Economic Inquiry

# INDUCING OVERCONFIDENCE

FRANCISCO SILVA*

*In this study, I present a theory which explains how the influence of others may generate overconfidence. The argument is built on the idea that the more help an agent receives when performing a task, the less informative the score on that task will be relative to the agent's ability to perform it. If an agent is confident, he tends to benefit from more cooperation opportunities simply because he is likely to be perceived as being more skilled. As a result, he remains confident because the future signals he will observe will contain very little information regarding his ability. On the contrary, if the agent is not confident, he will receive more informative scores, which will help him learn his true ability faster.* (*JEL* D81, D83, D84)

## I. INTRODUCTION

Overconfidence is a phenomenon which has been widely researched in a variety of subjects including economics. There have been several papers which claim to provide evidence in support of the idea that people are overconfident with respect to their own ability (see, e.g., Buehler, Griffin, and Ross 1994; Guthrie, Rachlinski, and Wistrich 2001; Svenson 1981). While there have been other papers that have not found any evidence of overconfidence (see Clark and Friesen 2009; Moore and Healy 2008), there is, at the very least, a justifiable suspicion that, in various settings, people are overconfident.

It is important to understand why overconfidence arises, as it may have serious consequences in people's lives. Overconfidence can be good—there is some evidence that confidence in one's ability improves performance (see Taylor and Brown 1988). It can also be tremendously bad. People who overestimate their ability will likely make poor decisions—for example, overconfident managers may be too willing to invest in risky projects (Malmendier and Tate 2005); overconfident entrepreneurs will start businesses they should not have (Camerer and Lovallo 1999); even overconfident researchers will immerse themselves into solving unsolvable mysteries.

Several theories have been presented to try to explain this phenomenon. In the majority of them, a person becomes overconfident only because of her own decisions.[1] In this article, I examine the link between overconfidence and the actions that other people take. There is a vast literature, mainly in applied psychology, that has documented external influences on people's expectations—Glasgow et al. (1997) and Smith and Powell (1990), among many others, highlight the impact of parents on their children's expectations; Meyer and Gellatly (1988) argue that setting external goals increases students' self-confidence; Hattie and Timperley (2007) discuss the influence that feedback provided by teachers or parents may have on one's confidence; and Klassen (2004) argues that the actions of one's peers may have a considerable impact on self-confidence. The traditional explanation for the influence of others' actions on self-confidence relies, at least to some extent, on the assumption that agents interpret the information they collect at face value. For example, imagine that some agent's father decides to take over his son's science project and the son ends up receiving an "A" for it. If the son is "naive" he might become overconfident about his skill simply because he does not take into account that most of the work was performed by his father. While arguments of this nature might sometimes be reasonable, one has to wonder whether, if this type of interaction

*I would like to thank Diego Amador, Lorenzo Braccini, Mustafa Dogan, Hanming Fang, Anqi Li, Daniel Neuhann, Andrew Postlewaite, Qiusha Peng, Yanhao Wei, and the anonymous referees, as well as UPenn's seminar participants for their useful comments.

*Silva:* Assistant Professor, Department of Economics, Pontificia Universidad Catolica de Chile, Santiago, 7820436, Chile. Phone (+562) 223547104, Fax (+562) 225525480, E-mail franciscosilva@uc.cl

1. Bénabou and Tirole (2003) is one notable exception, which I discuss below.



Economic Inquiry
(ISSN 0095-2583)
Vol. 55, No. 1, January 2017, 451–460

doi:10.1111/ecin.12388
Online Early publication August 24, 2016

happened often, the son would eventually realize that the only reason he was getting "A"'s was because of the help he was receiving and not because of his skill. I provide a theory of induced overconfidence where the agent properly discounts any outside influence on his performance.

As an example, consider the relationship between a graduate student and his academic adviser. Upon arrival at the university, the graduate student usually has to go through a series of tests and examinations during his first year. Once his scores are revealed, both he and his adviser use them to update their beliefs about the student's ability. It appears reasonable to assume the adviser will be more willing to provide guidance and advice to the student if she believes the student to be of high ability. Therefore, a student who has performed well in his first year will receive a substantial amount of help, unlike the student who has not.

The key to my argument is the way I model the impact of the second agent's action on the informativeness of the first agent's future signals: in the graduate student's story, the way I model the impact of the adviser's help on the signals that arrive after the first year. Certainly, more help means that the scores the student obtains in future assessments will probably be higher. However, the downside of receiving help would be that these future scores would also be less informative. If the student performs a task on his own and has a good score, he must think that his ability is high. Likewise, if his score is low, he will think his ability is low. However, when the student receives substantial help, he will be unsure whether the score he obtained was a product of his own ability or of the help he received. Therefore, he will not be as accurate in inferring his ability from the score he observes.

Take two students who happen to have the same ability but different scores in their first year. The one who received a high score will receive more help and will therefore place relatively little weight on future scores. By contrast, someone who has received a low score during his first year will not receive as much help and, as a consequence, will place considerable weight on future scores. Given that the first score was low, the student is likely to improve. It is this asymmetry that causes overconfidence: students who perform well in their first year put more weight on that score than those who performed worse.

There are a number of papers based on this same insight that, while less confident people will quickly leave that state, more confident people will not. Zabojnik (2004) argues that the opportunity cost of further investigating one's ability increases with ability and so more confident agents will likely stop investigating sooner than will less confident ones. Köszegi (2006) states that more confident people will sometimes obtain less information out of fear that a bad outcome may make them think their ability is low, which has a direct impact on their well-being. Bénabou and Tirole (2002) show that, with time-inconsistent preferences, a more confident agent may prefer not to acquire costless information. Brocas and Carrillo (2009) argue that more confident agents require less evidence to support their investment decisions, which may help explain why so many new businesses fail. What distinguishes this paper is that I employ a similar logic in a setup where there are two agents (a father and a son, an adviser and an advisee, etc.) and where one of them strategically tailors his actions to influence the confidence of the other. Bénabou and Tirole (2003) consider a setup similar to this, in that they also consider an external figure who influences the agent's beliefs. However, their model does not generate a systematic bias toward overconfident beliefs, which is a contribution of this study.

