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Jaime Casassus y Fernando Coloma.

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Asset Allocation and Pension Risk Hedging*

Jaime Casassus
Pontificia Universidad Católica de Chile

Fernando Coloma
Pontificia Universidad Católica de Chile

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Abstract

This paper develops a life-cycle asset allocation framework for target-date funds that incorporates the risk of financing retirement income. A hypothetical cohort-specific pension bond matches expected real pension payments conditional on survival. Although unavailable as a traded instrument, it provides a benchmark for measuring portfolio pension risk. A criterion based on the pension Sharpe ratio balances expected excess returns and pension risk across ages, weighted by account balances, while the market impact of changes in domestic asset demand links portfolio choices. Motivated by Chile's transition from multifunds to target-date funds, we use Chilean data to show how available hedging opportunities shape the glide path. For young cohorts, market instruments provide limited protection against distant pension payments, favoring higher expected returns. Long-duration government and corporate bonds enter around age 45, and their combined share increases toward age 65. Shorter-duration fixed income enters thereafter, reducing the fixed-income portfolio's duration as the remaining payment horizon shrinks. Around retirement, approximately 30% remains in growth assets, reflecting the substantial investment horizon still ahead. Replacement-rate and funding-ratio distributions illustrate pension outcomes and their dispersion over the life cycle, with the theoretical pension bond serving as a valuation reference rather than an available investment alternative.

Keywords: pension funds; asset allocation; pension risk; target-date funds; pension units.
JEL classification: G11, G23, J26.

1 Introduction

Pension fund investment aims to finance retirement income. This purpose should guide both risk measurement and asset selection. We study these decisions in target-date funds (TDFs), which pool investors with similar expected retirement dates and adjust their asset mix as the cohort ages. This age-dependent allocation schedule is the fund's *glide path*; the target date identifies expected retirement rather than the end of the investment horizon. The central question of this paper is how strategic asset allocation changes when performance is assessed by the ability to finance pensions, taking into account the horizon and timing of those payments.

Investment returns increase available resources, but their assessment must incorporate changes in the cost of converting savings into future income. A fund can earn a positive real return yet lose pension purchasing power if that cost rises faster. Two participants with identical balances and survival profiles may finance different incomes if they retire at different long-term interest rates. When rates fall, the present value of pension payments increases and more capital is needed to sustain a given monthly income. This distinction between accumulated wealth and retirement income is central to Merton (2003) and Merton (2007).

The pension horizon includes both the years before retirement and the subsequent period over which savings are spent. Retirement marks the start of payments that may extend over decades. Holding short-term assets can stabilize the current account balance while exposing future income to reinvestment conditions. By contrast, instruments with cash flows close to pension payments can appreciate when those payments become more expensive to finance, offsetting part of that risk. Asset volatility must therefore be evaluated alongside hedging capacity.

The investment literature has long recognized the importance of relating assets to the commitments they finance. Redington (1952) formalized immunization conditions based on matching present values and interest-rate sensitivities. Leibowitz (1986) and Leibowitz (1987) developed risk assessment through the surplus of assets over liabilities. This perspective was extended by Leibowitz and Henriksson (1988), Bookstaber and Gold (1988), Sharpe and Tint (1990), and Elton and Gruber (1992). Positive covariance between assets and commitments can reduce surplus risk even when assets are individually volatile. For

long-term investors, Campbell and Viceira (2002) also emphasize hedging demands in strategic portfolio choice. These contributions provide a basis for assessing defined-contribution funds relative to the cost of the income they must finance.

We organize this assessment around a cohort-specific pension bond: a benchmark asset whose real cash flows match a cohort's expected payments under specified actuarial assumptions. This retirement-income orientation relates to the instruments discussed by Merton and Muralidhar (2020). The bond summarizes changes in the cost of financing a pension and allows performance to be measured in units of that income. A portfolio's pension risk is the volatility of its return relative to the bond; it depends on both asset volatility and covariance with the benchmark at each age.

We define the *pension Sharpe ratio* (PSR) as expected return in excess of the pension bond per unit of pension risk, an information ratio relative to this benchmark. The objective aggregates age-specific PSRs using account-balance weights and incorporates portfolio constraints and the market impact of changes in domestic asset demand. These effects link choices across ages and require joint optimization. The criterion covers accumulation and retirement, with survival probabilities entering the pension benchmark. Simulations then assess pension outcomes. Following Mantilla-Garcia et al. (2024), the funding ratio compares managed balances with those financed by investing identical contributions exclusively in hypothetical pension bonds. This provides a measure of managers' performance in pension-income terms against a theoretical reference, evaluated through both expected gains and outcome dispersion; the matching bonds are not available as traded instruments.

Chile provides an illustrative application because its 2025 pension reform (Law No. 21.735) mandated a transition from the multifund system to target-date funds. The transition raises the policy question of how these funds should allocate assets over the life cycle to finance retirement income. We use Chilean data to illustrate how measuring risk relative to the pension benchmark affects portfolio choice.[1] The simulation

[1] Earlier research on pension risk includes several studies that use Chilean data. Walker (2006) studied optimal portfolios in units of future pension income, and Walker (2008) examined the implications of long-horizon risk for investment regulation. Castro (2009) proposed a new measure of pension fund performance, and Valdés, Castro and Torche (2010) discussed how an annuity price index could guide the selection of assets that hedge annuity price risk. Berstein, Fuentes and Villatoro (2013) incorporated the risk of converting accumulated balances into annuities when evaluating default life-cycle investment strategies in Chile. In related work on the Chilean annuity market, Casassus and Walker (2013) used a bond portfolio matching expected annuity payments as a valuation benchmark and accounted for illiquidity and insurer default risk.

considers a participant with uninterrupted contributions whose pension is financed exclusively from an individual account. The exercise is methodological: its assumptions isolate the effect of financial decisions, and the results are neither proposed regulatory portfolios nor pension forecasts.

The results show that the payment horizon shapes the TDF glide path. For young cohorts, available instruments provide little protection against distant commitments, favoring higher-expected-return assets. Long-duration domestic government and corporate bonds enter around age 45, and their combined share increases toward age 65. Shorter-duration fixed income enters thereafter, gradually reducing the fixed-income portfolio's duration. Age-specific frontiers without market impact clarify these hedging opportunities; the investment path incorporates the impact of domestic asset demand on expected returns.

Better hedging does not imply choosing the minimum-pension-variance portfolio. Around ages 60–65, approximately 30% remains invested in growth assets, reflecting the substantial investment horizon after retirement. Simulations trace outcomes through age 110, conditional on survival to each evaluated age. The initial replacement-rate distribution summarizes pension outcomes at retirement; the funding ratio extends the assessment over time. The theoretical bond strategy defines a funding ratio of one, but it is unavailable in the market. The simulated strategy's mean ratio of 1.57 at retirement corresponds to a mean pension level 57% above this hypothetical reference, using identical contributions and the same actuarial conversion factor. This comparison expresses pension outcomes in benchmark units; it is neither a 57% excess investment return nor an estimate of actual managers' outperformance over a feasible alternative.

The remainder of the paper is organized as follows. Section 2 develops the conceptual framework and objective function. Section 3 presents the construction and measurement of pension bonds. Section 4 describes the illustrative application, efficient frontiers, investment path, and simulated pension outcomes. Section 5 concludes. The appendices document the yield curve, asset data, and solution method.

2 Conceptual framework: asset allocation and pension risk

Asset allocation for a TDF requires defining the economic commitment its cohort's portfolio must finance. For a participant, this is a stream of real retirement income whose timing and length depend on age,

retirement date, and survival probabilities. We use this payment stream to define a pension benchmark and measure the risk of financing retirement income.

This section extends standard investment concepts, including the risk-free asset and the Sharpe ratio, to the financing of retirement income. It also presents an interpretation in pension units and accounts for the effect of asset demand on expected returns. These elements define the investment objective for TDFs.

2.1 The pension-risk-free asset

The definition of a risk-free asset depends on the currency and investment horizon. For an investor seeking a fixed nominal amount at a future date, a default-free zero-coupon bond in the same currency and maturing on that date is risk-free. If the objective is to preserve purchasing power, the payment must also be protected against inflation. For pensions, the objective is to finance a stream of real income throughout retirement. The relevant risk-free asset must therefore match the timing and purchasing power of those payments, as emphasized by Merton (2003) and Merton (2007).

Consider a 45-year-old individual who plans to retire at 65 and receive constant real pension income until age 85. In this certain-horizon example, the asset securing that objective is a stream of inflation-indexed payments beginning in twenty years and lasting another twenty years. A portfolio replicating those payments locks in the current cost of one unit of future pension income. We call the asset reproducing this cash-flow structure a *pension bond*. This construction is inspired by the retirement-income bonds proposed by Merton and Muralidhar (2020), sharing their focus on financing a stream of real retirement income.

The pension bond is a hypothetical benchmark, not necessarily a traded security. Its replicability depends on the cohort's payment horizon and the instruments available. At advanced ages, the shorter remaining expected payment stream may be replicated, or closely approximated, using liquid inflation-linked bonds with suitable maturities. For young cohorts, pension payments extend much further into the future, beyond the maturities spanned by available instruments, so full replication is generally infeasible. The benchmark remains useful for valuing the commitment and measuring pension risk even when it cannot be purchased or replicated.

A pension bond can fluctuate in market value while remaining risk-free relative to the income it represents. Expressing wealth in units of the bond clarifies this property: one unit continues to finance one unit of the promised stream. Real price volatility and stable pension purchasing power are therefore compatible.

For lifetime pensions, payments incorporate survival contingencies. The ideal instrument must reproduce both their timing and those contingencies. An actuarial representation using survival-weighted cash flows provides an interest-rate hedging benchmark.[2] The pension-risk-free asset is therefore a pension bond whose cash flows match the individual's remaining pension payments. Its duration generally declines as the individual ages and the remaining payment horizon shortens. Each cohort thus has its own pension benchmark, which evolves over time.

This benchmark changes the role of safe assets relative to traditional asset allocation. A short-term default-free bond is commonly treated as risk-free over its holding horizon, but financing a pension requires reinvesting its proceeds repeatedly over a much longer horizon. Changes in reinvestment rates can therefore alter the pension income financed even when the account balance has low volatility. A decline in real rates raises the cost of future pension payments, leaving a portfolio of short-term instruments exposed to pension risk. A portfolio matching those payments would instead appreciate with their financing cost. Thus, an asset that is safe in conventional return terms can be risky in pension terms, while a volatile long-duration bond can provide pension protection.

2.2 Pension Financing Cost Risk (PFCR)

Once the benchmark cash flow is defined, we measure pension financing cost risk: fluctuations in the amount needed to finance the specified pension payments. Let r_B denote the pension bond return over a given measurement horizon. Define

$$\mathrm{PFCR}_B = \sqrt{\mathrm{Var}[r_B]}. \tag{1}$$

[2] Valuing expected cash flows with a given mortality table does not itself eliminate individual longevity risk or mortality revision risk. A lifetime guarantee requires hedging or pooling those contingencies. Financial hedging below is conditional on the adopted actuarial assumptions.

PFCR captures changes in the amount needed to finance a given pension. When real interest rates fall, future pension payments become more expensive to finance. An individual holding the matching pension bond is protected against this change because the bond's value rises with the amount required to finance those payments.

The size of these changes depends on when pension payments are due. Payments further in the future are generally more sensitive to interest rates. Younger individuals therefore tend to face greater PFCR. As they age and the bond's duration declines, this sensitivity generally decreases.

The calculation uses a given mortality table and pension payment profile. Changes in contributions, earnings, or benefit rules are additional sources of uncertainty for retirement income.

