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Efficient Mechanisms for Multi-Asset Bilateral Trading

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ABSTRACT

We study when ex-post efficient bilateral trade is possible under two-sided asymmetric information with two agents and two assets, subject to balanced budget and voluntary participation constraints. We characterize how the interaction between asset characteristics and ownership structure determines the possibility of efficiency. Our main finding reveals a fundamental misalignment principle when goods are heterogeneous: efficiency is possible precisely when complementarity and ownership structure are *misaligned*. Under concentrated ownership (one agent owns both assets), efficiency is possible when assets are substitutes but impossible when they are complements. Under dispersed ownership (each agent owns one asset), the opposite holds: efficiency is possible when assets are complements but impossible when they are substitutes. This misalignment creates asymmetric valuations that overcome the participation constraints binding efficiency. Our results shed new light on the classic Myerson-Satterthwaite impossibility and Cramton–Gibbons–Klemperer possibility theorems to multi-asset environments, showing that the structure of asset bundles fundamentally alters the rents that we need to provide to ensure truthfulness and voluntary participation.

JEL Classification: C72, D82, D44, L14

1 | Introduction

Important economic and political decisions are determined through negotiations over multiple issues. Multilateral trade negotiations involve numerous goods and services; complex mergers and acquisitions allocate portfolios of assets;[1] technology licensing markets feature negotiations over patent portfolios that may cover overlapping or complementary technologies; labor contracts specify wages, benefits, and working conditions; and international agreements address interconnected policy dimensions. In most such negotiations, monetary transfers play a central role, as in mergers, acquisitions, and labor contracts.

An important economic insight is that asymmetric information seriously hinders the ability of negotiating parties to achieve mutually beneficial agreements. The seminal paper by Myerson and Satterthwaite (1983) demonstrates that in bilateral trade with double-sided private information, no feasible ex-post efficient mechanism exists when gains from trade are uncertain, establishing asymmetric information as a fundamental transaction cost in Coase's tradition. Yet the economics literature has provided limited analysis of multi-issue negotiations with monetary transfers and multidimensional private information, despite single-issue negotiations being more the exception than the rule.

The goal of this paper is to study multi-issue negotiations in the presence of asymmetric information, complementarities and substitutabilities among assets, and to characterize under which circumstances efficiency is possible. Since an agent's payoff from a given settlement is a function of his private information (which can be multidimensional), we use mechanism design tools to ask: Do efficient incentive-feasible mechanisms exist that satisfy voluntary participation without outside transfers? If the answer is no, then we know that no bargaining procedure–whether one-shot or sequential–can achieve efficiency under budget balance and voluntary participation.

We consider two agents negotiating over two assets, where each agent's type is three-dimensional. This represents a minimal departure from the classical bilateral trading frameworks of Myerson and Satterthwaite (1983) and Cramton et al. (1987) that allow us to study multi-asset negotiations. The three-dimensional specification—where each agent has valuations for asset A, asset B, and both assets together—is the most general type structure for this setting and isolates the essential economic forces: ownership structure, asset complementarity/substitutability, and information dimensionality.

Our analysis reveals a "misalignment" principle: with heterogeneous assets, efficiency is possible precisely when asset characteristics and ownership structure are *misaligned*. When assets are complements and ownership is dispersed, or when assets are substitutes and ownership is concentrated, efficiency can be achieved. When they are aligned–complements with concentrated ownership, or substitutes with dispersed ownership–impossibility prevails. This pattern reveals how participation costs and gains from trade interact with the dimensionality of private information to determine feasibility. For homogeneous assets, where private information is one-dimensional, efficiency is possible under dispersed ownership regardless of complementarity or substitutability, but never possible under concentrated ownership. For heterogeneous assets, where private information is multi-dimensional, the misalignment of ownership and asset characteristics becomes essential.

The intuition is as follows: With complements and dispersed ownership, each agent becomes a potential buyer who values "completing the bundle" more than the seller values retaining the individual asset. This alleviates the standard bilateral monopoly problem: buyers are willing to pay more than sellers require even after accounting for private information and voluntary participation rents. Conversely, with substitutes and concentrated ownership, the owner may prefer selling one asset while keeping the other, reducing participation costs. When asset characteristics and ownership are aligned, these beneficial forces disappear. The determining factor behind this pattern is the "critical type"–the type where gains from trade are minimized and participation constraints bind. Under concentrated ownership with homogeneous assets, the critical type is the seller's highest valuation, paralleling the Myerson and Satterthwaite (1983) impossibility. However, with heterogeneous substitutes, the critical type along the dimension of the asset with the lowest marginal utility can be interior or even the lowest valuation, decreasing participation costs. Under dispersed ownership, critical types can be interior with homogeneous assets, but with heterogeneous assets, each owner is a seller for their own asset and a buyer for the other, making critical types bind at extreme valuations. Only when assets are complements do the high marginal valuations from completing bundles overcome the participation costs.

1.1 | Related Literature

This paper relates to the extensive literature on efficient mechanism design, which includes the seminal papers by Vickrey (1961), Clarke (1971), and Groves (1973). A significant portion of this literature is concerned with the design of efficient *trading* mechanisms. The seminal contributions in this area are Myerson and Satterthwaite (1983) and Cramton et al. (1987). Important extensions, with methodological developments from which we borrow extensively, can be found in the works of Makowski and Mezzetti (1993, 1994), Williams (1999), Krishna and Perry (2000), and Schweizer (2006). None of these papers investigate the role of complementarities or substitutabilities vis-a-vis the status quo for efficiency. Segal and Whinston (2011) show that when the status quo is equal to the expectation of the efficient allocation, efficiency is possible. We ask a different question: Given the characteristics of assets and the status quo, when is efficiency possible? Efficiency is often possible in cases where the status quo is different from the expectation of the efficient allocation.

Our results also differ in spirit from papers investigating the extent to which inefficiencies can be alleviated by linking together a large number of independent decisions (Fang and Norman 2008; Jackson and Sonnenschein 2007). These papers exploit the law of large numbers and the ex-ante Pareto efficiency of the desired social choice function to achieve approximate efficiency. The idea of linking independent decisions, the main force behind those papers, differs from the forces we identify here. Here, we look at a small number of issues and investigate the joint role of their characteristics (whether they are complements or substitutes) and the initial ownership structure for efficiency.

We now describe our model.

2 | The Negotiation Setting

Two risk-neutral agents negotiate over two heterogeneous assets, A and B. Each agent's type is three-dimensional, $v_i = (v_i^A, v_i^B, v_i^{AB})$, where v_i^A is agent i's valuation for asset A alone, v_i^B for asset B alone, and v_i^{AB} for both. There are four possible allocations $Z = \{(AB, 0), (A, B), (B, A), (0, AB)\}$. Let $\pi_i^{z_j}$ denote agent i's payoff under allocation z_j. The following table lists all allocations and the corresponding payoffs for both agents:

Allocation	$\boldsymbol{\pi_1^z}$	$\boldsymbol{\pi_2^z}$
$z_1 = (AB, 0)$	v_1^{AB}	0
$z_2 = (A, B)$	v_1^A	v_2^B
$z_3 = (B, A)$	v_1^B	v_2^A
$z_4 = (0, AB)$	0	v_2^{AB}

Types $v = (v_1, v_2)$ have support $V = V_1 \times V_2$, where $V_i = [\underline{v}_i^A, \bar{v}_i^A] \times [\underline{v}_i^B, \bar{v}_i^B] \times [\underline{v}_i^{AB}, \bar{v}_i^{AB}]$ with $0 \leq \underline{v}_i^k \leq \bar{v}_i^k < \infty$ for $k \in \{A, B, AB\}$ and are independently distributed across agents according to F_i. If agent i decides not to participate, a *status quo* allocation $Q_i \in \Delta(Z)$ prevails. The payoff from non-participation is

$$\underline{U}_i(v_i) = \sum_{z \in Z} Q_i^z \pi_i^z(v_i).$$

Assets are heterogeneous because the value of owning each asset can differ, $v_i^A \neq v_i^B$, and because the value of owning the bundle, v_i^{AB}, can be greater or smaller than the sum of the values of owning each asset separately.

Definition 1. Complementarity and substitutability. We say that assets are *complements* if the set where $v_i^{AB} > v_i^A + v_i^B$ for $i = 1, 2$ has measure 1, and *substitutes* if the set where $v_i^{AB} < v_i^A + v_i^B$ for $i = 1, 2$ has measure 1.