## II. THE MODEL

There are two actors in the model: the agent and the person who provides help to the agent, whom I will call the adviser, following the story in the introduction. The agent is endowed with an ability level $y$ which neither he nor the adviser observes. I assume that $y \sim N\left(0, \sigma_y^2\right)$. There are two periods in the model. In the first period, the agent performs a task for which he receives a score $s_1$. This score will be a function not only of his ability $y$ but also of some independent random variable $\varepsilon_1$. In particular, I assume that

$$s_1 = y + \varepsilon_1$$

where $\varepsilon_1 \sim N(0, 1)$. Score $s_1$ is publicly available so the adviser is assumed to observe it. In the story in the introduction, the score $s_1$ represents the grades the graduate student receives during his first year.

At the end of period 1, there will be some interaction between the agent and the adviser, which will result in the agent receiving some level of help $h \geq 0$, which the agent is assumed to observe. In Section IV, I discuss this interaction in more detail; for now, I simply assume that the help the agent receives is a function of $s_1$. In particular, there is $g : \mathbb{R} \to \mathbb{R}_+$ such that $h = g(s_1)$.

In period 2, the student performs a second task which returns score $s_2$, where

$$s_2 = y + hx + \varepsilon_2.$$

I assume that $x \sim N\left(\mu_x, \sigma_x^2\right)$, $\varepsilon_2 \sim N(0, 1)$, and that $x$ and $\varepsilon_2$ are independent random variables. This second score still has the same two components as the first one (i.e., the agent's ability and "noise") but there is a third element, which depends on the help the agent receives. The random variable $x$ is interpreted as the ability of the adviser. The idea is that the help the agent receives determines whether the score depends more on the agent's ability or on the adviser's ability. If $h$ is small, then $s_2$ will depend mostly on the agent's ability; if $h$ is large, what matters the most is the adviser's ability.

## III. INDUCED OVERCONFIDENCE

I am interested in studying the circumstances under which the agent will be overconfident about his own ability $y$ at the end of the second period. Given the structure of the signals the agent observes and by simple Bayesian updating, it follows that

$$y|s_1, s_2 \sim N\left(\widehat{\mu}_y, \widehat{\sigma}_y^2\right)$$

for some $\widehat{\mu}_y$ and $\widehat{\sigma}_y^2$. The issue now is how to define "overconfidence."

One of the most common methods employed by the experimental literature on overconfidence is to ask people whether they believe they are better than the average (or more accurately the median).[2] If more than 50% of the subjects respond "yes," this is taken as evidence of overconfidence, as it is not possible that more than 50% of the people are actually better than the median. In my model, recall that $y$ is symmetric around 0, so there is no distinction between the mean and the median. Furthermore, the agents in my model also reach the end of period 2 believing their own ability is symmetric around $\widehat{\mu}_y$. So, when asked whether they believe their ability is greater than the mean/median, they would say "yes" if $\widehat{\mu}_y \geq 0$ and "no" otherwise, simply because $\widehat{\mu}_y$ represents the conditional expectation, median, and mode of their ability.[3]

2. See Dunning, Heath, and Suls (2004) for a review.

3. For simplicity, I have assumed that ties are solved in favor of the "yes." This assumption has no impact because the event where $\widehat{\mu}_y = 0$ has a 0 measure.

PROPOSITION 1. *If g is increasing, then*

$$\Pr\left\{\widehat{\mu}_y \geq 0\right\} \geq \frac{1}{2}.$$

*The inequality is strict if* g *is not constant.*

*Proof.* See Appendix.

Proposition 1 says that, if the help an agent receives is an increasing function of the first score, the probability that the agent believes his ability is greater than the median is larger than 50%. The intuition is the following. Let

$$\widehat{s}_2 = s_2 - h\mu_x$$

so that

$$\widehat{s}_2|y, s_1 \sim N\left(y, Var\left(y|s_1\right) + h^2\sigma_x^2 + 1\right).$$

It follows that the agent's conditional expectation $\widehat{\mu}_y$ is simply a weighted average of $s_1$ and $\hat{s}_2$ (and the prior mean 0). However, the weights depend on the help the agent receives. In particular, if help is increasing with $s_1$, the variance of the second signal $\hat{s}_2$ is larger, which makes it less informative. As a result, the weight of $s_1$ is larger when $s_1$ is larger and smaller when $s_1$ is smaller. In other words, if an agent draws a large $s_1$, he is more likely to end up with a conditional expectation close to it than if he draws a small $s_1$. Notice also that, if $g$ is constant, the weights on $s_1$ and $\hat{s}_2$ are also constant and so there is no bias. Furthermore, if $g$ is decreasing, the opposite happens and the probability that an agent's conditional expectation is below 0 is larger than 50%.

Benot and Dubra (2011) have criticized the previous method of documenting overconfidence. They argue that the fact that most of the population believes that they have a greater ability than the median is evidence of "apparent" overconfidence. The approach taken by those authors is that "real" overconfidence only exists when agents do not have rational expectations. Given that there are models with agents who have rational expectations where the same result is possible (such as this model), observing that most people believe they have a greater ability than the median is not indicative of overconfidence.

This type of criticism raises the question of whether there is a better, more accurate, way of defining overconfidence. Taken literally, when someone is overconfident, he is too confident with respect to something; in this case his own ability. So, whether an agent is overconfident or not will depend on some comparison between the agent's beliefs and his true ability. Of course, the challenge is to compare a distribution (the agent's beliefs) and a number (the agent's ability). However, in this model, the posterior belief of an agent will be a normal distribution with mean $\widehat{\mu}_y$, which is also the median and mode. Hence, it appears natural to think of $\widehat{\mu}_y$ as the agent's response to the question: "What do you think your ability is?" In this sense, it appears appropriate and intuitive to say that an agent is overconfident if $\widehat{\mu}_y > y$ and underconfident otherwise. Using this definition and maintaining the assumption that the agent indeed has rational expectations does preclude any systematic bias. In particular, if one defines $o$ to be such that

$$o \equiv \widehat{\mu}_y - y$$

it follows that $E(o) = \text{median}(o) = 0$. Therefore, if one is of the opinion that overconfidence exists if and only $E(o) > 0$ or if and only median$(o) > 0$, then it must be the case that overconfidence and rational expectations are incompatible, just as it is understood by Benot and Dubra (2011). However, it is not the case that overconfidence (defined as either $E(o) > 0$ or median$(o) > 0$) is incompatible with Bayes' updating.

