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Optimal screening with securities

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ABSTRACT

A liquidity-constrained asset owner screens an informed investor using financial securities. Information-insensitive securities reduce the investor's information rents. The optimal screening mechanism consists of a *monotone debt menu* where more optimistic investor types purchase larger amounts of debt. Interestingly, the issuer may benefit from trading with a better informed investor. For any monotone debt menu, as the asset's cash flows more accurately describe the investor's private information (Lehmann, 1988), the trade surplus unambiguously increases. Further, fixing the optimal debt menu, when the investor's private information originates from location experiments, increasing accuracy decreases (increases) information rents for low (high) types. We provide sufficient conditions that guarantee that the issuer strictly benefits from trading with a better informed investor. Our results imply that selling debt is an effective approach to raise funds in financial markets even when investors hold superior private information and provide a novel rationale for the emergence of venture debt.

1. Introduction

Firms frequently issue securities to raise financing and meet short-term obligations. In many settings, outside investors possess superior private information about the firm's underlying assets. This raises a central question: what type of securities should a liquidity-constrained firm optimally sell when facing informed investors? Should it trade with a better informed investor or, in turn, favor those with whom information asymmetry is less pronounced? Motivated by the prevalence of environments in which investors hold informational advantages—such as venture capital funds, management buyout groups, restructuring teams, and publishers—we examine how firms can use security design to screen informed investors and how the degree of asymmetric information shapes the firm's ability to raise funds.

To illustrate, consider a startup trying to raise funds from a venture capital fund (VC). VCs specialize in funding and coaching similar projects, and they usually have superior information about the potential for growth.[1] Startups, on the other hand, struggle to raise cash to fund their initial operations and are strongly liquidity-constrained. To raise liquid funds, startups usually sell claims on their future cash flows (i.e., securities) in exchange for cash. If the startup could design the securities purchased by the VC, which securities would it offer? Does the startup benefit from trading with a more specialized VC that holds more accurate information?

We show that an asset owner (henceforth, the issuer) optimally offers a *debt menu* when trading with an informed investor. By doing so, the issuer screens the investor's private information more effectively, reducing the investor's information rents. The optimality of

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[1] Several papers find evidence of VCs' repeatable skills, e.g., Kaplan and Schoar (2005) and Ewens and Rhodes-Kropf (2015). The performance persistence might originate from access to networks (Hochberg et al., 2007), industry experience (Hellmann and Puri, 2002), or screening skills (Sorensen, 2007).

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debt menus contrasts with the insights from the theoretical literature, largely drawn from the case with an informed issuer. Those models suggest that the informed party—the issuer in those models—should keep an information sensitive security, e.g., a levered equity stake or a warrant.[2] Our results suggest that selling debt is an effective approach to raise funds in financial markets regardless of which party holds superior private information.

To gain some intuition, consider an issuer and an investor who can be of two types, either H or L. Type H is more optimistic about the asset's future cash flows in the strong sense: type H's beliefs dominate type L's beliefs according to the monotone likelihood ratio order. We show that the issuer optimally screens the investor using debt securities.

Indeed, consider a menu of securities and payments, $\{(s^L, p^L), (s^H, p^H)\}$. Type H has incentives to understate their type to "buy cheap." We argue that the issuer can dissipate these incentives more effectively by selling debt to type L. If s^L is not debt, the issuer can construct a debt security, s^L_d, that makes type L indifferent. The security s^L_d promises a fixed payout $d > 0$ whenever possible and seniority when the asset's cash flows fall short of this amount. If type L is indifferent between s^L and s^L_d, then type H values s^L_d strictly less than s^L. Indeed, type H assigns more probability mass to high cash flows. For these realizations, however, s^L_d always offers the same flat payout, d. In other words, s^L_d minimizes the upside, which are also the states that type H deems more likely to occur, relative to type L. Thus, by modifying the original security s^L for s^L_d, the issuer reduces type H's incentives to under-report and can reduce the *information rents* captured by this type by raising price p^H. Furthermore, under truthful reporting, the issuer perfectly learns the investor's private information and therefore values both securities s^L_d and s_L equally. Thus, by modifying s^L for its equivalent debt security s^L_d, the issuer raises more funds (by increasing p^H) while keeping the expected value of the securities sold unchanged, and is therefore strictly better off.[3]

We extend the heuristic above to the general case with an arbitrary number of investor's types. The optimal mechanism consists of a *monotone debt menu* where more optimistic investor types purchase larger amounts of debt (i.e., debt with larger face values). The result, however, is subtler than the two-types example in that modifying arbitrary securities for their equivalent debt security need not preserve (global) incentive compatibility. Indeed, with more than two types, the intuition provided above may fail as the incentives of low types to overstate their private information are exacerbated when higher types are offered debt securities.

We first show in Lemma 2 that the investor's information rents comove with the securities' *information sensitivity*. Specifically, we show that among all securities that promise the same expected payoff (and hence that preserve gains from trade), a more information-sensitive security necessarily increases the investor's information rents. This result formalizes the intuition above that debt—the most information-insensitive security—is most effective at aligning the investor's incentives. The issuer thus strictly benefits from modifying each security in an arbitrary incentive-compatible mechanism for its equivalent debt security. However, as anticipated above, such modifications need not respect global incentive constraints.

In Theorem 1, we show that, starting from an arbitrary incentive-compatible mechanism that is not a debt menu, one can construct a dominating mechanism in two simple steps. First, for each investor type, we find the face value of the equivalent debt security, that is, the debt security that would make the type indifferent (as the construction of s^L_d above for type L). Second, we construct a new debt menu where the face values are given by the *upper monotone envelope* of the face values characterized in the first step. We show that the new mechanism is (globally) incentive compatible and strictly increases the issuer's payoff. This result implies that it is without loss of optimality to restrict attention to monotone debt menus. In particular, it reduces the issuer's problem over the infinite-dimensional allocation space of financial securities to the single-dimensional problem of characterizing the face value of each debt security.

This result shows that the informed party—the investor—purchases debt, the maximally information-insensitive security. This contrasts with the folk intuition that the informed party, typically the issuer, keeps an information-sensitive security like levered equity (Gorton and Pennacchi, 1990; Nachman and Noe, 1994; Biais and Mariotti, 2005). We argue that in those models with a privately-informed issuer, the nature of the incentives problem changes: the mechanism, in those cases, must prevent issuer types with pessimistic information from *overstating* their private information to "sell expensive." Stated differently, with an informed issuer, upward incentive constraints are typically more stringent. Interestingly, here too having the issuer selling debt securities (and therefore keeping levered equity), relaxes incentive constraints. Selling debt thus seems to be a robust prediction that holds regardless of the party that holds private information. Roughly, selling debt is effective at simultaneously (i) preventing investors with optimistic information to understate their private signals to buy cheap, and (ii) preventing issuers with negative information to overstate it to sell expensive.[4]

In the second part of the paper, we leverage the characterization of the optimal mechanism to explore how changes in the quality of the investor's private information affect the trading outcomes. Interestingly, we find that an issuer with the ability to flexibly design financial securities may benefit from facing a better informed investor. Formally, we consider the case where the asset's cash flows more *accurately* describe the investor's private information in the sense of Lehmann (1988). We argue that more accurate information induces two simultaneous effects that point in opposite directions. On the one hand, the asset's cash flows become a better signal

[2] Selling debt typically serves as a commitment device for the issuer not to exploit her future private information when trading with the investor (Nachman and Noe, 1994; DeMarzo and Duffie, 1999; Biais and Mariotti, 2005).

[3] This heuristic does not contradict the insight that more informed agents should expose themselves relatively more to the asset's cash flows to signal their private information (Leland and Pyle, 1977, Ross, 1977, Myers and Majluf, 1984, etc.). Under the optimal mechanism, more optimistic investor types purchase larger fractions of the underlying asset (i.e., debt with higher face values), which obviously exposes them to right-tail risk.

[4] The optimality of debt menus is not an artifact of the issuer having all the bargaining power. Instead, it originates in its effectiveness at maintaining incentives. We show in the Online Appendix that the investor's most-preferred incentive compatible mechanism also consists of a monotone debt menu.

of the investor's type, which might help the issuer to reduce the investor's rents. On the other hand, the investor becomes better informed about cash flows thereby exacerbating the extent of asymmetric information.

To get traction, we restrict attention throughout the analysis to two classes of accuracy-ranked information structures widely considered in the literature: *location experiments* and *copulas*. We show in Theorem 2 that, fixing *any* monotone debt menu, more accurate information unambiguously increases gains from trade. We show that, for location experiments, gains from trade increase for each investor type, whereas under the copula structure, more accurate information increases the *expected* gains from trade.

For location experiments, the result is a consequence of the concavity of debt securities and the fact that a more accurate experiment reduces the investor's ex-post residual uncertainty over cash flows. The effect is similar to the one experienced by a risk-averse agent who obtains a higher expected utility when exposed to less uncertainty. For copulas, in turn, the result follows from the observation that the accuracy ordering, which ranks experiments, induces the supermodular ordering, which ranks joint distributions. That is, more accurate information implies that the expected value of any supermodular function increases. The result then follows from the observation that any monotone debt menu is supermodular in cash flows and the investor's type.

We then show that accuracy has generally an ambiguous effect on information rents. Fixing the optimal debt menu, information rents display *single crossing differences* on the investor's type and accuracy when the investor's information originates from location experiments. That is, as accuracy increases, information rents decrease for low types and increase for high types. Roughly, we show that the effect of accuracy on information rents ultimately depends on the extent of "allocative distortion" experienced by each type. At the optimal mechanism, low types are distorted to a larger extent than high types (i.e., receive a lower face value) to prevent the latter from under-reporting. When the face value of the security offered to the type is sufficiently low, increasing accuracy tends to increase the type's expected value of the security relatively more than for higher types. Because the information rents generated by a given type to the adjacent higher type are given by the difference in the securities' expected value between the higher type and the original type, information rents decrease with accuracy.

We provide a sufficient condition on primitives that guarantees that aggregate informations rent decrease with accuracy. Roughly, the condition ensures that, at the optimal debt menu, the extent of allocative distortion remains large, especially for low types. The condition further implies that the issuer is better off when facing a more informed investor. Our results emphasize the stark differences in the economics of optimal screening between financial instruments and other types of assets, and call for prudence when extrapolating economic intuitions. The characterization of optimally-designed financial mechanisms appears to be critical in understanding the efficiency gains arising from enhanced transparency in financial markets.

Interestingly, the prediction that liquidity-constrained asset owners use debt instruments to raise funds from informed investors is consistent with some empirical regularities. In the case of VC funding, our predictions are consistent with the emergence of venture debt. Indeed, according to Tykvová (2017), approximately one-third of the current venture-backed companies use debt instruments to raise funds. Our theory shows that such startups can limit the VCs' information rents by choosing venture debt financing. Further, our analysis suggests that the issuer's preference for dealing with more informed investors is exacerbated by strong liquidity constraints, when gains from trade are largest. This is broadly consistent with the fact that startups, typically strongly liquidity-constrained, seek financing from VCs, highly specialized investors.

Our findings offer insights into how the informational environment affects the issuer's ability to raise external financing—contributing to the broader debate on how information dissemination shapes financial markets. In practice, the accuracy of investors' information is influenced by various factors, such as the introduction of new regulations (e.g., Regulation AB) which impose disclosure requirements for securities offerings, enabling investors to better assess the security's value.[5] Additionally, advancements in technological tools used by market participants to process information and the investors' due diligence efforts can also influence the accuracy of the available information. While our analysis remains agnostic on the determinants of the underlying information structure, it establishes sufficient primitive conditions to identify how changes in investors' information accuracy could impact the financial institutions' funding capacity.

The rest of the paper is organized as follows. Below, we wrap up the introduction by discussing how our paper connects with the rest of the literature. Section 2 provides a simple example which delivers the key messages in the paper and highlights the benefits of screening using debt menus in intuitive terms. Section 3 describes the primitives of the model. Section 4 contains the derivation of our first result establishing the optimality of *debt menus* and the characterization of the optimal mechanism. We further discuss the relation between securities' information sensitivity and the investor's information rents. In Section 5, we explore how changes in the quality of the investor's information affects the trading outcomes. All omitted proofs are relegated to the Appendix and Online Appendix.

Related literature

This paper relates to several strands of the literature. First, it contributes to the broad literature on security design under asymmetric information (see, e.g., Nachman and Noe, 1994, DeMarzo and Duffie, 1999, Biais and Mariotti, 2005, DeMarzo and Fishman, 2007, etc.).[6] We depart from those models by assuming that it is the investor, and not the issuer, who is endowed with private information.

[5] Under Regulation AB, the SEC imposes disclosure requirements for asset-backed securities offerings (https://www.sec.gov/corpfin/division scorpfinguidanceregulation-ab-interpshtm).

[6] Some recent work within this areas of study include Malenko and Tsoy (2020) and Lee and Rajan (2018). They explore optimal security design under robust requirements.

In the context of informed investors, DeMarzo et al. (2005) and Axelson (2007) are the closest to our paper. DeMarzo et al. (2005) consider an environment with multiple bidders. In contrast to our paper, they assume *no transfers*. Bidders compete by bidding securities and the winner obtains a security (according to the chosen protocol) and pays a fixed cost. They find that the first-price auction where buyers are restricted to bid call-options (i.e., the investor purchases debt) maximizes the issuer's revenue. Extrapolating their solution to the single-bidder case, the issuer offers a (take-it-or-leave-it) debt security to the buyer. This solution contrasts with the optimal debt *menu* in our paper.

Axelson (2007) considers multiple bidders, multiple securities, and transfers under a fixed selling protocol (uniform-price auctions). He finds that when the issuer cannot use the bidder's information, information-sensitive securities (e.g., equity stakes) can increase the issuer's payoff. In turn, when the issuer can use the bidders' information, it is optimal to sell debt securities. By focusing on uniform price, multi-unit auctions, the paper finds that debt minimizes the underpricing experienced by the issuer—the difference between the expected value of the security sold and the price paid by the investors. Importantly, under this auction, *all securities* are incentive compatible. This contrasts with our model where debt emerges as the most effective security at disciplining incentives (preventing optimistic investor types from understating their private information to buy cheap). Burkart and Lee (2016) study investor-initiated security design offers to a less informed issuer. In the Online Appendix, we explore a similar problem where we characterize the investor's most-preferred incentive compatible mechanism. Liu (2016) follows a mechanism design approach similar to the one proposed in this paper and study optimal auctions when the issuer is constrained to sell equity securities. Liu and Bernhardt (2021) provide sufficient conditions under which equity plus cash auctions achieve optimality in the context of target-initiated takeovers. Yuan (2020) tackles a problem where multiple issuers compete by selling securities to informed investors. Building on the fact that competition among sellers leads to the winner's curse, she finds that there is an equilibrium where all issuers sell debt securities. We show that the optimality of debt occurs even in the absence of competition.

Our paper complements these previous papers by (i) formalizing the connection between the securities' information sensitivity and the buyer's information rents; (ii) characterizing the (unrestricted) issuer's most-preferred, incentive compatible mechanism with securities and transfers; (iii) providing sufficient primitive conditions under which pointwise maximization is indeed IC and showing that even when these conditions are not met, the underlying economic forces still imply the optimality of debt menus; and (iv) leveraging this characterization to provide novel implications on how the extent of asymmetric information shapes the trading outcomes when the seller has the flexibility to screen using financial securities.

More recently, Gershkov et al. (2024) study the problem of screening investors with different degrees of risk aversion with financial securities and find the optimality of tranching. Luo and Yang (2023) rationalize tranching as a means of mitigating coordination frictions. Ball and Pekkarinen (2024) study optimal auctions with securities under costly verification. They find that the issuer optimally designs capped royalties.

Our paper is also related to the literature on information and security design. Yang (2020) studies a security design problem where the investor can acquire costly information. He shows that a debt contract is uniquely optimal and minimizes the incentives to produce private information. In an informed issuer model, Daley et al. (2022) show that reducing the degree of asymmetric information between the issuer and the investor leads the former to issue information-sensitive securities. Vanasco (2017), Szydlowski (2021) and Inostroza and Tsoy (2025) study the case where the issuer can design both the security and the information structure. Vanasco (2017) studies the case where issuer chooses both the security and the effort to improve the quality of the asset. She demonstrates that the adverse selection induced by the issuer's superior information mitigates the issuer's moral hazard problem when monitoring the quality of the pool of assets. Szydlowski (2021) shows that if the issuer's objective consists of raising a prespecified amount of funds, she is indifferent between all the securities yielding the same payoff. Inostroza and Tsoy (2025) show that when the issuer designs both the security and information structure, information-sensitive securities dominate debt instruments and asset sale maximizes the issuer's payoff.

2. Simple example

Example 1. We start with a simple example that highlights the benefits of screening using debt. The issuer (e.g., a startup) has an asset that promises future cash flows $\boldsymbol{y} \in \mathbb{R}_+$. Because of pressing liquidity needs, the issuer discounts future cash flows at rate $\delta \in [0,1)$. There is a single investor (e.g., a VC fund) who has private information about the asset's cash flows and does not discount the future. Both agents are risk neutral. The investor privately observes $\boldsymbol{\omega} \in \{L, H\} \equiv \{1/2, 3/2\}$ which correlates with $\boldsymbol{y}$, and we refer to it as the investor's type. We assume that there is a common prior $\mathbb{P}[\boldsymbol{\omega} = L] = \mathbb{P}[\boldsymbol{\omega} = H] = 1/2$, and $\boldsymbol{y} = \boldsymbol{\omega} + \frac{\boldsymbol{\varepsilon}}{\theta}$, where $\boldsymbol{\varepsilon} \sim U\left[-\frac{1}{2}, \frac{1}{2}\right]$ is independent of $\boldsymbol{\omega}$, and $\theta \in [1, \infty)$. We denote type ω's conditional distribution of cash flows as $F_\theta(y|\omega)$. Intuitively, parameter θ captures the quality of the investor's private signal. As $\theta \to \infty$, the investor becomes perfectly informed about $\boldsymbol{y}$. In turn, as $\theta \to 1$, the investor faces substantial residual uncertainty and believes that $\boldsymbol{y}$ is uniformly distributed over $\left[\omega - \frac{1}{2}, \omega + \frac{1}{2}\right]$. We consider two types of mechanisms that screen the investor using different types of securities.