2.3 Pension Relative Risk (PRR)

Let $\mathcal{A}$ be the set of all asset classes considered in the model. For each asset class $A \in \mathcal{A}$, let r_A denote its return, measured in the same unit and over the same interval as r_B. Define its Asset Pension Relative Risk (PRR_A) as the volatility of its return relative to the bond:

$$\mathrm{PRR}_A = \sqrt{\mathrm{Var}[r_A - r_B]}. \tag{2}$$

The pension bond represents the individual's pension payments and is therefore the corresponding pension-risk-free asset. Investing in this asset eliminates pension risk because its cash flows match those payments. Changes in the bond's value are exactly matched by changes in the amount needed to finance the pension. For any other asset, PRR measures the volatility of its return relative to this bond.

Expanding the variance identifies the determinants of this measure:

$$\mathrm{PRR}_A = \sqrt{\mathrm{Var}[r_A] - 2\,\mathrm{Cov}[r_A, r_B] + \mathrm{Var}[r_B]}. \tag{3}$$

For a given benchmark, PRR depends on both asset volatility and covariance with the bond. Positive covariance helps the asset appreciate when pension financing becomes more expensive. An instrument with higher standalone volatility may thus have lower pension risk if its hedging capacity is sufficiently stronger.

Correlation with the pension-risk-free asset is therefore central: for given volatilities, lower correlation means higher pension risk.

This property explains the role of long-duration real fixed income. When its cash flows approximate those of the pension commitment, changes in its value can offset much of the change in pension financing costs. Short-term fixed income offers different protection, potentially insufficient for cohorts with distant payments. The relevant comparison requires estimating covariances and exposure to the entire yield curve; the fixed-income label alone does not determine pension risk.

Portfolio assessment also incorporates covariance among holdings. Diversification can reduce relative risk even when individual assets have limited pension hedging capacity. The criterion applies to the combination and its joint relationship with the benchmark bond.

2.4 The Pension Sharpe Ratio (PSR)

The preceding risk measure supports an efficiency criterion aligned with the pension objective. The PSR relates expected return in excess of the bond to the volatility of that relative return:

$$\mathrm{PSR}_A = \frac{\mathbb{E}[r_A - r_B]}{\sqrt{\mathrm{Var}[r_A - r_B]}}. \tag{4}$$

Other things equal, an asset can be attractive because it has a higher expected return, lower return volatility, or higher covariance with the pension bond. The PSR brings these three features together: expected return determines the numerator, while volatility and covariance with the bond determine pension risk in the denominator.

The connection with the traditional Sharpe ratio is equally direct. Both measure expected return above a risk-free asset per unit of risk relative to that asset. The PSR uses the individual's pension-risk-free asset in place of the usual short-term risk-free asset.

If the corresponding pension bond were available, an individual could invest in it and eliminate pension risk. This makes the bond a natural benchmark. Investing in other assets creates the opportunity to earn

higher returns while taking risk relative to that benchmark. The PSR measures the expected compensation for taking this risk.

This is the logic of an *information ratio*. The pension bond is the benchmark to be tracked. The numerator measures expected return above the bond, and the denominator is the tracking error: the volatility of the return difference. A higher PSR means more expected return per unit of risk from deviating from the pension bond.[3] Maximizing the PSR therefore balances expected returns and pension hedging. The portfolio with the lowest pension risk need not offer the highest PSR: taking additional risk may provide enough expected return to improve the ratio. How much risk an individual should take also depends on available savings and desired retirement income.

2.5 Equivalence with the pension-unit approach

In discussing how households should assess investment risk over their retirement horizon, Merton (2003) emphasizes the distinction between preserving wealth and securing a stream of real consumption. When interest rates fall, more wealth is needed to finance the same income; an asset that protects retirement consumption must therefore increase in value as that financing cost rises. This changes both the relevant risk-free asset and the unit in which investment performance should be measured. Merton states the implication directly (p. 19):

> "Perhaps a simple version that picks up this effect could be introduced into the standard mean–variance model; namely, we should use an inflation-protected life annuity as *numeraire* for the investment portfolio."

Taking this annuity as numeraire means measuring wealth by the retirement income it can finance. The annuity is risk-free in these units because it secures the reference income stream, even though its market value can fluctuate. This change of numeraire connects the relative-return measures developed above to wealth expressed in pension units.

[3] The ratio is defined for positive tracking error. Investing exactly in the pension bond gives zero excess return and zero tracking error, so the ratio is undefined for the benchmark itself.

The interpretation of relative risk becomes direct when the unit of wealth is changed. Let A_t denote the market value at date t of the holdings under consideration, whether a position in a single asset class, a fund portfolio, or the participant's total invested wealth. Define a *pension unit* (PU) as entitlement to a monthly payment of one constant real monetary unit during retirement, under the relevant payment profile and actuarial assumptions. Let PU_t be its price at date t. If A_t and PU_t are expressed in the same real monetary unit, wealth in pension units is

$$A_t^* = \frac{A_t}{PU_t}. \tag{5}$$

This quantity measures how many units of the benchmark stream available wealth can finance. If the price of a PU rises while asset value remains constant, pension wealth falls even if the real balance is unchanged. If assets and the PU appreciate proportionately, pension purchasing power is preserved.

During accumulation, over an interval without contributions or withdrawals, the log return on wealth in pension units satisfies

$$\begin{aligned} r_{A,t}^* &= \log\left(\frac{A_t^*}{A_{t-1}^*}\right) = \log\left(\frac{A_t}{A_{t-1}}\right) - \log\left(\frac{PU_t}{PU_{t-1}}\right) \\ &= r_{A,t} - r_{PU,t} = r_{A,t} - r_{B,t}. \end{aligned} \tag{6}$$

The final equality uses the fact that the pension unit and pension bond represent the same payment profile up to a scale constant. Return volatility in pension units therefore equals pension risk as defined in equation (2). The PSR is the ratio of the mean to the standard deviation of asset returns expressed in this unit.

2.6 Asset demand and market impact

Pension fund demand can affect asset prices and expected returns when markets have limited depth. A large increase in demand for domestic assets may raise their prices and reduce the returns available to pension funds. Other investors may anticipate predictable changes in demand, further reducing expected returns. Portfolio choice should therefore account for the market impact of the investment path.

We represent market impact through a quadratic reduction in a domestic asset's expected return:

$$\mathbb{E}[r_A] = \mu_A - \phi(\Delta x_A)^2, \qquad \phi \geq 0, \tag{7}$$

where μ_A is expected return before market impact and Δx_A is a normalized measure of the change in demand between consecutive periods. The parameter ϕ determines the strength of the effect. The quadratic form imposes a larger return reduction for larger changes in demand. Because the term is symmetric, it reduces expected returns for both increases and decreases in exposure.

In the age-based formulation, Δx_{Ae} is approximated by changes in exposure between consecutive ages. This is a reduced-form representation of the effect of demand on returns. Relating these changes to market demand requires accounting for assets under management, contributions, withdrawals, and market depth. The specification makes expected returns depend on the chosen allocation path without solving for market-clearing prices.

Market impact links portfolio decisions across ages. A change in allocation may improve pension hedging while reducing expected returns through its effect on demand. Joint optimization balances these effects and can produce more gradual changes in portfolio composition.

2.7 Objective function for target-date funds

A target-date fund (TDF) must choose portfolios consistent with its evolving pension commitment. Let $\mathcal{E} = \{e_0, e_0 + 1, \ldots, e_{\max}\}$ be the set of ages, where e_0 is the age at which contributions begin and $e_{\max}$ is the highest age considered in the model. For $A \in \mathcal{A}$ and $e \in \mathcal{E}$, let x_{Ae} be the share invested in asset A at age e, and let r_{Be} be the return on the corresponding pension bond. Represent the TDF glide path by $X = (x_{Ae})$. Throughout, sums indexed by A or e run over $\mathcal{A}$ or $\mathcal{E}$, respectively. To aggregate decisions across ages, use economic weights $\omega_e \geq 0$ with $\sum_e \omega_e = 1$.

These weights reflect each age's importance in assessing the path. Wealth-based weights, for example, recognize that the same return change affects different amounts of savings. Proximity to retirement may enter through an additional explicit rule. The weighting rule is a design assumption that should be stated and subjected to sensitivity analysis.

Define aggregate pension efficiency as

$$F[X] = \sum_e \omega_e \frac{\mathbb{E}[r_{Pe} - r_{Be}]}{\sqrt{\mathrm{Var}(r_{Pe} - r_{Be})}}. \tag{8}$$

The illustrative application uses $e_0 = 25$ and $e_{\max} = 110$. The portfolio at each age is defined by

$$r_{Pe} = \sum_A x_{Ae} r_A, \qquad \sum_A x_{Ae} = 1, \quad \forall e \in \mathcal{E}. \tag{9}$$

This expression is exact for simple returns and given beginning-of-period weights. To preserve the pension-unit identity in equation (6), the mean–variance framework may use it as a first-order approximation for log returns; path evaluation should calculate the actual portfolio return.[4] All moments must use the same horizon and convention.

The budget constraint implies $r_{Pe} - r_{Be} = \sum_A x_{Ae}(r_A - r_{Be})$ under equation (9). Let $\mathcal{L} \subseteq \mathcal{A}$ denote the set of domestic assets. The allocation problem is

$$\begin{aligned} \max_X \quad & \sum_e \omega_e \frac{\mathbb{E}[\sum_A x_{Ae}(r_A - r_{Be})]}{\sqrt{\mathrm{Var}[\sum_A x_{Ae}(r_A - r_{Be})]}} \\ \text{with} \quad & \sum_e \omega_e = 1, \qquad \mathbb{E}[r_A \mid X, e] = \mu_A - \mathbf{1}_{\{A \in \mathcal{L}\}} \phi (\Delta x_{Ae})^2, \end{aligned} \tag{10}$$

subject to

$$\sum_A x_{Ae} = 1, \qquad \forall e \in \mathcal{E}, \tag{11}$$

$$x_{Ae} \geq 0, \qquad \forall A \in \mathcal{A},\ e \in \mathcal{E}, \tag{12}$$

$$R x_e \leq b, \qquad \forall e \in \mathcal{E}. \tag{13}$$

Conditional expected-return notation indicates that market impact depends on the chosen path. Matrix R and vector b represent linear limits on individual assets or groups. Together, the constraints define feasible portfolios, while the return penalty represents the effect of changes in demand.

The problem seeks maximum aggregate pension efficiency within this set. Without market impact or other cross-age constraints, the allocation separates into age-specific problems. The dependence of expected returns on Δx_{Ae} links decisions across ages and requires joint optimization. Gradual changes in allocation

[4] The exact identity is $r_{Pe} = \log(\sum_A x_{Ae} \exp(r_A))$ for log returns and nonnegative beginning-of-period weights. The difference from $\sum_A x_{Ae} r_A$ captures return aggregation.

can then emerge from the balance between pension hedging, expected returns, and market impact.

The objective function provides a reduced-form criterion for choosing a TDF glide path, motivated by the broader goal of maximizing participants' intertemporal utility from consumption. This perspective follows the life-cycle consumption and portfolio choice framework of Campbell and Viceira (2002). By aggregating age-specific pension Sharpe ratios, the criterion balances expected returns above the pension benchmark against pension financing risk across ages. Subsequent pension evaluation examines how the resulting glide path translates into retirement income and how it responds to constraints and alternative market assumptions.

3 Measuring pension risk

Measuring pension financing cost risk begins by defining the payments savings must finance. We consider an inflation-indexed life annuity paying a constant monthly amount in real terms after retirement for as long as the participant survives. Each payment is weighted by its survival probability and discounted using the real yield curve to construct the benchmark bond. The inputs are therefore cohort-specific survival probabilities and real interest rates at the relevant payment maturities. The life annuity is not replaced by an annuity certain ending at expected age at death: the full survival distribution up to the adopted maximum horizon is considered.

We use the Chilean case to illustrate this measurement framework for TDFs. The resulting synthetic asset tracks the cost of financing one unit of retirement income. The volatility of its return measures PFCR, while returns relative to this benchmark determine asset and portfolio pension risk. We next present cash-flow construction, valuation, and bond statistics by age and sex. Yield-curve specification and estimation are documented in Appendix A.