The rational expectations assumption can be thought of as three assumptions put together: first, the agent knows the prior distribution of his own ability $y$; second, the agent knows the distributions of scores $s_1$ and $s_2$; third, the agent updates his beliefs about his own ability by Bayes' rule. The first of these assumptions is particularly controversial. The assumption that the agent knows the distribution of his own ability is made mostly out of convenience and not because there is a particularly compelling reason to think that agents are magically born knowing such a thing. In this context, I believe that an equally (if not more) compelling way to model the agent's behavior is to assume that he has an uninformative prior rather than the correct one. In particular, it appears reasonable to think that, while $y \sim N\left(0, \sigma_y^2\right)$, the agent will believe $y \sim N\left(0, \delta_y^2\right)$, where $\delta_y > \sigma_y$, so that, in a way, the agent is more confused about his ability than he would be if he actually knew its prior distribution. Notice in particular that, as $\delta_y^2$ increases, the prior belief of the agent becomes more and more uniform.

Assuming that the agent's prior belief about $y$ is distributed according to $N\left(0, \delta_y^2\right)$, after having observed scores $s_1$ and $s_2$, the agent will believe

$$y|s_1, s_2 \sim N\left(\widetilde{\mu}_y, \widetilde{\sigma}_y^2\right).$$

Finally, let

$$\widetilde{o} = \widetilde{\mu}_y - y$$

so that $\tilde{o}$ represents the overconfidence of the agent under this new prior.

PROPOSITION 2. *If g is increasing and* $\delta_y > \sigma_y$, *then*

$$E\left(\widetilde{o}\right) \geq 0 \text{ and median} \left(\widetilde{o}\right) \geq 0.$$

*Both inequalities are strict if g is not a constant.*

*Proof.* See Appendix.

By simply removing the assumption that the agent knows the prior distribution of ability and instead assuming that the agent has a prior with a larger variance (and, in the limit, an uninformative prior), it is possible to have overconfidence in the most intuitive of definitions. Not only is this definition of overconfidence intuitive, but there are also examples of empirical papers which document the existence of "expected overconfidence" (where $E(\tilde{o}) > 0$). For example, in the study by Smith and Powell (1990), a survey of college seniors was conducted, in which they were asked their best guess about their own future earnings as well as the earnings of the rest of their cohort. The results were that the average of the answers regarding one's own earnings was higher than the average of the answers regarding the rest of their cohort—a clear indication of expected overconfidence. I believe that it is important to highlight that, even though the assumption of rational expectations must be removed for to hold, this does not imply the agent is "irrational." He is not endowed with any bias and does Bayes' updating as before. It is simply the case that he does not know much about his prior distribution of ability and, as a result, relies less on his prior beliefs and more on the signals he observes.

## IV. WHY IS HELP INCREASING?

In the previous section, I have argued that, if the help an agent receives is an increasing function of the initial score $s_1$, there will be overconfidence as I have defined it. The purpose of this section is to discuss several circumstances where such an assumption is reasonable and intuitive. I provide three separate models that result in an increasing help function. The common thread in all of them is that a large initial score $s_1$ leads to a public belief that the agent has high ability. As a result, in the different settings considered, there will be a desire on the part of others to help/cooperate with that agent for their own benefit.

### A. *Adviser/Graduate Student*

Consider the story from the introduction where a graduate student is completing his Ph.D. The student has some unknown ability $y$ and, after the first year, receives score $s_1$. At the end of the first year, the student is assigned to an adviser who must choose how much help $h \in \left[0, \overline{h}\right]$ to provide to the student. The help the student receives impacts his future score $s_2$ as in the previous section.

Suppose the adviser's utility is given by $v(s_2) - ch$, where $v'(s_2) > 0$ for all $s_2$ and $c > 0$. Assume $\mu_x > 0$ so that the adviser's help, at least on average, does not harm the student, and let $g(s_1)$ denote the optimal help choice made by the adviser.

PROPOSITION 3. *If $v''(s_2) > 0$ for all $s_2 \in \mathbb{R}$, then g is increasing.*

*Proof.* See Appendix.

By providing help, the adviser incurs a marginal cost $c$ and causes a constant marginal increase of $x$ of the score $s_2$. However, the adviser is impacted more from the constant increase in the score if the score is large. Thus, providing help to the student gives more benefits to the adviser if she believes the score is likely to be larger, which is why she is more willing to provide help if she believes the student is of high ability.

There are several different reasons why an adviser would have such preferences. Maybe the adviser receives recognition only for her better students, rather than for their mean quality. Or perhaps the adviser benefits only from cooperating with the students perceived to be better, so that there would be an added benefit associated with skilled students. Or it might even be that the adviser believes that highly skilled students are more likely to be in a position of influence in the future, which might benefit the adviser indirectly. As a result, the adviser might be particularly interested in developing a high-ability student's career by providing help in obtaining high scores. What is important for the argument to hold is that the desire to help those students who appear more skilled is greater than the desire to help those who appear less skilled.

### B. *Child/Parents*

Consider the interaction between a child and his parents. In particular, think of how parents choose to help their children succeed. Typically, the choice parents make is not so much whether to help their child but rather how to allocate such help.

Imagine that there are $N$ different activities or skills for which an individual is endowed with an ability level $y^n$, where $n = 1, \ldots N$. Assume that $y^n \sim^{iid} N\left(0, \sigma_y^2\right)$. In the first period, the child performs $N$ tests for which she receives the corresponding public scores $\left\{s_1^n\right\}_{n=1}^N$, where $s_1^n = y^n + \varepsilon_1^n$. As in the previous section, for all $n$, $\varepsilon_1^n \sim^{iid} N(0, 1)$ and is independent of all other random variables.

In the second period, the child performs another set of $N$ tests. However, the parents are able to help the child in each of these. In particular, for each task $n$, the score the child receives is given by $s_2^n = y^n + h^n x^n + \varepsilon_2^n$, where $h^n \geq 0$ is the amount of help the parents provide for task $n$ and $x^n$ is the parents' ability to help the child perform activity $n$. As before, $x^n \sim^{iid} N\left(\mu_x^n, \sigma_x^2\right)$, is independent of all other random variables and $\mu_x^n > 0$ for all $n$, while $\varepsilon_2^n \sim^{iid} N(0, 1)$ and is independent of all other random variables.

