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Efficiency in trading markets with multi-dimensional signals ☆

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Abstract

There is a continuum of agents, each of whom trades a divisible asset via demand function competition. Individual valuations are determined by payoff shocks that are correlated across agents. Agents observe multi-dimensional signals about the payoff shocks; it is only assumed that the signals are normally and symmetrically distributed. We give three results about this economy. First, an equilibrium exists. Second, the equilibrium is constrained inefficient; a higher total surplus could be attained if agents submitted different demands. Third, a constrained-efficient outcome can be implemented by setting an appropriate capital-gains tax. The second result identifies a new type of inefficiency that only arises when agents observe multi-dimensional signals; the third result identifies the taxation policy that allows correcting this inefficiency.

JEL classification: D41; D43; D82; D83; G12; G14

Keywords: Information aggregation; Multi-dimensional signals; Demand function competition; Supply function competition; Rational expectations equilibrium

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1. Introduction

In financial markets, an agent's valuation of an asset is determined by a combination of multiple asset-specific shocks (e.g., dividend shocks) and multiple agent-specific idiosyncratic shocks (e.g., shocks to the agent's hedging needs, liquidity needs, regulatory constraints, etc.). It is natural to assume that agents observe different signals about the different shocks that affect their valuation. Thus, it is also natural to assume that agents observe multi-dimensional signals.

The presence of multi-dimensional signals suggests that, in general, there will be a limit to how much information can be aggregated by the asset's price. If all information were public, then the equilibrium price would reflect the agents' average expected valuation, and the quantity they purchase would reveal the idiosyncratic component of their valuations. However, this outcome will seldom be achieved if agents observe multi-dimensional private signals about the various shocks that determine their valuation. For example, efficiently distributing the asset across agents requires that each agent's demand be perfectly correlated with the idiosyncratic component of their valuations. Yet, in this case, the asset's price will not reflect the agents' private information about asset-specific shocks; after all, such information is not reflected in the agents' demand for the asset. Hence, the price would not reveal the agents' average expected valuation.

In this paper, we address the following questions. Are the trading strategies used by agents efficient? What taxation policy (if any) can increase the total surplus?

The model consists of a continuum of agents trading a divisible asset under demand function competition. The asset is supplied by a competitive supplier that has increasing marginal costs. Agents bear a quadratic asset-holding cost and an agent's marginal valuation is determined by a shock, which we refer to as the payoff shock. The payoff shocks can exhibit any degree of correlation across agents; the fact that payoff shocks are not perfectly correlated captures the fact that the valuation is determined by asset- and agent-specific shocks. Each agent observes multi-dimensional private signals about each of the payoff shocks. Agents are symmetric, and the information structure is Gaussian; we impose no additional assumptions on the information structure. We study the symmetric linear equilibria of the demand function competition game. The model is mathematically equivalent to a rational expectations equilibrium; it is a classic model of trading under asymmetric information.

Our first theorem states that a linear equilibrium exists. This result does not rely on any assumptions about the information structure beyond symmetry and normality. To the best of our knowledge, the existence of an equilibrium in a linear-quadratic demand function competition game has been established only for the case of agents observing either one-dimensional signals or certain types of two-dimensional signals (examples from the literature are discussed in the paper). Our existence proof differs from those found in the literature because the multi-dimensional nature of the information structure impedes explicitly solving for the coefficients in a linear equilibrium. Instead, we write the equilibrium conditions as a polynomial system of equations, and then reduce these equations to finding the roots of a single-variable polynomial of degree $2 \times L - 1$, where L is the number of signals each agent observes.

We then examine whether the equilibrium outcome is efficient. To this end, we compare the total surplus generated by an equilibrium – that is, the sum of the agents' welfare and the profits of the competitive supplier – with the maximum surplus that can be generated across all linear demand functions. The set of all linear demand functions provide a benchmark in which the trading mechanism remains fixed, but agents' behavior is not constrained to be an equilibrium. This benchmark describes the set of outcomes that might be achieved by a policy, such as implementing tax instruments, that modifies agents' incentives but does not change the trading mechanism.

	Agents Submit Non-Contingent Demands	Agents Submit Price-Contingent Demands
Effect of Disclosing a Public Signal	Morris and Shin (2002) Angeletos and Pavan (2007)	Amador and Weill (2010)
Optimal Taxation	Angeletos and Pavan (2009)	Vives (2017) Our Paper

Fig. 1. Literature Review.

Our approach is akin to that of analyzing the solution to a game in which agents behave as a team (as in Vives (1988); Angeletos and Pavan (2007)).

Our second theorem shows that every linear equilibrium of the game is generically inefficient. More precisely, for almost every value of the supply's slope, the total surplus generated by an equilibrium demand function is strictly less than the maximum surplus that can be attained by linear demand functions. The equilibrium is inefficient because the weight that the agents place on the different private signals they observe is different than that in the surplus-maximizing demand function. This type of inefficiency arises only when agents observe multi-dimensional and is thus different from those identified by the literature (which we explain in due course).

Our third theorem shows that every surplus-maximizing demand function is the equilibrium of an economy in which the agents' payoffs are subject to a tax. Hence, the newly identified inefficiency can be corrected using an appropriate type of tax. This tax reduces an agent's profits (resp. losses) when he buys an asset at a price lower (resp. higher) than the first-best price. We interpret this instrument as a capital gains tax and as a tax benefit that allows for the discounting of capital losses. The intuition underlying this result is that a positive capital-gains tax reduces the gains from buying (or selling) an asset because the price does not perfectly reflect the average valuation. Thus, such a tax, increases how much agents trade to satisfy their idiosyncratic needs.

A remarkable property of this tax is its simplicity. The equilibrium is inefficient because the weights that the agents place on the different signals they observe is different than the socially optimal ones. Hence, one might expect that it would be necessary to impose taxes that depend on the realization of specific signals, which would lead to a number of tax instruments that increase with the number of signals. However, this is not the case; a single tax is sufficient to implement an efficient outcome.

1.1. Literature

Our paper contributes to the growing literature on the welfare analysis of economies where agents have private information. Fig. 1 tabulates some recent papers, grouping them in terms of their assumptions and the type of welfare exercise they perform. The table distinguishes between studies about economies where agents submit price-contingent demands (as in our paper) with studies about economies where agents submit non-contingent demands. It also distinguishes between the type of welfare exercise carried out by different papers; some papers study the effects of disclosing public signals, while others study optimal taxation.

Vives (2017) studies a competitive economy of agents trading a divisible good where agents condition the quantities they trade on the equilibrium price. He shows that the equilibrium is constrained inefficient (much like in our model). In his paper, the price fails to aggregate the agents' information because the supply is subject to shocks. Our paper identifies a different source of inefficiency in markets: the use of private signals in multi-dimensional environments. This difference leads to differences in terms of policy recommendations. In Vives (2017), efficiency is restored using a Tobin-like tax that is applied to the total amount traded by an agent, which is independent of the realization of the payoff shocks. In contrast, in our model, efficiency is restored using a capital gains tax that does depend on the realization of the shocks.

Amador and Weill (2010) study an economy where agents have private information, and they can condition their actions on the equilibrium price. In their model, each agent solves a prediction problem (i.e., the payoff of an agent does not depend on the actions taken by the other agents), but there are information externalities. Their model maps onto our model by considering a specific information structure and taking the limit in which the supply is perfectly elastic (we provide this mapping formally in due course). We show that the equilibrium is, in general, constrained inefficient. We also contribute by analyzing how taxes can correct these inefficiencies and by studying an environment that allows for richer information structures.

There is a rich literature performing welfare analysis in economies where agents have private information, but they must choose their actions without knowing the equilibrium price. This is akin to studying an economy where agents submit demands that are not contingent on the price. Morris and Shin (2002) demonstrate that, in the presence of private information, public signals may be detrimental to welfare. Angeletos and Pavan (2007) show that such inefficiencies can be understood as the difference between the equilibrium degree and the socially optimal degree of coordination.[1] Angeletos and Pavan (2009) show that some inefficiencies can be corrected by a tax that has the same form as the capital gains tax in (33). Although the *type* of tax needed to correct for inefficiencies is the same as in our model, the design of the tax rate is different. In the Angeletos and Pavan (2009) model, the optimal tax is designed by measuring the difference between the equilibrium degree and the socially optimal degree of coordination; in our model, the equilibrium degree of coordination plays no role in market efficiency. This is because, in our model, agents can condition their purchase on the equilibrium price; hence they face no strategic uncertainty. Therefore, the source of the inefficiency and the design of the tax in our model is different than that in Angeletos and Pavan (2009).

Additional related papers are discussed as we provide our results.

The rest of our paper is organized as follows. Section 2 describes the model. The set of equilibria are characterized in Section 3 and Section 4 gives a preview of the efficiency results with an example. In Section 5, we present our efficiency result, and Section 6 analyzes the impact of taxes. We conclude our paper in Section 7. All proofs are given in the Appendix.

2. Model

2.1. Payoff environment and information structure

There is a continuum of agents, indexed by $i \in [0, 1]$, who trade a divisible asset. Agent i's payoff depends on the quantity of the asset she buys (q_i), the asset's price (p), and the realization of a payoff shocks θ_i. Agent i's payoff function is given by

$$u(\theta_i, q_i, p) \triangleq q_i\theta_i - q_i p - \frac{1}{2}q_i^2. \tag{1}$$

[1] See also Bayona (2015) for a model in which agents compete in quantities, but there is an endogenous noisy public signal. In that model, agents have common values, and so the equilibrium outcome in this economy converges to the first-best outcome as the endogenous public signal becomes perfectly precise.

The quadratic term in this expression can be interpreted as a proxy for risk aversion or as a proxy for limits to asset holding.[2]

There is a *competitive* supplier whose cost C of supplying quantity Q is

$$C(Q) \triangleq \frac{bQ^2}{2},$$

with $b > -1$ and $b \neq 0$.[3] The supplier's marginal cost is equal to bQ, so the equilibrium price must satisfy $p = bQ$. Hence the market-clearing condition implies that:

$$p = b \int q_i \, di. \tag{2}$$

The parameter b determines the price's sensitivity to aggregate demand (i.e., to $\int q_i \, di$). In the limit $b \to \infty$, the asset is inelastically supplied. In the limit $b \to 0$, the price converges to zero.

Each agent observes a multi-dimensional signal $s_i \in \mathbb{R}^L$. The payoff and information structure are determined by the joint distribution of variables $(\theta_i, s_i)_{i \in [0,1]}$. We assume that for some $H \in \mathbb{N} \backslash \{1\}$[4]:

$$\forall \left(h \in \{1, ..., H\}, \; i, j \in (\frac{h-1}{H}, \frac{h}{H}] \right), \quad (\theta_i, s_i) = (\theta_j, s_j).$$

In other words, we assume the payoff shocks and signals of every agent within an interval $(\frac{h-1}{H}, \frac{h}{H}]$ are the same signals. We make this assumption to keep a finite number of random variables in our model, but H can be any natural number larger than 1.[5] Hereafter, whenever we refer to two typical agents i and j (e.g., in $\text{corr}(\theta_i, \theta_j)$) we assume that $|i - j| > 1/H$.[6]

We assume that the joint distribution of payoff shocks and signals is symmetric, jointly normally distributed, and all variables have zero mean. The payoff and information structure are entirely determined by the joint distribution of payoff shocks and signals of two agents $(\theta_i, s_i, \theta_j, s_j)$, which is the same for any two agents i, j. The normality and symmetry assumptions are important simplifying assumptions; they allow us to keep the model tractable. The assumption that all random variables have zero mean only simplifies the notation; the analysis goes through unchanged if the means differ from zero.[7]

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[2] The payoff function (1) can also be interpreted as a second-order approximation of an agent's utility from holding the asset. An alternative interpretation of the model is to view agents as firms supplying a good (e.g., in electricity markets); in that case, the quadratic term is a quadratic cost for supplying the good. For a textbook discussion of different interpretations of the model, see Vives (2010).

[3] The case $b < 0$ corresponds to situations in which the seller has decreasing marginal costs. The condition $b > -1$ states that the slope is smaller in absolute value than the slope of the agents' marginal utility; this condition is necessary and sufficient for the maximum total surplus to be well defined (i.e., less than infinity). We assume that $b \neq 0$ because when $b = 0$, the price does not co-move with the demand. To address the case $b = 0$, it is necessary to add some technical qualifications that do not change the analysis substantively, so we leave it out of our analysis.

[4] Agent $i = 0$ is subject to the same payoff shock and observes the same signals as agent $i = 1/H$.

[5] We could have assumed that every agent is subject to a different payoff shock and observes a different signal. This approach requires studying a continuum of random variables, which requires addressing technical issues regarding the measurability of these random variables. These additional considerations, obfuscate the analysis without providing any new economic insight.

