

Optimal Capital Structure with Stock Market Feedback*

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Abstract

This article studies optimal capital structure when firms learn from financial markets. We present a tractable model of stock market feedback with imperfect information aggregation. Debt issuance affects speculators' incentives to trade both directly, by changing the payoff structure of equity holders, and indirectly, through an asset substitution effect. We show that issuing debt can increase market informativeness and firm value, and may eliminate a coordination failure equilibrium with no provision of market information. We derive the optimal capital structure in this setting and present novel empirical predictions regarding the relationship between market frictions, market informativeness, and capital structure. Once the effect of debt on market informativeness is considered, risky debt does not necessarily lead to risk shifting.

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1. Introduction

By aggregating the views of a myriad of informed investors, asset prices convey information that guides firm managers in their investment decisions. This forms the basis of a feedback effect whereby prices not only reflect but also affect investment decisions (see [Bond, Edmans, and Goldstein \(2012\)](#) for a review of this literature). Trading frictions (e.g., capital constraints, position limits, or transactions costs), by hindering the incorporation of information into prices, reduce the efficiency gains generated by this feedback effect. In this article, we show how firms can adjust their capital structure in order to overcome such a detrimental effect of frictions.

We start from a tractable model of feedback which builds on [Dow, Goldstein, and Guembel \(2017\)](#), and in which trading in secondary financial markets is subject to frictions (trading costs and wealth constraints). The manager of a publicly traded firm must choose whether or not to undertake a risky investment opportunity.¹ The profitability of investment depends on underlying fundamentals, about which the manager has imprecise information. By looking at stock prices, the manager tries to learn about the return of the firm's investment opportunity. In the secondary financial market, a continuum of informed speculators receive private signals about fundamentals and may place orders to buy or sell the firm's equity claims (stocks). When deciding whether or not (and how much) to trade, speculators must form expectations about the behavior of other speculators in the market, and about the managerial reaction to stock prices.

Trading costs may lead speculators to trade less aggressively, and as a result, limit the incorporation of their information into prices. In such cases, we say that information aggregation is reduced. The equilibrium in the financial market often features an inefficiently low level of information aggregation. Moreover, the feedback effect may generate strategic complementarities in the trading behavior of speculators, potentially resulting in multiple equilibria, including one where a coordination failure shuts down the market provision of information. Importantly, equilibrium outcomes in the financial market depend on the firm's capital structure.

We then study what is the optimal security issuance for the firm in a previous stage, focusing at first on the equilibrium with the highest market provision of information (the best equilibrium in the firm's perspective). When setting its capital structure, the firm considers the impact it will have on market participants' incentives to trade and on the manager's incentives to invest, which are intertwined. How the firm's cash flow is divided between debt and equity claims affects speculators' incentives to trade the firm's stocks in secondary markets through two channels. First, raising debt has the direct effect of reducing the safe component of the stock dividend, increasing potential trading profits for informed speculators (up to some point). Second, higher debt indirectly boosts trading incentives since it increases managerial risk taking (an asset substitution effect), rendering the information traders have about the investment profitability more valuable. The latter effect is only present if debt is risky, in which case the firm faces a tradeoff between obtaining more information from the market and using market information more efficiently.

The firm can sometimes achieve the first best with the optimal capital structure: By issuing an appropriate amount of safe debt, the firm may obtain maximum information

1 The risky opportunity may be, for instance, launching a new product, adopting a new production technology, expanding into a new market, a merger or acquisition of another firm, etc.

aggregation without causing any distortion to the managerial use of information. This is the case when there are little market frictions preventing informed trading. When the first best is not achievable, the firm may still benefit from issuing debt. In this case, the optimal capital structure involves the minimum amount of risky debt that leads to maximum information aggregation. The reason is that any marginal gain in informativeness induced by higher risky debt more than compensates the distortion to the ex post use of information by the manager. If the informativeness of stock markets were kept unchanged following an increase in risky debt, the asset substitution effect (the risk-shifting behavior of the manager) would increase the probability of bad projects being undertaken, reducing firm value. However, once one takes into account the increase in market informativeness implied by the increase in debt, the probability of a bad project being undertaken does not actually increase, while the probability of a good project being undertaken rises—thus increasing firm value.

Next, we focus on the equilibrium with the lowest market informativeness, in which coordination failures among speculators may completely shut down the market provision of information, and characterize the optimal capital structure in that case. We show that the choice of an appropriate capital structure often eliminates the coordination failure equilibrium. If the prospects of a firm's investment opportunities are not so good ex ante, even speculators observing good news about that investment project might refrain from trading the firm's stock, anticipating that the firm is unlikely to invest. For an all-equity financed firm, an inefficient equilibrium arises where there is no informed trading and no learning from the market. By issuing enough debt, the firm can switch to a unique equilibrium with positive information transmission and higher firm value.

Our results on the optimal capital structure can help explain why firms issue risky debt despite it potentially leading to risk shifting: By boosting information aggregation in financial markets, risky debt can induce managers to make better decisions, even if it creates an agency conflict. Taking as given the information available to managers, the issuance of risky debt should lead to worse decision making within the firm, inducing excessive risk taking. But since risk taking encourages information production in financial markets, risky debt may end up improving managerial decisions.

In an extension, we also solve the problem of a firm that jointly determines its capital structure and commits to an investment policy before the observation of market activity—for instance, by offering certain managerial compensation contracts. We show that the solution to this problem can be achieved by simply inducing the manager to maximize equity value. An all-equity financed firm would like to deviate from equity value maximization, and would face a time-inconsistency problem in its investment decisions. However, under the optimal capital structure, this time-inconsistency problem disappears. There is no longer scope for increasing firm value by making the manager deviate from equity value maximization. Therefore, debt issuance is a useful instrument to boost information aggregation even if firms have other instruments available.

We also discuss several additional extensions to our setting. First, although in the main model we assume that the firm can only issue debt and equity securities, we show that the optimal capital structure derived is the solution to a more general security issuance problem. Second, we derive the optimal capital structure under an alternative specification of trading frictions (convex trading costs). Third, we show that our results extend to a setting where debt is allowed to be publicly traded. Fourth, we consider the possibility of debt overhang.

The model yields a number of empirical predictions. First, for firms with good investment (growth) opportunities, our model suggests that we should observe a positive correlation between firms' debt and common measures of informed trading in firms' stocks. The reason is two-fold: (i) debt boosts incentives for informed trading, and (ii) in markets where there is high potential for informed trading, firms have incentives to issue more debt. Second, in the presence of market frictions that increase trading costs, high-growth firms should issue more debt to counterbalance speculators' reduced incentives to trade. Third, short-selling restrictions should induce those firms to reduce debt. For firms with low growth potential, we show that the aforementioned relationships can be non-monotonic, as a consequence of local strategic complementarities in trading. Finally, one should expect firms with lower debt to rely more on managerial contracts that induce more risk taking (for instance, option-based compensation).

1.1 Related Literature

Empirical studies have suggested that activity in financial markets affects real decisions in different contexts (Luo, 2005; Chen, Goldstein, and Jiang, 2006; Bakke and Whited, 2010; Edmans, Goldstein, and Jiang, 2012; Phillips and Sertsios, 2017; Edmans, Jayaraman, and Schneemeier, 2017; Jayaraman and Wu, 2020). Motivated by this evidence, a growing body of theoretical work has been studying how information contained in market activity affects real decisions, what is often referred to as the *feedback effect*. Contributions on the topic include Dow and Gorton (1997); Goldstein and Guembel (2008); Bond and Eraslan (2010); Bond and Goldstein (2015); Edmans, Goldstein, and Jiang (2015); and Dow, Goldstein, and Guembel (2017).²

Several papers in this literature have emphasized that firms may undertake different measures to affect their ability to learn from stock prices. Examples of such measures include: the delegation of decisions to a biased agent (Boleslavsky, Kelly, and Taylor, 2017); product-market differentiation (Foucault and Frésard, 2019); managerial compensation (Lin, Liu, and Sun, 2019); and information disclosure (Goldstein and Yang, 2019; Xiong and Yang, 2021). We contribute to the literature by studying how firms' security issuance affects information aggregation in financial markets and their ability to use that information to guide real decisions.

In an independent work, Davis and Gondhi (2021) also point out that by distorting managerial decisions, firms' debt may affect incentives for information production in the presence of feedback from financial markets.³ Their focus is on the implications of agency conflicts that arise from debt on managerial learning, but they do not characterize the optimal issuance of debt and equity securities in the presence of feedback effects. Our results on capital structure then complement Davis and Gondhi (2021). We explain at the end of

2 Information transmission between investors and firm managers may also occur in the context of private equity (see, for instance, Brown and Davies, 2020).

3 In Chang and Yu (2010), debt also affects information acquisition in financial markets. However, differently from Davis and Gondhi (2021) and our article, in their setting debt does not affect managerial incentives to take risk, and information production in financial markets may decrease firm value due to adverse selection.

Section 2.3 further technical differences between our settings that make our model suitable to analyze the optimal capital structure.⁴

Our baseline model builds on [Dow, Goldstein, and Guembel \(2017\)](#). The critical difference is that our setting features both equity and debt claims that create the possibility of managerial risk shifting, and we introduce a previous stage in which firms optimally issue those. As in [Dow, Goldstein, and Guembel \(2017\)](#), there may be strategic complementarity or substitutability among speculators in our model. Strategic complementarities in trading are also present in [Goldstein, Ozdenoren, and Yuan \(2013\)](#); [Goldstein, Li, and Yang \(2014\)](#); and [Goldstein and Yang \(2015\)](#), for instance. We show that the mechanism through which firms' capital structure choices increase firm value may go through eliminating such strategic complementarities.

There is an extensive literature studying security design in financial markets subject to different types of frictions and how it affects market informativeness (e.g., [Allen and Gale, 1988](#); [Boot and Thakor, 1993](#); [Fulghieri and Lukin, 2001](#)). Here, we also look at a security design problem by studying the optimal balance between debt and equity. However, we abstract from typical adverse selection frictions in that literature and instead focus on managerial learning from financial markets. The key assumption here is that the firm wishes to obtain some information that market participants possess, and not the other way around.

More broadly, the article also relates to the extensive literature on how firm's capital structure affects firm value ([Modigliani and Miller, 1958](#); [Jensen and Meckling, 1976](#); [Myers and Majluf, 1984](#); [Harris and Raviv, 1990](#); and many others), as well as the market microstructure literature ([Kyle, 1985](#); [Glosten and Milgrom, 1985](#)).

The remainder of the article is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium and discusses the optimal issuance of securities in the presence of stock market feedback. Section 4 discusses several extensions. Section 5 summarizes the model's empirical predictions and Section 6 concludes. All proofs are relegated to the [Appendix](#).

2. The Model

There are three dates. At $t = 0$ (security issuance stage), a firm issues debt and equity claims to investors in a primary market. At $t = 1$ (trading stage), speculators receive private signals about the firm's fundamentals and trade the firm's equity claims (stocks) in a secondary financial market.⁵ At $t = 2$ (investment stage), a manager takes an action that affects the firm's cash flow (an investment decision), after observing prices and quantities traded. This generates the potential for feedback from financial markets to managerial decisions. The manager seeks to maximize equity value. Once all decisions have been made, the returns of the firm's projects realize and claim holders are paid. All agents in the model are risk-neutral.

4 In short, unlike this article, [Davis and Gondhi \(2021\)](#) assume that traders face a limit on the *number* of shares they can trade. However, for the specific objective of this article (namely, characterizing the optimal capital structure with feedback effects), such assumption would lead to misleading results.

5 For simplicity of exposition, we assume in the main text that only equity claims (stocks) are publicly traded. [Online Appendix B.4](#) allows for publicly traded bonds as well, and shows that our results extend.

2.1 Firm Value and Investment Decision

The value of the firm at the final period (cash flow) depends on a state variable $\theta \in \{\theta_L, \theta_H\}$ and on a managerial decision $a \in \{0, 1\}$, and is denoted by $v(\theta, a)$. Ex ante, each state θ is equally likely. The manager does not observe the realization of the state, but updates beliefs about it after observing activity in financial markets. If the manager chooses action 0, then the firm's cash flow is $v(\theta, 0) = V_0$. If action 1 is chosen, then firm value is state-contingent and is given by

$$v(\theta, 1) = \begin{cases} V_H & \text{if } \theta = \theta_H, \\ V_L & \text{if } \theta = \theta_L, \end{cases}$$

where $V_H > V_0 > V_L$. This implies that action 1 increases firm value only if the state is high ($\theta = \theta_H$). Action a affects the asset side of the firm's balance sheet. One can interpret action $a=0$ as choosing a safe project, or refraining from undertaking a risky investment opportunity. Action $a=1$ represents investment in a risky project. This action could represent the adoption of a new technology, launching a new product, a merger or acquisition, etc.

2.2 Security Issuance

At date $t=0$, an incumbent shareholder owns the firm. The incumbent shareholder issues debt and equity securities that are sold in a primary market at fair value at $t=0$, before the realization of the state θ . Therefore, when selling those securities, she obtains the expected firm value $\mathbb{E}_0[v(\theta, a)]$ —which possibly depends on the chosen capital structure for reasons to be discussed.⁶ We can summarize the incumbent shareholder's decision as the choice of the face value of debt $D \geq 0$ that maximizes expected firm value. Without loss of generality, we can normalize the quantity of equity claims issued (shares) to one, so that the dividend in the final period captures the whole equity value at $t=2$. Speculators are protected by limited liability and therefore stock dividends in the final period are

$$R(\theta, a) \equiv \max\{v(\theta, a) - D, 0\}. \quad (1)$$

To ease the exposition, we limit the securities available to debt and equity, but this is not critical for our results, as discussed in Section 4.2.c and [Online Appendix B.3](#).

2.3 Financial Market

The financial market consists of a continuum of informed speculators indexed by $i \in [0, \alpha]$, where $\alpha > 0$, liquidity (noise) traders, and a competitive market maker. At date $t=1$, each informed speculator can submit an order of up to one dollar to buy or (short) sell the firm's shares. These limits are justified by wealth constraints and short sale margin requirements, for instance. Formally, speculator i 's position in the stock market (in dollar amount) is denoted by $s_i \in [-1, 1]$.⁷ Speculators incur a trading cost $\tau > 0$ per dollar traded in stocks. One can interpret this cost as the marginal operating cost incurred by the competitive

6 Accordingly, the total amounts obtained by selling debt and equity securities at fair price at $t=0$ are $\mathbb{E}_0[\min\{D, v(\theta, a)\}]$ and $\mathbb{E}_0[\max\{v(\theta, a) - D, 0\}]$, respectively, which sum up to $\mathbb{E}_0[v(\theta, a)]$.

7 The specific assumption is that speculators have access to one dollar and face a margin requirement of 100% to short sell. The size of these limits is irrelevant: the results of the article remain unchanged if speculators can place buy orders of up to $\bar{K}$ and sell orders of up to $\underline{K}$ dollars, for any $\bar{K}, \underline{K} > 0$. What is important is that speculators cannot take unlimited positions.

market maker (which can be mapped, for instance, in the bid-ask spread or brokerage fees). Alternatively, one can interpret τ as the opportunity cost of trading the firms' stock, since for a wealth-constrained speculator, more trading in a security implies less trading in another security (in which case τ would represent the expected rate of return of the outside option). Finally, as further discussed at the end of this section, we can interpret τ as an information acquisition cost.

Before trading takes place, each speculator i receives a private signal $m_i \in \{m_L, m_H\}$ about the state, where $\Pr(m_i = m_H | \theta = \theta_H) = \Pr(m_i = m_L | \theta = \theta_L) = \lambda > 0.5$. Conditional on the realization of the state, signals are independently distributed across speculators. Each speculator i chooses an order $s_i \in [-1, 1]$ to maximize her expected trading profit

$$s_i \mathbb{E} \left[\frac{R(\theta, a)}{P} - 1 \middle| m_i \right] - \tau |s_i|, \quad (2)$$

where P denotes the stock price (to be determined endogenously). We define $R_H \equiv R(\theta_H, 1)$, $R_L \equiv R(\theta_L, 1)$, $R_0 \equiv R(\theta, 0)$, and $\Delta_R \equiv R_H - R_L$.

Besides the informed speculators, there are noise traders, who trade for exogenous reasons (to accommodate liquidity shocks, for instance). Noise traders submit an aggregate order of $\tilde{n}$ dollars for the firm's shares, where $\tilde{n}$ is drawn from a normal distribution with zero mean and variance σ^2 . The total order flow in the stock market, in dollars, is then

$$\tilde{X} = \tilde{n} + \int_0^x s_i di.$$

As in Kyle (1985), the market maker is competitive: after observing the total order flow, she meets the order at a price that equals the firm's expected cash flow.⁸ That is, $P = \mathbb{E}[R(\theta, a) | \tilde{X}]$. Naturally, the market maker rationally anticipates the managerial reaction to market activity when setting the price.

Throughout the article, we make the following parametric assumption:

Assumption 1. *Trading costs are sufficiently small, $\tau < 2\lambda - 1$.*

If Assumption 1 were violated, the equilibrium would trivially feature no informed trading and no information aggregation in financial markets. Table I summarizes the timing of events.

We conclude the presentation of the model with some remarks on our assumptions.

Remark 1. Our model is isomorphic to a model where traders need to pay a cost τ to obtain the signal m_i , but then face no trading costs after that, as in Dow, Goldstein, and Guembel (2017). This is because, as will be shown in the solution of the model, expected trading profits are equal for agents receiving good and bad news, and therefore the timing of the payment of the cost τ is not relevant. Hence, one can also think of τ as capturing an information acquisition cost.

Remark 2. Most of the literature on stock market feedback that assumes risk-neutral speculators imposes a limit on the *number of shares* that speculators can trade (e.g., Dow, Goldstein, and Guembel, 2017; Davis and Gondhi, 2021). We instead assume that there is

⁸ As usual in the market microstructure literature, the market maker is able to take any long or short position on the asset to clear the market.