3.1 Domestic real rates and payment discounting

Payments are expressed in UF, Chile's inflation-indexed unit of account, and discounted using domestic real rates in the same unit. This consistency measures the cost of income protected against Chilean inflation without introducing currency exposure into the benchmark commitment. Each date requires discount factors for every monthly payment maturity, including those beyond directly observed market tenors.

The interest-rate sample comes from RiskAmerica and covers December 2003–June 2024 at maturities of 2, 3, 5, 7, 10, 15, 20, 25, and 30 years. The specification of Svensson (1994) fits a continuous curve at each date and interpolates and extrapolates rates to pension cash-flow maturities. Table A1 in Appendix A reports the rate statistics.

Let $D_t(u)$ denote the UF discount factor for a payment u years ahead. With a continuously compounded real zero-coupon rate $y_t(u)$, $D_t(u) = \exp[-uy_t(u)]$. Equivalently, for an effective annual zero-coupon rate $z_t(u)$, $D_t(u) = [1 + z_t(u)]^{-u}$, with $z_t(u) = \exp[y_t(u)] - 1$. Curve fitting and cash-flow valuation must use consistent conventions.

Extrapolation is particularly important for young cohorts whose payments may lie decades beyond the longest observed tenor. The specification holds the asymptotic forward rate and both decay parameters constant across dates. Curves thus converge to a common very-long-term level, restricting estimated variability in that region. Risk estimates for the longest-duration pension bonds are conditional on this assumption and must be interpreted in light of sensitivity to very-long-term rate extrapolation.

3.2 Survival probabilities

Expected payments use the CB-H-2020 mortality table for men and RV-M-2020 for women, with 2025 as the cohort reference year. These tables contain age-specific mortality rates and improvement factors by age and projection year, following Superintendency of Pensions (2023). Relevant probabilities are conditional on survival to the valuation date and must follow the cohort as it ages. Differences in survival between men and women change payment timing and therefore pension bond cost and duration.

Valuation uses survival probabilities for each future month rather than concentrating the end of payments at a single age.

3.3 Pension bond cash flows and valuation

Let $S_t^{e,s}(u)$ be the probability that a person of sex s and age e, alive at t, survives another u years. Consider a monthly payment of c UF and retirement age J_s, set to 65 for men and 60 for women in this exercise. For m months ahead, the expected bond cash flow is

$$C_t^{e,s}(m) = c\, S_t^{e,s}(m/12)\, \mathbf{1}_{\{e+m/12>J_s\}}, \qquad m = 1, \ldots, M_e, \tag{14}$$

where $M_e = 12(e_{\max} - e)$ sets the maximum horizon at the upper age bound $e_{\max}$. The indicator places the first payment in the month after retirement age is reached. For retirees, payments start in the month following valuation. The pension bond value in UF is then

$$B_t^{e,s} = \sum_{m=1}^{M_e} D_t(m/12)\, C_t^{e,s}(m). \tag{15}$$

The level c only rescales bond value: any constant monthly amount can be used to compare returns, volatilities, and durations. Figure 1 normalizes expected payments using $c = 100$ UF.

A young cohort's payments begin after an extended accumulation period and subsequently decline with survival probabilities. For example, a man aged 25 in 2025 reaches 65 in 2065. A retired cohort has immediate payments and a shorter remaining horizon. Declining expected cash flows reflect cohort mortality, while the individual monthly payment remains constant in UF for survivors.

Equation (15) constructs an actuarial benchmark for pension financing costs. It is not a commercial annuity quote: it excludes expenses, margins, and other insurance terms. Nor does a portfolio of expected cash flows provide an individual longevity guarantee. Interest-rate hedging is conditional on the adopted survival structure, as discussed in Section 2.1.

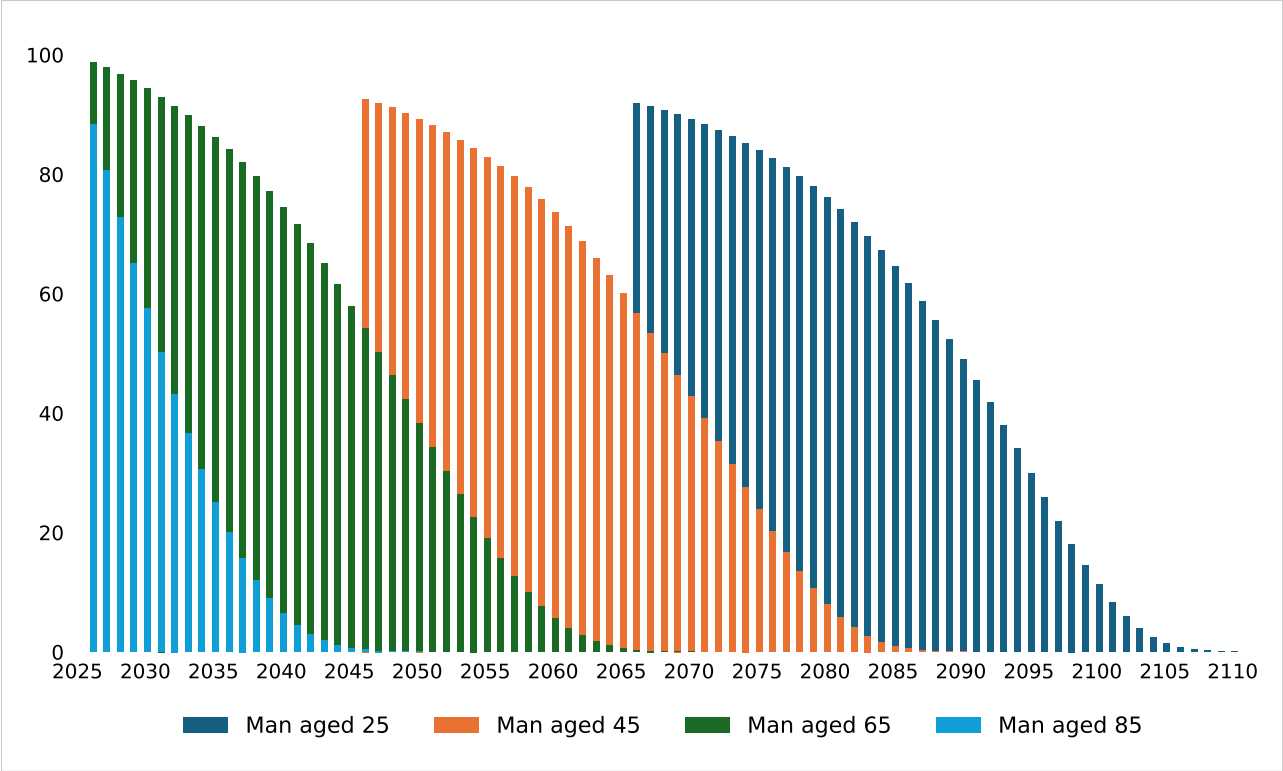


Figure 1: Expected pension bond cash flows for male cohorts

Note: cohorts aged 25, 45, 65, and 85 in 2025. Horizontal axis: calendar year. Vertical axis: expected payment in UF, for a monthly payment of 100 UF conditional on survival. Profiles are monthly payments weighted by survival; bars illustrate their evolution over time.

3.4 Returns, volatility, and duration

The sequence of real yield curves values cash flows at different dates and characterizes variation in their UF cost. Measuring a return requires following the same commitment over time: cohort age increases as time passes. During the accumulation phase before retirement, the pension bond makes no payments. For an interval without payments, with Δ measured in years, its log return is

$$r^{e,s}_{B,t,t+\Delta} = \log\left(\frac{B^{e+\Delta,s}_{t+\Delta}}{B^{e,s}_{t}}\right). \tag{16}$$

When payments occur during the period, total return must include them. For example, over a monthly interval with a payment at the end,

$$r^{e,s}_{B,t,t+1/12} = \log\left(\frac{B^{e+1/12,s}_{t+1/12} + C^{e,s}_{t}(1)}{B^{e,s}_{t}}\right), \tag{17}$$

for the expected cash-flow portfolio fixed at the beginning, without actuarial revisions during that month. For a surviving cohort, pooling effects must also be recorded and distinguished from rate-driven cost changes. Comparing indices that hold age fixed at each date describes an age-specific benchmark but does not automatically measure the return of a single aging cohort.

Table 1 reports available statistics for January 2004–June 2024. Returns and volatilities are annualized and expressed in UF. Bond return volatility measures PFCR as defined in equation (1). These statistics describe the historical behavior of the synthetic bonds under the adopted yield-curve and actuarial assumptions; they are not return forecasts and do not imply that longer duration guarantees higher expected returns.

Table 1: Pension bond statistics: January 2004–June 2024

	Men			Women		
Age	Return (% annual)	Volatility (% annual)	Duration (years)	Return (% annual)	Volatility (% annual)	Duration (years)
25	6.2	32.1	52.7	6.2	31.1	50.6
30	6.1	30.0	47.4	6.0	28.8	45.3
35	5.9	27.5	42.1	5.8	26.2	40.0
40	5.7	24.7	36.8	5.5	23.2	34.7
45	5.4	21.5	31.6	5.1	20.0	29.4
50	5.0	18.1	26.3	4.7	16.5	24.1
55	4.5	14.6	21.1	4.1	13.1	19.0
60	3.9	11.2	15.9	3.5	9.7	13.9
65	3.1	7.7	10.9	3.3	8.5	12.2
70	2.9	6.6	9.2	3.1	7.4	10.5
75	2.7	5.7	7.6	2.9	6.4	8.8
80	2.5	4.7	6.1	2.6	5.3	7.1
85	2.3	3.9	4.8	2.4	4.4	5.6
90	2.0	3.2	3.6	2.2	3.6	4.2
95	1.8	2.6	2.6	1.9	2.9	3.1
100	1.7	2.1	1.9	1.7	2.4	2.2

Note: annual returns and volatilities in UF; duration in years. Retirement ages are 65 for men and 60 for women. Historical synthetic-bond statistics under the stated discounting and survival assumptions.

Duration is particularly relevant for hedging at retirement. At age 60, the statutory retirement age for women used in this exercise, the pension bond has a duration of 13.9 years. If suitable UF-denominated fixed-income instruments were available, fully hedging the pension financing cost to first order against

parallel shifts in the real yield curve would require an asset portfolio of equal market value with this duration. This substantial duration reflects the long remaining payment horizon associated with women's life expectancy at retirement. For men retiring at age 65, the corresponding duration is 10.9 years. Retirement therefore still leaves a substantial interest-rate exposure to hedge. These are hypothetical duration-matching requirements, conditional on suitable instruments being available.

3.5 Relationship with asset classes

Having characterized pension bonds, we examine their relationship with the asset classes in Appendix B. Correlation with the bond indicates an asset's capacity to move with pension financing costs.

Figures 2 and 3 show historical correlations between pension bond returns and the asset classes in Appendix B. The sample covers January 2004–June 2024 using quarterly UF returns. Each row is an age- and sex-specific pension bond; each column is a financial exposure. Red indicates higher positive correlations, while blue indicates low or negative correlations. The maps reproduce the available historical exercise without re-estimating the series.

The highest correlations occur for domestic UF instruments: government bonds (GOB02-CL and GOB10-CL), corporate bonds (CORP02-CL and CORP10-CL), bank bonds (BANK-CL), and financial intermediation instruments (FIUF-CL). Long-duration indices stand out at ages 45–55. At older ages, correlations rise for shorter-duration instruments and, at very advanced ages, financial intermediation instruments. This pattern illustrates how hedging changes with age and relates to pension commitment duration.