2.1. Screening with equity

Consider an issuer who sells *equity* shares. The issuer asks the investor to report $\boldsymbol{\omega}$, and as a function of the report $\tilde{\omega}$, offers a security $s_{\tilde{\omega}}(\boldsymbol{y}) = \alpha_{\tilde{\omega}}\boldsymbol{y}$, $\alpha_{\tilde{\omega}} \in [0,1]$, at a price $p_{\tilde{\omega}}$. The issuer optimally offers the menu[7]

$$\mathcal{M}_{\star}^{\text{Equity}} = \left\{ \left(\alpha_L = 0, p_L = 0\right), \left(\alpha_H = 1, p_H = 3/2\right) \right\}.$$

In other words, the issuer makes a take-it-or-leave-it offer and sells pure equity (i.e., $s(\boldsymbol{y}) \equiv \boldsymbol{y}$) at a price of $3/2$, regardless of the specific value of δ. The offer is only accepted by type H and the issuer gets a payoff of $\Pi^{\text{Equity}} = \frac{1}{2} \times \frac{3}{2} + \frac{1}{2} \times \frac{\delta}{2} = \frac{3+\delta}{4}$. That is, with probability $1/2$, $\boldsymbol{\omega} = H$, and the investor pays $3/2$ for the whole asset, whereas with the complementary probability the issuer keeps the security, which is now valued at $\delta\mathbb{E}[\boldsymbol{y}|\boldsymbol{\omega} = L] = \delta/2$ as the refusal to accept the offer reveals the investor's private information. This corresponds to the standard *posted price* solution.[8] Intuitively, selling shares to type L is expensive because it forces the issuer to leave information rents to type H. Indeed, type L values α_L shares at $\mathbb{E}\left[\alpha_L \boldsymbol{y}\middle|L\right] = \alpha_L/2$, whereas type H values them at $\mathbb{E}\left[\alpha_L \boldsymbol{y}\middle|H\right] = 3\alpha_L/2$. Because the issuer can at most charge $\alpha_L/2$ for these shares, she is forced to leave at least $3\alpha_L/2 - \alpha_L/2 = \alpha_L$ in information rents which is larger than the trade surplus created from selling to type L, $(1-\delta)\alpha_L/2$. The issuer thus only sells to type H.

2.2. Screening with debt

Consider now the case where the issuer sells debt securities. The issuer offers a menu of securities and prices $\mathcal{M} = \left\{ \left(s_L, p_L\right), \left(s_H, p_H\right) \right\}$ where securities take the form $s_{\tilde{\omega}}(\boldsymbol{y}) = \min\left\{\boldsymbol{y}, D_{\tilde{\omega}}\right\}$, $\tilde{\omega} \in \{L, H\}$. That is, the issuer promises to pay a face value of $D_{\tilde{\omega}}$ whenever possible (i.e., $\boldsymbol{y} \geq D_{\tilde{\omega}}$) and to provide seniority otherwise. The mechanism described above, $\mathcal{M}_{\star}^{\text{Equity}}$, is feasible in this environment—it corresponds to debt securities with $D_L = 0$ and $D_H = \frac{3}{2} + \frac{1}{2\theta}$. However, the issuer can do strictly better by offering the alternative debt menu characterized by:

$$\mathcal{M}_{\star}^{\text{Debt}} = \left\{ \left(D_L^{\star} = \frac{1}{2} - \frac{\delta}{(2-\delta)\theta}, p_L^{\text{D}} \right), \left(D_H^{\star} = \frac{3}{2} + \frac{1}{2\theta}, p_H^{\text{D}} \right) \right\},$$

where $p_L^{\text{D}} = \mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^{\star}\right\}\middle|L\right]$ and $p_H^{\text{D}} = \frac{3}{2} + p_L^{\text{D}} - D_L^{\star}$. That is, the issuer sells the whole asset to type H, and sells debt with face value $D_L^{\star}$ to type L. $\mathcal{M}_{\star}^{\text{Debt}}$ corresponds to the issuer's most-preferred debt menu among all incentive compatible and individually rational mechanisms with debt securities.

Under the optimal debt menu, the issuer obtains

$$\Pi^{\text{Debt}} = 1 - \frac{1}{4\theta}\left(\frac{1-\delta}{2-\delta}\right) > \frac{3+\delta}{4} = \Pi^{\text{Equity}}, \quad \text{for all } \theta \in [1,\infty), \delta \in [0,1). \tag{1}$$

Thus, regardless of the quality of the investor's private signal θ, the issuer is strictly better off by screening with a debt menu that sells to both types.

Debt and information rents

To understand the advantages of debt menus, it is useful to contrast the information rents induced by debt and other securities. Consider an alternative menu $\mathcal{M} = \left\{ \left(s_L, p_L\right), \left(s_H, p_H\right) \right\}$ where the securities sold to each type preserve the gains from trade generated under $\mathcal{M}_{\star}^{\text{Debt}}$. That is, s_L is such that $\mathbb{E}\left[s_L\middle|L\right] = \mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^{\star}\right\}\middle|L\right]$ whereas $s_H(\boldsymbol{y}) \equiv \boldsymbol{y}$. Debt securities are more effective at curbing the investor's information rents. Intuitively, the information rents captured by type H are given by $I_\theta(H;\mathcal{M}) = \mathbb{E}\left[s_L\middle|H\right] - \mathbb{E}\left[s_L\middle|L\right]$—the expected value difference of s_L between the two types. Among all securities that preserve the gains from trade, debt minimizes the payoff upside, and therefore type H, who is most optimistic about cash flows, likes the debt security $\min\left\{\boldsymbol{y}, D_L^{\star}\right\}$ the least. That is,

$$I_\theta\left(H;\mathcal{M}_{\star}^{\text{Debt}}\right) = \mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^{\star}\right\}\middle|H\right] - \mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^{\star}\right\}\middle|L\right] < \mathbb{E}\left[s_L\middle|H\right] - \mathbb{E}\left[s_L\middle|L\right] = I_\theta(H;\mathcal{M}).$$

Fig. 1 contrasts type H's information rents under $\mathcal{M}_{\star}^{\text{Debt}}$ and the equity menu $\mathcal{M}^E$ constructed to preserve the gains from trade.

Debt and investor's information

Inequality (1) further shows that, when screening with the optimal debt menu, the issuer strictly benefits from trading with a better informed investor (larger θ). This observation contrasts with the case of linear instruments (e.g., equity shares) where the quality of the investor's private information does not play a role. In particular, this effect does not manifest with the optimal equity menu, the standard posted-price solution. To see the intuition, fix the optimal debt menu $\mathcal{M}_{\star}^{\text{Debt}}$ and note that improving the quality of the investor's information (increasing θ), simultaneously (i) increases gains from trade, and (ii) reduces information rents. Indeed, increasing θ strictly increases the gains from trading with type L, $\mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^{\star}\right\}\middle|L\right] = D_L^{\star} - \int_0^{D_L^{\star}} F_\theta(y|L)\mathrm{d}y$, whereas the gains from trading with type H, $\mathbb{E}[\boldsymbol{y}|H] = 3/2$, remain unchanged. This is a consequence of the *concavity* of debt. Type L's expected value of

[7] That the optimal mechanism does not depend on θ follows from the fact that, with linear securities $s_{\tilde{\omega}}(\boldsymbol{y}) = \alpha_{\tilde{\omega}}\boldsymbol{y}$ and a risk-neutral investor, the expected value of the security of each type, $\mathbb{E}\left[\alpha_{\tilde{\omega}}\boldsymbol{y}\middle|\omega\right] = \alpha_{\tilde{\omega}}\omega$, is invariant in θ.

[8] The investor's risk-neutrality implies that selling α shares of the asset (i.e., $s(\boldsymbol{y}) = \alpha\boldsymbol{y}$) can also be interpreted as committing to transfer the asset with probability α (similar to Mussa and Rosen (1978) but with interdependent values).

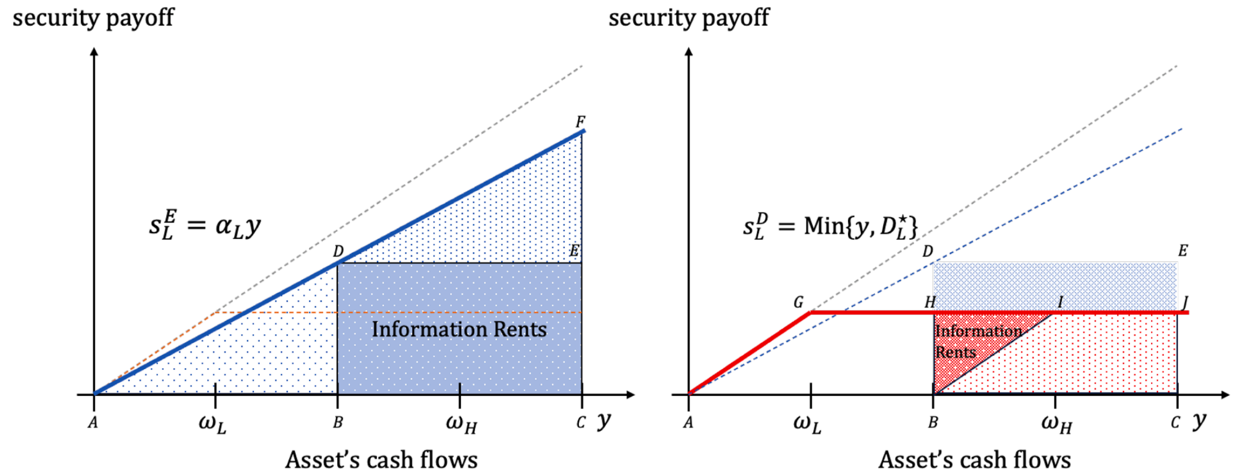

Fig. 1. Panel A: Information rents under equity menu $\mathcal{M}^E$. These are given by the difference in areas between the trapezoid BCFD ($\mathbb{E}\left[\alpha_L \boldsymbol{y}|H\right]$) and the triangle ABD ($\mathbb{E}\left[\alpha_L \boldsymbol{y}|L\right]$), and hence coincide with the area of the rectangle BCED. Panel B: Information rents under debt menu $\mathcal{M}_\star^{\text{Debt}}$. These are given by the difference in areas between the rectangle BCJH ($\mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^\star\right\}|H\right]$) and the trapezoid ABHG ($\mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^\star\right\}|L\right]$), and hence coincide with the area of the triangle BIH.

$\min\left\{\boldsymbol{y}, D_L^\star\right\}$ increases as the extent of residual uncertainty decreases. This effect is reminiscent of the fact that a risk averse investor's expected utility increases when facing less uncertainty.

Furthermore, increasing θ simultaneously decreases the investor's information rents. As explained above, Type H's information rents are given

$$
\begin{aligned}
I_\theta\left(H; \mathcal{M}_\star^{\text{Debt}}\right) &\equiv \mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^\star\right\}\middle|H\right] - \mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^\star\right\}\middle|L\right] \\
&= D_L^\star - \mathbb{E}\left[\min\left\{\boldsymbol{y}, D_L^\star\right\}\middle|L\right] \\
&= \int_0^{D_L^\star} F_\theta(y|L)\mathrm{d}y.
\end{aligned}
$$

This effect, again, is a consequence of the geometry of debt securities. Intuitively, type H's expected value of $\min\left\{\boldsymbol{y}, D_L^\star\right\}$, $D_L^\star$, is not affected by the extent of residual uncertainty as this type knows that cash flows are larger than $D_L^\star$ with certainty. In turn, type L's expected value of security $\min\left\{\boldsymbol{y}, D_L^\star\right\}$ strictly increases when the uncertainty about cash flows decreases. Thus, type H captures less information rents as the quality of the private signal improves (larger θ). The issuer thus fares strictly better when screening a more informed investor.[9]

Finally, it is instructive to consider the limit case where the investor is perfectly informed about cash flows ($\theta \to \infty$). Here, the issuer attains *full-surplus extraction*. Indeed, the optimal debt menu sets $D_\omega^\star = \omega$, $\omega \in \{L, H\}$, which simultaneously maximizes gains from trade and fully dissipates information rents. The case with $\theta \to \infty$ corresponds to a screening environment similar to the one in Mussa and Rosen (1978) (with interdependent values), under the assumption that the issuer can flexible design financial securities to share the asset's future cash flows. Our solution contrasts with the standard posted-price solution in that model and shows that the flexibility to sell financial securities allows the issuer to reach full-surplus extraction at the limit where the investor becomes perfectly informed about the asset's cash flows.

3. The model

3.1. Security design

The economy consists of an issuer (she) and an investor (he). The issuer owns a risky asset that delivers stochastic cash flows $\boldsymbol{y} \in \mathbb{R}_+$. There are two periods, $t \in \{1, 2\}$. In period 1, the issuer can sell a claim $s(\cdot)$ on the asset's period 2-cash flows to the investor at a price p. In period 2, the asset produces cash flows $\boldsymbol{y} \in \mathbb{R}_+$, and the investor obtains $s(\boldsymbol{y})$, whereas the issuer keeps the unsold claims of the asset $\boldsymbol{y} - s(\boldsymbol{y})$.

Securities. The issuer has the flexibility to design any arbitrary financial security satisfying limited liability and double-monotonicity in the asset's cash flows. These are standard assumptions in the security design literature. In their absence, one of

[9] By virtue of the Envelope Theorem, the marginal effect of varying the quality of the investor's private information on the issuer's profit at the optimal screening mechanism is negligible.

the agents has incentives to sabotage the success of the project.[10] The set of available securities is thus given by:

$$\mathcal{S} \equiv \{ s : \mathbb{R}_+ \to \mathbb{R}_+ \ \text{ s.t: } \ (\text{LL}): \ 0 \le s(y) \le y, \ \forall y \ge 0$$
$$(\text{M}): \ s(y) \text{and} y - s(y) \text{are nondecreasing} \}.$$

Information. The issuer and the investor share a common prior about $\boldsymbol{y}$, given by Ψ over $\mathcal{Y} \equiv \left[\underline{y}, \overline{y}\right] \subseteq \mathbb{R}_+$. In period 1, the investor receives private information about the cash flows summarized by $\boldsymbol{\omega} \in \Omega \equiv \left[\underline{\omega}, \overline{\omega}\right] \subseteq \mathbb{R}$. We refer to $\boldsymbol{\omega}$ as the investor's type. We denote by $F(\omega, y)$ the joint distribution of $\boldsymbol{\omega}$ and $\boldsymbol{y}$, and by Φ the marginal distribution of $\boldsymbol{\omega}$. We assume that both the conditional distribution of cash flows $F(\cdot|\omega)$ and the distribution of types Φ admit a strictly positive density, $f(\cdot|\omega)$ and $\phi(\cdot)$ respectively, and that $\left[\boldsymbol{y}|\boldsymbol{\omega} = \omega''\right] \succ_{\text{MLRP}} \left[\boldsymbol{y}|\boldsymbol{\omega} = \omega'\right]$ for all $\omega'' > \omega'$.[11] We further assume that $\mathbb{E}\left[\boldsymbol{y}|\overline{\omega}\right] < \infty$ and that $\mathbb{E}\left[\boldsymbol{y}\frac{\frac{\partial f(\boldsymbol{y}|\omega)}{\partial \omega}}{f(\boldsymbol{y}|\omega)}\middle|\omega\right] < \infty$ for all ω. The issuer does not observe any information before trading with the investor.

Preferences. The investor is risk-neutral. The expected utility of investor ω from buying security $s(\cdot)$ at price p is then given by:

$$\mathbb{E}[s(\boldsymbol{y})|\omega] - p = \int_{\mathcal{Y}} s(y) \mathrm{d}F(y|\omega) - p.$$

The issuer, on the other hand, is liquidity-constrained and discounts future cash flows at $\delta \in [0, 1)$. For any contract specifying security s and price p, her (ex-post) payoff is given by

$$v(y, s, p) \equiv p + \delta(y - s(y)).$$

The assumption that the issuer discounts future cash flows more than the investor guarantees that there are positive gains from trade, for any nontrivial security. In other words, the investor is the efficient asset holder.

Mechanisms. Without loss of generality, we restrict attention to incentive-compatible, direct mechanisms. The issuer asks the investor to report their type and offers and allocation and price according to the mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega \in \Omega}$.

Let $U_{\mathcal{M}}(\tilde{\omega}; \omega)$ represent the expected utility of an investor, under mechanism $\mathcal{M}$, whose true type is ω and reports $\tilde{\omega}$:

$$U_{\mathcal{M}}(\tilde{\omega}; \omega) \equiv \mathbb{E}[s(\boldsymbol{y}|\tilde{\omega})|\omega] - p(\tilde{\omega}), \quad \forall \tilde{\omega}, \omega.$$

Definition 1. We say that a mechanism $\mathcal{M}$ is *feasible* if it satisfies (i) individual rationality

$$[\text{IR}_\omega]: \quad U_{\mathcal{M}}(\omega; \omega) \ge 0, \quad \forall \omega \in \Omega,$$

and (ii) incentive compatibility

$$[\text{IC}_{\omega,\tilde{\omega}}]: \quad U_{\mathcal{M}}(\omega; \omega) \ge U_{\mathcal{M}}(\tilde{\omega}; \omega), \quad \forall \tilde{\omega}, \omega \in \Omega.$$

The issuer thus designs a mechanism $\mathcal{M} = \left\{s^\star[\omega], p^\star(\omega)\right\}_{\omega \in \Omega}$ that solves the program

$$\max_{\{s[\omega], p(\omega)\}_{\omega \in \Omega}} \mathbb{E}[v(\boldsymbol{y}, s[\boldsymbol{\omega}], p(\boldsymbol{\omega}))] = \int_{\Omega} \left(p(\omega) + \delta \int_{\mathcal{Y}} (y - s(y|\omega)) \mathrm{d}F(y|\omega) \right) \mathrm{d}\Phi(\omega)$$
$$\text{s.t.} \quad [\text{IR}_\omega], [\text{IC}_{\omega,\tilde{\omega}}], \forall \omega, \tilde{\omega} \in \Omega. \tag{2}$$

Importantly, program (2) clarifies that, when the investor (truthfully) reports to be type ω, the issuer uses the information conveyed by the investor's report to update the expected value of the unsold claims, $\boldsymbol{y} - s(\boldsymbol{y}|\omega)$.