[6] Otherwise, $\text{corr}(\theta_i, \theta_j)$ may be trivially equal to 1 (e.g., if $i = 1/H$ and $j = 1/2H$).

[7] If the payoff shocks do not have zero mean, then the only necessary adjustment to our analysis is that the equilibrium demands will have an intercept whose mean is not equal to zero. However, the intercept's mean does not depend on the information structure (as proved by Bergemann et al. (2015)). It follows that generalizing the model to allow for nonzero means would require that we carry, throughout the paper, extra terms that yield no economic insight.

To facilitate the mathematical presentation, we define[8]:

$$\bar{\theta} \triangleq \int \theta_i \, di \quad \text{and} \quad \bar{s} \triangleq \int s_i \, di. \tag{3}$$

We remark that $\bar{\theta}$ is a random variable with variance:

$$\text{var}(\bar{\theta}) = \frac{(1 + (H-1)\text{corr}(\theta_i, \theta_j))}{H} \text{var}(\theta_i), \tag{4}$$

and analogously, $\bar{s}$ is an L-dimensional random vector.[9] The variance–covariance matrix of the vector s_i is denoted $\text{var}(s_i) \in \mathbb{R}^L \times \mathbb{R}^L$, and the vector of covariances is denoted $\text{cov}(s_i, \theta_i) \in \mathbb{R}^L$.

Throughout the paper, all vectors are interpreted as column vectors. Hence the multiplication of a matrix by a vector is interpreted according to the standard conventions of matrix algebra, and the product of two vectors is interpreted as their dot product. For example, we write

$$\mathbb{E}[\theta_i | s_i] = \left(\text{var}(s_i)^{-1} \text{cov}(s_i, \theta_i)\right) \cdot s_i.$$

So here the term inside the larger parentheses is a vector of dimension L that when multiplied by the L-dimensional vector s_i, yields a scalar.

Examples of Information Structures. First consider an information structure in which an agent observes only a one-dimensional signal about his payoff shock:

$$s_i = \theta_i + \varepsilon_i, \tag{5}$$

where ε_i is a conditionally independent noise term. Under this information structure, agents observe a noisy signal about a payoff shock that is imperfectly correlated across agents (i.e., $\text{corr}(\theta_{i1}, \theta_{j1}) \in (0, 1)$). This one-dimensional information structure is commonly used in the literature (see e.g. Vives (2011); Rostek and Weretka (2012)).

Consider now an example in which agents' payoff shock is given by the sum of two independent payoff shocks:

$$\theta_i = \theta_{i1} + \theta_{i2}. \tag{6}$$

Agents observe one signal about each shock:

$$s_{i1} = \theta_{i1} + \varepsilon_i, \qquad s_{i2} = \theta_{i2}; \tag{7}$$

here ε_i is a conditionally independent noise term. An agent perfectly observes θ_{i2}, but he observes only a noisy signal about θ_{i1}. Assume that the correlation between the payoff shocks is such that $\text{corr}(\theta_{i1}, \theta_{j1}) = 1$ and $\text{corr}(\theta_{i2}, \theta_{j2}) \in (0, 1)$. The shock θ_{i1} is interpreted as a shock to the asset's dividends, which is a common shock to all agents, while the shock θ_{i2} is interpreted as a shock to

[8] We could alternatively write $\bar{\theta}$ in terms of a summation as follows: $\bar{\theta} = \frac{1}{H} \sum_{i \in \{\frac{1}{H}, \ldots, 1\}} \theta_i$, and analogously for $\bar{s}$.

[9] It is possible to verify (4) as follows:

$$\text{var}(\bar{\theta}) = \mathbb{E}[\bar{\theta}\bar{\theta}] = \mathbb{E}\Big[\int \theta_i \, di \int \theta_j \, dj\Big] = \frac{1}{H^2} \sum_{i,j \in \{\frac{1}{H}, \ldots, 1\}} \mathbb{E}[\theta_i \theta_j] = \frac{(1 + (H-1)\text{corr}(\theta_i, \theta_j))}{H} \text{var}(\theta_i);$$

here we have used that all random variables have zero mean (so $\mathbb{E}[\theta_i \theta_j] = \text{cov}(\bar{\theta}_i, \bar{\theta}_j)$) and that the distribution of variables is symmetric (so $\text{cov}(\bar{\theta}_i, \bar{\theta}_j)$ is independent of the indexes i, j).

agent i's hedging needs. This two-dimensional information structure is similar to that analyzed in Ganguli and Yang (2009) and Manzano and Vives (2011).[10]

Finally, consider the case in which agents' payoff shock is given by the sum of two independent payoff shocks (as in (6)). Agents observe two signals as follows:

$$s_{i1} = \theta_{i1} \quad \text{and} \quad s_{i2} = \theta_{i2} + \kappa \int \theta_{i2} di; \tag{8}$$

here $\kappa \in (-1, 0)$. By taking the limit $b \to 0$, this information structure gives the same equilibrium conditions as those in Amador and Weill (2010).[11] In the limit $b \to 0$ agents care about the aggregate demand only because this gives information about the payoff shocks but not because it changes the equilibrium price.

2.2. *Market competition and solution concept*

We assume that agents compete by submitting their respective demand functions to a Walrasian auctioneer. Agent i submits function $x_i : \mathbb{R} \to \mathbb{R}$, where $x_i(p)$ is the quantity that agent i is willing to buy at price p. The demand function submitted by agent i will often be written as $x_i(s_i, p)$ in order to make explicit that this demand function is measurable with respect to s_i, the agent's private information.[12] We use X to denote the family of all measurable functions $x(s_i, p)$.

Given the functions submitted by all agents, $\{x_i(s_i, p)\}_{i\in[0,1]}$, the Walrasian auctioneer sets the price so that:

$$b \int x_i(s_i, p)\, di = p. \tag{9}$$

We denote by p^x the market-clearing price when all agents submit the same demand x as a function of their private signals (i.e., all agents use the same function $x_i(s_i, p)$). The price p^x is implicitly defined as follows[13]:

[10] In Ganguli and Yang (2009) and Manzano and Vives (2011) agents have CARA utilities, they are subject to an endowment shock, and they observe a signal about the value of the asset. The endowment shock is effectively a second signal that gives an agent additional information about his own preferences. Our information structure (7) shares similar elements; each agent perfectly observes the realization of an idiosyncratic shock and additionally observes a noisy signal about a common shock.

[11] The equilibrium conditions in Amador and Weill (2010) correspond to equations (7)–(10) in their paper. In their model, an agent observes p_i and $p = \int p_i di$ and an agent's action is given by y_i. Equation (8) provides a linear relation between y_i and p_i, so one can think of agents choosing p_i instead of y_i. By replacing (7) into (8) and then (8) into (10), we obtain:

$$p_i = -\theta_i + \mathbb{E}[v + \frac{a_i}{(1+\delta_i)}] + \text{constants}.$$

Here θ_i is one of the shocks in their model, which we map onto θ_{i1} in our model. The second shock is given by $\theta_{i2} = v + a_i/(1+\delta_i)$. An agent observes $s_{i1} = \theta_{i1}$ and can deduce, from (7) and (8), signal $s_{i2} = v + a_i$; hence we may write $s_{i2} = \theta_{i2} - \delta_i \bar{\theta}_2/(1+\delta_i)$.

[12] In so doing, we abuse notation slightly given that $x_i(p)$ is an *action* in the demand function competition game whereas $x_i(s_i, p)$ is a *strategy* in that game.

[13] When agents submit linear demand functions, there exists a unique market-clearing price. Thus, the set of symmetric linear equilibria does not depend on the outcome of the game when there are multiple market-clearing prices, or a market-clearing price does not exist. Note that no single agent can deviate and change the number of market-clearing prices, so the outcome of the game when there is not a unique market-clearing price does not affect the agent's incentives.

$$b\int x(s_i, p^x)\,di \triangleq p^x. \tag{10}$$

We study the set of symmetric equilibria of the demand function competition game.

Definition 1 *(Symmetric Equilibrium).* The function $x : \mathbb{R}^{L+1} \to \mathbb{R}$ is a symmetric equilibrium if

$$x(s_i, p) \in \arg\max_{x_i \in X} \mathbb{E}\left[x_i(s_i, p^x)\left(\theta_i - p^x - \frac{1}{2}x_i(s_i, p^x)\right)\right], \tag{11}$$

where p^x is defined in (10).

The equilibrium in our model has two interpretations. First, it can be seen as the Nash equilibrium of the demand function competition game. The model is also mathematically isomorphic to a rational expectations equilibrium.[14]

2.3. Linear demands

Throughout the paper, we focus on equilibria in which agents submit linear demand functions. Such a demand function is denoted by the pair of constants $(\alpha, \beta) \in \mathbb{R}^L \times \mathbb{R}$, where agent i's demand function is given by

$$x_i(s_i, p) = \alpha \cdot s_i - \beta p. \tag{12}$$

Thus a linear demand function has two components: $\alpha \cdot s_i$ is the demand function's intercept, and β is the demand function's slope. We say an equilibrium is *linear* if x is linear (as in (12)).

We use the following notation for the quantity bought by agent i:

$$q_i^{(\alpha,\beta)} \triangleq x(s_i, p^{(\alpha,\beta)}).$$

The equilibrium price and the equilibrium quantities when agents submit demand (α, β) are:

$$p^{(\alpha,\beta)} = \frac{b}{1+b\beta}\alpha \cdot \bar{s} \quad \text{and} \quad q_i^{(\alpha,\beta)} = \alpha \cdot (s_i - \bar{s}) + \frac{1}{1+b\beta}\alpha \cdot \bar{s}, \tag{13}$$

respectively.

The conditions in (13) are obtain as follows. The first condition is obtained by writing the market clearing condition for demand (α, β):

$$p^{(\alpha,\beta)} = b\int q_i di = b\int \alpha s_i - \beta p^{(\alpha,\beta)} di.$$

Since $p^{(\alpha,\beta)}$ does not depend on i we can take it out of the integral, and solve for $p^{(\alpha,\beta)}$. This way, we obtain the first equation in (13). On the other hand, the second equation in (13) is obtained by noting that:

$$q_i^{(\alpha,\beta)} = \alpha \cdot s_i - \beta p^{(\alpha,\beta)} = \alpha \cdot s_i - \beta\frac{b}{1+b\beta}\alpha \cdot \bar{s}.$$

Rearranging terms, we obtain the second equation in (13).

[14] See Vives (2010) for a textbook discussion.

2.4. Extension of linear demands

To guarantee that a linear equilibrium exists, we extend the agents' strategy space. The objective is to consider a set of demand functions that lead to a closed set of equilibrium prices and quantities.

Perfectly elastic demand We allow agent i to submit demand functions that have infinite slope (i.e., (α, ∞)). An agent who submits demand (α, ∞) buys (resp., sells) an infinite amount of the asset if its price is less than (resp., greater than) zero; if the price is equal to zero then he buys $\alpha \cdot (s_i - \bar{s})$ of the asset. If every agent submits demand (α, ∞), then agent i buys quantity $q_i = \alpha \cdot (s_i - \bar{s})$ and the market-clearing price is zero. This outcome corresponds to taking the limit $\beta \to \infty$ of the market-clearing price and quantities in (13).

Demands that are collinear with the supply If an agent submits a demand equal to $x_i(p) = p/b$, then we allow her to transmit – to the Walrasian auctioneer – a one-dimensional statistic of the signals she observes. This extension of the demand $x_i(p) = p/b$ is denoted by $((0, -1/b), r)$. An agent who submits demand $((0, -1/b), r)$ buys p/b units of the asset. If every agent submits demand $((0, -1/b), r)$, then the market-clearing price is $p = r\bar{s}$ and every agent buys p/b units of the asset. The demand $((0, -1/b), r)$ attains the market-clearing price and quantities given by (13) in the limits $\alpha \to 0$, $\beta \to -1/b$ and $\alpha/(1 + b\beta) \to r$.

The extension of linear demand functions is a technical device for guaranteeing the existence of an equilibrium. These extensions are used only in knife-edge examples, but they are needed if we are to prove the existence of an equilibrium *without* imposing additional restrictions on the information structure.[15]

2.5. Idiosyncratic component of demands

We now introduce some notation that will help us simplify the analysis in our paper. For a given profile of quantities $\{q_i\}_{i\in[0,1]}$, we define:

$$\delta_i \triangleq q_i - \int q_j dj.$$

We call this the idiosyncratic component of the quantity demanded by agent i. We characterize the equilibrium values of $\{\delta_i\}_{i\in[0,1]}$ and p instead of characterizing the quantities $\{q_i\}_{i\in[0,1]}$ and p.