Finally, the parents' utility function is given by

$$u\left(\left\{s_2^n\right\}_{n=1}^N\right) = \sum_{n=1}^N v\left(s_2^n\right)$$

where $v'\left(s_2^n\right) > 0$ for all $s_2^n \in \mathbb{R}$. The parents' problem is to choose $\{h^n\}_{n=1}^N$ such that

$$_{n=1}^{N} h^n \leq \overline{h}$$

where $\overline{h} > 0$, in order to maximize their expected utility.

Finally, let $g^n\left(s_1^1, \ldots, s_1^N\right)$ denote the optimal help level allocated to task $n$, given the set of scores from the first period.

PROPOSITION 4. *For all n, if $v''(z) > 0$ for all $z \in \mathbb{R}$, then $g^n\left(s_1^1, \ldots, s_1^N\right)$ is increasing with $s_1^n$ for all $\left\{s_1^{\hat{n}}\right\}_{\hat{n} \neq n}$.*

*Proof.* See Appendix.

The idea behind the argument is the following. Any of these $N$ different skills could be the basis of a career for the child. For example, variable $y^1$ could refer to the child's musical ability, variable $y^2$ could refer to athletic ability, and so on. By obtaining a large score in the second period on a given task $n$, this might increase the chances of a successful career using skill $n$. So, for example, $s_2^1$ can be the score of an entry audition for the Juilliard music school, or $s_2^2$ can be the performance of the child in his high school football competition. Naturally, parents would care about these signals because they are positively correlated with the child's future success.

The assumption that $v$ is convex simply means that parents prefer the child to be very good at one activity than average at all activities. The parents' motivation is that, if one thinks of these activities as precursors of future careers, a successful career depends on having only one very good skill (if the child becomes a professional musician, his athletic ability is pretty irrelevant in terms of his career). The consequence of this assumption is that the more convinced the parents are that a child's skill $y^n$ is large, the more they will be willing to help the child in performing activity $n$.[4]

### C. Matching

In most circumstances, most of the influence exerted on an agent comes from his coworkers. However, the type of coworkers an agent has is likely to depend on the agent's perceived ability. In particular, the more skilled an agent is perceived to be, the more cooperation opportunities he is likely to attract.

Consider a similar scenario as before, where there is a community of $I$ agents all endowed with some ability level $y^i$, where $y^i \sim^{iid} N\left(0, \sigma_y^2\right)$. In the first period, each agent $i$ observes public signal $s_1^i = y^i + \varepsilon_1^i$, where $\varepsilon_1^i \sim^{iid} N(0,1)$ and is independent of all other random variables. In the second period, agents randomly form pairs and must decide whether to cooperate. Say a pair $(i', i'')$ is formed. If the agents cooperate, they receive a single joint signal $s_2 = y^{i'} + y^{i''} + \tilde{\varepsilon}_2$, where $\tilde{\varepsilon}_2 \sim N(0,1)$ and is independent of other random variables. If they do not, each of them receives his own signal $s_2^i = y^i + \varepsilon_2^i$ for $i = i', i''$, where $\varepsilon_2^i \sim^{iid} N(0,1)$ and is independent of other random variables.

I model the second period game between the two matched agents as follows. One of them is chosen with 50% probability to be given the opportunity to ask the other agent whether he wants to cooperate. If he has chosen to ask for cooperation, the other agent must choose whether or not to accept. Cooperation only occurs when the first agent chooses to ask for it, and the second agent accepts. Finally, I assume that each agent $i$'s utility is equal to $s_2$ if there is cooperation and equal to $s_2^i$ otherwise.

Notice that, after period 1, the public belief about any agent $i$'s ability is that

$$y^i | s_1^i \sim N\left(\frac{\sigma_y^2}{1+\sigma_y^2} s_1^i, \frac{\sigma_y^2}{1+\sigma_y^2}\right),$$

which then implies that each agent is only willing to cooperate with a partner for whom the first period's signal was positive.[5]

PROPOSITION 5. *For any match* (i′, i″),

*(i) If* $s_1^i > 0$ *for* $i = i', i''$, *the unique subgame perfect equilibrium outcome is to have cooperation.*

*(ii) If* $s_1^i < 0$ *for either* $i = i'$ *or* $i = i''$, *the unique subgame perfect equilibrium outcome is not to have cooperation.*

*Proof.* Notice that each agent prefers to cooperate if he believes the other agent's expected ability is positive and prefers to refuse to cooperate if he believes it is negative. Hence, if the conditions of (i) hold, the respondent strictly prefers to accept the cooperation offer and the proposer strictly prefers to propose cooperation. If the conditions of (ii) hold and if (a) the proposer's expected ability is negative, then the respondent will refuse cooperation, while if (b) the respondent's expected ability is negative but the proposer's is positive, then the proposer refuses to propose cooperation. ■

Agents only wish to cooperate if their match is skilled because only then does the match bring value to the project. Therefore, agents who receive a high initial score will have more cooperation opportunities than those who receive a low

4. Even though Propositions 1 and 2 do not follow directly, given that $g$ was assumed to be a function of only one variable, it is easy to show the same exact results hold. Basically, what follows directly from these is that there is overconfidence at activity $n$ (according to the respective definitions of propositions 1 and 2) conditional on the scores of all other activities $\left\{s_1^{\hat{n}}\right\}_{\hat{n} \neq n}$. By integrating out these other scores one obtains the unconditional overconfidence results of Propositions 1 and 2 applied to each activity $n$.

5. Notice that the same would be true even if the agents had an uninformative prior. In that case, the conditional expectation of $y^i$ would be exactly equal to $s_1^i$.

score. In particular, from the perspective of some agent $i'$, it is as if the second period score is equal to

$$y^{i'} + hx + \varepsilon_2$$

for some independent $\varepsilon_2 \sim N(0,1)$, where $x \sim N\left\{\left[\sigma^2/\left(\sigma^2+1\right)\right] s_1^{i''}, \left[\sigma^2/\left(\sigma^2+1\right)\right]\right\}$ and denotes the ability of the agent's match $i''$ and where $h$ is increasing with $s_1^{i'}$. As a result, the overconfidence results follow from the previous section.