4. The optimality of debt menus

We start the analysis by formally defining debt securities.

Definition 2. Security $s \in \mathcal{S}$ is said to be a debt security if there is $D > 0$ such that $s(\boldsymbol{y}) = \min\{\boldsymbol{y}, D\}$.

Intuitively, a security $s(\boldsymbol{y}) = \min\{\boldsymbol{y}, D\}$ promises a face value $D > 0$ whenever possible (i.e., when $\boldsymbol{y} \ge D$) and priority when cash flows fall short of D.

We show that the optimal feasible mechanism with securities consists of a *debt menu*. To prove this result, we first establish some basic properties that are instrumental to the proof of the main result. We first establish that information rents are monotone with the investor type and that the monotonicity is strict for strictly monotone securities. This property is important as it precludes the possibility of full surplus extraction. We further show that, consistent with the standard single-dimensional screening problem, it is without loss of optimality to restrict attention to mechanisms that leave no information rents to the lowest type (*no rents at the bottom*) and that do not distort the efficient allocation of the highest type (*no distortion at the top*).

Proposition 1. *For any feasible mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega \in \Omega}$, the following properties are true:*

[10] These are standard assumptions in the security design literature. When $s(y)$ is not monotone, the issuer can obtain (risk-free) credit from a third party to boost cash flows and thus decrease the amount owed to investor. In turn, when $y - s(y)$ is not monotone, the issuer can burn cash flows and improve her payoff.

[11] This is equivalent to requiring the conditional density $f(y|\omega)$ to be log-supermodular.

1. $U_{\mathcal{M}}(\omega';\omega') \geq U_{\mathcal{M}}(\omega;\omega) \geq 0$ *for any* $\omega' > \omega$*. Further, if* $s[\omega]$ *is strictly monotone, then* $U_{\mathcal{M}}(\omega';\omega') > U_{\mathcal{M}}(\omega;\omega)$ *for any* $\omega' > \omega$*.*
2. *If* $U_{\mathcal{M}}(\underline{\omega},\underline{\omega}) > 0$*, then* $\mathcal{M}$ *is strictly dominated.*
3. *If* $s[\overline{\omega}] \neq \boldsymbol{y}$ *a.e. (i.e.,* $s[\overline{\omega}]$ *is not pure equity), then* $\mathcal{M}$ *is strictly dominated.*

Property 1 in Proposition 1 says that information rents are monotone with the investor's type and strictly so for strictly monotone securities.[12] This result implies that the issuer cannot reach full surplus extraction as this would require to sell *pure equity*, a strictly monotone security. This result contrasts with the famous results by Crémer et al. (1987) and Crémer and McLean (1988) showing that when the issuer's and investor's information signals are correlated, the issuer can capture all the surplus. In our case, $\boldsymbol{y}$ and $\boldsymbol{\omega}$ are correlated; yet the structure of the problem prevents the issuer from appropriating the whole surplus. The results follows from the assumptions that (a) the issuer is liquidity-constrained and (b) sells financial securities $s \in \mathcal{S}$. The assumption that the issuer needs to raise cash in period 1 implies that the price paid by the investor, p, cannot be made contingent on the future realizations of the asset's cash flows. In turn, the assumption that issuer sells financial securities ($s \in \mathcal{S}$) implies that the security must exhibit cash flow monotonicity. This requirement imposes a constraint which precludes the design of the lotteries required for full-extraction (Crémer and McLean, 1988).[13]

Properties 2 and 3 are standard. Property 2 follows from the fact that the lowest type, $\underline{\omega}$, obtains the lowest information rents (property 1). If this type obtains positive rents under a given mechanism, the issuer can increase all prices $\{p(\omega)\}_{\omega\in\Omega}$ by the same amount until $[\text{IR}_{\underline{\omega}}]$ binds. Finally, property 3 obtains from the fact that the highest type, $\overline{\omega}$, values the asset more than the rest of types. The issuer can therefore sell the whole asset to type $\overline{\omega}$ and charge a price that only this type is willing to offer. Hence, it is optimal not to distort the allocation at the top.

4.1. Characterization of incentive compatible mechanisms

We leverage the tools developed by Milgrom and Segal (2002) and Pavan et al. (2014) to provide necessary and sufficient conditions for a mechanism to be (globally) incentive compatible.

Lemma 1. *Consider an arbitrary mechanism* $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$*. Suppose that* $\mathbb{E}\left[\boldsymbol{y}\frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\middle|\omega\right] < \infty$ *for all* $\omega \in \Omega$*. If* $\mathcal{M}$ *is incentive compatible, then*

$$U_{\mathcal{M}}(\omega,\omega) - U_{\mathcal{M}}(\underline{\omega},\underline{\omega}) = \int_{\underline{\omega}}^{\omega}\left(\int_{y} s(y|\tilde{\omega})\frac{\partial}{\partial\omega}f(y|\tilde{\omega})\mathrm{d}y\right)\mathrm{d}\tilde{\omega},\ \text{for }\Phi-\text{almost all }\omega. \tag{3}$$

In turn, if $\mathcal{M}$ *satisfies (3) and is such that*

$$\left(\int_{y}(s(y|\omega) - s(y|\hat{\omega}))\frac{\partial}{\partial\omega}f(y|\omega)\mathrm{d}y\right)\cdot(\omega-\hat{\omega}) \geq 0,\ \forall\omega,\hat{\omega}\in\Omega, \tag{4}$$

then, $\mathcal{M}$ *is incentive compatible.*

Eq. (3) is a *necessary* condition for $\mathcal{M}$ to be incentive compatible, whereas condition (4) is a *sufficient* condition, and is reminiscent to the standard *monotonicity* condition in single-dimensional, supermodular environments.

4.2. The benefits of screening with debt

We show next that the issuer has a strong preference to screen the investor using debt securities. Intuitively, among all securities that induce the same gains from trade, debt is the least preferred by high investor types because it minimizes their payout at high cash flows realizations. This feature of debt implies that the issuer can reduce the information rents she leaves to high types to prevent them from understating their private information by selling debt securities.

Consider an arbitrary incentive compatible mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$. Using the fact that, under truth-telling, $U_{\mathcal{M}}(\omega;\omega) = \mathbb{E}[s[\omega]|\omega] - p(\omega)$, together with the necessary condition in (3), we can write the issuer's (net) ex-ante payoff as[14]

$$\begin{aligned}&\mathbb{E}[p(\boldsymbol{\omega}) + \delta(\boldsymbol{y} - s[\boldsymbol{\omega}])] - \delta\mathbb{E}[\boldsymbol{y}]\\ &= \int_{\Omega}\int_{y}\left(s(y|\omega)\left(1-\delta-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right)\right)\mathrm{d}F(y|\omega)\mathrm{d}\Phi(\omega)\end{aligned}$$

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[12] The result is stronger under strict MLRP: the inequality is strict for any $s[\omega] \in \mathcal{S}$ that is not safe debt—that is, for any security that does not promise a constant payout below the smallest possible cash flow realization, $s(\boldsymbol{y}) \equiv c \in [0, \underline{y}]$. Safe debt is strictly suboptimal in our environment because there are strictly positive gains from trade. Note further that constant securities promising a payout above $\underline{y}$ are not feasible as they violate limited liability.

[13] Even if we dispense with the monotonicity assumption, the requirement that the securities must also satisfy limited liability would generally preclude full-extraction mechanisms (Crémer and McLean, 1988) that typically require *deep pockets*.

[14] By property 2 in Proposition 1, it is without loss to restrict attention to mechanisms that leave no rents at the bottom, $U_{\mathcal{M}}(\underline{\omega},\underline{\omega}) = 0$.

$$= \int_{\Omega} \left(\underbrace{(1-\delta)\mathbb{E}[s(\boldsymbol{y}|\omega)|\omega]}_{\text{Gains from Trade}} - \underbrace{\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\mathbb{E}\left[s(\boldsymbol{y}|\omega)\frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\middle|\omega\right]}_{\text{Information Rents}} \right) \mathrm{d}\Phi(\omega). \tag{5}$$

Following Inostroza and Tsoy (2025), we introduce the *information sensitivity* order as follows.

Definition 3. We say that security $s'' \in \mathcal{S}$ is more informationally sensitive than $s' \in \mathcal{S}$, if $s'' - \mathbb{E}\left[s''\right]$ crosses $s' - \mathbb{E}\left[s'\right]$ once from below.

In words, security s'' is more informationally sensitive than s', if after normalizing by the potential differences in means, s'' crosses s' once from below. We argue that more informationally sensitive securities increase the information rents captured by high types under any incentive compatible mechanism.

For any $\omega \in \Omega$, let

$$I_{\mathcal{M}}(\omega) \equiv \mathbb{E}\left[s(\boldsymbol{y}|\omega)\left(\frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\right)\middle|\omega\right] \tag{6}$$

represent the information rents *induced* by type ω. That is, $I_{\mathcal{M}}(\omega)$ represents the rents that need to be paid, under mechanism $\mathcal{M}$, to every type above ω to not pretend to be type ω. The next result shows that among all securities that promise the same expected payoff to the investor, $\mathbb{E}[s(\boldsymbol{y}|\omega)|\omega]$ (and hence that induce the same gains from trade), debt—the least information-sensitive security—minimizes the investor's information rents.

Lemma 2. *Consider two incentive compatible mechanisms* $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$, $\tilde{\mathcal{M}} = \{\tilde{s}[\omega], \tilde{p}(\omega)\}_{\omega\in\Omega}$. *Suppose that, for* $\omega \in \Omega$, *(i)* $\mathbb{E}[s[\omega] - \tilde{s}[\omega]|\omega] = 0$, *(ii)* s *is more information sensitive than* $\tilde{s}$. *Then,*

$$I_{\mathcal{M}}(\omega) - I_{\tilde{\mathcal{M}}}(\omega) = \mathbb{E}\left[(s(\boldsymbol{y}) - \tilde{s}(\boldsymbol{y}))\left(\frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\right)\middle|\omega\right] > 0.$$

Lemma 2 highlights the issuer's preference for *information-insensitive* securities. Indeed, the result above implies that, starting from an arbitrary incentive compatible mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$, the issuer strictly benefits from replacing the security $s[\omega]$ offered to type ω for a less information sensitive security $\tilde{s}[\omega]$ that promises the same expected payoff. The fact that among all securities that promise the same expected payoff, debt is the least informationally-sensitive security then implies that, abstracting from feasibility constraints, the issuer strictly benefits from offering debt securities to all types.

However, it is not clear whether starting from an arbitrary incentive compatible mechanism, the issuer can simply replace each security $s[\omega]$ with type ω's equivalent debt security $s^{\mathrm{D}}[\omega] \equiv \min\{\boldsymbol{y}, D(\omega)\}$ (with $\mathbb{E}\left[s[\omega] - s^{\mathrm{D}}[\omega]\middle|\omega\right] = 0$) without spoiling incentive compatibility constraints. While the argument above suggests that by replacing the original securities for their equivalent debt securities, the issuer relaxes downward incentive compatibility constraints (and reduces information rents captured by higher types), the same logic suggests that upward incentive compatibility constraints might be compromised. Below, we show that a variation of the argument above that accounts for this concern allows us to dominate any incentive-compatible mechanism with a debt menu.

4.3. Optimality of debt menus

Our next result shows that, starting from an arbitrary feasible mechanism $\mathcal{M}$, we can construct an alternative feasible mechanism $\hat{\mathcal{M}}$ that sells debt to all types and that dominates the original mechanism $\mathcal{M}$. The construction of the dominating mechanism is in two steps. In the first step, the issuer finds, for each $\omega \in \Omega$, the face value $D(\omega)$ characterizing type ω's equivalent debt security. That is, $D(\omega)$ is implicitly determined by $\mathbb{E}[s[\omega] - \min\{\boldsymbol{y}, D(\omega)\}|\omega] = 0$. In the second step, the issuer constructs $\hat{D}(\cdot)$ defined as the *upper monotone envelope* of $D(\cdot)$. That is, $\hat{D}(\cdot)$ is the smallest monotone function weakly above $D(\cdot)$. The issuer then offers the debt menu composed by the securities $\hat{s}[\omega] \equiv \min\left\{\boldsymbol{y}, \hat{D}(\omega)\right\}$, $\omega \in \Omega$. The resulting mechanism is globally incentive compatible and, unless $\mathcal{M}$ is a debt menu itself Φ–almost everywhere, strictly dominates $\mathcal{M}$.

Theorem 1. *Consider an arbitrary feasible mechanism* $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$. *Suppose that the set of types for which there is no* $D \geq 0$ *so that* $\mathbb{P}[s[\omega] = \min\{\boldsymbol{y}, D\}] = 1$ *has positive measure. Then, there is a feasible mechanism* $\hat{\mathcal{M}} = \{\hat{s}[\omega], \hat{p}(\omega)\}_{\omega\in\Omega}$ *with* $\hat{s}[\omega] \equiv \min\left\{\boldsymbol{y}, \hat{D}(\omega)\right\}$, *for all* $\omega \in \Omega$, *and* $\hat{D}(\cdot)$ *monotone, that strictly dominates* $\mathcal{M}$.

Theorem 1 shows that the solution to the issuer's program (2) is, in fact, a debt menu. Any feasible mechanism that incorporates securities other than debt over a positive measure set of types is strictly dominated by the debt menu characterized by $\hat{D}(\cdot)$.

The proof proceeds in two steps. Step 1 considers an arbitrary feasible mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$, and construct the auxiliary mechanism $\mathcal{M}^{\mathrm{D}} = \left\{s^{\mathrm{D}}[\omega], p(\omega)\right\}_{\omega\in\Omega}$, that replaces each security $s[\omega]$ for type ω's equivalent debt security $s^{\mathrm{D}}[\omega] \equiv \min\{\boldsymbol{y}, D(\omega)\}$, while maintaining the same prices. The new mechanism $\mathcal{M}^{\mathrm{D}}$ preserves the issuer's payoff but need not be feasible. In fact, if the schedule $D(\cdot)$ is strictly decreasing over a positive measure set, then $\mathcal{M}^{\mathrm{D}}$ violates incentive compatibility.

In Step 2, we construct the dominating mechanism $\hat{\mathcal{M}} = \{\hat{s}[\omega], \hat{p}(\omega)\}_{\omega\in\Omega}$ as follows. Suppose that there is an interval $\left(\omega^-, \omega^+\right)$ over which $D(\omega^-) > D(\omega)$ for all $\omega \in \left(\omega^-, \omega^+\right)$ (and let ω^+ be such that $\left(\omega^-, \omega^+\right)$ is the interval with maximal (Lebesgue) measure with that property). The new mechanism replaces the set of contracts $\left\{s^{\mathrm{D}}[\omega], p(\omega)\right\}_{\omega\in\left[\omega^-, \omega^+\right]}$ for the single contract $\left(s^{\mathrm{D}}[\omega^-], p(\omega^-)\right)$. In other

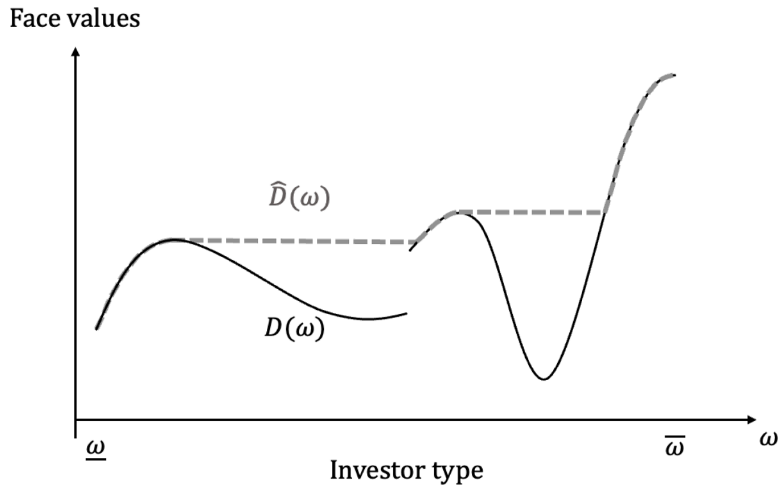

Fig. 2. Upper monotone envelope.

words, the new mechanism pools all types in the interval (ω^-, ω^+) under the contract $(s^{\mathrm{D}}[\omega^-], p(\omega^-))$ (see Fig. 2). We show that the resulting mechanism strictly increases the issuer's payoff. Intuitively, if type $\omega \in (\omega^-, \omega^+)$ *envies* type ω^-'s allocation under $\mathcal{M}^{\mathrm{D}}$ (i.e., $\mathbb{E}[s^{\mathrm{D}}[\omega] - s^{\mathrm{D}}[\omega^-]|\omega] < 0$), he must also envy type ω^-'s allocation under the original mechanism $\mathcal{M}$ (i.e., $\mathbb{E}[s[\omega] - s[\omega^-]|\omega] < 0$). This is because modifying type ω^-'s original security for type ω^-'s debt equivalent security $s^{\mathrm{D}}[\omega^-]$, reduces its attractiveness to all types above ω^-. This further implies that, under the original mechanism, the price paid by type ω must be small to prevent type ω from reporting to be type ω^-. That is, $p(\omega) \leq p(\omega^-) + \underbrace{\mathbb{E}[s[\omega] - s[\omega^-]|\omega]}_{<0}$. This, however, is suboptimal for the issuer. Roughly, under the original mechanism the issuer is selling inefficiently "small" securities to all types over (ω^-, ω^+), despite the fact that higher types increase the trade surplus for any given security. In the proof, we show that the issuer is strictly better off by replacing the contracts of all such types ω and under type ω^-'s contract under the new mechanism.