Table I. Timeline of events

$t = 0$: Security issuance	$t = 1$: Information and trade	$t = 2$: Learning and investment
• Firm issues debt and equity securities	• Informed speculators receive signals about the state • Orders are placed • Market maker sets price and meets orders	• Manager observes market activity and decides on investment • Cash flow is realized • Claim holders are paid

a limit on the *dollar amount* that can be traded. These alternative assumptions are in general analogous, but in the particular case of this article the choice for the former would not be appropriate—it would lack a proper economic meaning. Since we use the model to study the effects of debt issuance (which affects the value of equity outstanding), assuming the limit on the number of shares would lead to misleading results if we understand that limit as arising from wealth constraints and short sale margin requirements. For instance, an increase in a firm’s debt decreases the value of equity outstanding, all else equal. If we were to impose a limit on the number of shares, by increasing the firm’s debt we would be effectively tightening the wealth constraint of speculators in terms of dollars. That is, we would mechanically tie the speculators’ wealth constraint to the firm’s capital structure in an *ad hoc* manner.

Hence, our model features two key departures from [Dow, Goldstein, and Guembel \(2017\)](#): the trading limits in dollars and the possibility of firms (optimally) issuing debt, which together with the assumption that managers act on the interest of equity holders, may create a risk-shifting problem.

2.4 Equilibrium Definition

The equilibrium concept we adopt is Perfect Bayesian Equilibrium, which in our baseline model can be defined as follows.

Definition 1. *An equilibrium consists of a debt issuance policy $D \geq 0$; an investment policy $a(\tilde{X}) : \mathbb{R} \rightarrow \{0, 1\}$; a price function $P(\tilde{X}) : \mathbb{R} \rightarrow \mathbb{R}_+$; trading strategies $s_i(m_i) : \{m_L, m_H\} \rightarrow [-1, 1]$; beliefs $\mu(\tilde{X}) : \mathbb{R} \rightarrow [0, 1]$ for the market maker and the firm, and $\eta(m_i) : \{m_L, m_H\} \rightarrow [0, 1]$ for speculators (specifying the probabilities assigned to $\theta = \theta_H$), such that:*

- i. The debt issuance D maximizes expected firm value at $t = 0$, given the price function, trading strategies for speculators, and the investment policy of the manager.
- ii. Trading strategies for speculators maximize trading profits, given D , the price function, the investment policy of the manager, other traders’ strategies, and their private signals.
- iii. The investment policy of the manager maximizes equity value, given the information in the order flow $\tilde{X}$ and all other strategies.
- iv. The price function is such that the market maker breaks even in expectation, given the information in the order flow and all other strategies.
- v. Beliefs $\mu(\tilde{X})$ and $\eta(m_i)$ are consistent with Bayes’ rule.

Notice that we condition the manager's investment decision only on aggregate orders and not on prices. This is without loss of generality, since once the aggregate order is observed, the manager obtains no additional information from the price.

3. Equilibrium Characterization

We proceed by solving the model backwards. First, we fix some debt level $D \geq 0$ and characterize the equilibrium in the trading and investment stages of the game. Then, we study the firms' optimal choice of D in period $t = 0$.

3.1 Equilibrium in the Trading and Investment Stages

3.1.a. Beliefs

Speculators' beliefs are given by Bayes' rule: $\eta(m_H) = \lambda$ and $\eta(m_L) = 1 - \lambda$. We now construct the manager's and market maker's beliefs. Fix trading strategies $\{s_i(m_H), s_i(m_L)\}_{i \in [0,1]}$ for speculators after observing high or low signals. Denote by $S_H \equiv \int_0^x [\lambda s_i(m_H) + (1 - \lambda)s_i(m_L)] di$ the aggregate order of informed speculators when the state is high, and by $S_L \equiv \int_0^x [\lambda s_i(m_L) + (1 - \lambda)s_i(m_H)] di$ their aggregate order when the state is low. The aggregate order flow is then $\tilde{X} = \tilde{n} + S_H$ if $\theta = \theta_H$, and $\tilde{X} = \tilde{n} + S_L$ if $\theta = \theta_L$. Using Bayes' rule, the posterior probability assigned to the high state by the market maker (and the manager) when observing aggregate order $\tilde{X}$ can be written (with some abuse of notation) as

$$\mu(X) = \frac{1}{1 + \exp\{-X\}}, \quad (3)$$

where

$$X = \frac{1}{2\sigma^2} (S_H - S_L) [(\tilde{X} - S_H) + (\tilde{X} - S_L)]. \quad (4)$$

Given speculators' strategies, X is a sufficient statistic to update beliefs. Throughout the text we may refer to X as the "aggregate order," since it provides a summary of market activity. To save on notation, we may hereafter write functions of $\tilde{X}$ as functions of X instead—as we did for $\mu(\cdot)$ in Equation (3).

3.1.b. Investment decision

After observing market activity, the manager chooses whether or not to undertake the risky action ($a = 1$) to maximize equity value. Action $a = 1$ is optimal whenever

$$\mu(X)R_H + [1 - \mu(X)]R_L \geq R_0.$$

Since $\mu(X)$ is increasing, the manager uses a cutoff strategy: to invest if and only if $X \geq \bar{X}$, for some cutoff $\bar{X} \in [-\infty, \infty]$.⁹ The choice of debt D determines the cutoff $\bar{X}(D)$ adopted by the manager—recall that the dividends R_H , R_L , and R_0 depend on the level of debt, as shown in Equation (1). We often omit the argument D in $\bar{X}(D)$ to ease notation.

9 We adopt the tie-breaking convention that the manager chooses $a = 1$ when indifferent. Notice that always investing or never investing can also be seen as cutoff strategies, as they may be represented on the extended real line by $\bar{X} = -\infty$ and $\bar{X} = \infty$, respectively.

Whenever the manager observes a sufficiently large aggregate order X , she assigns a sufficiently high probability $\mu(X)$ to the high state, and consequently undertakes the risky project. Using the expression for dividends in Equation (1), it is easy to see that if $D \geq V_0$, the risky action is always undertaken ($\bar{X} = -\infty$). If $D \in (V_L, V_0)$, investment is undertaken if and only if

$$X \geq \ln\left(\frac{V_0 - D}{V_H - V_0}\right) \equiv X^*(D). \quad (5)$$

If $D \leq V_L$, investment is undertaken if and only if

$$X \geq \ln\left(\frac{V_0 - V_L}{V_H - V_0}\right) \equiv \bar{X}^*. \quad (6)$$

We refer to $\bar{X}^*$ as the *ex post* efficient investment strategy, since investing only when $X \geq \bar{X}^*$ is the decision rule that leads to maximum cash flow at $t=2$, for a *given* information set. However, that is not necessarily the investment rule that leads to maximum firm value *ex ante*, since, as will become clear, the amount of information obtained to guide investment decisions is endogenous to the investment rule.

Let $A \subseteq \mathbb{R}$ denote the set of aggregate orders X that lead the manager to invest. Since debt D pins down the manager's investment cutoff, in what follows, we characterize the equilibrium in the trading stage for a given $A = [\bar{X}(D), \infty)$. We postpone the discussion of the effects of changes in the debt level on the managerial action until Section 3.2, so we can focus here on the equilibrium characterization for a given D .

3.1.c. Prices

The market maker observes aggregate orders in financial markets and sets prices to reflect the expected value of the stock. We can then write

$$P(X) = \begin{cases} \mu(X)R_H + [1 - \mu(X)]R_L & \text{if } X \in A, \\ R_0 & \text{otherwise.} \end{cases} \quad (7)$$

If the market maker observes orders such that $X \in A$, she anticipates that the manager will undertake the risky action, in which case the price reflects the market maker's beliefs about the state. If instead $X \notin A$, the market maker anticipates that the manager will choose $a=0$, and there is no uncertainty about dividends.

3.1.d. Market informativeness

We define the variable κ , which is key for the equilibrium characterization:

$$\kappa \equiv \frac{1}{2\sigma^2}(S_H - S_L)^2. \quad (8)$$

One can think of κ as a measure of how aggressively speculators trade on their information. To see that, note that if speculators place the same orders regardless of the signals they observe, $\kappa=0$. If they adopt very different trading strategies when optimistic or pessimistic about the state, this is reflected in very different aggregate orders across high and low states, leading to a high κ . Moreover, $\kappa \rightarrow 0$ as the market becomes arbitrarily noisy ($\sigma \rightarrow \infty$), and $\kappa \rightarrow \infty$ if noise trading vanishes ($\sigma \rightarrow 0$) and $S_H \neq S_L$ (perfect information aggregation). Notice that $\kappa \in [0, \bar{\kappa}]$, where

$$\bar{\kappa} \equiv \frac{2x^2(2\lambda - 1)^2}{\sigma^2}. \quad (9)$$

If speculators' strategies when receiving good and bad signals are as different as possible—say, if everyone that receives a good (bad) signal buys (sells) as much as possible—we have that $\kappa = \bar{\kappa}$. We refer to κ as a measure of *market informativeness*, and $\bar{\kappa}$ represents the maximum possible level of information aggregation. In [Appendix A.5](#), we show that κ is a measure of informativeness of market activity in the sense of [Blackwell \(1951\)](#).

3.1.e. Trading profits

We now characterize trading profits, which only depend on other speculators' strategies through κ . Denote the standard normal density and cumulative distribution functions by $\phi(\cdot)$ and $\Phi(\cdot)$, respectively.

Lemma 1. *Suppose the manager undertakes investment if and only if $X \in A$, where A is any measurable subset of $\mathbb{R}$. Then, a speculator's trading profit disregarding trading costs only depends on other speculators' strategies through κ and is given by*

$$\Pi(s, m, \kappa) = s[2\eta(m) - 1] \int_A \left[\frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \right] \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du. \quad (10)$$

Since $\eta(m_H) = \lambda > 1/2 > 1 - \lambda = \eta(m_L)$, it follows from Lemma 1 that a speculator receiving a positive (negative) signal will necessarily buy (sell) or not trade the security (if it is not worth it given trading costs). More precisely, a positively informed speculator chooses $s = 1$ if $\Pi(1, m_H, \kappa) > \tau$, $s = 0$ if $\Pi(1, m_H, \kappa) < \tau$, and is indifferent between any $s \in [0, 1]$ if $\Pi(1, m_H, \kappa) = \tau$. Analogously, a negatively informed speculator chooses $s = -1$ if $\Pi(-1, m_L, \kappa) > \tau$, $s = 0$ if $\Pi(-1, m_L, \kappa) < \tau$, and any $s \in [-1, 0]$ if $\Pi(-1, m_L, \kappa) = \tau$.

Moreover, the broader the set of states where the manager is expected to undertake investment, the higher the speculators' incentives to trade. This is captured by the integration limit A in [Equation \(10\)](#). As the price rule in [Equation \(7\)](#) makes clear, speculators do not make any profit in the financial market in states where the firm does not undertake investment, since in those situations the information they have is not relevant to anticipate the firm's cash flow.

Importantly, Lemma 1 implies that each individual speculator best responds to κ . It is useful to define the expected trading profit *per dollar* traded, disregarding trading costs and taking into account that optimistic traders never sell and pessimistic traders never buy. Trading profits per dollar spent in buying (selling) shares for a speculator receiving a good (bad) signal are given by

$$\pi(\kappa) = (2\lambda - 1) \int_A \frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du. \quad (11)$$

3.1.f. Strategic effects in trading

The next lemma highlights some properties of the trading profit function $\pi(\cdot)$ given a cutoff investment strategy. Notice that, if D is such that the manager's investment cutoff satisfies $\bar{X} \leq 0$, she would invest if no additional information were obtained from the market, and if $\bar{X} > 0$, she would not invest under the prior alone.¹⁰

10 Inspecting [Equations \(4\) and \(8\)](#), one can verify that $\kappa = 0$ if and only if $S_H = S_L$ in which case X is zero regardless of the realization of the noise traders' orders. That is, when market activity does not add any relevant information, $X = 0$ with probability 1.

Lemma 2. Fix an investment cutoff $\bar{X}$, so that $A = [\bar{X}, \infty)$. Then, for any $R_H > R_L \geq 0$:

1. If $\bar{X} > 0$ (i.e., the manager would not invest under the prior), $\lim_{\kappa \rightarrow 0} \pi(\kappa) = 0$ and $\pi'(\kappa) > 0$ for low enough κ ;
2. If $\bar{X} \leq 0$ (i.e., the manager would invest under the prior), $\pi(\kappa)$ is strictly decreasing.

Lemma 2 implies that there can be both strategic complementarity and substitutability in speculators' strategies, as usual in models of feedback. There are two strategic effects at play: an *informational advantage effect* and a *feedback effect*. These are known effects in the feedback literature (e.g., see [Dow, Goldstein, and Guembel, 2017](#)) and therefore it should not be surprising that the lemma above holds in our setting. For completeness, we explain those effects below, but a reader familiar with them can skip ahead to Section 3.1.g.

The informational advantage effect is a source of strategic substitutability across speculators. As others trade more on their information, κ increases, market activity becomes more informative, and prices move closer to the real value of securities. This reduction in the informational advantage of informed speculators decreases potential trading profits on its intensive margin, and thus reduces incentives to trade.¹¹

The feedback effect operates through the managerial reaction to market activity. As κ increases, the manager puts more weight on information stemming from the market, and this affects the probability that the risky action is undertaken. Hence, κ also affects trading profits on the extensive margin: conditional on the manager choosing $a = 0$, expected profits are zero; conditional on $a = 1$, expected profits are positive and depend on the size of the informational advantage held by speculators.

The feedback effect may be a source of either strategic substitutability or complementarity across speculators. For instance, consider the case where the manager is ex ante prone to undertaking the risky action ($\bar{X} \leq 0$) and suppose $\kappa = 0$ is expected (no information aggregation). If speculators were to trade more aggressively on information (an increase in κ), the manager would put some weight on information stemming from the market and would be more likely to reverse its prior and not invest. In this situation, a speculator has lower incentives to trade on information if she expects others to trade more aggressively. Hence, both the informational advantage and the feedback effect go in the same direction, explaining why $\pi(\kappa)$ is decreasing in that case.

Now consider the opposite case, where the manager is not prone to undertaking the risky action under the prior ($\bar{X} > 0$), and again, suppose $\kappa = 0$ is expected. An increase in κ would make the manager rely on information gleaned from financial markets to some extent, and invest with some positive probability. In this case, speculators have more incentives to trade if others are trading on their information, and thus there are strategic complementarities. The latter situation may lead to a coordination problem among speculators: if a speculator expects others not to trade and the manager's prior is not favorable to investment, she has no incentives to trade either, and information provision by the market may be shut down altogether.

11 This sort of strategic substitutabilities are common in the literature on financial markets. See, for instance, [Grossman and Stiglitz \(1980\)](#).

3.1.g. Equilibrium in the trading stage

Proposition 1 characterizes equilibrium outcomes in the trading stage for a given investment cutoff $\bar{X}$ (which results from a choice of D in the first stage). We refer to the equilibrium value of informativeness by κ^* .

Proposition 1. *Fix a cutoff investment rule $\bar{X}$ and let $\underline{\tau} \equiv \pi(\bar{\kappa})$ and $\bar{\tau} \equiv \pi(0)$. Then, for any $R_H > R_L \geq 0$:*

1. *If $\bar{X} > 0$, an equilibrium with $\kappa^* = 0$ exists, and if τ is sufficiently low, there is an equilibrium with $\kappa^* > 0$;*
2. *If $\bar{X} \leq 0$, the equilibrium is (essentially) unique, with $\kappa^* = 0$ if $\tau \geq \bar{\tau}$, $\kappa^* \in (0, \bar{\kappa})$ if $\tau \in (\underline{\tau}, \bar{\tau})$, and $\kappa^* = \bar{\kappa}$ if $\tau \leq \underline{\tau}$.¹²*

The feedback effect explains why an equilibrium with zero information aggregation always exists when the manager is not willing to undertake the risky action under the prior ($\bar{X} > 0$). If speculators expect the manager not to invest, they have no incentives to trade. Then, the manager does not obtain any additional information and does not invest, confirming speculators' initial expectations.

Both the feedback effect and the informational advantage effect explain why the equilibrium is unique when the manager is willing to invest under the prior ($\bar{X} \leq 0$). For low κ , speculators know that the manager gives high weight to her own prior and is likely to undertake the risky investment opportunity. Then, trading on information is profitable (investment is likely and the market maker learns little). As more speculators trade on their information (higher κ), the market aggregates information better, pushing prices closer to the real security value and reducing incentives to trade. As is often the case, strategic substitutabilities lead to a unique equilibrium.

Figure 1 illustrates these results. In panel 1(a), consider $\kappa < \kappa^*$. For any such κ , each speculator individually would have incentives to trade as aggressively as possible (with $s_i(m_H) = 1$ and $s_i(m_L) = -1$). But this would lead the market to aggregate information much better, with $\kappa = \bar{\kappa}$, in which case expected trading profits would not compensate the trading cost τ , and speculators would not be willing to trade ($s_i(m_H) = s_i(m_L) = 0$, leading to $\kappa = 0$). The equilibrium is reached at an intermediate level of κ at which speculators are indifferent between any order size as long as $\kappa = \kappa^*$.

Panel 1(b) illustrates the result that $\pi(0) = 0$ when $\bar{X} > 0$. In that case, speculators do not have any incentives to trade on information if they expect others to refrain from trading. A coordination failure equilibrium exists ($\kappa^* = 0$), in which the market fails to aggregate information that would be useful to guide the managerial decision and increase firm value.

Notice that when $\bar{X} > 0$, due to strategic complementarities, there are multiple equilibria. In our main analysis, unless otherwise stated, we assume that Assumption 2 holds.

Assumption 2. *Whenever there are multiple equilibria in the trading stage, the one with the highest market informativeness (κ) is played. We refer to it as the “largest equilibrium.”*

12 An equilibrium with $\kappa^* \in (0, \bar{\kappa})$ can be either symmetric or asymmetric (for instance, a fraction of speculators may trade as much as possible and the rest not trade). We say the equilibrium is *essentially unique* in this case: there are multiple strategy profiles consistent with equilibrium, but market informativeness, stock prices, and firm value are uniquely pinned down.

