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Lending terms and aggregate productivity

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ABSTRACT

Several empirical studies suggest that lending terms are eased in expansions and tightened in recessions, thereby influencing the mix of financed entrepreneurs. We study a model of adverse selection in competitive financial markets and show that lending terms deteriorate with the aggregate state under two general conditions. If exogenous increments to entrepreneurs' productivity raise returns to investment and/or tighten the credit line needed to screen out a given entrepreneur type, competition results in contracts with less screening. Two endogenous effects on productivity emerge. Production scales grow closer to optimal, but lower productivity entrepreneurs enter the mix of producers. The positive (negative) effect dominates at low (high) aggregate states.

1. Introduction

Lending terms refer to loan contract terms, with an exception of interest rates, that are commonly used to screen out non-repaying borrowers.[1] Several empirical studies suggest that lending terms are eased in expansions and tightened in recessions, thereby exerting a systematic influence on the composition of financed entrepreneurs over the business cycle (see Asea and Blomberg, 1998; Berger and Udell, 2004; Lown and Morgan, 2006). In aggregate data, this is reflected in loan deliquency rates that begin rising while the economy is still expanding (see Fig. 1). In loan-level data, this is reflected in greater realized default rates on loans originated in booms (see Jiménez and Saurina, 2006).

While there is presently no adequate dataset that would allow one to cleanly parse out the separate effects of lending terms and aggregate conditions on the pool of financed entrepreneurs in the U.S., there is a growing need to better understand financial markets plagued with adverse selection.

In this paper, we develop a general model of financial markets for entrepreneurs of varying productivity, characterized by adverse selection and competitive banks. The aggregate state enters as an exogenous productivity component, common to

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[1] A loan size and time to loan maturity are typical examples.

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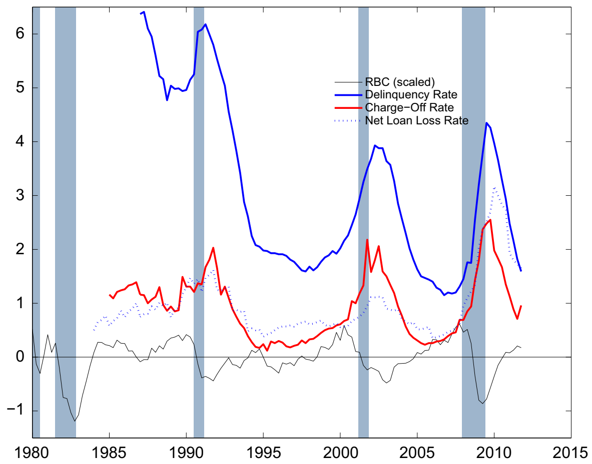

Fig. 1. Real business cycles, delinquency and charge-off rates on C&I loans. *Notes*: Charge-offs, which are the value of loans removed from the books and charged against loss reserves, are measured net of recoveries as a percentage of average loans, and annualized. The shadowed areas are NBER marked recessions.

all entrepreneurs. Our first goal is to explain under what general conditions screening weakens in the aggregate state. We refer to this regularity as the *lending terms easing*. Second, we examine the effect of the *lending terms easing* on productivity at a producer level (the *intensive margin*), the mix of financed entrepreneurs (the *extensive margin*), and the net effect.

Our assumptions best describe the markets for informationally opaque firms – relatively recent entrants and small business that lack collateralizable wealth. Several facts warrant our focus on this sector. Productivity of entering firms accounts for a major part of industry-level productivity dynamics (e.g. Foster et al., 2002). This sector is particularly important in accounting for the economic fluctuations. (Lee and Mukoyama (2015) document that job creation by manufacturing startups in booms exceeds the net job creation of preexisting firms.) Finally, the less established firms are most susceptible to fluctuations in lending terms.

To understand the key insights, consider the setup with only two types, the productive repaying type (G) and the unproductive defaulting type (B). Financial arrangements entail either financing of both types under a pooling contract or financing of type G alone under a separating contract. Separation is accomplished with a loan size offer that is too small for B to afford production. This provides an important insight: to credibly signal its type, G must suffer from the screening cost, which in our case entails operating at a less than optimal scale. The equilibrium contract maximizes G's utility subject to the financiers' zero profit condition (Hellwig, 1987).

The *lending terms easing* result emerges in the presence of two jointly sufficient effects. If exogenous increments to entrepreneurs' productivity raise returns to investment and tighten the credit line needed to screen out a given entrepreneur type, separation gives way to pooling in better aggregate states. Larger loans are then made to a worse mix of borrowers. Note that neither of the two effects is necessary: if one is missing, the other effect, if sufficiently strong, will ensure the result.

This qualitative shift in the financial market equilibrium influences the average productivity of entrepreneurs. On one hand, larger loans allow for more investment at the producer level, moving production scales closer to optimal and therefore increasing productivity at the *intensive margin*. On the other hand, low productivity entrepreneurs engage in production driving the average productivity down through the *extensive margin*. If type B is sufficiently unproductive, the *extensive margin* dominates, and pooling is inefficient from the standpoint of a social planner with the same informational constraints as the banks.

The *lending terms easing* result naturally generalizes to a setting with multiple types of entrepreneurs of varying productivity: every qualitative change in the financial market equilibrium, induced by better economic conditions, entails larger loans made to a worse pool of entrants.

Our setup with multiple producer types helps reconcile the two opposite views of financial frictions found in the literature (see Section 2 for details). One strand argues that financial frictions amplify exogenous productivity shocks, while the second strand argues the opposite. Both forces operate in our model. The amplification force, operating at the *intensive margin*, dominates in low aggregate states. The dampening force, operating at the *extensive margin*, dominates in high aggregate states.

An example with four types, given in Section 4.3, effectively illustrates the growing relative importance of the *extensive margin*. The *intensive margin* is at its strongest in low aggregate states because lending terms are particularly tight, returns to scale are high, and the average quality of the unfinanced producers is relatively high. When productivity is low, *lending terms easing* amplifies positive productivity shocks, as the benefit of relaxing investment constraints outweighs the negative impact from allocating some of the funds to the less productive types. Conversely, the *extensive margin* dominates at high aggregate states because production scales are already close to optimal and the pool of the unfinanced producers is particularly bad. Further easing of terms allows for their entry and dampens, or even reverses, positive productivity shocks.

The empirical implications of the mechanism underlying the *lending terms easing* result are in line with the data. Screening is relaxed during expansions. Loan size and entrepreneurs' investment levels rise. Laxer lending terms appearing at relatively high states entail financing of ex-ante unproductive projects. This helps interpret the observation that default rates begin rising during expansions (Berger and Udell, 2004; Lown and Morgan, 2006; Rajan, 1994; Fig. 1). Information asymmetry in the presence of ex-ante bad producers is essential for explaining the latter observation. (An alternative view that defaults are due to ex-ante good projects suffering from unfavorable realization of uncertainty is inconsistent with default rates rising in expansions if unfavorable shocks become less likely in expansions.) The model's implications for firm selection on productivity are also in line with the data: Lee and Mukoyama (2015) document that manufacturing plants entering under weaker economic conditions are of higher quality. In our model, the relatively high entrant quality in low aggregate states is a result of endogenously tight financing terms. Implications for the measure of productivity dispersion also conform to the data (see Section 4).

An important insight is contained in the intuition for the result that this economy is not always at its constrained-efficient output level. At the very top, returns to investment are particularly high and larger loans are particularly attractive. Competition for the productive types then leads to inefficiently lax lending terms because they allow for production at the optimal scale. Lemons enter as a side effect of abandoned screening. Interestingly, consistent with our model's insight, "intense competition to attract customers" is the only reason behind *lending terms easing* stated in the supervisory letter SR 98-18 of the Board of Governors. Conversely, when the economic conditions are bottomed out, the benefit to increasing the loan size, as seen by the most productive entrepreneurs, is simply not that high, and therefore lending terms are inefficiently tight.

In Section 2, we review related literature. In Section 3, we develop the model and derive the main results for the economy with two types of entrepreneurs. Section 4 generalizes the results to a setting with multiple types and formalizes the discussion of the amplification and dampening effects. Section 5 concludes.

2. Related literature

In order to understand the composition of financed producers on ex-ante unobservable characteristics, such as their ability to run a successful business, it is essential to study the problem of adverse selection in financial markets. This literature, however, is surprisingly small.

The two classic papers on adverse selection that can be used to study the composition of financed entrepreneurs are Stiglitz and Weiss (1981) and de Meza and Webb (1987) (henceforth, SW and DW). These papers differ only in the assumption on the project distribution. Considering pooling contracts, DW (SW) show that inefficiently many (few) projects are financed in equilibrium.[2] Our paper is more closely related to DW. The intuition for overinvestment in DW is as follows. Producer heterogeneity implies heterogeneity in expected profits for the bank. Since the overall profits are driven to zero by competition, the less productive entrepreneurs must be generating negative expected profits. A similar intuition applies in the case of pooling equilibria in our paper.

Relative to the model employed in SW and DW, we work with a more standard production structure, so that our entrepreneurs vary in optimal production scales and therefore investment needs, which better captures the salient features of markets for new entrants. Importantly, this assumption introduces the possibility of screening via loan size offers.

Our contribution is to study how the composition of entrepreneurs and its contribution to aggregate productivity evolve with the aggregate state. Another paper with a similar focus is House (2006) which embeds the static model used in SW and DW into an overlapping generations framework in order to endogenize the link between financial markets and productivity. As expected in light of the static results in DW and SW, it is shown that adverse selection can either amplify or dampen aggregate productivity shocks, depending on a particular parameterization of the project distribution.

In contrast, both forces operate in our model simultaneously, for a given parameterization. The amplification force, operating at the *intensive margin*, dominates in low aggregate states. The dampening force, operating through the composition of producers, dominates in high aggregate states. Therefore, our work can be viewed as bringing closer together the two opposing strands of literature on credit market frictions. The first strand argues that credit market imperfections amplify exogenous productivity shocks. The second strand argues the opposite.

Amplification forces are typically generated via moral hazard problems (e.g. Bernanke and Gertler, 1989 and variants) or contract enforcement frictions (Kiyotaki and Moore, 1997 and variants). A prominent amplification mechanism is due to Bernanke and Gertler (1989), where the entrepreneurs may lie about their output. Economic upturns improve the borrowers' net worth, which lowers agency costs of financing investment and increases investment. The amplification mechanism present in our model at the *intensive margin* is similar: exogenous productivity increments imply less screening and hence greater investment and output per producer. Also see Williamson (1987) and Rampini (2004).

The second strand of literature argues that credit market imperfections dampen exogenous productivity shocks. Moral hazard is typically behind the dampening forces. Among these are Suarez and Sussman (1997), Reichlin and Siconolfi

[2] We know from DW that the SW result is not robust to allowing for equity financing. In contrast, the DW and our results extend to equity contracts.

(2004).[3] and Favara (2012). Azariadis and Smith (1998) have proposed a model of adverse selection where the downturn is caused by a switch in savers' expectations of the future interest rate. Martin (2008) has also proposed a model of adverse selection, but with no action along the *extensive margin*. The two producers are always financed. But financial contracts arising at the top of the cycle entail strict lending terms, with low credit lines and high collateral requirements. Low investment then causes low output in the next period. The opposite is true in our model: lending standards are lax and entrepreneurs' investment levels are high when the economy is strong, consistent with empirical evidence.

Finally, our study also contributes to a strand of literature in finance that offers explanations of laxer financing terms in expansions. These include Berger and Udell (2004), Dell'Ariccia and Marquez (2006), Ruckes (2004), and Rajan (1994). Our explanation complements these by accurately capturing the "competitive pressure" explanation reported by the loan officers themselves.

3. A theory of lending standards

In this section, we discuss how lending terms depend on the state of the economy z in an economy with two (private) types of entrepreneurs and a competitive banking sector. The state of the economy z captures current economic conditions. It is convenient to think of it as the common exogenous component of productivity. We purposefully keep the environment very general so it can be nested in a variety of dynamic models.