Let $\mathcal{H}_{\mathrm{Mon}}$ be the set of monotone functions $h : \Omega \to \mathbb{R}_+$, then the face values of the dominating mechanism $\hat{\mathcal{M}}$ are given by:

$$\hat{D}(\cdot) \equiv \inf \left\{ h \in \mathcal{H}_{\mathrm{Mon}} : h(\omega) \geq D(\omega), \forall \omega \in \Omega \right\},$$

the *upper monotone envelope* of $D(\cdot)$. We finally show that the so-constructed mechanism $\hat{\mathcal{M}}$—a debt menu with monotone face values—satisfies the sufficient condition characterized in (4) and must therefore be globally incentive compatible.

4.4. Characterization of the optimal mechanism

Theorem 1 transforms the issuer's original problem over the infinite-dimensional allocation space of securities $\mathcal{S}$ into a problem with a single-dimensional allocation space, where the optimal security of each investor type is fully characterized by its face value.

Corollary 1. *The issuer's program can be re-stated as*

$$\max_{D(\cdot)} \int_{\Omega} \left(\mathbb{E}\left[\min\{\boldsymbol{y}, D(\omega)\} \left(1 - \delta - \left(\frac{1-\Phi(\omega)}{\phi(\omega)} \right) \frac{\frac{\partial}{\partial \omega} f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)} \right) \middle| \omega \right] \right) \mathrm{d}\Phi(\omega) \tag{7}$$
$$\text{s.t. } D(\cdot) \text{ monotone.}$$

The solution to the issuer's problem may still be involved and require non-trivial *ironing*. Below, we introduce standard assumptions that guarantee that the pointwise solution to program (7) is strictly monotone, and therefore does not feature pooling regions.

Assumption 1. The hazard rate $\frac{\phi(\omega)}{1-\Phi(\omega)}$ is nondecreasing in ω.

Assumption 2. The survival function $1 - F(y|\omega)$ is log-concave in ω, for all y.

Assumptions 1 and 2 guarantee that the virtual surplus,

$$\tilde{\Pi}(\omega; D) \equiv \mathbb{E}\left[\min\{\boldsymbol{y}, D\} \left(1 - \delta - \left(\frac{1-\Phi(\omega)}{\phi(\omega)} \right) \frac{\frac{\partial}{\partial \omega} f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)} \right) \middle| \omega \right],$$

that is, the difference between the gains from trading security $\min\{\boldsymbol{y}, D\}$ with type ω minus the induced information rents, have single-crossing differences in (D, ω). This means that, fixing type ω, if the marginal effect on the issuer's profit from increasing the face value of type ω's security,

$$\frac{\partial}{\partial D} \tilde{\Pi}(\omega; D) = (1-\delta)(1 - F(D|\omega)) - \left(\frac{1-\Phi(\omega)}{\phi(\omega)} \right) \frac{\partial}{\partial \omega}(1 - F(D|\omega))$$

is positive, so it is for any type $\omega' > \omega$ (i.e., $\frac{\partial}{\partial D}\tilde{\Pi}(\omega'; D) \geq 0$). This property implies that the pointwise-optimal choice of $D(\cdot)$ must be nondecreasing (and hence incentive compatible). Intuitively, under Assumption 2, $\frac{\frac{\partial}{\partial \omega}(1-F(D|\omega))}{1-F(D|\omega)}$ is decreasing in ω and hence the marginal effect of increasing face value D increases the gains from trade relatively more than the increase in information rents for larger values of ω. Interestingly, this condition is trivially satisfied for any location experiment satisfying MLRP—an standard assumption in the financial economics literature.

Our next result summarizes these arguments and shows that under the two assumptions, the issuer's pointwise-optimal solution is implementable. Furthermore, the optimal menu does not feature pooling.

Proposition 2. *Suppose that Assumptions (1) and (2) hold. Then, the optimal mechanism $\mathcal{M}^\star = \left\{s^\star[\omega], p^\star(\omega)\right\}_{\omega\in\Omega}$ is characterized by $s^\star(\omega) = \min\left\{y, D^\star(\omega)\right\}$, where $D^\star(\cdot)$ is strictly monotone and is implicitly characterized by the unique solution to*

$$(1-\delta)(1-F(D|\omega)) = \frac{1-\Phi(\omega)}{\phi(\omega)}\frac{\partial}{\partial\omega}(1-F(D|\omega)). \tag{8}$$

5. Accuracy and monotone comparative statics

We now explore how changes in the quality of the investor's private signal affects the optimal mechanism and the issuer's ability to raise funds. We remain agnostics about the determinants of the underlying information structure and, instead, consider exogenous changes to the quality of the investors' private information.

We show that, contrary to the classical intuition, an issuer who can flexible design financial securities may benefit from trading with a more informed investor. We show that, fixing any monotone debt menu, gains from trade increase as the investor's becomes more informed whereas information rents experience an ambiguous effect. We then provide primitive conditions under which the issuer strictly benefits from trading with a better informed investor.

5.1. Accuracy (Lehmann, 1988)

We first formalize the concept of improving the investor's information. After the investor observes $\boldsymbol{\omega} = \omega$, cash flows $\boldsymbol{y}$ are drawn from the conditional distribution $F(y|\omega)$ which we assume admits a positive density $f(y|\omega)$ over $\mathcal{Y}$ satisfying MLRP. We refer to the collection of conditional distributions $F \equiv \{F(\cdot|\omega)\}_{\omega\in\Omega}$ as an *experiment*. An experiment F, together with the prior distribution of types $\boldsymbol{\omega}$, Φ, uniquely determines the joint distribution of $(\boldsymbol{y}, \boldsymbol{\omega})$,

$$\mathbf{F}(y,\omega) \equiv \int_{\Omega\times\mathcal{Y}} F(y|\tilde{\omega})1\{\tilde{\omega}\leq\omega\}\mathrm{d}\Phi(\tilde{\omega}).$$

We introduce a natural ordering to rank the *accuracy* (Lehmann, 1988) of a family of experiments parameterized by $\theta \in \Theta \equiv \left[\underline{\theta}, \overline{\theta}\right]$, $\left\{F_\theta\right\}_{\theta\in\Theta}$, where for each θ, $\left[\boldsymbol{y}_\theta|\boldsymbol{\omega} = \omega\right]$ is drawn from $F_\theta(y|\omega)$.

Definition 4. We say that an experiment $F_{\theta'}$ is *more accurate* than F_θ (Lehmann, 1988), which we write $F_{\theta'} \succeq F_\theta$, if[15]

$$F_{\theta'}\left(F_{\theta'}^{-1}\left(p|\omega''\right)|\omega'\right) \geq F_\theta\left(F_\theta^{-1}\left(p|\omega''\right)|\omega'\right), \forall\omega'' > \omega'. \tag{10}$$

When $F_{\theta'} \succeq F_\theta$, cash flows $\boldsymbol{y}$ become a more accurate signal of $\boldsymbol{\omega}$. Provided that signals satisfy MLRP, accuracy is implied by the Blackwell order, but is less restrictive (i.e., it compares more signals).[16] Accuracy has been used in the information economics literature to perform monotone comparative statics in the context of auctions (Persico, 2000), moral hazard (Kim, 1995, Jewitt, 2007), and career concerns (Dewatripont et al., 1999).

Given an experiment F_θ and a distribution of types Φ, we refer to the induced joint distribution of $(\boldsymbol{y}, \boldsymbol{\omega})$ as $\mathbf{F}_\theta$, and to the marginal distribution of cash flows as $\Psi_\theta = \mathrm{marg}_{\boldsymbol{y}}\mathbf{F}_\theta \in \Delta\mathbb{R}_+$.

Remark 1. This approach treats cash flows $\boldsymbol{y}$ as a signal of the investor's type $\boldsymbol{\omega}$. A more accurate experiment thus allows the issuer to contract on a more accurate variable, thereby allowing the issuer to limit the investor's information rents. However, as we show in Appendix B, $\boldsymbol{y}$ being a more informative signal of $\boldsymbol{\omega}$ also implies that (after a natural normalization) the investor's type $\boldsymbol{\omega}$ becomes a more informative signal about $\boldsymbol{y}$ (according to the Monotone Information order (Athey and Levin, 2018)),[17] which

[15] Applying $F_{\theta'}^{-1}\left(\cdot|\omega'\right)$ to both sides in (10) and letting $y = F_\theta^{-1}\left(p|\omega''\right)$,

$$F_{\theta'}^{-1}\left(F_\theta\left(y|\omega''\right)\middle|\omega''\right) \geq F_{\theta'}^{-1}\left(F_\theta\left(y|\omega'\right)|\omega'\right), \forall\omega'' > \omega', \forall y, \tag{9}$$

another standard characterization of accuracy.

[16] Provided that signals satisfy MLRP, any two signals ordered according to Blackwell are also ordered according to Lehmann (1988). When the state space is binary, both notions of informativeness coincide (Jewitt, 2007). For recent work on properties of the Lehmann order, see Kim (2022) and Di Tillio et al. (2021).

[17] *A signal $\boldsymbol{\omega}_{\theta'}$ is more informative than $\boldsymbol{\omega}_\theta$ about $\boldsymbol{z}$ in the MIO-ND sense (Athey and Levin, 2018) if any decision maker with a supermodular payoff function in actions and states z, prefers signal $\boldsymbol{\omega}_{\theta'}$ over $\boldsymbol{\omega}_\theta$. The MIO-ND ordering is slightly less demanding than accuracy as the latter requires the preference above to extend to decision makers with preferences satisfying single-crossing differences (which include all supermodular functions). Notwithstanding this qualification, the MIO-ND ordering implies the standard notion of informativeness found in the information economics literature that the ex-ante distribution of posterior estimates is ranked in the mean-preserving spread ordering.*

may increase information rents. The current specification simplifies the exposition: it allows us to make assumptions directly on experiments $F_\theta(y|\omega)$, which shape the solution to the issuer's problem (see program 2).

5.1.1. Location experiments and copulas

We introduce two classes of accuracy-ranked experiments that have received much attention in the literature: location experiments and copulas. We say that the family of experiments $F \equiv \{F_\theta\}_{\theta\in\Theta}$ is a family of *location experiments* if there is a family of distributions $\{\tilde{F}_\theta\}_{\theta\in\Theta}$ such that

$$F_\theta(y|\omega) = \tilde{F}_\theta(y-\omega), \text{ for all } \theta, \omega, y.$$

That is, $\boldsymbol{y}_\theta \equiv \boldsymbol{\omega} + \boldsymbol{\varepsilon}_\theta$, with $\boldsymbol{\varepsilon}_\theta \sim \tilde{F}_\theta$ and $\boldsymbol{\omega} \perp \boldsymbol{\varepsilon}_\theta$. Location experiments are pervasive in the Financial Economics literature where $\boldsymbol{\varepsilon}_\theta$ is typically assumed to follow a log-concave distribution (e.g., normal distribution).

In turn, we say that the family of experiments $F \equiv \{F_\theta\}_{\theta\in\Theta}$ has a *copula structure* if, for any $\theta \in \Theta$, the induced joint distribution $\mathbf{F}_\theta$ belongs to the set of distributions $\mathcal{F}(\Psi, \Phi)$ with fixed marginals Ψ and Φ. Experiments with the copula structure are typical in the Statistics literature.[18] The assumption that all experiments within an accuracy-ranked family share the same marginals is appealing for our exercise as it allows us to vary the quality of the investor's information without changing the primitives of the problem (the distribution of cash flows Ψ and types Φ).[19]

Equipped with this taxonomy, we explore below how changes in the quality of the investor's information θ changes the trading outcomes. In particular, we study the effect of θ on the gains from trade and the information rents captured by the investor.

5.2. Gains from trade

We show first that, fixing an arbitrary feasible debt menu with monotone face values $\{s[\omega] = \min\{\boldsymbol{y}, D(\omega)\}, p(\omega)\}_{\omega\in\Omega}$ (henceforth, we refer to these menus as *monotone debt menus*), and considering any family of experiments $\{F_\theta\}_{\theta\in\Theta}$ with either the location or copula structure, gains from trade are monotone in accuracy. Specifically, for any $\omega \in \Omega$, let

$$G_\theta(\omega) \equiv (1-\delta)\mathbb{E}_{\mathbf{F}_\theta}\left[\min\left\{\boldsymbol{y}_\theta, D(\omega)\right\}\Big|\omega\right]$$

represent the gains from trading security $s(y|\omega) = \min\{y, D(\omega)\}$ with type ω, and let $\overline{G}_\theta \equiv \mathbb{E}\left[G_\theta(\boldsymbol{\omega})\right]$. We show that for location experiments, $G_\theta(\omega)$ is monotone in θ for each ω, whereas for experiments with the copula structure, $\overline{G}_\theta$ is monotone in θ.

Consider a family of location experiments $F_\theta(y|\omega) = \tilde{F}_\theta(y-\omega)$. Lehmann (1988) showed that $F_{\theta'} \succeq F_\theta$ if, and only if, $\tilde{F}_{\theta'}$ is less *dispersive* than $\tilde{F}_\theta$. Intuitively, as θ increases, cash flows become more concentrated around the investor's type thereby reducing their (ex-post) uncertainty over cash flows. In particular, each type ω experiences a mean-preserving contraction of their posterior beliefs $F_\theta[\omega] \succeq_{\text{MPS}} F_{\theta'}[\omega]$.[20] The result then follows from the concavity of debt on the asset's future cash flows. Fixing a debt security $s(\boldsymbol{y}) \equiv \min\{\boldsymbol{y}, D\}$, a reduction in ex-post uncertainty about $\boldsymbol{y}$, improves type ω's expected value of the security. The effect is similar to the one experienced by a risk-averse agent who obtains a higher expected utility when exposed to less uncertainty.

The result depends on the shape of the securities in the menu. Indeed, if instead of debt securities, the issuer offered an equity menu $\{\alpha(\omega)y\}_{\omega\in\Omega}$, then the gains from trade associated with each type ω would remain constant in accuracy:[21]

$$(1-\delta)\mathbb{E}\left[\alpha(\omega)\boldsymbol{y}_\theta\middle|\omega\right] = (1-\delta)\alpha(\omega)\underbrace{\mathbb{E}\left[\boldsymbol{y}_\theta\middle|\omega\right]}_{\equiv\overline{y}(\omega),\forall\theta},$$

In turn, if the issuer were to screen the investor using convex securities, (e.g., levered-equity or warrants, both of which have the form $\alpha\max\{y-K, 0\}$, $\alpha \in [0,1], K > 0$), then the associated gains from trade of each type ω would decrease with accuracy.

Consider next a family of experiments with the copula structure, $\{F_\theta\}_{\theta\in\Theta}$. We first introduce an auxiliary result that follows from Jewitt (2007) and Tchen (1980).

Lemma 3. *Fix any $\Phi \in \Delta\Omega$ and suppose that $\{F_\theta\}_{\theta\in\Theta}$ has the copula structure when coupled with Φ. Let $\boldsymbol{F}_{\theta'}(y,\omega)$ and $\boldsymbol{F}_\theta(y,\omega)$ be the joint distributions induced by Φ, and the experiments $F_{\theta'}$ and F_θ, respectively. Then, for any $\theta' > \theta$, and any supermodular function $v(y,\omega)$,*

$$\int v(y,\omega)\mathrm{d}\boldsymbol{F}_{\theta'}(y,\omega) \geq \int v(y,\omega)\mathrm{d}\boldsymbol{F}_\theta(y,\omega).$$

In other words, $\boldsymbol{F}_{\theta'}$ dominates $\boldsymbol{F}_\theta$ in the supermodular order, $\boldsymbol{F}_{\theta'} \succeq_{spm} \boldsymbol{F}_\theta$.

Lemma 3 shows that, provided that experiments have the copula structure, the accuracy order, which ranks experiments ($F_{\theta'} \succeq F_\theta$), induces the supermodular order, which in turn ranks joint distributions ($\mathbf{F}_{\theta'} \succeq_{\text{spm}} \mathbf{F}_\theta$). Roughly, supermodular functions reach a higher value "over the diagonal." That is, for any $y'' > y'$, and any $\omega'' > \omega'$, if v is supermodular then

$$v(y'', \omega'') + v(y', \omega') \geq v(y', \omega'') + v(y'', \omega').$$

[18] When two experiments F, G are such that (i) $F \succeq G$ and (ii) induce the same marginal distributions ($F, G \in \mathcal{F}(\Psi, \Phi)$), Lehmann (1966) refers to them as ordered in the *Positive Regression Dependent* order.

[19] The two classes of experiments are disjoint in that there are no two accuracy-ranked location experiments $\tilde{F}_{\theta'}$ and $\tilde{F}_\theta$ that induce the same marginal distribution of $\boldsymbol{y}$.

[20] The dispersive order implies the mean-preserving spread order (Shaked and Shanthikumar, 2007).

[21] The fact that $F_\theta[\omega] \succeq_{\text{MPS}} F_{\theta'}[\omega]$ implies, in particular, that $\mathbb{E}\left[\boldsymbol{y}_\theta|\omega\right] = \mathbb{E}\left[\boldsymbol{y}_{\theta'}|\omega\right]$.

When $F_{\theta'} \succeq F_\theta$, $\boldsymbol{y}$ and $\boldsymbol{\omega}$ become more associated and the ex-ante probability of them being closer increases (i.e., the probability mass over the diagonal increases). This increases the expected value of any supermodular function.