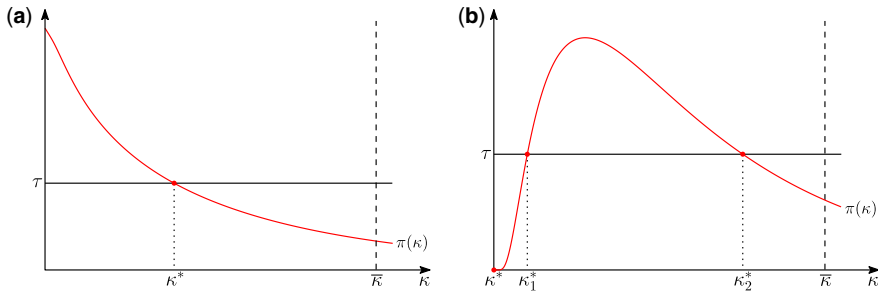


Figure 1. Equilibrium in the trading stage for a given $\bar{X}$. The figure illustrates the computation of equilibrium for different parameter cases, with κ^* , κ_1^* , and κ_2^* representing equilibrium values of κ . (a) Investing is ex ante optimal ($\bar{X} \leq 0$). (b) Investing is not ex ante optimal ($\bar{X} > 0$).

Assumption 2 simplifies the exposition, but we claim that our main insights regarding the effects of debt on information aggregation and firm value are more general. In Section 3.4, this becomes clear as we analyze our model under the opposite assumption, looking at the “smallest equilibrium”—the one with the lowest market informativeness. That analysis also allows us to discuss the potential role of risky debt in avoiding coordination failures in trading.

3.2 Optimal Debt Issuance

We now move on to the choice of the firm regarding the issuance of debt in the first stage ($t=0$) to maximize expected firm value. The firm anticipates that its security issuance will affect both the trading behavior of speculators and managerial decisions (at $t=2$, the manager maximizes equity value). Recall that, for simplicity of exposition, we have assumed that equity is publicly traded while debt is not. In [Online Appendix B.4](#), we show that the optimal capital structure presented here is actually optimal when both securities are (potentially) publicly traded.

Expected firm value at $t=0$ is given by

$$\begin{aligned} \mathcal{V} = & \frac{1}{2} [V_0 + \Pr(X \geq \bar{X}(D) | \theta = \theta_H)(V_H - V_0)] \\ & + \frac{1}{2} [V_0 - \Pr(X \geq \bar{X}(D) | \theta = \theta_L)(V_0 - V_L)], \end{aligned} \quad (12)$$

where those probabilities must be consistent with speculators' equilibrium strategies given D , and $\bar{X}(D)$ denotes the manager's best response to D (see Section 3.1). Debt is chosen at $t=0$ to maximize [Equation \(12\)](#).

We now discuss how debt affects the equilibrium at $t=1$ (trading stage) and $t=2$ (investment stage), highlighting the main tradeoffs regarding debt issuance, and then solve for the optimal choice of debt by the firm at $t=0$.

3.2.a. Debt and direct incentives to trade

Consider a fixed set of orders A for which the manager undertakes investment—this way, we can isolate the direct effect of D on trading profits by shutting down any indirect effects that operate through the managerial decision. Using the expression for dividends in [Equation \(1\)](#), trading profits in [Equation \(11\)](#) become

$$\pi(\kappa) = \begin{cases} (2\lambda - 1) \int_A \frac{\mu(u)(V_H - V_L)}{V_L - D + \mu(u)(V_H - V_L)} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du & \text{if } D < V_L, \\ (2\lambda - 1) \int_A \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du & \text{if } D \in [V_L, V_H), \\ 0 & \text{if } D \geq V_H. \end{cases} \quad (13)$$

If $D \geq V_H$, we have that $R_H = P = 0$ (i.e., debt is so high that the stock is worthless). There is no potential for trading profits, and any arbitrarily small trading cost eliminates all informed trading in stock markets.

For $D < V_L$, a marginal increase in D has a direct positive effect on $\pi(\cdot)$. The reason is the following: Every additional dollar in debt means a dollar less in returns to shareholders regardless of the state (the distance between R_H and R_L does not change), and the expected profit per share traded is not affected. However, the stock price falls, and hence a speculator can purchase more shares with a dollar, making higher trading profits in total.

For $D \in [V_L, V_H)$, direct incentives to trade are not affected by marginal changes in D . The reason is that, since the return in the low state is always zero for D in that range and the return in the high state is decreasing, expected profits per share conditional on investment are decreasing. But as before, a larger D also implies a lower stock price, so more shares can be purchased with a dollar, and these two effects exactly cancel out.

To better understand this cancellation result, notice that the direct incentives to trade stocks are maximized when $D \in [V_L, V_H)$. The intuition is that, in this case, the whole portion of the firm's cash flow that is certain (namely V_L) is removed from dividends and is thus not reflected in stock prices. Therefore, speculators do not have to "waste" part of their limited resources on a safe bet. Every dollar spent by informed speculators in this security is then converted into expected informational profits.¹³

3.2.b. Debt and managerial decisions

We now turn to the effect of debt on the managerial decision. Recall from Section 3.1.b that, for $D \geq V_0$, $\bar{X}(D) = -\infty$, that is, the risky action is always undertaken. For $D \in (V_L, V_0)$, $\bar{X}(D) = X^*(D)$ as given by Equation (5), which is decreasing in D . For $D \leq V_L$, $\bar{X}(D) = \bar{X}^*$.

We then have that safe debt ($D \leq V_L$) does not distort managerial incentives in relation to the ex post efficient decision, which is to invest according to $\bar{X}^*$. Risky debt ($D > V_L$), on the other hand, induces the manager to take more risk, as usual in the corporate finance literature (Jensen and Meckling, 1976). Importantly, in this setting, more risk taking by the manager implies a lower investment cutoff $\bar{X}$ (a broader set A where investment takes place), boosting incentives to trade and potentially increasing information aggregation. However, this distortion implies that whatever information is obtained from the market is used less efficiently ex post: whenever the realized X falls in the region $[\bar{X}(D), \bar{X}^*)$, the

13 If $D < V_L$, dividends are always at least $V_L - D$, and hence this certain component of the dividend is fully priced in by the market maker. In this case, buying a stock is equivalent to buying a portfolio consisting of a risk-free asset that pays $V_L - D$ plus a risky asset. All potential trading profits emerge from the risky portion of this portfolio.

manager invests but it would be ex post efficient from the perspective of the firm that investment were not undertaken. The next section characterizes the optimal capital structure in face of these effects. Note that putting together the optimal managerial decision and Equation (13), we can write trading profits (for a given D) as

$$\pi(\kappa) = \begin{cases} (2\lambda - 1) \int_{\bar{X}}^{\infty} \frac{\mu(u)(V_H - V_L)}{V_L - D + \mu(u)(V_H - V_L)} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du & \text{if } D < V_L, \\ (2\lambda - 1) \left[1 - \Phi\left(\frac{X^*(D) + \kappa}{\sqrt{2\kappa}}\right) \right] & \text{if } D \in [V_L, V_H], \\ 0 & \text{if } D \geq V_H. \end{cases} \quad (14)$$

3.2.c. Optimal debt under feedback effects

The next two propositions characterize the firm's optimal choice of debt at $t=0$, for two different cases of parameters. In Appendix A, we define the variables $\bar{\kappa}$, $\hat{\tau}_1$, $\hat{\tau}_2$, $\hat{\tau}_3$, $\underline{D}$, $\bar{D}$, $\hat{D}$, and $\hat{D}$, to which we refer in propositions that follow. Also, for future reference, we define the first best as the manager's and speculators' decisions that lead to the maximum possible firm value. As shown in Appendix A.2, the first best is achieved when (i) the manager does not distort its decision relative to the ex post efficient cutoff, $\bar{X} = \bar{X}^*$, and (ii) the market aggregates information as well as possible, $\kappa = \bar{\kappa}$.

Proposition 2. Suppose $\bar{X}^* \leq 0$. Then:

1. If $\tau \leq \hat{\tau}_2$, the first best can be achieved with safe debt: the firm chooses any $D \in [\max\{0, \underline{D}\}, V_L]$ and it leads to $\kappa^* = \bar{\kappa}$;
2. If $\tau > \hat{\tau}_2$, the optimal capital structure involves risky debt: $D = \bar{D} > V_L$ is the optimum and it leads to $\kappa^* = \bar{\kappa}$.

Proposition 2 implies that, whenever investment is profitable under the prior, maximum information aggregation is always achieved under the optimal capital structure. If trading costs are low enough, the firm can achieve the first-best outcome by issuing an appropriate amount of safe debt. In particular, for very low τ , the first best would be achieved even if $D=0$, while for τ closer to $\hat{\tau}_2$ the firm must issue enough debt ($D \geq \underline{D}$) to partially remove the safe portion of the firm's cash flow from the price of the stock, enhancing incentives to trade. As long as debt is safe ($D \leq V_L$), managerial incentives to take risk are not affected, and all the information extracted from market activity is efficiently used. Note that, in particular, setting $D = V_L$ is always optimal for $\tau \leq \hat{\tau}_2$.

For higher trading costs, the first best cannot be achieved. In this case, the firm is willing to increase debt beyond the range where there is no distortion to managerial incentives. That is, under the optimal debt, $\bar{X}$ will be strictly smaller than $\bar{X}^*$, meaning that the manager will not make the ex post efficient use of information (see Equation (5)). It turns out that increasing risky debt always pays off while there is scope for increasing information aggregation, despite the managerial distortion (the reason for this is discussed in the next subsection). The firm then issues the minimum amount of risky debt that yields maximum information aggregation ($\bar{D}$). Figure 2 illustrates the results in Proposition 2.

Proposition 3. Suppose $\bar{X}^* > 0$. Then:

1. If $\tau \leq \hat{\tau}_2$, the first best can be achieved with safe debt: the firm chooses any $D \in [\max\{0, \underline{D}\}, V_L]$ and it leads to $\kappa^* = \bar{\kappa}$;

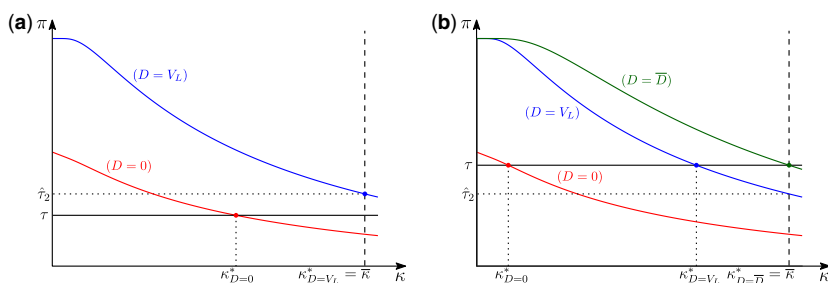


Figure 2. Optimal debt when $\bar{X}^* \leq 0$. For low τ (left panel), setting $D = V_L$ leads to the first best. For high τ (right panel), $D = V_L$ leads to an interior κ^* , and the firm optimally issues risky debt $\bar{D}$ to achieve $\kappa^* = \bar{\kappa}$. (a) Low trading costs ($\tau \leq \hat{\tau}_2$). (b) High trading costs ($\tau > \hat{\tau}_2$).

2. If $\hat{\tau}_2 < \tau < \hat{\tau}_3$, the optimal capital structure involves risky debt: $D = \bar{D} > V_L$ is the optimum and it leads to $\kappa^* = \bar{\kappa}$;
3. If $\tau > \hat{\tau}_3$, the firm chooses any $D < \bar{D} \leq \bar{D}$ and it leads to $\kappa^* = 0$.

Proposition 3 is very similar to Proposition 2, except for the third part. The intuition for Parts 1 and 2 is analogous to the case with $\bar{X}^* < 0$. For high enough trading costs, however, the firm gives up exploring the learning channel when the project is not ex ante profitable ($\bar{X}^* > 0$). To understand why, recall that if the firm receives no additional information ($\kappa = 0$), it would like not to invest. In order to induce information aggregation when τ is large, the firm must issue enough risky debt, inducing investment with high probability, which goes against the firms' prior beliefs. Given its negative prior, the expected informational gains of inducing $\kappa > 0$ do not compensate the expected costs in terms of managerial distortion. This contrasts with the case with $\bar{X}^* \leq 0$, in which the firm invests with probability 1 under the prior. In the latter, the firm would always prefer to induce some $\kappa > 0$ instead of acting based solely on the prior ($\kappa = 0$), even if that required inducing the manager to invest with very high probability.

Figure 3 summarizes the results in Propositions 2 and 3 for different sets of parameters. The left panel illustrates the case with ex ante profitable investment opportunities ($\bar{X}^* \leq 0$), and right panel, the case where investment is not ex ante profitable ($\bar{X}^* > 0$). In both cases, for low trading costs, the first best is achieved when the firm issues the right amount of safe debt. For a given $\bar{\kappa}$, as τ increases, the firm starts to optimally issue risky debt to achieve maximum information aggregation. The main difference between the two panels is that, if $\bar{X}^* \leq 0$, it is always optimal to issue enough debt to achieve maximum information aggregation, while for $\bar{X}^* > 0$, if trading costs become too large, the firm no longer has incentives to issue debt, as triggering information aggregation implies too large of a cost in terms of the (ex post) managerial distortion.¹⁴

14 Note from Figure 3 that the shape of the bound $\hat{\tau}_2$ (above which the firm starts to issue risky debt) is different for the cases of $\bar{X}^* \leq 0$ and $\bar{X}^* > 0$. As defined in Appendix A, $\hat{\tau}_2$ is the maximum value of trading cost for which the first best ($\kappa = \bar{\kappa}$ and $\bar{X} = \bar{X}^*$) can be achieved with safe debt $D = V_L$. When $\bar{X}^* > 0$ and $\bar{\kappa}$ is low, trading profits at the first best are increasing in κ . Hence, a marginal increase in $\bar{\kappa}$ makes it easier to sustain the first best and $\hat{\tau}_2$ increases. If instead $\bar{\kappa}$ is large, trading profits at the first best are decreasing in κ , and therefore a marginal increase in $\bar{\kappa}$ makes it harder to sustain the first best, and therefore $\hat{\tau}_2$ decreases.

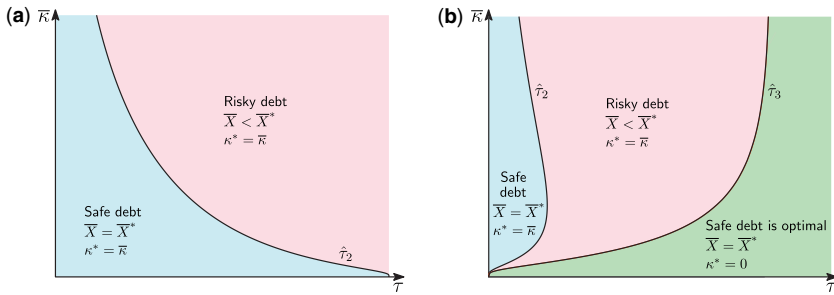


Figure 3. Optimal capital structure. The figure illustrates how the optimal capital structure—and the implied equilibrium values of κ and $\bar{X}$ —depend on τ and $\bar{\kappa}$, for a given λ . (a) Investing is ex ante profitable ($\bar{X}^* \leq 0$). (b) Investing is not ex ante profitable ($\bar{X}^* > 0$).

To shed further light on those results, we discuss next the effect of debt on investment probabilities.

3.3 Risky Debt and the Probability of Investment

It follows from the previous discussion that issuing safe debt never reduces firm value: it has a positive direct effect on speculators' incentives to trade, and it does not distort the use of market information by the manager ex post. Moreover, whenever the firm issues risky debt to achieve a positive level of information aggregation, it has incentives to further increase risky debt until it reaches *maximum* information aggregation—despite the distortion in the use of information. To help understand this result, the next proposition analyzes the effects of risky debt on the probabilities of investment in each state.

Proposition 4. *Suppose the firm issues an amount of risky debt $D > V_L$ that leads to an equilibrium informativeness $\kappa^* \in (0, \bar{\kappa})$. Then, a marginal increase in D :*

1. *Does not affect the probability of investment being undertaken in the bad state, $\Pr(a = 1 | \theta = \theta_L) = \frac{\tau}{2\lambda - 1}$;*
2. *Increases the probability of investment being undertaken in the good state.*

A marginal increase in risky debt increases incentives for the manager to undertake risk, which for a given level of market informativeness $\kappa > 0$ would lead to a higher probability of investment being undertaken in the bad state. However, precisely because the manager is more willing to undertake risk, speculators have larger incentives to trade and market informativeness increases, which contributes to a *reduction* in the probability of a bad investment being undertaken. In fact, these two effects cancel out: raising risky debt does not effectively lead to more investment in bad states once we take into account the gains in market informativeness.¹⁵

15 Although in a different setting, Proposition 4 is related to one of the findings of a recent paper by Davis and Gondhi (2021). They also find that a higher propensity of the manager to undertake risk does not necessarily translate into higher effective risk-shifting (i.e., higher probability of undertaking negative-NPV projects) in the presence of feedback effects.