Financial intermediation instruments (FIUF-CL) illustrate why short-term safety need not imply pension hedging. Their very short duration stabilizes their value, but they become one of the exposures most closely correlated with pension bonds only at ages around 95–100 for men and women. For longer pension horizons, the key problem is reinvestment risk: as deposits mature, the rates available for reinvesting are uncertain. Higher rates can improve the income financed, while lower rates can reduce it. These instru-

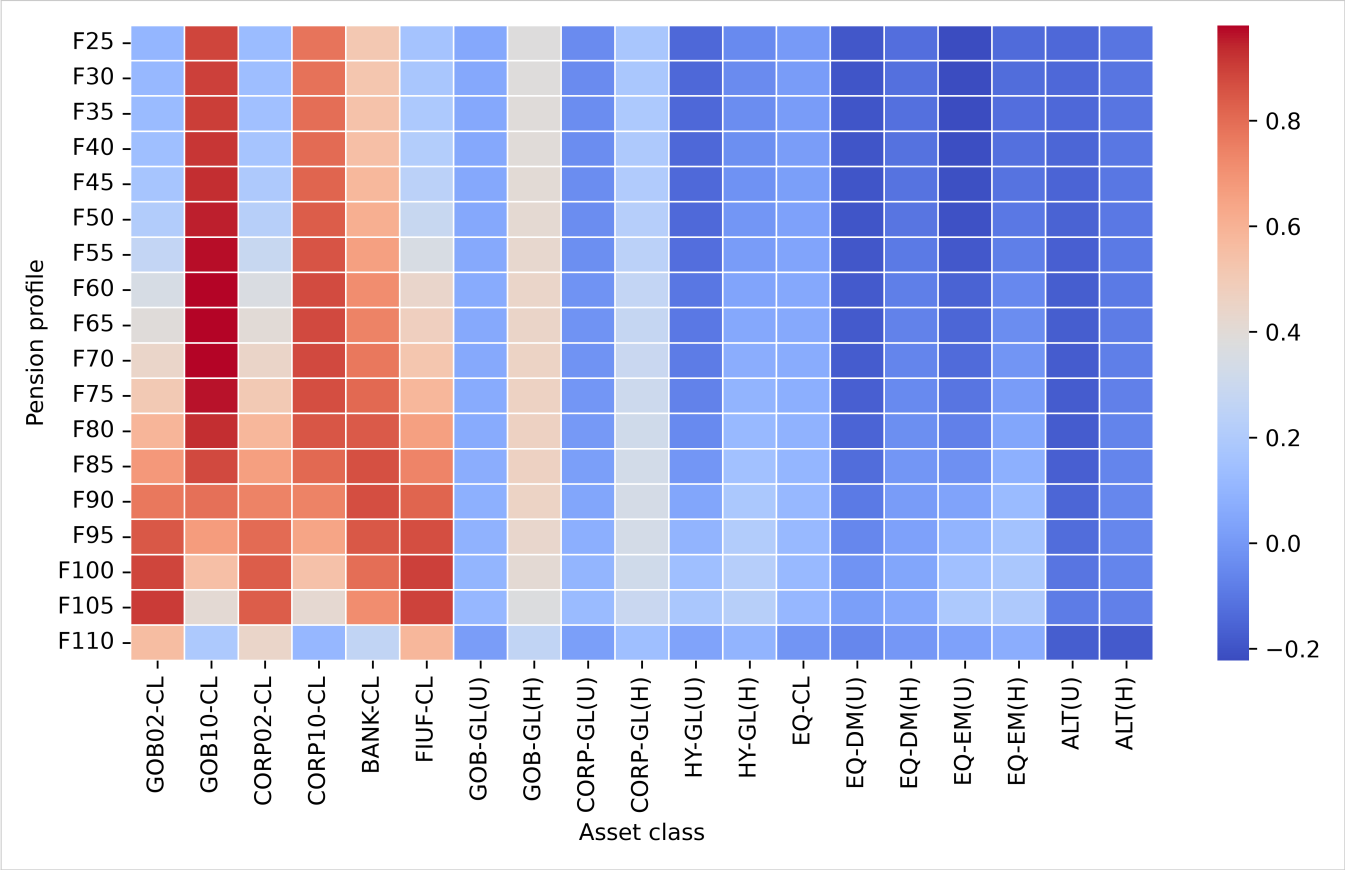


Figure 2: Correlations between female pension bonds at different ages and asset classes

Note: correlations of quarterly UF returns, January 2004–June 2024. Rows identify bond age and sex (M: men; F: women); columns use the asset abbreviations in Appendix B. Colors represent correlations, not volatilities or pension risk levels.

ments therefore leave participants exposed to interest-rate risk despite their low price volatility. Moving into such assets need not improve pension hedging and generally entails lower expected returns.

Lower correlations for international assets should be interpreted in light of index construction. Broad government bond, corporate bond, and equity benchmarks aggregate countries, currencies, and durations. They represent strategic asset-class exposures rather than instruments selected specifically to hedge Chilean pensions. A selection of long-duration foreign bonds might have a different relationship with domestic rates; the maps do not assess such a selection.

Duration disaggregation focuses on domestic fixed income, illustrating how its relationship with pension bonds changes with age. Alternatives use a standard composite alternative index combining multiple

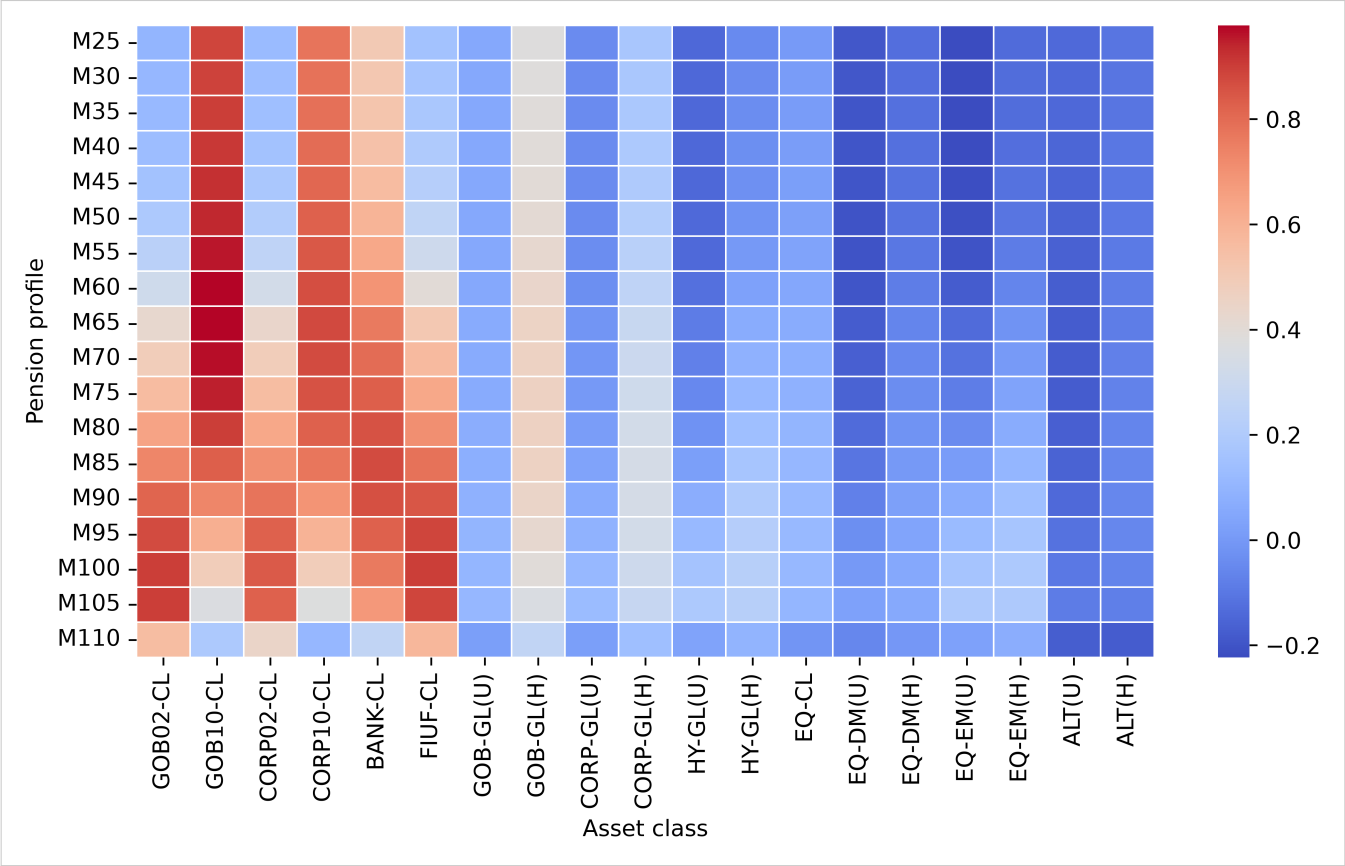


Figure 3: Correlations between male pension bonds at different ages and asset classes

Note: correlations of quarterly UF returns, January 2004–June 2024. Rows identify bond age and sex (M: men; F: women); columns use the asset abbreviations in Appendix B. Colors represent correlations, not volatilities or pension risk levels.

categories. Longer-duration infrastructure or real estate investments, for example, are not selected for particular cohorts. The correlations describe aggregate asset-class exposure rather than a hedge tailored to investment type and horizon.

4 Asset allocation and pension outcomes

Chile is used as an illustrative application of the framework. Under common assumptions and the stated investment limits, we examine the TDF glide path, the replacement-rate distribution at retirement, and the evolution of the funding ratio. The aim is to demonstrate the pension criterion and its implications, rather than provide forecasts.

4.1 Data and parameter estimation

The Chilean application uses quarterly UF returns for January 2004–June 2024. Sample means and covariances are consistently annualized, together with historical correlations between asset classes and pension bonds. Exposures and statistics are described in Appendix B. The exercise illustrates how the pension benchmark and investment constraints affect the TDF glide path. Historical moments are inputs to the exercise, not forecasts.[5]

We set the market impact parameter to $\phi = 10$ to smooth changes in domestic asset weights between adjacent ages without substantially reducing the objective function value. Weights ω_e are constructed from the smoothed, normalized distribution of accumulated pension account balances by age. June 2025 information comes from a representative 5% sample of active participants provided by Chile's Superintendency of Pensions. These weights reflect the importance of each age's assets in the aggregate criterion.

We impose the budget constraint, prohibit short sales, and apply the structural limits $Rx_e \leq b$ in equation (13). The bounds in Table 2 combine the broadest allocations permitted across Chile's multifunds for each asset class. This provides a relatively permissive set of restrictions grounded in the regulatory framework, rather than imposing the tighter limits of a particular fund. Applying the same bounds at all ages allows portfolio composition to respond to the pension commitment without an imposed age-declining sequence of limits.

[5] As emphasized by Michaud (1989), estimation error in historical means and covariances can lead to unstable, poorly diversified optimized portfolios. Robustness exercises therefore consider expected returns from a seven-factor model in the spirit of Fama and French (1992), as well as covariance matrices estimated using exponential weighting and the shrinkage method of Ledoit and Wolf (2004). Results are available upon request.

Table 2: Investment limits

Constraint	Limit
Maximum domestic debt	80%
Maximum foreign investment	100%
Maximum unhedged foreign debt	50%
Maximum foreign high-yield debt hedge	70%
Maximum foreign equity hedge	50%
Maximum foreign alternative-asset hedge	50%
Maximum restricted assets	20%
Maximum equity	80%
Minimum equity	0%
Maximum alternatives	20%

Note: structural limits apply by asset class. The calibration includes ceilings envisaged for August 2027 as exercise assumptions. The exercise does not reproduce the full investment regime, including limits by instrument, issuer, or instrument group.

The TDF glide path is obtained through successive quadratic approximation with relaxation, inspired by the homogenization idea of Cornuéjols and Tütüncü (2005). Risk normalization produces convex subproblems jointly determining portfolios at all ages, with market impact coefficients updated between iterations. Appendix C develops the solution method.