Lemma 3 implies that for experiments with the copula structure, increasing accuracy increases gains from trade in expectation. The conclusion follows from noting that, for any monotone debt menu, the associated debt securities $\min\{y, D(\omega)\}$ are supermodular in (y,ω). Indeed, for any $\omega'' > \omega'$, the monotonicity of $D(\cdot)$ implies that the *tranche* $\min\{y, D(\omega'')\} - \min\{y, D(\omega')\}$ is monotone in cash flows.

Theorem 2. *Consider an arbitrary monotone debt menu $\{s[\omega] = \min\{\boldsymbol{y}, D(\omega)\}, p(\omega)\}_{\omega\in\Omega}$. The following properties hold:*
(i) if $\{F_\theta\}_{\theta\in\Theta}$ is a location experiment, then for any $\theta' > \theta$, any $\omega \in \Omega$, $G_{\theta'}(\omega) \geq G_\theta(\omega)$;
(ii) if $\{F_\theta\}_{\theta\in\Theta}$ has the copula structure, then for any $\theta' > \theta$, $\overline{G}_{\theta'} \geq \overline{G}_\theta$.

Theorem 2 implies that, fixing any monotone debt menu, it is *efficient* for the issuer to trade with a better informed investor. Interestingly, this observation is not necessarily true for arbitrary mechanisms with financial securities. Indeed, as mentioned above, for location experiments, gains from trade are invariant in the quality of the investor's information when the issuer offers linear contracts, and decrease when the issuer screens using convex instruments (e.g., levered-equity, warrants). Secondly, the more stringent the liquidity constraints faced by the issuer are (i.e., smaller δ), the more sensitive the gains from trade are to information accuracy.

5.3. Information rents

As discussed in Section 5.1 (see also Appendix B), when F_θ becomes more accurate, two things are true at the same time: (i) the investor becomes more informed about cash flows, and (ii) the asset's cash flows $\boldsymbol{y}$—the contractible variable—more accurately describe the investor's type $\boldsymbol{\omega}$. This raises important questions, since a more informed investor can capture more information rents, but an issuer who can contract on a more informative signal may be more effective at curbing these rents. We thus explore how changes in accuracy affect the investor's information rents. We show that as accuracy improves, information rents tend to decrease for low types and to increase for high types.

More formally, fix any feasible debt menu $\{s[\omega] \equiv \min\{\boldsymbol{y}, D(\omega)\}, p(\omega)\}_{\omega\in\Omega}$ and let $U_\theta(\omega)$ represent the *information rents captured* by type ω. From property 2 in Proposition (1), it is without loss to restrict attention to mechanisms for which $U_\theta(\underline{\omega}) = 0$, and therefore, (3) implies that,

$$U_\theta(\omega) = \int_{\underline{\omega}}^{\omega} \left(\int_y s(y|\tilde{\omega}) \frac{\partial}{\partial \omega} f_\theta(y|\tilde{\omega}) \mathrm{d}y \right) \mathrm{d}\tilde{\omega}.$$

On the other hand, the information rents *induced* by type ω—the rents that must be provided to all types above ω to avoid mimicking (see 6)—are given by $I_\theta(\omega) = \int_y s(y|\omega) \frac{\partial}{\partial\omega} f_\theta(y|\omega)\mathrm{d}y$. Intuitively, $I_\theta(\omega)$ equals the difference in valuation for the security designed for type ω, $s[\omega]$, between type ω and the upward adjacent type $\omega + \mathrm{d}\omega$. Thus,

$$U_\theta(\omega) = \int_{\underline{\omega}}^{\omega} I_\theta(\tilde{\omega})\mathrm{d}\tilde{\omega}. \tag{11}$$

Below, we show that, for location experiments, both $U_\theta(\omega)$ and $I_\theta(\omega)$ have single crossing differences in (ω, θ) at the optimal debt menu. These results formalize the idea that increasing accuracy has distinct effects over different types and tend to decrease the information rents captured by low types and increase the ones captured by high types. Together, these results challenge the conjecture that the issuer gives up more information rents when screening a more informed investor.

To get some intuition, we first consider a slightly more general version of Example 1, with multiple types.

Example 2. Consider $\boldsymbol{y}_\theta = \boldsymbol{\omega} + \frac{\boldsymbol{\varepsilon}}{\theta}$, with $\boldsymbol{\varepsilon} \sim U[-\bar{\varepsilon}, \bar{\varepsilon}]$, $\theta \in \mathbb{R}_{++}$, and $\boldsymbol{\omega} \sim \Phi \in \Delta\Omega$ with $\Omega \equiv \{\omega_1 \equiv \underline{\omega}, \dots, \omega_N \equiv \overline{\omega}\}$, $\omega_{n+1} - \omega_n = d > 0$ for all $n \in \{1, \dots, N-1\}$, and $\boldsymbol{\omega} \perp \boldsymbol{\varepsilon}$. Assume that the issuer offers the security $s[\omega_n] = \min\{\boldsymbol{y}, D(\omega_n) = \hat{D}\}$ to type ω_n. In this case, the information rents induced by type ω_n are given by

$$I_\theta(\omega_n) = \mathbb{E}\left[\min\{\boldsymbol{y}_\theta, \hat{D}\}\middle|\omega_{n+1}\right] - \mathbb{E}\left[\min\{\boldsymbol{y}_\theta, \hat{D}\}\middle|\omega_n\right].$$

Whether increasing accuracy increases information rents $I_\theta(\omega_n)$ thus depends on whether or not the expected value of the security increases more for type ω_{n+1} or for type ω_n as θ increases.

Consider first the case $\hat{D} \in \left(\omega_n - \frac{\bar{\varepsilon}}{\theta}, \omega_{n+1} - \frac{\bar{\varepsilon}}{\theta}\right]$ where the intuition is straight-forward. Type ω_{n+1} believes that $\min\{\boldsymbol{y}_\theta, \hat{D}\} = \hat{D}$ with certainty and hence,

$$I_\theta(\omega_n) = \hat{D} - \mathbb{E}\left[\min\{\boldsymbol{y}_\theta, \hat{D}\}\middle|\omega_n\right].$$

The concavity of debt together with the fact that, as θ increases, type ω_n's posterior beliefs about $\boldsymbol{y}_\theta$ become less dispersed, imply that the information rents induced by type ω_n, $I_\theta(\omega_n)$, decrease with θ. Intuitively, type ω_n is exposed to the region where the security defaults ($\{y : y \leq \hat{D}\}$) and increasing accuracy reduces the ex-ante likelihood of default, i.e., $\mathbb{P}[\boldsymbol{y}_{\theta'} \leq \hat{D}|\omega_n] < \mathbb{P}[\boldsymbol{y}_\theta \leq \hat{D}|\omega_n]$, for all $\theta' > \theta$. In contrast, type ω_{n+1} believes that the security never defaults and always pays $\hat{D}$ regardless of θ.

More generally, we argue that when $\hat{D}$ is low enough, increasing accuracy increases the expected value of $s[\omega_n]$ relatively more for type ω_n than for type ω_{n+1}. Indeed, consider now the case where $\hat{D} \geq \omega_{n+1} - \frac{\overline{\epsilon}}{\theta}$—the case where type ω_{n+1} believes that $s[\omega_n]$ defaults with positive probability. In this case,

$$\frac{\partial}{\partial\theta} I_\theta(\omega_n) = \left(\frac{\omega_{n+1} - \omega_n}{2(\overline{\epsilon}/\theta)}\right)\left(\hat{D} - \frac{\omega_{n+1} + \omega_n}{2}\right),$$

and therefore $\frac{\partial}{\partial\theta} I_\theta(\omega_n) \geq 0$ if, and only if, $\hat{D} \geq \omega_n + \frac{d}{2}$. When $\hat{D}$ is sufficiently large, type ω_{n+1}'s expected value of $s[\omega_n]$ increases more than type ω_n's. This is because in this case type ω_{n+1} is more exposed to the concavity of debt, as captured by the threshold above which default is averted.

Interestingly, whether the investor's information rents decrease with accuracy ultimately depends on the extent of "allocative distortion" experienced by the investor type. Indeed, note that $\Delta_\theta(\omega) \equiv \omega + \overline{\epsilon}/\theta - D^\star(\omega)$ measures the extent of allocative distortions experienced by type ω—it captures the difference between the face value of the efficient allocation (i.e., selling the whole asset, or equivalently setting the face value at $\omega + \overline{\epsilon}/\theta$) and the issuer's privately optimal face value $D^\star(\omega)$. Because $D^\star(\omega)$ is implicitly characterized by

$$(1-\delta)\frac{\phi(\omega)}{1-\Phi(\omega)} = \frac{\frac{\partial}{\partial\omega}\left(1 - F_\theta\left(D^\star|\omega\right)\right)}{1 - F_\theta(D^\star|\omega)} = \frac{1}{\Delta_\theta(\omega)},$$

provided that Φ has a monotone hazard rate (Assumption (1)), the extent of distortion $\Delta_\theta(\omega)$ decreases with ω, and vanishes to 0 as ω approaches $\overline{\omega}$ (Proposition 1). A simple manipulation then shows that information rents increase with accuracy if, only if, $\Delta_\theta(\omega) \leq \frac{\overline{\epsilon}}{\theta} - \frac{d}{2}$. We conclude that the effect of increasing accuracy on information rents decreases for low types and increase for high types.

Interestingly, when θ is sufficiently large so that $\frac{\overline{\epsilon}}{\theta} < \frac{d}{2}$, $\Delta_\theta(\omega) \geq 0 > \frac{\overline{\epsilon}}{\theta} - \frac{d}{2}$ for all $\omega \in \Omega$, and hence the (induced) information rents $I_\theta(\omega)$ *decrease for all types*. Because the information rents *captured* by each type ω are the sum of the information rents *induced* by all types below ω (see Eq. (11)), $U_\theta(\omega)$ also decreases for all types. Furthermore, at the limit as $\theta \to \infty$, information rents vanish as the issuer attains full-surplus extraction. In the Online Appendix, we provide an alternative Example where the experiments have the copula structure. We show that, provided that θ is sufficiently large, information rents decrease and vanish as the experiments approach full information.

We show that the insights from the Example 2 above readily extend to arbitrary families of accuracy-ranked, location experiments. Consider $F_\theta(y|\omega) = \tilde{F}_\theta(y - \omega)$ for all θ, y, ω. Fix $\theta \in \Theta$ and any feasible debt menu $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$ with $s(\omega) \equiv \min\{\boldsymbol{y}_\theta, D(\omega)\}$. Our next result shows that the information rents induced by a given type ω, $I_\theta(\omega)$, have single crossing differences in (ω, θ). Further, because the information rents by type ω are the sum of the information rents induced by all types below ω, $U_\theta(\omega)$ also displays single crossing differences in (ω, θ).

Proposition 3. *Suppose that Φ satisfies Assumption (1). Fix any $\theta \in \Theta$ and the associated optimal debt menu $\mathcal{M} = \left\{\min\left\{\boldsymbol{y}, D_\theta^\star(\omega)\right\}, p_\theta^\star(\omega)\right\}_{\omega\in\Omega}$. Then, for any $\theta' > \theta$, $I_{\theta'}(\omega; \mathcal{M}) - I_\theta(\omega; \mathcal{M})$ and $U_{\theta'}(\omega; \mathcal{M}) - U_\theta(\omega; \mathcal{M})$ satisfy single crossing from below (SCFB) in ω.*

A natural question that follows from the analysis above is whether more accurate experiments decrease the *aggregate* information rents captured by the investor. Our next result provides sufficient conditions for this to be the case. Intuitively, a more informed investor captures less information rents under large allocative distortions. We formalize this idea below.

Condition 1. *Let $H_\Phi(\omega) \equiv \phi(\omega)/(1-\Phi(\omega))$ and $H_{\tilde{F}_\theta}(\epsilon) \equiv \tilde{f}_\theta(\epsilon)/\left(1 - \tilde{F}_\theta(\epsilon)\right)$ be the hazard rates of $\boldsymbol{\omega}$ and $\boldsymbol{\epsilon}_\theta$, respectively. The following properties hold:*

(a) $(1-\delta)H_\Phi\left(\underline{\omega}\right) \leq H_{\tilde{F}_\theta}\left(\underline{\epsilon}_\theta\right)$;

(b) $H_{\tilde{F}_\theta}^{-1}\left((1-\delta)H_\Phi(\omega)\right)$ *is (weakly) convex.*

Condition 1 captures the idea that the extent of "allocative distortions" at the optimal mechanism is large. Property (a) implies that type $\underline{\omega}$ faces maximal distortion and is completely excluded from trading ($D = 0$). Property (b), in turn, guarantees that low types are exposed to significant distortions. Intuitively, at the optimal mechanism, $D^\star(\omega) - \omega = H_{\tilde{F}_\theta}^{-1}\left((1-\delta)H_\Phi(\omega)\right)$ (from 8). As argued above, the extent of distortion is captured by $\Delta_\theta(\omega) = \overline{\epsilon}/\theta - \left(D^\star(\omega) - \omega\right)$. Property (b) then implies that the distortions are concave and hence decrease at an increasing rate with ω. Because distortions must vanish as ω approaches $\overline{\omega}$ (no distortion at the top), property (b) implies that extent of distortions remain high for low values of ω. The next result shows that Condition 1 is sufficient to imply that the aggregate information rents captured by the investor decrease with θ.

Lemma 4. *Suppose that Φ satisfies Assumption (1) and that Condition (1) holds. Fix any $\theta \in \Theta$ and the associated optimal debt menu $\mathcal{M} = \left\{\min\left\{\boldsymbol{y}, D_\theta^\star(\omega)\right\}, p_\theta^\star(\omega)\right\}_{\omega\in\Omega}$. Then, if we let $\overline{U}_\theta(\mathcal{M}) \equiv \int_\Omega U_\theta(\omega; \mathcal{M})\Phi(\mathrm{d}\omega)$, $\overline{U}_\theta(\mathcal{M}) \geq \overline{U}_{\theta'}(\mathcal{M})$ for any $\theta' > \theta$.*

While Condition 1 is sufficient, it is by no means necessary. Indeed, even if property (a) does not hold (i.e., type $\underline{\omega}$ purchases a nontrivial security), aggregate information rents decrease with θ provided that $H_{\tilde{F}_\theta}^{-1}\left((1-\delta)H_\Phi(\omega)\right)$ is sufficiently convex.

5.4. Issuer's profit

We finally explore whether the issuer benefits from screening a more informed investor. Throughout the analysis, we fix θ and the associated optimal mechanism $\mathcal{M}^\star = \left\{s^\star[\omega], p^\star(\omega)\right\}_{\omega\in\Omega}$, and explore the effect of marginally increasing accuracy. The Envelope Theorem implies that whether or not the issuer's benefits from marginally increasing θ depends on the direct effect that θ has on the issuer's profit function while keeping the optimal mechanism $\mathcal{M}^\star$ fixed. The results in Theorem 2 and Proposition 3 imply that whether the issuer's profit is monotone in accuracy depends on whether the increase in gains from trade and the reduction of information rents for a subset of types (low types for location experiments) compensates for the larger information rents captured by the rest of types.

The issuer's expected profit is given by the difference between aggregate gains from trade and aggregate information rents:

$$\begin{aligned}\overline{\Pi}_\theta &= \overline{G}_\theta - \overline{U}_\theta \\ &= \int_\Omega \big(G_\theta(\omega) - (1-\Phi(\omega))I_\theta(\omega)\big)\Phi(\mathrm{d}\omega).\end{aligned}$$

Lemma 4, together with Theorem 2, identifies a sufficient condition for the issuer to benefit from screening a more informed investor in the context of location experiments. Provided that the assumption in the lemma holds, the issuer's profit increases with accuracy, regardless of $\delta \in [0,1]$.

Proposition 4. *Suppose that Φ satisfies Assumption (1) and that Condition 1 holds. Then, for any $\delta \in [0,1]$, $\overline{\Pi}_\theta$ increases with θ.*

As explained above, the assumptions in the Proposition are sufficient but not necessary. In particular, the result above abstracts from the fact that gains from trade unambiguously increase with accuracy, and holds even in the case when $\delta = 1$ (i.e., when there are no gains from trade).

The fact that the issuer may benefit from trading with a better informed investor is broadly related to the *linkage principle* (Milgrom and Weber, 1982) according to which the asset owner benefits from disclosing information to the bidders in common value auctions. Ottaviani and Pratt, 2001 show that a version of the principle also emerges in the single-bidder case, where the asset owner benefits from the release of a public signal that correlates with the asset's value and that can be contracted upon. In our case, as the investor becomes better informed, the issuer cannot directly contract on the additional information conveyed by the increase in precision of the investor's signal. However, screening with financial securities allows the issuer to contract on a variable that more accurately describes the investor's private information. As we have shown, under some conditions, this might help reducing the investor's information rents.

5.4.1. Fully informative signals and full-extraction

We conclude our analysis by noting that, at the limit when the investor's type perfectly reveals the asset's future cash flows (i.e., $\boldsymbol{y} \equiv \boldsymbol{\omega}$), the issuer can attain full-surplus extraction. Indeed, consider the mechanism $\mathcal{M} \equiv \{s[\omega], p(\omega)\}_{\omega\in\Omega}$ with $s(\boldsymbol{y}|\boldsymbol{\omega}) \equiv \min\{\boldsymbol{y},\boldsymbol{\omega}\}$ and $p(\omega) \equiv \omega$. The mechanism is clearly incentive-compatible and individually rational.[22] By posting the debt menu $\mathcal{M}$, the issuer maximizes gains from trade and fully dissipates the investor's information rents. Interestingly, in this environment, full-extraction is achieved without using sophisticated lotteries (as in Crémer and McLean, 1988), but instead by selling debt securities which are ubiquitous in financial markets. As we discuss in the context of Example 1, it is useful to contrast this solution to the solution in the standard screening problem with linear instruments: posted prices. There, the seller does not benefit from accuracy and full extraction is not achieved. Provided that the investor's type corresponds to the contractible variable (e.g., cash flows), the issuer strictly benefits from her ability to sell financial securities and achieves full-surplus extraction.