It is illustrative to sketch the proof of this result. For any given investment cutoff $\bar{X}$, one can write

$$\Pr(a = 1 | \theta = \theta_L) = \Pr(X > \bar{X} | \theta = \theta_L) = 1 - \Phi\left(\frac{\bar{X} + \kappa^*}{\sqrt{2\kappa^*}}\right), \quad (15)$$

where the last equality follows from the conditional distribution of aggregate orders X (see proof of Lemma 1). Consider some $D > V_L$ that leads to an equilibrium with interior κ^* . In this case, $\bar{X} = X^*(D)$ and speculators are indifferent between trading or not, so trading profits equal trading costs. Using the trading profits in Equation (14), κ^* must satisfy

$$(2\lambda - 1) \left[1 - \Phi\left(\frac{X^*(D) + \kappa^*}{\sqrt{2\kappa^*}}\right) \right] = \tau, \quad (16)$$

which, using Equation (15), implies that in equilibrium $\Pr(a = 1 | \theta = \theta_L) = 1 - \Phi\left(\frac{X^*(D) + \kappa^*}{\sqrt{2\kappa^*}}\right) = \frac{\tau}{2\lambda - 1}$. Note that, all else constant, a marginal increase in D reduces $X^*(D)$, which increases $\Pr(a = 1 | \theta = \theta_L)$ and hence trading profits. But that incentivizes more informed trading until indifference is restored: $\pi(\kappa^*) = \tau$. As a result, trading profits and the probability of investment in the low state do not change in equilibrium.¹⁶

Regarding the probability of investment being undertaken in the good state, both the increase in market informativeness and the increase in incentives for the manager to take risk explain why higher risky debt raises the probability of investment in the good state.

In sum, a marginal increase in risky debt keeps unchanged the probability of investment taking place when it is not (ex post) efficient, and reduces the probability of good investment opportunities being missed. This is why firm value increases as the firm raises risky debt—up to the point where informativeness reaches its maximum, $\kappa^* = \bar{\kappa}$.

3.4 Optimal Debt Issuance in Face of Coordination Failures

In this section, we study how the firm's optimal capital structure changes if we adopt a pessimistic equilibrium selection procedure for the case with $\bar{X}^* > 0$. When $\bar{X}^* \leq 0$, the trading-stage equilibrium is unique, so nothing changes. Throughout the section, we replace Assumption 2 by the following.

Assumption 3. *Whenever there are multiple equilibria in the trading stage, the one with the lowest market informativeness (κ) is played. We refer to it as the “smallest equilibrium.”*

Under Assumption 3, as is clear from Proposition 1, any choice of debt D that leads to an investment cutoff $\bar{X}(D) > 0$ triggers a coordination-failure equilibrium where speculators have no incentives to trade, anticipating that others will not trade and the manager will not invest. Hence, no trading and no investment become a self-fulfilling prophecy.

16 In Online Appendix B.1, we show that with convex trading costs, higher risky debt increases the probability of investment in the bad state since the larger incentives to trade are attenuated by a higher marginal cost of trading. This implies that the gains in informativeness caused by an increase in risky debt will not fully offset the managerial distortion. Still, issuing risky debt can increase firm value, despite some risk shifting, since it also increases the probability of investment in the good state.

Proposition 5. *Suppose $\bar{X}^* > 0$. Then, positive levels of information aggregation are only achieved with risky debt. Moreover:*

1. *For $\bar{\kappa} \geq \tilde{\kappa}$ and $\tau \leq \hat{\tau}_3$, the firm issues the minimum amount of risky debt that leads to $\kappa^* = \bar{\kappa}$: the optimal D is $\hat{D}$ if $\tau \leq \hat{\tau}_1$ and $\bar{D} > \hat{D}$ if $\tau \in (\hat{\tau}_1, \hat{\tau}_3]$;*
2. *For $\bar{\kappa} < \tilde{\kappa}$ or $\tau > \hat{\tau}_3$, any $D < \hat{D}$ is optimal and leads to $\kappa^* = 0$.*

When investment is not optimal under the prior, the optimal capital structure can never lead to the first best under Assumption 3. Still, the firm might increase its value by issuing risky debt optimally at $t=0$, at least when the potential maximum level of informativeness is high enough ($\bar{\kappa} \geq \tilde{\kappa}$) and trading costs are low enough ($\tau \leq \hat{\tau}_3$). Recall that $\bar{\kappa}$ is larger the higher the precision of informed speculators' signals (λ), the larger the mass of informed speculators (α), and the smaller the amount of noise trading (σ)—see Equation (9).

If $\bar{\kappa} \geq \tilde{\kappa}$ and trading costs are sufficiently low ($\tau \leq \hat{\tau}_1$), the firm issues the precise amount of debt that implements a cutoff $\bar{X} = 0$ (namely $\hat{D}$), so that the economy switches from a coordination failure equilibrium (with $\kappa^* = 0$) to an equilibrium with maximum information aggregation, as illustrated in the left panel of Figure 4. Even though the information aggregated in the market is not used in the most efficient manner ex post (since $\bar{X} = 0 < \bar{X}^*$), given that the information revealed is of good enough quality, firm value is maximized.

If $\bar{\kappa} \geq \tilde{\kappa}$ but trading costs are higher ($\tau \in (\hat{\tau}_1, \hat{\tau}_3]$), by choosing the level of debt that implements $\bar{X} = 0$, the firm can no longer achieve the maximum level of information aggregation, even though it would eliminate the coordination failure equilibrium and lead to a positive κ in equilibrium. In this case, it pays off to further increase debt up to $\bar{D} > \hat{D}$ to reach maximum information aggregation as illustrated in the right panel of Figure 4. Despite higher debt further distorting the managerial decision rule ex post, the gain in informativeness always more than compensates the losses of potentially misusing the information obtained—as a consequence of Proposition 4.¹⁷

Finally, if $\bar{\kappa}$ is too low ($\bar{\kappa} < \tilde{\kappa}$) or trading costs are too high ($\tau > \hat{\tau}_3$), the firm opts for a low enough level of debt so that the market continues not to aggregate any information and the firm manager continues to act according to her prior only (foregoing the risky investment opportunity). If debt was too high ($D \geq \hat{D}$), the distortion in the managerial decision rule would be too large relative to the improvement in information aggregation, since the potential for information production is low. This is the situation described in the last part of Proposition 5.

Figure 5 summarizes the results in Proposition 5 for different sets of parameters. In the smallest equilibrium, the firm can never trigger information aggregation with safe debt. If the potential to aggregate information is low ($\bar{\kappa} < \tilde{\kappa}$) or trading costs are high ($\tau > \hat{\tau}_3$), the firm has no incentives to issue debt, as raising enough debt to leave the coordination failure ($\kappa^* = 0$) would be too costly in terms of the managerial distortion. If there is enough potential to aggregate information ($\bar{\kappa} > \tilde{\kappa}$), the firm finds it profitable to issue risky debt to achieve $\kappa^* = \bar{\kappa}$ (as long as τ is low enough).

17 Note that Proposition 4 was proved under Assumption 2. However, since $D \geq \hat{D}$ leads to a unique equilibrium, the equilibrium selection procedure is irrelevant for the argument. Moreover, Proposition 4 also holds under Assumption 3 (as proved in a previous version of this article, see Machado and Pereira, 2021).

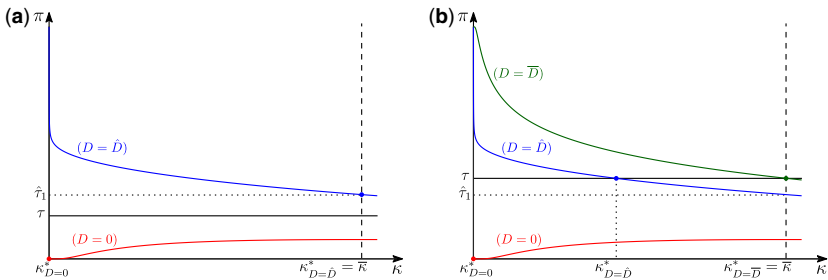


Figure 4. Optimal debt when $\bar{X}^* > 0$ and $\bar{\kappa} > \tilde{\kappa}$. For low τ (left panel), setting $D = \hat{D}$ eliminates the coordination failure and leads to $\kappa^* = \bar{\kappa}$. For higher τ (right panel), $D = \hat{D}$ leads to an interior κ^* , and the firm optimally issues risky debt $\bar{D}$ to achieve $\kappa^* = \bar{\kappa}$. (a) Low trading costs ($\tau < \hat{\tau}_1$). (b) Higher trading costs ($\tau \in (\hat{\tau}_1, \hat{\tau}_3)$).

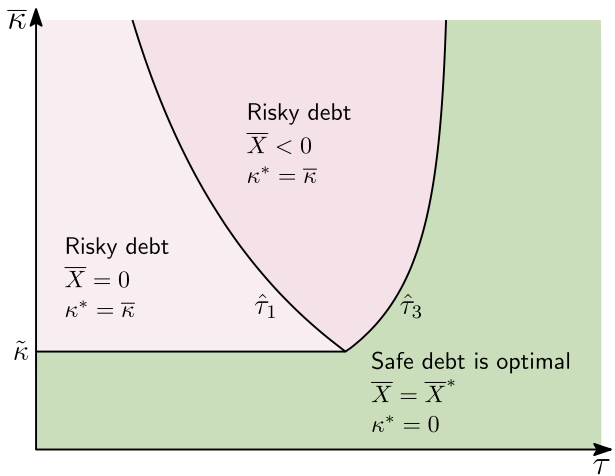


Figure 5. Optimal capital structure when investing is not ex ante profitable ($\bar{X}^* > 0$), under the smallest equilibrium. The figure illustrates how the optimal capital structure—and the implied equilibrium values of κ and $\bar{X}$ —depend on τ and $\bar{\kappa}$, for a given λ .

It is illustrative to compare the optimal capital structure under the two equilibrium selection assumptions (Assumptions 2 and 3) for $\bar{X}^* > 0$. There are two main differences. First, under Assumption 3, the firm always needs to issue risky debt if it wants to induce information aggregation. Under Assumption 2, on the other hand, safe debt suffices to yield $\kappa > 0$ when trading costs are low—and even the first best can be achieved. When coordination failures are possible, risky debt plays a role of not only incentivizing information production on the margin, but also of killing a bad equilibrium with no information provision. Since safe debt does not distort the managerial cutoff $\bar{X}$, it cannot eliminate the local strategic complementarities that arise for low κ and $\bar{X} > 0$ (see also Figure 1). Eliminating the bad equilibrium requires an amount of risky debt large enough to, at least, bring $\bar{X}$ down to zero, making trading profits positive even if no one else trades, since the manager invests under her prior.

Second, under Assumption 2, for every $\bar{\kappa}$ there exists a range of trading costs τ where risky debt is optimal—see the right panel of Figure 3. This is no longer true under Assumption 3. The reason is that, regardless of τ , triggering a positive κ requires a minimum amount of risky debt (the one that makes $\bar{X}(D) = 0$), and it does not pay off for the firm to issue that much debt if the potential for information production ($\bar{\kappa}$) is low.

4. Extensions

4.1 Debt and Investment Policy

When deriving the optimal capital structure, we have assumed that the firm manager maximizes equity value. In this section, we show that this is not key for our results in the following sense: If the firm can adjust its level of debt optimally, there is no reason for it to deviate from equity value maximization.

We consider the possibility of a firm jointly determining its optimal capital structure and investment policy, potentially committing to an investment policy that is not consistent with (ex post) equity value maximization. This could be done, for instance, through the design of an appropriate managerial compensation scheme. As an auxiliary result, we first study what would be the ex ante optimal investment policy, abstracting from security issuance considerations (assuming an exogenous $D \geq 0$).¹⁸

Lemma 3. *Fix an exogenous $D \geq 0$ and suppose that the firm can choose at $t = 0$ any measurable set $A \subseteq \mathbb{R}$ so that investment is undertaken if and only if $X \in A$. There is always a cutoff strategy $\bar{X}_c^*$ that is ex ante optimal. If there are gains from commitment, then $\bar{X}_c^* < \bar{X}^*$.*

When choosing its investment strategy under commitment, the firm potentially faces the following tradeoff: By deviating from the ex post efficient investment strategy, it makes worse use of the information obtained. On the other hand, it may lead to higher information aggregation. Market participants do not internalize the effect of their trading activity on information aggregation and firm value. To increase information aggregation, the firm needs to reward informed speculators when they trade more aggressively and reveal good or bad news. One way to reward speculators for their information provision is by investing in a broader range of states, since speculators only profit when investment is undertaken.

Lemma 3 shows that an ex ante optimal investment policy takes the form of a simple cutoff, possibly featuring investment in states where it is ex post inefficient to invest. Therefore, when looking for an ex ante optimal investment strategy, one can restrict attention to cutoff strategies $\bar{X}_c^*$ such that $\bar{X}_c^* \leq \bar{X}^*$. In Online Appendix B.6, we give further details on the characterization of ex ante optimal investment strategies.¹⁹

18 Two technical highlights are in order. First, it is convenient to adopt the same change of variables as in Section 3 and state the firm's problem in Lemma 3 in terms of X instead of the original aggregate order $\tilde{X}$, which is innocuous, as shown in Appendix A.3. Second, to simplify the exposition, here we allow the firm to condition its investment on orders (X) but not on prices (P). Appendix A.4 shows that this is without loss of generality, as there is always an optimal investment strategy that does not condition on the price when we allow for it.

19 Lemma 3 also holds if one assumes the smallest equilibrium is played, that is, if Assumption 2 is replaced by 3. In a previous version of the article, we prove that result and provide a characterization of ex ante optimal investment policies under Assumption 3 (see Machado and Pereira, 2021).

4.1.a. *The joint choice of capital structure and investment policy*

The idea that committing to invest in a broader range of states can boost information production and firm value has already been raised in the feedback literature (e.g., Boleslavsky, Kelly, and Taylor, 2017; Dow, Goldstein, and Guembel, 2017).²⁰ Here, we argue that by adjusting its capital structure, a firm can fulfill a similar objective, so policies on the liability side can be seen as substitutes for policies on the investment side when it comes to boosting information aggregation and firm value in the presence of stock market feedback.²¹ The next result speaks to that point.

Assume that at $t=0$, the firm can choose both $D \in \mathbb{R}_+$ and a measurable set $A \subseteq \mathbb{R}$ such that it invests if and only if $X \in A$. The next proposition discusses the solution to this more general problem.

Proposition 6. *Consider the problem where the firm can choose its capital structure and investment policy at $t=0$. The optimum can always be achieved with an investment policy consistent with ex post equity value maximization (i.e., $A = [\bar{X}(D), \infty)$ is optimal).*

In other words, when the firm can choose its level of debt optimally, there is no reason for the firm to deviate from equity value maximization (for instance, by designing certain managerial incentives as explained in Section 4.1.b). The intuition for this result is the following. First, the firm can often achieve the first best by simply issuing enough safe debt, boosting incentives to trade and keeping the managerial decision unaffected. If the first best is not achievable and it is optimal for the firm to induce more risk taking by the manager, this can be implemented by issuing an appropriate amount of risky debt—since raising debt beyond V_L does not hamper incentives to trade, but incentivizes investment in more states.

Proposition 6 partially follows from the result that if the firm wants to distort managerial decisions, it wants to do so in a monotonic way, specifying a different cutoff rule for the manager (as in Lemma 3). Therefore, the “optimal distortion” can be achieved with debt. Given that debt also directly boosts incentives to trade, there is no reason for the firm not to use it to achieve its desired outcome.

The corporate finance literature shows that debt can have a positive effect on firm value due to its role as a commitment device for firms in different contexts (see, for instance, Grossman and Hart, 1982; Zwiebel, 1996). As the results in this section indicate, in the context of managerial learning from financial markets, debt also operates as a commitment device—in our case, risky debt implies a commitment to invest more often, which improves managerial learning. This, together with the direct effect of debt on incentives to trade, explains why issuing debt can increase firm value.

We conclude this section with a remark on how to implement different investment policies through managerial compensation schemes.

20 Strobl (2014) makes a similar point, although in a model without managerial learning from the market. There, a firm manager has *perfect* information about its fundamentals. Due to incentives problems, shareholders may find it optimal to offer contracts that incentivize investment by the manager, as it induces market participants to convey information about the manager's hidden action.

21 Additionally, we make a technical contribution in determining that simple cutoff investment rules, often imposed in models of feedback effects, are actually the result of firms' optimization if they are allowed to commit to rather arbitrary investment strategies.

4.1.b. Implementation of time-inconsistent investment policies through managerial compensation

Notice that, whenever $\bar{X}_c^* < \bar{X}^*$, the ex ante optimal investment policy of Lemma 3 is time inconsistent: a decision maker seeking equity value maximization would like to deviate from it after observing market activity (and only invest to the right of $\bar{X}^*$). However, the firm's optimal investment policy under commitment can be easily implemented with a managerial compensation scheme that conditions the manager's payment on the firm's realized cash flow at the final period.

Suppose that, instead of maximizing equity value at $t=2$, a manager makes the investment decision to maximize its expected compensation. Denote the manager's compensation as a function of the firm's final cash flow by $w(v)$. For simplicity, we assume the manager's outside option is $\bar{U} \approx 0$ (her required compensation is arbitrarily low relative to the scale of the firm's projects). Define $w_H \equiv w(V_H)$, $w_L \equiv w(V_L)$, and $w_0 \equiv w(V_0)$, and consider $w_H \geq w_0 \geq w_L$. Given such a compensation scheme, the manager at $t=2$ will choose $a=1$ if and only if

$$X \geq \bar{X}_w = \ln \left(\frac{w_0 - w_L}{w_H - w_0} \right). \quad (17)$$

Note that any cutoff $\bar{X}_w \leq \bar{X}^*$ can be achieved by setting $w(\cdot)$ appropriately at $t=0$. Hence, the firm can implement its ex ante optimal investment policy $\bar{X}_c^*$ with such simple contracts. The cutoff $\bar{X}_w$ is a function of the ratio between the potential downside and upside in the manager's compensation scheme. Therefore, since $\bar{X}_c^* \leq \bar{X}^*$, to implement its commitment solution the firm would set a managerial contract that always incentivizes the manager to be weakly more *aggressive* (prone to investment) than shareholders would be (ex post), with a compensation scheme that offers a relatively lower downside risk (higher upside) than the company's assets, $\frac{w_0 - w_L}{w_H - w_0} \leq \frac{V_0 - V_L}{V_H - V_L}$.

4.2 Additional Extensions

We now briefly discuss some additional extensions, which are presented in more detail in the [Online Appendix](#).