When agents submit demand functions (α, β), the idiosyncratic component of the quantity purchased by agent i is denoted by

[15] If $\mathrm{corr}(\theta_i, \theta_j) = 0$ – that is, if payoff shocks are independently distributed – and if agents observe a signal of the form $s_i = \theta_i + \varepsilon$ (where ε is a common-value noise term), then the unique symmetric linear equilibrium is $(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}, \infty)$. In this case, there are no aggregate shocks to fundamentals, only aggregate shocks to the noise term; hence the price in equilibrium is zero. Now suppose that $\mathrm{corr}(\theta_i, \theta_j) = 1$ (i.e., payoff shocks are perfectly correlated), and that agents observe a signal of the form $s_i = \theta + \varepsilon_i$ for an independently distributed ε_i; then the unique symmetric linear equilibrium is $((0, -1/b), \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2})$. Here the only aggregate shock is to fundamentals, and there are no idiosyncratic payoff shocks; hence every agent buys the same quantity in equilibrium. In this case, the equilibrium price is equal to θ.

$$\delta_i^{(\alpha,\beta)} \triangleq q_i^{(\alpha,\beta)} - \int q_j^{(\alpha,\beta)} dj. \tag{14}$$

As before, it is possible to verify that:

$$\delta_i^{(\alpha,\beta)} = \alpha \cdot (s_i - \bar{s}). \tag{15}$$

Hence, $\delta_i^{(\alpha,\beta)}$ is proportional to the idiosyncratic component of agents' signals.

The reason why we characterize the idiosyncratic component of the demands instead of the demands themselves, is that this will simplify some of the algebra and some of the formulas we give. The reason this simplifies the formulas and algebra is that δ_i is always uncorrelated with p (i.e., $\text{cov}(\delta_i, p) = 0$). To see this, note that by construction $\int \delta_i di = 0$. The collinearity of the covariance implies that:

$$\int \text{cov}(\delta_i, p) di = \text{cov}(\int \delta_i di, p) = \text{cov}(0, p) = 0.$$

Since all distributions are symmetric, we have that

$$\forall \ell, j, \quad \text{cov}(\delta_\ell, p) = \text{cov}(\delta_j, p) = \int \text{cov}(\delta_i, p) di = 0.$$

We remark that this is not an equilibrium condition but a statistical condition that arises from the symmetry of the distribution of demands.

3. Equilibrium characterization

This section analyzes the demand function competition game. We begin by studying two benchmarks that correspond to the first-best and second-best outcomes. Then we discuss the implications of the market-clearing condition for the price and quantities. We continue by deriving the agents' first-order condition. Finally, we establish the existence of a linear equilibrium.

3.1. Benchmarks

We first characterize the competitive equilibrium when agents have complete information.

Proposition 1 *(First-Best: Equilibrium under Complete Information). If every agent observes $\{\theta_i\}_{i\in[0,1]}$, then the economy has a unique equilibrium in which the realized price and the quantity bought by agent i are, respectively, given by:*

$$p^* = \frac{b}{1+b}\bar{\theta} \quad \textit{and} \quad \delta_i^* = (\theta_i - \bar{\theta}). \tag{16}$$

This proposition characterizes the equilibrium when agents have complete information. The First Welfare Theorem guarantees that this equilibrium is Pareto efficient. Agents have quasi-linear utilities, so a Pareto optimal outcome maximizes the sum of the agents' utilities. Since agents are risk neutral, this also maximizes the sum of the expected utilities. Henceforth we refer to (δ_i^*, p^*) as the *first-best outcome*. Next, we characterize the competitive equilibrium when agents have symmetric information.

Proposition 2 *(Second-Best: Equilibrium under Symmetric Information). If every agent observes all the signals $\{s_i\}_{i\in[0,1]}$, then the economy has a unique equilibrium in which the asset's price and the quantity purchased by agent i are given by:*

$$p^{**}=\frac{b}{1+b}\mathbb{E}\big[\bar{\theta}|\{s_i\}_{i\in N}\big] \quad \textit{and} \quad \delta_i^{**}=\mathbb{E}\big[\theta_i-\bar{\theta}|\{s_i\}_{i\in N}\big]. \tag{17}$$

This second proposition characterizes the equilibrium when every agent observes the private signals of all other agents. We call (q_i^{**}, p^{**}) the *second-best outcome*.

3.2. Limits to information aggregation

In general, the equilibrium outcome will be different than the second-best outcome. This difference is not the result of agents using a "suboptimal" demand function, but it is because there is no outcome that can attain the second-best outcome. To gain some intuition, we write the equilibrium price and the idiosyncratic component of agents' demand:

$$p^{(\alpha,\beta)}=\frac{b}{1+b\beta}\alpha\cdot\bar{s} \quad \text{and} \quad \delta_i^{(\alpha,\beta)}=\alpha\cdot(s_i-\bar{s}). \tag{18}$$

The market-clearing price $p^{(\alpha,\beta)}$ is proportional to the product of α and the average of the signals. The idiosyncratic component of agents' demand $\delta_i^{(\alpha,\beta)}$ is equal to the product of α and the idiosyncratic component of the signals. In both cases, the signals are *projected* using the same vector α; this is not an optimality condition but rather a consistency condition implied by market clearing.

We now compare the market-clearing quantities and price given in (18) to their counterparts in the second-best outcome. We first define:

$$\alpha_\Delta^{**}\triangleq\big(\mathrm{var}(s_i-\bar{s})^{-1}\cdot\mathrm{cov}(s_i-\bar{s},\theta_i-\bar{\theta})\big) \quad ; \quad \alpha_p^{**}\triangleq\big(\mathrm{var}(\bar{s})^{-1}\cdot\mathrm{cov}(\bar{s},\bar{\theta})\big).$$

Using (17) along with standard formulas for Bayesian updating with normal random variables we compute explicitly the second-best outcome as follows:

$$\delta_i^{**}=\alpha_\Delta^{**}\cdot(s_i-\bar{s}) \quad ; \quad p^{**}=\frac{b}{1+b}\alpha_p^{**}\cdot\bar{s}.$$

Therefore the vectors α_Δ^{**} and $\alpha_p^{**}\in\mathbb{R}^L$ give the projection of the common and idiosyncratic component of the signals to compute the second-best outcome.

We can now understand when the outcome under some demand function (α,β) can coincide with the second-best outcome. If

$$\alpha_\Delta^{**}\not\propto\alpha_p^{**}, \tag{19}$$

then the projection of the average signals used to compute p^{**} will differ from the projection of the idiosyncratic component of the signals used to compute δ_i^{**}. Yet, for *any* demand function (α,β), the market clearing price and idiosyncratic component of the quantities are obtained using the same projection α (see (18)). Hence, when (19) is satisfied, then, for all demands (α,β) we have

$$\delta_i^{(\alpha,\beta)}\neq\delta_i^{**} \quad \text{or} \quad p^{(\alpha,\beta)}\neq p^{**}. \tag{20}$$

In other words, either the market-clearing quantities or the market-clearing price when agents submit demand function (α,β) differ from their respective values in the second-best outcome.

Since the equilibrium outcome cannot be the same as the second-best outcome, we are left with understanding the properties of the equilibrium outcome. In particular, we will study when the equilibrium outcome is efficient, in the sense that it maximizes total surplus. Importantly, the analysis will take into account the fact that no equilibrium outcome could possibly replicate the second-best outcome.

3.3. Demand of agents

We now characterize the optimality condition for agents in an equilibrium. For (11) to hold, the first-order condition must be satisfied, which implies that x is an equilibrium only if

$$x(s_i, p^x) = \mathbb{E}\left[\theta_i | s_i, p^x\right] - p^x. \tag{21}$$

Because the second-order condition for (11) is always satisfied, any function x that satisfies (21) is an equilibrium. In other words, x is an equilibrium if and only if (21) is satisfied.

In an equilibrium, each agent decides what quantity to buy (q_i) based on the market-clearing price (p). That price serves as a public signal of the realization of the payoff shocks $\{\bar{\theta}\}_{m\in M}$. Agent i's expectation in (21) contains p^x as a conditioning variable, which reflects that the equilibrium price contains information that is used by the agent to predict the payoff shocks.

Condition (21) can be written as[16]

$$\mathbb{E}[(\delta_i^{(\alpha,\beta)} - \delta_i^*) + \frac{1+b}{b}(p^{(\alpha,\beta)} - p^*)|s_i, p^{(\alpha,\beta)}] = 0. \tag{22}$$

In this representation, an agent's optimality condition is a prediction problem that explicitly gives two motives for trading. In any equilibrium, agent i's conditional expectation – of the difference between (a) the idiosyncratic component of the quantity he is buying and (b) the idiosyncratic component of the quantity under the first-best outcome *plus* $(1 + b)/b$ times the difference between (a) the equilibrium price and (b) the price under the first best – must be zero. The two terms being added correspond to agent i's dual trading motives. On the one hand, he wants the idiosyncratic component of his demand to be close to the efficient one because this will allow him to buy a greater quantity when his idiosyncratic shock is higher; on the other hand, he benefits from buying the asset when its price is lower than the first-best price (and vice versa). In equilibrium, then, there is no information the agent could possess that would lead him to improve his asset allocation *and* give him an opportunity to benefit from the asset's mispricing.

3.4. Existence of an equilibrium

We now establish that a linear equilibrium exists.

Theorem 1 *(Existence of an Equilibrium). A symmetric linear equilibrium exists.*

Theorem 1 states that an equilibrium exists. In general, there may be multiple equilibria.[17] We explain in due course the implications of equilibrium multiplicity for our analysis.

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[16] Equation (22) is obtained from (21) by replacing $x(s_i, p^x)$ and p^x with $q_i^{(\alpha,\beta)}$ and $p^{(\alpha,\beta)}$, respectively. We then place all the terms inside the expectation, which is possible because $q_i^{(\alpha,\beta)}$ is measurable with respect to $(s_i, p^{(\alpha,\beta)})$.

[17] For example, multiple equilibria may appear under information structure (7).

The classic approach to finding an equilibrium consists of conjecturing that a linear equilibrium exists and then finding the coefficients of this linear equilibrium in terms of the primitives of the model. We prove that such coefficients exist using the following approach. We reduce the system of equations that determine a linear equilibrium to a single-variable polynomial equation of degree $(2L-1)$. This polynomial equation determines the slope β of the demand functions submitted by agents; given an equilibrium β, the coefficients α are determined by linear equations.

The solution shows that following the standard solution method of finding closed-form solutions for the coefficients does not work for information structures in which the dimensionality of the signals is larger than 2. This difficulty arises because only up to two-dimensional signals the solution to the polynomial equation of degree $(2L-1)$ will have a closed-form solution; for higher-order dimensional information structures, a closed-form solution will, in general, not exist.

Theorem 1 guarantees the existence of a linear equilibrium. Henceforth, we will study the properties of linear equilibria. Du and Zhu (2017) show in a linear-quadratic setting like ours that there are no nonlinear equilibria when agents observe one-dimensional signals.[18] This suggests it is natural to focus on linear equilibria, but we do not know if every equilibrium is linear when agents observe multi-dimensional signals.

The study of trading models in informationally rich environments has been the focus of Lambert et al. (2018) and Lou et al. (2019). The proof of Proposition 1 shares common features with the existence proofs found in those papers. In particular, the strategy consists of conjecturing that a linear equilibrium exists and then verifying that the implied system of equations has a solution. Because the models are different in several respects, the results in those papers cannot be applied in our model. Most importantly, in our model, the price cannot aggregate all information because agents have private information about idiosyncratic shocks and not because there are noise traders.

4. An illustrative example

Before we give our results that concern the efficiency of our market, we provide an illustrative example. We use this example to illustrate and provide an intuition for the results we later provide. As we introduce the different elements in our example, we illustrate them in Fig. 2.

Information structures We assume that the payoff shock can be decomposed as the sum of three independent shocks:

$$\theta_i = \theta_{i1} + \theta_{i2} + \theta_{i3}.$$

Each agent observes one signal about each shock:

$$s_{i1} = \theta_{i1} + \varepsilon_i; \quad s_{i2} = \theta_{i2}; \quad s_{i3} = \theta_{i3}. \tag{23}$$

The noise term ε_i is independently distributed across agents. The correlations among payoff shocks of different agents satisfy $\text{corr}(\theta_{i1}, \theta_{j1}) = 1$ and $\text{corr}(\theta_{i2}, \theta_{j2})$, $\text{corr}(\theta_{i3}, \theta_{j3}) \in (0, 1)$. An agent perfectly observes θ_{i2} and θ_{i3} but observes only a noisy signal about θ_{i1}. We interpret θ_{i1} as a shock to an asset's dividends, which is a common shock to all agents. The shocks θ_{i2} and θ_{i3}

[18] They show that there are no nonlinear equilibria within the class of continuous-price ex-post equilibria.