Remark 1 Two modelling assumptions. Two features of the type space deserve explicit attention, as they play a key role in some of our results. (i) *No free disposal*. We do *not* impose free disposal on bundle valuations: v_i^{AB} is not required to satisfy $v_i^{AB} \geq \max\{v_i^A, v_i^B\}$. Hence it is possible that receiving both assets makes an agent *worse off* than receiving only one. This is consistent with the substitutes case ($v_i^{AB} < v_i^A + v_i^B$) and, in particular, allows v_i^{AB} to be substantially smaller than either individual valuation. The no-free-disposal assumption is appropriate in settings where holding an asset entails unavoidable costs or liabilities—for instance, patents that require maintenance fees, regulatory licences that impose compliance obligations, or assets bundled with debt. (ii) *No guaranteed retention*. Under any ex-post efficient mechanism, no agent can guarantee retaining their initial allocation: the efficient allocation depends on *both* agents' reports, and for any report of Agent i there exist type profiles of Agent j for which it is efficient to reassign Agent i's assets (c.f Cramton et al. 1987).

Definition 2 Ownership structure. We say that ownership is *concentrated* if one agent initially owns both assets, so that the status quo is $(AB, 0)$ or $(0, AB)$. We say that ownership is *dispersed* if each agent initially owns one asset, so that the status quo is (A, B) or (B, A).

The setting can nest the environments in Myerson and Satterthwaite (1983) and Cramton et al. (1987), making both objects identical, with $v_i^A = v_i^B$ and $v_i^{AB} = v_i^A + v_i^B$.

2.1 | Homogeneous Assets

We also examine a special case where assets are homogeneous. This corresponds to the case when $v_i^A = v_i^B$ (identical single-asset valuations) and bundle valuations satisfy $v_i^{AB} = (1 + \alpha)v_i$. There are three possible allocations: an agent obtains both assets, $z_1 = (2, 0)$ or $z_3 = (0, 2)$, or each agent receives one, $z_2 = (1, 1)$.

Agent types are one-dimensional, $v_i \in [\underline{v}_i, \bar{v}_i]$, distributed according to F_i. The payoffs are:

Allocation Z	π_1^z	π_2^z
$z_1 = (2, 0)$	$(1 + \alpha)v_1$	0
$z_2 = (1, 1)$	v_1	v_2
$z_3 = (0, 2)$	0	$(1 + \alpha)v_2$

Definition 3 Complements; substitutes. The two assets are *substitutes* if $\alpha < 1$ (diminishing marginal utility), *complements* if $\alpha > 1$ (increasing marginal utility), and *independent* if $\alpha = 1$ (constant marginal utility).

We assume gains from trade are uncertain in the sense that $\underline{v}_2 < \min\{\alpha\bar{v}_1, \bar{v}_1\}$, a natural generalization of the condition in Myerson and Satterthwaite (1983). The one-dimensional type structure simplifies the analysis but, as we show, can obscure some possibility results that emerge with richer type spaces.

2.2 | Ex-Post Efficient Mechanisms

By the revelation principle, we restrict attention to direct revelation mechanisms (DRMs) that satisfy truth-telling and voluntary participation. A DRM, $M = (p, x)$, consists of an *assignment rule* $p : V \to \Delta(Z)$ and a *payment rule* $x : V \to \mathbb{R}^2$. We denote by $p^z(v)$ the probability that outcome z is implemented when the vector of reports is v and $x_i(v)$ is the payment to agent i when reports are v.

The interim expected utility of agent i with type v_i who reports $\tilde{v}_i$ is

$$u_i(v_i, \tilde{v}_i; (p, x)) = E_{v_{-i}} \sum_{z \in Z} p^z(\tilde{v}_i, v_{-i})\pi_i^z(v_i) + x_i(\tilde{v}_i, v_{-i}). \quad (1)$$

Definition 4 Feasible mechanisms. For given outside options $\{Q_i\}_{i=1,2}$, a mechanism (p, x) is *feasible* if for all $v_i, \tilde{v}_i \in V_i$ and $v \in V$ it satisfies: (IC) $U_i(v_i) \equiv u_i(v_i, v_i; (p, x)) \geq u_i(v_i, \tilde{v}_i; (p, x))$, (VP) $U_i(v_i) \geq \underline{U}_i(v_i)$, (RES) $\sum_{z \in Z} p^z(v) = 1$ and $p^z(v) \geq 0$, (BB) $x_1(v) + x_2(v) = 0$.

Definition 5 Ex-post efficient mechanisms. A mechanism (p, x) is *ex-post efficient* if for all $v \in V, p^z(v) > 0$ implies $z \in \operatorname{argmax}_{z' \in Z}[\pi_1^{z'}(v_1) + \pi_2^{z'}(v_2)]$. The total social surplus at an ex-post efficient assignment is

$$W(v) = \max_{z \in Z}\left[\pi_1^z(v_1) + \pi_2^z(v_2)\right].$$

2.3 | The Transfer-Minimizing VCG

We now characterize when feasible ex-post efficient mechanisms exist. From the revenue equivalence theorem, all incentive compatible mechanisms implementing the same allocation rule generate the same expected payoff for each agent up to a constant. Therefore, we can work with the Vickrey-Clarke-Groves (VCG) mechanisms (which is known to be incentive compatible), which implements an ex-post efficient allocation given the reports and has payments $x_i(v) = \sum_{z \in Z} p^z(v)\pi_{-i}^z(v_{-i}) + T_i$, where T_i is an arbitrary constant. Therefore, agent i's interim payoff equals the expected gains from trade plus a constant:

$$U_i(v_i) = E_{v_{-i}}[W(v)] + T_i. \quad (2)$$

Agent i's expected payment is $X_i(v_i) = E_{v_{-i}}\left[\sum_{z \in Z} p^z(v)\pi_{-i}^z(v_{-i})\right] + T_i$. Voluntary participation requires

$$U_i(v_i) \geq \underline{U}_i(v_i), \text{(VP)}$$

which from (2) implies $T_i \geq \underline{U}_i(v_i) - E_{v_{-i}}[W(v)]$ for all $v_i \in V_i$. The type(s) of agent i least eager to participate then satisfy

$$v_i^* \in \operatorname*{argmax}_{v_i \in V_i}\{\underline{U}_i(v_i) - E_{v_{-i}}[W(v)]\}. \quad (3)$$

We call any such type a *critical type* of agent i.

Any critical type v_i^* corresponds to a type that makes Agent i *least willing to participate*: it maximises the gap between the agent's outside option $\underline{U}_i(v_i)$ and their expected contribution to social surplus $E_{v_{-i}}[W(v)]$. Intuitively, $\underline{U}_i(v_i)$ is the payoff Agent i can secure by refusing to trade, so it is large when the agent places a high value on whatever bundle they initially own. Meanwhile, $E_{v_{-i}}[W(v_i, v_{-i})]$ is increasing in v_i: a higher type contributes more to the efficient surplus and thus requires less compensation to be induced to trade. The critical type therefore combines the *highest* valuation for the status-quo bundle with the *lowest* valuations for all other bundles, making the outside option as attractive as possible while minimising the agent's expected contribution to trade. For example, under concentrated ownership with status quo $(AB, 0)$, Agent 1's outside option is $\underline{U}_1(v_1) = v_1^{AB}$, maximised at $\bar{v}_1^{AB}$. Setting $v_1^A = v_1^B = 0$ simultaneously minimises the expected efficient surplus involving Agent 1's individual valuations. The critical type is therefore $v_1^* = (0, 0, \bar{v}_1^{AB})$: Agent 1 values the bundle at its maximum but assigns zero value to either asset individually. Agent 2 owns nothing at the status quo, so $\underline{U}_2(v_2) = 0$ for all v_2, and any $v_2^* = (0, 0, 0)$ minimises the right-hand side of (3).

The lowest constant ensuring voluntary participation is therefore:

$$T_i^* = \underline{U}_i\left(v_i^*\right) - E_{v_{-i}}\left[W\left(v_i^*, v_{-i}\right)\right]. \tag{4}$$

The VCG mechanism with $T_i = T_i^*$ minimizes the sum of transfers among all efficient mechanisms satisfying (VP).

To find when budget balance holds, we sum the transfers across agents. Agent i's ex-post payment in the transfer-minimising VCG is

$$x_i(v) = \underbrace{\sum_{z \in Z} p^z(v)\pi_{-i}^z(v_{-i})}_{\text{Agent } -i\text{'s surplus under efficient rule}} + \underbrace{\underline{U}_i\left(v_i^*\right) - W\left(v_i^*, v_{-i}\right)}_{\text{participation correction}}. \tag{5}$$

The first term is the standard VCG pivot payment: Agent i receives the social surplus generated by the other agent under the efficient allocation, which internalises the externality their report imposes. The second term adjusts the constant so that the participation constraint binds exactly at the critical type v_i^*. Summing (5) over $i = 1, 2$ and using the fact that $\sum_i \sum_z p^z(v)\pi_{-i}^z(v_{-i}) = \sum_i [W(v) - \sum_z p^z(v)\pi_i^z(v_i)]$, which equals $W(v) - U_1(v_1) - U_2(v_2)$ under truth-telling, budget balance ($E_v[x_1(v) + x_2(v)] = 0$) is possible if and only if the expectation over v

$$\begin{aligned} S(v) &:= x_1(v) + x_2(v) = W(v) - \left[W(v) - W\left(v_1^*, v_2\right)\right] \\ &\quad - \left[W(v) - W\left(v_1, v_2^*\right)\right] + \underline{U}_1\left(v_1^*\right) + \underline{U}_2\left(v_2^*\right) \end{aligned} \tag{6}$$

is positive, that is $E_v[S(v)] \geq 0$ (a constant can be substracted to get budget balance without hurting voluntary participation). Therefore:

Proposition 1 Krishna and Perry 2000; Makowski and Mezzetti 1994. *There exists an ex-post efficient, incentive-compatible, individually rational, and budget-balanced mechanism if and only if the transfer-minimizing VCG satisfies* $E_v[S(v)] \geq 0$.