## V. CONCLUSION

In this article, I have presented a theory of induced overconfidence. I have argued that, in many circumstances, an individual's opportunities to cooperate and receive help are positively correlated with past signals of achievement. I gave the example of an academic adviser who prefers to help students she perceives to be more skilled, parents who prefer to help their children with the skills where they already distinguish themselves, and coworkers who prefer to cooperate only if they believe their partner is skilled. Overconfidence will then arise simply because those who are more confident attract cooperation and, as a result, receive less informative signals relative to their ability when compared with those who are less confident. The asymmetry in the cooperation opportunities is what causes the overconfidence bias in the agents' beliefs.

## APPENDIX

*Proof of Proposition 1*

By Bayesian updating the agent's beliefs, it follows that $y|s_1, s_2 \sim N\left(\widehat{\mu}_y, \widehat{\sigma}_y^2\right)$, where

$$\widehat{\mu}_y = \sigma_y^2 \frac{s_2 - h\mu_x + \left(h^2\sigma_x^2+1\right)s_1}{\sigma_y^2 + \left(h^2\sigma_x^2+1\right)\left(\sigma_y^2+1\right)}$$

and

$$\widehat{\sigma}_y^2 = \frac{\left(h^2\sigma_x^2+1\right)\sigma_y^2}{\sigma_y^2 + \left(h^2\sigma_x^2+1\right)\left(\sigma_y^2+1\right)}.$$

Therefore,

$$\Pr\left\{\widehat{\mu}_y \geq 0\right\} = \Pr\left\{q \geq 0\right\}$$
$$= \int_{-\infty}^{\infty} f_{s_1}\left(\widetilde{s}_1\right)\Pr\left\{q \geq 0 | s_1 = \widetilde{s}_1\right\} d\widetilde{s}_1$$

where

$$q\left(s_1, s_2\right) = s_2 - h\mu_x + \left(h^2\sigma_x^2+1\right)s_1.$$

Given that $s_1$ is symmetrically distributed around 0 it follows that

$$\Pr\left\{\widehat{\mu}_y \geq 0\right\} = \int_0^{\infty} f_{s_1}\left(\widetilde{s}_1\right)\begin{bmatrix}\Pr\left\{q\left(s_1,s_2\right)\geq 0|s_1=\widetilde{s}_1\right\}\\ +\Pr\left\{q\left(s_1,s_2\right)\geq 0|s_1=-\widetilde{s}_1\right\}\end{bmatrix} d\widetilde{s}_1.$$

Notice that, because $y|s_1$, $x|s_1$, and $\varepsilon_2|s_1$ are independent and normally distributed, it follows that $s_2|s_1$ is also normally distributed, which implies that

$$q|s_1 \sim N\left(\left(\frac{\sigma_y^2}{\sigma_y^2+1} + h^2\sigma_x^2 + 1\right)s_1, \frac{\sigma_y^2}{\sigma_y^2+1} + h^2\sigma_x^2 + 1\right).$$

Therefore,

$$\Pr\left\{q \geq 0|s_1 = \widetilde{s}_1\right\} + \Pr\left\{q \geq 0 | s_1 = -\widetilde{s}_1\right\}$$
$$= 2 - \Phi\left(-\widetilde{s}_1\sqrt{\frac{\sigma_y^2}{\sigma_y^2+1} + \left(g\left(\widetilde{s}_1\right)\right)^2\sigma_x^2 + 1}\right)$$
$$-\Phi\left(\widetilde{s}_1\sqrt{\frac{\sigma_y^2}{\sigma_y^2+1} + \left(g\left(-\widetilde{s}_1\right)\right)^2\sigma_x^2 + 1}\right).$$

Given that

$$\sqrt{\frac{\sigma_y^2}{\sigma_y^2+1} + h^2\sigma_x^2 + 1}$$

is strictly increasing with $h$ and $g$ is increasing it follows that

$$\Phi\left(-\widetilde{s}_1\sqrt{\frac{\sigma_y^2}{\sigma_y^2+1} + \left(g\left(\widetilde{s}_1\right)\right)^2\sigma_x^2 + 1}\right)$$
$$+\Phi\left(\widetilde{s}_1\sqrt{\frac{\sigma_y^2}{\sigma_y^2+1} + \left(g\left(-\widetilde{s}_1\right)\right)^2\sigma_x^2 + 1}\right) \leq 1$$

for all $\widetilde{s}_1 \geq 0$, which proves the result. If $g$ is not constant, it follows that the above inequality will be strict for all $\widetilde{s}_1 \geq \overline{s}_1$ for some $\overline{s}_1$, which implies that $\Pr\left\{\widehat{\mu}_y \geq 0\right\} > \frac{1}{2}$. ■

*Proof of Proposition 2*

Notice that

$$\widetilde{\mu}_y = \delta_y^2 \frac{s_2 - h\mu_x + \left(h^2\sigma_x^2+1\right)s_1}{\delta_y^2 + \left(h^2\sigma_x^2+1\right)\left(\delta_y^2+1\right)}$$

and so

$$\widetilde{o} = \delta_y^2 \frac{s_2 - h\mu_x + \left(h^2\sigma_x^2+1\right)s_1}{\delta_y^2 + \left(h^2\sigma_x^2+1\right)\left(\delta_y^2+1\right)} - y.$$

I start by showing the first part of the result. Notice that it is enough to show that

$$E\left(\widetilde{\mu}_y\right) \geq 0$$

if $g$ is increasing, with the inequality being strict if $g$ is not a constant.

Notice that

$$E\left(\widetilde{\mu}_y\right) = E_{s_1}\left(E\left(\widetilde{\mu}_y|s_1\right)\right)$$

where

$$E\left(\widetilde{\mu}_y|s_1\right) = d\left(h\right)s_1$$

and

$$d(h) = \frac{\delta_y^2}{\sigma_y^2+1}\frac{\sigma_y^2 + \left(h^2\sigma_x^2+1\right)\left(\sigma_y^2+1\right)}{\delta_y^2 + \left(h^2\sigma_x^2+1\right)\left(\delta_y^2+1\right)}.$$

Notice that $d(h)$ is strictly increasing with $h$ if $\delta_y > \sigma_y$, strictly decreasing if $\delta_y < \sigma_y$, and constant if $\delta_y = \sigma_y$.