We derive two jointly sufficient conditions, neither one of which is necessary, that ensure that lending terms weaken as the state of the economy z increases, the result we refer to as the *lending terms easing* throughout the paper. Since both conditions are likely to be satisfied across a variety of model specifications, we view the *lending terms easing* result as highly robust. Generality of our setup ensures transparency of assumptions underlying the main results.

3.1. A general setup with two types of entrepreneurs

Consider a partial equilibrium environment with a competitive banking sector and a sector of production that benefits from external finance. Production is carried out by entrepreneurs that hold no collateralizable wealth.

There are two privately known types of entrepreneurs, $i \in \{G, B\}$, present in proportions μ and $1-\mu$. With external financing available, they produce output according to the primary technology $f^i(x,z)$, where x is the size of the loan. We implicitly assume that formal production always takes place, which can be justified by the availability of a monitoring technology. Alternatively, both types can contribute $h(z)$ to output, which captures the outside option such as working for wages. We assume that with zero financing available, the outside option is preferred, and with a large enough loan size, formal production is preferred. We also assume type B is less productive than type G in his primary technology.

Assumption 1. $f^i(0,z) < h(z) < \lim_{x\to\infty} f^i(x,z)$ for all z; and $f^G(x,z) > f^B(x,z)$ for all x,z.

Standard technology assumptions are stated below.

Assumption 2. $f^i_x, f^i_z, h_z > 0$, and $f^G_{xx}(x,z) < 0$ whenever $f^G(x,z) > 0$.[4]

Note that both the state of the economy and the loan size increase production. One example of primary production technology is $f^i(x,z) = A^i z^\psi x^\alpha - F^i$, where it is assumed that the state of the economy z positively influences productivity, A^i and F^i are type-specific productivity and fixed cost.

In addition, we assume that type B agents have the ability to abscond with the proceedings of their production, and therefore never repay their loans.[5]

Finally, there is a competitive banking sector, with X_s as the amount of loanable funds. A risk-free savings technology is available to the bank at rate R_f. We assume banks do not have access to a technology that would allow them to learn the entrepreneur's type. This assumption is well grounded in the data, as banks rarely engage in new business idea evaluation.

3.2. Financial market equilibrium

In order to discuss the impact of the state of the economy z on lending terms, it suffices to define a financial market equilibrium, for a given z.

[3] We should note that, just as in House (2006), the model in Reichlin and Siconolfi (2004) could host either an amplification or a dampening force (but not both), depending on a particular parameterization.

[4] Note we do not assume that $f^G_{xx} < 0$ for all production levels. This allows us to accommodate fixed costs, e.g. $f^G(x,z) = g(x,z) - F$. Therefore, our setup accommodates both increasing and decreasing returns to scale at different production levels.

[5] For simplicity, we assume that type B defaults. It is straight-forward to endogenize this particular resolution of moral hazard by making the default decision a choice, assuming that banks seize a fraction of output in case of default and that B is sufficiently unproductive. The productive type has more to lose in case of default, and therefore repays.

We assume that loan contracts are of the form (x,R), where x is the loan size and R is the gross interest rate. The entrepreneurs' payoffs from a loan (x,R), for a given z, are

$$U^G(x,z,R) = f^G(x,z) - Rx, \tag{1}$$

$$U^B(x,z) = f^B(x,z). \tag{2}$$

Type B' s payoff is independent of loan interest rate because he never repays.

Definition 1. A financial market equilibrium, for a given aggregate state z, is given by loan contracts (x^G,R^G), (x^B,R^B) such that (1) they induce self-selection and participation by entrepreneurs, and participation by banks; (2) banks make zero profits; (3) (x^G,R^G) is the best contract for type G among all contract menus $\left\{(x^G,R^G),(x^B,R^B)\right\}$ satisfying (1) and (2).

In the definition above, we adopt the standard practice of finding equilibrium contracts in competitive markets with adverse selection as maximizers of type G's payoff subject to the zero-profit condition, a result due to Hellwig (1987). In models with private information and competition, Rothschild and Stiglitz (1976) showed that either the separating contract preferred by the good type is an equilibrium (if it dominates the best pooling contract), or there is no equilibrium at all. By explicitly modeling a three stage game played between banks and entrepreneurs, Hellwig (1987) showed that the contract preferred by G, whether pooling or separating, is the equilibrium contract. Our financial market setup differs from the setup in Hellwig (1987) in the presence of limited funds, which warrants further justification of the equilibrium concept. Precisely, we need to explain that the possibility of rationing does not alter Hellwig's conclusion. It suffices to show that a pooling contract withstands deviations by a bank seeking a profit opportunity.[6]

The assumption that type G repays and type B defaults implies that if both types are financed, they must be financed under the same (pooling) contract. This result is formalized in the lemma below. Intuitively, if types are financed through different contracts, to ensure self-selection by B, we must have $x^G < x^B$. It follows that to ensure bank participation, type G must pay a substantial cross-subsidy. A pooling contract that offers x^B to both types and ensures bank participation requires a smaller cross-subsidy, and therefore will be preferred by type G entrepreneurs.

Lemma 1. *In equilibrium, financing of both types occurs only under the same (pooling) contract. Formally, if $x^B > 0$, then $x^G = x^B$ and $R^G = R^B$.*

Proof. See the appendix.

In light of the previous lemma, we can restrict our attention to contract menus which either specify identical contracts for both types (a pooling contract), or specify different contracts with a zero contract offered to type B, $x^B = 0$ (separating contracts). In the latter case, type B self-selects into a zero contract, and hence produces output according to $h(z)$ (in light of Assumption 1). Thus, it suffices to restrict our attention to a menu of single contracts offered to type G that either pool or exclude type B, so we drop type-dependence to simplify notation. Considering the menu of such contracts satisfying (1) and (2) in the definition of the financial market equilibrium, it is clear that the interest rate is $R = R_f$ on separating contracts and $R = R_f/\mu$ on pooling contracts.

For any level of z, it is possible to exclude type B by simply setting the loan size sufficiently low. Formally, since $f^B(0,z) < h(z) < \lim_{x\to\infty} f^B(x,z)$ and $f_x^B > 0$ for all z (due Assumptions 1 and 2), there exists a unique cutoff level $\overline{x}(z) > 0$ of the loan size that equates B' s payoff from selecting type G' s loan contract to the outside option:

$$U^B(\overline{x}(z),z) = h(z). \tag{3}$$

It follows that type B is excluded for all $x \le \overline{x}(z)$ and financed under a pooling contract for all $x > \overline{x}(z)$. Therefore, among the contracts satisfying participation, self-selection and zero bank profit – the first two equilibrium conditions in Definition 1 – the relevant set is of the form

$$X(z) \equiv \left\{ (x,R) \middle| \begin{array}{ll} R = R_f & \text{if } x \le \overline{x}(z) \\ R = R_f/\mu & \text{if } x > \overline{x}(z) \end{array}, \quad G \text{ enters} \right\}.$$

To guarantee that the above set is always nonempty, we assume type G accepts the maximum loan size that excludes type B. It is essentially an assumption that type G is sufficiently productive to prefer formal production over the outside option.

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[6] To be explicit, consider the three stage game, where banks offer contracts, entrepreneurs apply to banks, and finally banks choose to accept or reject applications. The key assumption regarding the limited funds disbursement is that all borrowers, regardless of which bank they applied to, enter a queue at random. Funds are disbursed to borrowers in the queue according to their contract. We want to show that a pooling contract that generates zero profit and is preferred by G to a separating contract is the equilibrium contract. It suffices to show that it withstands two possible deviations by a bank seeking a profit opportunity. The first deviation is to offer a contract with a higher R. Such a contract will only attract the defaulting type (insensitive to R), and will therefore make losses. The second is to offer a contract with a smaller loan size and a smaller R in an attempt to disproportionately attract type G. In this case though, other banks will face a worse than average pool, and will reject all applicants. If the deviating contract dominates the outside option for B, the same line of argument as in Hellwig (1987) applies: anticipating rejection from the original contract, type B will also apply to the deviating contract, which will then make losses. If the outside option dominates the deviating contract for type B (the deviating bank offers a separating loan), he will not apply. It is then also clear that type G will not apply either, as he prefers the pooling contract to the best separating contract by assumption. Note that one cannot improve their chances of being financed by applying to the deviating contract, as everyone faces the same queue.

Assumption 3. $U^G(\overline{x}(z), z, R_f) > h(z)$ for all z.

Because the equilibrium contract must also be preferred by G – the last equilibrium condition – it is found as the best contract for type G among the set $X(z)$. The following proposition derives the equilibrium contract.

Proposition 1. *For a given z, there exists a unique equilibrium contract, given by*

$$\left(x^*(z), R^*(z)\right) = \arg\max_{(x,R) \in X(z)} U^G(x, z, R). \tag{4}$$

It is found as the contract that type G prefers among the optimal pooling contract $\left(x_p(z), R_f/\mu\right)$ and the optimal separating contract $\left(x_s(z), R_f\right)$, with the corresponding loan sizes defined by

$$x_s(z) = \arg\max_{x \le \overline{x}(z)} U^G(x, z, R_f) \tag{5}$$

and

$$x_p(z) = \arg\max_{x > \overline{x}(z)} U^G(x, z, R_f/\mu). \tag{6}$$

Proof. See the appendix.

Given our setup, and in particular, the lack of collateralizable wealth and idea screening technology, the contract form we are considering is most general. Screening by limiting the loan amount is the best the banks can do. Moreover, this type of screening is always effective: the lenders always have the ability to screen out bad entrepreneurs by setting the loan size sufficiently low.

The interesting tradeoff between financial regimes is present in the range of z such that $f_x^G(\overline{x}(z), z) \ge R_f/\mu$. The optimal pooling loan size is interior, whereas the optimal separating loan size is given by $\overline{x}(z)$. On one hand, a separating loan size, although offered at a low (risk free) rate, is too restrictive from the perspective of type G. On the other hand, a pooling contract provides the desired amount of financing but involves a cross-subsidy from the good to the bad type ($R > R_f$), since the former must subsidize the default risk of the latter.

An important and intuitively appealing insight emerges. The cost of effective screening is born by the most productive entrepreneurs. For separation to emerge, the good type must be willing to operate at a lower than the optimal scale of production. This is a simple way to capture the idea that the productive type must be willing to provide "hard evidence" to effectively signal their type.[7]

The costs and benefits entailed by the pooling and separating contracts are the direct counterparts of those characterizing competitive insurance contracts in Rothschild and Stiglitz (1976). In that classic paper, from the point of view of the low-risk agent, a pooling contract provides optimal coverage but entails greater insurance costs, while a separating contract keeps the cost of insurance low but provides an inefficiently low coverage.

Even though our focus is solely on the region of z for which the tradeoff between the two financial regimes exists, the proposition below characterizes all three regions for completeness. Note that if the function $f_x^G(\overline{x}(z), z)$ increases in z, then the three regions outlined above (interior separation, corner separation, and regime tradeoff) appear in that order as z increases.

Proposition 2. *The equilibrium contract $\left(x^*(z), R^*(z)\right)$ is given by*

$$\begin{cases} \left(x_s^{int}(z), R_f\right) & \text{if } f_x^G(\overline{x}(z), z) < R_f \text{ (interior separation)} \\ (\overline{x}(z), R_f) & \text{if } R_f < f_x^G(\overline{x}(z), z) < \dfrac{R_f}{\mu} \text{ (corner separation)} \\ \operatorname{argmax}\left\{U^G\left(\overline{x}(z), z, R_f\right), U^G\left(x_p^{int}(z), z, \dfrac{R_f}{\mu}\right)\right\} & \text{if } \dfrac{R_f}{\mu} \le f_x^G(\overline{x}(z), z) \text{ (regime tradeoff)} \end{cases}$$

with $x_s^{int}(z)$ and $x_p^{int}(z)$ implicitly defined by the first order conditions

$$f_x^G(x_s^{int}(z), z) = R_f, \tag{7}$$

$$f_x^G\left(x_p^{int}(z), z\right) = \frac{R_f}{\mu}. \tag{8}$$

Proof. See the appendix.