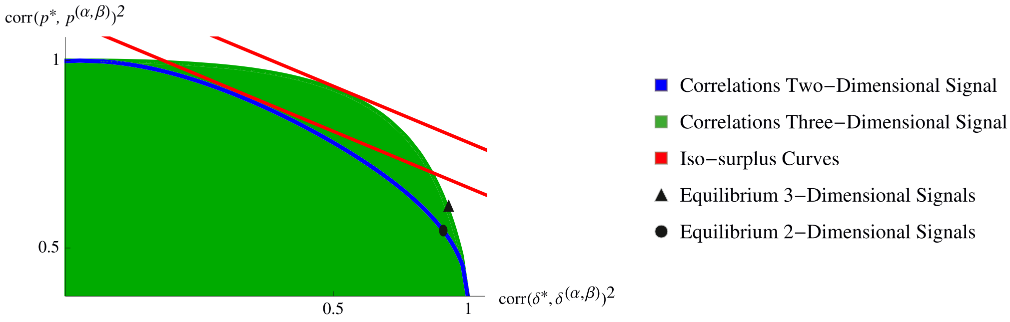

Fig. 2. This graph illustrates the set of squared correlations $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})^2$ and $\mathrm{corr}(p^*, p^{(\alpha,\beta)})^2$ that can be attained for two different information structures. The parametrization used to make this graph is: $\mathrm{var}(\theta_{i1}) = 1$; $\mathrm{var}(\theta_{i2}) = 0.68$; $\mathrm{var}(\theta_{i3}) = 0.68$; $\mathrm{var}(\varepsilon_i) = 9$; $\mathrm{corr}(\theta_{i1}, \theta_{j1}) = 1$; $\mathrm{corr}(\theta_{i2}, \theta_{j2}) = 0.97$; $\mathrm{corr}(\theta_{i3}, \theta_{j3}) = 0.0588$; H=1000; $b = -2/3$ (we use a negative slope for illustrative purposes, the main ideas do not hinge on this). (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

are interpreted as distinct shocks to the hedging needs of agent i; these shocks are mutually independent even though both are correlated across agents.

We compare the outcome under the three-dimensional signals given in (23) with the case where agents observe just two signals:

$$s_{i1} = \theta_{i1} + \varepsilon_i; \qquad s'_{i2} = \theta_{i2} + \theta_{i3}. \tag{24}$$

The information structure expressed here corresponds to an environment in which agents observe only the sum of the two shocks θ_{i2} and θ_{i3}.

Feasible correlations For each demand function (α, β) one can compute the following squared correlation:

$$\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})^2 = \mathrm{corr}(\theta_i - \bar{\theta}, \alpha(s_i - \bar{s}))^2 \quad \text{and} \quad \mathrm{corr}(\bar{\theta}, p^{(\alpha,\beta)})^2 = \mathrm{corr}(\bar{\theta}, \alpha\bar{s})^2. \tag{25}$$

Note that these correlations do not depend on the magnitude of α neither on β. Hence, the correlations are determined by the relative weight that a demand places on the different signals.

These correlations will play a crucial role in our analysis. Intuitively, $\mathrm{corr}(\bar{\theta}, p^{(\alpha,\beta)})^2$ measures how close is the aggregate quantity demanded by agents to the efficient one; $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})^2$ measures how close is the distribution of asset across agents to the efficient one. Fig. 2 plots the set of squared correlations $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})^2$ and $\mathrm{corr}(p^*, p^{(\alpha,\beta)})^2$ that can be attained across all α.

In a *two*-dimensional environment, the set of correlations is a curve. If agents place the same weight on both signals then the price co-moves exactly with $\bar{\theta}$ (recall that $\bar{\theta} = \bar{\theta}_1 + \bar{\theta}_2 + \bar{\theta}_3$) but the correlation $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})$ is low because an agent's allocation co-moves with the noise term ε_i. If agents place a zero weight on s_{i1}, then the idiosyncratic component of the demands will not co-move with the noise term and so the correlation $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})$ will be equal to 1; this case corresponds to the rightmost point of the curve. Yet, in this case, the price does not co-move with θ_1, which means that $\mathrm{corr}(p^*, p^{(\alpha,\beta)})$ is low.

In a *three*-dimensional environment, the set of feasible correlations is represented by an area rather than a curve. One might naïvely conjecture that the frontier of this area is the same in the two- and three-dimensional environments. After all, their only difference is that agents in the three-dimensional environment observe the two idiosyncratic shocks separately rather than merely their sum. However, that the frontier of this area *differs* from the curve we obtained in the two-dimensional environment will reveal that a surplus-maximizing demand function will not place equal weights on θ_{i2} and θ_{i3}.

Equilibrium demands The squared correlations induced by the equilibrium demands are indicated with a triangle and a dot. In the two-dimensional environment, the equilibrium demand function must be on the curve of feasible correlations. In the three-dimensional environment, the equilibrium demand function must be in the green area. In this example, the equilibrium demand function with three-dimensional signals is in the frontier of the set of feasible correlations.[19]

Surplus-maximizing demand functions We will show that the squared correlations (25) are a sufficient statistic for computing the total surplus generated in the economy (barring some minor requirements). In Fig. 2, the red lines are the iso-surplus curves, i.e., the set of squared correlations $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})^2$ and $\mathrm{corr}(p^*, p^{(\alpha,\beta)})^2$ that lead to the same total surplus. The iso-surplus curves are lines with slope $-(1+b)$. The surplus-maximizing demand is found by finding where the iso-surplus curve is tangent to the set of attainable correlations.

The slope of the iso-surplus curves is equal to $-(1+b)$. So changing b changes the slope of the iso-surplus curve, which, in turn, changes the surplus-maximizing demand. If b is large enough (resp. close enough to -1), the slope of the iso-surplus curves will be arbitrarily large (resp. arbitrarily close to zero). Hence, if b is large enough (resp. close enough to -1) the surplus-maximizing demand will maximize $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})^2$ (resp. $\mathrm{corr}(p^*, p^{(\alpha,\beta)})^2$).

Constrained inefficiency We will show that the value of b does not change the correlations induced by the equilibrium demands. Hence, changing b will change the iso-surplus curve, but not the equilibrium demands (i.e., the dot and the triangle in Fig. 2). Therefore, the surplus-maximizing demand function can be to the right or to the left of the equilibrium demands, which will depend on the value of b. However, generically (meaning for almost every value of b), the equilibrium demand will not be the same as the surplus-maximizing demand. Thus, a higher surplus could be attained if agents submitted demand functions that are different than the equilibrium demand function.

Effect of tax rate We show that a planner can use a tax instrument to increase total surplus. In particular, we consider a tax policy where each agent makes a tax payment:

$$\tau q_i^{(\alpha,\beta)}(p^* - p^{(\alpha,\beta)}).$$

Here p^* is the equilibrium price in the first-best outcome. We interpret this as a capital-gains tax; the details of this interpretation are relegated to Section 6. We now discuss the effect of this tax on the equilibrium outcome.

By appropriately setting the tax rate τ, the planner can implement the surplus-maximizing demand function (i.e., induce this demand function as an equilibrium of the economy). For example, if b is large enough, then the efficient demand function is implemented by a capital-gains

[19] In a previous version of this paper (cf. Heumann (2018)), we give sufficient conditions on the information structure so that the equilibrium demands will be in the frontier of the set of feasible correlations. However, for certain information structures, the equilibrium may not be on the frontier of this set.

tax close to 1. The intuition for this result is that the tax will prompt agents to ignore the aggregate shock $\bar{\theta}$, so in equilibrium, the correlation $\mathrm{corr}(\delta^*, \delta_i^{(\alpha,\beta)})^2$ will be maximized.

Fig. 2 allows appreciating the simplicity of the tax rate. A priori, when agents observe three-dimensional signals, the equilibrium demand function could be in the interior of the set of feasible correlations. Hence, the tax policies to correct the inefficiencies could be completely different than those needed when agents observe two-dimensional signals. In particular, one could have expected that it is necessary to tax agents differently depending on how they use their private information. However, this is not the case. The same tax allows correcting the inefficiencies, regardless of how complex is the information structure.

5. Efficiency analysis

We now analyze the efficiency of the equilibrium outcome. We first provide the benchmark used to analyze whether the equilibrium demands are efficient and then show that they are (in general) inefficient.

5.1. Total surplus

The *total surplus* is the sum of the agents' welfare and the seller's profits:

$$S \triangleq \int u\left(\theta_i, q_i, p\right) di + (pQ - C(Q)), \tag{26}$$

where $Q = \int q_i di$ and $p = bQ$. In this expression, the first term on the right-hand side is the agents' welfare and the second term is the supplier's profit. We define:

$$S^{(\alpha,\beta)} \triangleq \mathbb{E}\left[\int u(\theta_i, q_i^{(\alpha,\beta)}, p^{(\alpha,\beta)})\, di + p^{(\alpha,\beta)} \int q_i^{(\alpha,\beta)}\, di - \frac{b\left(\int q_i^{(\alpha,\beta)}\, di\right)^2}{2}\right]. \tag{27}$$

That is, $S^{(\alpha,\beta)}$ is the surplus generated when agents submit demand function (α, β). The maximum surplus that can be attained across all linear demand functions is denoted by:

$$\hat{S} \triangleq \max_{(\alpha,\beta)\in\mathbb{R}^{L+1}} S^{(\alpha,\beta)}. \tag{28}$$

We study when the equilibrium surplus is equal to $\hat{S}$.

We now characterize the set of demand functions that maximize total surplus. We express the total surplus as the sum of two terms: (i) the surplus generated by the aggregate quantity demanded by agents, and (ii) the surplus generated by efficiently distributing the aggregate supply across agents.

Proposition 3 *(Characterization of Maximum Surplus). A demand function* $\left(\alpha, \beta\right) \in \mathbb{R}^{L+1}$ *is solution to* (28)*, if and only if,*

$$\alpha \in \arg\max \quad \frac{\mathrm{var}(\theta_i - \bar{\theta})}{2} \mathrm{corr}(\delta_i^{(\alpha,\beta)}, \delta_i^*)^2 + \frac{\mathrm{var}(\bar{\theta})}{2(1+b)} \mathrm{corr}(p^{(\alpha,\beta)}, p^*)^2; \tag{29}$$

$$\mathrm{var}(\delta_i^{(\alpha,\beta)}) = \mathrm{cov}(\delta_i^{(\alpha,\beta)}, \delta_i^*); \tag{30}$$

$$\mathrm{var}(p^{(\alpha,\beta)}) = \mathrm{cov}(p^{(\alpha,\beta)}, p^*). \tag{31}$$

This proposition characterizes the set of optimal linear demand functions that maximize total surplus in terms of three conditions. The first condition (29) states that the relative weight on the different signals must maximize a linear combination of the squared correlations $\text{corr}(\delta_i^{(\alpha,\beta)}, \delta_i^*)$ and $\text{corr}(p^{(\alpha,\beta)}, p^*)^2$. This captures that the signals can be used to distributed the asset efficiently across agents (captured by $\text{corr}(\delta_i^{(\alpha,\beta)}, \delta_i^*)$) or the signals can be used to increase the efficiency of the aggregate quantity demanded by agents (captured by $\text{corr}(p^{(\alpha,\beta)}, p^*)^2$). As the supply becomes more inelastic (i.e., b increases), the weight on the correlation $\text{corr}(p^{(\alpha,\beta)}, p^*)^2$ decreases because the aggregate supply co-moves less with the aggregate shock. In the limit $b \to \infty$, the aggregate quantity demanded does not co-move at all with the aggregate shock, so the total surplus is independent of the correlation $\text{corr}(p^{(\alpha,\beta)}, p^*)^2$.

Note that the correlations do not change with the magnitude of α or with β. Hence, (29) imposes no restriction on the magnitude of α or the slope β. The magnitude of α and the slope β are determined by the conditions (30) and (31). These last two conditions will play essentially no role in our analysis because they are satisfied by every equilibrium demand and by every surplus-maximizing demand (we show this in the analysis that follows).

5.2. *Inefficiency*

We compare the equilibrium surplus with the maximum surplus that can be attained by some linear demand function. We first give the theorem stating that the equilibrium is generically constrained inefficient and then provide intuition for the result.

Theorem 2 *(Inefficiency of Equilibrium). If* (19) *is satisfied, then there exists at most* $2L-1$ *constants* $\{c_1, ..., c_{2L-1}\}$*, such that if* $b \notin \{c_1, ..., c_{2L-1}\}$*, then for every equilibrium demand function* (α, β)*,* $S^{(\alpha,\beta)} < \hat{S}$.