The term $S(v)$ in (6) equals the total gains from trade minus the sum of information rents and participation costs. When $S(v) \geq 0$ for all type profiles (not only in expectation), we have a *strong possibility result*: efficiency is achievable for any distribution F generating the critical types $\{v_1^*, v_2^*\}$. When $S(v) < 0$ for all type profiles, we have a *strong impossibility result*.

In what follows, we use Equation (6) to characterize when ex-post efficient mechanisms exist. We examine different specifications of the model–varying whether assets are complements or substitutes, and whether ownership is concentrated or dispersed–to understand how these features interact with the dimensionality of private information in determining the possibility of efficient trade.

3 | Negotiations Over Heterogeneous Assets

We study ex-post efficient mechanisms in negotiations over two heterogeneous assets. We characterize how the interaction between asset characteristics (complements vs. substitutes) and initial ownership structure (concentrated vs. dispersed) determines the possibility of efficient trade.

We present results for the baseline three-dimensional type specification, where agent i's type is $v_i = (v_i^A, v_i^B, v_i^{AB})$, capturing separate valuations for each asset individually and for the bundle. In the Online Appendix, we show that the main results extend to a simpler two-dimensional specification in which types take the form $v_i = (v_i^A, v_i^B)$ and bundle values are determined by a complementarity parameter, that is, $v_i^{AB} = v_i^A + \alpha v_i^B$.

3.1 | Concentrated Ownership

3.1.1 | *Complements*

We begin with the case where one agent (say, agent 1) initially owns both assets, so the status quo is $(AB, 0)$. When assets are complements, efficiency dictates that assets never be split between the two agents–either agent 1 keeps both or agent 2 acquires both. This creates a bilateral monopoly problem identical to Myerson and Satterthwaite (1983), yielding an expected impossibility result:

Theorem 2 Concentrated ownership: complements. *Suppose ownership is concentrated. Then there do not exist distributions F_1, F_2 for which assets are complements ($v_i^{AB} > v_i^A + v_i^B$ for $i = 1, 2$) and feasible and ex-post efficient mechanisms exist.*

This result is unsurprising: complementarity combined with concentrated ownership maximizes both the conflict between agents and the participation costs. The proof follows the Myerson-Satterthwaite logic by showing $S(v) \leq 0$ for all $v \in V$ and with a strict inequality for a non-null set of valuations. Note that assumption (ii) of Remark 1 is operative here: even if each agent tried to misreport so as to retain their own asset, the efficient mechanism would still sometimes reassign it.

3.1.2 | *Substitutes*

The case of substitutes with concentrated ownership yields one of our two most important results, which somewhat contradicts earlier intuitions. Here we find that even with concentrated ownership (as in Myerson and Satterthwaite 1983), efficiency is indeed possible. The key driver behind the result is that efficient allocations often involve splitting the assets, but the initial owner's critical type need not correspond to the highest bundle valuation–instead, efficiency may be possible precisely when the owner is better off after relinquishing one asset.

Proposition 2 Concentrated ownership: substitutes. *Suppose ownership is concentrated. Then there exist distributions F_1, F_2 for which assets are substitutes ($v_i^{AB} < v_i^A + v_i^B$ for $i = 1, 2$) and feasible and ex-post efficient mechanisms exist.*

Proof Sketch. When the status quo is $(AB, 0)$, the critical types are $v_1^* = (0, 0, \bar{v}_1^{AB})$ for agent 1 and $v_2^* = (0, 0, 0)$ for agent 2. For distributions placing sufficient mass on regions where efficiency dictates agent 1 retains only one asset (say, asset B with high valuation while transferring asset A), the surplus $S(v)$ from Equation (6) becomes positive. The crucial feature is that agent 1's participation cost is determined by $\bar{v}_1^{AB}$ rather than the sum $v_1^A + v_1^B$, which can be substantially lower when substitutability is strong. Full details are in Appendix A. Both assumptions of Remark 1 are operative here: the absence of free disposal allows $\bar{v}_1^{AB}$ to be low relative to $\bar{v}_1^A + \bar{v}_1^B$, reducing Agent 1's participation cost; and the inability to guarantee retention ensures that the efficient allocation still reassigns asset A to Agent 2 for some type profiles, keeping the surplus function $S(v)$ positive. □

Intuition: With substitutes, the initial owner may prefer giving away one asset while keeping the other, creating gains from trade that can overcome participation costs. The three-dimensional type structure is crucial: by declaring a low valuation for asset A, agent 1 does not simultaneously reveal information about v_1^B, allowing for efficient trade patterns impossible with one-dimensional types. As we see below in Section 4, this result runs counter to intuition from the homogeneous asset case where $v_i^B = v_i^A$.

3.2 | Dispersed Ownership

3.2.1 | Substitutes

The case where each agent initially owns one asset (status quo $z = (A, B)$), yields our second main result. For any distributions it is impossible to find an efficient and budget-balanced mechanisms that satisfies voluntary participation. This is different than the classical result in Cramton et al. (1987) where efficiency is possible under balanced ownership.

Theorem 3 Dispersed ownership: substitutes. *Suppose that ownership is dispersed. Then there do not exist distributions F_1, F_2 such that assets are substitutes ($v_i^{AB} < v_i^A + v_i^B$ for $i = 1, 2$) and feasible and ex-post efficient mechanisms exist.*

Proof Sketch. With status quo (A, B), the critical types are $v_1^* = (\bar{v}_1^A, 0, 0)$ and $v_2^* = (0, \bar{v}_2^B, 0)$. Each agent's participation cost reflects their highest valuation for their initial asset. When assets are substitutes, an agent acquiring the other asset gains marginal utility less than 1 (the marginal cost to the seller), making it impossible to compensate sellers adequately. The proof shows $S(v) \leq 0$ for all $v \in V$. □

This result contradicts conventional wisdom that "balanced" property rights facilitate efficiency. With substitutes, dispersed ownership increases total participation costs–each agent must be compensated as a seller for their own asset while valuing the other asset less than its owner does.

3.2.2 | Complements

We now turn to the case of complements with dispersed ownership, which yields a possibility result. Here, the marginal valuation of acquiring an asset to "complete" a bundle may exceed the seller's marginal valuation of losing it. This is in line with the main result in Cramton et al. (1987).

Proposition 3 Dispersed ownership: complements. *Suppose that ownership is dispersed. Then there exist distributions F_1, F_2 for which assets are complements ($v_i^{AB} > v_i^A + v_i^B$ for $i = 1, 2$) and feasible and ex-post efficient mechanisms exist.*

Proof Sketch. With status quo (A, B), the critical types are $v_1^* = (\bar{v}_1^A, 0, 0)$ and $v_2^* = (0, \bar{v}_2^B, 0)$. Efficiency is possible when complementarity is sufficiently strong that

$$v_i^{AB} \geq \bar{v}_1^A + \bar{v}_2^B \text{ for } i = 1 \text{ or } i = 2.$$

In these regions, the gains from assembling a bundle exceed the sum of the highest status quo payoffs, generating positive surplus $S(v)$. For distributions placing sufficient mass on these high-complementarity regions, $E[S(v)] \geq 0$. □

This is the paper's central positive result. When assets are strongly complementary, the buyer of an asset values "completing the bundle" more than the seller values retaining it individually. This reverses the standard bilateral monopoly problem: instead of sellers demanding too much compensation, buyers are willing to pay more than sellers require. Dispersed ownership is crucial–it ensures each agent can be a buyer for one asset, creating mutual gains from exchange.

Compared to the case of homogeneous assets that we examine below, complementarity here works *in favor of* efficiency because it creates asymmetric valuations: the bundle is worth substantially more than the sum of individual assets, giving potential assemblers higher willingness to pay than individual asset owners require.

Table 1 summarizes our findings:

The diagonal pattern reveals a striking interaction between asset characteristics and ownership structure. When these are *aligned*–complements with concentrated ownership, or substitutes with dispersed ownership–efficiency is impossible. When they are *misaligned*–complements with dispersed ownership, or substitutes with concentrated ownership–efficiency may be possible. This pattern reflects how participation costs and gains from trade interact in each configuration.

