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The value of aggregators in local electricity markets: A game theory based comparative analysis[☆]

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A B S T R A C T

Demand aggregators are expected to have a key role in future electricity systems. More specifically, aggregators can facilitate the harnessing of consumers' flexibility. This paper focuses on understanding the value of the aggregator in terms of aggregation of both flexibility and information. We consider the aggregation of flexibility as the ability to exercise a direct control over loads, while the aggregation of information refers to knowledge of the flexibility characteristics of the consumers. Several game theory formulations are used to model the interaction between the energy provider, consumers and the aggregator, each with a different information structure. We develop a potential game to obtain the Nash equilibrium of the non-cooperative game with complete information and we analyze the system dynamics of consumers using the adaptive expectations method in an incomplete information scenario. Several key insights about the value of aggregators are found. In particular, the value of the aggregator is mainly related to the aggregation of information rather than flexibility, and flexibility is valuable only when it can be coordinated. In this sense, prices are not enough to guarantee an effective coordination.

1. Introduction

In several countries an important political, social and environmental objective is the integration of a large amount of renewable energy sources [1]. In this new paradigm, several challenges emerge in both market design and system operations [2]. A source for these challenges is the volatile nature of several renewable energy sources, particularly the most widely developed (e.g., solar and wind). These sources impose new flexibility requirements for a reliable and economic operation [3]. Flexibility, defined as the ability of the system to cope with variations on generation and load [4], cannot only be provided by flexible generating units, e.g., hydro or gas, but also by the demand through Demand Side Management (DSM) [5], and Demand Response (DR) [6] schemes. According to [7], DR is a kind of DSM strategy that can be used to manage energy costs. In particular, DR is defined as a tariff or program that motivates changes in the electric usage by end-users, while DSM includes energy efficiency and/or load management programs.

Given the distributed and diverse nature of flexible loads, there has been extensive research regarding the impact of a new participant in charge of aggregating these loads, the so-called aggregator. The aggregator might play a key role in taking advantage of the demand flexibility and could be crucial in the integration of volatile energies resources in large-scale systems [8]. Several papers have studied the benefits of the aggregation of loads in several dimensions including: harnessing load flexibility, minimizing operations costs and reducing consumers' tariffs. For example, in [6] and [9], the market effects of an aggregator which takes advantage of operational flexibility of loads are examined. In particular, the equilibrium of a wholesale market in a Cournot game setup, where the players are the aggregator, the Independent System Operator (ISO) and generators, is analyzed. In [8], the participation of an aggregator as a strategic agent in the wholesale market is studied. The revenue of the aggregator can be transferred to end-users with the objective of encouraging participation in demand response programs. In [10] a market

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model in which a set of aggregators act as intermediaries between the utility and the consumers is introduced. Aggregators compete to sell demand response services to the utility and provide compensation to consumers in order to modify their consumption patterns. In [11] a smart pricing policy that encourages EVs to participate in frequency regulation through aggregators is presented and shown to benefit both consumers and the grid. In [12] an offering strategy for a risk-averse aggregator that participates in the balancing market is proposed. As flexibility product, the market accepts asymmetric block offers, which are defined by duration and power effect of the response part, duration and power effect of the rebound part, and the recovery time. A fixed-term DR contract market is proposed in [13]. The Distribution System Operator (DSO) buys flexibility services to DR aggregators in a competitive market, introducing scheduled and conditional contracts. The paper determines the optimal service for the system, allowing DSOs and DR aggregators ensure their profitability. In [14] a local coordinating mechanism for strategic aggregators is proposed. The goal of the mechanism corresponds to maximize the aggregators' utilities from their participation in the market, considering the physical limits of the distribution network.

In [15] a non-cooperative game is presented, in which aggregators compete between them to sell energy stored in consumers storage devices. In [16] a game theory model is proposed in order to find the equilibrium that maximizes the profit of aggregators and the distribution operator system (DSO). In [17] an offering strategy for wind power producers that includes DR aggregators is proposed. The problem is modeled using a bi-level programming approach, where the upper-level represents the producer's problem, and the lower-level models the aggregator behavior. In [18] a review of the value of the aggregator under different technological and regulatory scenarios is performed. Some benefits of the aggregator are related to the economies of scale and scope, uncertainty management and market complexities. In [19] a confidential information problem between aggregators and the balancing group manager (BGM) is presented. The paper proposes an iterative distributed algorithm based on non-cooperative game theory to address the confidential information problem considering strategic aggregators. In [20] the competition between DR aggregators to provide DR services to the system operator is modeled as a non-cooperative game with incomplete information. The goal of the aggregators is to obtain their optimal bidding strategies in a deregulated energy market. In [21] a two-stage stochastic bi-level programming approach is proposed to solve the interaction problem between aggregators and electricity retailers in a competitive environment. The upper-level problem corresponds to maximize the retailer's profit considering a two-stage stochastic programming scheme, while the lower-level problem corresponds to maximize the aggregator's profit. In [22] a multi follower bi-level framework is presented to model the interaction between the Distribution System Operator (DSO), the aggregator, and the Microgrid Owner (MGO). The upper-level problem minimizes the DSO's costs, while the lower-level problems maximize the MGO and aggregator's profits.

The existing literature has mainly focused on the case in which the aggregator is already on the market. In this paper, we study the conditions that might justify the existence of an aggregator in two key dimensions: the ability to exercise a direct control over flexible consumption, which we define as *flexibility aggregation*, and the ability to aggregate information about the load attributes of the flexible consumption, which we define as *information aggregation*. In order to understand what the value of the aggregator is, we consider three scenarios with different coordination schemes and levels of information. First, we consider a scenario with complete information in which each agent chooses its consumption knowing the preferences of other consumers, and has beliefs that perfectly anticipate other agent's actions. While this assumption is difficult to justify in the real world, we use it to study the case in which the only competitive advantage of the aggregator is the coordination of consumers' demand. Second, we consider a scenario where agents form beliefs about other agents' future actions based only on past price signals, which could be a result of incomplete information about the attributes of the other market participants. In this case, each agent relies only on its own preferences and price signals to choose consumption, based on the expectations about the actions of other consumers, which are formed based on history. Here, an aggregator may bring crucial advantages to the system by coordinating agents consumption and beliefs. Finally, in order to compare the previous decentralized scenarios, we implement as a benchmark the centralized problem which considers an aggregator that supplies energy to consumers using a direct control method. The optimal value of the centralized problem is compared with the two cases of competition. The higher the cost of the competition scenarios compared to the centralized problem, the greater the value of the aggregator. Regarding the main contributions of this paper, they include:

- Study of the value of the aggregator. In particular, we focus on understanding the value in terms of both aggregating flexibility and information.
- The development of a potential game as a solution method to find the Nash equilibrium of a non-cooperative game for demand response. The non-cooperative game is used to understand the value of the aggregator.
- The study and discussion of the effectiveness of prices as control signals to harness demand flexibility. In particular, we focus on understand conditions for which agents' response to prices might lead into cycles that remain far from an efficient outcome.