4.2 Illustrative participant and simulation assumptions

We consider a representative TDF participant to illustrate the relationship between pension risk and strategic asset allocation. The exercise uses an unmarried man without children, with uninterrupted contributions, whose pension is financed exclusively from his individual account. He is assumed not to receive the universal guaranteed pension (PGU) or other additional benefits. These conditions delimit the simulation and isolate the financial effect of the investment path.

The PSR is a relative efficiency measure that does not itself depend on accumulated wealth. With identical moments, pension commitments, and constraints, rescaling the account balance does not change the portfolio selection criterion. Income does determine contributions and simulated pensions. Other wealth components, including human capital and its correlation with traded assets, can affect financial allocation, as shown by Bodie et al. (1992) and Jagannathan and Wang (1996); such heterogeneity lies outside this illustrative profile.

The pension benchmark is the market value of a stream of constant real monthly payments conditional on survival. Each payment is weighted by the probability of survival to its payment date and discounted using the market real yield curve, as in equation (15). In the simulation, the participant bears longevity risk, and withdrawals are recalculated based on the remaining account balance and an age-specific actuarial factor, much like under Chile's programmed withdrawal (i.e., *retiro programado*).

The participant begins contributing at 25 and retires at 65. Initial monthly net income is 16.14 UF, corresponding to the reference median income for 2023; monthly gross income is 19.83 UF. Real wages grow at a constant 1.5% annually, contributions to the individual account equal 16% of gross income, and contribution density is 100%. Initial wealth is zero. The earnings path is simplified, without interruptions or idiosyncratic income variation.

The evaluation uses approximately 60,000 quarterly Monte Carlo paths. Log returns on asset classes and pension bonds are generated jointly from a multivariate normal distribution with historical moments. The participant's account follows the TDF glide path as the cohort ages, and domestic asset returns incorporate the market impact adjustment. Contemporaneous dependence between assets and the pension benchmark is preserved; outcomes are conditional on financial assumptions and survival to the evaluated age. The replacement-rate and funding-ratio figures use the original simulated glide path under the stated limits and parameters.

For a simple actuarial valuation, the simulation approximates discounting with a fixed real annual rate of 3% when calculating the unit capital requirement (*capital necesario unitario*, CNU, in Spanish). The initial self-financed pension equals the retirement balance divided by the cost of the benchmark monthly life-annuity payment. During decumulation, withdrawals are recalculated using the remaining balance and the age-specific actuarial factor. This simplifies market valuation: discount rates do not vary, and the simulation does not model an irrevocable annuity purchased from an insurer.

4.3 The pension efficient frontier without market impact

To understand the construction of TDF glide paths, we first examine age-specific pension efficient frontiers without market impact. This assumption separates the problem into independent static decisions by age, identifying the portfolio maximizing the PSR for each cohort. It provides an efficiency benchmark before introducing market impact into the joint choice of TDF portfolios across ages.

Figure 4 shows pension efficient frontiers for men aged 25, 45, 65, and 85 under the stated investment limits. All four panels use the same horizontal and vertical scales, allowing direct comparison of risk levels, expected returns, and frontier positions across ages. The horizontal axis measures portfolio pension risk as defined in equation (2), applied to the portfolio: $\sqrt{\text{Var}(r_P - r_B)}$. The vertical axis retains the absolute expected portfolio real return, $\mathbb{E}[r_P]$, for comparability across cohorts, without subtracting the pension bond's expected return. In each panel, the blue point is the corresponding pension bond, the risk-free asset from a pension perspective. The green point marks the frontier portfolio maximizing the PSR.

The individual asset points describe stand-alone exposures, whereas the frontier consists of portfolios satisfying the investment limits in Table 2. Investing entirely in a particular asset may violate those limits. An asset can therefore appear above or otherwise outside the constrained frontier without being an attainable portfolio. Its contribution to a feasible allocation depends on the permitted weight and its covariance with other holdings.

The age-25 panel shows high pension risk for both individual assets and feasible portfolios. None of the available exposures provides a sufficiently close hedge for the very distant pension payments of a young participant. This is consistent with the age patterns in Figures 3 and 2, read together with the long duration of young cohorts' pension bonds: positive correlation alone does not ensure that an asset offsets the magnitude of changes in pension financing costs. With limited scope to reduce the denominator of the PSR, differences in expected returns become more important. The portfolio maximizing pension efficiency consequently lies near the high-return end of the feasible frontier. This reflects limited hedging opportunities, rather than an assumed greater tolerance for risk among young participants.

By age 45, and especially by age 65, available fixed-income assets provide much more effective pension

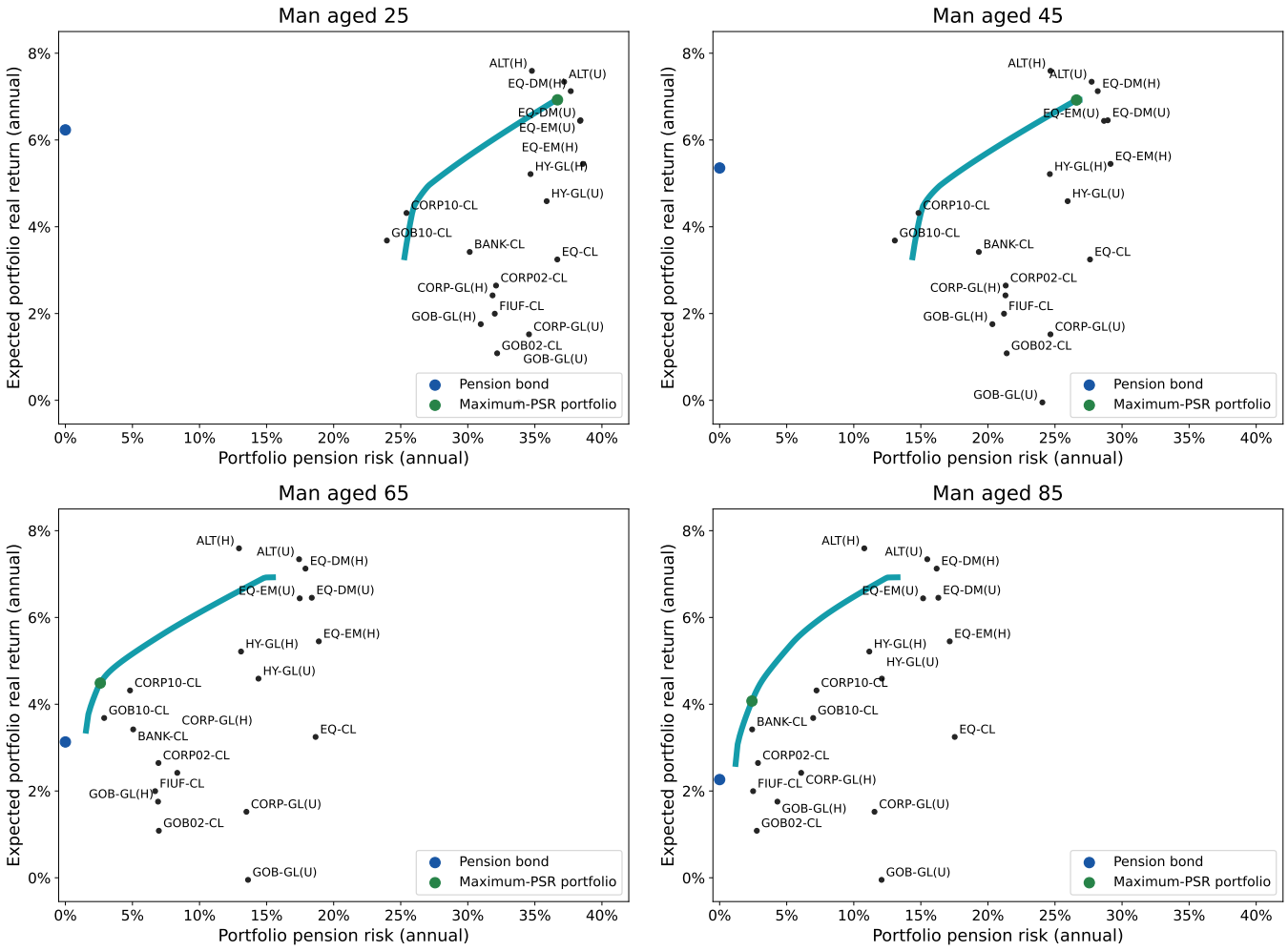


Figure 4: Pension efficient frontiers for different cohorts

Note: men aged 25, 45, 65, and 85; no market impact. All panels use common axis scales. The horizontal axis measures annual portfolio pension risk from equation (2), applied to the portfolio; the vertical axis retains absolute annual expected real return for comparison. Blue denotes the pension bond and green the portfolio maximizing the PSR.

hedging. On the common scales, the frontiers shift markedly to the left as feasible portfolios attain lower pension risk. At age 65, long-duration domestic government and corporate bonds (GOB10-CL and CORP10-CL) are among the assets with the lowest PRR. Their interest-rate exposure more closely matches the remaining pension commitment. This pattern accords with Figures 3 and 2, which show how the relationship between pension bonds and domestic fixed income evolves with age.

At age 85, the ordering changes again. The remaining pension horizon is shorter, and the long-duration government and corporate bonds that offered strong hedging at 65 move to the right: their PRR increases

because their duration is now too long relative to the pension payments to be financed. Shorter-duration instruments become better aligned with the commitment. Thus, the same bond can offer effective pension protection at one age and introduce a duration mismatch at another. The shift toward shorter-duration assets in Figure 5 follows this changing hedging need.

The frontier's shape also depends on relationships not visible in the individual asset points. As the pension benchmark changes with age, both asset pension relative risks and the correlations among returns expressed in pension units change. The historical moments of absolute asset returns are held fixed in this exercise; the changing dependence concerns returns relative to the age-specific pension bond. These covariances determine diversification opportunities, so movements in individual asset positions alone do not fully explain the frontier. Its location and curvature jointly reflect the benchmark, relative-return dependence, and investment constraints.

Better hedging does not imply that the portfolio maximizing the PSR minimizes pension risk. At age 65, the green point lies to the right of the frontier's lowest-risk endpoint: the additional expected return compensates for taking more risk and improves pension efficiency. The blue point is the ideal pension bond, which perfectly hedges the specified commitment; it differs from the minimum-risk portfolio of available assets, which retains residual risk. Even when hedging opportunities improve, efficient allocation combines pension protection with exposure to higher-expected-return assets.

4.4 Optimal asset demand over the life cycle

The age-specific frontiers in Section 4.3 clarify the balance between expected returns and pension hedging when asset demand has no market impact. We now solve the full optimization problem in equations (10)–(13), incorporating the effect of changes in domestic asset demand on expected returns. This market impact links portfolio choices across ages, requiring the TDF glide path to be determined jointly rather than through independent age-specific decisions.

Figure 5 presents the resulting TDF glide path under the stated limits and the parameters described in Section 4.1. The horizontal axis shows participant age; the vertical axis shows each asset class's portfolio

share, summing to 100% at every age.