Fig. 2, panel (a), illustrates graphically the selection of a separating contract in equilibrium in the presence of the regime tradeoff. The bold set gives the relevant menu of contracts $X(z)$. Among these, the contract that yields the highest utility to type G is chosen in equilibrium. The indifference curves of the productive type are defined by $f^G(x, z) - Rx = \overline{U}^G$, with the

[7] The idea is similar to "burning money" in game theory.

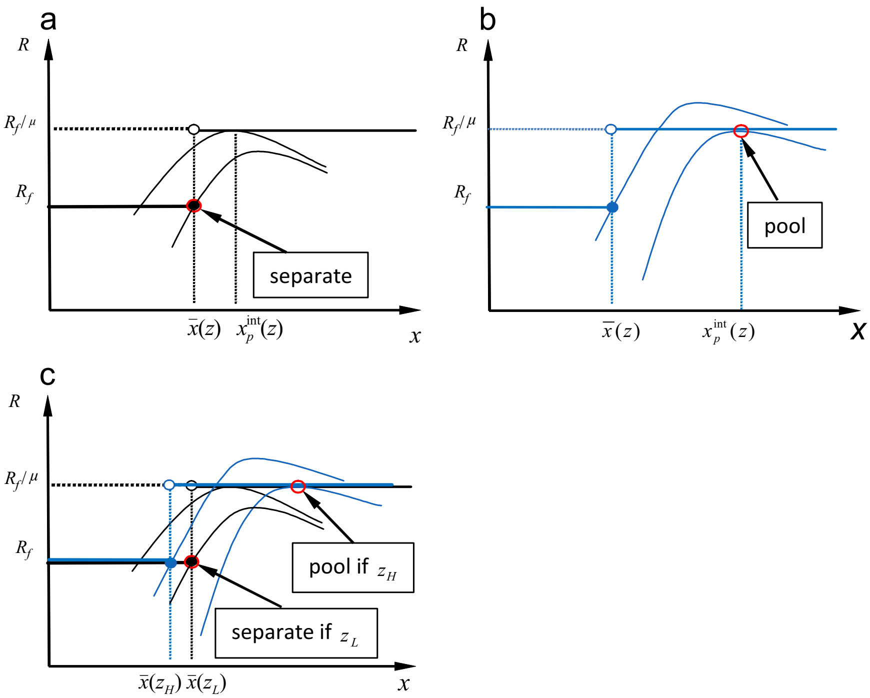

Fig. 2. Contract selection as a maximization of utility of type *G*. (a) Separating contract ($\bar{x}(z)$, R_f) dominates, (b) Pooling contract ($x_p(z)$, R_f/μ) dominates and (c) Equilibrium Contract for $z = z_L$ and $z = z_H$. *Notes*: This figure illustrates the banks' zero profit condition jointly with the indifference curves of type *G* in the space of loan size and interest rate. As the economy state *z* rises from z_L to z_H, the equilibrium loan contract switches from a separating one to a pooling one, allowing for the less productive type to enter production.

slope $\partial R/\partial x = (f_x^G(x,z) - R)/x$. Consider an indifference curve peaking at some given interest rate. The peak is associated with the optimal loan size at that interest. For smaller loan sizes, extra financing is desired and hence increases in *x* must be accompanied by increases in *R* to keep the utility level fixed. The opposite applies to loan sizes above the optimal level. Since the slope decreases in *R*, the optimal loan size is lower at higher interest rates. The two indifference curves in the figure represent the highest utility from separation $(\bar{x}(z), R_f)$ and pooling $\left(x_p^{int}(z), R_f/\mu\right)$. Separating contract is preferred. Similarly, Fig. 2, panel (b), illustrates selection of a pooling contract. In this case, the indifference curve corresponding to utility from the best pooling contract dominates.

The costs and benefits entailed by pooling and separating financial equilibria, as internalized by type *G*, clearly depend on the aggregate state *z*. It is then possible for a change in *z* to induce a qualitative shift in the financial regime. The implications formalized in the corollary below follow directly from Proposition 2.

Corollary 1. *At states z that induce a shift in the financial regime, i.e. z such that* $U^G(x_s(z), z, R_f) = U^G(x_p(z), z, R_f/\mu)$, *the regime tradeoff is present* $(f_x^G(\bar{x}(z), z) \geq R_f/\mu)$, *and the optimal pooling and separating loan sizes are given by* $x_p(z) = x_p^{int}(z)$ *and* $x_s(z) = \bar{x}(z)$.

Since, for a given *z*, the optimal pooling contract involves a larger loan size and financing of the unproductive/defaulting types along with the good type agents, we associate selection of pooling rather than separating contracts with laxer lending terms. Note that for an increase in loan size to imply laxer lending terms, it must be sufficiently large to induce type *B* to enter.

The cost of effective screening, as well as the benefit of pooling will depend on the state of the economy, and so will the choice of contracts by the productive type, thus determining whether or not screening is used in equilibrium. Our next step is to analyze under what conditions higher states *z* make the selection of pooling contracts more likely. These conditions would point to environments that accommodate relaxation of lending terms with better economic states.

3.3. Lending terms easing

Our objective is to characterize the set of environments that give rise to the *lending terms easing,* i.e. the result that lending terms deteriorate with *z*. We identify two jointly sufficient conditions, neither one of which is necessary. However, for the purpose of complete transparency, we begin by identifying the weakest sufficient (single crossing) condition.

Lemma 2. *Suppose that for all states z that induce a shift in the financial regime, i.e. z such that $U^G(x_s(z), z, R_f) = U^G(x_p(z), z, R_f/\mu)$, we have*

$$f_z^G(x_p(z), z) > f_z^G(\overline{x}(z), z) + [f_x^G(\overline{x}(z), z) - R_f]\overline{x}'(z). \tag{9}$$

Then there is at most one regime switching state $\overline{z}$, with a separating financial market equilibrium emerging for all $z \le \overline{z}$ and a pooling equilibrium emerging for all $z > \overline{z}$.

Proof. See the appendix.

Inequality (9) clearly reveals the effects needed for the existence of a threshold state of the economy above which type G prefers pooling and below which type G prefers separation. The left hand side reflects the increase in U^G from pooling as a result of a larger z. A larger z has a direct positive effect on productivity under pooling. The second effect through the change in the optimal loan size $x_p(z)$ disappears due to the envelope theorem.

The right hand side reflects the change in U^G from separation as a result of a larger z. The first term captures the direct positive effect of z on the amount of production under separation. The second term captures the change in U^G due to the change in the separating loan size. This term would be unambiguously negative if $\overline{x}'(z)$ were negative, that is, if credit lines had to be tightened with z to deter B' s entry. This term would then work in favor of generating weaker lending terms in better states z.

In fact, $\overline{x}'(z) \le 0$ can be ensured if we restrict attention to environments satisfying the following

Assumption 4. $h'(z) \le f_z^B(\overline{x}(z), z)$ for all z.

This assumption constitutes the first of the two sufficient conditions for the *lending terms easing* result. It states that, as the economy improves, the primary technology of type B realizes larger gains in output relative to the outside option. In the extreme case, the outside option could represent an income stream independent of z, such as income from menial jobs or welfare programs. Assumption 4 immediately implies that $\overline{x}'(z) \le 0$. Credit lines must be tightened with z to deter B' s entry, because becoming an entrepreneur becomes more attractive to type B relative to the outside option.

Inequality (9) helps identify the second sufficient condition for the *lending terms easing* result. The desired result emerges trivially in the case of complementarity of loan size and economic conditions ($f_{xz}^G > 0$). Such complementarity appears to be a plausible assumption, as it simply means the returns to investment rise in good times, and therefore a given unit of external finance increases its contribution to output. Most standard production functions are consistent with this type of complementarity. This is the case of our focus.[8]

Proposition 3 (*Lending Terms Easing*). *In the case of complementarity of loan size and productivity ($f_{xz}^G > 0$), there is at most one regime switching state $\overline{z}$, with a separating financial market equilibrium emerging for all $z \le \overline{z}$ and a pooling equilibrium emerging for all $z > \overline{z}$.*

Proof. Inequality (9) can be written as $f_z^G(x_p(z), z) - f_z^G(\overline{x}(z), z) > [f_x^G(\overline{x}(z), z) - R_f]\overline{x}'(z)$. The left hand side is positive due to $f_{xz}^G > 0$ and $x_p(z) > \overline{x}(z)$. The right hand side is negative due to Assumption 4.[9] The inequality holds and therefore the result in Lemma 2 applies. □

The intuition behind the *lending terms easing* hinges on two central effects, corresponding to the two sufficient conditions. First, as already discussed, the cost of screening endogenously rises with economic conditions: it takes a larger loan size restriction ($\overline{x}'(z) < 0$) to screen out type B. Second, the benefit of a pooling contract, which entails no investment restriction, increases because better economic conditions raise returns to investment. Therefore, as economic conditions improve, separating contracts give way to pooling ones.

Fig. 2, panel (c) illustrates equilibrium contract selection for two states of the economy. Separation is chosen under the lower state z_L. Under the higher state z_H, type G prefers pooling. The two effects discussed above are clearly seen in the diagram. First, the relevant menu of contracts $X(z)$ changes to the blue bold set, as credit must be restricted more ($\overline{x}(z_H) < \overline{x}(z_L)$) to keep separation viable. This implies that the indifference curve corresponding to the new best separation ($\overline{x}(z_H), R_f$)) is north-west of the original one, and separation is less attractive relative to pooling. Second, the slope of a given indifference curve increases in light of complementarity of x and z. Intuitively, as z increases, an extra unit of investment becomes more valuable. Therefore, for a given rise in x, R must increase more (or decrease less if in the range of the downward sloping part) to keep the agent indifferent. Consider the original best pooling contract $\left(x_p^{int}(z_L), R_f/\mu\right)$, corresponding to the optimal loan size at interest R_f/μ and given by the point of tangency of the black indifference curve. The optimal loan size rises, and the relevant pooling indifference curve is steeper and to the right of the original one. Meanwhile, the relevant separating indifference curve is steeper and rotated north-west relative to the original separating indifference curve. Both movements make pooling more attractive, as seen in the graph.

[8] Note that in the case of complementarity of x and z in production and under Assumption 4, the function $\rho f_x(\overline{x}(z), z)$ increases in z, and therefore the regions appearing in Proposition 2 (interior separation, corner separation and case of regime tradeoff) appear in that order as z increases.

[9] The term in the brackets is positive by Corollary 1.

Neither one of the two jointly sufficient conditions is necessary for the *lending terms easing* result. If one of the conditions fails, the other simply has to be strong enough to ensure inequality (9). Thus, even in the case of $\overline{x}'(z)>0$, the result will go through under sufficiently strong complementarity. Likewise, even in the less plausible case of substitutability of x and z in production ($f^G_{xz}\le 0$),[10] the result will hold as long as the effect due to Assumption 4 is dominant. In order for a pooling contract to be preferred in higher states of the economy, the negative effect of z on G's utility from separation, which is due to the greater tightening of the loan size ($\overline{x}'(z)<0$), must dominate. A sufficient condition for this is a bound on f^G_{xz} from below. Characterization of this bound is relegated to the appendix.

In our model, laxer lending terms appearing in relatively high states necessarily imply financing of ex-ante bad projects, which helps interpret two observations. Information asymmetry and the presence of ex-ante bad projects are critical ingredients of our model, and we argue both are essential for explaining these two observations.

First, there is direct evidence that ex-ante bad projects exist and get increasingly funded in expansions (Rajan, 1994). Expansive empirical literature that records weaker screening effort applied in expansions on the part of banks, cited in the Introduction, also makes it plausible that ex-ante bad projects would be able to get funded. In fact, the supervisory letter SR 98-18 of the Board of Governors estimates that nearly 80% of new loans made in 1997 were approved without any formal projection of the borrowers' future performance. Financing of ex-ante bad projects is probably best exemplified with the dot-com mania of the late nineties and the recent subprime mortgage lending.

The second observation that our model with ex-ante bad projects can help interpret is that the average delinquency and default rates begin rising during economic expansions (Fig. 1). Because we allow for private information regarding types, ex-ante bad projects can be financed in an equilibrium as a side effect of abandoned screening. An alternative model that views default as due to ex-ante good projects suffering from unfavorable realization of uncertainty is inconsistent with this fact: if unfavorable realizations become less likely in expansions, this model would imply that the average default rate declines in expansions.