Remark 4. The most surprising results are those for substitutes—Proposition 2 and Theorem 3. Unlike the case of complements, which extends the intuition of the Myerson and Satterthwaite (1983) and Cramton et al. (1987) to a multi-asset setting, the substitute case reverses conventional wisdom: impossibility occurs under dispersed ('balanced') ownership, while

TABLE 1 | Possibility of efficient trade with three-dimensional types.

	Concentrated Ownership	**Dispersed Ownership**
Complements	Impossible (Theorem 2)	Possible (Proposition 3)
Substitutes	Possible (Proposition 2)	Impossible (Theorem 3)

possibility arises under concentrated ownership. These reversals are driven by the behavior of critical types under multi-dimensional private information, which we discuss in detail below and again in the comparison with homogeneous assets in Section 4.

4 | Negotiations over Homogeneous Assets

We now examine a special case where the two assets are homogeneous–that is, agents view the assets as identical. This specification provides a bridge between the classic single-asset models of Myerson and Satterthwaite (1983) and Cramton et al. (1987) and our heterogeneous asset framework. While most results parallel those in Section 3, one finding–an impossibility result for substitutes even with dispersed ownership–offers novel insights into the limits of efficiency.

4.1 | Concentrated Ownership

When one agent (say, agent 1) initially owns both assets (status quo $z_1 = (2, 0)$), efficiency is impossible, regardless of whether assets are complements or substitutes. This extends the Myerson-Satterthwaite impossibility to multi-unit trade and holds for all $\alpha > 0$:

Theorem 5 Concentrated ownership; homogeneous assets. *If ownership is concentrated and gains from trade are uncertain, then for all $\alpha > 0$, there exists no feasible and ex-post efficient mechanism.*

The proof shows that $S(v) \leq 0$ for all valuation profiles where trade occurs. Even when α is small, so the owner places little additional value on the second unit and gains from trade seem large, the sum of transfers remains negative. For instance, when $\alpha < 1$ and efficiency dictates allocation $(1, 1)$, agent 1 receives $-v_2$ (the buyer's marginal valuation) while agent 2 pays αv_1 (the seller's marginal valuation). Since $v_2 > \alpha v_1$ in this region, total transfers equal $-v_2 + \alpha v_1 < 0$.

This impossibility result parallels Theorem 2 (complements + concentrated) for heterogeneous assets. However, unlike the heterogeneous case (Proposition 2), we cannot establish a general possibility result for substitutes under concentrated ownership. The one-dimensional type structure makes critical types depend endogenously on the distribution in ways that complicate the analysis.

4.2 | Dispersed Ownership

When each agent initially owns one asset (status quo $z_2 = (1, 1)$) and assets are complements ($\alpha \geq 1$), efficiency becomes possible under natural conditions. This extends the Cramton-Gibbons-Klemperer result to multi-unit trade:

Proposition 4 Dispersed ownership; homogeneous assets. *If ownership is dispersed and assets are complements ($\alpha \geq 1$), efficiency is achievable whenever the critical types satisfy*

$$\max\left\{\overset{*}{\underset{1}{v}}, v_2^*\right\} \leq \alpha \min\left\{\overset{*}{\underset{1}{v}}, \overset{*}{\underset{2}{v}}\right\}.$$

In particular, efficiency always obtains in symmetric environments. If assets are substitutes, then for any $\alpha < 1$ there exists an open set of environments–an open set of distributions (F_1, F_2) in the total variation metric–for which efficiency is achievable.

When $\alpha \geq 1$ efficiency dictates that both units go to the agent with the highest valuation. Proposition 4 shows that efficiency is possible when agents' payoffs at their critical types are close. In particular, if agents are ex-ante symmetric, efficiency is possible since critical types are the same.[2] Ceteris paribus, a higher α relaxes the condition $\max\left\{F_2^{-1}\left(\frac{1}{1+\alpha}\right), F_1^{-1}\left(\frac{1}{1+\alpha}\right)\right\} \leq \alpha\min\left\{F_2^{-1}\left(\frac{1}{1+\alpha}\right), F_1^{-1}\left(\frac{1}{1+\alpha}\right)\right\}$. This is because α increases the gains from trade, as the status quo $z_1 = (1, 1)$ is always inefficient.

The first part of Proposition 4 result parallels Proposition 3 for heterogeneous assets, and its intuition. The agent assembling both units values the second unit more (marginal utility $\alpha > 1$) than the seller values retaining it (marginal utility 1), generating gains that can offset participation costs. A higher α relaxes the condition by increasing these gains.

The most interesting result for homogeneous assets concerns substitutes with dispersed ownership. Unlike the heterogeneous case, where impossibility holds generally (Theorem 3), here efficiency is sometimes possible even under extreme substitutability. This is unexpected because as $\alpha \to 0$, the surplus in (6) simplifies to $S(v) \approx W(v) - \sum_{i=1,2}[W(v) - W(v_i^*, v_{-i})] - \sum_{i=1,2}\underline{U}_i(v_i^*)$, where $W(v) \approx \max\{v_1, v_2\}$, since owning two units adds negligible value. Notwithstanding the aforementioned result, for any α there exist distributions that admit efficient outcomes (see region (ii) in the proof of Proposition 4). This possibility arises because critical types can be arbitrarily small, thereby rendering participation constraints easily satisfiable. The fundamental distinction between the homogeneous and heterogeneous goods models lies in the behavior of critical types—and consequently, outside options: under dispersed ownership with single-dimensional private information, critical types are small and interior, whereas in the heterogeneous-goods case, they attain extreme values.

4.2.1 | *Summary and Comparison to Heterogeneous Assets*

Table 2 summarizes the results for homogeneous assets.

Relative to the heterogeneous-asset case (Table 1), the overall pattern is not identical. Under concentrated ownership, both models yield impossibility across all complementarity levels, though the heterogeneous case admits a possibility result for substitutes (Proposition 2), which cannot be replicated here. This is because with single-dimensional private information the critical type is equal to the highest type. Under dispersed ownership, both models yield a possibility result in the case of complements, coming from forces equivalent to the ones present in Cramton et al. (1987): critical types are lower than the highest possible valuation for the bundle, generating enough gains from trade allowing the buyer to compensate the seller after taking into account incentive and participation costs. The

TABLE 2 | Possibility of efficient trade with homogeneous assets.

	Concentrated Ownership	Dispersed Ownership
Complements ($\alpha > 1$)	Impossible (Theorem 5)	Possible (Proposition 4)
Substitutes ($\alpha < 1$)	Impossible (Theorem 5)	Possible (Proposition 4)

key difference arises with substitutes–while the heterogeneous model predicts impossibility for all degrees of substitutability (Theorem 3), the homogeneous case does not preclude efficiency (second part of Proposition 4).

5 | Concluding Remarks

We investigate when ex-post efficient bilateral trade, togeher with budget balance and voluntary participation, is possible, in a setting where two agents negotiate over two assets under two-sided asymmetric information. In our main setting, with hetrogeneous assets, our findings reveal a novel interaction between asset characteristics and ownership structure: efficiency becomes possible precisely when these are *misaligned*. When assets are complements and ownership is dispersed, or when assets are substitutes and ownership is concentrated, efficiency can be achieved. When they are aligned–complements with concentrated ownership, or substitutes with dispersed ownership–impossibility prevails. This diagonal pattern reveals how participation costs and gains from trade interact with the dimensionality of private information to determine feasibility.

The intuition is clear. With complements and dispersed ownership, each agent becomes a potential buyer who values "completing the bundle" more than the seller values retaining the individual asset. This reverses the bilateral monopoly problem: buyers are willing to pay more than sellers require. Conversely, with substitutes and concentrated ownership, the initial owner may prefer selling one asset while keeping the other, reducing participation costs. When asset characteristics and ownership are aligned, these beneficial forces disappear.

Our analysis clarifies the forces behind some classic results. The Myerson–Satterthwaite impossibility arises because the seller's participation constraint binds at the highest valuation while the buyer's binds at the lowest–maximal participation costs that gains from trade cannot cover. The Cramton–Gibbons–Klemperer possibility emerges because joint ownership creates interior critical types, lowering participation rents. Our results show these are manifestations of a more general principle: efficiency depends on how asset characteristics, ownership structure, and the dimensionality of private information jointly shape surplus to be split, critical types and participation costs.

The fundamental distinction between the homogeneous and heterogeneous models lies in the behavior of critical types. With heterogeneous assets and dispersed ownership (Theorem 2), the critical types are $v_1^* = (\overline{v}_1^A, 0, 0)$ and $v_2^* = (0, \overline{v}_2^B, 0)$—each agent's participation cost is pinned to the *highest* possible valuation for their initially owned asset, regardless of the degree of substitutability. No matter how strongly assets are substitutes, these extreme critical types generate participation costs that cannot be outweighed by the gains from trade, yielding a general impossibility.

With homogeneous assets and one-dimensional types, by contrast, the critical type is an interior quantile of the distribution. As substitutability increases (i.e., $\alpha \to 0$), participation costs shrink—because critical types become small—faster than the gains from trade disappear. Efficient trade therefore remains achievable for high levels of substitutability.