This paper is structured as follows: Section 2 presents the problem definition. Section 3 presents the characteristics of the system and explains the optimization models used. In Section 4 the solution methods for the game theoretical models are explained. The computational experiments and results are explained in Section 5. Section 6 is devoted to the discussion about the results and the impact on market design. Finally, Section 7 provides the conclusions.

2. Problem definition

We consider a market with three participants: consumers, an energy provider, and potentially, an aggregator. The energy provider establishes a tariff scheme based on price signals. We evaluate the effectiveness of demand coordination in centralized and decentralized scenarios. In the decentralized scenario, the energy provider communicates the price signals directly to consumers, while the centralized scenario considers that the energy provider communicates the price signals to the aggregator, and the aggregator is in charge of supplying energy to consumers. Note that the aggregator acts as an intermediary between the energy provider and the consumers. Also, we study the performance of a decentralized scenario considering partial and full availability of information for the consumers. Therefore, in order to understand what is the value of the aggregator, we formulate three scenarios based on different levels of information and interactions between the consumers:

1. **Competition scenario with complete information:** Consumers buy their energy requirements directly from the energy provider, minimizing their own costs and sharing information about attributes. Consumers receive the price signals communicated by the energy provider. Fig. 1 illustrates this scenario.

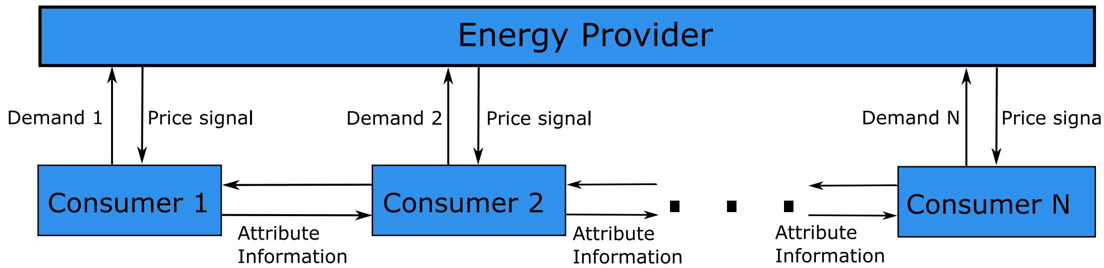

Fig. 1. Competition scenario with complete information.

Table 1
Scenario summary.

Scenarios	Competition	Centralized	Complete information	DLC
1	✓	✗	✓	✗
2	✓	✗	✗	✗
3	✗	✓	✓	✓

2. **Competition scenario with incomplete information:** Consumers buy their energy requirements directly from the energy provider, minimizing their own costs and not sharing information about attributes. Consumers receive the price signals communicated by the energy provider. Fig. 2 illustrates this scenario.
3. **Centralized scenario:** Consumers are under a direct control scheme of an aggregator, which minimizes the total energy cost[1] In this case, the aggregator receives the price signals communicated by the energy provider. Fig. 3 illustrates this scenario.

The Table 1 summarizes the different scenarios and their characteristics:

Regarding the third scenario, we assume that only the aggregator has complete information about all flexible loads, and not the consumers. Its goal is to minimize the total energy cost, and in order to supply energy to consumers, the aggregator buys energy from the energy provider following its tariff scheme. Because of the minimization of the total energy cost and the constraints related to all flexible loads, the aggregator obtains the optimal aggregate consumption trajectory. Note that in this scenario the consumers receive direct control signals instead of price signals. In order to model the interactions between the agents, we propose several game theoretical models and solution methods. We use the framework of non-cooperative games for the competition scenarios, and the centralized problem as the benchmark. In the non-cooperative game each agent minimizes its own cost, while in the centralized problem the aggregator minimizes the sum of the individual costs. Also, we consider different levels of information about the attributes of consumers. In the game with complete information, each agent knows the attributes of all agents, while in the game with incomplete information the agents only know the price signal. The price signals are constructed based on the tariff scheme given by the energy provider. It is worth mentioning that, in our scenarios, all agents have complete information about the tariff scheme and price signals. Also, it is assumed that the agents are fully rational, so they can perform an optimization problem to find their optimal consumption trajectory.

In the game with complete information, the consumers are able to solve a single optimization model called potential game [23], which optimal solution is equivalent to the Nash equilibrium of the non-cooperative game. For the incomplete information scenario, each agent has just the price signal as an input of information, therefore they are limited to a dynamic of best response, solving their individual optimization problem. We use the adaptive expectations method to process the price signal as available information. Note that to guarantee that consumers reach the Nash equilibrium, they need a large input of information (the attributes of all consumers) and being able to solve the potential game. On the other hand, a smaller input of information (price signal) does not allow to solve the potential game and limits to a dynamic of best response.

Therefore, the higher the cost of the non-cooperative game compared to the centralized problem, the greater the value that the aggregator has in the market because the aggregator would have a lower operating cost than consumers. On the other hand, if the cost of the consumers is similar to the cost of the aggregator, then the aggregator will not have operational value in the market. Note that in all these cases we do not consider communication or infrastructure costs associated with the implementation of any of these schemes.[2]

3. System and optimization models

The nomenclature used in the optimization models is presented in Sections 3.1 and 3.2 . We consider that consumers have duration-differentiated loads, which are defined by a fixed amount of energy in a fixed period of time. We present further information about this kind of loads in Section 3.3. Section 3.4 presents the market modeling considered in the optimization problems. Finally, Sections 3.5 and 3.6 present the non-cooperative game and centralized problem, respectively.

3.1. Indexes, sets, and parameters

- $i \in \mathcal{I}$: Index and set of consumers or agents.
- $h \in \mathcal{H}$: Index and set of time slots.
- $j \in \mathcal{J}_i$: Index and set of loads of agent i.
- $\mathcal{S}_{i,j}$: Set of time periods of agent i in which there is no consumption of j.
- N: Number of agents or consumers.
- $E_{i,j}$: Energy requirement of load j of agent i (kWh).
- $\theta_{i,j}^{max}$ Maximum power of load j of agent i (kW).
- $DL_{i,j}$: Deadline of load j of agent i.
- $EST_{i,j}$: Earliest start time of load j of agent i.
- a, b, γ: Coefficients of the artificial cost curve given by the energy provider.

[1] This problem can be considered equivalent to an aggregator's problem that maximizes its individual profit considering a fixed income. In other words, there could be contractual arrangements made previously by the aggregator and consumers that define a fixed tariff.

[2] We discuss in Section 6 about implementation costs issues.

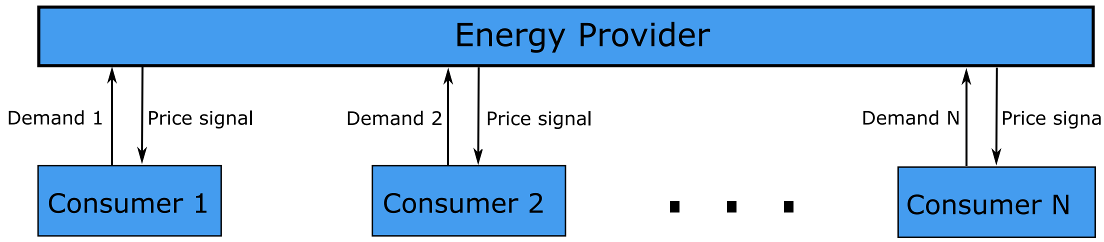

Fig. 2. Competition scenario with incomplete information.