However, as implied by Proposition 1, full-extraction only obtains at the limit. For any nondegenerate distribution of cash flows (that satisfies MLRP), property (i) in Proposition 1 implies that all investor types $\omega > \underline{\omega}$ obtain positive information rents.

6. Conclusions

We study how the flexibility of designing financial securities can help liquidity-constrained asset owners raise liquid funds from informed investors. We show, perhaps counterintuitively, that exploiting the information sensitivity of the securities within the menu is generally ineffective at screening the investors' private information and is dominated by simple debt menus. We show that information-insensitive securities allow the issuer to minimize the investors' information rents and therefore allow the former to raise larger amounts of funds.

Our contribution has an important methodological component. We show how, using the logic of a replication argument, the standard tools in the screening literature can be extended to the rich environment of financial securities, i.e., infinite-dimensional objects. One of the advantages of our direct mechanism approach vis-a-vis former results in the literature is the characterization of the optimal mechanism with securities and the investor's information rents within the mechanism. We leverage our characterization to perform novel comparative statics with respect to the primitives of the informational environment and the issuer's ability to raise external funding. We show that as quality of the investor's information improves, the trade surplus unambiguously increases, whereas information rents typically decrease for more pessimistic types and increase for more optimistic types. Our results underscore the fact

[22] Consider any type ω. By reporting $\tilde{\omega}$, she obtains $\min\{\omega,\tilde{\omega}\} - \tilde{\omega} \le 0$, where the equality only holds for $\tilde{\omega} = \omega$.

that, contrary to other type of goods, financial securities have a geometry associated with them which appears crucial for welfare analysis and should be accounted for when designing financial regulation.

The results in this paper are worth extending in several directions. The analysis assumes a single investor. It would be interesting to generalize our approach to the case with multiple bidders and provide a general characterization of the investors' information rents in such a case.

In our paper, we show how the optimal mechanism with securities change as we change the investor's exogenous private information. It is reasonable to conjecture that in practice, asset owners can manipulate the information asymmetry with respect to the investor. What is the optimal mechanism with securities when the issuer can design both the security and the information structure? Inostroza and Tsoy (2025) make progress in this direction and show that an issuer who can also design signal realizations prefers to sell highly information-sensitive securities.

CRediT authorship contribution statement

Nicolas Figueroa: Writing – review & editing, Writing – original draft; **Nicolas Inostroza:** Writing – review & editing, Writing – original draft.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

none.

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Appendix A. Proofs Section 3 (optimality of debt menus)

A.0.1. Proof of Proposition 1

To see (1), note that for any $\omega'' > \omega'$,

$$\begin{aligned} U_{\mathcal{M}}(\omega'';\omega'') = \mathbb{E}\left[s\left[\omega''\right]\middle|\omega''\right] - p(\omega'') &\geq U_{\mathcal{M}}(\omega';\omega'') \\ &= \mathbb{E}\left[s\left[\omega'\right]\middle|\omega''\right] - p(\omega') \\ &\geq \mathbb{E}\left[s\left[\omega'\right]\middle|\omega'\right] - p(\omega') \\ &= U_{\mathcal{M}}(\omega';\omega') \geq 0, \end{aligned}$$

where the first inequality follows from incentive compatibility, the second inequality follows from FOSD (implied by MLRP) and the monotonicity of $s\left[\omega'\right]$, and the last inequality follows from individual rationality. Furthermore, the second inequality is strict whenever $s\left[\omega'\right]$ is strictly monotone. Moreover, under strict MLRP, the inequality is strict as long as $s\left[\omega'\right]$ displays strict monotonicity over a positive measure of cash flows. A sufficient condition for this to be the case is that $s\left[\omega'\right]$ is not safe debt. Indeed, the fact that $s\left[\omega'\right] \in \mathcal{S}$ implies that it cannot promise a constant payout above $\underline{y}$.

Next, to see property (2), let $\xi_{\mathcal{M}} \equiv \min_{\omega\in\Omega} U_{\mathcal{M}}(\omega;\omega) = U_{\mathcal{M}}\left(\underline{\omega};\underline{\omega}\right)$, where the equality is a consequence of property (1). Suppose that $\xi_{\mathcal{M}} > 0$. Then, define $\tilde{\mathcal{M}} = \{\tilde{s}[\omega]\cdot, \tilde{p}(\omega)\}_{\omega\in\Omega}$ as follows. Let $\tilde{s}[\omega] \equiv s[\omega]$, and $\tilde{p}(\omega) \equiv p(\omega) + \xi_{\mathcal{M}}$ for all $\omega \in \Omega$. Increasing all prices by $\xi_{\mathcal{M}}$ does not spoil incentive compatibility. Moreover, by property (1), individual rationality constraints are satisfied. The new mechanism raises more funds than the original mechanism while it preserves the securities offered to the investor; therefore, $\tilde{\mathcal{M}}$ strictly dominates $\mathcal{M}$.

Finally, to see property (3), suppose that $\mathbb{P}\left[\boldsymbol{y} - s\left(\boldsymbol{y}|\overline{\omega}\right) > 0\right] > 0$. Consider the mechanism $\hat{\mathcal{M}} = \{\hat{s}[\omega], \hat{p}(\omega)\}_{\omega\in\Omega}$ where $\hat{s}[\omega] \equiv s[\omega]$, and $\hat{p}(\omega) \equiv p(\omega)$ for all $\omega < \overline{\omega}$, $\hat{s}(\boldsymbol{y}|\overline{\omega}) \equiv \boldsymbol{y}$ and $\hat{p}(\overline{\omega}) \equiv p(\overline{\omega}) + \mathbb{E}\left[\boldsymbol{y} - s\left(\boldsymbol{y}|\overline{\omega}\right)\middle|\overline{\omega}\right]$. By construction, under $\hat{\mathcal{M}}$, type $\overline{\omega}$'s payoff remains unchanged, whereas the payoff of any other type ω who chooses to report $\overline{\omega}$ is strictly smaller than under $\mathcal{M}$. In fact,

$$\begin{aligned} U_{\hat{\mathcal{M}}}(\overline{\omega};\omega) &= \mathbb{E}\left[\hat{s}\left[\overline{\omega}\right]\middle|\omega\right] - \hat{p}(\overline{\omega}) \\ &= \mathbb{E}\left[s\left[\overline{\omega}\right]\middle|\omega\right] - p(\overline{\omega}) + \underbrace{\left(\mathbb{E}\left[\boldsymbol{y} - s\left(\boldsymbol{y}|\overline{\omega}\right)\middle|\omega\right] - \mathbb{E}\left[\boldsymbol{y} - s\left(\boldsymbol{y}|\overline{\omega}\right)\middle|\overline{\omega}\right]\right)}_{<0} \end{aligned}$$

$$< U_{\mathcal{M}}(\overline{\omega};\omega),$$

where the inequality obtains from the fact that (i) $y - s\left(y|\overline{\omega}\right)$ is monotone and (ii) $\left[\mathbf{y}|\boldsymbol{\omega}=\overline{\omega}\right] \succ_{\text{MLRP}} [\mathbf{y}|\boldsymbol{\omega}=\omega]$, all $\omega < \overline{\omega}$. Therefore, the new mechanism $\hat{\mathcal{M}}$ is feasible. Finally, note that the issuer is better off (and strictly better off if $\overline{\omega}$ has positive measure) since

$$\begin{aligned}\mathbb{E}\left[v\left(\mathbf{y},\hat{p}(\overline{\omega}),\hat{s}[\overline{\omega}]\right)\middle|\overline{\omega}\right] = \hat{p}(\overline{\omega}) &= p(\overline{\omega}) + \mathbb{E}\left[\mathbf{y} - s\left(\mathbf{y}|\overline{\omega}\right)\middle|\overline{\omega}\right] \\ &> p(\overline{\omega}) + \delta\mathbb{E}\left[\mathbf{y} - s\left(\mathbf{y}|\overline{\omega}\right)\middle|\overline{\omega}\right]. \\ &= \mathbb{E}\left[v\left(\mathbf{y},p(\overline{\omega}),s[\overline{\omega}]\right)\middle|\overline{\omega}\right].\end{aligned}$$

where the strict inequality follows from $\delta < 1$.□

Definition 5 (Single crossing from below/above). We say that a function $h : \mathbb{R}_+ \to \mathbb{R}$ satisfies *single crossing from below* (SCFB) if there exists $y_0 \in \mathbb{R}_{++}$ so that $h(y) \leq 0$ for any $y < y_0$ and $h(y) \geq 0$ for any $y \geq y_0$. We say that h satisfies *strict single crossing from below* (SSCFB) if, in addition, the sets $\left\{y \in \mathbb{R}_+ : h(y) < 0\right\}$ and $\left\{y \in \mathbb{R}_+ : h(y) > 0\right\}$ have a positive (Lebesgue) measure. Similarly, we say that h satisfies *single crossing from above* (SCFA) or *strict single crossing from above* (SSCFA) if $-h$ satisfies SCFB or SSCFB, respectively.

Lemma 5. *Suppose that $(\mathbf{y},\boldsymbol{\omega})$ satisfy the MLRP and that h satisfies the SCFB property. If $\int_0^\infty h(y)\mathrm{d}F\left(y|\omega'\right) \geq 0$, then necessarily,*

$$\int_0^\infty h(y)\mathrm{d}F\left(y|\omega''\right) \geq 0,\ \forall \omega'' > \omega'.$$

If h satisfies $SSCFB$, then the second inequality is strict.

Proof. The proof follows from Lemma 5 in Athey (2002). □

A.0.2. Proof of Lemma 1

First note that, because $\mathbb{E}\left[\mathbf{y}\frac{\frac{\partial f(\mathbf{y}|\omega)}{\partial\omega}}{f(\mathbf{y}|\omega)}\middle|\omega\right] < \infty$ for all $\omega \in \Omega$, for any feasible mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$, there is an integrable function $b : \Omega \to \mathbb{R}$, satisfying

$$\left|\frac{\partial}{\partial\omega}U_{\mathcal{M}}(\tilde{\omega},\omega)\right| = \left|\mathbb{E}\left[s(\mathbf{y}|\tilde{\omega})\frac{\frac{\partial f(\mathbf{y}|\omega)}{\partial\omega}}{f(\mathbf{y}|\omega)}\right]\right| \leq b(\omega),\ \forall \tilde{\omega},\omega.$$

The arguments in Milgrom and Segal (2002) then imply that, because $U_{\mathcal{M}}(\hat{\omega},\omega)$ is absolutely continuous in ω, $U_{\mathcal{M}}(\omega,\omega)$ is absolutely continuous. Furthermore,

$$U_{\mathcal{M}}(\omega,\omega) - U_{\mathcal{M}}\left(\underline{\omega},\underline{\omega}\right) = \int_{\underline{\omega}}^{\omega}\left(\int_y s(y|\tilde{\omega})\frac{\partial}{\partial\omega}f(y|\tilde{\omega})\mathrm{d}y\right)\mathrm{d}\tilde{\omega},\ \text{for } \Phi - \text{almost all } \omega,$$

which proves the necessary condition (3).

The next result identifies a sufficient condition for incentive compatibility. Consider an arbitrary mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$ satisfying (3) and (4). We prove that $\mathcal{M}$ is incentive compatible.

For any $\omega, \hat{\omega} \in \Omega$, let

$$Q(\hat{\omega},\omega) \equiv U_{\mathcal{M}}(\omega,\omega) - U_{\mathcal{M}}(\hat{\omega},\omega).$$

Note that, for any $\hat{\omega}$, $Q(\hat{\omega},\cdot)$ is absolutely continuous and hence differentiable almost everywhere. Moreover, $Q(\hat{\omega},\hat{\omega}) = 0$ for all $\hat{\omega} \in \Omega$. This implies that, for (almost) any $\omega, \hat{\omega} \in \Omega$,

$$\begin{aligned}Q(\hat{\omega},\omega) &= Q(\hat{\omega},\omega) - Q(\hat{\omega},\hat{\omega}) \\ &= \int_{\hat{\omega}}^{\omega}\frac{\partial Q(\hat{\omega},z)}{\partial\omega}\mathrm{d}z \\ &= \int_{\hat{\omega}}^{\omega}\left\{\frac{\mathrm{d}}{\mathrm{d}\omega}U_{\mathcal{M}}(\omega,\omega)\bigg|_{\omega=z} - \frac{\partial}{\partial\omega}U_{\mathcal{M}}(\hat{\omega},\omega)\bigg|_{\omega=z}\right\}\mathrm{d}z \\ &= \int_{\hat{\omega}}^{\omega}\left\{\int_y (s(y|z) - s(y|\hat{\omega}))\frac{\partial}{\partial\omega}f(y|z)\mathrm{d}y\right\}\mathrm{d}z \\ &\geq 0,\end{aligned}$$

where the inequality follows from (4). We thus conclude that $U_{\mathcal{M}}(\omega,\omega) \geq U_{\mathcal{M}}(\hat{\omega},\omega)$, for any $\omega, \hat{\omega} \in \Omega$.

A.0.3. Proof of Theorem 1

For any $\omega'', \omega' \in \Omega$, we say that ω'' *envies* ω' if $\mathbb{E}\left[s\left[\omega''\right] - s\left[\omega'\right]\middle|\omega''\right] < 0$. For any given mechanism $\mathcal{M} = \{s[\omega], p(\omega)\}_{\omega\in\Omega}$, let $\Omega_{\mathrm{E}}[\mathcal{M}]$ be the set of types in Ω that envy the securities of some lower type under $\mathcal{M}$. Formally,

$$\Omega_{\mathrm{E}}[\mathcal{M}] \equiv \left\{\omega'' \in \Omega : \exists \omega' < \omega'' \text{such that } \mathbb{E}\left[s\left[\omega''\right] - s\left[\omega'\right]\middle|\omega''\right] < 0\right\}.$$

Next, for each $\omega \in \Omega$, we construct the debt security that makes type ω indifferent. That is, $s^{\mathrm{D}}[\omega] \equiv \min\{\boldsymbol{y}, D(\omega)\}$, where $D(\omega)$ is implicitly determined by $\mathbb{E}\left[s^{\mathrm{D}}[\omega] - s[\omega]\middle|\omega\right] = 0$. Let $\mathcal{M}^{\mathrm{D}} \equiv \left\{s^{\mathrm{D}}[\omega], p(\omega)\right\}_{\omega\in\Omega}$ be the resulting mechanism from replacing each security $s[\omega]$ in $\mathcal{M}$ for their debt equivalent $s^{\mathrm{D}}[\omega]$.

We note that, for any $\omega'' > \omega'$ such that $s\left[\omega'\right]$ is debt (i.e., $\mathbb{P}\left[s\left[\omega'\right] = s^{\mathrm{D}}\left[\omega'\right]\right] = 1$), $D\left(\omega''\right) < D\left(\omega'\right)$ if, and only if, ω'' envies ω' under $\mathcal{M}$. Indeed, by construction, $\mathbb{E}\left[s\left[\omega''\right] - s\left[\omega'\right]\middle|\omega''\right] < 0$ is equivalent to $\mathbb{E}\left[s^D\left[\omega''\right] - s^D\left[\omega'\right]\middle|\omega''\right] < 0$, which in turn is equivalent to $D\left(\omega''\right) < D\left(\omega'\right)$.

Claim 1. $\Omega\backslash\Omega_E[\mathcal{M}] \subseteq \Omega\backslash\Omega_E\left[\mathcal{M}^D\right]$. *In particular, for any $\omega'' \in \Omega\backslash\Omega_E[\mathcal{M}]$ and any $\omega' < \omega''$, $D\left(\omega''\right) \geq D\left(\omega'\right)$.*

We prove that $\omega'' \in \Omega\backslash\Omega_E[\mathcal{M}]$ implies that $\omega'' \in \Omega\backslash\Omega_E\left[\mathcal{M}^D\right]$. That is, if ω'' does not envy any lower type under $\mathcal{M}$, ω'' does not envy any lower type under $\mathcal{M}^D$, and therefore the face value of their equivalent debt security must be weakly larger than the ones of all lower types.

Consider any $\omega'' \in \Omega\backslash\Omega_E[\mathcal{M}]$, if $\omega' < \omega''$, then

$$0 \leq \mathbb{E}\left[s\left[\omega''\right] - s\left[\omega'\right]\middle|\omega''\right] = \mathbb{E}\left[s^D\left[\omega''\right] - s\left[\omega'\right]\middle|\omega''\right].$$

The fact that, by construction, $\mathbb{E}\left[s^D\left[\omega'\right] - s\left[\omega'\right]\middle|\omega'\right] = 0$, together with the fact that $s^D\left[\omega'\right] - s\left[\omega'\right]$ is SCFA in y, implies that Lemma 5 applies and therefore that $\mathbb{E}\left[s^D\left[\omega'\right] - s\left[\omega'\right]\middle|\omega''\right] \leq 0$. We conclude that $\mathbb{E}\left[s^D\left[\omega''\right] - s^D\left[\omega'\right]\middle|\omega''\right] \geq 0$. Thus, ω'' cannot envy any $\omega' < \omega''$ when security $s\left[\omega''\right]$ is replaced by $s^D\left[\omega''\right]$, and so $\omega'' \in \Omega\backslash\Omega_E\left[\mathcal{M}^D\right]$. As a result, for any $\omega' < \omega''$, $D\left(\omega''\right) \geq D\left(\omega'\right)$, as claimed. ■

Next, we argue that $\hat{D}(\omega) = D(\omega)$ if, and only if, $\omega \in \Omega\backslash\Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$. Indeed, since $\mathcal{M}^{\mathrm{D}}$ contains only debt securities, if $\omega'' \in \Omega\backslash\Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$, then $D\left(\omega''\right) \geq D\left(\omega'\right)$, all $\omega' < \omega''$. As a result, $D(\cdot)$ is monotone when restricted to $\Omega\backslash\Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$. Moreover, for any $\omega'' \in \Omega\backslash\Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$, there is no $\omega' < \omega''$ so that $D\left(\omega'\right) > D\left(\omega''\right)$. Thus, $\hat{D}(\omega) = D(\omega)$ for any $\omega \in \Omega\backslash\Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$. Consider now $\omega'' \in \Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$. The fact that there is $\omega' < \omega''$ such that $D\left(\omega'\right) > D\left(\omega''\right)$, implies that any monotone function h with $h \geq D$, is such that $h\left(\omega''\right) \geq h\left(\omega'\right) \geq D\left(\omega'\right) > D\left(\omega''\right)$, then implies that $\hat{D}\left(\omega''\right) > D\left(\omega''\right)$.