4.2.a. Convex trading costs

One key assumption for our results is that there are frictions hindering information aggregation in financial markets. We parsimoniously capture such frictions by a linear trading cost incurred by traders, together with a wealth constraint that prevents unlimited positions. In [Online Appendix B.1](#), we present an alternative version of our model with convex trading costs (and no exogenous trading limits). The message that, despite the usual risk-shifting channel, risky debt can increase information aggregation and firm value remains, and our results regarding the optimal capital structure are qualitatively similar.

4.2.b. Debt overhang

In [Online Appendix B.2](#), we present an extension where the firm has assets in place whose return is negatively correlated with the returns of the new investment opportunity the firm has available. This way, debt has the potential to lead to a debt-overhang problem, instead of risk shifting, as in [Davis and Gondhi \(2021\)](#). We show that the optimal capital structure is the same in the model with debt overhang.

4.2.c. General security design

In the main text, we assume that the securities the firm can issue are debt and equity claims. However, our results do not depend on that assumption. In [Online Appendix B.3](#), we solve a more general security design problem in which the firm can split its cash flow in any arbitrary way among two securities, as in [Allen and Gale \(1988\)](#), and show that it is optimal for the firm to issue debt and equity. Hence, the optimal capital structure derived is also a solution to this more general security design problem.

4.2.d. Publicly traded debt

[Online Appendix B.4](#) extends the model to allow for debt to be possibly publicly traded, assuming that aggregate trading activity in debt and equity markets are public information. We show that informed investors endogenously opt not to trade bonds, so the assumption that only equity is publicly traded is not crucial for our results.

Moreover, [Online Appendix B.5](#) considers the effect of short-selling restrictions, which is further discussed in the next section.

5. Comparative Statics and Empirical Predictions

We now derive some comparative statics that, together with our previous results, are useful to discuss some of the model's empirical implications.

Proposition 7. *For firms with good investment opportunities ($\bar{X}^* < 0$), the smallest and largest optimal levels of debt are increasing in the mass of informed traders α and in trading costs τ . For firms with bad investment opportunities ($\bar{X}^* > 0$), those relationships can be non-monotonic.*²²

As the proposition above indicates, some comparative statics depend on the potential of investment opportunities available to firms. The corporate finance literature has proposed different measures to proxy for firms' investment (growth) opportunities, such as the standard market-to-book ratio, or sales- and R&D-based measures (see [Goyal, Lehn, and Racic \(2002\)](#) and [Billett, King, and Mauer \(2007\)](#) for a discussion).

5.1 Informed Trading

Our model predicts that larger debt increases speculators' incentives to trade stocks. Therefore, after an exogenous increase in a firm's incentives to issue debt, we should expect an increase in measures of informed trading of their stocks—such as the probability of informed trading (PIN) and price nonsynchronicity (see [Chen, Goldstein, and Jiang, 2006](#)).

Moreover, we can interpret markets with large α as markets in which many traders possess information that is relevant for decisions makers (so there is a large potential for stock market feedback, $\bar{\kappa}$ is large).²³ For firms with good investment opportunities, higher market sophistication (α) raises incentives to issue debt to better explore the market provision of

22 We refer to the smallest and largest optima since for some parameters there are multiple optimal values of D . Another way to state the result is that the optimal D is increasing or decreasing in those parameters in the strong set order sense (see [Topkis, 1978](#); [Vives, 1999](#)).

23 For instance, we should expect α to be larger in industries where demand-side information (such as consumers' tastes) is important to forecast investment profitability, and α to be lower in industries where profitability is mostly determined by technological features mostly known by insiders.

information. Hence, among high-growth firms, one should expect more debt to be issued in markets with more informed traders. For low-growth firms, the same effect is there. However, there is an additional effect: Since there are local strategic complementarities in trading, an increase in α may make it easier to sustain an equilibrium with informed trading. Hence, a higher α may induce those firms to reduce debt, as it is less needed to sustain an informative equilibrium. This explains why the relationship between α and optimal debt can be non-monotonic for firms with bad investment opportunities.

5.2 Market Frictions

In markets with higher trading costs (τ), informed speculators have less incentives to trade, which, all else equal, tends to reduce market informativeness. For moderate trading costs, an increase in τ then incentivizes firms to issue more debt to counterbalance speculators' reduced incentives to trade and fully explore the market potential to aggregate information. This is true as long as trading costs are not so high so that firms give up trying to induce informed trading in their stock, which only happens for firms with bad investment opportunities ($\bar{X}^* > 0$) when $\tau \geq \hat{\tau}_3$ (see Figure 3). Hence, among a set of firms that have informative stock prices and whose real decisions react to market activity, we should observe an unambiguous positive relationship between trading frictions in secondary markets and firms' debt.

5.3 Managerial Compensation

From Proposition 6, under the optimal capital structure, firms have no incentives to induce the manager to depart from equity value maximization. However, if there are other factors that lead a firm to choose debt below the optimum predicted by our model, then such firm should rely more on incentive schemes that encourage risk taking. Hence, we should expect firms with lower debt to rely more on compensation schemes that protect managers from downside risk, inducing more risk taking (for instance, option-based compensation). Moreover, our results suggest that this relationship is stronger when market sophistication or market frictions are large. These predictions complement the analysis of managerial compensation under feedback effects in Lin, Liu, and Sun (2019). Extending their empirical setting to test these effects is a potential venue for future research.

5.4 Short-Selling Restrictions

Our model has implications for the effect of short-selling restrictions on market informativeness and firms' capital structure. In the model, short-selling constraints are isomorphic to a decrease in the maximum potential informativeness $\bar{\kappa}$, since those constraints prevent pessimistic investors with no initial position on the stock from trading. (For a formal treatment of this matter, see Online Appendix B.5). Therefore, short-selling restrictions have the same effect as a decrease in the mass of informed traders α .

For firms with good growth opportunities, we should then expect a decrease in their level of debt following a short-selling ban. For firms with bad investment opportunities, the effect of a short-selling ban on debt is ambiguous. Lin, Liu, and Sun (2019) study empirically the effect of a removal of short-selling restrictions on managerial compensation, exploring a channel of learning from financial markets. Our results suggest that, in the presence of such feedback effects, a removal of short-selling restrictions could also impact firm's funding decisions.

6. Final Remarks

This article studies how firms can issue debt and equity securities to exploit a learning-from-financial-markets channel and maximize firm value. Debt issuance affects information aggregation in financial markets both directly (by changing the payoff structure of equity holders) and indirectly (by affecting managerial incentives to take risks). Despite the usual asset substitution effect, risky debt can increase real efficiency: when the manager is expected to undertake more risk, financial markets aggregate information better, which more than compensates the distortion in managerial decisions. Safe debt, by reducing the safe portion of the firm's dividends, also increases incentives to trade and real efficiency, and its issuance sometimes leads to the first best.

The model presented highlights that the structure of firms' liabilities may have important implications for the real effects of financial markets. Its empirical predictions shed light on the relationship between firms' capital structure, market frictions, and market informativeness. Our theory can help guide future empirical work on the real effects of stock markets.

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Data Availability

No new data were generated or analyzed in support of this research.

Supplementary Material

[Supplementary data](#) are available at *Review of Finance* online.

A Technical Appendix

We define here some variables to which we refer in the main text:

$$\hat{\tau}_1 \equiv (2\lambda - 1) \left[1 - \Phi \left(\sqrt{\frac{\bar{\kappa}}{2}} \right) \right], \quad \hat{\tau}_2 \equiv (2\lambda - 1) \left[1 - \Phi \left(\frac{\bar{X}^* + \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) \right], \quad \hat{D} \equiv 2V_0 - V_H, \quad (\text{A.1})$$

$$\bar{D} = \bar{D}(\tau) \equiv V_0 - (V_H - V_0)e^{\Omega(\tau)}, \quad \text{where} \quad \Omega(\tau) \equiv \sqrt{2\bar{\kappa}}\Phi^{-1} \left(1 - \frac{\tau}{2\lambda - 1} \right) - \bar{\kappa}. \quad (\text{A.2})$$

Let $\hat{\tau}_3$ be implicitly defined by

$$1 - \Phi \left(\frac{X^*(\bar{D}(\hat{\tau}_3)) - \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) = \frac{V_0 - V_L}{V_H - V_0} \left[1 - \Phi \left(\frac{X^*(\bar{D}(\hat{\tau}_3)) + \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) \right], \quad (\text{A.3})$$

$\underline{D}$ by

$$\tau = (2\lambda - 1) \int_{\bar{X}^*}^{\infty} \frac{\mu(u)(V_H - V_L)}{V_L - \underline{D} + \mu(u)(V_H - V_L)} \frac{1}{\sqrt{2\bar{\kappa}}} \phi \left(\frac{u + \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) du, \quad (\text{A.4})$$

and $\tilde{D}$ by

$$\max_{\kappa \in [0, \bar{\kappa}]} (2\lambda - 1) \left[1 - \Phi \left(\frac{X^*(\bar{D}) + \kappa}{\sqrt{2\kappa}} \right) \right] = \tau. \quad (\text{A.5})$$

Finally, we define $\tilde{\kappa} \equiv 2 \left[\Phi^{-1} \left(\frac{V_0 - V_L}{V_H - V_L} \right) \right]^2$. We now present and prove two auxiliary lemmas that will be used in subsequent proofs.

Lemma A.1. *Let $\pi(\kappa)$ be as given by Equation (11). Then,*

$$\lim_{\kappa \rightarrow 0} \pi(\kappa) = \begin{cases} \frac{\Delta_R}{R_L + R_H} (2\lambda - 1) & \text{if } 0 \in A, \\ 0 & \text{if } 0 \notin A. \end{cases}$$

Proof. Using Equation (11), we can write

$$\pi(\kappa) = (2\lambda - 1) e^{-\frac{\kappa}{4}} \int_A \frac{\mu(u) \Delta_R}{R_L + \mu(u) \Delta_R} e^{-\frac{u}{\sqrt{2\kappa}}} \frac{1}{\sqrt{2\kappa}} \phi \left(\frac{u}{\sqrt{2\kappa}} \right) du.$$

Letting $\delta(u)$ be the Dirac delta function, we have that

$$\lim_{\kappa \rightarrow 0} e^{-\frac{\kappa}{4}} \int_A \frac{\mu(u) \Delta_R}{R_L + \mu(u) \Delta_R} e^{-\frac{u}{\sqrt{2\kappa}}} \frac{1}{\sqrt{2\kappa}} \phi \left(\frac{u}{\sqrt{2\kappa}} \right) du = \int_A \frac{\mu(u) \Delta_R}{R_L + \mu(u) \Delta_R} e^{-\frac{u}{\sqrt{2\kappa}}} \delta(u) du,$$

which equals $\frac{\mu(0) \Delta_R}{R_L + \mu(0) \Delta_R}$ if $0 \in A$ and 0 otherwise. Using the fact that $\mu(0) = 1/2$ yields the desired result. $\square$

Lemma A.2. *Suppose the firm plays a cutoff strategy $\bar{X}$ and let $\mathcal{V}(\bar{X}, \kappa)$ denote the ex ante firm value for a given $\bar{X}$ and $\kappa > 0$. Then,*

$$\text{sign} \left(\frac{\partial \mathcal{V}(\bar{X}, \kappa)}{\partial \kappa} \right) = \text{sign} \left(\bar{X} (e^{\bar{X}} - e^{\bar{X}^*}) + \kappa (e^{\bar{X}} + e^{\bar{X}^*}) \right), \quad (\text{A.6})$$

$$\text{sign} \left(\frac{\partial \mathcal{V}(\bar{X}, \kappa)}{\partial \bar{X}} \right) = \text{sign}(\bar{X}^* - \bar{X}). \quad (\text{A.7})$$

Proof. If the manager adopts a cutoff investment strategy $\bar{X}$, we can write the ex ante value of the firm as

$$\mathcal{V} = \frac{1}{2} \left[V_H - \Phi \left(\frac{\bar{X} - \kappa}{\sqrt{2\kappa}} \right) (V_H - V_0) \right] + \frac{1}{2} \left[V_L + \Phi \left(\frac{\bar{X} + \kappa}{\sqrt{2\kappa}} \right) (V_0 - V_L) \right]. \quad (\text{A.8})$$

(See the proof of Lemma 1 for the distribution of X conditional on each state). Therefore,

$$\frac{\partial \mathcal{V}}{\partial \kappa} = \frac{1}{2} \phi \left(\frac{\bar{X} - \kappa}{\sqrt{2\kappa}} \right) (V_H - V_0) \left(\frac{\bar{X} + \kappa}{2^{3/2} \kappa^{3/2}} \right) - \frac{1}{2} \phi \left(\frac{\bar{X} + \kappa}{\sqrt{2\kappa}} \right) (V_0 - V_L) \left(\frac{\bar{X} - \kappa}{2^{3/2} \kappa^{3/2}} \right), \quad (\text{A.9})$$

which has the same sign as

$$\phi \left(\frac{\bar{X} - \kappa}{\sqrt{2\kappa}} \right) (V_H - V_0) (\bar{X} + \kappa) - \phi \left(\frac{\bar{X} + \kappa}{\sqrt{2\kappa}} \right) (V_0 - V_L) (\bar{X} - \kappa).$$

Since $\phi \left(\frac{\bar{X} - \kappa}{\sqrt{2\kappa}} \right) / \phi \left(\frac{\bar{X} + \kappa}{\sqrt{2\kappa}} \right) = e^{\bar{X}}$ and $\frac{V_0 - V_L}{V_H - V_0} = e^{\bar{X}^*}$, the expression above has the same sign as $\bar{X} (e^{\bar{X}} - e^{\bar{X}^*}) + \kappa (e^{\bar{X}} + e^{\bar{X}^*})$. Moreover, we have that

$$\frac{\partial \mathcal{V}}{\partial \bar{X}} = -\frac{1}{2\sqrt{2\kappa}}(V_H - V_0)\phi\left(\frac{\bar{X} - \kappa}{\sqrt{2\kappa}}\right) + \frac{1}{2\sqrt{2\kappa}}(V_0 - V_L)\phi\left(\frac{\bar{X} + \kappa}{\sqrt{2\kappa}}\right). \quad (\text{A.10})$$

One can verify that the expression above has the same sign as $(\bar{X}^* - \bar{X})$. $\square$

A.1 Proofs

A.1.1 Proof of Lemma 1

Define

$$n = \frac{1}{\sigma^2}(S_H - S_L)\tilde{n}. \quad (\text{A.11})$$

Notice that $n \sim \mathcal{N}(0, 2\kappa)$. Using Equations (4) and (A.11), we can write

$$X = \begin{cases} n + \kappa & \text{if } \theta = \theta_H, \\ n - \kappa & \text{if } \theta = \theta_L. \end{cases} \quad (\text{A.12})$$

Let $N_H(\kappa, A) = \{n \in \mathbb{R} : (n + \kappa) \in A\}$ and $N_L(\kappa, A) = \{n \in \mathbb{R} : (n - \kappa) \in A\}$. For a speculator with beliefs $\eta = \eta(n)$, the expected profit from trading s dollars, ignoring transaction costs, is

$$\Pi = s \left\{ \eta \int_{N_H(\kappa, A)} \left[\frac{R_H}{P(n + \kappa)} - 1 \right] \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn + (1 - \eta) \int_{N_L(\kappa, A)} \left[\frac{R_L}{P(n - \kappa)} - 1 \right] \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn \right\}.$$

Using the price function (7) and applying the changes of variables $u = n + \kappa$ and $u = n - \kappa$, for the first and second integral, respectively,

$$\Pi = s \left\{ \eta \int_A \left[\frac{(1 - \mu(u))\Delta_R}{R_L + \mu(u)\Delta_R} \right] \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u - \kappa}{\sqrt{2\kappa}}\right) du - (1 - \eta) \int_A \left[\frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \right] \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du \right\}.$$

Using the fact that $(1 - \mu(u))\phi\left(\frac{u - \kappa}{\sqrt{2\kappa}}\right) = \mu(u)\phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right)$, we can write Π as in Equation (10). $\square$

A.1.2 Proof of Lemma 2

The first statement in the first part of Lemma 2 is a special case of Lemma A.1, with $A = [\bar{X}, \infty)$. That $\pi(\kappa)$ is increasing for low κ when $\bar{X} > 0$ follows from the fact that $\lim_{\kappa \rightarrow 0} \pi(\kappa) = 0$ and $\pi(\kappa) > 0$ for $\kappa > 0$ and $\Delta_R > 0$.