Theorem 2 states that every equilibrium is inefficient in the sense that it does not maximize the total surplus for almost every value of b. Note that, even if there are multiple equilibria, all of these equilibria will be inefficient for almost every value of b. The equilibrium demands are inefficient when compared with what can be achieved by agents submitting linear demand functions. This suggests that it is possible to increase the total surplus by changing agents' behavior.

Condition (19) is violated when agents observe one-dimensional signals. When agents observe one-dimensional signals, the terms in (19) are scalars, so the proportionality condition is always satisfied. In this case, the equilibrium outcome is the same as the second-best outcome.[20] In multi-dimensional environments, when (19) is violated, the signals can be projected onto a one-dimensional *sufficient* statistic so the information structure is isomorphic to a one-dimensional environment.[21] Of course, for generic multi-dimensional information structures, (19) will be satisfied

[20] Vives (2014) shows that this type of environment may provide a resolution of the Grossman-Stiglitz paradox of informationally efficient markets.

[21] For example, one can augment any one-dimensional environment by allowing agents to observe additional signals that are only noise without changing any qualitative feature of the equilibrium. For example, suppose agents observe a signal as in (5) and additionally a second signal $s_{i2} = \varepsilon_i'$, which is simply independent noise. Analyzing this situation would be equivalent to analyzing an environment where agents observe only (5); after all, the second signal is a noise term that can be ignored. More generally, any multi-dimensional signal that does not satisfy (19) can be written as a one-dimensional signal about the payoff shocks plus additional signals that only contain noise.

To understand why the equilibrium is inefficient, we begin by explaining what components of an equilibrium outcome are not inefficient.

Proposition 4 *(Magnitude of Intercept and Slope of Equilibrium Demands). If* (α, β) *is an equilibrium demand function, then it satisfies* (30) *and* (31).

This proposition states that the magnitude of the intercept α and the slope of the demand function β are determined the same way in equilibrium and in an efficient demand function. Hence, the equilibrium is *not* inefficient because of these two conditions. Hence, it must be that an equilibrium demand function does not satisfy (29). This means that the relative weights agents place on the different signals is inefficient.

To gain an intuition for why (29) is not satisfied, we give an alternative characterization of the intercept of an equilibrium demand function.

Proposition 5 *(Characterization of Intercepts). For a given* α*, there exists* β *such that* (α, β) *is an equilibrium demand function if and only if* α *satisfies* (29) *and for all* $\ell \in L$:

$$\operatorname{cov}\left(\theta_i - \bar{\theta} - \mathbb{E}[\theta_i - \bar{\theta} \mid \alpha(s_i - \bar{s})], s_{i\ell} - \bar{s}_\ell\right) + \operatorname{cov}\left(\left(\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha \bar{s}]\right), \bar{s}_\ell\right) = 0. \tag{32}$$

This proposition characterizes the set of equilibrium α. What is noteworthy is that the relative weight on the signals is determined by (32), which does not depend on b.[22] However, the intercept α of a linear demand function that maximizes total surplus is determined by (29), which depends on b. Hence, there is no reason why the equilibrium and the surplus-maximizing α would coincide. In fact, generically, they do not coincide.

The intuition for why the equilibrium α does not depend on b is as follows. Agents try to buy or sell the asset based on their payoff shock but they do not care about the decomposition in terms of an idiosyncratic and a common component separately. Thus, the elasticity of the supply does not play a role. For example, in the limit $b \to \infty$ the aggregate demand is constant, but agents still benefit from buying the asset when the price is higher than the average payoff shock.

6. Tax policies

In this section, we analyze how taxation policies allow one to implement a demand function that maximizes total surplus. We first give the payoff function for agents in the presence of the tax policy and then explain how this tax allows us to induce the surplus-maximizing demands.

6.1. Model with taxes

Here we assume that agent preferences are given by

$$u(\theta_i, q_i, p) = q_i\left(\theta_i - p - \frac{1}{2}q_i\right) - \tau q_i(p^* - p). \tag{33}$$

We refer to this environment as "an economy with taxes τ". Hence the tax *rate* is τ, and the total taxes paid by agent i amount to $\tau q_i(p^* - p)$. These total taxes depend on the equilibrium price

[22] Note that b appears in the first-order condition (22) but the mapping from the aggregate payoff shock to the price also depends on b. So, in equilibrium, one can write this first-order condition in such a way that it only depends on the payoff shocks and not the price (as in (32)), in which case it does not depend on b.

and also on the price that would prevail under complete information. We continue to compute the surplus generated by an economy with taxes τ as in (26). That is, we assume that the taxes are a net transfer from the agents to the government that does not change the computation of total surplus.

To interpret the tax itself, suppose that the rate satisfies $\tau \in [0, 1]$. Our interpretation is that agents initially trade under asymmetric information and that, in the long run, the average prevailing price is p^*. This scenario could be modeled as incorporating a second period in which the payoff shock becomes common knowledge and so yield the prevailing price p^*. Hence an agent's total tax payment is determined both by the price p at which he trades the asset and the second-period price. If the second-period price p^* is higher (resp. lower) than p, then agent i pays (resp. receives) $\tau q_i(p^* - p)$. Therefore, τ is akin to a capital gains tax for agents who profit from an asset's price change and to a tax deduction for agents who experience capital losses from an asset's price change. Although the tax can be more naturally interpreted when $\tau \in [0, 1]$, we allow for any $\tau \in \mathbb{R}$. For example, if $\tau < 0$, then an agent receives extra compensation upon realizing a capital gain. We remark that all our results go through if the tax payment in (33) is determined by the second-best price instead of the first-best price (that is, if the final term is $\tau q_i(p^{**} - p)$).[23]

6.2. Impact of taxes on outcomes

Under complete information, the equilibrium price is p^*, and so every agent pays zero tax, regardless of the value of τ. This implies that the presence of τ does not change the equilibrium outcome in the case of complete information. In other words, if every agent observes all the payoff shocks $\{\theta_i\}_{i\in[0,1]}$, then the outcome in the economy with tax rate τ is still the one given by (16). Similarly, if there is no private information among agents (i.e., if each agent observes $\{s_i\}_{i\in[0,1]}$) then the outcome is still the same as in (17).

Even though the tax rate τ changes neither the first-best nor second-best outcome, it nonetheless affects the equilibrium outcome when agents trade under asymmetric information. To explain how τ modifies an agent's incentives to trade, we rewrite his optimality condition. In a model with taxes, a linear demand function $(\alpha, \beta) \in \mathbb{R}$ is an equilibrium if and only if[24]

$$\mathbb{E}[(\delta_i^{(\alpha,\beta)} - \delta_i^*) + (1-\tau)\frac{b}{1+b}(p^{(\alpha,\beta)} - p^*)|s_i, p^{(\alpha,\beta)}] = 0. \tag{34}$$

That is, an agent still has two motives for trading: (i) he wants to benefit from the idiosyncratic component of his payoff shock; and (ii) he will look to benefit from any difference between the first-best price and the equilibrium price. However, the tax rate τ changes the weights that agents place on these two motives. On the one hand, if the tax rate is near unity (i.e., if $\tau \approx 1$) then the agent does not benefit from the difference between the equilibrium price and the price in the first-best outcome; hence he trades mostly so that the idiosyncratic component of his demand will be similar to the idiosyncratic component of his demand in the first-best outcome. On the other hand, a tax rate that is both large and negative (i.e., $\tau \to -\infty$) increases an agent's incentive to

[23] By construction $p^{**} = \mathbb{E}[p^*|\{s_i\}_{i\in[0,1]}]$ and so $\mathbb{E}[p^*|\mathcal{I}_i] = \mathbb{E}[\mathbb{E}[p^*|\mathcal{I}_i, \{s_i\}_{i\in[0,1]}]|\mathcal{I}_i] = \mathbb{E}[p^{**}|\mathcal{I}_i]$, regardless of what is the information $\mathcal{I}_i$ that is available to agent i (which includes his private signals and the information revealed by the equilibrium price). Thus, all computations in the paper remain unchanged if we replace p^* with p^{**}.

[24] The proof of (34) mimics the proof of (22); the only change is that all $(p^{(\alpha,\beta)} - p^*)$ terms become $(1-\tau)(p^{(\alpha,\beta)} - p^*)$.

benefit from the difference between the equilibrium price and the price in the first-best outcome; he is, therefore, less concerned about the idiosyncratic component of his demand.

Our final theorem shows that the tax can be used to implement a demand function that maximizes the total surplus.

Theorem 3 *(Optimal Taxation in Economy with Elastic Supply). If* (α, β) *is a solution to* (28)*, then this demand function is an equilibrium demand function in the economy with taxes:*

$$1 - \tau = \frac{\operatorname{cov}(\alpha \bar{s}, \frac{\bar{\theta}}{1+b})}{\operatorname{var}(\alpha \bar{s})}. \tag{35}$$

This theorem states that the tax rate τ allows implementing a surplus-maximizing demand function. The payments made by each agent depend only on the quantity of the asset they bought and the difference between the equilibrium price and the price in the first-best outcome. In this sense, the tax's design is remarkably simple; the tax need not have different effects depending on the various realizations of particular signals. While we show that an efficient demand function can be implemented as an equilibrium using an appropriate tax, we do not know whether this will be a unique equilibrium.

To compute the tax rate τ that implements a demand function that maximizes total surplus as an equilibrium, it is necessary to compute $\operatorname{cov}(\alpha \bar{s}, \bar{\theta})$. So, in general, to compute the optimal tax rate, precise knowledge of the information structure is required. A notable exception is when the asset is inelastically supplied; in this case, it is optimal to set a tax rate equal to 1.

Proposition 6 *(Minimizing the Price Distortion or the Allocative Distortion). In the limit* $b \to \infty$ *a tax rate* $\tau \to 1$ *implements the surplus-maximizing demand function.*

In the limit $b \to \infty$, the aggregate quantity demand must be equal to zero, so the only source of gains is the difference between the agents' payoff shocks. If the tax rate is close to 1, then the agents trade only to exploit the gains from the differences between their payoff shocks. For this reason, if the tax rate is close to 1, then the asset is efficiently allocated across agents.

7. Conclusions

We provided a welfare analysis of an economy where agents observe multi-dimensional signals. We showed that the equilibrium is inefficient and total surplus can be increased using a capital-gains tax. The main intuition is that this tax can dampen or exacerbate how much agents gain from their idiosyncratic needs relative to how much they gain from differences between the equilibrium price and the average shock. The results illustrate how optimal policy must incorporate the richness of the information structure.

An interesting direction for future research is performing welfare analysis in environments with ex-ante heterogeneous agents when agents observe multi-dimensional signals. This may bring up novelties that do not arise in symmetric environments because the price will reveal different amounts of information for different agents (see for example Rostek and Weretka (2012)).[25] In an environment where agents observe multi-dimensional signals, how an agent

[25] See also Rostek and Weretka (2015), Rahi and Zigrand (2018), and Rahi (2020) for the implications of asymmetric payoff environments for information acquisition and competition. See Dávila and Parlatore (2017) for the implications

uses his private information will depend on the agent's characteristics (e.g., uncertainty about his payoff shock or the size of the agent's quadratic asset-holding cost).

8. Appendix

The appendix is divided into four sections. The first three sections prove the first three theorems, and the final section gives the proofs of propositions, lemmas, and corollaries.

8.1. Proof of Theorem 1

To prove Theorem 1 we need to prove that there exists a demand function $(\alpha, \beta) \in \mathbb{R}^{L+1}$ that satisfied (21). Because (21) is equivalent to (22), we prove that there exists a linear demand (α, β) that satisfies:

$$\mathbb{E}[(\delta_i^{(\alpha,\beta)} - \delta_i^*) + \frac{1+b}{b}(p^{(\alpha,\beta)} - p^*)|s_i, p^{(\alpha,\beta)}] = 0.$$

We recall that $\theta_i = \delta_i^* + p^* b/(1+b)$. Throughout this proof we make the following two assumptions: (i) $\text{cov}(s_i, \theta_i)$ is *not* equal to an L-dimensional vector of zeros, and (ii) $\text{var}(\bar{s})$ is an invertible matrix. If the first of these assumptions is not satisfied (i.e., if $\text{cov}(s_i, \theta_i) = (0, ...0)$), then it is immediate to check that $x_i(p) = 0$ is an equilibrium. At the end of this proof, we explain how the proof we presented can be rewritten the same way when the second assumption is not satisfied (i.e., when $\text{var}(\bar{s})$ is not invertible).