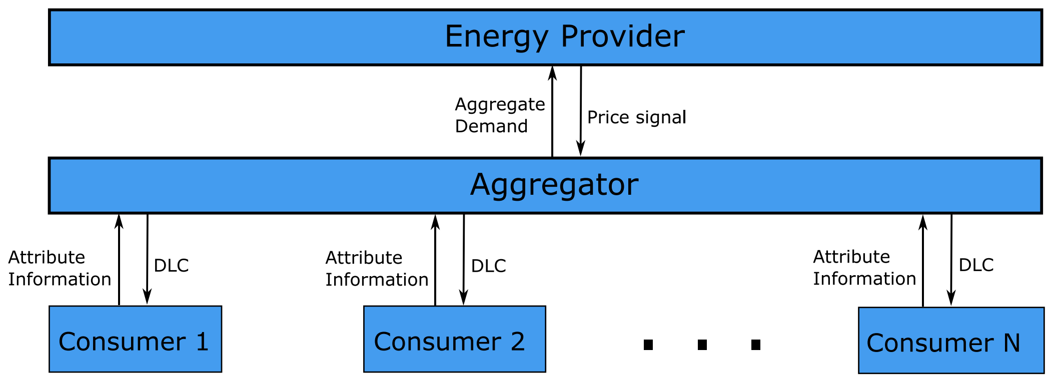

Fig. 3. Centralized scenario.

3.2. Variables

- $x_{i,h}$: Consumption of agent i in time h.
- $\theta_{i,j,h}$: Consumption of load j of agent i in time h.
- X_h: Aggregate consumption at time h.

3.3. Flexible loads

In this paper, the optimization models consider duration-differentiated loads [24] with maximum power constraint. The duration-differentiated loads have as attributes a fixed amount of energy in a fixed period of time. Some examples of these kind of loads are pool filters, EV charging, and other processes that can be interrupted without significant losses. The output level of each player i is the consumption $x_{i,h}$ at time h. The attributes of earliest start time, $EST_{i,j}$, and deadline, $DL_{i,j}$, establish a fixed period of time where the load j of agent i can shift its power consumption. In this sense, the variable $x_{i,j,h}$ can only be different from zero when it is inside of the time period defined by the earliest start time and deadline.

3.4. Market modeling

As we mentioned in Section 2, the energy provider communicates price signals to the consumers or the aggregator, depending on the scenario considered. The price signals are constructed based on a tariff scheme given by the energy provider to the other market participants. This tariff scheme is represented by an artificial cost curve following [25]. The energy price for end consumers, at the distribution level, corresponds to the marginal cost of the artificial cost curve. In this sense, in periods of high demand, there will also be a high price. Note that we suppose distribution lines with enough capacity to avoid congestion issues following the assumptions of [25,26]. Regarding the artificial cost function, we follow the assumptions of [25]:

- Assumption 1: The cost functions are increasing.
- Assumption 2: The cost functions are strictly convex.

We consider $c_h\left(P_h^d\right) = a\left(P_h^d\right)^2 + bP_h^d + \gamma$ as the artificial quadratic cost curve that satisfies the assumptions 1 and 2, where a, b and γ correspond to parameters of the energy provider cost function and P_h^d corresponds to the power demanded by the system in time slot h. The market price, λ_h, will be the marginal cost of the energy provider, $\frac{\partial c_h\left(P_h^d\right)}{\partial P_h^d} = b + 2aP_h^d$. Considering the power demanded as $P_h^d = \sum_{i\in I} x_{i,h} \quad \forall h$, the market price at time h corresponds to $\lambda_h = b + 2a\sum_{i\in\mathcal{I}} x_{i,h}$. Note that the quadratic cost curve considered to set the market price is an approximation of a tariff scheme based on a piecewise linear cost function. As in [25], we use this approximation in order to obtain a more tractable optimization problem. In [25], the authors use a quadratic cost function to approximate the tariff scheme of British Columbia Hydro in Canada, which adopts a tariff scheme based on a convex piecewise linear function.

3.5. Non-cooperative game: Individual problem

The individual problem for the agent n aims to minimize the cost paid for the energy consumed considering the price signal given by the energy provider. The objective function (1) corresponds to the minimization of the individual cost of the consumer n through load shifting, and the optimal value is represented by π_n^*. The first term of the cost function, $\left(b + 2a\sum_{i\in\mathcal{I}} x_{i,h}\right)$, corresponds to the price function λ_h, and the second term, $x_{n,h}$, corresponds to the consumption of the agent n at time h. Note that this tariff scheme induces a differentiation by consuming time. Unlike the modeling of [25], in which the objective function of the individual optimization problem is represented as a fixed value times the systemic energy cost, in our approach we model the objective function as the price times the consumption of the specific consumer.

$$\pi_n^* = \min_{\mathbf{x_n}} \sum_{h\in\mathcal{H}} \left(b + 2a\sum_{i\in\mathcal{I}} x_{i,h}\right) x_{n,h} \tag{1}$$

The constraint (2) represents the total consumption as the sum of the consumption of each load.

$$\text{s.t.}\ \ x_{n,h} = \sum_{j \in \mathcal{J}_n} \theta_{n,j,h} \quad \forall h \tag{2}$$

The Eq. (3) states the energy requirement of each load. Note that the power consumption is shifted but the total energy is the same.

$$\sum_{h \in \mathcal{H}} \theta_{n,j,h} = E_{n,j} \quad \forall j \tag{3}$$

The constraint (4) represents the attributes of earliest start time and deadline. In this sense, there is no load shifting in the time slots before the earliest start time and after the deadline.

$$\theta_{n,j,h} = 0 \quad \forall j,\ \forall h \in \mathcal{S}_{n,j} \tag{4}$$

Finally, the constraint (5) represents the minimum and maximum power. We assume a minimum power of zero in order to avoid using binary variables, resulting in a convex optimization problem,

$$0 \leq \theta_{n,j,h} \leq \theta_{n,j}^{max} \quad \forall j, h \tag{5}$$

Note that the cost function $\pi_n(\mathbf{x})$ is strictly convex, the payoff function $u_n(\mathbf{x}) = -\pi_n(\mathbf{x})$ is strictly concave with respect to vector $\mathbf{x}$, therefore, the individual problem is a strictly concave N-person game. Since the strategy spaces are compact and convex, and the payoff function is continuous and strictly concave, then a unique Nash equilibrium in pure strategies exists (see Theorems 1 and 3 of [27]).