We now prove Lemma 2, Part 2. Consider $\bar{X} \leq 0$ and $R_H > R_L \geq 0$. Using Lemma 1, we know that, conditional on κ , the specific strategies played by speculators do not matter to compute speculator's payoffs. We can then consider, without loss of generality, the following aggregate orders conditional on the state: $S_H = a\beta$, $S_L = -c\beta$, where $\beta \geq 0$, $a = \frac{R_H}{R_L + R_H}$, and $c = \frac{R_L}{R_L + R_H}$. Note that there is a one-to-one mapping from β to κ (the aggregate strategies considered imply $\kappa = \beta^2/2\sigma^2$). Bayesian updating implies

$$\mu(\tilde{X}) = \Pr(\theta = \theta_H | \tilde{X}) = \frac{\Pr(\tilde{X} | \theta = \theta_H) \Pr(\theta = \theta_H)}{\Pr(\tilde{X})},$$

which becomes

$$\mu(\tilde{X}) = \frac{\phi\left(\frac{\tilde{X}-S_H}{\sigma}\right)}{\phi\left(\frac{\tilde{X}-S_H}{\sigma}\right) + \phi\left(\frac{\tilde{X}-S_L}{\sigma}\right)} = \frac{\phi\left(\frac{\tilde{X}-a\beta}{\sigma}\right)}{\phi\left(\frac{\tilde{X}-a\beta}{\sigma}\right) + \phi\left(\frac{\tilde{X}+c\beta}{\sigma}\right)}.$$

We can write beliefs for a given $\tilde{n}$ in states θ_H and θ_L , respectively, as

$$\mu_H(\tilde{n}) = \frac{\phi\left(\frac{\tilde{n}}{\sigma}\right)}{\phi\left(\frac{\tilde{n}}{\sigma}\right) + \phi\left(\frac{\tilde{n}+\beta}{\sigma}\right)}, \quad \text{and} \quad \mu_L(\tilde{n}) = \frac{\phi\left(\frac{\tilde{n}-\beta}{\sigma}\right)}{\phi\left(\frac{\tilde{n}-\beta}{\sigma}\right) + \phi\left(\frac{\tilde{n}}{\sigma}\right)}. \quad (\text{A.13})$$

The manager invests whenever $X \geq \bar{X}$. Since $X = \frac{1}{2\sigma^2}\beta\left[2\tilde{X} - \beta\frac{\Delta_R}{R_L+R_H}\right]$ given the strategies assumed, she invests whenever

$$\tilde{X} \geq \frac{2\sigma^2\bar{X}}{\beta} + \beta\frac{\Delta_R}{R_L+R_H} \equiv \hat{X}(\beta).$$

Let $\tilde{\pi}(\beta)$ denote the expected trading profits for a speculator buying one dollar in stocks for a given belief η . Given that $\kappa = \beta^2/2\sigma^2$, to prove the second part of Lemma 2 it suffices to show that $\tilde{\pi}(\beta)$ is decreasing in β for $\eta > 0.5$ (and thus, given Lemma 1, trading profits from selling when $\eta < 0.5$ are decreasing in β). If the state is high, the manager invests whenever $\tilde{n} \geq \hat{X}(\beta) - a\beta$. If the state is low, she invests whenever $\tilde{n} \geq \hat{X}(\beta) + c\beta$. We can thus write

$$\tilde{\pi}(\beta) = \eta \int_{\hat{X}(\beta)-a\beta}^{\infty} \left[\frac{R_H}{R_L + \mu_H(\tilde{n})\Delta_R} - 1 \right] \frac{1}{\sigma} \phi\left(\frac{\tilde{n}}{\sigma}\right) d\tilde{n} + (1-\eta) \int_{\hat{X}(\beta)+c\beta}^{\infty} \left[\frac{R_L}{R_L + \mu_L(\tilde{n})\Delta_R} - 1 \right] \frac{1}{\sigma} \phi\left(\frac{\tilde{n}}{\sigma}\right) d\tilde{n}.$$

Using Equation (A.13) and performing changes of variables such that $u = \tilde{n} + a\beta$ in the first integral and $u = \tilde{n} - c\beta$ in the second, we have

$$\tilde{\pi}(\beta) = (2\eta - 1) \frac{1}{\sigma} \Delta_R \int_{\hat{X}(\beta)}^{\infty} M(u, \beta) du, \quad \text{where} \quad M(u, \beta) = \frac{\phi\left(\frac{u+c\beta}{\sigma}\right)\phi\left(\frac{u-a\beta}{\sigma}\right)}{R_L\phi\left(\frac{u+c\beta}{\sigma}\right) + R_H\phi\left(\frac{u-a\beta}{\sigma}\right)}.$$

Using the Leibniz rule, we have that

$$\frac{\partial \tilde{\pi}}{\partial \beta} = (2\eta - 1) \frac{1}{\sigma} \Delta_R \left[\int_{\hat{X}(\beta)}^{\infty} \frac{\partial M(u, \beta)}{\partial \beta} du - \frac{d\hat{X}(\beta)}{d\beta} M(\hat{X}(\beta), \beta) \right].$$

Since for $\bar{X} \leq 0$, $\frac{d\hat{X}(\beta)}{d\beta} > 0$, it suffices to show that $\frac{\partial M(u, \beta)}{\partial \beta} \leq 0$. $\frac{\partial M(u, \beta)}{\partial \beta}$ has the same sign as

$$aR_L(u - a\beta)\phi\left(\frac{u+c\beta}{\sigma}\right) - cR_H(u + c\beta)\phi\left(\frac{u-a\beta}{\sigma}\right),$$

which is zero if $R_L = 0$, since in this case $c = 0$. Consider now $R_L > 0$. It remains to show that the expression above is smaller than or equal to zero. Given that $aR_L = cR_H$, this is the case whenever

$$(u - a\beta)\phi\left(\frac{u+c\beta}{\sigma}\right) \leq (u + c\beta)\phi\left(\frac{u-a\beta}{\sigma}\right). \quad (\text{A.14})$$

If $u - a\beta = 0$, $u + c\beta > 0$ and Equation (A.14) is trivially satisfied. There are then two cases to be considered: $u - a\beta < 0$ or $u - a\beta > 0$. First, consider $u - a\beta > 0$. Inequality (A.14) can be written as

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} \leq \frac{(u+c\beta)}{(u-a\beta)},$$

which holds since

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} < 1 \quad \text{and} \quad \frac{(u+c\beta)}{(u-a\beta)} > 1.$$

Now, consider $u - a\beta < 0$. We need to show that

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} \geq \frac{(u+c\beta)}{(u-a\beta)}.$$

If $u + c\beta \geq 0$, the inequality above is satisfied, since the LHS > 0 and the RHS ≤ 0 . If instead $u + c\beta < 0$,

$$\frac{\phi\left(\frac{u+c\beta}{\sigma}\right)}{\phi\left(\frac{u-a\beta}{\sigma}\right)} > 1 \quad \text{and} \quad \frac{(u+c\beta)}{(u-a\beta)} \in (0, 1).$$

We have then established that function $M(u, \beta)$ is decreasing in β . Hence, for $\bar{X} \leq 0$, $\pi(\kappa)$ is strictly decreasing in κ . $\square$

A.1.3 Proof of Proposition 1

Consider $\pi(\kappa)$ as given by Equation (11), with $A = [\bar{X}, \infty)$. Suppose $\bar{X} > 0$. Since, from Lemma 2, $\lim_{\kappa \rightarrow 0} \pi(\kappa) = 0 < \tau$, if all other speculators are expected not to trade, it is a best response for each speculator not to trade, and hence there always exists an equilibrium with $\kappa^* = 0$. Also, since $\pi(\kappa) > 0$ for $\kappa > 0$, it is trivial that an interior equilibrium with $\pi(\kappa) = \tau$ exists for low enough τ .

Now, suppose $\bar{X} \leq 0$. From Lemma A.1, $\lim_{\kappa \rightarrow 0} \pi(\kappa)$ is strictly positive, and from Lemma 2, $\pi(\kappa)$ is strictly decreasing in κ . The best response to some κ from a speculator is: to trade as much as possible ($s_i(m_H) = 1, s_i(m_L) = -1$) if $\pi(\kappa) > \tau$; not to trade if $\pi(\kappa) < \tau$; and to place any orders $s_i(m_H) \in [0, 1]$ and $s_i(m_L) \in [-1, 0]$ if $\pi(\kappa) = \tau$. If $\tau \geq \bar{\tau} = \pi(0)$, $\tau > \pi(\kappa)$ for any $\kappa > 0$, and hence speculators' best response to any $\kappa > 0$ is not to trade, so the unique equilibrium features $\kappa^* = 0$. If $\tau \leq \bar{\tau} = \pi(\bar{\kappa})$, $\pi(\kappa) > \tau$ for any $\kappa < \bar{\kappa}$, and hence speculators' best response to any $\kappa \in [0, \bar{\kappa})$ is to trade as much as possible, that is, $s_i(m_H) = 1$ and $s_i(m_L) = -1$, so the unique equilibrium features $\kappa^* = \bar{\kappa}$. Finally, consider $\tau \in (\bar{\tau}, \bar{\tau})$. By continuity, there exists $\kappa' \in (0, \bar{\kappa})$ satisfying $\pi(\kappa') = \tau$. For any $\kappa < \kappa'$, speculators would like to trade as much as possible, leading to $\kappa = \bar{\kappa} > \kappa'$, but for every $\kappa > \kappa'$, speculators would like not to trade, leading to $\kappa = 0$. The equilibrium is reached when $\kappa^* = \kappa'$, and then any $s_i(m_H) \in [0, 1]$ and $s_i(m_L) \in [-1, 0]$ are best responses. Any strategy profile for speculators leading to $\kappa = \kappa'$ is consistent with equilibrium. The equilibrium is essentially unique in the sense that the level of informativeness κ' and firm value are uniquely determined. $\square$

A.1.4 Proof of Proposition 2

Suppose $\bar{X}^* \leq 0$. If $D \leq V_L$, as follows from the discussion in Section 3.1, investment is undertaken if and only if $X \geq \bar{X}^*$. Also, from Proposition 1, Part 2, we have that the unique

equilibrium in this case features $\kappa^* = \bar{\kappa}$ for $\tau \leq \pi(\bar{\kappa})$, which for $\bar{X} = \bar{X}^*$ and for a given $D \leq V_L$ becomes

$$\tau \leq (2\lambda - 1) \int_{\bar{X}^*}^{\infty} \frac{\mu(u)(V_H - V_L)}{V_L - D + \mu(u)(V_H - V_L)} \frac{1}{\sqrt{2\bar{\kappa}}} \phi\left(\frac{u + \bar{\kappa}}{\sqrt{2\bar{\kappa}}}\right) du.$$

Since the RHS of the inequality above is increasing in D and assumes value $\hat{\tau}_2$ at $D = V_L$, the interval of values of D satisfying the equation above is non-empty for any $\tau \leq \hat{\tau}_2$, and is given by $[\max\{0, \underline{D}\}, V_L]$, where $\underline{D}$ is given by Equation (A.4). The proof of Proposition 2, Part 1, is then concluded with the observation that, if $\tau \leq \hat{\tau}_2$, for any $D \in (V_L, V_H)$ the same level of information aggregation is achieved ($\kappa^* = \bar{\kappa}$) and the investment rule makes less efficient use of information, leading to strictly lower firm value. Formally, note that

$$\frac{dV}{dD} = \frac{\partial \bar{X}}{\partial D} \left[\frac{\partial V}{\partial \bar{X}} + \frac{\partial V}{\partial \kappa^*} \frac{\partial \kappa^*}{\partial \bar{X}} \right]. \quad (\text{A.15})$$

For $D \geq V_0$, $\bar{X} = -\infty$ (the manager always invests), $\frac{\partial \bar{X}}{\partial D} = 0$, and hence $\frac{dV}{dD} = 0$. For $D \in (V_L, V_0)$, $\bar{X} = X^*(D)$ (as given by Equation (5)), and hence $\frac{\partial \bar{X}}{\partial D} = -\frac{1}{V_0 - D}$. The other partial derivatives are as given in Lemma A.2. For $\tau \leq \hat{\tau}_2$, any $D \in (V_L, V_0)$ leads to $\kappa^* = \bar{\kappa}$, and hence $\frac{\partial \kappa^*}{\partial \bar{X}} = 0$. Also, since for $D \in (V_L, V_0)$ we have that $\frac{\partial \bar{X}}{\partial D} < 0$ and $\frac{\partial V}{\partial \bar{X}} > 0$ (given that $X^*(D) < \bar{X}^*$), $\frac{dV}{dD} < 0$ and $D > V_L$ cannot be optimal.

We now prove the second part of Proposition 2. From Lemma A.1:

$$\lim_{\kappa \rightarrow 0} \pi(\kappa) = \begin{cases} (2\lambda - 1) \frac{V_H - V_L}{V_L + V_H - 2D} & D < V_L, \\ 2\lambda - 1 & D \in [V_L, V_H], \end{cases} \quad (\text{A.16})$$

so the maximum value of $\lim_{\kappa \rightarrow 0} \pi(\kappa)$ is achieved when $D \in [V_L, V_H]$. Given that $\pi(\kappa)$ is strictly decreasing, if $\tau \geq 2\lambda - 1$, trading profits are always smaller than trading costs for any positive κ and for any D , and the unique equilibrium has $\kappa^* = 0$. This is the uninteresting case ruled out by Assumption 1. Consider then $\tau \in (\hat{\tau}_2, 2\lambda - 1)$. Inspection of Equation (13) shows that, for $D < V_L$, $\pi(\kappa)$ is increasing in D for any κ . Hence, $D < V_L$ is never optimal, since increasing D to V_L would increase κ^* without distorting the investment rule (as $\bar{X} = \bar{X}^*$ for all $D \leq V_L$). For $D \in [V_L, V_0)$, if the equilibrium value of informativeness κ^* is smaller than $\bar{\kappa}$, it satisfies

$$\pi(\kappa^*) = (2\lambda - 1) \left[1 - \Phi\left(\frac{\bar{X} + \kappa^*}{\sqrt{2\kappa^*}}\right) \right] = \tau,$$

and by the implicit function theorem, $\frac{\partial \kappa^*}{\partial \bar{X}} = -\frac{2\kappa^*}{(\kappa^* - \bar{X})}$. Also, in equilibrium $\bar{X} = X^*(D)$, and hence $\frac{\partial \bar{X}}{\partial D} = -\frac{1}{V_0 - D}$. $\frac{\partial V}{\partial \kappa}$ and $\frac{\partial V}{\partial \bar{X}}$ are given by Equations (A.9) and (A.10). After some algebra, one can verify that Equation (A.15) becomes

$$\frac{dV}{dD} = \frac{\kappa^*}{\kappa^* - X^*(D)} \frac{1}{\sqrt{2\kappa^*}} \phi\left(\frac{X^*(D) + \kappa^*}{\sqrt{2\kappa^*}}\right), \quad (\text{A.17})$$

for any $D \in [V_L, \bar{D})$, where $\bar{D}$ is the level of D that leads to $\kappa^* = \bar{\kappa}$. That is, $\bar{D}$ satisfies

$$\tau = (2\lambda - 1) \left[1 - \Phi\left(\frac{X^*(\bar{D}) + \bar{\kappa}}{\sqrt{2\bar{\kappa}}}\right) \right],$$

which solving for $\bar{D}$ yields the expression in Equation (A.2). Equation (A.17) is larger than zero since $X^*(D) \leq 0$ for $D \in [V_L, \bar{D}]$. Since $\frac{dV}{dD} > 0$ for all $D < \bar{D}$ and increasing D beyond $\bar{D}$ decreases firm value ($\frac{\partial V}{\partial X} \frac{\partial X}{\partial D} < 0$ and $\frac{\partial \kappa^*}{\partial X} = 0$), the firm chooses $D^* = \bar{D}$ for all $\tau \in (\hat{\tau}_2, 2\lambda - 1)$. $\square$

A.1.5 Proof of Proposition 3

In this proof, $\pi(\kappa, \bar{X}, D)$ represents the RHS of Equation (13) when setting $A = [\bar{X}, \infty)$. We also write firm value as a function of the cutoff $\bar{X}$ and κ , $V = V(\bar{X}, \kappa)$. Consider $\bar{X}^* > 0$. Suppose first that $\tau \leq \hat{\tau}_2$. Note that we can write $\hat{\tau}_2 = \pi(\bar{\kappa}, \bar{X}^*, V_L)$. Since safe debt yields $\bar{X}(D) = \bar{X}^*$, we have that for all $D \in [\max\{\underline{D}, 0\}, V_L]$, $\pi(\bar{\kappa}, \bar{X}(D), D) = \pi(\bar{\kappa}, \bar{X}^*, D) \geq \tau$ (see Equation (A.4)) and therefore $\kappa = \bar{\kappa}$ is an equilibrium (and it is not an equilibrium if $D < \max\{\underline{D}, 0\}$). Hence, by setting $D \in [\max\{\underline{D}, 0\}, V_L]$, the firm can achieve $\kappa^* = \bar{\kappa}$ and $\bar{X}(D) = \bar{X}^*$, which yields the first best (Proposition A.1). Any $D > V_L$ implies $\bar{X}(D) < \bar{X}^*$ and yields a strictly lower firm value (Lemma A.2). Also, any $D < \underline{D}$ yields $\bar{X}(D) = \bar{X}^*$ but $\kappa^* < \bar{\kappa}$, which by Lemma A.2 leads to lower firm value.

Now suppose that $\tau > \hat{\tau}_2$. In that case, by setting $D = V_L$, the firm cannot achieve $\kappa = \bar{\kappa}$: $\pi(\bar{\kappa}, \bar{X}^*, V_L) < \tau$. Note that in any largest equilibrium with $\kappa = \kappa^* \in (0, \bar{\kappa})$, we must have $\partial\pi/\partial\kappa' < 0$ at all $\kappa' \in (\kappa^*, \kappa^* + \epsilon)$ for $\epsilon > 0$ sufficiently small, otherwise there would be another equilibrium with larger κ . Hence, using Lemma A.2, increasing D in the interval $[0, V_L]$ cannot reduce firm value: it cannot reduce κ^* and it maintains $\bar{X}(D)$ at $\bar{X}^*$. Also, increasing D in that interval strictly increases firm value whenever D leads to an interior level of κ^* . Hence, when computing the optimal D we can restrict attention to $D \geq V_L$, taking into account that when $D = V_L$ is optimal and leads to $\kappa^* = 0$, any $D < V_L$ is also optimal, as well as any $D > V_L$ that also leads to $\kappa^* = 0$.