The multi-dimensional type structure of the heterogeneous model thus *sharpens* the impossibility for substitutes under dispersed ownership by locking critical types at extreme values, while the one-dimensional structure of the homogeneous model permits interior critical types that make efficiency easier to sustain.

Two modelling features deserve mention: We do not impose free disposal on bundle valuations: an agent's value for both assets together, v_i^{AB}, may fall below the value of either asset individually. This is the relevant specification for settings where holding an asset entails costs or obligations—maintenance fees on patents, compliance costs attached to regulatory licences, or debt bundled with productive assets. It is precisely in such settings that the possibility results we establish (Proposition 2 in particular) apply. In settings where free disposal is natural and costless, the possibility result for concentrated ownership with substitutes would not obtain. Additionally, under the ex-post efficient mechanisms we consider, no agent can guarantee retaining their initial allocation by strategic misreporting, since the efficient rule depends on both agents' reports simultaneously as in Myerson and Satterthwaite (1983) and Cramton et al. (1987). Together, these two features shape the critical types that determine feasibility throughout the paper, and we discuss their role explicitly as each result is presented.

The forces we identify–how marginal valuations shift with bundling, how critical types vary with dimensionality, how participation costs respond to ownership concentration–likely extend to settings with more than two assets, more than two agents, and richer ownership structures. Understanding these extensions would deepen our knowledge of when markets can achieve efficiency without external funds, a fundamental question in mechanism design and a necessary first step to address more practical questions in market design.

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Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Endnotes

[1] Empirical work by Boone and Mulherin (2007) suggests that about half of company sales are performed via negotiations.

[2] This is related to Figueroa and Skreta (2011), where we study asymmetric partnerships and show that the sum of transfers necessary for voluntary participation is minimized when critical types are equalized across agents.

[3] By symmetry, this is analogous to case 1. Note, also in case 4 it is not possible to have $v_2^A = \max\{\overline{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.
Online_Appendix.

Appendix A

Proofs

A.1 Proofs for Section 3

Proof of Theorem 2. We look at the case where the status quo allocation is $(AB, 0)$. The case of $(0, AB)$ is analogous. We first need to identify the critical types. This task is immediate for agent 2: Since he does not own any of the assets, his outside payoff is 0, so irrespective of his expected payoff at an ex-post efficient assignment, the type vector where the participation constraint binds (the critical type) is $(0, 0, 0)$. Now, since agent 1 owns both assets, his payoff from non-participation depends on his type, and it is given by v_1^{AB}. Therefore, regardless of the shape of the participation payoff determined by an ex-post efficient allocation and the distribution of types, the critical type is $(0, 0, \bar{v}_1^{AB})$.

Given these critical types, (6), for the environment under consideration, becomes

$$
\begin{aligned}
S\left(v_1^A, v_1^B, v_1^{AB}, v_2^A, v_2^B, v_2^{AB}\right) = &-\max\left\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\right\} \\
&+ \max\left\{\bar{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\right\} \\
&+ \max\left\{v_1^{AB}, v_1^A, v_1^B\right\} - \bar{v}_1^{AB} - 0.
\end{aligned}
$$

Because the assets are complements in the sense that $v_i^{AB} > v_i^A + v_i^B$ for $i = 1, 2$, the sum of transfers becomes

$$
-\max\left\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\right\} + \max\left\{\bar{v}_1^{AB}, v_2^{AB}\right\} + v_1^{AB} - \bar{v}_1^{AB}.
$$

We show that this is always less than or equal to zero: Suppose that $v_1^{AB} = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then if $\bar{v}_1^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ the sum of transfers is zero, whereas if $v_2^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ it is negative, since $v_2^{AB} - \bar{v}_1^{AB} \le v_2^{AB} - v_1^{AB} \le 0$.

Suppose that $v_1^A + v_2^B = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then if $\bar{v}_1^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ the sum of transfers is negative, since $-(v_1^A + v_2^B) + v_1^{AB} \le 0$, and if $v_2^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ it is negative, since $-(v_1^A + v_2^B) + v_2^{AB} + v_1^{AB} - \bar{v}_1^{AB} \le 0$.

Now, suppose that $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then if $\bar{v}_1^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ the sum of transfers is negative, since $-(v_1^B + v_2^A) + v_1^{AB} \le 0$, and if $v_2^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ it is negative, since $-(v_1^B + v_2^A) + v_2^{AB} + v_1^{AB} - \bar{v}_1^{AB} \le 0$.

Finally, suppose that $v_2^{AB} = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then if $\bar{v}_1^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ the sum of transfers is negative, since $-v_2^{AB} + v_1^{AB} \le 0$. If $v_2^{AB} = \max\{\bar{v}_1^{AB}, v_2^{AB}\}$ the sum of transfers is negative, since $-v_2^{AB} + v_2^{AB} + v_1^{AB} - \bar{v}_1^{AB} \le 0$. □

Proof of Proposition 2. We look at the case where the status quo allocation is $(AB, 0)$. It is direct to see that the critical type for agent 2 is $(0, 0, 0)$ and the one for agent 1 is $(0, 0, \bar{v}_1^{AB})$.

Given these critical types, (6), for the environment under consideration, becomes

$$
\begin{aligned}
S\left(v_1^A, v_1^B, v_1^{AB}, v_2^A, v_2^B, v_2^{AB}\right) = &-\max\left\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\right\} \\
&+ \max\left\{\bar{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\right\} \\
&+ \max\left\{v_1^{AB}, v_1^A, v_1^B\right\} - \bar{v}_1^{AB}.
\end{aligned}
$$

Suppose that $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then

- If $\bar{v}_1^{AB} = \max\{\bar{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$
-\left(v_1^B + v_2^A\right) + \bar{v}_1^{AB} + v_1^{AB} - \bar{v}_1^{AB} = -\left(v_1^B + v_2^A\right) + v_1^{AB} \le 0.
$$

- If $\bar{v}_1^{AB} = \max\{\bar{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$
-\left(v_1^B + v_2^A\right) + \bar{v}_1^{AB} + v_1^A - \bar{v}_1^{AB} = -\left(v_1^B + v_2^A\right) + v_1^A \le 0.
$$

- If $\bar{v}_1^{AB} = \max\{\bar{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$
-\left(v_1^B + v_2^A\right) + \bar{v}_1^{AB} + v_1^B - \bar{v}_1^{AB} = -v_2^A \le 0.
$$

- If $v_2^B = \max\{\bar{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$
-\left(v_1^B + v_2^A\right) + v_2^B + v_1^{AB} - \bar{v}_1^{AB} \le 0.
$$

- If $v_2^B = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^B + v_1^A - \overline{v}_1^{AB} \leq 0.$$

- If $v_2^B = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^B + v_1^B - \overline{v}_1^{AB} = -v_2^A + v_2^B - \overline{v}_1^{AB} \geq 0 \text{ if } v_2^B \geq \overline{v}_1^{AB} + v_2^A,$$

which because $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$ requires that

$$v_1^B \geq \overline{v}_1^{AB} + v_1^A.$$

- If $v_2^A = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^A + v_1^{AB} - \overline{v}_1^{AB} = -v_1^B + v_1^{AB} - \overline{v}_1^{AB} \leq 0.$$

- If $v_2^A = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^A + v_1^A - \overline{v}_1^{AB} = -v_1^B + v_1^A - \overline{v}_1^{AB} \geq 0 \text{ if } v_1^A \geq \overline{v}_1^{AB} + v_1^B$$

which because $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$ for the surplus to be positive it must be the case that

$$v_2^A - v_2^B \geq \overline{v}_1^{AB}.$$

- If $v_2^A = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^A + v_1^B - \overline{v}_1^{AB} = -\overline{v}_1^{AB} \leq 0.$$

- If $v_2^{AB} = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^{AB} + v_1^{AB} - \overline{v}_1^{AB} \leq 0.$$

- If $v_2^{AB} = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^{AB} + v_1^A - \overline{v}_1^{AB} \geq 0 \text{ if } v_1^A \geq \overline{v}_1^{AB} + \left(v_1^B + v_2^A\right) - v_2^{AB}.$$

Recall that also $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$ requires that

$$v_1^B + v_2^A - v_2^B \geq v_1^A,$$

so combining we get $v_2^{AB} - v_2^B \geq \overline{v}_1^{AB}$.