3.6. Centralized problem

The centralized problem minimizes the total energy cost of the consumers obtaining the optimal consumption trajectory. In the centralized scenario, the aggregator has complete information about all attributes of the agents. Regarding the optimization model, the objective function (6) is similar to (1), but it considers the aggregate consumption of all agents. The aggregate optimal cost is represented by Π^*. The constraint (7) defines the aggregate consumption at each time h.

$$\Pi^* = \min_{\mathbf{x_i}} \sum_{h \in \mathcal{H}} (b + 2aX_h) X_h \tag{6}$$

$$\text{s.t.}\ \ X_h = \sum_{i \in \mathcal{I}} x_{i,h} \quad \forall h \tag{7}$$

$$x_{i,h} \in \mathcal{X}_i \quad \forall i, h \tag{8}$$

In addition, the aggregator is subject to the flexibility constraints of each flexible load. The constraint (8) defines the domain of $x_{i,h}$, which is given by (2), (3), (4) and (5).

4. Solution methods for the non-cooperative game

4.1. Complete information: Potential game

We use the concept of potential game [23] to find the Nash equilibrium with asymmetric players in the non-cooperative game with complete information. Potential games have been used in several papers of electricity markets, such as [28–30]. The potential game used in the literature corresponds to that presented in [23], which models a Cournot game. In the literature, the potential game is used to find the Nash equilibrium for a Cournot game between generators. In this paper, the non-cooperative game for demand response is different of the Cournot game. We use the potential game to find the Nash equilibrium of a non-cooperative game between consumers. Therefore, the potential game that we propose and develop is different of the potential game used in the literature. The potential game allows to use a single optimization model that represents the *Karush–Kuhn–Tucker* (KKT) conditions of all individual optimization problems, i.e. the optimal solution of the potential game is equivalent to finding the Nash equilibrium of the non-cooperative game. The objective function of such potential game is known as *potential function*. Also, we need to consider the individual constraints of each agent [28]. Let $\mathbf{x}$ be the decision vector about the optimal consumption of all agents and $\pi_n(\mathbf{x})$ the individual cost function. In games where the cost $\pi_n(\mathbf{x})$ is differentiable, the potential function must satisfy the following equation [23]:

$$\frac{\partial P(\mathbf{x})}{\partial x_{n,h}} = \frac{\partial \pi_n(\mathbf{x})}{\partial x_{n,h}} \quad \forall n, h \tag{9}$$

Note that there is no general method process to find a potential function. Moreover, to represent a non-cooperative game as a potential game is not always possible [23]. A potential game that represents the non-cooperative game presented in 3.5 is described in Theorem 1. Proof is included in Appendix.

Theorem 1. *Based on the condition (9), the Eqs. (10) is the potential function of the non-cooperative game given by the objective function (1) and constraints (2)–(5).*

$$\begin{aligned} P(\mathbf{x}) = & \sum_{h \in \mathcal{H}} \left(b + 2a \sum_{i \in \mathcal{I}} x_{i,h} \right) \sum_{i \in \mathcal{I}} x_{i,h} \\ & - \sum_{h \in \mathcal{H}} a \sum_{i \in \mathcal{I}} x_{i,h} \sum_{m \neq i} x_{m,h} \end{aligned} \tag{10}$$

Thus, the model to be solved to find the Nash equilibrium corresponds to minimize $P(\mathbf{x})$, subject to the individual constraints of each agent $i \in \mathcal{I}$. The optimization problem is convex because the objective function and inequality constraints are convex and the equality constraints are affine [31]. Then, any local optimal point is also a global optimal.

4.2. Incomplete information: Adaptive expectations method

In the case of the non-cooperative game with incomplete information, an agent-based simulation is used to study the agent market dynamics. We use the adaptive expectations method as learning rule to study the dynamics of the non-cooperative game with incomplete information [32–34]. In general, an agent will play a best response against what he expects will be the other agents' actions. In Nash equilibrium these expectations coincides with the actions being played. However, this requires perfect forecast. We relax this assumption and consider expectations that adapt over time to take into account past actions of other agents. At game k, agents form their beliefs through a linear combinations of beliefs before game $k-1$ and the actions played in that game k. They play a best response based on these new beliefs.

Note that, based on standard game theoretic notation [35], we use $-i$ to represent all players except player i. Considering $x_{-i,h}(k)$ as the aggregate consumption of agents $-i$, in the iteration k, and $x^E_{-i,h}(k)$ as the expectation about the aggregate consumption of agents $-i$ in the iteration k, the adaptive expectations method defines $x^E_{-i,h}(k+1)$ by Eq. (11), where $\delta_{-i} \in [0,1]$ corresponds to the adjustment coefficient. Note that each new iteration corresponds to a new game, then, the iteration k is equivalent to the game k.

$$x^E_{-i,h}(k+1) = x^E_{-i,h}(k) + \delta_{-i}\left(x_{-i,h}(k) - x^E_{-i,h}(k)\right) \quad \forall h \tag{11}$$

The algorithm to simulate the non-cooperative game with incomplete information is the following:

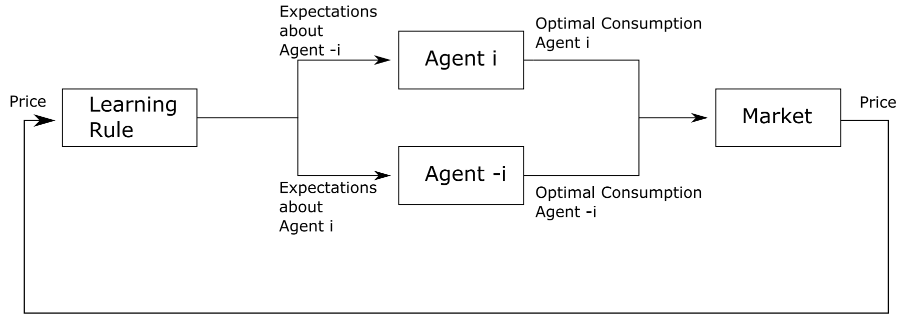

Fig. 4. Agent-based simulation with incomplete information.

1. Each agent i considers as a starting point $x^E_{-i,h}(0)$, which is an initial belief about the actions of $-i$.
2. Each agent i decides based on the solution of its individual optimization problem, (1)–(5), taking as input the expectation about the consumption of agents $-i$.
3. The energy provider communicates price signals to the agents at the end of the day. Each player i can know the aggregate consumption because the prices λ_h are known. Therefore, the agent i observes, in game k, an aggregate consumption of $-i$ defined by:

$$x_{-i,h}(k) = \frac{\lambda_h - b}{2a} - x_{i,h}(k)$$

.
4. Based on $x_{-i,h}(k)$ and the adaptive expectations method, the expectation about the aggregate consumption is obtained for the next iteration, $x^E_{-i,h}(k+1)$.

Fig. 4 shows two agents, i and $-i$, making their decisions based on a best response dynamic and the expectation that they have about the other agent. In this sense, the agent i obtains its optimal consumption using its expectation about the consumption of agent $-i$, which was obtained by the learning rule. The input of information for the agents is the price signal, from which the decision made by the other agent in the last game is known. In the simulation, each agent i has the process shown in Fig. 4, but we consider the agent $-i$ as the other agents of the market.