3.4. Implications of lending terms easing for productivity

In our model, the qualitative nature of the financial market equilibrium directly influences entrepreneurs' output. Precisely, financial markets determine two endogenous components of the average productivity: production scale and composition of producers in formal markets. In this section, we examine the effect of the *lending terms easing* result on entrepreneurs' average productivity and output. Whenever an increase in z implies a regime switch in financial markets, it induces an increase in productivity along the *intensive margin* and a decrease along the *extensive margin*. We relate the dominance of a given margin to the model primitives.

Total entrepreneurs' production is given by

$$y(z)=\begin{cases} y_s(z) & \text{if } U^G(x_s(z),z,R_f)>U^G(x_p(z),z,R_f/\mu) \\ y_p(z) & \text{otherwise}\end{cases}, \tag{10}$$

where regime-specific output is given by $y_s(z)=\nu_s(z)f^G(x_s(z),z)+(1-\nu_s(z))h(z)$ and $y_p(z)=\nu_p(z)\left[\mu f^G(x_p(z),z)+(1-\mu)f^B(x_p(z),z)\right]+(1-\nu_p(z))h(z)$, and regime-specific measures of entrepreneurs financed are given by $\nu_s(z)=\min\{X_s/x_s(z),\mu\}$ and $\nu_p(z)=\min\{X_s/x_p(z),1\}$. If all the available funds are employed in production, $\nu_i(z)x_i(z)=X_s$ where $i\in\{s,p\}$, then average productivity, defined as output per unit of investment, is given by $A(z)=y(z)/X_s$.

For any z, the selection of a pooling rather than a separating contract then implies a change in average productivity, which can be decomposed into productivity change at the *extensive* and *intensive margins*:

$$\frac{y_p(z)-y_s(z)}{X_s}=\left[\frac{[\mu f^G(x_p(z),z)+(1-\mu)f^B(x_p(z),z)]\nu_p(z)}{X_s}-\frac{f^G(x_p(z),z)\nu_p(z)}{X_s}\right] + \left[\frac{[f^G(x_p(z),z)-h(z)]\nu_p(z)}{X_s}-\frac{[f^G(x_s(z),z)-h(z)]\nu_s(z)}{X_s}\right]. \tag{11}$$

The first bracketed term in (11) represents the change in productivity at the *extensive margin*, which captures the influence of the shift in producer composition. It is evaluated keeping the loan size fixed at $x_p(z)$. This effect is unambiguously negative as a fraction of type G producers become crowded out by the less productive type B producers. The strength of the *extensive margin* clearly depends positively on the relative measure of the defaulting type and negatively on its relative type-specific productivity.

The second bracketed term represents the change in productivity at the *intensive margin*, evaluated keeping the composition of producers fixed. This effect depends on the returns to scale of the net output function $f^G(x,z)-h(z)$ for $x\in(x_s(z),x_p(z))$. Note that $f^G(x,z)-h(z)$ is the relevant production function as it describes the gain in output resulting from reallocating type G from engaging in the outside option to being an entrepreneur. Recall that $f^G(x,z)$ is concave whenever it

[10] One example is the case of z entering the primary technology only as an internal liquidity term, which substitutes for external financing, $f^i(x,z)=f^i(x+z)$. Because f^i is concave, the value of external funds falls in z. Thus, as z increases, the benefit of pooling, i.e. the unrestricted loan size, declines relative to separation: $\rho f^G_z(x_p(z),z)-\rho f^G_z(x_s(z),z)<0$.

is positive, and the outside option $h(z)$ is positive and independent of x, which gives rise to a region of increasing returns for $f^G(x,z)-h(z)$ even if $f^G(x,z)$ exhibits decreasing returns. Since, in addition, Kehrig (2011) documents non-decreasing returns to scale at a plant level among the manufacturing firms, we assume non-decreasing returns to scale of $f^G(x,z)-h(z)$, at the regime switching point $\overline{z}$. It ensures that an amplification mechanism, similar to the one in Bernanke and Gertler (1989), appears at the *intensive margin*. Laxer lending terms induced by an increase in z amplify its direct positive effect on productivity by lifting investment restrictions at the producer level. Lemma 3 formalizes this discussion.

Lemma 3 (*Positive intensive margin*). *If the net production function $f^G(x,\overline{z})-h(\overline{z})$ exhibits non-decreasing returns to scale,*

$$f_x^G(x,\overline{z}) \geq \frac{1}{x}\left[f^G(x,z)-h(z)\right], \tag{12}$$

for all $x \in [x_s(\overline{z}), x_p(\overline{z})]$, then the switch from separation to pooling (at $\overline{z}$) implies an increase in output and productivity at the intensive margin.

Proof. See the appendix.

Our contribution to the existing literature on credit market frictions and aggregate output is to highlight the presence of an adverse producer composition effect coexisting with an amplification effect: laxer lending terms that endogenously emerge in higher states of the economy necessarily imply a worsening of the producer pool.

Proposition 4 relates the dominance of the *extensive margin* to the model primitives, and in particular, characteristics of the producer pool and returns to scale of the net output function.

Proposition 4 (*Extensive margin dominance*). *Suppose at the regime switching point $\overline{z}$, the economies of scale of the net production function of G can be bounded, $(\partial/\partial x)f^G(x,\overline{z})-h(\overline{z})/x \leq M$, with the constant M satisfying*

$$M\left[x_p(\overline{z})-x_s(\overline{z})\right] \leq (1-\mu)\frac{f^G(x_p(\overline{z}),\overline{z})-f^B(x_p(\overline{z}),\overline{z})}{x_p(\overline{z})}. \tag{13}$$

Then the extensive margin dominates: the switch from separation to pooling at $\overline{z}$ implies a drop in output and productivity, $y_p(\overline{z}) \leq y_s(\overline{z})$ and $A_p(\overline{z}) \leq A_s(\overline{z})$.

Proof. See the appendix.

If the average productivity of type B is sufficiently low relative to that of type G and/or their measure $(1-\mu)$ is sufficiently high, as seen on the right hand side of (13), the negative effect on output at the *extensive margin* dominates the positive effect at the *intensive margin*, implying a drop in output and productivity under the switch from separation to pooling. The positive effect on output at the *intensive margin*, as seen on the left hand side of (13), captures output gains due to the increasing returns to scale, which are bounded by the increase in the loan size $(x_p(z)-x_s(z))$ and the size of economies of scale (bounded by M).

In the setup with two types, there is at most one regime switch, with one of the two opposing effects dominating. In the multiple types setting, however, studied in Section 4, the relative strength of the two effects varies across the multiple regime switching points, and therefore different margins may prevail at different states of the economy. The insight offered by the *lending terms easing* result therefore reconciles the findings of the two opposing strands of related literature, one viewing credit market imperfections as amplifying exogenous productivity shocks and one viewing them as responsible for reverting productivity trends (See Section 2).

4. An extension to multiple types of entrepreneurs

We generalize this environment to n types of entrepreneurs with varying productivity and show that the *lending terms easing* result naturally extends to this generalization: every qualitative change in financial market equilibrium induced by better economic conditions worsens the pool of entrants. The presence of multiple types, however, allows for multiple financial regime switches, which potentially entail different net effects on average productivity.

Consider types $i \in \{1,\dots,n\}$ of probability measures $\{\mu_i\}_{i=1}^n$, assume that types are ordered in their productivity according to $f^i(x,z) > f^j(x,z)$ if $i<j$, and that only the most productive type repays. Type 1 in our setup represents all repaying types, with $f^1(x,z)$ standing in for their average productivity.[11] Moreover, we assume that all types have the same outside option $h(z)$, and maintain, as in Assumption 1, that with no financing, the outside option is preferred, whereas it is dominated for a large enough loan: For all z,

$$f^i(0,z) < h(z) < \lim_{x\to\infty} f^i(x,z). \tag{14}$$

[11] While it is possible to also introduce heterogeneity of repaying types, doing so would not offer additional insight. From the point of view of the bank, what matters is whether an entrepreneur repays. From the point of view of the economy on aggregate, only the average productivity of repaying types matters.

Finally, as in Assumption 2, we impose monotonicity and concavity of production functions: $f^i_x, f^i_z, h_z \geq 0$, and $f^1_{xx}(x,z) < 0$ whenever $f^1(x,z) > 0$. The financial market equilibrium definition is generalized as follows:

Definition 2. A financial market equilibrium, for a given state of the economy z, is given by loan contracts $\{(x_i, R_i)\}_{i=1,\dots,n}$ such that (1) they induce self-selection and participation by entrepreneurs and banks; (2) banks make zero profits; (3) (x_1, R_1) is the best contract for type 1 among all contract menus $\{(x_i, R_i)\}_{i=1,\dots,n}$ satisfying (1) and (2).

As in Section 3, it is straightforward to prove that equilibrium contracts that entail participation of defaulting types must be pooling.

Lemma 4. *If type $i \geq 2$ is financed, it is financed along with other more productive entrepreneurs under a pooling contract. Formally, if $x_i > 0$ for some $i \geq 2$, then $(x_1, R_1) = \dots = (x_i, R_i)$, and $R_1 = \dots = R_i$.*

Proof. Analogous to the proof of Lemma 1.

In light of (14), any level of separation can be implemented: for any level of separation k, types $i \geq k$ can be excluded by offering a sufficiently low credit line $\overline{x}_{k-1}(z)$, implicitly defined by setting formal production of type k equal to the outside option:

$$f^k(\overline{x}_{k-1}(z), z) = h(z).$$

Since formal output f^k increases in x, type k is financed for all $x > \overline{x}_{k-1}(z)$ and excluded for $x \leq \overline{x}_{k-1}(z)$. Because types are ordered by their productivity, it is clear that $\overline{x}_i(z) < \overline{x}_j(z)$ for any $i < j$, that is, it takes a smaller loan size to exclude the more productive types. It follows that loans of size $x \in (\overline{x}_{k-1}(z), \overline{x}_k(z)]$ finance the defaulting types $i \leq k$, and that, provided type 1 accepts the contract, the zero banking profit condition reduces to $R \equiv R_f(\mu_1 + \dots \mu_k)/\mu_1$. In general, an increase in the loan size induces participation of the less productive defaulting types, inducing an unfavorable composition effect.

The relevant set among the contracts satisfying self-selection, participation and zero bank profit, i.e. conditions (1) and (2) from Definition 2, is of the form

$$X(z) \equiv \left\{ (x,R) | R = R_k \equiv \frac{R_f}{\mu_1} \sum_{i=1}^{k} \mu_i \text{if } x \in (\overline{x}_{k-1}(z), \overline{x}_k(z)], k \in \{1, \dots, n\}, \text{type 1 enters} \right\},$$

where we set $\overline{x}_0 = 0$ and $\overline{x}_n = \infty$. As in Assumption 3, we impose that a fully separating contract, which finances type 1 alone, induces entry of type 1 entrepreneurs, $U^1(\overline{x}_1(z), z, R_f) > h(z)$ for all z, which guarantees that $X(z)$ is non-empty.

The equilibrium contract must also satisfy requirement (3) from Definition 2. Hence, it can be found as the best contract for type 1 among the set $X(z)$. The following proposition characterizes the equilibrium contract.

Proposition 5. *For a given z, there exists a unique equilibrium contract. It is given by*

$$\left(x^*(z), R^*(z)\right) = \arg \max_{(x,R) \in X(z)} U^1(x,z,R). \tag{15}$$

It can be found as the contract that type 1 prefers among $\{x_k(z), R_k\}_{k=1}^n$, where $x_k(z)$ denotes the optimal loan size for a fixed level of separation $k \in \{1, \dots, n\}$:

$$x_k(z) \in \arg \max_{x \in (\overline{x}_{k-1}(z), \overline{x}_k(z)]} f^1(x,z) - R_k x. \tag{16}$$

Among these, we refer to $x_1(z)$ as a fully separating loan, to $x_k(z)$ with $k \in \{2, \dots, n-1\}$ as semi-pooling loans financing all $i \leq k$ entrepreneurs, and to $x_p(z) \equiv x_n(z)$ as a pooling loan.