If $\delta_y > \sigma_y$ and $g$ is increasing, it follows that

$$E\left(d\left(g\left(s_1\right)\right)s_1\right) = \int_{-\infty}^{\infty} f_{s_1}\left(\tilde{s}_1\right) d\left(g\left(\tilde{s}_1\right)\right)\tilde{s}_1 d\tilde{s}_1$$

$$= \int_0^{\infty} f_{s_1}\left(\tilde{s}_1\right)\left(d\left(g\left(\tilde{s}_1\right)\right) - d\left(g\left(-\tilde{s}_1\right)\right)\right)\tilde{s}_1 d\tilde{s}_1$$

$$\geq 0$$

where the inequality is strict if $g$ is not a constant.

As for the second part, notice that one can write

$$\Pr\left\{\tilde{o} \geq 0\right\} = \Pr\{p \geq 0\}$$

where

$$p = \delta_y^2 h\left(x-\mu_x\right) + \delta_y^2\varepsilon_2 + \delta_y^2\left(h^2\sigma_x^2+1\right)s_1 - \left(h^2\sigma_x^2+1\right)\left(\delta_y^2+1\right)y.$$

Notice that

$$\Pr\{p \geq 0\} = \int_0^{\infty} f_{s_1}\left(\tilde{s}_1\right)\begin{bmatrix}\Pr\left\{p \geq 0|s_1 = \tilde{s}_1\right\} \\ +\Pr\left\{p \geq 0|s_1 = -\tilde{s}_1\right\}\end{bmatrix} d\tilde{s}_1.$$

Because $x|s_1$, $y|s_1$, and $\varepsilon_2|s_1$ are all independent and normally distributed, it follows that

$$p|s_1 \sim N\left(\mu_p\left(h^2\right)s_1, \sigma_p^2\left(h^2\right)\right)$$

where

$$\mu_p\left(h^2\right) = \left(h^2\sigma_x^2+1\right)\left(\delta_y^2 - \sigma_y^2\frac{\delta_y^2+1}{\sigma_y^2+1}\right)$$

and

$$\sigma_p^2\left(h^2\right) = \delta_y^4 h^2\sigma_x^2 + \delta_y^4 + \left(h^2\sigma_x^2+1\right)^2\left(\delta_y^2+1\right)^2\frac{\sigma_y^2}{\sigma_y^2+1}.$$

Therefore,

$$\Pr\left\{p \geq 0|s_1 = \tilde{s}_1\right\} + \Pr\left\{p \geq 0|s_1 = -\tilde{s}_1\right\}$$

$$= 2 - \Phi\left(-\frac{\mu_p\left(\left(g\left(s_1\right)\right)^2\right)}{\sigma_p\left(\left(g\left(s_1\right)\right)^2\right)}\tilde{s}_1\right) - \Phi\left(\frac{\mu_p\left(\left(g\left(-s_1\right)\right)^2\right)}{\sigma_p\left(\left(g\left(-s_1\right)\right)^2\right)}\tilde{s}_1\right).$$

Notice that if $\delta_y > \sigma_y$, it follows that $\frac{\mu_p(h)}{\sigma_p(h)}$ is strictly increasing with $h$, if $\delta_y < \sigma_y$ it is strictly decreasing, and if $\delta_y = \sigma_y$ it is constant and equal to 0.

Therefore, if $g$ is increasing and $\delta_y > \sigma_y$, it is the case that

$$\Phi\left(-\frac{\mu_p\left(\left(g\left(s_1\right)\right)^2\right)}{\sigma_p\left(\left(g\left(s_1\right)\right)^2\right)}\tilde{s}_1\right) + \Phi\left(\frac{\mu_p\left(\left(g\left(-s_1\right)\right)^2\right)}{\sigma_p\left(\left(g\left(-s_1\right)\right)^2\right)}\tilde{s}_1\right) \leq 1$$

for all $\tilde{s}_1 > 0$, which shows the result. It is also clear that if $g$ is not a constant there will be some $\bar{s}_1 > 0$ such that the previous inequality holds strictly for all $\tilde{s}_1 \geq \bar{s}_1$. ■

*Proof of Proposition 3*

The problem of the adviser is to select $g(s_1)$ for all $s_1$ such that

$$g\left(s_1\right) \in \arg\max_{h\in\left[0,\bar{h}\right]} E\left(v\left(y+hx+\varepsilon_2\right)|s_1\right) - ch.$$

Notice that it must be that, for all $s_1$, $g\left(s_1\right) \in \{0, \bar{h}\}$ because, if not, it would have to be that

$$E\left(xv'\left(y+g\left(s_1\right)x+\varepsilon_1\right)|s_1\right) = c$$

and

$$E\left(x^2v''\left(y+g\left(s_1\right)x+\varepsilon_1\right)|s_1\right) \leq 0$$

which is not true because $v'' > 0$.

Therefore, it follows that $g\left(s_1\right) = \bar{h}$ if

$$E\left(v\left(y+\bar{h}x+\varepsilon_2\right)|s_1\right) - E\left(v\left(y+\varepsilon_2\right)|s_1\right) \geq c\bar{h}$$

and $g(s_1) = 0$ otherwise (where the assumption is that if there is a tie help is provided). The RHS is independent of $s_1$ so the proof is completed by showing the LHS is strictly increasing with $s_1$.

