat{\theta}_0;\overline{\ell}) & \text{if } \overline{\ell} \leq \tilde{r}, \\ \mathcal{W}_{n\uparrow}(\hat{\theta}_0;\overline{\ell}) & \text{if } \overline{\ell} > \tilde{r}, \end{cases} \tag{B.3}$$

where $W_{n\downarrow}(\cdot)$, $W_{n\uparrow}(\cdot)$ and $\tilde{r}$ are as in (A.7), (A.9) and (A.6), respectively.

We begin by establishing four useful properties for the characterization of the constrained planner's solution.

Property (i). From (A.7) and (A.9), $\mathcal{W}_{n\downarrow}(\hat{\theta}_0;0) = \mathcal{W}_{n\uparrow}(\hat{\theta}_0;0) = \hat{\kappa}/\rho$, while

$$\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0;1\Big) = \frac{\hat{\theta}_0 - \hat{\kappa} - \hat{\gamma}_{\text{out}}}{\rho+\delta} + \frac{\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}}}{\rho+2\delta} + \frac{\hat{\kappa}}{\rho} < \frac{\hat{\kappa}}{\rho}$$

and

$$\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0;1\Big) = \frac{\hat{\theta}_0 + \hat{\gamma}_{\text{in}}}{\rho} < \frac{\hat{\kappa}}{\rho},$$

where the inequalities follow from $\hat{\theta}_0 < \hat{\theta}^U$, after some algebra. Moreover,

$$\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0;1\Big) - \mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0;1\Big) > 0$$

since that difference is increasing in $\hat{\theta}_0$ and equals 0 for $\hat{\theta}_0 = \hat{\theta}^C(1)$.

Property (ii). $\mathcal{W}_{n\downarrow}(\cdot)$ and $\mathcal{W}_{n\uparrow}(\cdot)$ are U-shaped in $\overline{\ell}$ for $\overline{\ell} \in [0,1]$ and for any $\hat{\theta}_0$ in the considered range. To see this, note that both are strictly convex in $\overline{\ell}$, and from (A.8) and (A.10), $\frac{d\mathcal{W}_{n\downarrow}}{d\overline{\ell}}$ and $\frac{d\mathcal{W}_{n\uparrow}}{d\overline{\ell}}$ increase in $\hat{\theta}_0$. Then, since $\hat{\theta}_0 < \hat{\theta}^U$,

$$\left.\frac{d\mathcal{W}_{n\downarrow}}{d\overline{\ell}}\right|_{\overline{\ell}=0} = \frac{\hat{\theta}_0 - \hat{\kappa} - \hat{\gamma}_{\text{out}}}{\rho+\delta} < -\frac{\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}}}{\rho+\delta} < 0, \tag{B.4}$$

and

$$\left.\frac{d\mathcal{W}_{n\uparrow}}{d\overline{\ell}}\right|_{\overline{\ell}=0} = \frac{\hat{\theta}_0 - \hat{\kappa} - \hat{\gamma}_{\text{out}}}{\rho} < -\frac{\hat{\gamma}_{\text{out}} + \hat{\gamma}_{\text{in}}}{\rho} < 0. \tag{B.5}$$

Also, for any $\hat{\theta}' > \hat{\theta}^C(1)$, we have that

$$\left.\frac{d\mathcal{W}_{n\downarrow}}{d\bar{\ell}}\right|_{\bar{\ell}=1,\hat{\theta}_0=\hat{\theta}'} > \left.\frac{d\mathcal{W}_{n\downarrow}}{d\bar{\ell}}\right|_{\bar{\ell}=1,\hat{\theta}_0=\hat{\theta}^C(1)} = 0,$$

and

$$\left.\frac{d\mathcal{W}_{n\uparrow}}{d\bar{\ell}}\right|_{\bar{\ell}=1,\hat{\theta}_0=\hat{\theta}'} > \left.\frac{d\mathcal{W}_{n\uparrow}}{d\bar{\ell}}\right|_{\bar{\ell}=1,\hat{\theta}_0=\hat{\theta}^C(1)} = \frac{2\delta(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})}{\rho(2\delta+\rho)} > 0. \tag{B.6}$$