We first write (22) as a system of equations. Because s_i, $p^{(\alpha,\beta)}$, $\delta_i^{(\alpha,\beta)}$, p^* and δ_i^* have zero mean, demand (α, β) satisfies (22) if and only if[26]:

$$\forall z \in \{s_{i1}, ..., s_{iL}, p^{(\alpha,\beta)}\}, \quad \text{cov}(\delta_i^* - \delta_i^{(\alpha,\beta)} + \frac{b}{1+b}(p^* - p^{(\alpha,\beta)}), z) = 0. \tag{36}$$

Observe that (36) corresponds to $L+1$ equations and a linear demand (α, β) is defined by $L+1$ coefficients. We need to show that this system of equations always has a solution.

We now write separately the equations regarding the signals $\{s_{i1}, ..., s_{iL}\}$ and $p^{(\alpha,\beta)}$. We first observe that:

$$\text{cov}((\delta_i^* - \delta_i^{(\alpha,\beta)}) + \frac{1+b}{b}(p^* - p^{(\alpha,\beta)}), p^{(\alpha,\beta)}) = 0 \iff \text{cov}(\alpha \cdot \bar{s}, p^*) = \frac{b\text{var}(\alpha \cdot \bar{s})}{1+\beta b}. \tag{37}$$

This equivalence is proved by first observing that for all $(\alpha, \beta) \in \mathbb{R}^{L+1}$, $\text{cov}(\delta_i^*, p^{(\alpha,\beta)}) = \text{cov}(\delta_i^{(\alpha,\beta)}, p^{(\alpha,\beta)}) = 0$ and then replacing $p^{(\alpha,\beta)}$ with its expression in (13). We now write the other L equations in (36) in matrix form:

$$\text{cov}(\delta_i^* - \delta_i^{(\alpha,\beta)} + \frac{1+b}{b}(p^* - p^{(\alpha,\beta)}), s_{il}) = 0 \iff$$
$$\text{cov}(s_i, \theta_i) = \text{var}(s_i - \bar{s}) \cdot \alpha + \frac{(1+b)\text{var}(\bar{s}) \cdot \alpha}{1+b\beta}. \tag{38}$$

---

of heterogeneous agents for information aggregation and Babus and Kondor (2018) for a trading model where agents are heterogenous in their trading counterparts.

[26] For any normal random variables $(y, x_1, ..., x_K)$:

$$\mathbb{E}[y|x_1, ..., x_K] = 0 \iff \mathbb{E}[y] = 0 \text{ and } \text{cov}(y, x_1) = ... = \text{cov}(y, x_K) = 0.$$

.

To prove this equivalence we write $p^{(\alpha,\beta)}$ and $\delta_i^{(\alpha,\beta)}$ as in (13) and (15) and note that $\frac{b}{1+b}p^* + \delta_i^* = \theta_i$. It follows that a demand function (α, β) is an equilibrium if and only if it satisfies (37) and (38). Conditions (37) and (38) are interpreted in the natural way for the demands (α, ∞) and $((0, -1/b), r)$ (see Section 2.4 for the definitions of (α, ∞) and $((0, -1/b), r)$). For demand (α, ∞) these conditions reduce to:

$$\mathrm{cov}(\alpha \cdot \bar{s}, p^*) = 0 \text{ and } \mathrm{cov}(s_i, \theta_i) = \mathrm{var}(s_i - \bar{s}) \cdot \alpha. \tag{39}$$

For demand $((0, -1/b), r)$ these conditions reduce to[27]:

$$\mathrm{cov}(r\bar{s}, p^*) = \mathrm{var}(r\bar{s}) \text{ and } \mathrm{cov}(s_i, \theta_i) = \mathrm{var}(\bar{s})r.$$

We now solve for α using the second equation in (39) and then replace this expression in the first equation in (39); we conclude that an equilibrium with slope $\beta = \infty$ exists if

$$\mathrm{cov}\left((\mathrm{var}(s_i - \bar{s})^{-1} \cdot \mathrm{cov}(s_i, \theta_i)) \cdot \bar{s}, p^* \right) = 0. \tag{40}$$

Throughout the rest of this proof we assume that (40) is *not* satisfied (as otherwise the existence of an equilibrium is guaranteed to exists, as we just proved).

If $(\mathrm{var}(s_i - \bar{s}) + \frac{1+b}{1+\beta b}\mathrm{var}(\bar{s}))$ is invertible then we can write (38) as follows:

$$\alpha = \frac{1+\beta b}{1+b}\left(\frac{1+\beta b}{1+b}\mathrm{var}(s_i - \bar{s}) + \mathrm{var}(\bar{s})\right)^{-1} \mathrm{cov}(s_i, \theta_i). \tag{41}$$

We denote the determinant of matrix:

$$\left(\frac{(1+\beta b)}{1+b}\mathrm{var}(s_i - \bar{s}) + \mathrm{var}(\bar{s})\right)$$

by $det(\beta)$, which is equal to the sum of several terms, each term being the product of L elements of the matrix. Therefore, $det(\beta)$ – as a function of β – is a polynomial of degree less or equal than L.

We now prove that the degree of polynomial $det(\beta)$ is also no less than degree L. Observing that[28]:

$$\det(\beta) \geq \left(\frac{1+\beta b}{1+b}\right)^L det(s_i - \bar{s}) + det(\bar{s}) \tag{42}$$

This inequality can be satisfied in the limit $\beta \to \infty$ only if the degree of the polynomial $det(\beta)$ is no less than degree L. Therefore, $det(\beta)$ is a polynomial of degree L *exactly*. Because $det(\beta)$ is a polynomial of degree L, there exists at most L values of β such that $det(\beta) = 0$; we denote these values by $\{\tilde{\beta}_1, ..., \tilde{\beta}_L\}$.

We denote the adjugate of $(\frac{1+\beta b}{1+b}\mathrm{var}(s_i - \bar{s}) + \mathrm{var}(\bar{s}))$ by $A(\beta)$ and note that[29]:

$$\forall \beta \notin \{\tilde{\beta}_1, ..., \tilde{\beta}_L\}, \quad \left(\frac{1+\beta b}{1+b}\mathrm{var}(s_i - \bar{s}) + \mathrm{var}(\bar{s})\right)^{-1} = \frac{A(\beta)}{det(\beta)}. \tag{43}$$

[27] This last two equations correspond to taking the limits $\alpha \to 0$, $b \to -1/b$, and $\alpha b/(1+\beta b) \to r$ in (37) and (38).

[28] For any positive definite matrices A, B: $\det(A+B) \geq \det(A) + \det(B)$, which implies (42) (simply replace A and B with $\mathrm{var}(\frac{1+\beta b}{1+b}(s_i - \bar{s}))$ and $\mathrm{var}(\bar{s})$ and note that $\mathrm{var}(\frac{1+\beta b}{1+b}(s_i - \bar{s})) = \left(\frac{1+\beta b}{1+b}\right)^L \mathrm{var}(s_i - \bar{s})$.

[29] This is one of the standard formulas for writing the inverse of a matrix (see, for example, Schay (2012), p. 240).

For any $\beta \notin \{\tilde{\beta}_1, ..., \tilde{\beta}_L\}$, we can write (41) as follows:

$$\alpha = \frac{1+\beta b}{(1+b)det(\beta)} A(\beta) \cdot \text{cov}(s_i, \theta_i). \tag{44}$$

To make the notation more compact, we define:

$$v(\beta) \triangleq A(\beta) \cdot \text{cov}(s_i, \theta_i).$$

By replacing (44) into (37) and simplifying terms, we obtain the following equation that determines β:

$$cov(v(\beta) \cdot \overline{s}, p^*) = \frac{\text{var}(v(\beta) \cdot \overline{s})}{det(\beta)} \tag{45}$$

Multiplying both sides by $det(\beta)$ we write the equation as a polynomial equation:

$$cov(v(\beta) \cdot \overline{s}, p^*) det(\beta) = \text{var}(v(\beta) \cdot \overline{s}). \tag{46}$$

The rest of the proof proceeds in two steps: (i) we prove that there exists $\beta \in \mathbb{R}$ that solves (46), (ii) we prove that for any $\beta \in \mathbb{R}$ that solves (46), there exists an equilibrium in which agents submit a demand function (α, β), where α is determined by (44) (if $\beta \in \{\tilde{\beta}_1, ..., \tilde{\beta}_L\}$ then α is determined by taking the limit of (44), which we prove to be well defined).

(First Step) We first prove that the left-hand-side of (46) – as a function of β – is a polynomial of degree $2 \times L - 1$. Each element of $A(\beta)$ is equal to (or the negative of) the determinant of one of the principal minors of $(\frac{1+\beta b}{1+b}\text{var}(s_i - \overline{s}) + \text{var}(\overline{s}))$, so each element of $A(\beta)$ is a polynomial of degree $L - 1$ at most.[30] Thus, each element of $v(\beta)$ is also a polynomial of degree at most $L - 1$. We now prove that an element of $v(\beta)$ is a polynomial of degree $L - 1$ exactly. Note that:

$$\lim_{\beta \to \infty} \frac{\frac{1+\beta b}{1+b} cov(v(\beta) \cdot \overline{s}, p^*)}{det(\beta)}$$

$$= \lim_{\beta \to \infty} \text{cov}\left((\text{var}(s_i - \overline{s}) + \frac{1+b}{1+\beta b} \text{var}(\overline{s}))^{-1} \text{cov}(s_i, \theta_i)) \cdot \overline{s}, p^* \right) \tag{47}$$

$$= \text{cov}\left((\text{var}(s_i - \overline{s}) \cdot \text{cov}(s_i, \theta_i)) \cdot \overline{s}, p^* \right) \neq 0. \tag{48}$$

The first equality follows from the definition of $v(\beta)$ and using (43). The second equality is obtained by taking the limit $\beta \to \infty$, so the term that multiplies $b/(1 + \beta b)$ converges to zero. The last inequality follows from the fact that we are assuming that (40) is not satisfied (otherwise, we already showed that an equilibrium exists). Because $det(\beta)$ is a polynomial (as a function of β) of degree L it follows that $\text{cov}(v(\beta) \cdot \overline{s}, p^*)$ is a polynomial (as a function of β) of degree $L - 1$ exactly (otherwise, the limit (47) would not converge). Because $det(\beta)$ is a polynomial of degree L, the left-hand-side of (46) is a polynomial of degree $2 \times L - 1$.

Because each element of $A(\beta)$ is a polynomial of degree $L - 1$, the right-hand-side is a polynomial of degree $2 \times L - 2$ at most. It follows that solving (46) is equivalent to finding the roots of a polynomial of order $2L - 1$. Since $2L - 1$ is an odd number, a (real) root of a polynomial of order $2L - 1$ always exists. Thus, there exists a β that solves (46).

30 The proof is the same as the proof that $det(\beta)$ is a polynomial of degree L.

(Second Step) Let $\hat{\beta}$ be a solution to (46). If $det(\hat{\beta}) \neq 0$, then there exists an equilibrium $(\hat{\alpha}, \hat{\beta})$, where $\hat{\alpha}$ is determined by $\hat{\beta}$ via (44). If $det(\hat{\beta}) = 0$ we consider two cases. The first case is the one in which there exists $\hat{\alpha}$ such that:

$$\lim_{\beta \to \hat{\beta}} \frac{1+\beta b}{1+b} \frac{v(\beta)}{det(\beta)} = \hat{\alpha}.$$

In this case, $(\hat{\alpha}, \hat{\beta})$ by continuity solves (37) and (38), and so $(\hat{\alpha}, \hat{\beta})$ is a linear equilibrium.