- If $v_2^{AB} = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A, v_1^B\}$, then the sum of transfers becomes

$$-\left(v_1^B + v_2^A\right) + v_2^{AB} + v_1^B - \overline{v}_1^{AB} = -v_2^A + v_2^{AB} - \overline{v}_1^{AB} \geq 0 \text{ if } v_2^{AB} \geq \overline{v}_1^{AB} + v_2^A,$$

which because $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$ requires that

$$v_1^B \geq \overline{v}_1^{AB}.$$

To conclude surplus is positive in regions where $v_1^B \geq \overline{v}_1^{AB}$ or $v_1^A \geq \overline{v}_1^{AB}$. We construct distributions of valuations that guarantee positive expected surplus. Suppose that $\overline{v}_2^B = \overline{v}_1^A = \overline{v}_1^B = \overline{v}_2^A = 1$, whereas $\overline{v}_i^{AB} = 0.1$. All lowest valuations are zero. Pick any $\delta \in (0, 0.01)$. Consider the region of valuations where

$$v_1^A \in [0.01 \pm \delta],\ v_1^B \in [0.9 \pm \delta],$$
$$v_2^A \in [0.2 \pm \delta],\quad v_2^B \in [0.5 \pm \delta],$$
$$v_1^{AB}, v_2^{AB} \in [0.01 \pm \delta].$$

In that region, $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, also $v_2^B = \max\{\overline{v}_1^{AB}, v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A, v_1^B\}$. Then the sum of transfers becomes

$$S(v) = -v_2^A + v_2^B - \overline{v}_1^{AB} \geq -(0.2 + \delta) + (0.5 - \delta) - (0.1 + \delta) = 0.2 - 3\delta > 0.$$

It can be verified that a loose lower bound on $S(v)$ is -2.1 in this setting. Consider distributions F_i such that put mass greater than 21/22 in the region above, and uniformly distributed mass 1/22 in the complement of that region. Then the expected value of the surplus can be bounded as

$$E[S] \geq \frac{21}{22}(0.2 - 3\delta) + \frac{1}{22}(-2.1) > \frac{21}{22}0.1 + \frac{1}{22}(-2.1) = 0.$$

Finally, using the total variation metric in the space of distributions with a continuous and strictly positive density ($\|\mu - \nu\| = \sup_A|\mu(A) - \nu(A)|$), it is easy to see that there is an open set of distributions where the result holds. $\square$

Proof of Theorem 3. We look at the case where the status quo allocation is (A, B). The case of (B, A) is analogous. Since agent 1 owns asset A it is almost immediate to see that his critical type is $(\overline{v}_1^A, 0, 0)$, whereas, since agent 2 owns asset B, his critical type is $(0, \overline{v}_2^B, 0)$.

Given these critical types

$$S\left(v_1^A, v_1^B, v_1^{AB}, v_2^A, v_2^B, v_2^{AB}\right) = -\max\left\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\right\} + \max\left\{0, \overline{v}_1^A + v_2^B, v_2^A, v_2^{AB}\right\} + \max\left\{v_1^{AB}, v_1^A + \overline{v}_2^B, v_1^B\right\} - \overline{v}_1^A - \overline{v}_2^B.$$

We show that this is always less than or equal to zero. Note that because goods are substitutes the following holds:

$$v_i^A + v_i^B > v_i^{AB}. \tag{A1}$$

Case 1. Suppose that $v_1^{AB} = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then the $\max\{v_1^{AB}, v_1^A + \overline{v}_2^B, v_1^B\}$ is either v_1^{AB} or $v_1^A + \overline{v}_2^B$, so the relevant cases to consider are:

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-v_1^{AB} + \left(\bar{v}_1^A + v_2^B\right) + v_1^{AB} - \bar{v}_1^A - \bar{v}_2^B = v_2^B - \bar{v}_2^B \leq 0.$$

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-v_1^{AB} + \left(\bar{v}_1^A + v_2^B\right) + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B \\ &= -v_1^{AB} + v_2^B + v_1^A \leq 0.\end{aligned}$$

- If $v_2^A = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-v_1^{AB} + v_2^A - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} \\ &\leq -v_1^B - v_2^A + v_2^A - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} \\ &= -v_1^B - \bar{v}_1^A + v_1^{AB} - \bar{v}_2^B \leq 0,\end{aligned}$$

which follows by (A1).

- If $v_2^A = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-v_1^{AB} + v_2^A + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B = -v_1^{AB} + v_2^A + v_1^A - \bar{v}_1^A \leq 0.$$

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-v_1^{AB} + v_2^{AB} - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} &\leq -v_2^A - v_1^B + v_2^{AB} \\ &\quad - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} \\ &= -v_2^A - \bar{v}_2^B + v_2^{AB} - v_1^B - \bar{v}_1^A \\ &\quad + v_1^{AB} \leq 0,\end{aligned}$$

which follows by (A1).

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-v_1^{AB} + v_2^{AB} + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B = -v_1^{AB} + v_2^{AB} + v_1^A - \bar{v}_1^A \leq 0.$$

Case 2. Suppose that $v_1^A + v_2^B = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-\left(v_1^A + v_2^B\right) + \left(\bar{v}_1^A + v_2^B\right) + v_1^{AB} - \bar{v}_1^A - \bar{v}_2^B \\ &= -v_1^A + v_1^{AB} - \bar{v}_2^B \leq 0.\end{aligned}$$

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-\left(v_1^A + v_2^B\right) + \left(\bar{v}_1^A + v_2^B\right) + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B = 0.$$

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-\left(v_1^A + v_2^B\right) + \left(\bar{v}_1^A + v_2^B\right) + v_1^B - \bar{v}_1^A - \bar{v}_2^B \\ &= -v_1^A + v_1^B - \bar{v}_2^B \leq 0.\end{aligned}$$

Note that $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ holds if and only if $v_1^A = \bar{v}_1^A$. There are three subcases to consider, and the surplus is negative in all three of them is this case (case 2):

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$, we have

$$-\left(\bar{v}_1^A + v_2^B\right) + v_2^{AB} + v_1^{AB} - \bar{v}_1^A - \bar{v}_2^B \leq 0.$$

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$, we have

$$\begin{aligned}&-\left(\bar{v}_1^A + v_2^B\right) + v_2^{AB} + \bar{v}_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B \\ &= -\left(\bar{v}_1^A + v_2^B\right) + v_2^{AB} \leq 0.\end{aligned}$$

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$, we have

$$-\left(\bar{v}_1^A + v_2^B\right) + v_2^{AB} + v_1^B - \bar{v}_1^A - \bar{v}_2^B \leq 0.$$

It is impossible to have $v_2^A = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ in case 2.

Case 3. Suppose that $v_1^B + v_2^A = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-\left(v_1^B + v_2^A\right) + \left(\bar{v}_1^A + v_2^B\right) + v_1^{AB} - \bar{v}_1^A - \bar{v}_2^B \\ &= -\left(v_1^B + v_2^A\right) + v_1^{AB} + v_2^B - \bar{v}_2^B \leq 0.\end{aligned}$$

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-\left(v_1^B + v_2^A\right) + \left(\bar{v}_1^A + v_2^B\right) + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B \\ &= -\left(v_1^B + v_2^A\right) + v_2^B + v_1^A \leq 0.\end{aligned}$$

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-\left(v_1^B + v_2^A\right) + \left(\bar{v}_1^A + v_2^B\right) + v_1^B - \bar{v}_1^A - \bar{v}_2^B \\ &= -v_2^A + v_2^B - \bar{v}_2^B \leq 0.\end{aligned}$$

- If $v_2^A = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-\left(v_1^B + v_2^A\right) + v_2^A - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} &= -v_1^B - \bar{v}_1^A \\ &\quad - \bar{v}_2^B + v_1^{AB} \leq 0\end{aligned}$$

which follows by (A1).

- If $v_2^A = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-\left(v_1^B + v_2^A\right) + v_2^A + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B = -v_1^B + v_1^A - \bar{v}_1^A \leq 0$$

- If $v_2^A = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-v_1^B + v_1^B - \bar{v}_1^A - \bar{v}_2^B \leq 0.$$

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-\left(v_1^B + v_2^A\right) + v_2^{AB} - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} \leq 0$$

which follows by (A1).

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-\left(v_1^B + v_2^A\right) + v_2^{AB} + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B &\leq -v_2^{AB} + v_2^{AB} \\ &\quad + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B \\ &= v_1^A - \bar{v}_1^A \leq 0.\end{aligned}$$

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-\left(v_1^B + v_2^A\right) + v_2^{AB} + v_1^B - \bar{v}_1^A - \bar{v}_2^B &= -v_2^A + v_2^{AB} \\ &\quad - \bar{v}_1^A - \bar{v}_2^B \leq 0\end{aligned}$$

which follows by (A1).

Case 4. Suppose that $v_2^{AB} = \max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\}$, then[3]

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-v_2^{AB} + \left(\bar{v}_1^A + v_2^B\right) + v_1^{AB} - \bar{v}_1^A - \bar{v}_2^B &= -v_2^{AB} + v_1^{AB} + v_2^B \\ &\quad - \bar{v}_2^B \leq 0.\end{aligned}$$

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}&-v_2^{AB} + \left(\bar{v}_1^A + v_2^B\right) + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B \\ &= -v_2^{AB} + v_2^B + v_1^A \leq 0.\end{aligned}$$

- If $\bar{v}_1^A + v_2^B = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-v_2^{AB} + \left(\bar{v}_1^A + v_2^B\right) + v_1^B - \bar{v}_1^A - \bar{v}_2^B &= -v_2^{AB} + v_1^B \\ &\quad + v_2^B - \bar{v}_2^B \leq 0.\end{aligned}$$

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^{AB} = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-v_2^{AB} + v_2^{AB} - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} &\leq -v_1^B - v_2^A + v_2^{AB} \\ &\quad - \bar{v}_1^A - \bar{v}_2^B + v_1^{AB} \leq 0.\end{aligned}$$

which follows by (A1).