Proof. See the appendix.

As in Section 3, the interesting case is where the regime tradeoff exists, so that for a given level of separation k, a contract financing an additional type presents both a benefit and a cost to type 1 entrepreneurs:

$$f^1_x(\overline{x}_k(z), z) \geq R_{k+1}. \tag{17}$$

On one hand, the credit line that finances types $i \leq k+1$ is less restrictive than the credit line financing types $i \leq k$. On the other hand, it is offered at a higher interest rate, $R_k < R_{k+1}$. This tradeoff is always present at points of regime switches, and therefore the solution to (21) is $x_k(z) = \overline{x}_k(z)$, as formalized in the lemma below.

Lemma 5. *At any state z that induces a switch from financing i to financing $j > i$ entrepreneurs, the optimal loan size under the more restrictive regime i is given by $x_i(z) = \overline{x}_i(z)$.*

Proof. See the appendix.

We seek to identify general and transparent conditions that ensure that the *lending terms easing* result generalizes to the multiple types setting, i.e. that financial contracts become less and less exclusive with z. A fully separating contract $\left(x_1(z), R_f\right)$ is chosen only for a sufficiently low level of z, the equilibrium loan size increases as the contracts become more and more inclusive, with $\left(x_p(z), R/\mu_1\right)$ emerging for a sufficiently high level of z.

Analogous to Assumption 4, we assume that, as z rises, the primary technology of all types realizes larger gains in output relative to the outside option: for all z and $k \geq 2$,

$$h'(z) \leq f_z^k(\overline{x}_{k-1}(z), z). \tag{18}$$

This assumption implies that, as z rises, the formal production option becomes more attractive to the excluded types, therefore making it more difficult to keep a given level of separation viable. To maintain a given level of separation, a larger loan size restriction must be put in place to ensure that the excluded types continue choosing the outside option: $\overline{x}'_{k-1}(z) \leq 0$. Therefore, the cost of any given level of separation, as internalized by type 1, rises with z.

4.1. Lending terms easing

Lemma 6 identifies the general condition, analogous to inequality (9), that guarantees that any change in the financial regime necessarily entails a switch towards the less exclusive contracts, thus worsening the pool of entrants. The condition states that as z goes up, the less restrictive contracts appear marginally more attractive to type 1, relative to the more restrictive ones.[12]

Lemma 6. *Suppose that for all states z that induce a switch from financing i to financing $j > i$ entrepreneurs, i.e. z such that $U^1(x_i(z), z, R_i) = U^1(x_j(z), z, R_j)$, we have*

$$f_z^1(x_j(z), z) + [f_x^1(x_j(z), z) - R_j]x_j'(z) > f_z^1(x_i(z), z) + [f_x^1(x_i(z), z) - R_i]x_i'(z). \tag{19}$$

Then there exists at most one regime switching state $\overline{z}_{ij}$, with the more restrictive contract $(x_i(z), R_i)$ dominating for $z \leq \overline{z}_{ij}$ and the less restrictive contract $(x_j(z), R_j)$ dominating for $z > \overline{z}_{ij}$.

Proof. Condition (19) is essentially the single crossing property $\partial U^1(x_j(z), z, R_j)/\partial z > \partial U^1(x_i(z), z, R_i)/\partial z$. Whenever $U^1(x_j(z), z, R_j) = U^1(x_i(z), z, R_i)$, we also have $U^1(x_j(z'), z', R_j) > U^1(x_i(z'), z', R_i)$ for $z' > z$. Therefore, payoffs to type 1 under separation regimes i and j cross at most once, with contract j preferred for all z above the crossing point.□

We now provide more intuitive sufficient conditions that ensure that the single crossing property (19) holds and therefore the *lending terms easing* result generalizes to the environment with multiple types. In the more plausible case of complementarity between loan size and aggregate state ($f_{xz}^1 > 0$), one additional condition suffices, although it is not needed. It requires that, as z increases, excluding type $i+1$ becomes marginally less costly relative to excluding type i, that is $\overline{x}_i'(z) \leq \overline{x}_{i+1}'(z)$. This result can be justified with the more productive types experiencing greater gains in formal production, relative to their less productive peers.

Proposition 6 (*Lending terms easing with multiple types*). *If $f_{xz}^1 > 0$ and $\overline{x}_i'(z) \leq \overline{x}_{i+1}'(z) \leq 0$, then for any two separation levels i and $j > i$, there exists at most one regime switching state $\overline{z}_{ij}$, with the more restrictive contract $(x_i(z), R_i)$ dominating for $z \leq \overline{z}_{ij}$ and the less restrictive contract $(x_j(z), R_j)$ dominating for $z > \overline{z}_{ij}$.*

Proof. See the appendix.

The result above implies that, as z increases, the financial arrangements gradually become more inclusive. Again, complementarity $f_{xz}^1 > 0$ and $\overline{x}_k'(z) \leq 0$ drive the intuition. As the economy expands, the productive types increasingly value greater investment levels and suffer increasingly more from a fixed level of separation. Competition for the productive types then causes screening to gradually diminish.

Loan size and producers' investment levels grow in expansions, but the pool of producers worsens, thereby rationalizing the main empirical patterns in the market for loans. Moreover, provided the delay between contract entry and default is sufficiently short, our mechanism offers an interpretation of the empirical fact that default rates begin rising while the economy is still expanding.

Thus, we showed that the *lending terms easing* result naturally extends to the environment with multiple types. The result survives even in the less plausible case of substitutability between z and x, provided the effect due to $\overline{x}_k'(z) \leq 0$ is sufficiently strong. Conversely, the result survives even in the case of $\overline{x}_k'(z) > 0$ as long as the effect due to the complementarity between investment and aggregate state is sufficiently strong.

We proceed to examine implications of the *lending terms easing* for changes in the average productivity. We then investigate the dependence of this link on the model primitives and the state of the economy.

4.2. Implications of lending terms easing for productivity

The effect of the *lending terms easing* on entrepreneurs' productivity is decomposed into changes transpiring through the negative *extensive* and positive *intensive margins*. With multiple producer types present, there is a possibility of multiple financial regime shifts. In what follows, we compare the average productivity of entrepreneurs entering under different

[12] Note that in the case of complete pooling at $j=n$, the second term on the left hand side of (19) disappears due to the envelope theorem.

economic conditions, and therefore under different financial regimes. Our analysis reveals that the *intensive margin* is likely to dominate at low aggregate states, when lending terms are relatively tight, and the average productivity of the unfinanced producers is relatively high. Relaxation of lending terms then results in greater output and productivity (*productivity amplification*). The opposite is true at high aggregate states, and further relaxation of lending terms results in lower productivity of entrants (*productivity dampening*).

For any z, a switch in the financial market equilibrium from financing k to l producers, with $l > k \geq 2$, implies the following change in the average entrepreneurs' productivity, which we decompose into the *extensive* and *intensive margins*, denoted by $E_{k,l}(z)$ and $I_{k,l}(z)$:

$$\begin{aligned}\frac{y_l(z)-y_k(z)}{X_s} = & \sum_{i=1}^{l}\frac{\mu_i}{\mu_1+\cdots+\mu_l}\frac{f^i(x_l(z),z)}{x_l(z)} - \sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_k}\frac{f^i(x_l(z),z)}{x_l(z)} \\ & + \sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_k}\frac{f^i(x_l(z),z)-h(z)}{x_l(z)} - \sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_k}\frac{f^i(x_k(z),z)-h(z)}{x_k(z)}.\end{aligned} \tag{20}$$

On one hand, this shift towards laxer lending terms allows for lower productivity entrepreneurs to enter production, thereby reducing the average productivity through the *extensive margin,*stated in the first line. The *extensive margin* is given by the difference in the weighted average productivity level of the first l types and the first k types. This difference is unambiguously negative, reflecting the negative impact of *lending terms easing* on producer composition. The magnitude of the productivity change along the *extensive margin* depends on type-specific measures and productivity components of the additional types financed. On the other hand, this shift in financial markets lifts off investment restrictions for all of the financed types, positively affecting the average productivity at the *intensive margin*, stated in the second line. As the loan size increases from $x_k(z)$ to $x_l(z)$, each financed entrepreneur increases its production, but some of them are crowded out and switch to the outside option h. In the presence of non-decreasing returns to scale of the weighted net output function

$$\sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_k}\frac{f^i(x_l(z),z)-h(z)}{x_l(z)} \quad \text{for } x \in (x_k(z), x_l(z)),$$

the average productivity increases.[13] The greater the returns to scale are, the greater is the positive productivity change along the *intensive margin*. The relative strength of the two margins determines the overall impact on productivity.

We also aim to examine the behavior of output and productivity levels as z rises causing gradual shifts towards the less and less exclusive contracts. For tractability, we consider each margin separately and compare two consecutive financial regime shifts, both featuring entry of one additional type. Formally, we compare $E_{k,k+1}(\overline{z}_k)$ with $E_{k+1,k+2}(\overline{z}_{k+1})$ and $I_{k,k+1}(\overline{z}_k)$ with $I_{k+1,k+2}(\overline{z}_{k+1})$, where $\overline{z}_k < \overline{z}_{k+1}$. The next two propositions formalize conditions under which the *extensive margin* grows stronger and the *intensive margin* grows weaker with every shift in the financial regime. Note that such monotonic behavior is not necessary for the strength of the *extensive margin* to grow in terms relative to the *intensive margin*.

It is convenient to represent the productivity decline at the *extensive margin* as the sum of productivity declines due to the crowding out of each type $i \leq k$ by the entry of type $k+1$:

$$E_{k,k+1}(\overline{z}_k) = \sum_{i=1}^{k}\left(\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}} - \frac{\mu_i}{\mu_1+\cdots+\mu_k}\right)\frac{f^i(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k) - f^{k+1}(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)}.$$

This representation of the *extensive margin* helps identify two conditions sufficient for the strength of the *extensive margin* to increase with z. The proposition below formalizes the result.

Proposition 7. *The extensive margin strengthens as more types enter,* $0 > E_{k,k+1}(\overline{z}_k) > E_{k+1,k+2}(\overline{z}_{k+1})$, *if*

$$\frac{\mu_{k+1}}{\mu_1+\cdots+\mu_k}$$

and

$$\frac{f^i(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k) - f^{k+1}(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)},$$

with $i \leq k+1$, *both weakly increase in* k.

Proof. See the appendix.

The first condition requires that the measure of the less productive types that enter production grows sufficiently fast in k. If productivity across types were represented by a discretized version of a Pareto distribution, this condition would be naturally satisfied. Although the distribution of productivity across types is impossible to observe in the data due to selection issues, we know that the empirical distribution of existing firms can be closely approximated by a Pareto distribution (Luttmer, 2007).

[13] This condition is weaker than imposing non-decreasing returns to scale for each type, $f^i - h$.

The second condition requires that the difference in the average productivity of types i and $k+1$ at a loan level $\overline{x}_{k+1}(\overline{z}_k)$ and state $\overline{z}_k$ is smaller than the difference in the average productivity of types i and $k+2$ at a loan level $\overline{x}_{k+2}(\overline{z}_{k+1})$. This condition is natural for a fixed loan size x, although as the loan size increases with z, it introduces a counteracting force due to the concavity of f^i. In the example given below, with $f^i(x,z) = zA^i x^\alpha - F$, this condition reduces to $\overline{z}_k(A^i - A^{k+1})\overline{x}_{k+1}^{\alpha-1}(\overline{z}_k) < \overline{z}_{k+1}(A^i - A^{k+2})\overline{x}_{k+2}^{\alpha-1}(\overline{z}_{k+1})$, which is satisfied if the productivity of type $k+2$ is sufficiently low relative to that of type $k+1$.

Similarly, we represent the productivity increase at the *intensive margin* as the sum of productivity increments experienced by each type due to the increase in the loan size:

$$I_{k,k+1}(\overline{z}_k) = \sum_{i=1}^{k} \frac{\mu_i}{\mu_1 + \cdots + \mu_k} \left[\frac{f^i(\overline{x}_{k+1}(\overline{z}_k), \overline{z}_k) - h(\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)} - \frac{f^i(\overline{x}_k(\overline{z}_k), \overline{z}_k) - h(\overline{z}_k)}{\overline{x}_k(\overline{z}_k)} \right].$$

This representation enables us to identify two conditions sufficient for the strength of the *intensive margin* to decline with z.