The LHS can be written as

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\varepsilon_2}\left(\tilde{\varepsilon}_2\right)f_{y|s_1}\left(\tilde{y}\right)f_x\left(\tilde{x}\right) \times\left(v\left(\tilde{y}+\bar{h}\tilde{x}+\tilde{\varepsilon}_2\right) - v\left(\tilde{y}+\tilde{\varepsilon}_2\right)\right)d\tilde{x}d\tilde{y}d\tilde{\varepsilon}_2.$$

By doing a change of variables where $\hat{y} = \tilde{y} - \frac{\sigma^2}{\sigma^2+1}s_1$ and $\hat{x} = \tilde{x} - \mu_x$ it follows that the LHS is proportional to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times\begin{bmatrix} f_{\varepsilon_2}\left(\tilde{\varepsilon}_2\right)\exp\left(-\frac{\hat{y}^2}{2\frac{\sigma_y^2}{\sigma_y^2+1}}\right)\exp\left(-\frac{\hat{x}^2}{2\sigma_x^2}\right) * \\ \left(v\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1+\bar{h}\hat{x}+\bar{h}\mu_x+\tilde{\varepsilon}_2\right) \right. \\ \left. -v\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1+\tilde{\varepsilon}_2\right)\right)\end{bmatrix}d\hat{x}d\hat{y}d\tilde{\varepsilon}_2$$

which is equal to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times\begin{bmatrix} f_{\varepsilon_2}\left(\tilde{\varepsilon}_2\right)\exp\left(-\frac{\hat{y}^2}{2\frac{\sigma_y^2}{\sigma_y^2+1}}\right)\exp\left(-\frac{\hat{x}^2}{2\sigma_x^2}\right) * \\ \left[\begin{array}{c} v\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1+\bar{h}\hat{x}+\bar{h}\mu_x+\tilde{\varepsilon}_2\right) \\ -v\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1+\tilde{\varepsilon}_2\right) \\ +v\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1-\bar{h}\hat{x}+\bar{h}\mu_x+\tilde{\varepsilon}_2\right) \\ -v\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1+\tilde{\varepsilon}_2\right)\end{array}\right]\end{bmatrix}d\hat{x}d\hat{y}d\tilde{\varepsilon}_2.$$

Therefore, the derivative of the previous expression with respect to $s_1$ is strictly positive if $v'' > 0$ because, for all $\hat{x} > 0$, for all $\hat{y}$, and $\widetilde{\varepsilon}_2$

$$v'\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1+\overline{h}\hat{x}+\overline{h}\mu_x+\widetilde{\varepsilon}_2\right)$$
$$+v'\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1-\overline{h}\hat{x}+\overline{h}\mu_x+\widetilde{\varepsilon}_2\right)$$
$$>2v'\left(\hat{y}+\frac{\sigma_y^2}{\sigma_y^2+1}s_1+\widetilde{\varepsilon}_2\right)$$

which concludes the proof. ■

*Proof of Proposition 4*

The parent's problem is to choose $h^n \geq 0$ for all $n = 1,...,N$ such that $\sum_{n=1}^{N} h^n \leq \overline{h}$ in order to maximize $E\left[\sum_{n=1}^{N} v\left(s_2^n\right) \mid \left\{s_1^n\right\}_{n=1}^{N}\right]$.

First, I show that

$$\frac{\partial E\left[\sum_{n=1}^{N} v\left(s_2^n\right) \mid \left\{s_1^n\right\}_{n=1}^{N}\right]}{\partial h^{\hat{n}}}>0$$

for all $\hat{n}$, which implies that $\sum_{n=1}^{N} g^n = \overline{h}$. Notice that

$$\frac{\partial E\left[\sum_{n=1}^{N} v\left(s_2^n\right) \mid \left\{s_1^n\right\}_{n=1}^{N}\right]}{\partial h^{\hat{n}}}=E\left(x^{\hat{n}}v'\left(y^{\hat{n}}+h^{\hat{n}}x^{\hat{n}}+\varepsilon^{\hat{n}}\right)|s_1^{\hat{n}}\right)$$
$$=E\left[E\left(x^{\hat{n}}v'\left(y^{\hat{n}}+h^{\hat{n}}x^{\hat{n}}+\varepsilon^{\hat{n}}\right)|y^{\hat{n}},\varepsilon^{\hat{n}},s_1^{\hat{n}}\right)|s_1^{\hat{n}}\right].$$

Notice that letting $\widetilde{x}^{\hat{n}} = x^{\hat{n}} - \mu_{x^{\hat{n}}}$ it follows that

$$E\left(x^{\hat{n}}v'\left(y^{\hat{n}}+h^{\hat{n}}x^{\hat{n}}+\varepsilon^{\hat{n}}\right)|y^{\hat{n}},\varepsilon^{\hat{n}},s_1^{\hat{n}}\right)$$

is proportional to

$$\int_0^{\infty}\exp\left(-\frac{\left(\widetilde{x}^{\hat{n}}\right)^2}{\sigma_x^2}\right)\left[\begin{array}{c}\widetilde{x}^{\hat{n}}v'\left(y^{\hat{n}}+h^{\hat{n}}\widetilde{x}^{\hat{n}}+h^{\hat{n}}\mu_{x^{\hat{n}}}+\varepsilon^{\hat{n}}\right)\\ +\mu_{x^{\hat{n}}}v'\left(y^{\hat{n}}+h^{\hat{n}}\widetilde{x}^{\hat{n}}+h^{\hat{n}}\mu_{x^{\hat{n}}}+\varepsilon^{\hat{n}}\right)\\ -\widetilde{x}^{\hat{n}}v'\left(y^{\hat{n}}-h^{\hat{n}}\widetilde{x}^{\hat{n}}+h^{\hat{n}}\mu_{x^{\hat{n}}}+\varepsilon^{\hat{n}}\right)\\ +\mu_{x^{\hat{n}}}v'\left(y^{\hat{n}}-h^{\hat{n}}\widetilde{x}^{\hat{n}}+h^{\hat{n}}\mu_{x^{\hat{n}}}+\varepsilon^{\hat{n}}\right)\end{array}\right]d\widetilde{x}$$
$$>\int_0^{\infty}\exp\left(-\frac{\left(\widetilde{x}^{\hat{n}}\right)^2}{\sigma_x^2}\right)\widetilde{x}^{\hat{n}}\left[v'\left(y^{\hat{n}}+h^{\hat{n}}\widetilde{x}^{\hat{n}}+h^{\hat{n}}\mu_{x^{\hat{n}}}+\varepsilon^{\hat{n}}\right)\right.$$
$$\left.-v'\left(y^{\hat{n}}-h^{\hat{n}}\widetilde{x}^{\hat{n}}+h^{\hat{n}}\mu_{x^{\hat{n}}}+\varepsilon^{\hat{n}}\right)\right]\geq 0$$

for all $y^{\hat{n}}, \varepsilon^{\hat{n}}, s_1^{\hat{n}}$, where the last inequality follows from the fact that $v'' > 0$. This implies that

$$\frac{\partial E\left[_{n=1}^{N} v\left(s_2^n\right) \mid \left\{s_1^n\right\}_{n=1}^{N}\right]}{\partial h^{\hat{n}}}>0$$

and that $\sum_{n=1}^{N} g^n = \overline{h}$.