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^A + \bar{v}_2^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$-v_2^{AB} + v_2^{AB} + v_1^A + \bar{v}_2^B - \bar{v}_1^A - \bar{v}_2^B = v_1^A - \bar{v}_1^A \leq 0.$$

- If $v_2^{AB} = \max\{\bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\}$ and $v_1^B = \max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\}$ we have

$$\begin{aligned}-v_2^{AB} + v_2^{AB} + v_1^B - \bar{v}_1^A - \bar{v}_2^B &\leq -v_1^B - v_2^A + v_2^{AB} \\ &\quad + v_1^B - \bar{v}_1^A - \bar{v}_2^B \\ &= -v_2^A + v_2^{AB} - \bar{v}_1^A - \bar{v}_2^B \leq 0\end{aligned}$$

which follows by (A1).

Proof of Proposition 3. We treat the case in which the status quo allocation is (A, B); the case (B, A) is analogous. Since agent 1 initially owns asset A, his critical type is $(\bar{v}_1^A, 0, 0)$, whereas, since agent 2 initially owns asset B, his critical type is $(0, \bar{v}_2^B, 0)$.

Hence

$$\begin{aligned}S\left(v_1^A, v_1^B, v_1^{AB}, v_2^A, v_2^B, v_2^{AB}\right) = &-\max\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\} \\ &+\max\{0, \bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\} \\ &+\max\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\} - \bar{v}_1^A - \bar{v}_2^B.\end{aligned}$$

We now construct an open set of distributions for which expected surplus is strictly positive. Assume that

$$\bar{v}_1^A = \bar{v}_2^B = 1, \bar{v}_i^{AB} = 3 \text{ for } i = 1, 2,$$

and that the lowest valuations are zero. Thus

$$v_1^A, v_1^B, v_2^A, v_2^B \in [0, 1], v_1^{AB}, v_2^{AB} \in [0, 3].$$

Fix $\delta \in (0, 1)$, and define

$$T_i \equiv \{\left(v_i^A, v_i^B, v_i^{AB}\right) : v_i^{AB} \geq 2 + \delta\}, i = 1, 2.$$

Consider any profile $(v_1, v_2) \in T_1 \times T_2$. Then

$$v_1^{AB} \geq 2 + \delta, v_2^{AB} \geq 2 + \delta,$$

while, since $v_1^A, v_1^B, v_2^A, v_2^B \leq 1$,

$$v_1^A + v_2^B \leq 2, v_1^B + v_2^A \leq 2, \bar{v}_1^A + v_2^B \leq 2, v_1^A + \bar{v}_2^B \leq 2.$$

Therefore, on $T_1 \times T_2$,

$$\max\left\{v_1^{AB}, v_1^A + v_2^B, v_1^B + v_2^A, v_2^{AB}\right\} = \max\left\{v_1^{AB}, v_2^{AB}\right\},$$
$$\max\left\{0, \bar{v}_1^A + v_2^B, v_2^A, v_2^{AB}\right\} = v_2^{AB},$$
$$\max\left\{v_1^{AB}, v_1^A + \bar{v}_2^B, v_1^B\right\} = v_1^{AB}.$$

Substituting into the expression for S gives

$$\begin{aligned} S(v) &= -\max\left\{v_1^{AB}, v_2^{AB}\right\} + v_2^{AB} + v_1^{AB} - 2 \\ &= \min\left\{v_1^{AB}, v_2^{AB}\right\} - 2. \end{aligned}$$

Since $v_1^{AB}, v_2^{AB} \geq 2 + \delta$ on $T_1 \times T_2$, it follows that

$$S(v) \geq \delta \quad \text{for all } (v_1, v_2) \in T_1 \times T_2.$$

Now let

$$M \equiv \sup_v |S(v)| < \infty,$$

which is finite because the type space is bounded. Choose independent type distributions F_1 and F_2 such that, for each $i = 1, 2$,

$$F_i(T_i) > \sqrt{\frac{M}{\delta + M}}.$$

For example, one may take F_i to have a continuous strictly positive density and to assign mass greater than $\sqrt{M/(\delta + M)}$ to T_i, with the remaining mass placed on T_i^c.

Let $\mu = F_1 \times F_2$ denote the induced product distribution over type profiles. Then

$$\mu(T_1 \times T_2) = F_1(T_1)F_2(T_2) > \frac{M}{\delta + M}.$$

Using the lower bound $S(v) \geq \delta$ on $T_1 \times T_2$ and the trivial bound $S(v) \geq -M$ everywhere, we obtain

$$\begin{aligned} E_\mu[S] &= \mu(T_1 \times T_2)E_\mu[S|T_1 \times T_2] + (1 - \mu(T_1 \times T_2))E_\mu[S|(T_1 \times T_2)^c] \\ &\geq \delta\frac{M}{\delta + M} - M\left(1 - \frac{M}{\delta + M}\right) \geq 0 \end{aligned}$$

Finally, this inequality is strict, and therefore continues to hold for all joint distributions sufficiently close to μ in total variation. It follows that there is an open set of distributions with continuous and strictly positive densities for which the expected surplus is strictly positive. □

A.2 Proofs for Section 4

Proof of Theorem 5. We establish the result for the case that the status quo allocation is (2, 0). First, note that, given this status quo, it is easy to see that, regardless of the distributions of valuations, and of the exact form of the ex-post efficient assignment, the critical types for agent 1 and 2, are always given by $v_1^* = \bar{v}_1$ and $v_2^* = \underline{v}_2$, respectively. This is because the slope of the seller's payoff from non-participation is $1 + \alpha$, while the slope of his payoff from an ex-post efficient assignment is weakly less than $1 + \alpha$, for all $v_1 \in [\underline{v}_1, \bar{v}_1]$. Now, the slope of the buyer's non-participation payoff is 0, and the slope of his payoff from an ex-post efficient assignment is weakly greater than 0, for all $v_2 \in [\underline{v}_2, \bar{v}_2]$. With these critical types, $S(v_1, v_2, \alpha)$ from (6) becomes

$$\begin{aligned} S(v_1, v_2, \alpha) &= -W(v_1, v_2) + W(\bar{v}_1, v_2) + W(v_1, \underline{v}_2) - (1 + \alpha)\bar{v}_1 \\ &= -\max\{(1 + \alpha)v_1, v_1 + v_2, (1 + \alpha)v_2\} \\ &\quad +\max\{(1 + \alpha)\bar{v}_1, \bar{v}_1 + v_2, (1 + \alpha)v_2\} \\ &\quad +\max\{(1 + \alpha)v_1, v_1 + \underline{v}_2, (1 + \alpha)\underline{v}_2\} - (1 + \alpha)\bar{v}_1. \end{aligned} \tag{A2}$$

Notice that we explicitly note the dependence of the surplus on the degree of complementarity/substitutability of the assets α. In order to determine the sign of S, we examine the following scenarios:

If $(1 + \alpha)\bar{v}_1 = \max\{(1 + \alpha)\bar{v}_1, \bar{v}_1 + v_2, (1 + \alpha)v_2\}$, then the second and the last terms on the right-hand-side of (A2) cancel out, and we immediately have that

$$\begin{aligned} S(v_1, v_2, \alpha) = &-\max\{(1 + \alpha)v_1, v_1 + v_2, (1 + \alpha)v_2\} \\ &+\max\{(1 + \alpha)v_1, v_1 + \underline{v}_2, (1 + \alpha)\underline{v}_2\} \leq 0. \end{aligned}$$

If $(1 + \alpha)v_2 = \max\{(1 + \alpha)\bar{v}_1, \bar{v}_1 + v_2, (1 + \alpha)v_2\}$, then it follows that $(1 + \alpha)v_2 = \max\{(1 + \alpha)v_1, v_1 + v_2, (1 + \alpha)v_2\}$, which implies that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -(1 + \alpha)\bar{v}_1 + \max\{(1 + \alpha)v_1, v_1 + \underline{v}_2, (1 + \alpha)\underline{v}_2\} \\ &= \max\{(1 + \alpha)(v_1 - \bar{v}_1), (v_1 - \bar{v}_1) \\ &\quad + (\underline{v}_2 - \alpha\bar{v}_1), (1 + \alpha)(\underline{v}_2 - \bar{v}_1)\} \leq 0, \end{aligned}$$

where the last inequality follows from the fact that $\underline{v}_2 \leq \alpha\bar{v}_1$.