Property (iii). $\mathcal{W}_{n\downarrow}(\hat{\theta}_0;\bar{\ell}) > \mathcal{W}_{n\uparrow}(\hat{\theta}_0;\bar{\ell})$ if, and only if, $\bar{\ell} \in \left(0, \grave{\ell}\right)$, where

$$\grave{\ell} = \frac{(\rho+2\delta)\left(\hat{\kappa}-\hat{\theta}_0+\hat{\gamma}_{\text{out}}\right)}{2(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})(\rho+\delta)} \in (0,1)$$

is the only positive $\bar{\ell}$ that solves $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;\bar{\ell}\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;\bar{\ell}\right)$. The see why this property holds, recall that both functions are U-shaped (property (ii)). Also, from property (i), $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;0\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;0\right)$, and $\mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right) - \mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;1\right) > 0$. Finally, from (B.4) and (B.5), $\left.\frac{d\mathcal{W}_{n\uparrow}}{d\bar{\ell}}\right|_{\bar{\ell}=0} < \left.\frac{d\mathcal{W}_{n\downarrow}}{d\bar{\ell}}\right|_{\bar{\ell}=0} < 0$, and hence $\mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;\bar{\ell}\right) < \mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;\bar{\ell}\right)$ in a neighborhood of $\bar{\ell}=0$, and consequently, for any $\bar{\ell} \in \left(0,\grave{\ell}\right)$.

Property (iv). There exist values $r^\dagger, \check{r} \in (0,1)$, with $0 < r^\dagger < \check{r}$, satisfying $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;\check{r}\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;r^\dagger\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right)$. $\check{r}$ is given by (B.1), and $r^\dagger$ is given by (A.11). We now prove this property. Given that $\mathcal{W}_{n\downarrow}(\cdot)$ is U-shaped in $\bar{\ell} \in [0,1]$ (by property (ii)), and that $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;1\right) < \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right) < \mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;0\right)$ (by property (i)), there exists a single $\check{r} \in (0,1)$ satisfying $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;\check{r}\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right)$. Solving that equality for $\check{r}$ and picking the smallest solution yields, after some algebra, the expression in (B.1) (note that the other solution would be larger than one). Similarly, since by the same properties (i) and (ii) $\mathcal{W}_{n\uparrow}(\cdot)$ is U-shaped in $\bar{\ell} \in [0,1]$ and $\mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right) < \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;0\right)$, there exists a single $r^\dagger \in (0,1)$ satisfying $\mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;r^\dagger\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right)$. Solving that equality for $r^\dagger < 1$ yields (A.11). To see that $r^\dagger < \check{r}$, first note that

$$\grave{\ell} - r^\dagger = \frac{\rho\left(\hat{\theta}_0-\hat{\kappa}+\hat{\gamma}_{\text{out}}+2\hat{\gamma}_{\text{in}}\right)+2\delta(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})}{2(\rho+\delta)(\hat{\gamma}_{\text{out}}+\hat{\gamma}_{\text{in}})} > \frac{2\delta}{\rho+2\delta} > 0,$$

where the first inequality follows from $\hat{\theta}_0 > \hat{\theta}^C(1)$. Since $0 < r^\dagger < \grave{\ell}$, by property (iii) $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;r^\dagger\right) > \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;r^\dagger\right)$, and since by property (i) $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;1\right) < \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;r^\dagger\right)$, by continuity $\mathcal{W}_{n\downarrow}\left(\hat{\theta}_0;\bar{\ell}\right) = \mathcal{W}_{n\uparrow}\left(\hat{\theta}_0;1\right)$ at some $\bar{\ell} \in \left(r^\dagger,1\right)$. Since $\check{r}$ is precisely the value of $\overline{\ell} \in (0,1)$ that solves the last equality, $\check{r} > r^\dagger$.