5. Computational experiments

We present our different case studies and their sensitivities in Section 5.1. The Section 5.2 shows and explains the solution of the non-cooperative game with complete information, and compare this solution with the solution given by the centralized problem. Finally, Section 5.3 explains the solution given by the non-cooperative game with incomplete information, and compare it with the solution given by the centralized problem.

5.1. Case studies

The goal is to study how much cost effective is the centralized solution in comparison to competition in both complete and incomplete information scenarios. We analyze the performance of the market in these scenarios considering that the number of consumers and their flexibility increases. The game theoretical models are analyzed for three cases: 10, 30 and 50 consumers. We define two types of consumers in our experiments: type 1 and type 2. Agents type 1 have constraints of earliest start time and deadline in its loads, while agents type 2 are totally flexible with any of its loads (e.g. $EST = 1$ and $DL = H$). Considering a fixed number of consumers N, we evaluate the market performance under different market compositions. We evaluate 11 cases in which the participation of type 1 agents decreases and the participation of type 2 agents increases by $0.1N$ (10%). Considering $\pi_{competition}(n)$ as the aggregate cost of n agents in the non-cooperative game and $\pi_{centralized}(n)$ as the cost of the aggregator for n agents in the centralized problem, we define the aggregator ratio for a set of agents in Eq. (12).

$$r(n) = \frac{\pi_{competition}(n)}{\pi_{centralized}(n)} \tag{12}$$

Where $n \in \{10, 30, 50\}$. In the simulation with incomplete information, we have different values of $\pi_{competition}(n)$ in function of each δ, so we use the average $\pi_{competition}(n)$ value to calculate the aggregator ratio. Note that in case of knowing which δ value captures better the agents' behavior, the aggregator ratio could be calculated considering such δ value. The aggregator ratio corresponds to how many times greater is the aggregate cost of the agents, in a competitive scenario, with respect to the cost that the aggregator would have solving the centralized problem. Then, the higher the aggregator ratio, the greater the value of the aggregator in the market. Note that the aggregator ratio is similar to the concept of price of anarchy [36], in the sense that it compares the costs of the decentralized (Nash equilibrium) and centralized scenarios.

Regarding the parameters of the numerical examples, we consider that each agent has a set of six loads, with different earliest start time and deadline values. We obtain the earliest start time and deadline parameters from uniform distributions. In the case of type 1 agents, we consider that the difference between the earliest start time and deadline is less than two thirds of the total time horizon. Note that a greater difference between earliest start time and deadline means a higher level of flexibility. In case of type 2 agents, they have the maximum possible difference between their earliest start time and deadline parameters. The total energy requirement of the six loads corresponds to 20 kWh for each consumer. The simulation considers 24 h of operation in 15 min intervals (96 time slots). Finally, we consider that the parameters of the artificial cost function are similar to the parameters used in [25], being equal to $a = 0.3$ [c/kWh] and $b = \gamma = 0$.

5.2. Complete information results

The solutions given by the non-cooperative game and the centralized problem, with complete information, are analyzed. The solutions for $N = 10$ and in the case of all the agents being type 1 are shown in Fig. 5. We can see that the solution of the centralized problem filters the peaks of demand that provide the solution of the non-cooperative game. To illustrate what happens between the centralized problem and the non-cooperative game we suppose a simple example with two agents, $i = 1, 2$. They consume in a period of two hours and are in a competition scenario. The agent $i = 1$ is extremely flexible and can consume in any of the two hours, while the agent $i = 2$ can only consume in the first hour. Both agents consume an amount of energy E.

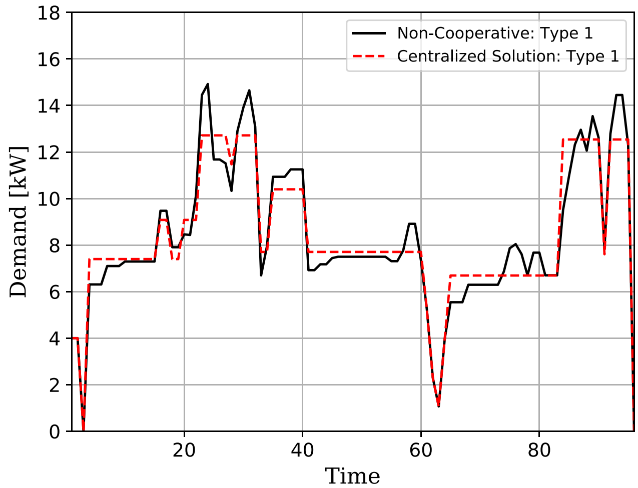

Fig. 5. Solution with complete information $N = 10$ and type 1 agents.

Proposition 1. *The optimal solution for agent $i = 1$ in the competition scenario corresponds to $x^*_{1,1} = \frac{1}{4}E$ and $x^*_{1,2} = \frac{3}{4}E$. Proof of this is presented in Appendix.*

Regarding the agent $i = 2$, its optimal solution clearly is $x^*_{2,1} = E$ and $x^*_{2,2} = 0$ because this agent can only consume in the first hour. We can see from the optimal solutions of both agents that there is a peak of demand in time $h = 1$ of $x_{1,1} + x_{2,1} = \frac{5}{4}E$. In order to understand the difference between the competition and centralized scenarios, assume now that the same two agents are under a direct load control of an aggregator. In that case, the aggregator minimizes the total cost of the system, i.e. the sum of the energy costs of both agents $i = 1, 2$.

Proposition 2. *The optimal solution for agent $i = 1$ in the centralized scenario corresponds to $x^*_{1,1} = 0$ and $x^*_{1,2} = E$. Proof of this is presented in Appendix.*

In the centralized scenario, as long as the demand flexibility allows it, the optimal solution equals the marginal energy cost of the aggregator in all time periods. In case of having a time period with a lower marginal energy cost, then, the aggregator increases the consumption in that time period until it reaches the same marginal energy cost than the others time periods. We can see that the competitive advantage in terms of flexibility of agent $i = 1$ produces a peak of demand in time $h = 1$ in the competition scenario. On the other hand, the centralized solution filters the peak of demand of $h = 1$. The same situation is happening in Fig. 5. Agents type 2 are taking advantage of their greater flexibility to minimize their individual energy costs. This competitive advantage of agents type 2 produces a greater number of peaks of demand in the non-cooperative game than in the centralized scenario. Regarding the results of the aggregator ratio, it is equal to 1.00 in most cases, and it has a maximum of 1.02 in the situation of $N = 10$ and considering a market composition of 0%, 10%, and 20% of type 2 agents. It means that the aggregation of flexibility by itself does not represent a great value for the aggregator in comparison to a non-cooperative game of fully rational agents in a complete information scenario. Therefore, the aggregator might not have value in a market where the consumers have complete information about the attributes of the other consumers.