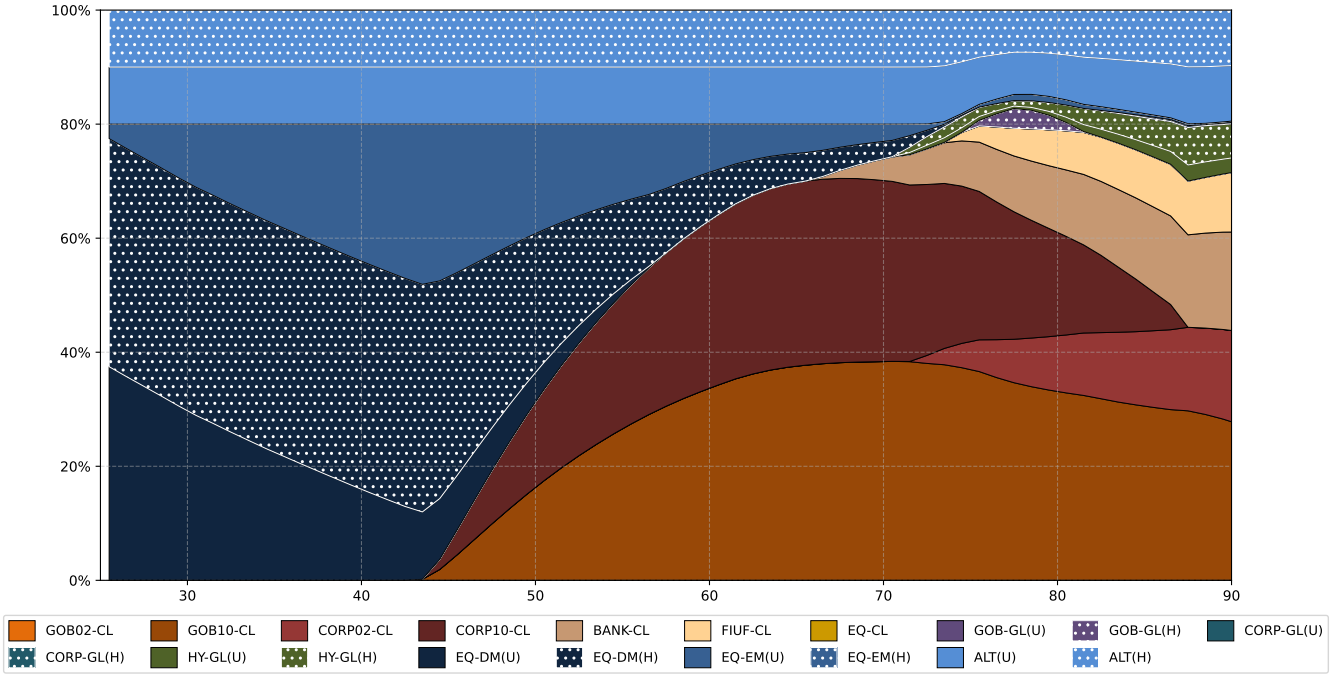


Figure 5: Optimal asset demand over the life cycle

Note: portfolio shares by age for the illustrative male participant. The solution uses unit increments of ϕ from 0 to 10 to initialize the iterative approximation at the target penalty. Abbreviations follow Appendix B; H and U denote hedged and unhedged variants.

The path reflects two findings related to age-specific hedging opportunities. For TDFs serving young cohorts, available assets do little to reduce the risk of distant pension payments. As the age-25 panel in Figure 4 shows, portfolio pension risk remains high across portfolios. Choice therefore focuses mainly on higher-expected-return assets, producing large foreign equity and alternative-asset shares in Figure 5.

Around age 45, long-duration domestic government and corporate bonds (GOB10-CL and CORP10-CL) enter the portfolio with an explicit pension-hedging role. Their combined portfolio share increases up to around age 65, consistent with the hedging opportunities illustrated by the age-45 and age-65 frontiers in Figure 4. Only from around age 65 do shorter-duration fixed-income classes begin to enter the path, including government and corporate indices with duration around two years (GOB02-CL and CORP02-CL) and bank bonds (BANK-CL), followed by financial intermediation instruments (FIUF-CL) at older ages. As these instruments join the mix, the duration of the fixed-income portfolio gradually declines, adapting

to the shorter remaining pension-payment horizon. Market impact smooths these changes in exposure.

The second finding is that pension hedging continues to coexist with investment for growth around retirement. Between ages 60 and 65, approximately 30% of the TDF portfolio remains allocated to growth assets, including equities and alternatives. Their primary role is to contribute growth and higher expected returns rather than to hedge pension financing costs. Participants still face a substantial investment horizon extending throughout retirement, leaving scope for these exposures alongside the fixed-income hedging component. This allocation is consistent with the distinction in Section 4.3: at age 65, the portfolio maximizing the PSR offers higher expected return and higher pension risk than the minimum-pension-variance portfolio. The additional expected return compensates for the additional risk under the exercise assumptions, so improved hedging does not imply eliminating growth assets at retirement.

Alternatives remain near the 20% ceiling for much of the path because of their high estimated Sharpe ratio. This requires caution: indices often rely on accounting or self-reported valuations that smooth returns, potentially understating volatility and correlations and overstating efficiency and diversification benefits. Illiquidity and heterogeneity of underlying assets also limit interpretation of a composite index as an investable exposure.

The exercise excludes public pension benefits such as Chile's Universal Guaranteed Pension (*Pensión Garantizada Universal*, PGU) and benefits under the new contributory social insurance scheme. For eligible participants, these benefits provide a relatively stable stream of retirement income with bond-like characteristics. By covering part of retirement income needs, they could reduce the need for fixed-income assets in the TDF and allow a higher allocation to growth assets. In this sense, the glide path reported here may be conservative in its allocation to growth assets.

4.5 Distribution of replacement rates at age 65

The initial replacement rate is the self-financed monthly pension at age 65 divided by the last monthly wage before retirement:

$$RR_{65} = \frac{p_{65}}{w_{65}}. \tag{18}$$

Only the illustrative participant's individual account resources are included. Simulation assumptions are given in Section 4.2.

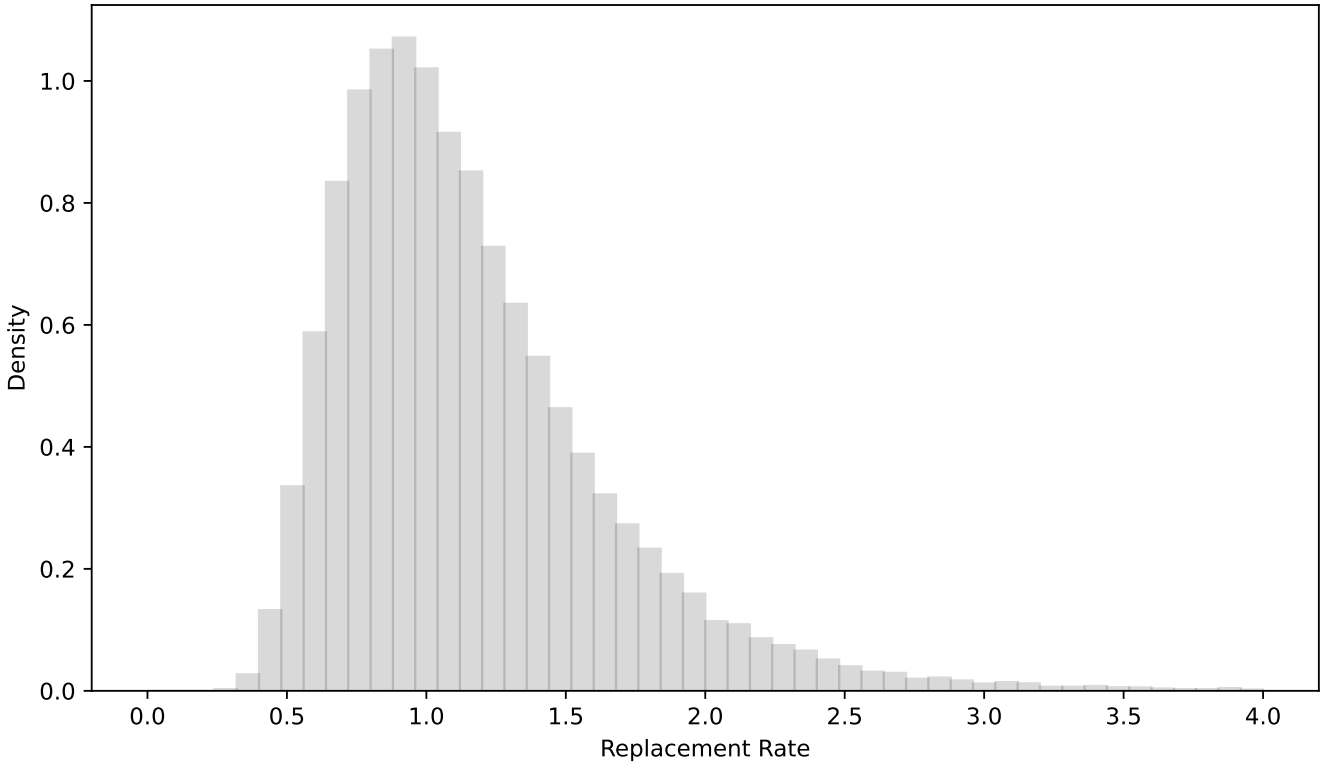


Figure 6: Distribution of replacement rates at age 65

Note: simulated distribution of the self-financed pension relative to the last wage, conditional on survival to age 65. A fixed real rate of 3% is used to calculate the CNU.

Figure 6 shows a right-skewed distribution concentrated near one with a tail of high outcomes. The mean replacement rate is 1.16, corresponding to an expected self-financed pension of 116% of the last wage. The figure also includes scenarios with replacement rates below one; the mean alone does not characterize adequacy across all paths.

The replacement-rate level reflects the exercise assumptions, notably 100% contribution density, a 16% individual-account contribution rate, and constant wage growth. These results illustrate financial performance under an uninterrupted earnings history. They are not projections for a median participant with contribution gaps or guarantees of retirement income.

Figure 6 thus summarizes the distribution of the initial pension relative to the last wage, making

both its expected level and the dispersion of possible outcomes visible. This is an illustrative assessment at retirement within a life-cycle framework. Our model does not maximize a utility function of the replacement rate at age 65: participants still have a substantial remaining lifetime, and their pension financing needs continue beyond that date. An objective defined solely over the initial replacement rate would leave the subsequent investment and pension-income risks outside its scope. Instead, the criterion in equations (10)–(13) balances expected returns and pension risk across ages up to 110, using the account-balance weights described in Section 4.1. At each age, the pension benchmark incorporates the remaining payments weighted by conditional survival probabilities. The initial pension distribution should therefore be read as one outcome of a TDF strategy covering both accumulation and retirement; the evolution of outcomes beyond age 65 is examined next through the funding ratio.

4.6 Evolution of the funding ratio

Following Mantilla-Garcia et al. (2024), the funding ratio measures investment performance relative to the pension income that the same contributions could have financed through exclusive investment in hypothetical matching pension bonds, which are not available as traded instruments:

$$FR_t = \frac{A_t}{L_t}, \qquad L_t > 0, \tag{19}$$

where A_t is the participant's balance under the TDF glide path and L_t is the pension-bond benchmark balance. Both accounts start without wealth and receive identical contributions during accumulation.

Consider the hypothetical world, inspired by the retirement-income perspective of Merton (2003) and Merton (2007), in which pension bonds matching the remaining payment commitments of participants at every age are available. Each contribution buys additional pension units. For the sole objective of securing this benchmark income, there is no separate role for a manager selecting among asset classes: purchasing the matching units gives $A_t = L_t$ and hence $FR_t = 1$ on every path once balances are positive.

The level $FR = 1$ is a theoretical reference, not an initial condition or a portfolio known to be attainable with available assets. A feasible portfolio may retain pension risk even when chosen solely to minimize that risk.

During accumulation, the benchmark of Mantilla-Garcia et al. (2024) can be written as

$$L_t = N_t B_t, \qquad N_t = \sum_{i=1}^{t} \frac{C_i}{B_i},$$
$$L_{t+1} = L_t \frac{B_{t+1}}{B_t} + C_{t+1}, \tag{20}$$

where C_i is the contribution at date i, paid at the end of the period, and B_i is the price of one pension unit for the same aging cohort. Thus N_t records the pension units purchased with all contributions received to date. The benchmark grows with their market value and each new contribution; it excludes future contributions not yet received.