We can now use these properties to characterize the social optimum. First note that, from properties (ii) and (iv), and using (B.3), $\mathcal{W}\Big(\hat{\theta}_0; 1\Big) > \mathcal{W}\Big(\hat{\theta}_0; \overline{\ell}\Big)$ for all $\overline{\ell} \in (\check{r}, 1)$, so $\overline{\ell} \in (\check{r}, 1)$ is never optimal.

Note that $\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \overline{\ell}\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)$ for all $\overline{\ell} \in (0, \check{r}]$. This is because for $\overline{\ell} \in \Big(r^\dagger, \check{r}\Big)$, $\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \overline{\ell}\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)$ (see property (iv)), and for $\overline{\ell} \leq r^\dagger < \grave{\ell}$, $\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \overline{\ell}\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)$ (see property (iii)). Now, recall that $\tilde{r}$ is the time limit that triggers a switch in the path of n_t (see (B.3)). Hence, whenever $\tilde{r} \leq \check{r}$, $\mathcal{W}\Big(\hat{\theta}_0; \tilde{r}\Big) = \mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \tilde{r}\Big) > \lim_{\overline{\ell} \to \tilde{r}^+} \mathcal{W}\Big(\hat{\theta}_0; \overline{\ell}\Big) = \lim_{\ell \to \tilde{r}^+} \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \overline{\ell}\Big)$, so the problem always has a well defined solution. Also, property (iii) implies that both $\mathcal{W}_{n\uparrow}(\cdot)$ and $\mathcal{W}_{n\downarrow}(\cdot)$ are decreasing in $\overline{\ell}$ whenever they are above $\mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big)$. Therefore, $\overline{\ell}^* \in \{\underline{r}, 1\}$. Given the properties described, the optimal policy depends on the position of $\underline{r}$ relative to $\tilde{r}$, $\check{r}$ and $r^\dagger$.

If $\underline{r} > \check{r}$, $\overline{\ell}^* = 1$. This follows automatically from the observation above that $\overline{\ell} \in (\check{r}, 1)$ is never optimal.

If $\underline{r} \in \Big(r^\dagger, \check{r}\Big)$, the optimum depends on $\tilde{r}$, since $\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \underline{r}\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \underline{r}\Big)$ – in words, it is only worth setting $\overline{\ell} = \underline{r}$ if that triggers a switch in the path of n_t. Thus, if moreover $\underline{r} < \tilde{r}$, then $\overline{\ell}^* = \underline{r}$, while for $\underline{r} > \tilde{r}$, $\overline{\ell}^* = 1$.

If $\underline{r} \in \Big(0, r^\dagger\Big)$, $\mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \underline{r}\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \underline{r}\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big)$, so $\overline{\ell}^* = \underline{r}$ (regardless of $\tilde{r}$).

We conclude with the characterization for some non-generic cases. Suppose $\underline{r} = \check{r}$. If $\underline{r} = \check{r} \leq \tilde{r}$, $\mathcal{W}\Big(\hat{\theta}_0; \underline{r}\Big) = \mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \underline{r}\Big) = \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big)$, and the planner is indifferent between any $\overline{\ell} \in \{\underline{r}, 1\}$. If $\underline{r} = \check{r} > \tilde{r}$, $\mathcal{W}\Big(\hat{\theta}_0; \underline{r}\Big) = \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \underline{r}\Big) < \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big)$, so $\overline{\ell}^* = 1$. Finally, suppose $\underline{r} = r^\dagger$. If $\underline{r} = r^\dagger > \tilde{r}$, the planner is indifferent between $\overline{\ell} \in \{\underline{r}, 1\}$ (since $\mathcal{W}\Big(\hat{\theta}_0; \underline{r}\Big) = \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; \underline{r}\Big) = \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big)$), and if $\underline{r} = r^\dagger \leq \tilde{r}$, $\overline{\ell}^* = \underline{r}$ (since $\mathcal{W}\Big(\hat{\theta}_0; \underline{r}\Big) = \mathcal{W}_{n\downarrow}\Big(\hat{\theta}_0; \underline{r}\Big) > \mathcal{W}_{n\uparrow}\Big(\hat{\theta}_0; 1\Big)$).

These results then imply that the optimal policy satisfies equation (B.2). $\square$

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