Finally, if $\bar{v}_1 + v_2 = \max\{(1 + \alpha)\bar{v}_1, \bar{v}_1 + v_2, (1 + \alpha)v_2\}$, this implies that $v_2 > \alpha\bar{v}_1$, so $\max\{(1 + \alpha)v_1, v_1 + v_2, (1 + \alpha)v_2\}$ is either $v_1 + v_2$ if $v_1 \geq \alpha v_2$, or $(1 + \alpha)v_2$ otherwise. If $v_1 \geq \alpha v_2$, then $v_1 + v_2 = \max\{(1 + \alpha)v_1, v_1 + v_2, (1 + \alpha)v_2\}$, and we have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -v_1 - \alpha\bar{v}_1 + \max\{(1 + \alpha)v_1, v_1 + \underline{v}_2, (1 + \alpha)\underline{v}_2\} \\ &= \max\{\alpha(v_1 - \bar{v}_1), \underline{v}_2 - \alpha\bar{v}_1, (\underline{v}_2 - \alpha\bar{v}_1) + (\alpha\underline{v}_2 - v_1)\} \leq 0, \end{aligned}$$

where the last inequality comes from the assumption that $\alpha\bar{v}_1 \geq \underline{v}_2$, and the fact that, in this case, $v_1 \geq \alpha v_2$. If $v_1 \leq \alpha v_2$, then $(1 + \alpha)v_2 = \max\{(1 + \alpha)v_1, v_1 + v_2, (1 + \alpha)v_2\}$, and we have that

$$\begin{aligned} S(v_1, v_2, \alpha) &= -\alpha v_2 - \alpha\bar{v}_1 + \max\{(1 + \alpha)v_1, v_1 + \underline{v}_2, (1 + \alpha)\underline{v}_2\} \\ &= \max\{(v_1 - \alpha v_2) + \alpha(v_1 - \bar{v}_1), (v_1 - \alpha v_2) \\ &\quad + (\underline{v}_2 - \alpha\bar{v}_1), (\underline{v}_2 - \alpha\bar{v}_1) + \alpha(\underline{v}_2 - v_2)\} \leq 0, \end{aligned}$$

where the last inequality comes from the assumption that $\alpha\bar{v}_1 > \underline{v}_2$, and the fact that, in this case, $v_1 \leq \alpha v_2$.

Therefore, $S(v_1, v_2, \alpha)$ is less than zero for all v_1, v_2. Moreover, there are regions of types with a non-empty interior where $S(v_1, v_2, \alpha) < 0$. These two observations together establish that it is impossible to design ex-post efficient mechanisms. □

Proof of Proposition 4. We first consider the case where $\alpha > 1$. Since each agent owns an asset, his payoff from non-participation is v_i which has slope 1. Participation payoffs can have any slope between 0 and $(1 + \alpha)$, therefore i's critical type can be any type v_i^*. Writing down (6) for this case we get

$$\begin{aligned} S(v, \alpha) = &-\max\{(1 + \alpha)v_1, (1 + \alpha)v_2\} \\ &+\max\{(1 + \alpha)v_1^*, (1 + \alpha)v_2\} \\ &+ \max\{(1 + \alpha)v_2^*, (1 + \alpha)v_1\} - v_1^* - v_2^*, \end{aligned} \tag{A3}$$

which can be rewritten as

$$S(v, \alpha) = \begin{cases} (1 + \alpha)(v_2 - v_1) + \alpha v_2^* - v_1^* & \text{if } v_2^* > v_1, v_1^* < v_2, v_1 > v_2 \\ (1 + \alpha)v_2 - v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_2 > v_1^*, v_1 > v_2 \\ \alpha v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_1^* > v_2, v_1 > v_2 \\ -(1 + \alpha)v_1 + \alpha v_1^* + \alpha v_2^* & \text{if } v_2^* > v_1, v_1^* > v_2, v_1 > v_2 \\ \alpha v_2^* - v_1^* & \text{if } v_2^* > v_1, v_1^* < v_2, v_1 < v_2 \\ (1 + \alpha)v_1 - v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_2 > v_1^*, v_1 < v_2 \\ (1 + \alpha)(v_1 - v_2) + \alpha v_1^* - v_2^* & \text{if } v_1 > v_2^*, v_1^* > v_2, v_1 < v_2 \\ -(1 + \alpha)v_2 + \alpha v_1^* + \alpha v_2^* & \text{if } v_2^* > v_1, v_1^* > v_2, v_1 < v_2 \end{cases}. \tag{A4}$$

Consider the situation where $v_1^* \geq v_2^*$ (the other case is analogous). Then, the first case is impossible, and the second, third and fourth yield a nonnegative surplus. Moreover, if $\alpha v_2^* \geq v_1^*$, the fifth to eighth cases are also nonnegative. The result follows directly from the assumption since $\alpha v_2^* = \alpha F_1^{-1}(\frac{1}{1+\alpha})$ and $v_1^* = F_2^{-1}(\frac{1}{1+\alpha})$.

We now consider the case $\alpha < 1$. Exactly as before, agent i's critical type can be any type v_i^*. We establish the possibility of efficiency for a symmetric environment. In such a case, (6) can be written as

$$
\begin{aligned}
S(v,\alpha) = &-\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} \\
&+\max\{(1+\alpha)v^*, (1+\alpha)v_2, v^*+v_2\} \\
&+\max\{(1+\alpha)v_1, (1+\alpha)v^*, v^*+v_1\} - 2v^*,
\end{aligned} \tag{A5}
$$

where v^* is the critical type of both agents. At that type, the slope of participation and non-participation payoffs are equal, which implies that v^* satisfies $\alpha F(\alpha v^*) + F(v^*/\alpha) = 1$.

For a point $\hat{v}$, consider a distribution F such that $F(\alpha\hat{v}) = \frac{1}{1+\alpha} - \epsilon$ and $F(\hat{v}/\alpha) = \frac{1}{1+\alpha} + \epsilon\alpha$. Then, $v^* = \hat{v}$ and, with probability close to 1, we have one of the following cases (note that the measure of the set $[\alpha\hat{v}, \hat{v}/\alpha]$ is of order ϵ):

i. $v_i \le \alpha v^*$ for $i \in \{1,2\}$, in which case $S(v,\alpha) = -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} + 2\alpha v^* \ge 0$.

ii. $v_i \ge \frac{v^*}{\alpha}$ for $i \in \{1,2\}$. Then, we immediately have that $S(v,\alpha) = -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} + (1+\alpha)v_1 + (1+\alpha)v_2 - 2v^* \ge 0$.

iii. $v_i \le \alpha v^*$ and $v_j \ge \frac{v^*}{\alpha}$. In this case, it is easy to see that $S(v,\alpha) = v^*(\alpha-1) < 0$.

Let

$$p_L \equiv \Pr(v \le \alpha v^*) = F(\alpha v^*) = \frac{1}{1+\alpha} - \epsilon$$

and

$$p_H \equiv \Pr\left(v \ge \frac{v^*}{\alpha}\right) = 1 - F\left(\frac{v^*}{\alpha}\right) = \frac{\alpha}{1+\alpha} - \alpha\epsilon.$$

Region (iii) consists of the two events

$$\{v_1 \le \alpha v^*, v_2 \ge v^*/\alpha\} \quad \text{and} \quad \{v_2 \le \alpha v^*, v_1 \ge v^*/\alpha\},$$

so its probability is

$$2p_L p_H = 2\left(\frac{1}{1+\alpha} - \epsilon\right)\left(\frac{\alpha}{1+\alpha} - \alpha\epsilon\right) = \frac{2\alpha}{(1+\alpha)^2} + O(\epsilon).$$

Since on region (iii) we have

$$S(v,\alpha) = v^*(\alpha-1),$$

the contribution of region (iii) is

$$\int_{(iii)} S(v,\alpha)dF = 2p_L p_H v^*(\alpha-1) = \frac{2\alpha}{(1+\alpha)^2}v^*(\alpha-1) + O(\epsilon).$$

On the other hand, region (i) has probability

$$p_L^2 = \left(\frac{1}{1+\alpha} - \epsilon\right)^2 = \frac{1}{(1+\alpha)^2} + O(\epsilon).$$

Moreover, if almost all mass in that region is concentrated near 0, then

$$S(v,\alpha) = -\max\{(1+\alpha)v_1, (1+\alpha)v_2, v_1+v_2\} + 2\alpha v^* \approx 2\alpha v^*,$$

so the contribution of region (i) is

$$\int_{(i)} S(v,\alpha)dF = \frac{1}{(1+\alpha)^2}2\alpha v^* + O(\epsilon).$$

Putting these two terms together yields

$$\frac{2\alpha v^*}{(1+\alpha)^2} + \frac{2\alpha}{(1+\alpha)^2}v^*(\alpha-1) = \frac{2\alpha^2 v^*}{(1+\alpha)^2} > 0.$$

Since $S(v,\alpha) \ge 0$ on region (ii), it follows that the overall expected surplus is strictly positive. □