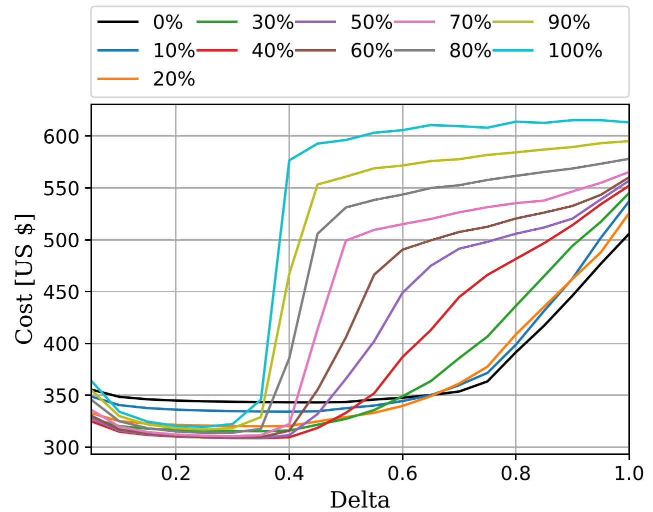

Fig. 6. Results of simulation with incomplete information considering a step-change of 10% in type 2 agents. $N = 10$.

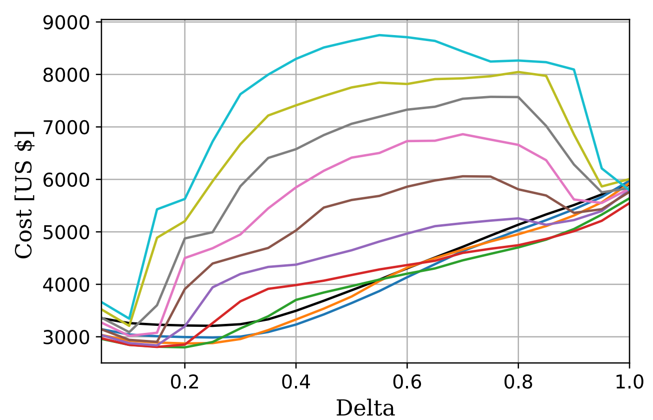

Fig. 7. Simulation results with incomplete information considering a step-change of 10% in type 2 agents. $N = 30$.

5.3. Incomplete information results

The parameter δ_i of Eq. (11) controls the adaptive expectation method. For the sake of simplicity, we assume that the adjustment coefficient $\delta_i = \delta, \quad \forall i$. We study the dynamics of the game using values of δ from 0.05 to 1.00 in steps of 0.05.

Figs. 6–8 show the total cost of the agents in function of the adjustment coefficient δ. Each curve represents a different combination between type 1 and type 2 agents. From Fig. 6, we can see two main regions in each curve. A first region where the total cost of the agents is lower, and a second region where the total cost of the agents is higher. In the first region, the total consumer costs tend to be lower because in that region the solution converges to the Nash equilibrium (region of convergence), while in the second region the response of the players oscillates and fails to converge to the Nash equilibrium, which increases the total cost. Note that in the second region the consumers do not reach the Nash equilibrium through the use of prices as the only available information. As we mentioned before, we consider the average cost of each curve to calculate the aggregator ratio. However, in case of knowing which δ value captures better the agents' behavior, then, the aggregator ratio could consider the cost of the competition scenario associated to such δ value.

Table 2 shows the maximum values of the adjustment coefficient δ where the game converges to the Nash equilibrium and

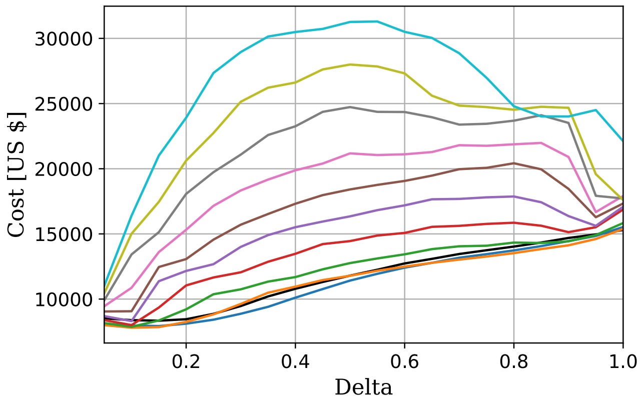

Fig. 8. Simulation results with incomplete information considering a step-change of 10% in type 2 agents. $N = 50$.

the aggregator ratio for the different case study. We can see that greater flexibility and number of agents increase the aggregator ratio, which means that the aggregator increases its value when there is a greater level of consumers' flexibility and/or there are a greater number of consumers in the market.

This can be explained in Fig. 6, where the second region shows a higher cost for agent sets with greater flexibility (greater presence of type 2). Note that in Figs. 7 and 8, with a greater number of consumers, the most flexible agents are always the most expensive ones. This happens because by having a greater set of strategies, agents tend to oscillate in their responses more easily and with more expensive trajectories of consumption in comparison to the Nash equilibrium. An oscillation in the agents' response means that the agents repeat their response pattern without converging to the Nash equilibrium (also known as best reply cycles [37]). Therefore, the best reply cycles increase the agents' cost in comparison to the cost of the agents when they reach the Nash equilibrium. This situation increases the aggregator ratio because the difference between the competition and centralized scenarios increases in terms of costs. Note that in the case that agents reach the Nash equilibrium in the competition scenario with incomplete information, then, the aggregator ratio would be close to a value of 1.00 as in the competition scenario with complete information.

In Figs. 9, 10, and 11 the consumer dynamics with different level of flexibility and δ are shown. Fig. 9 shows the Nash equilibrium and the simulation results of two games: one with all type 1 consumers and other with all type 2 consumers. We can see that the Nash equilibrium of the game with type 2 consumers is cheaper than the equilibrium of the game with type 1 consumers, which is expected because type 2 consumers are the most flexible agents. However, the simulation shows that type 2 consumers have a more expensive dynamic than type 1 consumers. Type 2 consumers, the most flexible agents, are more expensive because these consumers are not converging to the Nash equilibrium as type 1 consumers are doing. Instead of converging, the type 2 consumers stay in a best reply cycle, which makes the cost of the system more expensive. Fig. 10 shows a similar situation than in Fig. 9 but considering $\delta = 0.2$. We can see that in this case both types of consumers converge to the Nash equilibrium. Therefore, more flexible consumers (type 2) have a cheaper dynamic than consumers with a lower level of flexibility (type 1). Fig. 11 shows the dynamics with different values of delta. Using $\delta = 0.9$, the cost function oscillates without converging to Nash equilibrium. Using $\delta = 0.75$ there is still an oscillation but smaller than in the case with $\delta = 0.9$. Finally, using $\delta = 0.5$ the agents learn to play the game and they converge to Nash equilibrium.