Proposition 8. *The intensive margin weakens as more types enter*, $I_{k,k+1}(\overline{z}_k) > I_{k+1,k+2}(\overline{z}_{k+1}) > 0$, *if*

$$\frac{f^i(\overline{x}_{k+1}(\overline{z}_k), \overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)} - \frac{f^i(\overline{x}_k(\overline{z}_k), \overline{z}_k)}{\overline{x}_k(\overline{z}_k)}$$

strictly increases in k for all $i \le k+1$ *and*

$$\sum_{i=1}^{k} \frac{\mu_i}{\mu_1 + \cdots + \mu_k} \left[\frac{f^i(\overline{x}_{k+1}(\overline{z}_k), \overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)} - \frac{f^i(\overline{x}_k(\overline{z}_k), \overline{z}_k)}{\overline{x}_k(\overline{z}_k)} \right] \ge \frac{f^{k+1}(\overline{x}_{k+2}(\overline{z}_{k+1}), \overline{z}_{k+1})}{\overline{x}_{k+2}(\overline{z}_{k+1})} - \frac{f^{k+1}(\overline{x}_{k+1}(\overline{z}_{k+1}), \overline{z}_{k+1})}{\overline{x}_{k+1}(\overline{z}_{k+1})}.$$

Proof. See the appendix.

When comparing two consecutive regime switches, note that the second one occurs at a higher z and increments a larger loan. The first condition states that the gain the second switch implies for the average productivity of each type is smaller. This condition is plausible because the increasing returns to scale become less important at greater levels of x and z. The second condition states that the productivity increase of the least productive type at the second switch is smaller than the average increase in productivity at the first switch. Even though the less productive types exhibit greater returns to scale, the gain for the least productive type is evaluated at a larger loan size, which works in the right direction.

When the economic conditions are weak, the *intensive margin* is likely to dominate. With the positive *intensive margin* weakening and the negative *extensive margin* strengthening with every shift in the financial market equilibrium, the *extensive margin* is bound to dominate as lending terms get sufficiently lax, causing a drop in output and productivity.

From the point of view of a planner with the same informational constraints, financial regime switches characterized by the dominant *intensive margin*, occur “too late.”[14] Indeed, consider any switch point $\overline{z}_i$ with a dominant *intensive margin*. By continuity, the output level at $\overline{z}_i - \varepsilon$ is less than the constrained-efficient level, and in particular $Y_i(\overline{z}_i - \varepsilon) < Y_{i+1}(\overline{z}_i - \varepsilon)$, so output could be raised by imposing the laxer financial regime $i+1$ at $\overline{z}_i - \varepsilon$. The opposite is true at regime switches $\overline{z}_j$ characterized by the dominant *extensive margin*. The shift towards laxer terms at that point is inefficient.

An important insight is contained in the intuition for the result that this economy is not always at its constrained efficient output level. When the economic conditions are bottomed out, the benefit to increasing the loan size, as seen by the most productive entrepreneurs, is simply not that high, and therefore contracts with larger loan size offers do not survive in equilibrium. Conversely, at the very top, returns to investment are high and a larger loan size is highly attractive, allowing, as a side effect, for some lemons to engage in formal production.

Thus, in the multiple types setting, relaxation of lending terms can be beneficial at low economic states and detrimental at high economic states. It is therefore possible for the financial markets to both amplify exogenous movements in z, stimulating expansions when output is low, as well as dampen them, when output is high, thus helping explain the onset of recessions. The insight offered by the *lending terms easing* result therefore offers a way to reconcile the two opposing strands of related literature (See Section 2). It also suggests that an interaction term of some measure of lending terms and economic conditions may prove useful to include in empirical studies concerned with the influence of financial markets on the economy.

4.3. An example

A simple example effectively illustrates the main insights arising in the environment with multiple types. Following the business cycles literature, we assume the aggregate state positively affects the common productivity component in production technology of all types. Entrepreneurs' primary technology is given by $f^i(x,z) = A^i z x^\alpha - F$, with $A^i > A^j$ for $j > i$. The outside option is given by $h(z) = Bz - F$, which implies that the cutoff loan ensuring a given level of separation k, i.e.

[14] This planner cannot observe individual types and therefore cannot improve the outcome beyond enforcing the financial regime that maximizes total output for a given z.

Table 1
Parameterization for the numerical example.

Parameter	R_f	A_1	A_2	A_3	A_4	F	B	α	μ_1	μ_2	μ_3	μ_4	X_s
Value	1	2.38	1.5	1.27	0.92	0.37	1.49	0.7	0.53	0.09	0.12	0.24	0.53

Notes: This table summarizes the parameters used in the numerical example with four types of entrepreneurs.

financing types $i \le k$, is given by $\bar{x}_k = \left(B/A^{k+1}\right)^{1/\alpha}$. The cutoff is independent of z and decreases in A^{k+1}: it takes a smaller loan offer to exclude the more productive types.

With this technology, the assumptions from Section 4 are satisfied, and Proposition 5 applies. Therefore, the equilibrium contract is found as the one that maximizes utility of the most productive type among the optimal contracts for each level of separation, given by $(x_k(z), R_k)$, with

$$x_k(z) = \arg \max_{x \in (\bar{x}_{k-1}, \bar{x}_k]} A^1 z x^\alpha - R_k x. \tag{21}$$

A special case of the above is the case of complete pooling $k=n$, with the loan size given by $x_p(z) = \left[\alpha A^1 z \mu_1 / R_f\right]^{1/(1-\alpha)}$.

Table 1 summarizes the parameters used in our numerical example with four types of entrepreneurs.[15] To illustrate the growing relevance of the *extensive margin*, we assume that among the defaulting types, the less productive types appear in greater measures: the measure of type 3 is 4/3 of the measure of type 2, while the measure of the worst type is double the measure of type 3.

This example features three consecutive financial regime shifts. As z rises, complete separation, under which only the most productive type is financed, gives way to the less exclusive contracts, with types 2, 3, and 4 gradually included in financial contracts. The optimal loan size $x^*(z)$ rises through this transition from $\bar{x}_1$, to $\bar{x}_2$, to $\bar{x}_3$, and finally to $x_p(z)$.

Panel (a) of Fig. 3 depicts $U^1(x_k(z), z, R_k)$ as a function of z, for each financial regime k. Complementarity between z and investment, and therefore loan size, coupled with an increasing loan size across regimes implies that the slope of $U^1(\cdot)$ increases in k. This gives rise to *lending terms easing*, i.e. the ordering of regime switching points $\bar{z}_k$, $k \in \{1,2,3\}$, found by solving for levels of z that make type 1 indifferent between any two consecutive regimes.

Panel (b) depicts regime-specific loan size offers and the equilibrium loan size (in red) as functions of z. The green vertical lines mark the financial equilibrium shifts towards laxer terms: $\bar{z}_1 = 1.13$, $\bar{z}_2 = 1.24$ and $\bar{z}_3 = 1.9$. At low z, when returns to investment are relatively low, type 1 prefers complete separation, which is offered at a low risk-free rate. As z rises, so do the returns to investment. Type 1 gradually prefers the more expensive less exclusive contracts, which allow for the expansion of credit and therefore investment. The most pronounced growth in loan size happens at the point of the shift to complete pooling.

Output level implied by the financial regime that finances k producers is given by

$$Y_k(z) = \frac{X_s}{x_k(z)} \bar{A}^k z x_k^\alpha(z) + \left(1 - \frac{X_s}{x_k(z)}\right) Bz - F, \tag{22}$$

where $\bar{A}^k = \sum_{i=1}^k (\mu_k/(\mu_1 + \cdots + \mu_k)) A^k$. Because z enters as a common productivity component in all technologies, output level associated with any given financial regime increases in z. Output levels differ across financial regimes because of the endogenously different producer composition and investment levels. The difference in the total output of entrepreneurs financed under regimes k and $k+1$, that is $Y_{k+1}(z) - Y_k(z)$, can be stated as a sum of the *extensive* and *intensive margins*, that is, $X_s/x_{k+1}(z)\left[\bar{A}^{k+1} - \bar{A}^k\right] z x_{k+1}^\alpha(z)$ and $X_s/(x_{k+1}(z))[\bar{A}^k z x_{k+1}^\alpha(z) - Bz] - X_s/(x_k(z))[\bar{A}^k z x_k^\alpha(z) - Bz]$.

The *extensive margin* is unambiguously negative because the average productivity term $\bar{A}^k$ decreases in k. It strengthens with k, for a given z, because the (negative) difference in the average productivity term $\bar{A}^{k+1} - \bar{A}^k$ increases. The *intensive margin* is positive because the net output function $\bar{A}^k z x^\alpha - Bz$ exhibits increasing returns to scale. It weakens in k, for a given z, because increasing returns become less important at larger scales of production. Although the negative *extensive margin* grows in strength across all three regime shifts, whereas the positive *intensive margin* weakens, the *extensive margin* dominates only at the last regime shift. Panels (c) and (d) report the equilibrium level of output (in red) as a function of z, that is $Y(z) = Y_k(z)$ for $z \in [\bar{z}_{k-1}, \bar{z}_k)$, $k \le 4$, along with regime-specific output levels $Y_k(z)$. For a given z, output levels are ordered across regimes according to $Y_2(z) > Y_1(z)$, $Y_3(z) > Y_2(z)$, but $Y_4(z) < Y_3(z)$, reflecting the *intensive margin* dominance when comparing the first two regime pairs and the *extensive margin* dominance when comparing the last two regimes. One reason why output drops at the last switching point is that a large fraction of funds is allocated to type 4 entrepreneurs (Panel f).

The constrained efficient level of output arises under the financial contract that funds the first three types.[16] It follows that at low levels of z, lending terms are inefficiently tight because returns to investment are particularly low, and therefore

[15] The parameters must satisfy the equivalent of Assumption 3 , $A^1 z - R_f > A^2 z$, which implies participation of type 1 under complete separation in the range of z we consider.

[16] Note that this planner could not improve the outcome beyond enforcing the regime that maximizes total output for a given z. An attempt to concentrate production among a smaller measure of type 1 entrepreneurs would result in the entry of the less productive types.