Finally, we consider the case in which $det(\hat{\beta}) = 0$ and

$$\lim_{\beta \to \hat{\beta}} \frac{1+\beta b}{1+b} \frac{v(\beta)}{det(\beta)} \to \infty. \tag{49}$$

We prove that in this case there exists $\hat{\beta}' \neq \hat{\beta}$ that solved (46). To prove this, note that the left-hand-side of (46) converges to 0 as $\beta \to \hat{\beta}$ and so the right-hand-side also converges to 0. This implies that:

$$\lim_{\beta \to \hat{\beta}} v(\beta) = (0, ..., 0). \tag{50}$$

In other words, in the limit $\beta \to \hat{\beta}$ we must have that $v(\beta)$ converges to 0, otherwise $\text{var}(v(\beta) \cdot \bar{s}) \neq 0$ (recall that we assume that $\text{var}(\bar{s})$ is invertible, which implies it is strictly positive definite). Therefore, there exist integers $t', t \in \mathbb{R}$ such that:

$$v(\beta) = (\beta - \hat{\beta})^{t'} \hat{v}(\beta) \text{ and } det(\beta) = (\beta - \hat{\beta})^{t} \widehat{det}(\beta),$$

and $\lim_{\beta \to \hat{\beta}} \hat{v}(\beta) \neq 0$ and $\lim_{\beta \to \hat{\beta}} \widehat{det}(\beta) \neq 0$. That is, we factor out all the terms of the polynomials that are of the form $(\beta - \hat{\beta})$. Dividing both sides of (46) by $(\beta - \hat{\beta})^{2t'}$ we get:

$$cov(\hat{v}(\beta) \cdot \bar{s}, p^*) \widehat{det}(\beta) (\beta - \hat{\beta})^{t-t'} = \text{var}(\hat{v}(\beta) \cdot \bar{s}). \tag{51}$$

Note that the term $(\beta - \hat{\beta})^{t'}$ appears in $cov(v(\beta) \cdot \bar{s}, p^*)$ and squared in $\text{var}(v(\beta) \cdot \bar{s})$, so we get the term $(\beta - \hat{\beta})^{t-t'}$ in (51). Because (49) implies that $t > t'$ the left-hand-side of (51) is a polynomial. Moreover, this is a polynomial of degree $2L - 1 - 2t'$.[31] As before, solving (51) is equivalent to finding the roots of a polynomial of degree $2L - 1 - 2t'$, which always exists. Thus, there exists $\hat{\beta}'$ that solves (51). Moreover, $\hat{\beta}' \neq \hat{\beta}$ because the right-hand-side of (51) does not converge to zero in the limit $\beta \to \hat{\beta}'$.

We now repeat the argument for $\hat{\beta}'$. If there exists $\hat{\alpha}'$ such that:

$$\lim_{\beta \to \hat{\beta}'} \beta \frac{v(\beta)}{det(\beta)} = \lim_{\beta \to \hat{\beta}'} \beta \frac{\hat{v}(\beta)}{(\beta - \hat{\beta})^{t-t'} \widehat{det}(\beta)} = \hat{\alpha}',$$

then $(\hat{\alpha}', \hat{\beta}')$ is an equilibrium. If such $\hat{\alpha}'$ does not exists, following the same argument as before, we can reduce the order of the polynomials $\hat{v}(\beta)$ and $\widehat{det}(\beta)$ once again, and find a different root $\hat{\beta}''$. By repeating the argument we must find an equilibrium, as we cannot reduce the order of the polynomials more than L times. Therefore, we conclude that an equilibrium exists.

Analysis of the Case that $\text{var}(\bar{s})$ **is not Invertible.** In the proof previously presented we assumed that $\text{var}(\bar{s})$ was invertible (we used this fact right after (50)); we now explain how to

[31] The left-hand-side of (46) is a polynomial of degree $2L - 1$ and (51) is obtained by dividing both sides by $(\beta - \hat{\beta})^{2t'}$. Thus, the left-hand-side of (51) is a polynomial of order $2L - 1 - 2t'$.

analyze the case in which it is not invertible. Let the range of $\text{var}(\bar{s})$ be $\tilde{L} < L$ and let M be a $L \times L$ matrix such that $M \cdot \text{var}(\bar{s}) \cdot M^T$ is a block matrix with an invertible $\tilde{L} \times \tilde{L}$ matrix in the first block and a matrix of zeros in the other block. Now define $(\tilde{s}_i, t_i) \in \mathbb{R}^{\tilde{L}} \times \mathbb{R}^{L-\tilde{L}}$ as follows $(\tilde{s}_i, t_i) = M \cdot s_i$. Observe that $\text{var}((\int \tilde{s}_i di, \int t_i di)) = M\text{var}(\bar{s})M^T$ and so $\text{var}(\int \tilde{s}_i di)$ is invertible and $\int t_i di = (0, ..., 0)$ by construction. We can now repeat the proof previously presented line-by-line in the same way by considering only the signals $\tilde{s}_i$ and by replacing the shocks θ_i with the shocks $\tilde{\theta}_i = \theta_i - \mathbb{E}[\theta_i | t_i]$. In particular, it is simple to check that, if $\tilde{x}(\tilde{s}_i, p)$ is a symmetric equilibrium when the payoff and information structure is $(\tilde{\theta}_i, \tilde{s}_i, \tilde{\theta}_j, \tilde{s}_j)$, then $x(s_i, p) = \tilde{x}(\tilde{s}_i, p) + \mathbb{E}[\theta_i | t_i]$ is a symmetric equilibrium when the payoff and information structure is $(\theta_i, s_i, \theta_j, s_j)$. □

8.2. *Proof of Theorem 2*

We begin by computing the first-order condition of the maximization problem (29). Let $e_\ell \in \mathbb{R}^L$ be a vector of zeros with a one in the ℓ-th component and let $\epsilon \in \mathbb{R}$. We note that:

$$\delta_i^{(\alpha+\epsilon e_\ell,\beta)} = \delta_i^{(\alpha,\beta)} + \epsilon(s_{i\ell} - \bar{s}_\ell) \quad \text{and} \quad p^{(\alpha+\epsilon e_\ell,\beta)} = p^{(\alpha,\beta)} + \frac{b}{1+b\beta}\epsilon\bar{s}_\ell .$$

$$\text{var}(\delta_i^*)\frac{d}{d\epsilon}\text{corr}(\delta_i^{(\alpha+\epsilon e_\ell,\beta)}, \delta_i^*)^2 \Big|_{\epsilon=0} = \frac{2}{\text{var}(\delta_i^{(\alpha,\beta)})}\Bigg(\text{cov}(\delta_i^*, \delta_i^{(\alpha,\beta)})\text{cov}(\delta_i^*, (s_{i\ell} - \bar{s}_\ell)) \tag{52}$$

$$- \frac{\text{cov}(\delta_i^*, \delta_i^{(\alpha,\beta)})^2}{\text{var}(\delta_i^{(\alpha,\beta)})}\text{cov}(\delta_i^{(\alpha,\beta)}, (s_{i\ell} - \bar{s}_\ell))\Bigg) \tag{53}$$

$$= 2\text{cov}(\delta_i^* - \delta_i^{(\alpha,\beta)}, (s_{i\ell} - \bar{s}_\ell)) \tag{54}$$

where in the last equation we have used that $\text{cov}(\delta_i^*, \delta_i^{(\alpha,\beta)}) = \text{var}(\delta_i^{(\alpha,\beta)})$ (see (30)). Similarly for the other correlation, we get:

$$\text{var}(\bar{\theta})\frac{d}{d\epsilon}\text{corr}(p^{(\alpha+\epsilon e_\ell,\beta)}, p^*)^2 \Big|_{\epsilon=0} = \frac{(1+b)^2}{b^2}\frac{d}{d\epsilon}\frac{\text{cov}(p^{(\alpha,\beta)} + \epsilon\frac{b}{1+b\beta}\bar{s}_\ell, p^*)^2}{\text{var}(p^{(\alpha,\beta)} + \epsilon\frac{b}{1+b\beta}\bar{s}_\ell)} \tag{55}$$

$$= \frac{(1+b)^2}{b^2}\frac{b}{1+b\beta}2\text{cov}(p^* - p^{(\alpha,\beta)}, \bar{s}_\ell) \tag{56}$$

Thus, we get that the first-order condition is given by:

$$\text{cov}(\delta_i^* - \delta_i^{(\alpha,\beta)}, (s_{i\ell} - \bar{s}_\ell)) + \frac{(1+b)}{b(1+b\beta)}\text{cov}(p^* - p^{(\alpha,\beta)}, \bar{s}_\ell) = 0.$$

Using the same arguments as in Section 2.5, it is simple to show that:

$$\text{cov}(\delta_i^*, \bar{s}_\ell) = \text{cov}(\delta_i^{(\alpha,\beta)}, \bar{s}_\ell) = \text{cov}(p^*, \ (s_{i\ell} - \bar{s}_\ell)) = \text{cov}(p^{(\alpha,\beta)}, \ (s_{i\ell} - \bar{s}_\ell)) = 0.$$

Hence, we can write the first-order condition as follows,

$$\text{cov}\left((\delta_i^* - \delta_i^{(\alpha,\beta)}) + \frac{(1+b)}{b(1+b\beta)}(p^* - p^{(\alpha,\beta)}), s_{i\ell}\right) = 0$$

We now use (30) and (31) to write the first-order condition as follows:

$$\text{cov}\left((\theta_i - \bar{\theta}) - \mathbb{E}[\theta_i - \bar{\theta} \mid \alpha(s_i - \bar{s})] + \frac{1}{(1+b\beta)}(\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}]), s_{i\ell}\right) = 0. \tag{57}$$

Hence, any demand function (α, β) that solves (28) must satisfy (57).

Any equilibrium demand function satisfies (30), (31) and (32). An equilibrium demand function (α, β) solves (28) only if it satisfies (57), which can only happen if (32) and (57) are both satisfied. This can happen in two situations. The first situation is that $\beta = 0$. The second situation is that:

$$\operatorname{cov}\left((\theta_i - \bar{\theta}) - \mathbb{E}[\theta_i - \bar{\theta} \mid \alpha(s_i - \bar{s})], s_{i\ell}\right) = \operatorname{cov}(\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}]), s_{i\ell}\big).$$

However, this condition implies that (19) is violated. Hence, if an equilibrium demand function (α, β) solves (28), then it must be the case that $\beta = 0$

We now note that (31) implies that any equilibrium demand function (α, β) satisfies:

$$\frac{b}{1+\beta b} = \frac{\operatorname{cov}(\alpha\bar{s}, p^*)}{\operatorname{var}(\alpha\bar{s})}. \tag{58}$$

Since an equilibrium demand function solves (28) only if $\beta = 0$, we conclude that an equilibrium demand function (α, β) solves (28) only if

$$b = \frac{\operatorname{cov}(\alpha \cdot \bar{s}, p^*)}{\operatorname{var}(\alpha\bar{s})}. \tag{59}$$

Replacing $p^* = b\bar{\theta}/(1+b)$, we obtain:

$$1 + b = \frac{\operatorname{cov}(\alpha \cdot \bar{s}, \bar{\theta})}{\operatorname{var}(\alpha\bar{s})}. \tag{60}$$

Hence, an equilibrium demand function solves (28) only if (60) is satisfied.

We rewrite (30) as follows:

$$\operatorname{var}(\alpha(s_i - \bar{s})) = \operatorname{cov}(\alpha(s_i - \bar{s}), \delta_i^*). \tag{61}$$

We note that α is determined by (61) and (32). Therefore, whether for a given α, there exists a β such that (α, β) is an equilibrium demand function is independent of b. In the proof of Theorem 1 we proved that the set of equilibria is determined by the roots of a polynomial of degree $(2L - 1)$. Hence, there exists at most $(2L - 1)$ equilibria. Let $\{\tilde{\alpha}_1, ..., \tilde{\alpha}_{2L-1}\}$ the set of possible equilibrium intercepts. We remark that this set is independent of b.