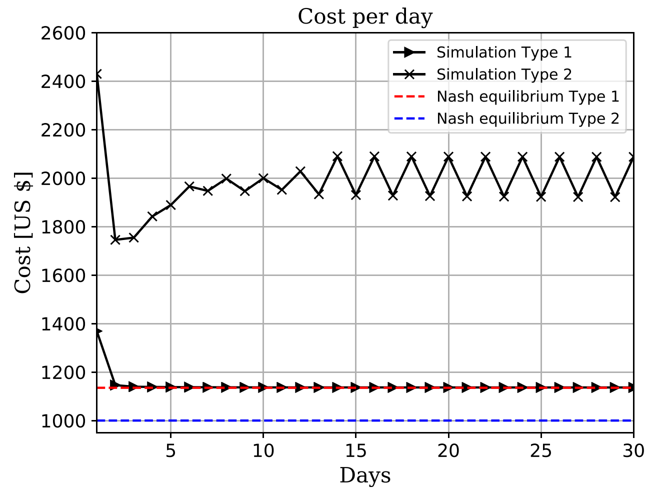

Fig. 9. Dynamics with different level of flexibility. $N = 10$ and $\delta = 0.5$.

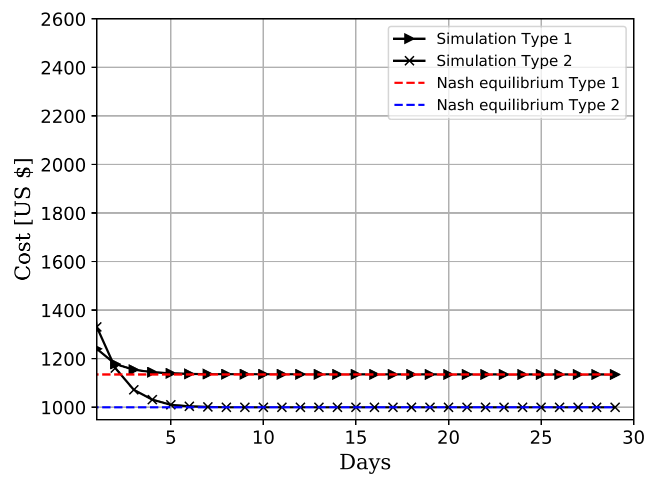

Fig. 10. Dynamics with different level of flexibility. $N = 10$ and $\delta = 0.2$.

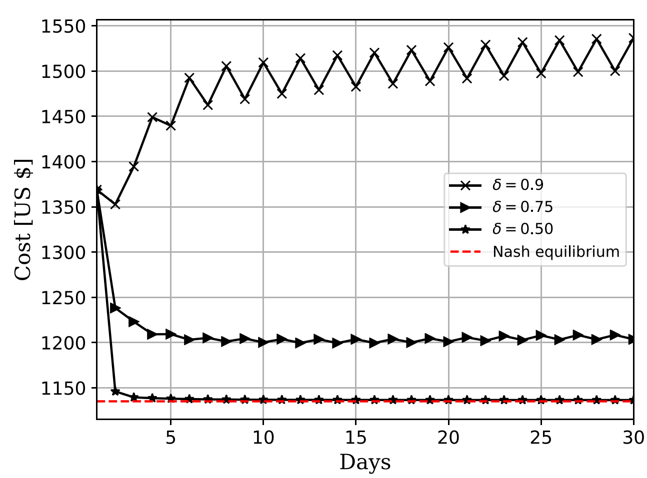

Fig. 11. Dynamics with different delta. $N = 10$ and type 1 agents.

6. Discussion

The analyzed cases allow us to obtain several insights about market design and demand response implementation schemes. Based on the previous results, we have three cases:

Table 2
Convergence and aggregator ratio: incomplete information.

Type 2	$\delta^{max}_{N=10}$	$\delta^{max}_{N=30}$	$\delta^{max}_{N=50}$	r (10)	r (30)	r (50)
0%	0,50	0,25	0,15	1,11	1,34	1,43
10%	0,45	0,25	0,15	1,16	1,38	1,46
20%	0,40	0,20	0,10	1,18	1,44	1,52
30%	0,40	0,20	0,10	1,21	1,46	1,63
40%	0,40	0,15	0,10	1,29	1,52	1,81
50%	0,35	0,15	0,05	1,35	1,67	2,01
60%	0,35	0,15	0,05	1,40	1,86	2,23
70%	0,35	0,10	0,05	1,46	2,06	2,48
80%	0,35	0,10	0,05	1,54	2,26	2,79
90%	0,35	0,10	0,05	1,61	2,47	3,08
100%	0,30	0,10	0,05	1,70	2,69	3,46

- **First Case:** Consumers know the attributes of all flexible loads.
- **Second Case:** Consumers have price signals as available information.
- **Third Case:** Aggregator uses a Direct Load Control with full information about the flexible loads.

In the first and second case, the consumers could reach the Nash equilibrium of the market, while in the third case, the aggregator reaches the cheapest consumption trajectory of the aggregate demand. We note that in the evaluated cases the aggregator's trajectory and Nash equilibrium have similar costs, so in the cases that consumers reach the Nash equilibrium, the aggregator does not add operational value in the market. Regarding the first case, the results showed that with complete information the consumers reach the Nash equilibrium, which allows them to have a similar performance with respect to the aggregator. However, note that each consumer needs to know the attributes of all flexible loads in the market. This is difficult to implement in real markets due to privacy and security of information concerns [38]. Not all agents are willing to share their attributes, preferences, and personal information with all market participants. An alternative to reduce the information burden on consumers is to consider an information aggregator. For example, an implementation in which only the information aggregator knows the attributes of all flexible loads. In this implementation, only the information aggregator knows the personal information about consumption preferences, which avoids sharing information among all consumers. With complete information, the information aggregator is able to solve the potential game. Then, instead of doing direct control over loads, the information aggregator communicates to each agent about the aggregate consumption of the other market participants in the Nash equilibrium. If each consumer knows the aggregated demand in the equilibrium, they will be able to reach the Nash equilibrium in a decentralized way, which is similar to the centralized solution in terms of system costs.

In this sense, the information aggregator is an alternative to an aggregator that exercises direct load control. In addition, the communication costs of an information aggregator should be lower than an aggregator which aggregates information and flexibility resources.

A popular decentralized scheme, in current market designs, is the use of prices as control signals for harnessing demand flexibility [39–41]. However, our results of the second case illustrate that the price signal could not be enough to reach the Nash equilibrium. In a scenario of incomplete information agents might not reach the equilibrium (even if such equilibrium exists), and their expectation formation leads to cycles that remain far from an efficient outcome. These results imply that market schemes based on prices as the only information mechanism, are not able to harness all the advantages of the demand flexibility, even with fully rational consumers. More importantly, the system cost could increase when the flexibility of the agents is greater.

We note that there are other costs and benefits, in different time scales, that we are not considering and could also matter to assess the value of the aggregator in an holistic way. For example, we assume that the communications infrastructure for direct load control already exists and its associated costs are sunk. Similarly, there are other benefits regarding the aggregator, beyond operational ones, that we are not capturing. For instance, the aggregator could watchdog the operation in the distribution system or it could generate contracts that consider more attributes than just the energy consumption [42].