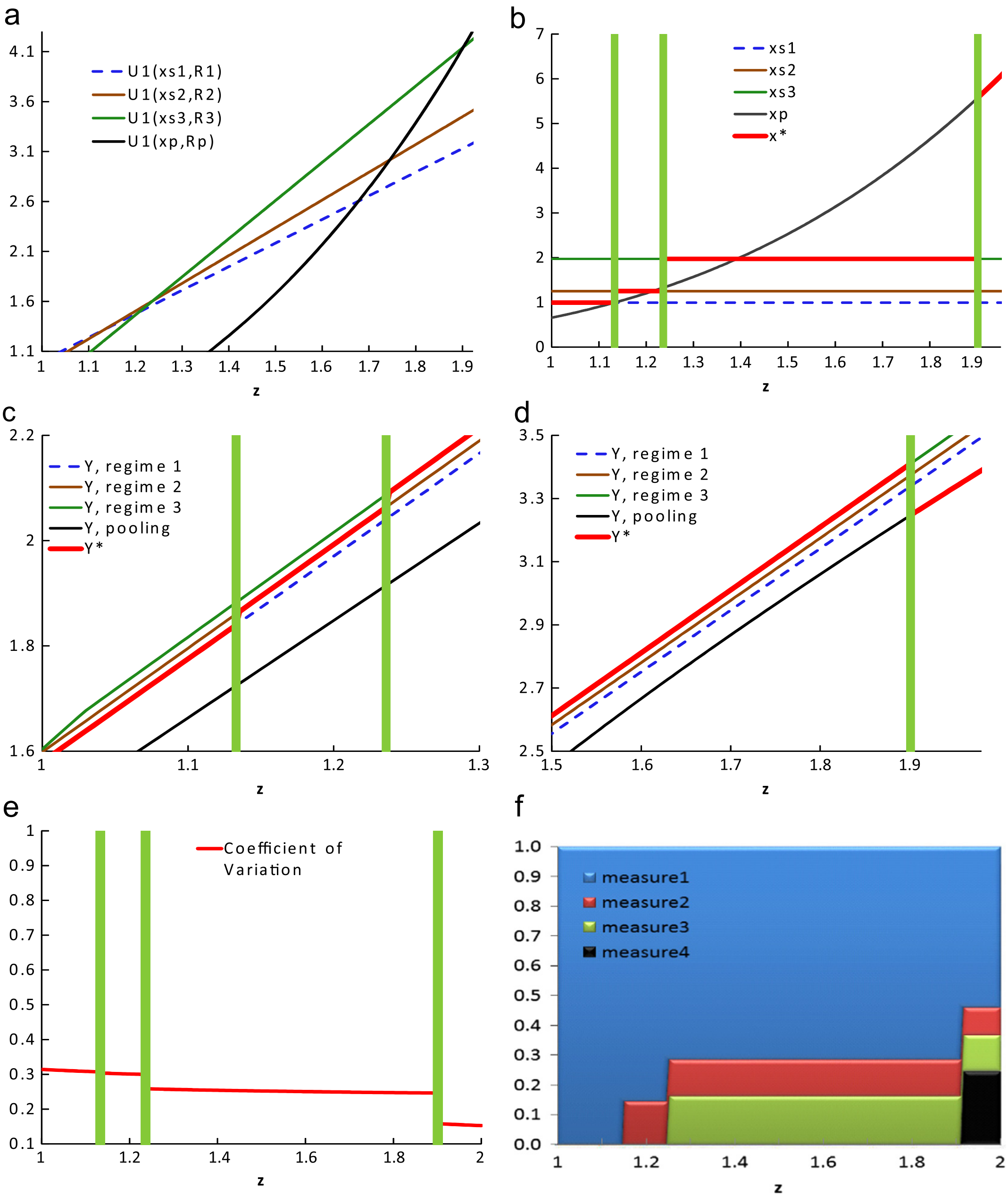

Fig. 3. An example with four types of entrepreneurs. (a) U^1, (x_k (z), z, R_K) for each regime k, (b) Regime-specific and equilibrium loans, (c) Regime-specific and equilibrium output, (d) Regime-specific and equilibrium output, (e) Dispersion of output production and (f) Distribution of funds across types. *Notes*: This example features three consecutive financial regime shifts. Vertical lines mark the regime switching points $\bar{z}_{12}, \bar{z}_{23}$, and $\bar{z}_{34}$. As z rises, complete separation, under which only the most productive type is financed, gives way to the less exclusive contracts, with types 2, 3, and 4 gradually included in financial contracts. The optimal loan size and investment per producer rises, but the composition of producers deteriorates. (For the colored version of this figure, the reader is referred to the web version of the paper.)

contracts that survive competition are cheap small loans preferred by the best type. Increasing the loan size to allow for the second and third types to enter increases output. Conversely, when the economic conditions are particularly strong, lending terms are inefficiently lax because returns to investment are particularly high, and therefore contracts offering larger credit lines dominate. A switch to complete pooling, which funds even the least productive entrepreneur, implies a substantial drop in total output.

Thus, the example clearly illustrates most of the results in the paper. We associate an economic expansion with an exogenous rise in the common productivity component, and study its implication for financial markets and output. An economic expansion brings about an endogenous easing of lending terms. To allow for greater investment, contracts become less and less exclusive. Loan size and investment levels rise, and default rates may begin to rise during an expansionary period, provided there is not much delay between loan origination and default. When the exogenous productivity component is low, less screening boosts output, as the benefit of easing investment constraints outweighs the negative impact from allocating some of the funds to the less productive types. Thus, at low levels of z, positive movements in the exogenous productivity component are amplified via the financial channel. At expansion heights, lending terms are relatively weak. Further easing of lending terms allows for the entry of the relatively large measure of the most unproductive type, and leads to the drop in output. In this case, positive movements in z are dampened via the same financial channel.

There is an important implication for output (or productivity) dispersion across types, which is consistent with the empirical finding, documented in Kehrig (2011), that the cross-sectional productivity dispersion across the U.S. manufacturing firms is countercyclical, and is primarily driven by changes at the bottom end of the productivity distribution. Although our setup is only appropriate for understanding productivity of the new entrants, the implications for productivity dispersion are empirically plausible. Panel (e) of Fig. 3 reveals that the coefficient of variation of output across types declines with z.[17] The less productive types are increasingly given the opportunity to use the superior primary production technology, implying the dispersion across types declines with z.

5. Conclusions

We examined lending terms determination in competitive financial markets for informationally opaque firms. Screening is accomplished by offering a low enough credit line to keep the defaulting types out. Therefore, the cost of screening always falls on the productive repaying types in the form of curtailed production scales. We showed that, across a variety of settings, lending terms deteriorate with the state of the economy. Stronger economic conditions raise productivity and returns to investment. Both effects raise the relative benefit of contracts that allow for larger credit lines, even if more expensive. Contracts become less and less exclusive, thereby moving production scales closer to optimal via the positive *intensive margin*, but inadvertently worsening the composition of entrants via the negative *extensive margin*.

We have argued that the mechanism behind the *lending terms easing* is well grounded in the data. We also made comparison of outcomes with constrained-efficient outcomes. When the economic conditions are weak, credit lines and production scales may be inefficiently low. This is because the benefit to increasing loan size, as seen by the most productive entrepreneurs, is simply not that high, and therefore contracts offering larger credit lines, although desirable from the planner's point of view, do not survive in equilibrium. Conversely, at the very top, returns to investment are particularly high, and a large credit line is highly attractive to the best types. Contracts that abandon screening altogether and provide plenty of credit, although undesired from the planner's perspective, beat the competition. As a side effect, the least productive types enter.

We showed that adverse selection can be responsible for both amplifying and dampening exogenous productivity shocks. The presence of both forces operating in our model reconciles the two opposing views on the role of financial frictions, discussed in Section 2. We investigated the dependence of the relative strength of these two forces on the model primitives and the aggregate state of the economy and argued that the amplification force is likely to dominate at low aggregate states, while the dampening force is likely to dominate at high aggregate states. These findings suggest that future empirical work should allow for the possibility that the link between the current state of financial markets and future outcomes closely depends on the current aggregate state.

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Appendix A

A.1. Proof of Lemma 1

Consider an arbitrary element in the contract menu, (x^G, R^G), (x^B, R^B) satisfying equilibrium conditions (1), (2) and (3). Since B' s payoff increases in x^B (due to $f_x^B > 0$) and is independent of R^B, self-selection for type B implies that $x^G \le x^B$. Suppose that $x^G < x^B$. Since B defaults, voluntary participation of the bank implies that revenue exceeds the opportunity cost of funds: $\mu x^G R^G \ge [\mu x^G + (1-\mu)x^B]R^f$. Combining this inequality with the supposition that $x^G < x^B$ we get that

[17] The behavior of the coefficient of variation of productivity $A(z)$ looks similar.

$R^G \geq (\mu x^G + (1-\mu)x^B/\mu x^G)R_f > R_f/\mu$. But this means a menu element that improves the position of type G can be offered. In fact, suppose that only one contract is offered: $(x^B, R_f/\mu)$. Then banks make zero profits, both types select the contract, and type G is strictly better off. Hence, we arrived at a contradiction. Therefore, $x^G = x^B$. In turn, self-selection for type G implies $R^G = R^B$.

A.2. Proof of Proposition 1

It is clear that $x_s(z)$ exists since it is given by maximizing a continuous function over a compact set. However, $x_p(z)$ is not well defined for z such that $f_x^G(\overline{x}(z), z) < R_f/\mu$, i.e. in the case the constraint in (6) binds. For these levels of z, however, the optimal separating contract strictly dominates all pooling contracts; and therefore, equilibrium always exists. Intuitively, if the marginal revenue of external funds at $\overline{x}(z)$ is smaller than R_f/μ, type G prefers a smaller loan at the pooling rate, $(\overline{x}(z), R_f/\mu)$, but this loan size is available at the risk-free rate under separation. Since the contract $(\overline{x}(z), R_f)$ can be chosen as an optimal separating contract, and we know it induces G' s participation by Assumption 3, it follows that the optimal contract also induces G' s participation and is therefore in $X(z)$. Uniqueness is due to the strict concavity of U^G in x.

A.3. Proof of Proposition 2

From the definition of U^G, it is clear that $x_s(z)$ is interior if $f_x(\overline{x}(z), z) < R_f$, and the constraint in (5) binds otherwise. To see that the optimal separating contract dominates all pooling contracts under the first two cases, note that for z such that $f_x^G(\overline{x}(z), z) < R_f/\mu$, we have $x_p(z) = \overline{x}(z)$. It follows that the equilibrium contract is $(x_s(z), R_f)$ since

$$\begin{aligned} U^G\left(x_p(z), z, R_f/\mu\right) &= f^G(\overline{x}(z), z) - \frac{R_f}{\mu}\overline{x}(z) \\ &< f^G(\overline{x}(z), z) - R_f\overline{x}(z) \\ &\leq U^G(x_s(z), z, R_f). \end{aligned}$$

For z in the case of regime tradeoff, i.e. such that $f_x^G(\overline{x}(z), z) \geq R_f/\mu$, it follows from the definition of U^G that the optimal pooling loan size is characterized by the first order condition. Moreover, $f_x^G(\overline{x}(z), z) \geq R_f/\mu > R_f$, and therefore the constraint $x \leq \overline{x}(z)$ in (5) binds, $x_s(z) = \overline{x}(z)$.

A.4. Proof of Lemma 2

Suppose that for any z such that $U^G(x_s(z), z, R_f) = U^G(x_p(z), z, R_f/\mu)$ we have

$$\frac{\partial}{\partial z}\left[U^G\left(x_p(z), z, R_f/\mu\right) - U^G\left(x_s(z), z, R_f\right)\right] > 0. \tag{23}$$

This means that if $U^G(x_p(z), z, R_f/\mu) - U^G(x_s(z), z, R_f)$ crosses zero as a result of increasing z, it crosses it from below. It follows that there exists at most one such crossing, say at $\overline{z}$, and that a separating financial market equilibrium emerges for $z \leq \overline{z}$ and a pooling one emerges for $z > \overline{z}$.

It suffices to show that (23) is equivalent to (9). Indeed, condition (23) can be rewritten as

$$U_x^G(x_p(z), z, R_f/\mu)x_p'(z) + U_z^G(x_p(z), z, R_f/\mu) > U_x^G(x_s(z), z, R_f)x_s'(z) + U_z^G(x_s(z), z, R_f).$$

By Corollary 1, we have that at points of regime switching, $x_p(z) = x_p^{int}(z)$ and $x_s(z) = \overline{x}(z)$. Therefore, the term $U_x^G(x_p(z), z, R_f/\mu)x_p'(z)$ cancels due to the envelope theorem, and condition (23) can be rewritten as (9).

A.5. Lending terms easing in the case of substitutibility

Proposition 9. *Consider the case of substitutability of loan size and aggregate state ($f_{xz}^G \leq 0$). Suppose the effect due to the separating loan size is bounded by some constant N, so that $|\overline{x}'(z)| \geq N$. If $|f_{xz}^G| \leq M$ with the bound M satisfying $M \leq R_f/\mu(1-\mu)N/f_x^G - 1(R_f/\mu; 0)$, there is at most one regime switching state $\overline{z}$, with a separating financial market equilibrium emerging for all $z \leq \overline{z}$ and a pooling equilibrium emerging for all $z > \overline{z}$.*

Proof. Condition (9) can be rewritten as $f_z^G(\overline{x}(z), z) - f_z^G(x_p(z), z) \leq (f_x^G(\overline{x}(z), z) - R_f)(-\overline{x}'(z))$. Given the assumptions and applying the mean value theorem, we have

$$\begin{aligned} f_z^G(\overline{x}(z), z) - f_z^G(x_p(z), z) &\leq M(x_p(z) - \overline{x}(z)) \\ &\leq Mx_p(z) \leq Mf_x^G - 1(R_f/\mu; 0) \end{aligned}$$

and

$$(f_x^G(\overline{x}(z),z)-R_f)\left(-\overline{x}'(z)\right) \geq -\left(f_x^G(x_p(z),z)-R_f\right)N = R_f(1-\mu)N/\mu.$$

Since $R_f(1-\mu)N/\mu \geq Mf_x^G - 1(R_f/\mu;0)$, condition (9) holds.□

A.6. Proof of Lemma 3

First, consider the case where $\nu_p(z) = X_s/x_p(z)$. Then, since $\mu \leq X_s/x_s(z)$, output change along the *intensive margin* at $\overline{z}$ is greater or equal than

$$X_s\left[L^G(x_p(\overline{z}),\overline{z})-L^G(x_s(\overline{z}),\overline{z})\right] = X_s\int_{x_s(\overline{z})}^{x_p(\overline{z})} L_x^G(x,\overline{z})\,dx \geq 0. \tag{24}$$

The last inequality follows from $L_x^G(x,z) = -f^G(x,z)-h(z)/x^2 + f_x^G(x,z)/x \geq 0$ and $x_p(\overline{z}) > x_s(\overline{z})$.