Note that V in Equation (12) only depends on parameters and on the probability of investment conditional on each state. It then follows from Equation (12) and the results in Proposition 4 that, if the firm issues $D \geq V_L$ and it leads to an interior κ^* , then it is optimal to keep increasing D until $\kappa^* = \bar{\kappa}$ is achieved. One can see that the smallest value of D that leads to $\kappa^* = \bar{\kappa}$ is $\bar{D}$, defined in Equation (A.2), and $\bar{D} < V_0$ since $\tau < 2\lambda - 1$. Hence, if $D = V_L$ leads to an interior κ^* , the optimum is $D = \bar{D}$ (note that $D > \bar{D}$ is suboptimal given Lemma A.2). Now suppose $D = V_L$ leads to $\kappa^* = 0$. In that case, the firm compares issuing $D = \bar{D}$ (in which case $\bar{X} = X^*(\bar{D})$ and $\kappa^* = \bar{\kappa}$) with issuing $D \leq V_L$ (in which case $\bar{X} = \bar{X}^*$, $\kappa^* = 0$, and the firm does not invest). The former is optimal whenever

$$\begin{aligned} V(X^*(\bar{D}), \bar{\kappa}) &= \frac{1}{2} \left[V_H - \Phi \left(\frac{X^*(\bar{D}) - \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) (V_H - V_0) \right] \\ &+ \frac{1}{2} \left[V_L + \Phi \left(\frac{X^*(\bar{D}) + \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) (V_0 - V_L) \right] \geq V_0 = V(\bar{X}^*, 0). \end{aligned} \quad (\text{A.18})$$

Note that in cases where $D = \bar{D}$ is optimal but $D = V_L$ leads to an interior equilibrium with $\kappa = \kappa' \in (0, \bar{\kappa})$, Equation (A.18) must hold, since in that case $V(X^*(\bar{D}), \bar{\kappa}) > V(\bar{X}^*, \kappa') > V(\bar{X}^*, 0)$ (Lemma A.2 and Proposition 4). Hence, issuing $D = \bar{D}$ is optimal iff Equation (A.18) holds, and issuing $D = V_L$ is optimal iff it is violated or holds with equality.

Consider the limit as $\tau \rightarrow (2\lambda - 1)^-$. In this case, $\bar{D} \rightarrow V_0$ (see Equation (A.2)), $X^*(\bar{D}) \rightarrow -\infty$, and $V(X^*(\bar{D}), \bar{\kappa})$ approaches $\frac{V_H + V_L}{2} < V_0$, so it is optimal to set $D = V_L$, which yields $\kappa^* = 0$. Now consider the limit as $\tau \rightarrow \hat{\tau}_2^+$. Since $\lim_{\tau \rightarrow \hat{\tau}_2^+} \bar{D} = V_L$,

$\lim_{\tau \rightarrow \hat{\tau}_2^+} \mathcal{V}(X^*(\bar{D}), \bar{\kappa}) = \mathcal{V}(\bar{X}^*, \bar{\kappa}) > V_0$, and hence $D = \bar{D}$ is optimal for τ slightly above $\hat{\tau}_2$. We now claim that as we increase τ , $\mathcal{V}(X^*(\bar{D}), \bar{\kappa})$ in Equation (A.18) decreases. $\bar{D}$ is increasing in τ and $X^*(\bar{D})$ is decreasing in $\bar{D}$. Hence, an increase in τ decreases $X^*(\bar{D})$, which reduces $\mathcal{V}(X^*(\bar{D}), \bar{\kappa})$ by Lemma (A.2). Hence, $D = \bar{D}$ is optimal iff $\tau \leq \tau'$, where τ' is the value of τ such that Equation (A.18) holds with equality. As one can verify using the definition of $\hat{\tau}_3$ in Equation (A.3), $\tau' = \hat{\tau}_3$. If $\tau \geq \hat{\tau}_3$, $D = V_L$ is optimal, and it yields $\kappa^* = 0$.

To conclude, consider $\tau \geq \hat{\tau}_3$ and notice that the values of D such that $\pi(\kappa, \bar{X}(D), D) < \tau$ for all $\kappa \in [0, \bar{\kappa}]$ are $D \in [0, \tilde{D})$, where $\tilde{D}$ is defined by Equation (A.5). Note that if $X^*(\tilde{D}) < 0$, the LHS of Equation (A.5) is strictly larger than τ , since $\lim_{\kappa \rightarrow 0} (2\lambda - 1) \left[1 - \Phi\left(\frac{X^*(\tilde{D}) + \kappa}{\sqrt{2\kappa}}\right) \right] = 2\lambda - 1 > \tau$. Hence, to satisfy Equation (A.5), it must be that $X^*(\tilde{D}) \geq 0$. Thus, any $D' \in [0, \tilde{D})$ leads to $\kappa^* = 0$ and $\bar{X}(D') > 0$, and hence firm value is $\mathcal{V}(\bar{X}(D'), 0) = V_0$. Therefore, for $\tau \geq \hat{\tau}_3$ not only $D = V_L$ is optimal, but in fact any $D \in [0, \tilde{D})$ is optimal. $\square$

A.1.6 Proof of Proposition 4

Given an investment cutoff $\bar{X}$, using Equation (A.12) we have that

$$\Pr(a = 1 | \theta = \theta_L) = 1 - \Phi\left(\frac{\bar{X} + \kappa^*}{\sqrt{2\kappa^*}}\right) \quad \text{and} \quad \Pr(a = 1 | \theta = \theta_H) = 1 - \Phi\left(\frac{\bar{X} - \kappa^*}{\sqrt{2\kappa^*}}\right).$$

Consider some $D \in (V_L, \bar{D})$ that leads to a largest equilibrium with interior κ^* (note that $D \geq \bar{D}$ leads to $\bar{\kappa}$). In equilibrium, $\bar{X} = X^*(D)$ and κ^* satisfies Equation (16), which automatically implies that $\Pr(a = 1 | \theta = \theta_L) = 1 - \Phi\left(\frac{\bar{X} + \kappa^*}{\sqrt{2\kappa^*}}\right) = \frac{\tau}{2\lambda - 1}$. Moreover,

$$\frac{d\Pr(a = 1 | \theta = \theta_H)}{dD} = \frac{\partial \bar{X}}{\partial D} \left[\frac{\partial \Pr(a = 1 | \theta = \theta_H)}{\partial \bar{X}} + \frac{\partial \Pr(a = 1 | \theta = \theta_H)}{\partial \kappa^*} \frac{\partial \kappa^*}{\partial \bar{X}} \right].$$

Computing all the partial derivatives above and rearranging yield:

$$\frac{d\Pr(a = 1 | \theta = \theta_H)}{dD} = \frac{1}{V_0 - D} \frac{1}{\sqrt{2\kappa^*}} \phi\left(\frac{\bar{X} - \kappa^*}{\sqrt{2\kappa^*}}\right) \left[1 + \frac{\kappa^* + \bar{X}}{\kappa^* - \bar{X}} \right].$$

Note that $D < \bar{D} < V_0$. If $\bar{X} = X^*(D) \leq 0$, the expression above is strictly positive. Now suppose $\bar{X} > 0$. We claim that in this case $\kappa^* \geq \bar{X}$, which makes the expression above strictly positive. To see that, notice that for $D \geq V_L$, $\pi(\kappa) = (2\lambda - 1) \left[1 - \Phi\left(\frac{\bar{X} + \kappa}{\sqrt{2\kappa}}\right) \right]$ is a single-peaked function and achieves its maximum at $\kappa = \bar{X}$. Hence, in an interior equilibrium with the highest κ among all equilibria, it must be that $\kappa^* > \bar{X}$, otherwise there would be another equilibrium with higher κ . $\square$

A.1.7 Proof of Proposition 5

Consider $\bar{X}^* > 0$. For any $D < V_L$, in equilibrium $\bar{X} = \bar{X}^* > 0$. For $D \in [V_L, V_0)$, $\bar{X} = X^*(D)$, and for $D \geq V_0$, investment is always undertaken. Since $X^*(D)$ is decreasing, for any $D < \hat{D} \equiv 2V_0 - V_H$, $\bar{X} > 0$, and from Proposition 1, $\kappa^* = 0$. Hence, $\mathcal{V} = V_0$ for

$D < \hat{D}$. For $D \geq \hat{D}$, $\bar{X} = X^*(D) \leq 0$, and from Proposition 1, the equilibrium features $\kappa^* > 0$ —since for $D \geq \hat{D} > V_L$, $\bar{\tau} = \pi(0) = 2\lambda - 1 > \tau$ (see Assumption 1 and Equation (A.16)).

Suppose $\tau \leq \hat{\tau}_1$ (defined in Equation (A.1)). In this case, setting $D \geq \hat{D}$ leads to $\kappa^* = \bar{\kappa}$, since $\pi(\kappa)$ is decreasing for $X^*(D) \leq 0$, and

$$\pi(\bar{\kappa}) = (2\lambda - 1) \left[1 - \Phi \left(\frac{X^*(D) + \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) \right] \geq (2\lambda - 1) \left[1 - \Phi \left(\frac{\bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) \right] = \hat{\tau}_1 \geq \tau.$$

Since for any positive κ we have $\partial\mathcal{V}/\partial\bar{X} > 0$ for $\bar{X} < \bar{X}^*$ (Lemma A.2), $D > \hat{D}$ cannot be optimal (it distorts the ex post use of information without further increasing κ). Since $X^*(\hat{D}) = 0$, setting $D = \hat{D}$ yields

$$\mathcal{V} = \frac{V_L + V_0}{2} + \frac{V_H - V_L}{2} \Phi \left(\sqrt{\frac{\bar{\kappa}}{2}} \right), \quad (\text{A.19})$$

which, one can verify, is larger than V_0 if and only if $\bar{\kappa} \geq \tilde{\kappa} = 2 \left[\Phi^{-1} \left(\frac{V_0 - V_L}{V_H - V_L} \right) \right]^2$. Therefore, for $\tau \leq \hat{\tau}_1$ and $\bar{\kappa} \geq \tilde{\kappa}$, it is optimal to set $D = \hat{D}$. If $\tau \leq \hat{\tau}_1$ and instead $\bar{\kappa} < \tilde{\kappa}$, any $D \in [0, \hat{D})$ is optimal as it yields V_0 .

Now, notice that if $\bar{\kappa} < \tilde{\kappa}$ and $\tau > \hat{\tau}_1$, the optimal capital structure is also $D \in [0, \hat{D})$: any $D \geq \hat{D}$ would lead to an equilibrium with $\kappa^* > 0$, but the ex ante firm value would be strictly smaller than the one in Equation (A.19), which is itself smaller than V_0 . Hence, the optimal D is $D^* \in [0, \hat{D})$ for $\bar{\kappa} < \tilde{\kappa}$.

In the remainder of the proof, consider $\bar{\kappa} \geq \tilde{\kappa}$ and $\tau > \hat{\tau}_1$. In this case, optimal debt is either some $D^* \in [\hat{D}, V_0)$ that leads to $\kappa^* > 0$, or any $D \in [0, \hat{D})$, leading to $\kappa^* = 0$ and $\mathcal{V} = V_0$. Recall that $\bar{D}$, as defined in Equation (A.4), is the minimum D that leads to $\kappa^* = \bar{\kappa}$. For $D \in [\hat{D}, \bar{D})$, $\bar{X} = X^*(D) \leq 0$ and $\partial\mathcal{V}/\partial D$ (which is as given in Equation (A.17)) is strictly positive. For $D > \bar{D}$, using Equation (A.15), $\partial\mathcal{V}/\partial D < 0$ (since as implied by Lemma A.2, $\frac{\partial\mathcal{V}}{\partial\bar{X}} < 0$ and $\frac{\partial\kappa^*}{\partial\bar{X}} = 0$, as $\kappa^* = \bar{\kappa}$). The candidates for optimal debt are then $D < \hat{D}$, in which case $\mathcal{V} = V_0$ and $D = \bar{D}$, in which case

$$\mathcal{V} = \frac{1}{2} \left[V_H - \Phi \left(\frac{X^*(\bar{D}) - \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) (V_H - V_0) \right] + \frac{1}{2} \left[V_L + \Phi \left(\frac{X^*(\bar{D}) + \bar{\kappa}}{\sqrt{2\bar{\kappa}}} \right) (V_0 - V_L) \right]. \quad (\text{A.20})$$

As $\tau \rightarrow \hat{\tau}_1^+$, $\bar{D} \rightarrow \hat{D}$, $\bar{X}^*(\bar{D}) \rightarrow 0$, and $\mathcal{V}$ approaches Equation (A.19), which is larger than V_0 since $\bar{\kappa} \geq \tilde{\kappa}$, so $D^* = \bar{D}$. Now consider the limit where $\tau \rightarrow (2\lambda - 1)^-$. In this case, $\bar{D} \rightarrow V_0$ and $X^*(\bar{D}) \rightarrow -\infty$, and $\mathcal{V}$ approaches $\frac{V_H + V_L}{2} < V_0$, so it is optimal to set any $D \in [0, \hat{D})$. Since $\bar{D}$ is increasing in τ , $X^*(\cdot)$ is decreasing, and $\partial\mathcal{V}/\partial\bar{X}$ is positive for $\bar{X} < \bar{X}^*$, firm value in Equation (A.20) is decreasing in τ . Hence, there exists $\hat{\tau}_3 \in (\hat{\tau}_1, 2\lambda - 1)$ such that the optimal D is $\bar{D}$ for $\tau \leq \hat{\tau}_3$, and any $D < \hat{D}$ is optimal otherwise. Writing $\bar{D}$ as a function of τ (see Equation (A.2)), $\hat{\tau}_3$ is the value of τ that equates Equation (A.20) to V_0 , as given by Equation (A.3). $\square$

A.1.8 Proof of Lemma 3

Let $A \subseteq \mathbb{R}$ be a measurable set such that the firm invests if and only if $X \in A$. Note that if $D \geq V_H$, $\pi(\kappa) = 0$ for all κ , and hence setting $A = [\bar{X}^*, \infty)$ is trivially optimal. We hereafter restrict attention to $D < V_H$. By inspection of Equation (10), one can easily verify that a trading stage equilibrium exists for any A . Let $N_H(\kappa, A) = \{n \in \mathbb{R} : (n + \kappa) \in A\}$ and $N_L(\kappa, A) = \{n \in \mathbb{R} : (n - \kappa) \in A\}$. Using Equation (A.12), the firm payoff can be written as a function of κ and A with a small abuse of notation:

$$\mathcal{V}(A, \kappa) = V_0 + \frac{1}{2} \Pr((n + \kappa) \in A)(V_H - V_0) - \frac{1}{2} \Pr((n - \kappa) \in A)(V_0 - V_L),$$

where

$$\begin{aligned} \Pr((n + \kappa) \in A) &= \int_{N_H(\kappa, A)} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn \quad \text{and} \\ \Pr((n - \kappa) \in A) &= \int_{N_L(\kappa, A)} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{n}{\sqrt{2\kappa}}\right) dn. \end{aligned}$$

Applying the change of variables $u = n + \kappa$ in the first probability above and $u = n - \kappa$ in the second one, we have

$$\mathcal{V}(A, \kappa) = V_0 + \frac{1}{2} \int_A (V_H - V_0) \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u - \kappa}{\sqrt{2\kappa}}\right) du - \frac{1}{2} \int_A (V_0 - V_L) \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du,$$

which using the fact that $\phi\left(\frac{u - \kappa}{\sqrt{2\kappa}}\right) / \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) = e^u$ becomes

$$\mathcal{V}(A, \kappa) = V_0 + \frac{1}{2} \int_A J(u) \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du, \quad (\text{A.21})$$

where $J(u) \equiv e^u(V_H - V_0) - (V_0 - V_L)$. Notice that $J(\cdot)$ is strictly increasing. Moreover, from Equation (11), $\pi(\cdot)$ can also be written, with a small abuse of notation, as a function of κ and A :

$$\pi(\kappa; A) = (2\lambda - 1) \int_A Q(u) \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u + \kappa}{\sqrt{2\kappa}}\right) du,$$

where $Q(u) \equiv \frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R}$. Notice that $Q(\cdot)$ is increasing.

The problem of the firm is to choose a measurable set $A \subseteq \mathbb{R}$ and $\kappa \in [0, \bar{\kappa}]$ to maximize Equation (A.21) subject to $\kappa = f(A)$, where $f(A)$ denotes the highest value of κ consistent with equilibrium in the trading stage for a given A . We first analyze a relaxed problem in which the firm chooses κ and A to maximize Equation (A.21) subject to $\kappa \leq f(A)$, showing that any strategy that is not a cutoff strategy is weakly dominated by a cutoff strategy (Step 1). We then show that any cutoff strategy in which the constraint $\kappa \leq f(A)$ does not bind is weakly dominated by a strategy in which it binds (Step 2).

Step 1. Let (A^*, κ^*) be an optimal strategy for the firm in the relaxed problem. Suppose first that $f(A^*) = 0$, which implies $\kappa^* = 0$. This implies that $X = 0$ with probability 1. Then, there exists a cutoff strategy (A', κ^*) with $A' = [\bar{X}', \infty)$ that is optimal: if $0 \in A^*$, any $\bar{X}' \leq 0$ and $\kappa^* = 0$ is optimal; if $0 \notin A^*$, any $\bar{X}' > 0$ and $\kappa^* = 0$ is optimal.

Now, suppose that (A^*, κ^*) is such that: (i) $f(A^*) > 0$; (ii) there exists bounded sets B_1 and B_2 with strictly positive measure satisfying $B_1 \subseteq A^*$, $B_2 \subseteq (A^*)^c$, and $X_2 > X_1$ for every $X_1 \in B_1$ and $X_2 \in B_2$. (Note that if a non-cutoff strategy does not satisfy (ii), trivially there is a cutoff strategy that yields the same firm payoff.) Consider a deviation to (A', κ^*) such that $A' = (A^* \setminus C_1) \cup C_2$, where C_1 and C_2 are measurable subsets of B_1 and B_2 , respectively, with

$$\int_{C_1} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du = \int_{C_2} \frac{1}{\sqrt{2\kappa}} \phi\left(\frac{u+\kappa}{\sqrt{2\kappa}}\right) du.$$

Such deviation increases firm value since $J(u)$ is strictly increasing. It remains to check that such deviation is feasible, meaning that $\kappa^* \leq f(A')$. It then suffices to verify that $f(A') \geq f(A^*)$. To see that, first notice that $\pi(\kappa; A') \geq \pi(\kappa; A^*)$ for every κ , since $Q(u)$ is increasing. If $f(A^*) = \bar{\kappa}$, by the equilibrium definition, $\pi(\bar{\kappa}; A^*) \geq \tau$, and hence $\pi(\bar{\kappa}; A') \geq \pi(\bar{\kappa}; A^*) \geq \tau$, so $f(A') = \bar{\kappa}$ as well. If instead $f(A^*) \in (0, \bar{\kappa})$, it must be that for $\epsilon > 0$ sufficiently small, $\partial \pi(\kappa'; A^*) / \partial \kappa' < 0$ for all $\kappa' \in (f(A^*), f(A^*) + \epsilon)$, or else there would be another equilibrium with larger κ , contradicting the definition of $f(\cdot)$. Hence, since $\pi(f(A^*); A') \geq \pi(f(A^*); A^*) = \tau$, the highest value of κ consistent with equilibrium under A' must be at least $f(A^*)$, and so A^* is dominated. Therefore, when looking for a maximum in the relaxed problem we can restrict attention to cutoff strategies.