The funding ratio is therefore a useful measure of TDF managers' performance in pension-income units, with the theoretical bond strategy providing a common reference. It shows how financial-market allocation, balancing expected returns, diversification, and pension risk, changes the resources available to finance pensions relative to that reference. Its distribution describes both the expected pension-income multiple and the risk of falling below the benchmark. In this exercise, these are simulated strategy outcomes, rather than observed manager results or outperformance relative to an investable portfolio.

During retirement, the simulation extends the comparison using withdrawals recalculated in both accounts with the same actuarial rule and fixed real rate of 3%. Subsequent ratio dynamics reflect relative investment returns and decumulation. Equation (20) describes the contribution phase; the retirement comparison uses the withdrawal convention in Section 4.2.

Figure 7 shows the funding ratio mean and 10th and 90th percentiles. The percentile band is the central range of simulated outcomes, rather than a confidence interval for the estimated mean. Dispersion increases in the early decades, when foreign equity and alternative exposure is high. From around age 45, fixed income with duration closer to pension commitments contains dispersion, which stabilizes and even declines toward retirement. After retirement, the percentile interval widens again while the mean continues rising.

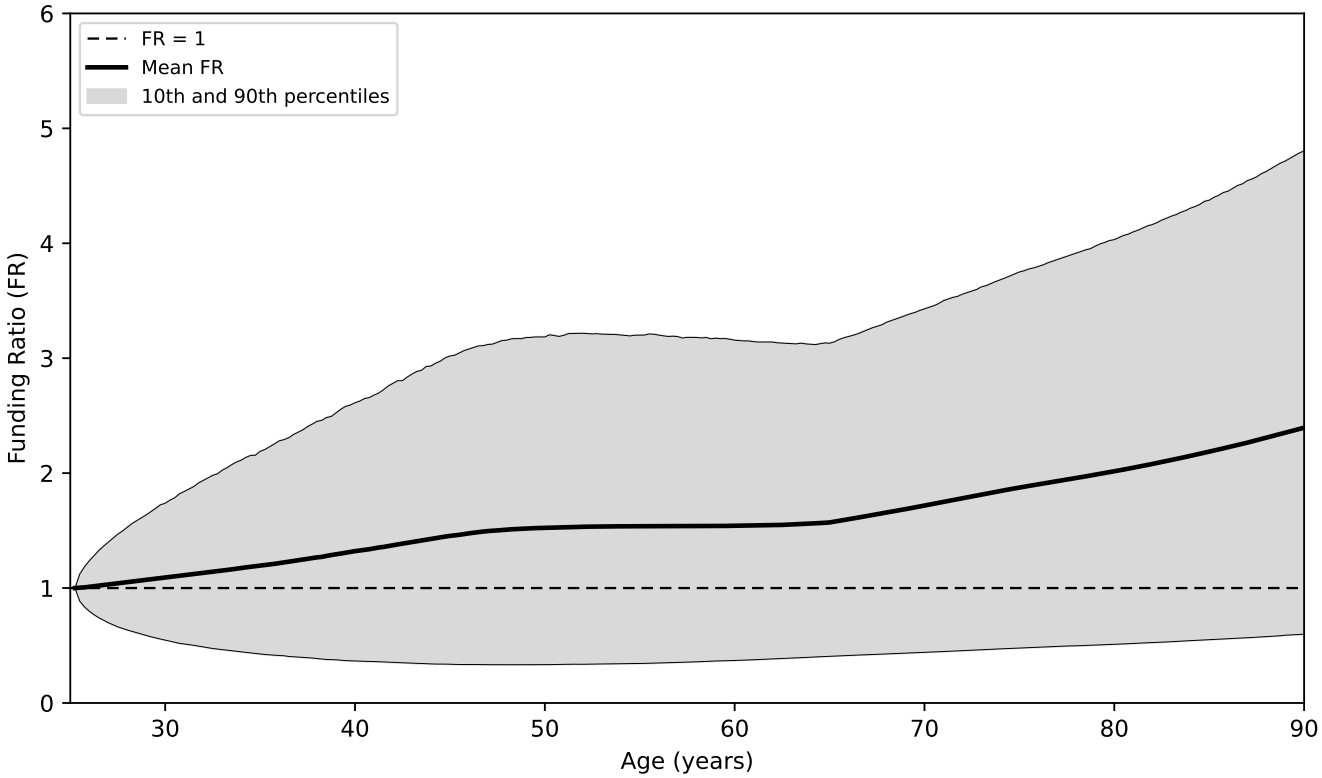


Figure 7: Evolution of the funding ratio

Note: mean and 10th and 90th percentiles of the simulated distribution. The dashed line is $FR = 1$, the theoretical full-hedging strategy, unavailable as a traded investment. Decumulation uses recalculated withdrawals at a fixed real rate of 3%; the ratio is not a guaranteed pension.

Table 3: Mean funding ratio at different ages

45 years	65 years	85 years
1.45	1.57	2.19

Note: means of simulated ratios between strategy balances and the hypothetical pension-bond benchmark, which is not a traded investment. Both accounts receive identical contributions and follow the same retirement withdrawal rule.

Table 3 summarizes the ratio at three ages. At age 65, its mean is 1.57: the simulated strategy finances, on average, 1.57 times the pension represented by the hypothetical bond account. Both balances use identical contributions and the same actuarial conversion factor, so along each path, $p^A_{65}/p^L_{65} = A_{65}/L_{65} = FR_{65}$. The 57% difference is a mean proportional pension comparison against a theoretical reference that cannot be purchased in the market. It does not measure an excess investment return or actual managers' outperformance over a feasible full-hedging strategy; such a strategy need not exist with the available

instruments.

At age 85, the mean funding ratio reaches 2.19 under the stated decumulation assumptions. These expected gains are conditional on the exercise's return and risk assumptions; individual paths can deliver lower outcomes, as the dispersion in Figure 7 illustrates.

5 Conclusions

Asset allocation for target-date funds (TDFs) requires defining risk relative to the income savings must sustain. The hypothetical cohort-specific pension bond represents that commitment and makes expected returns and hedging capacity comparable in pension terms. The objective aggregates age-specific pension Sharpe ratios using account-balance weights. Incorporating the market impact of changes in domestic asset demand makes portfolio selection a joint life-cycle problem, covering both accumulation and retirement, with survival probabilities reflected in the benchmark payments.

The illustrative application shows how each asset's role changes along the TDF glide path. Limited opportunities to hedge young cohorts' distant commitments favor higher expected returns. Long-duration government and corporate bonds enter around age 45, their share increases toward age 65, and shorter-duration instruments subsequently reduce the fixed-income portfolio's duration. Approximately 30% remains in growth assets around retirement, when a substantial investment horizon still lies ahead. Hedging and growth thus remain complementary after retirement.

The initial replacement-rate distribution illustrates the level and dispersion of the pension at retirement; it is one outcome of the life-cycle strategy. Following Mantilla-Garcia et al. (2024), the funding ratio assesses TDF managers' performance in pension-income terms against a theoretical pension-bond account receiving identical contributions. These matching bonds are not available as traded instruments, so a ratio of one is a reference level rather than an assured feasible allocation. The simulated mean ratio of 1.57 at age 65 places the pension level, on average, 57% above this hypothetical reference; it does not establish actual managers' outperformance over an available investment alternative. The distribution reveals the accompanying risk, and the comparison continues into retirement under the stated withdrawal assumptions.

The contribution is methodological. Portfolio shares and simulated outcomes depend on historical moments, constraints, contributions, and actuarial valuation. Alternative-asset measurement and very-long-term rate extrapolation require particular caution. The results show how evaluating performance in pension-income terms connects investment decisions to participants' financing needs throughout the life cycle.

Appendix A The real term structure: specification and estimation

We use the model of Svensson (1994), which extends Nelson–Siegel with a second curvature component. For maturity $u > 0$, the instantaneous real forward rate is

$$f_t(u) = \beta_0 + \beta_{1t} e^{-u/\tau_1} + \beta_{2t} \frac{u}{\tau_1} e^{-u/\tau_1} + \beta_{3t} \frac{u}{\tau_2} e^{-u/\tau_2}, \qquad \tau_1, \tau_2 > 0. \tag{A1}$$

The continuously compounded zero-coupon rate is the average instantaneous forward rate between zero and u:

$$\begin{aligned} y_t(u) &= \frac{1}{u} \int_0^u f_t(v)\, dv \\ &= \beta_0 + \beta_{1t} \frac{1 - e^{-u/\tau_1}}{u/\tau_1} + \beta_{2t} \left[\frac{1 - e^{-u/\tau_1}}{u/\tau_1} - e^{-u/\tau_1} \right] \\ &\quad + \beta_{3t} \left[\frac{1 - e^{-u/\tau_2}}{u/\tau_2} - e^{-u/\tau_2} \right]. \end{aligned} \tag{A2}$$

Parameters β_{1t}, β_{2t}, and β_{3t} vary by date; β_0, τ_1, and τ_2 are common across the sample. The parameter set is $\Theta = \{\beta_0, \tau_1, \tau_2, (\beta_{1t}, \beta_{2t}, \beta_{3t})_{t=1}^N\}$. It is estimated by least squares over the nine observed maturities:

$$\min_{\Theta:\, \tau_1, \tau_2 > 0} \frac{1}{9N} \sum_{t=1}^{N} \sum_{j=1}^{9} \left[y_t(u_j; \Theta) - y_t^{\text{obs}}(u_j) \right]^2, \tag{A3}$$

where $u_j \in \{2, 3, 5, 7, 10, 15, 20, 25, 30\}$ and observed and fitted rates must use the same compounding convention. The constant $1/(9N)$ converts the sum of squared errors into its mean without changing the minimizer.

Table A1 summarizes the observed real rates by maturity used in estimation.

Table A1: Domestic real rates used in estimation: December 2003–June 2024

Maturity (years)	Mean rate (% annual)	Standard deviation (% annual)
2	1.47	1.37
3	1.66	1.20
5	1.89	1.06
7	2.06	1.00
10	2.24	0.97
15	2.44	0.95
20	2.57	0.94
25	2.64	0.93
30	2.67	0.93

Rates in UF. RiskAmerica data; statistics from the reference exercise.

Both forward and zero-coupon rates converge to β_0 as maturity increases. Imposing a common β_0 makes time variation in very-long-maturity rates converge to zero. This estimation assumption affects pension bond sensitivities and volatilities; it is not evidence of an absence of risk at those horizons.

Interpretation depends on rate conventions, estimation restrictions, and extrapolation assumptions. In particular, young-cohort risk measurement is sensitive to the dynamics assigned beyond observed maturities. The reported statistics come from the historical exercise described above.

Appendix B Asset-class data and indices

The investment universe comprises domestic and foreign fixed income and equities, together with alternative assets. Indices represent broad strategic exposures, providing comparable building blocks for estimating returns, volatilities, and covariances and studying their relationship with pension bonds. They are not operational portfolios that must be replicated literally.

B.1 Investment universe and sources

Table B1 identifies indices and abbreviations. RiskAmerica provides domestic fixed-income indices separating government and corporate bonds by duration, as well as bank bonds and UF financial intermediation instruments. This disaggregation links hedging to sensitivity to domestic rates. Chilean equities are represented by the IPSA.