We construct the dominating mechanism $\hat{\mathcal{M}} = \{\hat{s}[\omega], \hat{p}(\omega)\}_{\omega\in\Omega}$ as follows. For any $\omega \in \Omega\backslash\Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$, let $(\hat{s}[\omega], \hat{p}(\omega)) \equiv \left(s^{\mathrm{D}}[\omega], p(\omega)\right)$. For any $\tilde{\omega} \in \Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$, let

$$r(\tilde{\omega}) \equiv \sup\left\{\omega < \tilde{\omega} \,:\, \hat{D}(\omega) = D(\omega)\right\}$$

be the closest non-envious type under $\mathcal{M}^{\mathrm{D}}$ to the left of $\tilde{\omega}$. Then, for any $\omega \in \Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$, let $(\hat{s}[\omega], \hat{p}(\omega)) \equiv \left(\lim_{\tilde{\omega}\downarrow r(\omega)} s^{\mathrm{D}}[\tilde{\omega}], \lim_{\tilde{\omega}\downarrow r(\omega)} p[\tilde{\omega}]\right)$. That is, the new mechanism $\hat{\mathcal{M}}$ pools all envious types under $\hat{\mathcal{M}}$, $\Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$, with their closest non-envious type from the left.

We show that the issuer strictly prefers the alternative mechanism $\hat{\mathcal{M}}$. For any $\omega \in \Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$, define $p^{\#}(\omega)$ as the price that the most patient issuer ($\delta = 1$) would require to be indifferent between $(\hat{s}[\omega], \hat{p}(\omega))$ and $(s[\omega], p(\omega))$. That is,

$$p^{\#}(\omega) = p(\omega) + \mathbb{E}\left[\lim_{\tilde{\omega}\downarrow r(\omega)} s^{\mathrm{D}}[\tilde{\omega}] - s[\omega]\middle|\omega\right].$$

We show that $\lim_{\tilde{\omega}\downarrow r(\omega)} p(\tilde{\omega}) > p^{\#}(\omega)$ for all $\omega \in \Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$. This means that, regardless the value of δ, the issuer prefers to sell the security $\lim_{\tilde{\omega}\downarrow r(\omega)} s^{\mathrm{D}}[\tilde{\omega}]$ to type ω at price $\lim_{\tilde{\omega}\downarrow r(\omega)} p(\tilde{\omega})$ rather than selling security $s[\omega]$ at price $p(\omega)$. Therefore, $\hat{\mathcal{M}}$ strictly dominates $\mathcal{M}$. To see this, note that because $\mathcal{M}$ is incentive compatible,

$$\mathbb{E}\left[s[\omega] - \lim_{\tilde{\omega}\downarrow r(\omega)} s[\tilde{\omega}]\middle|\omega\right] \geq p(\omega) - \lim_{\tilde{\omega}\downarrow r(\omega)} p(\tilde{\omega})$$

The inequality above implies that, for $\omega \in \Omega_{\mathrm{E}}\left[\mathcal{M}^{\mathrm{D}}\right]$,

$$\begin{aligned}
p^{\#}(\omega) &= p(\omega) + \mathbb{E}\left[\lim_{\tilde{\omega}\downarrow r(\omega)} s^{\mathrm{D}}[\tilde{\omega}] - s[\omega]\middle|\omega\right] \\
&\leq \lim_{\tilde{\omega}\downarrow r(\omega)} p(\tilde{\omega}) + \mathbb{E}\left[s[\omega] - \lim_{\tilde{\omega}\downarrow r(\omega)} s[\tilde{\omega}]\middle|\omega\right] + \mathbb{E}\left[\lim_{\tilde{\omega}\downarrow r(\omega)} s^{\mathrm{D}}[\tilde{\omega}] - s[\omega]\middle|\omega\right] \\
&= \lim_{\tilde{\omega}\downarrow r(\omega)} p(\tilde{\omega}) + \lim_{\tilde{\omega}\downarrow r(\omega)} \mathbb{E}\left[s^{\mathrm{D}}[\tilde{\omega}] - s[\tilde{\omega}]\middle|\omega\right] \\
&\leq \lim_{\tilde{\omega}\downarrow r(\omega)} p(\tilde{\omega}),
\end{aligned}$$

where the last inequality follows from the fact that, for any $\tilde{\omega}$, $s^{\mathrm{D}}[\tilde{\omega}] - s[\tilde{\omega}]$ is SCFA in y and such that and $\mathbb{E}\left[s^{\mathrm{D}}[\tilde{\omega}] - s[\tilde{\omega}]\middle|\tilde{\omega}\right] = 0$. Therefore, for any $\tilde{\omega} < \omega$, $\mathbb{E}\left[s^{\mathrm{D}}[\tilde{\omega}] - s[\tilde{\omega}]\middle|\omega\right] \leq 0$ with strict inequality whenever $\mathbb{P}\left[s^{\mathrm{D}}[\tilde{\omega}] \neq s[\tilde{\omega}]\right] > 0$ (i.e., whenever $s[\tilde{\omega}]$ is not debt). This proves that $\hat{\mathcal{M}}$ strictly dominates $\mathcal{M}$.

Finally, the fact that, by construction, $\hat{\mathcal{M}}$ is a monotone debt menu implies that it must be incentive compatible. Indeed, for any $\omega'', \omega' \in \Omega$, with $\omega'' > \omega'$, the fact that $\hat{s}\left[\omega''\right] - \hat{s}\left[\omega'\right]$ is monotone and non-negative, together with the fact that $\frac{\partial}{\partial\omega} f(y|\omega)/f(y|\omega)$ is monotone and satisfies $\mathbb{E}\left[\frac{\frac{\partial}{\partial\omega} f(\boldsymbol{y}|\omega)}{f(\boldsymbol{y}|\omega)}\middle|\omega\right] = 0$, jointly imply that

$$\mathbb{E}\left[\left(\hat{s}\left[\omega''\right] - \hat{s}\left[\omega'\right]\right)\frac{\frac{\partial}{\partial\omega} f(\boldsymbol{y}|\omega'')}{f(\boldsymbol{y}|\omega'')}\middle|\omega''\right] = \operatorname{cov}\left[\hat{s}\left[\omega''\right] - \hat{s}\left[\omega'\right], \frac{\frac{\partial}{\partial\omega} f(\boldsymbol{y}|\omega'')}{f(\boldsymbol{y}|\omega'')}\middle|\omega''\right] \geq 0,$$

which proves that the condition (4), which is sufficient for incentive compatibility, is satisfied. □

A.0.4. Proof of Proposition 2

Let

$$\tilde{\Pi}(\omega, D) \equiv \int_{\underline{y}} \min\{y, D\}\left\{1-\delta-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right\}\mathrm{d}F(y|\omega).$$

The issuer then chooses $\{D(\omega)\}_{\omega\in\Omega}$ to maximize $\int_\Omega \tilde{\Pi}(\omega, D(\omega))\mathrm{d}\Phi(\omega)$ subject to feasibility constraints. We show that when (1) and (2) hold, the function

$$\begin{aligned}\tilde{\Pi}(\omega, D) &\equiv \int_{\underline{y}} \min\{y, D\}\left\{1-\delta-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\frac{\partial}{\partial\omega}f(y|\omega)}{f(y|\omega)}\right\}\mathrm{d}F(y|\omega)\\ &= (1-\delta)\int_{\underline{y}}^{D}(1-F(y|\omega))\mathrm{d}y-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\int_{\underline{y}}^{D}\frac{\partial}{\partial\omega}(1-F(y|\omega))\mathrm{d}y\end{aligned}$$

has single crossing differences. Indeed,

$$\frac{\partial\tilde{\Pi}}{\partial D} = (1-\delta)(1-F(D|\omega))-\left(\frac{1-\Phi(\omega)}{\phi(\omega)}\right)\frac{\partial}{\partial\omega}(1-F(D|\omega)).$$

Thus, if there is ω' so that $\frac{\partial\tilde{\Pi}}{\partial D}(\omega', D) \geq 0$, then for any $\omega'' > \omega'$,

$$1-\delta \geq \left(\frac{1-\Phi(\omega')}{\phi(\omega')}\right)\frac{\frac{\partial}{\partial\omega}\left(1-F\left(D|\omega'\right)\right)}{1-F(D|\omega')} \geq \left(\frac{1-\Phi(\omega'')}{\phi(\omega'')}\right)\frac{\frac{\partial}{\partial\omega}\left(1-F\left(D|\omega''\right)\right)}{1-F(D|\omega'')},$$

where the first inequality follows from the assumption that $\frac{\partial\tilde{\Pi}}{\partial D}(\omega', D) \geq 0$, and the second inequality from Assumptions (1) and (2), together with the observation that $\frac{\partial}{\partial\omega}(1-F(D|\omega)) > 0$, by MLRP. We conclude that $\frac{\partial\tilde{\Pi}}{\partial D}(\omega'', D) \geq 0$. Milgrom and Shannon (1994) then implies that the value of D that (pointwise) maximizes $\tilde{\Pi}(\omega, D)$, $D^\star(\omega)$, must be non-decreasing in ω. This further implies that

$$\begin{aligned}&\mathbb{E}\left[\left(\min\left\{\boldsymbol{y}, D^\star\left(\omega''\right)\right\}-\min\left\{\boldsymbol{y}, D^\star\left(\omega'\right)\right\}\right)\frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega'')}{f(\boldsymbol{y}|\omega'')}\middle|\omega''\right] = \\ &\mathrm{cov}\left[\min\left\{\boldsymbol{y}, D^\star\left(\omega''\right)\right\}-\min\left\{\boldsymbol{y}, D^\star\left(\omega'\right)\right\}, \frac{\frac{\partial}{\partial\omega}f(\boldsymbol{y}|\omega'')}{f(\boldsymbol{y}|\omega'')}\middle|\omega''\right] \geq 0,\end{aligned}$$

and therefore the sufficiency condition in (4) is satisfied. The set of pointwise optimal securities is thus feasible and hence optimal.□

Appendix B. Accuracy ordering

B.1. Explanation why θ parameterizes the quality of the investor's information

For any $\theta \in \Theta$, define $\boldsymbol{z}_\theta \equiv \Psi_\theta(\boldsymbol{y}_\theta) \sim U[0,1]$. Intuitively, $\boldsymbol{z}_\theta$ informs the observer the quantile where the realization of $\boldsymbol{y}_\theta$ is located. The distribution of the normalized signal can be obtained from Φ and F_θ: $G_\theta(z|\omega) \equiv F_\theta\left(\Psi_\theta^{-1}(z)\middle|\omega\right)$. Clearly, learning $\boldsymbol{z}_\theta$ conveys the same information about $\boldsymbol{\omega}$ as learning $\boldsymbol{y}_\theta$. Naturally, this normalization preserves both the MLRP and the accuracy ordering.

Lemma 6. *Suppose that $F_{\theta'} \succeq F_\theta$, then (a) for any $\theta \in \Theta$, $[\boldsymbol{z}_\theta|\boldsymbol{\omega}]$ satisfies MLRP, and (b) $G_{\theta'} \succeq G_\theta$.*

The proof is standard and hence relegated to the Online Appendix. Importantly, as the next result shows, if the (normalized) experiments $\{G_\theta(z|\omega)\}$ are ordered in accuracy, then if we interpret $\boldsymbol{\omega}$ as a signal of $\boldsymbol{z}$ (and hence of $\boldsymbol{y}$), the induced experiments, say $H_\theta(\omega|z)$, are ordered according to the *Monotone Information order* (MIO-ND) introduced by Athey and Levin (2018). Thus, a more accurate experiment leads to a better informed investor.

Proposition 5. *Suppose that $G_{\theta'} \succeq G_\theta$. For any $\Phi \in \Delta\Omega$, let $\boldsymbol{G}_{\theta'}(z,\omega)$ and $\boldsymbol{G}_\theta(z,\omega)$ be the joint distributions induced by Φ, and the experiments $G_{\theta'}$ and G_θ, respectively. Fix any $\Psi \in \Delta\mathbb{R}_+$ and let $H_\theta(\omega|z) \equiv \int_{\underline{\omega}}^{\omega}\mathrm{d}\boldsymbol{G}_\theta(z,\tilde{\omega})$. Let $\boldsymbol{y} \sim \Psi$ and $\hat{\boldsymbol{\omega}}_\theta$ be such that $[\hat{\boldsymbol{\omega}}_\theta|\Psi(\boldsymbol{y}) = z] \sim H_\theta[z]$. Then, $\hat{\boldsymbol{\omega}}_{\theta'}$ dominates $\hat{\boldsymbol{\omega}}_\theta$ in the Monotone Information Order MIO-ND (Athey and Levin, 2018).*[23] *In particular, $\mathbb{E}[\Psi(\boldsymbol{y})|\hat{\boldsymbol{\omega}}_{\theta'}] \succeq_{cvx} \mathbb{E}_{\boldsymbol{G}_\theta^\Phi}[\Psi(\boldsymbol{y})|\hat{\boldsymbol{\omega}}_\theta]$. That is, if we denote by $J_{\theta'}$ and J_θ the distributions of $\mathbb{E}[\Psi(\boldsymbol{y})|\hat{\boldsymbol{\omega}}_{\theta'}]$ and $\mathbb{E}[\Psi(\boldsymbol{y})|\hat{\boldsymbol{\omega}}_\theta]$, respectively, then, for any convex function $v : \mathbb{R}_+ \to \mathbb{R}$,*

$$\int v(x)\mathrm{d}J_{\theta'}(x) \geq \int v(x)\mathrm{d}J_\theta(x).$$

[23] We say that a signal $\boldsymbol{\omega}_{\theta'}$ is more informative than $\boldsymbol{\omega}_\theta$ about $\boldsymbol{z}$ in the MIO-ND sense if *any* decision maker with supermodular payoff function in actions and states z, prefers signal $\boldsymbol{\omega}_{\theta'}$ over $\boldsymbol{\omega}_\theta$. The MIO-ND ordering is slightly less demanding than the accuracy ordering as the latter requires the preference above to extend to decision makers with payoff functions satisfying *single-crossing differences* (which include all supermodular functions).

Proof of Proposition 5. Suppose we first draw $\tilde{\boldsymbol{y}}$ from some arbitrary $\Psi \in \Delta\mathbb{R}_+$ and let $\tilde{z} \equiv \Psi(\tilde{\boldsymbol{y}})$, and then, for each $\theta \in \Theta$, obtain $\left[\tilde{\boldsymbol{\omega}}_\theta \middle| \tilde{z} = z\right]$ from the experiment $H_\theta^\Phi(\omega|z) \equiv \int_{\underline{\omega}}^{\omega} \mathrm{d}\mathbf{G}_\theta^\Phi(\tilde{\omega}|z)$. Let $\Phi_\theta\left[H_\theta, U\right] \equiv \mathrm{marg}_{\tilde{\boldsymbol{\omega}}_\theta}\mathbf{G}_\theta^\Phi \in \Delta\Omega$ be the marginal distribution induced of $\tilde{\boldsymbol{\omega}}_\theta$. Note that for any $\theta \in \Theta$, $\Phi_\theta\left[H_\theta, U\right] = \Phi$.

Next, for any nondecreasing function $u(\cdot)$, and any $q \in [0,1]$, we note that the function $\mathbb{I}\{\Phi(\omega) \geq q\}u(z)$ is supermodular in (z, ω) and, therefore, from property (i), for any $\theta'' > \theta'$,

$$\int_{\mathbb{R}_+}\int_{\Omega} \mathbb{I}\{\Phi(\omega) \geq q\}u(z)\mathrm{d}\mathbf{G}_{\theta''}^\Phi(z,\omega) \geq \int_{\mathbb{R}_+}\int_{\Omega} \mathbb{I}\{\Phi(\omega) \geq q\}u(z)\mathrm{d}\mathbf{G}_{\theta'}^\Phi(z,\omega).$$

This means that, for any nondecreasing function $u(\cdot)$,

$$\mathbb{E}\left[u(\tilde{z})\middle|\Phi_{\theta''}\left(\tilde{\boldsymbol{\omega}}_{\theta''}\right) \geq q\right] \geq \mathbb{E}\left[u(\tilde{z})\middle|\Phi_{\theta'}\left(\tilde{\boldsymbol{\omega}}_{\theta'}\right) \geq q\right],$$

and, therefore,

$$\left[\tilde{z}\middle|\Phi_{\theta''}\left(\tilde{\boldsymbol{\omega}}_{\theta''}\right) \geq q\right] \succeq_{\mathrm{FOSD}} \left[\tilde{z}\middle|\Phi_{\theta'}\left(\tilde{\boldsymbol{\omega}}_{\theta'}\right) \geq q\right],\ \forall q \in [0,1].$$

This implies that

$$H_{\theta''}^\Phi\left(\Phi_{\theta''}^{-1}(q)\middle|\tilde{z} \leq z\right) \geq H_{\theta'}^\Phi\left(\Phi_{\theta'}^{-1}(q)\middle|\tilde{z} \leq z\right).$$

Next, if we let $p = \Phi_{\theta'}^{-1}(q)$, we conclude that

$$\Phi_{\theta''}^{-1}\left(\Phi_{\theta'}(q)\right) \geq \left(H_{\theta''}^\Phi\right)^{-1}\left(H_{\theta'}^\Phi(p|\tilde{z} \leq z)\middle|\tilde{z} \leq z\right)$$

In other words, $\tilde{\boldsymbol{\omega}}_{\theta''}$ dominates $\tilde{\boldsymbol{\omega}}_{\theta'}$ in the Monotone Information Order for Nondecreasing objective functions (MIO-ND) sense (Athey and Levin, 2018). Theorem 1 in Ganuza and Penalva (2010) then implies that $\mathbb{E}\left[\tilde{z}|\tilde{\boldsymbol{\omega}}_{\theta''}\right] \succeq_{\mathrm{cvx}} \mathbb{E}\left[\tilde{z}|\tilde{\boldsymbol{\omega}}_{\theta'}\right]$.□

Proof of Lemma 3. The proof follows the arguments in Jewitt (2007). We add it here for the reader's convenience. We first establish an intermediate result. We argue that $F_{\theta'} \geq F_\theta$ implies that for any y and $\tilde{y}$, $F_{\theta'}(y|\cdot)$ crosses $F_\theta(\tilde{y}|\cdot)$ once from above. To see this, pick any y and $\tilde{y}$, and let p and $\hat{\omega}$ be such that $y = F_{\theta'}^{-1}(p|\hat{\omega})$ and $\tilde{y} = F_\theta^{-1}(p|\hat{\omega})$. That $F_{\theta'} \geq F_\theta$ implies that, for any $\omega < \hat{\omega}$,

$$F_{\theta'}(y|\omega) = F_{\theta'}\left(F_{\theta'}^{-1}(p|\hat{\omega})|\omega\right) \geq F_\theta\left(F_\theta^{-1}(p|\hat{\omega})|\omega\right) = F_\theta(\tilde{y}|\omega).$$

A similar argument implies that, for any $\omega > \hat{\omega}$,

$$F_{\theta'}(y|\omega) = F_{\theta'}\left(F_{\theta'}^{-1}(p|\hat{\omega})|\omega\right) \leq F_\theta\left(F_\theta^{-1}(p|\hat{\omega})|\omega\right) = F_\theta(\tilde{y}|\omega),$$

therefore proving that $F_{\theta'}(y|\cdot)$ crosses $F_\theta(\tilde{y}|\cdot)$ once from above.