7. Conclusions

We study the value of the aggregator in terms of aggregation of flexibility and information. The analysis is carried out by defining several interaction schemes between the consumers, aggregator, and the energy provider. The strategic interactions among the agents were characterized by using game theoretical models. We conclude that information plays an important role in the operational evaluation of the aggregator. Based on the comparison between the performance of the agents and the aggregator, we obtain that the less information available to consumers, the greater the operational value of the aggregator. This is because the consumers reach the Nash equilibrium in scenarios with full information, while in incomplete information scenarios they could oscillate in their responses without converging to the Nash equilibrium. In particular, we present in the computational experiments that a greater number of consumers and a higher level of flexibility could decrease the region of convergence of the system to the Nash equilibrium. In this sense, flexible agents could increase the cost of the system when prices are the only way to convey information. Thus, prices are not enough to guarantee an effective flexibility coordination. Regarding the optimization models, we propose a potential game as solution method to find the Nash equilibrium of the non-cooperative game presented in the paper.

Future work might include the consideration of other flexible load attributes, such as interruptible and interdependent loads, different modes, drain/losses, downtime, or uptime [43]. Also, more detailed EV formulations can be considered [44]. Note that including other flexible load attributes with non-convex formulations could imply the existence of more than one Nash equilibrium. In addition, we propose to include a more detailed communication system and study how its associated costs could impact in the optimal size of the aggregator. Also, future work includes other information schemes, uncertainty, the consideration of prosumers with local generation, other market designs, distribution network constraints, and more complex interactions between the aggregator and energy provider.

CRediT authorship contribution statement

Rafael Rodríguez: Investigation, Writing, Formal analysis, Visualization, Software. **Matías Negrete-Pincetic:** Conceptualization, Supervision, Project administration, Methodology , Writing - Review editing, Funding acquisition. **Nicolás Figueroa:** Conceptualization, Methodology, Writing, Editing. **Álvaro Lorca:** Conceptualization, Writing, Editing. **Daniel Olivares:** Conceptualization, Writing, Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Proofs

Proof. Theorem 1. We need to show that the derivative of the potential function with respect to $x_{i,h}$ is equivalent to the marginal cost of any agent i at any time h. First, we obtain the marginal cost of an agent i at time h.

$$\frac{\partial \pi_i(\mathbf{x})}{\partial x_{i,h}} = b + 2a \sum_{w \in \mathcal{I}} x_{w,h} + 2a\, x_{i,h} \tag{13}$$

Differentiating $P(\mathbf{x})$ with respect to $x_{i,h}$:

$$\frac{\partial P(\mathbf{x})}{\partial x_{i,h}} = b + 2a \sum_{w \in \mathcal{I}} x_{w,h} + 2a \sum_{w \in \mathcal{I}} x_{w,h} - 2a \sum_{m \neq i} x_{m,h} \tag{14}$$

We can rewrite (14) as:

$$\frac{\partial P(\mathbf{x})}{\partial x_{i,h}} = b + 2a \sum_{w \in \mathcal{I}} x_{w,h} + 2a\, x_{i,h} \tag{15}$$

We can see in Eqs. (13) and (15) that the condition (9) holds. □

Proof. Proposition 1. We can model the optimization problem of the agent 1 as the following:

$$\pi_1^* = \min_{x_1} \sum_{h=1}^{2} \left(b + 2a \sum_{i=1}^{2} x_{i,h} \right) x_{1,h} \tag{16}$$

$$\text{s.t.} \quad \sum_{h=1}^{2} x_{1,h} = E \tag{17}$$

Considering λ as the dual variable of the constraint (17), the Lagrangian of the optimization problem given by (16) and (17) corresponds to:

$$\mathcal{L} = \sum_{h=1}^{2} \left(b + 2a \sum_{i=1}^{2} x_{i,h} \right) x_{1,h} + \lambda \left(E - \sum_{h=1}^{2} x_{1,h} \right) \tag{18}$$

Based on Eq. (18), the KKT conditions are the following:

$$2ax_{1,h} + b + 2a(x_{1,h} + x_{2,h}) - \lambda = 0 \quad h = 1, 2 \tag{19}$$

$$E - \sum_{h=1}^{2} x_{1,h} = 0 \tag{20}$$

Using equation (19), we obtain Eq. (21).

$$2x_{1,1} + x_{2,1} = 2x_{1,2} + x_{2,2} \tag{21}$$

Considering that agent $i = 2$ can only consume energy in time $h = 1$, its optimal solution clearly is $x_{2,1}^* = E$ and $x_{2,2}^* = 0$. Using the optimal solution of agent $i = 2$ and replacing (20) in (21), we obtain (22).

$$2\left(E - 2x_{1,1}\right) = E \tag{22}$$

Therefore, the optimal solution of agent $i = 1$ corresponds to $(x_{1,1}^*, x_{1,2}^*) = (\frac{1}{4}E, \frac{3}{4}E)$. □

Proof. Proposition 2. In this particular case, the centralized problem can be formulated as:

$$\min_{\mathbf{x_i}} \sum_{h=1}^{2} (b + 2aX_h)\, X_h \tag{23}$$

$$\text{s.t.} \quad X_h = \sum_{i=1}^{2} x_{i,h} \quad \forall h \tag{24}$$

$$\sum_{h=1}^{2} x_{i,h} = E \quad \forall i = 1, 2 \tag{25}$$

$$x_{2,2} = 0 \tag{26}$$

As we mentioned before, agent $i = 2$ can only consume energy in time $h = 1$, therefore, its optimal solution clearly is $x_{2,1}^* = E$ and $x_{2,2}^* = 0$. Considering the optimal solution of agent $i = 2$, and for the sake of simplicity, we can formulate (23), (24), (25), and (26) as (27) and (28).

$$\min_{\mathbf{x_i}} \left(b + 2a(x_{1,1} + E)\right)\left(x_{1,1} + E\right) + \left(b + 2a\, x_{1,2}\right) x_{1,2} \tag{27}$$

$$\text{s.t.} \quad x_{1,1} + x_{1,2} = E \tag{28}$$

Considering μ as the dual variable of the constraint (28), the Lagrangian of the optimization problem given by (27) and (28) corresponds to:

$$\mathcal{L} = \left(b + 2a(x_{1,1} + E)\right)\left(x_{1,1} + E\right) + \left(b + 2a\, x_{1,2}\right) x_{1,2} + \mu\left(E - x_{1,1} - x_{1,2}\right) \tag{29}$$

Based on Eq. (29), the KKT conditions are the following:

$$2a\left(x_{1,1} + E\right) + b + 2a\left(x_{1,1} + E\right) - \mu = 0 \tag{30}$$

$$2a\, x_{1,2} + b + 2a\, x_{1,2} - \mu = 0 \tag{31}$$

$$E - \sum_{h=1}^{2} x_{1,h} = 0 \tag{32}$$

We can see from Eqs. (30) and (31) that the optimal solution in the centralized scenario equals the marginal energy costs in all time periods. In case of having a time slot with a lower marginal energy cost, then, the aggregator would increase the consumption in that time period until it equals the marginal energy cost of the other time periods. Note that it is possible as long as the demand flexibility allows it. Therefore, the optimal solution of the aggregator corresponds to $(x_{1,1}^*, x_{1,2}^*, x_{2,1}^*, x_{2,2}^*) = (0, E, E, 0)$. □

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