Table B1: Asset-class indices: January 2004–June 2024

Representative index	Abbreviation	Return (% annual)	Volatility (% annual)	Sharpe ($r = 0$; %)
Domestic fixed income (RiskAmerica)				
Government: duration 2	GOB02-CL	1.1	3.4	31.6
Government: duration 10+	GOB10-CL	3.7	9.9	37.0
Corporate: duration 2	CORP02-CL	2.6	3.3	80.6
Corporate: duration 10+	CORP10-CL	4.3	9.9	43.7
Bank bonds: aggregate	BANK-CL	3.4	4.9	69.4
UF financial intermediation	FIUF-CL	2.0	3.0	67.0
Domestic equity				
IPSA	EQ-CL	3.2	17.5	18.5
Foreign fixed income (Bloomberg)				
Global Treasury Total Return Index Value Unhedged	GOB-GL(U)	0.0	11.7	-0.4
Global Treasury Total Return Index Value Hedged	GOB-GL(H)	1.8	4.3	41.2
Global Agg Corp Total Return Index Value Unhedged	CORP-GL(U)	1.5	11.0	13.9
Global Aggregate Corporate Total Return Index Hedged	CORP-GL(H)	2.4	6.1	39.6
Global High Yield Total Return Index Value Unhedged	HY-GL(U)	4.6	11.5	40.0
Global High Yield Total Return Index Value Hedged	HY-GL(H)	5.2	11.1	46.9
Foreign equity (MSCI)				
MSCI World Index	EQ-DM(U)	6.5	15.4	42.0
MSCI World 100% Hedged to USD Index	EQ-DM(H)	7.1	15.7	45.5
MSCI Emerging Markets (EM) Index	EQ-EM(U)	6.4	14.7	43.9
MSCI Emerging Markets 100% Hedged to USD Index	EQ-EM(H)	5.4	17.1	31.9
Alternative assets				
Standard composite alternative index	ALT(U)	7.3	14.3	51.3
Standard composite alternative index	ALT(H)	7.6	9.8	77.7

Note: all returns are in UF. Annual statistics are computed from quarterly observations. H and U denote hedged and unhedged variants. The final column retains the original percentage presentation of mean return divided by volatility with a zero UF return benchmark: 31.6% corresponds to a ratio of 0.316. This is not the PSR, whose risk and benchmark depend on the cohort bond. Differences between ratios calculated from displayed columns and reported values reflect rounding. Historical figures are retained without re-estimation.

Foreign fixed income uses Bloomberg total-return indices for global government bonds, global corporate bonds, and global high-yield debt. Foreign equities use MSCI developed-market (World) and emerging-market indices. Hedged and unhedged variants are denoted H and U. The distinction between hedging the index's underlying currencies and hedging its conversion into pesos is explained below.

Alternative assets are represented by standard composite alternative indices combining weighted subindices for categories such as infrastructure, real estate, and private equity. Aggregation makes them a strategic asset class but may smooth component fluctuations.

B.2 Frequency, currency, and currency hedging

The sample covers January 2004–June 2024. Quarterly frequency is determined by alternative-asset data availability. Asset classes must be aligned to common dates, and pension bond returns use the same frequency for covariance estimation. Returns are logarithmic; return and volatility statistics are annualized.

Indices are converted into UF before performance is compared. For a domestic peso index I_t^{CLP} and the peso value of UF, U_t, the real index is $I_t^{\mathrm{UF}} = I_t^{\mathrm{CLP}}/U_t$. For a dollar-denominated foreign index I_t^{USD} and exchange rate E_t in pesos per dollar, unhedged CLP/USD conversion satisfies

$$I_t^{\mathrm{UF}} = \frac{I_t^{\mathrm{USD}} E_t}{U_t}, \qquad r_t^{\mathrm{UF}} = r_t^{\mathrm{USD}} + \Delta \log E_t - \Delta \log U_t. \tag{B1}$$

The identity explains why foreign index risk may change when measured in UF: dollar performance, the exchange rate, and domestic inflation all enter.

Currency hedging operates at two levels. First, some global indices hedge their underlying currencies into dollars, as in the MSCI *100% Hedged to USD* indices. Second, CLP/USD exposure on a dollar investment is hedged using a synthetic forward constructed from the spot exchange rate and peso and dollar interest rates. This incorporates the hedge payoff, including financing costs, when converting performance into domestic currency. Hedging underlying currencies into dollars does not itself eliminate a Chilean investor's CLP/USD exposure.

No specific Chilean inflation hedge is included. Expressing returns in UF measures real performance but does not amount to purchasing inflation protection. Even a dollar-hedged investment may therefore have variable real returns in UF. Currency treatment must be preserved when estimating correlations with pension bonds.

B.3 Using the data to measure pension risk

Table B1 describes the individual performance of the asset classes. Hedging assessment uses covariances with age- and sex-specific pension bonds and the covariance matrix among assets. A common sample of quarterly UF returns is used with consistent return and currency-hedging conventions. The comparison thus incorporates both standalone risk and the capacity to offset pension financing cost changes.

Using historical means to estimate expected returns assumes stability in average behavior over the relevant horizon. The sample may contain episodes unlikely to recur and omit relevant scenarios. Aggregation and valuation practices for

alternatives also require caution in interpreting volatilities and correlations. These considerations motivate sensitivity analysis using alternative estimators and assumptions; they do not change the asset-class definitions in this appendix.

Appendix C Solution method for the investment path

Inspired by the Sharpe-ratio homogenization idea of Cornuéjols and Tütüncü (2005), we use risk normalization. Fixing scale through variance allows a linear objective with a convex quadratic constraint and supports joint portfolio selection. We first present the problem without market impact and then incorporate the return penalty through an iterative approximation.

C.1 Risk normalization without penalties

Let $j = 1, \ldots, J$ index TDFs ordered by representative age e_j, with budget-normalized portfolio weights x_j. Define moments of returns relative to the corresponding bond as

$$m_j = \mathbb{E}[r - r_{Bj}\mathbf{1}], \qquad S_j = \mathrm{Cov}(r - r_{Bj}\mathbf{1}), \qquad \sigma_j(x_j) = \sqrt{x_j' S_j x_j}. \tag{C1}$$

Both moments incorporate the pension benchmark. The transformation therefore does not require zero real volatility of the bond. We retain the linear return approximation in equation (9).

Without market impact, fund decisions are not linked. The aggregate problem separates into

$$\max_{x_j \in \mathcal{X}_j} \frac{m_j' x_j}{\sqrt{x_j' S_j x_j}}, \qquad \mathcal{X}_j = \{x_j \geq 0 : \mathbf{1}' x_j = 1,\ R_j x_j \leq b_j\}. \tag{C2}$$

The ratio is not a quadratic function. Its numerator and denominator are nevertheless homogeneous of the same degree: for any $a > 0$,

$$\frac{m_j'(a x_j)}{\sqrt{(a x_j)' S_j (a x_j)}} = \frac{m_j' x_j}{\sqrt{x_j' S_j x_j}}. \tag{C3}$$

This invariance allows a convenient scale without changing the Sharpe ratio.

For positive-risk portfolios, choose the scale normalizing relative variance to one:

$$w_j = \frac{x_j}{\sigma_j(x_j)}, \qquad k_j = \mathbf{1}' w_j = \frac{1}{\sigma_j(x_j)}. \tag{C4}$$

At this scale,

$$w_j' S_j w_j = 1, \qquad m_j' w_j = \frac{m_j' x_j}{\sigma_j(x_j)}, \qquad R_j w_j \leq b_j(\mathbf{1}' w_j). \tag{C5}$$

Thus, without market impact, solve

$$\max_{w_j \geq 0} m_j' w_j \quad \text{subject to} \quad w_j' S_j w_j \leq 1, \qquad R_j w_j \leq b_j(\mathbf{1}' w_j). \tag{C6}$$

If a feasible portfolio has positive expected relative return and risk is positive along the relevant directions, increasing scale

improves the objective until the risk constraint binds. The inequality therefore preserves the maximum-PSR solution while keeping the feasible set convex. Recover weights using $x_j = w_j/(\mathbf{1}'w_j)$. Unlike the return normalization in Cornuéjols and Tütüncü (2005), this formulation normalizes risk: it replaces the ratio with a linear objective and a convex quadratic constraint. Both exploit invariance of the Sharpe ratio to scale.

C.2 Introducing the return penalty

Let $\ell_A = 1$ for domestic assets and zero otherwise, and $h_j = e_j - e_{j-1} > 0$ be the distance between representative ages. Market impact reduces asset expected returns and, after weighting by portfolio shares, produces the portfolio return reduction

$$C_j(X) = \frac{\phi}{h_j} \sum_A \ell_A x_{Aj}(x_{Aj} - x_{A,j-1})^2, \qquad j \geq 2, \qquad C_1 = 0. \tag{C7}$$

For annual age intervals, $h_j = 1$. The penalty is quadratic in the asset exposure change; the portfolio return reduction also includes its portfolio weight. The objective becomes

$$F_{\text{net}}(X) = \sum_j \omega_j \frac{m_j' x_j - C_j(X)}{\sigma_j(x_j)}. \tag{C8}$$

A choice now affects the return reduction for both its own fund and the next fund. Portfolios must be determined jointly. This link describes adjustments along an age-based path; a market-impact interpretation requires calibration to trading amounts and market depth.

Retaining the exact risk normalization in equation (C4), the objective is

$$\sum_j \omega_j m_j' w_j - \phi \sum_{j=2}^{J} \frac{\omega_j}{h_j} \sum_A \ell_A w_{Aj} \left(\frac{w_{Aj}}{k_j} - \frac{w_{A,j-1}}{k_{j-1}} \right)^2, \qquad k_j = \mathbf{1}'w_j. \tag{C9}$$

This expression explains why a single application of the earlier transformation is insufficient: scales, weights, and differences depend on the variables. The resulting problem is not the quadratic program without market impact.

C.3 Successive quadratic approximation with relaxation

To recover quadratic subproblems, iteration t holds $k_j^{(t)} = \mathbf{1}'w_j^{(t)} > 0$ and penalty weights $w_{Aj}^{(t)}$ fixed. The joint subproblem is

$$\begin{aligned} \max_{\{w_j \geq 0\}} \quad & \sum_j \omega_j m_j' w_j - \sum_{j=2}^{J} \frac{\omega_j \phi}{h_j} \sum_A \ell_A w_{Aj}^{(t)} \left(\frac{w_{Aj}}{k_j^{(t)}} - \frac{w_{A,j-1}}{k_{j-1}^{(t)}} \right)^2 \\ \text{subject to} \quad & R_j w_j \leq b_j(\mathbf{1}'w_j), \qquad w_j' S_j w_j \leq 1, \quad \forall j. \end{aligned} \tag{C10}$$

Both funds' weights inside each difference are optimized. The scale and penalty weight are fixed, not the preceding fund's portfolio. With positive semidefinite covariance matrices, positive distances, and nonnegative coefficients, each subproblem maximizes a concave function over a convex set.

Let $W^{(0)}$ be an initial path feasible for the homogeneous and risk constraints, with positive scales. Given the subproblem's joint solution $\widehat{W}^{(t+1)}$, apply the relaxed update

$$w_j^{(t+1)} = (1-\tau)w_j^{(t)} + \tau\widehat{w}_j^{(t+1)}, \qquad 0 < \tau \leq 1. \tag{C11}$$

Convexity preserves feasibility of this combination. Budget-normalized portfolios and scales for the next iteration are

$$k_j^{(t+1)} = \mathbf{1}'w_j^{(t+1)}, \qquad x_j^{(t+1)} = \frac{w_j^{(t+1)}}{k_j^{(t+1)}}. \tag{C12}$$

Moments, economic weights, and limits remain fixed; scales and penalty weights are updated. For tolerance $\varepsilon > 0$, stabilization of the path can be expressed by the relative distance

$$d_t = \frac{\|X^{(t+1)} - X^{(t)}\|_F}{\max\{1, \|X^{(t)}\|_F\}} \leq \varepsilon, \qquad \|X\|_F = \left(\sum_j \sum_A x_{Aj}^2\right)^{1/2}. \tag{C13}$$

Exact recovery of the net Sharpe ratio requires binding risk constraints: $w_j = k_j x_j$ implies $k_j\sigma_j(x_j) \leq 1$, with $k_j = 1/\sigma_j(x_j)$ only at equality. With market impact, this must be checked for the resulting path. Fixing scales and penalty weights also omits some derivatives of the original objective. Stabilization is a numerical convergence criterion and does not itself establish stationarity or global optimality of equation (C8).

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