To prove property (i), note first that the fact that $\boldsymbol{y}_\theta \sim \Psi$ for all θ implies that $\mathbf{F}_{\theta'}\left(y, \overline{\omega}\right) = \mathbf{F}_\theta\left(y, \overline{\omega}\right)$. This observation together with the fact that $F_{\theta'}(y|\cdot) - F_\theta(y|\cdot)$ satisfies SCFA implies that

$$\mathbf{F}_{\theta'}(y, \tilde{\omega}) - \mathbf{F}_\theta(y, \tilde{\omega}) = \int_{\underline{\omega}}^{\tilde{\omega}} \left(F_{\theta'}(y|\omega) - F_\theta(\tilde{y}|\omega)\right)\mathrm{d}\Phi(\omega) \geq 0.$$

This means that $\mathbf{F}_{\theta'}(y, \tilde{\omega}) \geq \mathbf{F}_\theta(y, \tilde{\omega})$ for all $\tilde{\omega}$ and all y. Thus, the induced joint distributions $\mathbf{F}_{\theta'}$ and $\mathbf{F}_\theta$ are ranked in the positive quadrant dependence (PQD) order.[24] For random vectors of dimension $N = 2$, the PQD order, in turn, is equivalent to the supermodular order (Tchen, 1980).□

Proof of Proposition 3. We prove first an auxiliary result.

Lemma 7. *Consider a family of location experiments with $F_\theta(y|\omega) = \tilde{F}_\theta(y - \omega)$ for all θ, y, ω. Then, $\tilde{F}_{\theta'}$ crosses $\tilde{F}_\theta$ once from below.*

Proof. For any $\theta' > \theta$, the fact that $F_{\theta'} \geq F_\theta$, together with assumption that $\left\{F_\theta\right\}_{\theta\in\Theta}$ is a location experiment, implies that $\tilde{F}_\theta$ is more dispersive than $\tilde{F}_{\theta'}$, and therefore, for any $q, p \in [0,1]$, $q > p$,[25]

$$\tilde{F}_{\theta'}^{-1}(q) - \tilde{F}_{\theta'}^{-1}(p) \leq \tilde{F}_\theta^{-1}(q) - \tilde{F}_\theta^{-1}(p), \tag{B.1}$$

where $\tilde{F}_\theta^{-1}(z) \equiv \sup\left\{\varepsilon : \tilde{F}_\theta(\varepsilon) \geq z\right\}$. Suppose there is $\hat{\varepsilon}$ such that $\tilde{F}_{\theta'}(\hat{\varepsilon}) = \tilde{F}_\theta(\hat{\varepsilon}) \equiv q$. Consider any $r > q$, inequality (B.1) implies that

$$\tilde{F}_{\theta'}^{-1}(r) - \underbrace{\tilde{F}_{\theta'}^{-1}(q)}_{=\hat{\varepsilon}} \leq \tilde{F}_\theta^{-1}(r) - \underbrace{\tilde{F}_\theta^{-1}(q)}_{=\hat{\varepsilon}}.$$

Applying $\tilde{F}_\theta(\cdot)$ to both sides, we obtain that

$$\tilde{F}_\theta\left(\tilde{F}_{\theta'}^{-1}(r)\right) \leq \tilde{F}_\theta\left(\tilde{F}_\theta^{-1}(r)\right) = r = \tilde{F}_{\theta'}\left(\tilde{F}_{\theta'}^{-1}(r)\right).$$

[24] A distribution $P \in \Delta\mathbb{R}^N$ dominates $Q \in \Delta\mathbb{R}^N$ in the PQD order, if $P\left(z_1, \ldots, z_N\right) \geq Q\left(z_1, \ldots, z_N\right), \forall\left(z_1, \ldots, z_N\right) \in \mathbb{R}^N$. See, e.g., Shaked and Shanthikumar (2007).

[25] This result was first proved by Bickel and Lehmann (2011).

Because $r > q$ can be chosen arbitrarily, we conclude that $\tilde{F}_\theta(\varepsilon) \le \tilde{F}_{\theta'}(\varepsilon)$ for any $\varepsilon > \hat{\varepsilon}$. Further, suppose there is $\varepsilon' > \hat{\varepsilon}$ so that $\tilde{F}_\theta(\varepsilon') < \tilde{F}_{\theta'}(\varepsilon') \equiv r'$. Let $\varepsilon'' > \varepsilon'$ be such that $\tilde{F}_\theta(\varepsilon'') = r'$. For any $r'' > r'$,

$$\tilde{F}_{\theta'}^{-1}(r'') - \underbrace{\tilde{F}_{\theta'}^{-1}(r')}_{=\varepsilon'} \le \tilde{F}_\theta^{-1}(r'') - \underbrace{\tilde{F}_\theta^{-1}(r')}_{=\varepsilon''}.$$

We conclude that for any $r'' > r'$, $\tilde{F}_{\theta'}^{-1}(r'') < \tilde{F}_\theta^{-1}(r'')$, and as a result that

$$\tilde{F}_\theta\big(\tilde{F}_{\theta'}^{-1}(r'')\big) < \tilde{F}_\theta\big(\tilde{F}_\theta^{-1}(r'')\big) = r'' = \tilde{F}_{\theta'}\big(\tilde{F}_{\theta'}^{-1}(r'')\big).$$

Thus, $\tilde{F}_\theta(\varepsilon) < \tilde{F}_{\theta'}(\varepsilon)$ for any $\varepsilon = \tilde{F}_{\theta'}^{-1}(r'') > \varepsilon' = \tilde{F}_{\theta'}^{-1}(r')$.

Similarly, for any $p < q$,

$$\underbrace{\tilde{F}_{\theta'}^{-1}(q)}_{=\hat{\varepsilon}} - \tilde{F}_{\theta'}^{-1}(p) \le \underbrace{\tilde{F}_\theta^{-1}(q)}_{=\hat{\varepsilon}} - \tilde{F}_\theta^{-1}(p),$$

which implies that

$$\tilde{F}_\theta\big(\tilde{F}_\theta^{-1}(p)\big) = p = \tilde{F}_{\theta'}\big(\tilde{F}_{\theta'}^{-1}(p)\big) \ge \tilde{F}_{\theta'}\big(\tilde{F}_\theta^{-1}(p)\big).$$

Because the inequality holds for any $p < q$, we conclude that $\tilde{F}_\theta(\varepsilon) \ge \tilde{F}_{\theta'}(\varepsilon)$ for any $\varepsilon < \hat{\varepsilon}$, and that if there is ε_0 so that $\tilde{F}_\theta(\varepsilon_0) > \tilde{F}_{\theta'}(\varepsilon_0)$, $\tilde{F}_\theta(\varepsilon) > \tilde{F}_{\theta'}(\varepsilon)$ for any $\varepsilon < \hat{\varepsilon}_0$. □

Next, any location experiment F_θ that satisfies MLRP also satisfies Assumption 2. Thus, provided that Φ satisfies Assumption 1, $D^\star(\omega)$ solves (8) and is therefore implicitly characterized by:

$$(1-\delta)\frac{\phi(\omega)}{1-\Phi(\omega)} = \frac{\frac{\partial}{\partial\omega}\big(1-\tilde{F}_\theta(D-\omega)\big)}{1-\tilde{F}_\theta(D-\omega)} = \frac{\tilde{f}_\theta}{1-\tilde{F}_\theta}(D-\omega).$$

That the hazard rates associated with Φ and $\tilde{F}_\theta$ are both monotone then implies that $D^\star(\omega) - \omega$ must be monotone as a function of ω. Next, note that the last observation, together with Lemma (7), jointly imply that, for any $\tilde{\theta}$, selling security $\min\{y, D_\theta^\star(\omega)\}$ to type ω induces information rents to all types above according to

$$\begin{aligned} I_{\tilde{\theta}}(\omega;\mathcal{M}) &= \int_{\underline{y}_\theta(\omega)}^{\overline{y}_\theta(\omega)} \min\{y, D_\theta^\star(\omega)\}\frac{\partial}{\partial\omega} f_{\tilde{\theta}}(y|\omega)\mathrm{d}y \\ &= \int_{\underline{y}_\theta(\omega)}^{D_\theta^\star(\omega)} \frac{\partial}{\partial\omega}\big(1-\tilde{F}_{\tilde{\theta}}(y-\omega)\big)\mathrm{d}y \\ &= \tilde{F}_{\tilde{\theta}}\big(D_\theta^\star(\omega)-\omega\big). \end{aligned}$$

The monotonicity of $D_\theta^\star(\omega) - \omega$ together with Lemma (7) then imply that, for any $\theta' > \theta$,

$$I_{\theta'}(\omega;\mathcal{M}) - I_\theta(\omega;\mathcal{M}) = \tilde{F}_{\theta'}\big(D_\theta^\star(\omega)-\omega\big) - \tilde{F}_\theta\big(D_\theta^\star(\omega)-\omega\big)$$

satisfies SCFB in ω. In other words, $I_\theta(\omega;\mathcal{M})$ has single crossing differences in (ω,θ). That $U_\theta(\omega;\mathcal{M})$ also has single crossing differences in (ω,θ) then follows from (11). To see this, suppose that $I_{\theta'}(\omega) - I_\theta(\omega)$ is different from 0 over a positive (Lebesgue) measure of types and that it turns from negative to positive at some $\omega_0 \in (\underline{\omega}, \overline{\omega})$. Because, $U_{\theta'}(\omega) - U_\theta(\omega) = \int_{\underline{\omega}}^{\omega}\big(I_{\theta'}(\tilde{\omega}) - I_\theta(\tilde{\omega})\big)\mathrm{d}\tilde{\omega}$, if there is any $\hat{\omega}$ for which $U_{\theta'}(\hat{\omega}) - U_\theta(\hat{\omega}) \ge 0$, then it has to be the case that $\hat{\omega} \ge \omega_0$—otherwise, if $\hat{\omega} < \omega_0$, $\int_{\underline{\omega}}^{\hat{\omega}}\underbrace{\big(I_{\theta'}(\tilde{\omega}) - I_\theta(\tilde{\omega})\big)}_{\le 0}\mathrm{d}\tilde{\omega} < 0$. But then, because $I_{\theta'}(\omega) - I_\theta(\omega) \ge 0$ for any $\omega > \hat{\omega} \ge \omega_0$, it must be the case that $U_{\theta'}(\omega) - U_\theta(\omega) \ge 0$ for any $\omega > \hat{\omega}$, which proves that $U_{\theta'}(\omega) - U_\theta(\omega)$ indeed satisfies SCFB.□

Proof of Lemma 4. The arguments in the proof of Proposition 3 imply that, for location experiments $\boldsymbol{y}_\theta = \boldsymbol{\omega} + \boldsymbol{\varepsilon}_\theta$, with $\boldsymbol{\varepsilon}_\theta \sim \tilde{F}_\theta$ over $[\underline{\varepsilon}_\theta, \overline{\varepsilon}_\theta]$, the induced information rents, under the debt menu $\mathcal{M}$, are given by $I_\theta(\omega) = \tilde{F}_\theta\big(D_\theta^\star(\omega)-\omega\big)$, for all $\omega \in \Omega$.

Next, note that the aggregate information rents are given by

$$\begin{aligned} \overline{U}_\theta(\mathcal{M}) &\equiv \int_{\underline{\omega}}^{\overline{\omega}} U_\theta(\omega;\mathcal{M})\mathrm{d}\Phi(\omega) \\ &= \int_{\underline{\omega}}^{\overline{\omega}} \left(\int_{\underline{\omega}}^{\omega} I_\theta(\tilde{\omega};\mathcal{M})\mathrm{d}\tilde{\omega}\right)\mathrm{d}\Phi(\omega) \\ &= \int_{\underline{\omega}}^{\overline{\omega}} (1-\Phi(\omega))I_\theta(\omega;\mathcal{M})\mathrm{d}\omega \\ &= \int_{\underline{\omega}}^{\overline{\omega}} (1-\Phi(\omega))\big(\tilde{F}_\theta\big(D_\theta^\star(\omega)-\omega\big)\big)\mathrm{d}\omega. \end{aligned}$$

where the second equality follows from (11), and the third equality follows from applying integration by parts.

Next, let $\varphi_\theta(\omega) \equiv D^\star_\theta(\omega) - \omega$ for all ω. Proposition 2 implies that

$$\varphi_\theta(\omega) = H^{-1}_{\tilde{F}_\theta}\big((1-\delta)H_\Phi(\omega)\big).$$

Further, assumption (1) and property (ii) in Condition (1) implies that $\varphi_\theta(\cdot)$ in monotone and convex. Finally, property (i) in Condition (1) implies that $\varphi_\theta\big(\underline{\omega}\big) = \underline{\varepsilon}_\theta$ (maximal distortion).

Claim 2. *For any $\theta' > \theta$, any decreasing function α, $\int_{\underline{\varepsilon}_\theta}^{\bar{\varepsilon}_\theta} \big(\tilde{F}_{\theta'}(z) - \tilde{F}_\theta(z)\big)\alpha(z)\mathrm{d}z \le 0$.*

Proof of the Claim. Applying integration by parts,

$$\int_{\underline{\varepsilon}_\theta}^{\bar{\varepsilon}_\theta} \big(\tilde{F}_{\theta'}(z) - \tilde{F}_\theta(z)\big)\alpha(z)\mathrm{d}z = -\int_{\underline{\varepsilon}_\theta}^{\bar{\varepsilon}_\theta}\left(\alpha'(z)\int_{\underline{\varepsilon}_\theta}^{z}\big(\tilde{F}_{\theta'}(e) - \tilde{F}_\theta(e)\big)\mathrm{d}e\right)\mathrm{d}z \le 0,$$

where the equality follows the fact that $\int_{\underline{\varepsilon}_\theta}^{\bar{\varepsilon}_\theta} \big(\tilde{F}_{\theta'}(z) - \tilde{F}_\theta(z)\big)\mathrm{d}z = \mathbb{E}\big[\varepsilon_\theta - \varepsilon_{\theta'}\big] = 0$, whereas the inequality obtains from (i) the observation that α is decreasing and (ii) the fact that $\int_{\underline{\varepsilon}_\theta}^{z} \big(\tilde{F}_{\theta'}(e) - \tilde{F}_\theta(e)\big)\mathrm{d}e \le 0$ for all z, which in turn is a consequence from the fact that $\tilde{F}_{\theta'} - \tilde{F}_\theta$ satisfies SCFB (by Lemma (7)) and $\int_{\underline{\varepsilon}_\theta}^{\bar{\varepsilon}_\theta} \big(\tilde{F}_{\theta'}(z) - \tilde{F}_\theta(z)\big)\mathrm{d}z = 0$.■

Note then that, for any $\theta' > \theta$,

$$\begin{aligned}\overline{U}_{\theta'}(\mathcal{M}) - \overline{U}_\theta(\mathcal{M}) &= \int_{\underline{\omega}}^{\overline{\omega}} (1-\Phi(\omega))\big(\tilde{F}_{\theta'}\big(D^\star_\theta(\omega) - \omega\big) - \tilde{F}_\theta\big(D^\star_\theta(\omega) - \omega\big)\big)\mathrm{d}\omega \\ &= \int_{\underline{\varepsilon}_\theta}^{\bar{\varepsilon}_\theta} \big(\tilde{F}_{\theta'}(z) - \tilde{F}_\theta(z)\big)\left(\frac{1-\Phi\big(\varphi_\theta^{-1}(z)\big)}{\varphi'_\theta\big(\varphi_\theta^{-1}(z)\big)}\right)\mathrm{d}z \le 0,\end{aligned}$$

where the second equality follows from the change of variables $z = \varphi_\theta(\omega)$ and the fact that the optimal debt menu satisfies no-distortion at the top, and full-distortion at the bottom (by property (i) in Condition (1)). The inequality then obtains from applying Claim 2 above by noting that, as argued above, φ_θ is monotone and convex, and therefore $\varphi'_\theta\big(\varphi_\theta^{-1}(z)\big)$ is also monotone.

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