Step 2. Let (A^*, κ^*) be a cutoff strategy in the relaxed problem with $\kappa^* < f(A^*)$. Denote by $\tilde{X}^*$ the cutoff associated to that strategy, that is, $A^* = [\tilde{X}^*, \infty)$. If $\tilde{X}^* = \bar{X}^*$, firm value is strictly increasing in κ (by Lemma A.2), and hence choosing $\kappa = f(A^*)$ increases firm value. If $\tilde{X}^* \neq \bar{X}^*$, then, by Lemma A.2, the firm could increase its payoff with a marginal reduction (increase) in $\tilde{X}^*$ if $\tilde{X}^* > \bar{X}^*$ ($\tilde{X}^* < \bar{X}^*$) without violating $\kappa^* \leq f(A^*)$. Hence, (A^*, κ^*) is dominated.

We have then shown that finding a maximum of the relaxed problem is equivalent to finding a maximum of the original problem restricted to cutoff strategies. This concludes the proof that if there is an optimum, it can be achieved with a cutoff strategy $\bar{X}_c^*$ (first result). It remains to show that an optimum exists and $\bar{X}_c^* \leq \bar{X}^*$ (second result). This follows immediately from the characterization in Propositions B.5 and B.6, that can be found in the [Online Appendix](#) (note that we only use the first result to prove those propositions). $\square$

A.1.9 Proof of Proposition 6

Using Lemma 3, we restrict attention to the problem of choosing D and a cutoff $\bar{X}_c \leq \bar{X}^*$ at $t=0$. Recall from Section 3.2.b that, for a given level of debt D , the investment cutoff that maximizes equity value (ex post) is

$$\hat{X}(D) = \bar{X}(D) = \begin{cases} \bar{X}^* & \text{if } D \leq V_L, \\ X^*(D) & \text{if } D \in (V_L, V_0), \\ -\infty & \text{if } D \geq V_0. \end{cases}$$

(To avoid confusion, in this proof, we refer to $\bar{X}(D)$ as $\hat{X}(D)$, since in the general problem considered the firm can play a cutoff $\bar{X} \neq \bar{X}(D)$). Suppose that $(\bar{X}_c^*, D^*)$ is a solution to the problem considered. First, suppose that it leads to $\kappa^* = 0$. If $\bar{X}_c^* \leq 0$, the firm always invests in equilibrium, and if $\bar{X}_c^* > 0$, it never does. Consider the strategy $(\bar{X}^*, D)$, with

$D \leq V_L$, which is consistent with ex post equity value maximization. If that strategy leads to $\kappa^* > 0$, then $(\bar{X}_c^*, D^*)$ cannot be optimal since $\frac{\partial V}{\partial \kappa} |_{\bar{X}=\bar{X}^*} > 0$, by Lemma A.2, a contradiction. Then, $(\bar{X}^*, D)$, with $D \leq V_L$, also leads to $\kappa^* = 0$, and it is also optimal since $\bar{X}^*$ maximizes ex ante firm value for a given level of κ .

Hereafter, assume $(\bar{X}_c^*, D^*)$ leads to $\kappa^* > 0$. Suppose $\bar{X}_c^* = \bar{X}^*$. By Lemma A.2, firm value is increasing in κ at $\bar{X} = \bar{X}^*$. Recall from Equation (13) that for any κ , $\pi(\kappa)$ achieves its maximum for any $D \in [V_L, V_H]$. Hence, setting $D = V_L$ and keeping the cutoff at $\bar{X}^*$ cannot decrease κ and therefore cannot decrease firm value, so $(\bar{X}^*, V_L)$ is optimal (and $\hat{X}(V_L) = \bar{X}^*$). Now consider $\bar{X}_c^* < \bar{X}^*$ and $\kappa^* > 0$. Optimality implies that $\frac{dV}{d\bar{X}} = \frac{\partial V}{\partial \bar{X}} + \frac{\partial V}{\partial \kappa} \frac{\partial \kappa}{\partial \bar{X}}$ must be zero at $\bar{X} = \bar{X}_c^*$. Using that expression for $\frac{dV}{d\bar{X}}$ and Lemma A.2, and since at $\bar{X} = \bar{X}_c^* < \bar{X}^*$ we have $\frac{\partial V}{\partial \bar{X}} > 0$, $\frac{dV}{d\bar{X}} = 0$ is only possible if $\frac{\partial V}{\partial \kappa} > 0$ and $\frac{\partial \kappa}{\partial \bar{X}} < 0$ (recall that an increase in $\bar{X}$ cannot increase κ in the largest equilibrium). Hence, since for any κ , $\pi(\kappa)$ achieves its maximum for any $D \in [V_L, V_H]$, it must be that $D^* \in [V_L, V_H]$, otherwise there is an increase in D and $\bar{X}$ that does not change κ and increases firm value. But note that, keeping the cutoff $\bar{X}_c^*$ fixed, any $D \in [V_L, V_H]$ leads to the same κ . Finally, notice that there exists $D' \in [V_L, V_0]$ that leads to $\hat{X}(D') = \bar{X}_c^*$. Therefore, setting $(\hat{X}(D'), D')$ is optimal since it leads to the same cutoff and the same κ^* as $(\bar{X}_c^*, D^*)$ (as for a given cutoff, $\pi(\kappa)$ is independent of D for $D \in [V_L, V_0]$). We have then shown that the optimum can always be achieved with ex post equity value maximization. $\square$

A.1.10 Proof of Proposition 7

Case 1. We start looking at the case in which $\bar{X}^* \leq 0$. Suppose first that $\tau \leq \hat{\tau}_2$ and hence the firm initially chooses $D^* \in [\max\{0, \underline{D}\}, V_L]$ (see Proposition 2). Note that $\underline{D}$ is strictly increasing in τ (see Equation (A.4)). Therefore, following an increase in τ to some $\tau' \leq \hat{\tau}_2$, the optimal values of D are in an interval $[\max\{0, \underline{D}'\}, V_L]$, where $\underline{D}' > \underline{D}$. Moreover, following an increase in τ to some $\tau'' > \hat{\tau}_2$, the optimal level of debt is given by $\bar{D}(\tau'') > V_L$. Now suppose that initially $\tau > \hat{\tau}_2$. Any increase in τ increases the optimal level of debt since $\bar{D}$ is strictly increasing in τ (see Equation (A.2)). We have then shown that, for $\bar{X}^* \leq 0$, the largest and smallest optimal debt are always increasing in τ .

Now consider the effect of an increase in α , which affects the optimal debt only through $\bar{\kappa}$. Suppose that initially $\tau \leq \hat{\tau}_2$. Notice that $\hat{\tau}_2$ in Equation (A.1) is decreasing in $\bar{\kappa}$. Also, it follows from Lemma 2 that the RHS of Equation (A.4) is decreasing in $\bar{\kappa}$, and hence $\underline{D}$ is increasing in $\bar{\kappa}$. Therefore, an increase in α (and hence in $\bar{\kappa}$) that reduces $\hat{\tau}_2$ to some $\hat{\tau}_2' \geq \tau$ changes the set of optimal levels of debt from $[\max\{0, \underline{D}\}, V_L]$ to some $[\max\{0, \underline{D}'\}, V_L]$ with $\underline{D}' > \underline{D}$. An increase in α that, instead, reduces $\hat{\tau}_2$ to some $\hat{\tau}_2'' < \tau$ leads to an optimal debt $\bar{D} > V_L$. Finally, if initially $\tau > \hat{\tau}_2$, any increase in α increases the optimal level of debt since $\bar{D}$ is increasing in $\bar{\kappa}$ for every $\tau > \hat{\tau}_2$, as can be verified using Equation (A.2) and the definition of $\hat{\tau}_2$.

Case 2. We now consider the case where $\bar{X}^* > 0$. Suppose $\tau < \hat{\tau}_2$ and consider an increase in τ such that after the increase $\tau \in (\hat{\tau}_2, \hat{\tau}_3)$. By Proposition 3, that increases the debt optimally chosen by the firm, since $\bar{D} > V_L$. Now suppose that initially $\hat{\tau}_2 < \tau < \hat{\tau}_3$ and an

increase in τ makes $\tau > \hat{\tau}_3$. Such an increase in τ reduces the debt optimally chosen by the firm, by Proposition 3.

For the proof of the non-monotonicity in the comparative statics on α , we differentiate $\hat{\tau}_2$ with respect to $\bar{\kappa}$ to conclude that $\frac{d\hat{\tau}_2}{d\bar{\kappa}}$ has the same sign as $(\bar{X}^* - \bar{\kappa})$. Hence, for low values of $\bar{\kappa}$, we have $\frac{d\hat{\tau}_2}{d\bar{\kappa}} > 0$ and for high values, we have $\frac{d\hat{\tau}_2}{d\bar{\kappa}} < 0$. Consider a pair $(\tau, \bar{\kappa})$ such that $\frac{d\hat{\tau}_2}{d\bar{\kappa}} > 0$ and τ is slightly above $\hat{\tau}_2$ (and hence, $\hat{\tau}_2 < \tau < \hat{\tau}_3$). Then, there must be an increase in $\bar{\kappa}$ that increases $\hat{\tau}_2$ enough so that after the change in $\bar{\kappa}$ we have $\tau < \hat{\tau}_2$, which implies that the level of debt chosen by the firm must decrease (by Proposition 3). As $\bar{\kappa} \rightarrow \infty$, $\hat{\tau}_2 \rightarrow 0$. Hence, if $\bar{\kappa}$ keeps increasing, eventually $\hat{\tau}_2$ will be such that $\hat{\tau}_2 < \tau < \hat{\tau}_3$ again, increasing the level of debt chosen by the firm (relative to the case where $\bar{\kappa}$ yields $\tau < \hat{\tau}_2$). $\square$

A.2 First Best

Proposition A.1. *The ex ante firm value only depends on speculators' strategies through κ and is strictly increasing in κ under the ex post optimal investment strategy $\bar{X}^*$. The maximal ex ante firm value is achieved when $\kappa = \bar{\kappa}$ and the manager invests if and only if $X \geq \bar{X}^*$.*

Proof. Suppose the firm invests if and only if $X \in A$ for any measurable subset $A \subseteq \mathbb{R}$. Firm value at $t = 0$ can be written as

$$V = \frac{1}{2} [V_0 + \Pr(X \in A | \theta = \theta_H)(V_H - V_0)] + \frac{1}{2} [V_0 - \Pr(X \in A | \theta = \theta_L)(V_0 - V_L)]. \quad (\text{A.22})$$

By Equations (A.11) and (A.12), the distribution of X conditional on a given state only depends on κ , which concludes the proof of the first statement. It follows from Lemma A.2 that under the ex post optimal strategy $\bar{X} = \bar{X}^*$, $\frac{\partial V}{\partial \kappa^*} > 0$. Lemma A.2 also implies that, for a given κ , the ex ante optimal investment strategy is the ex post optimal cutoff $\bar{X}^*$. Since firm value is strictly increasing in κ under that cutoff strategy, the firm value can never be larger than when $\kappa = \bar{\kappa}^*$ and $\bar{X} = \bar{X}^*$. $\square$

A.3 Change of Variables in the Commitment Problem

Suppose the firm can commit to an investment strategy such that it invests if and only if $\tilde{X} \in \tilde{A}$, where $\tilde{A}$ is a measurable subset of the real line. Given its choice of $\tilde{A}$, the firm anticipates aggregate orders conditional on the state, S_H and S_L , that are consistent with the equilibrium with the highest κ . If $S_H \neq S_L$, then there is a one-to-one continuous map from X to $\tilde{X}$, and hence there exists a measurable subset A such that $X \in A \iff \tilde{X} \in \tilde{A}$. If $S_H = S_L$, then $\kappa = 0$ and $X = 0$ for every $\tilde{X}$. Then, the firm obtains no information from the market, so it must be that either (i) not investing regardless of market activity ($\tilde{A} = \emptyset$) is optimal or (ii) always investing ($\tilde{A} = \mathbb{R}$) is optimal. Therefore, the firm can attain the same payoff as the optimal $\tilde{A}$ by investing iff $X \in A$, where A is such that it does not include 0 (if (i) is optimal) or it includes 0 (if (ii) is optimal).

A.4 Ex Ante Investment and Prices

In this section, for convenience, we write the price setting rule P and beliefs μ as the original functions of $\tilde{X}$ (see Section 2). Under the most general formulation, the problem of the firm

is to choose the strategies of all agents (manager, informed traders, and market maker) subject to those being consistent with equilibrium in the trading stage. More precisely, the firm chooses a measurable subset $Z \subseteq \mathbb{R}^2$, trading strategies for informed speculators, and a price-setting rule $P(\tilde{X})$ such that: (i) the manager invests iff $(\tilde{X}, P(\tilde{X})) \in Z$; (ii) speculators' strategies are consistent with the equilibrium with the highest κ in the trading stage for a given Z and $P(\cdot)$; and (iii) the market maker breaks even in expectation for every $\tilde{X}$. We call this the general ex ante investment problem. In Section 4.1, we claimed that the solution presented there would also be a solution to this more general problem, which we now prove.

Lemma A.3. *There is an ex ante investment strategy that only conditions investment on X that is a solution to the general ex ante investment problem.*

Proof. For the market maker to break even, the price-setting rule must satisfy

$$P(\tilde{X}) = \begin{cases} R_L + \mu(\tilde{X})\Delta_R & \text{if } (\tilde{X}, P(\tilde{X})) \in Z, \\ R_0 & \text{otherwise.} \end{cases} \quad (\text{A.23})$$

Suppose that at the optimum $Z = Z^*$. Since, given Z^* , $P(\tilde{X})$ is uniquely pinned down for each $\tilde{X}$, the manager invests if and only if $\tilde{X} \in \tilde{A}$, where $\tilde{A} = \{y : (y, P(y)) \in Z^*\}$. Therefore, the firm could achieve the same outcome by choosing a set $\tilde{A}$ such that the manager invests iff $\tilde{X} \in \tilde{A}$ and the same speculator's strategies, with the market maker setting prices according to $P(\tilde{X}) = R_L + \mu(\tilde{X})\Delta_R$ if $\tilde{X} \in \tilde{A}$ and $P(\tilde{X}) = R_0$ otherwise. $\square$

A.5 Informativeness Measure

We claim that the variable κ is the relevant measure of market informativeness since market activity is more informative in the sense of Blackwell when κ is larger. That is, a larger κ would increase the payoff of a decision maker with any arbitrary payoff structure. We now demonstrate this result. When there is a binary state, one can verify Blackwell dominance by restricting attention to a simple class of decision problems with the following payoff matrix:

	$\theta = \theta_L$	$\theta = \theta_H$
Action A	$-(1 - y)$	y
Action B	0	0

A signal z is more informative than a signal z' if and only if it leads to a higher expected payoff for a decision maker facing these payoffs for any $y \in (0, 1)$ (see Blackwell and Girshick (1954) or the proof of Lemma 1 in Börgers, Hernando-Veciana, and Kräbmer, 2013). Consider the signals X observed in our model and recall from the proof of Lemma 1 that $X|\theta = \theta_H \sim \mathcal{N}(\kappa, 2\kappa)$ and $X|\theta = \theta_L \sim \mathcal{N}(-\kappa, 2\kappa)$. Action A is strictly preferred whenever

$$\Pr(\theta = H|X)y - \Pr(\theta = L|X)(1 - y) > 0,$$

which is equivalent to

$$\Pr(\theta = H|X) = \mu(X) > 1 - y \iff X > \ln\left(\frac{1-y}{y}\right) \equiv X^\dagger,$$

where $\mu(X)$ is given by Equation (3). The decision maker strictly prefers B when $X < X^\dagger$ and is indifferent between the two actions when $X = X^\dagger$. Hence, under the optimal choice her expected utility is

$$\begin{aligned} U &= \frac{1}{2} \Pr(X > X^\dagger | \theta = \theta_H) y - \frac{1}{2} \Pr(X > X^\dagger | \theta = \theta_L) (1-y) \\ &= \frac{y}{2} \left[1 - \Phi\left(\frac{X^\dagger - \kappa}{\sqrt{2\kappa}}\right) \right] - \frac{(1-y)}{2} \left[1 - \Phi\left(\frac{X^\dagger + \kappa}{\sqrt{2\kappa}}\right) \right]. \end{aligned}$$

Now note that

$$\frac{dU}{d\kappa} = \frac{y}{2} \phi\left(\frac{X^\dagger - \kappa}{\sqrt{2\kappa}}\right) \left(\frac{X^\dagger + \kappa}{2\kappa\sqrt{2\kappa}}\right) - \frac{(1-y)}{2} \phi\left(\frac{X^\dagger + \kappa}{\sqrt{2\kappa}}\right) \left(\frac{X^\dagger - \kappa}{2\kappa\sqrt{2\kappa}}\right).$$

Dividing the expression above by $\phi\left(\frac{X^\dagger + \kappa}{\sqrt{2\kappa}}\right)$ and using the fact that $\phi\left(\frac{X^\dagger - \kappa}{\sqrt{2\kappa}}\right) / \phi\left(\frac{X^\dagger + \kappa}{\sqrt{2\kappa}}\right) = e^{X^\dagger} = \frac{1-y}{y}$, one can verify that $\frac{dU}{d\kappa}$ has the same sign as $\kappa(1-y) > 0$. Therefore, the utility of such decision maker is increasing in κ , so the larger κ , the more informative market activity is in the sense of Blackwell.

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