Now consider the case where $\nu_p(z)=1$. Then, output change along the *intensive margin* is given by $f^G(x_p(z),z)-h(z)/X_s-[f^G(x_s(z),z)-h(z)]\mu/X_s \geq (f^G(x_s(z),z)-h(z))(1-\mu)/X_s \geq 0$.

A.7. Proof of Proposition 4

It is useful to introduce the following shorthand for the average output: $L^G(x,z) = 1/x[f^G(x,z)-h(z)]$. It suffices to show that the absolute value of output change along the *extensive margin* is greater than the positive output change along the *intensive margin*. The output change along the *extensive margin* is given by $X_s(1-\mu)f^G(x_p(\overline{z}),\overline{z})-f^B(x_p(\overline{z}),\overline{z})/x_p(\overline{z})$, while the output change along the *intensive margin* is given by

$$\begin{aligned}
&X_s\left[\frac{f^G(x_p(\overline{z}),\overline{z})-h(\overline{z})}{x_p(\overline{z})}-\frac{[f^G(x_s(\overline{z}),\overline{z})-h(\overline{z})]}{x_s(\overline{z})}\right]\\
&\quad = X_s\left[L^G(x_p(\overline{z}),\overline{z})-L^G(x_s(\overline{z}),\overline{z})\right]\\
&\quad = X_s\int_{x_s(\overline{z})}^{x_p(\overline{z})} L_x^G(x,\overline{z})dx \leq X_sM[x_p(\overline{z})-x_s(\overline{z})].
\end{aligned}$$

The result follows directly. If output drops at the regime switching point $\overline{z}$, i.e. $y_p(\overline{z}) < y_s(\overline{z})$, then productivity change is also negative: $y_p(\overline{z})/X_s < y_s(\overline{z})/X_s$.

A.8. Proof of Proposition 5

Each optimal contract $x_k(z)$ is found by maximizing a continuous, strictly concave function over an interval with one open end at $\overline{x}_{k-1}(z)$. Therefore, $x_k(z)$ is not well defined only in the case of z such that the constraint $x > \overline{x}_{k-1}(z)$ binds, which happens whenever $f_x^1(\overline{x}_{k-1}(z),z) < R_k$. But this case is not relevant, because type 1 prefers a smaller loan size $\overline{x}_{k-1}(z)$ which excludes type k and is available at a lower interest rate R_{k-1}, the contract $(\overline{x}_{k-1}(z),R_{k-1})$. Finally, because the contract $(\overline{x}_1(z),R_f)$, which induces type 1 to participate, is in the choice set when selecting the optimal fully separating contract $(x_1(z),R_f)$, the optimal contract must also induce type 1 to participate, and it is therefore in $X(z)$. Therefore, equilibrium always exists, and it is unique.

A.9. Proof of Lemma 5

Suppose not, i.e. suppose that $x_i(z) < \overline{x}_i(z)$. In light of (21), it follows that $f_x^1(\overline{x}_i(z),z) < R_i$. Since $\overline{x}_j(z) > \overline{x}_i(z)$, strict concavity of f^1 in x implies that $f_x^1(\overline{x}_j(z),z) < R_i < R_j$. In other words, at the interest rate R_j, type 1 prefers a loan smaller than $\overline{x}_j(z)$, which is also available at a lower interest R_i. It follows that $U^1(x_i(z),z,R_i) > U^1(x_j(z),z,R_j)$, a contradiction.

A.10. Proof of Lemma 6

Consider $i<j$ and note that in light of Lemma 5, at any z such that $U^1(x_i(z),z,R_i) = U^1(x_j(z),z,R_j)$, we have $x_i(z) = \overline{x}_i(z)$. Then

$$\begin{aligned}
&[f_x^1(x_j(z),z)-R_j]x_j'(z)+f_z^1(x_j(z),z) \geq [f_x^1(x_j(z),z)-R_j]x_j'(z)+f_z^1(x_i(z),z)\\
&\quad \geq [f_x^1(x_i(z),z)-R_i]x_i'(z)+f_z^1(x_i(z),z),
\end{aligned}$$

where the first inequality comes from $f_{xz}^1 \geq 0$, $x_i(z) < x_j(z)$, and the second from the fact that $R_i < R_j$, $f_{xx}^1 < 0$ and $\overline{x}_i'(z) \leq \overline{x}_j'(z)$.

If $f_{xz}^1 > 0$ and $\overline{x}_i'(z) \leq \overline{x}_{i+1}'(z)$, then for any two separation levels i and $j > i$, there exists at most one regime switching state $\overline{z}_{ij}$, with the more restrictive contract $(x_i(z),R_i)$ dominating for $z \leq \overline{z}_{ij}$ and the less restrictive contract $(x_j(z),R_j)$ dominating for $z > \overline{z}_{ij}$.

A.11. Multiple types: lending terms easing in the case of substitutibility

Proposition 10. *Suppose that $f^1_{xz}<0$ and $\overline{x}'_i(z)\le\overline{x}'_{i+1}(z)$. Suppose also that there exist constant bounds M_1 and M_2, $f^1_{xx}\le M_1$, $|f^1_{xz}|\le M_2$, such that $M_2\le M_1\overline{x}'_i(z)$. Then for any two separation levels i and $j>i$, there exists at most one regime switching state $\overline{z}_{ij}$, with the more restrictive contract $(x_i(z),R_i)$ dominating for $z\le\overline{z}_{ij}$ and the less restrictive contract $(x_j(z),R_j)$ dominating for $z>\overline{z}_{ij}$.*

Proof. It suffices to prove that inequality (19) holds. In light of Lemma 5, we have $x_{i,s}(z)=\overline{x}_i(z)$ at z that induces a regime switch from i to j. It suffices to prove that

$$[f^1_x(x_j(z),z)-R_j]x'_j(z)-[f^1_x(\overline{x}_i(z),z)-R_i]\overline{x}'_i[z]\ge f^1_z(\overline{x}_i(z),z)-f^1_z(x_j(z),z).$$

Note that

$$\begin{aligned}[f^1_x(x_j(z),z)-R_j]x'_j(z)-[f^1_x(\overline{x}_i(z),z)-R_i]\overline{x}'_i(z)&\ge[f^1_x(\overline{x}_j(z),z)-R_j]\overline{x}'_j(z)-[f^1_x(\overline{x}_i(z),z)-R_i]\overline{x}'_i(z)\\&\ge[f^1_x(\overline{x}_j(z),z)-f^1_x(\overline{x}_i(z),z)]\overline{x}'_i(z)\\&\ge M_1(\overline{x}_j(z)-\overline{x}_i(z))\overline{x}'_i(z).\end{aligned}\tag{25}$$

The first inequality is due to $x_j<\overline{x}_j$. The second inequality is due to $f_{xx}<0$, $R_i<R_j$ and $0>\overline{x}'_j(z)>\overline{x}'_i(z)$. The third is direct from the mean value theorem. Finally, directly from the assumptions, we have $f^1_z(\overline{x}_i(z),z)-f^1_z(x_j(z),z)\le M_2(x_j(z)-\overline{x}_i(z))\le M_2(\overline{x}_j(z)-\overline{x}_i(z))$.□

A.12. Proof of Proposition 7

Because the first condition implies that

$$\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}-\frac{\mu_i}{\mu_1+\cdots+\mu_k}\ge\frac{\mu_i}{\mu_1+\cdots+\mu_{k+2}}-\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}},$$

we obtain that

$$\begin{aligned}E_{k,k+1}(\overline{z}_k)&=X_s\sum_{i=1}^{k}\left(\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}-\frac{\mu_i}{\mu_1+\cdots+\mu_k}\right)\frac{f^i(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)-f^{k+1}(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)}\\&\ge X_s\sum_{i=1}^{k}\left(\frac{\mu_i}{\mu_1+\cdots+\mu_{k+2}}-\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}\right)\frac{f^i(\overline{x}_{k+2}(\overline{z}_{k+1}),\overline{z}_{k+1})-f^{k+2}(\overline{x}_{k+2}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+2}(\overline{z}_{k+1})}\\&>X_s\sum_{i=1}^{k+1}\left(\frac{\mu_i}{\mu_1+\cdots+\mu_{k+2}}-\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}\right)\frac{f^i(\overline{x}_{k+2}(\overline{z}_{k+1}),\overline{z}_{k+1})-f^{k+2}(\overline{x}_{k+2}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+2}(\overline{z}_{k+1})}\\&=E_{k+1,k+2}(\overline{z}_{k+1}),\end{aligned}$$

where the first inequality is due to the two conditions, and the second is due to adding an extra negative term.

A.13. Proof of Proposition 8

Output change at the *intensive margin* at $\overline{z}_k$ can be rewritten as

$$\begin{aligned}I_{k,k+1}(\overline{z}_k)&=X_s\sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_k}\left[\frac{f^i(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)}-\frac{f^i(\overline{x}_k(\overline{z}_k),\overline{z}_k)}{\overline{x}_k(\overline{z}_k)}\right]\\&=X_s\sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}\left[\frac{f^i(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)}-\frac{f^i(\overline{x}_k(\overline{z}_k),\overline{z}_k)}{\overline{x}_k(\overline{z}_k)}\right]\\&\quad+X_s\sum_{i=1}^{k}\left[\frac{\mu_i}{\mu_1+\cdots+\mu_k}-\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}\right]\left[\frac{f^i(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)}-\frac{f^i(\overline{x}_k(\overline{z}_k),\overline{z}_k)}{\overline{x}_k(\overline{z}_k)}\right]\\&>X_s\sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}\left[\frac{f^i(\overline{x}_{k+2}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+2}(\overline{z}_{k+1})}-\frac{f^i(\overline{x}_{k+1}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+1}(\overline{z}_{k+1})}\right]\\&\quad+X_s\frac{\mu_{k+1}}{\mu_1+\cdots+\mu_{k+1}}\sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_k}\left[\frac{f^i(\overline{x}_{k+1}(\overline{z}_k),\overline{z}_k)}{\overline{x}_{k+1}(\overline{z}_k)}-\frac{f^i(\overline{x}_k(\overline{z}_k),\overline{z}_k)}{\overline{x}_k(\overline{z}_k)}\right]\\&>X_s\sum_{i=1}^{k}\frac{\mu_i}{\mu_1+\cdots+\mu_{k+1}}\left[\frac{f^i(\overline{x}_{k+2}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+2}(\overline{z}_{k+1})}-\frac{f^i(\overline{x}_{k+1}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+1}(\overline{z}_{k+1})}\right]\\&\quad+X_s\frac{\mu_{k+1}}{\mu_1+\cdots+\mu_{k+1}}\left[\frac{f^{k+1}(\overline{x}_{k+2}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+2}(\overline{z}_{k+1})}-\frac{f^{k+1}(\overline{x}_{k+1}(\overline{z}_{k+1}),\overline{z}_{k+1})}{\overline{x}_{k+1}(\overline{z}_{k+1})}\right]=I_{k+1,k+2}(\overline{z}_{k+1})\end{aligned}$$

where the first and second inequalities arise from the first and second conditions respectively.

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