We now conclude the proof by noting that an equilibrium demand function solves (28) only if b satisfies (60). Yet, we proved that there are at most $2L - 1$ equilibrium α, and this values are independent of b. Hence, an equilibrium demand function solves (28) only if:

$$1 + b = \frac{\operatorname{cov}(\tilde{\alpha}_\ell \cdot \bar{s}, \bar{\theta})}{\operatorname{var}(\tilde{\alpha}_\ell \bar{s})},$$

for some $\tilde{\alpha}_\ell \in \{\tilde{\alpha}_1, ..., \tilde{\alpha}_{2L-1}\}$. Hence, there are at most $2L - 1$ values of b for which the equilibrium demand function solves (28). □

8.3. Proof of Theorem 3

We denote by (α', β') a solution to (28). In the proof of Theorem 2, we proved that this demand function satisfies (57). Moreover, (58) implies that $1/(1+\beta b) = \frac{\operatorname{cov}(\alpha\bar{s}, \frac{\bar{\theta}}{1+b})}{\operatorname{var}(\alpha\bar{s})}$. Thus, we can write (57) as follows:

$$\mathrm{cov}\left((\theta_i-\bar{\theta})-\mathbb{E}[\theta_i-\bar{\theta}\mid\alpha'(s_i-\bar{s})]+\frac{\mathrm{cov}(\alpha\bar{s},\frac{\bar{\theta}}{1+b})}{\mathrm{var}(\alpha\bar{s})}(\bar{\theta}-\mathbb{E}[\bar{\theta}\mid\alpha'\bar{s}]),s_{i\ell}\right)=0.$$

However, if the tax rate is given by (35), then we can write this equation as follows:

$$\mathrm{cov}\left((\theta_i-\bar{\theta})-\mathbb{E}[\theta_i-\bar{\theta}\mid\alpha'(s_i-\bar{s})]+(1-\tau)(\bar{\theta}-\mathbb{E}[\bar{\theta}\mid\alpha'\bar{s}]),s_{i\ell}\right)=0.$$

Using the arguments in Section 2.5 we conclude that:

$$\mathrm{cov}\left((\theta_i-\bar{\theta})-\mathbb{E}[\theta_i-\bar{\theta}\mid\alpha'(s_i-\bar{s})],(s_{i\ell}-\bar{s}_\ell)\right)+(1-\tau)\mathrm{cov}\left((\bar{\theta}-\mathbb{E}[\bar{\theta}\mid\alpha'\bar{s}]),\bar{s}_\ell\right)=0.$$

If we apply Proposition 5 to the economy with tax (35), we conclude that there exists $\hat{\beta}$ such that $(\alpha',\hat{\beta})$ is an equilibrium demand function in the economy with tax (35). However Proposition 3 and Proposition 4 imply that:

$$\frac{b}{1+\hat{\beta}b}=\frac{b}{1+\beta'b}=\frac{\mathrm{cov}(\alpha'\bar{s},(1-\tau)p^*)}{\mathrm{var}(\alpha'\bar{s})}$$

Thus, $\beta'=\hat{\beta}$, which leads us to conclude that (α',β') is an equilibrium demand function in the economy with tax (35). □

8.4. *Proof of lemmas, propositions and corollaries*

Proofs of Proposition 1 and Proposition 2. We prove Proposition 2 explicitly; Proposition 1 may be proven line-by-line the same way by simply taking out the expectations. The first order condition of (11) imply:

$$q_i^{**}=\mathbb{E}[\theta_i|\{s_i\}_{i\in[0,1]}]-p^{**}. \tag{62}$$

Market clearing imply that:

$$b\int q_i^{**}di=b\int\mathbb{E}[\theta_i|\{s_i\}_{i\in[0,1]}]-p^{**}di=p^{**}.$$

Solving for p^{**} and using Fubini to bring the integral inside the expectation we obtain:

$$p^{**}=\mathbb{E}[\frac{b}{1+b}\overline{\theta}|\{s_i\}_{i\in[0,1]}].$$

Replacing into (62) and using that $q_i^{**}=\delta_i^{**}+p^{**}/b$, we obtain:

$$\delta_i^{**}=\mathbb{E}[(\theta_i-\overline{\theta})|\{s_i\}_{i\in[0,1]}].$$

Thus, we prove the result. □

Proof of Proposition 3. The total surplus is given by (27). By canceling out the terms that have the price $\hat{p}^{(\alpha,\beta)}$ and noting that $\theta_i=\delta_i^*+(1+b)p^*/b$ we get:

$$S^{(\alpha,\beta)}=\int q_i^{(\alpha,\beta)}(\delta_i^*+\frac{1+b}{b}p^*-\frac{1}{2}q_i^{(\alpha,\beta)})di-b\frac{(\int q_i^{(\alpha,\beta)}di)^2}{2}. \tag{63}$$

We now note that:

$$q_i^{(\alpha,\beta)}=\delta_i^{(\alpha,\beta)}+\frac{1}{b}p^{(\alpha,\beta)}.$$

Replacing this expression for $\hat{q}_i^{(\alpha,\beta)}$ into (63) we obtain:

$$S^{(\alpha,\beta)} = \mathbb{E}\Big[\int \delta_i^{(\alpha,\beta)}(\delta_i^* - \frac{1}{2}\delta_i^{(\alpha,\beta)})di + \frac{(1+b)}{b^2}p^{(\alpha,\beta)}(p^* - \frac{1}{2}p^{(\alpha,\beta)})\Big]. \tag{64}$$

Here we used the following equalities to simplify the expression:

$$\int \delta_i^{(\alpha,\beta)}di = \int \delta_i^{(\alpha,\beta)}p^{(\alpha,\beta)}di = \int \delta_i^{(\alpha,\beta)}p^*di = \int \delta_i^* p^{(\alpha,\beta)}di = 0.$$

We now observe that:

$$\mathbb{E}[\int \delta_i^{(\alpha,\beta)}(\delta_i^* - \frac{1}{2}\delta_i^{(\alpha,\beta)})di] = \text{cov}(\delta_i^*, \delta_i^{(\alpha,\beta)}) - \frac{1}{2}\text{var}(\delta_i^{(\alpha,\beta)}); \tag{65}$$

$$\mathbb{E}[p^{(\alpha,\beta)}(p^* - \frac{1}{2}p^{(\alpha,\beta)})] = \text{cov}(p^*, p^{(\alpha,\beta)}) - \frac{1}{2}\text{var}(p^{(\alpha,\beta)}). \tag{66}$$

We now note that:

$$\text{cov}(\delta_i^*, \delta_i^{(\alpha,\beta)}) - \frac{1}{2}\text{var}(\delta_i^{(\alpha,\beta)}) \le \frac{\text{var}(\delta_i^*)}{2}\text{corr}(\delta_i^{(\alpha,\beta)}, \delta_i^*)^2; \tag{67}$$

$$\text{cov}(p^*, p^{(\alpha,\beta)}) - \frac{1}{2}\text{var}(p^{(\alpha,\beta)}) \le \frac{\text{var}(p^*)}{2}\text{corr}(p^{(\alpha,\beta)}, p^*)^2, \tag{68}$$

with equality if and only if (30) and (31) are satisfied. Hence, any demand function that solves (27) satisfies (30) and (31). Hence, we get that, for any demand function satisfying (30) -(31):

$$S^{(\alpha,\beta)} = \frac{\text{var}(\delta_i^*)}{2}\text{corr}(\delta_i^{(\alpha,\beta)}, \delta_i^*)^2 + \frac{\text{var}(p^*)(1+b)}{2b^2}\text{corr}(p^{(\alpha,\beta)}, p^*)^2.$$

Thus, any demand function (α, β) that solves (27) will satisfy that α maximizes this equation. Finally, replacing the expressions for δ_i^* and p^* found in (16) we obtain that:

$$\text{var}(\delta_i^*) = \text{var}(\theta_i - \bar{\theta}) \quad \text{and} \quad \text{var}(p^*) = \frac{b^2}{(1+b)^2}\text{var}(\bar{\theta})$$

Hence, we obtain that any demand function that solves (27) must satisfy (29). □

Proof of Proposition 4. We already proved that every equilibrium demand function satisfies (31) (note that (37) is the same as (31), but multiplying times $b/(1+\beta b)$ on both sides of the equation). We now prove in an analogous way that every equilibrium demand function satisfies (30). Note that:

$$\delta_i^{(\alpha,\beta)} = \alpha s_i - \beta p^{(\alpha,\beta)} - \frac{1}{b}p^{(\alpha,\beta)}.$$

Hence, (22) implies that:

$$\mathbb{E}[(\delta_i^{(\alpha,\beta)} - \delta_i^*) + \frac{1+b}{b}(p^{(\alpha,\beta)} - p^*)|s_i, p^{(\alpha,\beta)}, \delta_i^{(\alpha,\beta)}] = 0.$$

Here we added $\delta_i^{(\alpha,\beta)}$ as a conditioning variable, which is possible because we showed that $\delta_i^{(\alpha,\beta)}$ is measurable with respect to s_i and $p^{(\alpha,\beta)}$. This implies that:

$$\text{cov}((\delta_i^* - \delta_i^{(\alpha,\beta)}) + \frac{b}{1+b}(p^* - p^{(\alpha,\beta)}), \delta_i^{(\alpha,\beta)}) = 0 \iff$$
$$\text{cov}(\alpha \cdot (s_i - \bar{s}), \delta_i^*) = \text{var}(\alpha \cdot (s_i - \bar{s})).$$

The proof is completely analogous to the proof of (37). This implies that every equilibrium demand function satisfies (30). □

Proof of Proposition 5. Recall that any equilibrium demand function satisfies (30) and (31) (this is formally stated in Proposition 4). This implies that,

$$p^{(\alpha,\beta)} = \mathbb{E}[p^* \mid p^{(\alpha,\beta)}] \quad \text{and } \delta_i^{(\alpha,\beta)} = \mathbb{E}[\delta_i^* \mid \delta_i^{(\alpha,\beta)}]. \tag{69}$$

Since $p^{(\alpha,\beta)} \propto \alpha\bar{s}$, conditioning on $p^{(\alpha,\beta)}$ is the same as conditioning on $\alpha\bar{s}$. Similarly, conditioning on $\delta_i^{(\alpha,\beta)}$ is the same as conditioning on $\alpha(s_i - \bar{s})$. Using (16), we write (69) as follows:

$$p^{(\alpha,\beta)} = \mathbb{E}[\frac{b}{1+b}\bar{\theta} \mid \alpha\bar{s}] \quad \text{and } \delta_i^{(\alpha,\beta)} = \mathbb{E}[\theta_i - \bar{\theta} \mid \alpha(s_i - \bar{s})]. \tag{70}$$

Replacing this on (22) we get:

$$\mathbb{E}[(\theta_i - \bar{\theta} - \mathbb{E}[\theta_i - \bar{\theta} \mid \alpha(s_i - \bar{s})]) + (\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}])|s_i, p^{(\alpha,\beta)}] = 0. \tag{71}$$

This implies that:

$$\forall \ell \in L, \quad \text{cov}\left((\theta_i - \bar{\theta} - \mathbb{E}[\theta_i - \bar{\theta} \mid \alpha(s_i - \bar{s})]) + (\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}]), s_i\right) \tag{72}$$

Finally, using the same arguments as in Section 2.5 we conclude that:

$$\text{cov}(\theta_i - \bar{\theta}, \bar{s}_\ell) = 0 \quad \text{and} \quad \text{cov}(\bar{\theta}, s_i - \bar{s}_\ell) = 0.$$

Hence, we can write (72) as (32), which proves the result. □

Proof of Proposition 6. We first note that in the limit $b \to \infty$, the maximization problem (29) converges to:

$$\alpha \in \arg\max \quad \frac{\text{var}(\theta_i - \bar{\theta})}{2}\text{corr}(\delta_i^{(\alpha,\beta)}, \delta_i^*)^2.$$

Note that $\delta_i^{(\alpha,\beta)} \propto \alpha(s_i - \bar{s})$, so we can write this as follows:

$$\alpha \in \arg\max \quad \frac{\text{var}(\theta_i - \bar{\theta})}{2}\text{corr}(\alpha(s_i - \bar{s}), \delta_i^*)^2.$$

It is clear that the correlation is maximized when $\alpha(s_i - \bar{s}) \propto \mathbb{E}[q^* \mid (s_i - \bar{s})]$. Hence, in the limit $b \to \infty$, the demand function that solves (28) satisfies that $\delta^{(\alpha,\beta)} \to \delta_i^{**}$. This implies that in the limit $b \to \infty$, the demand function (α, β) that solves (28) satisfied:

$$\alpha \to \left(\text{var}(s_i - \bar{s})^{-1} \cdot \text{cov}(s_i - \bar{s}, \delta_i^*)\right). \tag{73}$$

We now note that for a linear demand function (α, β) that solves (29) in the limit $b \to \infty$, it must solve:

$$\text{cov}\left(\theta_i - \bar{\theta} - \mathbb{E}[\theta_i - \bar{\theta} \mid \alpha(s_i - \bar{s})], s_{i\ell} - \bar{s}_\ell\right) + (1-\tau)\text{cov}\left(\left(\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}]\right), \bar{s}_\ell\right) = 0.$$

This is condition (32) for an economy with tax τ. Yet, if α satisfies (73), the first term converges to zero. Hence, we must have that either $(1-\tau)$ converge to zero or $\text{cov}\left(\left(\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}]\right), \bar{s}_\ell\right)$ converge to zero. If $\text{cov}\left(\left(\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}]\right), \bar{s}_\ell\right)$ does not converge to zero, then we must have that $\tau \to 1$, and we obtain the desired result. If $\text{cov}\left(\left(\bar{\theta} - \mathbb{E}[\bar{\theta} \mid \alpha\bar{s}]\right), \bar{s}_\ell\right)$ converges to zero, it means that $\mathbb{E}[\bar{\theta} \mid \alpha\bar{s}] \to \mathbb{E}[\bar{\theta} \mid \bar{s}]$. In other words, $\alpha\bar{s}$ is a sufficient statistic to compute $\mathbb{E}[\bar{\theta} \mid \bar{s}]$, which means (19) is violated. However, in this case, we have that in the limit $\delta_i^{(\alpha,\beta)} = \delta_i^{**}$ and $p^{(\alpha,\beta)} = p^{**}$ for every τ, which is obviously the solution to (28). Hence, we obtain the result. □

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