Now I show that, for any arbitrary $\hat{n}$, $g^{\hat{n}} \in \left\{0, \overline{h}\right\}$. Suppose not. Then there must be at least $n'$, $n''$ such that $g^{n'} \in (0,1)$ and $g^{n''} \in (0,1)$. WLOG, say $n' = 1$ and $n'' = 2$. Let $\widetilde{h} = \overline{h} -_{n=3}^{N} g^n$. It follows that

$$g^1\in\arg\max_{h^1\in\left[0,\widetilde{h}\right]}\ E\left[v\left(y^1+h^1x^1+\varepsilon_2^1\right)|s_1^1\right]$$
$$+E\left[v\left(y^2+\widetilde{h}x^2-h^1x^2+\varepsilon_2^2\right)|s_1^2\right].$$

Given that $g^1 \notin \left\{0, \widetilde{h}\right\}$ it follows that

$$E\left[x^1v'\left(y^1+g^1x^1+\varepsilon_2^1\right)|s_1^1\right]$$
$$-E\left[x^2v'\left(y^2+g^2x^2+\varepsilon_2^2\right)|s_1^2\right]=0$$

and

$$E\left[\left(x^1\right)^2v''\left(y^1+g^1x^1+\varepsilon_2^1\right)|s_1^1\right]$$
$$+E\left[\left(x^2\right)^2v''\left(y^2+g^2x^2+\varepsilon_2^2\right)|s_1^2\right]\leq 0.$$

The second inequality is false given that $v'' > 0$ which is a contradiction. Therefore, $g^{\hat{n}} \in \left\{0, \overline{h}\right\}$.

Finally, given that, for all $n$, $g^n \in \left\{0, \overline{h}\right\}$ and $_{n=1}^{N} g^n = \overline{h}$, it must be that the parents choose some $\tilde{n}$ for which $g^{\widetilde{n}} = \overline{h}$ such that

$$\widetilde{n}\in\arg\max_{n=1,\ldots,N}\ \left\{E\left[v\left(y^n+\overline{h}x^n+\varepsilon_2^n\right)|s_1^n\right]\right.$$
$$\left.-E\left[v\left(y^n+\varepsilon_2^n\right)|s_1^n\right]\right\}.$$

Therefore, to complete the proof, it suffices to show that, for all $n$,

$$\left\{E\left[v\left(y^n+\overline{h}x^n+\varepsilon_2^n\right)|s_1^n\right]-E\left[v\left(y^n+\varepsilon_2^n\right)|s_1^n\right]\right\}$$

is increasing with $s_1^n$, which I have on the proof of Proposition 3. ■

## REFERENCES


Bénabou, R., and J. Tirole. "Self-Confidence and Personal Motivation." *Quarterly Journal of Economics*, 117(3), 2002, 871–915.

———. "Intrinsic and Extrinsic Motivation." *Review of Economic Studies*, 70(3), 2003, 489–520.

Benot, J.-P., and J. Dubra. "Apparent Overconfidence." *Econometrica*, 79(5), 2011, 1591–625.

Brocas, I., and J. D. Carrillo. "Information Acquisition and Choice under Uncertainty." *Journal of Economics & Management Strategy*, 18(2), 2009, 423–55.

Buehler, R., D. Griffin, and M. Ross. "Exploring the "Planning Fallacy": Why People Underestimate Their Task Completion Times." *Journal of Personality and Social Psychology*, 67, 1994, 366–81.

Camerer, C., and D. Lovallo. "Overconfidence and Excess Entry: An Experimental Approach." *American Economic Review*, 89(1), 1999, 306–18.

Clark, J., and L. Friesen. "Overconfidence in Forecasts of Own Performance: An Experimental Study." *The Economic Journal*, 119(534), 2009, 229–51.

Dunning, D., C. Heath, and J. M. Suls. "Flawed Self-Assessment Implications for Health, Education, and the Workplace." *Psychological Science in the Public Interest*, 5(3), 2004, 69–106.

Glasgow, K. L., S. M. Dornbusch, L. Troyer, L. Steinberg, and P. L. Ritter. "Parenting Styles, Adolescents' Attributions, and Educational Outcomes in Nine Heterogeneous High Schools." *Child Development*, 68(3), 1997, 507–29.

Guthrie, C., J. J. Rachlinski, and A. J. Wistrich. "Inside the Judicial Mind." *Cornell Law Review*, 86, 2001, 777–830.

Hattie, J., and H. Timperley. "The Power of Feedback." *Review of Educational Research*, 77(1), 2007, 81–112.

Klassen, R. M. "A Cross-Cultural Investigation of the Efficacy Beliefs of South Asian Immigrant and Anglo Canadian Nonimmigrant Early Adolescents." *Journal of Educational Psychology*, 96(4), 2004, 731–42.

Köszegi, B. "Ego Utility, Overconfidence, and Task Choice." *Journal of the European Economic Association*, 4(4), 2006, 673–707.

Malmendier, U., and G. Tate. "CEO Overconfidence and Corporate Investment." *Journal of Finance*, 60(6), 2005, 2661–700.

Meyer, J. P., and I. R. Gellatly. "Perceived Performance Norm as a Mediator in the Effect of Assigned Goal on Personal Goal and Task Performance." *Journal of Applied Psychology*, 73(3), 1988, 410.

Moore, D. A., and P. J. Healy. "The Trouble with Overconfidence." *Psychological Review*, 115(2), 2008, 502.

Smith, H. L., and B. Powell. "Great Expectations: Variations in Income Expectations among College Seniors." *Sociology of Education*, 63(3), 1990, 194–207.

Svenson, O. "Are We All Less Risky and More Skillful Than Our Fellow Drivers?" *Acta Psychologica*, 47(2), 1981, 143–48.

Taylor, S. E., and J. D. Brown. "Illusion and Well-Being: A Social Psychological Perspective on Mental Health." *Psychological Bulletin*, 103(2), 1988, 193–210.

Zabojnik, J. "A Model of Rational Bias in Self-Assessments." *Economic Theory*, 23(2), 2004, 259–82.